Ad Blocking, Whitelisting, and Advertiser Competition*

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Abstract

We consider a market environment with a monopoly ad blocker which offers exclusive or non-exclusive whitelisting to two publishers. Advertisers post ads on the publishers websites to attract the attention of consumers (who visit both publishers). Since advertisers are competing in the marketplace, an advertiser may have an incentive to foreclose its competitor through excessive advertising. We fully characterize the equilibrium in which ad blocker, publishers, and advertisers make strategic pricing decisions. Under some conditions, the ad blocker profitably sells whitelisting to one publisher and both publishers are strictly better off than without the ad blocker. Under other conditions, not only publishers but also advertisers or consumers may be worse off.

Keywords: advertising, advertiser competition, ad blocker, whitelisting, imperfect competition

JEL-classification: L12, L13, L15, M37

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1 Introduction

Internet advertising is a main source of revenue for digital media.However, ad funding has come under the attack from ad blockers. According to a survey from PageFair, 40 % of US internet users claim to avoid seeing ads by using an "ad-blocker" - a third-party software that prevents advertisements being displayed on websites (PageFair, 2020, p. 10). This number has been growing for mobile users since 2014. The market structure for ad blockers is often monopolistic. For example, in Germany, Adblock Plus is the largest ad-blocking firm with a 95 market share in 2017 (OLG München, 2017, para. 20).

What is the business model of ad blockers? Ad-blocking firms earn money by allowing some select publishers to show ads, a practice called whitelisting.[1] To be part of the whitelist, large publishers (defined as those publishers that generate more than 10 million additional advertising impressions through whitelisting per month) have to pay 30 % of their additional revenue to the ad-blocker (Adblock Plus, n.d.).

In this paper, we model the strategic and welfare effects of whitelisting by a monopoly ad blocker, when users multi-home with publishers, advertisers may multi-home with publishers, and advertisers compete with each other in the product market. More precisely, we consider a parsimonious model with two publishers, two advertisers and a continuum of consumers. In the presence of the ad blocker some consumers use the ad blocker, while others do not. Consumers only consider buying a product that was advertised. Thus, it is essential for advertisers to reach consumers with their ads.

Advertisers are engaged in duopoly competition for consumers. There is one ad slot per publisher. Since consumers visit both publishers, an advertiser can foreclose the other advertiser by buying the ad slot at both publishers and thus achieve a monopoly position in the product market.

[1]Not all types of ads are eligible. The two ad-block firms Adblock Plus and Adblock jointly run the Acceptable Ads Committee (AAC), a committee that determines criteria that define which ads are non-intrusive enough to be shown on whitelisted publishers' websites (AAC, 2019, p. 27). The criteria refer to size and distinctiveness from the text (Adblock Plus, n.d.). While users can also change the settings on their ad blocker and see no ads at all, most do not, at least in Germany: 90 % of the users of Adblock Plus keep the default settings and see the filtered ads (Bundesgerichtshof, 2019, para. 3).

Depending on the intensity of competition between advertisers, one advertiser buys both ad slots or each advertiser buys one. An ad blocker will sell whitelisting to one or both publishers. When advertisers price discriminate between consumers who use the ad blocker and consumers who do not, some of the surplus that advertisers and publishers obtain without ad blocking is extracted by the ad blocker.

If advertisers can not price discriminate and thus resort to uniform pricing, the picture is richer. When publishers suffer due to ad blocking, the ad blocker's surplus is extracted only from publishers, from publishers and advertisers, or from publishers and consumers. However, it is also possible that publishers do better with the ad blocker in which case either advertisers or consumers are worse off.

Our model shows the importance of product market competition for the economic effects of an ad blocker. Our main insight that an ad blocker may sell whitelisting to only one advertiser is robust to endogenous ad blocker installation where some consumers install an ad blocker only if the overall exposure to ads is reduced. Importantly, the ad blocker will only operate if only one of the publishers is whitelisted. Several extensions complement the main analysis.

Related literature. Previous work on the role of ad blockers focused on the interaction between ad blocker and publishers abstracting from advertiser competition in the product market (Anderson and Gans, 2011; Despotakis, Ravi, and Srinivasan, 2021; Gritckevich, Katona, and Sarvary, 2022).[2]

We follow the advertising literature that views advertising as a way to increase the probability that consumers become aware of a product (because they did not know about it or because it was no longer part of their consideration set). According to this informative view of advertising, a consumer only considers to buy a product if it has been exposed to an ad about this product (Butters, 1977; Grossman and Shapiro, 1984).[3]

[2]We do not address the interaction between ad targeting and ad blocking; see Johnson (2013) for an analysis. In a different vein, Chen and Liu (2022) focus on the signaling role of advertising following Nelson (1974) and Milgrom and Roberts (1986) and analyze how ad blocking affects the advertising cost.

[3]Starting with Grossman and Shapiro (1984), one stream of this literature considers advertiser competition with differentiated products restricting attention to symmetric settings that have symmetric equilibrium outcomes; see Soberman (2004), Christou and Vettas (2008), and Amaldoss and He (2010).

One may suspect that ad blocking benefits consumers since everything else given consumers can reduce the intake of advertising, which reduces ad nuisance. However, Anderson and Gans (2011) and Gritckevich, Katona, and Sarvary (2022) show that ad-blocking can have a negative indirect effect on consumer welfare as it may lead to lower quality. In particular, Anderson and Gans (2011) show that publishers may increase ad volume in the presence of ad blockers because those with highest nuisance cost of advertising install and thus the presence of the ad blocker changes the composition of those consumers who still see all the ads and makes if more attractive for publishers to increase the ad volume. Our analysis has a different focus: by construction ad volumes can not increase with the introduction of an ad blocker and quality is exogenous. We uncover a different mechanism by which consumers can suffer from the presence of an ad blocker. The ad blocker may limit the exposure of consumers to ads from different advertisers and thereby lead to a less competitive outcome in the product market. As a result, consumers have to pay higher prices in the product market and thereby suffer from the presence of the ad blocker (under uniform pricing in the product market this holds for consumer who installed the ad blocker and those who did not).

One may also suspect that ad blocking hurts publishers. In particular, one may think that an ad blocker extracts rents without providing additional benefits to publishers and, thus, ad blocking hurts publishers. However, as shown by Despotakis, Ravi, and Srinivasan (2021), competing publishers sometimes benefit from ad blocking in a setting with heterogeneous consumers as ad blocking enables them to discriminate between consumers with different sensitivity to advertising; for a similar finding with a monopoly publisher, see Aseri et al. (2020). We also find that ad blocking can be beneficial for publishers; in contrast to Despotakis, Ravi, and Srinivasan (2021), our argument again hinges on the competitive effects in the product market in which advertisers compete in prices for consumers, as this affects the rent extraction possibility of publishers and ad blocker.

More broadly our paper relates to the work on two-sided platforms that cater to two sides, say sellers and buyers, and allow for competition between sellers (e.g.,Nocke, Peitz, and Stahl, 2007; Hagiu, 2009; Belleflamme and Peitz, 2019; Karle, Peitz, and Reisinger, 2020; Teh, 2022) and analyze how this affects market outcomes. Our paper also relates to work on media platforms since publishers do not make profits directly from consumer, but charge advertisers (Anderson and Coate, 2005). In these works, in contrast to this paper, network effects figure prominently. Advertiser competition not only features in the literature on the economics of advertising (Butters, 1977; Grossman and Shapiro, 1984), but it has also been introduced in models of competing media platforms (Dukes and Gal-Or, 2003). A key economic mechanism in our model is that an advertiser may advertise with both publishers and thus foreclose its competitor in the product market, which is reminiscent of Prat and Valletti (2022) who have analyzed the competitive effects of media mergers when consumers multi-home. All that literature does not consider the role of ad blockers.

For illustration of our analysis with reduced-form profits we draw on imperfect competition models with differentiated products (Hotelling, 1929; Perloff and Salop, 1985). If only one publisher whitelists, ensuing product market competition may turn out to be asymmetric, which means that we have to further develop those analyses.

The paper proceeds as follows. In section 2 we present the base model in absence of the ad blocker and analyze how publishers, advertisers and consumers interact; here, we introduce two specific models of product market competition that we use throughout for illustration. In section 3 we introduce the model with the ad blocker and an exogenous fraction of consumers installing the ad blocker. This model is analyzed in 4 distinguishing between two product market setting: in one setting advertisers can price discriminate between consumers who installed the ad blocker and those who did not; in the other setting advertisers must set uniform retail prices. In section 5 we endogenize the consumers' decision whether or not to install the ad blocker. In section 6 we discuss several extensions and argue that our main insight is robust. Section 7 concludes.

2 Preliminaries: Model and analysis without an ad blocker

In a given product category, each publisher can post at most one ad that can be seen by a unit mass of consumers.[4] A publisher bundles own content with advertising and makes money by

[4]The limit to one ad slot per publisher can be motivated by consumers' limited attention for ads when visiting a publisher's website. If consumers dislike advertising and can pay attention to at most one ad on a

selling consumer attention to advertisers. Advertisers can only make a sale if they attract attention through at least one of the publishers. More specifically, there are two advertisers competing in the same product category. There are two publishers that are both frequented by consumers – in other word, consumers are multi-homers. Thus, for an advertiser it is sufficient to show an ad on one publisher to get the consumer's attention. If one advertiser advertises on one of the publisher's website and the other advertiser on the other publisher's website, consumers learn about both products and there is duopoly competition. In the equilibrium in the product market, the gross profit of each advertiser is denoted by π^d. If an advertiser does not advertise its gross profit will be equal to zero. Thus, each advertiser is willing to pay up to π^d to show the ad, provided the other advertisers shows an ad with the other publisher.

We consider the timing in which first publishers simultaneously set a price for the ad, second advertisers sequentially decide whether to accept the offer,[5] third advertisers simultaneously set product prices, and fourth consumers make purchase decisions.

Suppose that publishers have set the same fee f. If each one of the advertisers agrees to pay to advertise with one publisher, net profit of each advertiser is $\pi^d - f$. Instead, one advertiser could exclude the other advertiser by buying the ad slot on both websites. This would give net profit $\pi^m - 2f$, where π^m is the maximal gross profit when the advertiser is a monopolist in the product market. If $f > \pi^d$, it does not pay for advertiser 2 to buy the second slot. Thus, advertiser 1 will not buy the second slot at a fee above π^d. Hence, if $\pi^m > 2\pi^d$, publishers set $f = \pi^d$ and advertiser A buys both ad slots. Here, advertiser A obtains net surplus $\pi^m - 2\pi^d$.

By contrast, if $\pi^m < 2\pi^d$, publishers extract the full gross profit from advertisers by setting $f = \pi^d$. Each advertiser buys one slot and there will be duopoly competition between advertisers.

Duopoly industry profits are larger than monopoly profits if the advertisers' products are sufficiently differentiated. In this case both advertisers buy an ad slot. Otherwise, one

website, a publisher does best by posting only one ad.

[5] If a slot has been taken by the first advertiser, the second advertiser is excluded from the respective publisher. By assuming sequential acceptance decisions, we avoid mixed-strategy equilibria.

advertiser advertises on both websites. We illustrate the relationship between monopoly profits and duopoly industry profits in the well-known Hotelling model of price competition with differentiated products.

Example: (Hotelling, 1929). *We first compare monopoly to industry duopoly profits in the familiar Hotelling model with linear transport cost. Consumers are uniformly distributed on the unit interval, are of mass 1, and demand one unit of one of the products or do not buy in the market. Consumer x obtains net utility $v - p_i - t|x - l_i|$ from product i sold at price p_i at location l_i; the net utility of not buying is normalized to 0. Advertisers sell products located at locations 0 and 1, respectively. They have constant marginal costs of production c. Advertisers set retail prices and consumers make purchasing decisions.*

If one of the advertisers operates as a monopolist it makes profit

$$\pi(p) = (p - c) \min \left\{ \frac{v - p}{t}, 1 \right\}.$$

Solving for the profit-maximizing price we obtain $p^m = (v+c)/2$ for $v \leq c+2t$ and $p_m = v - t$ else. The monopoly profit is

$$\pi^m = \begin{cases} \frac{(v-c)^2}{4t}, & \text{if } c < v \leq c + 2t, \\ v - t - c, & \text{if } v > c + 2t. \end{cases}$$

In duopoly, we restrict attention to equilibria in which the market is fully covered; for the complete characterization, see B. Suppose first that advertisers compete for the marginal consumer and this consumer obtains a strictly positive surplus. The profit of firm i is $\pi_i = (p_i - c)\left(\frac{1}{2} + \frac{p_d - p_i}{2t}\right) = \frac{1}{2t}(p_i - c)(t + p_d - p_i)$. The first-order condition implies that $t + p_d - 2p_i + c = 0$ and the equilibrium price is $p_d = c + t$. The equilibrium demand is equal to $1/2$ and, thus, equilibrium duopoly industry profits are $2\pi^d = t$. The marginal consumer obtains strictly positive surplus if and only if $v - c - t - t/2 > 0$, which is equivalent to $v > c + \frac{3}{2}t$.

Suppose that $v < c + \frac{3}{2}t$. For v not too low, the indifferent consumer obtains zero surplus and the market is fully covered in equilibrium – that is, $p_d = v - t/2$. Note that it is not profit-maximizing for any advertiser to increase its price if and only if $v \geq c + t$. If a firm were to increase its price above p^d it would serve demand $(v - p)/t$. Maximizing profits for this demand gives price $(v + c)/2$. Such a higher price can not be profit-maximizing in the *range $v - t \geq c$. Similarly, it is not profit-maximizing to set the price below p^d if and only if $v \leq c + \frac{3}{2}t$. Thus, if $v \in \left[c + t; c + \frac{3}{2}t\right]$ we have that $p^d = v - t/2$ is the equilibrium.*

For $v > c + 2t$, monopoly profits are larger than industry duopoly profits if and only if $v - t - c > t$; which trivially holds. For $v \in (c + \frac{3}{2}t, c + 2t)$, monopoly profits are larger than industry duopoly profits if and only if $\frac{(v-c)^2}{4t} > t$, which requires that $v - c > 2t$ and cannot be satisfied in this parameter range. For $v \in \left[c + t; c + \frac{3}{2}t\right]$, for monopoly profits to be larger than industry duopoly profits it must hold that $v - t - c > v - t/2 - c$, which can not be satisfied.

Hence, under the assumption that $v \geq c + t$, the market will be covered in duopoly, and we have that $\pi^m < 2\pi^d$ for $v \in [c + t, c + 2t)$ and $\pi^m > 2\pi^d$ for $v > c + 2t$. In other words, industry duopoly profits are larger than monopoly profits, if the degree of product differentiation is sufficiently large.

This is a discrete choice example with perfectly negatively correlated match values and full consumer participation. In Appendix C we develop a discrete choice with independent match values due to Perloff and Salop (1985) in which there is only partial market coverage. Also in that example, industry duopoly profits are larger than monopoly profits, if the degree of product differentiation is sufficiently large.[6]

In the absence of an ad blocker, we have the following result on pure-strategy subgame-perfect Nash equilibria.

Proposition 1. *Consider an environment without an ad blocker. If $\pi^m < 2\pi^d$, in any equilibrium, both publishers set fees $f_1 = f_2 = \pi^d$ and each advertiser buys an ad slot. If $\pi^m > 2\pi^d$, in any equilibrium, both publishers set fees $f_1 = f_2 = \pi^d$ and advertiser A buys the ad slot on each publisher's website. In the borderline case, both publishers set fees $f_1 = f_2 = \pi^d$ and either advertiser A buys both slots or each advertisrer buys one slot.*

Proof. If advertiser A buys both ad slots its net profit will be $\pi^m - f_1 - f_2$ because it will operate as a monopolist. Instead, if advertiser A does not buy both slots, then it buys

[6] The comparison of monopoly and duopoly industry profits can be analyzed in other imperfect competition models where a parameter different than the degree of product differentiation differs across industries; see, for instance, the discussion in Karle et al. (2020).

the slot at the lowest fee and advertiser B either buys the remaining slot or foregoes the possibility to advertise. Advertiser A makes profit $\pi^d - \min\{f_1, f_2\}$ and advertiser B makes $\max\{0, \pi^d - \max\{f_1, f_2\}\}$.

If $\pi^m < 2\pi^d$, both publishers will set $f_i = \pi^d$ and each advertiser buys one slot. At a higher fee they are not able to fill the ad slot and they reduce revenues when setting a lower fee. If in an equilibrium at least one publisher set a fee strictly less than π^d, it has an incentive to increase its fee. If in an equilibrium at least one publisher set a fee strictly higher than π^d, the publisher with the (weakly) highest fee would not be able fill its ad slot with probability 1; it increases its profit by undercutting the other publisher (and never charging above π^m) if that publisher's fee is strictly above π^d and by setting the fee equal to π^d otherwise.

If $\pi^m > 2\pi^d$, both publishers will also set $f_i = \pi^d$; now advertiser A buys both slots. Publishers do not have an incentive to fill their slot at a lower fee. If one of the publishers were to increase its fee above π^d, advertiser A would decide not to buy this slot. At this fee, advertiser B prefers not to buy this slot as well since it can only make pi^d. This the unique equilibrium, as we argue next. If there were an equilibrium in which at least one publisher charged strictly more than π^d and publisher set different fees, advertiser A would not buy the more expensive, nor would advertiser B giving zero profit to the more expensive publisher; if publishers charged the same fee and this fee is larger than π^d each publisher increases its profit by undercutting (and never charging above π^m) because this implies that the ad slot will be filled with probability 1 instead of a probability in $(0, 1)$. If there were an equilibrium in which at least one publisher charged strictly less than π^d, the publisher with the weakly lower fee can increase its fee and continue to sell the ad slot. □

If $\pi^m > 2\pi^d$, advertiser A buys both ad slots. We note that buying the second ad slot may look like a wasted expense since all consumers are reached in any case. However, advertiser A buys the second slot to foreclose advertiser B; as alluded to in the introduction, this logic reminiscent of Prat and Valletti (2022).[7]

Table 1 reports the surplus for the different market participants depending on whether or not monopoly profits are larger than duopoly industry profits in the product market.

[7] In their setting an incumbent firm can reach consumers in any case; it may then take the ad slot of each publisher to foreclose a potential competitor for whom advertising is necessary to reach consumers.

Table 1: Net surplus without an ad blocker

	$\pi^m > 2\pi^d$	$\pi^m \leq 2\pi^d$
Publisher surplus	$2\pi^d$	$2\pi^d$
Advertiser surplus	$\pi^m - 2\pi^d$	0
Consumer surplus	$CS(p^{m*}, \infty)$	$CS(p^{d*}, p^{d*})$

In both cases, each publisher makes profit π^d. In the latter case, publishers fully extract the advertisers' gross profit; in the former, advertisers obtain net surplus $\pi^m - 2\pi^d$.

The intuition for the lack of full rent extraction by publishers in the former case is that both publishers provide access to the consumers' attention. If a publisher asked for a higher fee, advertiser 1 would stop buying the ad without endangering its monopoly position in the product market. Up until π^d the Bertrand undercutting logic applies to publishers because for any fees f_1, f_2 with $\max\{f_1, f_2\} > \pi^d$ advertiser A will drop the publisher with the higher fee, while, at equal fees above π^d it would randomize between the two; advertiser B will not buy a slot at those fees.

3 The model with an ad blocker

We now introduce an ad blocker that offers whitelisting to publishers and asks for a uniform fee to be whitelisted. A publisher who buys whitelisting makes sure that the ad on its website is shown to all consumers, including those who installed the ad blocker. By contrast, an ad from a publisher who does not pay for whitelisting will not be visible to those consumers.

For an ad to be visible to a consumer without an ad blocker, the corresponding advertiser must obtain an ad slot with at least on publisher. For an ad to be visible to a consumer with an ad blocker, the corresponding advertiser must obtain an ad slot with at least on publisher and at least one of those publisher must have bought whitelisting from the ad blocker.

The timing is the following

1. The ad blocker sets a uniform fee.

2. Publishers simultaneously decide whether to accept the ad blocker's offer.

3. Publishers simultaneously set the advertising fee.

4. Advertisers arrive in sequential order and decide on which publishers to advertise.

5. Advertisers simultaneously set retail prices: (version a) advertisers price discriminate between consumers who use an ad blocker and those who do not or b) advertisers set the same retail price to all consumers.

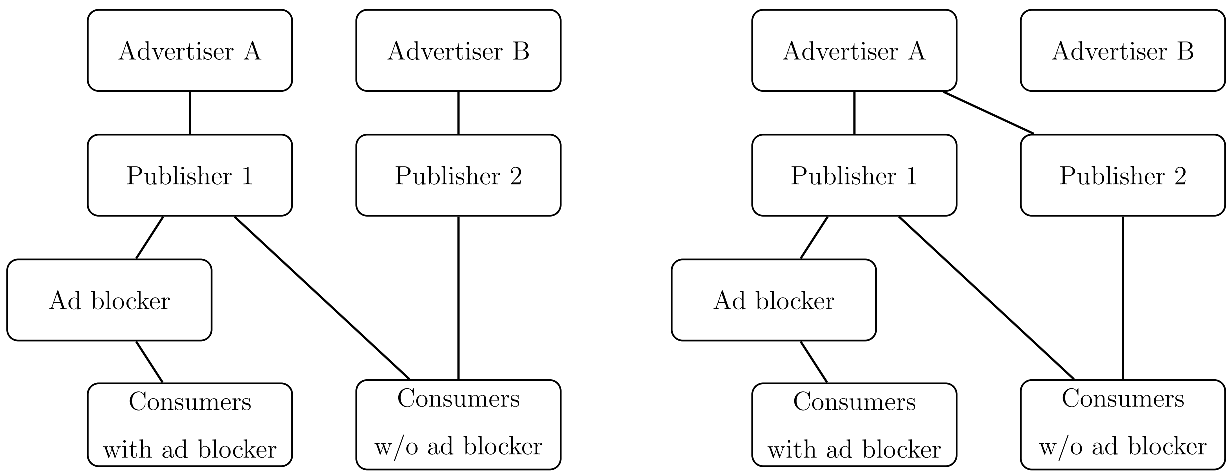

Figure 1: Consumer choice sets when one publisher whitelists

Figure 1 illustrates the consumers' choice sets in two cases in which one publisher has bought whitelisting and the other has not. In the figure, a consumer can only buy those products for which there is a connecting line between advertiser and consumer. In the figure on the left-hand side, each advertiser buys one ad slot; thus consumers without an ad blocker can choose between the two products, whereas consumers with the ad blocker can only buy the product from the advertiser who bought the ad slot on the whitelisted publisher. In the figure on the right-hand side, advertiser A buys both ad slots and thus no consumer can buy from advertiser B. In the following sections we establish conditions for such choice sets to emerge in equilibrium.

A fraction α of consumers use the ad blocker. We consider three different models of how consumers use the ad blocker. In the main model ("Fixed ad blocker installation"), we treat α as an exogenous parameter. Both, one, or none of the publishers buys whitelisting. If both publishers do, we are back to the outcome as in the previous section. If none of them does, publishers make money only from consumers who are not using the ad blocker.

In the second model ("Upfront ad blocker installation"), a fraction α of consumers experiences an ad nuisance and install the ad blocker if they expect fewer ads with an ad blocker. Here, consumers make the installation decision at stage 0.

In the third model ("Committed ad blocker and subsequent ad blocker installation"), a fraction α of consumers experiences an ad nuisance and install the ad blocker if the ad blocker has committed to reduce the amount of advertising. Here,

4 Fixed ad blocker installation

4.1 Fixed ad blocker installation and retail price discrimination

A fraction α of consumers use the ad blocker. We treat α as an exogenous parameter; ad blocker installation becomes endogenous in Section XXX. Both, one, or none of the publishers buys whitelisting. If both publishers do, we are back to the outcome as in the previous section. If none of them does, publishers make money only from consumers who are not using the ad blocker.

The novel case arises if the ad of one advertiser (e.g., advertiser A) appears on the whitelisted website, whereas the ad of the other advertiser (advertiser B) is shown on the other website. In this case, advertiser A has exclusive access to the fraction α of consumers with an ad blocker.

If advertisers can price-discriminate between those consumers who use an ad blocker and those who do not, advertiser A makes per-consumer profit π^m from consumers with an ad blocker; both advertisers make per-consumer profit π^d on consumers without an ad blocker (by Proposition 1). Thus, advertiser A obtains profit $\alpha\pi^m + (1-\alpha)\pi^d$ and advertiser B obtains $(1-\alpha)\pi^d$. This implies that advertiser A is willing to pay the increment $\alpha\pi^m$ to advertise on website 1 instead of 2. In other words, publishers set fees $f_1 = \alpha\pi^m + (1-\alpha)\pi^d$ and $f_2 = (1-\alpha)\pi^d$. Whitelisting is worth $\alpha\pi^m$, which can be extracted by the ad blocker. This is the fee set by the ad blocker in equilibrium if the ad blocker does not prefer to whitelist both advertisers. Now, the ad blocker can charge a whitelisting fee of $\alpha\pi^d$ and induce both

Table 2: Net surplus under price discrimination

	$\pi^m > 2\pi^d$	$\pi^m \leq 2\pi^d$
Ad blocker surplus	$2\alpha\pi^m$	$2\alpha\pi^d$
Publisher surplus	$2(1-\alpha)\pi^d$	$2(1-\alpha)\pi^d$
Advertiser surplus	$(1-\alpha)(\pi^m - 2\pi^d)$	0
Consumer surplus	$CS(p^{m*}, \infty)$	$CS(p^{d*}, p^{d*})$

publishers to accept the offer. The reason is that the publisher's gross profit would drop from π^d to $(1-\alpha)\pi^d$ if one of the publishers refused the offer. Hence, the ad blocker could make $2\alpha\pi^d$. The ad blocker prefers to admit only one publisher if $\alpha\pi^m > 2\alpha\pi^d$, which is equivalent to monopoly profits being larger than industry duopoly profits. Otherwise, if $\pi^m < 2\pi^d$, for given α, the ad blocker would provide whitelisting to both publishers. Overall we see that the ad blocker can extract all profits made from consumers who have installed the ad blocker.

The following proposition summarizes this discussion.

Proposition 2. *Consider an environment with an ad blocker and price-discriminating advertisers. If $\pi^m < 2\pi^d$, then the ad blocker provides whitelisting to both publishers at a price $\alpha\pi^d$, both publishers buy whitelisting and set $f_1 = f_2 = \pi^d$, and each advertiser buys an ad slot.*
If $\pi^{m*} \geq 2\pi^d$, then the ad blocker offers whitelisting at price $\alpha\pi^m$ and one publisher accepts. The whitelisted publisher sets its fee equal to $f_1 = \alpha\pi^m + (1-\alpha)\pi^d$ and the non-whitelisted publisher sets $f_2 = (1-\alpha)\pi^d$. Advertiser A buys the ad slot on each publisher's website.

***FORMAL PROOF TO BE ADDED

Table 2 reports the resulting net surplus for ad blocker, publishers, advertisers, and consumers. We recall that the condition $\pi^m > 2\pi^d$ holds if there is not too much product differentiation in the Hotelling example.

If $\pi^m < 2\pi^d$, there is no difference to the model without an ad blocker except that now each publisher pays $(1-\alpha)\pi^d$ to the ad blocker and, thus, parts of the publishers' rents are shifted to the ad blocker. Advertisers and consumers are not affected by the presence of the ad blocker, and total surplus is unchanged.

Consider now the opposite case $\pi^m \geq 2\pi^d$. With the ad blocker there is exclusive whitelisting and advertiser A obtains gross profits π^m and the other advertiser is not active. Again, consumers are not affected by the presence of the ad blocker, and total surplus is unchanged. Absent the ad blocker, advertiser A obtains net profit $\pi^m - 2\pi^d$, while with the ad blocker it obtains $\pi^m - [\alpha\pi^m + 2(1-\alpha)\pi^d] = (1-\alpha)(\pi^m - 2\pi^d)$. Advertiser A is better off without the ad blocker if $\pi^m - 2\pi^d > (1-\alpha)(\pi^m - 2\pi d)$, which always holds. Each publisher makes net profit $(1-\alpha)\pi^d$ with the ad blocker and thus is worse off than without the ad blocker. Overall, the ad blocker makes a profit at the expense of publishers *and* advertisers; total surplus is not affected.

Corollary 1. *Consider an environment with price-discriminating advertisers. When an ad blocker enters it extracts fraction α of the surplus from publishers and advertisers (in case the latter have any surplus at all).*

4.2 Fixed ad blocker installation and uniform retail prices

If advertisers can not price-discriminate between consumers with and without an ad blocker and thus have to set uniform retail prices, a richer picture emerges. In particular, we will provide conditions under which publishers overall benefit from ad blocking.

When only one publisher whitelists and both advertisers buy one ad slot each, product market competition is asymmetric: the advertiser with the whitelisted publisher enjoys a monopoly position regarding the consumers who installed the ad blocker, while there is duopoly competition for consumers without an ad blocker. Under uniform retail pricing these two market segments are interdependent and compared to the setting with price discrimination, the advertiser with the whitelisted publisher is a less aggressive competitor in the competitive consumer segment.[8]

[8]Such asymmetric competition with uniform pricing has been looked at in the context of universal service obligations; see Anton, Vander Weide, and Vettas (2002) and Valletti, Hoernig, and Barros (2002); the latter considers a Hotelling duopoly similar to our example.

We need to introduce some extra notation. We denote profits as a function of prices by $\pi(p_A, p_B) = \pi_A(p_A, p_B) = \pi_B(p_B, p_A)$. If the competitor cannot reach consumers, an advertiser is in a monopoly position and makes gross profit $\pi(p, \infty)$ at price p. At the profit-maximizing monopoly price p^m we have $\pi(p^m, \infty) = \pi^m$. Under symmetric duopoly competition, assuming that there is a unique price equilibrium, Nash duopoly prices are p_A^* and p_B^* with $p_A^* = p_B^* = p^{d*}$ and yield gross profits $\pi(p^{d*}, p^{d*}) = \pi^d$ for each advertiser.

A novel element of the analysis under uniform pricing is what happens when one advertiser has access to all consumers and the other only to consumers without an ad blocker (as we show, this is off the equilibrium path). This tends to relax competition for consumers without an ad blocker if the monopoly price is above the symmetric duopoly equilibrium price. When advertiser A is visible to all consumers and advertiser B to consumers without an ad blocker, the gross profit of advertiser A is $\alpha\pi(p_A, \infty) + (1-\alpha)\pi(p_A, p_B)$ and advertiser B's gross profit is $(1-\alpha)\pi(p_B, p_A)$. Again, we assume that there is a unique price equilibrium with prices p^{w*} for the advertiser with the whitelisted publisher and p^{nw*} for the other. Equilibrium gross profits of advertiser A can be written as $\pi^w \equiv \alpha\pi(p^{w*}, \infty) + (1-\alpha)\pi(p^{w*}, p^{nw*})$ and advertiser B's gross profit as $\pi^{nw} \equiv (1-\alpha)\pi(p^{nw*}, p^{w*})$. Clearly, $\pi^d(p^{nw*}, p^{w*}) > \pi^d$ and $\pi^{m*} > \pi(p^{w*}, \infty)$ for $p^{w*} \in (p^{d*}, p^m)$ and $\alpha \in (0, 1)$.

We confirm the uniqueness of the asymmetric duopoly equilibrium in our Hotelling example (see Appendix B).[9]

Proposition 3. *Consider an environment with an ad blocker and advertisers setting uniform prices.*

- *If* $\pi^w + 2\pi^{nw} < (3-\alpha)\pi^d$*, then the ad blocker provides whitelisting to both publishers at price* $\pi^d - \pi^{nw}$*, both publishers buy whitelisting and set* $f_1 = f_2 = \pi^d$*.*
- *If* $\pi^w + 2\pi^{nw} > (3-\alpha)\pi^d$*, then the ad blocker whitelists a single publisher at price* $\pi^w - (1-\alpha)\pi^d$*. The whitelisted publisher sets its fee equal to* π^w *and the non-whitelisted*

[9] More generally, we note that a sufficient condition to use first-order conditions to characterize the price equilibrium is the log-concavity of demand. The complication in our model is that the joint demand of the advertiser with the whitelisted publisher is the sum of its demand in the monopoly and the duopoly segment. Since log-concavity is not an additive property it is not sufficient to show that the demand in each segment is log-concave.

publisher sets π^{nw}.

If $\pi^m > f_1 + f_2$*, advertiser A buys the ad slot on each publisher's website and otherwise each advertiser buys one slot each.*

In the proof, which is relegated to Appendix A, we show that, in equilibrium, all ad slots will be filled and that advertiser A takes at least one of the two slots. This result can be seen as follows. Suppose that advertiser A did not buy any slots because it deemed them too expensive. Then advertiser B will buy only one ad slot, as it does not pay to reach consumers without an ad blocker through both publishers. It will buy ad slot 1 if $f_1 \leq \max\{\pi^m, f_2 + \alpha\pi^m\}$ and ad slot 2 if $f_2 \leq \max\{(1-\alpha)\pi^m, f_1 - \alpha\pi^m\}$, there would be asymmetric Bertrand competition between publishers for advertiser B resulting in $f_2 = 0$ and $f_1 = \alpha\pi^m$. Clearly, at those fees, advertiser A has a strict incentive to buy at least one ad slot. This implies that for any fees such that advertiser A does not buy any ad slot, at least one of the publishers has an incentive to reduce its fee. Thus, in equilibrium, advertiser A buys at least one slot. Suppose that A takes slot 1 only. Advertiser B is willing to pay $(1-\alpha)\pi(p^{nw*}, p^{w*})$ for slot 2 and publisher 2 has an incentive sell the slot. Suppose that A takes slot 2 only. Advertiser A is willing to pay $\alpha\pi(p^{w*}, \infty) + (1-\alpha)\pi(p^{w*}, p^{nw*})$ for slot 1 and publisher 1 has an incentive sell the slot. Hence, if advertiser A buys only one slot, advertiser B Will buy the second. As a result, one of the following situations must arise in equilibrium: advertiser A buys both slots; advertiser A buys slot 1 and advertiser B slot 2; or advertiser A buys slot 2 and advertiser B slot 1.

If advertiser A buys both slots and both publishers whitelist, all consumers are reached through both publishers and the advertiser could still reach all consumers if it dropped one of the publishers. Similarly, if advertiser A buys both slots and one publisher whitelists, all consumers without the ad blocker are reached through both publishers and the advertiser could still reach these consumers if it dropped the non-whitelisted publishers. The only reason advertiser A may still want to buy both slots is to foreclose advertiser B.

Whether there is whitelisting of one or both publishers depends on product market competition among advertisers. If $\alpha\pi(p^{w*}, \infty) + (1-\alpha)\pi(p^{w*}, p^{nw*}) + 2(1-\alpha)\pi(p^{nw*}, p^{w*}) > (3-\alpha)\pi^d$, then, in equilibrium, there is whitelisting by one publisher. In such an equilibrium, publishers set fees equal to $f_1 = \alpha\pi(p^{w*}, \infty) + (1-\alpha)\pi(p^{w*}, p^{nw*})$ and $f_2 = (1-\alpha)\pi(p^{nw*}, p^{w*})$, respectively. The product market outcome depends on whether inequality 1 is satisfied. If it is, advertiser A buys both slots and operates as a monopolist in the product market (and obtains a strictly positive net surplus). If it is not, there is some competition for users with the ad blocker and all consumers are better off than in the reverse case.

Gaining commitment power to the ad blocker to sell exclusive whitelisting (i.e. a commitment to sign with at most one publisher does not help the ad blocker; if more than one asks for exclusive whitelisting, a random draw determines which publisher is selected) is not in the interest of the ad blocker. If one publisher is willing to accept the offer of exclusive whitelisting the other does as well. This implies that a deviation by a publisher not to accept will imply that the other one obtains exclusive whitelisting. The deviating publisher can then get $(1-\alpha)\pi(p^{nw*}, p^{w*})$, which is greater than $(1-\alpha)\pi^d$ for $p^{w*} > p^{d*}$. Thus, the ad blocker would be strictly worse off if it committed to exclusive whitelisting. We can also see this by taking a look at the payments received by the ad blocker. Under exclusive whitelisting, when both publishers ask for exclusive whitelisting, the ad blocker can extract $t^u \equiv \alpha\pi(p^{w*}, \infty) + (1-\alpha)\pi(p^{w*}, p^{nw*}) - (1-\alpha)\pi(p^{nw*}, p^{w*})$ because, when not accepting the ad blocker's fee the other publisher will be whitelisted and, thus, the deviating publisher makes $(1-\alpha)\pi(p^{nw*}, p^{w*})$, which constitutes the publisher's outside option.[10]

Table 3 reports equilibrium surplus for consumers, advertisers, publishers, and the ad blocker under all possible constellations. Consumer surplus per unit mass of consumes is a function of the prices p_A, p_B set by the advertisers that reach consumers, denoted by $CS(p_A, p_B)$.

Comparison to no ad blocking Who pays for the ad blocker? We compare how surpluses change with the introduction of an ad blocker. We have to make the comparison in the different parameter region. To reduce the number of regions, we make an assumption on industry profits:

Assumption 1: For any given $\alpha \in (0, 1)$, asymmetric duopoly industry profits satisfy

[10] In our model the ad blocker sets a price for whitelisting. Enriching the strategy of the ad blocker, one could allow the ad blocker to commit to exclusive whitelisting. However, the ad blocker is better off not committing to do so.

Table 3: Net surplus under uniform pricing

	$\pi^w + 2\pi^{nw} < (3-\alpha)\pi^d$		$\pi^w + 2\pi^{nw} > (3-\alpha)\pi^d$	
	$\pi^m > 2\pi^d$	$\pi^m \leq 2\pi^d$	$\pi^m > \pi^w + \pi^{nw}$	$\pi^m \leq \pi^w + \pi^{nw}$
Ad blocker surplus	$2(\pi^d - \pi^{nw})$	$2(\pi^d - \pi^{nw})$	$\pi^w - (1-\alpha)\pi^d$	$\pi^w - (1-\alpha)\pi^d$
Publisher surplus	$2\pi^{nw}$	$2\pi^{nw}$	$\pi^{nw} + (1-\alpha)\pi^d$	$\pi^{nw} + (1-\alpha)\pi^d$
Advertiser surplus	$\pi^m - 2\pi^d$	0	$\pi^m - (\pi^w + \pi^{nw})$	0
Consumer surplus	$CS(p^{m*}, \infty)$	$CS(p^{d*}, p^{d*})$	$CS(p^{m*}, \infty)$	$\alpha CS(p^{w*}, \infty)$ $+(1-\alpha)CS(p^{w*}, p^{nw*})$

Table 4: Net surplus under uniform pricing and Assumption 1

	$\pi^w + 2\pi^{nw} < (3-\alpha)\pi^d$	$\pi^w + 2\pi^{nw} > (3-\alpha)\pi^d$ and $\pi^m \leq 2\pi^d$	$\pi^m > 2\pi^d$
Ad blocker surplus	$2(\pi^d - \pi^{nw})$	$\pi^w - (1-\alpha)\pi^d$	$\pi^w - (1-\alpha)\pi^d$
Publisher surplus	$2\pi^{nw}$	$\pi^{nw} + (1-\alpha)\pi^d$	$\pi^{nw} + (1-\alpha)\pi^d$
Advertiser surplus	0	0	$\pi^m - (\pi^w + \pi^{nw})$
Consumer surplus	$CS(p^{d*}, p^{d*})$	$\alpha CS(p^{w*}, \infty)$ $+(1-\alpha)CS(p^{w*}, p^{nw*})$	$CS(p^{m*}, \infty)$

$\pi^w + \pi^{nw} \in (\min\{2\pi^d, \pi^m\}, \max\{2\pi^d, \pi^m\})$.

This assumption says that asymmetric duopoly industry profits lie between symmetric duopoly industry profits and monopoly profits. Under this assumption, we can not be in the first column of Table 3: if $2\pi^d < \pi^w + \pi^{nw} < \pi^m$, then $\pi^w + 2\pi^{nw} > 2\pi^d + \pi^{nw} > 2\pi^d + (1-\alpha)\pi^d = (3-\alpha)\pi^d$. Under Assumption 1, we obtain the simpler Table 4 (note that we reversed the order of the last two columns).

We verify Assumption 1 in our Hotelling example and provide the conditions on primitives for the three different cases: The first lemma shows that in a parameter region for which there exists a pure strategy equilibrium the ad blocker always finds it optimal to whitelist a single publisher. We show in Appendix B that if $\frac{v-c}{t} < 7/2$, then there is always a pure-strategy equilibrium in the asymmetric model.

Lemma 1. *In the Hotelling model with linear transportation cost and a positive fraction of consumers using the ad blocker, the inequality* $\pi^m + 2\pi^{nw} > (3-\alpha)\pi^d$ *is satisfied.*

The following proposition fully describes the outcome in the Hotelling model.

Proposition 4. *Consider an environment with an ad blocker and advertising setting uniform prices in the Hotelling model with linear transportation cost. The ad blocker whitelists a single publisher at price* $\pi^m - (1-\alpha)\pi^d$*. The whitelisted publisher sets its fee equal to* π^w *and the non-whitelisted publisher sets* π^{nw}*. If* $v \geq c + 2t$*, then advertiser* A *buys the ad slot on each publisher's website and otherwise each advertiser buys one slot each.*

Returning to the general case, Assumption 1 implies that $\pi^m > 2\pi^d$ if and only if $\pi^m > \pi^w + \pi^{nw}$. Then, as follows from Propositions 1 and 3, in equilibrium, advertiser A operates as a monopolist in the product market with ad blocking under uniform pricing if and only if it does so without the ad blocker.

Corollary 2. *Consider an environment with advertisers setting uniform prices. When an ad blocker enters and both publishers buy whitelisting, the ad blocker extracts surplus from publishers only. When an ad blocker enters and one publisher buys whitelisting, the ad blocker extracts some surplus either from advertisers or consumers. Furthermore, publisher surplus can be higher or lower when the ad blocker is present.*

Take a look at the second column in Table 4. The condition for publisher surplus to be higher with the ad blocker is that $\pi^{nw} + (1-\alpha)\pi^d > 2\pi^d$, which is equivalent to

$$\pi(p^{nw*}, p^{w*}) > \frac{1+\alpha}{1-\alpha}\pi^d.$$

We note that this implies that $\pi^w + 2\pi^{nw} > (3-\alpha)\pi^d$ is satisfied (under our assumption that $\pi^w + \pi^{nw} < 2\pi^d$).

In the Hotelling example we checked numerically that the condition $\pi(p^{nw*}, p^{w*}) > \frac{1+\alpha}{1-\alpha}\pi^d$ is satisfied if and only if $v > c + 2t$, which implies that $\pi^m > 2\pi^d$. Thus, publishers are better off whenever consumers are worse off from the presence of the ad blocker.

Consumers do not care about the presence of the ad blocker as long as product market competition does not change (i.e. when $\pi^{m*} \geq 2\pi^{d*} + \max\{0, -\frac{1-\alpha}{\alpha})(\pi^d(p^{w*}, p^{nw*}) - \pi^{d*})\}$ or $\pi^{m*} \geq 2\pi^{d*} + \min\{0, -\frac{1-\alpha}{\alpha})(\pi^d(p^{w*}, p^{nw*}) - \pi^{d*})\})$. Otherwise, consumers benefit from the presence of the ad blocker if $2\pi^{d*} - \frac{1-\alpha}{\alpha}(\pi^d(p^{w*}, p^{nw*}) - \pi^{d*}) > \pi^{m*} > 2\pi^{d*}$ (which requires that $\pi^d(p^{w*}, p^{nw*}) < \pi^{d*}$); consumers suffer from the presence of the ad blocker if $2\pi^{d*} > \pi^{m*} > 2\pi^{d*} - \frac{1-\alpha}{\alpha}(\pi^d(p^{w*}, p^{nw*}) - \pi^{d*})$ (which requires that $\pi^d(p^{w*}, p^{nw*}) > \pi^{d*}$).

The introduction of an ad blocker either reduces the total surplus or leaves it unchanged. For the former, conditions spelled out in the second column of Table 4 must be satisfied: while there would be symmetric duopoly competition in the absence of an ad blocker, the ad blocker will induce an allocation in the product market such that consumers with the ad blocker will only consume advertiser A's product at price $p^{w*} > p^{d*}$ and consumers with the ad blocker face prices (p^{nw*}, p^{w*}) instead of p^{d*}, p^{d*}. Under full participation at p^{w*}, total surplus regarding consumers with an ad blocker is not affected, while there is a misallocation regarding consumers without an ad blocker (as $p^{nw*} \neq p^{w*}$).

5 Ad blocker installation

If consumers install ad blockers to reduce the amount of advertising they are exposed to, the question arises which of our previous results are robust to endogenous ad blocker installation.

Suppose that a fraction α of consumers have "high" nuisance cost $\eta > 0$ per ad they are exposed to and the remaining $1 - \alpha$ fraction of consumers do not mind seeing ads (or have sufficiently "low" nuisance costs). We assume that the opportunity cost of installing the ad blocker, F_I is such that consumers with a high nuisance cost will install it if they reduce ad exposure by at least one ad (that is, $F_I < \eta$), while consumers with a low nuisance cost will not for any reduction in ad exposure.

According to Propositions 2 and 3 the ad blocker provides whitelisting to one or to both publishers in equilibrium. If monopoly profits are larger than duopoly industry profits, advertiser A buys the ad slot from both publishers and thereby avoids retail price competition; under Assumption 1 this holds both under uniform and discriminatory pricing. In this case, the ad blocker offers the advantage that consumers are exposed to advertising on one publisher only. If both advertisers buy an ad slot and only one publisher whitelists, a trade-off arises for consumers, as the installation of the ad blocker implies that they are exposed to one advertiser only and thus forego the opportunity to buy from the other advertiser. Whether this has an impact on ad blocker installation depends on whether consumers rationally anticipate that their experience in the product market depends on their installation decision.

We consider two environments. First, we consider the consumers' ad blocker installation to be an inflexible decision and thus postulate that consumers move before the ad blocker sets its fee. Second, we consider the reverse situation in which consumers install the ad blocker after the ad blocker has committed to its price. As before, we distinguish the setting with retail price discrimination from the one with uniform retail prices.

5.1 Upfront ad blocker installation and retail price discrimination

Consider an extension in which consumers decide whether to install the ad blocker before the first period of the game.

With retail price discrimination, consumers correctly foresee that both publishers will be whitelisted if $2\pi^d > \pi^m$. This implies that ad blocking does not reduce ad exposure and, therefore, no consumer will install the ad blocker. Thus implies that the ad blocker can only be active if $2\pi^d \leq \pi^m$ in which case only one publisher will be whitelisted. Since advertiser A buys the ad slot from each publisher, the ad with the non-whitelisted publisher does not affect consumer choice in the product market and merely adds to the ad nuisance. For this reason, consumers with high nuisance cost have a strict incentive to install the ad blocker. Proposition 2 can thus be reformulated as follows:

Proposition 5. *Consider an environment with endogenous ad blocker installation and price-discriminating advertisers. If $\pi^m < 2\pi^d$, then none of the consumers installs the ad blocker; both publishers set $f_1 = f_2 = \pi^d$; and each advertiser buys an ad slot.*
If $\pi^{m*} \geq 2\pi^d$, then the consumers with high nuisance costs install the ad blocker; the ad blocker offers whitelisting at price $\alpha\pi^m$; and one publisher accepts. The whitelisted publisher sets its fee equal to $f_1 = \alpha\pi^m + (1-\alpha)\pi^d$ and the non-whitelisted publisher sets $f_2 = (1-\alpha)\pi^d$. Advertiser A buys the ad slot on each publisher's website.

The proposition tells us that one should observe ad blockers in environments in which duopoly competition in the product market is intense.

5.2 Upfront ad blocker installation and uniform retail prices

We now turn to the case in which advertisers have to set uniform retail prices. Suppose that Assumption 1 holds. Let us first assume that consumers have limited cognition when deciding whether to install the ad blocker in the sense that they do not internalize that this decision affects their experience in the product market. This means that the adoption decision is purely based on the comparison between nuisance from advertising and the opportunity cost of ad blocker installation.

Proposition 6. *Consider an environment with endogenous ad blocker installation and advertisers setting uniform prices.*

- *If $\pi^w + 2\pi^{nw} < (3-\alpha)\pi^d$, then none of the consumers installs the ad blocker and both publishers set $f_1 = f_2 = \pi^d$; and each advertiser buys one slot.*
- *If $\pi^w + 2\pi^{nw} > (3-\alpha)\pi^d$, then consumers with high nuisance cost install the ad blocker and the ad blocker whitelists a single publisher at price $\pi^w - (1-\alpha)\pi^d$. The whitelisted publisher sets its fee equal to π^w and the non-whitelisted publisher sets π^{nw}. If $\pi^m > 2\pi^d$, advertiser A buys the ad slot on each publisher's website and otherwise each advertiser buys one slot each.*

If consumers are fully rational, they take into account that ad blocker installation will lead to a worse experience in the product market since they can only buy advertiser A's product. Thus, ad blocker installation reduces a consumer's net surplus in the product market by $CS(p^{w*}, \infty) - CS(p^{w*}, p^{nw*})$. For the our purposes so far, we only needed one group of consumers with high nuisance costs. More generally, there may be consumers who are mostly concerned about ad nuisance (consumers with very high nuisance costs), while others find the product market experience relatively more important. The former will install the ad blocker, while the latter may not. Thus, there is a fraction of consumers strictly less than α who will install the ad blocker; this are consumers with very high nuisance costs. The take-away from this discussion is that a model with rational consumers has the feature that fewer consumers will install the ad blocker.

5.3 Committed ad blocker and retail price discrimination

We now turn to an environment in which the ad blocker commits to its fee and consumers correctly infer the subsequent market structure that will prevail. This implies that the ad blocker will never profitably sell whitelisting to both publishers, as this would imply that no consumers installs the ad blocker. In other words, the ad blocker will have to set its fee such that only one publisher will buy whitelisting; hence, in contrast to our result under exogenous ad blocker installation, even if $\pi^m < 2\pi^d$, only one publisher will buy whitelisting. Whitelisting gives the advertiser on the website of the whitelisted publisher a monopoly position over consumers with an ad blocker. This increase in profits of $\alpha\pi^m$ can be charged as the increment in the advertising fee by the whitelisted publisher on top of the fee charged by the non-whitelisted publisher. Thus, whitelisting is worth $\alpha\pi^m$ to the publisher.

Proposition 7. *Consider an environment with endogenous ad blocker installation after the ad blocker committed to its whitelisting fee and price-discriminating advertisers. The ad blocker offers whitelisting at price $\alpha\pi^m$ and one publisher accepts. The whitelisted publisher sets its fee equal to $f_1 = \alpha\pi^m + (1-\alpha)\pi^d$ and the non-whitelisted publisher sets $f_2 = (1-\alpha)\pi^d$. If $\pi^m > 2\pi^d$, advertiser A buys the ad slot on each publisher's website and otherwise each advertiser buys one slot each.*

5.4 Committed ad blocker and uniform retail prices

Consider the setting in which advertisers set uniform prices in the product market. Also in this setting and for the same reason as with discriminatory pricing, the ad blocker will never profitably sell whitelisting to both publishers. Again, the ad blocker will have to set its fee such that only one publisher will buy whitelisting. Whitelisting gives the advertiser on the website of the whitelisted publisher a monopoly position over consumers with an ad blocker.

If both advertisers buy one ad slot each, under uniform pricing the gross profit of the advertiser with the whitelisted publisher is π^w, and the gross profit of the other advertiser is π^{nw}, which the publishers can fully extract. If the whitelisted publisher deviated and did not buy whitelisting, its gross profit would be $(1-\alpha)\pi^d$. Hence, the ad blocker can extract $\pi^w - (1-\alpha)\pi^d$.

If advertiser A buys both ad slots, it makes profit π^m. If the fee charged by the non-whitelisted publisher is above π^{nw} it will not bother to buy slot 2 since advertiser B will not buy the slot at such a fee given that advertiser A bought slot 1. Correspondingly, if the fee charged by the whitelisted publisher is above π^w it will not bother to buy slot 1 since advertiser B will not buy the slot at such a fee given that advertiser A bought slot 2. Thus, fees are $f_1 = \pi^w$ and $f_2 = \pi^{nw}$. As above, if the whitelisted publisher deviated and did not buy whitelisting, its gross profit would be $(1-\alpha)\pi^d$. Hence, the ad blocker can extract $\pi^w-(1-\alpha)\pi^d$. Advertiser A prefers to buy both ad slots if $\pi^m - f_1 - f_2 > \max\{\pi^w - f_1, \pi^{nw}, 0\}$. The condition is $\pi^m - \pi^w - \pi^{nw} > 0$, which, under Assumption 1, is equivalent to $\pi^m > 2\pi^d$.

Proposition 8. *Consider an environment with endogenous ad blocker installation after the ad blocker committed to its whitelisting fee and advertisers setting uniform prices. The ad blocker whitelists a single publisher at price $\pi^w - (1-\alpha)\pi^d$. The whitelisted publisher sets its fee equal to π^w and the non-whitelisted publisher sets π^{nw}. If $\pi^m > 2\pi^d$, advertiser A buys the ad slot on each publisher's website and otherwise each advertiser buys one slot each.*

Comparing the surpluses under ad blocking versus no ad blocking, our insights from Corollary 2 boil down to the following result: the ad blocker extracts some surplus either from advertisers or consumers. Furthermore, publisher surplus can be higher or lower when the ad blocker is present. The condition for ad blocker and publisher interests to be aligned is that $\pi^{nw} + (1-\alpha)\pi^d > 2\pi^d$.

6 Discussion

TO BE ADDED

7 Conclusion

While ad-blocking spares users from viewing annoying ads, it complicates publishers commercializing website traffic by showing ads to users. In Germany, the publishing company Axel Springer has been trying to defend itself legally since 2014 without success. They accused Adblock Plus' business model of violating the right of freedom of the press. Their lawsuit was dismissed in 2019 by the Bundesgerichtshof, Germany's highest court of civil and criminal jurisdiction.

In this paper, we evaluated the equilibrium effects of ad blocking when an ad blocker can whitelist certain publishers and take a cut in the publishers revenues from advertising. Our analysis applies to product markets operating as narrow oligopolies and sheds light on the endogenous prices in product and advertising markets. Our analysis confirms the view that publishers may be harmed by ad blocking. As we show, there may also be harm to consumers or harm to advertisers. However, as we show in the context of uniform prices in the product market, publishers are not necessarily worse off, as the presence of an ad blocker may relax price competition between advertisers. The ensuing higher advertiser revenues allow publishers to charge more to advertisers. The ad blocker can not fully extract these increased publisher revenues because also the publisher that does not whitelist makes higher revenues.

Appendix

A Relegated proofs

Lemma 2. *Consider an environment with an ad blocker and advertisers setting uniform retail prices. Suppose that one publisher bought whitelisting. Then, in equilibrium, advertiser A buys at least one ad slot.*

Proof. Recall that $\pi^w = \alpha\pi(p^{w*}, \infty) + (1-\alpha)\pi(p^{w*}, p^{nw*})$ and $\pi^{nw} = (1-\alpha)\pi(p^{nw*}, p^{w*})$.

Suppose that publisher 1 and publisher 2 set f_1 and f_2 respectively. We assume for a contradiction that advertiser A does not buy any slot in equilibrium.

In such an equilibrium we must have that advertiser A does not find it profitable to buy either slot 1 or slot 2. If advertiser A buys only slot 1, then its profit is equal to $\pi^w - f_1$ if advertiser B buys slot 2, and is equal to $\pi^m - f_1$ otherwise. This implies that $f_1 > \pi^w$ as otherwise advertiser A would find it profitable to buy only slot 1.

Now consider advertiser A buying only slot 2. Since $f_1 > \pi^w$ we have that advertiser B does not buy slot 1 and advertiser A makes monopoly profit from consumers who do not use ad blocker resulting in profits $(1-\alpha)\pi^m - f_2$. In equilibrium advertiser A does not find it profitable to buy only slot 2 implying that $f_2 > (1-\alpha)\pi^m$.

Note that $f_2 > (1-\alpha)\pi^m > \pi^{nw}$ as $\pi^m > \pi(p^{nw*}, p^{w*})$. Thus, $f_2 > (1-\alpha)\pi^m$ implies that advertiser B would not buy slot 2 in case advertiser A decides to buy slot 1 only. This implies that advertiser A would be a monopoly if it decides to buy only slot 1. In the equilibrium, this deviation is unprofitable implying that $f_1 > \pi^m$.

We showed that $f_1 > \pi^m$ and $f_2 > (1-\alpha)\pi^m$ which implies that advertiser B does not buy any slot in the equilibrium either. This leads to non-positive profits for both publishers that cannot be in equilibrium, a contradiction.

□

Proof of Proposition 3. We have to distinguish between two possible pure-strategy equilibrium outcomes of the full game: either one publisher buys whitelisting or both publishers do so. It cannot be an equilibrium that none buys whitelisting because the ad blocker would make zero profit, which is dominated by selling whitelisting at any positive price.

Consider the subgame in which both publishers bought whitelisting. Then Proposition 1 applies.

Consider now the subgame in which one publisher bought whitelisting (without loss of generality, publisher 1) and publishers have set fees f_1 and f_2. Recall that first advertiser A decides which ad slots to buy and then the remaining slots are offered to advertiser B. If advertiser A does not buy any slot it makes profit zero. Three cases remain to be considered.

Suppose that advertiser A has bought both slots. It thus operates as a monopolist and makes profit $\pi^m - f_1 - f_2$.

Suppose that advertiser A has bought slot 2 only. Then advertiser B either buys slot 1 or foregoes the possibility to advertise. Advertiser A makes $(1-\alpha)\pi(p^{nw*}, p^{w*}) - f_2$ if advertiser B buys the slot and $(1-\alpha)\pi^m - f_2$ otherwise. Advertiser B buys slot 1 if and only if $\alpha\pi(p^{w*}, \infty) + (1-\alpha)\pi(p^{w*}, p^{nw*}) - f_1 \geq 0$.

Suppose that advertiser A has bought slot 1 only. If advertiser B buys slot 2, advertiser A makes $\alpha\pi(p^{w*}, \infty) + (1-\alpha)\pi(p^{w*}, p^{nw*}) - f_1$ and otherwise $\pi^m - f_2$. Advertiser B buys slot 2 if and only if $(1-\alpha)\pi(p^{nw*}, p^{w*}) - f_2 \geq 0$. In Lemma 2 we have shown that there cannot be an equilibrium in which advertiser A does not buy any ad slot. Furthermore, in equilibrium, advertiser B buys any remaining slot if advertiser A left a slot vacant.

Thus, there are three possible equilibrium allocations of ad slots: advertiser A buys both slots; advertiser A buys slot 1 and advertiser B slot 2, or advertiser A buys slot 2 and advertiser B slot 1.

As shown above, if at fee $f_1 \leq \alpha\pi(p^{w*}, \infty) + (1-\alpha)\pi(p^{w*}, p^{nw*})$ advertiser A drops slot 1, advertiser B will buy this slot. Thus, in equilibrium of the subgame starting with publishers simultaneously setting fees, f_1 cannot be lower than $\alpha\pi(p^{w*}, \infty) + (1-\alpha)\pi(p^{w*}, p^{nw*})$. Correspondingly, f_2 cannot be lower than $(1-\alpha)\pi(p^{nw*}, p^{w*})$.

If any publisher sets a higher fee, advertiser B would not buy this ad slot. This implies that if one publisher sets a higher fee, whereas the other does not, the former cannot sell its ad slot. If both set higher fees with $f_1 \leq \pi^m$ and $f_2 \leq (1-\alpha)\pi^m$, advertiser A will select the ad slot that gives it the largest net surplus leading to asymmetric Bertrand competition between publishers. This implies that indeed $f_1 = \alpha\pi(p^{w*}, \infty) + (1-\alpha)\pi(p^{w*}, p^{nw*})$ and $f_2 = (1-\alpha)\pi(p^{nw*}, p^{w*})$.

For advertiser A to buy both slots, it must be that at those fees $\pi^m - f_1 - f_2$ is non-negative. For this to be the case, we must have

$$\pi^m > \alpha\pi(p^{w*}.\infty) + (1-\alpha)(\pi(p^{w*}, p^{nw*}) + \pi^d(p^{nw*}, p^{w*})) > 0 \tag{1}$$

or, equivalently,

$$\alpha(\pi^m - \pi(p^{w*}, \infty)) + (1-\alpha)[\pi^m - (\pi(p^{w*}, p^{nw*}) + \pi(p^{nw*}, p^{w*}))] > 0.$$

If inequality (1) is violated, publishers continue to set the same fees, but advertiser A will buy only one slot (it is indifferent as to which one). In this case, both advertisers are active and both make zero net surplus. ***TO BE EXTENDED

When one publisher asks for whitelisting at payment t, this publisher makes $\alpha\pi(p^{w*}, \infty) + (1-\alpha)\pi(p^{w*}, p^{nw*}) - t$, while it would make $(1-\alpha)\pi^d$ if it were to reject the whitelisting offer. Thus, for any $t \leq t^{ux} \equiv \alpha\pi(p^{w*}, \infty) + (1-\alpha)(\pi(p^{w*}, p^{nw*}) - \pi^d)$, each publisher is better off accepting the whitelisting offer given that the other publisher rejects it.

When both publisher ask for whitelisting at payment t, each publisher makes $\pi^d - t$, while a publisher would make $(1-\alpha)\pi(p^{nw*}, p^{w*})$ if it were to reject the whitelisting offer given the other publisher accepted the offer. Thus, for any $t \leq \pi^d - (1-\alpha)\pi(p^{nw*}, p^{w*})$, both publishers accept the whitelisting offer. First note that $\alpha\pi(p^{w*}, \infty) + (1-\alpha)(\pi(p^{w*}, p^{nw*})) - \pi^d) > \pi^d - (1-\alpha)(\pi(p^{nw*}, p^{w*})$, which implies that for sufficiently high t only one publisher asks for whitelisting.

If $\alpha\pi(p^{w*}, \infty) + (1-\alpha)(\pi(p^{w*}, p^{nw*})) - \pi^d) > 2[\pi^d - (1-\alpha)(\pi(p^{nw*}, p^{w*})]$ or, equivalently, $\alpha\pi(p^{w*}, \infty) + (1-\alpha)\pi(p^{w*}, p^{nw*}) + 2(1-\alpha)\pi(p^{nw*}, p^{w*}) > (3-\alpha)\pi^d$, the ad blocker decides to set the price for whitelisting such that one publisher decides to whitelist.

Under this condition, there are two asymmetric pure-strategy equilibria of the subgame starting with the publishers' decision whether to accept the whitelisting offer. Either publisher 1 or publisher 2 asks for whitelisting and the maximal payment to get such an agreement is t^{ux}. □

B Full analysis of the Hotelling model

In this section, we thoroughly analyze the symmetric equilibrium in the Hotelling (1929) model as well as all pure strategy equilibria in the asymmetric Hotelling model in which only one firm has access to consumers who use the ad blocker.[11]

We start our analysis with the monopoly problem.

Monopoly seller Suppose that there is one seller located at 0. A consumer located at x buys a product at price p if $v - p - tx \geq 0$. The profit of this seller setting price p is

$$\pi(p) = (p - c)\min\left\{\frac{v-p}{t}, 1\right\}.$$

By solving for the optimal price we obtain that $p_m = (v + c)/2$ if $v \leq c + 2t$ and $p_m = v - t$. The monopoly profit is

$$\pi^m = \begin{cases} \frac{(v-c)^2}{4t}, \text{ if } v \leq c + 2t, \\ v - t - c, \text{ if } v > c + 2t. \end{cases}$$

Symmetric advertiser competition Consider a standard Hotelling model with linear transportation cost and fully covered market. Suppose that firm 1 is located at 0 and firm 2 is located at 1. Consumers' reservation value is v. The transportation cost is equal to t.

We start by deriving the demand function of firm 1 setting price p_1. Suppose that firm 2 charges price p_2. A consumer located at x buys from firm 1 if and only if $v - p_1 - tx \geq v - p_2 - t(1 - x)$. This implies that all consumers located closer to firm 1 than the marginal consumer with $\hat{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t}$ choose between firm 1 and the outside option. If $v - p_1 - t\hat{x} > 0$ then $D_1 = \hat{x}$, otherwise if $p_1 \leq v$, then all consumers with $x < \frac{v-p_1}{t} < \hat{x}$ buy from firm 1 and $D_1 = \frac{v-p_1}{t}$. Thus, the demand function of firm 1 is given by

$$D_1(p_1, p_2) = \max\left\{0, \min\left\{\frac{1}{2} + \frac{p_2 - p_1}{2t}, \frac{v - p_1}{t}, 1\right\}\right\}.$$

We consider three possibilities: *i)* both firms act as local monopolists, *ii)* firms compete and the indifferent consumer located at $\hat{x}$ obtains positive surplus. *iii)* firms compete and the indifferent consumer obtains zero surplus.

[11] While a large IO literature has used the Hotelling model as a building block, we are not aware of a full analysis of this asymmetric model.

First, suppose that both firms act as local monopolists and the demand of firm i in a small neighborhood of the equilibrium price p_d is $D_i = \frac{v-p_i}{t}$. Then, the optimal price is $p_d = (c+v)/2$. The equilibrium demand is equal to $\frac{v-c}{2t}$. This constitutes an equilibrium if and only if $\frac{v-c}{2t} < \frac{1}{2}$ or equivalently $v < c+t$.

Second, suppose that firms compete for the marginal consumer and this consumer obtains a strictly positive surplus. The profit of firm i is $\pi_i = (p_i - c)\left(\frac{1}{2} + \frac{p_d - p_i}{2t}\right) = \frac{1}{2t}(p_i - c)(t + p_d - p_i)$. The first-order condition implies that $t + p_d - 2p_i + c = 0$ and the equilibrium price is $p_d = c + t$. The equilibrium demand is equal to $1/2$. The marginal consumer obtains strictly positive surplus if and only if $v - c - t - t/2 > 0$, which is equivalent to $v > c + \frac{3}{2}t$.

Third, suppose that the indifferent consumer obtains zero surplus and the market is fully covered in equilibrium –that is, $p_d = v - t/2$. Note that it is not optimal for any firm to increase its price if and only if $v \geq c+t$. If a firm were to increase its price above p_d it would serve demand $(v-p)/t$. From our analysis of case *i)* it follows that for $v \geq c+t$ the profit function is increasing on $[c, 1/2]$. Similarly, it is not optimal to set the price below p_d if and only if $v \leq c + \frac{3}{2}t$ (case *ii)*). Thus, if $v \in \left[c+t; c+\frac{3}{2}t\right]$ we have that $p_d = v - t/2$ is the equilibrium.

To sum up, we obtain that

$$p_d = \begin{cases} \frac{v+c}{2}, \text{ if } v < c+t, \\ v - \frac{t}{2}, \text{ if } v \in \left[c+t; c+\frac{3}{2}t\right], \\ c+t, \text{ if } v > c+\frac{3}{2}t. \end{cases}$$

The equilibrium duopoly profit is

$$\pi^d = \begin{cases} \frac{(v-c)^2}{4t}, \text{ if } v < c+t, \\ \frac{1}{2}\left(v - \frac{t}{2}\right), \text{ if } v \in \left[c+t; c+\frac{3}{2}t\right], \\ \frac{t}{2}, \text{ if } v > c+\frac{3}{2}t. \end{cases}$$

Note that $\pi^m < 2\pi_d$ for $v < c+t$.

We show that $\pi^m < 2\pi^d$ for any $v \in [c+t, c+3/2t]$. The difference in profits $\pi^m - 2\pi^d$ can be represented as

$$\frac{(v-c)^2}{4t} - (v-c) - \left(c + \frac{t}{2}\right).$$

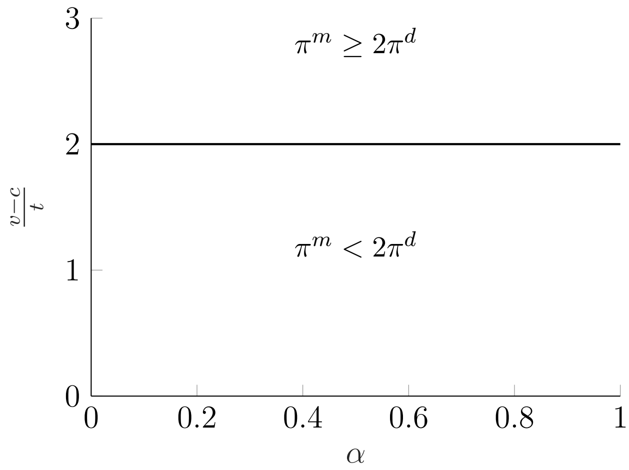

Figure 2: The regions of parameters in the symmetric Hotelling (1929) model for which $\pi^m \geq (<) 2\pi^d$.

By dividing this by t we obtain

$$\begin{aligned}\pi^m - 2\pi^d &= \frac{1}{4}\left(\frac{v-c}{t}\right)^2 - \frac{v-c}{t} - \left(\frac{c}{t} + \frac{1}{2}\right) \\ &= \left(1 - \frac{1}{2}\frac{v-c}{t}\right)^2 - \frac{3}{2} - \frac{c}{t} < \frac{1}{4} - \frac{3}{2} - \frac{c}{t} < 0.\end{aligned}$$

for all $v \in [c+t, c+3/2t]$.

Suppose that $v \in \left(c + \frac{3}{2}t, c + 2t\right]$. Since $\frac{v-c}{t} \in (1, 2]$ we obtain that the monopoly profit is weakly lower than the duopoly profit – that is, $\frac{(v-c)^2}{4t} < t$.

If $v > c + 2t$, then we have that the monopoly profit, $v - t - c$, is always greater than the duopoly profit that is equal to t.

We summarize this discussion in the following proposition.

Proposition 9. *In the Hotelling model with linear transportation cost we have that $\pi^m > 2\pi^d$ for $\frac{v-c}{t} > 2$ and $\pi^m \leq 2\pi^d$ otherwise.*

Asymmetric advertiser competition Suppose that a fraction α of consumers buy either from firm 1 or take the outside option. This is the situation in which the fraction α of consumers is informed about firm 1 but not firm 2, whereas the remaining fraction is informed about both firms.[12]

We suppose that firms play a pure strategy equilibrium and verify the conditions under which this is true in the end. Denote p_w and p_{nw} as the equilibrium prices of firm 1 and firm 2 respectively.

The demand function for firm 1 setting price p_1 when firm 2 price is p^{nw} is given by

$$D_1(p_1, p_{nw}) = \alpha \max\left\{0, \min\left\{\frac{v - p_1}{t}, 1\right\}\right\} + (1-\alpha) \max\left\{0, \min\left\{\frac{1}{2} + \frac{p_{nw} - p_1}{2t}, \frac{v - p_1}{t}, 1\right\}\right\}.$$

The demand of firm 2 setting price p_2 playing against firm 1 setting price p_w is given by

$$D_2(p_2, p_w) = (1 - \alpha) \max\left\{0, \min\left\{\frac{1}{2} + \frac{p_w - p_2}{2t}, \frac{v - p_2}{t}, 1\right\}\right\}.$$

We explore pure strategy equilibria.

Note that in any equilibrium firm 2 sells to a positive measure of consumers in the competitive market. If it were not the case, then the effective price that a consumer located at 1 pays, $p_w + t$, would have to be weakly lower than the lowest price firm 2 can charge which is equal to c. Clearly, for any $t > 0$ there is no such a price p_w that would result in positive profits for firm 1. This implies that firm 2 always sells in the competitive market.

Therefore, it is sufficient to consider four different possibilities: *i)* firm 1 sells in the competitive market and the competitive market is not fully covered, *ii)* firm 1 does not sell in the competitive market *iii)* firm 1 sells in the competitive fully covered market and the marginal consumer obtains a positive surplus, *iv)* firm 1 sells in the competitive fully covered market and the marginal consumer obtains zero surplus.

***i)* Firm 1 sells in the competitive market; the competitive market is not fully covered.** In this case firms act as local monopolies. The profit of firm 1 setting price p_1 at which there are still some consumers in the competitive market who prefer to take the outside option equals $\pi_1(p_1, p_{nw}) = (p_1 - c)(v - p_1)/t$. The optimal price is $p_w = (v + c)/2$.

[12] Several articles on informative advertising and differentiated products starting with Grossman and Shapiro (1984) and including Soberman (2004), Christou and Vettas (2008), and Amaldoss and He (2010) focused on symmetric settings. The asymmetric Hotelling model with a monopoly and a competitive segment has been analyzed by Valletti, Hoernig, and Barros (2002) under some parameter restrictions.

The profit of firm 2 setting price p_2 is $(1-\alpha)(p_2-c)(v-p_2)/t$ which is also maximized at price $p_{nw} = (v+c)/2$. The necessary condition for this to be in equilibrium is that the demand of firm 1 and the demand of firm 2 in the competing segment do not overlap – that is, $\frac{v-c}{2t} < 1 - \frac{v-c}{2t}$ implying $\frac{v-c}{t} < 1$. Note that no firm finds it optimal to deviate and lower its price as it would not do so even if there was no competitor present in the competing segments.

To sum up, if $\frac{v-c}{t} < 1$ there is an equilibrium in which firms act as local monopolists setting the monopoly prices

$$p_w = p_{nw} = \frac{v+c}{2}.$$

The respective profits are

$$\pi^w = \frac{(v-c)^2}{4t} \text{ and } \pi^{nw} = (1-\alpha)\frac{(v-c)^2}{4t}.$$

***ii)* Firm 1 does not sell in the competitive market.** If firm 1 does not sell in the competitive market, then firm 2 must fully serve it in the equilibrium. We consider two cases depending on whether or not the monopolistic market of firm 1 is fully covered.

Assume for a contradiction that firm 1 does not serve all consumers in the monopolistic market. This can only occur if $\frac{v-c}{t} < 2$ as otherwise firm 1 would find it optimal to deviate and serve the entire monopolistic market. But if $\frac{v-c}{t} < 2$, then firm 2 serving all consumers in the competitive market would find it profitable to lower its price, a contradiction. This implies that if firm 1 does not sell in the competitive market, then it must serve all consumers in its monopolistic market.

Suppose now that firm 1 serves all consumers in the monopolistic market. If this scenario occurs in equilibrium, then the consumer located at 1 in the monopolistic market cannot enjoy a positive surplus as firm 1 could increase its price and make higher profits. This pins down the equilibrium price of firm 1, $p_w = v - t$. In order to solve for p_{nw} we note that the consumer located at zero has to be indifferent between firm 1 and firm 2 – that is, $v - t - p_{nw} = v - p_w$. By plugging in p_w and solving for p_{nw} we find that the possible equilibrium is represented by the following prices

$$p_w = v - t \text{ and } p_{nw} = v - 2t.$$

If this constitutes an equilibrium, then neither firm finds it profitable to deviate. We first establish the conditions under which firm 2 does not have incentives to deviate to a higher price. The profit of firm 2 deviating to $p_2 > p_{nw}$ is $\pi_2(p_2, p_{nw}) = \frac{1}{2t}(1-\alpha)(p_2-c)(t+p_w-p_2)$. The derivative of this profit function is

$$c + t + p_w - 2p_2 < c + t + p_w - 2p_{nw} = -v + c + 4t \leq 0,$$

if and only if $\frac{v-c}{t} \geq 4$. Under this condition the profit function decreases for all $p_2 > p_{nw}$ and firm 2 does not deviate to a higher price.

Next, we explore firm 1's incentive to deviate. Condition $\frac{v-c}{t} \geq 4$ implies that firm 1 does not deviate to higher prices (see the monopoly problem in the symmetric case). Thus, it remains to establish conditions under which firm 1 does not deviate to lower prices. If firm 1 sets a lower price, $p_1 < v - t$, then it would serve some consumers from the competitive markets resulting in total profits

$$\pi_1(p_1, p_{nw}) = (p_1 - c)\left(\frac{1+\alpha}{2} + \frac{1-\alpha}{2t}(p_{nw} - p_1)\right).$$

The derivative of this profit (multiplied by $2t/(1-\alpha)$) is

$$\begin{aligned} c + \frac{1+\alpha}{1-\alpha}t + p_{nw} - 2p_1 &> c + \frac{1+\alpha}{1-\alpha}t + p_{nw} - 2v + 2t \\ &= c + \frac{1+\alpha}{1-\alpha}t - v \geq 0, \end{aligned}$$

if and only if $\frac{v-c}{t} \leq \frac{1+\alpha}{1-\alpha}$. This conditions ensures that firm 1 does not deviate to lower prices.

To sum up, we conclude that for $\alpha \geq \frac{3}{5}$ and $\frac{v-c}{t} \in \left[4, \frac{1+\alpha}{1-\alpha}\right]$ there exists an equilibrium in which firms set prices

$$p_w = v - t \quad \text{and} \quad p_{nw} = v - 2t,$$

all consumers in the monopolistic market buy from firm 1, all consumers in the competitive market buy from firm 2. The respective profits are given by

$$\pi^w = \alpha(v - t - c) \quad \text{and} \quad \pi^{nw} = (1-\alpha)(v - 2t - c).$$

***iii)* Firm 1 sells in the competitive fully covered market and the marginal consumer obtains a positive surplus.** Suppose that all $1-\alpha$ consumers in the competing market buy from either of the firms, the marginal consumer is in the interior (each firm has a positive market share) and enjoys a positive surplus. The profit of firm 1 setting price p_1 is given by

$$\pi_1(p_1, p_{nw}) = (p_1 - c)\left(\alpha \min\left\{\frac{v - p_1}{t}, 1\right\} + (1-\alpha)\left(\frac{1}{2} + \frac{p_{nw} - p_1}{2t}\right)\right).$$

We consider two cases taking into account whether the monopolistic market for firm 1 consisting of α consumers is fully covered or not.

Case 1.1: Monopolistic segment of firm 1 is fully covered. The consumer located at 1 **in the monopolistic segment obtains a positive surplus.** If this case can occur in equilibrium, then we have that $v - p_w - t > 0$. The profit of firm 1 setting a price p_1 at which all α consumers in the monopolistic market continue to buy and the marginal consumer in the competitive market remains in the interior is given by

$$\pi_1(p_1, p_{nw}) = (p_1 - c)\left(\frac{1+\alpha}{2} + \frac{1-\alpha}{2t}(p_{nw} - p_1)\right).$$

The first-order condition at $p_1 = p_w$ is

$$\frac{1+\alpha}{2} + \frac{1-\alpha}{2t}(p_{nw} - p_w) - \frac{1-\alpha}{2t}(p_w - c) = 0,$$

implying that

$$p_w = \frac{1}{2}\left(c + p_{nw} + \frac{1+\alpha}{1-\alpha}t\right).$$

Since $v - p_w - t > 0$, then the profit of firm 2 does not have kinks (if a consumer does not buy from firm 2 she always buys from firm 1 rather than taking the outside option). Therefore, the profit of firm 2 can be written as

$$\pi_2(p_2, p_{nw}) = (1-\alpha)(p_2 - c)\left(\frac{1}{2} + \frac{p_w - p_2}{2t}\right).$$

By solving the first-order condition we find that $p_{nw} = \frac{1}{2}(c + t + p_w)$. By plugging this back into the expression for p_w we find that

$$p_w = c + \frac{3+\alpha}{3(1-\alpha)}t \quad \text{and} \quad p_{nw} = c + \frac{3-\alpha}{3(1-\alpha)}t.$$

The marginal consumer in the competitive market is in the interior if $p_w - p_{nw} < t$. This is the case whenever $\alpha < \frac{3}{5}$.

The prices constitute an equilibrium if *i)* the monopolistic market of firm 1 is fully covered (this, in turn, would imply that the marginal consumer in the competitive market enjoys a positive surplus and firm 2 does not have incentives to deviate) and *ii)* firm 1 does not want to set a higher price such that some consumers from the monopolistic market do not buy.[13]

The first condition is satisfied if $v - p_w - t = v - c - \frac{2(3-\alpha)}{3(1-\alpha)}t > 0$, or equivalently, $\frac{v-c}{t} > \frac{2(3-\alpha)}{3(1-\alpha)}$. It remains to explore the second condition.

Consider firm 1 deviating to a price $p_1 \in (v-t, p_{nw}+t)$. Note that $p_1 < p_{nw}+t$ ensures that the marginal consumer in the competitive market is in the interior. One can show that $v-t < p_{nw}+t$ if and only if $\frac{v-c}{t} < \frac{9-7\alpha}{3(1-\alpha)}$. By taking into account the first condition we obtain that $\frac{2(3-\alpha)}{3(1-\alpha)} < \frac{v-c}{t} < \frac{9-7\alpha}{3(1-\alpha)}$ which is non-empty for $\alpha < \frac{3}{5}$. We show that such a deviation is unprofitable. The profit of firm 1 is given by

$$\pi_1(p_1, p_{nw}) = (p_1 - c)\left(\frac{2\alpha v + (1-\alpha)t}{2t} + \frac{1-\alpha}{2t}p_{nw} - \frac{1+\alpha}{2t}p_1\right).$$

By taking the derivative of this profit function (multiplied by $2t$) and plugging in p_{nw} we find

$$\begin{aligned}
&2\alpha v + (1-\alpha)t + (1-\alpha)p_{nw} + (1+\alpha)c - 2(1+\alpha)p_1 \\
&\quad < 2\alpha v + (1-\alpha)t + (1-\alpha)p_{nw} + (1+\alpha)c - 2(1+\alpha)v + 2(1+\alpha)t \\
&\quad = (1-\alpha)t + 2c + \frac{3-\alpha}{3}t + 2(1+\alpha)t - 2v.
\end{aligned}$$

For all v satisfying the first condition (i.e. $v - c - \frac{2(3-\alpha)}{3(1-\alpha)}t > 0$) we have that the derivative can be evaluated from above by

$$\begin{aligned}
(1-\alpha)t + 2c + \frac{3-\alpha}{3}t + 2(1+\alpha)t - 2c - \frac{4(3-\alpha)}{3(1-\alpha)}t \\
= (3+\alpha)t + \frac{3-\alpha}{3}\left(1 - \frac{4}{1-\alpha}\right)t \\
= (3+\alpha)t - \frac{(3-\alpha)(3+\alpha)}{3(1-\alpha)}t = -\frac{2\alpha(3+\alpha)}{3(1-\alpha)}t < 0,
\end{aligned}$$

implying that the profit of firm 1 strictly decreases for all prices in $(v-t, p_{nw}+t)$. Thus, a deviation to such p_1 is never optimal.

Next, consider a deviation of firm 1 to a price $p_1 > \max\{p_{nw}+t, v-t\}$. For these prices, the demand of firm 1 in the competitive market drops down to zero and the profit is equal

[13] Obviously, firm 1 does not find it profitable to deviate to too low prices to serve the entire competitive market.

to $\pi_1(p_1, p_{nw}) = \alpha(p_1 - c)(v - p_1)/t$. Note that $\frac{v-c}{t} > \frac{2(3-\alpha)}{3(1-\alpha)} = \frac{2}{3} + \frac{4}{3(1-\alpha)} > \frac{2}{3} + \frac{4}{3} = 2$. Thus, the analysis of the monopoly problem suggests that the profit of firm 1 decreases in p_1 and is maximal at $p_1 = \max\{p_{nw} + t, v - t\}$. First, suppose that $\frac{2(3-\alpha)}{3(1-\alpha)} < \frac{v-c}{t} < \frac{9-7\alpha}{3(1-\alpha)}$ (which holds true for $\alpha < \frac{3}{5}$) and, therefore, $v - t < p_{nw} + t$. In this case, we have shown that the equilibrium profit is higher than the profit at $p_1 = p_{nw} + t$. We conclude that a deviation to a price $p_1 > p_{nw} + t$ is unprofitable. Second, consider the case in which $\frac{v-c}{t} \geq \frac{9-7\alpha}{3(1-\alpha)}$ which ensures that $v - t \geq p_{nw} + t$. The maximal profit from such a deviation is equal to $\alpha(v - t - c)$. The equilibrium profit of firm 1 is weakly larger than the profit from this deviation if and only if $\frac{(3+\alpha)^2}{18(1-\alpha)} t \geq \alpha(v - t - c)$. By rearranging we obtain

$$\frac{v-c}{t} \leq 1 + \frac{(3+\alpha)^2}{18\alpha(1-\alpha)}.$$

One can show that function

$$g(\alpha) = 1 + \frac{(3+\alpha)^2}{18\alpha(1-\alpha)} - \frac{9-7\alpha}{3(1-\alpha)}$$

strictly decreases on $\alpha \in (0, 3/5)$ and is equal to 0 at $\alpha = \frac{3}{5}$. This implies that for all $\alpha < 3/5$ we have that for all $\frac{9-7\alpha}{3(1-\alpha)} < \frac{v-c}{t} < 1 + \frac{(3+\alpha)^2}{18\alpha(1-\alpha)}$ and a deviation to price $v - t$ is unprofitable.

To sum up, we can conclude that if $\alpha < \frac{3}{5}$ and $\frac{2(3-\alpha)}{3(1-\alpha)} < \frac{v-c}{t} < 1 + \frac{(3+\alpha)^2}{18\alpha(1-\alpha)}$, then there is an equilibrium in which firms set prices

$$p_w = c + \frac{3+\alpha}{3(1-\alpha)} t \quad \text{and} \quad p_{nw} = c + \frac{3-\alpha}{3(1-\alpha)} t,$$

all consumers in the monopoly segment buy from firm 1 and the market in the competitive segment is fully covered. Moreover, the marginal consumer in the competitive segment enjoys a positive surplus. The most remotely located consumer in the monopolistic market also enjoys a positive surplus. The respective profits are

$$\pi^w = \frac{(3+\alpha)^2}{18(1-\alpha)} t \quad \text{and} \quad \pi^{nw} = \frac{(3-\alpha)^2}{18(1-\alpha)} t.$$

Case 1.2: Monopolistic segment of firm 1 is fully covered. The consumer located at 1 in the monopolistic segment obtains zero surplus. Consider the possibility that a consumer located at 1 in the monopolistic market is indifferent between buying from firm 1 and taking the outside option. This implies that firm 1 sets a price $p_w = v - t$. From the analysis of the previous case, the best response price of firm 2 is $p_{nw} = \frac{1}{2}(c + t + p_w)$, implying that

$$p_w = v - t \;\text{ and }\; p_{nw} = \frac{v+c}{2}.$$

To ensure that firm 1 sells in the competitive market we must have that the location of the marginal consumer in the competitive market is in the interior, $|p_w - p_{nw}| < t$, implying that $\frac{v-c}{t} < 4$. Note that under this condition firm 2 does not find it profitable to deviate.

It remains to check firm 1's incentives to deviate. If firm 1 sets a price $p_1 > p_w$ and the marginal consumer in the downstream market is still in the interior, then its profit is given by

$$\begin{aligned}\pi_1(p_1, p_{nw}) &= (p_1 - c)\left(\alpha\frac{v-p_1}{t} + (1-\alpha)\left(\frac{1}{2} + \frac{p_{nw}-p_1}{2t}\right)\right)\\ &= (p_1 - c)\left(\frac{2\alpha v + (1-\alpha)t}{2t} + \frac{1-\alpha}{2t}p_{nw} - \frac{1+\alpha}{2t}p_1\right).\end{aligned}$$

The derivative of this profit (multiplied by $2t/(1+\alpha)$) is

$$\begin{aligned}&c + \frac{2\alpha v}{1+\alpha} + \frac{1-\alpha}{1+\alpha}t + \frac{1-\alpha}{1+\alpha}p_{nw} - 2p_1 < c + \frac{2\alpha v}{1+\alpha} + \frac{1-\alpha}{1+\alpha}t + \frac{1-\alpha}{1+\alpha}p_{nw} - 2p_w\\ &= -\frac{3+\alpha}{2(1-\alpha)}(v - c - 2t) \le 0,\end{aligned}$$

if and only if $\frac{v-c}{t} \ge 2$. The analysis of the monopoly problem implies that under this condition firm 1 does find it profitable to deviate to even higher prices at which it does not sell in the competitive market.

If firm 1 deviates to a price $p_1 < p_w$, then it does not increase sales in the monopolistic market and its profit function is given by

$$\pi_1(p_1, p_{nw}) = (p_1 - c)\left(\frac{1+\alpha}{2} + \frac{1-\alpha}{2t}(p_{nw} - p_1)\right).$$

The derivative of this profit function (multiplied by $2t/(1-\alpha)$) is

$$\begin{aligned}&c + \frac{1+\alpha}{1-\alpha}t + p_{nw} - 2p_1 > c + \frac{1+\alpha}{1-\alpha}t + p_{nw} - 2p_w\\ &= -\frac{3}{2}\left(v - c - \frac{2(3-\alpha)}{3(1-\alpha)}\right) \le 0,\end{aligned}$$

if and only if $\frac{v-c}{t} \le \frac{2(3-\alpha)}{3(1-\alpha)}$. This condition ensures that firm 1 does not deviate to lower prices.

To sum up, we conclude that for $\alpha \le \frac{3}{5}$ and $\frac{v-c}{t} \in \left[2, \frac{2(3-\alpha)}{3(1-\alpha)}\right]$ as well as for $\alpha > \frac{3}{5}$ and $\frac{v-c}{t} \in [2, 4]$ there exists an equilibrium in which firms set prices

$$p_w = v - t \ \text{ and } \ p_{nw} = \frac{v+c}{2},$$

all consumers in the monopoly segment buy from firm 1 and the market in the competitive segment is fully covered. The marginal consumer in the competitive market enjoys a positive surplus. The most remotely located consumer in the monopolistic market obtains zero surplus. The respective profits are

$$\pi^w = (v - c - t)\left(1 - (1-\alpha)\frac{v-c}{4t}\right) \ \text{ and } \ \pi^{nw} = (1-\alpha)\frac{(v-c)^2}{8t}.$$

Case 2: Monopolistic segment of firm 1 is not fully covered. If this case can occur in the equilibrium, then we have that the most remotely located consumer in the monopolistic market does not buy from firm 1 – that is, $v - p_w - t < 0$. The profit of firm 1 setting price p_1 at which the monopoly segment of firm 1 is not fully covered and the marginal consumer in the competitive market is in the interior is

$$\pi_1(p_1, p_{nw}) = (p_1 - c)\left(\frac{2\alpha v + (1-\alpha)t}{2t} + \frac{1-\alpha}{2t}p_{nw} - \frac{1+\alpha}{2t}p_1\right).$$

By taking the first-order condition and solving for the optimal price of firm 1 we find that

$$p_w = \frac{1}{2}\left(c + \frac{1-\alpha}{1+\alpha}p_{nw} + \frac{2\alpha v + (1-\alpha)t}{1+\alpha}\right).$$

The problem of firm 2 is exactly the same as in the previous case implying that $p_{nw} = \frac{1}{2}(c + t + p_w)$. By solving this system of equations with respect to p_w and p_{nw} we find

$$p_w = \frac{3+\alpha}{3+5\alpha}c + \frac{3(1-\alpha)}{3+5\alpha}t + \frac{4\alpha}{3+5\alpha}v \ \text{ and } \ p_{nw} = \frac{3(1+\alpha)}{3+5\alpha}c + \frac{3+\alpha}{3+5\alpha}t + \frac{2\alpha}{3+5\alpha}v.$$

We characterize conditions on parameters such that at this price the monopolistic market is not fully covered, the marginal consumer's location in the competitive market is in the interior.

The monopolistic market is not fully covered if and only if $v - p_w - t < 0$ which is equivalent to

$$\frac{3+\alpha}{3+5\alpha}v - \frac{3+\alpha}{3+5\alpha}c - \frac{2(3+\alpha)}{3+5\alpha}t < 0 \quad \Longleftrightarrow \quad \frac{v-c}{t} < 2.$$

Next, we check that the marginal consumer in the competitive market is in the interior. This occurs if and only if $|p_w - p_{nw}| < t$. Since the monopolistic market is not fully covered we have that

$$p_{nw} - p_w = \frac{4\alpha}{3+5\alpha}t - \frac{2\alpha}{3+5\alpha}(v-c) = \frac{2\alpha}{3+5\alpha}(2t-(v-c)) > 0$$

and moreover, $p_{nw} - p_w < \frac{4\alpha}{3+5\alpha}t < t$, implying that the marginal consumer is indeed in the interior.

The surplus of the marginal consumer is positive if

$$\begin{aligned} v - p_w - t\left(\frac{1}{2} + \frac{p_{nw}-p_w}{2t}\right) &= v - \frac{t}{2} - \frac{p_{nw}+p_w}{2} \\ &= \frac{3+2\alpha}{3+5\alpha}\left(v - c - \frac{9+3\alpha}{6+4\alpha}t\right) > 0 \quad \iff \quad \frac{v-c}{t} > \frac{9+3\alpha}{6+4\alpha}. \end{aligned}$$

Both conditions imply that if this equilibrium exists, then it must be that $\frac{v-c}{t} \in \left(\frac{9+3\alpha}{6+4\alpha}, 2\right)$.

To show that p_w and p_{nw} constitute an equilibrium it remains to show that *i)* firm 1 does not have incentives to set a lower price to fully serve either of the markets and *ii)* firm 2 does not have incentives to increase its price such that some consumers from the competitive market do not buy.

We start by exploring condition *i)* accounting for $\frac{v-c}{t} \in \left(\frac{9+3\alpha}{6+4\alpha}, 2\right)$. Suppose that firm 1 deviates from p_w and lowers its price to $p_1 \in (p_{nw} - t, v - t]$. In this case, it corners the monopoly market but the competitive market remains to be covered such that both firms have positive market shares. The profit from such a deviation is given by

$$\pi_1(p_1, p_{nw}) = (p_1 - c)\left(\frac{1+\alpha}{2} + \frac{1-\alpha}{2t}(p_{nw} - p_1)\right).$$

By taking the derivative (multiplied by $2t/(1-\alpha)$) we have that

$$\begin{aligned} c + \frac{1+\alpha}{1-\alpha}t + p_{nw} - 2p_1 &\geq c + \frac{1+\alpha}{1-\alpha}t + p_{nw} - 2v + 2t \\ &= -\left(v - p_{nw} - \frac{1+\alpha}{1-\alpha}t\right) + (2t - (v-c)). \end{aligned}$$

Note that the second term is positive since $\frac{v-c}{t} < 2$. The second term $v - p_{nw} - \frac{1+\alpha}{1-\alpha}t$ can be bound from above by $v - p_w - t$ which is negative as the most remotely located consumer is not served. Thus, we showed that the profit function of firm 1 is increasing on $(p_{nw}, v - t]$.

Moreover, at price p_{nw} firm 1 fully serves both markets and setting a price lower than it cannot be optimal. Therefore, we established the fact that any deviation to a price lower than $v-t$ is unprofitable implying that firm 1 does not find it optimal to deviate from p_w.

Next, we show that *ii)* is satisfied and firm 2 does not find it profitable to set a price $p_2 > 2v - t - p_w$ implying that consumers with locations $\frac{v-p_w}{t} + \varepsilon$, when $\varepsilon > 0$ is small, do not buy from either firm – that is, $v - p_2 - t\left(1 - \frac{v-p_w}{t}\right) < 0$. In this case, the profit of firm 2 strictly decreases if $p_2 > \frac{v+c}{2}$ where the latter expression represents the price a local monopoly would optimally set. Note that

$$\begin{aligned} p_2 - \frac{v+c}{2} &> v - t - p_w + \frac{v-c}{2} \\ &> \left(v - \frac{t}{2} - \frac{p_{nw}+p_w}{2}\right) + \frac{v-c-t}{2} > 0. \end{aligned}$$

The first term in brackets represents the surplus of the marginal consumer in the equilibrium and is always positive. The second term is also positive since $\frac{v-c}{t} > \frac{9+3\alpha}{6+4\alpha} > 1$. Hence, the profit function of firm 2 decreases for p_2 higher than $2v-t-p_w$ implying that firm 2 deviation to such a p_2 is unprofitable.

To sum up, we have established that for $\frac{v-c}{t} \in \left(\frac{9+3\alpha}{6+4\alpha}, 2\right)$ there is an equilibrium in which firms set prices

$$p_w = \frac{3+\alpha}{3+5\alpha}c + \frac{3(1-\alpha)}{3+5\alpha}t + \frac{4\alpha}{3+5\alpha}v \quad \text{and} \quad p_{nw} = \frac{3(1+\alpha)}{3+5\alpha}c + \frac{3+\alpha}{3+5\alpha}t + \frac{2\alpha}{3+5\alpha}v,$$

the monopolistic market is not fully covered; the competitive market is fully covered and the marginal consumer enjoys a positive surplus. The corresponding equilibrium profits are

$$\pi^w = \frac{1+\alpha}{2t}\left(\frac{4\alpha}{3+5\alpha}(v-c) + \frac{3(1-\alpha)}{3+5\alpha}t\right)^2 \text{ and } \pi^{nw} = \frac{1-\alpha}{2t}\left(\frac{2\alpha}{3+5\alpha}(v-c) + \frac{3+\alpha}{3+5\alpha}t\right)^2.$$

We have explored all the cases in which there is a marginal consumer in the competitive market who is located in the interior and enjoys a positive surplus. We come to the next possible equilibrium structure.

***iv)* Firm 1 sells in the competitive fully covered market and the marginal consumer obtains zero surplus.** Note that if this type of equilibrium occurs, then firm 1 serves exactly the same fraction of consumers in both markets implying that the monopolistic market cannot be fully covered.

Define the location of marginal consumer in the competitive market as $\hat{x} \in (0,1)$. This consumer is indifferent between both firms and the outside option. This implies that

$$p_w = v - t\hat{x} \ \text{ and } \ p_{nw} = v - t + t\hat{x}.$$

We characterize all possible $\hat{x}$ that can constitute an equilibrium. First, firms do not have incentives to increase their prices if and only if $\max\left\{0, 1 - \frac{v-c}{2t}\right\} \le \hat{x} \le \min\left\{\frac{v-c}{2t}, 1\right\}$. This condition is satisfied for some $\hat{x} \in (0,1)$ if and only if $\frac{v-c}{t} \ge 1$. Second, consider a deviation of firm 2 to a price $p_2 < p_{nw}$. The profit of firm 2 is

$$\pi_2(p_2, p_w) = (1-\alpha)(p_2 - c)\left(\frac{1}{2} + \frac{p_w - p_2}{2t}\right).$$

This profit function increases for prices $p_2 < p_w$ if and only if its derivative at p_2 is positive. The derivative of the profit function of firm 2 (multiplied by $2t$) is

$$\begin{aligned} c + t + p_w - 2p_2 &> c + t + p_w - 2p_{nw} \\ &= c + t + v - t\hat{x} - 2v + 2t - 2t\hat{x} \\ &= 3t\left(1 - \hat{x} - \frac{v-c}{t}\right) \ge 0, \end{aligned}$$

if and only if $\hat{x} \le 1 - \frac{1}{3}\frac{v-c}{t}$. Next, we establish the conditions on $\hat{x}$ to ensure that firm 1 does not deviate to lower prices. The profit of firm 1 deviating to $p_1 < p_w$ is given by

$$\pi_1(p_1, p_{nw}) = (p_1 - c)\left(\frac{2\alpha v + (1-\alpha)t}{2t} + \frac{1-\alpha}{2t}p_{nw} - \frac{1+\alpha}{2t}p_1\right).$$

The derivative of this profit function (multiplied by $2t/(1-\alpha)$) is

$$\begin{aligned} & c + \frac{2\alpha}{1+\alpha}v + \frac{1-\alpha}{1+\alpha}t + \frac{1-\alpha}{1+\alpha}p_{nw} - 2p_1 > c + \frac{2\alpha}{1+\alpha}v + \frac{1-\alpha}{1+\alpha}t + \frac{1-\alpha}{1+\alpha}p_{nw} - 2p_w \\ &= c + \frac{2\alpha}{1+\alpha}v + \frac{1-\alpha}{1+\alpha}t + \frac{1-\alpha}{1+\alpha}v - \frac{1-\alpha}{1+\alpha}t + \frac{1-\alpha}{1+\alpha}t\hat{x} - 2v + 2t\hat{x} \\ &= \frac{3+\alpha}{1+\alpha}t\left(\hat{x} - \frac{1+\alpha}{3+\alpha}\frac{v-c}{t}\right) \ge 0, \end{aligned}$$

if and only if $\hat{x} \ge \frac{1+\alpha}{3+\alpha}\frac{v-c}{t}$.

Thus, $\hat{x} \in (0,1)$ can be supported in an equilibrium if and only if

$$\begin{cases} \max\left\{0, 1 - \frac{v-c}{2t}\right\} \le \hat{x} \le \min\left\{\frac{v-c}{2t}, 1\right\}, \\ \hat{x} \le 1 - \frac{1}{3}\frac{v-c}{t}, \\ \hat{x} \ge \frac{1+\alpha}{3+\alpha}\frac{v-c}{t}. \end{cases}$$

Suppose that $\frac{v-c}{t} > 2$, then the first condition is always satisfied. Note that there exists $\hat{x}$ satisfying condition 2 and condition 3 if

$$\left(\frac{1+\alpha}{3+\alpha}+\frac{1}{3}\right)\frac{v-c}{t}=\frac{6+4\alpha}{9+3\alpha}\frac{v-c}{t}<1.$$

It is straightforward to see that this condition is never satisfied for $\frac{v-c}{t} > 2$.

Next, suppose that $\frac{v-c}{t} \in [1,2]$. Then, condition 1 simplifies to $1-\frac{v-c}{2t} \leq \hat{x} \leq \frac{v-c}{2t}$. We consider two cases of whether $\frac{v-c}{t} \in [1;6/5]$ or $\frac{v-c}{t} \in (6/5;2]$ separately. First, suppose that $\frac{v-c}{t} \in [1;6/5]$. Note that in this case $1-\frac{1}{3}\frac{v-c}{t} \geq 1-\frac{1}{3}\times\frac{6}{5} = \frac{1}{2}\frac{6}{5} \geq \frac{1}{2}\frac{v-c}{t}$. Moreover, since $\frac{1+\alpha}{3+\alpha} \leq \frac{1}{2}$ for all $\alpha \in [0,1]$ we have that there exists a non-empty interval of $\hat{x}$ satisfying all of the conditions. Note that $1-\frac{1}{2}\frac{v-c}{t} \geq (<)\frac{1+\alpha}{3+\alpha}\frac{v-c}{t}$ if and only if $\frac{v-c}{t} \leq (>)\frac{6+2\alpha}{5+3\alpha}$. Therefore, we can conclude that $\hat{x}$ that satisfies all the conditions

$$\begin{cases}\hat{x} \in \left[1-\frac{1}{2}\frac{v-c}{t}, \frac{1}{2}\frac{v-c}{t}\right], \text{ if } \frac{v-c}{t} \in \left[1, \frac{6+2\alpha}{5+3\alpha}\right] \\ \hat{x} \in \left[\frac{1+\alpha}{3+\alpha}\frac{v-c}{t}, \frac{1}{2}\frac{v-c}{t}\right], \text{ if } \frac{v-c}{t} \in \left(\frac{6+2\alpha}{5+3\alpha}, \frac{6}{5}\right].\end{cases}$$

Next, consider the case in which $\frac{v-c}{t} \in (6/5;2]$. By following the above argumentation we can show that $\frac{1}{2}\frac{v-c}{t} > 1-\frac{1}{3}\frac{v-c}{t}$. Moreover, $\frac{1+\alpha}{3+\alpha}\frac{v-c}{t} > \frac{1}{3}\times\frac{6}{5} = 1-\frac{1}{2}\times\frac{6}{5} > 1-\frac{1}{2}\frac{v-c}{t}$. This implies that $\hat{x}$ satisfying all the conditions belongs to $\left[\frac{1+\alpha}{3+\alpha}\frac{v-c}{t}, 1-\frac{1}{3}\frac{v-c}{t}\right]$. This interval is non empty if $\frac{v-c}{t} \leq \left(\frac{6}{5}, \frac{9+3\alpha}{6+4\alpha}\right]$.

To sum up, we conclude that for $\frac{v-c}{t} \in \left[1, \frac{9+3\alpha}{6+4\alpha}\right]$ there are multiple equilibria characterized by the location of the marginal consumer $\hat{x} \in (0,1)$. In particular possible equilibrium location of the marginal consumer can be summarized as follows

$$\begin{cases}\hat{x} \in \left[1-\frac{1}{2}\frac{v-c}{t}, \frac{1}{2}\frac{v-c}{t}\right], \text{ if } \frac{v-c}{t} \in \left[1, \frac{6+2\alpha}{5+3\alpha}\right] \\ \hat{x} \in \left[\frac{1+\alpha}{3+\alpha}\frac{v-c}{t}, \frac{1}{2}\frac{v-c}{t}\right], \text{ if } \frac{v-c}{t} \in \left(\frac{6+2\alpha}{5+3\alpha}, \frac{6}{5}\right], \\ \hat{x} \in \left[\frac{1+\alpha}{3+\alpha}\frac{v-c}{t}, 1-\frac{1}{3}\frac{v-c}{t}\right], \text{ if } \frac{v-c}{t} \leq \left(\frac{6}{5}, \frac{9+3\alpha}{6+4\alpha}\right].\end{cases}$$

The equilibrium prices are given by

$$p_w = v - t\hat{x} \text{ and } p_{nw} = v - t + t\hat{x},$$

the monopolistic market is not fully covered, the competitive market is fully covered and the marginal consumer obtains zero surplus. The corresponding resulting profits are

$$\pi^w = \frac{1}{t}\left(\frac{v-c}{t}-\hat{x}\right)\hat{x} \text{ and } \pi^{nw} = \frac{1-\alpha}{t}\left(\frac{v-c}{t}-(1-\hat{x})\right)(1-\hat{x}).$$

Note that $\hat{x}$ is always weakly lower than $\frac{1}{2}\frac{v-c}{t}$ implying that π^w increases in $\hat{x}$ and π^{nw} decreases in $\hat{x}$ for all $\hat{x}$ that might constitute an equilibrium.

The key results First, we establish the lemma ensuring that assumption 1 is satisfied.

Lemma 3. *In the pure strategy equilibrium of the Hotelling model with linear transportation cost and a positive fraction of consumers using the ad blocker Assumption 1 is always satisfied.*

Proof. To be added. □

Next, we show that for all parameters in the model for which there exists a pure strategy equilibrium, we have that the ad blocker whitelists only one publisher

Lemma 4. *In the pure strategy equilibrium of the Hotelling model with linear transportation cost and a positive fraction of consumers using the ad blocker we always have that* $\pi^m + 2\pi^{nw} > (3-\alpha)\pi^d$ *is satisfied implying that the ad blocker whitelists a single publisher.*

Proof. To be added. □

C Product market competition in the model by Perloff and Salop (1985)

Two advertisers produce differentiated products at marginal costs $c \geq 0$. There is a unit mass of consumers, and each consumer has unit demand. The match values of consumers are identically and independently distributed across consumers and the advertisers according to the uniform distribution on $[\underline{v}, \overline{v}]$. The outside option of consumers is normalized to zero.

As in the Hotelling example, advertisers simultaneously choose prices p_A, p_B. Then, consumers choose between buying the advertised product that provides the highest net surplus and choosing the outside option.

Monopoly If only advertiser A advertises, advertiser A operates as a monopolist. Its profit at price $p_A \in [\underline{v}, \overline{v}]$, $\overline{v} > c$, is given by $(p_A - c)(\overline{v} - p_A)/(\overline{v} - \underline{v})$. Thus, the monopoly price is equal to $p_m = \max\{(\overline{v}+c)/2, \underline{v}\}$, the monopoly profit is $\pi^m = \frac{(\overline{v}-c)^2}{4(\overline{v}-\underline{v})}$ if $\overline{v} - \underline{v} \geq \underline{v} - c$ and $\pi^m = \underline{v} - c$ otherwise.

Symmetric duopoly competition If both advertisers advertise and there is symmetric duopoly competition, we can show that there is a unique symmetric equilibrium of the symmetric advertiser competition game that is given by

$$p_d = \begin{cases} \underline{v} + \sqrt{2((\overline{v}-\underline{v})^2 - (\overline{v}-\underline{v})(\underline{v}-c))} - (\overline{v}-\underline{v}) & \text{if } \overline{v}-\underline{v} \geq 2(\underline{v}-c), \\ c + (\overline{v}-\underline{v})/2 & \text{if } \overline{v}-\underline{v} < 2(\underline{v}-c). \end{cases}$$

We characterize the symmetric equilibrium in which both advertisers set p_d.

First, suppose that $p_d \in [\underline{v}, \overline{v}]$. Then, the profit of firm i setting price $p_i \in [p^d, \overline{v}]$ when firm $j \neq i$ sets p_d is given by

$$\pi_i(p_i, p_d) = (p_i - c)\mathbb{P}[v_i - p_i \geq \max\{v_j - p_d, 0\}] = (p_i - c)\int_{p_i}^{\overline{v}} \frac{(v_i - p_i + p_d) - \underline{v}}{\overline{v}-\underline{v}} \frac{1}{\overline{v}-\underline{v}} dv_i.$$

The first-order condition (multiplied by $(\overline{v}-\underline{v})^2$) at the symmetric equilibrium price $p_i = p^d$ is given by

$$\begin{aligned} 0 &= \int_{p_d}^{\overline{v}} (v - \underline{v})dv + (p_d - c)(-(p_d - \underline{v}) - (\overline{v} - p_d)) \\ &= \frac{1}{2}((\overline{v}-\underline{v})^2 - (p_d - \underline{v})^2) - (p_d - c)(\overline{v}-\underline{v}) = (\overline{v}-\underline{v})^2 - (\overline{v}-\underline{v})(\underline{v}-c) - \frac{1}{2}(\overline{v}-\underline{v}+p_d-\underline{v})^2, \end{aligned}$$

which implies that

$$p_d = \underline{v} + \sqrt{2((\overline{v}-\underline{v})^2 - (\overline{v}-\underline{v})(\underline{v}-c))} - (\overline{v}-\underline{v}). \tag{2}$$

Note that $p_d \geq \underline{v}$ if and only if $\overline{v}-\underline{v} \geq 2(\underline{v}-c)$. If $c < \underline{v}$ and $\overline{v}$ is sufficiently close $\underline{v}$ such that $\overline{v}-\underline{v} < 2(\underline{v}-c)$, then there is no symmetric equilibrium price $p^d > \underline{v}$. Otherwise, if $c \geq \underline{v}$ or if $c < \underline{v}$, but $\overline{v}$ is high enough so that $p_d > \underline{v}$, then p_d characterized in (2) can be a candidate for symmetric equilibrium.

To establish whether p_d defined in (2) can be an equilibrium it remains to consider *i)* a deviation to $p_i \in [\underline{v}, p_d)$ and *ii)* a deviation to $p_i < \underline{v}$.

First, consider a deviation to a price $p_i \in [\underline{v}, p_d)$. The profit from this deviation is equal to

$$\begin{aligned} \pi_i(p_i, p_d) &= (p_i - c)\left(\int_{p_i}^{\overline{v}-p_d+p_i} \frac{(v_i - p_i + p_d) - \underline{v}}{\overline{v}-\underline{v}} \frac{1}{\overline{v}-\underline{v}} dv_i + \int_{\overline{v}-p_d+p_i}^{\overline{v}} 1 \times \frac{1}{\overline{v}-\underline{v}} dv_i\right) \\ &= (p_i - c)\left(\int_{p_d}^{\overline{v}} \frac{u - \underline{v}}{\overline{v}-\underline{v}} \frac{1}{\overline{v}-\underline{v}} du + \frac{p_d - p_i}{\overline{v}-\underline{v}}\right) \end{aligned}$$

The derivative of the profit function with respect to p_i is

$$\left(\int_{p_d}^{\overline{v}} \frac{u-\underline{v}}{\overline{v}-\underline{v}}\frac{1}{\overline{v}-\underline{v}}du + \frac{p_d-p_i}{\overline{v}-\underline{v}}\right) - \frac{p_i-c}{\overline{v}-\underline{v}} = \left(\frac{p_d-c}{\overline{v}-\underline{v}} + \frac{p_d-p_i}{\overline{v}-\underline{v}}\right) - \frac{p_i-c}{\overline{v}-\underline{v}} = \frac{2(p_d-p_i)}{\overline{v}-\underline{v}} > 0,$$

implying that the profit function strictly increases on $[\underline{v}p_d]$, where p_d solves (2).

Second, consider a deviation to a price $p_i < \underline{v}$. The profit of a firm setting p_i is given by

$$\begin{aligned}\pi_i(p_i,p_d) &= (p_i-c)\left(\int_{\underline{v}}^{\overline{v}-p_d+p_i} \frac{(v_i-p_i+p_d)-\underline{v}}{\overline{v}-\underline{v}}\frac{1}{\overline{v}-\underline{v}}dv_i + \int_{\overline{v}-p_d+p_i}^{\overline{v}} 1\times\frac{1}{\overline{v}-\underline{v}}dv_i\right)\\ &= (p_i-c)\left(\int_{p_d+(\underline{v}-p_i)}^{\overline{v}} \frac{u-\underline{v}}{\overline{v}-\underline{v}}\frac{1}{\overline{v}-\underline{v}}du + \frac{p_d-p_i}{\overline{v}-\underline{v}}\right).\end{aligned}$$

By taking the derivative of the profit function with respect to p_i we find that

$$\begin{aligned}&\left(\int_{p_d+(\underline{v}-p_i)}^{\overline{v}} \frac{u-\underline{v}}{\overline{v}-\underline{v}}\frac{1}{\overline{v}-\underline{v}}du + \frac{p_d-p_i}{\overline{v}-\underline{v}}\right) + \left(\frac{p_d-p_i}{(\overline{v}-\underline{v})^2} - \frac{1}{\overline{v}-\underline{v}}\right)(p_i-c)\\ &> \left(\int_{p_d}^{\overline{v}} \frac{u-\underline{v}}{\overline{v}-\underline{v}}\frac{1}{\overline{v}-\underline{v}}du + \frac{p_d-p_i}{\overline{v}-\underline{v}}\right) - \frac{(p_i-c)}{\overline{v}-\underline{v}} > 0,\end{aligned}$$

which follows from the argument made for a deviation to a price in $[\underline{v}, p_d]$.

This establishes the result that if $\overline{v}-\underline{v} \geq 2(\underline{v}-c)$ there is a unique symmetric equilibrium price p_d given in (2).

Now, suppose that $p_d < \underline{v}$. Consider firm i setting price $p_i > p_d$. First, suppose that $p_i < \underline{v}$. The profit from such a deviation is

$$\pi_i(p_i,p_d) = (p_i-c)\int_{\underline{v}}^{\overline{v}} \frac{(v_i-p_i+p_d)-\underline{v}}{\overline{v}-\underline{v}}\frac{1}{\overline{v}-\underline{v}}dv_i.$$

The first-order condition for firm i at $p_i = p_d$ implies that

$$\int_{\underline{v}}^{\overline{v}} \frac{v-\underline{v}}{\overline{v}-\underline{v}}\frac{1}{\overline{v}-\underline{v}}dv = \frac{p_d-c}{\overline{v}-\underline{v}}.$$

By solving for p_d we obtain

$$p_d = c + \frac{\overline{v}-\underline{v}}{2}. \tag{3}$$

Note that p_d is less than $\underline{v}$ if and only if $\overline{v}-\underline{v} < 2(\underline{v}-c)$.

Next, we show that this is indeed the unique symmetric equilibrium in this parameter range. First, consider a deviation to a price $p_i \in [\underline{v}, \overline{v}]$. The profit from this deviation is equal to

$$\pi_i(p_i,p_d) = (p_i-c)\int_{p_i}^{\overline{v}} \frac{(v_i-p_i+p_d)-\underline{v}}{\overline{v}-\underline{v}}\frac{1}{\overline{v}-\underline{v}}dv_i.$$

The derivative of this profit with respect to p_i is

$$\int_{p_i}^{\overline{v}} \frac{(v_i - p_i + p_d) - \underline{v}}{\overline{v} - \underline{v}} \frac{1}{\overline{v} - \underline{v}} dv_i + (p_i - c)\left(-\frac{p_d - \underline{v}}{(\overline{v} - \underline{v})^2} - \frac{\overline{v} - p_d}{(\overline{v} - \underline{v})^2}\right)$$
$$< \int_{p_i}^{\overline{v}} \frac{(v_i - p_d + p_d) - \underline{v}}{\overline{v} - \underline{v}} \frac{1}{\overline{v} - \underline{v}} dv_i - \frac{p_i - c}{\overline{v} - \underline{v}}$$
$$\int_{\underline{v}}^{\overline{v}} \frac{v_i - \underline{v}}{\overline{v} - \underline{v}} \frac{1}{\overline{v} - \underline{v}} dv_i - \frac{p_i - c}{\overline{v} - \underline{v}} = \frac{p_d - c}{\overline{v} - \underline{v}} - \frac{p_i - c}{\overline{v} - \underline{v}} = \frac{p_d - p_i}{\overline{v} - \underline{v}} < 0.$$

This implies that the profit function strictly decreases on $[\underline{v}, \overline{v}]$ and a deviation to any price in this region is unprofitable.

It remains to consider a downward deviation to prices below p_d. Firm i setting price $p_i < p_d$ makes profits

$$\pi(p_i, p_d) = (p_i - c)\left(\int_{\underline{v}}^{\overline{v} - p_d + p_i} \frac{(v_i - p_i + p_d) - \underline{v}}{\overline{v} - \underline{v}} \frac{1}{\overline{v} - \underline{v}} dv_i + \int_{\overline{v} - p_d + p_i}^{\overline{v}} 1 \times \frac{1}{\overline{v} - \underline{v}} dv_i\right)$$
$$= (p_i - c)\left(\int_{\underline{v} + (p_d - p_i)}^{\overline{v}} \frac{u - \underline{v}}{\overline{v} - \underline{v}} \frac{1}{\overline{v} - \underline{v}} du + \frac{p_d - p_i}{\overline{v} - \underline{v}}\right).$$

The derivative of the profit function with respect to p_i is

$$\left(\int_{\underline{v} + (p_d - p_i)}^{\overline{v}} \frac{u - \underline{v}}{\overline{v} - \underline{v}} \frac{1}{\overline{v} - \underline{v}} du + \frac{p_d - p_i}{\overline{v} - \underline{v}}\right) + \left(\frac{p_d - p_i}{(\overline{v} - \underline{v})^2} - \frac{1}{\overline{v} - \underline{v}}\right)(p_i - c)$$
$$= \left(\int_{\underline{v} + (p_d - p_i)}^{\overline{v}} \frac{u - \underline{v}}{\overline{v} - \underline{v}} \frac{1}{\overline{v} - \underline{v}} du + \int_{\underline{v}}^{\underline{v} + (p_d - p_i)} \frac{\overline{v} - \underline{v}}{\overline{v} - \underline{v}} \frac{1}{\overline{v} - \underline{v}} dv_i\right) + \left(\frac{p_d - p_i}{(\overline{v} - \underline{v})^2} - \frac{1}{\overline{v} - \underline{v}}\right)(p_i - c)$$
$$= \left(\int_{\underline{v} + (p_d - p_i)}^{\overline{v}} \frac{u - \underline{v}}{\overline{v} - \underline{v}} \frac{1}{\overline{v} - \underline{v}} du + \int_{\underline{v}}^{\underline{v} + (p_d - p_i)} \frac{\overline{v} - \underline{v}}{\overline{v} - \underline{v}} \frac{1}{\overline{v} - \underline{v}} dv_i\right) + \left(\frac{p_d - p_i}{(\overline{v} - \underline{v})^2} - \frac{1}{\overline{v} - \underline{v}}\right)(p_i - c)$$
$$> \int_{\underline{v}}^{\overline{v}} \frac{u - \underline{v}}{\overline{v} - \underline{v}} \frac{1}{\overline{v} - \underline{v}} du - \frac{p_i - c}{\overline{v} - \underline{v}} = \frac{p_d - c}{\overline{v} - \underline{v}} - \frac{p_i - c}{\overline{v} - \underline{v}} > 0,$$

which implies that the profit function strictly increases on $[0, p_d]$ and downward deviations from p_d are unprofitable.

Thus, we have shown that if $\overline{v} - \underline{v} < 2(\underline{v} - c)$, then there exists a unique symmetric equilibrium p_d that is given by (3).

Note that if $\overline{v} - \underline{v} \geq 2(\underline{v} - c)$, then the market is not fully covered. The total industry profit is given by

$$2\pi^d = (p_d - c)\left(1 - \left(\frac{p_d - \underline{v}}{\overline{v} - \underline{v}}\right)^2\right).$$

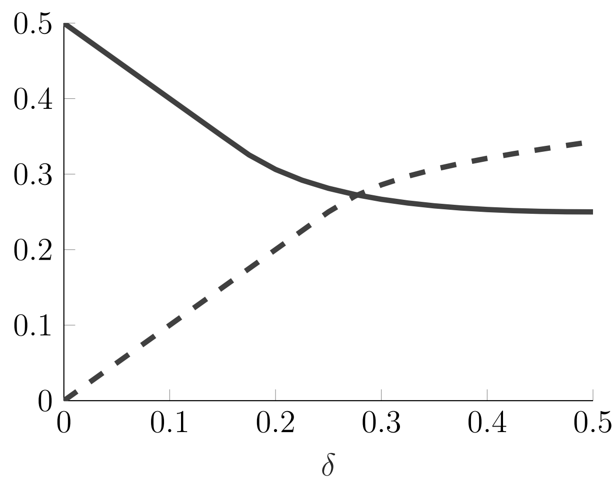

Figure 3: Monopoly and duopoly industry profits in Example 2 with π^m (solid) and $2\pi^d$ (dashed) for $c = 0$ and $v \sim U[0.5 - \delta, 0.5 + \delta]$.

Otherwise, if $\overline{v} - \underline{v} < 2(\underline{v} - c)$, then all consumers buy from some firm and the total industry profit is equal to $p_d - c$.

Figure C shows the profit under monopoly (solid) and under duopoly (dashed) for the uniform distribution on $[\underline{v}, \overline{v}] = [0.5 - \delta, 0.5 + \delta]$, where $\delta \in [0, 0.5]$ and marginal cost $c = 0$.

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