Should Platforms be Held Liable for Defective Third-Party Goods?*

Yusuke Zennyo†

March 12, 2023

Abstract

This paper presents an examination of issues related to liability for harm to consumers caused by goods sold by third-party sellers on e-commerce platforms. A model of platform design is presented in which a platform chooses its liability design, specifying the percentages of consumer harm that should be compensated respectively by the defective good seller and the platform. The model shows that the platform assumes no liability. Full liability is imposed on sellers, who enhance their defect-reducing investments. Regulations requiring minimum standards of platform liability improve consumer surplus under conditions of moderate seller competition and weak platform market power.

JEL Classifications: L1, L4, K13
Keywords: platform liability, ex-post compensation, product liability, marketplace, harmful goods

*The author thanks Zhijun Chen, Chongwoo Choe, Jay Pil Choi, Federico Etro, Xinyu Hua, Noriaki Matsushima, Hiroyuki Odagiri, Martin Peitz, Kathryn Spier, and participants in talks at the Competition Policy Research Center (CPRC), Okayama University, Monash University, Keio University, Kwansei Gakuin University, Sixth Asia-Pacific Industrial Organization Conference (APIOC), and NUS Economics of Platform Workshop for valuable comments and suggestions. This work was partially supported by JSPS KAKENHI Grant Numbers 20H01551 and 22H00043. The usual disclaimers apply.

†Graduate School of Business Administration, Kobe University, 2-1 Rokkodai, Nada, Kobe, Hyogo, 657-8501, Japan. E-mail: xyzennyo@b.kobe-u.ac.jp

1 Introduction

Many unsafe goods, including physical products and digital content, are distributed through online platforms (OECD, 2016).[1] Some platforms operate marketplaces that enable buyers and sellers to make direct transactions, such as sites for e-commerce (e.g., Amazon, eBay, Rakuten, JD.com, and Flipkart), app store (e.g., Apple App Store and Google Play Store), online travel agency (e.g., Booking.com and Expedia), and peer-to-peer trading (e.g., Poshmark and Mercari). Numerous goods are traded on those platforms, including harmful, defective, fraudulent, and illegal goods. Heated controversy has arisen with respect to the liability of platforms for such illicit goods (Buiten, De Streel and Peitz, 2020; Lefouili and Madio, 2022).

Some sellers might *intentionally* list patent-infringing or pirated products on marketplaces. These illicit goods should be eliminated *before* they cause some harm (say, *ex-ante* platform liability). The European Commission enacted the Digital Services Act (DSA), which requires digital gatekeeper platforms to detect and remove illicit goods.[2] The obligations of ex-ante platform liability are also designated as content moderation. They have been studied extensively (e.g., Jeon, Lefouili and Madio, 2021; Liu, Yildirim and Zhang, 2022).

Another type of platform liability is the main point examined for this study. For instance, if a consumer sustains some damage *accidentally*, not intentionally, from a third-party seller's product purchased through a platform, then should the platform compensate that consumer? We designate this question as one of *ex-post* platform liability because it is incurred *after* consumers have been harmed by defective goods.

An ongoing debate surrounds ex-post platform liability because online platforms have been immune from liability under current rules, including the Product Liability Act in Japan, Section 230 of the Communications Decency Act in the US, and e-Commerce Directive in the EU, which were enacted respectively in 1994, 1996, and 2000: all in the early days of the internet. These outdated rules include liability exemption clauses for platform intermediaries with the purpose of protecting them

[1] For a recent investigation, one can refer to the following website: `https://www.oecd.org/digital/consumer/put-product-safety-first/`

[2] Similar attempts are apparent around the world. For example, the Australian Competition and Consumer Commission (ACCC) released its fourth interim report on general online retail marketplaces as part of its Digital Platform Services Inquiry. The report, available at `https://www.accc.gov.au/publications/serial-publications/digital-platform-services-inquiry-2020-2025/digital-platform-services-inquiry-march-2022-interim-report`, examines the responsibilities of online marketplaces for illicit goods traded via their platforms.

from endless litigation that might hinder innovation (Lefouili and Madio, 2022).

Recently, the tide has been turning, as some platforms have come to establish dominant market power. In the US, for example, some courts have found platforms liable for defective third-party goods, e.g., Bolger v. Amazon.com (laptop battery), State Farm Fire & Cas. Co. v. Amazon.com (thermostat), Oberdorf v. Amazon.com (dog collar), and Loomis v. Amazon.com (hoverboard). Accordingly, in August 2021, Amazon announced a change of its 'A-to-Z Guarantee' policy and started compensating consumers up to $1,000 for harm caused by third-party goods sold via Amazon Marketplace.

On the policy side, an international movement to amend the current outdated rules has been growing. However, a gap persists in attitudes toward ex-post platform liability among jurisdictions. In the EU, subsequent to the DSA, the European Commission announced a new proposal related to liability for defective products in September 2022.[3] However, exemption clauses for platforms remain: platforms are not held liable if they do not have actual knowledge about (or are not aware of) facts or circumstances from which the harmful or illegal content is apparent.[4] By contrast, in the US, a more stringent bill without such exemptions was proposed by California Assembly in February 2021.[5] The bill described a need to "make an electronic place [...] strictly liable for all damages proximately caused by a defective product that is purchased or sold through the electronic place."

From the perspective of economics, it remains unclear whether regulations for ex-post platform liability are necessary, or not. The purpose of this study is to provide formal analyses that not only elucidate platforms' private incentive for ex-post liability, but which also help the assessment of regulations that hold platforms liable.

To this end, we develop a model of two-sided markets in which a platform operates a marketplace to earn commissions by facilitating direct transactions between consumers and third-party sellers. Some product-liability aspects are incorporated into the model: With some probability, sellers' goods might cause defects that hurt the

[3] The proposal is available at https://single-market-economy.ec.europa.eu/document/download/3193da9a-cecb-44ad-9a9c-7b6b23220bcd_en?filename=COM_2022_495_1_EN_ACT_part1_v6.pdf.

[4] For the DSA, see Article 5(1). As for the new proposal, it is described that "[t]he proposal does not affect the conditional liability exemption under the Digital Services Act."

[5] The California Assembly Bill, formally "California AB 1182 (Stone) Product liability: products purchased online," is available at https://leginfo.legislature.ca.gov/faces/billHistoryClient.xhtml?bill_id=202120220AB1182.

consumer. To reduce the probability and damage of defects, sellers can make costly investments. However, preventing the occurrence of defects completely is difficult. If some defect occurs, then the harm sustained by the consumer is compensated according to the liability design of the marketplace, which describes the percentages of compensation covered by the platform and by the defective goods seller. The platform optimizes its liability design and the commission fee imposed on sellers.

Using the model, we first examine the platform's private incentive for its liability design. The model shows that the platform has no incentive to assume liability for defective third-party goods. Instead, full liability is imposed on third-party sellers, i.e., all harm suffered by consumers must be compensated completely by the defective goods sellers. Consequently, in equilibrium, sellers make the socially optimal level of investment, minimizing the net social loss associated with defects.

Next, the model is used to assess the effects of policy interventions related to ex-post platform liability. Specifically, we allow a policymaker to set a minimum standard for the level of platform liability. Platform-liability regulation has both direct and indirect effects on consumer surplus. The direct effect derives from changes in the degree of liability made by the platform and by a defective goods seller. Facing regulation, the platform must assume some liability to meet the minimum standard, implying that part of the seller liability is shouldered by the platform. Reduced compensation costs of sellers dampen their incentive for defect-reducing investments, while encouraging more sellers to join the platform's marketplace, thereby expanding the product variety available to consumers. The latter network expansion effect is shown to be dominant, implying that the direct effect of regulation is beneficial to consumers.

The indirect effect stems from changes in commission. The platform-liability regulation increases the platform's compensation costs, which then compels the platform to raise its commission. Increased commissions mean an increase in the sellers' marginal cost, which negatively affects consumer surplus in several ways. First, the increased marginal cost simply exacerbates the double-marginalization problem, resulting in higher prices. Second, it hinders seller investment, making the marketplace more dangerous. Finally, increased cost discourages sellers from entering the marketplace, thereby leading to poor product variety.

If the direct effect dominates the aggregate indirect effects, then the platform-liability regulation can enhance consumer surplus. The indirect effects tend to be small in the following environments: (i) the platform's market power is weak because it cannot increasingly raise its commission; (ii) sellers face less elastic demand because they do not raise prices considerably; (iii) harms from defects are insufficiently large because reduced seller investment does not engender steep hikes in consumer damage; and (iv) seller competition is moderate because the seller profitability is not reduced so much that entry becomes markedly lower. Therefore, when these conditions are satisfied, the negative indirect effects are dominated by the positive direct effect, consequently implying that platform-liability regulation can enhance consumer surplus at the expense of the platform's profit. Otherwise, however, regulation might lead to undesirable consequences.

Additionally, to accommodate further policy discussions, the model is extended in three ways: (1) allowing the platform to sell its own goods (so-called hybrid platform and dual-role platform); (2) considering platform competition; and (3) adding the presence of irresponsible sellers who are not willing (or who are not able) to provide compensation (also called judgment-proof sellers). Consequently, our first result, that the platform has no private incentive to assume liability, is shown to be robust under all extensions.

Regarding the second result on the platform-liability regulation, with extensions (1) and (2), it remains qualitatively unchanged. However, the condition under which the platform-liability regulation can enhance consumer surplus has changed quantitatively. For extension (1), when the platform plays a dual role, the regulation's negative consequences are shown to become more likely to occur because the dual-role platform has greater incentives to raise its commission than the pure-marketplace platform has (Anderson and Bedre-Defolie, 2021). By contrast, extension (2) shows that platform competition makes the positive consequences more likely to occur because fiercer competition makes it difficult for the platforms to raise commissions considerably.

As for extension (3), if the marketplace is full of irresponsible sellers, then the platform-liability regulation unambiguously reduces consumer surplus. This result might emphasize the complementarity between ex-ante and ex-post platform liability, although the former is not considered explicitly in the model. As the ex-ante platform liability which induces platforms to eliminate irresponsible sellers becomes greater, the ex-post platform liability that is able to enhance consumer surplus becomes increasingly effective. Ex-ante and ex-post liability regulations would be mutually complementary.

2 Literature

The contents of this paper lie at the intersection of Industrial Organization (IO) and Law & Economics. Among the IO literature on two-sided markets, earlier studies have examined optimal pricing for platforms to leverage positive feedback loops associated with indirect network externalities and to solve chicken-and-egg problems related to coordination failures in the formation of large networks (Caillaud and Jullien, 2003; Rochet and Tirole, 2003, 2006; Armstrong, 2006), without taking much care of microfoundations for detailed transactions between users of different types. Later studies incorporated some microfounded models for competition (e.g., Cournot and Bertrand) in an effort at more precise analysis of how buyers and sellers make transactions in platforms (Belleflamme and Peitz, 2019; Karle, Peitz and Reisinger, 2020).

Recent studies have increasingly devoted attention to platform governance, specifically investigating how platforms design their marketplaces from various perspectives, e.g., first-party selling and self-preferencing (Anderson and Bedre-Defolie, 2021; Etro, 2023; Hagiu, Teh and Wright, 2022; Zennyo, 2022),[6] managing seller competition (Casner, 2020; Teh, 2022), and shrouding additional information about goods (Johnen and Somogyi, 2022).

Issues of platform liability, which can be regarded as a direction of platform governance, have recently attracted much attention (Buiten, De Streel and Peitz, 2020; Busch, 2021; Lefouili and Madio, 2022). Many studies have specifically examined *ex-ante* platform liability, especially regarding content moderation by social media platforms that earn revenues from either advertising or user subscriptions, or both (e.g., De Chiara et al., 2021; Jain, Hazra and Cheng, 2020; Liu, Yildirim and Zhang, 2022; Madio and Quinn, 2021).[7] Differently, Jeon, Lefouili and Madio (2021) consider that an e-commerce platform, which earns commission from sellers, can invest in screening technology to delist patent-infringing products that are harmful to the brand owners, but not to consumers. Although higher screening intensity encourages more brand owners to list their original products on the platform, regulations that require the platform to increase its screening intensity would lead the platform to raise its commissions, consequently discouraging the entry of brand owners. They

[6] One can refer to recent survey papers about self-preferencing by digital platforms (Etro, 2022; Kittaka, Sato and Zennyo, 2022; Peitz, 2022).

[7] Empirical and experimental studies have examined the effects of content moderation of social media platforms (e.g., Jiménez-Durán, 2022; Jiménez-Durán, Müller and Schwarz, 2022).

demonstrate that, if the latter effect dominates the former one, then the platform-liability regulation reduces the entry of brand owners, which in turn decreases social welfare.

In contrast, remarkably few studies have addressed *ex-post* platform liability, i.e., compensation for people who have sustained damage from goods distributed through platforms. To address this important issue, we import a modeling approach of product liability from the Law & Economics literature into a model of platforms used in the IO literature.

In product-liability models, goods are presumed to cause harm to consumers with some probability. Depending on product-liability rules (e.g., strict liability, negligence, and no liability), sellers decide how much effort to put into reducing the probability of defects occurring (e.g., Polinsky and Rogerson, 1983; Daughety and Reinganum, 1995, 2006). Some studies allow sellers to choose their liability policy as a means of marketing, e.g., enhancing consumer demand, screening heterogeneous consumers, and performing price discrimination (e.g., Choi and Spier, 2014; Hua and Spier, 2020). Similarly, we assume for this study that platforms are allowed to determine a liability design for their marketplaces.

The present study is closely related to recent studies by Hua and Spier (2023) and Yasui (2022), both of which examine ex-post compensation for consumers who have sustained damage from defective third-party goods. Hua and Spier (2023) examine a model with a monopoly platform, into which sellers of two types are incorporated: harmful and safe. Compared to safe sellers, harmful sellers have a lower marginal cost, but their goods cause harm to buyers with higher probability. Harm sustained by buyers is (partially) compensated by the platform and the seller according to an exogenously given liability rule.

Their interest lies in content moderation. That is, given the liability rule, the platform attempts to exclude harmful sellers from its marketplace. If the seller liability is so high that harmful sellers gain less profit than safe sellers, then the platform can easily prevent the entry of harmful sellers simply by raising its commission. Alternatively, if the seller liability is low, then an increased commission does not work as intended, driving out safe sellers instead. Consequently, the platform must make costly investments in technology to detect invading harmful sellers (i.e., ex-ante content moderation). Hua and Spier (2023) show that, although greater platform liability encourages the platform to invest in its screening technology, it can be socially excessive in some cases.

The present study differs greatly from Hua and Spier (2023) in that we model a detailed microfoundation for buyer–seller transactions. Hua and Spier (2023) simply assume that the benefits which a buyer and a seller derive from interaction are fixed exogenously, independently of the respective levels of platform liability and seller liability. With this simplifying assumption, changes in liability levels affect neither buyer behaviors (e.g., purchasing decisions) nor seller behaviors (e.g., pricing and investment decisions).[8] For this reason, unlike the present study, the welfare effect of policy interventions made through changes in buyer and seller behaviors is not considered completely in their study.

In that sense, Yasui (2022) models buyer–seller interactions on a platform over a continuous and infinite horizon time. In his model, sellers are allowed to exit the platform's marketplace without making any compensation if their goods are somehow defective. Even with no seller liability, sellers still have incentives to undertake costly efforts at reducing the probability of defects to enhance consumer demand through reputation building. Yasui (2022) shows that the sellers' effort level decreases with the platform's liability and monitoring levels. Therefore, if the platform can choose both the levels of platform liability and monitoring, then no liability is assumed. The resulting monitoring level is insufficient in terms of total welfare.

Additionally, Yasui (2022) allows a government to set a platform-liability level to maximize total welfare, thereby implying that the platform only chooses its monitoring level given the government's decision-making. Full platform liability can be the optimal policy, but it is not necessarily the optimal policy. Depending on the circumstances, partial liability is desirable because excessive platform liability dampens the sellers' effort level.

Results obtained in Yasui (2022) seem, to some degree, analogous to those of this paper. A crucially important difference is that we allow, endogenously, both consumers and sellers to decide whether to use the platform or not, i.e., the transaction volume and resulting network benefits are determined endogenously in the model, whereas the number of transactions is fixed in Yasui (2022). The presence of indirect network externalities would alter the influence of policy intervention effects. In fact, in our model, platform-liability regulation is shown to reduce the transaction volume

[8] As an extension, Hua and Spier (2023) also examine a case in which sellers can offer a price to buyers endogenously. However, for simplicity, every seller is assumed to be matched randomly with a buyer. The seller then offers a take-it-or-leave-it price to that buyer. This assumption enables sellers to extract all surplus from buyers, i.e., the buyer surplus is always equal to zero. In other words, both buyer and seller behaviors are determined independently of the liability rule.

and the resulting network benefits. This negative aspect not only makes the regulation less effective; in some cases, it might even make regulation detrimental. Our analyses are expected to contribute to the literature and policy debates by presenting the benefits and shortcomings of platform-liability regulation, with careful consideration devoted to indirect network externalities.

3 Model

This section presents a description of the model examined for this study, which is built based on Anderson and Bedre-Defolie (2021).

A monopoly platform operates a marketplace in which consumers and third-party sellers can make transactions directly. The sellers must incur a fixed entry cost of e to list goods for sale in the marketplace. One can consider that the entry cost is necessary to prepare to start selling through a new channel (e.g., building inventory, logistics, and shipping systems). We assume free entry of sellers: the number of sellers participating in the platform, denoted as n, is determined by the zero-profit condition.

There is a unit mass of consumers, each of whom has unit demand for the sellers' goods. Unlike sellers, consumers need not pay any fee or bear any cost to visit the platform's marketplace, implying that all consumers visit the platform for shopping. Not all of them actually shop therein: consumers have an outside option, as explained below.

In the marketplace, each seller $i \in \{1, \ldots, n\}$ chooses price p_i and investment level x_i (explained later). Following the literature, we assume sellers to be "fringe" so that any change in strategy by individual sellers does not affect the other sellers' payoff. In other words, seller competition is modeled as monopolistic competition (Anderson and Bedre-Defolie, 2021; Etro, 2023). The utility that a consumer derives from seller i's good is given below.

$$u_i = v_i - p_i - (1 - x_i)d(1 - l_p - l_s) + \mu\epsilon_i \tag{1}$$

Therein, the first term v_i represents the standard value of seller i's good. For simplicity, we assume $v_i = v$ for all $i \in \{1, \ldots, n\}$. The second term is the price of the good.

The third term represents the net disutility the consumer suffers from defects of the good. After the purchase, the good might be defective and might cause harm to the consumer with probability of $1-x_i$. The probability is negatively associated with the seller's investment level x_i (i.e., defect-reducing investment), which is assumed to be observable to consumers (Hamada, 1976; Hua and Spier, 2020).[9] A variety of defects might occur. The expected amount of harm sustained by the consumer, conditional upon the incidence of defects, is expressed as d (Polinsky and Rogerson, 1983; Daughety and Reinganum, 1995). The harm is compensated by the platform and the seller according to the platform's liability design.[10] Variables l_p and l_s respectively represent the liability shares which are shouldered by the platform and the seller. One can consider that the total compensation does not exceed the harm, as assumed in Hua and Spier (2023). That is, $l_p + l_s \leq 1$ holds for $l_p \in [0,1]$ and $l_s \in [0,1]$. For simplicity, we assume that there are no litigation costs.

The fourth term in Equation (1) denotes a match value expressing the idiosyncratic value of seller i's good to the consumer, where ϵ_i is a random variable and μ is an exogenous parameter.

Consumers have an outside option of not purchasing in the marketplace, from which they gain utility of $u' = \mu\epsilon'$. Assuming that ϵ_i and ϵ' are independently and identically distributed according to Gumbel distribution (also called Type-I Extreme Value distribution), we are led to the following Logit demand function of

$$q_i = \frac{\exp(b_i/\mu)}{1 + \sum_{j=1}^{n} \exp(b_j/\mu)}, \tag{2}$$

where $b_i \equiv v - p_i - (1-x_i)d(1-l_p-l_s)$. Parameter μ represents the degree of product differentiation among sellers.

It is worth emphasizing that choosing the outside option means that the consumer visited the marketplace but did not buy anything therein. The consumer might have chosen not to buy the goods, or might have purchased them through other retail channels such as rival platforms and brick-and-mortar retailers. Therefore, one can infer that the consumer behavior and resulting demand system (2) allow for

[9]This observability assumption is made for ease of exposition. The main results presented below remain unchanged even if seller investment x_i is unobservable by consumers. Details are available upon request to the author.

[10]Sellers are assumed to be so sound that they follow the liability design chosen by the platform and that they have *deep pockets* to provide compensation for the harm which the defect has caused. In Section 5.3, we allow for the presence of irresponsible sellers who are not willing (or not able) to provide compensation for the harm which the defects cause. Such sellers are called *judgment-proof* sellers in the Law & Economics literature.

consumers' endogenous decision-making on whether to use the platform or not.

Seller i chooses p_i and x_i to maximize its profit as

$$\pi_i = \{(1-\tau)p_i - c(x_i) - (1-x_i)dl_s\}\, q_i - e \ , \tag{3}$$

where $\tau \in [0,1]$ is an ad valorem commission set by the platform and $c(x_i)$ denotes the marginal cost of selling a unit of goods. We assume that $c'(\cdot) > 0$ and $c''(\cdot) > 0$. The former assumption implies that sellers must incur higher marginal costs to produce safer goods. The latter ensures the existence and uniqueness of an interior equilibrium.

The term $(1-x_i)dl_s$ in Equation (3) represents the seller's expected compensation cost per unit sold, which depends on the degree of seller liability l_s chosen by the platform. Following the Law & Economics literature (e.g., Daughety and Reinganum, 1995; Polinsky and Rogerson, 1983), we refer to $c(x_i) + (1-x_i)dl_s$ as the seller's *full* marginal cost.

We let $\overline{x}$ be the socially desirable level of investment which minimizes the total cost associated with defects. Formally, $\bar{x} = \arg\min_x c(x) + (1-x)d$, or equivalently $\bar{x} = c'^{-1}(d)$.

The platform chooses a commission rate (i.e., τ) and a liability design (i.e., a pair of l_p and l_s) to maximize its profit as shown below.

$$\Pi = \sum_{i=1}^{n} \left(\tau p_i - (1-x_i)dl_p\right) q_i \tag{4}$$

Therein, τp_i stands for the commission revenue collected from third-party sellers. Also, $(1-x_i)dl_p$ represents the platform's expected compensation cost.

The timing of the game is the following. At Stage 1, the platform chooses a liability design for its marketplace as l_p and l_s. At Stage 2, the platform sets a commission rate of τ.[11] At Stage 3, sellers make an entry decision about whether to list their goods on the marketplace. If they choose to enter, then they incur the fixed entry cost of e; they then simultaneously choose their price and investment level, respectively, as p_i and x_i.

[11] The decisions made by the platform are, for ease of exposition, divided into two stages. Because the decisions are made by a single player, by envelop theorem, the results remain unchanged even if they are made simultaneously.

4 Analyses

Next, we analyze the model described in the preceding section. Section 4.1 presents the platform's private incentive for its liability design. To come to the point, the profit-maximizing platform can be shown to have no incentive to assume liability for defective third-party goods. Given this result, Section 4.2 presents our examination of if (and when) regulations that hold the platform liable can enhance consumer surplus. Specifically, we allow a policymaker to set a minimum standard of platform liability.

The omitted proofs are presented in Appendix.

4.1 Platform Private Incentive

We here solve the game with no policy intervention.

At Stage 3, given (τ, l_p, l_s), sellers choose p_i and x_i simultaneously. We specifically examine the symmetric equilibrium in which all sellers choose the same price of p and investment level of x. If seller i deviates from this equilibrium by setting p_i and x_i, then the demand of the deviant seller is given as

$$q_i(p_i, x_i; l_p, l_s) = \frac{\exp(b_i/\mu)}{1 + n \cdot \exp(b/\mu)} = \frac{\exp(b_i/\mu)}{A}, \tag{5}$$

where $b = v - p - (1-x)d(1 - l_p - l_s)$.

Following the terminology of aggregative games, we refer to the denominator of Equation (5) as the "aggregate" and let $A = 1 + n \cdot \exp(b/\mu)$. The aggregate is unaffected by any change in strategy by individual "fringe" sellers, as assumed in Anderson and Bedre-Defolie (2021). In other words, A is independent of p_i and x_i.

The aggregate A has some nice properties. First, it can be regarded as a proxy for consumer surplus because consumer surplus is expressed as $CS = \ln A$. Second, the aggregate captures the extent of indirect network externalities between consumers and sellers. By the definition of A, a greater number of sellers n, all else being equal, is associated with greater aggregate A. The number of consumers who use the platform, which is given as $1 - 1/A$, also increases with aggregate A.

Given the aggregate, deviant seller i sets p_i and x_i to maximize

$$\pi_i = \{(1-\tau)p_i - c(x_i) - (1-x_i)dl_s\} \frac{\exp(b_i/\mu)}{A} - e. \tag{6}$$

Solving the first-order conditions with respect to p_i and x_i, one can derive that the symmetric subgame equilibrium $(p(\tau, l_p, l_s), x(\tau, l_p, l_s))$ satisfies the following system.

$$p(\tau, l_p, l_s) = \frac{c\left(x(\tau, l_p, l_s)\right) + \left(1 - x(\tau, l_p, l_s)\right) dl_s}{1 - \tau} + \mu \tag{7}$$

$$c'\left(x(\tau, l_p, l_s)\right) = d\left(\tau l_s + (1 - \tau)(1 - l_p)\right) \tag{8}$$

One can also confirm that the second-order condition is satisfied.

The price presented in Equation (7) consists of two terms. The first term represents the *effective* full marginal cost of sellers. The second term is the standard Logit markup. It is worth mentioning that, because of the nature of revenue sharing, a \$1 increase in the full marginal cost of $c(x) + (1 - x)dl_s$ compels the seller to raise its price by $\$1/(1 - \tau)$. That is, the cost pass-through ratio is greater than unity, unless $\tau = 0$.

The seller strategy has the following properties.

Lemma 1. *Investment level $x(\tau, l_p, l_s)$ and price $p(\tau, l_p, l_s)$ are affected by changes in the platform design (τ, l_p, l_s) in the following manner:*

(i) An increase in commission τ reduces the seller investment level (i.e., $\partial x/\partial \tau < 0$). Its effect on the price is ambiguous (i.e., $\partial p/\partial \tau$ can be either zero, positive, or negative).

(ii) Greater platform liability decreases both the investment level and the price (i.e., $\partial x/\partial l_p < 0$ and $\partial p/\partial l_p < 0$).

(iii) Greater seller liability increases both the investment level and the price (i.e., $\partial x/\partial l_s > 0$ and $\partial p/\partial l_s > 0$).

Lemma 1 presents how changes in the platform design affect seller behavior. Point (i) shows that a higher commission hinders sellers' incentive for defect-reducing investments. The price effect is ambiguous because a higher commission raises the price directly by exacerbating the double marginalization while lowering the costs associated with defect-reducing investments.

Next, Points (ii) and (iii) demonstrate that changes in l_p and l_s have the opposite effect on seller behavior. Greater platform liability attenuates sellers' incentives for investments. Reduced investment costs induce them to charge a lower price. In other words, greater platform liability might result in the proliferation of low-quality and low-price goods in the marketplace. The opposite happens for greater seller liability.

The resulting profit of sellers is computed as $\pi(\tau, l_p, l_s) = \mu(1-\tau)\frac{V(\tau,l_p,l_s)}{A} - e$, where

$$V(\tau, l_p, l_s) = \exp\left(\frac{v - p(\tau, l_p, l_s) - ((1 - x(\tau, l_p, l_s))\, d(1 - l_p - l_s)}{\mu}\right). \tag{9}$$

The zero-profit condition of free entry, $\pi(\tau, l_p, l_s) = 0$, pins down the aggregate as

$$A = \frac{\mu(1-\tau)}{e} V(\tau, l_p, l_s) \equiv A(\tau, l_p, l_s). \tag{10}$$

The following lemma summarizes the properties of $V(\tau, l_p, l_s)$ and $A(\tau, l_p, l_s)$.

Lemma 2. *Changes in the platform design (τ, l_p, l_s) affect $V(\tau, l_p, l_s)$ and $A(\tau, l_p, l_s)$ in the following manner:*

(i) An increase in commission τ results in lower $V(\tau, l_p, l_s)$ and $A(\tau, l_p, l_s)$.

(ii) Greater platform liability increases both $V(\tau, l_p, l_s)$ and $A(\tau, l_p, l_s)$.

(iii) Greater seller liability decreases both $V(\tau, l_p, l_s)$ and $A(\tau, l_p, l_s)$.

Because $CS = \ln A$, Point (i) implies that a higher commission is associated with lower consumer surplus. This is true because an increase in commission requires an increase in the sellers' marginal cost, which lowers their profitability and which therefore discourages them from participating in the marketplace. The reduction in product variety worsens consumer surplus.

Points (ii) and (iii) show that higher platform liability and lower seller liability increase consumer surplus, provided that τ is fixed. The former helps reduce the consumers' expected harm from defective goods, whereas the latter decreases the sellers' compensation cost. Both encourage consumers and sellers to participate in the marketplace. Increased transactions generate greater consumer surplus.

It is also noteworthy that, given a fixed degree of product safety, higher seller liability raises the sellers' prices and raises the consumers' willingness to pay. In the literature, these two effects are presented as offsetting (e.g., Hamada, 1976). By contrast, in the present model, because the platform charges an ad valorem commission, the sellers' pass-through rate is higher than unity, making the price increase greater than the increase in consumers' willingness to pay. Therefore, these effects overall engender lower output and lower consumer surplus.

The platform's profit can be rewritten as presented below.

$$\Pi(\tau, l_p, l_s) = (\tau p(\tau, l_p, l_s) - (1 - x(\tau, l_p, l_s))\, dl_p) \cdot \left(1 - \frac{1}{A(\tau, l_p, l_s)}\right) \tag{11}$$

Therein, because $1/A$ represents the number of consumers who use the outside option of not buying in the marketplace, $1 - 1/A$ is the total quantity traded through the platform. For each unit sold, the platform collects the commission revenue of τp, but the platform must incur the compensation cost of $(1 - x)dl_p$ in terms of expectation.

At Stage 2, the platform sets an ad valorem commission to maximize its profit presented in Equation (11). One can infer that the platform will neither charge a low commission such that $\tau p - (1 - x)dl_p < 0$ nor a high commission such that $A < 1$. Following Anderson and Bedre-Defolie (2021), we assume the existence of an optimal commission that satisfies the following first-order condition.

$$\frac{\partial \Pi(\tau, l_p, l_s)}{\partial \tau} = \left(p + \tau \frac{\partial p}{\partial \tau} + \frac{\partial x}{\partial \tau} dl_p\right)\left(1 - \frac{1}{A}\right) + (\tau p - (1 - x)dl_p)\, \frac{1}{A^2} \frac{\partial A}{\partial \tau} = 0 \tag{12}$$

In the equation above, p, x, A, and all the derivatives are evaluated at (τ, l_p, l_s). The first term of Equation (12) represents the marginal revenue from an increased commission rate. The second term is associated with the marginal cost of reducing the total quantity traded in the marketplace. We denote by $\tau(l_p, l_s)$ the optimal commission which solves the first-order condition (12). Moreover, we let $\Pi(l_p, l_s) \equiv \Pi\left(\tau(l_p, l_s), l_p, l_s\right)$.

At Stage 1, the platform chooses a pair of (l_p, l_s) to maximize $\Pi(l_p, l_s)$. One can obtain the following proposition for the equilibrium of the model.

Proposition 1. *At equilibrium, the platform does not assume any liability for defects caused by third-party sellers (i.e., $l_p^* = 0$). The platform requires that sellers assume full liability: $l_s^* = 1$. The equilibrium commission τ^* is set at the level which solves Equation (12) with (l_p^*, l_s^*). Consequently, sellers make the optimal level of defect-reducing investment (i.e., $x^* = \bar{x}$, or equivalently $c'(x^*) = d$), and set the price as $p^* = \frac{c(x^*)+(1-x^*)d}{1-\tau^*} + \mu$.*

This proposition shows that the platform has no private incentive to assume liability for defective third-party goods. Higher liability increases the platform's liability cost directly. In addition to this direct effect, an increase in platform liability exerts several indirect effects on the platform's profit. As shown in Lemma 1 (ii), greater platform liability reduces the sellers' price, which in turn decreases the platform's revenue, provided that the commission rate is fixed. It also reduces the sellers' investment level, which increases the platform's expected liability cost. Moreover, unlike these negative effects, increased platform liability has the positive characteristic of increasing consumer demand, which however is shown to be dominated by the direct effect under the Logit demand system examined in this paper. Overall, platform liability reduces the profit of the platform unambiguously, leading to $l_p^* = 0$ in equilibrium.

The platform imposes full liability on third-party sellers, which compels them to engage in defect-reducing investment at the socially optimal level. Consequently, in equilibrium, the platform design is desirable in terms of product safety in the marketplace.

One can regard the equilibrium outcome presented in Proposition 1 as that of another scenario in which liability is basically imposed on the seller side, but in which the platform can shoulder part of the responsibility. Formally stated, one can consider a game in which the platform chooses only l_p at Stage 1. Accordingly, the seller liability is set at $l_s = 1 - l_p$. Results show that, even in this game, the platform has no incentive to shoulder the seller's liability.

The platform's private incentive for its liability design will be confirmed as highly robust with several extensions in Section 5. Specifically, the equilibrium liability design remains unchanged ($l_p^* = 0$ and $l_s^* = 1$), even if the platform is allowed to sell its own goods (Section 5.1), if platform competition exists (Section 5.2), and if irresponsible sellers are present as well (Section 5.3).

4.2 Platform-Liability Regulation

So far, we have seen that the platform has no incentives to assume liability for defective third-party goods. In reality, however, a growing policy discussion has arisen about regulations that hold platforms liable for defective goods sold through their marketplaces. Assessing the consequences of a platform-liability regulation is not an easy task because the regulation might affect various parties' decision-making. Earlier results have indicated that, all else being equal, greater platform liability hinders seller investment (Lemma 1[ii]), but enhances consumer surplus (Lemma 2[ii]) because it reduces defect-related burdens for consumers and sellers, encouraging them to use the platform's marketplace, consequently creating greater network externalities. At the same time, however, greater platform liability brings forth higher compensation costs to the platform, compelling the platform to raise its commission. Increased commissions are shown to reduce consumer surplus (Lemma 2[i]).

Therefore, we use the model to provide formal analyses for the assessment of platform-liability regulation in terms of consumer protection. Specifically, we allow a policymaker to impose a minimum standard of platform liability, denoted as L_p. Under the regulation, the platform must set its assumed level of liability as greater than or equal to the minimum standard as $l_p \geq L_p$.

First, we can describe the platform's response to the regulation. Proposition 1 shows that the platform has incentives to decrease l_p and increase l_s to the greatest degree possible. Therefore, the regulation leads the platform to set $l_p = L_p$ and $l_s = 1 - L_p$ at Stage 1. At Stage 2, subsequently, the platform adjusts its commission rate to maximize $\Pi(\tau, L_p, 1 - L_p)$. The first-order condition can be given as that presented in Equation (12) with $l_p = L_p$ and $l_s = 1 - L_p$, or equivalently

$$\frac{\partial \Pi(\tau, L_p, 1 - L_p)}{\partial \tau} = \frac{p - \tau\mu}{1 - \tau}\left(1 - \frac{1}{A}\right) + (\tau p - (1 - x)dL_p)\,\frac{1}{A^2}\frac{\partial A}{\partial \tau} = 0, \tag{13}$$

where p, x, and A are evaluated at $(\tau, L_p, 1 - L_p)$.

Denoting by $\tau(L_p)$ the solution of the first-order condition (13), one can derive

$$\tau'(L_p) = \frac{\frac{\partial^2 \Pi(\tau, L_p, 1-L_p)}{\partial \tau \partial l_p} - \frac{\partial^2 \Pi(\tau, L_p, 1-L_p)}{\partial \tau \partial l_s}}{-\frac{\partial^2 \Pi(\tau, L_p, 1-L_p)}{\partial \tau^2}}, \tag{14}$$

which represents the extent to which the platform changes its commission rate in response to a marginal increase in L_p.

Next, we shift our attention to the effect of an increase in L_p on consumer surplus. With the regulation, consumer surplus is expressed as $CS(L_p) \equiv CS(\tau(L_p), L_p, 1 - L_p) = \ln A(\tau(L_p), L_p, 1 - L_p)$. The derivative of $CS(L_p)$ is

$$CS'(L_p) = \frac{1}{A}\left(\underbrace{\frac{\partial A}{\partial \tau} \cdot \tau'(L_p)}_{\text{indirect effect}} + \underbrace{\frac{\partial A}{\partial l_p} - \frac{\partial A}{\partial l_s}}_{\text{direct effect}}\right), \tag{15}$$

where A and its derivatives are evaluated at $(\tau(L_p), L_p, 1 - L_p)$.

Equation (15) shows that the regulation has direct and indirect effects on consumer surplus. The regulation directly increases the platform liability of $l_p = L_p$ and decreases the seller liability of $l_s = 1 - L_p$, both of which are shown to enhance consumer surplus in Lemma 2 (i.e., $\frac{\partial A}{\partial l_p} > 0$ and $\frac{\partial A}{\partial l_s} < 0$).

In addition, the regulation affects consumer surplus indirectly through changes in commission. This indirect effect can be detrimental to consumers because the regulation compels the platform to raise its commission (i.e., $\tau'(L_p) > 0$). The increased commission reduces consumer surplus (i.e., $\frac{\partial A}{\partial \tau} < 0$, as shown in Lemma 2 [i]).

To ascertain whether introduction of the platform-liability regulation increases or decreases consumer surplus, we specifically examine the sign of $CS'(l_p^*)$.

Proposition 2. *Platform-liability regulations can enhance consumer surplus when sellers have a per-unit margin greater than their per-unit cost in equilibrium. Formally stated,* $CS'(l_p^*) > 0$ *if and only if*

$$\mu(1-\tau^*) > c(x^*) + (1-x^*)d, \tag{16}$$

or equivalently

$$\varepsilon^* < 2, \tag{17}$$

where $\varepsilon^* \equiv -\frac{\partial q_i(p^*, x^*; l_p^*, l_s^*)}{\partial p_i} \frac{p^*}{q_i(p^*, x^*; l_p^*, l_s^*)}$ *denotes the price elasticity of demand in equilibrium.*

This proposition provides the necessary and sufficient conditions under which platform-liability regulation can be an effective policy intervention in terms of consumer surplus. Conditions (16) and (17) imply that policymakers should consider imposing a minimum standard of platform liability in the following situations: [i] the platform's market power is weak (i.e., τ^* is small); [ii] seller competition is moderate in terms of high margin and low cost (i.e., μ is large and $c(\cdot)$ is small); [iii] sellers' goods are not so harmful (i.e., d is small); and [iv] sellers face less elastic demand (i.e., ε^* is small).[12]

The intuition underlying Proposition 2 is the following. When Points [i]–[iv] are satisfied, the negative indirect effect of the regulation is more likely to be sufficiently small and therefore dominated by its positive direct effect, as explained below. First, with weak market power (Point [i]), the platform cannot considerably raise its commission rate in response to the regulation, i.e., $\tau'(L_p)$ is small.

Next, an increase in commission τ reduces consumer surplus in several ways. First, the increased commission exacerbates the double-marginalization problem, resulting in a higher price. This effect is small if sellers face less elastic demand (Point

[12]Condition (17) could be generalized, to some degree, by the application of the model of Etro (2023). One might infer that the results would remain qualitatively unchanged.

[iv]). Secondly, the increased commission hinders sellers' incentive for defect-reducing investments, leaving consumers exposed to dangerous goods. This effect is not significant if sellers' goods are not so harmful (Point [iii]). Finally, increased commission diminishes sellers' profit and thereby discourages them from entering the platform's marketplace, resulting in poor product variety available to consumers. This effect can be reduced if the seller competition is not so fierce (Point [ii]). In total, Points [ii]–[iv] render the extent of $\frac{\partial A}{\partial \tau}$ smaller.

In other words, if any of Points [i]–[iv] is not met, the platform-liability regulation would be harmful to consumers. One can argue that, to prevent such negative, and unintended, consequences of the regulation, policymakers should make decisions on a case-by-case basis. In that sense, Proposition 2 also suggests that not only platform-level assessment based on Point [i], but also product-level assessment based on Points [ii]–[iv] would be necessary.

5 Robustness and Policy Implications

For robustness of the main results obtained earlier (i.e., Propositions 1 and 2), three extensions are examined in this section. Specifically, the original model is extended in ways that allow the platform to sell its own goods in Section 5.1, which allow for platform competition in Section 5.2, and which allow for the presence of irresponsible sellers in Section 5.3.

5.1 Hybrid platform

We allow the platform to sell its first-party goods in competition with third-party sellers, as in Anderson and Bedre-Defolie (2021), Etro (2023), and Zennyo (2022). The formal presentation of analyses for the hybrid platform model is given in Supplementary Appendix A.

Denoting the price and investment level of the platform's first-party good by p_0 and x_0, one can assume that the consumers' indirect utility from purchasing a unit of the first-party good is given as

$$u_0 = v - p_0 - (1 - x_0)d(1 - l_p - l_s) + \mu\epsilon_0 \ , \tag{18}$$

where the match value ϵ_0 follows the Gumbel distribution, like ϵ_i and ϵ'. Consequently, by letting $b_0 = v - p_0 - (1 - x_0)d(1 - l_p - l_s)$, one can derive the demand for the first-party good as presented below.

$$q_0 = \frac{\exp(b_0/\mu)}{1 + \exp(b_0/\mu) + \sum_{i=1}^{n} \exp(b_i/\mu)} \tag{19}$$

Following Anderson and Bedre-Defolie (2021), we assume that the platform chooses p_0 and x_0 at Stage 2, simultaneously with its commission choice. The hybrid platform's profit is given as shown below.

$$\Pi_h = (p_0 - c(x_0) - (1 - x_0)d(l_p + l_s))\, q_0 + \sum_{i=1}^{n} (\tau p_i - (1 - x_i)dl_p)\, q_i \tag{20}$$

The first term represents the profit from first-party selling. It is noteworthy that, unlike third-party sellers, when the first-party good causes some defect, the platform must compensate the consumers both as the operator of the marketplace and as the seller of the defective good. The second term is the profit collected from third-party sellers, as in the original model.

By solving the hybrid platform model, one can demonstrate that the ability of first-party selling enables the hybrid platform to charge a higher commission to third-party sellers.

Lemma 3. *Given (l_p, l_s), in comparison with the original model, the hybrid platform imposes a higher commission on third-party sellers. Formally, $\tau_h(l_p, l_s) > \tau(l_p, l_s)$ holds for any $(l_p, l_s) \in [0, 1]^2$.*

As demonstrated in Anderson and Bedre-Defolie (2021), the hybrid platform has incentives to increase its commission, raising rivals' costs and enhancing the cost advantage of its first-party good. This result holds under any liability design (l_p, l_s).

The following proposition demonstrates that the platform's private incentive for its liability design remains unchanged even in the hybrid platform model.

Proposition 1a. *Even if the platform sells its first-party goods, the equilibrium liability design remains unchanged, i.e., $l_p = 0$ and $l_s = 1$.*

This proposition confirms the robustness of Proposition 1. Therefore, irrespective of the presence or absence of first-party selling, the platform has no incentive to assume liability for defective third-party goods.

Next, in a similar vein to that of the original analysis, we investigate the effects of a regulation that imposes a minimum standard of L_p for the platform liability level. We use $CS_h(L_p)$ to denote consumer surplus under the regulation in the hybrid platform model.

Proposition 2a. *In the hybrid platform model, as compared with the pure-marketplace model, the platform-liability regulation is more likely to reduce consumer surplus. Formally, $CS_h'(L_p) < CS'(L_p)$ holds for all $L_p \in [0, 1]$.*

First-party selling by the platform is shown to make the negative consequence of the regulation more likely to occur. As in the original model, the platform-liability regulation reduces the sellers' investment level. However, it does not affect the investment level of the hybrid platform (i.e., $x_0(L_p) = \overline{x}$). The difference in safety between first-party and third-party goods incentivizes the platform to steer consumers toward its safer goods in an effort to save the compensation payment. To this end, the hybrid platform increases its commission more highly than the pure-marketplace platform would do, as shown in Lemma 3, which amplifies the negative indirect effect of the platform-liability regulation.

Proposition 2a indicates that regulations holding platforms liable can be more effective in product categories in which platforms have not started first-party selling. For example, Amazon sells its own products in some categories, but not all (Zhu and Liu, 2018). Rakuten in Japan sells no first-party products: they are a pure marketplace. Policymakers should consider the presence or absence of first-party selling when deliberating about the introduction of platform-liability regulation.

5.2 Platform competition

We examine competition between M platforms (M stands for *marketplace*). Detailed analyses are presented in Supplementary Appendix B.

Platform $m \in \{1, \ldots, M\}$ decides a triplet of (τ^m, l_p^m, l_s^m). Consumers choose a platform to visit. As in the original model, consumers need not incur any cost for the visit. The utility of a consumer joining platform m is given as

$$U^m = \ln A^m + \eta\sigma^m, \tag{21}$$

where $A^m = 1 + \sum_{i=1}^{n^m} \exp(b_i^m/\mu)$ and $b_i^m = v - p_i^m - (1 - x_i^m)d(1 - l_p^m - l_s^m)$. The first term, $\ln A^m$, represents the expected benefit that the consumer gains from purchasing a unit of goods from one of n^m sellers active on platform m. The second term denotes a match value that expresses the consumer's benefit of visiting platform m. We assume that σ^m follows the Gumbel distribution. Consequently, parameter η represents the degree of platform differentiation. The probability that consumers choose platform m to visit can be given as the following Logit system with no outside option.

$$\mathbb{P}^m = \frac{A^m}{\sum_{l=1}^{M} A^l} \tag{22}$$

We specifically examine the symmetric equilibrium in which all platforms choose the same strategy of $(\tau(M), l_p(M), l_s(M))$. Moreover, for the sake of simplicity, we assume that consumers are unaware of changes in strategy by platforms, as in Teh and Wright (2022) and Teh et al. (2023). In other words, at the time of deciding which platform to visit, consumers believe that all platforms follow the equilibrium strategy, even if some platforms have deviated from the equilibrium. Therefore, we look for the perfect Bayesian equilibrium of the game with platform competition.

Decisions made by fringe sellers remain unchanged. Given $(\tau(M), l_p(M), l_s(M))$, seller i on platform m chooses p_i^m and x_i^m to maximize

$$\pi_i^m = \{(1-\tau(M))p_i^m - c(x_i^m) - (1-x_i^m)dl_s(M)\}\, \mathbb{P}^m q_i^m - e, \tag{23}$$

where $q_i^m = \frac{\exp\left(b_i^m/\mu\right)}{A^m}$. By solving the maximization problem, one can derive that all sellers choose the same price of $p\left(\tau(M), l_p(M), l_s(M)\right)$ and the same investment level of $x\left(\tau(M), l_p(M), l_s(M)\right)$, which are the same functions as those presented in Equations (7) and (8).

The resulting profit is computed as $\pi^m = \mu\left(1-\tau(M)\right) \cdot \mathbb{P}^m \cdot \frac{V(\tau(M), l_p(M), l_s(M))}{A^m} - e$. In the symmetric equilibrium, because of $\mathbb{P}^m = 1/M$, the zero-profit condition of free entry pins down the aggregate as

$$A^m = \frac{\mu\left(1-\tau(M)\right)}{Me} \cdot V\left(\tau(M), l_p(M), l_s(M)\right) = \frac{A\left(\tau(M), l_p(M), l_s(M)\right)}{M}, \tag{24}$$

which implies that the sum of aggregates of all platforms is equal to that of the monopoly model (i.e., $MA^m = A$).

Next, we turn to the decisions made by platforms. If platform m deviates from the equilibrium by setting (τ^m, l_p^m, l_s^m), then its profit is expressed as

$$\Pi^m(\tau^m, l_p^m, l_s^m) = \left(\tau^m p^m - (1-x^m)dl_p^m\right) \mathbb{P}^m \left(1 - \frac{1}{A^m}\right), \tag{25}$$

where $p^m = p(\tau^m, l_p^m, l_s^m)$ and $x^m = x(\tau^m, l_p^m, l_s^m)$.

Because of the assumption that consumers are unaware of deviation by the platform, $\mathbb{P}^m$ remains as $1/M$ independently of (τ^m, l_p^m, l_s^m). Therefore, the deviant platform's maximization problem is highly analogous to that of the original model. The only difference is the inclusion of parameter M in A^m.

With $\mathbb{P}^m = 1/M$, the first-order condition with respect to τ^m is given as

$$\frac{\partial \Pi^m}{\partial \tau^m} = \left(p^m + \tau^m \frac{\partial p^m}{\partial \tau^m} + \frac{\partial x^m}{\partial \tau^m} dl_p^m\right)\left(\frac{1}{M} - \frac{1}{A}\right) + \left(\tau^m p^m - (1-x^m)dl_p^m\right)\frac{1}{A^2}\frac{\partial A}{\partial \tau} = 0, \tag{26}$$

where $p^m = p(\tau^m, l_p^m, l_s^m)$, $x^m = x(\tau^m, l_p^m, l_s^m)$, and $A = A(\tau^m, l_p^m, l_s^m)$. We assume $\tau(l_p^m, l_s^m; M)$ as the interior solution for the first-order condition (26). When $M = 1$, as one would expect, the first-order condition (26) is equivalent to that of the original model presented in Equation (12), i.e., $\tau(l_p^m, l_s^m; 1) = \tau(l_p, l_s)$.

It is noteworthy that $p^m = p(\tau^m, l_p^m, l_s^m)$, $x^m = x(\tau^m, l_p^m, l_s^m)$, $A = A(\tau^m, l_p^m, l_s^m)$, and their derivatives in Equation (26) are independent of M. Consequently, it follows that $\frac{\partial}{\partial M}\left(\frac{\partial \Pi^m}{\partial \tau^m}\right) < 0$, leading to the following lemma.

Lemma 4. *Given that (l_p^m, l_s^m) is fixed, $\tau(l_p^m, l_s^m; M)$ is a decreasing function in $M \geq 1$.*

This lemma is somewhat intuitive. Platform competition engenders a lower commission.

The liability rule of (l_p^m, l_s^m) is set to maximize $\Pi^m(l_p^m, l_s^m) \equiv \Pi^m(\tau(l_p^m, l_s^m; M), l_p^m, l_s^m)$. In the same vein as that of the original model, one can derive the equilibrium of the game with platform competition.

Proposition 1b. *With platform competition ($M \geq 2$), in the symmetric equilibrium, no platform assumes liability for defective third-party goods; all platforms impose full liability on third-party sellers, i.e., $l_p(M) = 0$ and $l_s(M) = 1$. The equilibrium commission rate is set at $\tau(M) = \tau(0, 1; M)$, which is lower than that of the monopoly case: $\tau(M) < \tau^*$.*

This proposition confirms the robustness of Proposition 1. Although platform competition lowers the equilibrium commission, it does not change the platforms' private incentive for their liability design.

Next, one can observe the consequences of introducing a minimum platform-liability standard of L_p. One can assume that this minimum standard is applied to all platforms. Against the regulation, at Stage 1, all the competing platforms set their liability level at the minimum standard (i.e., $l_p(M) = L_p$). The remaining liability share is imposed on the sellers (i.e., $l_s(M) = 1 - L_p$). At Stage 2, subsequently, their commission rate is set at the one which solves Equation (26) with $l_p^m = L_p$ and $l_s^m = 1 - L_p$. We use $\tau(L_p; M)$ to denote the solution. Formally, $\tau(L_p; M) \equiv \tau(L_p, 1 - L_p; M)$.

The resulting consumer surplus is computed as presented below.

$$CS = \ln\left(\sum_{m=1}^{M} A^m\right) \tag{27}$$

$$= \ln\left(A(\tau(L_p; M), L_p, 1 - L_p)\right) \equiv CS(L_p; M) \tag{28}$$

We see how the introduction of the platform-liability regulation affects consumer surplus.

Proposition 2b. *With platform competition, the platform-liability regulation enhances consumer surplus, $CS'(0; M) > 0$, if and only if*

$$\mu(1 - \tau(M)) > c(x(M)) + (1 - x(M))d, \tag{29}$$

where $x(M)$ represents the equilibrium investment level in the model with M competing platforms, i.e., $x(M) = x(\tau(M), l_p(M), l_s(M))$.

Condition (29) is qualitatively the same as Condition (16). If the sellers' per-unit margin is greater than their full marginal cost, then the regulation can be an effective policy intervention in terms of consumer surplus.

The following remark states whether an increase in the number of competing platforms makes Condition (29) more or less likely to hold.

Remark 1. *The fiercer the platform competition becomes (i.e., as M increases), the more likely the positive consequence of the platform-liability regulation (i.e., Condition [29]) is to hold.*

Fierce competition suppresses the market power of individual platforms, making it difficult for them to pass increased compensation costs associated with the regulation onto the commission fees they charge to sellers. This competitive effect serves to curb the negative indirect effects of the regulation. Consequently, one can infer that facilitating platform competition can make platform-liability regulation more effective.

5.3 Irresponsible sellers

For the original model, all sellers are presumed to have *deep pockets* for compensation, and to compensate the consumers if their goods cause some defect. In reality, however, this is not necessarily the case. Some sellers might not be willing (or might not have the financial ability) to pay compensation. Against this background, we add irresponsible sellers (also called *judgment-proof* sellers) into the original model. Detailed proofs are relegated from the text to Supplementary Appendix C.

There are sellers of two types: responsible and irresponsible. Responsible sellers are those considered in the original model. We presume that irresponsible sellers do not compensate the consumers even if their goods caused some defect. Therefore, they do not invest in product safety, i.e., $x^J = 0$, where superscript 'J' is used to represent the model with judgment-proof sellers.

The profit of a judgment-proof seller i is given as

$$\pi_i^J = \{(1-\tau)p_i^J - c(0)\}q_i^J - e, \tag{30}$$

where

$$q_i^J = \frac{\exp\left(\frac{v-p_i^J-d(1-l_p)}{\mu}\right)}{A}. \tag{31}$$

As in the original model, given the equilibrium aggregate A, judgment-proof seller i chooses price p_i^J to maximize its profit π_i^J, implying that

$$p_i^J = \frac{c(0)}{1-\tau} + \mu \equiv p^J(\tau). \tag{32}$$

The price depends on commission τ, not on the level of platform liability l_p. The resulting profit is $\pi^J(\tau, l_p) = \mu(1-\tau) \cdot \frac{V^J(\tau,l_p)}{A} - e$, where

$$V^J(\tau, l_p) = \exp\left(\frac{v - p^J(\tau) - d(1-l_p)}{\mu}\right). \tag{33}$$

Comparing the profits of responsible and irresponsible sellers, described respectively in Equations (9) and (33), one can derive the following outcome.

Lemma 5. *Given l_p and l_s, there exists a threshold value of the commission rate, denoted by $\hat{\tau}(l_p, l_s)$, such that $\pi(\tau, l_p, l_s) > \pi^J(\tau, l_p)$ if and only if $\tau < \hat{\tau}(l_p, l_s)$. Moreover, the threshold value decreases in l_p and l_s, i.e., $\frac{\partial \hat{\tau}(l_p,l_s)}{\partial l_p} < 0$ and $\frac{\partial \hat{\tau}(l_p,l_s)}{\partial l_s} < 0$.*

This lemma shows that responsible sellers can earn greater profits than irresponsible sellers when the platform sets a commission rate lower than the threshold value $\hat{\tau}$. With the free-entry assumption, in equilibrium, the marketplace is full of responsible sellers with greater profitability. There is no space for irresponsible sellers with lower profitability to enter. Therefore, in this case, the subsequent analyses and main results (i.e., Propositions 1 and 2) remain the same.

In contrast, if the commission rate becomes higher than the threshold, then the situation changes drastically. Irresponsible sellers take over the marketplace in place of responsible sellers. This is true because a higher commission rate amplifies the cost advantage of irresponsible sellers over responsible ones, i.e., the cost difference $\frac{c(x)+(1-x)dl_s}{1-\tau} - \frac{c(0)}{1-\tau}$ increases with τ. A greater cost advantage enables irresponsible sellers to attract consumers with lower prices, whereas their dangerous goods cause harm to consumers more frequently. Additional analyses would be necessary if this were to happen in equilibrium. In what follows, therefore, we check the validity of Propositions 1 and 2 even when the marketplace is full of judgment-proof sellers.

First, the following proposition confirms the robustness of Proposition 1.

Proposition 1c. *Even if the marketplace is full of judgment-proof sellers in equilibrium, the platform has no incentive to assume liability for defective third-party goods, i.e.,* $l_p^J = 0$.

This proposition implies that the proliferation of irresponsible sellers does not change the platform's private incentive for its liability design. That is, the platform has no incentive to assume liability for defective goods sold at its marketplace. Therefore, in equilibrium, harm suffered by consumers is not compensated.

Next, we turn to effects of the platform-liability regulation of L_p. We denote by $CS_J(L_p)$ consumer surplus with the minimum standard; then we evaluate the sign of $CS_J'(0)$.

Proposition 2c. *With the proliferation of judgment-proof sellers, the platform-liability regulation unambiguously reduces the consumer surplus. That is,* $CS_J'(0) < 0$ *holds.*

This proposition demonstrates that the regulation is detrimental to consumers in cases where the marketplace is full of judgment-proof sellers. Compared with Proposition 2, one might infer that the presence of judgment-proof sellers would make the regulation less effective. This is true because judgment-proof sellers make no investment (i.e., $x^J = 0$). Consequently, the marketplace is flooded with illicit products, leading to greater harm to consumers than when the marketplace was full of responsible sellers. Regulation forces the platform to incur greater compensation costs. Increased compensation costs lead the platform to charge a higher commission, which is detrimental to consumer surplus.

Finally, one can note the following remark related to whether regulation encourages or discourages entry into the marketplace by judgment-proof sellers.

Remark 2. *Platform-liability regulation might help judgment-proof sellers prevail in place of responsible sellers.*

This remark derives from the following two outcomes. First, as in the original model, regulation leads the platform to raise its commission. Therefore, $\tau^J(L_p)$ is an increasing function. This feature is confirmed in Proposition 2c and its proof. Second, when $x > \tau$ holds in equilibrium (i.e., the equilibrium commission rate is not too high), the regulation can lower the threshold of the commission rate, $\hat{\tau}$, above which the marketplace is full of judgment-proof sellers. These two outcomes imply that a more stringent regulation makes the inequality of $\tau^J(L_p) > \hat{\tau}(L_p, 1 - L_p)$ more likely to hold, which implies that, from Lemma 5, judgment-proof sellers are more likely than responsible sellers to earn greater profits and to occupy the marketplace.

Proposition 2c and Remark 2 show that the presence of judgment-proof sellers would make the platform-liability regulation less effective, without content moderation or screening. In other words, encouraging platforms to make greater efforts in content moderation and screening (i.e., ex-ante liability) helps regulations that hold platforms liable for defective goods (i.e., ex-post liability) work as intended. Ex-ante and ex-post liability regulations would be mutually complementary.

6 Conclusion

This study was conducted to provide formal analyses for the following question: *Should platforms be held liable for defective third-party goods?* The relevant findings present novel insights into ex-post platform liability. First, platforms apparently have no private incentive to assume liability for defective goods traded in their marketplaces. Next, we identify the necessary and sufficient conditions under which regulations that hold platforms liable can enhance consumer surplus. The platform-liability regulation is more likely to be desirable in the following market environment: [i] the platform's market power is weak; [ii] seller competition is moderate in terms of high margin and low cost; [iii] sellers' goods are not so harmful; and [iv] sellers face less elastic demand. Consequently, not only platform-level assessment based on Point [i], but also product-level assessment based on Points [ii]–[iv] would be necessary for policy intervention.

Moreover, three extended analyses are examined for additional policy implications. Results imply that the following attempts might be helpful for making platform-liability regulation more effective: banning platforms from having a dual role, facilitating platform competition, and encouraging the elimination of irresponsible sellers. These results can be regarded as valuable contributions to recent competition policy debates.

Additionally, this study has yielded theoretical contributions to the literature, which are greater than mere extensions of work by Anderson and Bedre-Defolie (2021). An analytical framework for platform competition is presented in Section 5.2. The framework is expected to be useful to consider platform competition in different contexts, including static competition between asymmetric platforms and dynamic competition between incumbent and entrant platforms.

Finally, it would be worth mentioning some limitations of our analyses and some points of complementarity with some recent studies. Because of the purpose of this paper, we specifically examine ex-post platform liability, but do not address ex-ante platform liability (i.e., content moderation). For this reason, we assume that product safety, as represented by the seller investment level, is observable. Moreover, we assume that no heterogeneity exists in product safety among sellers. Although seller heterogeneity (i.e., responsible and irresponsible sellers) is examined in Section 5.3, either type of seller joins the platform eventually in equilibrium. This outcome would depend on the assumption of observability of product safety and on the free-entry assumption. It would be interesting to consider extensions where sellers are heterogeneous with respect to unobservable product safety for the joint consideration of ex-ante and ex-post platform liability. However, that point is beyond the scope of this study.

As for the relation between ex-ante and ex-post platform liability, one can refer to recent studies by Hua and Spier (2023) and by Yasui (2022), which allow for ex-ante screening in addition to ex-post compensation by platforms. Table 1 presents a summary of the relation among the three papers. Hua and Spier (2023) and Yasui (2022) provide novel insights into how liability for ex-post compensation affects the incentive of platforms for ex-ante screening. This paper, instead of disregarding ex-ante screening, develops the model with indirect network externalities to provide

Table 1. Complementarity with other similar studies

		Hua and Spier (2023)	Yasui (2022)	This paper
Platform design	Commission	endogenous	fixed	endogenous
	Compensation	fixed	endogenous	endogenous
	Screening	endogenous	endogenous	n/a
Seller strategy	Pricing	fixed	endogenous	endogenous
	Investment	fixed	endogenous	endogenous
Indirect network externalities		fixed	fixed	endogenous

further insights into issues of ex-post platform liability.

Appendix Omitted Proofs

Proof of Lemma 1 Differentiating Equations (7) and (8) with respect to τ, l_p, and l_s, one can derive the following outcomes.

$$\frac{\partial x(\tau, l_p, l_s)}{\partial \tau} = -\frac{d(1-l_p-l_s)}{c''(x)} < 0 \tag{34}$$

$$\frac{\partial p(\tau, l_p, l_s)}{\partial \tau} = \frac{\partial x(\tau, l_p, l_s)}{\partial \tau} \cdot d(1-l_p-l_s) + \frac{p(\tau, l_p, l_s) - \mu}{1-\tau} \tag{35}$$

$$\frac{\partial x(\tau, l_p, l_s)}{\partial l_p} = -\frac{d(1-\tau)}{c''(x)} < 0 \tag{36}$$

$$\frac{\partial p(\tau, l_p, l_s)}{\partial l_p} = \frac{\partial x(\tau, l_p, l_s)}{\partial l_p} \cdot d(1-l_p-l_s) < 0 \tag{37}$$

$$\frac{\partial x(\tau, l_p, l_s)}{\partial l_s} = \frac{d\tau}{c''(x)} > 0 \tag{38}$$

$$\frac{\partial p(\tau, l_p, l_s)}{\partial l_s} = \frac{\partial x(\tau, l_p, l_s)}{\partial l_s} \cdot d(1-l_p-l_s) + \frac{d(1-x(\tau, l_p, l_s))}{1-\tau} > 0 \tag{39}$$

■

Proof of Lemma 2 With the results presented in the proof of Lemma 1 above, by differentiating Equations (9) and (10) with respect to τ, l_p, and l_s, one can derive the following outcomes.

$$\frac{\partial V(\tau, l_p, l_s)}{\partial \tau} = -\frac{V(\tau, l_p, l_s)}{\mu} \cdot \frac{p(\tau, l_p, l_s) - \mu}{1-\tau} < 0 \tag{40}$$

$$\frac{\partial A(\tau, l_p, l_s)}{\partial \tau} = -\frac{A(\tau, l_p, l_s)}{\mu} \cdot \frac{p(\tau, l_p, l_s)}{1-\tau} < 0 \tag{41}$$

$$\frac{\partial V(\tau, l_p, l_s)}{\partial l_p} = \frac{V(\tau, l_p, l_s)}{\mu} \cdot d(1 - x(\tau, l_p, l_s)) > 0 \tag{42}$$

$$\frac{\partial A(\tau, l_p, l_s)}{\partial l_p} = \frac{A(\tau, l_p, l_s)}{\mu} \cdot d(1 - x(\tau, l_p, l_s)) > 0 \tag{43}$$

$$\frac{\partial V(\tau, l_p, l_s)}{\partial l_s} = -\frac{V(\tau, l_p, l_s)}{\mu} \cdot d(1 - x(\tau, l_p, l_s)) \cdot \frac{\tau}{1-\tau} < 0 \tag{44}$$

$$\frac{\partial A(\tau, l_p, l_s)}{\partial l_s} = -\frac{A(\tau, l_p, l_s)}{\mu} \cdot d(1 - x(\tau, l_p, l_s)) \cdot \frac{\tau}{1-\tau} < 0 \tag{45}$$

■

Proof of Proposition 1 First, we show that $\frac{\partial \Pi(l_p,l_s)}{\partial l_p} = \frac{\partial \Pi(l_p,l_s)}{\partial l_s} \cdot \left(-\frac{1-\tau}{\tau}\right)$ holds. The derivative of $\Pi(l_p, l_s)$ with respect to l_p is computed as follows.

$$\frac{\partial \Pi(l_p, l_s)}{\partial l_p} = \left(\tau \frac{\partial p}{\partial l_p} + \frac{\partial x}{\partial l_p} dl_p - (1-x)d\right)\left(1 - \frac{1}{A}\right) + (\tau p - (1-x)dl_p) \cdot \frac{1}{A^2}\frac{\partial A}{\partial l_p} \tag{46}$$

$$= \left(-\frac{d(1-\tau)}{c''(x)} d\left(\tau(1-l_s) + (1-\tau)l_p\right) - (1-x)d\right)\left(1 - \frac{1}{A}\right) + \frac{\tau p - (1-x)dl_p}{A\mu} \cdot (1-x)d \tag{47}$$

Similarly, the derivative of $\Pi(l_p, l_s)$ with respect to l_s is expressed as shown below.

$$\frac{\partial \Pi(l_p, l_s)}{\partial l_s} = \left(\tau \frac{\partial p}{\partial l_s} + \frac{\partial x}{\partial l_s} dl_p\right)\left(1 - \frac{1}{A}\right) + (\tau p - (1-x)dl_p) \cdot \frac{1}{A^2}\frac{\partial A}{\partial l_s} \tag{48}$$

$$= \left(\frac{d\tau}{c''(x)} d\left(\tau(1-l_s) + (1-\tau)l_p\right) + \frac{(1-x)d\tau}{1-\tau}\right)\left(1 - \frac{1}{A}\right) - \frac{\tau p - (1-x)dl_p}{A\mu} \cdot \frac{(1-x)d\tau}{1-\tau} \tag{49}$$

By comparing Equations (47) and (49), one can realize that $\frac{\partial \Pi(l_p,l_s)}{\partial l_p} = \frac{\partial \Pi(l_p,l_s)}{\partial l_s} \cdot \left(-\frac{1-\tau}{\tau}\right)$ holds.

Next, we demonstrate that $\frac{\partial \Pi(l_p,l_s)}{\partial l_p} < 0$ holds for any $(l_p, l_s) \in [0,1]^2$. Rearranging Equation (12) yields the following expression.

$$\left(p + \tau \frac{\partial p}{\partial \tau} + \frac{\partial x}{\partial \tau} dl_p\right)\left(1 - \frac{1}{A}\right) = \frac{\tau p - (1-x)dl_p}{A\mu} \cdot \frac{p}{1-\tau} \tag{50}$$

Using this equation, Equation (47) can be rewritten as follows.

$$\frac{\partial \Pi(l_p, l_s)}{\partial l_p} = \left(\tau \frac{\partial p}{\partial l_p} + \frac{\partial x}{\partial l_p} dl_p - (1-x)d\right)\left(1 - \frac{1}{A}\right) + \frac{\tau p - (1-x)dl_p}{A\mu} \cdot (1-x)d \tag{51}$$

$$= \left(\tau \frac{\partial p}{\partial l_p} + \frac{\partial x}{\partial l_p} dl_p - (1-x)d\right)\left(1 - \frac{1}{A}\right) + \frac{1-\tau}{p}\left(p + \tau \frac{\partial p}{\partial \tau} + \frac{\partial x}{\partial \tau} dl_p\right)\left(1 - \frac{1}{A}\right) \cdot (1-x)d \tag{52}$$

$$= \left(1 - \frac{1}{A}\right)\left(\tau \frac{\partial p}{\partial l_p} + \frac{\partial x}{\partial l_p} dl_p + \frac{1-\tau}{p}\left(p + \tau \frac{\partial p}{\partial \tau} + \frac{\partial x}{\partial \tau} dl_p - \frac{p}{1-\tau}\right)(1-x)d\right) \tag{53}$$

From Lemma 1, $\frac{\partial p}{\partial l_p} < 0$ and $\frac{\partial x}{\partial l_p} < 0$ hold. Moreover, we have the following computation.

$$p + \tau \frac{\partial p}{\partial \tau} + \frac{\partial x}{\partial \tau} dl_p - \frac{p}{1-\tau} = -\frac{\tau}{1-\tau}\mu - \frac{d^2(1 - l_p + l_s)}{c''(x)} \cdot (\tau(1-l_s) + (1-\tau)l_p) < 0 \tag{54}$$

In total, one can show that $\frac{\partial \Pi(l_p, l_s)}{\partial l_p} < 0$, implying that, in equilibrium, l_p is set at the lowest level, i.e., $l_p^* = 0$. Furthermore, because of the first result above, it follows that $\frac{\partial \Pi(l_p, l_s)}{\partial l_s} > 0$, implying that l_s is set at the highest level, i.e., $l_s^* = 1$. ■

Proof of Proposition 2 Here, we derive the condition for which $CS'(0) < 0$ holds.

From Equation (15), $CS'(L_p) < 0$ holds if and only if

$$\frac{\partial A}{\partial \tau} \cdot \tau'(L_p) + \frac{\partial A}{\partial l_p} - \frac{\partial A}{\partial l_s} < 0, \tag{55}$$

where $\frac{\partial A}{\partial \tau} = -\frac{pV}{e}$, $\frac{\partial A}{\partial l_p} = (1-\tau) \cdot \frac{Vd(1-x)}{e}$, and $\frac{\partial A}{\partial l_s} = -\tau \cdot \frac{Vd(1-x)}{e}$. Thus, using Equation (14), one can have

$$CS'(L_p) < 0 \iff p\left(\frac{\partial^2 \Pi}{\partial \tau \partial l_p} - \frac{\partial^2 \Pi}{\partial \tau \partial l_s}\right) - d(1-x)\left(-\frac{\partial^2 \Pi}{\partial \tau^2}\right) > 0, \tag{56}$$

where p, x, and all the derivatives are evaluated at $(\tau(L_p), L_p, 1 - L_p)$. Differentiating the first-order condition (12) with respect to τ, l_p, and l_s, one can derive the following expressions.

$$
\begin{aligned}
\frac{\partial^2\Pi}{\partial\tau^2} =& \frac{1}{1-\tau}\frac{2(p-\mu)}{1-\tau}\left(1-\frac{1}{A}\right) \\
&- \frac{p}{\mu(1-\tau)A}\left[2\cdot\frac{p-\tau\mu}{1-\tau} + (\tau p-(1-x)dL_p)\frac{1}{1-\tau}\left(2-\frac{\mu}{p}+\frac{p}{\mu}\right)\right]
\end{aligned} \tag{57}
$$

$$
\begin{aligned}
\frac{\partial^2\Pi}{\partial\tau\partial l_p} - \frac{\partial^2\Pi}{\partial\tau\partial l_s} =& -\frac{1}{1-\tau}\frac{d(1-x)}{1-\tau}\left(1-\frac{1}{A}\right) \\
&+ \frac{p}{\mu(1-\tau)A}\left[\begin{array}{l} \frac{2p-\tau\mu}{p}\cdot\frac{d(1-x)}{1-\tau} - \left(\frac{\partial x}{\partial l_p}-\frac{\partial x}{\partial l_s}\right)dL_p \\ +(\tau p-(1-x)dL_p)\frac{d(1-x)}{1-\tau}\left(\frac{1}{p}+\frac{1}{\mu}\right) \end{array}\right]
\end{aligned} \tag{58}
$$

Using these expressions, one can obtain the following outcome.

$$
\begin{aligned}
& p\left(\frac{\partial^2\Pi}{\partial\tau\partial l_p} - \frac{\partial^2\Pi}{\partial\tau\partial l_s}\right) - d(1-x)\left(-\frac{\partial^2\Pi}{\partial\tau^2}\right) \\
&= \frac{p-2\mu}{1-\tau}\cdot\frac{d(1-x)}{1-\tau}\left(1-\frac{1}{A}\right) + \frac{p}{\mu(1-\tau)A}\left[\begin{array}{l} \tau\mu\frac{d(1-x)}{1-\tau} - p\left(\frac{\partial x}{\partial l_p}-\frac{\partial x}{\partial l_s}\right)dL_p \\ +(\tau p-(1-x)dL_p)\frac{d(1-x)}{1-\tau}\left(\frac{\mu}{p}-1\right) \end{array}\right]
\end{aligned} \tag{59}
$$

When $l_p = L_p$ and $l_s = 1 - L_p$, from the first-order condition (12), one can derive the following equation.

$$
1-\frac{1}{A} = \frac{1-\tau}{p-\tau\mu}\cdot(\tau p-(1-x)dL_p)\cdot\frac{p}{\mu(1-\tau)A} \tag{60}
$$

Substituting this equation into Equation (59) yields the following result.

$$
\begin{aligned}
& p\left(\frac{\partial^2\Pi}{\partial\tau\partial l_p} - \frac{\partial^2\Pi}{\partial\tau\partial l_s}\right) - d(1-x)\left(-\frac{\partial^2\Pi}{\partial\tau^2}\right) \\
&= \frac{p}{\mu(1-\tau)A}\left[\begin{array}{l} \tau\mu\frac{d(1-x)}{1-\tau} - p\left(\frac{\partial x}{\partial l_p}-\frac{\partial x}{\partial l_s}\right)dL_p \\ -(\tau p-(1-x)dL_p)\frac{d(1-x)}{1-\tau}\frac{\mu\{(1-\tau)p+\tau\mu\}}{p(p-\tau\mu)} \end{array}\right] \qquad (61) \\
&= \frac{p}{\mu(1-\tau)A}\left[\begin{array}{l} \left(\tau\mu - \tau p\cdot\frac{\mu\{(1-\tau)p+\tau\mu\}}{p(p-\tau\mu)}\right)\frac{d(1-x)}{1-\tau} \\ +dL_p\left\{-p\left(\frac{\partial x}{\partial l_p}-\frac{\partial x}{\partial l_s}\right) + (1-x)\cdot\frac{d(1-x)}{1-\tau}\cdot\frac{\mu\{(1-\tau)p+\tau\mu\}}{p(p-\tau\mu)}\right\} \end{array}\right] \qquad (62)
\end{aligned}
$$

For $L_p = 0$, it follows that

$$\left[p\left(\frac{\partial^2\Pi}{\partial\tau\partial l_p} - \frac{\partial^2\Pi}{\partial\tau\partial l_s}\right) - d(1-x)\left(-\frac{\partial^2\Pi}{\partial\tau^2}\right)\right]_{L_p=0}$$
$$= \frac{p}{\mu(1-\tau)A} \cdot \frac{d(1-x)}{1-\tau} \cdot \tau\mu\left(1 - \frac{(1-\tau)p + \tau\mu}{p - \tau\mu}\right). \tag{63}$$

Therefore, one can derive the condition for $CS'(0) < 0$ as shown below.

$$CS'(0) > 0 \iff \left[p\left(\frac{\partial^2\Pi}{\partial\tau\partial l_p} - \frac{\partial^2\Pi}{\partial\tau\partial l_s}\right) - d(1-x)\left(-\frac{\partial^2\Pi}{\partial\tau^2}\right)\right]_{L_p=0} < 0 \tag{64}$$
$$\iff 1 - \frac{(1-\tau)p + \tau\mu}{p - \tau\mu} < 0 \tag{65}$$
$$\iff \mu(1-\tau) > c(x) + (1-x)d \tag{66}$$

■

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Supplementary Appendix A Hybrid Platform

Here, we present the derivation of the equilibrium of the hybrid platform model.

At Stage 3, given the platform's strategy $(\tau, l_p, l_s, p_0, x_0)$, fringe sellers choose p_i and x_i simultaneously. As in the original model, we confine our attention to the symmetric equilibrium, where all sellers choose the same price p and the same investment level x. One can confirm that the sellers' optimal strategy (p, x) is the same as that of the original model presented in Equations (7) and (8), as explained below.

If seller i deviates from the symmetric equilibrium, its demand is given as

$$q_i = \frac{\exp(b_i/\mu)}{1 + \exp(b_0/\mu) + n \cdot \exp(b/\mu)} \equiv \frac{\exp(b_i/\mu)}{A_h}, \tag{A.1}$$

where A_h represents the aggregate in the hybrid platform model. Seller i maximizes its profit as $\pi_i = ((1-\tau)p_i - c(x_i) - (1-x_i)dl_p) \cdot \frac{\exp(b_i/\mu)}{A_h}$. In the maximization problem of fringe sellers, the only difference between the original model and the hybrid platform model is the content of the aggregate (A or A_h). In the original analysis, the optimal strategy (p, x) is determined independently of the aggregate. Thus, in the same vein, one can derive the same outcome (p, x) also in the hybrid platform model.

The resulting seller profit is also the same as that of the original model, that is, $\pi_i(\tau, l_p, l_s) = \mu(1-\tau)\frac{V(\tau, l_p, l_s)}{A_h} - e$. The zero-profit condition pins down the aggregate A_h as shown below.

$$A_h = \frac{\mu(1-\tau)}{e} V(\tau, l_p, l_s) = A(\tau, l_p, l_s) \tag{A.2}$$

Therefore, the aggregate remains unchanged even if the platform sells its own good, i.e., $A_h(\tau, l_p, l_s) = A(\tau, l_p, l_s)$. The platform profit can be rewritten as

$$\Pi_h = (p_0 - c(x_0) - (1-x_0)d(l_p + l_s)) \frac{V_0}{A} + (\tau p - (1-x)dl_p) \frac{A - V_0 - 1}{A} \tag{A.3}$$

$$= \Pi(\tau, l_p, l_s) + \frac{V_0}{A} (p_0 - c(x_0) - (1-x_0)d(l_p + l_s) - \tau p + (1-x)dl_p) \tag{A.4}$$

where p, x, and A are evaluated at (τ, l_p, l_s) and $V_0 = \exp\left(\frac{v - p_0 - (1-x_0)d(1-l_p-l_s)}{\mu}\right)$.

The first term is equal to the platform's profit in the original model. The second term represents the additional profit generated from first-party selling. Because the first term is independent of (p_0, x_0), the platform chooses p_0 and x_0 to maximize the second term of Equation (A.4). Solving the maximization problem, one can derive that the platform's optimal strategy for its first-party good, denoted as $p_0(\tau, l_p, l_s)$ and $x_0(\tau, l_p, l_s)$, satisfies the following system:

$$x_0(\tau, l_p, l_s) = \bar{x} \tag{A.5}$$

$$p_0(\tau, l_p, l_s) = \underbrace{c(\bar{x})}_{\text{selling cost}} \underbrace{+ (1 - \bar{x})\, d(l_p + l_s)}_{\text{compensation cost}} \underbrace{+ \tau p - (1 - x) d l_p}_{\text{opportunity cost}} \underbrace{+\mu}_{\text{Logit markup}} \tag{A.6}$$

where p and x are evaluated at (τ, l_p, l_s). Equation (A.5) implies that, irrespective of the liability design, the platform makes the optimal level of defect-reducing investment in its first-party good. Moreover, Equation (A.6) shows that the price is set at the sum of the selling cost, compensation cost, opportunity cost, and the standard Logit markup.

Using those results, one can rewrite the platform's profit function as

$$\Pi_h(\tau, l_p, l_s) = \Pi(\tau, l_p, l_s) + \mu \cdot \frac{V_0(\tau, l_p, l_s)}{A(\tau, l_p, l_s)}, \tag{A.7}$$

where $V_0(\tau, l_p, l_s) = \exp\left(\frac{v - p_0(\tau, l_p, l_s) - (1-\bar{x}) d(1 - l_p - l_s)}{\mu}\right)$. One can notice that $\Pi_h(\tau, l_p, l_s) > \Pi(\tau, l_p, l_s)$ holds, implying that the platform gains from first-party selling.

The hybrid platform chooses a commission rate to maximize the profit presented in Equation (A.7). The first-order condition is given as the following.

$$\frac{\partial \Pi_h(\tau, l_p, l_s)}{\partial \tau} = \frac{\partial \Pi(\tau, l_p, l_s)}{\partial \tau} + \mu \cdot \frac{\partial}{\partial \tau}\left(\frac{V_0(\tau, l_p, l_s)}{A(\tau, l_p, l_s)}\right) = 0 \tag{A.8}$$

As in the original model, we suppose that the maximization problem has a unique solution that satisfies Equation (A.8), which we denote by $\tau_h(l_p, l_s)$.

As presented in Lemma 3, one can show that the hybrid platform charges a higher commission than the pure-marketplace platform, i.e., $\tau_h(l_p, l_s) > \tau(l_p, l_s)$ holds for any $(l_p, l_s) \in [0, 1]^2$. The proof is as follows.

Proof of Lemma 3 We show that $\frac{\partial \Pi_h(\tau, l_p, l_s)}{\partial \tau} > \frac{\partial \Pi(\tau, l_p, l_s)}{\partial \tau}$ holds for any (τ, l_p, l_s). From Equation (A.8), it suffices to show $\frac{\partial}{\partial \tau}\left(\frac{V_0(\tau, l_p, l_s)}{A(\tau, l_p, l_s)}\right) > 0$.

$$\frac{\partial}{\partial \tau}\left(\frac{V_0(\tau, l_p, l_s)}{A(\tau, l_p, l_s)}\right) = \frac{1}{A^2}\left(\frac{\partial V_0}{\partial \tau} A - V_0 \frac{\partial A}{\partial \tau}\right) \tag{A.9}$$

$$= \frac{1}{A}\left(-\frac{V_0}{\mu}\left(p + \tau\frac{\partial p}{\partial \tau} + \frac{\partial x}{\partial \tau}dl_p\right)\right) - \frac{V_0}{A^2}\left(-\frac{pA}{\mu(1-\tau)}\right) \tag{A.10}$$

$$= -\frac{V_0}{A\mu}\left(p + \tau\frac{\partial p}{\partial \tau} + \frac{\partial x}{\partial \tau}dl_p - \frac{p}{1-\tau}\right) \tag{A.11}$$

$$= \frac{V_0}{A\mu}\left(\frac{\tau}{1-\tau}\mu + \frac{d^2(1-l_p+l_s)}{c''(x)}\left(\tau(1-l_s)+(1-\tau)l_p\right)\right) \tag{A.12}$$

$$> 0 \tag{A.13}$$

Therefore, it follows that $\frac{\partial \Pi_h(\tau,l_p,l_s)}{\partial \tau} > \frac{\partial \Pi(\tau,l_p,l_s)}{\partial \tau}$, implying that $\tau_h(l_p,l_s) > \tau(l_p,l_s)$. ■

Under the hybrid platform model, imposing a higher commission not only increases the per-unit commission revenue, but also generates a greater profit from first-party selling by raising the marginal cost of third-party sellers. Therefore, the hybrid platform has a greater incentive to impose a high commission on sellers.

Finally, at Stage 1, the platform chooses its liability design of (l_p, l_s) to maximize $\Pi_h(l_p, l_s) = \Pi_h(\tau_h(l_p, l_s), l_p, l_s)$. Solving the problem, one can derive Proposition 1a presented in Section 5.1. The proof of the proposition is the following.

Proof of Proposition 1a First, we show that $\frac{\partial \Pi_h(l_p,l_s)}{\partial l_p} = \frac{\partial \Pi_h(l_p,l_s)}{\partial l_s} \cdot \left(-\frac{1-\tau}{\tau}\right)$. The derivative of $\Pi_h(l_p, l_s)$ with respect to l_p is given as follows.

$$\frac{\partial \Pi_h(l_p,l_s)}{\partial l_p} = \frac{\partial \Pi(l_p,l_s)}{\partial l_p} + \mu \cdot \frac{\partial}{\partial l_p}\left(\frac{V_0}{A}\right) \tag{A.14}$$

$$= \frac{\partial \Pi(l_p,l_s)}{\partial l_p} + \frac{V_0}{A}\left(-\tau\frac{\partial p}{\partial l_p} - \frac{\partial x}{\partial l_p}dl_p\right) \tag{A.15}$$

Similarly, the derivative of $\Pi_h(l_p, l_s)$ with respect to l_s is given as follows.

$$\frac{\partial \Pi_h(l_p,l_s)}{\partial l_s} = \frac{\partial \Pi(l_p,l_s)}{\partial l_s} + \mu \cdot \frac{\partial}{\partial l_s}\left(\frac{V_0}{A}\right) \tag{A.16}$$

$$= \frac{\partial \Pi(l_p,l_s)}{\partial l_s} + \frac{V_0}{A}\left(-\tau\frac{\partial p}{\partial l_s} - \frac{\partial x}{\partial l_s}dl_p + (1-x)d\frac{\tau}{1-\tau}\right) \tag{A.17}$$

$$= \frac{\partial \Pi(l_p,l_s)}{\partial l_s} + \frac{V_0}{A}\left(\tau\frac{\partial p}{\partial l_p} + \frac{\partial x}{\partial l_p}dl_p\right)\frac{\tau}{1-\tau} \tag{A.18}$$

As shown in Proposition 1, it follows that $\frac{\partial \Pi(l_p,l_s)}{\partial l_p} = \frac{\partial \Pi(l_p,l_s)}{\partial l_s} \cdot \left(-\frac{1-\tau}{\tau}\right)$, which in turn ensures that $\frac{\partial \Pi_h(l_p,l_s)}{\partial l_p} = \frac{\partial \Pi_h(l_p,l_s)}{\partial l_s} \cdot \left(-\frac{1-\tau}{\tau}\right)$ also holds.

Next, we show $\frac{\partial \Pi_h(l_p,l_s)}{\partial l_p} < 0$ for $l_p \in [0,1]$. Using Equation (47), Equation (A.15) can be rewritten as the following.

$$\begin{aligned}\frac{\partial \Pi_h(l_p,l_s)}{\partial l_p} =& \left(\tau\frac{\partial p}{\partial l_p}+\frac{\partial x}{\partial l_p}dl_p-(1-x)d\right)\left(1-\frac{1}{A}\right)\\ &+(\tau p-(1-x)dl_p)\cdot\frac{d(1-x)}{\mu A}+\frac{V_0}{A}\left(-\tau\frac{\partial p}{\partial l_p}-\frac{\partial x}{\partial l_p}dl_p\right) \qquad \text{(A.19)}\end{aligned}$$

Here, from the first-order condition with respect to τ (i.e., Equation (A.8)), one can derive the following expression.

$$\begin{aligned}\frac{\tau p-(1-x)dl_p}{\mu A} =& \frac{1-\tau}{p}\left(p+\tau\frac{\partial p}{\partial \tau}+\frac{\partial x}{\partial \tau}dl_p\right)\left(1-\frac{1}{A}\right)\\ &-\frac{1-\tau}{p}\frac{V_0}{A}\left(p+\tau\frac{\partial p}{\partial \tau}+\frac{\partial x}{\partial \tau}dl_p-\frac{p}{1-\tau}\right) \qquad \text{(A.20)}\end{aligned}$$

Substituting Equation (A.20) into Equation (A.19) yields the following.

$$\begin{aligned}\frac{\partial \Pi_h(l_p,l_s)}{\partial l_p} =& \left(\tau\frac{\partial p}{\partial l_p}+\frac{\partial x}{\partial l_p}dl_p-(1-x)d\right)\left(1-\frac{1}{A}\right)+\frac{V_0}{A}\left(-\tau\frac{\partial p}{\partial l_p}-\frac{\partial x}{\partial l_p}dl_p\right)\\ &+d(1-x)\frac{1-\tau}{p}\left(p+\tau\frac{\partial p}{\partial \tau}+\frac{\partial x}{\partial \tau}dl_p\right)\left(1-\frac{1}{A}\right)\\ &-d(1-x)\frac{1-\tau}{p}\frac{V_0}{A}\left(p+\tau\frac{\partial p}{\partial \tau}+\frac{\partial x}{\partial \tau}dl_p-\frac{p}{1-\tau}\right) \qquad \text{(A.21)}\\ =& \left[\tau\frac{\partial p}{\partial l_p}+\frac{\partial x}{\partial l_p}dl_p+d(1-x)\frac{1-\tau}{p}\left(p+\tau\frac{\partial p}{\partial \tau}+\frac{\partial x}{\partial \tau}dl_p-\frac{p}{1-\tau}\right)\right]\\ &\times\left(1-\frac{1}{A}-\frac{V_0}{A}\right) \qquad \text{(A.22)}\end{aligned}$$

Here, from Lemma 1, one can show that $\frac{\partial p}{\partial l_p}<0$, $\frac{\partial x}{\partial l_p}<0$, and $p+\tau\frac{\partial p}{\partial \tau}+\frac{\partial x}{\partial \tau}dl_p-\frac{p}{1-\tau}<0$. Moreover, because $A=V_0+nV+1$, it holds that $1-\frac{1}{A}-\frac{V_0}{A}>0$. Thus, it follows that $\frac{\partial \Pi_h(l_p,l_s)}{\partial l_p}<0$.

In total, as in the original model, both $\frac{\partial \Pi_h(l_p,l_s)}{\partial l_p}=\frac{\partial \Pi_h(l_p,l_s)}{\partial l_s}\cdot\left(-\frac{1-\tau}{\tau}\right)$ and $\frac{\partial \Pi_h(l_p,l_s)}{\partial l_p}<0$ hold. Therefore, in equilibrium, the hybrid platform chooses the lowest value for l_p and the highest value for l_s, i.e., $l_p^{**}=0$ and $l_s^{**}=1$. ■

Finally, we investigate the effect of a regulation holding the hybrid platform liable. As in the original model, we let a policymaker impose a regulation that requires the hybrid platform to take liability greater than or equal to L_p. The platform responds by choosing $l_p=L_p$ and $l_s=1-L_p$, because $\frac{\partial \Pi_h(l_p,l_s)}{\partial l_p}<0$ and $\frac{\partial \Pi_h(l_p,l_s)}{\partial l_s}>0$ hold for all $(l_p,l_s)\in[0,1]^2$, as shown in the proof of Proposition 1a. Then, at Stage 2, the hybrid platform chooses τ to maximize

$$\Pi_h(\tau, L_p, 1-L_p) = \Pi(\tau, L_p, 1-L_p) + \mu \cdot \frac{V_0(\tau, L_p, 1-L_p)}{A(\tau, L_p, 1-L_p)}. \tag{A.23}$$

Let $\tau_h(L_p)$ solve the first-order condition of $\frac{\partial \Pi_h(\tau, L_p, 1-L_p)}{\partial \tau} = 0$. Thus, in a similar way to the derivation of Equation (14), one can obtain the following.

$$\tau_h'(L_p) = \frac{\frac{\partial^2 \Pi_h(\tau, L_p, 1-L_p)}{\partial \tau \partial l_p} - \frac{\partial^2 \Pi_h(\tau, L_p, 1-L_p)}{\partial \tau \partial l_s}}{-\frac{\partial^2 \Pi_h(\tau, L_p, 1-L_p)}{\partial \tau^2}} \tag{A.24}$$

Consumer surplus is given as $CS_h(L_p) \equiv \ln A(\tau_h(L_p), L_p, 1-L_p)$. Its derivative with respect to L_p is computed as

$$CS_h'(L_p) = \frac{1}{A}\left(\underbrace{\frac{\partial A}{\partial \tau} \cdot \tau_h'(L_p)}_{\text{indirect effect}} + \underbrace{\frac{\partial A}{\partial l_p} - \frac{\partial A}{\partial l_s}}_{\text{direct effect}}\right), \tag{A.25}$$

where A and all the derivatives are evaluated at $(\tau_h(L_p), L_p, 1-L_p)$.

The proof of Proposition 2a is the following.

Proof of Proposition 2a We here prove that $CS'(L_p) > CS_h'(L_p)$ holds for all $L_p \in [0, 1]$.

Comparison between Equations (15) and (A.25) implies that $CS'(L_p) > CS_h'(L_p)$ if and only if $\tau_h'(L_p) > \tau'(L_p)$, because $\frac{\partial A}{\partial \tau} = -\frac{pV}{e} < 0$. Thus, it suffices to show that $\tau_h'(L_p) > \tau'(L_p)$ holds for all $L_p \in [0, 1]$.

Differentiating the first-order condition (A.8) with respect to τ, l_p, and l_s, one can derive the following expressions.

$$\frac{\partial^2 \Pi_h}{\partial \tau^2} = \frac{\partial^2 \Pi}{\partial \tau^2} + \mu \frac{V_0}{A} \frac{1+\tau^2}{(1-\tau)^2} \tag{A.26}$$

$$\frac{\partial^2 \Pi_h}{\partial \tau \partial l_p} - \frac{\partial^2 \Pi_h}{\partial \tau \partial l_s} = \left(\frac{\partial^2 \Pi}{\partial \tau \partial l_p} - \frac{\partial^2 \Pi}{\partial \tau \partial l_s}\right) - \frac{V_0}{A}\left(\frac{\partial x}{\partial l_p} - \frac{\partial x}{\partial l_s}\right) dL_p \frac{\tau}{1-\tau} \tag{A.27}$$

From Equation (A.26), it follows that $\frac{\partial^2 \Pi_h}{\partial \tau^2} > \frac{\partial^2 \Pi}{\partial \tau^2}$. Moreover, from Equation (A.27), $\left(\frac{\partial^2 \Pi_h}{\partial \tau \partial l_p} - \frac{\partial^2 \Pi_h}{\partial \tau \partial l_s}\right) \geq \left(\frac{\partial^2 \Pi}{\partial \tau \partial l_p} - \frac{\partial^2 \Pi}{\partial \tau \partial l_s}\right)$ holds because $\frac{\partial x}{\partial l_p} - \frac{\partial x}{\partial l_s} = -\frac{d}{c''(x)} < 0$. Using these results, one can show $\tau_h'(L_p) > \tau'(L_p)$ as presented below.

$$\tau_h'(L_p) = \frac{\frac{\partial^2 \Pi_h}{\partial \tau \partial l_p} - \frac{\partial^2 \Pi_h}{\partial \tau \partial l_s}}{-\frac{\partial^2 \Pi_h}{\partial \tau^2}} \tag{A.28}$$

$$> \frac{\frac{\partial^2 \Pi}{\partial \tau \partial l_p} - \frac{\partial^2 \Pi}{\partial \tau \partial l_s}}{-\frac{\partial^2 \Pi}{\partial \tau^2}} = \tau'(L_p) \tag{A.29}$$

Therefore, $CS'(L_p) > CS_h'(L_p)$ holds for all $L_p \in [0, 1]$. ■

Supplementary Appendix B Platform Competition

We here present the proofs for Propositions 1b and 2b, respectively.

Proof of Proposition 1b First, the derivative of Π^m with respect to l_p^m is given as

$$\begin{aligned}
\frac{\partial \Pi^m(l_p^m, l_s^m)}{\partial l_p^m} &= \left(\tau^m \frac{\partial p^m}{\partial l_p^m} + \frac{\partial x^m}{\partial l_p^m} dl_p^m - (1-x^m)d\right) \mathbb{P}^m \left(1 - \frac{1}{A^m}\right) \\
&\quad + \left(\tau^m p^m - (1-x^m)dl_p^m\right) \mathbb{P}^m \left(\frac{1}{A^m}\right)^2 \frac{\partial A^m}{\partial l_p^m} && \text{(B.1)} \\
&= \left(\tau^m \frac{\partial p^m}{\partial l_p^m} + \frac{\partial x^m}{\partial l_p^m} dl_p^m - (1-x^m)d\right) \mathbb{P}^m \left(1 - \frac{1}{A^m}\right) \\
&\quad + \left(\tau^m p^m - (1-x^m)dl_p^m\right) \mathbb{P}^m \frac{1}{A^m} \frac{(1-x^m)d}{\mu}\ , && \text{(B.2)}
\end{aligned}$$

where $\tau^m = \tau(l_p^m, l_s^m; M)$ and p^m, x^m, and A^m are evaluated at $(\tau(l_p^m, l_s^m; M), l_p^m, l_s^m)$. From the first-order condition (26), one can derive the following expression.

$$\frac{\tau^m p^m - (1-x^m)dl_p^m}{A^m \mu} = \frac{1-\tau^m}{p^m} \left(p^m + \tau^m \frac{\partial p^m}{\partial \tau^m} + \frac{\partial x^m}{\partial \tau^m} dl_p^m\right) \left(1 - \frac{1}{A^m}\right) \quad \text{(B.3)}$$

Substituting this expression into Equation (B.2) yields

$$\frac{\partial \Pi^m(l_p^m, l_s^m)}{\partial l_p^m} = \mathbb{P}^m \left(1 - \frac{1}{A^m}\right) \left[\begin{array}{l} \tau^m \frac{\partial p^m}{\partial l_p^m} + \frac{\partial x^m}{\partial l_p^m} dl_p^m \\ + \frac{1-\tau^m}{p^m} \left(p^m + \tau^m \frac{\partial p^m}{\partial \tau^m} + \frac{\partial x^m}{\partial \tau^m} dl_p^m - \frac{p^m}{1-\tau^m}\right)(1-x^m)d \end{array} \right], \quad \text{(B.4)}$$

where $p^m = p(\tau^m, l_p^m, l_s^m)$ and $x^m = x(\tau^m, l_p^m, l_s^m)$. Thus, from Lemma 1, $\frac{\partial p^m}{\partial l_p^m} < 0$ and $\frac{\partial x^m}{\partial l_p^m} < 0$ hold. Moreover, in the same vein as that of Inequality (54) presented in the proof of Proposition 1, one can prove $p^m + \tau^m \frac{\partial p^m}{\partial \tau^m} + \frac{\partial x^m}{\partial \tau^m} dl_p^m - \frac{p^m}{1-\tau^m} < 0$. In total, $\frac{\partial \Pi^m(l_p^m, l_s^m)}{\partial l_p^m} < 0$ holds, implying $l_p(M) = 0$ in equilibrium.

In addition, in the same vein as that of the proof of Proposition 1, one can show that $\frac{\partial \Pi^m(l_p^m, l_s^m)}{\partial l_s^m} = \frac{\partial \Pi^m(l_p^m, l_s^m)}{\partial l_p^m} \cdot \left(-\frac{\tau^m}{1-\tau^m}\right) > 0$ holds, leading to $l_s(M) = 1$. ■

Proof of Proposition 2b The proof is highly analogous to that of Proposition 2.

As in the main model, $CS'(L_p;M)>0$ holds if and only if

$$CS'(L_p;M)>0 \iff \frac{\partial A}{\partial \tau}\cdot \tau'(L_p;M)+\frac{\partial A}{\partial l_p}-\frac{\partial A}{\partial l_s}>0 \tag{B.5}$$

$$\iff p^m\left(\frac{\partial^2 \Pi^m}{\partial \tau^m \partial l_p^m}-\frac{\partial^2 \Pi^m}{\partial \tau^m \partial l_s^m}\right)-d(1-x^m)\left(-\frac{\partial^2 \Pi^m}{\partial \tau^{m2}}\right)<0, \tag{B.6}$$

where $p^m=p(\tau(L_p;M),L_p,1-L_p)$ and $x^m=x(\tau(L_p;M),L_p,1-L_p)$, and all the derivatives are evaluated at $(\tau(L_p;M),L_p,1-L_p)$.

Differentiating the first-order condition (26) with respect to τ, l_p, and l_s, one can derive the following expressions.

$$\begin{aligned}\frac{\partial^2 \Pi^m}{\partial \tau^{m2}} =&\frac{1}{1-\tau}\frac{2(p-\mu)}{1-\tau}\left(\frac{1}{M}-\frac{1}{A}\right)\\ &-\frac{p}{\mu(1-\tau)A}\left[2\cdot\frac{p-\tau\mu}{1-\tau}+(\tau p-(1-x)dL_p)\frac{1}{1-\tau}\left(2-\frac{\mu}{p}+\frac{p}{\mu}\right)\right]\end{aligned} \tag{B.7}$$

$$\begin{aligned}\frac{\partial^2 \Pi^m}{\partial \tau^m \partial l_p^m}-\frac{\partial^2 \Pi}{\partial \tau^m \partial l_s^m} =&-\frac{1}{1-\tau}\frac{d(1-x)}{1-\tau}\left(\frac{1}{M}-\frac{1}{A}\right)\\ &+\frac{p}{\mu(1-\tau)A}\left[\begin{array}{l}\frac{2p-\tau\mu}{p}\cdot\frac{d(1-x)}{1-\tau}-\left(\frac{\partial x}{\partial l_p}-\frac{\partial x}{\partial l_s}\right)dL_p\\ +(\tau p-(1-x)dL_p)\frac{d(1-x)}{1-\tau}\left(\frac{1}{p}+\frac{1}{\mu}\right)\end{array}\right]\end{aligned} \tag{B.8}$$

One might notice that the only difference from the main model is the term $\left(\frac{1}{M}-\frac{1}{A}\right)$ replacing the original one of $\left(1-\frac{1}{A}\right)$. Using these expressions, one can obtain the following.

$$\begin{aligned}&p\left(\frac{\partial^2 \Pi}{\partial \tau \partial l_p}-\frac{\partial^2 \Pi}{\partial \tau \partial l_s}\right)-d(1-x)\left(-\frac{\partial^2 \Pi}{\partial \tau^2}\right)\\ &=\frac{p-2\mu}{1-\tau}\cdot\frac{d(1-x)}{1-\tau}\left(\frac{1}{M}-\frac{1}{A}\right)+\frac{p}{\mu(1-\tau)A}\left[\begin{array}{l}\tau\mu\frac{d(1-x)}{1-\tau}-p\left(\frac{\partial x}{\partial l_p}-\frac{\partial x}{\partial l_s}\right)dL_p\\ +(\tau p-(1-x)dL_p)\frac{d(1-x)}{1-\tau}\left(\frac{\mu}{p}-1\right)\end{array}\right]\end{aligned} \tag{B.9}$$

When $l_p=L_p$ and $l_s=1-L_p$, from the first-order condition (26), one can derive the following equation.

$$\frac{1}{M}-\frac{1}{A}=\frac{1-\tau}{p-\tau\mu}\cdot(\tau p-(1-x)dL_p)\cdot\frac{p}{\mu(1-\tau)A} \tag{B.10}$$

Substituting this equation into Equation (B.9) yields

$$\begin{aligned}
&p\left(\frac{\partial^2\Pi}{\partial\tau\partial l_p}-\frac{\partial^2\Pi}{\partial\tau\partial l_s}\right)-d(1-x)\left(-\frac{\partial^2\Pi}{\partial\tau^2}\right)\\
&=\frac{p}{\mu(1-\tau)A}\left[\begin{array}{l}\tau\mu\frac{d(1-x)}{1-\tau}-p\left(\frac{\partial x}{\partial l_p}-\frac{\partial x}{\partial l_s}\right)dL_p\\ -\left(\tau p-(1-x)dL_p\right)\frac{d(1-x)}{1-\tau}\frac{\mu\{(1-\tau)p+\tau\mu\}}{p(p-\tau\mu)}\end{array}\right] &\text{(B.11)}\\
&=\frac{p}{\mu(1-\tau)A}\left[\begin{array}{l}\left(\tau\mu-\tau p\cdot\frac{\mu\{(1-\tau)p+\tau\mu\}}{p(p-\tau\mu)}\right)\frac{d(1-x)}{1-\tau}\\ +dL_p\left\{-p\left(\frac{\partial x}{\partial l_p}-\frac{\partial x}{\partial l_s}\right)-(1-x)\cdot\frac{d(1-x)}{1-\tau}\cdot\frac{\mu\{(1-\tau)p+\tau\mu\}}{p(p-\tau\mu)}\right\}\end{array}\right], &\text{(B.12)}
\end{aligned}$$

which is completely the same as Equation (62). That is, the remaining proof can be completed in the same way as the corresponding part of the proof of Proposition 2. Therefore, one can confirm that $CS'(0;M)>0$ holds if and only if $\mu(1-\tau(M))>c(x(M))+(1-x(M))d$. ■

Proof of Remark 1 Condition (29) holds if and only if

$$f(M)\equiv\mu(1-\tau(M))-c(x(M))-(1-x(M))d>0. \tag{B.13}$$

One derives the following.

$$\begin{aligned}
f'(M)&=-\mu\tau'(M)-c'(x(M))\cdot x'(M)+x'(M)d &\text{(B.14)}\\
&=-\mu\tau'(M)-\{c'(x(M))-d\}\,x'(M) &\text{(B.15)}
\end{aligned}$$

From Equation (8), $c'(x)=d$ holds in equilibrium, because $l_p(M)=0$ and $l_s(M)=1$. Thus, it follows that $f'(M)=-\mu\tau'(M)$. Moreover, from Lemma 4, one can see that $\tau'(M)=\frac{\partial\tau(0,1;M)}{\partial M}<0$ holds.

In total, $f'(M)>0$, which implies that an increase in M makes Inequality (B.13) more likely to hold. ■

Supplementary Appendix C   Irresponsible Sellers

Here, we allow for the presence of irresponsible sellers as well as responsible sellers. As in the main model, all sellers are assumed to be fringe. Therefore, taking the equilibrium aggregate as given, every seller chooses its price and investment level to maximize its own profit. The resulting profits of irresponsible and responsible sellers are given as $\pi(\tau, l_p, l_s)$ and $\pi^J(\tau, l_p)$, respectively. Because of the free-entry assumption, if $\pi(\tau, l_p, l_s) > \pi^J(\tau, l_p)$, then the marketplace is full of responsible sellers. Otherwise, only irresponsible sellers participate in the marketplace.

Whether inequality $\pi(\tau, l_p, l_s) > \pi^J(\tau, l_p)$ holds or not depends on the platform's decision on commission τ, as illustrated in Lemma 5. The proof is the following.

Proof of Lemma 5 Comparison between $\pi(\tau, l_p, l_s)$ and $\pi^J(\tau, l_p)$ yields the following relationship.

$$\pi(\tau, l_p, l_s) \gtreqless \pi^J(\tau, l_p) \tag{C.1}$$

$$\iff V(\tau, l_p, l_s) \gtreqless V^J(\tau, l_p) \tag{C.2}$$

$$\iff x(\tau, l_p, l_s)d(1-l_p) - \frac{\tau}{1-\tau}(1-x(\tau, l_p, l_s))dl_s - \frac{c(x(\tau, l_p, l_s)) - c(0)}{1-\tau} \gtreqless 0, \tag{C.3}$$

Using $J(\tau, l_p, l_s)$ to denote the left-hand side of inequality (C.3), one can show that its derivative with respect to τ takes a negative value.

$$\frac{\partial J(\tau, l_p, l_s)}{\partial \tau} = -\frac{(1-x)dl_s + c(x) - c(0)}{(1-\tau)^2} < 0 \tag{C.4}$$

This result implies that there exists a threshold value of τ below which $\pi(\tau, l_p, l_s) > \pi^J(\tau, l_p)$ holds. We use $\hat{\tau}(l_p, l_s)$ to denote this threshold value. That is, $\hat{\tau}(l_p, l_s)$ solves $J(\tau, l_p, l_s) = 0$. Thus, one can compute $\frac{\partial \hat{\tau}(l_p,l_s)}{\partial l_p}$ and $\frac{\partial \hat{\tau}(l_p,l_s)}{\partial l_s}$ as follows.

$$\frac{\partial \hat{\tau}(l_p, l_s)}{\partial l_p} = -\frac{\partial J(\tau, l_p, l_s)}{\partial l_p} \Big/ \frac{\partial J(\tau, l_p, l_s)}{\partial \tau} = -\frac{(1-\tau)^2 xd}{(1-x)dl_s + c(x) - c(0)} < 0 \tag{C.5}$$

$$\frac{\partial \hat{\tau}(l_p, l_s)}{\partial l_s} = -\frac{\partial J(\tau, l_p, l_s)}{\partial l_s} \Big/ \frac{\partial J(\tau, l_p, l_s)}{\partial \tau} = -\frac{\tau(1-\tau)(1-x)d}{(1-x)dl_s + c(x) - c(0)} < 0 \tag{C.6}$$

■

Lemma 5 implies that, if the equilibrium commission rate is set lower than the threshold value $\hat{\tau}(l_p, l_s)$, then the analyses regarding Stages 1 and 2 remain the same as those of the original model. Otherwise, we need to re-analyze the platform's decision-making in Stages 1 and 2, as presented below.

For $\tau > \hat{\tau}(l_p, l_s)$, the marketplace is full of irresponsible sellers. The zero-profit condition of free entry pins down the aggregate as

$$A^J(\tau, l_p) = \frac{\mu(1-\tau)}{e} \cdot V^J(\tau, l_p), \tag{C.7}$$

where

$$V^J(\tau, l_p) = \exp\left(\frac{v - p^J(\tau) - d(1-l_p)}{\mu}\right). \tag{C.8}$$

The platform chooses l_p and τ in Stages 1 and 2, respectively, to maximize its profit below.

$$\Pi^J(\tau, l_p) = \left(\tau p^J(\tau) - dl_p\right)\left(1 - \frac{1}{A^J(\tau, l_p)}\right) \tag{C.9}$$

The equilibrium outcome is summarized in Proposition 1c. The proof is as follows.

Proof of Proposition 1c At Stage 2, the platform chooses τ to maximize $\Pi^J(\tau, l_p)$. The first-order condition is given as follows.

$$\begin{aligned} \frac{\partial \Pi^J(\tau, l_p)}{\partial \tau} &= \left(p^J(\tau) + \tau \frac{\partial p^J(\tau)}{\partial \tau}\right)\left(1 - \frac{1}{A^J(\tau, l_p)}\right) + \left(\tau p^J(\tau) - dl_p\right)\frac{1}{(A^J)^2}\frac{\partial A^J(\tau, l_p)}{\partial \tau} \\ &= 0 \end{aligned} \tag{C.10}$$

$$\iff \left(p^J(\tau) - \tau\mu\right)\left(1 - \frac{1}{A^J(\tau, l_p)}\right) = \left(\tau p^J(\tau) - dl_p\right)\frac{p^J(\tau)}{\mu A^J(\tau, l_p)} \tag{C.11}$$

We let $\tau^J(l_p)$ solve the first-order condition (C.11).

At Stage 1, the platform chooses l_p to maximize $\Pi^J(l_p) \equiv \Pi^J(\tau^J(l_p), l_p)$. Using envelop theorem, one can derive the derivative of $\Pi^J(l_p)$ with respect to l_p as

$$\frac{d\Pi^J(l_p)}{dl_p} = \frac{\partial \Pi^J(\tau^J(l_p), l_p)}{\partial l_p} \tag{C.12}$$

$$= -d\left(1 - \frac{1}{A^J}\right) + \left(\tau^J(l_p)p^J(\tau^J(l_p)) - dl_p\right)\frac{1}{(A^J)^2}\frac{\partial A^J(\tau^J(l_p), l_p)}{\partial l_p} \tag{C.13}$$

$$= -d\left(1 - \frac{1}{A^J}\right) + \left(\tau^J(l_p)p^J(\tau^J(l_p)) - dl_p\right) \cdot \frac{d}{\mu A^J}, \tag{C.14}$$

where A^J is evaluated at $(\tau^J(l_p), l_p)$. With the first-order condition (C.11), the above expression (C.14) can be simplified as presented below.

$$\frac{d\Pi^J(l_p)}{dl_p} = -d\left(1-\frac{1}{A^J}\right) + \left(p^J(\tau^J(l_p)) - \tau^J(l_p)\mu\right)\left(1-\frac{1}{A^J}\right)\frac{d}{p^J(\tau^J(l_p))} \tag{C.15}$$

$$= -d\left(1-\frac{1}{A^J}\right)\frac{\tau^J(l_p)\mu}{p^J(\tau^J(l_p))} < 0 \tag{C.16}$$

Therefore, the platform chooses $l_p = 0$ in equilibrium. ■

Next, we turn to the consumer-surplus effect of a regulation that imposes a minimum standard of L_p. Proposition 1c implies that, under the regulation, the platform sets $l_p = L_p$ in Stage 1. Accordingly, the platform adjusts its commission rate. That is, in Stage 2, the platform faces the following maximization problem: $\max_\tau \Pi^J(\tau, L_p)$. We let $\tau^J(L_p)$ denote the optimal commission rate, which satisfies the first-order condition of $\frac{\partial \Pi^J(\tau^J(L_p), L_p)}{\partial \tau} = 0$. Moreover, differentiating this first-order condition with respect to L_p, one can derive the following expression.

$$\frac{d\tau^J(L_p)}{dL_p} = \frac{\partial^2 \Pi(\tau^J(L_p), L_p)}{\partial\tau\partial l_p} \Big/ \left(-\frac{\partial^2\Pi(\tau^J(L_p), L_p)}{\partial\tau^2}\right) \tag{C.17}$$

Consumer surplus is expressed as $CS_J(L_p) \equiv \ln A^J(\tau^J(L_p), L_p)$. The impact of the regulation on consumer surplus is computed as

$$CS_J'(L_p) = \frac{1}{A^J(\tau^J(L_p), L_p)}\left(\underbrace{\frac{\partial A^J(\tau^J(L_p), L_p)}{\partial\tau}\cdot\frac{d\tau^J(L_p)}{dL_p}}_{\text{indirect effect}} + \underbrace{\frac{\partial A^J(\tau^J(L_p), L_p)}{\partial l_p}}_{\text{direct effect}}\right), \tag{C.18}$$

which can be decomposed to direct and indirect effects, as in the main model.

For the proof of Proposition 2c, in what follows, we prove that $CS_J'(0) < 0$.

Proof of Proposition 2c Throughout this proof, for ease of exposition, we write $\tau^J = \tau^J(L_p)$, $p^J = p^J(\tau^J(L_p))$, $A^J = A^J(\tau^J(L_p), L_p)$, $V^J = V^J(\tau^J(L_p), L_p)$, and $\Pi^J = \Pi^J(\tau^J(L_p), L_p)$.

From Equation (C.18), it follows that

$$CS_J'(L_p) < 0 \iff \frac{\partial A^J}{\partial\tau}\cdot\frac{d\tau^J}{dL_p} + \frac{\partial A^J}{\partial l_p} < 0, \tag{C.19}$$

where $\frac{\partial A^J}{\partial\tau} = -\frac{p^J A^J}{e}$ and $\frac{\partial A^J}{\partial l_p} = \frac{\mu(1-\tau)V^J}{e}\cdot\frac{d}{\mu}$. Thus, it suffices to show the following inequality.

$$p^J \cdot \frac{\partial^2 \Pi^J}{\partial \tau \partial l_p} - (1-\tau^J) d \left(- \frac{\partial^2 \Pi^J}{\partial \tau^2} \right) > 0 \tag{C.20}$$

In the same vein as that of the proof of Proposition 2, one can obtain the following.

$$\begin{aligned} \frac{\partial^2 \Pi^J}{\partial \tau^2} =& \frac{1}{1-\tau^J} \cdot \frac{2\left(p^J - \mu\right)}{1-\tau^J} \cdot \left(1 - \frac{1}{A^J}\right) \\ & - \frac{p^J}{\mu(1-\tau^J)A^J} \left[2 \cdot \frac{p^J - \tau^J \mu}{1-\tau^J} + (\tau^J p^J - dL_p) \cdot \frac{1}{1-\tau^J} \cdot \left(2 - \frac{\mu}{p^J} + \frac{p^J}{\mu} \right) \right] \end{aligned} \tag{C.21}$$

$$\frac{\partial^2 \Pi^J}{\partial \tau \partial l_p} = \frac{d}{\mu(1-\tau^J)A^J} \left(2p^J - \tau^J \mu + (\tau^J p^J - dL_p) \cdot \frac{p^J}{\mu} \right) \tag{C.22}$$

With these outcomes, one can compute the left-hand side of Inequality (C.20) as follows.

$$\begin{aligned} & p^J \cdot \frac{\partial^2 \Pi^J}{\partial \tau \partial l_p} - (1-\tau^J) d \left(- \frac{\partial^2 \Pi^J}{\partial \tau^2} \right) \\ =& \frac{2d(p^J - \mu)}{1-\tau^J} \left(1 - \frac{1}{A^J} \right) + \frac{dp^J}{\mu(1-\tau)A^J} \left[\tau^J \mu + (\tau^J p^J - dL_p) \left(-2 + \frac{\mu}{p^J} \right) \right] \end{aligned} \tag{C.23}$$

From Equation (C.11), it follows that $1 - \frac{1}{A^J} = \frac{1-\tau^J}{p^J - \tau^J \mu} (\tau^J p^J - dL_p) \frac{p^J}{\mu(1-\tau^J)A^J}$. With this expression, Equation (C.23) can be further rewritten as shown below.

$$\begin{aligned} & p^J \cdot \frac{\partial^2 \Pi^J}{\partial \tau \partial l_p} - (1-\tau^J) d \left(- \frac{\partial^2 \Pi^J}{\partial \tau^2} \right) \\ =& \frac{dp^J}{\mu(1-\tau^J)A^J} \left[\tau^J \mu + (\tau^J p^J - dL_p) \left(\frac{2(p^J - \mu)}{p^J - \tau^J \mu} - 2 + \frac{\mu}{p^J} \right) \right] \end{aligned} \tag{C.24}$$

Therefore, for $L_p = 0$, one can derive the following relationship.

$$\left[p^J \cdot \frac{\partial^2 \Pi^J}{\partial \tau \partial l_p} - (1-\tau^J) d \left(- \frac{\partial^2 \Pi^J}{\partial \tau^2} \right) \right]_{L_p = 0} \tag{C.25}$$

$$= \frac{dp^J}{\mu(1-\tau^J)A^J} \left[\tau^J \mu + \tau^J p^J \left(\frac{2(p^J - \mu)}{p^J - \tau^J \mu} - 2 + \frac{\mu}{p^J} \right) \right] \tag{C.26}$$

$$= \frac{dp^J}{\mu(1-\tau^J)A^J} \cdot 2\tau^J (p^J - \mu) \left(\frac{p^J}{p^J - \tau^J \mu} - 1 \right) \tag{C.27}$$

$$> 0 \tag{C.28}$$

Therefore, it is shown that $CS'_J(0) < 0$ holds. ■

Proof of Remark 2 It is already shown in the proof of Proposition 2c that $\frac{d\tau^J(L_p)}{dL_p} > 0$ holds.

In addition, we here show that a more stringent regulation reduces the threshold $\hat{\tau}(l_p, l_s)$. With regulation of L_p, the platform sets $l_p = L_p$ and $l_s = 1 - L_p$, as shown in Proposition 1c. Thus, it is sufficient to prove $\frac{\partial\hat{\tau}(L_p,1-L_p)}{\partial l_p} - \frac{\partial\hat{\tau}(L_p,1-L_p)}{\partial l_s} < 0$.

Using Equations (C.5) and (C.6), one can derive the following.

$$\frac{\partial\hat{\tau}(L_p, 1 - L_p)}{\partial l_p} - \frac{\partial\hat{\tau}(L_p, 1 - L_p)}{\partial l_s}$$

$$= -\frac{(1-\tau)^2 xd}{(1-x)d(1-L_p) + c(x) - c(0)} - \left(-\frac{\tau(1-\tau)(1-x)d}{(1-x)d(1-L_p) + c(x) - c(0)}\right) \tag{C.29}$$

$$= -\frac{d(1-\tau)(x-\tau)}{(1-x)d(1-L_p) + c(x) - c(0)} \tag{C.30}$$

$$< 0 \iff x > \tau \tag{C.31}$$

Therefore, if $x > \tau$ holds in equilibrium, a more stringent regulation reduces the threshold $\hat{\tau}$.