Labor Market Power in the Presence of Informality and Self-Employment

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Abstract

I propose a theory of labor market power when informality and self-employment are simultaneously present - a salient characteristic of developing economies' labor markets. I construct a model of heterogeneous firms with endogenous entry and calibrate it with Brazilian data. Firms self-select into formality or informality and workers have the outside option of working as self-employed. From the firm's perspective, labor supply elasticity is endogenously determined by competition across firms, across sectors and against self-employment. I find that informality increases firm entry, but those firms are less productive and less competitive, lowering equilibrium wages, aggregate productivity and welfare. Self-employment is an effective alternative for workers, increasing average wages due to a competition channel. However, labor allocated to self-employment drastically impairs aggregate productivity and welfare. Compared to first best allocation, the market equilibrium features 26% lower welfare, of which 7% is explained by firm informality and 13% by self-employment.

Keywords: informality, self-employment, labor market power, heterogeneous firms

JEL Codes: J31, J42, J46, O17, O54

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1 Introduction

Informality is a prevalent characteristic of developing economies' labor markets. Both from the perspective of undocumented firms, or from being an undocumented employee, informality in general is associated with lower productivity, resource misallocation and lower wages [1]. Even though informal markets may represent a considerable portion of a country's labor market and a non-negligible fraction official output, the literature on its impact on market power is limited and recent.

Among wage workers, those with a formal contract earn higher wages even when controlling by observable characteristics or, more noticeable, when controlling by individual (Engbom et al. (2022)). However, firms pay similar formal and informal workers the same (Ulyssea (2018)). Taken together, these evidences point towards firm-driven, instead of worker-driven, formality premium among wage workers. In other words, the evidence suggests that high-paying firms select into formality, and low-paying firms selecting into informality. Moreover, worker transitions between formality, informality and self-employment are high (Narita (2020),Gomes et al. (2020)), which arguably indicates a certain degree of substitutability between these dimensions.

Leveraging these facts, the present work seeks to shed light on the potential interactions that emerge from considering informal firms and self-employment in an environment with imperfect competition for labor. The approach is two-fold: empirically, I construct a novel dataset tying together measures of wages, market concentration and informality rates, and explore the relationships therein. Theoretically, I develop a model that incorporates firm self-selection into (in)formality when employers, both formal and informal, enjoy a certain degree of labor market power. Formal firms are subject to taxes while informal firms face a productivity wedge that is increasing in demanded labor. In equilibrium, firms formalize if they expect to be productive (and therefore large) enough, after considering the costs and taxes. Workers, on the other hand, will supply labor to formal and informal firms alike, or become self-employed. As a result, competition in wage posting will take in consideration competitor firms' behavior in both formal and informal sections of the market, and workers' outside option as self-employed.

The model is calibrated to match the relevant moments in an average sector in the Brazil-

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[1] see Ulyssea (2020) for a comprehensive review

ian labor market, and is able to reproduce the most salient features of data and establishes some relationships that allow for theoretical predictions. Initial counterfactual analysis show that the possibility of informal wage work is responsible for one fifth of the gap between the market solution and the first best. The existence of self-employment work as an outside option, on the other hand, explains two thirds of the gap. In both cases, labor is allocated to worse technologies than the one present in formal firms, which directly worsens average productivity and welfare. The existence of self-employment increases competition from firm perspective, which goes in the direction of improving allocation of wage workers, but this improvement is dominated by the productivity effect. The existence of informality worsens both competition for wage work and productivity.

2 Related Literature

This work lies in the intersection of two strands of literature.

First, it relates to a growing literature on market power in labor markets in the spirit of nested CES preferences established in Berger et al. (2022). A substantial amount of recent work has been dedicated to identifying, estimating and studying the implications of labor market power (Felix (2021), Azar et al. (2022), Azar et al. (2022), Benmelech et al. (2022), Herkenhoff et al. (2020)). When it comes to developing countries, Muralidharan et al. (2017), Brooks et al. (2021) and Amodio and De Roux (2021) find evidence of relevant oligopsony power in the wage sector in India, China and Colombia, respectively.

Second, it focuses on informal labor markets with special emphasis in firm selection a la Ulyssea (2018), which I build on. Several recent contributions have been dedicated to dissecting the particularities of informal labor markets and their micro and aggregate interactions. In the specific case of Brazil, some have focused on earnings and switching dynamics of formal and informal workers (Engbom et al. (2022), Gomes et al. (2020), Narita (2020)), while others seek to address the potential drivers of informality (Almeida and Carneiro (2012), Seminario Amez (2021), Haanwinckel and Soares (2021)). Utilizing a similar theoretical framework as I do for firm selection and informality, Corbellini (2023) studies the impact of tax enforcement on aggregate productivity.

Some authors tackle the same intersection of the present work. Amodio et al. (2022) and Alvarez-Cuadrado et al. (2020) address labor market power while focusing on the issue of selection between wage work and self-employment. In addition to self-employment, my work

features firm selection and informality in wage work, adding to these authors' findings. By developing a search model, Meghir et al. (2015) rationalizes facts regarding informality and worker transitions, indirectly addressing competition via a wage-posting framework.

3 Data

In this section I empirically document and investigate labor market relationships in Brazilian data. First, I describe the three different data sources employed; Second, I describe the procedure utilized to construct a novel panel on informal labor and market concentration; Third, I go through the empirical findings.

3.1 Data

The panel I construct spans the period of 2012 (1st quarter), when most variables of interest become harmonized, to 2019 (4th quarter), thus avoiding noise from the COVID pandemic. Given the focus on labor market power, I also restrict measures to those in metropolitan regions. Non-metropolitan areas are included in the survey but their geographic cut is not explicitly defined and I am thus unable to properly establish a notion local labor markets in these cases and match them across data sources.

3.1.1 PNAD

The National Household Survey (*Pesquisa Nacional por Amostra de Domicilio*, PNAD) is held by the Brazilian Bureau of Statistics (IBGE), which conducts a continuous interview scheme and is published quarterly. Each household sampled participates in 5 consecutive quarters, then is rotated out. Every quarter has approximately one fifth of households in its first interview, one fifth in its second, and so forth. This survey collects individual-level information including education level, labor force attachment, other demographics, hours worked and earnings. Most importantly, it contains information about its formality status, if it self-employed or not, and in which sector and occupation the individual is in. Since individual identification is not maintained across interviews (only household identification), only the first interview of a household will be taken into the empirical exercises. Given the selection on metropolitan regions, in the period 2012-2019 and selected sectors, I am left with over 400,000 individual observations split among 21 metropolitan areas.

3.1.2 RAIS

The Annual Registry of Social Information (*Relação Anual de Informações Sociais*, RAIS) is collected yearly by the Ministry of Labor and covers the universe of formal firms and formal workers. Each firm that is formally registered must submit detailed information on its workforce (dataset *"vínculos"*, at the worker-firm linkage level) and firm details (dataset *"estb"*, at establishment level). Compliance is high, as firms that do not submit their information in time are subject to fines that increase in firm size and report delay. In the period 2012-2019, for the 21 metropolitan regions and selected sectors, I am left with over 9.7 million establishment-year observations, covering over 137 million worker-firm-year linkages.

3.1.3 ECINF

The Urban Informal Economy Survey (*Pesquisa de Economia Informal Urbana*, ECINF) is a cross section survey of small firms up to five employees. It was collected by the IBGE in 1997 and 2003, of which I use the latter edition in this work. This survey directly asked a representative sample of small firm entrepreneurs about the characteristics of their enterprise (sector, whether it is registered, revenues and expenses, etc.) and their employees (if registered, wages, demographics, etc.). The crucial feature of this survey is that it mostly covers a similar number of formal and informal establishments, and features information on formal and informal workers.

3.2 A Panel on Informal Labor and Market Concentration

The final objective of this panel is to obtain, for each year-metro-sector, measures of formality rates, level (amount) of formal and informal employment, as well as self-employment, their distribution among firm sizes, wage bills and, finally, market concentration. Table 1 summarizes on which dataset, or combination of datasets, each variable is obtained from. Some measures are straightforward to obtain, such as level and distribution of formal work (from RAIS) and informality rates (from PNAD). In what follows, I follow the usual nomenclature in the literature: **formal workers** are those who hold a formal contract and are registered before authorities and subject to taxes; **self-employed** are those who have non-wage earnings and do not employ other individuals; **formal firms** are those registered to fiscal authorities and subject to taxes and labor legislation. Additionaly, denote informal workers in formal firms the **intensive** margin of informality and informal workers in informal

firms the **extensive** margin. Although this margin is not employed in the main exercise, it is available for extensions and future work.

Table 1: Data sources for construction of the panel on informal labor and market concentration

Variable	**Level**	**Distribution (Firms and Intensive x Extensive)**
Informality Rates	PNAD	-
Formal Workers	RAIS	RAIS
Formal Firms	RAIS	RAIS
Informal Workers (All)	Informality Rate (PNAD) and Formal Work Level (RAIS)	(see below)
Informal Workers (Intensive)	RAIS + ECINF	RAIS + ECINF
Informal Workers (Extensive)	Total Informality (RAIS + PNAD) - Intensive Informality (RAIS + ECINF)	ECINF
Informal Firms	Extensive Informality and ECINF distribution	ECINF

For the distributional properties of informal workers, I rely on the patterns present in the ECINF. Namely, from the ECINF I uncover two crucial measures: intensive informality conditional on formal firm size (i.e. how many informal workers we expect to have in a formal firm of a given size) and the distribution of extensive informality (i.e. the size distribution of informal firms).

Table 2: Intensive Informality

Dependent Variable:	log(# Informal Workers)
log(# Total Workers)	0.3671***
	(0.0090)
Observations	1,416
R^2	0.26272
Adjusted R^2	0.26272

IID standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

For the intensive margin, I employ a parsimonious relationship between the number of informal workers in formal firms, and their total size, reported in table 2. This relationship indicates that in formal firms, on average, a 1% increase in total employment (formal + informal) is associated with a 0.36% increase in informal employment. This means, for example, that firms with 1, 5 and 50 formal workers employ on average 1.3, 2 and 4.3

informal workers.

For the extensive margin, I select all observations on informal workers in informal firms up to 5 workers - the level at which the survey is meant to be representative for this measure. I then fit the observations on a truncated geometric distribution, obtaining a probability parameter of 0.41. This means that an informal worker in an informal firm taken at random has around 41% probability to work on a firm with 1 employee, 24% to work at a firm with 2 employees, and so forth.

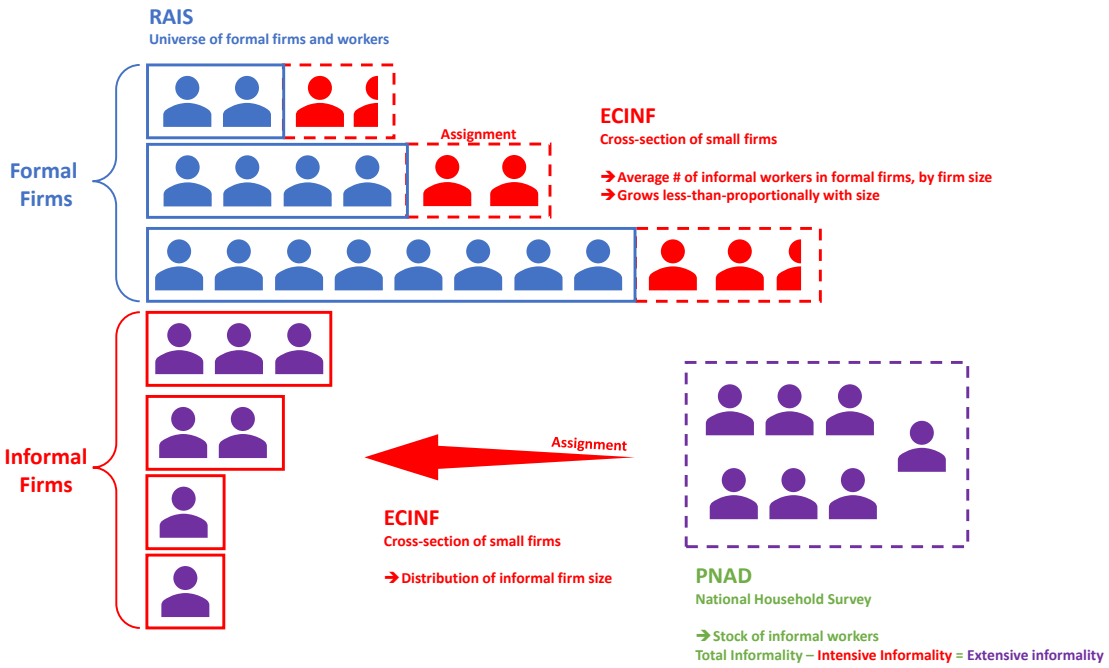


Figure 1: Schematics for the construction of the panel on informal labor and market concentration

With these two distributional measures at hand, I proceed to infer the total level and distributions in the universe of formal and informal workers using the PNAD and RAIS[2]. From the PNAD we have informality rate measures for each 2-digit sector, year and metropolitan region group. Taken together with total formal employment from RAIS, this allows us to have a measure on the total level of informal work. Then, each formal firm in the RAIS is assigned an expected level of informal employment based on its number of formal workers and the relationship found in the ECINF shown in table 2. The sum of all such workers in

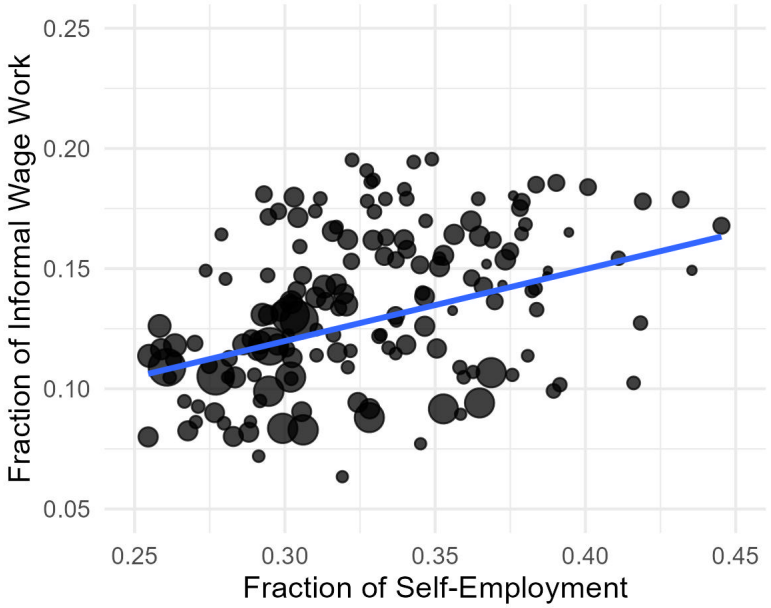
[2]The ECINF was conducted in 2003, 9 years before the start of the RAIS and PNAD samples utilized. Thus, this work relies on the assumption that the distribution pattern in both intensive and extensive margins are sufficiently constant throughout the period of interest.

a given group of firms gives us a measure of the level of intensive informality in that group. Finally, the level of extensive informality (informal workers in informal firms) is taken to be the difference between total informality and intensive informality. These informal workers at the extensive margin are then assigned to a hypothetical informal firm according to the geometric distribution relationship identified in the ECINF.

In the end we are left with a data set that contains, for each year, 2-digit sector and metropolitan region, measures on the level and distribution of observed formal employment and its inferred informal counterpart at the intensive and extensive margin. Figure 1 presents a schematic for the dataset construction. Formal workers are represented in blue, intensive informality in red and extensive informality in purple.

Figure 2 plots the fractions of self-employment and wage-work informality for the 21 metropolitan regions for the period 2012-2019. It shows that both informal wage work and self-employment are prevalent characteristics throughout different regions, and those measures are positivelly correlated.

Figure 2: Informality and Self-Employment



Each dot represents a metropolitan region (21 in total) in a year (2012-2019). Dot size represents the relative size, in employment terms.

With the information on wages and employment at hand, I am able to construct a measure on market concentration. For this purpose I employ the Herfindahl-Hirschman

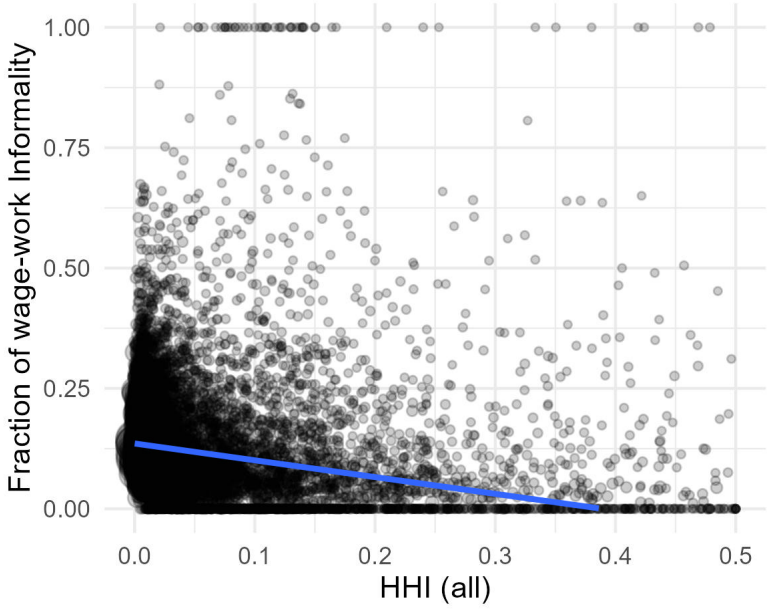
Index (HHI) of firms' wage-bills. This index is given by

$$HHI_j = \sum_i s_{ij}^2, \tag{1}$$

where $s_{ij} = n_{ij}w_{ij} / \sum_k n_{kj}wkj$ is the wage-bill share of firm i in sector j. In what follows, two possible HHI measures are considered in all empirical exercises. The first considers only formal firms in formal workers, and is the usual measure employed in most works. This measure mechanically overestimates concentration by not considering informal firms, and features an additional bias due to mismeasurement on firm sizes. As seen in table 2, smaller formal firms employ a larger fraction of informal workers, thus its size is usually undereported to a greater extent. The second HHI measure makes use of the entire panel constructed, and considers all dimensions of informality, both extensive (informal firms) and intensive (informal workers in formal firms).

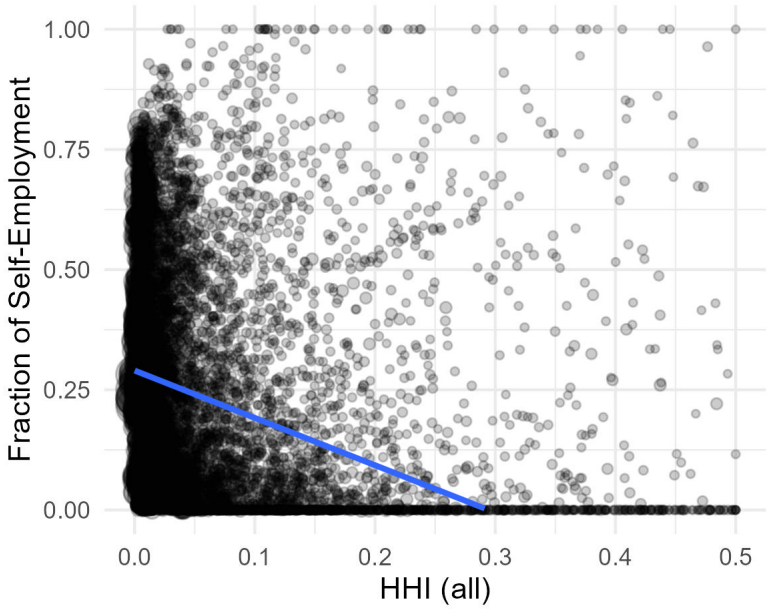
Figures 3 and 4 show the relationship between market concentration, informality and self-employment. Each dot represents 2-digit sector in a metropolitan region in a given year in the data [3], and the blue line is the linear fit of the data. It shows that, on average, more concentrated markets feature lower informality and self-employment rates - in other words, they are more formal.

Figure 3: Informality vs Market Concentration



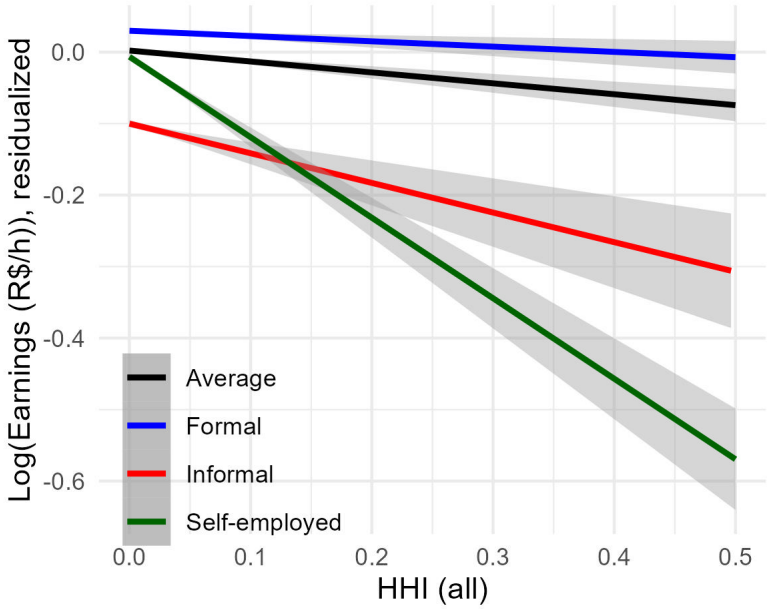
[3]The complete regressions can be found in table 7 of the appendix.

Figure 4: Self-Employment vs Market Concentration



Finally, I explore the relationship between individual earnings and market concentration. Figure 5 plots the relative earnings of each group after controlling for observable characteristics such as occupation, age, gender, education and race [4].

Figure 5: Earnings vs Market Concentration



[4]Complete estimates can be found in tables 8 and 9 in the appendix.

In it, we can see that even though average and formal earnings are quite flat across different market concentrations, the same is not true for the informal and self-employment worker. Moving from markets with very low concentration (near 0 in the x-axis) towards very concentrated ones (near 0.5 in the x-axis), one observes an average relative decrease of 13% and 41% in earnings for the informal and the self-employed, respectively. This provides addition evidence that markets that feature worse options outside of formal labor (i.e. lower potential earnings in informality and self-employment) are also the ones that are more concentrated.

4 Model

In this section I develop a theoretical model in order to incorporate the heterogeneity present in the empirical sections and study its predictions from counterfactual exercises. I build on Berger et al. (2022) and Ulyssea (2018), joining the nested CES labor supply and oligopolistic competition of the first with firm selection into formality of the second. By doing so, I allow for interactions between labor market power and the choice of formality by employers. Additionally, self-employment exists as a relevant outside option for workers, expanding on these previous works.

4.1 Firms

A local labor market is endowed with a continuum J of sectors. Each sector contains a number $M_j < \infty$ of risk-neutral potential entrant firms. Firms operate in each sector in 3 steps. First, a firm i in sector j draws a pre-entry productivity signal ν_{ij} from a distribution F_ν. Second, the firm chooses whether to enter the market, and to do so as a formal or an informal firm, based on the expected profits in each case:

$$\max \left\{ \quad 0 \quad , \quad E\left[\pi^F | \nu_{ij}\right] - \kappa^F, \quad E\left[\pi^I | \nu_{ij}\right] - \kappa^I \right\} \tag{2}$$

Entering each market has a fixed cost, κ^F for formal firms and κ^I for informal ones. I denote the pre-entry expected profits as $E\left[\pi_{ij}\right]$.

Finally, once a firm has chosen its formality status and paid its entry cost, it draws its *de facto* productivity z_{ij} from a distribution $F_z(z|\nu)$. Finally, the firm decides how many workers to hire and produces $y(z_{ij}, n_{ij}) = z_{ij} n_{ij}^{\alpha}$. Hiring choices are made in order to

maximize profits. Informal firms only hire informally, and formal firms only hire formally[5]. Wages $w(n_{ij}, n_{-ij})$ are endogenous and depend on own and competitors' labor demand.

$$\pi_{ij}^F(z_{ij}, n_{-ij}) = \max_n \left\{ \underbrace{(1-\tau_\pi)}_{\text{Revenue tax}} y(z_{ij}, n) - \underbrace{(1+\tau_w)}_{\text{Payroll tax}} nw(n, n_{-ij}) \right\} \tag{3}$$

$$\pi_{ij}^I(z_{ij}, n_{-ij}) = \max_n \left\{ y(z_{ij}, n) - nw(n, n_{-ij}) - \underbrace{\phi^I \frac{n^{1+\gamma}}{1+\gamma}}_{\substack{\text{Informality} \\ \text{wedge}}} \right\} \tag{4}$$

Equations 3 and 4 show formal and informal firm's problem, respectively. Formal firms are subject to revenue and payroll taxes, τ_π and τ_w, whereas informal firms are not. On the other hand, informal firms are subject to an informality wedge, increasing and convex in the number of hired workers and parameterized by a level ϕ^I and elasticity γ. This wedge captures institutional differences faced by informal firms, relative to formal ones, e.g. less access to credit and good markets, risk of detection by authorities, etc.

4.1.1 Firm Labor Demand

Firms compete for labor à la Cournot, and thus pick their optimal labor demand conditional on their competitors' demand. The optimization of (3) and (4) results in the following labor demand schedule:

$$w_{ij}^F = \mu_{ij}^F mrpl_{ij} \qquad \text{(Formal firm)} \tag{5}$$

$$w_{ij}^I = \mu_{ij}^I mrpl_{ij} \qquad \text{(Informal firm)} \tag{6}$$

[5]In data, there is also the case of formal firms hiring informally. Those are approximately 5% of the workforce, and are not considered in the main exercise. Considering them, as in Ulyssea (2018), does not change substantially the results.

where $mrpl_{ij}$ denotes the marginal revenue productivity of labor. The terms μ^F and μ^I denote the markdown of a formal and informal firm, and are given by:

$$\mu_{ij}^F = \left(\frac{1+\varepsilon_{ij}}{\varepsilon_{ij}}\right)^{-1} \frac{(1-\tau_\pi)}{(1+\tau_w)} \qquad \text{(Formal firm)} \tag{7}$$

$$\mu_{ij}^I = \left(\frac{1+\varepsilon_{ij}}{\varepsilon_{ij}}\right)^{-1} \qquad \text{(Informal firm)} \tag{8}$$

where $\varepsilon_{ij} := \frac{\partial n_{ij}}{\partial w_{ij}} \frac{w_{ij}}{n_{ij}}$ is the wage elasticity of labor. The markdown pressed by firms follows a usual specification in the literature, and depends on both labor supply elasticity and, for formal firms, marginal taxation.

4.2 Labor Supply

The labor supply is structured in a similar fashion as Berger et al. (2022), where a representative household allocates wage labor n_{ij} at each firm i in sector j, where they earn an equilibrium wage w_{ij} and self-employed workers n_{sj}, who earn a sector-specific w_j^S. Self-employment acts as a de facto outside option to wage work that is not unemployment and is not subject to taxes or other institutional constraints. A continuum of sectors exist, thus each firm sees itself contributing to its sector size, but not to overall market size. Three accounting aggregations are established: sector level wage employment n_j^W, sector level total labor n_j and aggregate labor N.

$$N = \left(\int_0^1 n_j^{\frac{\theta+1}{\theta}} dj\right)^{\frac{\theta}{\theta+1}} \qquad \text{Aggregate Labor} \tag{9}$$

$$n_j = \left(n_j^W{}^{\frac{\delta+1}{\delta}} + \beta_S^{-1} n_j^S{}^{\frac{\delta+1}{\delta}}\right)^{\frac{\delta}{\delta+1}} \qquad \text{Sector Labor} \tag{10}$$

$$n_j^W = \left(\sum_i \beta_{ij}^{-1} n_{ij}^{\frac{\eta+1}{\eta}}\right)^{\frac{\eta}{\eta+1}} \qquad \text{Wage-work Employment} \tag{11}$$

where $\beta_{ij} \in \{\beta_F, \beta_I\}$ is a taste-shifter for self-employment, informal work and formal work. A higher β_{ij} represents a lower disutility (i.e. a relative higher preference for) from providing labor to firm i.

Denote the elasticity of labor supply to firm i in sector j as $\varepsilon_{ij} := \partial \log(n_{ij}) / \partial \log(w_{ij})$. In equilibrium, the inverse elasticity is equal to the weighted average of elasticities η, δ and

θ, where the weights are the relevant market shares at each level. Namely,

$$\varepsilon_{ij}^{-1} = \underbrace{(1-s_{ij})\eta^{-1}}_{\text{Across-firm}} + s_{ij}\left(\underbrace{\left(1-s_j^S\right)\theta^{-1}}_{\text{Across-Sector}} + \underbrace{s_j^S\delta^{-1}}_{\text{Wage vs Self-Empl.}}\right) \tag{12}$$

where $s_{ij} := n_{ij}w_{ij}/\sum_k n_{kj}w_{kj}$ denotes the firm wage-bill share among all firms in sector j, and $s_j^S := n_j^S w_j^S/(n_j^W w_j^W + n_j^S w_j^S)$ denotes the self-employment earnings share in the sector.

The representative household has separable utility in consumption and leisure. Optimal labor supply thus follows

$$w_{ij} = \frac{1}{\beta_{ij}}\left(\frac{n_{ij}}{n_j^W}\right)^{\frac{1}{\eta}}\left(\frac{n_j^W}{n_j}\right)^{\frac{1}{\delta}}\left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} W \tag{13}$$

where the accounting relationships are given by $\sum w_{ij}n_{ij} = w_j^W n_j^W$, $w_j n_j = n_j^W w_j^W + n_j^S w_j^S$ and $\sum w_j n_j = WN$. Conversely, one may express the labor supply as a function of wages and aggregate labor supply:

$$n_{ij} = \left(\beta^i\right)^\eta \left(\frac{w_{ij}}{w_j^W}\right)^\eta \left(\frac{w_j^W}{w_j}\right)^\delta \left(\frac{w_j}{W}\right)^\theta N \tag{14}$$

$$\tag{15}$$

4.3 Sector-level equilibrium

Given aggregate wage and employment level, a sector-level equilibrium in market J is an allocation of labor $\{n_{iJ}\}_{i=1,2,\ldots,M_J,S}$ satisfying

$$\overbrace{\mu_{ij}mrpl_{ij}}^{\substack{\text{From Firm}\\ \text{Labor Demand}}} = \underbrace{\frac{1}{\beta_{ij}}\left(\frac{n_{ij}}{n_j^W}\right)^{\frac{1}{\eta}}\left(\frac{n_j^W}{n_j}\right)^{\frac{1}{\delta}}\left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} W}_{\text{From Labor Supply}} \qquad \text{Firms} \tag{16}$$

$$\underbrace{z_{sj}}_{\substack{\text{Self-employment}\\ \text{earnings}}} = \overbrace{\frac{1}{\beta^S}\left(\frac{n_j^S}{n_j}\right)^{\frac{1}{\delta}}\left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} W} \qquad \text{Self-employed} \tag{17}$$

Here we have a critical departure from Berger et al. (2022) and similar works in the literature. Since the informality wedge is scale dependent by construction, we are unable to achieve an equilibrium allocation in which wage bill shares ($s_{ij} = n_{ij}w_{wij}/\sum_k n_{kj}w_{wkj}$) are independent of aggregate measures of employment N and wage W. Nonetheless, it can be shown that solutions under different aggregates can be mapped in identity by the means of scaling relevant parameters and levels of employment and wage. Thus, normalizing N and W is still possible when studying a single sector in a local labor market, but not when comparing two sectors or when conducting counterfactual analyses that have an impact on aggregates.

4.4 Market-level equilibrium

Firms and the self-employed produce tradable goods that are perfect substitutes, and compete perfectly in a national market at a homogeneous price, normalized to one. Given market structure, firms behave strategically at sector level, but are price takers at a market level. A market-level equilibrium thus is given by an allocation of labor $\{\{n_{ij}\}_{i=1,2,...,M_j,S}\}_{j\in J}$ such that equilibrium is satisfied in each sector given aggregate measures.

In what follows, most analyses will be confined to a single sector, with aggregate measures exogenously set. Since sectors are atomistic, firms do not see themselves contributing to aggregate measures. Thus, this approach is equivalent to studying the equilibrium conditions of a sector for a given market-level equilibrium, the details of which are irrelevant in these analyses.

5 Calibration

Calibration is conducted via a Method of Simulated Moments (MSM). The parameter set Γ^* is obtained by solving

$$\Gamma^* = \text{argmin}_\Gamma \left(Q_{model}(\Gamma) - Q_{data}\right)' W \left(Q_{model}(\Gamma) - Q_{data}\right) \tag{18}$$

where Q stands for a vector of moments (e.g. firm informality rate, market concentration as measured by HHI, etc.). Empirical values for moments and their model counterparts are normalized and the weighting matrix W is the identity matrix. In what follows, I normalize the variables whose level are not relevant for identifying the baseline equilibrium. The

following variables are set to 1: the ex-ante expected productivity; aggregate labor supply and one of the preference shifters. I then match selected moments of the average sector in the economy [6].

The elasticity parameters η, δ and θ are crucial for any calibration, but to the best of my knowledge there is no empirical estimate that considers the three dimensions (across-firm, across-sector and wage versus self-employment) simultaneously. Although estimating such objects is desirable, for now I leverage the existing empirical results in order to input them in a reasonable ordering and magnitude. Regarding wage work, there is sufficient evidence (Berger et al. (2022), Felix (2021)) that establish across-sector elasticity to be consistently lower than across-firm, within-sector, elasticities, i.e. $\eta > \theta$. When it comes to wage-work versus self-employment, some works have pointed to either a relatively higher elasticity than across-firms (Amodio et al. (2022) finds this elasticity to be on average 1.6, compared to an across-firm estimate of 1.0 in Felix (2021)) or, more noticeable, that firms faces higher elasticities when self-employment is more prevalent (Felix (2021)). Taken together with the elasticity composition derived in equation 12, these pieces of evidence point towards self-employment elasticity being the highest, followed by across-firm and across-sector, respectively. In other words, $\delta > \eta > \theta$.

5.0.1 The Average Sector

There are 9 parameters to be calibrated, $\Gamma = \{\xi, \sigma^2, \phi^I, \gamma, \kappa^F, \kappa^I, \beta^I, \beta^F, w^s\}$. Table 3 presents the calibrated moments. Moments that are dependent of other targeted moments (such as share of formal workers, and total share of informal wage workers) are shown but not targeted. The model is flexible enough to successfully match all targeted moments.

Table 4 presents the values of all parameters, both externally supplied (top half) and estimated (bottom half). Some interesting patterns emerge when comparing the estimates. Entry costs are estimated to be 25% higher in the formal sector, which goes in line with previous findings in the literature [7]. There is a noticeable difference between the preference shifters - 55% larger for formal firms[8]. This parameter is responsible for disciplining the size

[6] Aggregate labor is relevant for counterfactuals that affect the whole market, and will naturaly vary in such circumstances. For calibration purposes, this is merely a scaling issue.

[7] Ulyssea (2018) estimates an entry cost for formal firms more than twice that of informal firms. In his work, other dynamic dimensions (per-period cost, exit probability) exist that do not allow for a direct comparison with the present model. Nonetheless, the idea of a higher cost of formality is present in the same fashion.

[8] This implies in practice that an informal firm offering the same wage as a formal firm is able to

Table 3: Calibrated moments (Average Sector)

Moment	Data	Model
Share of informal firms	**44%**	40%
Share of self-employment	**32%**	31%
Share of formal wage workers	56%	56%
Share of informal wage workers	**12%**	12%
HHI (formal only)	**0.14**	0.15
HHI (with informality)	**0.11**	0.11
$E[\log(w_I)] - E[\log(w_F)]$	**-0.12**	-0.12
$E[\log(w_S)] - E[\log(w_F)]$	**-0.04**	-0.04
Wage Dispersion*	0.18	0.15
Informal Workers*	**0.22**	0.17
Formal Workers*	**0.17**	0.14

(*) Empirical wage dispersion equals to the variance from residuals of log hourly earnings on individual and market-level fixed-effects
Targeted moments in boldface

a firm is able to reach for a given offered wage. Thus, the difference can be interpreted not only as actual idiosyncratic preference of the worker for a formal status, but the collection of all elements that make a formal job preferable (and not explicitly modelled) such as pension contribution, stability, severance, amenities and other benefits [9].

Figures 6 and 7 show the equilibrium distribution of labor and wages next to their empirical counterparts. It manages to reproduce two important features of data. First, formal firms are larger and pay higher wages on average. Second, there is considerable overlap between the two distributions.

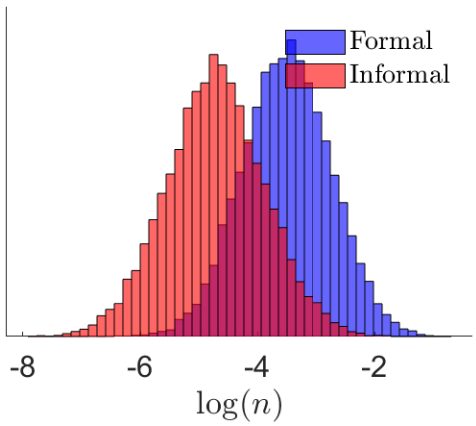
sustain an employment level of approximately 2/5 of that of a formal firm in equilibrium - Fixing wages, the relative employment is $n_{ij}/n_{kj} = (\beta^I/\beta^F)^\eta = 0.41$.

[9] See Meghir et al. (2015) for a search model with informality where formal benefits such as unemployment insurance and severance payment play a significant role rationalizing compensating differentials.

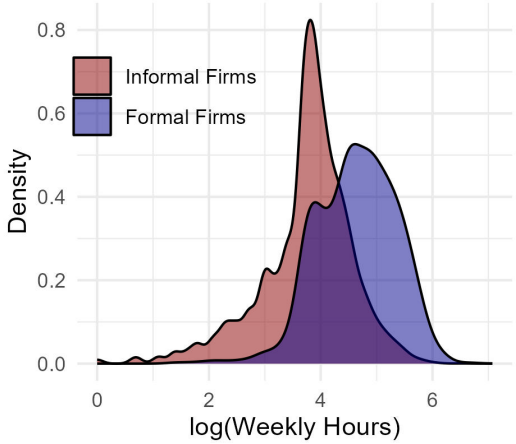
Table 4: Model calibration of the average sector in the economy - Parameters

Parameter	Description	Value	Source
α	Returns to scale	0.6	Ulyssea (2018)
η	Between-firm elasticity of labor	2	Indirect Evidence
δ	Wage-work/Self-employment elasticity of labor	4	Indirect Evidence
θ	Inter-sector elasticity of labor	1	Indirect Evidence
ν_0	Productivity signal scale parameter	1	Norm. $E[z] = 1$
μ	Effective productivity scale parameter	-0.44	Norm. $E[z] = 1$
β^S	Preference shifter (self-employed)	1	Normalization
$1/\xi$	Productivity signal dispersion	0.21	Estimated
σ	Effective productivity dispersion	0.69	Estimated
ϕ^I	Wedge level	8.26	Estimated
γ	Wedge elasticity	0.33	Estimated
κ^I	Entry cost (informal)	0.036	Estimated
κ^F	Entry cost (formal)	0.045	Estimated
β^F	Preference shifter (formal)	0.31	Estimated
β^I	Preference shifter (informal)	0.20	Estimated
z_s	Self-employment earning level	1.05	Estimated

Figure 6: Labor distribution: model vs data



Source: Model

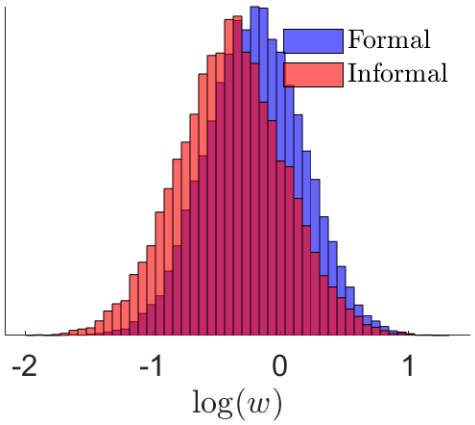


Source: ECINF (2003)

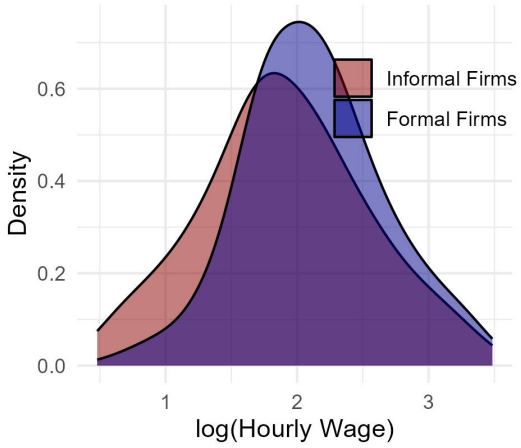
6 Counterfactual Exercises

Given the baseline calibration, I have a starting point for assessing counterfactuals and decompose the partial contributions of informality, self-employment and market power to aggregate productivity and welfare. Throughout this analysis, it is useful to keep track of externalities and the kinds of distortions they cause in the model economy.

Figure 7: Wage distribution: model vs data



Source: Model



Source: ECINF (2003)

The choice of labor supply is centered at the representative household, and thus its allocative choice is undistorted given wages posted by firms. The heterogeneous firms, on the other hand, are responsible for and subject to two externalities. First, firms do not retain the totality of surplus resulting from its economic activity - part of it is retained by the worker in the form of wages and, in the case of formal frims, part is subtracted in the form of taxes. This externality leads to firms under-hiring and discouraging entry. The second externality firms face is market stealing. When posting wages, firms do not internalize the impact of its hiring decision on its competitors. This mechanism in isolation leads to over-hiring and encourages too much entry in the market. In very specific setups[10], these externalities offset each other, making entry and allocation efficient. In general this is not the case, and, as will be shown next, in the particular setup of this work we have that surplus grabbing externality dominates, making entry, wages and hiring inefficiently low.

The reflections of these two externalities show up as two distortions in the model, namely an allocative distortion and an entry distortion. Given a set of entrant firms under formal or informal status, firms post wages and hire in quantities that are not socially optimal. This is the allocative distortion. On top of a distorted allocation, firms respond to private incentives when deciding whether to enter a market, and whether they should do so formally or informally. Since profits are but a fraction of the total social benefit of an entering firm, entry decisions are naturally not aligned with the social otpimum.

[10]See Dhingra and Morrow (2019) for a thorough discussion in the context of market power in the goods market.

In what follows I will present alternative equilibria departing from the baseline calibration. First, as a reference point, I evaluate what is the first best in this environment. Although not feasible in any constrained way, this planner solution serves as an upper bound in welfare terms to all alternative allocations and entry that may arise from a market solution. In this planner solution, labor allocation and thresholds are chosen in order to maximize expected utility. This means the social planner, although optimizing for total social benefit, does so under the same kind of information setup firms face pre-entry. Thus, it does not choose firm entry for each sector draw of productivity signals, but instead establish the rule (thresholds) that firms must follow in order to maximize expected social surplus.

In this version of the project, there are two additional exercises that serve to decompose the contributions of informality and self-employment to productivity and welfare[11]. In the first, informality is removed as an option for firms [12] and the resulting equilibrium is a market one, both in entry and allocation. In the second one, self-employment is removed as an option for workers, and the equilibrium is also the market one.

At this point, we must define a functional form for the utility function. It follows a standard structure shown in equation 19, with parameters σ, φ and $\bar{\varphi}$. The two first will have a value of 2, commonly adopted in the literature. The last one is a residual from the allocation found in the baseline calibration, and kept constant for the counterfactuals. The budget constraint features the totality of wages paid (WN), profits (Π), taxes (T) and entry costs paid (K).

$$U(C,N) = \max_{C,N} \left[\frac{C^{1-\sigma}}{1-\sigma} - \bar{\varphi} \frac{N^{1+\varphi}}{1+\varphi} \right] \tag{19}$$
$$\text{s.t.} \quad C = WN + \Pi + T - K$$

Table 5 summarizes the results. In the first two columns we have the aggregate values for consumption and labor, the welfare relevant measures. Under "Allocation" I show the wage index W next to an average labor productivity $\overline{y/n}$. These two variables serve as proxies for how efficiently the market is employing labor, and how competitive the decentralized market is. Under "Entry", I show the average number of firms per sector, both informal and formal,

[11] The counterfactual exercises tackling market power is work in progress and its exposition is left for future versions of this work.

[12] This can be endogenously obtained by making the entry cost for informal firms prohibitively high, and/or making the informality wedge much more penalizing.

in each configuration. In the last column I report the consumption-equivalent welfare change of each scenario, with respect to the baseline.

Table 5: Counterfactual Exercises - Welfare and Distortions

			Allocation		Entry		
	C	N	W	$\overline{y/n}$	# Firms (F)	# Firms (I)	ΔC_{eq}
Baseline	3.74	1	1.35	4.03	17	12	-
No Informality	3.95	0.96	1.39	4.2	25	0	**+7%**
No Self-Employment	4.13	0.88	1.29	5.07	28	3	**+15%**
First-Best	5.49	1.14	3.82	4.71	40	0	**+35%**

The First-Best shows us to what extend both allocative and entry distortions affect this economy. In this planner solution, total firm entry is much greater (40 vs 29 in baseline), and they are all formal, pointing towards the non-optimality of informality. Wages are almost 3 times higher, meaning firms operate at a much bigger scale. Even in the presence of decreasing returns of wage work, average productivity is 17% higher due to massive reallocation from self-employment, which is considerably less productive. In table 6 one may notice that the increase in total entry costs K is small relative to the increase in total output.

Table 6: Counterfactual Exercises - Additional Measures

	Y	Π	T	K	% I	% S	HHI	$\bar{\mu}$
Baseline	4.93	2.21	1.37	1.2	12	31	0.11	0.41
No Informality	5.07	2.05	1.69	1.12	0	27	0.11	0.35
No Self-Employment	5.47	2.43	1.89	1.34	3	0	0.1	0.36
First-best	7.28	-0.6	3.51	1.8	0	0	0.08	1

The alternative scenario with no informality improves welfare in 7%, or 1/5 of the gap between the baseline and the first best. In this equilibrium, the total number of entering firms decreases, but the amount of formal ones increases. Formal firms not only are more productive, as seen in the higher average worker productivity, but they are also more competitive. In terms of competing for labor, the increase in formal firms more than makes up for the reduced number of total firms, and wages are 3% higher.

The scenario with no self-employment features even greater gains to welfare: 15% in consumption equivalent terms, or 2/5 of the gap between the baseline and the first best. Firm entry is slightly higher, and those that enter are mostly formal due to increased labor supply making operating at a larger scale more desirable. Here we can also see the opposite

forces from self-employment: on one hand average wages *decrease* due to the absence of an outside option for workers; on the other hand, the remaining option, wage work, is much more productive. The second force dominates and we have simultaneously an increase in output, consumption and a decrease in total labor N.

7 Conclusion and Next Steps

Labor market power, informality and self-employment have been thoroughly explored as individual mechanisms, both theoretically and empirically. This work contributes to providing a better understanding of the interactions between these forces when they are simultaneously present in an economy, which is the case of most of the developing world.

By leveraging different data sources from Brazil, I build a dataset on the distribution of firms, wages and workers for 21 metropolitan regions in the span of 8 years. I then build a model of heterogeneous firms and calibrate it to match the most relevant aspects of an average sector in the Brazilian economy. The model successfully reproduces the patterns seen in the data and provides the basis for counterfactual analysis. Initial results show that the market equilibrium is 26% below the first best, in consumption equivalent terms. Decomposing the contribution of informality and self-employment to this measure, I find that individually they contribute to about 1/5 and 2/5 of this gap.

In the continuation of this project I aim to make improvements in two fronts. First, to develop and implement a strategy of indirectly estimating the novel elasticity structure of my model, using existing partial measures as targets. Second, I will conduct counterfactual analysis on the relevance of competition in this environment, providing a complete picture of the mechanisms explored and potentially leading to policy implications.

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Appendices

A Tables

Table 7

Dependent Variables:	Fraction Informality (all)		Fraction Wage-Work Informality		Fraction Self-Employment	
Model:	(1)	(2)	(3)	(4)	(5)	(6)
Constant	0.4194***	0.4207***	0.1341***	0.1345***	0.2853***	0.2862***
	(0.0403)	(0.0403)	(0.0109)	(0.0109)	(0.0380)	(0.0381)
HHI (Formal Workers)	-0.7403***		-0.1785***		-0.5618***	
	(0.1877)		(0.0523)		(0.1695)	
HHI (All)		-1.038***		-0.2576***		-0.7807***
		(0.2219)		(0.0607)		(0.2010)
Observations	8,728	8,728	8,728	8,728	8,728	8,728

Observations at the Year x Metro Area x 2-dig sector level
Clustered (Sector (2-digit) & Metro Area) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table 8

Dependent Variable:	$\log(HourlyEarnings)$			
	HHI measure: Formal Only		HHI measure: All Workers	
Baseline: Formal Worker	(2)	(3)	(4)	(5)
HHI	-0.1435***	-0.0227	-0.1056*	0.0176
	(0.0514)	(0.0488)	(0.0591)	(0.0556)
HHI × Informal Wage Worker		-0.2516***		-0.2800**
		(0.0862)		(0.1064)
HHI × Self-Employed		-0.8027***		-1.059***
		(0.1308)		(0.1448)
Fixed-effects	Yes	Yes	Yes	Yes
Observations	331,146	331,146	331,146	331,146

Clustered (Occupation (2-digit)) standard-errors in parentheses
Fixed effects: Year × Quarter, Individual Controls
Individual Controls: Work status, Occupation (2-dig), age, gender, education, race
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table 9

Dependent Variable:	$\log(HourlyEarnings)$			
	HHI measure: Formal Only		HHI measure: All Workers	
Baseline: Formal Worker	(2)	(3)	(4)	(5)
HHI	-0.0256	0.0541	-0.0086	0.0714
	(0.0452)	(0.0539)	(0.0546)	(0.0596)
HHI × Informal Wage Worker		-0.1506		-0.1772
		(0.1046)		(0.1292)
HHI × Self-Employed		-0.4460***		-0.5626***
		(0.1249)		(0.1581)
Fixed-effects	Yes	Yes	Yes	Yes
Observations	331,146	331,146	331,146	331,146

Clustered (Occupation (2-digit) & Sector (2-digit) & Metro Area) standard-errors in parentheses
Fixed effects: Year × Quarter, Individual Controls, Sector (2-dig), Metropolitan Region
Individual Controls: Work status, Occupation (2-dig), age, gender, education, race
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*