Information Design with Competing Receivers*

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March 15, 2024
Preliminary and incomplete version

Abstract

We study information design in a model with one sender and many receivers who compete to form a match with the sender. The sender commits to a Blackwell experiment which provides information about the profitability of the match. The receivers then make offers to the sender and the sender can accept at most one of these offers. We provide a condition – which we call *competitiveness* – under which a public experiment is optimal and the sender does not benefit from being able to commit to an acceptance decision ex-ante.

Keywords: Information design, commitment
JEL Classification: D82, D83

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1 Introduction

In many economic environments, a sender provides information to several receivers but can match with at most one of these. On labor markets, workers apply to several potential employers but accept only one job offer. Governments who seek a private company to extract a natural resource typically provide information and solicit offers from several companies before assigning the exploitation rights to one of these. Entrepreneurs pitch their business ideas to different potential investors but often require only a single investor to fund their company.

In this paper, we study optimal information design for a sender who seeks to persuade competing receivers. In our model, a sender chooses an information structure which provides receivers with some information about an underlying state of the world. Upon observing the information, receivers can make an offer to the sender. The sender can then accept one of these offers or remain unmatched.

As our main result, we provide a condition – which we call *competitiveness* – under which public information design is optimal and the sender does not benefit from being able to commit to an acceptance decision ex-ante. We prove our result constructively and characterize the optimal public experiment and the associated equilibrium. In particular, finding an optimal experiment boils down to solving a linear program.

In practice, public information design may benefit a sender due to the following two robustness properties. First, under public information design the sender does not need to worry about the exchange of informative cheap-talk messages between receivers, which is a potential caveat under private information design. Second, with public information design, the players play a game under symmetric information so that out-of-equilibrium beliefs – and potential issues of equilibrium selection – are of no concern.

Similarly, the sequential rationality of the sender's acceptance decision may viewed as a desirable robustness property. In particular, it requires less commitment power on the sender's side. Informational commitment can fully substitute for decision commitment in competitive environments.

Our environment captures a generalized first-price auction as the receivers make offers to the sender which they need to fulfill in case the sender picks them. Our model generalizes the standard first-price auction in that we do not impose a restriction to quasilinear preferences and moreover allow preferences to depend arbitrarily on the state of the world, capturing both cases of independent and correlated values. Moreover, we impose no restrictions on the set of offers which the receivers can make. This allows us to capture both multidimensional offers as well as situations in which the set of offers is very small, for instance, because the sender and the winning receiver negotiate the terms of the contract later on.

The key condition of competitiveness requires that for each public belief regarding the state of the world, any offer by any sender can be matched in terms of the sender's expected value by some offer from some other receiver. Under this condition, there exist equilibria in which the receivers are *price-takers*. If they lower the quality of their offer, they lose the auction with certainty.

Our paper therefore contributes to the literature on information design in first-price auctions. Following the seminal contribution of Milgrom and Weber (1982), Bergemann and Pesendorfer (2007) study the optimal design of information and auction rules if the bidders' values are independent and each bidder can only receive information about their own value. They show that it is optimal to induce asymmetric distributions over the bidders' valuations. Thus, as Myerson (1981) suggests, discriminating auctions rules are optimal. Bergemann, Brooks and Morris (2017) consider fully general forms of information design in the context of a standard first-price auction. In their Theorem 2, they show that the auctioneer's optimal payoff can be achieved through a public information structure and an efficient outcome of the auction. We consider a much more general environment and show that their result extends to all competitive environments.

In two seminal contributions, Kamenica and Gentzkow (2011) and Rayo and Segal (2010) consider situations in which a sender communicates with a single receiver.[1] A growing literature on information design analyzes situations in which the sender communicates with a group of receivers (Bergemann and Morris, 2013, 2016; Taneva, 2019; Mathevet, Perego and Taneva, 2020). The characterization of optimal information design is challenging in general. Some contributions – such as Alonso and Camara (2016) focus on public experiments in a voting context. Under unanimity voting, Bardhi and Guo (2018) show that public and private persuasion coincide. For other voting rules, they focus on experiments that generate conditionally independent signals. Arieli and Babichenko (2019) show that private information design outperforms public information design in a context where receivers make independent, binary decisions. In their context, private and public information design may only be equivalent if receivers are homogeneous. We contribute to this literature by showing that public information design performs as well as private information design in our setting with competing receivers. We add the analysis regarding the role of decision commitment which naturally arises in our setting.

The remainder of this paper is organized as follows. In Section 2, we introduce our model. In Section 3, we state the main result. Section 4 shows how we arrive at this result and Section 5 concludes.

[1] Kolotilin (2018) studies this setup using a linear programming approach, as we do in this paper.

2 Model

2.1 Environment

There is one sender and a finite set $I = \{1, \ldots, n\}$ of receivers with $n \geq 2$. The sender can match with at most one receiver. Prior to the matching, each receiver i makes an offer $a_i \in A_i$ that influences the sender's value from matching with receiver i. We denote a receiver's decision not to offer a match by $a_0 \in A_i$. The sender can also decide to remain unmatched which we also denote by a_0. The match payoffs of the sender and the receivers depend also on an unknown state of the world $\omega \in \Omega$ and are given by the functions

$$u : \Omega \times A \to \mathbb{R} \quad \text{and} \quad v_i : \Omega \times A \to \mathbb{R},$$

where $A \equiv \cup_{i=1}^{n} A_i \cup \{a_0\}$ is the set of possible outcomes of the matching. The payoff of any unmatched player is zero, i.e., for all $\omega \in \Omega$, $u(\omega, a_0) = 0$ and $v_i(\omega, a) = 0$ for $a \notin A_i$ or $a = a_0$. Finally, we assume Ω and A_i to be finite sets for all $i \in I$.

2.2 Information Structure

All players share a common prior $p \in \text{int}(\Delta\Omega)$ regarding the state of the world. The sender can generate additional information about ω by selecting a Blackwell experiment $\sigma = (S, \mu)$ where S is a set of signal realizations with the product structure $S = S_0 \times S_1 \times \cdots \times S_n$ and $\mu : \Omega \to \Delta S$ assigns to each state ω a conditional distribution over S. The sender only observes the signal realization $s_0 \in S_0$ and receiver i observes only the signal realization $s_i \in S_i$. We assume all sets S_0, S_1,..., S_n to be finite but sufficiently large. We denote the set of all such Blackwell experiments by Σ. An experiment is *public* if it provides the same information to the sender and all receivers, or – formally – if for any $s \in S$, each element s_i of s is a sufficient statistic of the whole vector s.

2.3 Base Game

For a given experiment $\sigma = (S, \mu)$, the players play the following Bayesian *game*:

t=0: A signal realization $s \in S$ realizes.

t=1: Each receiver i observes $s_i \in S_i$ and chooses an offer $a_i \in A_i$.

t=2: The sender observes $s_0 \in S_0$ and the vector of offers $\mathbf{a} \in \mathbf{A} \equiv \times_{i=1}^{n} A_i$. The sender selects one of the offers or chooses a_0.

We denote a strategy and a belief of receiver i be $\alpha_i : S_i \to \Delta A_i$ and $\rho_i : S_i \to \Delta(\Omega \times S_{-i})$. A strategy and a belief of the sender are given by $\beta : S_0 \times \mathbf{A} \to \Delta A$ and $\rho_0 : S_0 \times \mathbf{A} \to \Delta(\Omega \times S_{-0})$. The sender's strategy β needs to satisfy the restriction that $\text{supp}(\beta(s_0, \mathbf{a})) \in \{a_0\} \cup \mathbf{a}$, i.e., the sender can only accept offers that have been made. We denote by B the set of all strategies of the sender that satisfy this constraint. We use the equilibrium concept of (weak) perfect Bayesian equilibrium (PBE).[2] We denote by $\mathcal{E}^*(\sigma)$ the set of all PBEs for a given information structure σ, with a generic element $\varepsilon^* = ((\alpha_1, \dots, \alpha_n, \beta), (\rho_0, \dots, \rho_n))$.

2.4 Benchmark: Decision Commitment

As a benchmark, we also consider the setting in which the sender's strategy β is fixed and commonly known before the receivers make an offer. For a given information structure σ and a fixed strategy β, the receivers play the following *game under decision commitment*:

t=0: A signal realization $s \in S$ realizes.

t=1: Each receiver i observes $s_i \in S_i$ and chooses an offer $a_i \in A_i$.

t=2: An outcome is chosen according to the strategy β.

We denote the set of all PBEs of the game under decision commitment for the information structure σ and the sender strategy β by $\mathcal{E}^{**}(\sigma, \beta)$. As the game is static, this set coincides with the set of all Bayes Nash equilibria of the game.[3] Let $\varepsilon^{**} = ((\alpha_1, \dots, \alpha_n), (\rho_1, \dots, \rho_n))$ be a generic element of $\mathcal{E}^{**}(\sigma, \beta)$.

2.5 Sender's Information Design Problem

The sender's information design problem is to choose an experiment σ which maximizes the sender's expected payoff across all possible PBEs under σ. Denote the probability of a vector of receiver offers $\mathbf{a} = (a_1, \dots, a_n)$ for the vector of receiver signal realizations $s_{-0} = (s_1, \dots, s_n)$ by $\alpha(\mathbf{a}|s_{-0}) \equiv \prod_{i=1}^{n} \alpha_i(\mathbf{a}_i|s_i)$. We can then formally define the sender's optimal expected payoff as

$$U^* \equiv \sup_{\sigma \in \Sigma} \sup_{\varepsilon^* \in \mathcal{E}^*(\sigma)} \sum_{\omega \in \Omega} \sum_{s \in S} \sum_{\mathbf{a} \in \mathbf{A}} p(\omega)\mu(s|\omega)\alpha(\mathbf{a}|s_{-0}) \sum_{a \in A} \beta(a|s_0, \mathbf{a}) u(\omega, a).$$

Thus, we assume that for a given experiment σ, the players coordinate on a sender-optimal equilibrium in the set $\mathcal{E}^*(\sigma)$. We thereby follow the standard approach in the

[2] For a formal definition, see Definition 9.C.3 in Mas-Colell, Whinston and Green (1995).
[3] For a formal definition, see Definition 8.E.1 in Mas-Colell et al. (1995).

literature on information design (Bergemann and Morris, 2013, 2016; Taneva, 2019).[4] We say that an *experiment* σ *is optimal* if the sender attains the optimal payoff U^* under σ for some equilibrium in $\mathcal{E}^*(\sigma)$.

An upper bound of the sender's optimal payoff U^* can be derived from the benchmark with decision commitment. If the sender can ex-ante commit not only to an experiment σ but also to a strategy β, the sender can attain at most a payoff of

$$U^{**} \equiv \sup_{\sigma\in\Sigma,\,\beta\in B} \sup_{\varepsilon^{**}\in\mathcal{E}^{**}(\sigma,\beta)} \sum_{\omega\in\Omega}\sum_{s\in S}\sum_{\mathbf{a}\in\mathbf{A}} p(\omega)\mu(s|\omega)\alpha(\mathbf{a}|s_{-0})\sum_{a\in A}\beta(a|s_0,\mathbf{a})u(\omega,a).$$

Of particular interest is the situation in which the optimal payoff U^* coincides with its upper bound U^{**}.[5] In this case, the sender does not gain from committing ex-ante to some strategy β.

Definition 1. *Decision commitment has no value for the sender if* $U^* = U^{**}$.

2.6 Competitiveness

We now define a key condition on the environment to which we refer to as *competitiveness*. To this purpose, let $\iota : A \to I \cup \{0\}$ be the function which assigns each outcome in A to the matched receiver, i.e., we have $\iota(a) = i \iff a \in A_i$ for $a \neq a_0$ and $\iota(a_0) = 0$ where we define $v_0 : A \times \Delta\Omega \to \mathbf{R}$ by $v(\cdot,\cdot) = 0$ to simplify notation.

Definition 2. *The environment is competitive if for each* $a \in A$ *and* $q \in \Delta\Omega$ *with*

$$\sum_{\omega\in\Omega} q(\omega)v_{\iota(a)}(\omega,a) \geq 0$$

there is a competing offer $a' \in A$ *such that (i)* $\iota(a) \neq \iota(a')$ *and (ii) for all* $a'' \in A_{\iota(a)}$ *with*

$$\sum_{\omega\in\Omega} q(\omega)v_{\iota(a)}(\omega,a) < \sum_{\omega\in\Omega} q(\omega)v_{\iota(a)}(\omega,a'') \quad \text{and} \quad \sum_{\omega\in\Omega} q(\omega)u(\omega,a) > \sum_{\omega\in\Omega} q(\omega)u(\omega,a'')$$

it holds that

$$\sum_{\omega\in\Omega} q(\omega)u(\omega,a'') \leq \sum_{\omega\in\Omega} q(\omega)u(\omega,a') \leq \sum_{\omega\in\Omega} q(\omega)u(\omega,a).$$

Imagine a receiver i who conditional on acceptance of his offer a updates his posterior regarding the state ω to q. Suppose further that receiver i prefers making the offer a to

[4]Information design under alternative equilibrium selection criteria is studied in Mathevet et al. (2020).

[5]Formally, $U^* \leq U^{**}$ follows from the observation that $((\alpha'_1,\ldots,\alpha'_n,\beta'),(\rho'_0,\ldots,\rho'_n)) \in \mathcal{E}^*(\sigma) \implies ((\alpha'_1,\ldots,\alpha'_n),(\rho'_1,\ldots,\rho'_n)) \in \mathcal{E}^{**}(\sigma,\beta')$.

not making any offer at the posterior q. Let receiver i now consider to make another offer a'' which receiver i would prefer over a at the posterior q. In a competitive environment, there is another offer a' made by a different receiver $j \neq i$ which the sender prefers at the posterior q over any such offer a'' by receiver i that would hurt the sender at q. Thus, – provided that receiver j offers a' – the sender does not accept the offer a'' by receiver i. In line with the analogy of competitive markets, receiver i would lose all "demand" if he were to deviate to an offer that is worse for the sender. In the next section, we show that the condition of competitiveness is satisfied in many natural applications of the model.

2.7 Applications

Note that the base game of our model shares an important element of the standard first-price auction in that the receivers fulfill their own offer as the winning bidder in the first-price auction pays their own bid. However, our model can capture situations in which the receivers can only make very coarse offers – for instance due to a lack of commitment – or can make multidimensional offers. Moreover, we allow for a wide set of preferences, subject to the condition of competitiveness. In the following, we describe three specific applications of our model, as well as the role of the condition of competitiveness in each of them.

Labor market platforms Consider n firms who may hire a worker. The worker can either be a good or a bad fit for each firm. If the worker is a good fit, hiring the worker generates a payoff of $g_i > 0$ for firm i. Hiring a bad fit leads to a negative payoff of $-\ell_i < 0$ for firm i. Not hiring the worker gives a payoff of zero. Independently of the quality of the fit, the worker has a utility of $u_i > 0$ from being hired at firm i and a utility of zero from remaining unemployed. A labor market platform, such as Linkedin or Upwork, can provide the firms with information about the match values captured by the state $\omega \in \times_{i=1}^{n} \{g_i, -\ell_i\}$. The platform seeks to attract more workers to the platform and therefore maximizes the expected payoff of the worker. After observing the information, each firm i decides whether to offer a job to the worker, $a_i = a_i^1$, or not $a_i = a_0$.

Note that this environment is competitive. As the worker always weakly prefers to receive an offer from some firm, the only deviation a'' which could satisfy $\sum_{\omega \in \Omega} q(\omega)u(\omega, a) > \sum_{\omega \in \Omega} q(\omega)u(\omega, a'')$ is $a'' = a_0$. However, at this deviation the firm's payoff is zero. Thus, condition (ii) in Definition 2 is void and the environment is competitive.

Exploitation of a natural resource Consider n firms who are interested in exploiting a natural resource. A government can grant one of these firms the right to exploit the resource. Firms' offers consist of a financial payment t to the government as well as commitments for environmental protection e. Each firm i's payoff depends on the offer $a_i = (t_i, e_i)$ and the state of the world ω which may capture the richness of the reservoir, the costs of exploiting it, as well as the expenses needed to protect the environment. The government can provide information about the state by conducting test drills or commissioning studies regarding the environmental impact of exploiting the resource.

Note that the environment is competitive if it is symmetric, i.e., all firms have access to symmetric sets of offers $A_i = (a_i^1, \ldots, a_i^K)$ and the government's payoff is symmetric with respect to the firms' offers: $u(\omega, a_i^k) = u(\omega, a_j^k)$ for all i, j and k. In this case, a competing offer is belief-independent and matches the original offer, i.e., $c(a_i^k) = a_j^k$ such that the conditions of Definition 2 are trivially satisfied.

Entrepreneurial financing Consider an entrepreneur who seeks an investment from one of n potential investors to develop a product. The gross payoff from developing the product depends on the product's quality which is initially unknown and captured by the state ω. The entrepreneur can generate information by developing a prototype of the product. After being presented with the prototype, investors make offers to the entrepreneur which specify financial aspects – such as debt and equity – as well as aspects of authority – such as board representation and voting rights. The environment is competitive if the investor can offer the entrepreneur a buyout at price p. For any other offer a, a competing offer $c(a, q)$ is then given by a buyout offer where the price p is set equal to the certainty equivalent of the offer a.[6]

3 Result

We can now present our result.

Theorem 1. *If the environment is competitive, there is a public experiment which is optimal and decision commitment has no value for the sender.*

Under the condition of competitiveness, the sender can rely on public information provision. Apart from the advantage of being able to focus on a smaller set of experiments, there are two more benefits. First, the sender does not have to worry about the exchange of cheap-talk messages between receivers. Second, with public experiments, the players play a game under symmetric information. Thus, the equilibrium notion does not hinge on out-of-equilibrium beliefs.

The condition of competitiveness also implies that the sender does not gain from commitment to an acceptance decision. Thus, commitment to an information structure

[6]As we assume all sets A_i to be finite, we require that the price p can be chosen from a sufficiently fine grid on $\mathbb{R}$.

substitutes decision commitment. In first-price auctions with asymmetric bidders and an exogenous information structure, it is well-known that the auctioneer benefits from discriminating between bidders (Myerson, 1981). Such discrimination clearly requires commitment power at the decision stage as a non-maximal bid may win. As our environment includes the first-price auction with asymmetric bidders as a special case, we can deduce that information commitment again alleviates the need for commitment at the decision stage.

We prove our result constructively and characterize the optimal public experiment and the associated equilibrium. To this purpose, we find the optimal outcome rule – a mapping from the set of states Ω to distributions over the set of offers A – as the solution to a linear program. We then show how to implement this outcome with a public experiment.

4 Optimal experiment and equilibrium

We prove the result in two steps. First, we establish an upper bound on the sender's payoff in the game with decision commitment. This upper bound is the value of a linear programme. In a second step, we provide a public experiment and an PBE for this experiment in the game without commitment decision commitment under which the sender's expected payoff attains the upper bound. This demonstrates that the given public experiment is optimal among all experiments and that the sender does not gain from being able to commit to her strategy in the beginning of the game.

4.1 An Upper Payoff Bound

In this subsection, we derive an upper bound on the sender's payoff in the game with decision commitment. To this purpose, it is helpful to define an *outcome rule* by the mapping $\lambda : \Omega \to \Delta A$ which assigns to each state $\omega \in \Omega$ a conditional distribution over matching outcomes. Note that any given combination of an experiment σ and strategies for all players induces an outcome rule

$$\lambda(a|\omega) = \sum_{s \in S} \sum_{\mathbf{a} \in \mathbf{A}} \mu(s|\omega)\alpha(\mathbf{a}|s_{-0})\beta(a|s_0, \mathbf{a}).$$

As the payoffs of all players depend only on the state and the matching outcome, an outcome rule captures all payoff relevant information from the ex-ante perspective. We denote by Λ the set of all outcome rules.

Lemma 1. *The sender's optimal expected payoff in the game with decision commitment U^{**} is bounded from above by*

$$U^{***} \equiv \max_{\lambda \in \Lambda} \sum_{\omega \in \Omega} \sum_{a \in A} p(\omega)\lambda(a|\omega)u(\omega, a) \quad s.t. \quad \sum_{\omega \in \Omega} p(\omega)\lambda(a|\omega)v_{\iota(a)}(\omega, a) \geq 0, \; \forall a \in A.$$

Proof. Fix an information structure σ, a strategy of the sender β, and an equilibrium in the game with decision commitment $\varepsilon^{**} \in \mathcal{E}^{**}(\sigma, \beta)$. Let λ be the induced conditional distributions over accepted offers. Take some offer $a \in A$ such that $\lambda(a|\omega) > 0$ for some $\omega \in \Omega$. Thus, receiver $\iota(a) = i$ has some signal realization s_i which is send with strictly positive probability after which the receiver offers a with strictly positive probability, i.e., $\alpha_i(a|s_i) > 0$. As receiver i can guarantee a payoff of zero by not making any offer, i.e., by choosing $a_0 \in A_i$, the receiver's expected payoff from offering a needs to be weakly positive. Thus,

$$\sum_{\omega \in \Omega} \sum_{s_{-i} \in S_{-i}} \rho_i(\omega, s_{-i}|s_i) \sum_{\mathbf{a}_{-i} \in \mathbf{A}_{-i}} \prod_{j \neq i} \alpha_j(a_j|s_j)\beta(a|s_0, (a, \mathbf{a}_{-i}))v_i(\omega, a) \geq 0 \tag{1}$$

In any equilibrium, the belief of receiver i is determined by Bayes' rule as

$$\rho_i(\omega, s_{-i}|s_i) = \frac{p(\omega)\mu(s_i, s_{-i}|\omega)}{\sum_{\omega \in \Omega} \sum_{s_{-i} \in S_{-i}} p(\omega)\mu(s_i, s_{-i}|\omega)}. \tag{2}$$

The conditions (1) and (2) imply

$$\sum_{\omega \in \Omega} \sum_{s_{-i} \in S_{-i}} p(\omega)\mu(s_i, s_{-i}|\omega) \sum_{\mathbf{a}_{-i} \in \mathbf{A}_{-i}} \prod_{j \neq i} \alpha_j(a_j|s_j)\beta(a|s_0, (a, \mathbf{a}_{-i}))v_i(\omega, a) \geq 0$$

As $\beta(a|s_0, \mathbf{a}) = 0$ if $a \notin \mathbf{a}$, it follows that

$$\sum_{\omega \in \Omega} \sum_{s_{-i} \in S_{-i}} p(\omega)\mu(s_i, s_{-i}|\omega) \sum_{\mathbf{a} \in \mathbf{A}} \alpha(\mathbf{a}|s_{-0})\beta(a|s_0, \mathbf{a})v_i(\omega, a) \geq 0.$$

We can now note that this inequality holds across all $s_i' \in S_i$, as either $\alpha_i(a|s_i') > 0$ for which the argument above applies, or $\alpha_i(a|s_i') = 0$, such that $\beta(a|s_0, \mathbf{a}) = 0$ as $a \notin \mathbf{a}$.

Thus,

$$
\begin{aligned}
&\sum_{s_i'\in S_i}\sum_{\omega\in\Omega}\sum_{s_{-i}\in S_{-i}} p(\omega)\mu(s_i', s_{-i}|\omega)\sum_{\mathbf{a}\in\mathbf{A}}\alpha(\mathbf{a}|(s_i', s_{-(0,i)}))\beta(a|s_0,\mathbf{a})v_i(\omega,a)\geq 0\\
\iff\quad &\sum_{\omega\in\Omega}\sum_{s\in S} p(\omega)\mu(s|\omega)\sum_{\mathbf{a}\in\mathbf{A}}\alpha(\mathbf{a}|(s_{-(0)}))\beta(a|s_0,\mathbf{a})v_i(\omega,a)\geq 0\\
\iff\quad &\sum_{\omega\in\Omega} p(\omega)\lambda(a|\omega)v_i(\omega,a)\geq 0.
\end{aligned}
$$

It follows that the last inequality needs to hold in any mapping λ which is induced by some experiment and an associated equilibrium in the game with decision commitment. Thus, $U^{**}\leq U^{***}$. □

4.2 Attaining the upper bound with a public experiment

We now provide a public experiment and an associated PBE of the game in which the sender attains the upper payoff bound of U^{***} if the environment is competitive. Clearly, this requires to implement an outcome rule λ which solves the linear program in Lemma 1.

We start by constructing the public experiment. Say that the offer $a\in A$ is in the support of the outcome rule λ – denoted by $\text{supp}(\lambda)$ – if $\lambda(a|\omega)>0$ for some $\omega\in\Omega$. Fix some solution λ^* to the linear program in Lemma 1. We can now define the public experiment $\sigma^*=(S^*,\mu^*)$ by

$$
S^*\equiv\{\mathbf{a}\in\mathbf{A}:\exists a\in\text{supp}(\lambda^*)\text{ s.t. }\mathbf{a}=a\mathbf{1}_{n+1}\}
$$

where $\mathbf{1}_{n+1}=(1,\dots,1)\in\mathbb{R}^{n+1}$, and $\mu^*:\Omega\to\Delta S^*$ with

$$
\mu^*(a\mathbf{1}_{n+1}|\omega)=\lambda^*(a|\omega),\ \forall(a,\omega)\in\text{supp}(\lambda^*)\times\Omega.
$$

In the next step, we define beliefs and strategies for all players which constitute a PBE under σ^*. Under the public experiment σ^*, the players know the signal realizations of all other players. Thus, they only need to form a belief regarding the state $\omega\in\Omega$. Following the signal realization $s^*=a\mathbf{1}^{n+1}$, Bayes' rule implies that all players have the belief

$$
q^*(\cdot|a)\equiv\frac{p(\cdot)\lambda^*(a|\cdot)}{\sum_\omega p(\omega)\lambda^*(a|\omega)}.
$$

To specify the receivers' strategies, we define a mapping which assigns to each pair of an offer a and a belief q a competing offer a' as defined in Definition 2. In particular, let the mapping $c:\{(a,q)\in A\times\Delta\Omega:\sum_{\omega\in\Omega}q(\omega)v_{\iota(a)}(\omega,a)\geq 0\}\to A$ satisfy the following two conditions. First, for each pair (a,q) in the domain of c we have $\iota(c(a,q))\neq\iota(a)$.

Second, if $D(a) \neq \emptyset$ for

$$D(a) \equiv \Big\{a'' \in A_{\iota(a)} : \sum_{\omega\in\Omega} q(\omega)v_{\iota(a)}(\omega, a) < \sum_{\omega\in\Omega} q(\omega)v_{\iota(a)}(\omega, a''),$$
$$\sum_{\omega\in\Omega} q(\omega)u(\omega, a) > \sum_{\omega\in\Omega} q(\omega)u(\omega, a'')\Big\},$$

then

$$\sum_{\omega\in\Omega} q(\omega)u(\omega, a'') \leq \sum_{\omega\in\Omega} q(\omega)u(\omega, c(a, q)) \leq \sum_{\omega\in\Omega} q(\omega)u(\omega, a),$$

and if $D(a) = \emptyset$, then $c(a, q) = a_0$. By definition, such a mapping c exists in a competitive environment. Let the strategy of receiver i be given by

$$\alpha_i^*(a) = \begin{cases} \delta_a & \text{if} \quad a \in A_i, \\ \delta_{c(a,q^*(a))} & \text{if} \quad c(a, q^*(a)) \in A_i, \\ \delta_{a_0} & \text{otherwise.} \end{cases}$$

Finally, let the sender's strategy satisfy

$$\beta^*(a, \mathbf{a}) \in \Delta\{\arg\max_{x\in\mathbf{a}} \sum_{\omega\in\Omega} q^*(\omega|a)u(\omega, x)\}$$

with the restrictions that

$$\beta^*(a, \mathbf{a}) = \begin{cases} \delta_a & \text{if} \quad a \in \arg\max_{x\in\mathbf{a}} \sum_{\omega\in\Omega} q^*(\omega|a)u(\omega, x), \\ \delta_{c(a,q^*(a))} & \text{if} \quad a \notin \arg\max_{x\in\mathbf{a}} \sum_{\omega\in\Omega} q^*(\omega|a)u(\omega, x) \\ & \text{and} \quad c(a, q^*(a)) \in \arg\max_{x\in\mathbf{a}} \sum_{\omega\in\Omega} q^*(\omega|a)u(\omega, x). \end{cases}$$

Lemma 2. *Given the experiment σ^*, the strategy combination $(\alpha_1^*, \ldots, \alpha_n^*, \beta^*)$ and the beliefs $\rho_i^* = \{q^*(a)\}_{a\in\text{supp}(\lambda^*)}$ for all $i = 0, 1, \ldots, n$ constitute a PBE and induce the optimal outcome rule λ^*, thereby generating the optimal expected payoff $U^* = U^{***}$ for the sender.*

Proof. We want to prove that $(\alpha_1^*, \ldots, \alpha_n^*, \beta^*)$ and the beliefs $\rho_i^* = \{q^*(a)\}_{a\in\text{supp}(\lambda^*)}$ for all $i = 0, 1, \ldots, n$ constitute a PBE. Note that the information structure σ^* clearly induces the beliefs $\{\rho_i^*\}_{i\in I\cup\{0\}}$ by Bayes' rule. Moreover, the sender's strategy β^* is obviously sequentially rational given the common belief as it puts for all signal realizations and offer profiles full mass on those offers that are maximizing the sender's expected payoff at the joint belief.

Fix a signal realization $s^* = a\mathbf{1}^{n+1}$. Suppose at first that $a \neq a_0$ and $a \in A_i$. Note that the expected payoff of receiver i from offering a is weakly positive by the constraint of the linear program in Lemma 1. Thus, receiver i does not benefit from offering a_0 instead of a. Consider now the possibility for the receiver to make a different offer a''. This deviation can only be profitable if it is accepted with strictly positive probability. Given that some other receiver offers $c(a, q^*(a))$, the sender only accepts the other offer if the sender strictly prefers a'' to $c(a, q^*(a))$ at the belief $q^*(a)$. The deviation to a'' can therefore only be strictly profitable if

$$\sum_{\omega \in \Omega} q^*(\omega|a) v_i(\omega, a) < \sum_{\omega \in \Omega} q^*(\omega|a) v_i(\omega, a'')$$

and

$$\sum_{\omega \in \Omega} q^*(\omega|a) u(\omega, a'') > \sum_{\omega \in \Omega} q^*(\omega|a) u(\omega, c(a, q^*(a)))$$

which contradicts the definition of c. Thus, receiver i has no profitable deviation.

Next, consider receiver k with $k \neq i$ for $a \neq a_0$ or $k \in I$ for $a = a_0$. This receiver makes a payoff of zero as her offer – $c(a, q^*(a))$ for $k = \iota(c(a, q^*(a)))$ and a_0 for $k \neq \iota(c(a, q^*(a)))$ – is never accepted. Any strictly profitable deviation $a' \in A_k$ for receiver k needs to be accepted by the sender, i.e.,

$$\sum_{\omega \in \Omega} q^*(\omega|a) u(\omega, a') > \sum_{\omega \in \Omega} q^*(\omega|a) u(\omega, a)$$

and needs to induce a strictly positive profit to receiver k, i.e.,

$$\sum_{\omega \in \Omega} q^*(\omega|a) v_k(\omega, a') > 0.$$

Toward a deviation, suppose that such a strictly profitable deviation exists. Then we can define a new outcome rule λ' given by $\lambda'(a|\omega) = 0$, $\lambda'(a'|\omega) = \lambda^*(a'|\omega) + \lambda^*(a|\omega)$, and $\lambda'(a''|\omega) = \lambda^*(a''|\omega)$ for $a'' \in A \setminus \{a, a'\}$, for all $\omega \in \Omega$. The outcome rule λ' obviously satisfies the constraints of the linear program in Lemma 1 for a and $a'' \in A \setminus \{a, a'\}$. It also satisfies the constraint for a' as

$$\sum_{\omega \in \Omega} p(\omega) \lambda'(a'|\omega) v_k(\omega, a') = \sum_{\omega \in \Omega} p(\omega) \lambda^*(a'|\omega) v_k(\omega, a') + \sum_{\omega \in \Omega} p(\omega) \lambda^*(a|\omega) v_k(\omega, a')$$

where the first term on the right-hand side is weakly positive by feasibility of λ^*, and the second term satisfies

$$\sum_{\omega \in \Omega} p(\omega) \lambda^*(a|\omega) v_k(\omega, a') = \sum_{\omega \in \Omega} p(\omega) \lambda^*(a'|\omega) \sum_{\omega \in \Omega} q^*(\omega|a) v_k(\omega, a') > 0.$$

Moreover, the expected payoff of the sender under λ' is

$$\begin{aligned}
&\sum_{\omega\in\Omega}\sum_{a''\in A} p(\omega)\lambda'(a''|\omega)u(\omega,a'') \\
&\quad= \sum_{a''\in A\setminus\{a,a'\}}\sum_{\omega\in\Omega} p(\omega)\lambda^*(a''|\omega)u(\omega,a) + \sum_{\omega\in\Omega} p(\omega)(\lambda^*(a|\omega)+\lambda^*(a'|\omega))u(\omega,a') \\
&\quad= \sum_{\omega\in\Omega}\sum_{a''\in A} p(\omega)\lambda^*(a''|\omega)u(\omega,a) + \sum_{\omega\in\Omega} p(\omega)\lambda^*(a|\omega)(u(\omega,a')-u(\omega,a))
\end{aligned}$$

where the second term in the last line is strictly positive as

$$\begin{aligned}
&\sum_{\omega\in\Omega} p(\omega)\lambda^*(a|\omega)(u(\omega,a')-u(\omega,a)) \\
&\quad= \sum_{\omega\in\Omega} p(\omega)\lambda^*(a'|\omega)\Big\{\sum_{\omega\in\Omega} q^*(\omega|a)u(\omega,a') - \sum_{\omega\in\Omega} q^*(\omega|a)u(\omega,a)\Big\} > 0
\end{aligned}$$

by our initial assumptions regarding a'. Thus, λ^* is not a solution to the linear program, which gives us a contradiction. Therefore, no strictly profitable deviation a' exists.

It is now straightforward to show that the combination of σ^* and the equilibrium above induces the outcome rule λ^* and therefore the expected payoff U^**. □

5 Concluding remark

Theorem 1 is not a revelation principle as some suboptimal outcome rules may be only implementable with non-public experiments. To see this, reconsider the example of a labor market platform in Section 2.7. Suppose that there are two firms and two possible states of the world, i.e., $\Omega = \{(g,g),(-1,-1)\}$, which describe the firms' payoffs from hiring the worker in this state. Suppose that both states are equally likely, i.e., $p(g,g) = 0.5$, and that $g \in (0,1)$. Suppose that the worker's payoff from being hired by the two firms are $u_1 = 1$ and $u_2 = 10$. As both firms have identical payoffs, the worker only tries to persuade firm 2 to make an offer. In particular, an optimal public experiment is given by a public recommendation to either hire the worker (H) or not hiring the worker (N) where the positive recommendation H is sent with certainty in state $\omega = (g,g)$ and with probability g in state $\omega = (-1,-1)$.

Suppose now that the labor market platform wants to maximize the probability of the worker being hired by firm 1. Under a public experiment, the worker can never be hired by firm 1 as firm 2 is willing to hire whenever firm 1 is willing and the worker always prefers firm 2. However, if the platform shows the outcome of the optimal public experiment only to firm 1, firm 1 is willing to make an offer after the recommendation

H whereas firm 2 refrains from making an offer as the expected payoff from hiring is $0.5g + 0.5(-1) < 0$. Thus, the platform can effectively exclude firm 2 from the market and ensure that the worker is only hired by firm 1.

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