Bargaining and Market Power in Bilateral Monopoly*

Harold Houba†

This version is prepared for AERIE 2024

Abstract

We study the impact of bargaining power on market power in bilateral monopoly. Bargaining weights of the Nash bargaining solution quantify bargaining power, while a surplus above the competitive equilibrium level measures market power. Seller market power emerges if the seller bargaining weight exceeds a pivotal threshold and buyer market power if this weight falls short. This threshold is contingent on the marginal cost and price curves. We identify three contract types and characterize the bargaining solution for each. Price contracts with endogenous right-to-manage assign this right to the buyer in case of seller market power and to the seller for buyer market power. Our results have fundamental implications for empirical Industrial Organization. We extend our results to indivisible goods.and Nash-in-Nash negotiation markets.

JEL Classification: D43, F12, L13, L14, J5

Keywords: Bargaining Power, Market Power, Bilateral Monopoly, Nash-in-Nash, Right to Manage, Unionized Wage Bargaining

*I thank Hans-Peter Weikard and Erik Ansink for a discussion on bargaining versus market power that started this project. I also thank Evgenia Motchenkova and Jose Luis Moraga Gonzalez for their comments.

†School of Business and Economics and Tinbergen Institute, Vrije Universiteit Amsterdam, De Boelelaan 1105, 1081 HV Amsterdam, Netherlands. ORCID: 0000-0001-9085-7339. Email: harold.houba@vu.nl. Tel: +31 20 598 6014.

1 Introduction

Market power and bargaining dynamics are central themes in economic inquiry. Seller market power exhibits elevated profits, higher prices, and markups. Conversely, buyer market power reveals itself through increased consumer surplus, lower prices, and markdowns. In the realm of bilateral trades, where negotiations play a pivotal role, bargaining power serves as the origin of market power. These negotiations constitute a third form of competition, complementing the traditional realms of quantity and price competition. Our theoretical exploration delves into the emergence of seller and buyer market power from bargaining power, shedding light on the character of negotiated agreements.

The urgency behind our investigation stems from the growing adoption of the Nash-in-Nash approach within applied Industrial Organization (IO). Under this paradigm, all market transactions are the result of bilateral negotiations. The initial invocation of Nash pertains to bilateral bargaining and the resulting agreements. These agreements are outcomes of the bilateral asymmetric Nash bargaining solution (ANBS), as axiomatized by Kalai (1977a) and Svejnar (1986). Notably, since buyers can engage with multiple sellers, and vice versa, our second reference to Nash delves into the stability of bilateral contracts—a pioneering concept introduced by Horn and Wolinsky (1988) in the context of upstream-downstream duopoly markets. Collard-Wexler et al. (2019) further enrich this landscape by embedding a general two-sided market within a strategic bargaining framework, providing a robust theoretical foundation for the Nash-in-Nash paradigm.

The empirical exploration of the Nash-in-Nash approach within IO takes as its perspective: each seller-buyer pair engages in negotiations to determine a unit price per trade. In this context, two distinct scenarios emerge: The seller sets the quantity, operating along the supply curve, versus the buyer determines the quantity, positioned on the demand curve. The latter scenario, known as the "right to manage", draws parallels with the unionized wage bargaining literature, exemplified by works such as Svejnar (1986) and Layard et al. (1991). Here, the union and employers negotiate wages, while the firm retains its fundamental authority to manage employment—residing on its labor demand curve.[1]

In the context of wage bargaining, it might seem intuitive to assume that the demand side naturally holds the right to manage. Yet, this is no longer intuitive in empirical IO. Consider the case of business-to-business (B2B) interactions: Does a must-have brand like Coca-Cola wield the right to manage, or do major retail companies dictate terms during sales-promotion negotiations? This emphasizes that the choice of which side attains the right deserves both empirical and theoretical scrutiny. Our study introduces a groundbreaking innovation: we endogenize the right to manage. Not only do sellers and buyers negotiate unit prices, but they also determine who attains this critical right. As a result, bargaining power becomes the decisive factor in allocating this right, influencing negotiated prices, and linking to either seller or buyer market power.

In this theoretical study, we study a parsimonious bilateral monopoly—a market structure featuring a sole seller and a sole buyer. Our analysis centers on the interplay of bargaining weights, which quantify the relative power of each party, and the fundamental market primitives: a continuous nondecreasing marginal cost curve and a continuous decreasing price (or inverse demand) curve. The buyer's objective is to maximize consumer surplus, while the seller aims to maximize producer surplus. Any surplus beyond the competitive equilibrium level serves as a measure of market power.

To unravel the interplay between bargaining power, the fundamentals and market power, we adopt a layered approach, considering three distinct contract types. Each subsequent contract type adds complexity to our analysis and results:

1. Quantity-Transfer Contracts: The negotiations are equivalent to a divide-an-endogenous-dollar bargaining problem in which endogenous social welfare represents the dollar. These contracts optimize social welfare and specify the competitive equilibrium quantity. The lumpsum transfer distributes social welfare proportional to the bargaining weights. For this simple case, we derive a threshold for the seller bargaining weight that is pivotal: below this threshold, buyer market power prevails, while above it, seller market power dominates. The threshold itself

[1] Note that the "right to manage" framework is inherently inefficient, precluding the possibility of achieving optimal wage-employment contracts.

corresponds to the seller bargaining weight results in the competitive equilibrium outcome.

2. Price-Quantity Contracts: The outcome of the negotiations coincides with the outcome for quantity-transfer contracts because of the equivalence between revenue and the transfer. The negotiated price is the average price of the quantity-transfer contract. The same pivotal threshold for the seller bargaining weight ties markups to seller market power and markdowns to buyer market power. Finally, we obtain the vertical contract curve as characterized in e.g., Bowley (1928) and Blair et al. (1989).

3. Price Contracts with Endogenous Right-to-Manage: The inefficiency introduced by this type of contract transforms the vertical contract curve into a contract curve that exhibits a shape akin to $\succ$, where the upper part corresponds to the downward sloping price curve, while the lower part corresponds to the upward sloping marginal cost curve. As both curves meet in the competitive equilibrium, the pivotal threshold for the seller bargaining weight ties buyer right-to-manage to seller market power and seller right-to-manage to buyer market power.

Surprisingly, it is the weaker side of the market that attains this managerial privilege. The underlying intuition is that, say in case of seller market power, excess producer surplus above the level in the competitive equilibrium requires negotiating a markup. Markups, however, materialize only at prices exceeding the competitive equilibrium level, necessitating strategic positioning along the demand curve. In the context of negotiated markups, the precise magnitude adheres to a modified Lerner index. This index is bounded from below by zero (for bargaining weights precisely at the threshold) and bounded from above by the classic bilateral monopoly markup (when the seller wields dictator-like power). Our analysis extends beyond sellers to encompass buyer bargaining power and negotiated markdowns.

To summarize, negotiations of price contracts with endogenous right-to-manage offer a unifying theory that endogenizes which side of the market holds market power and, consequently, whether there will be a markup or markdown. The pivotal threshold on the seller bargaining weight holds the answer. Notably, our model unifies competition theories, encompassing both quantity and price competition as special cases. As Alviarez et al. (2023) eloquently put it (for exogenous right-to-manage), these findings "subsume standard pricing theories as limit cases."

Our characterization of price contracts with endogenous right-to-manage have empirical implications. When the data set includes information about buyer right to manage, it serves as a clear signal of the seller's bargaining weight surpassing the critical threshold, indicative of seller market power. However, in most data sets this crucial information is unobserved and the challenge of linking price and quantity is inherently based on the econometrician's prior beliefs about the market. These beliefs necessitate robustness checks ex-post. Our findings propose an alternative approach to assessing robustness: examining whether the estimated seller bargaining weight falls within the theoretically correct range. For example, marginal costs that are constant are a popular assumption in empirical IO. Then, the lower part of the $\succ$-shaped curve is horizontal and the critical threshold will be zero. Then buyer market power is impossible, and this justifies seller market power and markups. We will also briefly discuss how an empirical strategy recently proposed in Tomori et al. (2024) may provide opportunities to estimate which side holds market power.

2 Bilateral Monopoly

In this section, we introduce a parsimonuous model of a bilateral monopoly with a single product in which a seller (S) and a buyer (B) negotiate a binding contract. The asymmetric Nash bargaining solution (ANBS) in e.g. Kalai (1977a) and Svejnar (1986) is assumed to describe the outcome of these negotiations. This bargaining solution has the advantage that its bargaining weights provide a measure for bargaining power that is well understood. Formally, the seller has bargaining weight $\beta \in [0, 1]$ and the buyer has bargaining weight $1 - \beta$. A key question to be addressed is how bargaining power will impact market power and social welfare.

Since the outcome of the negotiations is a contract, the answer to the previous question depends crucially upon the type of contract that is feasible. In our study, we distinguish the following types of contracts:

Lump-sum transfer contracts specify a nonnegative quantity that the seller agrees to supply to the buyer, who in return receives a nonnegative lump-sum transfer as compensation. The transfer distributes the surpluses generated by the supply. Such contracts are denoted (q, t), where $q, t \geq 0$. It will be convenient to treat the null contract $(0, 0)$ as the disagreement otucome.

Price-quantity contracts specify a nonnegative quantity that the seller agrees to sell to the buyer, who in return receives a nonnegative unit price from the buyer. The corresponding revenue distributes surpluses. Such contracts are denoted (p, q), where $p, q \geq 0$. The null contract is the disagreement outcome.

Right-to-manage price contracts specify a nonnegative unit price and a right-to-manage to set quantity for either the seller or buyer. Such contracts are denoted (p, i), where $p \geq 0$ and $i = B, S$ obtains the right to set quantity. The contract $(0, S)$ is a natural disagreement outcome as it specifies that the supply is for free and, without loss of generality, the seller sets supply equal to zero.

The first two types of contracts are common and often implicitly defined in the literature. In these cases the negotiations are about to specify both a quantity and financial compensation. The third type of contract is also common, but novel is that the negotiations will not only specify a price but also the market side that will have the right to manage in setting quantity, which is a binary variable.

The economic fundamentals of any bilateral monopoly are the demand and supply curves, which we model as the marginal cost function, respectively, the price or inverse demand function. Formally, $MC : \mathbb{R}_+ \to \mathbb{R}_+$ is the marginal cost function, where $MC(q)$ denotes the marginal cost of producing quantity q. In the absence of fixed costs, the cost function is given by $C(q) = \int_0^q MC(\hat{q})\, d\hat{q}$. Next, $P : \mathbb{R}_+ \to \mathbb{R}_+$ is the price function, where $P(q)$ denotes the willingness to pay for quantity q. For our parsimonuous benchmark model, we impose the following assumption.

Assumption 1 *Marginal cost function $MC(q)$ is continuous and nondecreasing in q with intercept $MC(0) \geq 0$; the fixed costs are zero; and price function P is continuous and decreasing in q with finite intercept $P(0) > MC(0)$ and $\lim_{q\to\infty} P(q) = 0$.*

This assumption includes the most studied case in the literature, namely constant marginal costs, as a boundary case. It excludes indivisibilities and corresponding step functions, which will be addressed in the section discussing extensions.

Assumption 1 is a sufficient condition to guarantee existence of a unique $q^* > 0$ such that $P(q^*) = MC(q^*)$ and this quantity also maximizes social welfare. We denote the corresponding price as p^* and define it as the marginal cost price, i.e. $p^* = MC(q^*)$. In terms of competitive equilibrium, we may interpret p^* as the equilibrium price at which demand equals supply and q^* as the corresponding competitive equilibrium quantity.

Consumer and producer surpluses and social welfare are defined in the standard way but the exact specification will depend upon the type of contract under consideration. For the first two types of contract we have that

$$CS(q,t) = \int_0^q P(\hat{q})\,d\hat{q} - t, \qquad PS(q,t) = t - \int_0^q MC(\hat{q})\,d\hat{q},$$
$$CS(p,q) = \int_0^q P(\hat{q})\,d\hat{q} - pq, \qquad PS(p,q) = pq - \int_0^q MC(\hat{q})\,d\hat{q}.$$

The surpluses attained in the competitive equilibrium will play an important role in our analysis. As shorthand notation, we write PS^* for $PS(p^*, q^*)$ and CS^* for $CS(p^*, q^*)$, which are both nonnegative under Assumption 1. Since the disagreement outcome is an important component of the ANBS, we will use shorthand notation CS_0 for $CS(0,0)$ and PS_0 for $PS(0,0)$ in our main derivations even though $CS(0,0) = PS(0,0) = 0$ under Assumption 1. Finally, for contract (p,i) and quantity q, the surpluses are given by

$$CS(p,i) = \int_0^q P(\hat{q})\,d\hat{q} - pq, \qquad PS(p,i) = pq - \int_0^q MC(\hat{q})\,d\hat{q}$$

and we will also use shorthand notation CS_0 and PS_0 to denote the surpluses corresponding to the disagreement outcome $(0, S)$. Social welfare is the difference between consumer and producer surplus and will cancel either the financial transfer or revenue. Partial derivatives with respect to continuous contract variables are well defined.

Convenient measures of market power that apply to every type of contract in our study are the consumer and producer surpluses. We define seller market power as the seller's ability to negotiate a producer surplus that is larger than the counterfactual producer surplus in the competitive equilibrium. Similarly, buyer market power is measured as a consumer surplus exceeding that of the counterfactual competitive equilibrium. We will define market power for each type of contract.

Definition 2 (Market power) *Contract (q,t) features seller market power if $PS(q,t) > PS^*$ and buyer market power if $CS(q,t) > CS^*$. Similar for contracts (p,q) and (p,i).*

This definition is intuitive, flexible and easy to apply theoretically. It also captures the essence of market power: The side with market power strategically manipulates market conditions in order to become better off. Since the aggregate of both surpluses is bounded by maximal social welfare, it follows that at most one side of the market can have market power.[2] The latter is a desirable property because it would be odd if both sides could have market power at simultaneously. This definition is also flexible enough to eveluate contracts in which both sides are worst off, which is the case for contracts resulting in sufficiently small quantities. The latter will not occur under the ANBS. As we will make clear later, in case of either price-quantity or right-to-manageprice contracts, this definition implies a price markup in case of seller market power and a markdown for buyer market power. The latter are also important and popular measures of market power.

3 The Nash bargaining solution

In this section, we characterize the ANBS for each of the types of contract defined in the previous section. For all types, the bilateral monopoly is equivalent to a divide-the-dollar problem with an endogenous size of the dollar that is equal to social welfare, which depends upon quantity supplied. The outcome of the negotiations either specifies a quantity directly or, for right-to-manage price contracts, indirectly via either demand or supply. The analysis for each type of contract is delegated to dedicated subsections.

3.1 Lumpsum Transfer Contracts

Negotiations concerning lump-sum transfer contracts aim to precisely define both a quantity and a transfer. In these negotiations, the involved parties directly influence the endogenous social

[2] Formally, from $PS(q,t) + CS(q,t) \leq PS^* + CS^*$ and seller market power $PS^* < PS(q,t)$ it follows that $CS(q,t) < CS^*$. Similarly, $CS(q,t) > CS^*$ implies $PS(q,t) < PS^*$.

welfare, which subsequently gets distributed through the transfer. Specifically, when dealing with lump-sum transfer contracts, we express the (ANBS) in logarithmic form, along with the ANBS contract:

$$(q(\beta), t(\beta)) \in \arg\max_{q,t \geq 0} (1-\beta)\ln(CS(q,t) - CS_0) + \beta \ln(PS(q,t) - PS_0). \tag{1}$$

Our primary finding establishes a threshold bargaining weight, denoted as β^*, which distinguishes seller market power from buyer market power. Formally, we define β^* as follows:

$$\beta^* = \frac{PS^* - PS_0}{SW^* - PS_0 - CS_0}. \tag{2}$$

The detailed proofs for subsequent propositions are provided in the appendix.

Proposition 1 *Let $\beta \in [0,1]$ and $CS_0 + PS_0 < SW^*$. The ANBS lump-sum transfer contract $(q(\beta), t(\beta))$ is given by $q(\beta) = q^*$ and $t(\beta) = C(q^*) + PS_0 + \beta[SW^* - PS_0 - CS_0]$. The contract features buyer market power for $\beta \in [0, \beta^*)$ and seller market power for $\beta \in (\beta^*, 1]$. Moreover, $\beta^* \in (0,1)$ if and only if both $PS_0 < PS^*$ and $CS_0 < CS^*$. Finally, consumer and producer surpluses are given by*

$$CS(q(\beta), t(\beta)) = CS_0 + (1-\beta)[SW^* - PS_0 - CS_0],$$
$$PS(q(\beta), t(\beta)) = PS_0 + \beta[SW^* - PS_0 - CS_0].$$

As mentioned, social welfare can be seen as the endogenous value of the "dollar" to be distributed by the transfer. The axiom of Pareto efficiency, which underlies the ANBS, hinges on maximizing this endogenous value. Therefore, quantity has to maximize social welfare and it coincides with the competitive equilibrium quantity. This conclusion echoes well-established findings by Bowley (1928) and Blair et al. (1989). Furthermore, this result extends to various axiomatic bargaining solutions that adhere to the Pareto efficiency axiom. Notable examples include the Kalai-Smorodinsky solution (Kalai and Smorodinsky, 1975), the egalitarian solution (Kalai, 1977b), and the unified bargaining solution (Haake and Qin, 2018).

Svejnar (1986) axiomatizes the ANBS by substituting the axiom of Pareto efficiency for the axiom of Stong Individual Rationality and the axiom of Symmetry by an axiom that equates the Fear of Disagreement relative to Bargaining Power. In our context, contracts have two dimensions, quantity and the transfer. Fear is present in both dimensions and differs across dimensions. This follows from the first-order conditions of Equation (1) when these are rewritten as

$$\frac{\frac{CS(q,t)-CS_0}{\frac{\partial}{\partial q}CS(q,t)}}{1-\beta} = \frac{\frac{PS(q,t)-PS_0}{\frac{\partial}{\partial q}PS(q)}}{\beta} \quad \text{and} \quad \frac{\frac{CS(q,t)-CS_0}{\frac{\partial}{\partial t}CS(q,t)}}{1-\beta} = \frac{\frac{PS(q,t)-PS_0}{\frac{\partial}{\partial t}PS(q)}}{\beta}.$$

The expressions on each side of these equalities extend the one-dimensional definition of fear provided in Svejnar (1986). Since the partial derivatives in each dimension differ, the fear of disagreement in the quantity dimension differs from the fear in the monetary dimension. In the ANBS contract, the fears in both dimensions have to be equal.

We continue our discussion by interpreting the expressions associated with the ANBS from the perspective of the divide-the-dollar problem. First, the net surplus is defined as the difference between the dollar value (represented by maximal social welfare SW^*) and the disagreement payoffs (CS_0 and PS_0) when no agreement is reached. Second, each of the ANBS consumer and producer surpluses combines the disagreement payoff with a share of the net surplus. The share is both proportional to and increasing in a market side's bargaining weight (β or $1-\beta$). Third, the lump-sum transfer achieves these consumer and producer surpluses. It pays the producer surplus to the seller and compensates her for the costs associated with supplying the social welfare maximizing quantity ($C(q^*)$). Notably, extreme cases of dictator bargaining power constrain the lump-sum transfers within the range from $C(q^*) + PS_0$ to $C(q^*) + SW^* - CS_0$. Furthermore, the negotiated transfer is the price-of-a-contract in Collard-Wexler et al. (2019) for contracts that specify the social welfare maximizing quantity.

The concept of bargaining power is intrinsically tied to market power. Bargaining power directly translates into market power. The market side wielding more bargaining power — relative to the other side— holds a stronger position. Quantifying power through bargaining weights allows us to establish a clear partition. Specifically, a bargaining weight equal to the threshold serves as the decisive boundary between seller and buyer market power. This threshold aligns with the seller's bargaining weight needed to ensure that negotiations are concluded with the social welfare maximum. The latter outcome reflects the economic fundamentals of demand

and supply in the bilateral monopoly. The seller's bargaining weight and these fundamentals pinpoint which side exerts market power. Importantly, demand and supply shocks can shift this threshold, potentially altering the balance of market power.

In the extensively studied scenario of constant marginal costs, the producer surplus in the competitive equilibrium is zero. Consequently, seller market power prevails across all bargaining weights. When marginal costs are increasing, the producer surplus will be positive. This mirrors the concept of Ricardian rents in standard competitive markets. Buyer market power may emerge, potentially displacing seller market power, especially when bargaining power on the seller's side is low. Remarkably, even a threshold close to one —indicative of an almost flat demand curve— can theoretically lead to buyer market power dominance.

John Nash (1950) axiomatized the bargaining solution named after him by imposing the axiom of symmetry.[3] The interpretation of this axiom, discussed in Nash (1953), posits that rational players must have equal bargaining skills. Asymmetric bargaining outcomes can only be explained by differences in the players' preferences. In bilateral monopolies, the price and marginal cost functions express the buyer's and seller's preferences. Equal bargaining skills translate to equal bargaining weights. Importantly, the threshold —lying anywhere between zero and one— determines whether the seller's symmetric bargaining weight is above or below it. Therefore, equal bargaining weights should not be confused with equal market power.

In the field of Industrial Organization, the pass-through of supply shocks is a crucial empirical strategy in estimating bargaining power. To focus our discussion, let's consider the multiplicative (marginal) cost shift parameter denoted by ϕ. Here, $\phi = 1$ represents the situation before the shock, and $\phi \approx 1$ represents the situation after the shock. Formally, we define the marginal cost function as $\phi MC(q)$, where $\phi C(q)$ represents the cost function. This shift in costs impacts the maximum social welfare, specifically the direction of impact can be expressed as $\frac{d}{d\phi} q^*(\phi) < 0$ and $\frac{d}{d\phi} CS^*(\phi) < 0$. Introducing additional notation, we rewrite the transfer as:

$$t(\beta, \phi) = (1 - \beta)\left[\phi C(q^*(\phi)) + PS_0\right] + \beta\left[CS^*(\phi) - CS_0\right].$$

[3] Note that the Kalai-Smorodinsky bargaining solution in Kalai and Smorodinsky (1975) and the egalitarian solution in Kalai (1977b) also satisfy the axiom of symmetry.

Taking derivatives with respect to the cost shift parameter ϕ on both sides, we obtain:[4]

$$\frac{d}{d\phi}t\left(\beta,\phi\right)\Big|_{\phi=1} = \left(1-\beta\right)C\left(q^{*}\right) + MC\left(q^{*}\right)\frac{d}{d\phi}q^{*} > 0, \tag{3}$$

where the expression on the right-hand side suppresses $\phi = 1$. For a one percent increase in (marginal) costs, the pass-through is less than $1-\beta$ percent. Buyer bargaining power acts as a countervailing force, restricting the seller's ability to fully pass through cost shocks. This direct effect is complemented by an indirect effect operating through a shift in the maximum social welfare outcome. Upward shifts in the marginal cost curve impact the social welfare maximum by constricting quantity. This creates a boomerang effect that reduces marginal costs, variable costs and the consumer surplus, influencing both buyer and seller market power dynamics. The implication for empirical strategies based on pass-through is that neglecting the boomerang effect introduces a systematic bias that would overestimates buyer bargaining power.

3.2 Price-Quantity Contracts

Negotiations concerning price-quantity contracts aim to precisely define both a quantity and a unit price. Also in these negotiations, the involved parties directly influence the endogenous social welfare, which subsequently gets distributed through the revenue. Specifically, when dealing with price quantity contracts, we express the (ANBS) in logarithmic form, along with the ANBS contract:

$$\left(\hat{p}\left(\beta\right),\hat{q}\left(\beta\right)\right) \in \arg\max_{p,q\geq 0}\left(1-\beta\right)\ln\left(CS\left(p,q\right)-CS_0\right) + \beta\ln\left(PS\left(p,q\right)-PS_0\right) \tag{4}$$

Our second finding establishes results that are similar to those in Proposition 1. In order to express this similarity, we state such results as in the previous proposition.

[4] In this footnote we take a more general perspective. Consider the marginal cost function $MC\left(q,\theta\right)$, where we assume that $\frac{d}{d\theta}MC\left(q,\theta\right) > 0$ and for the corresponding cost function $\frac{d}{d\theta}C\left(q,\theta\right) \equiv C'_{\theta}\left(q,\theta\right) > 0$. Taking derivatives with respect to the cost shift parameter θ on both sides, we obtain:

$$\begin{aligned}\frac{d}{d\theta}t\left(\beta,\theta\right)\big|_{\theta=1} &= \left(1-\beta\right)C'_{\theta}\left(q^{*}\left(\theta\right),\theta\right) + \left[\left(1-\beta\right)MC\left(q^{*}\left(\theta\right),\theta\right) + \beta P\left(q^{*}\left(\theta\right)\right)\right]\frac{d}{d\theta}q^{*}\left(\theta\right)\ \big|_{\theta=1} \\ &= \left(1-\beta\right)C_{\theta}\left(q^{*}\left(1\right),1\right) + MC\left(q^{*}\left(1\right),1\right)\frac{d}{d\theta}q^{*}\left(1\right),\end{aligned}$$

because $P\left(q^{*}\left(1\right)\right) = MC\left(q^{*}\left(1\right),1\right)$ by definition of the competitive equilibrium. The imposed assumptions hold for both multiplicative shocks $\theta MC\left(q\right)$ and additive shocks $MC\left(q\right)+\theta$.

Proposition 2 *Let $\beta \in [0,1]$ and $CS_0 + PS_0 < SW^*$. The ANBS price-quantity contract $(\hat{p}(\beta), \hat{q}(\beta))$ is given by $\hat{p}(\beta) = t(\beta)/q^*$ and $\hat{q}(\beta) = q^*$. This contract features buyer market power for $\beta \in [0, \beta^*)$ and seller market power for $\beta \in (\beta^*, 1]$. Moreover, $\beta^* \in (0,1)$ if and only if both $PS_0 < PS^*$ and $CS_0 < CS^*$. Finally, consumer surplus $CS(\hat{p}(\beta), \hat{q}(\beta)) = CS(p(\beta), t(\beta))$ and producer surplus $PS(\hat{p}(\beta), \hat{q}(\beta)) = PS(q(\beta), t(\beta))$.*

The previous result aligns closely with Proposition 1. To satisfy the axiom of Pareto efficiency, it is essential that the negotiated quantity coincides with the social welfare maximizing quantity. Additionally, we can view revenue as a lump-sum transfer, implying that revenue must equal the lump-sum transfer described in Proposition 1. Subsequently, the unit price emerges by dividing revenue by quantity. Since the lump-sum transfer distributes social welfare, the unit price plays this crucial role.

The multi-dimensional interpretation of the axiom of Equality of Fear of Disagreement relative to Bargaining Power in Svejnar (1986), as given in the previous subsection, also holds for price-quantity contracts. Fear is present in both dimensions and differs across dimensions.

Furthermore, the preceding equivalences imply that both consumer and producer surpluses align with those observed for lump-sum transfers. Consequently, the same threshold for seller and buyer market power, as outlined in Proposition 1, holds true.

However, it is crucial to emphasize that the unit price represents an *average* price and should not be confused with an allocative price. Allocative prices, commonly assumed in consumer and producer theory, equate to either willingness to pay or marginal costs, leading to the determination of the optimal quantity.

In the context of a bilateral monopoly with price-quantity contracts, where the quantity is fixed, deviations arise when $\beta \neq \beta^*$. Specifically, the average price diverges from the price function or marginal cost in the competitive equilibrium (i.e., $p(\beta) \neq MC(q^*(\beta))$). Consequently, depending on bargaining weights and thresholds, we encounter either a markup or markdown that remains unrelated to Lerner indices and elasticities.

Moreover, extreme cases of dictator bargaining power constrain average prices within the range of $[C(q^*) + PS_0]/q^*$ to $[C(q^*) + SW^* - CS_0]/q^*$. Combining this fixed quantity with the specified price range yields the vertical contract curve, a concept characterized by Bowley (1928) and Blair et al. (1989). The proof of Proposition 1 provides a rather convenient derivation of this important result. Importantly, this vertical contract curve emerges for any other axiomatic bargaining solution that adheres to the axiom of Pareto efficiency.

To conclude this subsection, we delve into the concept of pass-through once more. Given that revenue aligns with the lump-sum transfer described in Proposition 1, our previous discussion already encompasses the pass-through of costs into revenue. Leveraging this equivalence streamlines our analysis of pass-through into the average price.

Following the approach from the preceding subsection, we model the multiplicative cost shock as $\phi MC(q)$. By taking the derivative on both sides of the equivalence $\hat{p}(\beta,\phi) \cdot q^*(\phi) = t(\beta,\phi)$ and rearranging, we arrive at the following expression:

$$\left.\tfrac{d}{d\phi}\hat{p}(\beta,\phi)\right|_{\phi=1} = \left.\frac{\tfrac{d}{d\phi}t(\beta,\phi) - \hat{p}(\beta,\phi)\tfrac{d}{d\phi}q^*(\phi)}{q^*(\phi)}\right|_{\phi=1} = \frac{\tfrac{d}{d\phi}t(\beta,1)}{q^*} - \hat{p}(\beta)\frac{\tfrac{d}{d\phi}q^*}{q^*} > 0.$$

The first term links pass-through in the average price to the first-order change in revenue per unit in the social welfare-maximizing quantity prior to the cost shock, q^*. The second term represents the negated first-order loss in revenue due to the change in quantity per unit in the same social welfare-maximizing quantity, $\tfrac{d}{d\phi}q^*/q^*$, evaluated against the negotiated price prior to the cost shock, $\hat{p}(\beta)$. Both effects contribute positively, resulting in an overall positive pass-through.

From an econometric estimation perspective, two strategies emerge for developing pass-through: either via revenue or through the average price, or perhaps a combination of both.

3.3 Negotiating Right-to-Manage Price Contracts

In this subsection, we delve into the intricacies of right-to-manage price contracts. These contracts aim to specify a unit price and determine which side of the market holds the authority to set the quantity. Much like price-quantity contracts, the price in right-to-manage price contracts plays a crucial role in distributing endogenous social welfare. The party with the right to manage will then strategically choose the quantity, either along the demand or supply curve. However, an important distinction arises: while the negotiating parties possess perfect foresight regarding how quantity will respond to price, they no longer wield direct control over social welfare.

Formally, within the context of right-to-manage price contracts, we express the (ANBS) in logarithmic form and define the ANBS contract as follows:

$$(\tilde{p}(\beta), \tilde{\imath}(\beta)) \in \arg\max_{p\geq 0, i\in\{B,S\}} (1-\beta)\ln(CS(p,i) - CS_0) + \beta \ln(PS(p,i) - PS_0), \tag{5}$$

$$\text{s.t. } p = MC(q) \text{ if } i = S; \quad \text{and } p = P(q) \text{ if } i = B.$$

Our subsequent finding diverges significantly from our prior two results, with the exception of the threshold as outlined in Proposition 1. In this upcoming result, the quantity that satisfies the following equation will assume a pivotal role:

$$MC(q) + \left(1 - \frac{\beta}{1-\beta}\frac{CS(MC(q), q) - CS_0}{PS(MC(q), q) - PS_0}\right) MC'(q)\, q = \quad P(q) \tag{6}$$

and similar for the quantity that satisfies

$$P(q) + \left(1 - \frac{1-\beta}{\beta}\frac{PS(P(q), q) - PS_0}{CS(P(q), q) - CS_0}\right) \quad P'(q)\, q = MC(q). \tag{7}$$

For expository reasons, we impose a more stringent condition than in the previous propositions. We denote CS_M and PS_m for the consumer surplus in the classic monopoly (M) and the producer surplus in the classic monopsony (m) outcome, respectively.

Proposition 3 *Let $\beta \in [0,1]$, $CS_0 < CS_M$ and $PS_0 < PS_m$. The ANBS right-to-manage price contract $(\tilde{p}(\beta), \tilde{\imath}(\beta))$ and corresponding right-to-manage quantity $\tilde{q}(\beta)$ are given by*

$$\begin{cases} \tilde{p}(\beta) = MC(\tilde{q}(\beta)), & \tilde{\imath}(\beta) = B, & \tilde{q}(\beta) \text{ solves (6)}, & \text{if } \beta \in [0, \beta^*), \\ \tilde{p}(\beta) = p^*, & \tilde{\imath}(\beta) \in \{B, S\}, & \tilde{q}(\beta) = q^*, & \text{if } \beta = \beta^*, \\ \tilde{p}(\beta) = P(\tilde{q}(\beta)), & \tilde{\imath}(\beta) = S, & \tilde{q}(\beta) \text{ solves (7)}, & \text{if } \beta \in (\beta^*, 1]. \end{cases}$$

The contract features buyer market power for $\beta \in [0, \beta^*)$ and seller market power for $\beta \in (\beta^*, 1]$, where $\beta^* \in (0,1)$. Finally, consumer surplus $CS(\tilde{p}(\beta), \tilde{\imath}(\beta)) = CS(\tilde{p}(\beta), \tilde{q}(\beta))$ and producer surplus $PS(\tilde{p}(\beta), \tilde{\imath}(\beta)) = PS(\tilde{p}(\beta), \tilde{q}(\beta))$.

In examining right-to-manage price contracts, our initial observation reveals a departure from maximal social welfare, except in a specific non-generic scenario. This qualitative difference sets right-to-manage price contracts apart from both lump-sum transfer and price-quantity contracts. The fundamental reason is that right-to-manage price contracts introduce allocative prices, which present an opportunity for strategic manipulation. Specifically, parties can influence the right-to-manage quantity by skillfully navigating these prices. The constraints in Equation (5) put price-quantity combinations on either the price curve or curve the marginal costs curve.[5] Consequently, the contract curve of such combinations exhibits a shape akin to $\succ$, rendering the social welfare maximizing vertical-bar contract curve (as seen in works by Bowley, 1928, and Blair et al., 1989) unattainable —except for the non-generic case where both curves coincide.

Analogous to classic monopoly and monopsony models, price manipulation through negotiations creates a wedge between the negotiated allocative price and the marginal costs associated with producing the right-to-manage quantity. This wedge is expressed by the second term on the left-hand sides of Equation (7) and (6). These wedges are zero if the seller's bargaining weight is equal to the threshold —the same threshold as derived before. The wedges are increasing in the distance of the seller's bargaining weight from the threshold and are at their widest at the extreme cases associated with dictator bargaining power, i.e., $\beta = 1$, respectively, $\beta = 0$. Then, the wedge will be equal to the one attained in the classic monopoly and monopsony models. In summary, as the seller's bargaining weight varies from zero to one, we observe a continuous transition of price-quantity combinations along the $\succ$-shaped contract curve, moving from the monopsony outcome to the monopoly outcome. From an economic perspective, bargaining power can be seen as a countervailing power in reducing the wedge.

Bargaining power plays a pivotal role in determining which side of the market secures the right to manage. While it may appear advantageous and indicative of strength, the side holding market power willingly relinquishes this right. Let's illustrate this with an example involving seller market power: To boost profit beyond the competitive equilibrium level, the seller must negotiate a price exceeding the competitive equilibrium price. Given that right-to-manage price contracts confine both parties to the $\succ$-shaped contract curve, achieving this higher price necessitates that the final price-quantity combination lies on the price curve. Consequently, the buyer must obtain the right to manage, allowing the seller to manipulate the price during negotiations, leveraging her bargaining power. Therefore, the side holding the right to manage typically lacks market

[5] One can show that both constraints in Equation (5) can be relaxed to $MC(q) \leq p \leq P(q)$. Notably, either the lower or upper bound must be binding (for detailed proof, we refer to the appendix).

power.

Market power in the current study is defined in terms of the ability for one of the market sides to raise her surplus above the competitive equilibrium surplus. As before, seller market power arises whenever the seller's bargaining weight exceeds the threshold and buyer market power arise otherwise. Again, this insight ties the concept of bargaining power to market power.

The following well-known and alternative measures for market power —often applied as alternative definitions— can be derived and shown to be equivalent to Definition 2 for right-to-manage price contracts. In case of seller market power, an allocative price above the competitive equilibrium price is indicative for the presence of this power; and similar for buyer market power. Consequently, a markup is indicative for seller market power and a markdown for buyer market power. Furthermore, the first-order conditions for the right-to-manage quantity can be rewritten into generalized Lerner indices and expressed in elasticities. Formally, consider the simplified notation of Equation (6) and (7) given by

$$\begin{cases} MC(q) + \theta\ MC'(q)\, q = \quad P(q), & \text{if } \beta < \beta^*, \\ \quad P(q) + \xi \quad P'(q)\, q = MC(q), & \text{if } \beta > \beta^*, \end{cases} \tag{8}$$

where the Greek symbols $\theta, \xi \in [0, 1]$ represent the wedges. Since the negotiated price represents $MC(q)$ in case of buyer market power, we obtain the following expressions for the generalized Lerner indices:

$$\begin{cases} \frac{P(q)-MC(q)}{MC(q)} = \theta \frac{MC'(q)q}{MC(q)}, & \text{if } \beta < \beta^*, \\ \frac{P(q)-MC(q)}{P(q)} = -\xi \frac{P'(q)q}{P(q)}, & \text{if } \beta > \beta^*. \end{cases}$$

Given the restrictions on θ and ξ and how these respond to β, the right-hand sides are bounded by the extreme distributions of dictator bargaining power.

The theoretical results above show that the negotiation approach to bilateral monopoly with the right-to-manage price contracts serves as an overarching umbrella theory, bridging centuries of economic thought. By understanding the interplay of bargaining power, market structure, and welfare implications, we gain deeper insights into the emergence of market power and how it shapes market outcomes.

Implications for Empirical Industrial Organization: A Theoretical Perspective

In this subsection, we delve into several critical implications for empirical Industrial Organization. Specifically, we address the challenge of linking price and quantity in the context of bilateral monopoly, particularly when such monopolies are thought to distort social welfare a priori. Our analysis centers around the application of the ANBS in Nash-in-Nash, a widely used framework. Traditionally, researchers have resolved this issue by assuming either the buyer or the seller determines the quantity. Both scenarios imply a right-to-manage price contract.

However, this assumption is inherently based on the econometrician's prior beliefs about the market. These beliefs necessitate robustness checks ex-post. Our theoretical findings propose an alternative approach to assessing robustness: examining whether the estimated seller bargaining weight falls within the theoretically correct range. To illustrate, consider the work of Bonnet et al. (2024), who investigate manufacturer-retail negotiations in the French soft drink industry. Their assumption is that manufacturers operate with constant marginal costs, and retailers order downstream market demand. In this setup, the retailer holds the right-to-manage. Crucially, the manufacturer's threshold remains non-positive (as discussed in the subsection on lump-sum transfers), ensuring that the manufacturer wields seller market power regardless of her bargaining weight. Consequently, the retailer's position aligns with her demand curve. Our results provide theoretical justification for the a priori assumption made by Bonnet et al. (2024). By focusing on the estimated seller bargaining weight, we offer a principled approach to assessing robustness in bilateral monopoly models.

By endogenizing which side attains the right to manage, we open up the possibility of developing an empirical strategy that identifies the party holding this right based on available data. Coincidentally, such an empirical strategy has been proposed by Tomori et al. (2024) in their study of the Californian water market, which is a thin market in which most traders trade at most once. Their approach centers around a standard Cournot model. Specifically, in one of their robustness checks they endogenously estimate which side holds market power. Let's translate their check to our context. The observed price is set as in (8). Following the footsteps of Tomori et al. (2024), we replace the functions specifying price with the actual observed price.

Neglecting the seller bargaining weight, this leads us to the following system of equations:

$$\begin{aligned} p &= \quad P(q) - \theta \; MC'(q)\, q, \\ p &= MC(q) - \xi \quad P'(q)\, q. \end{aligned}$$

where p represents the observed price. The parameters θ and ξ hold the key to buyer, respectively, buyer market power. This system can be directly estimated under the restrictions $\theta \in [0,1]$ and $\xi \in [0,1]$ without imposing a priori which side holds power. In the case of the Californian water market, Tomori et al. (2024) do not impose these constraints. Their reported estimates reveal a significant $\theta \gg 1$ (with a p-value below 1%) and an insignificant ξ, suggesting that $\theta = 1$ and $\xi = 0$. From our theoretical perspective, this implies not only buyer market power but also a scenario where buyers wield dictator bargaining power.

4 Extensions

In this section, we discuss several extensions of the parsimonous bilateral monopoly analyzed in the previous section.

4.1 Indivisible Goods

In many markets indivisible goods are traded, f.e. housing, cars, consumer electronics, luxureous yachts, cruise ships, chip-making equipment and other "big-ticket" items. Indivisibilities can be modeled as demand and supply functions that are step functions in price. The Nash bargaining approach can easily accomodate indivisibilities.

To formalize our discussion, suppose items are identical and numbered. Quantity $q \in \mathbb{Z}_+$ is the supply measured in the number of items while price remains $p \in \mathbb{R}_+$. The demand function is defined as $d : \mathbb{R}_+ \to \mathbb{Z}_+$ and the supply function as $s : \mathbb{R}_+ \to \mathbb{Z}_+$.[6] The discontinuities of these functions represent reservation prices and are functions of quantity. The buyer's reservation price function is defined as $r^B : \mathbb{Z}_+ \to \mathbb{R}_+$ given by $r^B(q) = \sup_p \{p \in \mathbb{R}_+ | \, d(p) \geq q\}$. Similarly, $r^S : \mathbb{Z}_+ \to \mathbb{R}_+$ given by $r^S(q) = \inf_p \{p \in \mathbb{R}_+ | \, s(p) \geq q\}$ is the seller's reservation price function. We impose the following assumption.

[6] Strictly speaking, demand and supply are both correspondences of price as they map into two (or more) quantities at reservation prices for $q \in \mathbb{Z}_+$. Also, the equivalent of constant marginal costs $MC(q) = c$ in case of indivisibilities would translate in $s(p) = \{0\}$ for $p < c$, $s(c) = \mathbb{Z}_+$ and $s(p)$ is either empty or $\{\infty\}$ for $p > c$.

Assumption 3 *Demand function d is nonincreasing in p with finite intercept $d(0) \geq 1$ and $\lim_{p\to\infty} d(p) = 0$; supply function $s(p)$ is nondecreasing in p with intercept $s(0) = 0$ and $\lim_{p\to\infty} s(p) > d(0)$; the fixed costs are zero; and $r^B(1) \geq r^S(1) \geq 0$.*

This assumption is a sufficient condition to guarantee existence of a unique and positive competitive equilibrium quantity $q^* \in \mathbb{N}$ and the range of competitive equilibrium prices p^* bounded from below by $\underline{p}^* = \max\left\{r^S(q^*), r^B(q^*+1)\right\}$ and from above by $\overline{p}^* = \min\left\{r^S(q^*+1), r^B(q^*)\right\}$.[7] As before, this quantity maximizes social welfare and each equilibrium price distributes this welfare in a different way through the revenue it generates.

The multiplicity of equilibrium prices requires a minor modification of the definition of market power. A contract features seller market power if its producer surplus is larger than the seller's *highest* producer surplus in any competitive equilibrium, which is attained at upper bound $\overline{p}^*$. Similarly, buyer market power features a consumer surplus that exceeds the buyer's *highest* equilibrium consumer surplus, which is attained at lower bound $\underline{p}^*$. For later reference, we define the seller's bargaining weight $\overline{\beta}^*$ as the weight that yields her an ANBS producer surplus that matches her highest competitive equilibrium surplus. Similarly, the seller's bargaining weight $\underline{\beta}^*$ yields an ANBS consumer surplus equal to her highest competitive equilibrium consumer surplus, where $\underline{\beta}^* \leq \overline{\beta}^*$. By definition, any seller bargaining weight $\beta \in \left[\underline{\beta}^*, \overline{\beta}^*\right]$ yields ANBS surpluses that corresponds to a competitive equilibrium and vice versa. This range replaces threshold β^*. Furthermore, there is absence of market power for seller weights in this range as neither the seller nor buyer is able to negotiate a surplus above her highest comeptitive equilibrium surplus. Note that $\underline{\beta}^* = 0$ and $\overline{\beta}^* = 1$ are theoretically possible, for example in case of a single indivisible good. In that case, there is no market power, only bargaining power that distributes social welfare.

For later reference, we also define $\overline{\overline{p}}^* = \max\left\{r^S(q^*+1), r^B(q^*)\right\}$ and $\overline{\overline{\beta}}^*$ as the seller's largest bargaining weight that achieves this price (and q^*). And similarly, $\underline{\underline{p}}^* = \min\left\{r^S(q^*), r^B(q^*+1)\right\}$ and $\underline{\underline{\beta}}^*$ as the corresponding weight.

Lumpsum Transfer and Price-Quantity Contracts

[7] The equilibrium condition $s(p) = q^*$ imposes $r^S(q^*) \leq p \leq r^S(q^*+1)$ and $d(p) = q^*$ imposes $r^B(q^*+1) \leq p \leq r^B(q^*)$. Combining both conditions yields the stated bounds.

The intuition underlying Proposition 1 and 2 extends in a straightforward way. For both lump-sum transfer and price-quantity contracts, Pareto efficiency implies that the parties agree upon the social welfare maximizing quantity, while revenue depends upon the bargaining weights and will distribute maximal welfare. Consequently, the expressions of Proposition 1 and 2 remain valid after straightforward modifications of the producer and consumer surpluses.[8] Furthermore, bargaining power induces buyer market power if $\beta < \underline{\beta}^*$ and seller market power if $\beta > \overline{\beta}^*$. For bargaining weights between these bounds, there is absence of market power. Notice that market power will drive the average price outside the range of competitive equilibrium prices. This implies the occurence of a markup with respect to $r^B(q^*)$ or a markdown with respect to $r^S(q^*)$. We summarize these findings in the following result.

Proposition 4 *Let $\beta \in [0,1]$ and $CS_0 + PS_0 < SW^*$. The closed form solutions of Proposition 1 and 2 are valid in case of indivisible goods. The contract features buyer market power for $\beta \in [0, \underline{\beta}^*)$, seller market power for $\beta \in (\overline{\beta}^*, 1]$ and absence of market power for $\beta \in \left[\underline{\beta}^*, \overline{\beta}^*\right]$.*

Right-to-Manage Price Contracts

For right-to-manage price contracts, the smooth trade-off between price and continuous quantity of Proposition 3 may break down in the presene of indivisibilities. Along any vertical bar, strictly between reservation prices of subsequent quantities, either demand or supply has become perfectly inelastic with respect to price. Then, marginal changes in the seller's bargaining weight has no effect on quantity at all and only marginal effects on the negotiated price (and revenue). At a reservation price, however, quantity is perfectly elastic and marginal changes triggers a discrete change in either supply (for a price decrease only) or demand (for a price increase only). This implies a countervailing effect for the weakest side of the market that will obtain the right to manage. Also, this can enhance market efficiency.

An interesting question is what range of seller bargaining weights supports the social welfare

[8] The consumer and producer surpluses for price-quantity contracts are defined as

$$CS(p,q) = \int_0^{r^b(1)} \min\{d(\hat{p}), q\}\, d\hat{p} - pq, \quad PS(p,q) = \int_0^{p} \min\{s(\hat{p}), q\}\, d\hat{p},$$

which are continuous in p and q.

maximizing quantity as part of the ANBS outcome. Consider a seller's bargaining weight β just above $\overline{\beta}^*$. Should the seller negotiate a price at or above $\overline{p}^*$? There are two cases to consider:

1. $\overline{\overline{p}}^* = r^B(q^*) > r^S(q^*+1) = \overline{p}^*$: For prices slightly above $\overline{p}^*$, demand is perfectly inelastic. Similar as for price-quantity contracts, the contract curve is locally vertical and, consequently, $\tilde{p}(\beta) = \hat{p}(\beta)$. This argument holds for all $\beta \leq \overline{\overline{\beta}}^*$. At the upperbound, similar arguments as for the second case apply.

2. $\overline{\overline{p}}^* = \overline{p}^* = r^B(q^*) \leq r^S(q^*+1)$: Then demand is perfectly elastic for marginal price increases above $r^B(q^*)$ and any marginal price increase will drop demand by one unit. Such drop also reduces the seller's reservation price from $r^S(q^*)$ to $r^S(q^*-1)$. Because quantity q^* maximizes profits at price $r^B(q^*)$, the seller will be worse off under a negotiated price slightly above $r^B(q^*)$.[9] So, the negotiated price $\tilde{p}(\beta)$ is bounded by $\overline{\overline{p}}^* = \overline{p}^*$ for β slightly above $\overline{\overline{\beta}}^* = \overline{\beta}^*$.[10]

Similar arguments apply for seller's bargaining weights β just below $\underline{\beta}^*$. It shows that maximization of social welfare and right-to-manage prices outside the range of competitive prices is possible. We summarize these insights in the following proposition.

Proposition 5 *Let $\beta \in [0,1]$ and $CS_0 + PS_0 < SW^*$. If $\beta \in \left[\underline{\underline{\beta}}^*, \overline{\overline{\beta}}^*\right]$, then the negotiated right-to-manage price contract $(\tilde{p}(\beta), \tilde{\imath}(\beta))$ and corresponding right-to-manage quantity $\tilde{q}(\beta)$ maximize social welfare. Furthermore, $\underline{\underline{p}}^* \leq \tilde{p}(\beta) \leq \overline{\overline{p}}^*$ and $\tilde{q}(\beta) = q^*$. The right to manage $\tilde{\imath}(\beta) = S$ if $\underline{\underline{\beta}}^* \leq \beta < \underline{\beta}^*$; $\tilde{\imath}(\beta) \in \{S, B\}$ if $\underline{\beta}^* \leq \beta \leq \overline{\beta}^*$; and $\tilde{\imath}(\beta) = B$ if $\overline{\beta}^* < \beta \leq \overline{\overline{\beta}}^*$.*

The bounds $\underline{\underline{\beta}}^*$ and $\overline{\overline{\beta}}^*$ on the the seller's bargaining weight form sufficient conditions for this result. Necessary conditions and exact bounds for the seller's bargaining weight will be

[9] At price $r^B(q^*)$, the price $r^B(q^*) + \varepsilon$, $\varepsilon > 0$, makes the seller weakly worse off if

$$\left[r^B(q^*) + \varepsilon - r^S(q^*-1)\right](q^*-1) \leq \left[r^B(q^*) - r^S(q^*)\right] q^*.$$

After rewriting, we obtain $r^B(q^*) \geq r^S(q^*)\, q^* + r^S(q^*-1)\,(q^*-1)$, when evaluated at $\varepsilon = 0$.This condition states that the incremental revenue of the last unit exceeds the incemental cost of supplying the last unit, which holds in q^* and $r^S(q^*)$.

[10] To retrieve the upper bound on seller bargaining weights, one may apply the vertical contract curve for price-quantity contracts fixed at quantity q^*-1 to calculate the weight from the average price $\hat{p}(\beta)$ that yields the upper bound. The caveat is that the buyer may be better off q^*-2 (or less) at $\hat{p}(\beta)$, which suggests an iterative process.

case specific. Example 4 illustrates that it is theoretically possible that right-to-manage price contracts can maximize social welfare for every bargaining weight. It also demonstrates how the perfectly elastic demand and supply can act as a countervailing power against the strongest side bargaining power.

Example 4 *Consider the demand and supply step functions on the price range* $[1,4]$ *given by*

$$d(p) = \begin{cases} 3, & \text{if } p \in [1,2], \\ 2, & \text{if } p \in [2,3], \\ 1, & \text{if } p \in [3,4], \end{cases} \qquad \text{and} \qquad s(p) = \begin{cases} 1, & \text{if } p \in [1,2], \\ 2, & \text{if } p \in [2,3], \\ 3, & \text{if } p \in [3,4]. \end{cases}$$

The underlying reservation prices are given by $r^S(1) = 1$, $r^S(2) = 2$ and $r^S(3) = 3$ for the seller and $r^B(1) = 4$, $r^B(2) = 3$ and $r^B(3) = 2$ for the buyer.

Next, $\underline{p}^* = \max\left\{r^S(2), r^B(3)\right\} = 2$ and $\overline{p}^* = \min\left\{r^S(3), r^B(2)\right\} = 3$ yields the range of competitive equilibrium price given by $[2,3]$. The equilibrium supports quantity $q^* = 2$ and generates social welfare $SW(p^*, q^*) = 4$.

In our analysis, only q equal to either 1 or 2 matters. The consumer surplus $CS(p,q)$, expressed for price-quantity contracts, is given by $CS(p,1) = 4-p$ and $CS(p,2) = 7-2p$, while the producer surplus $PS(p,q)$ is given by $PS(p,1) = p-1$ and $PS(p,2) = 2p-3$.

We analyze price-quantity contracts first. Application of Proposition 1 and 2 yields the ANBS price, revenue, consumer surplus and producer surplus given by

$$\hat{p}(\beta) = \tfrac{3}{2} + 2\beta, \qquad \hat{t}(\beta) = 3 + 4\beta, \qquad CS(\hat{p}(\beta), 2) = 4(1-\beta), \qquad PS(\hat{p}(\beta), 2) = 4\beta.$$

The average price $\hat{p}(\beta)$ lies between $\frac{3}{2}$ and $\frac{7}{2}$. The negotiated price is a competitive equilibrium price if and only if $\beta \in \left[\frac{1}{4}, \frac{3}{4}\right]$. Hence, $\underline{\beta}^* = \frac{1}{4}$ and $\overline{\beta}^* = \frac{3}{4}$.

We continue with analyzing right-to-manage price contracts. Then, $\underline{\underline{p}}^* = \min\left\{r^S(2), r^B(3)\right\} = 2$ and $\overline{\overline{p}}^* = \max\left\{r^S(3), r^B(2)\right\} = 3$ and these bounds coincides with $\underline{p}^*$, respectively, $\overline{p}^*$. Hence, $\underline{\underline{\beta}}^* = \frac{1}{4}$ and $\overline{\overline{\beta}}^* = \frac{3}{4}$. Consider $\beta > \frac{3}{4}$. Then, a negotiated price of 3 yields a demand of 2 units and a producer surplus of 3. Furthermore, a price above 3 reduces demand to 1 unit, generates a social welfare of 3. The maximizer of the Nash product $CS(p,1)^{1-\beta} PS(p,1)^{\beta}$ is the price

$1+3\beta$ and generates a producer surplus of $3\beta \leq 3$ for all $\beta \in [0,1]$. Hence, the seller is better off settling on the price $\overline{\overline{p}}^* = \overline{p}^* = 2$. Similarly for $\beta < \frac{1}{4}$, a negotiated price of 2 yields a supply of 2 units and a consumer surplus of 2, while a price below 2 reduces supply to 1 unit, also generates a social welfare of 3 and the price $1+3\beta$ is the maximizer of the same Nash product that generates a consumer surplus of $3(1-\beta) \leq 3$ for all $\beta \in [0,1]$. This implies that the buyer is better off settling on the price $\underline{\underline{p}}^* = \underline{p}^* = 2$. To summarize, the ANBS right-to-manage price contract is given by

$$(\tilde{p}(\beta), \tilde{\imath}(\beta)) = \begin{cases} (2, B) & \text{if } 0 \leq \beta < \frac{1}{4}, \\ \left(\frac{3}{2} + 2\beta, B \text{ or } S\right), & \text{if } \frac{1}{4} \leq \beta \leq \frac{3}{4}, \\ (3, S), & \text{if } \frac{3}{4} < \beta \leq 1. \end{cases}$$

More intesting is that the right-to-manage quantity $\tilde{q}(\beta) = 2 = q^*$ maximizes social welfare for all $\beta \in [0,1]$. This demonstrates how the perfectly elastic supply at $\underline{\underline{p}}^*$ and perfectly elastic demand at $\overline{\overline{p}}^*$ can act as a countervailing power against the strongest side bargaining power.

5 Appendix: Mathematical Proofs

Proof of Proposition 1

The ANBS is Pareto efficient. Furthermore, only the competitive equilibrium quantity maximizes social welfare. Hence, $q(\beta) = q^*$ for all $\beta \in [0,1]$. Since all functions are partially differentiable, we investigate the first-order conditions (FOCs) of (1) presuming $q, t > 0$. Because $\frac{\partial}{\partial q} CS(q,t) = P(q)$, $\frac{\partial}{\partial t} CS(q,t) = -1$, $\frac{\partial}{\partial q} PS(q,t) = -MC(q)$ and $\frac{\partial}{\partial t} PS(q,t) = 1$, the FOCs are given by[11]

$$\frac{(1-\beta) P(q)}{CS(q,t) - CS_0} - \frac{\beta MC(q)}{PS(q,t) - PS_0} = 0,$$
$$\frac{-(1-\beta)}{CS(q,t) - CS_0} + \frac{\beta}{PS(q,t) - PS_0} = 0.$$

After making use of the second condition, we obtain $P(q) = MC(q)$. Hence, $q(\beta) = q^*$. Evaluating the second condition in $q = q^* > 0$ and rewriting yields

$$\beta = \frac{PS(q^*,t) - PS_0}{CS(q^*,t) + PS(q^*,t) - PS_0 - CS_0} = \frac{t - C(q^*) - PS_0}{SW^* - PS_0 - CS_0}.$$

[11] An alternative derivation of $q(\beta) = q^*$ is obtained by substituting the second equation into the first equation, which then reduces to $P(q) = MC(q)$.

After solving for t, we obtain

$$t(\beta) = C(q^*) + PS_0 + \beta[SW^* - PS_0 - CS_0],$$

where $t(\beta) \geq 0$ and $t(\beta) = 0$ for the nongeneric case $\beta = C(q^*) = PS_0 = 0$. The stated consumer and producer surpluses follow immediately. Finally, the contract features seller market power if $PS(q(\beta), t(\beta)) > PS^*$, which can be rewritten as $\beta > \frac{PS^* - PS_0}{SW^* - PS_0 - CS_0} = \beta^*$. Similarly, buyer market power $CS(q(\beta), t(\beta)) > CS^*$ implies $1 - \beta > \frac{CS^* - CS_0}{SW^* - PS_0 - CS_0} = 1 - \beta^*$. If both $PS_0 < PS^*$ and $CS_0 < CS^*$ hold, then $\beta^* \in (0, 1)$. Furthermore, the weight $\beta = \beta^*$ yields the competitive equilibrium surpluses. However, it may also be the case that either $PS_0 > PS^*$ or $CS_0 > CS^*$ hold (but never both, which is excluded by imposing $PS_0 + CS_0 < SW^*$ in this proposition). In the former case, by individual rationality we have that $PS(q(\beta), t(\beta)) \geq PS_0 > PS^*$ trivially implies seller market power for all $\beta \in [0, 1]$ and also that the condition $\beta > \frac{PS^* - PS_0}{SW^* - PS_0 - CS_0}$ remains valid for all $\beta \in [0, 1]$. In the latter case, $CS(q(\beta), t(\beta)) \geq CS_0 > CS^*$ implies that there is buyer market power for all $\beta \in [0, 1]$ and also that $1 - \beta > \frac{CS^* - CS_0}{SW^* - PS_0 - CS_0}$ remains valid for all $\beta \in [0, 1]$. To summarize, seller market power arises if β belongs to the possibly empty interval $[0, \beta^*)$ and buyer market power if β belongs to the possibly empty interval $(\beta^*, 1]$. ■

Proof of Proposition 2

As in the proof of Proposition 2, the axiom of Pareto efficiency implies $\hat{q}(\beta) = q^*$. Next, because $\frac{\partial}{\partial q} CS(p, q) = P(q) - p$, $\frac{\partial}{\partial p} CS(p, q) = -q$, $\frac{\partial}{\partial q} PS(p, q) = p - MC(q)$ and $\frac{\partial}{\partial p} PS(p, q) = q$, the FOCs of (4) while presuming $p, q > 0$ are given by[12]

$$\frac{(1-\beta)(P(q) - p)}{CS(p, q) - CS_0} + \frac{\beta(p - MC(q))}{PS(p, q) - PS_0} = 0,$$
$$\frac{-(1-\beta)q}{CS(p, q) - CS_0} + \frac{\beta q}{PS(p, q) - PS_0} = 0.$$

After making use of the second condition, we obtain $P(q) = MC(q)$. Hence, $\hat{q}(\beta) = q^* > 0$. Evaluating the second equation in $q = q^*$ and rewriting yields

$$\beta = \frac{PS(p, q^*) - PS_0}{CS(p, q^*) + PS(p, q^*) - PS_0 - CS_0} = \frac{pq^* - C(q^*) - PS_0}{SW^* - PS_0 - CS_0}.$$

[12] An alternative derivation of $\hat{q}(\beta) = q^*$ is obtained by substituting the second equation (after canceling q) into the first equation, which then reduces to $P(q) = MC(q)$.

After solving for revenue pq^*, we obtain

$$\hat{p}(\beta)\, q^* = PS_0 + C(q^*) + \beta\left[SW^* - PS_0 - CS_0\right] = t(\beta),$$

where $t(\beta) \geq 0$ and $t(\beta) = 0$ for the nongeneric case $\beta = C(q^*) = PS_0 = 0$. So, $\hat{p}(\beta) = t(\beta)/q^* > 0$ is the generic case. Hence, the consumer and producer surpluses coincide with those stated in Proposition 1. Consequently, the bounds on the seller's bargaining weight for either seller or buyer market power coincide with those stated in the latter proposition. ■

Proof of Proposition 3

To get rid of the binary variable $i \in \{B, S\}$, we consider price-quantity contracts (p, q) and relax the equality restrictions on price in (5) into two weak inequality restrictions such that the price function specifies the upper bound on p and the marginal cost curve the lower bound on p. Formally, the modified ANBS is given by

$$(\check{p}(\beta), \check{q}(\beta)) \in \arg\max_{p,q \geq 0} (1-\beta)\ln(CS(p,q) - CS_0) + \beta \ln(PS(p,q) - PS_0), \tag{9}$$

$$\text{s.t. } MC(q) \leq p \quad (\lambda), \qquad \text{and } p \leq P(q) \quad (\mu),$$

where λ and μ denote shadow prices. The setup of this proof is that we first show elementary properties of the solution to (9) in Claim A, then we characterize the solution to (9) and, finally, we show Claim B that the solution to (9) is also the solution to (5) and vice versa.

Claim A: $\check{q}(\beta) \leq q^*$ and either $\check{p}(\beta) = MC(\check{q}(\beta))$, or $\check{p}(\beta) = P(\check{q}(\beta))$, or both.

Proof of the Claim A

Combining both constraints imposes $MC(q) \leq p \leq P(q)$ and hence feasibility requires $q \leq q^*$. In particular, $\check{q}(\beta) \leq q^*$. Suppose both constraints do not bind, i.e. $MC(\check{q}(\beta)) < \check{p}(\beta) < P(\check{q}(\beta))$. Then, necessarily $\check{q}(\beta) < q^*$. We distinguish two cases: 1. $\check{p}(\beta) \leq p^*$ and 2. $\check{p}(\beta) \geq p^*$.

Case 1. We have $MC(\check{q}(\beta)) < \check{p}(\beta) \leq p^*$. Increasing quantity to the amount $q > \check{q}(\beta)$ for which $MC(q) = \check{p}(\beta)$ (and $q \leq q^*$ still holds), would increase both surpluses and, therefore, this would increase the Nash product, contradicting that $(\check{q}(\beta), \check{p}(\beta))$ is the solution of (9).

Case 2. Then $p^* \leq \check{p}(\beta) < P(\check{q}(\beta))$ and, similar as for Case 1, increasing quantity to the amount $q > \check{q}(\beta)$ for which $P(q) = \check{p}(\beta)$ would increase the Nash product above the maximum, a contradiction. This completes the proof of Claim A.

By the Maximum Theorem, see e.g. Ok (2007), the solution to (9) is continuous in $\beta \in [0,1]$. The ANBS also satisfies the axiom of Strict Individual Rationality for $\beta \in (0,1)$, see e.g. Svejnar (1986), and individual rationality at either $\beta = 0$ and $\beta = 1$. Without loss of generality, we may assume that $CS(p,q) \geq CS_0$ and $PS(p,q) \geq PS_0$ are both non-binding. Then, the Lagrangian function of (9) is given by

$$(1-\beta)\ln(CS(p,q) - CS_0) + \beta \ln(PS(p,q) - PS_0) - \lambda(MC(q) - p) - \mu(p - P(q))$$

and the Karush-Kuhn-Tucker (KKT) conditions are given by

$$\begin{aligned}
\frac{-(1-\beta)q}{CS(p,q) - CS_0} + \frac{\beta q}{PS(p,q) - PS_0} + \lambda - \mu &= 0, \\
\frac{(1-\beta)(P(q) - p)}{CS(p,q) - CS_0} + \frac{\beta(p - MC(q))}{PS(p,q) - PS_0} - \lambda MC'(q) + \mu P'(q) &= 0, \\
\lambda(MC(q) - p) &= 0, \\
\mu(p - P(q)) &= 0.
\end{aligned}$$

By Claim A, there are three cases to investigate.

1. $p = P(q) > MC(q)$ and $\lambda = 0$. The remaining unknowns are $q \geq 0$ and $\mu \geq 0$. The first term of the second line is equal to 0. This line pins down μ, which can be substituted into the second line to obtain

$$P(q) + \left(1 - \frac{(1-\beta)}{\beta} \frac{PS(P(q),q) - PS_0}{CS(P(q),q) - CS_0}\right) P'(q)\, q = MC(q). \tag{10}$$

   For $\beta = 1$, this equality reduces to the FOC of the classic monopoly: $P(q) + P'(q)\,q = MC(q)$ with monopoly quantity $q_M < q^*$ and monopoly price $p_M = P(q_M) > p^*$, etc. The associated surplusses are feasible as these satisfy the imposed conditions $CS_0 < CS_M$ and $PS_0 < PS_m$. For $\beta < 1$, the product of fractions is positive and we obtain $P(q) + P'(q)\,q < MC(q)$ as a necessary condition for optimality, which implies a quantity above the classic monopoly quantity and a price below the classic monopoly price. Next, $\mu \geq 0$ in the second line of the KKT conditions implies

$$\frac{(1-\beta)}{\beta}\frac{PS\left(P\left(q\right),q\right)-PS_0}{CS\left(P\left(q\right),q\right)-CS_0} \leq 1.$$

This implies that the coefficient of $P'\left(q\right)q$ is nonnegative and that (10) reduces to the weak version of this case's assumption $P\left(q\right) \geq MC\left(q\right)$, i.e. $q \leq q^*$. Consequently $p \in [p^*, p_M]$. Every such pair (p, q) satisfies the imposed conditions $CS_0 < CS_M$ and $PS_0 < PS_m$, because $CS\left(p,q\right) > CS_M > CS_0$ and $PS_0 < PS_m < PS^* \leq PS\left(p,q\right)$.

Next, note that *i*) (9) is a constrained version of (4), *ii*) (q^*, p^*) is feasible in both maximization problems and *iii*) for $\beta = \beta^*$ it holds that (q^*, p^*) is the solution to (4) by Proposition 2. Application of the axiom of irrelevant alternatives underlying the ANBS to the case $\beta = \beta^*$ yields that the ANBS of (9) must be (q^*, p^*) for $\beta = \beta^*$. Hence, $\left(\check{p}\left(\beta^*\right), \check{q}\left(\beta^*\right)\right) = (q^*, p^*)$. By continuity of $\left(\check{p}\left(\beta\right), \check{q}\left(\beta\right)\right)$ in the bargaining weight β and $\left(\check{p}\left(1\right), \check{q}\left(1\right)\right)$ corresponds to the classic monopoly outcome, it must hold that $\check{p}\left(\beta\right) \geq p^*$ for all $\beta \in [\beta^*, 1]$. Moreover, $\left(\check{p}\left(\beta\right), \check{q}\left(\beta\right)\right)$ lies on the price curve for all $\beta \in [\beta^*, 1]$.

2. $p = MC\left(q\right) < P\left(q\right)$ and $\mu = 0$. The remaining unknowns are $q \geq 0$ and $\lambda \geq 0$. The second term of the second line is equal to 0. This line pins down λ, which can be substituted into the second line to obtain

$$P\left(q\right) - MC\left(q\right) - \left(1 - \frac{\beta}{1-\beta}\frac{CS\left(MC\left(q\right),q\right)-CS_0}{PS\left(MC\left(q\right),q\right)-PS_0}\right)MC'\left(q\right)q = 0. \tag{11}$$

For $\beta = 0$, this equality reduces to the FOC for the classic monopsony: $MC\left(q\right) + MC'\left(q\right)q = P\left(q\right)$ with monopsony quantity $q_m \leq q^*$ and monopsony price $p_m = MC\left(q_m\right) \leq p^*$, etc.[13] The associated surplusses are feasible as these satisfy the imposed conditions $CS_0 < CS_M$ and $PS_0 < PS_m$. For $\beta > 0$, the product of fractions is positive and we obtain $P\left(q\right) > MC\left(q\right) + MC'\left(q\right)q$ as a necessary condition for optimality. Next, $\lambda \geq 0$ in the second line of the KKT conditions implies

$$\frac{\beta}{1-\beta}\frac{CS\left(MC\left(q\right),q\right)-CS_0}{PS\left(MC\left(q\right),q\right)-PS_0} \leq 1.$$

[13] Equality only holds in case of constant margainal costs, which we allow.

This implies that the coefficient of $P'(q)\,q$ is nonnegative and that (11) reduces to the weak version of this case's assumption $MC(q) \leq P(q)$, i.e. $q \leq q^*$ and consequently $p \in [p_m, p^*]$. For $\beta \geq 0$, every such pair (p, q) satisfies the imposed conditions $CS_0 < CS_M$ and $PS_0 < PS_m$.

Next, for reasons similar to the previous case, application of the axiom of irrelevant alternatives implies $(\check{p}(\beta^*), \check{q}(\beta^*)) = (q^*, p^*)$. By continuity of $(\check{p}(\beta), \check{q}(\beta))$ in the bargaining weight β and $(\check{p}(0), \check{q}(0))$ corresponds to the classic monopsony outcome, it must hold that $\check{p}(\beta) \leq p^*$ for all $\beta \in [0, \beta^*]$. Furthermore, all $(\check{p}(\beta), \check{q}(\beta))$ lie on the marginal cost curve for all $\beta \in [0, \beta^*]$.

3. $p = P(q) = MC(q)$. The only feasible solution is $(\check{p}(\beta), \check{q}(\beta)) = (q^*, p^*)$. As a consequence of the previous two cases, this can only occur in case of $\beta = \beta^*$.

This completes the characterization of the solution to program (9). We conclude the proof by showing that the solution to program (9) is a solution to program (5).

Claim B:

For $\beta \leq \beta^*$: $(\tilde{p}(\beta), S)$ is a solution of (5) if and only if $(\check{p}(\beta), \check{q}(\beta)) = (\tilde{p}(\beta), MC^{-1}(\tilde{p}(\beta)))$ is a solution of (9).

For $\beta \geq \beta^*$: $(\tilde{p}(\beta), B)$ is a solution of (5) if and only if $(\check{p}(\beta), \check{q}(\beta)) = (\tilde{p}(\beta), P^{-1}(\tilde{p}(\beta)))$ is a solution of (9).

Proof of the Claim B

$\Longrightarrow$ Consider $\beta \leq \beta^*$ first. Suppose $(\tilde{p}(\beta), S)$ is the solution of (5), then $\tilde{q}(\beta)$ solves $MC(q) = \tilde{p}(\beta)$, or $\tilde{q}(\beta) = MC^{-1}(\tilde{p}(\beta))$. Consider the modified maximization program

$$\max_{p,q \geq 0} (1-\beta)\ln(CS(p,q) - CS_0) + \beta \ln(PS(p,q) - PS_0), \quad \text{s.t. } MC(q) = p. \tag{12}$$

Then, $(\tilde{p}(\beta), MC^{-1}(\tilde{p}(\beta)))$ is the solution to (12) also. For $\beta < \beta^*$: the lower bound on price in (9) is binding and the upper bound is non-binding. At $\beta = \beta^*$, both are binding. Hence, for any $\beta \leq \beta^*$ it must hold that $(\tilde{p}(\beta), MC^{-1}(\tilde{p}(\beta)))$ is the solution to program (9). Similarly for

$\beta \geq \beta^*$, $(\tilde{p}(\beta), B)$ induces $(\tilde{p}(\beta), P^{-1}(\tilde{p}(\beta)))$ as the solution to both (5) and

$$\max_{p,q \geq 0} (1-\beta) \ln (CS(p,q) - CS_0) + \beta \ln (PS(p,q) - PS_0), \quad \text{s.t. } P(q) = p, \tag{13}$$

whenever $\beta \geq \beta^*$.

$\Longleftarrow$ Consider $\beta < \beta^*$ first. For solution $(\check{p}(\beta), \check{q}(\beta))$ to program (9) it holds that $p \leq P(\check{q}(\beta))$ is not binding and $\check{q}(\beta) < q^*$ solves $MC(q) = \check{p}(\beta)$, where $\check{p}(\beta) < p^*$. Then, $(\check{p}(\beta), \check{q}(\beta)) = (\check{p}(\beta), MC^{-1}(\check{p}(\beta)))$ is the solution to (12) and $(\check{p}(\beta), S)$ to (5). This argument extends to $\beta = \beta^*$. Similarly, let $\beta > \beta^*$. Then, for solution $(\check{p}(\beta), \check{q}(\beta))$ to program (9) it holds that $p \geq P(\check{q}(\beta))$ is not binding and $\check{q}(\beta) < q^*$ solves $P(q) = \check{p}(\beta)$, where $\check{p}(\beta) > p^*$. Then, $(\check{p}(\beta), \check{q}(\beta)) = (\check{p}(\beta), P^{-1}(\check{p}(\beta)))$ is the solution to (12) and $(\check{p}(\beta), S)$ to (5). This completes the proof of Claim B.

This completes the proof of Proposition 3. ■

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A Supplementary Material: demand and supply functions

The analysis stated in $P(q)$ and $MC(q)$ is less demanding in terms of mathematics than the equivalent analysis in terms of $d(p)$ and $s(p)$ to which we turn our attention in this section. The main reason is that the expressions for consumer and producer surplus of price-quantity contracts are easier to state in terms of the price and marginal cost curve than the demand and supply function. Also, producer surplus expressed in terms of the marginal cost function can accomodate constant marginal costs, while inverting this function to obtain the supply function requires monotonic increasingness. Therefore, we strengthen Assumption 1 as follows.

Assumption 5 *Marginal cost function $MC(q)$ is continuous and increasing in q with intercept $MC(0) \geq 0$; the fixed costs are zero; and price function P is continuous and decreasing in q with finite intercept $P(0) > MC(0)$ and $\lim_{q\to\infty} P(q) = 0$.*

For the price-quantity contract (p, q), we rewite the definition of the consumer surplus as follows

$$\begin{aligned} CS(p,q) &= \int_0^q P(\hat{q})\, d\hat{q} - pq = \int_0^{d^{-1}(0)} \min\{d(\hat{p}), q\}\, d\hat{p} - pq \\ &= \int_{d^{-1}(q)}^{d^{-1}(0)} d(\hat{p})\, d\hat{p} + \left[d^{-1}(q) - p\right] q, \end{aligned}$$

where the integral is the consumer surplus at fictitious price $d^{-1}(q)$ and the second term can be either positive in case the contract price p lies below this fictitious price or negative if the contract price lies above. The partial derivatives are given by

$$\begin{aligned} &\tfrac{\partial}{\partial p} CS(p,q) = -q, \\ &\tfrac{\partial}{\partial q} CS(p,q) = -d\left(d^{-1}(q)\right) \tfrac{d}{dq} d^{-1}(q) + q \tfrac{d}{dq} d^{-1}(q) + d^{-1}(q) - p = d^{-1}(q) - p. \end{aligned}$$

Similar, for the producer surplus it follows

$$\begin{aligned} PS(p,q) &= pq - \int_0^q MC(\hat{q})\, d\hat{q} = \int_0^q [p - MC(\hat{q})]\, d\hat{q} = \int_0^p \min\{s(\hat{p}), q\}\, d\hat{p} \\ &= \int_0^{s^{-1}(q)} s(\hat{p})\, d\hat{p} + \left[p - s^{-1}(q)\right] q, \end{aligned}$$

where the integral is the producer surplus at fictitious price $s^{-1}(q)$ and the second term can be either positive in case the contract price p lies above this fictitious price or negative if the contract price lies below. The partial derivatives are given by

$$\frac{\partial}{\partial p} PS(p,q) = q,$$
$$\frac{\partial}{\partial q} PS(p,q) = s\left(s^{-1}(q)\right)\frac{d}{dq}s^{-1}(q) - q\frac{d}{dq}s^{-1}(q) + p - s^{-1}(q) = p - s^{-1}(q).$$

For completeness, social welfare for $q \leq q^*$ is given by

$$SW(p,q) = \int_{d^{-1}(q)}^{d^{-1}(0)} d(\hat{p})\, d\hat{p} + \int_{0}^{s^{-1}(q)} s(\hat{p})\, d\hat{p} + \left[d^{-1}(q) - s^{-1}(q)\right] q.$$

As in the main text, we define the surpluses for contract (p,i) and right-to-manage quantity q. These surpluses are given by the expressions stated above:

$$CS(p,i) = \int_{d^{-1}(q)}^{d^{-1}(0)} d(\hat{p})\, d\hat{p} + \left[d^{-1}(q) - p\right] q, \qquad PS(p,i) = \int_{0}^{s^{-1}(q)} s(\hat{p})\, d\hat{p} + \left[p - s^{-1}(q)\right] q$$

Formally, within the context of right-to-manage price contracts, we express the (ANBS) in logarithmic form and define the ANBS contract as follows:

$$\left(\tilde{p}(\beta), \tilde{\imath}(\beta)\right) \in \arg\max_{p \geq 0, i \in \{B,S\}} (1-\beta)\ln\left(CS(p,i) - CS_0\right) + \beta \ln\left(PS(p,i) - PS_0\right), \tag{14}$$
$$\text{s.t. } q = s(p) \text{ if } i = S; \quad \text{and } p = d(q) \text{ if } i = B.$$

Our subsequent finding diverges significantly from our prior two results, with the exception of the threshold as outlined in Proposition 1. In this upcoming result, the quantity that satisfies the following equation will assume a pivotal role:

$$p + \left(1 - \frac{\beta}{1-\beta}\frac{CS(p,s(p)) - CS_0}{PS(p,s(p)) - PS_0}\right)\frac{s(p)}{s'(p)} = d^{-1}(s(p)) \tag{15}$$

and similar for the quantity that satisfies

$$p + \left(1 - \frac{1-\beta}{\beta}\frac{PS(p,d(p)) - PS_0}{CS(p,d(p)) - CS_0}\right)\frac{d(p)}{d'(p)} = s^{-1}(d(p)). \tag{16}$$

Proposition 6 *Let $\beta \in [0,1]$ and $CS_0 + PS_0 < SW^*$. The negotiated right-to-manage price contract $(\tilde{p}(\beta), \tilde{\imath}(\beta))$ and corresponding right-to-manage quantity $\tilde{q}(\beta)$ are given by*

$$\begin{cases} \tilde{p}(\beta) \text{ solves (15)}, & \tilde{\imath}(\beta) = B, & \tilde{q}(\beta) = s(\tilde{p}(\beta)), & \text{if } \beta \in [0,\beta^*), \\ \tilde{p}(\beta) = p^*, & \tilde{\imath}(\beta) \in \{B,S\}, & \tilde{q}(\beta) = q^*, & \text{if } \beta = \beta^*, \\ \tilde{p}(\beta) \text{ solves (16)}, & \tilde{\imath}(\beta) = S, & \tilde{q}(\beta) = d(\tilde{p}(\beta)), & \text{if } \beta \in (\beta^*, 1]. \end{cases}$$

The contract features buyer market power for $\beta < \beta^*$ and seller market power for $\beta > \beta^*$. Moreover, consumer surplus $CS\left(\tilde{p}\left(\beta\right), \tilde{\imath}\left(\beta\right)\right) = CS\left(\tilde{p}\left(\beta\right), \tilde{q}\left(\beta\right)\right)$ and producer surplus $PS\left(\tilde{p}\left(\beta\right), \tilde{\imath}\left(\beta\right)\right) = PS\left(\tilde{p}\left(\beta\right), \tilde{q}\left(\beta\right)\right)$.

Proof of Proposition 6

Similar as in the proof of Proposition 3, we first get rid of the binary variable $i \in \{B, S\}$ by rewriting (5) as follows

$$\left(\check{p}\left(\beta\right), \check{q}\left(\beta\right)\right) \in \arg\max_{p,q\geq 0} \left(1-\beta\right) \ln\left(CS\left(p,q\right) - CS_0\right) + \beta \ln\left(PS\left(p,q\right) - PS_0\right), \tag{17}$$

$$\text{s.t. } q \leq s\left(p\right) \quad \left(\lambda\right), \qquad \text{and } d\left(p\right) \leq q \quad \left(\mu\right),$$

where λ and μ denote shadow prices. We will not repeat Claim A and B and their proofs as given in Proposition 3, We investigate the KKT conditions only.

The Lagrangian function of (17) is given by

$$\left(1-\beta\right) \ln\left(CS\left(p,q\right) - CS_0\right) + \beta \ln\left(PS\left(p,q\right) - PS_0\right) - \lambda\left(q - s\left(p\right)\right) - \mu\left(d\left(p\right) - q\right)$$

and the Karush-Kuhn-Tucker (KKT) conditions are given by

$$\begin{aligned}
\frac{-\left(1-\beta\right) q}{CS\left(p,q\right) - CS_0} + \frac{\beta q}{PS\left(p,q\right) - PS_0} + \lambda s'\left(p\right) - \mu d'\left(p\right) &= 0,\\
\frac{\left(1-\beta\right)\left(d^{-1}\left(q\right) - p\right)}{CS\left(p,q\right) - CS_0} + \frac{\beta\left(p - s^{-1}\left(q\right)\right)}{PS\left(p,q\right) - PS_0} - \lambda + \mu &= 0,\\
\lambda\left(q - s\left(p\right)\right) &= 0,\\
\mu\left(d\left(p\right) - q\right) &= 0.
\end{aligned}$$

There are three cases to investigate.

1. $q = d\left(p\right) > s\left(p\right)$ and $\lambda = 0$. The remaining unknowns are $p \geq 0$ and $\mu \geq 0$. The first term of the second line is equal to 0. This line pins down μ, which can be substituted into the first line to obtain

$$\frac{-\left(1-\beta\right) q}{CS\left(p,q\right) - CS_0} + \frac{\beta\left(q + \left[p - s^{-1}\left(d\left(p\right)\right)\right] d'\left(p\right)\right)}{PS\left(p,q\right) - PS_0} = 0.$$

$$\frac{-\left(1-\beta\right) q}{CS\left(p,q\right) - CS_0} + \frac{\beta}{PS\left(p,q\right) - PS_0}\left(q + p - s^{-1}\left(q\right)\right) d'\left(p\right) = 0,$$

After substituting q out and some rewriting, we obtain

$$p+\left(1-\frac{1-\beta}{\beta}\frac{PS\left(p,d\left(p\right)\right)-PS_{0}}{CS\left(p,d\left(p\right)\right)-CS_{0}}\right)\frac{d\left(p\right)}{d'\left(p\right)}=s^{-1}\left(d\left(p\right)\right), \tag{18}$$

which is equivalent to (6) after substitution of $p=P\left(q\right)$, $s^{-1}\left(d\left(p\right)\right)=MC\left(q\right)$ and $1/d'\left(p\right)=P'\left(q\right)$. The remainder of this case follows Case 1 of the proof of Proposition 3.

2. $q=s\left(p\right)<d\left(p\right)$ and $\mu=0$. The remaining unknowns are $q\geq 0$ and $\lambda\geq 0$. The second term of the second line is equal to 0. This line pins down λ, which can be substituted into the first line to obtain

$$\frac{\beta q}{PS\left(p,q\right)-PS_{0}}+\frac{\left(1-\beta\right)}{CS\left(p,q\right)-CS_{0}}\left(-q+\left(d^{-1}\left(q\right)-p\right)s'\left(p\right)\right)=0.$$

After substituting q out and some rewriting, we obtain

$$p+\left(1-\frac{\beta}{1-\beta}\frac{CS\left(p,s\left(p\right)\right)-CS_{0}}{PS\left(p,s\left(p\right)\right)-PS_{0}}\right)\frac{s\left(p\right)}{s'\left(p\right)}=d^{-1}\left(s\left(p\right)\right), \tag{19}$$

which is equivalent to (7) after substitution of $p=P\left(q\right)$, $s^{-1}\left(d\left(p\right)\right)=MC\left(q\right)$ and $1/s'\left(p\right)=MC'\left(q\right)$. The remainder of this case follows Case 2 of the proof of Proposition 3.

3. $q=d\left(p\right)=s\left(p\right)$. The only feasible solution is $\left(\check{p}\left(\beta\right),\check{q}\left(\beta\right)\right)=\left(q^{*},p^{*}\right)$. As a consequence of the previous two cases, this can only occur in case of $\beta=\beta^{*}$.

This completes the proof of Proposition 6. ■

Bonnet et al. (2024)

For contract $\left(p,i\right)$ and quantity equal to either $d\left(p\right)$ or $s\left(p\right)$, the consumer surplus is given by

$$CS\left(p,i\right)=\begin{cases}\int_{p}^{P(0)}d\left(\hat{p}\right)d\hat{p}, & \text{if } i=B,\\ \int_{P(s(p))}^{P(0)}d\left(\hat{p}\right)d\hat{p}+\left[P\left(s\left(p\right)\right)-p\right]s\left(p\right), & \text{if } i=S,\end{cases}$$

with derivative with respect to price given by

$$CS'(p,i)=\begin{cases}-d(p), & \text{if } i=B,\\ -d(P(s(p)))\,P'(s(p))\,s'(p)+ & \text{if } i=S.\\ \qquad [P'(s(p))\,s'(p)-1]\,s(p)+[P(s(p))-p]\,s'(p), & \end{cases}$$

Similar for the producer surplus

$$PS(p,i)=\begin{cases}\int_0^{MC(d(p))} s(\hat{p})\,d\hat{p}+[p-MC(d(p))]\,d(p), & \text{if } i=B,\\ \int_0^{p} s(\hat{p})\,d\hat{p}, & \text{if } i=S,\end{cases}$$

with derivative with respect to price given by

$$PS'(p,i)=\begin{cases}s(MC(d(p)))\,MC'(d(p))\,d'(p) & \\ \qquad [1-MC'(d(p))\,d'(p)]\,d(p)+[p-MC(s(p))]\,d'(p), & \text{if } i=B,\\ s(p), & \text{if } i=S.\end{cases}$$

Formally, within the context of right-to-manage price contracts, we express the (ANBS) in logarithmic form and define the ANBS contract as follows:

$$(\tilde{p}(\beta),\tilde{\imath}(\beta))\in\arg\max_{p\geq 0,i\in\{B,S\}}(1-\beta)\ln(CS(p,i)-CS_0)+\beta\ln(PS(p,i)-PS_0).$$

Skipping the derivation some of the steps in the proof of Proposition 3, we directly investigate the first-order condition of this program while fixing i. Given either $i=B,S$, we obtain as the first-order condition

$$(1-\beta)\frac{CS'(p,i)}{CS(p,i)-CS_0}+\beta\frac{PS'(p,i)}{PS(p,i)-PS_0}=0.$$

B Derivation Example Indivisibilities

The competitive equilibrium is reached at equilibrium quantity $q^*=2$, supported by the price range $\left[r^S(q^*),r^B(q^*)\right]=[2,3]$ and generates social welfare $SW(p^*,q^*)=4$.

Only $q=1$ or 2 matter in the analysis. The consumer surplus $CS(p,q)$ is given by

$$CS(p,1)=\left\{\;4-p,\quad\text{if } p\in[1,4],\right.,\qquad CS(p,2)=\left\{\;7-2p,\quad\text{if } p\in[1,4],\right.$$

the producer surplus $PS(p,q)$ is given by

$$PS(p,1)=\left\{\;p-1,\quad\text{if } p\in[1,4],\right.,\qquad PS(p,2)=\left\{\;2p-3,\quad\text{if } p\in[1,4].\right.$$

The Pareto efficient ANBS price-quantity contract (p, q) will be computed assuming competitive equilibrium quantity $q^* = 2$ and $CS_0 = PS_0 = 0$. The asymmetric Nash product is given by

$$(1-\beta)\ln CS(p,2) + \beta \ln PS(p,2) = (1-\beta)\ln(7-2p) + \beta\ln(2p-3).$$

Maximization of this Nash product yields the ANBS price, ANBS consumer surplus and ANBS producer surplus given by

$$\hat{p}(\beta) = \tfrac{3}{2} + 2\beta, \qquad CS(\hat{p}(\beta), 2) = 4(1-\beta), \qquad PS(\hat{p}(\beta), 2) = 4\beta.$$

The negotiated price is a competitive equilibrium price if $\hat{p}(\beta) \in [2,3] \iff \beta \in \left[\frac{1}{4}, \frac{3}{4}\right]$. Hence, $\left[\underline{\beta}^*, \overline{\beta}^*\right] = \left[\frac{1}{4}, \frac{3}{4}\right]$. There is seller market power for $\beta > \frac{1}{4}$ and buyer market power for $\beta < \frac{1}{4}$.