Warm up by Revelation to Cool down in Competition: Strategic Provision of Relationship-sensitive Information*

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Abstract

In markets where variety-seeking consumers hold horizontally heterogeneous preferences over competing brands, product differentiation enhances brand loyalty while product complementarity stimulates multi-brand purchase. In a world of complete information, when the latter force exceeds the former, firms switch their pricing strategies from being "responsive" as Bertrand competitors, to being "independent" as local monopolies. However, when the information about product complementarity becomes asymmetric, the uninformed firm is unable to price conditional on the above two types of relationships. We characterize the pricing equilibrium and information sharing incentives between rival firms when such relationship-sensitive information is asymmetric. At the pricing stage, the informed seller can choose to charge a monopolistic price independently, or respond to the rival's price — the latter option is more attractive if a higher price can be induced. We show that, the informed seller is willing to share (resp., conceal) the information if products turn to be substitutes (resp., complements) such that competition (resp., independence) is going to take place. Moreover, the informed seller keeps silent unconditionally if the degree of complementarity is possibly high enough. Consequently, it is socially efficient to make the product complementarity information public. Our study provides new insights on data sharing strategies between platform retail verticals and third-party sellers when they supply complementary services.

Keywords: Horizontal differentiation, Multi-purchase, Asymmetric information, Information revelation, Brand complementarity

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1 Introduction

In the digital era, users' data provide valuable information for market players. In platform markets, as a marketplace, the digital platform is able to collect fine-grained information about users from both sides. Compared with individual sellers, the platform has a better understanding about the whole picture of the entire market, especially the interactions among different players. Data services like Amazon Brand Analytics or Google Play Console, indeed provide patronized sellers/developers with a plenty of raw data about their own business, and *some* "insights" about the market trends. However, providing firm-specific information is not sufficient to satisfy the needs of the third-party sellers who demand more knowledge about how their competitors are performing, the overall listings, and interactions among market players. The latter type of information, typically referred to as the inferred data processed at the platform-aggregated level, are exclusively possessed by the platforms, who claim that sharing data containing relevant information about one's competitor might be involved with legal barriers. Nevertheless, platforms' reluctance in "promoting data transparency" leads to public and regulatory concerns that the informed platforms might leverage their dominant positions by conducting self-preferencing such that the third-party sellers/developers cannot viably compete with the marketplace controller's retail verticals, resulting in anti-competitive outcomes.[1]

For regulatory issues, the concept of market power and the boundary of competition exhibit varying features in platform markets, especially with the presence of multi-homing and complementary services, which deserves new measures and relevant regulations (Montjoye, Schweitzer and Crémer, 2019). Currently, different from the attitude towards the customer-specific profiles, regulation policies, including GDPR and TFEU Article 102, do not impose general restrictions on sharing aggregated data, a subset of which describes precisely the interactions among potential competitors. To this point, how to evaluate the pro-/anti-competition effects of sharing relationship-sensitive information, is left to be unanswered, calling for a cautious examination.

Therefore, studying the market-based incentives and welfare consequences of sharing relationship-sensitive information (which describes the degree of interdependence among market players) is necessary. To address the issue, we need to model firms' interdependence in a broadened sense. For that reason, we consider an extended version of the horizontally differentiated market, where duopolists supply differentiated services that could be either substitutes or complements: differentiation enhances consumers' loyalty while complementarity may gen-

[1] The investigation by Gineikytė, Barcevičius and Cibaitė (2021) shows that Amazon has a clear understanding of consumers' purchase behavior and firms' interactions. However, data services including Brand Analytic of Amazon and Google Play Console, provide "no" data about one's competitor, whereby only "some" insights about user behavior and market trends. The survey shows that 52% of the sellers patronized in digital platforms cannot access the information demanded, especially the information about the entire market competition. An interviewed representative of a developer claims that the data provided by Google Play is helpful to understand how the developer's own apps are performing, but it does not necessarily show how other business are doing. Some interviewees argued that Brand Analytics is "simply not enough considering what access to data Amazon Retail has."

erate variety-seeking.[2] In a traditional Hotelling model without variety-seeking (Hotelling, 1929), duopolists compete head-on for the brand-switching consumer, engaging into a price competition. In contrast, when the effect of complementarity dominates that of differentiation, as suggested by some previous studies (Anderson, Foros and Kind, 2017; Jeitschko, Jung and Kim, 2017; Kim and Serfes, 2006), duopolists are freed from competition by those who choose to buy from both sellers (multi-purchase), which allow them to price independently as local monopolies.[3] In this way, the degree of interdependence can be captured not only by an intensive measure (price competition), but also along the extensive, relationship-sensitive margin: with or without competition (competition vs. independence). Correspondingly, as the relative forces between differentiation and complementarity vary, it requires duopolists adopting relationship-specific pricing patterns conditional on their interdependence (see Table 1).

However, the information about duopolists interdependence could be asymmetric between the two firms. To see this, note that by definition, as a measure of product correlations, brand complementarity is bilaterally determined by a pair of coexistent brands. Therefore, complementarity cannot be unilaterally defined by an individual firm alone, giving rise to the incomplete information problem. In practice, brand complementarity can be reflected by consumers' marginal utility from using a **different** product supplied by the **rival** firm, relative to buying one brand only. This implies that, as a marketplace, the platform has superior knowledge about consumers' shopping behavior, and therefore is better informed about product correlations compared to third-party sellers.[4] When the competition takes place between the informed platform (represented by its retail subsidiary) and an uninformed third-party seller, it leads to the asymmetric information problem (which is sometimes accused of "self-preferencing").[5]

In this paper, we characterize the equilibrium pricing and information revelation strategies in markets where the information about complementarity is asymmetric. Our study answers the following questions: (1) When one seller cannot access the relationship-sensitive informa-

[2]Examples include (but not limited to) Surface supplied by Microsoft vs. iPad sold by Apple, fashion dresses, apps exhibited in platforms, video games displayed in Steam, etc.

[3]Consumers located around the midpoint of a Hotelling line have a weak preference for either brand. When they start to buy from both sellers, all the other single-purchase consumers are isolated as two disjoint groups. Such a scenario is referred to as a relationship of "independence" between the two sellers. In that case, when a seller cuts its own price, it merely attracts more variety-seeking consumers who are going to buy an additional, different product without giving up using the rival's product. Hence such unilateral price adjustment is orthogonal to the demand of the other firm.

[4]A rich set of evidence (Gineikytė, Barcevičius and Cibaitė, 2021; Li, Tian and Zheng, 2021; Montjoye, Schweitzer and Crémer, 2019; Tsunoda and Zennyo, 2021) shows the superior capabilities of processing the information about product complementarities by digital platforms, e.g., inferred through customer purchase behavior, click-though rates of listings, brows histories, subscriptions and follow-ups, search keywords, etc.

[5]For instance, the dual role as a retailer and a marketplace of Amazon who is suspected to use third-party seller data to enhance its market power is under anti-trust investigation by the European Commission. Amazon claimed that, in contrast, it uses "aggregated data" from third-party sellers to improve their overall business. See Hu, K. (2019), "Revealed: how Amazon uses seller data to build a private label juggernaut," Yahoo! Finance. Available at https://finance.yahoo.com/news/amazon-uses-thirdparty-sellers-data-to-build-private-labels-145813238.html, and Dastin. J., and Bose, N. (2019), "Amazon uses aggregated seller data to help business, it tells lawmakers," Reuters. Available at https://www.reuters.com/article/us-usa-antitrust-amazon-com/amazon-uses-aggregated-seller-data-to-help-business-it-tells-lawmakers-idUSKBN1XT2N9.

tion about the interdependence between duopolists while the other seller can, what are the sellers' optimal pricing strategies? (2) Will the informed seller (e.g., the subsidiary of the vertically integrated platform) be willing to share the information with the uninformed competitor (e.g., third-party seller) and on what grounds? (3) What is the socially optimal provision of the relationship-sensitive data?

To address to the first question ("asymmetric information"), we assume that the true value of the relationship-sensitive variable is privately known by one seller while a prior distribution is commonly known between the duopolists.[6] For the uninformed seller, price cannot be charged conditional on a particular type of relationship, but instead a flat price will be charged based on expectation/prior belief. The informed seller, in contrast, by making use of the observed level of complementarity, is able to respond to its rival under price competition, or set a monopolistic price independently.

For the second question ("strategic revelation"), we argue that there exist some more sophisticated concerns in strategic and selective provision of relationship-sensitive information. Since all players know that the platform has information, the action of hiding information itself, could be informative. To see this, consider a three-stage game: At stage 1, the informed seller can choose to conceal or reveal the information truthfully. Upon concealment being observed, the uninformed rival updates its belief by inferring that such action is intended to be made in the interests of the informed seller. Then at stage 2, both sellers offer prices simultaneously playing best responses to the updated beliefs, and consumers make purchase decisions at stage 3.

In contrast to the general insights that "the informed platform may avoid data openness for the purpose of preventing patronized sellers from becoming competitors," we show that: (i) When the maximum level of complementarity is not high enough, the informed seller is eager to share the information and remind its rival to engage into an upcoming price competition, while concealing the information under independence. (ii) When the upper bound of complementarity exceeds a threshold, the informed seller will never reveal anything.

The primary purpose of selective revelation, is to induce the rival to charge a high price at competition, while the rival's actions are irrelevant when they stay independently. To better understand this, consider how firms will price conditional on the following three states with full information: (1) When differentiation dominates complementarity such that competition occurs, the payoff of a seller is increasing in the rival's price (i.e., *competition effect*). (2) With a moderate size of complementarity that begins to dominate differentiation, the emergence of multi-purchase allows sellers to offer lower prices independently in exchange for promoting sales, without affecting the rival's payoff (i.e., *demand-expansion effect*);[7] As complemen-

[6] According to Gineikytė, Barcevičius and Cibaitė (2021), since Amazon shares some consumers' information with Amazon.com, Inc. and subsidiaries, the third-party sellers are not on the level playing field with Amazon private label brand, while the latter has "all the data in the world to know what products to create and exactly what keywords to target."

[7] At the threshold which makes the first and second type of equilibrium equally profitable, a lower price-sum is needed to make the brand-switching consumer (who pays one price) replaced by those who are willing to purchase

tarity increases, the monopolistic prices are increasing in consumers' willingness to pay for buying a different product. And when (3) complementarity becomes high enough such that all consumers are captive (to both firms), it is optimal to charge an even higher price (than the level under competition) extracting consumers' willingness to pay (i.e., *price-raising effect*). However, without information, a flat price will be offered, by "averaging" the above three effects. Then, if the degree of complementarity is not believed to be possibly high enough, the demand-expansion effect is relatively magnified, giving rise to a lower price. In that case, if the true degree of complementarity turns to be low such that competition is going to take place, by reminding the rival about this, it leads to a softened competition that makes both parties better-off.[8]

Since higher prices will be induced under asymmetric information environment, and therefore, for the third question, we find that it is socially optimal to make the relationship-sensitive information public. Again, the relationship-sensitive margin combining with the demand-expansion effect plays an important role. With full information, when complementarity exceeds the "competition-independence" threshold, duopolists have their prices cut symmetrically in order to serve more customers. However, the asymmetric information environment makes the threshold unobservable, preventing a price cut that would allow more consumers to enjoy product variety.

Our findings suggest new strategies for the optimal "data-as-a-service" established by platforms, and provide distinct explanations about why it is not wise to make data entirely transparent.[9] We make theoretical contributions in the sense that the information revelation setting here is verifiable without pre-commitment, which is suitable for analyzing the provision of inferred data or data insights containing information about correlations among competitors.[10] In particular, since the information about firms' interdependence can be processed anonymously in the platform-aggregated form, such data can be shared subject to much less legal or privacy concern.[11] We also provide policy implications for data regulation in markets with complementary goods. The above implications also allow us to have a better understanding about multi-homing, co-opetition (the conflicts between platforms and its business users), and relevant legal issues.

The rest of the paper is structured as follows. Section 2 reviews related works. The baseline model is built in Section 3, where full access of information is assumed (labeled as benchmark

from both sellers (pay twice).

[8] In a similar sense, Guo and Wu (2018) discuss the intensity of price competition when competing sellers can share symmetric/asymmetric capacities. Chen, Narasimhan and Zhang (2001) and Kim and Choi (2010) address the incentives of sharing information for avoiding an intensified competition, but within the scope of price competition (in each period).

[9] As claimed by Google Play: "To provide the developers with the optimal amount of data."

[10] Tsunoda and Zennyo (2021) and Li, Tian and Zheng (2021) consider platform data sharing in a pre-commitment setting, which might work perfectly for the situations where there exist some fixed agreements (e.g., through APIs) of accessing data about one's own business. Zhu and Liu (2018) empirically study Amazon's strategies in competing with complementors.

[11] As a separate issue, another type of data containing customer-specific profiles, are involved with privacy regulations. See Acquisti, Taylor and Wagman (2016) and Hoffmann, Inderst and Ottaviani (2020), for instance.

Brand complementarity (relative to differentiation)	Low	Medium	High
Firms' relationship	Competition	Independence	
Price responses	Yes	No	
# multi-purchase	None	Some but not all	All
Price level	Medium	Low	High
Pricing incentives	Differentiation	Demand-expansion	Price-raising
Uninformed seller (A)	Flat price		
Informed seller (A)	Respond to flat price	Demand-expansion	Price-raising
Uninformed seller (R)	Contingent upon information received		
Informed seller (R)	Induce a high price	Conceal*	

Table 1: Relationship-specific pricing under complete/incomplete information

* Appendix A.2 provides alternative refinements to pin down the revelation behavior and the corresponding beliefs when the informed firm is "indifferent" between revealing and concealing under independence (where the rival's actions are irrelevant).

F). In Section 4, asymmetric information is introduced (benchmark A), where one firm is informed but the opponent is not (self-preferencing), and revealing information is not available. The incentive of strategic revelation of the informed seller is studied in Section 5, which is labeled as benchmark R. The equilibrium welfare generated by the above three information structures (F, A and R) are compared in Section 6.[12] Section 7 concludes.

2 Related Literature

We characterize the equilibrium pricing and information sharing strategies between duopolists when the information about product complementarity is asymmetric. Without such information, firms' relationship (competition or independence) is interim-unobservable. Hence, our paper enriches the following branches of literature.

First, the demand information emphasized here, is the preference for brand variety (Kim, Allenby and Rossi, 2002; Sajeesh and Raju, 2010; Thomassen et al., 2017), with which, consumers may make multi-purchase from competing sellers. However, to our knowledge, previous studies involving multi-purchase in a simultaneous-move game do not consider the issue of incomplete information about consumers' willingness to pay for making multi-purchase, and we provide a first attempt to pursue this direction. Gabszewicz and Wauthy (2003) and Zeithammer and Thomadsen (2013) investigate multi-purchase and variety seeking in vertically differentiated markets. Kim and Serfes (2006), Jeitschko, Jung and Kim (2017), Anderson, Foros and Kind (2017) and Dou and Ye (2018) introduce multi-brand purchase in horizontally differentiated markets (Hotelling, 1929). Ambrus, Calvano and Reisinger (2016) consider a more general demand function in studying multi-home and single-home in platform competition. Using the Hotelling framework, Klemperer (1987), Seetharaman and Che (2009) and

[12] We also fully characterized the equilibrium for the scenario where neither firm has information. The result was presented in an earlier version of the paper (available upon request), and we choose not to include it in the current version as that scenario is irrelevant to our analysis.

Sajeesh and Raju (2010) model consumers' variety seeking but in a sequential setting, and each customer is assumed to make single-purchase per period. Kim, Allenby and Rossi (2002) and Thomassen et al. (2017) empirically examine the demand for variety and product complementarities.

In a similar sense, indeed there are some literature studying the effect of information on complementarities and competition. For example, Ke and Lin (2020) study the complementary effect on competing products (where the uncertainty comes from the buyers' side: search). Liu and Serfes (2006) and Kim and Choi (2010) analyze information sharing among rival firms in a sequential game. In particular, Kim and Choi (2010) consider firms' incentives of sharing consumers' information about product complementarity but each consumer can only buy from one seller per-period, hence "multi-purchase" in their work is made sequentially where previous purchase reveals some information. Chen, Narasimhan and Zhang (2001) consider the effect of information sharing on switchers' choices without the option for multi-purchase. In contrast to the above studies, we focus on multi-purchase that is made in one purchase occasion such that the information about complementarity is interim-unobservable.

The information transmission mechanism in our paper is assumed to be verifiable without ex-ante commitment. The rules defining the way of how information is shared can be generally categorized by the following dimensions. First, the demand information that is going to be shared for multi-purchase in our work, is assumed to be verifiable (Bertomeu and Cianciaruso, 2018; Gal-Or, 1985; Okuno-Fujiwara, Postlewaite and Suzumura, 1990), rather than unverifiable signals (Crawford and Sobel, 1982). Second, information revelation in our paper should be regarded as interim strategy (Ford et al., 2020), which differ from the ex-ante committed strategy (Raith, 1996). The latter case can be further divided into type-independent and type-dependent commitment, e.g., see Shapiro (1986) for the Cournot game, Kamenica and Gentzkow (2011) for a sender-receiver game and Wu and Zheng (2017) for contests.

In order to address the role of relationship-sensitive information, it is useful to clarify different types of information in oligopolistic competition. The way of how asymmetric information affects the intensity of competition differs significantly between the common demand-side uncertainty and firm-specific private information, and our work belongs to the former category. (1) For the uncertainty about a common demand factor, it could be the demand intercept in a Cournot model (Gal-Or, 1985; Goyal and Netessine, 2007; Jansen, 2008; Novshek and Sonnenschein, 1982; Raith, 1996; Vives, 1984); or the heterogeneities/locations in a Hotelling model (Balvers and Szerb, 1996; Bounie, Dubus and Waelbroeck, 2021; Jentzsch, Sapi and Suleymanova, 2013; Liu and Serfes, 2004; Meagher and Zauner, 2004; Shy and Stenbacka, 2016; Villas-Boas and Schmidt-Mohr, 1999). (2) For the information about firm-specific characters, it could be private information about production cost (Gal-or, 1986; Jeitschko, Liu and Wang, 2018; Shao, Wu and Zhang, 2020) or product qualities (Guo and Zhao, 2009; Levin, Peck and Ye, 2009), etc.[13] For the Hotelling model, in particular, without considering multi-purchase,

[13] In addition, Guo and Iyer (2010),Gal-Or, Geylani and Dukes (2008), Tsunoda and Zennyo (2021) and Li, Tian and Zheng (2021) consider demand information sharing in a vertical relationship/supply chain.

the demand uncertainty is captured by consumers' locations and the degree of differentiation, which cannot describe the degree of interdependence along the extensive margin emphasized here.

Finally, our results shed lights on some practical and policy issues, especially on platform economics and privacy regulations. The motivation for making multi-purchase in our work, is similar with a parallel issue — multi-homing in two-sided markets (Ambrus, Calvano and Reisinger, 2016; Bakos and Halaburda, 2020; Barua and Mukherjee, 2021; Belleflamme and Peitz, 2019; Chao and Derdenger, 2013; Choi, 2010; Choi and Jeon, 2021; Jullien and Sand-Zantman, 2021), and complementarities in platform economics (Cennamo, Ozalp and Kretschmer, 2018; Hagiu, Jullien and Wright, 2020; Li and Zhu, 2021). The information game discussed in our work, adds new features to the issue of co-opetition between a platform with its complementors (Casadesus-Masanell and Yoffie, 2007; Dukes and Liu, 2016; Li and Zhu, 2021; Panico and Cennamo, 2015; Parker and Van Alstyne, 2018; Yoffie and Kwak, 2006; Zhu and Liu, 2018). Jullien and Pavan (2019) provide a comprehensive discussion about the role of information and pricing strategies in platform markets. Lastly, although we focus on the provision of relationship-sensitive data instead of personal-specific data, the process during data collecting, processing and provision might be somewhat involved with privacy issues (Casadesus-Masanell and Hervas-Drane, 2015; Choi, Jeon and Kim, 2019; Hoffmann, Inderst and Ottaviani, 2020; Montes, Sand-Zantman and Valletti, 2019). Acquisti, Taylor and Wagman (2016) provide a review with respect to the economics of privacy.

Compared with the existing literature, we provide new insights in predicting the equilibrium pricing, information sharing strategies, and policy implications under asymmetric information about a particular type of demand that can potentially make duopolists independent. Without such relationship-sensitive information, competition and independence are interim-unobservable for simultaneous movers.

3 Model Setup

The essential information addressed in this study shall capture a varying degree of interdependence between potential competitors, which can be generated by a purchase pattern that could make duopolists either independent or not. The introduction of brand complementarity and variety-seeking preference acts as a buffer in price competition by counteracting differentiation and brand loyalty. In order to capture the relative forces between brand differentiation and complementarity by using simple and separable parameters, the discrete choice model class (Anderson, De Palma and Thisse, 1992), e.g., Hotelling model becomes a workhorse.[14]

Consider two brand owners selling horizontally differentiated goods/services. The brands are located at opposite ends of a "Hotelling line" with length 1. Seller 0 is at the far left (point 0) and seller 1 at the far right (point 1). Within the line, there are one unit mass of consumers,

[14]In contrast, if the Cournot-style model is adopted, then it is technically messier to isolate the issue of multi-brand purchase relative to buying one brand exclusively.

whose tastes are denoted by $x \in [0,1]$. The personal profile of a particular consumer is reflected by his/her location x, and hence is not available by sellers, but the distribution of consumer tastes is assumed to be common knowledge. Assume that x is uniformly distributed along the line. Sellers provide one-shot prices p_0 and p_1 simultaneously.

Given the two brands that can be chosen, there are two types of utilities involved: The first is the consumption value in terms of quantity purchased, represented by $V(k)$, where $k = 1, 2$ is the number of **brands** used. The second is the traditionally defined "travel disutility" that is incurred from buying a particular brand which deviates from one's idiosyncratic tastes x, and such disutility is measured by tx (resp., $t(1-x)$) if buying from 0 (resp., buying from 1). In other words, the size of t represents the degree of brand differentiation, while the distance between a consumer and the brand reveals consumers' heterogeneous tastes over brand-specific attributes, e.g., product positioning, seller's reputation, and pre-/after-sale service, etc. Therefore, buying one unit from seller 0, and buying one unit from 1, gives

$$U_0 = V(1) - tx - p_0, \text{ and } U_1 = V(1) - t(1-x) - p_1,$$

respectively. We call such purchase pattern "single-purchase" as $k = 1$.

Assume that the marginal utility of using a second unit of the same product is zero (such that nobody purchases from the same seller twice or more[15]), but it is not necessarily true for the marginal utility of using a different brand. Without loss of generality, let $V(2) = V(1) + \beta$, where β is the marginal utility of using a product supplied by a **different** brand. That is, using a combination of both brands generates some consumption value measured by β which cannot be obtained by using one product only.[16] It could be possible that β is sufficiently small, meaning that conditional on owning one brand already, the incremental utility of buying from the other brand is low (or even negative), such that nobody purchases from both sellers simultaneously because the two brands are substitutes, as assumed in the traditional Hotelling model. For instance, $\beta = 0$ can be considered as a special case where the functionalities of two products completely overlap such that it is totally unnecessary to buy two units. In contrast, a higher β is associated with a higher degree of brand complementarity, making some consumers more willing to buy from both sellers. For instance, if $\beta = V(1)$ such that $V(2) = 2V(1)$, it can be regarded as a special case where every consumer is willing to buy from both. Combining brand-specific tastes and prices to be paid, buying two units — one unit for each seller, gives

$$U_{01} = V(2) - tx - t(1-x) - p_0 - p_1 \xlongequal{V(2)=V(1)+\beta} V(1) + \beta - t - p_0 - p_1.$$

[15] The possibility that purchasing multiple units from one seller is ruled out because if a consumer buys $\forall N > 1$ units from seller 0, then Np_0 will be paid without obtaining additional benefits. E.g., a user could install either Mathematica or Maple, or both; but it is unnecessary to purchase one of them twice. The reason not to discuss within-brand multi-unit purchase here is that, such type of purchase is less related to the inter-dependence between two sellers that is addressed in this paper. See Shao (2020), for example.

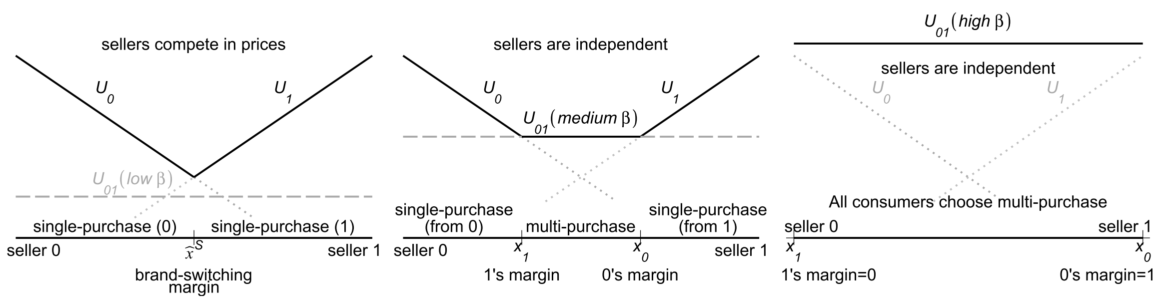
[16] That the variety-seeking preference β and heterogeneity x are assumed to be additive can be seen in Kim and Serfes (2006) and Jeitschko, Jung and Kim (2017). Kim and Serfes (2006) assume $V(2) = V(1)q_j + \beta q_A q_B$, where $q_j \in \{0,1\}$ indicates the number of units purchased from brand $j \in \{A, B\}$. Jeitschko, Jung and Kim (2017) assume $V(2) = 2V(1) - \psi$, and our β is equivalent to $V(1) - \psi$, where ψ is allowed to take negative values.

We call such purchase pattern "multi-purchase" as $k = 2$.

The market is assumed to be fully covered and each consumer chooses one of the three options in $\{U_0, U_1, U_{01}\}$ that gives the highest surplus:

$$\max\{\underbrace{V(1) - tx - p_0}_{\text{buy 1 unit from 0 only}}, \underbrace{V(1) - t(1-x) - p_1}_{\text{buy 1 unit from 1 only}}, \underbrace{V(1) + \beta - t - p_0 - p_1}_{\text{buy from both 0 and 1}}\}. \tag{1}$$

From the buyers' side, compared with single-purchase, the parameter $\beta = V(2) - V(1)$ captures the marginal utility of using a different product relative to using one brand alone, and the size of β measures the desire for variety. Assume that β is homogeneous for all consumers.[17]



(a) single-purchase: all (b) multi-purchase: some (c) multi-purchase: all

Figure 1: Single- and multi-purchase in the Hotelling line

More importantly, a homogeneous β is reasonable from the sellers' side: a low (resp., high) β indicates that the two brands are substitutes (resp., complements). By construction, brand complementarity is generated by the coexistence of both brands instead of being unilaterally determined — as we will illustrate it later, β here also measures interdependence between the two sellers. For that reason, different from the raw data x that represents heterogeneous personal profiles, the information about β, as a platform-aggregate form, can be inferred by the digital platform who is better informed about the interactions between patronized sellers. In addition, since a homogeneous β is orthogonal to consumers' personal profiles (x), sharing the information about β does not violate the anonymity rule required by most privacy regulations (Montjoye, Schweitzer and Crémer, 2019).

Assumption 1. *Assume that $\beta \in [\underline{\beta}, \overline{\beta}]$, where $\underline{\beta} = 0$ and $\overline{\beta} \geq 2t$.*[18]

[17] Since consumers are already differentiated along their locations, hence the "gross" marginal utility from buying an additional, different product (including brand-specific tastes x), is heterogeneous. For example, for a consumer located at x, from consuming 0 only to consuming both, the change in utility is $\beta - t(1-x)$, which is heterogeneous for different x. Therefore, to keep the subsequent analysis simple without losing the key insights, assuming a homogeneous β is sufficient.

[18] In this model, as we will show later, the impact of β on equilibrium outcomes will be measured by the size of β *relative* to differentiation t. By allowing β to range from a sufficiently small lower bound to an upper bound that is no less than $2t$, all possible equilibrium patterns can be covered: i) every consumer chooses single-purchase; ii) some but not all consumers choose multi-purchase; and iii) all consumers choose multi-purchase. In the main-text,

In this section, we start from the simplest setting "full information (benchmark F)" meaning that both sellers are assumed to have full and equal access to the information about β. For instance, consumers buy from the two sellers through a digital platform who is assumed to be fully informed about consumers' shopping habits.[19] The digital platform is assumed to authorize sellers APIs, or publicizes such data insights without reservation. To address the knowledge structure of firms specified in subsequent sections, it is innocuous to consider the model as a two-stage game. At stage 1, two firms set prices simultaneously and at stage 2, given prices, each consumer chooses one of the three options expressed in equation (1) to maximize his/her surplus.

To specify each seller's demand in stage 2, we need to find consumers' indifference conditions determined by each pair of options in (1). Notice that in (1), given β, if the price sum $(p_0 + p_1)$ to be paid for multi-purchase is relatively high, then single-purchase is more attractive. Then, every consumer makes a single-purchase: a marginal, brand-switching consumer who is indifferent between the two sellers locates at

$$U_0 = U_1 \Leftrightarrow \widehat{x}^S(p_0, p_1) = \frac{1}{2} + \frac{p_1 - p_0}{2t}. \tag{2}$$

The superscript "S" denotes for "single-purchase," i.e., the typical margin defined in textbook models.

In contrast, when the sum of the prices is relatively low, a positive number of consumers choose to make a multi-purchase. Then, firm 0's demand consists of consumers who buy from 0 only, and those who choose multi-purchase. In other words, firm 0's demand margin is determined by the consumer who is "struggling" between buying from seller 1 only, and buying from both. The gross marginal utility from consuming 1 only to both, is $U_{01} - U_1 = \beta - tx - p_0$, the value of which is positive (resp., negative) if the consumer buys both (resp., from 1 only), and is equal to zero to make the consumer indifferent between buying from 0 or not. Hence, firm 0's demand margin, denoted by $\widehat{x}_0$, is solved by

$$U_{01} = U_1 \Leftrightarrow \widehat{x}_0(p_0, \beta) = \frac{\beta - p_0}{t}. \tag{3}$$

Note that under multi-purchase, firm 0's demand depends on its own price and the demand for multi-purchase, and is orthogonal to the rival's price p_1. Similarly, firm 1's demand margin, denoted by $\widehat{x}_1$, is the one who is indifferent between buying from 0 only and making a multi-purchase:

$$U_0 = U_{01} \Leftrightarrow \widehat{x}_1(p_1, \beta) = 1 - \frac{\beta - p_1}{t}. \tag{4}$$

we normalize $\underline{\beta} = 0$ for expositional simplicity. For instance, from single-purchase to multi-purchase, additional travel disutilities have already been incurred even when $\beta = 0$, making two brands substitutes. We will show in Appendix A.4 that all results will continue to go through by allowing $\underline{\beta}$ to be negative, e.g., installing a pair or incompatible/conflicted software — a special case for "nobody makes multi-purchase."

[19] In addition to the raw data of consumer preferences, the information about β could also be inferred through analyzing click-though rates of listings, brows histories, subscriptions and follow-ups, search keywords, etc.

To solve the equilibrium with complete information on β, note that in this model, there exist three mutually exclusive cases:

Single-purchase (labeled by regime superscript S) Every consumer chooses single-purchase. A necessary condition for single-purchase equilibrium is that $\widehat{x}_0 < \widehat{x}_1$ and the true margin is (2), where 0's demand is $\widehat{x}^S(p_0, p_1)$ and 1's demand is $1 - \widehat{x}^S(p_0, p_1)$, and apparently, the prices are *strategic complements*. See Figure 1a.

Interior case of multi-purchase (labeled by regime superscript M) Some but not all consumers choose multi-purchase. Multi-purchase emerges when

$$\widehat{x}_1(p_1, \beta) < \widehat{x}_0(p_0, \beta) \Leftrightarrow p_0 + p_1 < 2\beta - t \tag{5}$$

holds, for a relatively high β or when the price sum is low enough. When (5) holds, 0's payoff is $p_0\widehat{x}_0(p_0, \beta)$ and 1's payoff is $p_1(1 - \widehat{x}_1(p_1, \beta))$, both of which are orthogonal to the rival's price. Hence, an important feature under multi-purchase, is that duopolistic prices are *strategically independent*. See Figure 1b.

Boundary case of multi-purchase (labeled by regime superscript B) All consumers choose multi-purchase. Then, $\widehat{x}_0 = 1$, $\widehat{x}_1 = 0$, and the demand of each seller is 1. See Figure 1c.

At stage 2, given prices, if condition (5) does not hold, i.e., no consumers chooses multi-purchase, then the payoffs of both sellers are dependent on the rivals' prices and each seller charges a price as a best response to the rival's choice (i.e., competition scenario S); if condition (5) holds, i.e., a positive number of consumers choose multi-purchase, then it results in an independent scenario (M or B) where each seller charges a price as a local monopoly. That is, the optimization problems solved by each seller are different across the two scenarios that are mutually exclusive, and whether the competition scenario or independent scenario occurs is **jointly** determined by the pricing choices of both sellers at stage 1. In previous studies where β is common knowledge, the equilibrium strategies (and the conditions for switching strategies between competition and independence) are symmetric. As we will show later that, such a claim shall be challenged and re-examined when incomplete information on β is introduced.

At stage 1, the equilibrium pricing strategies are determined by the intersections of firms' best responses. At the competition scenario (S), each seller solves

$$\begin{aligned} p_0^S &= \arg\max_{p_0} p_0 \widehat{x}^S(p_0, p_1) = \frac{p_1 + t}{2} \Rightarrow \pi_0^S = \frac{(p_1 + t)^2}{8t} \\ p_1^S &= \arg\max_{p_1} p_1 \left(1 - \widehat{x}^S(p_0, p_1)\right) = \frac{p_0 + t}{2} \Rightarrow \pi_1^S = \frac{(p_0 + t)^2}{8t} \end{aligned}. \tag{6}$$

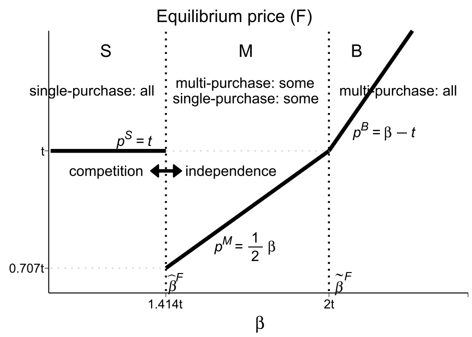


Figure 2: Equilibrium price (conditional on β)

At the independence scenario (M), each seller solves

$$p_0^M = \arg\max_{p_0} p_0 \widehat{x}_0(p_0, \beta) = \frac{\beta}{2} \Rightarrow \pi_0^M = \frac{\beta^2}{4t}$$
$$p_1^M = \arg\max_{p_1} p_1 \left(1 - \widehat{x}_1(p_1, \beta)\right) = \frac{\beta}{2} \Rightarrow \pi_1^M = \frac{\beta^2}{4t}. \tag{7}$$

Let $-i$ be seller i's rival. Each firm's payoff expressed by (6) is monotonically increasing in the rival's price, whereby the payoff expressed by (7) is orthogonal to the rival's price. Hence the two options are equally profitable evaluated at $p_{-i} = \sqrt{2}\beta - t$. The best response of each firm is given by

$$p_i^{BR} = \begin{cases} \frac{p_{-i}+t}{2}, & p_{-i} > \sqrt{2}\beta - t \\ \frac{\beta}{2}, & p_{-i} < \sqrt{2}\beta - t \end{cases}. \tag{8}$$

Lastly, when β is large enough such that all consumers choose multi-purchase (B), the demand of each firm is fixed and firm 0's (resp., 1's) profit function is $p_0 \cdot 1$ (resp., $p_1 \cdot 1$). By equating $\widehat{x}_0 = 1$ and $\widehat{x}_1 = 0$, it gives a boundary solution $p_i^B = \beta - t$.

Combining the above three cases, the symmetric Bertrand-Nash equilibrium under "full information" (denoted by F) can be summarized by

Lemma 1 (Equilibrium under Full Information (F))**.** *When both firms are informed about the demand for multi-purchase, i.e., β, there exists a unique and symmetric equilibrium such that*

$$\begin{aligned} (p_0^S, p_1^S) &= (t, t),\ 0 \leq \beta < \sqrt{2}t \\ (p_0^M, p_1^M) &= \left(\frac{\beta}{2}, \frac{\beta}{2}\right),\ \sqrt{2}t \leq \beta \leq 2t \\ (p_0^B, p_1^B) &= (\beta - t, \beta - t),\ 2t \leq \beta \leq \overline{\beta} \end{aligned} \tag{9}$$

Proof. See Appendix A, where a refinement at $\beta = \sqrt{2}t$ is also provided. □

The full information environment described in Lemma 1 serves as our first benchmark, in which the equilibrium price expressed by (9) is a piecewise linear function contingent upon the level of the demand for multi-purchase β, relative to product differentiation t. To facilitate the subsequent analysis, the two important equilibrium thresholds (vertical lines in Figure 2) that deserve to be addressed here:

(1) The "competition-independence threshold," defined as $\widehat{\beta}$, is equal to $\widehat{\beta}^F = \sqrt{2}t$ under full information. When β is below the threshold, no consumer makes multi-purchase and firms compete in prices. When β is above the threshold, some but not all consumers make multi-purchase and each firm charges its own price independently.

(2) The "no single-purchase threshold," is defined as $\widetilde{\beta}$, above which no consumer makes single-purchase. With full information, $\widetilde{\beta}^F = 2t$.

Based on the two thresholds, the equilibrium is divided into three regimes: single-purchase equilibrium (S) occurs when $\beta \in [0, \widehat{\beta}^F)$; the interior solution of multi-purchase equilibrium (M) occurs when $\beta \in [\widehat{\beta}^F, \widetilde{\beta}^F]$; and the boundary solution of multi-purchase equilibrium (B) occurs when $\beta \in [\widetilde{\beta}, \overline{\beta}]$.

There are some worth-mentioning observations regarding the level of equilibrium price as a function of β (see Figure 2): First, around the "competition-independence" threshold $\widehat{\beta}^F$, when β starts to increase from below $\sqrt{2}t$ to above $\sqrt{2}t$, the equilibrium price jumps downward from t to $\frac{\beta}{2}\big|_{\beta=\sqrt{2}t} \approx 0.707t < t$. That is, when the equilibrium switches from single-purchase (competition) to the interior solution under multi-purchase (independence), both firms turn to charge lower prices. The reason behind, is that evaluated at the threshold such that both types of equilibrium are equally profitable, a lower price sum induces more sales by making more consumers choose multi-purchase, and firms are willing to do so when β exceeds the threshold.

Second, under multi-purchase equilibrium, as β increases within the range $[\widehat{\beta}^F, \widetilde{\beta}^F]$, the equilibrium price is adjusted according to a higher willingness to pay for buying the second unit (i.e., demand-expansion effect), and hence is increasing in β.

Finally, when β exceeds the second threshold $\widetilde{\beta}^F$ such that every consumer chooses multi-purchase, the boundary solution $\beta - t$ exceeds the price charged under single-purchase, t, and is increasing in β with an even faster speed compared to the slope of the interior solution under multi-purchase. When all consumers choose multi-purchase, all consumers are captive (no marginal consumers) and therefore, a full proportion of the additional willingness to pay can be extracted (i.e., price-raising effect). The incentives of raising prices are rather restricted when some but not all consumers choose multi-purchase, because a seller faces a trade-off due to a marginal increase in price: on one hand, a higher β associated with a higher price brings about more infra-marginal revenue (among those who buy from both sellers); on the other hand, however, a higher price also makes the seller lose the marginal consumer who is indifferent between buying from both sellers and buying only one unit from its rival.

The main rationale to introduce (the information about) β, is to capture the degree of interdependence, which can be reflected around the "competition-independence threshold." When β exceeds the threshold, the relationship between duopolists switches from competition and independence, calling for a downward jump in equilibrium prices. However, the problem of choosing the right price for simultaneous movers arises if somehow, the information about β is incomplete such that "competition" and "independence" are interim indistinguishable.

4 Asymmetric Information

The information about consumers' willingness to pay for making additional purchases from the rival firm, is generated by a pair of coexistent brands, each of whom might not necessarily be able to get access to such information. Instead, the platform, as a marketplace, is better informed about such aggregated data. In reality, the data service is only made partly or not available to third-party business users, whereas the first-party sellers may benefit from self-preferencing. When the competition takes place between a first-party seller owned by a vertically integrated platform and the third-party seller, the asymmetric information problem arises.

In this section, we introduce asymmetric information about β, which is labeled as "benchmark A." Assume that seller 0 (i.e., the first-party) is provided with preferential treatment, and hence is fully informed about β, but seller 1 (i.e., third-party) is not. Let μ be firm 1's prior about β, and is assumed to be uniformly distributed over $[0, \overline{\beta}]$.[20] Seller 1 is aware of the fact that seller 0 is fully informed, and the above claims are assumed to be common knowledge. For now, we assume that the information about β is not allowed to be revealed, and the incentive of information revelation will be considered in Section 5.[21]

A critical difference between asymmetric and full information is the role played by the "competition-independence threshold." With full information, both sellers are able to choose between competing in prices and charging one's own price independently, conditional whether β is observed to be below or above the threshold $\widehat{\beta}^F$. Under asymmetric information, without information about a particular β, firm 1 has to charge a flat price.[22] Instead, firm 0 is able to charge a price optimally contingent upon not only β, but also firm 1's flat price. Therefore, owning the information about β provides firm 0 with an addition advantage (compared with firm 1), i.e., by choosing between charging a price that is a best response to firm 1's fixed price to induce single-purchase, and charging a price independently to induce multi-purchase. That

[20] We prefer to use uniform prior for the purpose of properly restricting the range of β within $[0, \overline{\beta}]$, such that: (1) All three types of purchase (S, M, B) are likely to emerge while no preferential probabilistic weight is put on a particular purchase type; (2) Because the equilibrium price is a piecewise linear function of β, and hence by using uniform prior, we can underline the features of pricing incentives *per se*, without being involved with unnecessary complexities due to unequal densities; (3) By using uniform prior, the comparative statics with respect to $\overline{\beta}$ is sufficiently informative in comparing the equilibrium outcomes and welfare. See Narayanan, Raman and Singh (2005) as an example for uniformly distributed demand.

[21] Beyond the economics incentives, there also exist legal and technical constraints that make data unavailable to individual third-party sellers.

[22] Readers should keep in mind that in this paper, prices are assumed to be offered simultaneously. At the moment when prices are going to be announced, the rival's price is unobservable, e.g., interim unobservability (Gaudin, 2019). In other words, there is no way for firm 1 to obtain additional information about β. The revelation incentives of firm 0 is considered in Section 5, where firm 0's revelation behavior reveals some information that firm 1 can use to update its belief.

is, firm 0 is able to implement relationship-specific pricing based on an observed true β, while firm 1 cannot.

Let p_1^A be firm 1's equilibrium price, which is not a function of β (but a function of the distribution of β, i.e., $\overline{\beta}$). Firm 0's optimal price, is a function of β and p_1. Given p_1, by observing a particular β, the relative positions of the demand margins $\widehat{x}_1(p_1, \beta)$ and $\widehat{x}_0(p_0, \beta)$ are predictable and manipulable by firm 0. Therefore, firm 0 has the following options:

(i) By charging p_0 such that $\widehat{x}_1(p_1, \beta) > \widehat{x}_0(p_0, \beta) \Leftrightarrow p_0 + p_1 > 2\beta - t$, no consumer makes multi-purchase and firm 0 solves

$$\max_{p_0} p_0 \widehat{x}^S(p_0, p_1) \Rightarrow p_0^S = \frac{p_1 + t}{2},\ \pi_0^S = \frac{(p_1 + t)^2}{8t}, \tag{10}$$

as a best response to firm 1's price.[23] Plug the solution into $\widehat{x}_1 > \widehat{x}_0$, we have $\beta < \frac{3}{4}(p_1 + t)$.

(ii) By charging p_0 such that $\widehat{x}_1(p_1, \beta) < \widehat{x}_0(p_0, \beta) < 1 \Leftrightarrow p_0 + p_1 < 2\beta - t$ and $\beta < p_0 + t$, some but not all consumers make multi-purchase and firm 0 solves the following problem independently:

$$\max_{p_0} p_0 \widehat{x}_0(p_0, \beta) \Rightarrow p_0^M = \frac{\beta}{2}, \pi_0^M = \frac{\beta^2}{4t}. \tag{11}$$

Plug the solution into $\widehat{x}_1 < \widehat{x}_0 < 1$, we have $\frac{2}{3}(p_1 + t) < \beta < 2t$.

(iii) Under $\widehat{x}_1(p_1, \beta) < 1 \leq \widehat{x}_0(p_0, \beta) \Leftrightarrow p_0 + p_1 < 2\beta - t$ and $\beta \geq p_0 + t$, all consumers make multi-purchase and firm 0 solves the boundary case:

$$\begin{aligned} &\max_{p_0} p_0 \cdot 1 \\ &s.t.\ \widehat{x}_0(p_0, \beta) \geq 1 \end{aligned} \Rightarrow p_0^B = \beta - t,\ \pi_0^B = \beta - t. \tag{12}$$

Plug the solution into $\widehat{x}_0 \geq 1$, we have $\beta \geq 2t$.[24]

Note that, both the option (i) and (ii) are feasible when $\frac{2}{3}(p_1 + t) < \beta < \frac{3}{4}(p_1 + t)$. In option (i), single-purchase equilibrium is chosen where firm 0's payoff (10) is orthogonal to β, but is a function of a given p_1 only; In option (ii), multi-purchase is realized and firm 0's payoff (11) is increasing in β. Therefore, there exists a threshold value, denoted as $\widehat{\beta}$, which makes the two options equally profitable for firm 0:

$$\underbrace{\frac{(p_1 + t)^2}{8t}}_{\pi_0^S} = \underbrace{\frac{\widehat{\beta}^2}{4t}}_{\pi_0^M},\ p_0 = \begin{cases} p_0^S = \frac{p_1+t}{2},\ \beta < \widehat{\beta} \Rightarrow \text{single-purchase (S) and } \widehat{x}_1 > \widehat{x}_0 \\ p_0^M = \frac{\beta}{2}, \quad \beta > \widehat{\beta} \Rightarrow \text{multi-purchase (M) and } \widehat{x}_1 < \widehat{x}_0 \end{cases}. \tag{13}$$

[23] With a slight abuse of notation, here we use p_0^S as an abbreviation, which is short for the functional form $p_0^S(p_1) = \frac{p_1+t}{2}$, i.e., firm 0's best response at single-purchase. In benchmark F or equation (9), when (p_0^S, p_1^S) hold simultaneously, we can directly solve (p_0^S, p_1^S) as (t, t) by the two equations.

[24] The restriction for the relationship between p_1 and β is redundant in case (iii) because when $\widehat{x}_0 \geq 1$, $\widehat{x}_1$ cannot be greater than $\widehat{x}_0$; otherwise, firm 1's demand $(1 - \widehat{x}_1)$ is negative.

Now consider firm 1's strategy. Although firm 1 cannot observe β, firm 0's whole plan (described above) that is going to be enforced can be commonly inferred. That is, when $\beta < \widehat{\beta} \Rightarrow \widehat{x}_1 > \widehat{x}_0$, single-purchase will be realized (with probability $\Pr(\beta < \widehat{\beta})$), then the corresponding profit function of firm 1 is $p_1 \left(1 - \widehat{x}^S\right)$; when $\widehat{\beta} \leq \beta < p_1 + t \Rightarrow 0 < \widehat{x}_1 < \widehat{x}_0$, multi-purchase will be realized (with probability $\Pr(\widehat{\beta} \leq \beta < p_1 + t)$), then firm 1's profit function is $p_1 \left(1 - \widehat{x}_1\right)$; when $\beta \geq p_1 + t \Rightarrow \widehat{x}_1 \leq 1 < \widehat{x}_0$, firm 1's demand is 1 (with probability $\Pr(\beta \geq p_1 + t)$) and its payoff is $p_1 \cdot 1$.

Integrating the above cases, firm 1 maximizes its expected payoff:

$$\max_{p_1} \underbrace{\int_0^{\widehat{\beta}} \frac{1}{\overline{\beta}} p_1 \left(1 - \widehat{x}^S(p_0, p_1)\right) d\beta}_{\text{single-purchase}} + \underbrace{\int_{\widehat{\beta}}^{p_1+t} \frac{1}{\overline{\beta}} p_1 \left(1 - \widehat{x}_1(p_1, \beta)\right) d\beta}_{\text{multi-purchase:interior}} + \underbrace{\int_{p_1+t}^{\overline{\beta}} \frac{1}{\overline{\beta}} p_1 d\beta}_{\text{multi-purchase:boundary}}, \quad (14)$$

which is a function of the rival's price p_0, the threshold $\widehat{\beta}$ and the upper bound $\overline{\beta}$. The first-order condition of (14) gives the best response of p_1 with respect to p_0, which is shown to be increasing in p_0.

There are five unknowns that are of our interest: $(p_1, p_0^S, p_0^M, p_0^B)$ and a threshold $\widehat{\beta}$, all of which are simultaneously determined by equation (10), (11), (12), (13) and the first-order condition of (14).[25] Let $\widehat{\beta}^A$ be the equilibrium competition-independence threshold. The equilibrium is obtained by solving the above system.

Proposition 1 (Equilibrium under Asymmetric Information (A))**.** *When β is known to firm 0 only and firm 1 holds a uniform prior with support $[0, \overline{\beta}]$, there exists a unique and asymmetric equilibrium such that firm 1 charges a flat price:*

$$p_1^A(\overline{\beta}) \equiv \frac{3\sqrt{2} - 50}{73} t + \frac{1}{511} \sqrt{(138\sqrt{2} + 547)\left((44\sqrt{2} + 344)\overline{\beta} + 49t\right) t}. \quad (15)$$

Meanwhile firm 0 charges a price conditional on β:

$$p_0 = \begin{cases} \frac{p_1^A + t}{2}, & 0 \leq \beta < \widehat{\beta}^A \\ \frac{\beta}{2}, & \widehat{\beta}^A \leq \beta \leq \widetilde{\beta}^A \\ \beta - t, & \widetilde{\beta}^A \leq \beta \leq \overline{\beta} \end{cases}, \quad (16)$$

where the competition-independence threshold is $\widehat{\beta}^A \equiv \frac{p_1^A + t}{\sqrt{2}}$, and the boundary condition which makes all consumers buy from firm 1 is $\widetilde{\beta}^A = 2t$.[26]

[25] By independence, p_0^M and p_0^B are actually directly solved from (11) and (12), respectively. In addition, notice that in our simultaneous game, $\widehat{\beta}$ is interim unobservable and is determined jointly by both sellers, hence firm 1 cannot plug $\widehat{\beta}$ as a function of p_1 expressed in (13) into its objective; otherwise, firm 1 becomes a "leader."

[26] In Appendix A, the relative positions of marginal consumers (3) and (4) are verified to be consistent with the equilibrium purchase patterns: $\beta < \widehat{\beta}^A$ (resp., $\beta > \widehat{\beta}^A$) is a sufficient condition for single-purchase (resp., multi-purchase). In addition, we can also solve the equilibrium by finding the intersections between best responses, and then verify the associated parametric conditions, by using sub-graphs 8a, 8b and 8c in Figure 8.

Comparing full information and asymmetric information, the equilibrium in the latter benchmark exhibits a sharp difference in pricing structure: The uninformed seller charges a single price given by (15), whereas the whole plan of the informed seller (16) consists of three pricing patterns. When β is relatively low such that single-purchase is more profitable, the informed seller charges a price that is a best response to its uninformed rival, engaging into a price competition. When β is relatively high, the informed seller prices independently as what he/she does under full information. Therefore, a critical decision for both sellers is to form a threshold $\widehat{\beta}^A$ that isolates competition and independence.

Quantitatively, both the competition-independence threshold $\widehat{\beta}^A$, as well as the flat price p_1^A, are strictly increasing in $\overline{\beta}$ for $\overline{\beta} \geq 2t$.[27] That is, when the uninformed seller 1 charges a higher (resp., lower) price, competition (resp., independence) is more profitable for the informed seller 0. In addition, let $\widetilde{\beta}^A$ be the equilibrium boundary threshold such that $\widehat{x}_1(p_1^A, \widetilde{\beta}^A) = 0$, we find that when $p_1^A = t$, $\widehat{\beta}^A = \sqrt{2}t$ or $\widetilde{\beta}^A = 2t$, it gives $\overline{\beta} = (5 - 2\sqrt{2})t \approx 2.172t$.

Proposition 2 (Monotonicity under Asymmetric Information (A))**.** *The equilibrium price p_1^A, and two thresholds, $\widehat{\beta}^A$ and $\widetilde{\beta}^A$, are increasing in $\overline{\beta}$. In addition, $p_1^A = t$, $\widehat{\beta}^A = \sqrt{2}t$ and $\widetilde{\beta}^A = 2t$ if and only if $\overline{\beta} = (5 - 2\sqrt{2})t$.*

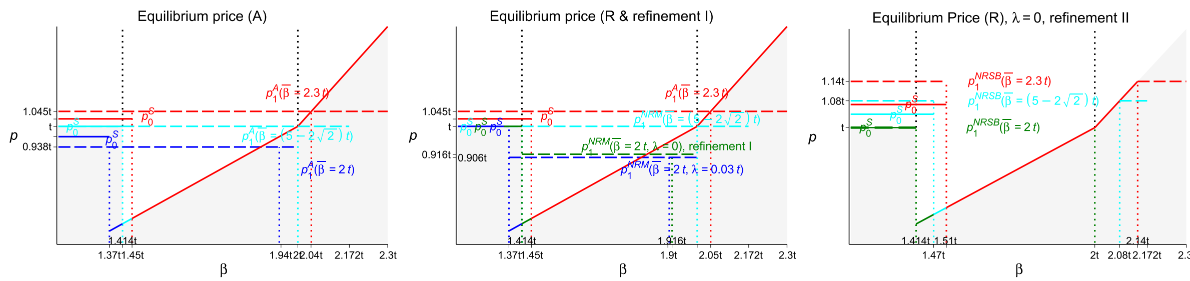
Proposition 1 and 2 show how the informed seller can take the advantage of asymmetric information: the informed seller is able to choose whether or not to respond optimally in price competition. When β is relatively low such that being engaged into price competition is more attractive, then a higher (resp., lower) price charged by the rival benefits (resp., harms) the informed seller, because prices are strategic complements in a Bertrand game.[28] Consequently, the price charged by the uninformed rival, and the competition-independence threshold moves to the same direction.

The pricing incentives under full information and asymmetric information deliver different welfare implications. Under full information, prices of both sellers jump downward symmetrically at the competition-independence threshold $\widehat{\beta}^F = \sqrt{2}t$, implying an upward jump for a greater level of market coverage, thus creating more social values. Under asymmetric information, however, since the uninformed seller cannot charge different prices conditional on the threshold $\widehat{\beta}^A$, then it results in a slow and continuous process of market expansion when β exceeds $\widehat{\beta}^A$. In other words, when the market switches from single- to multi-purchase equilibrium, the total volume of transactions is lower under asymmetric information compared with that under full information.

[27] The sign of the derivative of p_1^A with respect to $\overline{\beta}$ can be heuristically seen from (13) and (14). Since both $\widehat{\beta}^A$ and $\widetilde{\beta}^A$ are increasing in p_1^A, hence as $\overline{\beta}$ changes, the equilibrium firm 1's price and the two thresholds move to the same direction.

[28] In Figure 3a, the price charged by the informed seller follows the changes of price charged by the uninformed rival: and is less dispersed around t.

[29] The equilibrium price under full information is denoted by the upper-bound of the gray-shaded regions (for the purpose of comparing with prices plotted by lines under other benchmarks). The solid-lines (resp., dashed-lines) represent the prices charged by the informed (resp., uninformed) firm. The competition-independence threshold and the no single-purchase threshold are marked by the vertical dotted-lines.



(a) Asymmetric information A

(b) Strategic revelation R

(c) Refinement II in R

Figure 3: Equilibrium prices[29]
Gray-shaded area & solid lines: prices conditional on β. Dashed lines: flat price.

5 Strategic Revelation: Optimal Information Sharing

In benchmark A, firm 0 is fully informed about β and revealing the true β to firm 1 is assumed to be not allowed. In reality, the platform indeed provides "data-as-a-service" to some extent. For example, one of the aims of Google Play is "to provide the developers with the optimal amount of data." Nevertheless, some business users complain that the platform data service only gives a miniscule part of the full picture.

In this section, we allow firm 0 to choose between revealing or concealing the information about β to firm 1 (labeled as benchmark R). By investigating the revelation incentives, we provide supplemental explanations for the conflicts incurred between the informed platform and the uninformed third-party sellers. We argue that, in addition to the obstacles (e.g., legal barriers) claimed by the data providers, there exist some more sophisticated concerns and incentives for sharing relationship-sensitive data. Since all parties know that the platform owns relevant data, a selective provision, or concealment of some part of data, reveals some information about their interdependence.

Assume that the game lasts for three stages.

Stage 1 Upon a true β is observed, firm 0 can choose to reveal the true β to firm 1, or choose to conceal (i.e., keeping silent). Firm 1 observes firm 0's revelation action and updates its belief about β accordingly. We assume that if firm 0 chooses to reveal, then telling firm 1 a β that differs from the true β is not allowed, i.e., we focus on verifiable revelation.[30]

Stage 2 Both sellers offer prices simultaneously.

Stage 3 Consumers make purchase decisions

To unveil the revelation incentives, it is helpful to keep the following two points in mind. First, when duopolists stay independently, then revealing β is unnecessary. Second, when sellers compete in prices, each seller benefits from a high price charged by the rival. The main

[30] Sending a signal that is not verifiable, i.e., cheap talk (Crawford and Sobel, 1982), belongs to a setting that is completely different, which is not discussed in this paper.

complexity arises at how to induce the belief about the uninformed seller in order to benefit the informed seller by revealing/concealing some particular level of β. The above claims can be commonly inferred by both firms, and at equilibrium, both firms charge prices that are best responses to the beliefs.

5.1 Analysis

"Nature" assigns a particular $\beta \in [0, \overline{\beta}]$. Firm 1's prior belief about β is distributed over $[0, \overline{\beta}]$ according to uniform distribution. Firm 0 is informed about the true β, and can choose to reveal the true β or conceal. The above information is assumed to be common knowledge. At stage 1, firm 0's action set A_0 consists of two strategies: revealing the true β, denoted as $R = \beta$, or conceal, denoted as NR.

At stage 2, firms charge prices (p_0, p_1) simultaneously. If R is observed at stage 1, then duopolists charge stage-2 prices that are identical to those under full information (benchmark F) given by (9); If NR is observed, then firm 1 updates its belief about β, denoted as μ_1, for which the support is a subset of $[0, \overline{\beta}]$. Then duopolists offer prices simultaneously with firm 0 being fully informed of β and firm 1 having a posterior belief μ_1. Let p_1^{NR} be firm 1's optimal price upon $A_0 = NR$ is observed.

The equilibrium strategies are defined as

$$\{A_0^*(\beta), \mu_1, p_0^*, p_1^*\}.$$

At equilibrium, firm 1's belief about β upon concealment, μ_1, should be consistent with firm 0's optimal choice NR. In particular, it is worth noticing that when firm 0 chooses to conceal, firm 1's belief about β is formed by inferring that such concealment and the associated consequences bring firm 0 about a payoff that is no less than that under revelation. If β is concealed, firm 0 might not necessarily price according to the true β, but instead respond optimally to firm 1's updated belief about β.

To figure out firm 0's revelation incentives and the updated beliefs, we need to compare firm 0's realized payoff at stage 3, with and without revelation. With full information, the outcomes have been stated in Lemma 1. If the information is not revealed, for a given price p_1^{NR}, firm 0 can charge a price to change the relative positions of $\widehat{x}_1$, $\widehat{x}_0$ and $\widehat{x}^S$, which is similar with the analysis conducted under asymmetric information: (i) charging $p_0^S = \frac{p_1^{NR}+t}{2}$ as an optimal response to firm 1's price to induce single-purchase, gives a payoff $\pi_0^S = \frac{(p_1^{NR}+t)^2}{8t}$, which is increasing in the rival's price p_1^{NR}; (ii) charging $p_0^M = \frac{\beta}{2}$ independently, gives $\pi_0^M = \frac{\beta^2}{4t}$ or (iii) charging $p_0^B = \beta - t$ independently, gives $\pi_0^B = \beta - t$. In the first case, firm 0 has a strong incentive to make p_1^{NR} as high as possible while in the latter two cases, firm 1's actions are irrelevant.

The above incentives can be generalized as the following intuitive rule for revelation strategy at single-purchase:

Lemma 2. *Firm 0's payoff at single-purchase equilibrium is strictly increasing in firm 1's price (regardless of revealing or concealing), i.e.,* $\frac{d}{dp_1}\left[\frac{(p_1+t)^2}{8t}\right] > 0$. *Revealing* β *gives* $p_1 = t \Rightarrow \pi_0^S = \frac{t}{2}$. *Therefore, at single-purchase, inducing any* p_1 *that is charged below* t *is strictly dominated.*

Notice by Lemma 2 that the payoff of firm 0 at single-purchase equilibrium is not a function of β, but merely a function of the rival's price (hence eventually, a function of parameters t and $\overline{\beta}$). Such an observation simplifies our analysis in the sense that when single-purchase equilibrium is going to be realized, all β that supports single-purchase equilibrium should be subjected to the same revelation action, i.e., there is no reason to reveal one β but hide another within the subset that supports single-purchase equilibrium.

Finally, consider firm 0's choice at multi-purchase equilibrium. Suppose that β is high enough such that keeping to be independent is optimal, then the issue of revelation becomes irrelevant. This brings about a potential problem in pinning down firm 1's belief about firm 0's behavior upon concealment is observed. If firm 0's preference over the two "indifferent" options is formed in a fashion that is entirely random and unpredictable, then firm 1's belief cannot be well defined which results in a variety of trivial outcomes.

To make the equilibrium more tractable, some restrictions shall be imposed on firm 0's behavior at independence. As we mentioned earlier, the process of data sharing could be costly, and therefore, in the following Subsection 5.2, we introduce a positive revelation cost for firm 0, denoted as $\lambda > 0$, i.e., "losses" incurred by information revelation. In reality, the costs incurred during the data sharing process include but is not limited to, for instance, the cost of data interoperability and the revision of back-end code; the investment made to make the shared data anonymous with safe storage and portability; legal uncertainties caused by diverging interpretations and enforcement of GDPR, which makes data sharing risky, etc.

Relevant discussions — equilibrium refinements under zero revelation cost ($\lambda = 0$), and the situation when the cost is incurred by concealing instead of revealing information, are provided in Appendix A.2.[31]

5.2 Equilibrium Revelation with Positive Cost

Clearly, when β is high enough such that keeping to be independent is optimal for firm 0, due to a positive cost λ, revealing is strictly dominated by concealing. That is,

$$\underbrace{\pi_0^M(A_0 = NR)}_{\text{conceal at multi-purchase}} = \frac{\beta^2}{4t} > \underbrace{\pi_0^M(A_0 = R)}_{\text{reveal at multi-purchase}} = \frac{\beta^2}{4t} - \lambda,\ \forall \lambda > 0.$$

For the boundary case, $\pi_0^B(A_0 = NR) = \beta - t > \pi_0^B(A_0 = R) = \beta - t - \lambda$, and firm 0 conceals.

[31] Conceptually, when the equilibrium is characterized by the two-dimensional space (β, λ), the case for $\lambda = 0$ occurs with zero probability under certain specifications. In addition, in Appendix A.2, one of the refinements shows that the equilibrium profiles become discontinuous as λ approaches to zero. Hence, the case for $\lambda = 0$ is displayed separately in Appendix.

However, the revelation strategy chosen followed by price competition should be subject to careful inspection. Since we have equipped with a two-dimensional space with axes β and λ, consider the strategies taken conditional on different combinations of the two parameters. Given a positive cost λ, when β is relatively high such that keeping to be independent is more profitable, we have shown that firm 0 conceals definitely; otherwise, competition occurs with possible revelation actions. Given a particular β, as the revelation cost λ increases, firm 0 is less likely to reveal. Therefore, let $\widehat{\beta}$ be the competition-independence threshold that makes the payoff from multi-purchase and single-purchase equilibrium equally profitable for firm 0, and let $\widehat{\lambda}$ be a threshold such that firm 0 is indifferent between revealing and concealing.[32] Then, all the possible revelation strategies can be categorized into the remaining three cases:

Case (NRM) When $\beta \geq \widehat{\beta}$, no information is revealed at multi-purchase as shown above. Then firm 0 charges its prices independently, and

$$\left(p_0^M, p_1^{NR}\right) = \left(\frac{\beta}{2}, p_1^{NR}\right), \text{ or } (p_0^B, p_1^{NR}) = (\beta - t, p_1^{NR}).$$

Case (NRS) When $0 \leq \beta < \widehat{\beta}$ and $\lambda > \widehat{\lambda}$, no information is revealed at single-purchase. Then sellers charge

$$\left(p_0^S, p_1^{NR}\right) = \left(\frac{p_1^{NR} + t}{2}, p_1^{NR}\right).$$

Case (RS) When $0 \leq \beta < \widehat{\beta}$ and $0 < \lambda < \widehat{\lambda}$, β is revealed at single-purchase. Then sellers charge

$$(p_0, p_1) = (t, t).$$

The critical values $\widehat{\beta}$ and $\widehat{\lambda}$ are endogenously determined by solving the indifference conditions when comparing each pair of the three cases above.

First, consider firm 0's revelation incentives in case (RS) and (NRS) for some $\beta \in [0, \widehat{\beta})$ such that single-purchase is chosen. If firm 0 chooses to incur a cost by revealing β and induce single-purchase, then firms charge $(p_0, p_1) = (t, t)$ as stated in (9), and the payoff of firm 0 is

$$\underbrace{\pi_0^S(A_0 = R)}_{\text{reveal at single-purchase}} = p_0 \widehat{x}^S(p_0, p_1)\big|_{(p_0,p_1)=(t,t)} = \frac{t}{2} - \lambda. \tag{17}$$

Instead, if firm 0 conceals, then firm 1 charges p_1^{NR}.[33] And firm 0's payoff becomes

$$\max_{p_0} p_0 \widehat{x}^S(p_0, p_1^{NR}) \Rightarrow \underbrace{\pi_0^S(A_0 = NR)}_{\text{conceal at single-purchase}} = \frac{(p_1^{NR} + t)^2}{8t}. \tag{18}$$

[32] It should be noted that, though, the threshold $\widehat{\lambda}$ is endogenously determined under equilibrium.

[33] Because firm 0 definitely conceals β at multi-purchase, and hence if firm 0 also conceals β at single-purchase, then p_1^{NR} corresponds to firm 1's price supported by the belief μ_1 such that all βs are possible.

Since firm 0's payoff at single-purchase is not a function of β, and therefore, the choice between case (NRS) and case (RS) is simply based on the size of revelation cost λ by comparing (17) and (18):

$$\begin{cases} \frac{t}{2} - \lambda > \frac{(p_1^{NR}+t)^2}{8t}, & \text{reveal at single-purchase (RS)} \\ \frac{t}{2} - \lambda < \frac{(p_1^{NR}+t)^2}{8t}, & \text{conceal at single-purchase (NRS)} \end{cases}.$$

Evaluated at the critical value $\widehat{\lambda}$, firm 0 is indifferent between the above two options:

$$\widehat{\lambda} = \frac{t}{2} - \frac{(p_1^{NR}|_{\lambda > \widehat{\lambda}} + t)^2}{8t}. \tag{19}$$

Next, the subsequent analysis for finding the critical value $\widehat{\beta}$ is conditional on whether λ is greater or fewer than $\widehat{\lambda}$. And we will solve the closed-form solution of $\widehat{\lambda}$ after completing the analysis of the following two situations.

Under $0 < \lambda < \widehat{\lambda}$**: Comparing (NRM) and (RS)** By constraining $\lambda < \widehat{\lambda}$, we need to find $\widehat{\beta}$ that makes firm 0 indifferent between strategy (NRM) and (RS). When the cost of revelation is given to be low, revealing information to induce single-purchase gives (17), whereby concealing at multi-purchase gives

$$\max_{p_0} p_0 \widehat{x}_0(p_0, \beta) \Rightarrow \pi_0^M(A_0 = NR) = \frac{\beta^2}{4t}. \tag{20}$$

Then, firm 0's choice between case (NRM) and (RS) is determined by comparing (17) and (20):

$$\begin{cases} \frac{t}{2} - \lambda > \frac{\beta^2}{4t}, & \text{reveal at single-purchase (RS)} \\ \frac{t}{2} - \lambda < \frac{\beta^2}{4t}, & \text{conceal at multi-purchase (NRM)} \end{cases}$$

The payoff from the option (17) is decreasing in λ (for a given p_1^{NR}), whereby the payoff from option (20) is increasing in β. Hence, there exists a threshold of β (as a function of λ), that makes the two options equally profitable. That is

$$\underbrace{\pi_0^S(A_0 = R)}_{\text{reveal at single-purchase}} = \underbrace{\pi_0^M(A_0 = NR)}_{\text{conceal at multi-purchase}} \Rightarrow \beta = \widehat{\beta}(\lambda) \equiv \sqrt{2}\sqrt{t^2 - 2\lambda t}, \text{ given } \lambda < \widehat{\lambda}. \tag{21}$$

As long as $\lambda < \widehat{\lambda}$ and the action of concealing information are observed, firm 1's belief about β is updated such that firm 1 believes that β is distributed within the subset $[\widehat{\beta}(\lambda), \overline{\beta}]$, i.e., independence. Then firm 1 eliminates the possibility of single-purchase and solves

$$\begin{aligned} p_1^{NR}|_{\lambda<\widehat{\lambda},\beta\in[\widehat{\beta}(\lambda),\overline{\beta}]} =& \arg\max_{p_1} \underbrace{\int_{\widehat{\beta}(\lambda)}^{p_1+t} \frac{1}{\overline{\beta}-\widehat{\beta}(\lambda)} p_1\left(1-\widehat{x}_1(p_1,\beta)\right) d\beta}_{\text{multi-purchase:interior}} + \underbrace{\int_{p_1+t}^{\overline{\beta}} \frac{1}{\overline{\beta}-\widehat{\beta}(\lambda)} p_1 d\beta}_{\text{multi-purchase:boundary}} \\ \equiv& \frac{2}{3}\left(\sqrt{2}\sqrt{t^2-2\lambda t}-t\right)+\frac{1}{3}\sqrt{6\overline{\beta}t-4\lambda t+3t^2-8\sqrt{2}\sqrt{t^2-2\lambda t}} \end{aligned} \tag{22}$$

Because the price charged in (22) assigns zero probability in single-purchase, and hence let $p_1^{NRM} = p_1^{NR}|_{\lambda<\widehat{\lambda},\beta\in[\widehat{\beta}(\lambda),\overline{\beta}]}$ for short ("no-revelation at multi-purchase"). Notice that p_1^{NRM} is decreasing in λ and is increasing in $\overline{\beta}$, and evaluated at $\overline{\beta} = (5-2\sqrt{2})t$, $p_1^{NRM} = t$.

Under $\lambda > \widehat{\lambda}$: Comparing (NRS) and (NRM) Instead, when $\lambda > \widehat{\lambda}$, a high cost makes firm 0 never reveals for any β. Then, the scenario becomes similar to benchmark A. Firm 0 can choose to charge $p_0 = \frac{p_1^{NR}|_{\lambda>\widehat{\lambda}}+t}{2}$ to induce single-purchase which gives $\pi_0^S(A_0 = NR) = \frac{(p_1^{NR}|_{\lambda>\widehat{\lambda}}+t)^2}{8t}$, or to charge $\frac{\beta}{2}$ to induce multi-purchase that gives $\pi_0^M(A_0 = NR) = \frac{\beta^2}{4t}$. The above two options are equally profitable evaluated at $\beta = \widehat{\beta}$, where $\widehat{\beta}$ can be solved from

$$\underbrace{\pi_0^S(A_0 = NR) = \frac{(p_1^{NR}|_{\lambda>\widehat{\lambda}}+t)^2}{8t}}_{\text{conceal at single-purchase}} = \underbrace{\pi_0^M(A_0 = NR) = \frac{\widehat{\beta}^2}{4t}}_{\text{conceal at multi-purchase}}, \text{ given } \lambda > \widehat{\lambda}. \tag{23}$$

From the view of firm 1, $p_1^{NR}|_{\lambda>\widehat{\lambda}}$ is solved by

$$p_1^{NR}|_{\lambda>\widehat{\lambda},\beta\in[0,\overline{\beta}]} = \arg\max_{p_1} \int_0^{\widehat{\beta}} \frac{1}{\overline{\beta}} p_1\left(1-\widehat{x}^S(p_0,p_1)\right) d\beta + \int_{\widehat{\beta}}^{p_1+t} \frac{1}{\overline{\beta}} p_1\left(1-\widehat{x}_1(p_1,\beta)\right) d\beta + \int_{p_1+t}^{\overline{\beta}} \frac{1}{\overline{\beta}} p_1 d\beta. \tag{24}$$

Meanwhile, firm 0 also solves (18). Combining (24), (23) and (18), the three unknowns $p_1^{NR}|_{\lambda>\widehat{\lambda}}$, p_0^S and $\widehat{\beta}$ are solved as

$$\begin{aligned} &\widehat{\beta} = \widehat{\beta}^A,\ \lambda > \widehat{\lambda} \\ &p_1^{NR}|_{\lambda>\widehat{\lambda},\beta\in[t,\overline{\beta}]} = p_1^A, \\ &p_0^S = \frac{p_1^A+t}{2} \end{aligned} \tag{25}$$

which are equivalent to (16) derived in benchmark A. Therefore, benchmark A is a special case of benchmark R under $\lambda > \widehat{\lambda}$.

Plug (25) into (19), the critical value of the revelation cost which makes firm 0 indifferent between revealing or not at single-purchase has the explicit form:

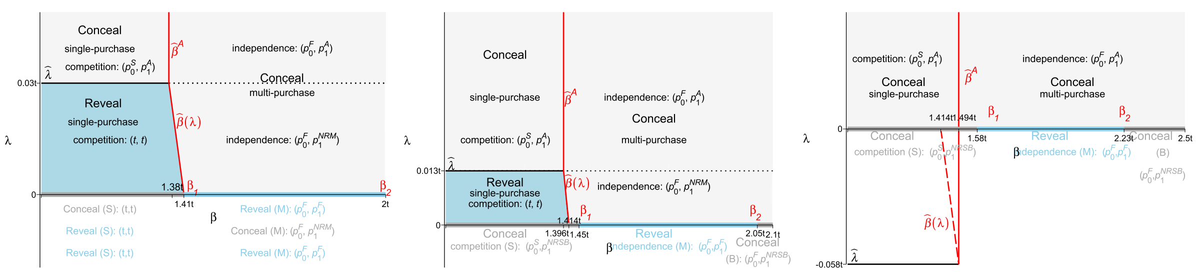
$$\widehat{\lambda} = \frac{t}{2} - \frac{(p_1^A+t)^2}{8t} \text{ (indifferent between (RS) and (NRS)).} \tag{26}$$

In (26), p_1^A is given by (15). Because $\overline{\beta} = (5-2\sqrt{2})t \Leftrightarrow p_1^A = t$, the cutoff value of revelation cost becomes $\widehat{\lambda} = 0$ evaluated at $\overline{\beta} = (5 - 2\sqrt{2})t$. That is, if $\overline{\beta} > (5 - 2\sqrt{2})t$ such that $\widehat{\lambda} < 0$, firm 0 never reveals any β unconditionally.

Combining the case when $0 < \lambda < \widehat{\lambda}$ (partial revelation), and the case when $\lambda > \widehat{\lambda}$ (no revelation or benchmark A), the equilibrium competition-independence threshold is chosen conditional on whether the revelation cost exceeds $\widehat{\lambda}$, and is expressed by

$$\widehat{\beta}^R \equiv \begin{cases} \widehat{\beta}^A,\ \lambda > \widehat{\lambda} \text{ (indifferent between (NRS) and (NRM))} \\ \widehat{\beta}(\lambda),\ \lambda < \widehat{\lambda} \text{ (indifferent between (RS) and (NRM))} \end{cases}, \tag{27}$$

where $\widehat{\beta}(\lambda)$ is given by (21). Also, we can verify that when $\lambda = \widehat{\lambda}$, $\widehat{\beta}^A$ and $\widehat{\beta}(\lambda)$ are identical. The threshold defined in (27) implies that evaluated at $\beta = \widehat{\beta}^R$, firm 0 is indifferent between charging a price to induce single-purchase and charging a price to induce multi-purchase. In particular, when $\lambda < \widehat{\lambda}$ and $\beta = \widehat{\beta}(\lambda)$, firm 0 is indifferent between revealing β to induce single-purchase, and concealing β at multi-purchase. When $\lambda = \widehat{\lambda}$ and $\beta < \widehat{\beta}^R$, firm 0 is indifferent between revealing β at single-purchase and concealing β at single-purchase. When $\lambda > \widehat{\lambda}$, firm 0 never reveals and the equilibrium reduces to benchmark A.



(a) $\overline{\beta} = 2t \Leftrightarrow \widehat{\lambda} = 0.0305t$ (b) $\overline{\beta} = 2.1t \Leftrightarrow \widehat{\lambda} = 0.013t$ (c) $\overline{\beta} = 2.5t \Leftrightarrow \widehat{\lambda} = -0.058t$

Figure 4: Equilibrium Revelation and Pricing

Proposition 3 (Equilibrium under Strategic Revelation with Positive Cost)**.** *Consider $\lambda > 0$. When $\beta \geq \widehat{\beta}^R$, firm 0 does not reveal β and charges a price to induce multi-purchase equilibrium. When $\beta < \widehat{\beta}^R$, firm 0 reveals β if and only if $\lambda < \widehat{\lambda}$ to induce single-purchase equilibrium. That is,*

(1) At stage 1, firm 0's equilibrium revelation action is

$$A_0^* = \begin{cases} R,\ \beta < \widehat{\beta}(\lambda), \lambda < \widehat{\lambda} \\ NR,\ \textit{otherwise} \end{cases}.$$

If firm 0 chooses to conceal, firm 1's belief is updated as

$$\mu_1 \in \{\beta | NR\} = \begin{cases} U\left[\widehat{\beta}(\lambda), \overline{\beta}\right],\ 0 < \lambda < \widehat{\lambda} \\ U\left[0, \overline{\beta}\right], \qquad \lambda > \widehat{\lambda} \end{cases},$$

where $\widehat{\beta}(\lambda)$ *is given by* (21)*, and* $\widehat{\lambda}$ *is given by* (26)*.*

(2) At stage 2, the equilibrium prices are offered as

$$(p_0^*, p_1^*) = \begin{cases} (t, t), & 0 \leq \beta < \widehat{\beta}(\lambda), 0 < \lambda < \widehat{\lambda} \\ \left(\frac{p_1^A + t}{2}, p_1^A\right), & 0 \leq \beta < \widehat{\beta}^A, \lambda > \widehat{\lambda} \\ \left(\frac{\beta}{2}, p_1^{NRM}\right), & \widehat{\beta}(\lambda) \leq \beta \leq 2t, 0 < \lambda < \widehat{\lambda} \\ \left(\frac{\beta}{2}, p_1^A\right), & \widehat{\beta}^A \leq \beta \leq 2t, \lambda > \widehat{\lambda} \\ \left(\beta - t, p_1^A\right), & 2t \leq \beta \leq \overline{\beta} \end{cases},$$

where p_1^{NRM} *is give by* (22)*,* $\widehat{\beta}^A = \frac{p_1^A + t}{\sqrt{2}}$ *and* p_1^A *is given by* (15)*.*[34]

The optimal data sharing strategy can be intuitively seen in Figure 4, where the informed seller chooses to reveal (resp., conceal) the information when the combination of the two parameters β and λ falls into the blue-shaded (resp., gray-shaded) region.[35]

One technical feature regarding the competition-independence threshold $\widehat{\beta}(\lambda)$ is that it is decreasing in the data sharing cost. To see this, consider the scenario where the true information is $\beta = 1.4t < \sqrt{2}t$. If data is free to be shared, then the informed seller is willing to reveal to avoid a low price from the rival and then they compete in prices. Fixing $\beta = 1.4t$, now assume that the cost of sharing data increases, which makes revelation at single-purchase less profitable. Then, the informed seller needs to compare the following two options: conceal the information but still compete in single-purchase; or price independently to induce multi-purchase. However, without reminding the rival about the true information, the former choice induces a low price and an intensified competition, and hence the latter choice becomes more attractive.

Another interesting feature in our equilibrium revelation strategy is that the asymmetric information equilibrium (benchmark A) studied previously serves as a special case here. That is, it could be possible that although the informed seller can choose to reveal or not, he/she chooses never to do so at any state. This happens when consumers' demand for multi-purchase is possible to be drawn within a sufficiently large interval such that the price-raising effect dominates the demand-expansion effects, then such prior belief itself is sufficient to support a high price. We have shown that p_1^A is increasing in $\overline{\beta}$, and when $\overline{\beta}$ exceeds a threshold, p_1^A becomes high enough to make the payoff of the informed seller greater than that if the information is revealed under competition. In that case, the informed seller never reveals information even

[34] In Appendix A, we show the uniqueness and check the necessary conditions for Proposition 3. The best-price-reply correspondences for parametric values associated with Figure 4 are plotted in Figure 8. Sub-graphs 8a, 8b and 8c have been shown to be equivalent to benchmark A at $\lambda > \widehat{\lambda}$. The curves of best-price-reply correspondences plotted in sub-graphs 8d and 8e include only the "off-equilibrium" candidate strategies evaluated around the neighborhood of the critical value $\widehat{\lambda}$; and 8f, 8g and 8h list the candidate responses around the neighborhood of the critical value $\widehat{\beta}(\lambda)$.

[35] The equilibrium outcomes along the horizontal line are refined in Appendix A.2.

when the cost of doing so is small (see Figure 4c). The minimum value of $\overline{\beta}$ that makes the informed seller conceal for all β irrespective of the cost levels, is determined by letting $\widehat{\lambda} = 0$ in equation (26), which implies $\overline{\beta} = (5 - 2\sqrt{2})t$.

Corollary 1. *Firm 0 never reveals for any β when $\lambda > \widehat{\lambda}$. $\widehat{\lambda}$ is decreasing in $\overline{\beta}$. In particular, for $\overline{\beta} > (5 - 2\sqrt{2})t$ such that $\widehat{\lambda} < 0$, then the equilibrium stated in Proposition 3 reduces to that under benchmark A, where the competition-independence threshold becomes $\widehat{\beta}^A$ and $p_1^{NR} = p_1^A$.*

In summary, we can characterize the information sharing incentives as:

(i) The informed seller is willing to reveal the information for an upcoming competition in order to prevent the rival from charging a low price, and the information will be revealed provided that the cost of doing so is low enough.

(ii) The informed seller chooses to conceal the information when he/she finds it optimal to keep independent, or when the informed seller is willing to reveal for an upcoming competition but doing so is too costly.

(iii) When the prior belief of the uninformed rival supports a price that is high enough, the informed seller never reveals any information.

6 Welfare Analysis

In this section, we investigate socially optimal information structure in data regulation by comparing the equilibrium welfare under benchmark A, R, and F. We consider two types of comparisons: (1) For a given β, which benchmark gives the highest welfare? (2) For a given $\overline{\beta}$, which benchmark gives the highest expected welfare? The candidate policies are: full access (full information F), unequal access or preferential treatment (asymmetric information A) or the market-based solution (strategic revelation R) whereby manipulating the sharing cost λ.

6.1 Total Surplus

Let W^j be total surplus under benchmark $j = F, A, R$, which is the sum of the consumer surplus and firms' profits. Because the price paid by a buyer is equal to the amount received by the seller, hence W^j can be alternatively expressed as the sum of: consumers' reservation values (and the benefits from multi-purchase, if any), total travel disutility from differentiation, then minus the revelation cost (if any). Appendix B provides the detailed expressions of W^j.

(1) Under single-purchase equilibrium (S), the volume of transactions is fixed to be 1. Therefore, without considering revelation cost, the remaining term that affects total surplus is the sum of travel disutility, which is minimized if prices are symmetric: $\widehat{x}^S = \frac{1}{2} \Leftrightarrow p_0 = p_1$.[36]

[36] See equation (B.1) in Appendix B.

(2) For the interior case of multi-purchase (M), the market for the additional consumption of a different product $\widehat{x}_0(p_0, \beta) - \widehat{x}_1(p_1, \beta)$, is not fully covered, and therefore, a lower price sum means a greater market coverage:

$$\text{a higher } \widehat{x}_0(p_0,\beta) - \widehat{x}_1(p_1,\beta) \Leftrightarrow \text{a lower } p_0 + p_1, \text{ and } p_0^j + p_1^j = \begin{cases} \beta, & j = F \\ \frac{\beta}{2} + p_1^A, & j = A \\ \frac{\beta}{2} + p_1^{NRM}, & j = R \end{cases} . \quad (28)$$

When all consumers make multi-purchase (B), both the market coverage and the travel disutilities are fixed.

(3) Total surplus under multi-purchase equilibrium is higher than that under single-purchase equilibrium. That is, consider two benchmarks j and j' (where $j, j' \in \{F, A, R\}$), then $W^j > W^{j'}$ evaluated at $\widehat{\beta}^j < \beta < \widehat{\beta}^{j'}$.

Therefore, within the multi-purchase equilibrium, the problem of comparing equilibrium welfare can be reduced to a problem of comparing equilibrium prices. For comparing single- and multi-purchase equilibrium, we need to use the equilibrium competition-independence thresholds. The following subsection provides rankings of prices and thresholds.

6.2 Comparisons: Equilibrium Prices and Thresholds

In equation (22), p_1^{NRM} is decreasing in λ and is increasing in $\overline{\beta}$, and is reduced to p_1^A for $\overline{\beta} \geq (5 - 2\sqrt{2})t$. Using the closed-form solutions, we can verify that as λ approaches to zero, $p_1^{NRM} < p_1^A$.[37] Therefore, partial revelation gives the lowest price for the uninformed seller. The orders of prices are shown in Figure 5a and associated thresholds are shown by Figure 5b.[38]

Proposition 4 (Price Comparison)**.** *For any $\overline{\beta} \geq 2t$ and $\lambda > 0$, $p_1^{NRM} \leq p_1^A$.*

For comparing the competition-independence thresholds, using the results obtained above, we have

Proposition 5 (Competition-independence Threshold Comparison)**.** *When $2t \leq \overline{\beta} \leq (5 - 2\sqrt{2})t$, then $\widehat{\beta}^A \leq \widehat{\beta}(\lambda) \leq \sqrt{2}t$. When $\overline{\beta} > (5 - 2\sqrt{2})t$, then $\sqrt{2}t < \widehat{\beta}^A$.*[39]

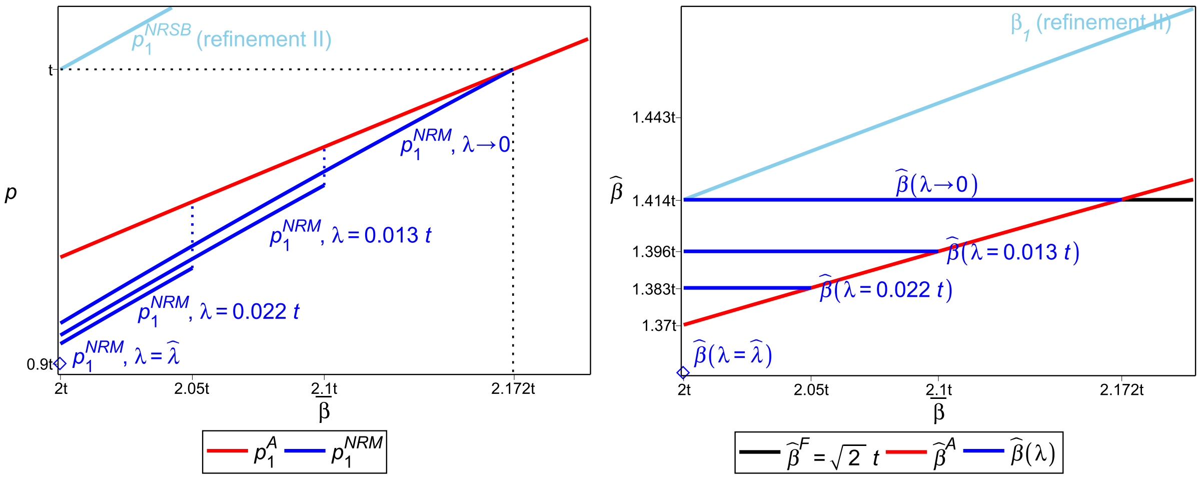
6.3 Preferential Treatment (Benchmark A)

The market is realized when consumers with a true β make purchase decisions according to the equilibrium prices given by Proposition 1, and the equilibrium thresholds are given by

[37] The limiting case for $\lambda \to 0^+$ refers to refinement I in Appendix A.2. We can show that the other refinement expressed in Proposition 9 gives the highest price for firm 1 (by construction), and hence is dominated by other benchmarks in comparing welfare.

[38] When $\lambda > \widehat{\lambda}$, and firm 0 conceals for all β, then p_1^{NRM} jumps up to p_1^A.

[39] For comparing the equilibrium no single-purchase thresholds, not that if firm 1 is not provided with information, then $\widehat{x}_1 = 0 \Leftrightarrow \widetilde{\beta} = p_1 + t$. Then, the relative positions of $\widetilde{\beta}$s are consistent with the rankings of prices stated in Proposition 4.



(a) Equilibrium prices

(b) Comeptition-independence thresholds

Figure 5: Equilibrium Prices and Thresholds in Benchmark A and R

Proposition 2. The true indifferent margins are $\widehat{x}^S(p_1^A, p_0^S)$, $\widehat{x}_0(p_0^F, \beta)$ and $\widehat{x}_1(p_1^A, \beta)$, which are obtained by plugging the equilibrium prices into (2), (3) and (4), respectively. In particular, when $\beta > \widehat{\beta}^A$, firm 0's prices are identical to those under benchmark F. If $\widehat{x}_0(p_0^F, \beta) \geq 1$ and $\widehat{x}_1(p_1^A, \beta) \leq 0$, every consumer chooses multi-purchase (boundary case). The following proposition summarizes the comparison between W^A and W^F (see Appendix A.3 for details).

Proposition 6 (Compare W^A and W^F)**.** *Total surplus under benchmark A is higher than that under benchmark F if and only if $2t \leq \overline{\beta} < (5 - 2\sqrt{2})t$, $\beta \in (2p_1^A, 2t)$ and $\beta \in (\widehat{\beta}^A, \sqrt{2}t)$.*

Compared with full information, within the multi-purchase equilibrium, a lower price charged by the uninformed seller gives rise to a greater market coverage (when $p_1^A < \frac{\beta}{2}$). Besides, evaluated at $\widehat{\beta}^A < \beta < \widehat{\beta}^F$, multi-purchase is realized in benchmark A while no consumer makes multi-purchase in benchmark F.

6.4 Market-Based Solution (Benchmark R)

When $\overline{\beta} > (5 - 2\sqrt{2})t$ or $\lambda > \widehat{\lambda}$, benchmark R reduces to benchmark A, then the corresponding comparisons between W^R and W^F are identical to those stated in Proposition 6. When it is possible for firm 0 to reveal at equilibrium, single-purchase in benchmark R incurs a cost, whereby at multi-purchase, a lower p_1^{NRM} is more socially desirable.

Proposition 7 (Compare W^R and W^F)**.** *When $\overline{\beta} > (5 - 2\sqrt{2})t$, benchmark R reduces to benchmark A and $W^R \leq W^F$ for all β and λ. When $\overline{\beta} < (5 - 2\sqrt{2})t$, benchmark R gives a higher total surplus than that under benchmark F at (1) $\beta \in (\widehat{\beta}^A, \sqrt{2}t)$ where $\lambda^{-1}(\widehat{\beta}) < \lambda < \widehat{\lambda}$; and (2) $\beta \in (2p_1^{NRM}, 2t)$.*

Proposition 6 and 7 can be summarized by Figure 6.

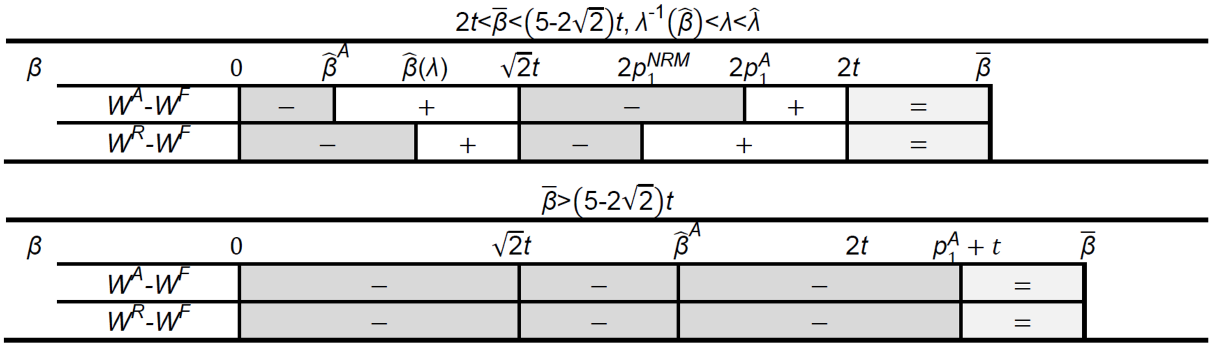


Figure 6: Total surplus: benchmarks F, A and R

The next question is which information structure gives the highest total surplus, for a given β. By Proposition 6 and 7, when the upper bound of complementarity exceeds the threshold $(5-2\sqrt{2})t$, the regulator should make the data public (Figure 12 in Appendix B).

Corollary 2. *Total surplus under incomplete information (i.e., benchmark A and R) can never exceed that under full information (benchmark F) if* $\overline{\beta} > (5-2\sqrt{2})t$.

The comparison for W^F, W^A and W^R for the case when $\overline{\beta} < (5-2\sqrt{2})t$ can be conducted by using a similar logic described above. More details and numerical examples are provided in Appendix A.3 and Appendix B.

6.5 Comparing the Expected Total Surplus

The above comparisons provide normative implications when a particular β is given. However, as β changes, switching from one information structure to another might reveal some information. In practice, the data policy is typically fixed before a particular party observes any information about β. Hence in the following, we consider a more reasonable comparison by considering the expected value of total surpluses, given the upper bound $\overline{\beta}$.

Figure 7 plots the expected total surplus under benchmark F, A, R (including two refinements) as functions of $\overline{\beta}$ and λ, where $\overline{\beta} \geq 2t$ and $0 \leq \lambda \leq \widehat{\lambda}$. It shows that the expected welfare keeps to be the highest when both sellers are fully informed. With full information, as β exceeds the competition-independence threshold, prices of both sellers jump downward, and the equilibrium switches from single- to multi-purchase, giving rise to a huge upward-jump in welfare, whereby with incomplete information, the price of the uninformed seller keeps to be constant (the dashed-lines in Figure 3), restricting the market-expansion effect.[40] More detailed illustrations with numerical examples are provided in Appendix B.

[40]Surprisingly, when the data sharing cost is strictly zero, the second refinement (see Proposition 9 in Appendix A.2), although induces the highest price of the uninformed seller (see Proposition 4), gives the highest expected welfare among the benchmarks with incomplete information. In that refinement, the informed seller reveals the information within the interior case of multi-purchase, and hence as β exceeds the competition-independence threshold, prices of both sellers jump downward like the scenario under full information.

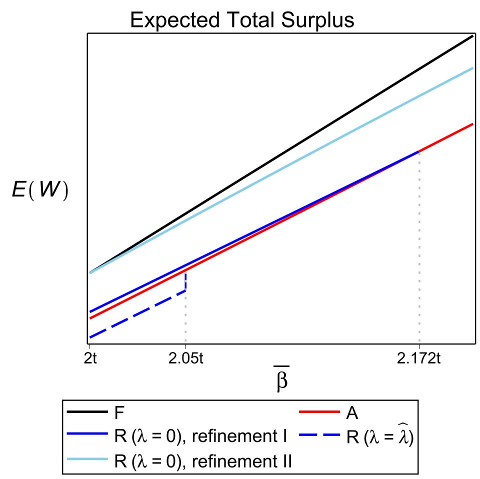


Figure 7: The Expected Total Surplus

Therefore, if the regulator is going to maximize the expected total surplus by picking one of the above information structures, the answer is full information, i.e., letting duopolists get full and equal access to the information about their interdependence.

Proposition 8. *Full information gives the highest expected total surplus for any $\overline{\beta} \geq 2t$.*

7 Concluding Remarks

The definition of market power and the boundary of competition exhibit varying forms in the digital era, especially in markets supplying complementary services. The concept of competition, thereof, deserves a broadened interpretation that is not limited to consider the interactions within the intensive measure only, but specifying the extensive, relationship-sensitive margin is equally important. Without the information about interdependence, it is problematic to choose a right price conditional on a specific relationship. We extend the horizontally differentiated duopoly model by introducing asymmetric information about firms' interdependence, and provide characterizations of the equilibrium pricing strategies. In addition, we also solve the equilibrium revelation and pricing strategies when the informed firm can choose to reveal verifiable information selectively to its rival. Our work can extend our knowledge in better understanding the structure of competition in a more comprehensive way.

By introducing the incomplete information about competition vs. independence, our results provide an additional angle to understand the relationship between competition vs. cooperation, and the conflict of interests between a platform and its complementors. For example, under certain conditions, reminding the uninformed rival about the true information that they are going to compete results in a win-win situation. In that sense, price competition and "cooperation by truthful reporting" coexist, which suggests a new information management strategy for platform data services.

It goes without saying that the current model captures only a tiny part of the overall picture in terms of market strategies, information management and welfare. For instance, the platforms' motives for providing data services are not restricted to competition incentives as we addressed, but also include attracting more business users and optimize revenue management. In addition, if dynamics and sequential strategies are taken into account, the demand information will be revealed in a different way that allows sellers to adopt price discrimination based on histories — itself a hotly debated topic (Choe, King and Matsushima, 2018; Conitzer, Taylor and Wagman, 2012; Fudenberg and Villas-Boas, 2006; Rhee and Thomadsen, 2017; Taylor, 2004). Nevertheless, we provide normative implications about the role of relationship-sensitive information in affecting interdependence between simultaneous movers, and offer distinct explanations and new insights for platform information strategies and relevant regulations.

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Appendix A Proofs of Main Results

A.1 Full and Asymmetric Information

Proof of Lemma 1. First, we briefly summarize the proof for Lemma 1 where the details can be seen in Kim and Serfes (2006) or Jeitschko, Jung and Kim (2017). Then, we refine the equilibrium around the relationship-sensitive margin $\widehat{\beta}^F = \sqrt{2}t$.

The equilibrium can be obtained by checking the intersections between the best responses given by (8). Note that, (8) is a piecewise function contingent upon a threshold of the rival's price $\sqrt{2}\beta - t$. Plug the threshold into the best response $p_i^S = \frac{p_{-i}+t}{2}$, then $p_i^S|_{p_{-i}=\sqrt{2}\beta - t} = \frac{\sqrt{2}}{2}\beta > \frac{\beta}{2}$. That is, when the rival's price is below $\sqrt{2}\beta - t$, the best response is a constant $\frac{\beta}{2}$, i.e., independence. When the rival charges a price that is slightly above $\sqrt{2}\beta - t$, firm i's price jumps up to $\frac{\sqrt{2}}{2}\beta$. And when the rival further increases its price, firm i also increases its price (strategic complements). Hence the intersections are determined by the relative positions of $\sqrt{2}\beta - t$, $\frac{\beta}{2}$ and $\frac{\sqrt{2}}{2}\beta$.

When $\sqrt{2}\beta - t < \frac{\beta}{2} < \frac{\sqrt{2}}{2}\beta$, or $\beta < \frac{2}{2\sqrt{2}-1}t$ holds, there is a unique intersection (t, t) occurred on the first lines of (8).

When $\frac{\beta}{2} < \sqrt{2}\beta - t < \frac{\sqrt{2}}{2}\beta$, there are two intersections. For the first lines of (8), the intersection is (t, t), which gives the payoff $\frac{t}{2}$; For the second lines of (8), the intersection is $\left(\frac{\beta}{2}, \frac{\beta}{2}\right)$ which gives payoff $\frac{\beta^2}{4t}$. Evaluated at $\sqrt{2}\beta - t < \frac{\sqrt{2}}{2}\beta \Leftrightarrow \beta < \sqrt{2}t$, the former option is more profitable for both sellers, and is survived in this case.

When $\frac{\beta}{2} < \frac{\sqrt{2}}{2}\beta < \sqrt{2}\beta - t$, or $\beta > \sqrt{2}t$ holds, the unique intersection is $\left(\frac{\beta}{2}, \frac{\beta}{2}\right)$.

Refinement at $\beta = \sqrt{2}t$ Evaluated at the threshold $\beta = \sqrt{2}t$, both single-purchase (requiring to charge t) and multi-purchase (requiring to charge $\frac{\beta}{2}|_{\beta=\sqrt{2}t} = \frac{\sqrt{2}}{2}t < t$) are equally profitable. Hence we shall provide a non-cooperative solution for an optimal interim strategy taken at the threshold.

When both firms charge the same price (either $\frac{\sqrt{2}}{2}t$ or t), then the payoffs of both are $\frac{t}{2}$. Now assume that one of them (say, firm 1) charges $p_1^M = \frac{\sqrt{2}t}{2}$ whereby the other (firm 0) charges $p_0^S = t$. Then the relative positions of demand margins are

$$\widehat{x}_1\left(\frac{\sqrt{2}t}{2}, \sqrt{2}t\right) = \frac{2-\sqrt{2}}{2} \approx 0.293 < \widehat{x}_0\left(t, \sqrt{2}t\right) = (\sqrt{2}-1) \approx 0.414.$$

That is, multi-purchase is realized. Then firm 1's payoff is still $\frac{t}{2}$ but firm 0's payoff is $t \cdot \widehat{x}_0 \approx 0.414t < 0.5t$. Therefore, when $\beta = \sqrt{2}t$, charging $p_0^S = t$ is dominated by charging a lower price at $p_i^M = \frac{\beta}{2} = \frac{\sqrt{2}t}{2} < t$.

When two sellers charge different prices around $\beta = \sqrt{2}t$, one for independence while the other for competition, then the seller who charges a higher price (competition) suffers a loss due to the "business stealing effect."

□

Supplemental Proof for Proposition 1. The first-order condition of (14) is given by

$$p_1^{BR}(p_0) = \frac{\widehat{\beta}^A - 2t}{3} + \frac{1}{3}\sqrt{3\widehat{\beta}^A p_0 + t^2 + 6\overline{\beta}t - \widehat{\beta}^A t - 2(\widehat{\beta}^A)^2}. \tag{A.1}$$

p_1^A, p_0^S and $\widehat{\beta}^A$ are solved from (10), (13) and (A.1). The explicit solutions are given by

$$\begin{aligned} p_1^A(\overline{\beta}) &= \frac{3\sqrt{2}-50}{73}t + \frac{1}{511}\sqrt{(138\sqrt{2}+547)\left((44\sqrt{2}+344)\overline{\beta}+49t\right)t}. \\ \widehat{\beta}^A(\overline{\beta}) &= \frac{p_1^A + t}{\sqrt{2}} \\ p_0^S(\overline{\beta}) &= \frac{p_1^A + t}{2} \end{aligned} \quad . \tag{A.2}$$

In the remaining, we complete the proof by showing that evaluated at asymmetric equilibrium (A.2), the necessary conditions (the relative positions of indifference margins (3) and (4) are consistent with the equilibrium outcome) are satisfied.

When $\beta < \widehat{\beta}^A$, single-purchase is more profitable for firm 0, whereby no consumers should choose multi-purchase evaluated at prices given by (16), or $\widehat{x}_1\left(p_1^A, \beta\right) > \widehat{x}_0\left(p_0^S, \beta\right)$, which is equivalent to

$$\beta < \frac{9\sqrt{2}+69}{292}t + \frac{3}{2044}\sqrt{(138\sqrt{2}+547)\left((44\sqrt{2}+344)\overline{\beta}+49t\right)t}. \tag{A.3}$$

It can be confirmed that for any $\overline{\beta} \geq 2t$, (A.3) is implied by $\beta < \widehat{\beta}^A$.

When $\beta \geq \widehat{\beta}^A$, multi-purchase equilibrium is more profitable for firm 0, whereby a positive number of consumers choose multi-purchase evaluated at prices given by (16), or $\widehat{x}_1\left(p_1^A, \beta\right) < \widehat{x}_0\left(\frac{\beta}{2}, \beta\right)$, which is equivalent to

$$\beta > \frac{2(3\sqrt{2}+23)}{219}t + \frac{2}{1533}\sqrt{(138\sqrt{2}+547)\left((44\sqrt{2}+344)\overline{\beta}+49t\right)t}. \tag{A.4}$$

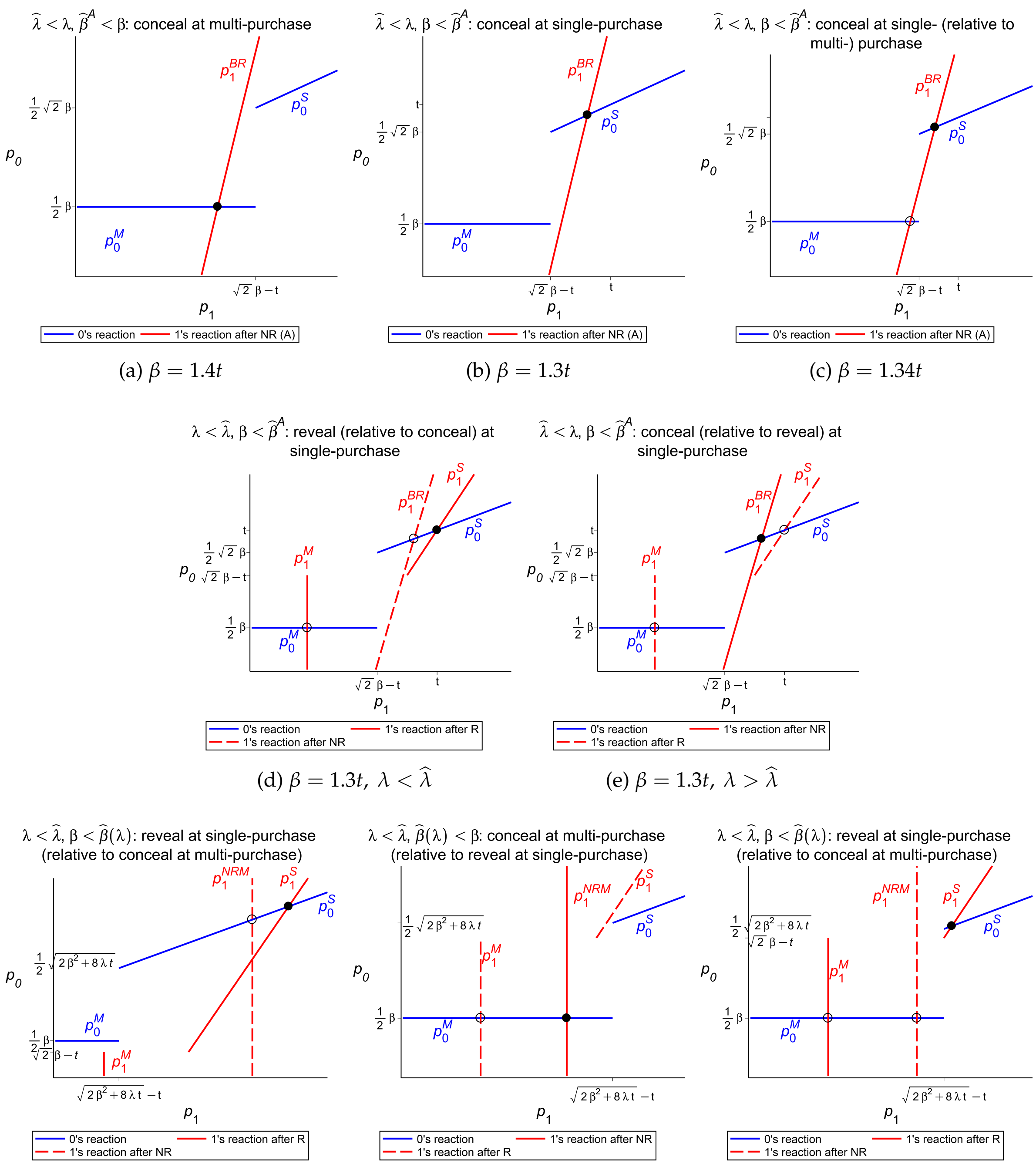
Evaluated at $\beta \geq \widehat{\beta}^A$, condition (A.4) holds. □

A.2 Strategic Revelation: Proofs and Extensions

A.2.1 Supplemental Proofs for Positive Revelation Cost

Supplemental Proof for Proposition 3. First we verify that the relative positions of $\widehat{x}_1$ and $\widehat{x}_0$ are consistent with the pricing equilibrium (necessary conditions). Then, we show the uniqueness of the equilibrium by checking the number of intersections between the best-price-reply correspondences shown by Figure 8.

When $\lambda > \widehat{\lambda}$, the process is equivalent to the proof of Proposition 1. Next, when $\lambda < \widehat{\lambda}$, the critical value between single- and multi-purchase is $\widehat{\beta}(\lambda)$. For $\beta < \widehat{\beta}(\lambda)$, single-purchase



(a) $\beta = 1.4t$

(b) $\beta = 1.3t$

(c) $\beta = 1.34t$

(d) $\beta = 1.3t,\ \lambda < \widehat{\lambda}$

(e) $\beta = 1.3t,\ \lambda > \widehat{\lambda}$

(f) $\beta = 1.05t,\ \lambda = 0.03t$

(g) $\beta = 1.386t,\ \lambda = 0.03t$

(h) $\beta = 1.386t,\ \lambda = 0.01t$

Figure 8: Best responses under asymmetric information & revelation: $\overline{\beta} = 2t$, $\widehat{\lambda} = 0.0305t$, $\widehat{\beta}^A = 1.37t$

is realized, which requires $\widehat{x}_1(t,\beta) > \widehat{x}_0(t,\beta)$, or equivalently, $\beta < \frac{3}{2}t = 1.5t$. Note that, since $\widehat{\beta}'(\lambda) < 0$, combining $\widehat{\beta}(\lambda)|_{\lambda=0} = \sqrt{2}t < 1.5t$, the condition $\beta < \frac{3}{2}t$ is satisfied evaluated at $\beta < \widehat{\beta}(\lambda)$ for all λ.

For $\beta > \widehat{\beta}(\lambda)$, multi-purchase is realized, which requires that $\widehat{x}_1\left(p_1^{NRM},\beta\right) < \widehat{x}_0\left(\frac{\beta}{2},\beta\right)$, or equivalently,

$$\beta > \frac{4\sqrt{2}\sqrt{t^2-2\lambda t}+2}{9}t + \frac{2}{9}\sqrt{6\overline{\beta}t + 3t^2 - 4\lambda t - 8\sqrt{2}t\sqrt{t^2-2\lambda t}}. \tag{A.5}$$

The proof will be completed if the right-hand-side of (A.5) is shown to be lower than $\widehat{\beta}(\lambda)$ for any $\lambda < \widehat{\lambda}$. It is true indeed because the maximum value of the right-hand-side of (A.5) is lower than $\widehat{\beta}(\lambda)$. To see this, note that the right-hand-side of (A.5) is increasing in $\overline{\beta}$ and suppose that its maximum, i.e.,

$$\frac{4\sqrt{2}\sqrt{t^2-2\lambda t}+2}{9}t + \frac{2}{9}\sqrt{6(5-2\sqrt{2})t^2 + 3t^2 - 4\lambda t - 8\sqrt{2}t\sqrt{t^2-2\lambda t}}, \tag{A.6}$$

is greater than $\widehat{\beta}(\lambda)$. Since $\widehat{\beta}(\lambda)$ is decreasing in λ, and therefore $\widehat{\beta}(\lambda)$ is equal to (A.6) if and only if

$$\lambda = \frac{1}{2}\left(1 - \left(\frac{1}{7}\sqrt{142-56\sqrt{2}} - \frac{\sqrt{2}}{7}\right)^2\right)t \approx 0.067t,$$

evaluated at which, $\widehat{\beta}(\lambda) \approx 1.315t < \widehat{\beta}^A|_{\overline{\beta}=(5-2\sqrt{2})t} = \sqrt{2}t$, which implies $\lambda > \widehat{\lambda}$ that contradicts with the case considered within $\lambda < \widehat{\lambda}$, and hence condition (A.5) holds when $\beta > \widehat{\beta}(\lambda)$ and $\lambda < \widehat{\lambda}$.

Uniqueness of the Equilibrium In the remaining, we verify the uniqueness of the equilibrium expressed by Proposition 3. Figure 8 shows the number of intersections of best responses: 8a to 8c compare single- and multi-purchase for $\lambda > \widehat{\lambda}$, i.e., benchmark A; 8d and 8e compare (RS) and (NRS); 8f to 8h compare (RS) and (NRM), for different levels of β and λ.

First, consider the strategies taken around the neighborhood of $\widehat{\beta}^A$ when $\lambda > \widehat{\lambda}$, i.e., benchmark A in the first-line-graph in Figure 8. Given $\lambda > \widehat{\lambda}$, firm 0 will not reveal for any β. Without revelation, firm 1 responds by charging (A.1). Given p_1^{BR}, firm 0 has two options (around the neighborhood of $\widehat{\beta}^A$): (1) Charging $p_0^M = \frac{\beta}{2}$ independently (the constant part of firm 0's best response) to achieve multi-purchase; or (2) charging p_0^S as strategic complements (the upward sloping part of firm 0's best response). The two options are equally profitable evaluated at $p_1 = \sqrt{2}\beta - t$. Then, the intersection(s) between the best responses of duopolists could be divided into three cases:

(i) A unique intersection between p_0^M and p_1^{BR} as shown by Figure 8a: this corresponds to $p_1^{BR}\left(p_0 = p_0^S\right) = p_1^A < \sqrt{2}\beta - t \Leftrightarrow \beta > \widehat{\beta}^A$.

(ii) A unique intersection between p_0^S and p_1^{BR} as shown by Figure 8b: this corresponds to

$p_1^{BR}\left(p_0 = \frac{1}{2}\beta\right) > \sqrt{2}\beta - t \Leftrightarrow \beta < \widehat{\text{thres}}$, where

$$\widehat{\text{thres}} = \frac{22\sqrt{2}+3}{137}t + \frac{7}{6713}\sqrt{(132\sqrt{2}+977)\left((64\sqrt{2}+340)\overline{\beta}+49t\right)t}. \tag{A.7}$$

(iii) Two intersections, between p_0^M and p_1^{BR} as shown by the hollow circle in Figure 8c, and between p_0^S and p_1^{BR} shown by the solid circle in Figure 8c: this corresponds to $p_1^{BR}\left(p_0 = \frac{1}{2}\beta\right) < \sqrt{2}\beta - t < p_1^A$, or equivalently, $\widehat{\text{thres}} < \beta < \widehat{\beta}^A$, evaluated at which, the payoff under single-purchase (solid point) is higher.

Therefore, under $\lambda > \widehat{\lambda}$, the competition-independence threshold is $\beta = \widehat{\beta}^A$, i.e., the condition which makes firm 0 indifferent between choosing Figure 8a and 8c.

Next, consider the strategies taken around the neighborhood of $\widehat{\lambda}$ within single-purchase, i.e., fixing a particular β, comparing the payoff of (RS) for a low cost $\lambda < \widehat{\lambda}$ as shown by 8d, and (NRS) for a high cost $\lambda > \widehat{\lambda}$ as shown by 8e. When firm 0 chooses to reveal, the dashed line in Figure 8d represents firm 1's best response (expressed by (A.1)) taken at the "off-equilibrium path" (conceal); similarly, when firm 0 chooses to conceal, the dashed lines in Figure 8e become firm 1's actions under firm 0's off-equilibrium path strategy (reveal).

(i) If firm 0 decides to reveal, then according to Lemma 1, both firms respond by charging the first line of (8). Evaluated at $\beta < \sqrt{2}t$, there could be a unique intersection at single-purchase (the solid circle in Figure 8d), or multiple intersections including multi-purchase (hollow circles in Figure 8e). We have shown in Lemma 1 that within the range of a low β, single-purchase is more profitable (hence the hollow circle for multi-purchase is eliminated). The payoff of firm 0 is $\frac{t}{2} - \lambda$.

(ii) If firm 0 chooses to conceal, the best price response of firm 1 becomes (A.1), where the associated equilibrium is expressed as the solid cross-over point in Figure 8e.[41] The payoff of firm 0 is given by equation (18).

Because firm 0 can choose to reveal or conceal, whereby the payoff from the former option is decreasing in λ, and hence firm 0 is indifferent between Figure 8d and 8e if and only if $\lambda = \widehat{\lambda}$ as shown by equation (19).

Finally, consider the strategies taken around the neighborhood of $\widehat{\beta}(\lambda)$ when $\lambda < \widehat{\lambda}$. Fixing a cost λ that is relatively high (but is still lower than $\widehat{\lambda}$), Figure 8f shows a unique intersection under (RS) when β is low; when β becomes higher, the intersection occurs at (NRM). In particular, for $\lambda < \widehat{\lambda}$ and when concealment is observed, firm 1 updates its belief to $\beta > \widehat{\beta}(\lambda)$, and then the best response becomes a constant. Fixing the same β in 8g, if λ is reduced such that $\beta < \widehat{\beta}(\lambda)$, then it is optimal to choose (RS) as shown in Figure 8h. In particular, the parametric values associated with Figure 8g satisfy $p_1 < \sqrt{2\beta^2 + 8\lambda t} - t$, which is transformed from $\frac{(p_1+t)^2}{8t} - \lambda < \frac{\beta^2}{4t}$, and firm 0's payoff is $\frac{\beta^2}{4t}$. In Figure 8f or 8h, (RS) gives $\frac{t}{2}$,

[41] Note that, (A.1) and the best responses of firm 0 can also intersect twice, and we have analyzed those cases in benchmark A.

and hence firm 0 is indifferent between (RS) and (NRM) provided that $\beta = \widehat{\beta}(\lambda)$ expressed by (21). The solid intersection in Figure 8f or 8h is valid if the point locates at the right-side of $\sqrt{2\beta^2 + 8\lambda t} - t$, or equivalently, $t > \sqrt{2\beta^2 + 8\lambda t} - t \Leftrightarrow \beta < \sqrt{2}\sqrt{t^2 - 2\lambda t} = \widehat{\beta}(\lambda)$, i.e., choosing (RS) is consistent with the condition $\beta < \widehat{\beta}(\lambda)$. The solid intersection at Figure 8g is valid provided that $p_1 < \sqrt{2\beta^2 + 8\lambda t} - t$, which is true because within $\overline{\beta} < (5 - 2\sqrt{2})t$, $p_1 < t \Rightarrow \frac{(p_1+t)^2}{8t} - \lambda < \frac{t}{2} - \lambda$, and if firm 0 prefers the intersection at Figure 8g to those in 8f or 8h, it must be that $\frac{t}{2} - \lambda < \frac{\beta^2}{4t}$, such that $\frac{(p_1+t)^2}{8t} - \lambda < \frac{\beta^2}{4t} \Leftrightarrow p_1 < \sqrt{2\beta^2 + 8\lambda t} - t$ holds.

Combining the above cases, neither firm has an incentive to deviate (from one to another graph in Figure 8) provided that all the conditions stated in Proposition 3 hold simultaneously.

□

A.2.2 Refinements of Equilibrium Revelation with Zero Cost

When the cost of revealing the information is strictly zero ($\lambda = 0$), there is no restriction on the choice of the informed seller when duopolists are independent. In this case, there are infinitely many possibilities, including pure-strategy and mixed-strategy equilibria. For example, the limiting case of Proposition 3, evaluated at $\lambda = 0$, is an equilibrium. However, except such limiting case, there exist other equilibria, e.g., since firm 0 becomes indifferent between revealing and concealing β at $\beta \geq \widehat{\beta}^R$, firm 0 can choose to randomize between revealing and concealing when a particular β is drawn at $\beta \in [\widehat{\beta}^R, \overline{\beta}]$, and consequently, firm 1's belief is not unique, depending on the "types" of firm 0's revelation behavior.

One possible refinement for the limiting case of Proposition 3 is by assuming that when firm 0 is indifferent between revealing and concealing, then let firm 0 conceal. Then, Proposition 3 is valid for all $\lambda \geq 0$, including $\lambda = 0$. That is, the revelation strategies taken at the horizontal axis in Figure 4 are consistent with the limits of those in the shaded regions located around the neighborhood of the horizontal axis.

Corollary 3 (Refinement I: Equilibrium Revelation with Zero Cost)**.** *If firm 0 chooses to conceal rather than reveal when firm 0 is indifferent between revealing and concealing, then Proposition 3 is valid for all $\lambda \geq 0$.*

In the following, we propose an alternative refinement under $\lambda = 0$ by considering whether firm 0 can do better by selectively reveal some β at multi-purchase, and assume that such profit-seeking behavior (i.e., firm 0's type) looks reasonable for firm 1, and therefore can be supported by firm 1's belief.

According to Lemma 2, firm 0 reveals (resp., conceals) β at single-purchase if and only if the rival's price p_1^{NR} is induced to be below (resp., above) t. Actually, in addition to simply comparing p_1^{NR} and t, firm 0's profit maximization problem is equivalent to maximizing the rival's price p_1^{NR} (under price competition). Although firm 0 can choose to ignore the issue after β is somehow realized to be high such that keeping independent is optimal, still, the "whole plan" of firm 0's revelation strategy that is going to be solved is a "credible threat" to firm 1 who believes that the plan is ready to be played as long as single-purchase is potentially feasible (i.e., the lower bound of β is 0 such that single-purchase is possible to occur).

To locate the intervals that support higher prices, recall that, both the "competition" and "price-raising" effects result in higher prices than those under the "demand-expansion" effect. More precisely, in benchmark F, the equilibrium price is charged above t when β is below $\sqrt{2}t$ or above $2t$ (see Figure 2). Hence, a higher price can be induced by "eliminating" the possibility of "demand-expansion" effect in firm 1's belief, i.e., reveal β only when β is of a moderate size. Precisely, let firm 0 conceals β for $\beta \in [0, \beta_1] \cup [\beta_2, \overline{\beta}]$, and reveals β for $\beta \in [\beta_1, \beta_2]$. The first threshold β_1 is the competition-independence threshold; the second threshold, β_2, is chosen such that when β is concealed, p_1 is induced to be greater than t and is increasing in β_2.[42]

When expressing firm 1's expected payoff, due to $\widehat{x}_1(p_1, \beta) = 0 \Leftrightarrow \beta = p_1 + t$, there are two possibilities depending on the relative positions of β_2 and $p_1 + t$. Therefore, for the second threshold β_2, if $\beta_2 < p_1 + t$, it implies that not all β will be revealed under multi-purchase (M); Otherwise, if $\beta_2 > p_1 + t$, then all β are revealed under multi-purchase (M).

For the first case ($\beta_2 \leq p_1 + t$), the induced p_1^{NR} is solved by

$$\begin{cases} p_1^{NR} = \arg\max_{p_1} \int_0^{\beta_1} \frac{p_1\left(1-\widehat{x}^S\right)}{\beta_1+\overline{\beta}-\beta_2} d\beta + \int_{\beta_2}^{p_1+t} \frac{p_1(1-\widehat{x}_1)}{\beta_1+\overline{\beta}-\beta_2} d\beta + \int_{p_1+t}^{\overline{\beta}} \frac{p_1}{\beta_1+\overline{\beta}-\beta_2} d\beta \\ \frac{(p_1^{NR}+t)^2}{8t^2} = \frac{\beta_1^2}{4t} \\ p_0 = \frac{p_1^{NR}+t}{2} \end{cases}. \tag{A.8}$$

The solution of p_1 in (A.8), is shown to be increasing in β_2, and satisfies $\beta_2 \leq p_1^{NR}(\beta_2) + t$. Therefore, the maximum of p_1^{NR} is solved by $p_1^{NR}(\beta_2) + t = \beta_2$.

For the second case ($p_1 + t \leq \beta_2 \leq \overline{\beta}$), the induced p_1^{NR} is solved by

$$\begin{cases} p_1^{NR} = \arg\max_{p_1} \int_0^{\beta_1} \frac{p_1\left(1-\widehat{x}^S\right)}{\beta_1+\overline{\beta}-\beta_2} d\beta + \int_{\beta_2}^{\overline{\beta}} \frac{p_1}{\beta_1+\overline{\beta}-\beta_2} d\beta \\ \frac{(p_1^{NR}+t)^2}{8t^2} = \frac{\beta_1^2}{4t} \\ p_0 = \frac{p_1^{NR}+t}{2} \end{cases}. \tag{A.9}$$

The solution of p_1 in (A.9), is shown to be decreasing in β_2, and satisfies $p_1^{NR}(\beta_2) + t \leq \beta_2$. Therefore, the maximum of p_1^{NR} is solved by $p_1^{NR}(\beta_2) + t = \beta_2$.

Combining the above two cases, the maximum value of p_1^{NR} that can be induced when firm 0 conceals β, is the solution of firm 1's profit maximization problem where the support of firm 1's expected payoff consists of only the possibility of single-purchase (S) and the boundary case of multi-purchase (B). Replacing $p_1 + t$ by β_2 in (A.8) or (A.9), it gives the solution

[42] Such refinement is not valid for $\lambda > 0$ studied in the previous subsection, because "revealing information when firms are independent" is incredible due to a positive cost, and hence firm 0 will definitely conceal at multi-purchase. The reason for letting β_1 be the competition-independence threshold can be verified by the following logic: If $\beta_1 < \widehat{\beta}$, then the 0's payoff at $\beta \in (\beta_1, \widehat{\beta})$ is $\frac{(p_1+t)^2}{8t} = t$, which could have been increased by concealing β in that interval to induce a higher p_1; If $\widehat{\beta} < \beta_1$, then firm 1 will charge a lower p_1 than the level under $\beta_1 = \widehat{\beta}$ because price jumps downward evaluated at that interval with full information.

for $\max\{p_1^{NR}\}$, denoted as p_1^{NRSB}, where the superscript is short for "no revelation at single-purchase and at the boundary case for multi-purchase." The equilibrium is given by

$$\begin{aligned} p_1^{NRSB} &= -\frac{2}{3}\sqrt{2}t + \frac{1}{3}\sqrt{(12\sqrt{2}+17)\left((204\sqrt{2}+288)\overline{\beta}+t\right)t} \\ p_0^S &= \frac{p_1^{NRSB}+t}{2} \\ \beta_2 &= p_1^{NRSB}+t,\ \beta_1 = \frac{\beta_2}{\sqrt{2}} \end{aligned} . \tag{A.10}$$

We can verify that p_1^{NRSB} is increasing in $\overline{\beta}$, and is equal to t if $\overline{\beta} = 2t$. That is, as long as $\overline{\beta} \geq 2t$, firm 0 is able to induce $p_1 \geq t$. The following proposition explicitly states such a pure-strategy.

Proposition 9 (Refinement II: Equilibrium Revelation with Zero Cost)**.** *When $\lambda = 0$, firm 0 reveals β evaluated at $\beta \in [\beta_1, \beta_2]$, such that some but not all consumers make multi-purchase. Otherwise, either when all consumers make single-purchase, or when all consumers make multi-purchase, i.e., $\beta \in [0, \beta_1] \cup [\beta_2, \overline{\beta}]$, firm 0 chooses to conceal. That is*

$$\begin{aligned} A_0^* &= \begin{cases} R, & \beta_1 \leq \beta \leq \beta_2 \\ NR, 0 \leq \beta \leq \beta_1, \beta_2 \leq \beta \leq \overline{\beta} \end{cases},\ \mu_1 \in \{\beta | NR\} = U\left[[0,\beta_1] \cup [\beta_2, \overline{\beta}]\right], \\ (p_0^*, p_1^*) &= \begin{cases} \left(\frac{p_1^{NRSB}+t}{2}, p_1^{NRSB}\right), 0 \leq \beta \leq \beta_1 \\ \left(\frac{\beta}{2}, \frac{\beta}{2}\right), \quad \beta_1 \leq \beta \leq \beta_2 \\ \left(\beta - t, p_1^{NRSB}\right), \quad \beta_2 \leq \beta \leq \overline{\beta} \end{cases} , \end{aligned} \tag{A.11}$$

where p_1^{NRSB}, β_1 and β_2 are given by (A.10). In addition, $p_1^{NRSB} \geq t$ for any $\overline{\beta} \geq 2t$, and firm 0 must reveal for $\beta \in [\beta_1, \beta_2]$ with a strictly positive measure.

In the following, we verify that the relative positions of marginal consumers are consistent with the revelation strategy stated in Proposition 9. First, when $\beta < \beta_1$ such that single-purchase equilibrium occurs, no consumer should make multi-purchase. That is, $\widehat{x}_1(p_1^{NRSB}, \beta) > \widehat{x}_0\left(p_0^S, \beta\right)$, which is equivalent to

$$\beta < \frac{3}{4}t + \frac{1}{4}\sqrt{\left(12\sqrt{2}\overline{\beta} + (25 - 12\sqrt{2})t - 4\sqrt{24\sqrt{2}\overline{\beta}t + (34 - 24\sqrt{2})t^2}\right)t}. \tag{A.12}$$

Evaluated at $\beta < \beta_1$, condition (A.12) holds

When $\beta \geq \beta_1$ such that some consumers make multi-purchase, then $\widehat{x}_1\left(\frac{\beta}{2}, \beta\right) < \widehat{x}_0\left(\frac{\beta}{2}, \frac{\beta}{2}\right)$, or

$$\beta > t. \tag{A.13}$$

Evaluated at $\beta \geq \beta_1$, condition (A.13) holds.

Corollary 4. *When $\lambda = 0$, then Refinement II stated in Proposition 9 gives the highest possible payoff* *for firm 0 at single-purchase equilibrium. In other words, if λ can be chosen by firm 0, then Refinement II is firm 0's optimal equilibrium.*

The second refinement for equilibrium revelation under $\lambda = 0$ can be visualized by looking at the bold-colored lines plotted exactly at the horizontal axes in Figure 4, where the black-colored (resp., blue-colored) lines represent for concealment (resp., revelation).

In addition, note that according to (A.10), the highest possible value of p_1^{NRSB} is t when $\overline{\beta} = 2t$, evaluated at which, $\beta_1 = \sqrt{2}t = \widehat{\beta}^F$, and $\beta_2 = 2t = \overline{\beta}$. That is, when $\overline{\beta} = 2t$, the revelation strategy stated in Proposition 9 is not unique. The following example proposes three possible strategies for $\overline{\beta} = 2t$ and $\lambda = 0$.

Example 1. *When $\overline{\beta} = 2t$ and $\lambda = 0$, the following three revelation strategies are equally profitable for firm 0:*

(1) $A_0^* = \begin{cases} R, & \sqrt{2}t \leq \beta \leq 2t \\ NR, & otherwise \end{cases}$, $\mu_1 = U\left[0, \sqrt{2}t\right]$, *and the stage-2 prices* (p_0^*, p_1^*) *are equivalent with those in benchmark F.*

(2) $A_0^* = \begin{cases} R, & 0 \leq \beta \leq \sqrt{2}t \\ NR, & \sqrt{2}t \leq \beta \leq 2t \end{cases}$, $\mu_1 = U\left[\sqrt{2}t, 2t\right]$, *and the stage-2 prices are*

$$(p_0^*, p_1^*) = \begin{cases} (t, t), & 0 \leq \beta \leq \sqrt{2}t \\ \left(\frac{\beta}{2}, p_1^{NRM}\right), & \sqrt{2}t \leq \beta \leq 2t \end{cases}.$$

(3) $A_0^* = R$ *for all* $\beta \in [0, 2t]$ *and* (p_0^*, p_1^*) *are equivalent with those in benchmark F.*

The second refinement, suggests that although it looks unnecessary to share the data when duopolists operate independently, still some information will be provided on purpose because the whole plan that is going to be played is a credible threat. This result is consistent with the common concerns (beliefs) from the third-party business users who feel that they are put into a disadvantageous position, and the claimant from the informed platforms insisting that they have no intention to harm their business users. Arguably, the informed platform, indeed provide some "insights" that are thought to be uninformative when there's no conflicts of interests, but such "insights" could be interpreted as a signal for "being ready to compete."

A.2.3 Concealing Cost

As a robustness check, in this subsection, we assume that there is no cost for revealing β, but concealing β incurs a cost $c > 0$ (e.g., the disutility incurred from being accused of not making data transparent). Clearly, at multi-purchase equilibrium ($M\&B$), due to independence, firm 0 will reveal β, because revealing gives a payoff $\frac{\beta^2}{4t}$ for the interior case (or $\beta - t$ for the boundary case), which is greater than the payoff under concealing $\frac{\beta^2}{4t} - c$ (or $\beta - t - c$ under the boundary case). In the remaining, we check whether firm 0 has an incentive to conceal β at certain intervals in order to induce firm 1 to charge a high price under single-purchase equilibrium.

Similar with the notations adopted in the main text, let $\widehat{\beta}$ be the competition-independence threshold. Evaluated at $\beta < \widehat{\beta}$, single-purchase equilibrium will realize. At single-purchase equilibrium, if firm 0 reveals, its payoff is $\frac{t}{2}$; if firm 0 conceals, its payoff is $\frac{(p_1+t)^2}{8t} - c$, where p_1 is charged based on the updated belief of firm 1 who expects that the support of β should be consistent with the ranges when firm 0 chooses to conceal. As shown above, firm 0 will definitely reveal under multi-purchase, (which can also be inferred by firm 1), and therefore, the only remaining possibility is single-purchase, i.e., setting $p_1 = t$ evaluated at $\beta < \widehat{\beta}$. Meanwhile, since concealing always incurs a positive cost, firm 0 will choose to reveal β for the single-purchase equilibrium.

To summarize, if concealing information incurs a positive cost, then firm 0 reveals for all β and the equilibrium reduces to "full information."

A.3 Welfare

Proof for Proposition 6. Under asymmetric information, due to asymmetric prices, the conditions for "all consumers are captive" are different for firm 0 and firm 1. From firm 0's side, the boundary condition is $\beta = 2t$. From firm 1's side, the boundary condition (the second threshold $\widetilde{\beta}^A$) is obtained by equating $\widehat{x}_1(p_1^A, \beta)$ to 0. Proposition 2 implies that

$$\widehat{x}_1(p_1^A, \widetilde{\beta}^A) = 0 \Rightarrow \widetilde{\beta}^A = p_1^A + t \Rightarrow \frac{d\widetilde{\beta}^A}{d\overline{\beta}} > 0$$

and $\widetilde{\beta}^A = 2t$ if and only if $\overline{\beta} = (5 - 2\sqrt{2})t$. That is, when $2t \leq \overline{\beta} < (5 - 2\sqrt{2})t$ and evaluated at $\beta = \widetilde{\beta}^A < 2t$, then $0 = \widehat{x}_1 < \widehat{x}_0 < 1$, i.e., firm 1's demand is exactly 1, whereas the demand for firm 0's product is less than 1. Conversely, when $\overline{\beta} > (5 - 2\sqrt{2})t$ so that $\widetilde{\beta}^A > 2t$, then evaluated at $\beta = 2t < \widetilde{\beta}^A$, $0 < \widehat{x}_1 < \widehat{x}_0 = 1$, i.e., firm 0's demand is exactly 1 but firm 1's demand is less than 1. Therefore, consider two possibilities:

(i) For $\overline{\beta} \leq (5 - 2\sqrt{2})t$ such that $\widehat{\beta}^A < \sqrt{2}t$, $p_1^A < t$ and $\widetilde{\beta}^A < 2t$:

At single-purchase equilibrium or $\beta \leq \widehat{\beta}^A$, by asymmetry, $p_0^S \neq p_1^A \Rightarrow W^A \leq W^F$, and $p_0^S = p_1^A$ if and only if $p_1^A = t \Leftrightarrow \overline{\beta} = (5 - 2\sqrt{2})t$. Evaluated at $\widehat{\beta}^A \leq \beta < \sqrt{2}t$, $W^A > W^F$.[43] Evaluated at $\sqrt{2}t < \beta < 2t$, $W^A > W^F$ if $p_1^A < \frac{\beta}{2}$, which is feasible provided that $\beta < \widetilde{\beta}^A \Rightarrow \overline{\beta} < (5 - 2\sqrt{2})t$.

(ii) For $\overline{\beta} > (5 - 2\sqrt{2})t$ such that $\widehat{\beta}^A > \sqrt{2}t$, $p_1^A > t$ and $\widetilde{\beta}^A > 2t$, then $W^A < W^F$ at $\beta \leq \sqrt{2}t$ due to asymmetry. And by $\widehat{\beta}^A > \sqrt{2}t$ and $p_1^A > t$, then $W^A < W^F$ for any β.

□

[43] Multi-purchase is realized in benchmark A while it is still single-purchase equilibrium in benchmark F.

Proof for Proposition 7. The case for $\lambda > \widehat{\lambda}$ is equivalent to Proposition 6. Now consider the market outcome under $\lambda < \widehat{\lambda}$ and $\overline{\beta} < (5 - 2\sqrt{2})t$. Clearly, as long as firm 0 reveals a true β (at single-purchase), the equilibrium reduces to benchmark F. However, firm 0 incurs a non-negative cost upon revelation. Therefore, when $0 \leq \lambda < \widehat{\lambda}$ and $\beta < \widehat{\beta}(\lambda)$, then $W^R \leq W^F$. Within the single-purchase equilibrium, the difference between W^R and W^F is simply the size of λ incurred.

When $\lambda < \widehat{\lambda}$ and $\beta \geq \widehat{\beta}(\lambda)$, multi-purchase is realized in benchmark R, and the difference between benchmark R and F is determined by firm 1's price (22) only. Therefore, when $p_1^{NRM} < \frac{\beta}{2} \Leftrightarrow \widehat{x}_1(p_1^{NRM}, \beta) < \widehat{x}_0\left(\frac{\beta}{2}, \beta\right)$, i.e., when β exceeds a threshold, total surplus under concealment is higher than that under full information.

From equation (22), firm 1's price is decreasing in λ and is increasing in $\overline{\beta}$. Let $\widetilde{\beta}^R$ be a threshold that solves $\widehat{x}_1(p_1^{NRM}, \widetilde{\beta}^R) = 0$, and it can be verified that $p_1^{NRM} = t$ and $\widetilde{\beta}^R = 2t$ if and only if

$$\overline{\beta} = 5t - 2\lambda - 2\sqrt{2}\sqrt{t^2 - 2\lambda t} \geq (5 - 2\sqrt{2})t, \tag{A.14}$$

where the left-hand-side of the inequality is increasing in λ and is equal to $(5 - 2\sqrt{2})t$ when $\lambda = 0$. However, as we have shown in Corollary 1 that firm 0 never reveal for $\overline{\beta} > (5 - 2\sqrt{2})t$, (A.14) is a redundant condition for the feasibility of $p_1^{NRM} < \frac{\beta}{2}$. Under $\overline{\beta} < (5 - 2\sqrt{2})t$ and when firm 0 conceals, the price charged by firm 1 is lower than that under full information evaluated at $\beta \in (p_1^{NRM}, 2t)$.

Finally, consider the subtle differences between W^F and W^R for the range $\widehat{\beta}(\lambda) < \beta < \sqrt{2}t$, where $\widehat{\beta}(\lambda)$ is a decreasing in λ, and is equal to $\widehat{\beta}^A$ when $\lambda = \widehat{\lambda}$. Let $\lambda^{-1}(\widehat{\beta})$ be the inverse of $\widehat{\beta}(\lambda)$ expressed by (21). When $0 \leq \lambda < \lambda^{-1}(\widehat{\beta})$, firm 0 reveals at single-purchase, incurring a positive cost and therefore, $W^R \leq W^F$. When $\lambda^{-1}(\widehat{\beta}) < \lambda < \widehat{\lambda}$, firm 0 conceals at multi-purchase, and multi-purchase is realized in benchmark R whereby it keeps to be single-purchase in benchmark F, and therefore, $W^R > W^F$. □

Rankings of Total Surpluses When $\overline{\beta} < (5 - 2\sqrt{2})t$ Using the first rule established by Subsection 6.1, within the single-purchase equilibrium, benchmark F is the best choice. By the third rule, multi-purchase equilibrium is better than single-purchase equilibrium, and hence the relative positions of the equilibrium competition-independence thresholds matter. We have shown in Proposition 5 that under $\overline{\beta} < (5 - 2\sqrt{2})t$,

$$\widehat{\beta}^A \leq \widehat{\beta}(\lambda) \leq \sqrt{2}t. \tag{A.15}$$

The outcomes within multi-purchase equilibrium can be compared by using the second rule. (1) The prices are symmetric in F but it is not true for A or R. Therefore, we use equation (28); (2) Since $p_1^{NRM} \leq p_1^A$, then benchmark A can be dropped from the comparison; (3) Both p_1^{NRM} and $\widehat{\beta}(\lambda)$ are decreasing in λ, and assume that the size of λ can be chosen.

Therefore, for a given β and by manipulating λ, the ranking of total surpluses can be summarized as

(i) For $0 < \beta < \widehat{\beta}^A$: $W^F = W^R > W^A$, where $\lambda = 0$ (refinement I).

(ii) For $\widehat{\beta}^A < \beta < \widehat{\beta}(\lambda)$: $W^A > W^F = W^R$, where $\lambda = 0$ (refinement I).

(iii) For $\widehat{\beta}(\lambda) < \beta < \sqrt{2}t$: $W^R > W^A > W^F$, where $0 < \lambda < \widehat{\lambda}$.

(iv) For $\sqrt{2}t < \beta < 2p_1^{NRM}$: $W^F > W^R > W^A$, where $0 < \lambda < \widehat{\lambda}$.

(v) For $2p_1^{NRM} < \beta < 2p_1^A$: $W^R > W^F > W^A$, where $0 < \lambda < \widehat{\lambda}$.

(vi) For $2p_1^A < \beta < 2t$: $W^R \geq W^A > W^F$, where $0 < \lambda < \widehat{\lambda}$.

(vii) For $2t < \beta < \overline{\beta}$: $W^R = W^A = W^F$, where $0 < \lambda < \widehat{\lambda}$.

A.4 Robustness Check for $\underline{\beta} < 0$

In the main-text, we normalize the lower bound of β to be zero. In theory, when β is below a positive threshold, single-purchase equilibrium will be realized, and the fact that "the equilibrium outcomes under single-purchase are functions of t rather than β" renders some degrees of freedom in picking a lower bound — as long as the range of β is set to be wide enough such that all three types of equilibrium are included. Technically, in the following, we replace the lower bound by $\underline{\beta}$, which is allowed to be strictly negative, to check the robustness of the model. Then, the equilibrium under benchmark A, i.e., equation (A.2) becomes

$$p_1^A(\underline{\beta}, \overline{\beta}) = \frac{3\sqrt{2} - 50}{73}t + \left(\frac{15\sqrt{2}}{146} + \frac{21}{73}\right)\underline{\beta}$$
$$+ \frac{1}{146}\sqrt{\underline{\beta}\left(-6876\sqrt{2}t - 20304t + (1260\sqrt{2} + 2214)\underline{\beta}\right) + 16352\overline{\beta}t + \sqrt{2}(5840\overline{\beta}t + 552t^2) + 2188t^2}.$$
$$\widehat{\beta}^A = \frac{p_1^A + t}{\sqrt{2}},\ p_0^S = \frac{p_1^A + t}{2} \tag{A.16}$$

In addition, fixing $\overline{\beta}$, firm 1's equilibrium price in (A.16), is decreasing in $\underline{\beta}$, because as the range of β increases, the possibility for the interior case of multi-purchase (M) decreases, reducing the incentives for charging a low price.

For benchmark R, notice that, as long as "partial revelation" is possible, the "competition-independence" threshold $\beta(\lambda)$, given by equation (21), is orthogonal to $\underline{\beta}$. That is, a lower $\underline{\beta}$ does not change the payoff under single-purchase equilibrium when firm 0 reveals the information. Besides, when firms are independent such that firm 0 conceals, firm 1's price, given by (22), is also orthogonal to $\underline{\beta}$. Finally, the condition for "firm 0 never reveals β" is solved by letting $p_1^A = t$, which corresponds to $\overline{\beta} = (5 - 2\sqrt{2})t$, which does not depend on $\underline{\beta}$.

Appendix B Numerical Examples

In this section, in order to compare the equilibrium welfare and the corresponding purchase patterns, some numerical examples are provided. When $\overline{\beta}$ is normalized to $2t$, the critical value of revelation cost is $\widehat{\lambda} \approx 0.0305t$.

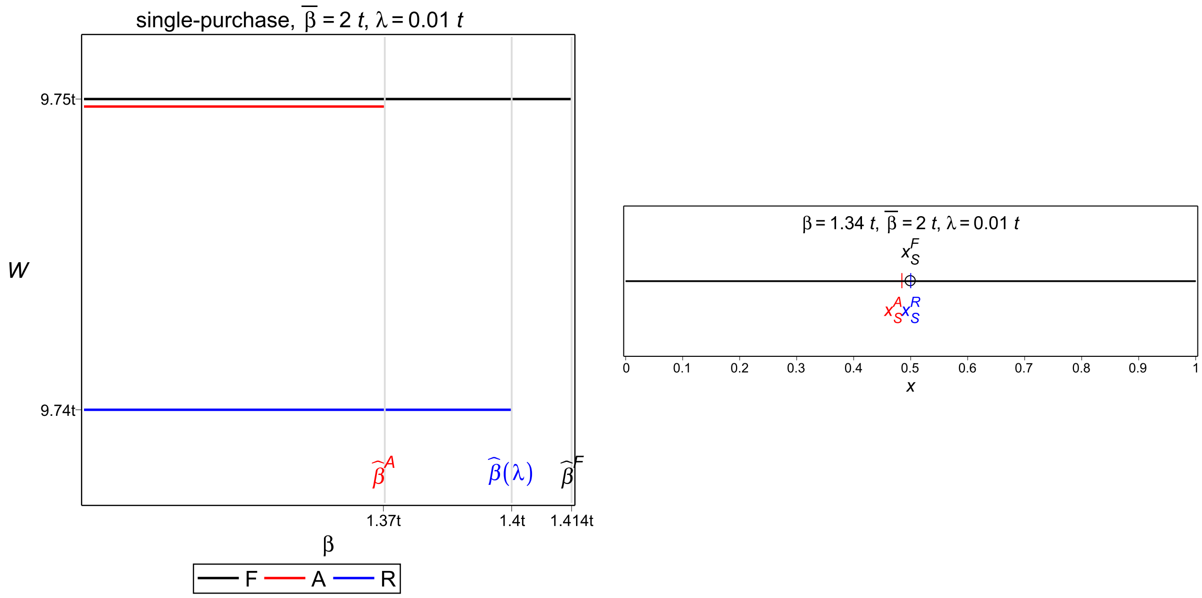


Figure 9: Total surplus (S) and travel distuilities ($\overline{\beta} = 2t$, $\lambda = 0.01t$)

Total surplus under single-purchase equilibrium for benchmarks $j = F, A, R$ is given by

$$\begin{aligned} W^j &= \int_0^{\widehat{x}^S(p_0^j, p_1^j)} (V(1) - tx)\, dx + \int_{\widehat{x}^S(p_0^j, p_1^j)}^1 (V(1) - t(1-x))\, dx - \Lambda(j) \\ &= \frac{4V(1)t - t^2 - (p_0^j - p_1^j)^2}{4t} - \Lambda(j) \end{aligned}, \quad 0 \leq \beta < \widehat{\beta}^j. \tag{B.1}$$

where the last term (i.e., revelation cost) is $\Lambda(j) = \lambda \geq 0$ if $j = R$, and is equal to 0 for $j = F, A$. (B.1) is decreasing in $|p_0 - p_1|$. Therefore, within the single-purchase equilibrium, symmetric prices with zero revelation cost (i.e., $j = F$) is socially desirable.

Within the multi-purchase equilibrium, including both the interior and the boundary cases, total surplus for benchmarks F is given by

$$W^F = \begin{cases} \int_0^{\widehat{x}_1(p_1^F, \beta)} (V(1) - tx)\, dx + \int_{\widehat{x}_1(p_1^F, \beta)}^{\widehat{x}_0(p_0^F, \beta)} (V(2) - t)\, dx + \int_{\widehat{x}_0(p_0^F, \beta)}^1 (V(1) - t(1-x))\, dx, & \sqrt{2}t \leq \beta \leq 2t \\ \int_0^1 (V(2) - t)\, dx, & 2t \leq \beta \leq \overline{\beta} \end{cases}. \tag{B.2}$$

For benchmarks A, since $\widehat{x}_1(p_1^A, \widetilde{\beta}^A) = 0 \Leftrightarrow \widetilde{\beta}^A = 2t \Leftrightarrow \overline{\beta} = (5 - 2\sqrt{2})t$, the expression of total surplus under multi-purchase is divided into two cases. For $2t \leq \beta < (5 - 2\sqrt{2})t \Leftrightarrow$

$\widetilde{\beta}^A < 2t$,

$$W^A = \begin{cases} \int_0^{\widehat{x}_1(p_1^A,\beta)} (V(1)-tx)\,dx + \int_{\widehat{x}_1(p_1^A,\beta)}^{\widehat{x}_0\left(\frac{\beta}{2},\beta\right)} (V(2)-t)\,dx + \int_{\widehat{x}_0\left(\frac{\beta}{2},\beta\right)}^1 (V(1)-t(1-x))\,dx, & \widehat{\beta}^A \le \beta < \widetilde{\beta}^A \\ \int_0^{\widehat{x}_0\left(\frac{\beta}{2},\beta\right)} (V(2)-t)\,dx + \int_{\widehat{x}_0\left(\frac{\beta}{2},\beta\right)}^1 (V(1)-t(1-x))\,dx, & \widetilde{\beta}^A \le \beta < 2t \\ \int_0^1 (V(2)-t)\,dx, & 2t \le \beta \le \overline{\beta} \end{cases}.$$

For $\beta \ge (5-2\sqrt{2})t \Leftrightarrow \widetilde{\beta}^A \ge 2t$,

$$W^A = \begin{cases} \int_0^{\widehat{x}_1(p_1^A,\beta)} (V(1)-tx)\,dx + \int_{\widehat{x}_1(p_1^A,\beta)}^{\widehat{x}_0\left(\frac{\beta}{2},\beta\right)} (V(2)-t)\,dx + \int_{\widehat{x}_0\left(\frac{\beta}{2},\beta\right)}^1 (V(1)-t(1-x))\,dx, & \widehat{\beta}^A \le \beta < 2t \\ \int_0^{\widehat{x}_1(p_1^A,\beta)} (V(1)-tx)\,dx + \int_{\widehat{x}_1(p_1^A,\beta)}^1 (V(2)-t)\,dx, & 2t \le \beta < \widetilde{\beta}^A \\ \int_0^1 (V(2)-t)\,dx, & \widetilde{\beta}^A \le \beta \le \overline{\beta} \end{cases}.$$

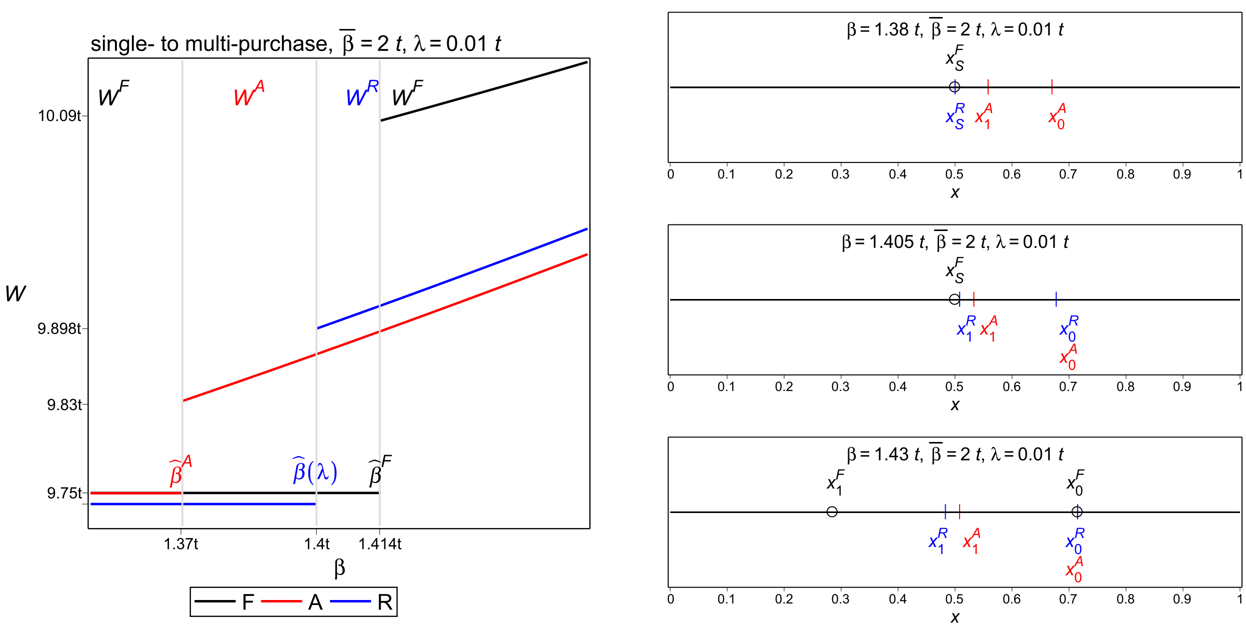


Figure 10: Total surplus (from (S) to (M)) and market coverage ($\overline{\beta} = 2t$, $\lambda = 0.01t$)

For benchmark R, according to (A.14), $\widehat{x}_1(p_1^{NRM}, \widetilde{\beta}^R) = 0 \Leftrightarrow \widetilde{\beta}^R = 2t \Leftrightarrow \overline{\beta} = 5t - 2\lambda - 2\sqrt{2}\sqrt{t^2 - 2\lambda t}$. For $2t \le \beta < 5t - 2\lambda - 2\sqrt{2}\sqrt{t^2-2\lambda t} \Leftrightarrow \widetilde{\beta}^R < 2t$ and $0 < \lambda < \widehat{\lambda}$,

$$W^R = \begin{cases} \int_0^{\widehat{x}_1(p_1^{NRM},\beta)} (V(1)-tx)\,dx + \int_{\widehat{x}_1(p_1^{NRM},\beta)}^{\widehat{x}_0\left(\frac{\beta}{2},\beta\right)} (V(2)-t)\,dx + \int_{\widehat{x}_0\left(\frac{\beta}{2},\beta\right)}^1 (V(1)-t(1-x))\,dx, & \widehat{\beta}(\lambda) \le \beta < \widetilde{\beta}^R \\ \int_0^{\widehat{x}_0\left(\frac{\beta}{2},\beta\right)} (V(2)-t)\,dx + \int_{\widehat{x}_0\left(\frac{\beta}{2},\beta\right)}^1 (V(1)-t(1-x))\,dx, & \widetilde{\beta}^R \le \beta < 2t \\ \int_0^1 (V(2)-t)\,dx, & 2t \le \beta \le \overline{\beta} \end{cases}$$

For $\beta \geq 5t - 2\lambda - 2\sqrt{2}\sqrt{t^2 - 2\lambda t} \Leftrightarrow \widetilde{\beta}^R \geq 2t$ and $0 < \lambda < \widehat{\lambda}$,

$$W^R = \begin{cases} \int_0^{\widehat{x}_1(p_1^{NRM},\beta)} (V(1) - tx)\, dx + \int_{\widehat{x}_1(p_1^{NRM},\beta)}^{\widehat{x}_0\left(\frac{\beta}{2},\beta\right)} (V(2) - t)\, dx + \int_{\widehat{x}_0\left(\frac{\beta}{2},\beta\right)}^{1} (V(1) - t(1-x))\, dx, & \widehat{\beta}(\lambda) \leq \beta < 2t \\ \int_0^{\widehat{x}_1(p_1^{NRM},\beta)} (V(1) - tx)\, dx + \int_{\widehat{x}_1(p_1^{NRM},\beta)}^{1} (V(2) - t)\, dx, & 2t \leq \beta < \widetilde{\beta}^R \\ \int_0^1 (V(2) - t)\, dx, & \widetilde{\beta}^R \leq \beta \leq \overline{\beta} \end{cases}.$$

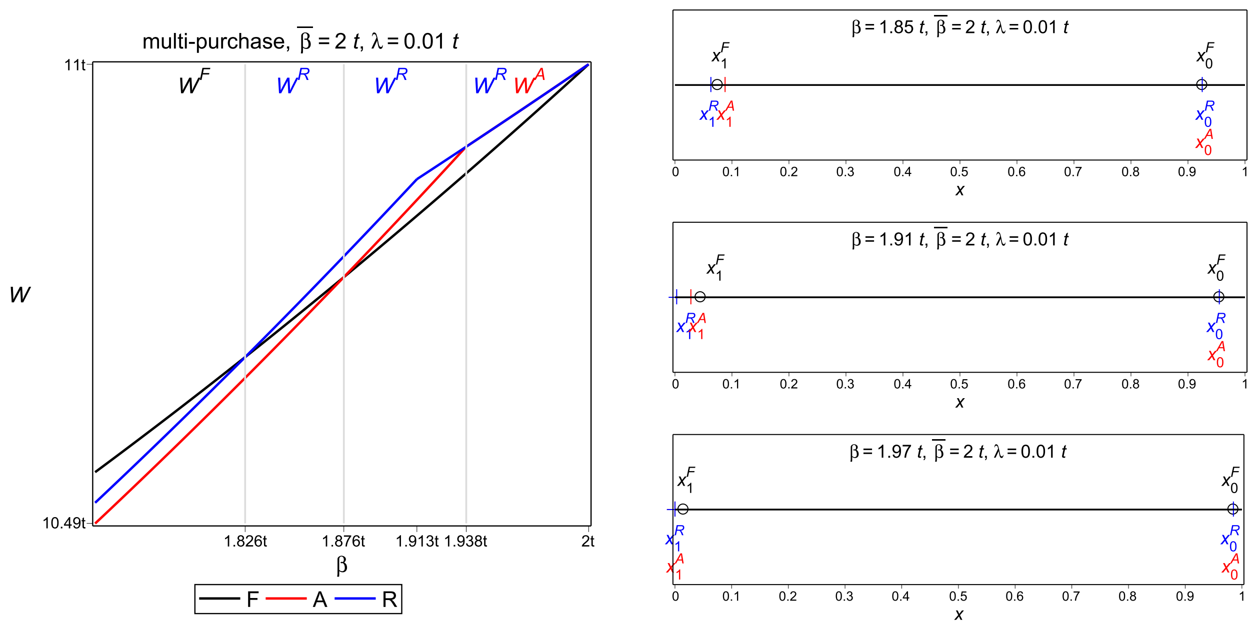


Figure 11: Total surplus (M) and market coverage ($\overline{\beta} = 2t$, $\lambda = 0.01t$)

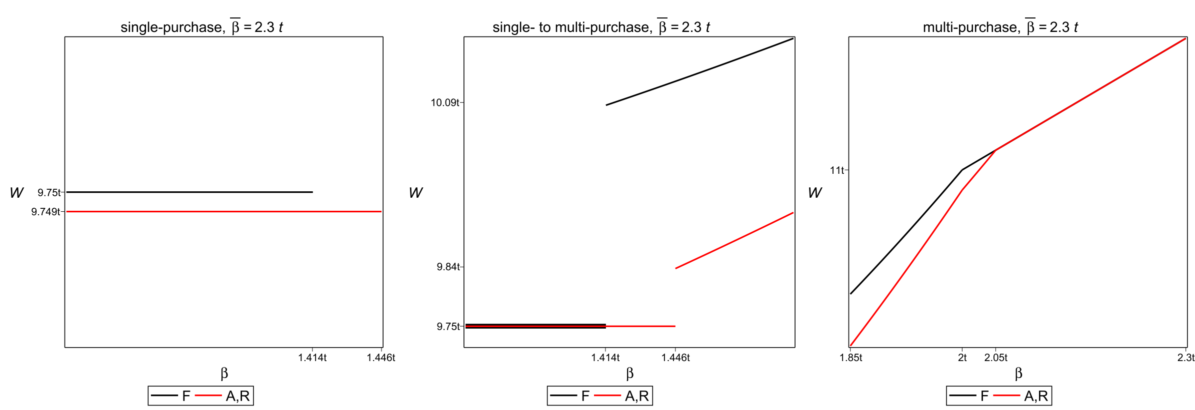


Figure 12: Total surplus: $\overline{\beta} = 2.3t > (5 - 2\sqrt{2})t$, $W^F \geq W^A = W^R$