STRATEGIC YET NOT DEMANDING: THE EMPIRICAL REACH OF OLIGOPOLY WITH CES PREFERENCES*

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March 15, 2024

[Preliminary and incomplete]

Abstract

We consider the behavior of heterogeneous firms in oligopolistic markets when preferences are CES. We provide an explicit characterization of the implied demand manifold, a smooth curve representing the demand function in the space of the elasticity and curvature of demand. We show that Bertrand and Cournot competition with nested CES preferences imply the same demand manifold, but equilibrium location on the manifold depends on conduct. Through the demand manifold, we identify how Betrand competition with CES is more restrictive in terms of predicted passthrough than Cournot competition with CES. The demand manifold also highlights testable predictions on the elasticity-curvature relationship which can be used to discriminate between monopolistic competition with variable elasticity demands and oligopoly with CES demands.

Keywords: Heterogeneous Firms; Oligopoly with CES Preferences; Pass-Through

JEL Classification: F12, L11, F23

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*We are grateful to David Argente, Mayara Felix, Keith Head, Oleg Itskhoki, Volker Nocke, and Steve Redding for stimulating discussions and comments. Email: monika.mrazova@unige.ch.

†Peter Neary sadly passed away in June 2021.

1 Introduction

The workhorse model that combines constant-elasticity-of-substitution (CES) preferences or technology with monopolistic competition continues to dominate many fields of economics, including macroeconomics and international trade. However, its deficiencies are well-known – all firms in a sector must have the same constant markup, and cost increases are always passed on 100% to consumers – and have been highlighted by the increasing availability of micro-level data on markups and pass-through. In response, at least two approaches have been explored. On the one hand, the implications of going "beyond the CES" while staying within the general-equilibrium monopolistically-competitive paradigm have been considered by, among others, Kimball (1995), Zhelobodko et al. (2012), Bertoletti and Epifani (2014), Mrázová and Neary (2017), and Arkolakis et al. (2018). On the other hand, the study of oligopolistic markets combined with CES preferences, initiated by Atkeson and Burstein (2008), has inspired many applications including Edmond et al. (2015), Hottman et al. (2016), Gaubert and Itskhoki (2020), and Breinlich et al. (2020). Both of these approaches avoid the deficiencies of the workhorse model, but they clearly model market structure in very different ways.

How can we compare these two different approaches? In this paper we show how they can be related to one another as well as to empirical estimates of pass-through and markups. We build on a methodology introduced in Mrázová and Neary (2017), which exploits the fact that demand functions are simpler to compare when illustrated not in price-quantity space but in the space of the elasticity and convexity of demand. In our earlier paper we showed that any well-behaved demand function can be represented by a smooth curve in this space, which we called the "demand manifold" corresponding to the demand function. However, our previous applications related to the case of monopolistic competition only. Here we provide an explicit characterization of the demand manifold implied by CES preferences in oligopolistic markets.

We show that the manifold representing the demand function perceived by oligopolistic firms when preferences are CES takes a simple and elegant form. In effect, it is a natural out-growth of the locus of elasticity and convexity values consistent with constant-elasticity demand (and hence with CES preferences in monopolistic competition). It deviates from the constant-elasticity case by more, the more the elasticity of substitution between goods diverges from that between sectors (or, in the non-nested CES case, the more the elasticity of substitution diverges from one).

In the case of Bertrand competition, the interpretation of points along the manifold is particularly convenient because market shares are linear in the elasticity of demand. We show that demand is always convex in the benchmark case of symmetric duopoly, which implies that all firms with market shares less than 50% also operate on the convex part of their perceived demand curve, in a range that is close to the constant-elasticity locus. Hence, except for at most one dominant firm per sector (with a market share above 50%), the oligopoly manifold does not diverge "too far" from the constant-elasticity locus.

In the case of Cournot competition, the demand manifold takes the same form as in the Bertrand case. However, market shares are no longer linear in elasticity. In general, they are higher for the same parameter values, reflecting the fact that markets are less competitive, with firms facing lower elasticities and enjoying higher markups than in Bertrand competition. In principle this opens up the possibility of a wider range of convexity values being consistent with equilibrium bahavior, though as we show the range is still relatively restricted except for firms that are very large in their market.

As well as examining the properties of the oligopoly demand manifold, we are able to compare it with empirical evidence. We do this by illustrating in the same space the range of estimated values of the elasticity of demand (taken from some of the extensive empirical evidence on demand), as well as the estimated values of demand convexity implied by the literature on pass-through, especially exchange-rate pass-through. Our overall conclusion is that, although Betrand and Cournot competition with CES preferences appear qualitatitely similar, the range of empirical values of passthrough is far greater than can be easily reconciled with the potential range of values consistent with Betrand competition and CES demands echoing some findings from previous literature (e.g. Amiti et al. (2019)).

Finally, comparing the demand manifolds implied by CES oligopoly and monopolistic competition with widely-used demands with variable elasticity, we identify testable differences between these two approaches.

2 The CES Demand Manifold

2.1 Preliminaries: Consumers and Firms

The starting point is a nested CES preference structure.[1] The aggregate consumer cares about consumption of goods, x_{ki}, that are grouped into sectors, each with a sub-utility function (or true quantity index) X_k. There is a continuum of sectors, indexed by $k \in [0,1]$, with a finite number of goods, n_k, in each. The upper-level utility function U, defined over the X_k, is CES with elasticity of substitution η, while each lower-level sub-utility function X_k, defined over the corresponding x_{ki}, is CES with elasticity of substitution σ:

$$U = \left(\int_0^1 X_k^{\frac{\eta-1}{\eta}} dk \right)^{\frac{\eta}{\eta-1}} \qquad X_k = \left(\sum_{i=1}^{n_k} x_{ki}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad k \in [0,1]. \tag{1}$$

As is standard in nested CES models, we assume that consumers view goods within a sector as more substitutable than composite outputs across sectors: $\sigma > \eta \geq 1$. We allow for the possibility that $\eta = 1$, the "Dixit-Stiglitz-lite" case where (taking limits appropriately) the upper-tier utility function is Cobb-Douglas.[2] Note that we assume that preferences are symmetric across all goods. This assumption can easily be relaxed by adding a "taste shifter" for each good to (1); for ease of exposition we dispense with this.

[1] We follow the Dixit-Stiglitz approach of assuming CES preferences, rather than the Ethier one of assuming a single final good produced from CES aggregates of intermediate goods as in Atkeson and Burstein (2008) and Edmond et al. (2015). The two approaches are formally identical.

[2] See Neary (2003b). Setting $\eta = 1$ also represents the textbook case of a non-nested CES utility function: $U = \left(\sum_{i=1}^{n} x_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$.

We assume that the aggregate consumer maximizes utility (1) subject to the budget constraint:

$$\int_0^1 \sum_{i=1}^{n_k} p_{ki} x_{ki} dk \leq I, \tag{2}$$

where I denotes total consumer expenditure. This yields the direct demand functions:

$$x_{ki} = \left(\frac{p_{ki}}{P_k}\right)^{1-\sigma} \left(\frac{P_k}{P}\right)^{1-\eta} \frac{I}{p_{ki}} = p_{ki}^{-\sigma} P_k^{\sigma-\eta} P^{\eta-1} I, \tag{3}$$

where P_k and P denote the true price indices for sector k and for total consumer expenditure respectively:

$$P_k = \left(\sum_{i=1}^{n_k} p_{ki}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \qquad \text{and} \qquad P = \left(\int_0^1 P_k^{1-\eta} dk\right)^{\frac{1}{1-\eta}}. \tag{4}$$

Define the market share of firm i in sector k:

$$\omega_{ki} \equiv \frac{p_{ki} x_{ki}}{\sum_{j=1}^{n_k} p_{kj} x_{kj}} \tag{5}$$

Using the direct demand, the market share can be re-expressed as a function of prices

$$\omega_{ki} = \frac{p_{ki}^{1-\sigma}}{\sum_{j=1}^{n_k} p_{kj}^{1-\sigma}} = \left(\frac{p_{ki}}{P_k}\right)^{1-\sigma} \tag{6}$$

Note that changes in sectoral market share are linked to changes in price:

$$\hat{\omega}_{ki} = (1-\sigma)(1-\omega_{ki})\hat{p}_{ki} \tag{7}$$

where $\hat{x} \equiv d \log x$.

Turning to firms, we assume that each has a unit cost c_{ki} drawn from a distribution $G(c_{ki})$, and seeks to maximize profits. The preferences in (1) allow for a "GOLE" ("OLigopoly in General Equilibrium") approach to market structure: we assume that firms are "large in the small but small in the large", in the sense that they compete strategically against a finite number of rivals in their own sector, but take economy-wide variables as given. (See Neary (2003a).)

2.2 The Space of Elasticity and Convexity of Demand

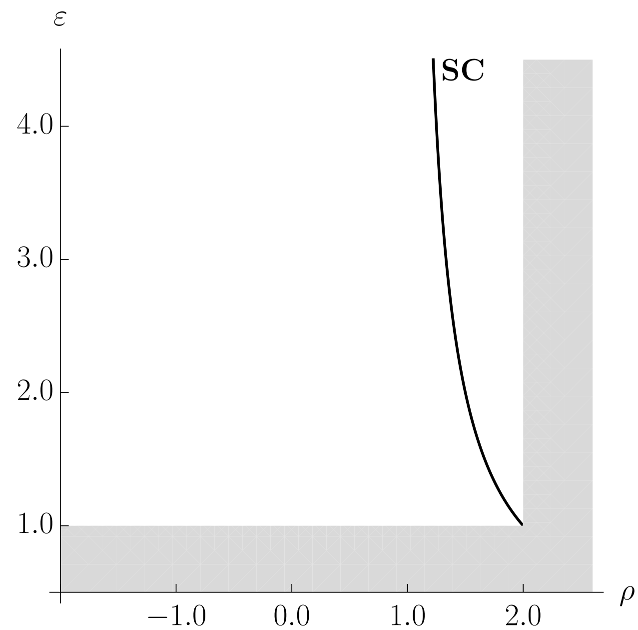

Figure 1: The Space of Elasticity and Convexity of Demand
SC: The sub-superconvex boundary

Our goal is to compare the equilibria that can arise in oligopolistic markets with CES preferences with those than can arise in monopolistically competitive markets for any class of preferences. As we showed in Mrázová and Neary (2017), the latter are conveniently illustrated in the space of the elasticity and convexity of demand, as in Figure 1. It should be emphasized that we are taking a "firm's-eye view" throughout: these are the elasticity and convexity of demand perceived by each firm, so in general they differ between firms:[3]

$$\varepsilon_i \equiv \frac{p_i}{x_i}\frac{dx_i}{dp_i} \quad \text{and} \quad \rho_i \equiv x\frac{d^2x_i}{dp_i^2} / \left(\frac{dx_i}{dp_i}\right)^2 \tag{8}$$

[3] ρ is the convexity of the inverse demand function: $\rho = -x_i \frac{d^2p_i}{dx_i^2} / \frac{dp_i}{dx_i}$. See Mrázová and Neary (2017), Appendix A.

Not all points in elasticity-convexity space are consistent with the first- and second-order conditions for profit maximization. In oligopoly with differentiated products as here, these conditions imply respectively that ε must be greater than one and ρ must be less than two.[4] Just as in the case of monopolistic competition, this defines an admissible region in the space as shown by the red loci: firms can only be in equilibrium at points that lie below and to the right of the Cobb-Douglas benchmark, where $\varepsilon = 1$ and $\rho = 2$.

Another key reference in Figure 1 is the "superconvex" locus, denoted "SC", defined by $\rho = (\varepsilon + 1)/\varepsilon$. Each point on this locus represents a particular constant-elasticity demand function, to which corresponds a particular value of the convexity of demand. As indicated in Figure 1, points above this locus exhibit more convexity – we say that they lie in the (strictly) superconvex region; while points below this locus exhibit less convexity – we say that they lie in the subconvex region. In monopolistic competition, the SC locus is also the locus of equilibria that are consistent with firms maximizing profits facing a CES demand function. We wish to establish how this changes when we allow for oligopolistic behavior.

A key result we use, from Mrázová and Neary (2017), is that, to every three-times differentiable demand function in price-quantity space, there corresponds a smooth curve in elasticity-convexity space. (In the constant-elasticity case, this curve collapses to a point at the appropriate values of ε and ρ.) We call this curve the "demand manifold" of the demand function in question. This makes it easy to compare the properties of different demand functions, especially since many demand manifolds are invariant to changes in some or all of the parameters of the demand function.

2.3 Bertrand Competition

We consider the Bertrand case in this section, so firms choose their price, taking as given the prices of all their competitors; they take account of their impact on the sectoral price index P_k but rationally take total consumer spending I and the aggregate price index P as

[4] In oligopoly with homogeneous products, the boundaries depend on market shares and so are firm-specific; see Mrázová and Neary (2017), Appendix B.

given, so there are no “Ford Effects”.[5]

In what follows we can suppress the sector subscript k.

The first-order condition for profit maximization implies that the markup or price-cost margin depends on the elasticity of demand:

$$\frac{p_i}{c_i} = \frac{\varepsilon_i}{\varepsilon_i - 1}. \tag{9}$$

Crucially, the elasticity is that perceived by the firm, which in Bertrand competition, where the firm takes account of its effect on the sectoral price index, takes the form:

$$\varepsilon_i \equiv \frac{p_i}{x_i}\frac{dx_i}{dp_i} = \frac{p_i}{x_i}\left(\frac{\partial x_i}{\partial p_i} + \frac{\partial x_i}{\partial P_i}\frac{\partial P_i}{\partial p_i}\right). \tag{10}$$

Using (3) to evaluate this yields, as in Atkeson and Burstein (2008):

$$\varepsilon_i = \sigma(1 - \omega_i) + \eta\omega_i \tag{11}$$

Thus the elasticity of demand is a weighted average of the upper- and lower-tier elasticities of substitution, where the weights depend on the firm’s market share. Differentiating logarithmically the elasticity yields

$$\hat{\varepsilon}_i = (\eta - \sigma)\frac{\omega_i}{\varepsilon_i}\hat{\omega}_i \tag{12}$$

Substituting $\hat{\omega}_i$ from (7) and ω_i from (11) into (12) yields

$$\hat{\varepsilon}_i = \underbrace{\frac{(1-\sigma)}{(\eta-\sigma)}\frac{(\varepsilon_i - \sigma)(\eta - \varepsilon_i)}{\varepsilon_i}}_{S_i}\hat{p}_i \tag{13}$$

$$\tag{14}$$

[5]For alternative approaches to Ford Effects, where firms’ decisions affect the total spending they face, see d’Aspremont et al. (1996), Eaton et al. (2013), and Azar and Vives (2020).

where S_i is Kimball's (1995) super-elasticity. Ad shown in Mrázová and Neary (2017) $S_i = \varepsilon_i + 1 - \varepsilon_i \rho_i$, and so we can now solve for the demand manifold $\rho_i(\varepsilon_i)$:

$$\rho(\varepsilon) = \underbrace{\frac{\varepsilon + 1}{\varepsilon}}_{(SC)} - \underbrace{\frac{\sigma - 1}{\sigma - \eta} \frac{(\varepsilon - \eta)(\sigma - \varepsilon)}{\varepsilon^2}}_{(*)} \tag{15}$$

The first term is the same as in monopolistic competition, and defines the SC locus. The second term $(*)$ shows that ρ is lower than this whenever $\eta < \varepsilon < \sigma$. Note that the manifold is invariant with respect to the number of firms in the market. It is also the same for all firms, so we have dropped the subscript "i".

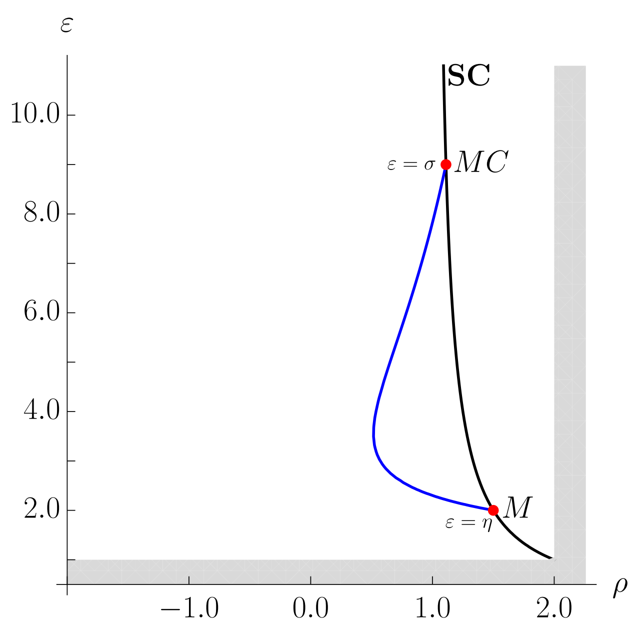

Figure 2: CES Demand Manifold, $\sigma = 9$, $\eta = 2$

MC: Monopolistic competition; M: Monopoly

Figure 2, drawn for $\sigma = 9$ and $\eta = 2$, shows a representative example of an oligopoly manifold. Recall that its location does not depend on firms' market shares. Rather, each point on the manifold corresponds to a particular market share; this is linearly decreasing in the elasticity of demand from expression (11) above:

$$\omega_i = \frac{\sigma - \varepsilon_i}{\sigma - \eta} \tag{16}$$

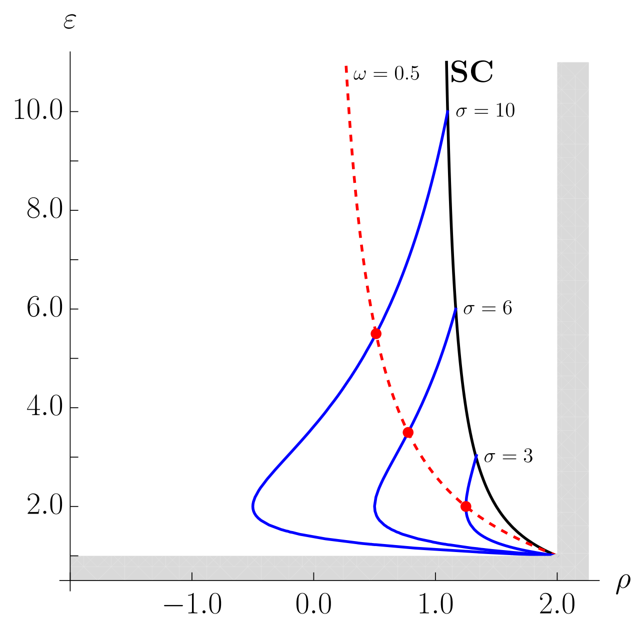

Figure 3: CES Demand Manifold, $\eta = 1$ and $\sigma = 3, 6, 10$

This simple property reflects the fact that the game is an aggregative one: in equilibrium, a firm's profits depend only on its own market share and not on the composition of its rivals' market shares.[6] The oligopoly manifold starts at point MC, where $\varepsilon_i = \sigma$ and $\omega_i = 0$. Here firms are infinitesimal, and behave just as all firms do in a monopolistically competitive equilibrium, taking the market demand curve as given. It then curves away from the SC locus into the subconvex region, before curving back again to rejoin the SC locus at point M, where $\varepsilon_i = \eta$ and $\omega_i = 1$. This represents the monopoly equilibrium: a single firm fully controls the market, and exercises its market power by operating along the sectoral demand function.

Figure 3 illustrates how the demand manifold changes with σ. The red dotted curve labelled $\omega = 0.5$ traces the location on the manifold for a firm with 50% market share. At most one firm can be to the left of this curve.

It is clear from (15) that the degree of convexity along a given manifold is decreasing

[6] The implication that Coca-Cola should be indifferent between facing Pepsi-Cola or facing a continuum of tiny firms with the same aggregate market share may seem paradoxical at first; but it makes sense in this model where by assumption every firm produces a single distinct product that is valued by the diversity-loving consumer.

in the upper-level elasticity of substitution σ and increasing in the lower-level elasticity of substitution η. Figure 3 throws light on how much this matters quantitatively.

It shows that, for higher values of σ, the manifold can become highly concave, in principle extending the range of observed elasticity-convexity combinations that can be rationalized by the model. However, this is relevant for at most only a single firm in each market. This implies that, in any market, there can be at most one firm that faces a concave demand function, and it must have a market share greater than 50%. Putting this differently, all firms with market shares less than 50% lie in the convex region and, except for very high values of σ, relatively close to the SC locus.

2.4 Cournot Competition

Consider next the Cournot case where firms compete on quantity. From Atkeson and Burstein (2008) the elasticity of demand in this case is:[7]

$$\varepsilon_i = \left(\sigma^{-1}(1-\omega_i) + \eta^{-1}\omega_i\right)^{-1} \tag{17}$$

This is a market-share-weighted hyperbolic average of σ and η. For given values of σ, η and ω_i, it is necessarily less than the Bertrand arithmetic average in (11). Logarithmically differentiating:

$$\widehat{\varepsilon}_i = -\frac{\sigma-\eta}{\sigma\eta}\omega_i\varepsilon_i\widehat{\omega}_i \tag{18}$$

In this case the budget share is no longer linear in ε_i (compare (16)):

$$\omega_i = \frac{\eta}{\varepsilon_i}\frac{\sigma-\varepsilon_i}{\sigma-\eta} \tag{19}$$

[7]This result was previously derived by Yang and Heijdra (1993) for the case of symmetric firms. See also the response of Dixit and Stiglitz (1993).

Solving for the demand manifold yields

$$\rho(\varepsilon) = \underbrace{\frac{\varepsilon + 1}{\varepsilon}}_{(SC)} - \underbrace{\frac{\sigma - 1}{\sigma - \eta} \frac{(\varepsilon - \eta)(\sigma - \varepsilon)}{\varepsilon^2}}_{(*)} \tag{20}$$

which is identical to the Bertrand case. So with nested CES preferences, Betrand competition and Cournot competition imply the same relationship between the elasticity and curvature of the perceived demand. However, location on the manifold is different under Cournot competition which has implications for the passthrough properties.

[TO BE WRITTEN]

3 Monopolistic Competition with Variable Elasticity versus CES Oligopoly

[TO BE WRITTEN]

4 Conclusion

We hope our results will be of interest to at least three groups of researchers. First, applied theorists in industrial organization and international trade should welcome our explicit characterization of the demand implications of models of oligopoly under CES preferences, which allows a direct comparison with previous results on the implications of alternative preference and demand assumptions when markets are monopolistically competitive. Second, empirical scholars in fields such as exchange-rate pass-through and international trade should be interested in our unified framework which allows a direct comparison between empirical estimates and the implications of alternative theoretical frameworks. Third, scholars wishing to calibrate models of oligopoly with CES preferences can console themselves with the thought that our results, while suggestive, by no means rule out the empirical relevance of this approach; and can use our framework to select values of the upper- and lower-tier elasticities of substitution which come closest to the values of pass-through and markups that they wish to match.

Appendices

[TO BE WRITTEN]

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