Firm productivity and derived factor demand under variable markups

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Abstract

Under perfect competition, productivity growth leads firms to expand production and demand more inputs. If firms have market power, I prove that this is not always the case. At high levels of markups, firms may reduce their input demand when they become more productive. This decoupling of input demand from productivity growth is driven by the incomplete pass-through of productivity to output. I characterize the theoretical conditions that lead to this result in terms of the shape of output demand and market structure. Many widely-used demand functions meet these conditions in workhorse models of monopolistic and oligopolistic competition. I also discuss under what circumstances this decoupling can be detected in the data. In an empirical illustration based on Chinese manufacturing firms, I find patterns consistent with this result in many narrowly-defined industries.

Keywords: *productivity, derived factor demand, variable markups, incomplete pass-through.*

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1 Introduction

Productivity growth is one of the main drivers of firm and industry dynamics (Hopenhayn, 2014). In perfectly competitive markets, it is well understood that there is a clear relationship between the productivity of a firm, its output level, and its derived factor demand, i.e. the amount of input factors employed to maximize its profits. When a firm becomes more productive, it has the incentive to expand production and demand more inputs. For this reason, firms' employment and intermediate inputs are used in many applications as a *proxy* for firm productivity, both in levels (Moscarini & Postel-Vinay, 2012) and in changes (Levinsohn & Petrin, 2003).

In recent years, however, new evidence and concerns emerged regarding a decline in the responsiveness of input demand to firm-level productivity shocks (Decker, Haltiwanger, Jarmin, & Miranda, 2020). A recent strand of research at the intersection of industrial organization and macroeconomics has started to explore the role of firms' market power in driving this decline and influencing the transmission of productivity growth to output and job creation (De Loecker, Eeckhout, & Mongey, 2021; Edmond, Midrigan, & Xu, 2018; Eeckhout, 2021; Syverson, 2019).

In this paper, I prove that if a firm increases its markup when it becomes more productive, its derived factor demand becomes gradually less responsive to productivity. At high levels of markups, this mechanism can be strong enough that firms reduce their input use after a productivity shock. In this regard, the ability of firms to vary their markups can lead to a decoupling of derived factor demand from productivity growth, which is a consequence of market power that has remained overlooked so far. The contribution of my paper is to shed light on this decoupling, to characterize its theoretical determinants in workhorse models of imperfect competition, and to show under which conditions it is possible to detect it in the data.

To establish this result, I consider how the derived factor demand of a firm reacts to a *Hicks-neutral* productivity shock. In general, a positive productivity shock generates two effects. A higher productivity level allows the firm to produce the same level of output with fewer inputs. At the same time, a higher productivity level lowers the firm's marginal costs, raising its incentive to produce more. Thus, the firm needs more inputs to meet its higher production targets. Whether the firm increases or decreases its derived factor demand depends on how these opposing effects balance out. I show that this is primarily driven by how much the firm decides to expand its output, which is an equilibrium outcome influenced by the features of output demand and the nature of competition.

Suppose that the firm exerts a certain degree of market power over its customers and faces a lower price elasticity of demand if it produces more. In that case, its incentive to expand production when it becomes more productive gradually declines. This occurs because its marginal revenue is strongly diminishing in output. When demand becomes nearly satiated at high levels of output, the firm finds it optimal to expand production less than proportionally to the productivity improvement. As a result, the latter is more than enough to cover the additional production, and fewer inputs are needed. Thus, the mechanism that leads to the decoupling of derived factor demand from productivity is the incomplete pass-through of productivity to output.

The features of output demand are key to determining whether this mechanism occurs in practice. Starting from the case of a monopoly, I prove that the pass-through of productivity to output becomes incomplete when the price elasticity of demand is low enough. Still, the exact value depends on the convexity (or curvature) of demand. This is because the rate at which marginal revenue diminishes with output depends on the values of both the elasticity and the convexity. To identify which demand systems lead to this outcome, I bring this insight into the demand manifold framework recently developed by Mrázová and Neary (2017, 2019, 2020). This framework allows comparing demand functions based only on the values of their elasticity and convexity. Building on it, I prove that a decoupling of derived factor demand from productivity can arise in many demand specifications that are widely used in the literature.[1]

Characterizing this decoupling in terms of the elasticity and convexity of output demand allows me to link it with many other comparative statics predictions that are also driven by these two fundamental features of demand. In particular, I show that a decoupling of derived factor demand from productivity occurs in correspondence with high levels of markups and low pass-through rates of cost to prices. Since markups and pass-through rates can be estimated at the firm level, this connection is particularly useful to infer whether a firm operates in a region of demand where it would scale back its input use following a productivity shock.

Beyond monopoly, I demonstrate that a similar mechanism is also at play in workhorse models of monopolistic competition and oligopolistic competition in quantity. Intuitively, this is because each

[1] Among others, it arises under linear demand, the Klenow and Willis (2016) specification of Kimball demand, the Bulow and Pfleiderer (1983) demand, the Linear expenditure system, the CARA demand used by Behrens, Mion, Murata, and Suedekum (2020), the Logistic demand (Cowan, 2016), the Multinomial and Mixed Logit demand (Miravete, Seim, & Thurk, 2022), and many other functional forms of demand that satisfy Marshall's Second Law of Demand. This property is considered theoretically more plausible and consistent with empirical evidence showing an incomplete pass-through of costs to prices (Nakamura & Zerom, 2010) and the fact that firms producing more exhibit lower pass-through rates (Berman, Martin, & Mayer, 2012) and higher markups (De Loecker & Goldberg, 2014).

firm behaves as a monopolist on its residual demand curve in these settings. For this reason, I show that the responsiveness of each firm to productivity depends on the price elasticity of its *residual* demand. Of course, the latter is influenced by the shape of market demand but also by any element that determines the competitive pressures perceived by each firm, such as the number of competitors, their conduct, and the differences in their productivity.

Based on these insights, I derive the following important results. First, firms can react very differently to the same productivity shock even within the same market. While less productive low-markup firms always increase their inputs, more productive high-markup firms have a lower - and potentially negative - elasticity of derived factor demand to productivity. In this sense, variable markups lead to a gradient of responsiveness to firm-level productivity shocks. Second, the relationship between the levels of productivity and input use can be non-monotonic under monopolistic competition. In other words, the most productive firms within a market are not necessarily the largest firms in terms of input use. Third, for a given functional form of demand, a decoupling of factor demand from productivity growth is more likely to emerge in less competitive markets. Fourth, the range of demand functions leading to this decoupling is wider under oligopoly compared to monopolistic settings. Notably, I find that even a Constant Elasticity of Substitution (CES) demand can lead to this result under Cournot competition.[2]

While I investigate the role of demand in great generality, throughout my analyses I consider a rather stylized supply side, featuring a production function with Hicks-neutral productivity, a single input factor, constant returns to scale, and price-taking behavior in the factor market. However, when I relax these assumptions one at a time under monopoly, I prove that the decoupling of derived factor demand from productivity occurs also with multiple input factors, with a technology displaying non-constant returns to scale, and with monopsony power in the factor market.

After establishing the theoretical determinants of the decoupling of derived factor demand from productivity, I discuss how to assess its empirical relevance. To detect this result it would be enough to observe two monopolistically competitive firms that face the same demand conditions and have the same technology, except for their Hicks-neutral productivity levels. If the more productive firm produces more but employs fewer inputs, this implies that it operates in the range of demand where input demand decreases with productivity. However, this approach is difficult to implement with real-world data because of the identification challenges it poses. To begin with, I show that other

[2]CES demand does not lead to this decoupling under monopolistic competition but it does under oligopolistic competition. This is because strategic interactions lead to a variable elasticity of *residual* demand and thus variable markups.

sources of firm heterogeneity may hide this non-monotonicity, even when it is actually there. This may occur because favorable demand or cost shifters always lead firms to increase both their output and input use. In addition to that, estimating productivity is very challenging from a methodological point of view in contexts with variable markups, multiple sources of firm heterogeneity, and output data reported in terms of revenues instead of physical quantities. Furthermore, the control function approach, which is the state-of-the-art method to estimate production functions, critically relies on a monotonic relationship between input demand and productivity (Ackerberg, Caves, & Frazer, 2015).

To overcome these challenges, I derive two cross-sectional predictions under monopolistic competition that are informative about the decoupling but do not require estimating firms' productivity or taking a stance on the specific functional form of demand faced by the firms. The logic behind these predictions is that the equilibrium distributions of input use (in levels or changes), revenue, and markups convey some information about the decoupling. First, I show that if the relationship between input use and productivity is non-monotonic in a cross-section, this must be observable also in the relationship between revenue and input levels. Second, I prove that if a firm increases its revenue and markups but reduces its input use, then it is operating in the range of price elasticities of demand where the decoupling takes place. Across firms, this is more likely to occur among those setting higher markups.

As an illustration, I bring these two predictions to the data on Chinese manufacturing firms. Building on previous work by Brandt, Van Biesebroeck, and Zhang (2012, 2014), I use data from the census of manufacturing firms conducted by the National Bureau of Statistics of China. This enables me to analyze the input decisions of large single-product firms in more than 350 narrowly-defined industries during a period of intense productivity growth. Overall, I find patterns in the data in line with these predictions in more than one-third of the industries analyzed. This suggests that the non-monotonicity between derived factor demand and productivity is empirically relevant.

Since the interest in firm productivity and derived input demand spans a wide range of fields, my paper relates and contributes to various strands of the literature. First, my paper relates to the literature that uses employment (or intermediate inputs) as an indirect measure of firm productivity. This practice is common in both academic (Levinsohn & Petrin, 2003; Moscarini & Postel-Vinay, 2012) and policy work (Bassi et al., 2019). While theoretically sound under perfect competition or monopolistic competition with CES demand (Melitz, 2003), my contribution is to demonstrate that this is not necessarily the case in contexts with variable markups. In fact, most productive firms are not necessarily the largest in terms of input use. Most models of monopolistic competition with variable markups lead to this prediction, especially when productivity is assumed to be unbounded. Among others, this is the case with demands that satisfy Marshall's Second Law in the models developed by Zhelobodko, Kokovin, Parenti, and Thisse (2012), Dhingra and Morrow (2019), and Mayer, Melitz, and Ottaviano (2021). However, this result has remained neglected in the literature. To my knowledge, Bakhtiari (2009) and Matsuyama and Ushchev (2022) are the only working papers that acknowledge the non-monotonic relationship between input use and productivity. On the contrary, most papers focusing on variable markups assume straight away that more productive firms are the largest in terms of input. Among others, prominent examples of this practice can be found in De Loecker and Syverson (2021) or in Autor, Dorn, Katz, Patterson, and Van Reenen (2020). Ruling out this non-monotonicity by assumption remains an admissible option. However, my paper demonstrates that this comes at the cost of severely restricting the range of markups and pass-through rates that a model can predict and accommodate.

Second, the results of my paper are relevant to the literature on production function estimation with the control function approach. My contribution is to extend the theoretical bases for using this approach in imperfectly competitive settings with variable markups. Beyond perfect competition (Levinsohn & Petrin, 2003) and monopolistic competition with CES demand (De Loecker, 2011), a more general proof was lacking indeed. On the downside, my findings on the non-monotonicity between input demand and productivity indicate that there are limits to the validity of the control function approach. In this regard, my paper provides clear theoretical conditions that need to be checked before applying this approach in settings with variable markups. In addition, the two predictions that I derive about the non-monotonicity provide practitioners with an implementable strategy to detect this pattern in standard firm-level data.

Third, my paper links the responsiveness of firms' factor demand to the features of output demand, building on the demand manifold framework introduced by Mrázová and Neary (2017). I develop their insights further by extending the range of applications of the manifold to input decisions of firms and to oligopolistic settings. In doing so, I revisit a relatively old literature on the Hicks–Marshall laws of derived demand under imperfect competition. In particular, my analyses relate to those of Maurice and Ferguson (1973) and Foran (1976) under monopoly, and to Waterson (1980) and de Meza (1982) concerning oligopoly. While this literature already made clear that the relevant elasticity to analyze the behavior of firms with market power is the elasticity of marginal revenue to output, my contribution is to express it in terms of meaningful features of output demand through the lens of the demand manifold framework.[3] In this regard, my analysis is also related to the work by Saint-Paul (2006), who explores the role of demand satiation in the relationship between wages and labor demand. More generally, my paper analyzes how the derived factor demand changes after a productivity shock, while the focus of this literature has been on the reactions to factor price changes.

From this perspective, another contribution of my work is to enlarge the scope of the theoretical literature on pass-through (Weyl & Fabinger, 2013). While the standard focus is on changes in marginal costs (due to taxes, input price shocks, exchange rates, tariffs, etc.), I look at productivity, a particular input-saving marginal cost shifter. Moreover, rather than limiting the analysis to the price reactions, I look at the changes in output and, as a by-product, in input demand.

Finally, my paper relates to the literature on the declining responsiveness of labor demand to firm-level productivity shocks. To explain this decline, Decker et al. (2020) mainly focus on the role of adjustment costs. Instead, my paper and De Loecker et al. (2021) argue that higher markups make firms' output and employment less responsive to productivity. Still, while their analysis is centered around the role of strategic interactions among firms in a model with nested-CES demand, my paper proves that this is a much more general result in terms of demand, market structure, and magnitude. Moreover, I show that the presence of variable markups generates a gradient of responsiveness across firms also in the case of demand and cost shocks. However, only a productivity shock can lead to a contraction of derived factor demand at high levels of markups.

The paper is structured as follows. Section 2 provides the main theoretical results on the decoupling of derived factor demand from productivity under imperfect competition. In Section 3, I discuss how to assess its empirical relevance. Section 4 contains an empirical illustration based on Chinese manufacturing firm-level data. Section 5 briefly discusses the implications of this overlooked result and directions for future research. Section 6 concludes. The Appendices contain proofs of all my propositions, further technical and data details, as well as robustness checks.

[3]Contrary to Maurice and Ferguson (1973), for example, who argue that the elasticity of marginal revenue "*is unquestionably related to the elasticity of commodity demand. Yet the relation is a tenuous one, and it cannot be stated explicitly in meaningful economic terms*" (p. 185).

2 Theory

Why do firms with market power decrease their derived factor demand after a productivity shock? In this section, I identify the theoretical conditions that answer this question and characterize them both in terms of the features of output demand and the nature of competition. In Section 2.1, I define the theoretical settings and provide the intuition of the key mechanism leading to this result. In Section 2.2, I present my main theoretical result under monopoly and in Section 2.3 I describe the features of output demand that determine it. In Section 2.4, I show that this result occurs also in workhorse models of monopolistic and oligopolistic competition, and discuss how it is influenced by the market structure and degree of competition across firms.

2.1 Derived factor demand and productivity shocks

To build the intuition, I consider a generic profit-maximizing firm i that produces and sells a single product under the following assumptions.

Assumption A1 (Input factor). *The firm produces its output q_i according to a standard production function $q_i = f(x_i)\,\omega_i$ where x_i is a single input factor, which is static and fully flexible, and ω_i denotes its Hicks-neutral productivity level.*

Assumption A2 (Technology). *The productive technology of the firm is linear and exhibits constant returns to scale, i.e. $q_i = x_i\,\omega_i$.*

Assumption A3 (Input price). *The firm is price-taker on the input market and $w > 0$ is the prevailing market price at which the firm can employ the input x.*

I assume this stylized setting to highlight in a parsimonious way the key economic mechanism underlying the main result of the paper. However, in Section 2.5 I relax these assumptions one at a time and show that the main results of the paper hold in more general environments with multiple input factors, with non-constant returns to scale, and with market power in the input market.

Under the assumptions (**A1-A2-A3**), I consider a situation where firm i becomes more productive. How would this firm adjust its derived factor demand, i.e. the amount of input used to maximize its profits? This is the key comparative static I focus on throughout the paper.[4] Underlying this question,

[4] In this regard, whenever I mention the outcome of a productivity shock experienced by a firm, in fact I mean the comparison of the equilibrium outcomes of two identical firms with a different level of productivity. Since my analysis is essentially static, the latter would be more correct. For expositional convenience, however, I intentionally refer to productivity shocks.

two opposing forces are at play. A higher level of productivity means that the firm can produce the same level of output with a lower amount of input. At the same time, a higher productivity level implies that its marginal costs are now lower, which raises its incentive to expand output. Thus, the firm needs more input to meet its higher production targets. The relative strength of these two effects is theoretically ambiguous. In general, the net effect depends on the firm's optimal rate of output expansion. To highlight this trade-off, we can start with the profit-maximizing levels of output and input,

$$q_i^* = x_i^* \, \omega_i \, , \tag{1}$$

express it in logs and isolate the derived factor demand x^* so that

$$\log(q_i^*) = \log(x_i^*) + \log(\omega_i)$$
$$\log(x_i^*) = \log(q_i^*) - \log(\omega_i) \, .$$

By taking the total derivatives with respect to (log) productivity, the relationship between optimal input and output changes after a productivity shock can be expressed in terms of elasticities[5], i.e.

$$\frac{d\log(x_i^*)}{d\log(\omega_i)} = \frac{d\log(q_i^*)}{d\log(\omega_i)} - \frac{d\log(\omega_i)}{d\log(\omega_i)} \tag{2}$$

$$\eta_{x_i^*,\omega_i} \quad = \quad \eta_{q_i^*,\omega_i} \quad - \quad 1 \ . \tag{3}$$

The elasticity of derived factor demand to productivity, i.e. $\eta_{x^*,\omega}$, depends on the elasticity of optimal output to productivity $\eta_{q^*,\omega}$. For this reason, after a +1% productivity shock, the derived factor demand decreases if and only if the firm decides to increase its output by less than 1%. Therefore,

$$\eta_{x_i^*,\omega_i} < 0 \iff \eta_{q_i^*,\omega_i} < 1 \, . \tag{4}$$

This incomplete pass-through of productivity to output is the key mechanism that leads a firm to reduce its derived factor demand after a positive productivity shock. As this decision is an equilibrium outcome, it is influenced by all the structural features of the market in which the firm operates. In the next sections, I analyze how the elasticity of derived factor demand to productivity is related to the shape of output demand and the nature of competition.

[5]Throughout the paper, I use the notation $\eta_{g,\,y} \equiv \frac{\partial g(y,h)}{\partial y} \frac{y}{g(y,h)}$ to denote the elasticity of the function $g(y,h)$ with respect to y. The function $g(y,h)$ can have other arguments h, but they are not always reported (unless it avoids possible confusion).

2.2 Monopoly

To shed light on the role of output demand in influencing the responsiveness of derived factor demand to productivity, I consider the simplest setting of imperfect competition: a monopoly. This allows me to (temporarily) abstract away from any market structure consideration, which I analyze in Section 2.4. In a monopolized market, there is a single firm producing and selling the product.[6] The buyers' willingness to pay for this product is assumed to satisfy the following properties.

Assumption A4 (Demand). *The market demand for the output q is described by the inverse demand function $p(q)$, which is continuous, three-times differentiable, and strictly decreasing in q, i.e. $p'(q) < 0$.*

To formalize how the features of output demand influence the elasticity of derived factor demand to productivity, I follow Mrázová and Neary (2017) in defining the following unit-free measures of the elasticity and convexity (or curvature) of demand:

$$\boldsymbol{\varepsilon}(q) \equiv -\frac{p(q)}{p'(q)\,q} \quad \text{and} \quad \boldsymbol{\rho}(q) \equiv -\frac{p''(q)\,q}{p'(q)}\,. \tag{5}$$

I intentionally express the elasticity and convexity as a function of the output level because, with the exception of CES demand, both of them vary along a demand curve.

Under the assumptions on technology, costs and demand (**A1-A2-A3-A4**), a monopolist optimally chooses the output level to maximize its operating profits

$$\max_q \quad \pi = r(q) - mc\,q = (p(q) - mc)\;q\;,$$

where $r(q) = p(q)\;q$ denotes its revenue and $mc = \frac{w}{\omega}$ its marginal cost, which depends only on the input price and its productivity level. Profit-maximization imposes restrictions on the possible values that $\boldsymbol{\varepsilon}$ and $\boldsymbol{\rho}$ can take at a profit-maximizing level of output q^*. From the first-order condition, a markup greater than one implies that the elasticity must be greater than one:

$$p(q^*) + p'(q^*)\,q^* = mc \quad \Rightarrow \quad \mu = \frac{p(q^*)}{mc} = \frac{\boldsymbol{\varepsilon}(q^*)}{\boldsymbol{\varepsilon}(q^*) - 1} > 1 \quad \Rightarrow \quad \boldsymbol{\varepsilon}(q^*) > 1\,. \tag{6}$$

From the second-order condition, the marginal revenue $mr(q) = (p + p'q)$ decreasing in output implies that the convexity must be strictly less than two:

$$2p'(q^*)\; + p''(q^*)\,q^* < 0 \quad \Rightarrow \quad \boldsymbol{\rho}(q^*) < 2\,. \tag{7}$$

[6]For this reason, in this section, I omit the subscript i for notational convenience.

The first-order condition in terms of x leads to the standard result that the marginal revenue product of the input $mrp(x)$ must be equated to its price:

$$\frac{\partial \pi}{\partial x} = \frac{\partial \pi}{\partial q}\frac{\partial q}{\partial x} = \left(p + p'\,q - \frac{w}{\omega}\right)\omega = 0 \quad \Leftrightarrow \quad \underbrace{(p + p'\,x\,\omega)\,\omega}_{mrp(x)} \;=\; w\,. \tag{8}$$

Based on this, the monopolist's derived factor demand is

$$x^* = -\frac{p(q^*)}{p'(q^*)}\frac{1}{\omega} + \frac{w}{p'(q^*)\,\omega^2}\,. \tag{9}$$

After multiplying and dividing it by the price, the derived factor demand can be expressed in terms of the Lerner index which is a standard measure of a firm's market power, i.e.

$$x^* = \frac{1}{\omega}\,\frac{p(q^*)}{-p'(q^*)}\,\underbrace{\frac{p(q^*) - mc}{p(q^*)}}_{\text{Lerner index}} = \frac{1}{\omega}\,\frac{p(q^*)}{-p'(q^*)}\,\frac{1}{\varepsilon(q^*)} \tag{10}$$

where $\varepsilon(q^*)$ is the value of the price elasticity of demand, evaluated at the profit-maximizing output level. By taking the derivative of x^* with respect to ω, we obtain equation (11) which shows that reaction of the monopolist's derived factor demand to a productivity change is *ex-ante* ambiguous:

$$\frac{\partial\, x^*}{\partial\, \omega} = -\frac{1}{\omega^2}\underbrace{\frac{p(q^*)}{-p'(q^*)}\,\frac{1}{\varepsilon(q^*)}}_{q^*} + \frac{1}{\omega}\frac{\partial}{\partial\,\omega}\underbrace{\left(\frac{p(q^*)}{-p'(q^*)}\,\frac{1}{\varepsilon(q^*)}\right)}_{q^*} = \underbrace{-\frac{1}{\omega^2}\,q^*}_{<0} + \underbrace{\frac{1}{\omega}\frac{\partial\, q^*}{\partial\,\omega}}_{>0} \gtreqless 0\,. \tag{11}$$

The sign of $\frac{\partial\, x^*}{\partial\, \omega}$ depends on how on two opposing terms balance out. The first term is always negative because a higher productivity level means that the monopolist can produce the same level of output with a lower amount of input. Instead, the second term is always positive because a higher productivity level lowers the monopolist's marginal costs decrease, which raises its incentive to expand output and demand more input.

Whenever the price elasticity of demand decreases in output, i.e. $\frac{\partial\,\varepsilon(q)}{\partial\, q} < 0$, the second term remains positive but decreases in productivity. This is because, as the firm becomes more productive and expands production, it moves to a portion of demand with a lower price elasticity of demand. Facing a less elastic demand, the monopolist has a lower incentive to pass on this marginal cost reduction through higher output at a lower price. The declining incentive to expand production after a productivity improvement is the fundamental mechanism through which market power dampens the responsiveness of derived factor demand to productivity. At low levels of $\varepsilon(q)$, this effect can be so strong that the derived factor demand becomes completely unresponsive to or may even decline after a productivity shock, i.e. $\frac{\partial\, x^*}{\partial\, \omega} < 0$.

Proposition 1. *Under the assumptions on technology (**A1-A2**), input costs (**A3**) and demand (**A4**), a monopolist reacts to a productivity shock by decreasing its input use if and only if either of these equivalent conditions holds at its profit-maximizing level of output q^*:*

$$\eta_{x^*,\omega} < 0 \iff \eta_{q*,\omega} < 1 \iff \eta_{mr,q}(q^*) < -1 \iff \boldsymbol{\varepsilon}(q^*) < 3 - \boldsymbol{\rho}(q^*)$$

(a) *the elasticity of its optimal output with respect to productivity $\eta_{q*,\omega}$ is lower than 1;*

(b) *the elasticity of marginal revenue with respect to output $\eta_{mr,q}$ is lower than -1;*

(c) *the price elasticity of demand is lower than 3 minus the value of the convexity of demand.*

Proof reported in Appendix A.1.

Why would a profit-maximizing firm expand its output less than proportionally to a +1% productivity shock, i.e. *(a)* $\eta_{q*,\omega} < 1$? A firm reacts in this way because it starts experiencing strongly diminishing returns to increase its output. Whenever the price elasticity of demand declines in output, indeed, the marginal revenue generated by an additional unit of output diminishes as well. As formalized by condition *(b)* $\eta_{mr,q} < -1$, the firm decides to expand output by less than 1% exactly when its marginal revenue starts decreasing by more than 1%. This is a direct consequence of profit-maximization when output demand becomes *nearly-satiated*, i.e. when the price decline required to induce consumers to purchase 1% more output is so large that the marginal revenue decreases by more than 1%. Instead of selling more at a very low price, the firm takes "its foot off the gas" and decides to expand output to a lesser extent to prevent its margins from declining too rapidly.

In general, this occurs at lower values of the price elasticity of demand. However, the elasticity is not the only feature of output demand that determines whether and when this near-satiation arises. Indeed, the rate at which marginal revenue declines in output, i.e. $\frac{\partial mr(q)}{\partial q} = 2p' + q\,p''$, depends not only on the first but also on the second derivative of demand.[7] Therefore, both the elasticity - i.e. the slope of the demand curve - and the convexity - i.e. the rate at which the slope decline with output -, determine the elasticity of marginal revenue to output:

$$\eta_{mr,q}(q^*) \equiv \frac{(2p' + q^*\,p'')q^*}{p + q^*\,p'} = -\frac{2 - \boldsymbol{\rho}(q^*)}{\boldsymbol{\varepsilon}(q^*) - 1}\,. \tag{12}$$

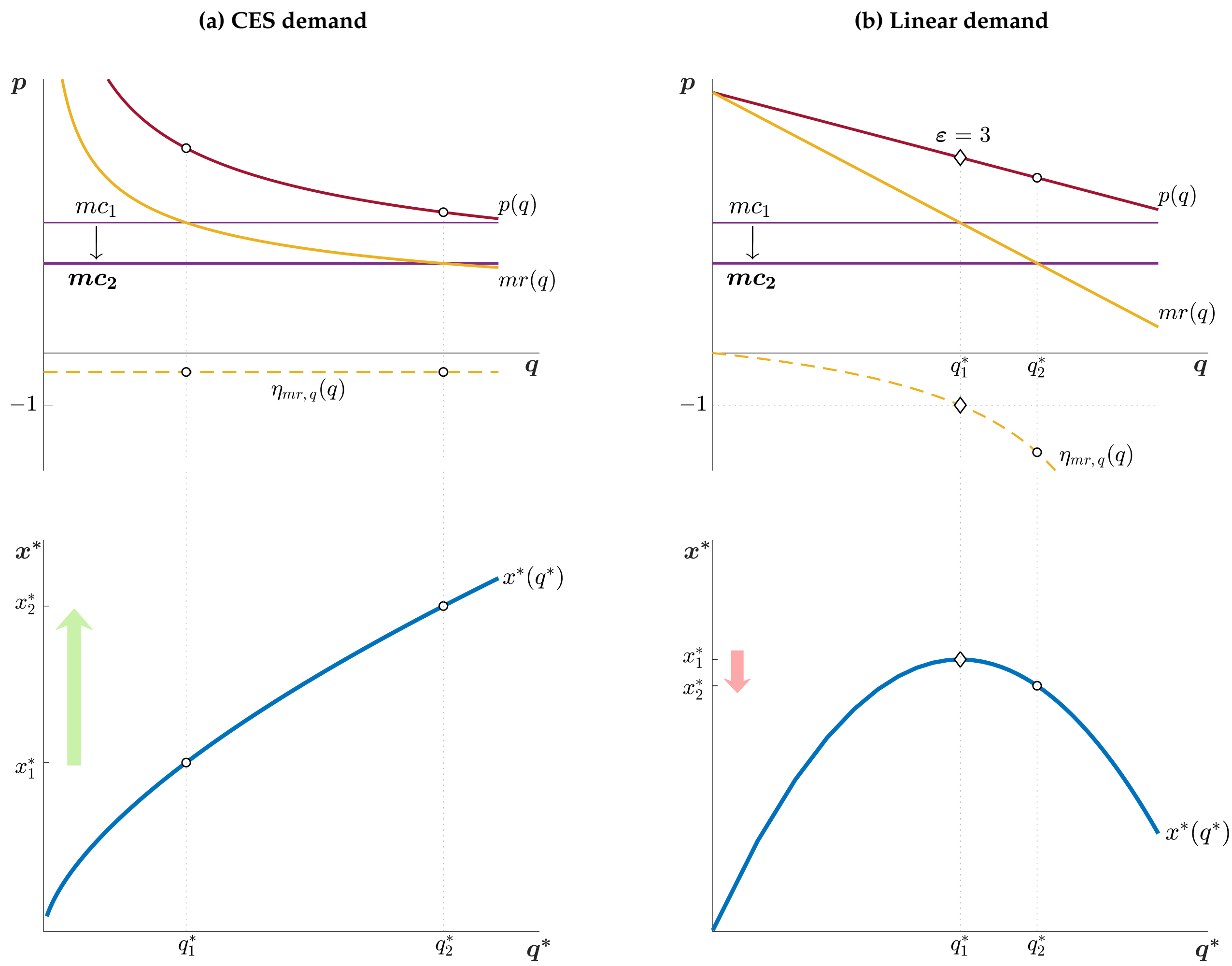
Based on this, condition *(c)* $\boldsymbol{\varepsilon}(q^*) < 3 - \boldsymbol{\rho}(q^*)$ characterizes the level of price elasticity of demand below which $\eta_{mr,q}(q^*) < -1$. Below this threshold, a 1% productivity increase in productivity is

[7]This is because it combines both the response of the price of the additional unit of output as well as the impact on revenue from the change in price on infra-marginal units.

more than enough to cover an optimal increase in output of less than 1%, which is why the derived factor demand starts decreasing and becomes negative, i.e. $\eta_{x^*,\omega} < 0$.

Illustration. To provide further intuition for this result, I illustrate in Figure 1 the comparative statics for the reaction of a monopolist to a positive productivity shock under two commonly used demand systems. The first is CES, defined by the inverse demand function $p(q) = \beta q^{-1/\sigma}$ with $\sigma > 1$ and $\beta > 0$, and the second is liner demand, defined by $p(q) = \alpha - \beta\, q$ with $\alpha, \beta > 0$. While the price elasticity is constant with CES, it declines with output in the case of linear demand. The comparison of these two cases shows that the derived factor demand (blue line in the bottom panels) does not always increase with productivity if the price elasticity of demand varies with output.

Figure 1. Reaction of a monopolist to a productivity shock $\uparrow \omega$.



From the two panels at the top, we can see that if the monopolist's productivity increases from ω_1 to ω_2, its marginal cost falls from $mc_1 = \frac{w}{\omega_1}$ to $mc_2 = \frac{w}{\omega_2}$. This induces an output expansion from

q_1^* to q_2^* and a price reduction under both demand systems. However, the derived factor demand of the monopolist looks very different depending on the output demand it faces. This is illustrated in the two bottom panels where the blue lines depict the optimal amount of input employed x^* in correspondence with the profit-maximizing levels of output q^*. When demand is CES (panel a), the derived factor demand is always increasing in output and productivity. On the contrary, for linear demand (panel b) the derived factor demand starts decreasing with productivity beyond q_1^*.

Based on the conditions of Proposition 1, we can rationalize the behavior of the monopolist under the two different demand systems. First, focus on the size of output expansion from q_1^* to q_2^* when productivity increases. When demand is CES, the firm expands output with an elasticity of $\eta_{q^*,\omega} = \sigma - 1 > 1$, which is larger than 1 and, crucially, does not depend on output. On the contrary, when demand is linear, the same elasticity $\eta_{q^*,\omega}$ is a decreasing function of q. Therefore, productivity leads to smaller expansions in output. This reflects the values of the elasticity of marginal revenue to output, which is illustrated by the yellow dashed line in the top panels. In the case of CES, it remains constant at $\eta_{mr,q}(q) = -\frac{1}{\sigma}$ and always higher than -1. With linear demand, instead, it decreases with output and falls below -1 exactly at q_1^*. Finally, to map condition *(c)* of Proposition 1 to Figure 1, note that in the case of CES the price elasticity of demand never falls below 3 minus the convexity. On the contrary, in the case of linear the derived factor demand halts and starts declining in correspondence of $\varepsilon(q) = 3$ since the convexity is $\rho(q) = 0$ for any level of output.

The visualization of the monopolist's first order condition from Equation (8) in terms of the marginal revenue product - i.e. the amount of revenue a firm can generate by purchasing one additional unit of input - offers a complementary perspective on Proposition 1.

Figure 2. Reaction of $mrp(x)$ to a productivity shock ($\uparrow \omega$): outward shift *vs.* rotation.

(a) CES **(b)** Linear

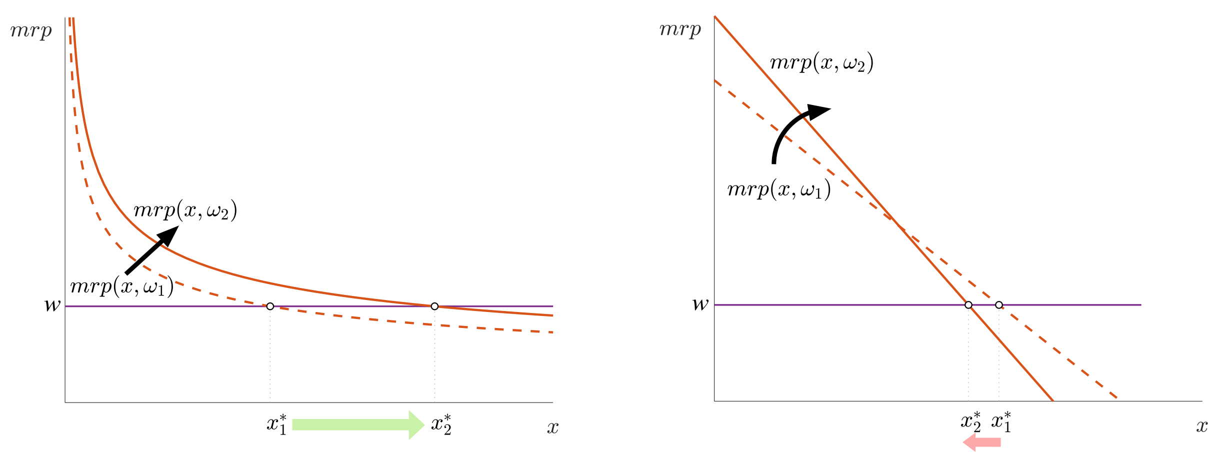


Figure 2 shows how the marginal revenue product (mrp) changes after a productivity increase under CES demand in panel (a) and under linear demand in panel (b). Since the firm is assumed to be *price-taker* on the input market, the intersection between the $mrp(x)$ curve and the factor price w determines the profit-maximizing level of input x^*. The left panel shows that a productivity increase leads to an *outward shift* of the $mrp(x)$ if the demand is CES. Differently, the right panel shows that the same shock generates a *rotation* of the $mrp(x)$ if demand is linear. As a result, derived factor demand is increasing in productivity for any level of output and input in the case of CES demand, but it can decrease under linear demand if the level of output and input is high enough.

2.3 Features of output demand

In this section, I show that, beyond linear, many commonly-used demand functions lead to a non-monotonic relationship between derived factor demand and productivity. To do that, I bring the insight from Proposition 1*(c)* into a general framework that allows comparing different demand functions based only on their elasticity and convexity. Moreover, I link the responsiveness of derived factor demand to productivity to firm-level predictions on markups and pass-through rates.

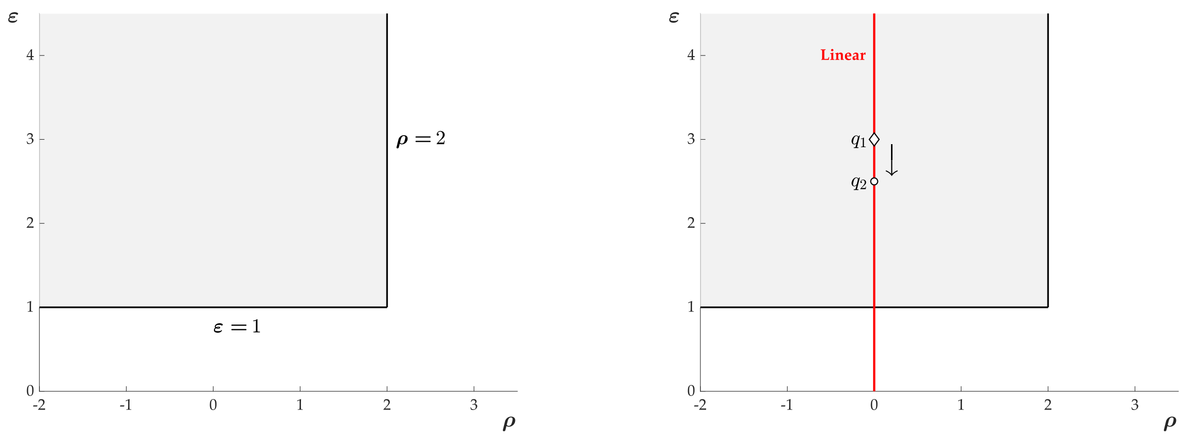
2.3.1 Demand manifold framework

Mrázová and Neary (2017) show that any well-behaved demand function can be represented by its *demand manifold*, a smooth curve relating the values of the elasticity $\varepsilon(q)$ of demand to the values of the convexity $\boldsymbol{\rho}(q)$.[8] Figure 3 illustrates the $(\varepsilon, \boldsymbol{\rho})$-space where the demand manifold of every demand system satisfying (**A4**) can be represented. As shown in Equation (6) and Equation (7), the first- and second-order conditions restrict the possible values of $(\varepsilon, \boldsymbol{\rho})$ in which a profit-maximizing monopolist with constant marginal costs would operate. The shaded area in panel (a) of Figure 3 illustrates the resulting admissible region in the $(\varepsilon,\ \boldsymbol{\rho})$-space. As an example, in panel (b) I plot the manifold of the linear demand function $p(q) = \alpha - \beta q$. Along any linear demand, the elasticity $\varepsilon(q)$ declines with output since $\varepsilon(q) = \frac{\alpha}{\beta q} - 1$. As p'' is zero for any level of q, the convexity is always $\boldsymbol{\rho}(q) = 0$. This is why the corresponding manifold is a vertical line at $\boldsymbol{\rho} = 0$. When a firm expands production, it faces a lower value of the price elasticity of demand, which is represented by a downward movement along the manifold from q_1 to q_2.

[8] For the sake of clarity, note that the definitions of $\varepsilon(q)$ and $\boldsymbol{\rho}(q)$ are not entirely consistent with each other. The measure of convexity $\boldsymbol{\rho}(q)$ equals the elasticity of the slope of inverse demand. Mrázová and Neary follow this standard practice and work throughout with the price elasticity of direct demand, given its greater intuitive appeal (at least in industrial organization) and its focus on the region of parameter space where comparative statics results are ambiguous.

Figure 3. Overview of the Convexity-Elasticity Space.

(a) Admissable region **(b)** Linear demand manifold



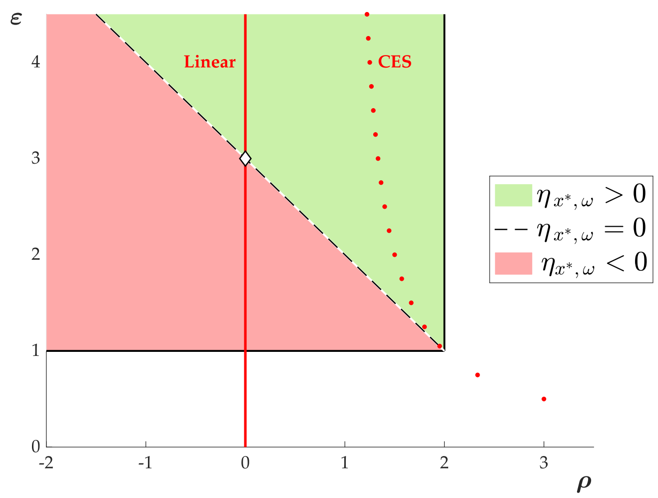
Notes: the admissible region is in fact $\{(\varepsilon, \boldsymbol{\rho}) : 1 \leq \boldsymbol{\varepsilon} < \infty \text{ and } -\infty < \boldsymbol{\rho} < 2\}$. Following Mrázová and Neary (2017), I highlighted only a subset of the admissible region, which is where most interesting issues arise and is also consistent with available empirical evidence. Consumers may be willing to consume outside this region, but such values of $(\boldsymbol{\varepsilon}, \boldsymbol{\rho})$ cannot represent a profit-maximizing equilibrium.

An advantage of working with the demand manifolds rather than directly with the demand functions is the degree of generality that the $(\varepsilon, \boldsymbol{\rho})$-space enables. For example, in the case of linear demand, a different value for the parameters α or β would shift the perceived demand curve, but it would never affect the corresponding demand manifold. Mrázová and Neary call this property "manifold invariance". When it holds, exogenous shocks lead only to movements along the manifold, not to shifts thereof. In this regard, the manifold framework allows me to provide a unified representation of the result of Proposition 1 *(c)* at a high level of generality.

A monopolist decreases its derived factor demand after a productivity shock when it faces a low price elasticity of demand, depending on the values of the convexity. Specifically, condition *(c)* establishes that this occurs when $\boldsymbol{\varepsilon}(q) < 3 - \boldsymbol{\rho}(q)$. The red region in Figure 4 represents the corresponding combinations of elasticity and convexity values that lead to $\eta_{x^*,\omega} < 0$. As shown before, if demand is linear this occurs when $\boldsymbol{\varepsilon}(q) < 3$ since $\boldsymbol{\rho}(q) = 0$. The demand manifold for CES demand, instead, lays in the green region because a profit-maximizing monopolist facing a CES demand reacts to a productivity shocks by increasing its derived factor demand, i.e. $\eta_{x^*,\omega} > 0$.[9]

[9]This is because $\boldsymbol{\varepsilon}(q) = \sigma \;\forall q$ and $\boldsymbol{\rho}(q) = \frac{\sigma+1}{\sigma} \;\forall q$, which implies that $\sigma > 3 - \frac{\sigma+1}{\sigma}$ which holds if $\sigma > 1$. The demand manifold for CES demand is represented by a dotted line, where every dot corresponds to a different value for σ. However, this is an exception. Along the manifold of all the other demands, the elasticity and/or the convexity vary as output changes.

Figure 4. Illustration of Proposition 1*(c)* in the manifold space.



Mrázová and Neary show that the CES manifold is very special and represents an important knife-edge case. It divides the admissible region into two subregions that classify any demand based on its convexity relative to a CES demand with the same elasticity. Demand functions with higher convexity than CES are located to the right of the CES manifold and called "super-convex", while those with lower convexity than CES are on the left and called "sub-convex". This taxonomy has important implications for the properties of the price elasticity. $\varepsilon(q)$ is increasing in q if a demand is super-convex, while decreases with output (as with linear demand) if it is sub-convex. This represents an important boundary for several comparative static predictions. Among others, it determines the relationship between markups and output (see Appendix A.2.2). Moreover, this taxonomy motivates the following result.

Corollary 1. *The elasticity of derived factor demand to productivity of a monopolist can become negative, i.e.* $\eta_{x^*,\omega} < 0$*, if only if output demand is sub-convex, i.e. the price elasticity of demand declines with output.*

The sub-convexity property is often called "Marshall's Second Law of Demand", but other terminologies are common as well.[10] This property is considered theoretically more plausible because it implies that consumers are more responsive to price changes the greater their consumption.[11] More-

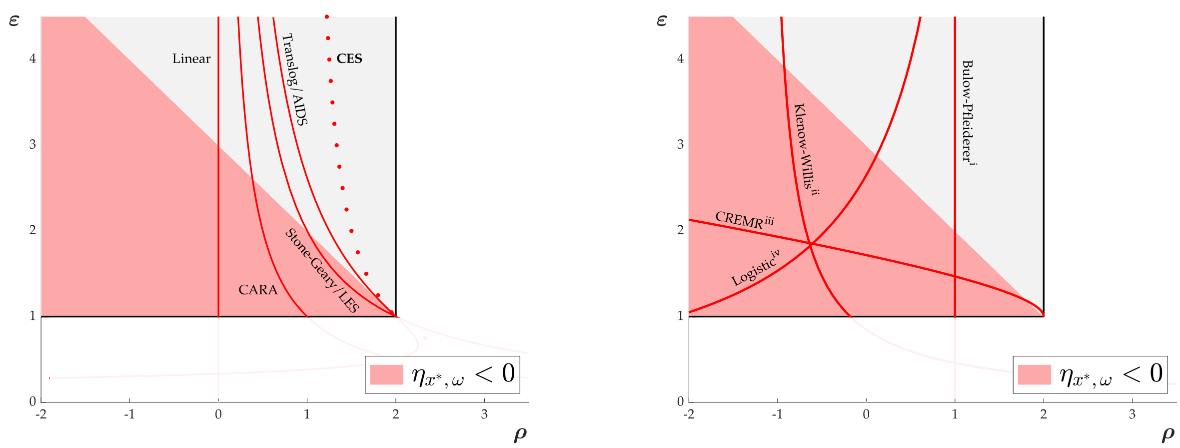
[10] As it was originally introduced by Marshall (1890) in his *Principles of Economics*, where he argued that "*the elasticity of demand is great for high prices, and great, or at least considerable, for medium prices; but it declines as the price falls; and gradually fades away if the fall goes so far that satiety level is reached. This rule appears to hold with regard to nearly all commodities and with regard to the demand of every class*" (Book III). Although many other terminologies are used in the literature for sub-convex demands. Arkolakis, Costinot, Donaldson, and Rodríguez-Clare (2019) describe such demand functions as "log concave in log prices", Zhelobodko et al. (2012) describe them through the concept of "increasing relative love of variety", while Kimball (1995) defines this property as "positive super-elasticity of demand".

[11] In addition, it serves as a sufficient condition for the existence - and sometimes uniqueness - of equilibrium in standard models of imperfect competition. Caplin and Nalebuff (1991) show that it is a sufficient condition for unique equilibrium to exist in common models of Cournot competition and differentiated products Bertrand competition.

over, sub-convexity is consistent with empirical findings on incomplete pass-through (Nakamura & Zerom, 2010), and the fact that firms producing more exhibit lower pass-through rates (Berman et al., 2012) and higher markups (De Loecker & Goldberg, 2014). For these reasons, I mainly focus on sub-convex demand functions from now onwards in the paper. Figure 5 illustrates the manifolds of several sub-convex demand functions that are commonly used in the literature. The fact that relevant portions of their manifolds fall in the red region implies that a non-monotonic relationship between derived factor demand and productivity arises in many demand specifications.[12] Beyond linear, this occurs for example with the negative exponential or CARA demand (Behrens et al., 2020), the Logistic demand (Cowan, 2016), the Klenow and Willis (2016) specification of Kimball demand, the CREMR demands ("Constant-Revenue-Elasticity-of-Marginal-Revenue") introduced by Mrazova, Parenti, and Neary (2021), in the Bulow and Pfleiderer (1983) demand, and in the Stone-Geary/Linear Expenditure System (LES).[13] Recently, Miravete et al. (2022) characterize the demand manifold also of discrete choice demand models. Although not illustrated in Figure 5, also Multinomial and Mixed Logit demand can lead to $\eta_{x^*,\omega} < 0$.

Figure 5. Demand Manifolds for common sub-convex demand functions.

(a) Invariant Manifolds **(b)** Manifolds for given parameters



Notes: in **(b)** I consider certain demand functions in which the location of the manifold depends on specific parameter values. For illustration, I take these values from previous calibrations in the literature. In (i) the Bulow and Pfleiderer (1983) demand is such that the absolute pass-through from cost to price is 1 (i.e. euro-for-euro). In (ii) the value of super-elasticity is 2.18 based on the calibration by Edmond et al. (2018). In (iii) the parameter for CREMR is set to $\sigma = 1.11$ following Mrazova et al. (2021). In (iv) I follow the specification of Cowan (2016). For additional details, see Appendix A.2.1.

[12] These are just a few examples. For details on the manifolds of other demand functions, I refer the reader to Mrázová and Neary (2017) and their rich Appendix for additional material. Moreover, Proposition 1 is fundamentally related to Proposition 4 in Mrázová and Neary (2019). With a different focus, they find that the same conditions determine the super-modularity of firms' profits in their own marginal cost and the iceberg transport cost in models where firms choose between two alternative ways of serving a market (exports vs. foreign direct investment). In light of this, any demand function that is shown to be super-modular in their paper will also lead to $\eta_{x^*,\omega} < 0$.

[13] On the contrary, in the case of Translog demand, which from the firm's perspective is consistent also with the Almost Ideal or "AIDS" model of Deaton and Muellbauer (1980), I find that $\eta_{x^*,\omega} = 0$ at most.

2.3.2 Corresponding values of markups and pass-through

The representation of Proposition 1 in the manifold framework allows me to link these new results on derived factor demand to other firm-level outcomes, such as markups and pass-through behaviors. The following corollary formalizes what else we should expect in correspondence of the decoupling of derived factor demand from productivity.

Corollary 2. *The derived factor demand of a monopolist halts and starts decreasing after a productivity shock if and only if the level of its markups is high enough and its pass-through of cost to prices is low enough. In particular,*

$$\eta_{x^*,\omega} \leq 0 \quad \Longleftrightarrow \quad \mu \geq 1 + \frac{1}{2-\boldsymbol{\rho}} \quad \Longleftrightarrow \quad \frac{\partial p}{\partial mc} \leq \frac{\varepsilon}{\varepsilon - 1} - 1$$

In the case of linear demand, the derived factor demand becomes unresponsive to productivity shock in correspondence of a markup $\mu = 1.5$ and a cost-to-price pass-through of 0.5€. In general, the values of markups and pass-through rates at which $\eta_{x^*,\omega} = 0$ depend on the convexity of demand. To illustrate this, I report in Table 1 the values of markups and two pass-through measures (absolute and proportional) in correspondence of $\eta_{x^*,\omega} = 0$ for three different values of convexity, (i) $\boldsymbol{\rho} = -1$, (ii) $\boldsymbol{\rho} = 0$ (i.e. linear demand), (iii) $\boldsymbol{\rho} = +1$.[14] The lower is $\boldsymbol{\rho}$, the lower the values of markups and pass-through rates at which $\eta_{x^*,\omega} = 0$. This is because in less convex demands a firm reaches sooner - at a relatively higher level of elasticity - the point where demand becomes nearly satiated.

Table 1. Correspondence between $\eta_{x^*,\omega} = 0$ and other firm-level outcomes.

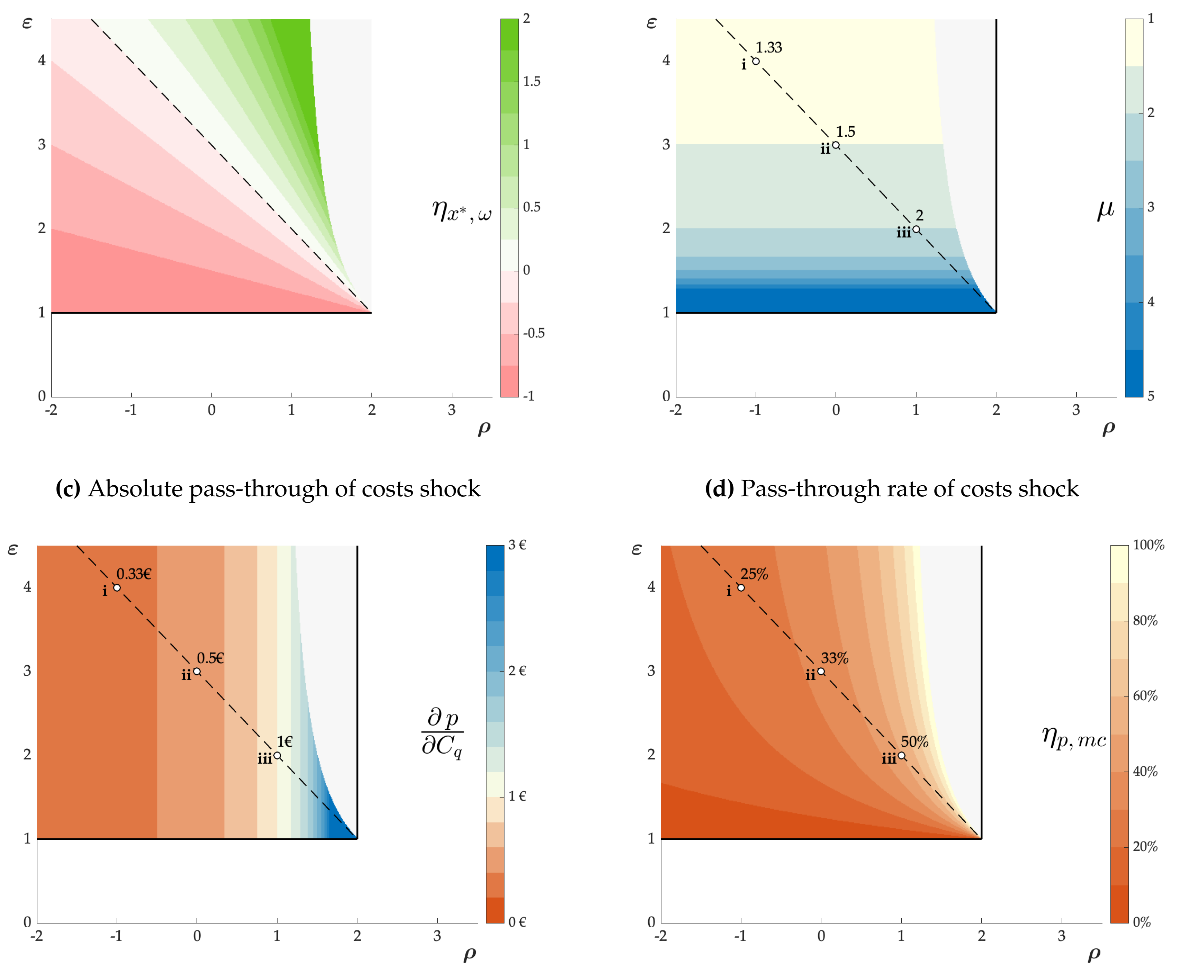
		(i)	(ii)	(iii)
		$\boldsymbol{\rho} = -1$	$\boldsymbol{\rho} = 0$	$\boldsymbol{\rho} = 1$
(a) Elasticity of x^* to ω	$\eta_{x^*,\omega} = \dfrac{\varepsilon + \boldsymbol{\rho} - 3}{2 - \boldsymbol{\rho}}$	0	0	0
(b) Markups	$\mu = \dfrac{\varepsilon}{\varepsilon - 1}$	1.33	1.5	2
(c) Cost-*to*-price pass-through *(€-to-€)*	$\dfrac{\partial p}{\partial mc} = \dfrac{1}{2 - \boldsymbol{\rho}}$	0.33€	0.5€	1€
(d) Cost-*to*-price pass-through rate *(in %)*	$\eta_{p,\,mc} = \dfrac{\varepsilon - 1}{\varepsilon(2 - \boldsymbol{\rho})}$	25%	33%	50%

To put these values in context, in Figure 6 I illustrate within the demand manifold framework the overall correspondence between the values of $\eta_{x^*,\omega}$, markups and pass-through rates for sub-convex

[14] In industrial organization the focus is on the absolute pass-through (i.e. by how much a firm raises its price if its marginal costs increase by one euro), while in macro/international economics on the proportional pass-through.

demands. Panel (a) reports the values of the elasticity of derived factor demand to productivity in the (ε, ρ)-space. Panel (b) shows the corresponding values of markups, while panel (c) and panel (d) displays the values of pass-through in absolute and in relative terms, respectively. The content of Corollary 2 can be seen in the fact that regions where $\eta_{x^*,\omega} < 0$ - areas in red in panel (a) - correspond to high-markup - areas in darker blue in panel (b) - and low pass-through regions - areas in darker orange in panels (c) and (d).

Figure 6. Range of firm-level outcomes in the (ε, ρ)-space.



(a) Elasticity of derived factor demand

(b) Markups

(c) Absolute pass-through of costs shock

(d) Pass-through rate of costs shock

It is noteworthy that higher markups and incomplete pass-through rates represent a necessary and sufficient condition for the decoupling of factor demand from productivity. To the best of my knowledge, this is the first paper to formalize this clear link between the nature of factor demand and pricing behavior. Since firm-level markups can be estimated in the data, in Section 3 I exploit this mapping to infer whether a firm operates in a region of demand where the decoupling takes place.

2.4 Role of market structure

The analysis under a monopoly provides many insights on the influence of the features of output demand on the elasticity of derived factor demand to productivity shocks. Beyond monopoly, however, the results of Proposition 1 need some degree of refinement because the relevant elasticity for firms' behavior is the elasticity of each firm's *residual* demand, rather than the elasticity of market demand. For this reason, any additional force that affects the residual demand faced by each firm will influence its responsiveness to productivity. In the following sections, I turn to standard models of monopolistic competition with free-entry and oligopoly competition in quantity, and I examine the effect of competition, competitors' characteristics, and market structure.

2.4.1 Monopolistic competition

A monopolistic competitive market consists of many small firms, each producing a different variety of a product. Every such firm continues to satisfy the assumptions (**A1**-**A2**-**A3**), but firms may have different productivity levels ω_i. Mrázová and Neary (2017) prove that all comparative statics predictions developed in the manifold framework for a monopolist carry through to the case of a monopolistically competitive firm in a general equilibrium model à la Melitz (2003) or its generalization to non-CES demands by Zhelobodko et al. (2012) and Dhingra and Morrow (2019). The requirements for this result to hold are that (i) consumers' preferences need to be symmetric and (ii) the elasticity of demand for a variety depends on its level of consumption only.

These conditions are verified, among others, by additively separable preferences, which are very common in the literature (Mayer et al., 2021). In this setting, the following assumption formalizes such requirements and adapts assumption (**A4**) with a micro-foundation of consumer preferences.

Assumption A5 (Demand). *Let $i \in [0, N]$ be the continuum of horizontally differentiated varieties available to L consumers, whose preferences are represented by the utility function*

$$\mathcal{U} = \int_0^N u(q_i)di \ ,$$

where $u(q_i)$ is the sub-utility associated with the consumption of q_i units of product i. The function $u(\cdot)$ is strictly increasing and concave, i.e. $u'(q_i) > 0$ and $u''(q_i) < 0$ for $q_i \geq 0$, and $u(0) = 0$.

Given the prices for each variety, each consumer chooses his/her optimal demand for each i by maximizing his/her utility subject to a budget constraint. This leads to a strictly decreasing residual (inverse) demand for each variety defined by

$$p_i(q_i) = \frac{u'(q_i)}{\lambda}\,, \tag{13}$$

where λ is the marginal utility of income of the consumer. The sub-utility function $u(\cdot)$ is such that the residual demand is strictly decreasing in output.[15]

Being negligible to the market, each firm chooses its output level to maximize its operating profits by taking the residual demand function it faces as given. This includes the aggregate demand conditions captured by λ. In Appendix A.3 I describe the conditions that determine the value of λ at the unique free-entry equilibrium.

The standard prediction of this class of models is that the levels of profits, output, and revenues are monotonic increasing in firm's productivity. With a sub-convex demand, this is the case also for markups. While in the literature this prediction is usually extended also to input use, I prove this is not necessarily the case. The following formulae summarize these statements:

$$\frac{\partial \pi_i(\omega_i,\,\lambda)}{\partial \omega_i} > 0,\; \frac{\partial q_i(\omega_i,\,\lambda)}{\partial \omega_i} > 0,\; \frac{\partial r_i(\omega_i,\,\lambda)}{\partial \omega_i} > 0,\; \frac{\partial \mu_i(\omega_i,\,\lambda)}{\partial \omega_i} > 0 \;\text{ but }\; \frac{\partial x_i(\omega_i,\,\lambda)}{\partial \omega_i} \gtreqqless 0\,.$$

In Proposition 2, I prove that input use is not always monotonic increasing in productivity and the results of Section 2.3 extend to monopolistic competition. Intuitively, this is because a monopolistic competitive firm behaves like a monopolist on the residual demand for its variety. Formally because, even if the residual demand and marginal revenues curves depend on aggregate conditions through λ, their elasticities do not depend directly on it.[16] It follows that their relationship (i.e. the demand manifold) and the elasticity of marginal revenue $\eta_{mr_i,\,q_i}(q_i^*)$ are independent of λ too. Therefore, all the results of Proposition 1 remain valid from the perspective of each firm.

Proposition 2. *Under the assumptions on technology (**A1-A2**), input costs (**A3**) and demand (**A5**), a monopolistic competitive firm reacts to a productivity shock by decreasing its input use if and only if either of these equivalent conditions holds at its profit-maximizing level of output $q_i^*(\omega_i, \lambda)$:*

$$\eta_{x_i^*,\omega_i} < 0 \iff \eta_{q_i^*,\omega_i} < 1 \iff \eta_{mr_i,\,q_i}(q_i^*) < -1 \iff \boldsymbol{\varepsilon}(q_i^*) < 3 - \boldsymbol{\rho}(q_i^*)$$

[15] Zhelobodko et al. (2012) show that the inverse demand inherits the properties of $u(\cdot)$. In particular, $p_i(q_i)$ is strictly decreasing because $u(\cdot)$ is strictly concave.

[16] In particular, $mr_i(q_i,\,\lambda) = \frac{u'(q_i)+u''(q_i)q_i}{\lambda}$. However, the elasticity $\boldsymbol{\varepsilon}(q_i) = -\frac{p(q_i)}{p'(q_i)q_i} = -\frac{u'(q_i)}{u''(q_i)q_i}$ and the convexity $\boldsymbol{\rho}(q_i) = -\frac{p''(q_i)q_i}{p'(q_i)} = -\frac{u'''(q_i)q_i}{u''(q_i)}$ of residual demand do not depend directly on λ.

If the price elasticity of residual demand $\varepsilon(q_i(\omega_i,\, \lambda))$ declines with q_i, the most productive firms within a market:

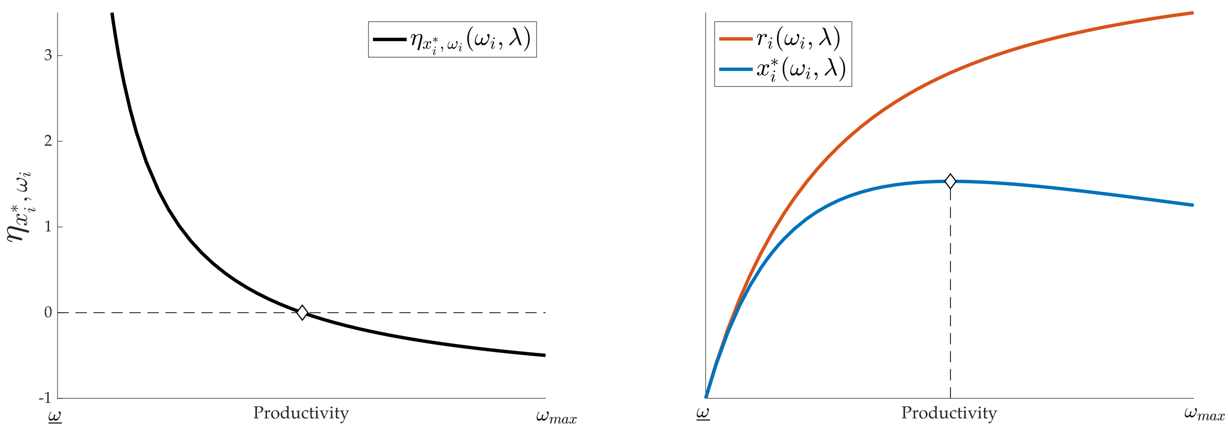
(a) have a lower elasticity of factor demand to productivity than less productive competitors;

(b) are not necessarily the largest in terms of input use.

In Figure 7, I plot both these cross-sectional predictions in an illustrative setting with linear demand. In panel (a), I plot the elasticity of derived factor demand to productivity along the distribution productivity levels. Variable markups lead to a gradient in the responsiveness of firms to productivity: from left to right, $\eta_{x_i^*,\omega_i}$ is higher among less productive/low-markup firms and gets lower (and even negative) among more productive/high-markup firms. Panel (b) shows the levels of revenue (orange line) and input use (blue line) in a cross-section of firms with different productivity levels. While more productive firms in a market are always the largest in terms of revenue, this is not necessarily the case in terms of input use.[17]

Figure 7. Cross-firm predictions within a monopolistic competitive market.

(a) $\eta_{x_i^*,\omega_i}$ at different productivity levels. **(b)** Revenue and input use in levels.



Notes: equilibrium outcome with linear demand $p_i(q_i) = \alpha - \beta\, q_i$. The white point corresponds to the firm with productivity level where $\eta_{x_i^*,\omega_i} = 0$. The productivity values range from the minimum cut-off $\underline{\omega}$ to ω_{max}. See Appendix A.3 for details about the simulation.

So far, I focused on cross-firm predictions within a market for a given level of competition λ. The following predictions, instead, are aimed at a comparison of different markets. Keeping constant the preferences of consumers and thus the shape of demand, I analyze what happens when firms with the same level of productivity face a different level of λ.[18]

[17]This cross-sectional prediction follows from considering two firms 1 and 2 with different productivity levels $\omega_2 > \omega_1$. The more productive firm will be larger in terms of input if and only if $\frac{log(q_2^*)-log(q_1^*)}{log(\omega_2)-log(\omega_1)} > 1$. This is the case if at $q_2^*(\omega_i, \lambda)$ either of the conditions of Proposition 2 are met.

[18]As λ reflects all the structural features of the assumed economic environment, there are many reasons (e.g., the number

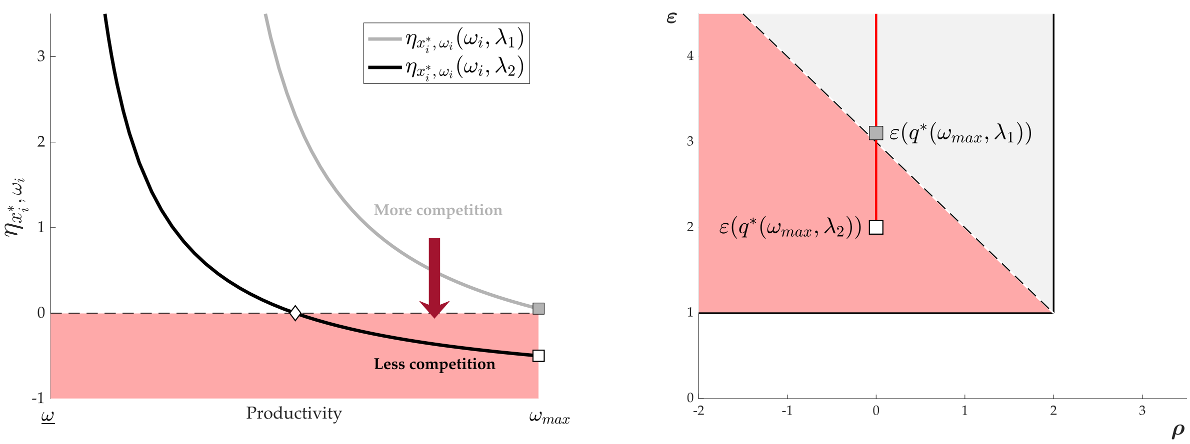
Whenever the equilibrium value of λ is lower, the output of a firm with productivity ω_i, i.e. $q_i^*(\omega_i, \lambda)$, will be higher since its residual demand curve shifts outward. At a higher output, the value of the elasticity of residual demand $\varepsilon(q_i^*)$ is lower. In turn, this has a direct impact on firm-level markups and the elasticity of derived factor demand to productivity $\eta_{x_i^*,\omega_i}$. Although with a different objective, Mayer et al. (2021) show that the shape of the marginal revenue curve determines the gradient of this change, since $\eta_{q_i,\lambda} = \frac{1}{\eta_{mr_i,q_i}}$. Because $\eta_{q_i,\lambda} = \eta_{x_i,\lambda}$, I can characterize in general the effect of competition for the elasticity of derived factor demand as follows.

Corollary 3. *The elasticity of derived factor demand to productivity of a monopolistic competitive firm i is positively related to the degree of competition in the market: less competitive pressures (lower λ) induce a lower $\eta_{x_i^*,\omega_i}$, while more competition (higher λ) increases it.*

Figure 8 illustrates this result. Under linear demand, I compare two market equilibria that differ in the degree of competition perceived by firms. In panel (a), I plot the elasticity of derived factor demand along the productivity distribution and show that lower competitive pressures (i.e. lower λ) lead to a lower elasticity of derived factor demand to productivity at any given level of productivity. In panel (b), I show the values of the price elasticity of demand for the most productive firm in both equilibria. As the price elasticity of residual demand declines with less competition, i.e. $\varepsilon(q_i^*(\omega_{max}, \lambda_2)) < \varepsilon(q_i^*(\omega_{max}, \lambda_1))$, this corresponds to a downward shift along the demand manifold.

Figure 8. Impact of lower competitive pressures (under linear demand).

(a) Two equilibria with $\lambda_1 > \lambda_2$.   **(b)** Movement along the manifold



Notes: equilibrium outcome with quadratic preferences leading to linear demand $p_i(q_i) = \alpha - \beta\, q_i$. The squares denote the firm with the highest productivity level. The two equilibria differ in terms of entry costs, which influences the degree of competition. See Appendix A.3 for details about the simulation.

of consumers, the entry costs, the distribution of firm productivity, etc.) for it to be different across markets.

2.4.2 Oligopoly

In this section, I investigate how strategic interactions between firms influence the elasticity of derived factor demand to productivity. I consider a market with a limited number of firms $i = 1, ..., N$ which produce a homogeneous good q_i, with heterogenous levels of productivity ω_i. The assumptions on their technology (**A1**-**A2**) and input price (**A3**) remain the same as previous settings. This implies that their marginal costs are equal to $mc_i = \dfrac{w}{\omega_i}$ and depend on the input price w and their productivity level ω_i. In this setting, the assumption on demand is a version of (**A4**) in terms of aggregate output $Q = \sum_{i=1}^{N} q_i$.

Assumption A6 (**Demand**). *The market demand for the homogeneous good is described by the inverse demand function $p(Q)$, which is continuous, three-times differentiable, and strictly decreasing in Q.*

The key strategic interaction that I focus on is the extent to which a firm's quantity choice q_i affects other firms' profits through aggregate output Q. To model these interactions, I follow the conduct parameter approach and assume that the effect of each firm's quantity choice on aggregate output Q is summarized by the parameter θ. This assumption nests a number of well-known special cases. The standard Cournot oligopoly model corresponds to $\theta = 1$. The case of perfect collusion corresponds to $\theta = N$, while a perfectly competitive outcome emerges if $\theta \to 0$.[19] In this class of models, the first-order condition of profit maximization for each firm i is

$$\underbrace{p + \theta\, p'(Q)\, q_i}_{mr_i} = mc_i\,. \tag{14}$$

If we divide and multiply the left-hand side by Q, Equation (14) can be expressed in terms of the elasticity of market demand $\varepsilon(Q)$ and the market shares of each firm $s_i = \dfrac{q_i}{Q}$:

$$p\left(1 - \frac{\theta\, s_i}{\varepsilon(Q)}\right) = mc_i\,.$$

Summing over the first-order conditions across all competitors $j \neq i$, we can see that the firm i's market share s_i depends on the elasticity of market demand, the number of its competitors and their

[19]When firms compete à la Cournot, each firm takes the quantities of the other firms as given, conjecturing that total output increases by the same amount as its own quantity. Under perfect collusion, each firm conjectures that each rival will fully match a quantity increase. If $\theta \to 0$, each firm conjectures that the rivals contract their quantities in response to a change in its own quantity so that Q output remains constant. As discussed by Verboven and Van Dijk (2009), "*outside these special cases this framework has little game-theoretic appeal since it aims to capture dynamic responses within a static model. It has, however, often been used in empirical work to estimate the conduct or average collusiveness of firms without having to specify a fully dynamic model*".

average marginal costs ($\overline{mc}_j$):

$$s_i = 1 - (N-1)\frac{\varepsilon(Q)}{\theta}\left(1 - \frac{\overline{mc}_j}{p}\right) . \tag{15}$$

Similarly to monopoly, profit-maximization imposes restrictions on the possible values that $\varepsilon(Q)$ and $\boldsymbol{\rho}(Q)$ can take at a profit-maximizing equilibrium. From the first-order condition, a markup greater than one implies that the price elasticity of residual demand must be greater than one. From the second-order condition, the marginal revenue decreasing in its own output implies that the convexity of the residual demand must be strictly less than two. In terms of elasticity and convexity of market demand, this implies that for each active firm i it must be that

$$\varepsilon(Q) \geq \theta\, s_i \quad \text{and} \quad \boldsymbol{\rho}(Q) < \frac{2}{\theta s_i} .$$

Following Seade (1980), an additional restriction on the convexity of market demand is $\boldsymbol{\rho}(Q) < \frac{N}{\theta} + 1$, which originates from the stability criterion. Within this oligopolistic competitive environment, I show that the results from Proposition 1 extend, with few adjustments, also to a setting with strategic interactions.

Proposition 3. *Under the assumptions on technology (**A1**-**A2**), input price (**A3**) and demand (**A6**), an oligopolistic firm i with a market share s_i defined by Eq. (15) reacts to a shock to its own productivity by decreasing its input use if and only if either of these equivalent conditions holds at its optimal level of output q_i^*:*

(a) the elasticity of its output with respect to its productivity $\eta_{q_i^*,\omega_i}$ is lower than 1;

(b) the elasticity of its marginal revenue curve with respect to its output η_{mr_i,q_i} is lower than -1;

(c) the price elasticity of its residual demand is lower than $3 - \boldsymbol{\rho}(Q)\,\theta\, s_i$.

$$\eta_{x_i^*,\omega_i} < 0 \iff \eta_{q_i^*,\omega_i} < 1 \iff \eta_{mr_i,q_i} < -1 \iff \frac{\varepsilon(Q)}{\theta\, s_i} < 3 - \boldsymbol{\rho}(Q)\,\theta\, s_i$$

Proof reported in Appendix A.4.

As an oligopolist acts as a monopolist on its *residual* demand, it should not surprise that these conditions look very similar to those of Proposition 1. As a matter of fact, the economic mechanism leading to a decoupling of derived factor demand from productivity is the same. At lower levels of price elasticity of demand, a firm that becomes more productive has a lower incentive to further expand production due to strongly diminishing marginal revenue from doing so. However, condition *(c)* is expressed in terms of the elasticity and convexity of the residual demand of each firm. As both depend on its market share s_i and on the conduct parameter θ, we can see how the market structure plays an important role in the responsiveness of derived factor demand to productivity.

Corollary 4. *The elasticity of firm i's derived factor demand to own productivity $\eta_{x_i^*,\omega_i}$ is lower, the fewer the competitors in the market (i.e. lower N) and the higher the degree of collusion among them (i.e. higher θ).*

To provide some intuition, consider the case in which firms have the same productivity and thus marginal costs. In this situation, market shares are symmetric $s_i = \frac{1}{N}\ \forall i$ and Eq. (A3) simplifies to

$$\eta_{x_i^*,\omega_i} = -1 - \frac{1 - \frac{\varepsilon(Q)N}{\theta}}{2 - \frac{\rho(Q)\,\theta}{N}}\,.$$

Ceteris paribus, fewer competitors ($\downarrow N$) and/or less competitive pressures among them ($\uparrow \theta$) reduce $\eta_{x_i^*,\omega_i}$. On the contrary, if $N \to \infty$ and/or $\theta \to 0 \Rightarrow \eta_{x_i^*,\omega_i} > 0\,\forall i$. This result highlights why in perfect competition the elasticity of derived factor demand to productivity is always positive.

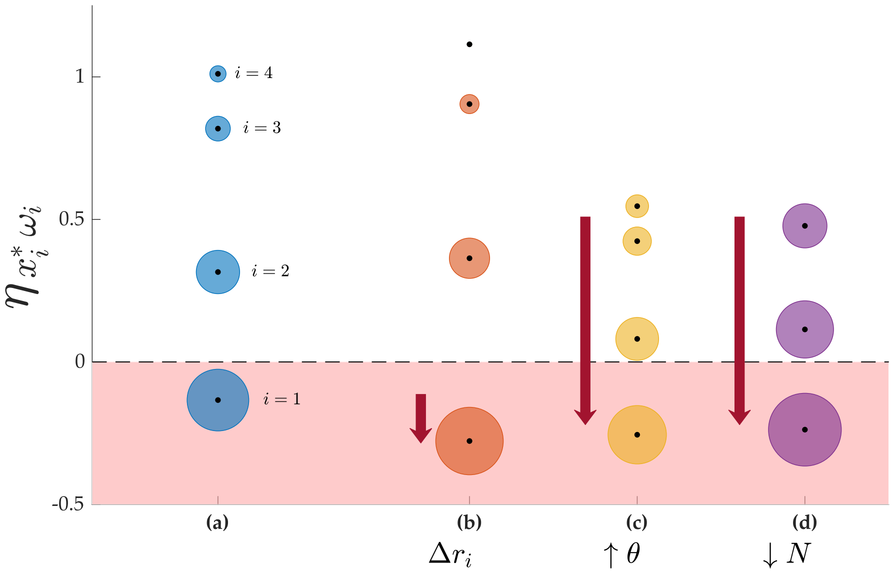
Whenever firms are heterogeneous, instead, the elasticity of derived factor demand to own productivity varies considerably between small and larger firms, and the relative differences in their productivity/marginal costs levels turn out to be important determinants of $\eta_{x_i^*,\omega_i}$ as well.

Corollary 5. *The firm with the largest market share s_i has the lowest elasticity of derived factor demand to productivity and this is accentuated by its cost advantage relative to the competitors.*

Proof reported in Appendix A.4.

To illustrate these results, in Figure 9 I show the predicted elasticities of derived factor demand to productivity in different market scenarios. In the baseline **(a)**, there are $N = 4$ firms that compete à la Cournot ($\theta = 1$) facing a linear demand. Firms are ranked based on their productivity $i = 1, 2, 3, 4$. The black dots represent the values of $\eta_{x_i^*,\omega_i}$ for each firm, while the size of the circles is proportional to each firm's market share s_i. The most productive firm - and largest in terms of market shares - has the lowest and even negative elasticity of derived factor demand to productivity, i.e. $\eta_{x_1^*,\omega_1} < 0$. In scenario **(b)**, I assume that the largest firm has a higher productivity level, so the relative cost advantage (defined as $r_i \equiv \dfrac{mc_i}{\overline{mc}}$) changes for all the firms. A higher cost advantage of the dominant firm leads to an even lower $\eta_{x_1^*,\omega_1}$. In **(c)**, I consider the effect of more collusive conduct among firms. A value of $\theta > 1$ decreases the $\eta_{x_i^*,\omega_i} < 0$ of all firms. In the last scenario **(d)**, I assume that the least productive firm does not operate anymore in the market ($N = 3$). As a result, all the other firms face a lower competitive pressure, which reduces their elasticity of derived factor demand to productivity.

Figure 9. The influence of market structure on $\eta_{x_i^*,\omega_i}$.



Notes: own simulation based on $w = 4$ and productivity values $\omega_i = [1.2; 1.05; 0.97; 0.95]$.
In **(b)** the productivity advantage of the market's leader is higher ($\omega_1 = 1.3$) so that $\downarrow r_1$.
In **(c)** the conduct is relatively more collusive among firms and set equal to $\theta = 1.25 > 1$.
In **(d)** the least productive is not operating anymore after a merger or its exit, so $N = 3$.

Demand manifolds in oligopoly. Mrázová and Neary (2017) suggest that the demand manifold framework may also be applied to oligopoly, but they left it to future research. Another contribution of my paper is to extend their framework to a setting of oligopolistic competition in quantity. The complexity of this extension originates from the fact the restrictions on elasticity and convexity implied by Equation (15) and the stability condition make the admissible region endogenous to firms' market shares. I highlight here the two main results, while in Appendix A.4.1 I provide more details.

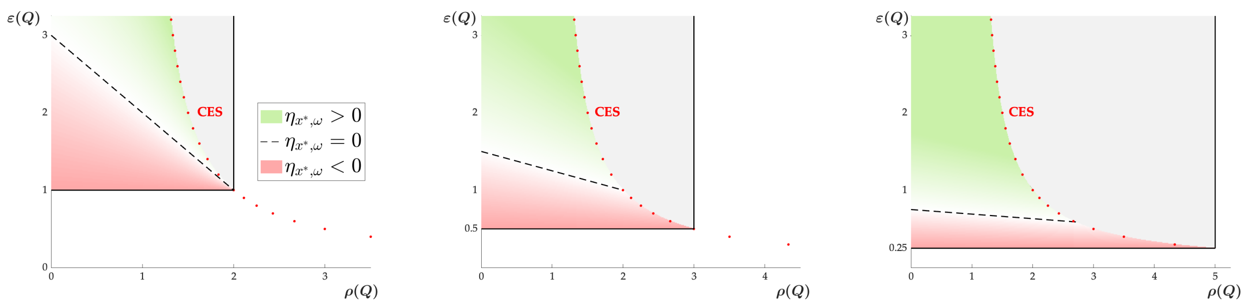
First, I find that the manifold can be re-formulated in terms of elasticity and convexity of the *residual* demand of each firm. Within it, I prove that the comparative statics predictions on the elasticity of derived factor demand to productivity derived under monopoly carry on to oligopoly. I illustrate this result in Figure A2.

Second, the admissible region in terms of elasticity and convexity of market demand becomes larger under oligopoly compared to monopolistic competition. This stems from the fact that the elasticity of residual demand is higher than the elasticity of market demand, i.e. $\frac{\varepsilon(Q)}{\theta\, s_i} \geq \varepsilon(Q)$. In particular, the elasticity of market demand can be lower than 1 in an oligopolistic equilibrium, even though the price elasticity of the residual demand of each firm remains greater than 1. As a result, the range of elasticity and convexity values leading to $\eta_{x_i^*,\omega_i} < 0$ changes. I illustrate this result in

Figure 10 in an oligopolistic setting à la Cournot ($\theta = 1$) with symmetric firms.[20] As the number of firms increases from panel (a) to panel (c), the red region in the manifold space changes.

Figure 10. Demand manifold regions in monopoly *vs.* duopoly *vs.* oligopoly.

(a) $N = 1$ **(b)** $N = 2$ **(c)** $N = 4$



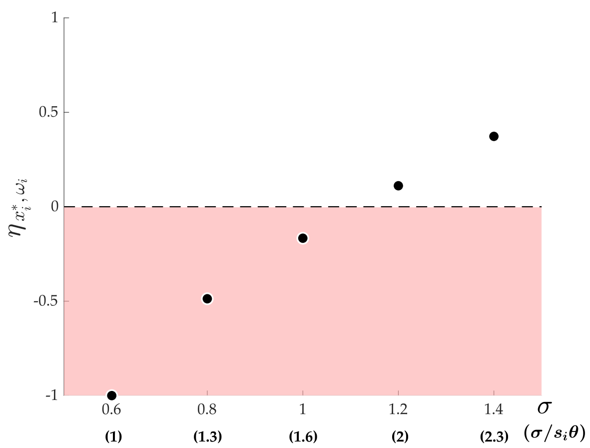
This leads to the following result.

Corollary 6. *A demand function can lead to $\eta_{x_i^*,\omega_i} \leq 0$ in oligopoly, even if this is not the case under monopoly or monopolistic competition. Notably, this is the case for CES demand.*

Figure 11 illustrates this point in the case of a setting à la Cournot and CES demand. In panel (a), I consider a firm with market share $s_i = 60\%$ and I plot the values of the elasticities of its derived factor demand to productivity for different values of market demand elasticity $\varepsilon(Q) = \sigma$. With a less elastic market demand (i.e. lower σ), the elasticity of derived factor demand to productivity of this firm decreases. In the table in (b), I report the threshold values of a firm's market share and markup above which $\eta_{x_i^*,\omega_i}$ turns negative and a decoupling of factor demand from productivity occurs - even with CES demand.

Figure 11. Non-monotonicity in an oligopoly à la Cournot with CES demand.

(a) $\eta_{x_i^*,\omega_i}$ for $s_i = 60\%$



(b) Thresholds above which $\eta_{x_i^*,\omega_i} \leq 0$

$\varepsilon(Q) = \sigma$	$s_i \geq \ldots$	$\mu_i \geq \ldots$
0.5	21%	1.7
0.6	26%	1.76
0.75	34%	1.83
0.8	37%	1.85
1	50%	2
1.2	70%	2.4
1.25	83%	3
1.4	-	-

[20] A similar result is shown in Figure A3 for different levels of θ while keeping N fixed.

2.5 Extensions

So far, the analysis maintained quite restrictive assumptions on a production technology with a single input factor (**A1**), constant returns to scale (**A2**), and on price-taking behavior in the input market (**A3**). In this section, I relax these assumptions one at a time and I prove that, even in more general settings, the elasticity of input demand to productivity shocks can become negative at higher levels of output. While the key economic mechanism remains the same, the set of structural determinants of the elasticity of derived factor demand to productivity becomes richer. To shed light on each new determinant while preserving intuition, I go back to the case of a single-product monopolist (so the subscript i is dropped). I describe here the main results, while details and proofs are reported in Appendix A.5.

2.5.1 Multiple input factors

To keep the problem tractable, I consider that the firm has to hire two static and fully flexible inputs, labor (l) and material (m), in order to produce its output as follows.[21]

Assumption A1Ext (Inputs). *The firm produces its output q according to a standard production function $q = \varphi(\boldsymbol{x}, \omega) = f(\boldsymbol{x})\,\omega$ where ω denotes its Hicks-neutral productivity level and $\boldsymbol{x} = [l, m]$ is a vector of fully flexible input factors. The function f is assumed to be increasing, concave, and twice continuously differentiable in each input.*

The prices of both inputs are given to the firm and are denoted by w_l and w_m, respectively. Therefore, the profit maximization problem of the monopolist is

$$\max_{l,m} \quad \pi = p(q)\, q - w_l\, l - w_m\, m\,.$$

Extending the results from one input to multiple inputs makes the analysis more involved but the main intuition still holds. While the prices and the marginal products of each input (φ_l, φ_m) play a role in determining the optimal combination of inputs for a given level of output, they do not determine when labor and material become unresponsive to a Hicks-neutral productivity shock. As Proposition 4 shows, this is still driven by the elasticity of marginal revenue to output. If the marginal revenue starts decreasing more than proportionally to output, the firm decides to expand production less than proportionally to productivity, which reduces the demand for all the inputs.

[21] While the result can be generalized to numerous factor inputs, the key insight can be gained by examining just two factors. As in De Loecker (2011), dynamic capital is not part of the analysis.

Proposition 4. *Under the assumptions on technology (**A1Ext**), input prices (**A3**) and demand (**A4**), a monopolist reacts to a Hicks-neutral productivity shock by decreasing the use of its input factors if and only if the elasticity of marginal revenue at its profit-maximizing level of output $\eta_{mr,q}(q^*)$ is lower than -1.*

$$\frac{\partial l^*}{\partial \omega} < 0 \quad \Leftrightarrow \quad \underbrace{\left(-\frac{\varphi_l}{\omega}\varphi_{mm} + \frac{\varphi_m}{\omega}\varphi_{lm}\right)}_{>0}(1+\eta_{mr,q}) < 0 \quad \Leftrightarrow \quad \eta_{mr,q} < -1$$

$$\frac{\partial m^*}{\partial \omega} < 0 \quad \Leftrightarrow \quad \underbrace{\left(-\frac{\varphi_m}{\omega}\varphi_{ll} + \frac{\varphi_l}{\omega}\varphi_{ml}\right)}_{>0}(1+\eta_{mr,q}) < 0 \quad \Leftrightarrow \quad \eta_{mr,q} < -1\,.$$

Proof reported in Appendix A.5.1.

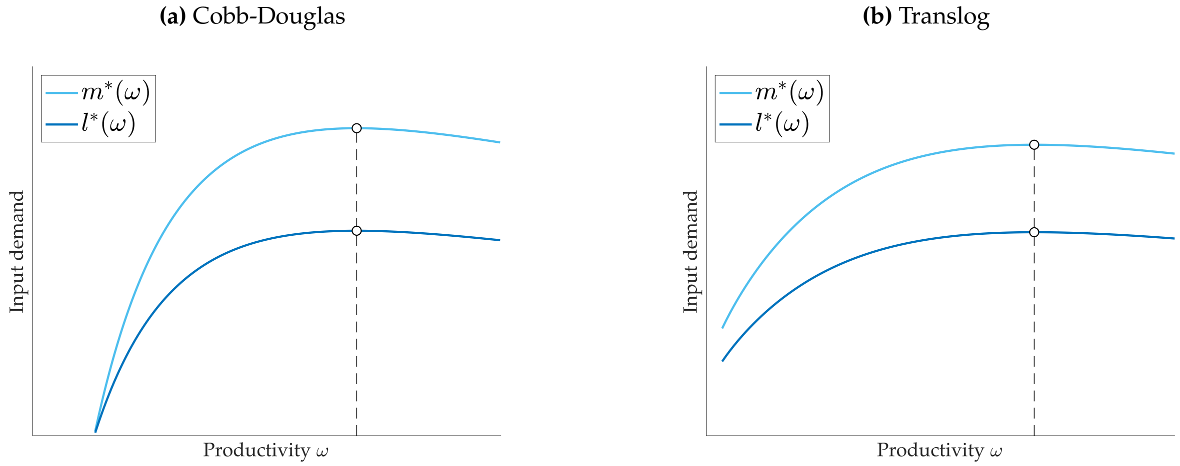
Having established that the mechanism highlighted with a single input is at play also with multiple inputs, the following result sheds light on the relative behavior of the different inputs.

Corollary 7. *For a given production function and level of input prices, if the relationship between derived factor demand and productivity is non-monotonic for any of the two inputs, it is non-monotonic for both of them. Also, the non-monotonicity occurs at the same level of output and productivity for both inputs.*

Figure 12 illustrates the derived material (light blue line) and labor (dark blue line) demands at different levels of productivity in the case of two common production functions. Panel (a) represents the case of a Cobb-Douglas production function $q = l^{\beta_l} m^{\beta_m} \omega$, for which the condition in Proposition 4 simplifies to $(\beta_l + \beta_m)(1 + \eta_{mr,q})$. Panel (b) illustrates the case for a more general Translog production function. For both production functions, if output demand is linear the relationship between derived factor demand and productivity is non-monotonic and it becomes downward-sloping when productivity is higher than a threshold value (dotted line), which is symmetric across inputs.

This result relates to Levinsohn and Petrin (2003) and De Loecker (2011), who prove that the demand for a variable input is always monotonically increasing in productivity. In their analyses, this result is crucial to use the control function approach in the estimation of production functions. If monotonicity holds, the demand for a variable input can be inverted and used as a *proxy* for the unobservable productivity term. The proof by Levinsohn and Petrin is valid under perfect competition (where markups are one), while De Loecker extends it to a monopolistic competitive setting with CES demand (where markups are constant and $\eta_{mr,q} > -1 \,\forall\, q$). My paper confirms these findings, but also shows that in settings featuring variable markups the relationship between input demand and productivity is not necessarily monotonic. Proposition 4 provides the conditions that need to be checked before applying the control function approach if markups are variable.

Figure 12. Non-monotonicity with multiple input factors.



Notes: simulations with a linear demand and identical factor costs $w_l = w_m$. In **(a)** I consider a Cobb-Douglas production function with $\beta_l = 0.4$ and $\beta_m = 0.6$, while in **(b)** a translog production function $log(q) = \beta_l log(l) + \beta_m log(m) + \beta_{ll} log(l)^2 + \beta_{mm} log(m)^2 + \beta_{ml} log(l) log(m) + log(\omega)$ with the same β_l and β_m, $\beta_{ll} = -0.02$, $\beta_{mm} = -0.03$ and $\beta_{ml} = 0.01$.

2.5.2 Non-constant returns to scale

In this section, I consider the role of returns to scale in affecting the elasticity of derived factor demand to productivity shocks. To focus my analysis only on the role of technology, I consider a single input (**A1**) but allow for a more general production function, which is defined as follows.

Assumption A2Ext (Technology). *The technology of the firm is described by a homothetic production function $q = \varphi(x, \omega) = f(x)\,\omega$ which is assumed to be strictly increasing ($f' > 0$), concave ($f'' < 0$), and twice continuously differentiable in the input x.*

Under the assumptions on technology, costs and demand (**A1-A2Ext-A3-A4**), the monopolist optimally chooses the output level to maximize its operating profits:

$$\max_{q} \quad \pi = p(q)\,q - mc(q, \omega, w)\,q = r(q) - \underbrace{C(q, \omega, w)}_{\text{Cost function}} .$$

In this setting, the marginal cost and the cost function depend also on the level of production, in addition to input price and productivity level. Following Equation (1), I start from its optimal output and input use, take logs and differentiate them with respect to ω in order to obtain:

$$\begin{aligned} q^* &= f(x^*)\,\omega \\ \frac{d\log(q^*)}{d\log(\omega)} &= \frac{d\log(f)}{d\log(x^*)}\,\frac{d\log(x^*)}{d\log(\omega)} + \frac{d\log(\omega)}{d\log(\omega)} \\ \eta_{q^*,\omega} &= \quad \eta_{f,x^*} \quad\;\; \eta_{x^*,\omega} \quad + \;\; 1 \;\; . \end{aligned}$$

Compared to Equation (2), the elasticity of derived factor demand to productivity is also influenced by the returns to scale, in particular by the scale elasticity $\eta_{f,\,x^*}$. As a result,

$$\eta_{x^*,\,\omega} = \frac{\eta_{q^*,\,\omega} - 1}{\eta_{f,\,x^*}} \,. \tag{16}$$

In fact, also the elasticity of optimal output to productivity $\eta_{q^*,\,\omega}$ is different from a situation with constant returns to scale because the fact the marginal costs depend now on output changes the firm's incentive to expand output after a productivity shock. Because of this, I show that the rate at which the cost changes, i.e. $\eta_{mc,q} \equiv \frac{\partial mc(q,\omega,w)}{q}\frac{q}{mc}$, matters. In particular, if returns to scale are decreasing, marginal costs increase as a firm produces more, i.e. $\eta_{mc,q} > 0$. Therefore, when a firm expands output, not only its price elasticity of demand declines but also its marginal costs rise.

If we consider the cost function dual to the production function, we can shed additional light on Equation (16). The assumptions of homotheticity of the production function and Hicks-neutral productivity ensure that relative changes in cost can be decoupled into output and productivity effects. Following Bakhtiari (2009), the dual cost function can be represented by two components, $c_1(q)$ and $c_2(\omega)$, defined as follows $C(q,\omega,w) = x(q,\omega)w = c_1(q)c_2(\omega)\,w$ with $c_1' > 0$ and $c_2' < 0$. As discussed in more detail in Appendix A.5.2, this is useful to determine the elasticity of optimal output to productivity. In particular, the latter becomes $\eta_{q^*,\,\omega} = \frac{\eta_{c_2,\,\omega}}{\eta_{mr,\,q} - \eta_{mc,\,q}}$ where $\eta_{c_2,\,\omega} = \frac{c_2'\,\omega}{c_2}$ is the elasticity of the component of the cost function directly related to productivity. Moreover, for homothetic production functions, the scale elasticity $\eta_{f,\,x^*}$ equals the returns to scale (RTS) of the production function and is equal to the inverse of the elasticity of the cost function with respect to quantity $\eta_{C,\,q} \equiv \frac{\partial C}{\partial q}\frac{q}{C} = \frac{C_q\,q}{C}$.[22] Based on these results, I extend Proposition 1 to non-constant returns to scale as follows.

Proposition 5. *Under the assumptions on technology (**A1-A2Ext**), input prices (**A3**) and demand (**A4**), the elasticity of derived factor demand to productivity of a monopolist is*

$$\eta_{x^*,\omega} = \eta_{c_2,\omega}\frac{\eta_{C,q} + \eta_{mr,q} - \eta_{mc,q}}{\eta_{mr,q} - \eta_{mc,q}} \tag{17}$$

and therefore it also depends on the characteristics of the cost function, i.e $\eta_{c_2,\omega}$, $\eta_{C,q}$, *and* $\eta_{mc,q}$.

Proof reported in Appendix A.5.2.

[22] As discussed by Panzar (1989) in a setting with multiple input factors, under mild regularity conditions the technology-based definition of scale economies $\tilde{S} = \frac{\sum x_i f_i(\mathbf{X})}{f(\mathbf{X})}$ and the cost based definition $S = \frac{C(q,\omega,w)}{q\,C_q(q,\omega,w)}$ are equivalent. This has been recently revisited also by Syverson (2019). In the context under analysis, this is the case since

$$\frac{1}{\eta_{C,\,q}} = \frac{C}{q\,C_q} = \frac{w\,x^*}{f(x)\omega\,\frac{w}{\omega f'}} = \frac{f'\,x^*}{f} = \eta_{f,\,x_i^*}\,.$$

In general, this result has the following implication.

Corollary 8. *Decreasing returns to scale reduce the elasticity of a firm's derived factor demand to productivity, while increasing returns have the opposite effect.*

With a standard Cobb-Douglas production function, $q = x^{\beta}\omega$, Equation (17) leads to

$$\eta_{x^*,\omega} = -\frac{1}{\beta}\frac{\frac{1}{\beta} + \eta_{mr,q} - \left(\frac{1}{\beta} - 1\right)}{\eta_{mr,q} - \left(\frac{1}{\beta} - 1\right)} = \frac{1 + \eta_{mr,q}}{1 - \beta - \beta\eta_{mr,q}} .$$

I illustrate this in Figure A5 for different degrees of returns to scale (i.e. values of β) with CES and linear demand. Decreasing returns to scale (i.e. $\beta < 1$) lead to a lower $\eta_{x^*,\omega}$ compared to a technology with constant returns to scale (i.e. $\beta = 1$). This implies that with a Cobb-Douglas production function - and many other homogeneous production functions - the degree of scale economies enjoyed by the firm affects the level of $\eta_{x^*,\omega}$, but it does not change the level of output at which $\eta_{x^*,\omega} = 0$. In Appendix A.5.2, I discuss the implications for $\eta_{x^*,\omega}$ when returns to scale vary with output.

2.5.3 Monopsonistic power in input market

This last extension relates to the price paid by the firm to employ the input factor x. I relax the assumption (**A3**) that the input market is perfectly competitive and assume, instead, that the monopolist can exert a certain degree of market power also on its suppliers.[23]

Assumption A3Ext (**Input price**). *The firm faces an upward-sloping inverse supply curve for the input factor x, i.e. $w(x)$ with $w' > 0$.*

In such a setting, the first-order condition of the profit-maximization problem of the firm is

$$\frac{\partial \pi}{\partial x} = \frac{\partial \pi}{\partial q}\frac{\partial q}{\partial x} = 0 \quad \Leftrightarrow \quad \underbrace{(p + p'\, q)\,\omega}_{mrp} = \underbrace{w + w'(x)x}_{me} .$$

The marginal revenue product of the input (mrp) is set equal to its marginal expenditure (me). The latter includes the input price w plus an extra term that captures the fact that a monopsonist must raise the input price when it demands and purchases additional units of input.[24]

[23] Since the seminal work of Robinson (1933), the concept of monopsony has been predominantly used when referring to market power in the labor markets. However, it can be applied to any factor market in which a firm manages to set a price below the marginal product of the input.

[24] In a static framework, the degree of monopsony power is measured by the wedge between the marginal revenue product and the factor price $\frac{mrp}{w} = (1 + \eta_{w,x})$, which ultimately depends on the elasticity of inverse supply $\eta_{w,x} \equiv \frac{w'(x)\,x}{w(x)}$. The higher is $\eta_{w,x}$, the higher will be the monopsony power exercised by the firm.

In the presence of market power also in the input market, the derived factor demand of the firm is

$$x^* = \frac{w - p\,\omega}{p'\omega^2 - w'} \,. \tag{18}$$

Based on this, I extend the results of Proposition 1 to the presence of monopsony power.

Proposition 6. *Under the assumptions on technology (**A1**-**A2**), input prices (**A3Ext**) and demand (**A4**), the elasticity of derived factor demand to productivity of a monopolist is*

$$\eta_{x^*,\omega} = -\frac{1+\eta_{mr,q}}{\eta_{mr,q} - \eta_{me,x}} \,, \tag{19}$$

where $\eta_{me,x} \equiv \frac{me'(x)\,x}{me(x)}$ *is the elasticity marginal expenditure with respect to input use.*

Proof reported in Appendix A.5.3.

Since $\eta_{x^*,\omega}$ is negatively related to $\eta_{me,x}$, this has the following implication.

Corollary 9. *The presence of monopsonistic power in the input market reduces the elasticity of a firm's derived factor demand to productivity but it does not change the level of output at which* $\eta_{x^*,\omega} = 0$*. Therefore,*

$$\eta_{x^*,\omega} = 0 \quad \Leftrightarrow \quad \eta_{mr,q} = -1 \,.$$

I prove this result in Appendix A.5.3, where I also illustrate it within the manifold framework in Figure A7. The key mechanism at play remains centered around the pass-through of productivity to output. In the presence of monopsony power, productivity improvements lead to smaller increases in output because the firm faces an additional trade-off compared to a situation where it is a price-taker in the input market. As a firm produces more, indeed, its marginal cost increase due to monopsonistic pecuniary effects, which further refrains it from expanding its output after a productivity shock.[25] In turn, this has a negative effect on the responsiveness of derived factor demand to productivity.

In the Appendix A.5.3, I show how $\eta_{me,x}$ reflects the shape of the inverse supply function $w(x)$. In particular, I prove that Equation (19) can be expressed in terms of the elasticity and convexity of the inverse supply, mirroring what happens with output demand. A higher elasticity of inverse supply curve $\eta_{w,x}$ (i.e. higher monopsony power) refrains a firm from getting even larger, while the convexity determines the rate at which its marginal expenditures increase as it employs more inputs.

[25] A direct comparison of the elasticity of optimal output to productivity in both cases makes it apparent. Since $\eta^{Monopsony}_{q^*,\omega} = \frac{\eta_{me,x}\,\eta_{x^*,\omega}}{\eta_{mr,q}} - \frac{1}{\eta_{mr,q}}$ and $\eta_{me,x} \geq 0$, the following result holds: $\eta^{Monopsony}_{q^*,\omega} \leq \eta^{Competitive}_{q^*,\omega}$.

3 From theory to empirics

Having established from a theoretical point of view that productivity growth can lead to a reduction in the derived factor demand of a firm, the next step is to assess the empirical relevance of this result. In this section, I discuss what are the challenges to identifying this mechanism in the data and describe two approaches that can be implemented to overcome them.

In theory, to verify whether this decoupling occurs in a given market would be enough to observe two monopolistic competitive firms that face the same demand and have the same technology, except for their productivity levels. Denoting these two firms by $i = 1, 2$ and assuming that $\omega_2 > \omega_1$, the direct comparison of their output and input use would be informative. As discussed in Section 2.4.1 and shown in Figure 7, if the more productive firm produces more but has a lower input use, i.e.

$$\begin{cases} q_2^*(\omega_2) > q_1^*(\omega_1) \\ \boldsymbol{x}_2^*(\omega_2) < \boldsymbol{x}_1^*(\omega_1)\,, \end{cases} \tag{20}$$

this a necessary and sufficient condition for $\eta_{x_2^*, \omega_2} < 0$. In other words, Equation (20) implies that firm 2 is operating in the range of price elasticity of demand where it finds it optimal to not fully pass its productivity advantage to output and to scale back its input use.[26]

To bring Equation (20) to the data, however, a number of identification challenges arise. The first one is posed by the fact that in most datasets, firms' production is reported in terms of revenue ($r_i = p_i\, q_i$) rather than physical quantities (q_i). As a result, productivity estimated as a residual from a production function is likely to suffer from the omitted output price bias. This is a well-known issue in the literature on firm productivity (De Loecker & Goldberg, 2014) and the standard solution is to deflate revenues with a price index. However, this solves the problem only if all firms set the same price, as it is the case under perfect competition, but not when firms set different prices. To overcome this issue, Klette and Griliches (1996) and De Loecker (2011) develop a solution to recover output elasticities and productivity when only revenue is available. However, this works only under monopolistic competition with CES demand. Without committing to a specific functional form of demand, estimating productivity with revenue data and variable markups remains an unresolved challenge. Nonetheless, in the next sections, I show that it is possible to infer whether firms react to productivity by decreasing their input even when firm-level productivity can not be estimated.

[26] Although with a different background, this is the approach adopted by Bakhtiari (2012) to show a non-monotonic relationship between employment and productivity in the ready-mixed concrete industry in the US.

3.1 Testable prediction in *levels*

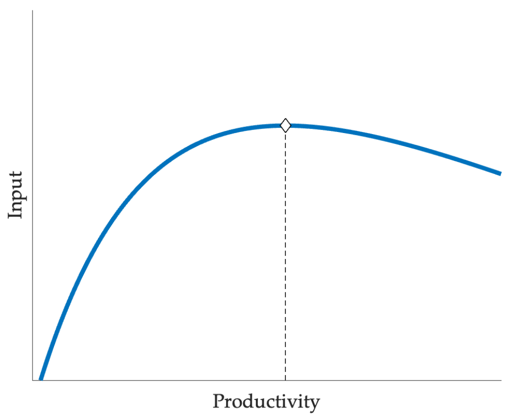
In this section, I show that data on firms' revenue and input in levels can be informative, under certain assumptions, about the decoupling of derived factor demand from productivity. The reason for this is that under monopolistic competition a non-monotonic relationship between input use and productivity, when it is there, gets reflected also in the relationship with revenue.

Prediction 1 ***(in levels)***: if the relationship between input use x_i^* and productivity ω_i is non-monotonic, the relationship between x_i^* and revenue r_i is non-monotonic too.

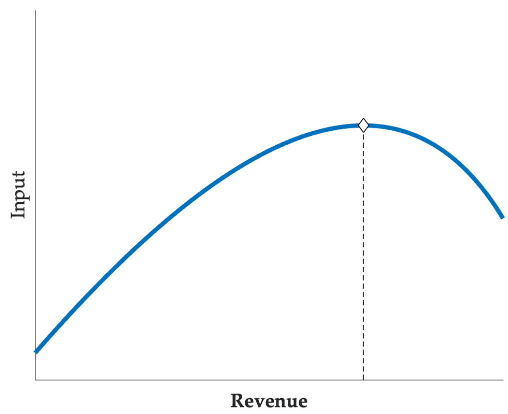
I illustrate this prediction below in a monopolistic competitive setting with linear demand. Panel (a) shows the level of input used by firms with different productivity levels. As depicted in panel (b), this non-monotonicity is reflected also in the equilibrium relationship between the levels of input and revenue of these firms. This holds also with multiple input factors, as shown in Figure 14(a).

Figure 13. Non-monotonicity between input use and productivity or revenues.

(a) Prediction with productivity.



(b) Prediction based on revenue.



Notes: equilibrium outcome as in Figure 7. The diamond indicates where $\eta_{x^*,\omega} = 0$.

While Prediction 1 does not require estimating productivity or taking a stance on the functional form of demand faced by the firms, it is important to acknowledge that it comes with a set of demanding assumptions which I list below and label with **(Alev)**. All the firms under consideration must be profit-maximizing and

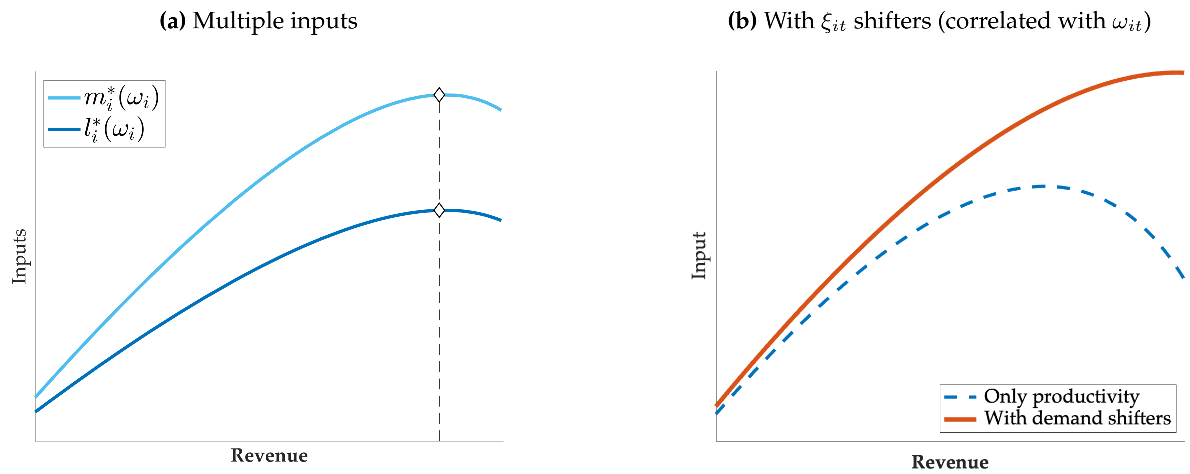
(Alev)
- **(i)** have the same production technology, up to Hicks-neutral productivity differences ω_i,
- **(ii)** face the same input prices,
- **(iii)** face the exact same demand schedule, i.e. $p(q_i) = p(q_j) \, \forall \, i \neq j$.

If any of these assumptions do not hold in the data, the relationship between revenue and productivity would appear monotonic even if the most productive firms operate in the range of demand where their derived factor demand is decreasing in productivity, i.e. where $\varepsilon(q) < 3 - \boldsymbol{\rho}(q)$. This is because any violation of assumptions **(Alev)(i)-(ii)-(iii)** implies that the input and output levels of different firms are influenced by other sources of firm heterogeneity, in addition to productivity. To convey the intuition, I denote by $\boldsymbol{z_{it}}$ a vector of unobservables fundamentals that lead a firm to produce more and use more input factors:

$$q^*_{it}(\omega_{it}, \underbrace{\boldsymbol{z_{it}}}_{+}) \quad \text{and} \quad x^*_{it}(\omega_{it}, \underbrace{\boldsymbol{z_{it}}}_{+}). \tag{21}$$

Examples of $\boldsymbol{z_{it}}$ are standard demand shifters. The demand for a firm's product, indeed, can be higher because its product has a higher appeal to consumers (given its brand or its perceived higher quality) or because it operates in a larger market. For a given functional form of demand $p(q_i)$, in the first case, the price consumers are willing to pay changes by the same factor (denoted by ξ_i) for all quantities. The resulting demand for the firm is equal to $\xi_i \, p(q_i)$. In the case of market size differences, demand varies by the same factor ψ_i for any price so that $p(\frac{q_i}{\psi_i})$. This may be due to more consumers being present in a market or the firm having a larger geographical scope. Also cost shifters may be part of $\boldsymbol{z_{it}}$, for example when a firm can purchase its input at a lower price compared to its competitors. In Appendix B, I prove that these favorable demand and cost shifters, differently from productivity, always lead to higher output, revenue and input use.[27]

Figure 14. Prediction 1 with multiple inputs and with correlated unobservables.



To illustrate the identification challenge posed by the presence of $\boldsymbol{z_{it}}$, in Figure 14(b) I illustrate a

[27]Differences in technology would be another potential source of $\boldsymbol{z_{it}}$, but their effect on the non-monotonicity is not trivial to assess in general.

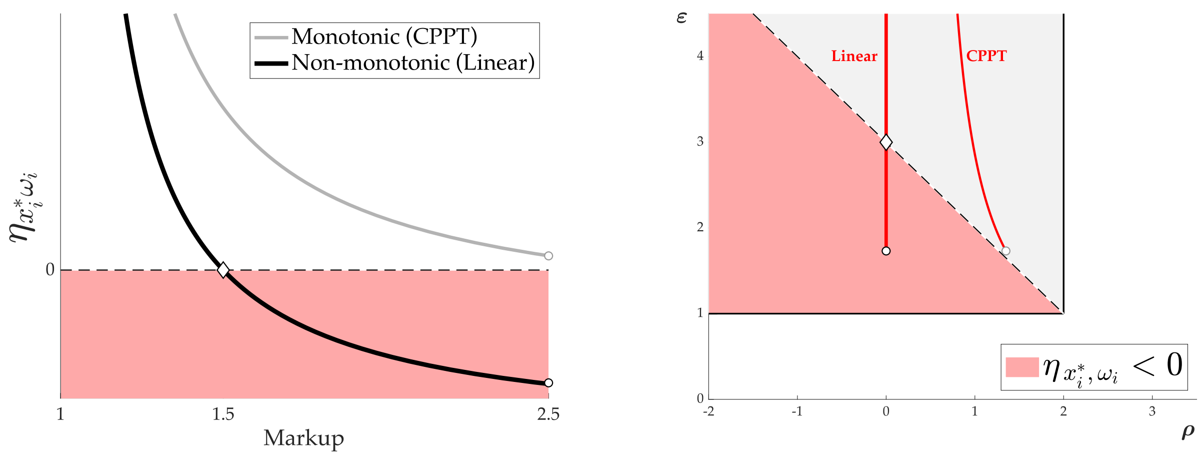
setting where firms differ both in their productivity and their products' appeal, and these two sources of heterogeneity are positively related. Although here firms with higher revenue operate in a range of price elasticity of demand where $\eta_{x_i^*,\omega_i} < 0$, the fact that they have also higher demand shifters lead to monotonic relationship between input and revenue (orange line). In such a situation, since the assumption **(Alev)(iii)** does not hold, the outcome of Prediction 1 would not be informative about the non-monotonicity between input use and productivity. Therefore, before bringing Prediction 1 to the data, it would be ideal to verify either the absence of additional sources of heterogeneity, i.e. $\boldsymbol{z_{it}} \approx \boldsymbol{z_{jt}}\ \forall i \neq j$, or that these shifters are not correlated with productivity, i.e. $\mathbb{E}\left[\omega_i\, \boldsymbol{z_{it}}\right] = 0$.

3.2 Testable prediction in *changes*

Looking at how firms change their input use between two consecutive periods can be informative about the decoupling of derived factor demand from productivity even in the presence of multiple sources of heterogeneity across firms. Indeed, by focusing on within-firm variation, we can condition on these additional sources of heterogeneity - namely on some elements of $\boldsymbol{z_{it}}$ - that may confound the cross-sectional predictions in levels. Another difference relative to the previous section comes from ranking firms based on their markups, instead of their revenues. This is more informative because the markup set by each firm is directly related to the price elasticity of its residual demand, which is what determines how it responds to a productivity change.

Figure 15. Theoretical prediction with two different demands.

(a) $\eta_{x_i^*,\omega_i}$ along the markup distribution **(b)** Range of elasticity in the manifold.



Notes: the diamond indicates where $\eta_{x^*,\omega} = 0$. The circles indicate the value of elasticity where $\mu_i = 2.5$. The Constant Proportional Pass-through (CPPT) demand has a PT rate of 65%.

To illustrate the idea of looking at input changes of firms that set different markups, in panel (a) of Figure 15 I plot the predicted elasticities of derived factor demand to productivity along the markup distribution in the case of linear demand and another demand that does not lead to a decoupling of factor demand from productivity. In both cases, firms setting higher markups have a lower $\eta_{x_i^*,\omega_i}$, but they reduce their input use if and only if they operate in a region of demand where $\varepsilon(q) < 3 - \rho(q)$. This is the case with linear demand for firms setting markups $\mu_i > 1.5$, but it is never the case for the other demand. This is illustrated in panel (b) by the red region in the manifold framework. Based on this insight, the following prediction is testable in the data.

Prediction 2 *(in changes)*: if a monopolistic competitive firm increases both its revenue and markups but reduces its input use over two consecutive periods, this is because $\eta_{x_i^*,\omega_i} < 0$. Across firms, this is more likely to take place among those setting higher markups.

$$\eta_{x_i^*,\omega_i} < 0 \quad \iff \quad \Delta x_i^*|_{\Delta r_i^* > 0 \;\&\; \Delta\mu_i > 0} < 0\,.$$

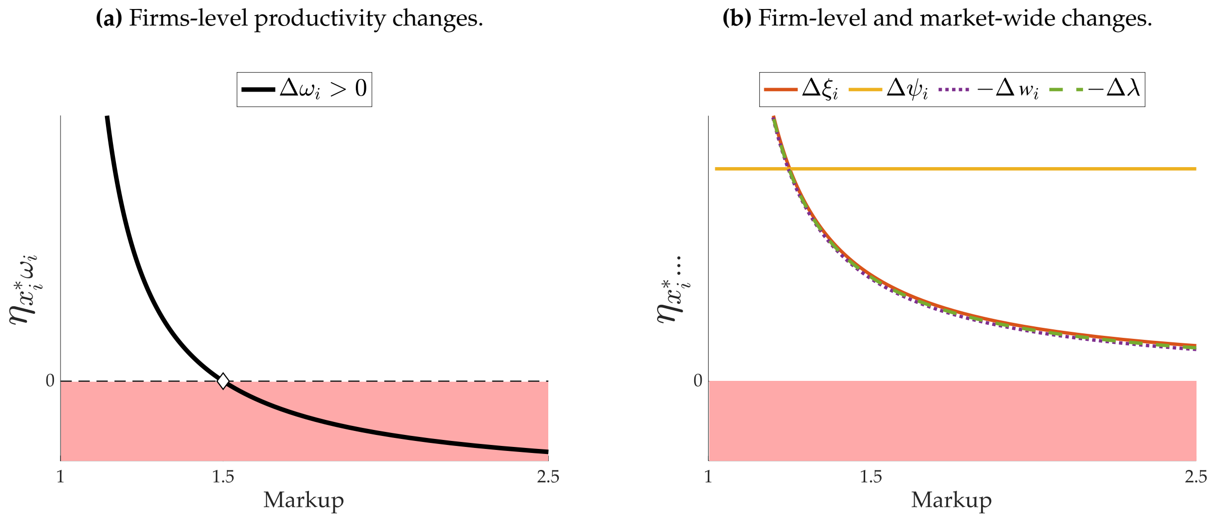
The rationale for looking only at firms that increased their revenue over two years is the following. If a profit-maximizing firm increases its revenue between two consecutive periods ($\Delta r_i^* > 0$), something must have changed in its fundamentals. In a monopolistic competitive setting, this can be the result of a positive productivity change ($\uparrow \omega_i$), but also of a positive demand shock ($\uparrow \xi_i$ or $\uparrow \psi_i$), of a reduction in input prices ($\downarrow w_i$), of a reduction in competitive pressure in the market ($\downarrow \lambda$), or even of a mix of them. While it is difficult to distinguish which type of shock has hit a firm, in Appendix B I prove that (i) only a productivity shock can ultimately lead to $\Delta x_i^* < 0$ and (ii) this happens if and only if a firm faces a low price elasticity such that $\varepsilon < 3 - \rho$. As I show in Table 2, all the other shocks lead to higher revenue, output, and derived factor demand.

Table 2. Comparative statics predictions leading to higher revenues.

<table>
<tr><th></th><th>$\eta_{r_i^*,\dots}$</th><th>$\eta_{q_i^*,\dots}$</th><th colspan="2">$\eta_{x_i^*,\dots}$</th></tr>
<tr><td>Higher demand appeal ($\uparrow \xi_i$)</td><td>> 0</td><td>> 0</td><td colspan="2">> 0</td></tr>
<tr><td>Larger market size ($\uparrow \psi_i$)</td><td>> 0</td><td>> 0</td><td colspan="2">> 0</td></tr>
<tr><td>Lower input costs ($\downarrow w_i$)</td><td>> 0</td><td>> 0</td><td colspan="2">> 0</td></tr>
<tr><td>Less competition ($\downarrow \lambda$)</td><td>> 0</td><td>> 0</td><td colspan="2">> 0</td></tr>
<tr><td>Productivity ($\uparrow \omega_i$)</td><td>> 0</td><td>> 0</td><td>> 0</td><td>$\mathbf{< 0}$</td></tr>
<tr><td></td><td></td><td></td><td>if
$\varepsilon > 3 - \rho$
Lower μ_i</td><td>if
$\varepsilon < 3 - \rho$
Higher μ_i</td></tr>
</table>

While I report here only the sign of these comparative static predictions, in Appendix B I show how to derive the elasticity of both revenue and input to these different shocks and in Table A2 how their values relate to the elasticity and convexity of output demand. This allows me to link them to the values of markups and provide direct guidance on whether the decoupling of factor demand to productivity has taken place in a given market. In Figure 16, I illustrate how firms setting different markups respond to all these changes in the case of linear demand. Firms setting higher markups will reduce their input use after a productivity shock, but this is never the case for other firm-level and market-wide changes. As shown in panel (a), the elasticity of derived factor demand to productivity, which is represented by a black line, declines so much that it becomes negative (red region). Instead, panel (b) illustrates how the elasticity of derived factor demand to all the other favorable firm-level and market-wide changes never becomes negative and never ends up in the red region.

Figure 16. Productivity shocks *vs.* other sources of changes (linear demand).



Compared to the prediction in levels, Prediction 2 relies on a less restrictive set of assumptions which I denote by **(Ach)**. Looking at input changes over the markups distribution remains informative about the decoupling of factor demand to productivity even when firms differ also in terms of demand shifters, technology, and costs.[28] Overall, with Prediction 2 we gain flexibility with respect to many cross-sectional unobservable differences between firms. However, since all the comparative statics in Section 2 are based on static models, I have to explicitly rule out adjustment costs.[29]
In particular, firms must be profit-maximizing in both periods and

[28]In the empirical illustration, I will impose that firms have common input prices and technology. However, this is just for estimation purposes.

[29]Future extensions of my theoretical results to dynamic settings can relax this, or at least characterize it in more detail.

$$
\textbf{(Ach)}\begin{cases}\textbf{(i)}\text{ can have different production technologies, but for each firm it must be constant over time,}\\ \textbf{(ii)}\text{ face potentially different input prices } w_{it},\\ \textbf{(iii)}\text{ face the same demand system, up to firm-level and time-varying shifters } (\xi_{it} \text{ and } \psi_{it}),\\ \textbf{(iv)}\text{ do not face adjustment costs,}\\ \textbf{(v)}\text{ the effect of their positive productivity shocks are not outweighed by other shocks.}\end{cases}
$$

The last assumption relates to the fact, in the presence of multiple shocks, the observed input change is the net effect of all shocks and changes experienced by each firm over two periods:

$$
\eta_{x_i^*,\dots} = \underbrace{\eta_{x_i^*,\xi_i} + \eta_{x_i^*,\psi_i} + \eta_{x_i^*,-w} + \eta_{x_i^*,-\lambda}}_{>0} + \underbrace{\eta_{x_i^*,\omega_i}}_{\gtreqless 0}\ .
$$

Prediction 2 remains informative about the decoupling of factor demand to productivity if the negative effect on input demand driven by a productivity shock is not outweighed by other contemporaneous firm- or industry-level changes. Put it differently, assumption **(v)** requires that

$$
\eta_{x_i^*,\omega_i} - \left(\eta_{x_i^*,\xi_i} + \eta_{x_i^*,\psi_i} + \eta_{x_i^*,-w} + \eta_{x_i^*,-\lambda}\right) < 0 \ \text{ when } \ \eta_{x_i^*,\omega_i} < 0.
$$

Otherwise, even if $\eta_{x_i^*,\omega_i} < 0$, this would not be observable since it would be masked by other shocks. For this reason, when testing Prediction 2 in different markets, it is likely to provide a *lower bound* of the prevalence of the decoupling result.

4 Empirical illustration

In this section, I bring predictions 1 and 2 to the data on Chinese manufacturing firms during a period of structural transformation in which China emerged as the “world's factory”. Given my interest in analyzing how productivity changes influence firms' output and input decisions, focusing on a period of intense productivity growth is ideal.

4.1 Data

I use the data from the Chinese surveys on the ”above-scale” industrial firms conducted by the National Bureau of Statistics (NBS), building on previous work by Brandt et al. (2012). During the 1998-2008 period, the NBS implemented yearly a census of all state-owned enterprises (SOEs) and all non-state firms with sales exceeding RMB 5 million, or about $600,000 at the exchange rate over that period. Compared to the universe of enterprises in the Economic Census, the sample of above-scale establishments covers the manufacturing and mining sectors and represents the bulk of industrial activity in China. I refer to Brandt et al. (2014) for a comprehensive description of this dataset.

To focus on firms of comparable size, I impose the threshold for inclusion in the survey for private firms also to the SOEs (revenue $>$ RMB 5 million). This dataset provides detailed firm-level information, including industry affiliation at the 4-digit level based on the Chinese Industry Classification (CIC), geographic location, and all operations and performance items from their accounting statements. In addition, it provides textual descriptions for (up to) three main products produced by each firm and information about the output value generated by newly introduced products. I use these two sources of information to narrow my analysis to firms that reported only one (main) product and have not introduced new products in the past year. I do so to minimize the potential source of bias generated by multi-product firms and the introduction of new products. As standard, I keep only firms with non-negative revenue, inputs, and value-added.

All the results in the paper are based on the 1999-2000 period. I focus on these years for several reasons. First, the year-on-year average productivity growth is extremely high, around 8%.[30] Second, the average output and input prices are relatively stable in the first years of the sample, as shown in Figure A11. Third, the large demand shock generated by the WTO accession had not taken place yet. Such a relatively stable macroeconomic environment reduces the likelihood that productivity shocks are outweighed by aggregate demand and input price changes.

In my analyses, I use firm-level information about the value of total production (revenue), use of materials, intermediates and service inputs (materials), total employment (labor), and the real capital stock (capital) constructed by Brandt et al. (2012). I deflate all monetary variables using the output, input, and investment deflators of each 4-digit industry.[31] In doing so, I assume that firms within each narrowly-defined industry face the same input prices. In the analysis of year-on-year changes, by construction, I restrict my focus to firms that remained active in the same industry over the two years. Moreover, to have a minimum number of observations in each industry, I consider those with at least 10 observations. Table A3 provides the summary statistics of the sample used, which (after the filtering procedure) covers 55,717 firms operating in 370 narrowly-defined industries.

[30]These are the benchmark estimates for (value-added weighted) average productivity growth rates estimated with the same data by Brandt et al. (2014). Figure A12 reports the evolution of year-on-year productivity growth for the other years.

[31]The industry concordance as well as deflators, and programs to construct the firm panel and real capital stock are available online here.

4.2 Estimation of the composite input and markups

To keep my empirical analysis as close as possible to the theoretical section, I estimate a single input as a composite of multiple input factors. To do so, I assume that the firm i, which operates in the industry j, produces at time t its output q_{ijt} according to a Hicks-neutral production function in line with Assumption **(A1Ext)**. As standard, I assume that the technology is represented by a Cobb-Douglas production function with three types of input factors: labor (l), materials (m), and capital (k). Under these assumptions, the composite input $\boldsymbol{x}_{ijt}$ is defined as follows

$$q_{ijt} = \underbrace{l_{ijt}^{\beta_j^l} \, m_{ijt}^{\beta_j^m} \, k_{ijt}^{\beta_j^k}}_{\boldsymbol{x}_{ijt}} \, \omega_{ijt} \, . \tag{22}$$

The output elasticities of the three input factors are denoted by β_j^l, β_j^m, and β_j^k.[32] I estimate these elasticities with the cost-share approach because, as I discuss in detail in Appendix C.2, the assumptions for the production function approach appear too restrictive in the settings considered in my paper. Under the assumption that returns to scale are constant, the condition for static cost minimization implies that an input's output elasticity equals the input's cost share. To measure the cost shares for labor and material, I use the total wage bill ($w_l l$) and the costs of materials, intermediate and service input ($w_m m$) directly from the accounting statements of each firm. Cost shares for capital are notably more difficult, since capital is owned (and hence rental rates are implicit) rather than rented. In line with the literature, I estimate the user cost of capital as a function of the real interest rates (RIR_t) plus a depreciation rate (δ). For the yearly real interest rates, I use those reported by the World Bank data, while for the depreciation rate I follow Brandt et al. (2014) and set $\delta = 9\%$.[33]

Following Collard-Wexler and Loecker (2016) and De Loecker, Eeckhout, and Unger (2020), I take the median of the cost share by industry to mitigate potential misspecification errors due to adjustment costs and/or optimization errors. Whenever individual producers are operating with idiosyncratically high or low inputs, the link between observed cost shares and the needed output elasticities does not hold at any given moment. However, by averaging over time and across producers, one can smooth out idiosyncratic misalignments between actual and optimal input levels. As a result, the output elasticities for materials, for example, are measured as follows

$$\hat{\beta}_j^m = median\left(\frac{w_{mt} m_{ijt}}{w_{lt} l_{ijt} + w_{mt} m_{ijt} + w_{kt} k_{ijt}}\right) \, .$$

[32] In the robustness checks, I allow the output elasticities to vary also by year, by province, and by ownership status.

[33] In particular, RIR_t is 7.4% in 1998, 7.2% in 1999, and 3.7% in 2000.

I report the resulting output elasticities for each 4-digit industry in Table A5. Over my entire sample, the average output elasticity for labor is 0.08, 0.86 for material, and 0.06 for capital. However, there is substantial heterogeneity across industries.[34] Equipped with these elasticities, I estimate the composite input $\boldsymbol{x}_{ijt}$, both in levels and in changes, according to

$$log\left(\boldsymbol{x}_{ijt}\right) = \hat{\beta}_j^l log(l_{ijt}) + \hat{\beta}_j^m log(m_{ijt}) + \hat{\beta}_j^k log(k_{ijt})$$
$$\Delta_{t,t-1} log\left(\boldsymbol{x}_{ij}\right) = log\left(\boldsymbol{x}_{ijt}\right) - log\left(\boldsymbol{x}_{ij(t-1)}\right) \ .$$

To estimate firm-level markups, I follow the approach of De Loecker and Warzynski (2012) which does not require specifying conduct or a particular demand system. Under the assumption that a firm is cost-minimizing, its markup is equal to the ratio of the output elasticity for a variable input to the corresponding revenue share. Accordingly, I estimate the markups in the following way:

$$\mu_{ijt} = \hat{\beta}_j^m \frac{r_{ijt}}{w_{mt} m_{mjt}} \ .$$

In line with Brandt et al. (2017), I use materials as variable input in the estimation because they can be adjusted more flexibly than either capital or labor use. In the analysis with markups, I do not consider observations with values lower than 1 because this would not be compatible with a profit-maximizing firm as shown in Equation (6). I also filter out firm observations with abnormal growth rates in employment and the composite input (higher or lower than 500%, probably driven by mergers or acquisitions) and exclude observations for which the composite input, deflated revenues, and markups are in the 1% upper and lower tail of the 4-digit industry-year distribution. Table A4 provides the summary statistics of all the variables used in analyses.

4.3 Results

I leverage the richness of this data to assess the empirical relevance of the two predictions on the decoupling of derived factor demand and productivity developed in Section 3. I begin with the prediction in levels about the cross-sectional relationship between firms' input use and their revenue. To compare firms facing similar demand conditions, I analyze each narrowly-defined 4-digit CIC industry separately, and I restrict the focus to firms producing only one main product. Overall, I find suggestive evidence of a non-monotonic relationship in 16 out of the 370 narrowly-defined industries

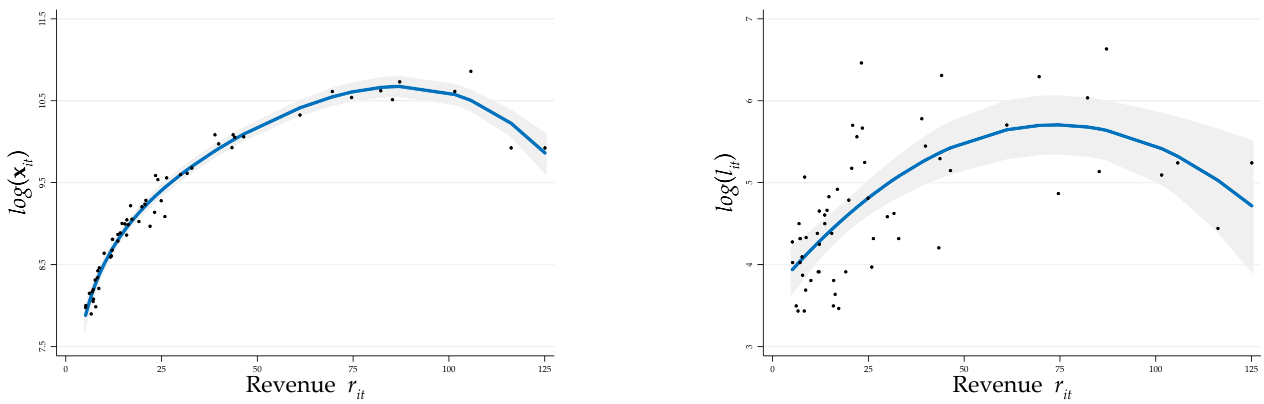
[34] My estimates are comparable to those estimated by Brandt, Van Biesebroeck, Wang, and Zhang (2017) separately by 2-digit industry with Cobb-Douglas production function using the methodology of De Loecker and Warzynski (2012). See Table A.2 in their online Appendix.

when I focus on the composite input and in 35 when I look directly at employment. In all the others, I estimate a monotonic relationship between input use and revenue. I report the results for all the industries in Table A5. Below I illustrate the estimated relationship for two of them, both in terms of the levels of (a) the composite input and (b) employment. Inevitably, the precision of the estimates decreases as the number of firms shrinks at higher levels of revenue. This is the case in all industries and in line with a monopolistically competitive model when the productivity distribution is right-skewed. In the top panels, I show how input use and revenue are related across the single-product firms in the manufacturing of pigments industry (in blue). The largest firms in terms of revenue are not necessarily the largest in terms of input use. In light of Prediction 1, this pattern is consistent with a non-monotonic relationship between productivity and input.

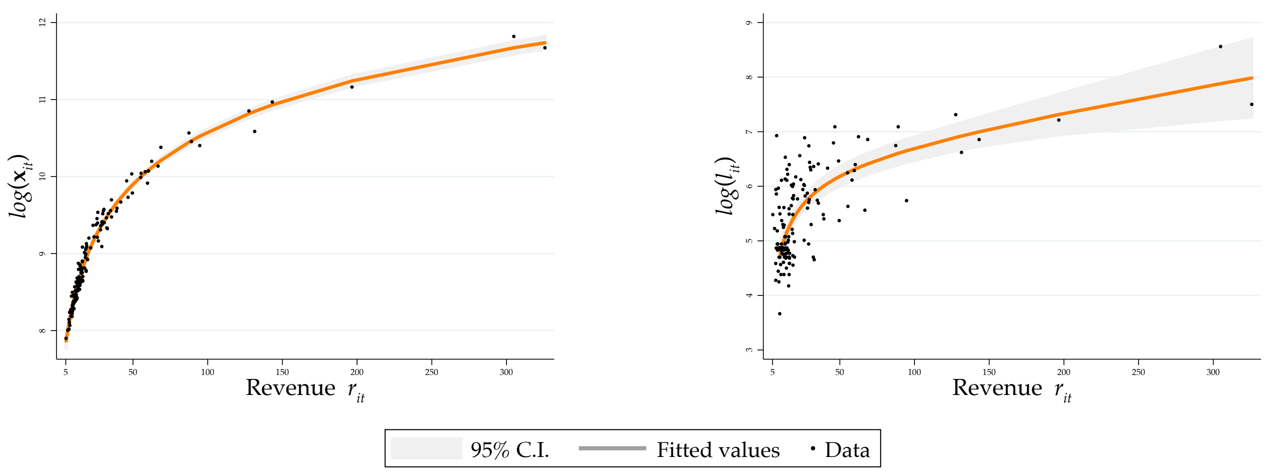
Figure 17. Relationship *in levels* in two illustrative industries.

1. Manufacturing of pigments (CIC 2643).

(a) Composite input **(b)** Employment



2. Manufacturing of rubber boots (CIC 2960).



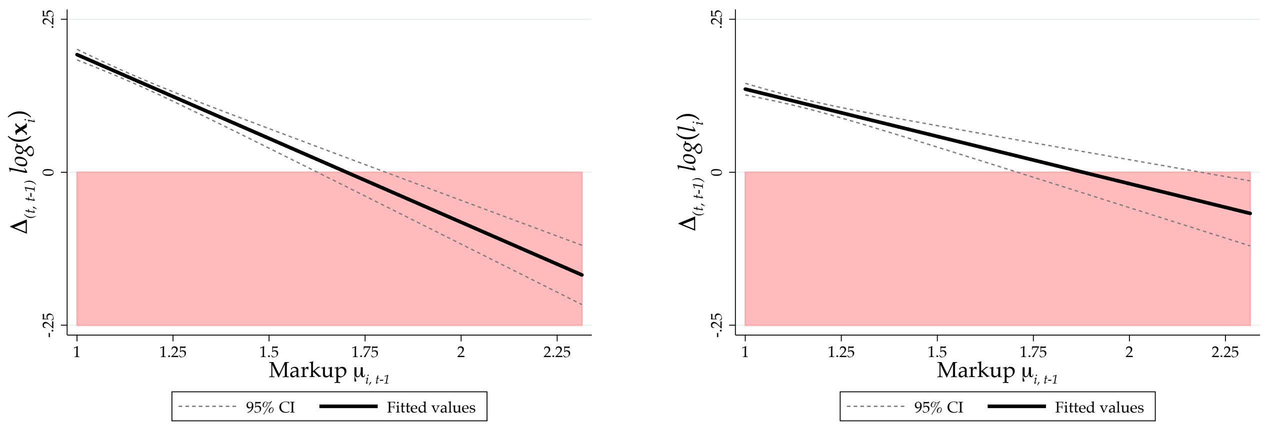
Notes: fitted values are based on a fractional polynomial of degree 2 estimated with the `fpfitci` package in Stata. Data for $t = 2000$. Data for composite input and employment are reported in log values to ease comparability. Revenues are deflated and expressed in millions of RMB.

The two panels at the bottom, instead, show the results for the manufacturing of rubber boots industry where the estimated relationship (in orange) between revenue and input is undoubtedly monotonic. It is noteworthy that this is the case for most of the industries analyzed. Does this result imply that the relationship between input use and productivity is monotonic in all these industries? Not necessarily. As discussed in Section 3.1, some caution is warranted in interpreting these results in level given the restrictive assumptions in **(Alev)**. If any of these assumptions do not hold, indeed, a positive relationship between input use and revenue may be driven by other sources of firm heterogeneity, without implying that the relationship between input demand and productivity is monotonic too. Industry and institutional details can provide suggestive information in support of these assumptions, at least in some industries. However, formally testing for this at scale would be very demanding in terms of data.[35] This is not the path I follow in this paper.

I now analyze how firms that raised their revenue and increased their markups between $t-1=1999$ and $t=2000$ changed their input.[36] Before presenting the results of my industry-by-industry analysis, in Figure 18 I provide an overview of how input changes are related to firm-level markups (in $t-1$) by pooling all the industries together.

Figure 18. Input changes over the markup distribution of Chinese manufacturing firms.

(a) Changes in composite input. **(b)** Changes in employment.



Notes: firms growing in terms of revenue and markups over 1999-2000. Fitted values based on a linear regression. In Figure A14 I provide an overview of how the underlying data are distributed.

The black lines depict the estimated relationships among all firms between the relative changes in their composite input (panel a) or their employment (panel b) and their markups. This is based on a

[35]One would need data on consumer purchases, the geographical scope of firm activity, and potential variations in input prices. Moreover, one should take a stance on a particular demand function in order to estimate firm-level demand shifters.

[36]This is the case for 36% of the firms in my sample.

linear regression but in Figure A13 I report similar results with a flexible parametrization. Overall, I find that the input changes tend to be lower and even negative for firms with higher markups.

While this pattern is informative about the existence of a gradient of responsiveness of firms setting different markups, Prediction 2 specifies that firms with higher markup values should be more likely to reduce their input use (while increasing revenue and markups). This is because if firms set higher markups it means that they face a lower price elasticity of demand, which raises the likelihood of a decoupling of derived factor demand from productivity. To verify that, I estimate with a logit model how the probability that a firm i in the industry j reduced its input use is related to its markup level in $t-1$.[37] To control for the cross-industry variation in the values of markups, I also estimate a version of each specification that includes industry dummies.[38] Finally, in order to minimize potential measurement errors in my estimates of the markup levels, I also consider the relative ranking (in terms of quintiles) of each firm in the markup distribution within each industry, instead of its markup values. In Table 3 I report the estimated average marginal effects, in columns (1-3) for the composite input and in (4-6) for employment.

Table 3. Probability of reducing input use over the level of markups.

	$\Delta log(\boldsymbol{x}_{ij}) < 0$			$\Delta log(l_{ij}) < 0$		
Variables	(1)	(2)	(3)	(4)	(5)	(6)
μ_{ijt-1}	0.28***	0.24***		0.12***	0.13***	
	(0.026)	(0.029)		(0.033)	(0.036)	
2nd quintile μ_{ijt-1}			0.02			0.00
			(0.013)			(0.015)
3rd quintile μ_{ijt-1}			0.03*			0.02
			(0.014)			(0.015)
4th quintile μ_{ijt-1}			0.03**			0.04***
			(0.014)			(0.015)
5th quintile μ_{ijt-1}			0.09***			0.07***
			(0.014)			(0.016)
Observations *N*	10,237	10,045	10,237	10,237	10,101	10,237
Industry FEs by j	-	Yes	-	-	Yes	-

Notes: Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

In line with Prediction 2, a higher level of markup in $t-1$ is associated with a higher probability of reducing the input use while increasing revenue and markups. On average, a higher markup (by

[37]Note that a firm here can set a higher markup than a competitor (and thus being on the right of the distribution) not only because of its productivity advantage but also because of its higher product appeal.

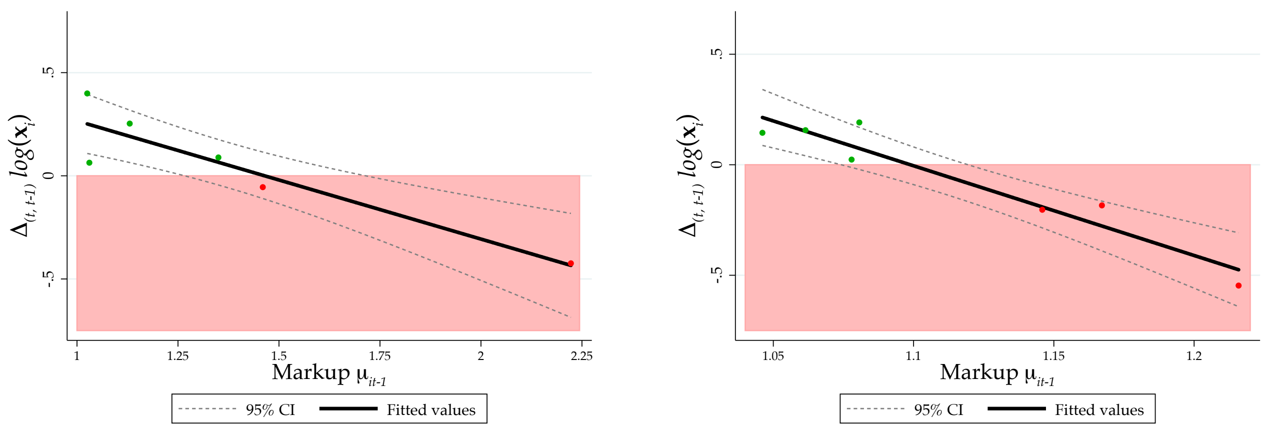
[38]In columns (2) and (5), the number of observations *N* is slightly smaller as few industries are dropped since their corresponding dummies perfectly predict the outcome.

one unit) is associated with a higher probability of reducing the composite input by 28 percentage points and employment by 12. These estimates are robust to the inclusion of industry dummies in columns (2) and (5). Moreover, the results based on quintiles in columns (3) and (6) show that this is particularly salient at the highest quintiles of the markup distribution within each industry.

Beyond this general overview, I analyze how widespread this pattern is by checking, within each narrowly-defined industry, whether the input changes are declining in markups and to what extent they become negative at higher values of markups. In Figure 19, I show this for the same two industries already considered for the results in levels in Figure 17. In both of them, I find a pattern consistent with Prediction 2: firms setting higher markups in 1999 tended to reduce their input use in the following year. For the manufacturing of pigments industry (panel a), the results in changes are in line with the results in levels. This industry was, indeed, one of the few in which I estimated a non-monotonic relationship in levels between input use and revenue. For the firms manufacturing rubber boots, instead, I find that results in changes (panel b) lead to the opposite interpretation of the results in levels.

Figure 19. Illustrative results *in changes*.

(a) Manufacturing of pigments (CIC 2643). **(b)** Manufacturing of rubber boots (CIC 2960).



Notes: firms growing in terms of revenue and markups over 1999-2000. Fitted values based on a linear regression.

Besides these illustrative industries, this is the case for many industries analyzed. In particular, I find a pattern consistent with Prediction 2 in around one-third of the industries analyzed. In particular, I find that the input changes are declining with markups in 197 out of the 282 industries tested.[39] In 107 of them, I also find that input changes are predicted to be negative at the highest quintile of

[39]The number of industries is less than 370 because focusing on changes reduces significantly the number of firms. Moreover, for estimation purposes, I analyze input changes only in industries with at least 5 firms that grew in terms of revenue and markups.

the markup distribution within each industry. Results are very similar when I look at employment changes: 192 with a declining pattern, whose 128 with negative input changes at the highest quintile. As discussed in Section 3.2, these numbers are likely to represent a lower bound because the effects of productivity shocks may be outweighed and masked by other shocks. While these estimates are based on the linearly fitted values over the markup distribution, I show in Appendix C.3.1 that more flexible fits lead to analogous results. Moreover, these results are robust to using output elasticities estimated at more disaggregated levels and to considering different sample compositions.

Table 4. Industries with a pattern in line with Prediction 2.

	$\Delta log(\boldsymbol{x})$	$\Delta log(l)$
Number of industries analyzed	282	
- with input changes ($\searrow$) declining in markup	197	192
- with input changes < 0 at highest 5ile of $\mu_{i,t-1}$	107	128

Under the assumptions **(Ach)**, these results can be rationalized only by productivity growth and the fact that the price elasticity of demand is such that these firms chose to scale back in terms of input to maximize their profits. It is noteworthy that this is very different from what we would have concluded by looking only at the results in levels. Taken as face value, this means that cross-sectional differences across firms (in addition to productivity) are likely to mask the non-monotonic relationship between input used and productivity, as foreseen in Section 3.1.

The aim of this illustration has been to investigate whether the testable predictions derived in Section 3 have some ground in the data. This seems to be the case for a non-negligible number of narrowly-defined industries in the Chinese manufacturing sector. However, since I am not testing the maintained assumptions of Prediction 2, this evidence remains at the moment only suggestive. Building on this exploratory analysis, future research with a narrower industry-specific focus (and the data/information to verify these assumptions) can move forward and provide more conclusive evidence about the result put forward by this paper.

5 Implications and way forward

The theoretical prediction that firms with market power are less responsive to productivity shocks and may even scale back in terms of input use has many wide-ranging implications. In this section, I summarize the most important ones discussed throughout the paper and delineate a number of research directions that I intend to explore in the future.

1. **Firm size as a *proxy* for productivity? Not necessarily.** Firm size measured in terms of input use, in particular employment, is often used as a proxy for productivity. While this is theoretically grounded under perfect competition or under monopolistic competition with CES demand (Melitz, 2003), in models with variable markups this is not necessarily the case, even when productivity is the only source of firm heterogeneity. Conversely, revenue and output remain valid proxies for productivity even in these settings.

2. **Industry equilibrium models with heterogeneous productivity.** In most theoretical models of monopolistic competition with sub-convex demands (Zhelobodko et al. (2012), Dhingra and Morrow (2019), Behrens et al. (2020), Mayer et al. (2021)), the non-monotonicity between input use and productivity eventually arises. In particular, if the distribution of firm productivity is assumed to be unbounded. Whether desired or not, this is a prediction of these widely-used models. Ruling it out by assumption, as is often the case, is possible but it implies severely restricting the range of markups and pass-through rates.

3. **Control function approach for production function estimation.** The monotonic relationship between derived factor demand for a variable input and a (scalar Hicks-neutral) productivity term is pivotal for the control function approach. My theoretical result about $\eta_{x_i^*,\omega_i} < 0$ identifies the conditions that need to be checked by practitioners before applying this approach in a context with variable markups. Retrospectively, it is fundamental to evaluate from an econometric point of view the impacts of the violation of the monotonicity assumption on the estimates of output elasticities and the measures that are built on them. As this non-monotonicity concerns mainly "granular" firms, any potential mismeasurement of their productivity and/or markups can be significant also from a macroeconomic point of view.

4. **Measurement of industry aggregates and reallocation.** Aggregate performance in a given industry (be it productivity, markups, or other indicators) is often measured as a weighted av-

erage of each firm's performance, with their shares of total inputs or revenue used as weights. The results of this paper imply that using input shares is likely to deliver a different result compared to revenue shares. The less convex output demand is, the more significant the difference between revenue and input share weighting. This is likely to matter for the interpretation of static and dynamic decompositions based on Baily, Hulten, and Campbell (1992), in particular for their reallocation components. Beyond measurement, the mechanism highlighted in the paper can also hinder the process of reallocation of productive resources across firms. As this is generally in a direction conducive to higher aggregate efficiency, further analysis on this along the lines of Edmond et al. (2018) seems of first-order importance.

5. **Declining responsiveness of labor demand to firm-level shocks.** The presence of variable markups leads to a gradient of the responsiveness of firms' output and factor demand to productivity and other shocks. In a companion paper (Biondi, Inferrera, Mertens, & Miranda, 2022), we find evidence of this in many European countries. We build on this insight to investigate the role of firms' market power in the decline in the responsiveness of labor demand to firm-level shocks and job reallocation rates. These alternative mechanisms offer a complementary explanation to the existing one by Decker et al. (2020) which is centered around the role of adjustment costs.

6. **Distributional consequences of productivity growth.** The fact that productivity growth is not fully transmitted into higher output and input, but it is turned into higher markups and profits, has an impact on the distribution of value added generated across factors of production within each firm. While this has been analyzed by Autor et al. (2020) and Kehrig and Vincent (2021), the non-monotonicity of derived factor demand further accentuates this mechanism and leads to a lower variable input factor share (not only for labor). Beyond a firm's boundaries, the incomplete pass-through of productivity to higher output and lower prices clearly affects consumers negatively. This is the standard output-reducing effect of market power but examined here in *first-differences*. , Along the lines of Eeckhout (2021), the results of my paper suggest that also factor suppliers may not benefit from higher productivity growth in the downstream sectors. Since the firms potentially reducing their derived factor demand after a productivity shock are the dominant firms in their markets, this may represent an overlooked channel through which market power leads to a disconnection between productivity and wages.

6 Conclusions

In this paper, I show that variable markups can lead firms to reduce their input use when they become more productive. The key mechanism leading to this result is the incomplete pass-through of productivity to output, which can arise from the exertion of market power in the output market. I characterize the conditions for this result to emerge and link it to the primitives of output demand and the nature of competition. I find that this mechanism is at play in workhorse models of imperfect competition, which are the core of most theoretical analyses related to firm productivity not only in industrial organization but also in macroeconomics, international trade, and public economics. For this reason, the result identified in this paper has wide-ranging implications, many of which still need to be explored.

To assess the empirical relevance of this overlooked result, I derive two predictions under monopolistic competition that can be easily brought to the data. Under certain conditions, I show that, even without estimating productivity or demand, they can be used by researchers to infer whether the decoupling of factor demand from productivity takes place in the market under analysis. As an illustration, I analyze the input decisions of Chinese manufacturing firms during a period of strong productivity growth and find patterns in the data in line with these predictions in many narrowly-defined industries. Building on this exploratory cross-industry analysis, future research can provide additional and more solid evidence for this result with a narrower industry-specific focus. In this regard, combining the analysis based on markups with empirical estimates of pass-through - which contains information about the convexity of demand - may represent a more powerful approach to detecting the decoupling result in the data.

My theoretical analyses characterize in full generality the role of output demand, but they are admittedly stylized for what concerns the supply side. This has been essential to identify the conditions that lead to the decoupling of factor demand from productivity growth. However, enrichment of the supply side is clearly desirable. My extensions under monopoly to non-constant returns to scale technology and the presence of monopsony power in the factor market go in this direction. Further extensions to adjustment costs, capacity constraints, non-Hicks neutral productivity shocks, and oligopolistic settings with differentiated products constitute all interesting paths for future research.

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Appendices

A Additional theoretical results

A.1 Monopoly

Proof of Proposition 1.

Proof. Start from the derived factor demand of the monopolist x^* reported in Eq. (10) and take its derivative with respect to productivity:

$$\frac{\partial x^*}{\partial \omega} = \frac{1}{p' \omega^2} (p - 2c) + \frac{p''}{p'^2 \omega} \frac{\partial q^*}{\partial \omega} (p - c) - \frac{1}{\omega} \frac{\partial q^*}{\partial \omega} \gtreqless 0 \,.$$

Express it in terms of elasticity by multiplying by $\dfrac{\omega}{x^*}$

$$\eta_{x^*, \omega} = \frac{\omega}{x^*} \frac{1}{p' \omega^2} (p - 2c) + \frac{\omega}{x^*} \frac{p''}{p'^2 \omega} \frac{\partial q^*}{\partial \omega} (p - c) - \frac{\omega}{x^*} \frac{1}{\omega} \frac{\partial q^*}{\partial \omega} \,.$$

The conditions for profit maximization imply that the marginal cost must be equal to marginal revenue, so that $(p - 2mc) = -p - 2\,p'q$ and $(p - mc) = -p'q$. As a result,

$$\eta_{x^*, \omega} = \frac{1}{x^*} \frac{(-p - 2\,p'q)}{p' \omega} + \frac{1}{x^*} \frac{p''}{p'^2} \frac{\partial q^*}{\partial \omega} (-p'q) - \frac{1}{x^*} \frac{\partial q^*}{\partial \omega} \,.$$

Substitute $x^* \omega = q^*$ and apply the definitions of elasticity and convexity from Eq. (5)

$$\eta_{x^*, \omega} = \frac{(-p - 2\,p'q)}{p'q} - \frac{1}{x^*} \frac{p''\,q}{p'} \frac{\partial q^*}{\partial \omega} - \frac{1}{x^*} \frac{\partial q^*}{\partial \omega} = \boldsymbol{\varepsilon}(q^*) - 2 + (\boldsymbol{\rho}(q^*) - 1) \frac{1}{x^*} \frac{\partial q^*}{\partial \omega} \,. \tag{A1}$$

To get $\dfrac{\partial q^*}{\partial \omega}$, take the derivative of the first-order condition with respect to ω so that

$$\frac{\partial\,(p + p'q)}{\partial q} \frac{\partial q}{\partial \omega} = \frac{\partial\, mc}{\partial \omega} \quad \Rightarrow \quad \frac{\partial q}{\partial \omega} = \frac{\dfrac{\partial\, mc}{\partial \omega}}{\dfrac{\partial\,(p + p'q)}{\partial q}} = \frac{-\dfrac{w}{\omega^2}}{2p' + p''q} = -\frac{c}{\omega(2p' + p''q)} \,.$$

At q^*, it must be that $mc = p + p'q^*$ and $\dfrac{1}{\omega} = \dfrac{x^*}{q^*}$. Therefore,

$$\frac{\partial q^*}{\partial \omega} = -x^* \frac{p + p'q^*}{q^*(2p' + p''q^*)} = x^* \frac{-p\,(1 + \frac{p'q^*}{p})}{q^*\,p'\,(2 + \frac{p''q}{p'})} = x^* \frac{\boldsymbol{\varepsilon}(q^*) \left(1 - \frac{1}{\boldsymbol{\varepsilon}(q^*)}\right)}{2 - \boldsymbol{\rho}(q^*)} = x^* \frac{\boldsymbol{\varepsilon}(q^*) - 1}{2 - \boldsymbol{\rho}(q^*)} > 0 \,.$$

Plug $\dfrac{\partial q^*}{\partial \omega} > 0$ back into Eq. (A1) to obtain

$$\eta_{x^*,\omega} = \boldsymbol{\varepsilon}(q^*) - 2 + (\boldsymbol{\rho}(q^*) - 1)\ \frac{\boldsymbol{\varepsilon}(q^*) - 1}{2 - \boldsymbol{\rho}(q^*)} = \frac{\boldsymbol{\varepsilon}(q^*) + \boldsymbol{\rho}(q^*) - 3}{2 - \boldsymbol{\rho}(q^*)}\,. \tag{A2}$$

Since the denominator is always positive for the second-order condition to hold (i.e. $\boldsymbol{\rho}(q^*) < 2$), condition *(c)* is proved:

$$\eta_{x^*,\omega} < 0 \quad \overset{(c)}{\Longleftrightarrow} \quad \boldsymbol{\varepsilon}(q^*) < 3 - \boldsymbol{\rho}(q^*)\,.$$

Based on this, substitute $\dfrac{1}{x^*} = \dfrac{q^*}{\omega}$ into Eq. (A1) to obtain $\eta_{q^*,\omega} = \frac{\partial q^*}{\partial \omega}\frac{\omega}{q^*}$ and

$$\eta_{x^*,\omega} = \boldsymbol{\varepsilon}(q^*) - 2 + (\boldsymbol{\rho}(q^*) - 1)\,\eta_{q^*,\omega}\,.$$

Express $\eta_{x^*,\omega}$ in terms of demand primitives from Eq. (A2)

$$\begin{aligned}
(\boldsymbol{\rho}(q^*) - 1)\,\frac{\boldsymbol{\varepsilon}(q^*) - 1}{2 - \boldsymbol{\rho}(q^*)} &= (\boldsymbol{\rho}(q^*) - 1)\,\eta_{q^*,\omega} \\
\frac{\boldsymbol{\varepsilon}(q^*) - 1}{2 - \boldsymbol{\rho}(q^*)} &= \eta_{q^*,\omega} \\
\eta_{x^*,\omega} + 1 &= \eta_{q^*,\omega}\,.
\end{aligned}$$

It follows that

$$\eta_{x^*,\omega} < 0 \quad \overset{(a)}{\Longleftrightarrow} \quad \eta_{q^*,\omega} < 1.$$

Similarly, condition *(b)* is proved by expressing $\eta_{mr,q}$ in terms of its demand primitives:

$$\eta_{x^*,\omega} < 0 \quad \overset{(b)}{\Longleftrightarrow} \quad \eta_{mr,q}(q^*) = \frac{(2p' + q^*\,p'')q^*}{p + q^*\,p'} = -\frac{2 - \boldsymbol{\rho}(q^*)}{\boldsymbol{\varepsilon}(q^*) - 1} = -\frac{1}{\eta_{q^*,\omega}} < -1\,.$$

As marginal revenue is equal to marginal cost, the elasticity of marginal revenue with respect to output is indeed the inverse of the elasticity of output with respect to productivity (and more general marginal cost). Condition *(b)* can also be proved by reshuffling Eq. (A2) so that

$$\eta_{x^*,\omega} = \frac{\boldsymbol{\varepsilon}(q^*) + \boldsymbol{\rho}(q^*) - 3}{2 - \boldsymbol{\rho}(q^*)} = -\frac{\eta_{mr,q}(q^*) + 1}{\eta_{mr,q}(q^*)} < 0 \quad \overset{(b)}{\Longleftrightarrow} \quad \eta_{mr,q}(q^*) < -1\,.$$

■

A.2 Demand Manifold

A.2.1 Overview of demand manifold for common functional forms

In this section, I report the demand manifold for commonly-used functional forms of demand that lead to $\eta_{x^*,\omega} < 0$ in Proposition 1(c). Note that this is not an exhaustive list. For the families of demand functions whose manifold depends on the values of some parameters, I specify the values that lead to $\eta_{x^*,\omega} < 0$, building on previous derivations by Mrázová and Neary (2017).

Table A1. Manifolds of commonly used demand functions leading to $\eta_{x^*,\omega} < 0$.

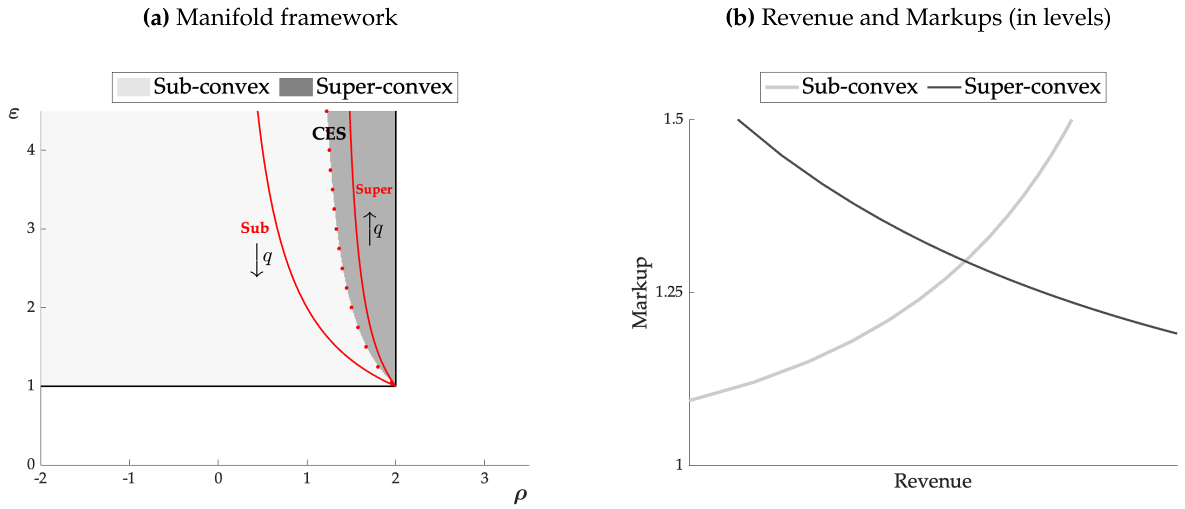
Demand functions	**Manifold**	**Parameters s.t.** $\eta_{x^*,\omega} < 0$
Manifold invariant		
Linear	$\boldsymbol{\rho} = 0$	$\forall$
CARA	$\boldsymbol{\rho}(\boldsymbol{\varepsilon}) = \frac{1}{\boldsymbol{\varepsilon}}$	$\forall$
Linear Expenditure System / Stone-Geary	$\boldsymbol{\rho}(\boldsymbol{\varepsilon}) = \frac{2}{\boldsymbol{\varepsilon}}$	$\forall$
Manifolds that depend on parameters		
(i) Bulow-Pfleiderer	$\boldsymbol{\rho} = 2 - \frac{1}{\kappa}$	$\kappa < 2$
(ii) CEMR	$\boldsymbol{\rho}(\boldsymbol{\varepsilon}) = 2 - \frac{1}{\kappa}(\boldsymbol{\varepsilon} - 1)$	$\kappa < 1$
(iii) CREMR	$\boldsymbol{\rho}(\boldsymbol{\varepsilon}) = 2 - \frac{1}{\kappa}\frac{(\boldsymbol{\varepsilon} - 1)^2}{\boldsymbol{\varepsilon}}$	$\kappa < 1$
(iv) Klenow-Willis	$\boldsymbol{\rho}(\boldsymbol{\varepsilon}) = \frac{(1-b)\boldsymbol{\varepsilon}+1}{\boldsymbol{\varepsilon}}$	$b > 0$
(v) Logistic	$\boldsymbol{\varepsilon} = \frac{a - log(1-\boldsymbol{\rho})}{2-\boldsymbol{\rho}}$	$\forall$

Notes: (i) in the family of Bulow and Pfleiderer (1983) demands the absolute pass-through from cost to price is constant ($\frac{\partial p}{\partial c} = \kappa$). In (ii) the family of CEMR (Constant Elasticity of Marginal Revenue) the proportional pass-through from cost to output is constant ($\frac{d\,log(q)}{d\,log(mc)} = -\kappa$). In (iii) the family of CREMR (Constant Revenue Elasticity of Marginal Revenue) the proportional pass-through from cost to revenue is constant ($\frac{d\,log(r)}{d\,log(mc)} = -\kappa$). Both of them have been put forward and used by Mrazova et al. (2021). Klenow and Willis (2016) introduced a parametric family of Kimball's demand (iv) in which the superelasticity of demand is a linear function of the elasticity: $S = b\varepsilon$. For Logistic demand (v) I follow the specification of Cowan (2016) where a is the price that induces a 50% market share. For additional details, I refer the reader to the Mrázová and Neary (2017).

A.2.2 Predictions with sub. *vs.* super-convex demands

As mentioned in the main text, the CES loci divide the admissible region in two: at an arbitrary point, any demand will be either more or less convex at that point than a CES demand function with the same elasticity. Demand functions whose manifold are located to the right of CES are called super-convex, while sub-convex those to the left. I represent the two regions in Figure A1(a).

Figure A1. Comparison of sub- *vs.* super-convex demands.



Notes: comparative statics for a monopolist facing a Constant Proportional Pass-through (CPPT) demand with two different % PT rates (determined by the parameter k, set at 0.5 and 1.5 respectively). With a 50% PT rate (in light grey) the demand is sub-convex, while with a 150% PT rate (darker grey) the demand is super-convex. Note that CES demand loci is exactly in the middle with a 100% PT rate.

The critical difference between these two types of demands is how the price elasticity of demand $\varepsilon(q)$ varies with output. In particular, the price elasticity of demand $\varepsilon(q)$ increases with output if demand is super-convex, while it decreases with output if it is sub-convex. In between, $\varepsilon(q) = \sigma$ is independent of output along the CES locus. The two arrows in the manifold indicate the (opposite) direction of movement as output increases.

This different relationship between the price elasticity of demand and output determines several comparative static predictions. Among others, it determines the relationship between markups and output (and revenue), both in levels and in changes. I illustrate this in panel (b) for two different demands, one in the sub-convex region and another one in the super-convex. If demand is sub-convex, when a firm produces more and increases its revenue, it will move to a portion of its demand where it faces a lower $\varepsilon(q)$ and thus it will set higher a higher markup. The opposite occurs if demand is super-convex.

A.3 Monopolistic competition

As explained by Mrázová and Neary (2019), the specification in (**A5**) is consistent also with a very broad class of demands that Pollak (1972) calls "generalized additive separability", such that the inverse demand for each good depends on its own quantity and on a single aggregate. In addition to (directly and indirectly) additive preferences, this class includes quasi-linear quadratic preferences as in Melitz and Ottaviano (2008), where λ equals the total sales of all firms; and the family of choke-price demands considered by Arkolakis et al. (2019), where λ is an aggregate price index.

Additional details about the setting. Prior to entry, firms face uncertainty about their productivity and entry requires a sunk cost f_E. Once the entry cost is paid, firms observe their productivity, which is drawn from a distribution $G(\omega)$ with support $[\omega_{min}, \omega_{max}]$. Last, after observing its type, each entrant decides to produce or not based on its operating profits:

$$\pi_i(\omega_i, \lambda) = \max_{q_i} \left(p_i(q_i, \lambda) - \frac{w}{\omega_i} \right) L\, q_i \ .$$

As these ultimately depend on the productivity term, this implies that there will be the minimum level of productivity $\underline{\omega}$ to remain profitably active. This is determined by two conditions. First, a break-even condition that all producers make nonnegative operating profits. Second, a zero-expected-profit condition, which drives the entry decision and requires that entry occurs until the expected value of taking a productivity draw is zero. The unique free-entry equilibrium determines the productivity cut-off $\underline{\omega}$, the mass of firms N and the marginal utility of income λ, which can be interpreted as a measure of the degree of competition each firm faces. In this regard, λ is the counterpart of the price index when demand is CES.

Details about the simulation in Fig. 7 and Fig. 8 The equilibrium values of the elasticity of derived factor demand to productivity and the other cross-sectional outcomes are obtained by assuming a mass of consumers $L = 100$ with quadratic preferences $u(q) = \alpha q - \frac{\beta}{2} q^2$ where $\alpha = 5$ and $\beta = 1$. The productivity distribution is assumed to be a bounded Pareto with $k = 5$ and $\omega_i \in [1, 4]$. The fixed entry cost is $f_E = 1$ in the equilibrium leading to $\lambda_2 = 6.7$, while it is equal to $f_E = 0.1$ in the equilibrium leading to $\lambda_1 = 10.2$ in Fig. 8.

A.4 Oligopoly

Proof of Proposition 3.

Proof. Start from $x_i^* = \dfrac{q_i^*}{\omega_i}$ and take its derivative with respect to productivity, so that

$$\frac{\partial x_i^*}{\partial \omega_i} = \frac{1}{\omega_i}\frac{\partial q_i^*}{\partial mc_i}\frac{\partial mc_i}{\partial \omega_i} - \frac{q_i^*}{\omega_i^2} = -\frac{1}{\omega_i}\frac{\partial q_i^*}{\partial mc_i}\frac{w}{\omega_i^2} - \frac{x_i^*}{\omega_i}\,.$$

Expressing it in terms of elasticity, it follows that

$$\eta_{x_i^*,\omega_i} \equiv \frac{\partial x_i^*}{\partial \omega_i}\frac{\omega_i}{x_i^*} = -1 - \frac{1}{\underbrace{\omega_i\, x_i^*}_{q_i^*}}\underbrace{\frac{w}{\omega_i}}_{mc_i}\frac{\partial q_i^*}{\partial mc_i} = -1 - \frac{c_i}{q_i^*}\frac{\partial q_i^*}{\partial mc_i} = -1 - \eta_{q_i^*,mc_i}\,. \tag{A3}$$

Noting that $\eta_{q_i^*,mc_i} = -\eta_{q_i^*,\omega_i}$, condition *(a)* holds

$$\eta_{x_i^*,\omega_i} < 0 \iff \eta_{q_i^*,\omega_i} < 1\,.$$

To analyze how a firm in oligopoly adjusts its output, derive its first-order condition with respect to mc_i

$$p' + \frac{\partial Q}{\partial q_i^*}\frac{\partial q_i^*}{\partial mc_i} + p''\,\theta\, q_i^*\,\frac{\partial Q}{\partial q_i^*} + p'\,\theta\,\frac{\partial q_i^*}{\partial mc_i} = 1\,.$$

Isolate $\frac{\partial q_i^*}{\partial mc_i}$ and express it in terms of elasticity to obtain

$$\eta_{q_i^*,mc_i} \equiv \frac{\partial q_i^*}{\partial mc_i}\frac{mc_i}{q_i^*} = \frac{1}{2\,p'\,\theta + p''\,\theta^2\, q_i^*}\frac{p + p'\,\theta\, q_i^*}{q_i^*} = ... = \frac{1 - \frac{\boldsymbol{\varepsilon}(Q)}{\theta\, s_i}}{2 - \boldsymbol{\rho}(Q)\,\theta\, s_i}\,.$$

By plugging this back into Eq. (A3)

$$\eta_{x_i^*,\omega_i} = -1 - \eta_{q_i^*,mc_i} = -1 - \frac{1 - \frac{\boldsymbol{\varepsilon}(Q)}{\theta\, s_i}}{2 - \boldsymbol{\rho}(Q)\,\theta\, s_i} = \frac{-3 + \boldsymbol{\rho}(Q)\,\theta\, s_i + \frac{\boldsymbol{\varepsilon}(Q)}{\theta\, s_i}}{2 - \boldsymbol{\rho}(Q)\,\theta\, s_i}\,, \tag{A4}$$

condition *(c)* holds since the denominator must be always positive

$$\eta_{x^*,\omega} < 0 \iff \frac{\boldsymbol{\varepsilon}(Q)}{\theta\, s_i} + \boldsymbol{\rho}(Q)\,\theta\, s_i < 3.$$

Condition *(b)* follows from deriving the mr_i with respect to q_i

$$\eta_{mr_i,q_i} \equiv \frac{\partial mr_i}{\partial q_i}\frac{q_i}{mr_i} = \left(p'\theta + \theta p''\frac{\partial Q}{\partial q_i}q_i + p'\theta\right)\frac{q_i}{mr_i} = ... = \frac{2 - \boldsymbol{\rho}(Q)\,\theta\, s_i}{1 - \frac{\boldsymbol{\varepsilon}(Q)}{\theta\, s_i}} = \frac{1}{\eta_{q_i^*,mc_i}}\,.$$

Thus,

$$\eta_{x_i^*,\omega_i} < 0 \iff \eta_{mr_i,q_i} < -1.$$

■

Proof of Corollary 5.

Proof. Sum the first-conditions of all firms to obtain $p = \frac{\bar{c}}{1-\frac{\theta}{N\,\varepsilon}}$. Substitute it into Eq. (15) and, after a few simplifications, the market share of firm i can be expressed as

$$s_i = \frac{\varepsilon}{\theta} - \frac{mc_i}{\overline{mc}}\left(\frac{\varepsilon}{\theta} - \frac{1}{N}\right)$$

The ratio $r_i \equiv \dfrac{mc_i}{\overline{mc}}$ is an inverse measure of the cost advantage of firm i: the lower it is, the greater the cost advantage of firm i with respect to average marginal costs in the industry. Plug it into Eq. (A3), to obtain

$$\eta_{x_i,\omega_i} = -1 + \frac{\frac{\varepsilon}{\theta\left[\frac{\varepsilon}{\theta} - r_i\left(\frac{\varepsilon}{\theta} - \frac{1}{N}\right)\right]} - 1}{2 - \boldsymbol{\rho}\,\theta\left[\frac{\varepsilon}{\theta} - r_i\left(\frac{\varepsilon}{\theta} - \frac{1}{N}\right)\right]} = -1 + \frac{\frac{1}{1 - r_i\boldsymbol{\varepsilon} + r_i\frac{\theta}{\boldsymbol{\varepsilon} N}} - 1}{2 - \boldsymbol{\rho}\,\boldsymbol{\varepsilon}\left(1 - r_i + r_i\frac{\theta}{\boldsymbol{\varepsilon} N}\right)}. \tag{A5}$$

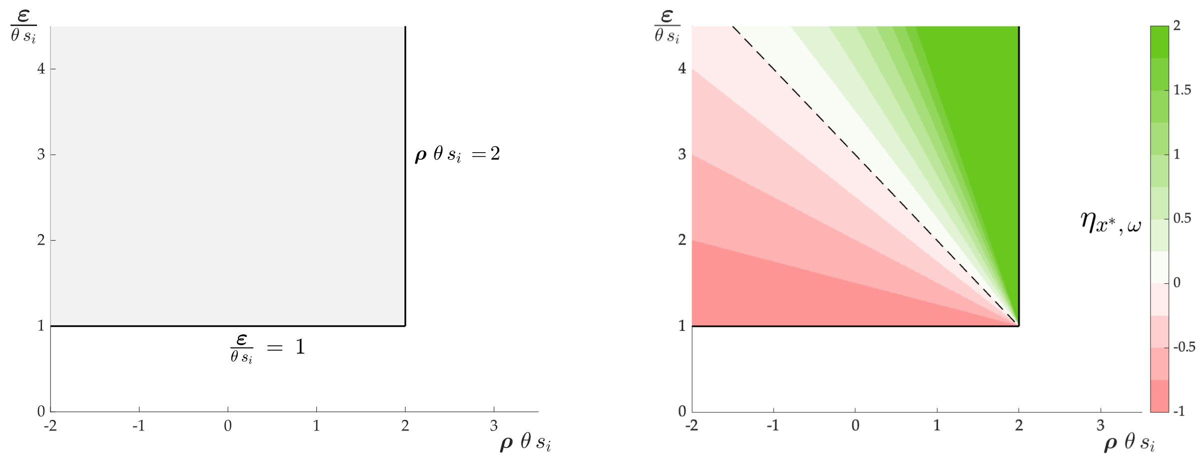
Based on this, the smaller is r_i, the lower is η_{x_i,ω_i}. This is because a firm with a substantial cost-advantage is likely to control a large share of the market. If so, it faces a low price elasticity of its residual demand and has a lower incentive to increase its output after a productivity shock. ■

A.4.1 Demand manifold in oligopoly.

In an oligopolistic setting, I find that the manifold can be re-formulated in terms of elasticity and convexity of the *residual* demand of each firm. This can be useful because most comparative statics predictions derived for a monopolist can be translated into the following *residual* demand manifold. In particular, the prediction about the elasticity of derived factor demand to productivity ($\eta_{x_i^*,\omega_i}$) is isomorphic to those of a monopolist.

Figure A2. The *residual* demand manifold of firm i.

(a) Admissible region. **(b)** Values of $\eta_{x_i^*,\omega_i} = \frac{-3+\boldsymbol{\rho}(Q)\,\theta\,s_i + \frac{\boldsymbol{\varepsilon}(Q)}{\theta\,s_i}}{2-\boldsymbol{\rho}(Q)\,\theta\,s_i}$

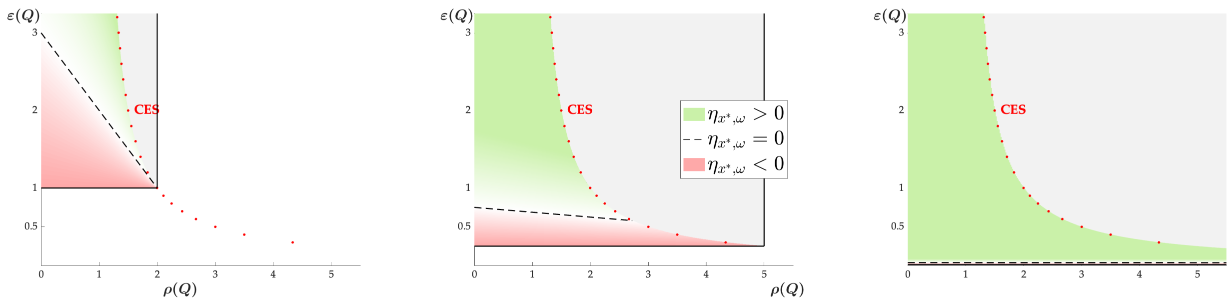


However, note that this is valid for a given s_i and θ. So this *residual* demand manifold representation is not informative about movements along the manifolds as is the case for monopoly or monopolistic competitive settings.

To complement the result of Figure 10, below I show how the admissible region changes with different values of the conduct parameter θ. If $\theta \to 0$, a perfectly competitive outcome emerges and the elasticity of derived factor demand to productivity is positive for any (admissible) value of elasticity and convexity. With higher values of θ, instead, the range of values in the manifold leading to $\eta_{x_i^*,\omega_i} < 0$ increases. This is because, with less aggressive/more collusive behavior, the firms will be able to exert more market power, which reduces their responsiveness to shocks.

Figure A3. Demand manifold regions by the degree of competition.

(a) Perfect cartel ($\theta = N$) **(b)** Cournot competition (θ = 1) **(c)** Perfect competition ($\theta \to 0$)



Notes: results are for a market with N =4 competitors.

A.5 Extensions

A.5.1 Multiple inputs

Proof of Proposition 4

Proof. Start from the first-order conditions for both inputs

$$\begin{cases} \dfrac{\partial \pi}{\partial l} = mr(q)\varphi_l - w_l = 0 \\ \dfrac{\partial \pi}{\partial m} = mr(q)\varphi_m - w_m = 0\,. \end{cases}$$

Differentiate them with respect to productivity to obtain the following system

$$\begin{bmatrix} mr\,\varphi_{ll} + \varphi_l^2\,\frac{\partial mr}{\partial q} & mr\,\varphi_{lm} + \varphi_l\,\varphi_m\,\frac{\partial mr}{\partial q} \\ mr\,\varphi_{ml} + \varphi_m\,\varphi_l\,\frac{\partial mr}{\partial q} & mr\,\varphi_{mm} + \varphi_m^2\,\frac{\partial mr}{\partial q} \end{bmatrix} \begin{bmatrix} \frac{\partial l^*}{\partial \omega} \\ \frac{\partial m^*}{\partial \omega} \end{bmatrix} = \begin{bmatrix} -mr\,\varphi_{l\omega} - \varphi_l\,\varphi_\omega\,\frac{\partial mr}{\partial q} \\ -mr\,\varphi_{m\omega} - \varphi_m\,\varphi_\omega\,\frac{\partial mr}{\partial q} \end{bmatrix}. \tag{A6}$$

Solve it by using Cramer's rule and collect mr in order to express it in terms of elasticity. The derived labor demand changes with ω according to

$$\frac{\partial l^*}{\partial \omega} = \frac{mr^2\left[(-\varphi_{l\omega} - \frac{\varphi_l \varphi_\omega}{q}\eta_{mr,q})(\varphi_{mm} + \frac{\varphi_m^2}{q}\eta_{mr,q}) - (\varphi_{lm} + \frac{\varphi_l\varphi_m}{q}\eta_{mr,q})(\varphi_{ml} + \frac{\varphi_m\varphi_l}{q}\eta_{mr,q})\right]}{mr^2\left[(\varphi_{ll} + \frac{\varphi_l^2}{q}\eta_{mr,q})(\varphi_{mm} + \frac{\varphi_m^2}{q}\eta_{mr,q}) - (\varphi_{lm} + \frac{\varphi_l\varphi_m}{q}\eta_{mr,q})(\varphi_{ml} + \frac{\varphi_m\varphi_l}{q}\eta_{mr,q})\right]}\,.$$

As the denominator is always positive under the profit-maximizing conditions, the sign of $\frac{\partial l^*}{\partial \omega}$ ultimately depends on the numerator. After a few simplifications, this becomes

$$-\varphi_{lm}\varphi_{mm} - \frac{\varphi_l\varphi_\omega}{q}\varphi_{mm}\eta_{mr,q} - \varphi_{l\omega}\frac{\varphi_m^2}{q}\eta_{mr,q} + \varphi_{m\omega}\varphi_{lm} + \frac{\varphi_l\varphi_m}{q}\varphi_{m\omega}\eta_{mr,q} + \varphi_{lm}\frac{\varphi_m\varphi_\omega}{q}\eta_{mr,q}\,.$$

This can be further simplified since the productivity term is assumed to be *Hicks-neutral*. In this case, indeed, $\varphi_{l\omega} = \frac{\varphi_l}{\omega}$, $\varphi_{m\omega} = \frac{\varphi_m}{\omega}$ and $\varphi_\omega = \frac{q}{\omega}$. Thus, the sign of $\frac{\partial l^*}{\partial \omega}$ depends on the value of $\eta_{mr,q}$:

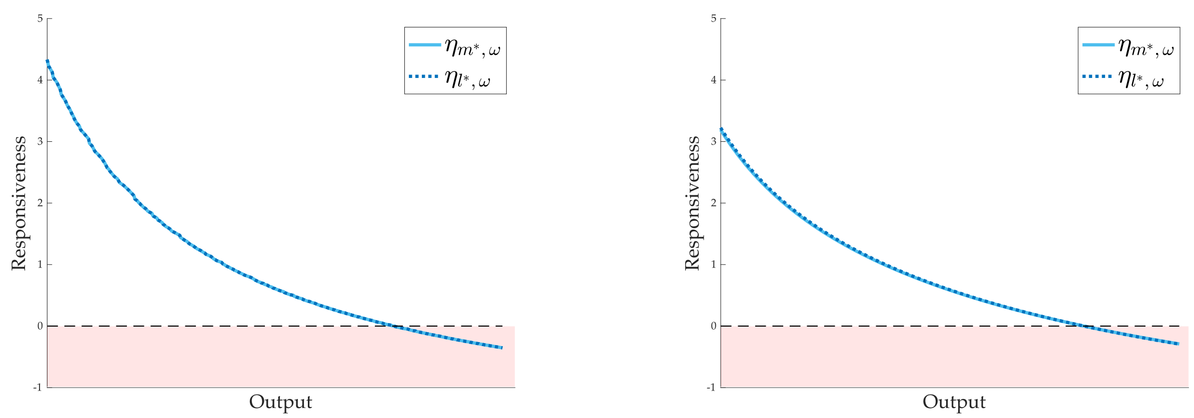
$$\frac{\partial l^*}{\partial \omega} < 0 \quad \Leftrightarrow \quad \underbrace{\left(-\frac{\varphi_l}{\omega}\varphi_{mm} + \frac{\varphi_m}{\omega}\varphi_{lm}\right)}_{>0}(1 + \eta_{mr,q}) < 0\,. \tag{A7}$$

Following the same steps, a similar result holds for m^*. The equivalent conditions in terms of $\eta_{q^*,\omega} < 1$ and $\boldsymbol{\varepsilon}(q^*) < 3 - \boldsymbol{\rho}(q^*)$ follows directly from Proposition 1. ■

Below I illustrate the values of $\eta_{l^*,\omega}$ and $\eta_{m^*,\omega}$ in the simulations reported in Figure 12. In both cases, there is no difference between the elasticity of derived factor demand to productivity among the two inputs. This is an implication of considering Hicks-neutral productivity shocks.

Figure A4. Non-monotonicity with multiple input factors.

(a) Cobb-Douglas production function **(b)** Translog production function



A.5.2 Technological returns to scale

Proof of Proposition 5.

This is a reformulation of previous derivations by Bakhtiari (2009), which I extend in the last part.

Proof. Starting from the cost function $C(q,\omega,w) = c_1(q)c_2(\omega)\,w$, isolate the derived factor demand

$$x^* = \frac{C(q,\omega,w)}{w} = c_1(q)c_2(\omega)\,.$$

Take its derivative with respect to ω

$$\frac{\partial x^*}{\partial \omega} = c_1'(q)\frac{\partial q^*}{\partial \omega}c_2(\omega) + c_1(q)c_2'(\omega)\,,$$

then express $\dfrac{\partial x^*}{\partial \omega}$ in terms of elasticity

$$\eta_{x^*,\omega} \equiv \frac{\partial x^*}{\partial \omega}\frac{\omega}{x^*} = \frac{c_1'(q)c_2(\omega)\omega}{c_1(q)c_2(\omega)}\frac{\partial q^*}{\partial \omega} + \frac{c_1(q)c_2'(\omega)\omega}{c_1(q)c_2(\omega)} = \frac{c_1'(q)\omega}{c_1(q)}\frac{\partial q^*}{\partial \omega} + \frac{c_2'(\omega)\omega}{c_2(\omega)}\,. \tag{A8}$$

To obtain $\frac{\partial q^*}{\partial \omega}$, take the derivative of the first-order condition $R_q(q) - C_q(q,\omega,w) = 0$ with respect to ω so that

$$(R_{qq} - C_{qq})\frac{\partial q^*}{\partial \omega} = C_{q\omega} \quad\Rightarrow\quad \frac{\partial q^*}{\partial \omega} = \frac{C_{q\omega}}{R_{qq} - C_{qq}} = \frac{c_1'(q)\,c_2'(\omega)\,w}{R_{qq} - C_{qq}} = \frac{c_2'(\omega)}{c_2(\omega)}\frac{C_q}{R_{qq} - C_{qq}}\,.$$

Plug it back into Eq. (A8)

$$\eta_{x^*,\omega} = \frac{c_1'(q)\omega}{c_1(q)}\frac{c_2'(\omega)}{c_2(\omega)}\frac{C_q}{R_{qq} - C_{qq}} + \frac{c_2'(\omega)\omega}{c_2(\omega)}\,.$$

Rearrange, multiply and divide by q the first term so that

$$\eta_{x^*,\omega} = \frac{c_1'(q)q}{c_1(q)}\frac{c_2'(\omega)\omega}{c_2(\omega)}\frac{C_q}{q(R_{qq} - C_{qq})} + \frac{c_2'(\omega)\omega}{c_2(\omega)}\,.$$

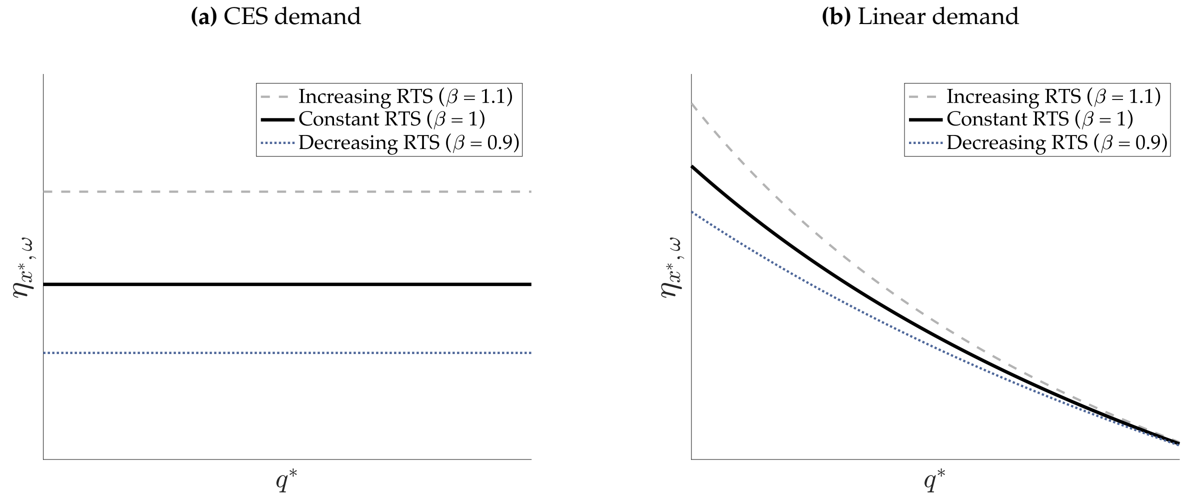
By the first-order condition $C_q = R_q$, so this simplifies to

$$\eta_{x^*,\omega} = \eta_{c_1,q}\,\eta_{c_2,\omega}\frac{1}{\eta_{mr,q} - \eta_{mc,q}} + \eta_{c_2,\omega} = \eta_{c_2,\omega}\frac{\eta_{c_1,q} + \eta_{mr,q} - \eta_{mc,q}}{\eta_{mr,q} - \eta_{mc,q}}\,.$$

■

Varying returns to scale. In the main text, I considered a Cobb-Douglas production function in which returns to scale take the same value for any level of output, either < 1 if RTS are decreasing or > 1 when RTS are increasing. In these cases, these are the corresponding different levels of $\eta_{x^*,\omega}$.

Figure A5. Influence of returns to scale on $\eta_{x^*,\omega}$.



This is the case also with CES production function and other homogeneous production functions with are usually applied in empirical studies to ease the estimation of output elasticities.

However, in theory, returns to scale can take different values depending on the output level. For example, a U-shaped average cost curve requires the scale elasticity $S(q) \equiv \frac{C(q,\omega,w)}{q\, C_q(q,\omega,w)}$ to vary: to be larger than one at low output levels and to decrease below one as a firm produces more. In general, the rate at which the returns to scale vary with output is described by

$$\eta_{S,q} \equiv \frac{\partial S(q)}{\partial q} \frac{q}{S(q)} .$$

This leads to the following result.

Corollary 10. *If returns to scale decline with output, the elasticity of derived factor demand to productivity decreases with output. Moreover, the level of output at which $\eta_{x^*,\omega} = 0$ is lower, further restricting the range of price elasticities of output demand for a monotonic relationship.*

Proof. Note that the sign of $\eta_{x^*,\omega}$ in Equation (17) depends on the sign of its numerator. This is because $\eta_{c_2,\omega} < 0$ and $(\eta_{mr,q} - \eta_{mc,q}) < 0$ under profit-maximization. Moreover, since $\eta_{c_1,q} = \eta_{C,q}$ and $\eta_{C,q} - \eta_{mc,q} - 1 = \eta_{S,q}$, it holds that

$$\eta_{x^*,\omega} = \eta_{c_2,\omega} \frac{1 + \eta_{mr,q} + \eta_{S,q}}{\eta_{mr,q} - \eta_{mc,q}} .$$

Therefore,

$$\eta_{x^*,\omega} \geq 0 \quad \Leftrightarrow \quad 1 + \eta_{mr,q} + \eta_{S,q} \geq 0 \quad \Leftrightarrow \quad \eta_{mr,q} \geq -1 - \eta_{S,q}$$

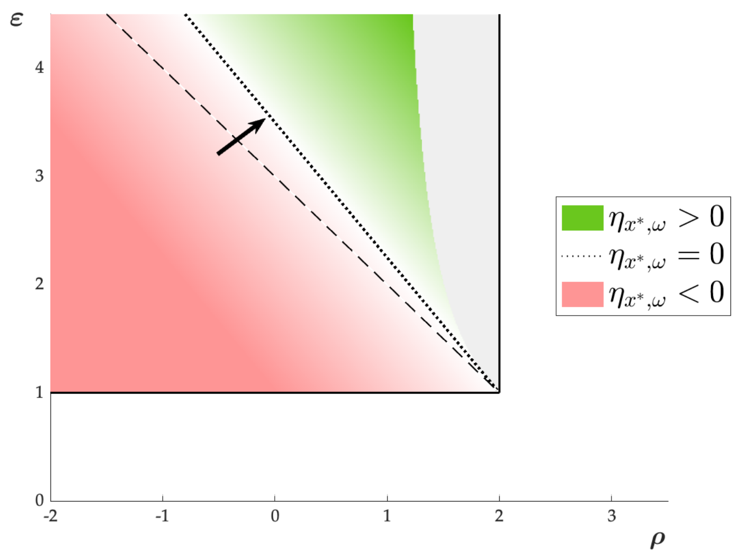
If returns to scale decline with output, i.e. $\eta_{S,q} < 0$, $\eta_{x^*,\omega} = 0$ must occur at a lower level of q. ■

As a result, the range of possibilities for a monotonic relationship is restricted whenever technological scale economies get exhausted at a higher level of output. This is the case, for example, for production technologies that lead to a U-shaped average cost function, which is a common assumption in many settings. The prediction of Corollary 10 is clearly visible in the manifold framework since

$$\eta_{x^*,\omega} < 0 \quad \Leftrightarrow \quad -\frac{2 - \boldsymbol{\rho}(q)}{\boldsymbol{\varepsilon}(q) - 1} < -1 - \eta_{S,q}(q) \quad \Leftrightarrow \quad \boldsymbol{\varepsilon} < \frac{3 - \boldsymbol{\rho} + \eta_{S,q}}{1 + \eta_{S,q}}.$$

Figure A6 shows that the region of elasticity and convexity values at which $\eta_{x^*,\omega} < 0$ expands. This means that, whenever technological returns to scale decline with output, higher values of ε can lead to a decoupling of derived factor demand from productivity growth.

Figure A6. Monotonicity in the manifold with declining RTS ($\eta_{S,q} < 0$).



Notes: illustrative example with $\eta_{S,q} = -0.25$. The dashed line represents the threshold with constant RTS, i.e. $\eta_{S,q} = 0$.

In parallel with the price elasticity and convexity of demand, I find that Proposition 5 can also be expressed directly in terms of elasticity and convexity of the cost function defined as $\rho_{C,q} \equiv \frac{C_{qq}\, q}{C_q} = \eta_{mc,q}$. Since $\rho_{C,q} = \eta_{mc,q}$, we have that

$$\eta_{x^*,\omega} = \eta_{c_2,\omega} \frac{\eta_{C,q} + \eta_{mr,q} - \rho_{C,q}}{\eta_{mr,q} - \rho_{C,q}}.$$

A.5.3 Monopsonistic power in input market

Proof for Proposition 6

Proof. Following the same logic of Proposition 1, take the derivative of Eq. (18) with respect to ω

$$\frac{\partial x^*}{\partial \omega} = \frac{\frac{\partial (w - p\,\omega)}{\partial \omega}(p'\omega^2 - w') - (w - p\,\omega)\frac{\partial (p'\omega^2 - w')}{\partial \omega}}{(p'\omega^2 - w')^2}\,.$$

After expressing it in terms of elasticity and a few other manipulations, it becomes

$$(p'\omega^2 - 2w' - x^* w'')\,\eta_{x^*,\omega} = \eta_{q^*,\omega}\,(-p' - p''\,q)\omega^2 - 2p'\omega^2 - \frac{p\omega^2}{q}\,.$$

To know how optimal output q^* is adjusted, derive also the first-order condition with respect to ω. As a result,

$$mr + mr'\,\frac{\partial q^*}{\partial \omega}\omega = me'\,\frac{\partial x^*}{\partial \omega}\,.$$

Isolate $\frac{\partial q^*}{\partial \omega}$ and express it in terms of elasticity

$$\frac{\partial q^*}{\partial \omega}\frac{\omega}{q^*} = \frac{me'}{mr'\,q^*}\frac{\partial x^*}{\partial \omega} - \frac{mr}{mr'\,q^*}\,.$$

Then, multiply the second term by $\frac{\omega\,mr}{me}\frac{x^*\omega}{x^*\omega}$ and rearrange so that

$$\eta_{q^*,\omega} = \frac{\eta_{me,x}\,\eta_{x^*,\omega}}{\eta_{mr,q}} - \frac{1}{\eta_{mr,q}}\,. \tag{A9}$$

Then plug it back into the equation

$$(p'\omega^2 - 2w' - x^* w'')\,\eta_{x^*,\omega} = \frac{\eta_{me,x}\,\eta_{x^*,\omega}}{\eta_{mr,q}} - \frac{1}{\eta_{mr,q}}\,(-p' - p''\,q)\,\omega^2 - 2p'\omega^2 - \frac{p\,\omega^2}{q}$$

and multiply both sides by $\eta_{mr,\,q}$

$$\eta_{mr,q}\,(p'\omega^2 - me')\,\eta_{x^*,\omega} = \eta_{me,x}\,\eta_{x^*,\omega} - (-p'\,\omega^2 - p''\,q\,\omega^2) - \eta_{mr,q}\left(2p'\omega^2 + \frac{p\,\omega^2}{q}\right)\,.$$

Finally, isolate $\eta_{x^*,\omega}$ and collect $(-p'\,\omega^2 - p''\,q\,\omega^2)$

$$\eta_{x^*,\omega} = \frac{(p'\,\omega^2 + p''\,q\,\omega^2) - \left(2p'\omega^2 + \frac{p\,\omega^2}{q}\right)\eta_{mr,q}}{\eta_{mr,q}(p'\omega^2 - me') - \eta_{me,x}(-p'\,\omega^2 - p''\,q\,\omega^2)} = \frac{-1 - \eta_{mr,q}\frac{\left(2p'\omega^2 + \frac{p\,\omega^2}{q}\right)}{(-p'\,\omega^2 - p''\,q\,\omega^2)}}{\eta_{mr,q}\frac{(p'\omega^2 - me')}{(p'\,\omega^2 + p''\,q\,\omega^2)} - \eta_{me,x}}\,.$$

Since the term highlighted in orange becomes

$$\frac{\left(2p'\omega^2 + \frac{p\,\omega^2}{q}\right)}{(-p'\,\omega^2 - p''\,q\,\omega^2)} = \frac{\frac{p\,\omega^2}{q}\left(\frac{2p'\omega^2 q}{p\,\omega^2} + 1\right)}{\frac{p\,\omega^2}{q}\left(\frac{-p'\,q}{p} - \frac{p''\,q^2}{p}\right)} = \frac{-\frac{2}{\varepsilon} + 1}{\frac{1}{\varepsilon} - \frac{\rho}{\varepsilon}} = -\frac{\varepsilon - 2}{\rho - 1}\,,$$

the numerator simplifies to $-1 + \frac{\varepsilon-2}{\boldsymbol{\rho}-1}\,\eta_{mr,\,q}$. The term highlighted in blue, instead, simplifies to

$$\frac{mr'\,q}{mr}\,\frac{mr}{q}\,\frac{\left(\frac{p'\,\omega^2\,q}{mr} - \frac{me'\,x\,\omega^2}{me}\right)}{p'\,\omega^2\,\left(-1 - \frac{p''\,q\,\omega^2}{p'\,\omega^2}\right)} = mr'\,\frac{\left(\frac{p'\,q}{mr} - \frac{me'\,x}{me}\right)}{p'\,\left(-1 - \frac{p''\,q}{p'}\right)} = \frac{mr'}{p'}\,\frac{\left(\frac{p'\,q}{mr} - \eta_{me,\,x}\right)}{(-1+\boldsymbol{\rho})}\,.$$

Therefore, the denominator becomes

$$\begin{aligned}
&= \frac{mr'}{p'}\,\frac{\left(\frac{p'\,q}{mr} - \eta_{me,\,x}\right) - (\boldsymbol{\rho}-1)\eta_{me,\,x}}{(\boldsymbol{\rho}-1)} = \frac{\eta_{mr,\,q} - \frac{2p'+p''q}{p'}\,\eta_{me,\,x} - \eta_{me,\,x} - (\boldsymbol{\rho}-1)\eta_{me,\,x}}{(\boldsymbol{\rho}-1)} \\
&= \frac{\eta_{mr,\,q} - (2-\boldsymbol{\rho})\,\eta_{me,\,x} - \eta_{me,\,x} - (\boldsymbol{\rho}-1)\eta_{me,\,x}}{(\boldsymbol{\rho}-1)} = \frac{\eta_{mr,\,q} - \eta_{me,\,x}}{\boldsymbol{\rho}-1}\,.
\end{aligned}$$

Bringing them back together, we obtain

$$\eta_{x^*,\omega} = \frac{-(\boldsymbol{\rho}-1) + (\boldsymbol{\varepsilon}-2)\,\eta_{mr,\,q}}{\eta_{mr,\,q} - \eta_{me,\,x}}\,.$$

Since $\eta_{mr,\,q} = -\dfrac{2-\boldsymbol{\rho}}{\boldsymbol{\varepsilon}-1}$, the numerator can be further simplified to

$$-(\boldsymbol{\rho}-1) - (\boldsymbol{\varepsilon}-2)\frac{2-\boldsymbol{\rho}}{\boldsymbol{\varepsilon}-1} = \frac{-\boldsymbol{\rho}(\boldsymbol{\varepsilon}-1) + \boldsymbol{\varepsilon} - 1 - (\boldsymbol{\varepsilon}-2)(2-\boldsymbol{\rho})}{\boldsymbol{\varepsilon}-1} = -\frac{\boldsymbol{\rho}+\boldsymbol{\varepsilon}-3}{\boldsymbol{\varepsilon}-1} = -(1+\eta_{mr,q})\,.$$

As a result, it holds that

$$\eta_{x^*,\omega} = -\frac{1+\eta_{mr,q}}{\eta_{mr,\,q} - \eta_{me,\,x}}\,.$$

■

Proof of Corollary 9

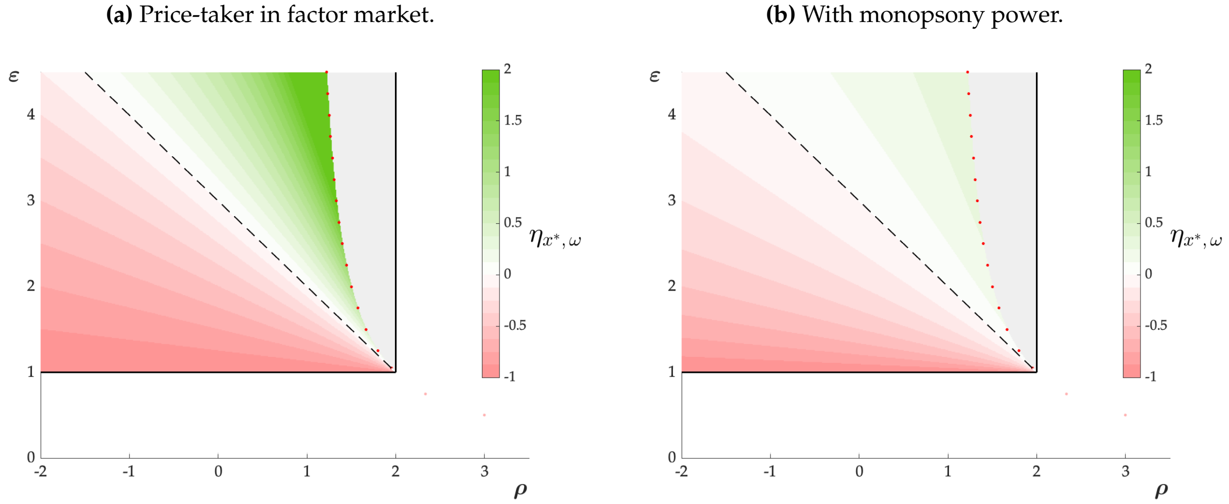
Proof. This follows from the fact that the denominator in Equation (19) has to be negative for the second-order condition to hold. Alternatively, by simply rewriting Equation (A9) in terms of $\eta_{x^*,\,\omega}$, it holds that

$$\eta_{x^*,\,\omega} = \frac{\eta_{q^*,\,\omega}\,\eta_{mr,\,q}}{\eta_{me,\,x}} + \frac{1}{\eta_{me,\,x}} = 0 \;\;\Leftrightarrow\;\; \eta_{q^*,\,\omega} = -\frac{1}{\eta_{mr,\,q}}\,.$$

■

To illustrate Corollary 9 in the manifold space, I consider the simplest case of an isoelastic inverse supply curve $w(x) = g\,x^m$, which leads to a constant $\eta_{me,x} = m \geq 0$. In the two figures below, I compare the elasticity of derived factor demand to productivity with and without monopsony power in the factor market.

Figure A7. Lower values of $\eta_{x^*,\omega}$ with monopsony power.



Notes: In the case of monopsony power, $\eta_{x^*,\omega}$ is illustrated for a constant $\eta_{me,x} = m = 1.5$.

Relationship with the features of the inverse supply curve. Similarly to the elasticity of marginal revenue with the demand curve, the elasticity of marginal expenditure ($\eta_{me,x}$) depends on the shape of the inverse supply curve. In particular, I find that it is jointly determined by its elasticity $\eta_{w,x}$ and convexity $\rho_{w,x}$ according to

$$\eta_{me,x} = -\frac{\eta_{w,x}\,(2+\rho_{w,x})}{1+\eta_{w,x}} \quad \text{where} \quad \eta_{w,x} \equiv \frac{w'(x)\,x}{w(x)} \quad \text{and} \quad \rho_{w,x} \equiv \frac{w''(x)\,x}{w'(x)}\,.$$

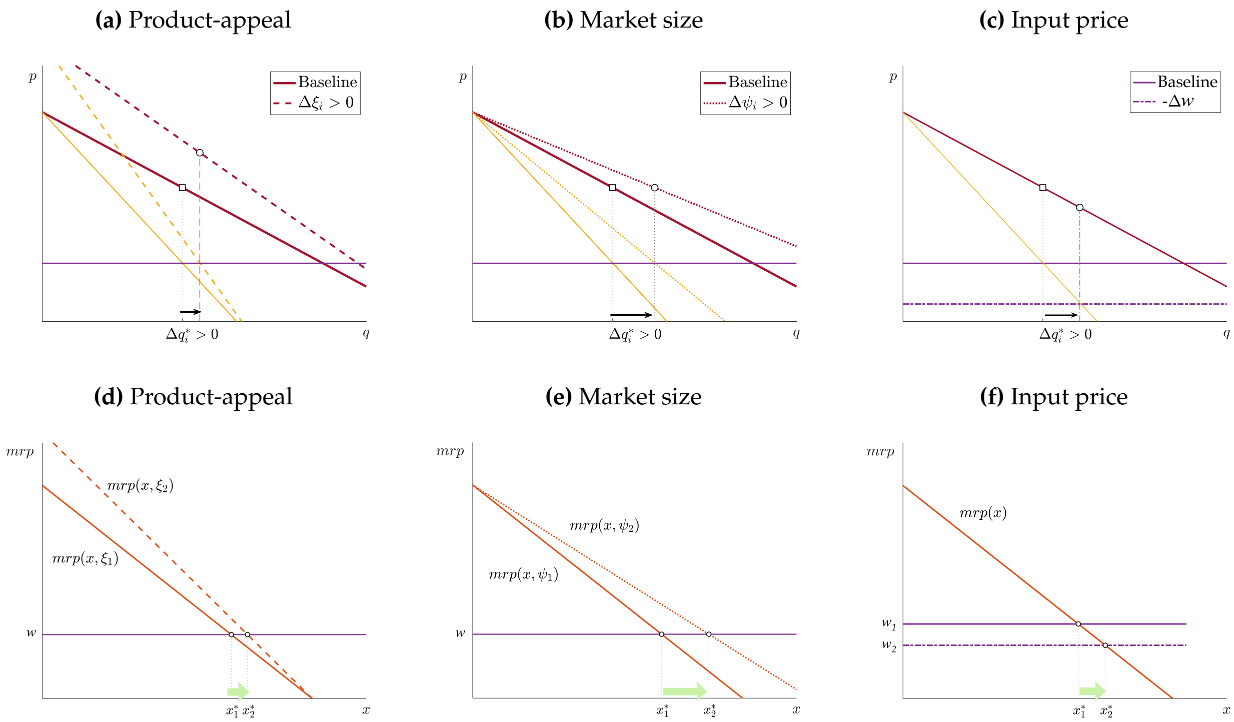
This show that a higher elasticity of inverse supply curve $\eta_{w,x}$ (i.e. higher monopsony power) refrains a firm from getting even larger, as marginal factor costs increase due to monopsonistic pecuniary effects. On top of it, also the convexity of the inverse supply curve matters as it determines the rate at which these effects on marginal costs increase as a firm gets larger and move along the inverse supply curve.

B Different impacts of shocks

B.1 Comparative statics of demand and cost shifters.

To highlight the difference with productivity changes, I illustrate below the comparative statics of derived factor demand to other firm-level shifters. In particular, in Figure A8 I plot the different comparative static predictions in the case of a monopolist facing a linear demand. In (a) and (d), I plot the difference in output and derived factor demand with a higher product appeal ξ_i. In (b) and (e), I show the comparative statics of a larger market size ψ_i. Differently from productivity, these demand shifters always lead to higher output and derived factor demand $x_2^* > x_1^*$. As can be seen in (c) and (f), this is the case also for a reduction in the input price *w*.

Figure A8. Comparative statics of demand and cost shocks.



Beyond linear, I derive the elasticity of derived factor demand and revenue with respect to productivity, demand, and favourable cost shocks in the case of a monopolist under the assumptions **(A1-A4)**. As reported in the Table A2, all these elasticities end up being a function of the values of elasticity and convexity of demand.

Table A2. Elasticities of derived factor demand and revenue to various shocks.

	$\eta_{x_i^*,\ldots}$	$\eta_{r_i^*,\ldots}$
Productivity (ω_i)	$\dfrac{\boldsymbol{\varepsilon}+\boldsymbol{\rho}-3}{2-\boldsymbol{\rho}}$	$\dfrac{2-\boldsymbol{\rho}}{\boldsymbol{\varepsilon}}$
Demand appeal (ξ_i)	$\dfrac{\boldsymbol{\varepsilon}-1}{2-\boldsymbol{\rho}}$	$\dfrac{(\boldsymbol{\varepsilon}-1)^2}{\boldsymbol{\varepsilon}(2-\boldsymbol{\rho})}+1$
Market size (ψ_i)	$2-\boldsymbol{\rho}$	$\dfrac{(2-\boldsymbol{\rho})(\boldsymbol{\varepsilon}-1)}{\boldsymbol{\varepsilon}}$
Input costs ($-w_i$)	$\dfrac{\boldsymbol{\varepsilon}-1}{2-\boldsymbol{\rho}}$	$\dfrac{(\boldsymbol{\varepsilon}-1)^2}{\boldsymbol{\varepsilon}(2-\boldsymbol{\rho})}$

To get to these results, the starting point is always the optimal output or revenue expressed in *logs*:

$$\log(x_i^*) = \log(q_i^*) - \log(\omega_i)$$
$$\log(r_i^*) = \log(q_i^*) + \log(p_i)\,.$$

Then, building on the proof of Proposition 1, I derive all the results reported in Table A2 as follows.

Elasticity of revenue to productivity:

$$\eta_{r_i^*,\omega_i} = \frac{d\log(q_i^*)}{d\log(\omega_i)} + \frac{d\log(p_i)}{d\log(\omega_i)} = \eta_{q_i^*,\omega_i} + \underbrace{\frac{\partial p_i}{\partial \omega_i}\frac{\omega_i}{p_i}}_{\eta_{p_i,q_i^*}\eta_{q_i^*,\omega_i}} = \eta_{q_i^*,\omega_i}\,(1+\underbrace{\eta_{p_i,q_i^*}}_{-\frac{1}{\varepsilon}}) = -\frac{1}{\eta_{mr_i,q_i}}\left(1-\frac{1}{\varepsilon}\right)$$
$$= \frac{2-\boldsymbol{\rho}}{\boldsymbol{\varepsilon}-1}\left(\frac{\boldsymbol{\varepsilon}-1}{\boldsymbol{\varepsilon}}\right) = \frac{2-\boldsymbol{\rho}}{\boldsymbol{\varepsilon}}$$

Elasticity of derived factor demand to demand appeal shocks:

$$\eta_{x_i^*,\xi_i} = \frac{d\log(q_i^*)}{d\log(\xi_i)} - \frac{d\log(\omega_i)}{d\log(\xi_i)} = \eta_{q_i^*,\xi_i} - 0 = \frac{\boldsymbol{\varepsilon}-1}{2-\boldsymbol{\rho}}$$

I recover this from the first-order condition and by taking the derivative of $q_i^* = \frac{mc_i-\xi_i p}{\xi_i p_i'}$ w.r.t. ξ_i

$$\frac{\partial q_i^*}{\partial \xi_i} = \frac{\partial}{\partial \xi_i}\left(\frac{mc_i}{\xi_i p_i'}\right) - \frac{\partial}{\partial \xi_i}\left(\frac{p}{\xi_i p_i'}\right) =$$
$$= -\frac{mc}{(\xi_i p_i')^2}\left(p_i' + \xi_i p''\frac{\partial q_i^*}{\partial \xi_i}\right) - \frac{\left(p_i'\frac{\partial q_i^*}{\partial \xi_i}p_i' - p_i p_i''\frac{\partial q_i^*}{\partial \xi_i}\right)}{p_i'^2}\,.$$

Exploiting the fact that $mc_i = \xi_i p_i + \xi_i p_i' q_i^*$ at the optimal level of output q_i^*, it follows that

$$\frac{\partial q_i^*}{\partial \xi_i} = -\frac{\xi_i p + \xi_i p_i' q_i^*}{(\xi_i p_i')^2}\left(p_i' + \xi_i p_i'' \frac{\partial q_i^*}{\partial \xi_i}\right) - \frac{\partial q_i^*}{\partial \xi_i} + \frac{p_i\, p_i''}{p_i'^2}\frac{\partial q_i^*}{\partial \xi_i}$$

Then, divide by $\frac{\partial q_i^*}{\partial \xi_i}$ and simplify so that

$$\begin{aligned} 1 &= \frac{\boldsymbol{\varepsilon}}{\eta_{q_i^*,\xi_i}} - \frac{1}{\eta_{q_i^*,\xi_i}} - \frac{\xi_i\, p_i\, p_i''}{\xi_i\, p_i'^2} - \frac{\xi_i\, q_i^*\, p_i'\, p_i''}{\xi_i\, p_i'^2} - 1 + \frac{p_i}{p_{ii}'^*}\frac{p_{i\,i}''^*}{p_i'} = \frac{\boldsymbol{\varepsilon}-1}{\eta_{q_i^*,\xi_i}} - \boldsymbol{\varepsilon\rho} + \boldsymbol{\rho} - 1 + \boldsymbol{\varepsilon\rho} \\ \iff \eta_{q_i^*,\xi_i} &= \frac{\boldsymbol{\varepsilon}-1}{2-\boldsymbol{\rho}}. \end{aligned}$$

Elasticity of revenue to demand appeal shocks:

$$\eta_{r_i^*,\xi_i} = \frac{d\log(q_i^*)}{d\log(\xi_i)} + \frac{d\log(p_i)}{d\log(\xi_i)} + \frac{d\log(\xi_i)}{d\log(\xi_i)} = \eta_{q_i^*,\xi_i}(1+\eta_{p_i,q_i^*}) + 1 = \frac{\boldsymbol{\varepsilon}-1}{2-\boldsymbol{\rho}}\left(\frac{\boldsymbol{\varepsilon}-1}{\boldsymbol{\varepsilon}}\right) + 1 = \frac{(\boldsymbol{\varepsilon}-1)^2}{\boldsymbol{\varepsilon}(2-\boldsymbol{\rho})} + 1$$

Elasticity of derived factor demand to market size shocks. I recover this from the first-order condition and by taking the derivative of $q_i^* = \frac{\psi_i\left(mc_i - p\left(\frac{q_i^*}{\psi_i}\right)\right)}{p'\left(\frac{q_i^*}{\psi_i}\right)}$ with respect to ψ_i:

$$\frac{\partial q_i^*}{\partial \psi_i} = \frac{mc_i - p\left(\frac{q_i^*}{\psi_i}\right)}{p'\left(\frac{q_i^*}{\psi_i}\right)} + \frac{q_i^*\left(mc_i - p\left(\frac{q_i^*}{\psi_i}\right)\right)p''\left(\frac{q_i^*}{\psi_i}\right)}{\psi_i p'\left(\frac{q_i^*}{\psi_i}\right)^2} + \frac{q_i^*}{\psi_i}$$

Multiplying it by $\frac{\psi_i}{q_i^*}$, after a few rearrangements this simplifies to

$$\eta_{q_i^*,\psi_i} = 2 + \frac{q_i^* p''\left(\frac{q_i^*}{\psi_i}\right)}{\psi_i p'\left(\frac{q_i^*}{\psi_i}\right)} = 2 - \boldsymbol{\rho}$$

Elasticity of revenue to market size shocks:

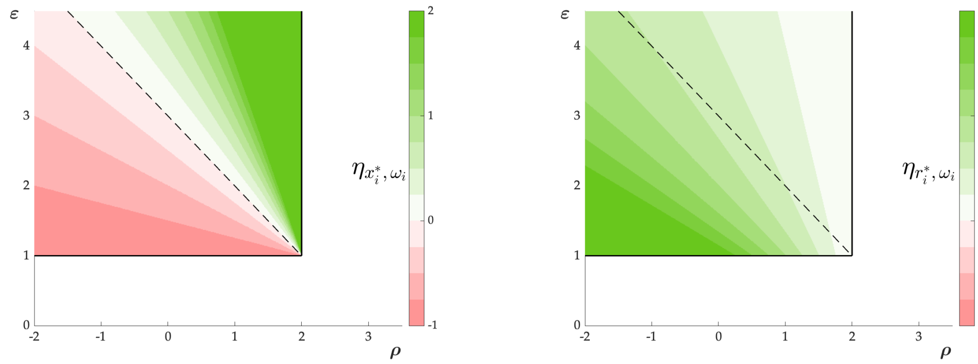
$$\eta_{r_i^*,\psi_i} = \eta_{q_i^*,\psi_i}(1+\eta_{p_i,q_i^*}) = (2-\boldsymbol{\rho})\left(\frac{\boldsymbol{\varepsilon}-1}{\boldsymbol{\varepsilon}}\right) = \frac{(2-\boldsymbol{\rho})(\boldsymbol{\varepsilon}-1)}{\boldsymbol{\varepsilon}}$$

To derive the **elasticities of derived factor demand and revenue to input price shocks** I simply rewrite the equations (25) and (28) in Mrázová and Neary (2020) with the opposite sign and in terms of w_i instead of c_i.

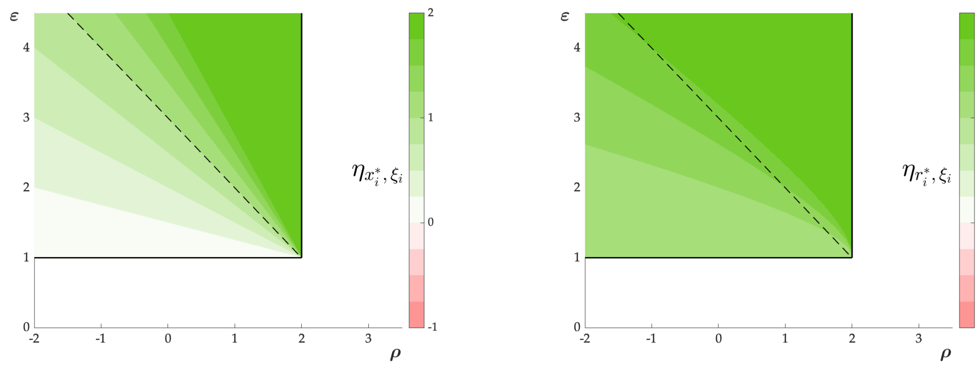
In Figure A9, I illustrate the values of all these elasticities in the manifold framework. As pointed out in Section 3, the only favorable shock that increases revenue but leads to a lower derived factor demand is productivity.

Figure A9. Values of $\eta_{x_i^*,\ldots}$ and $\eta_{r_i^*,\ldots}$ to favourable shocks in the manifold space.

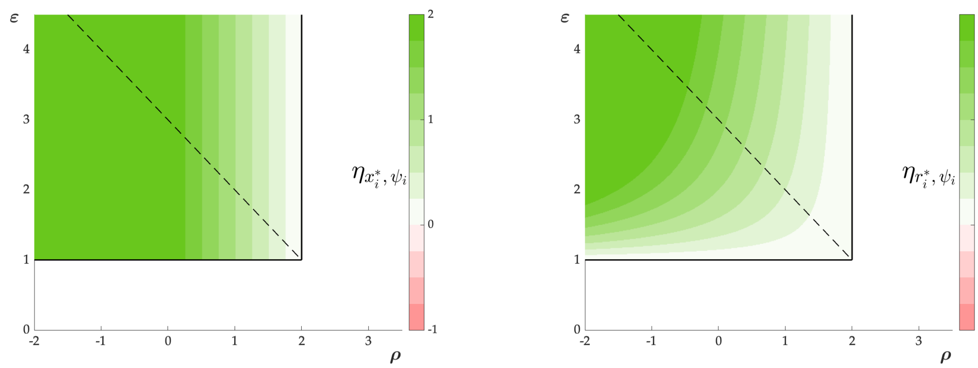
(a) Productivity shock ($\uparrow \omega_i$)



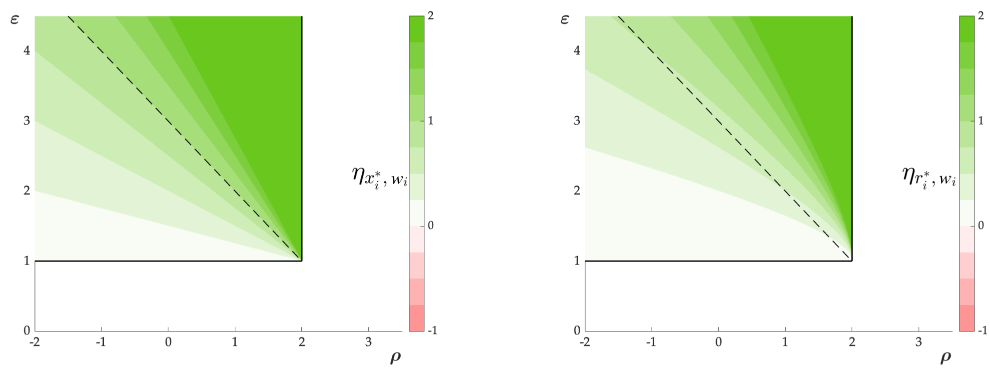
(b) Demand appeal shock ($\uparrow \xi_i$)



(c) Market size shock ($\uparrow \psi_i$)



(d) Input cost shock ($\downarrow w_i$)

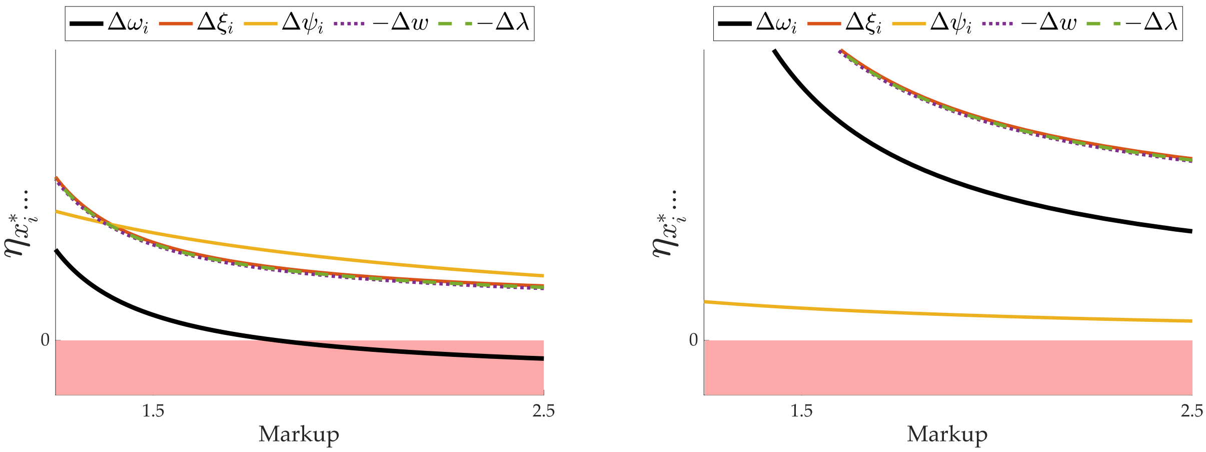


B.1.1 Elasticities for sub-convex and super-convex demands.

To complement the illustration with linear demand in Figure 16, I plot the values of $\eta_{x_i^*,\ldots}$ for different shocks along the markup distribution in the case of the two sub-convex and super-convex demands already considered in Appendix A.2.2.

Figure A10. Values of $\eta_{x_i^*,\ldots}$ for different shocks along the markup distribution.

(a) Sub-convex demand. **(b)** Super-convex demand.



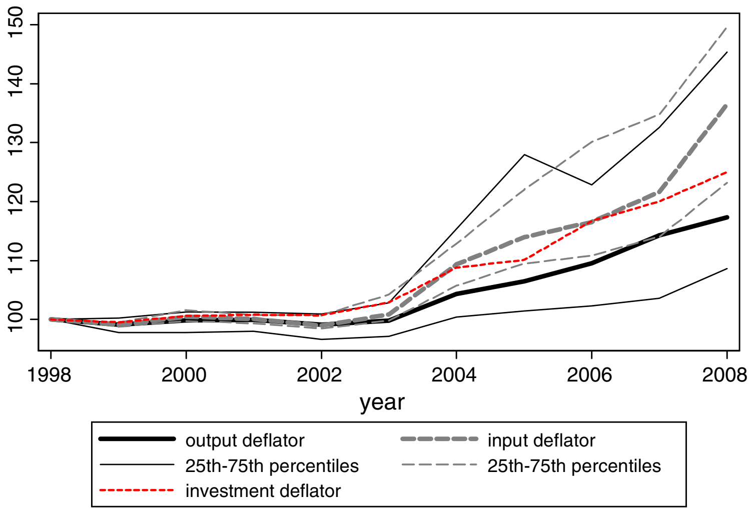
Notes: Constant Proportional Pass-Through (CPPT) demands with (a) 50% and (b) 150% Pass-through rates.

C Additional material of the empirical illustration

C.1 Data

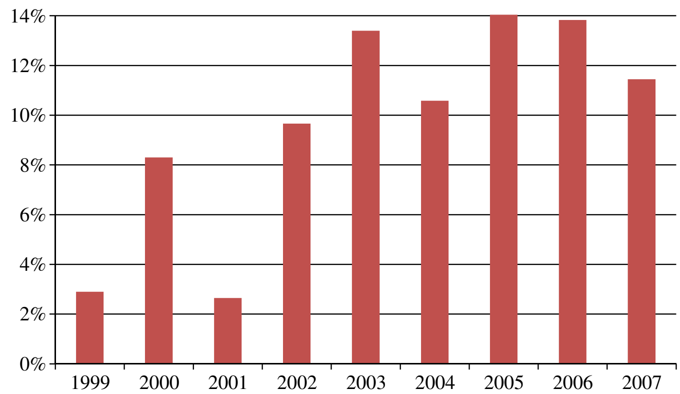
In this section, I report selected summary statistics of the data used in the empirical illustration. Figure A11 shows the evolution over time of price indices used to deflate revenue and input expenditures and in the construction of the real capital stock. While these are available for each 4-digit industry, the lines in the figure report the median values and inter-quartile range across industries. As mentioned in the main text, in the first part of the sample, including the years considered in my analysis (1999-2000), output and input prices have remained relatively stable.

Figure A11. Evolution of output, input, and investment deflators.



Source: Brandt et al. (2014) - Figure 1.

Figure A12. Productivity growth of Chinese manufacturing firms.



Source: Brandt et al. (2012) - Figure 2. Value-added weighted averages of firm-level year-on-year TFP growth estimates for the full unbalanced sample of Chinese manufacturing firms.

Table A3 provides an overview of the data used in the estimation of the output elasticities and the analysis in levels, while Table A4 provides a summary of the data used in the analysis *in changes* after the cleaning and filtering process described in Section 4.1.

Table A3. Summary statistics of data used.

	Mean	St.Dev.	Median	Min	Max	N
1999						
Revenue	37,916	156,909	15,349	4,365	17,926,378	55,717
Employment	295	1,269	148	1	166,857	55,717
Labor costs	2,840	18,195	1,170	1	2,879,647	55,717
Intermediate inputs expenditures	30,970	119,158	12,765	17	12,921,365	55,717
Real capital stock	22,058	317,424	5,139	4	53,578,120	55,717
2000						
Revenue	43,480	184,696	16,972	3,644	21,174,938	55,717
Employment	295	1,209	150	1	161,654	55,717
Labor costs	3,125	19,560	1,272	1	3,408,158	55,717
Intermediate inputs expenditures	34,690	135,933	13,792	1	14,745,580	55,717
Real capital stock	22,865	313,572	5,497	4	48,756,088	55,717

Note: all monetary variables are deflated and reported in thousands of RMB.

Table A4. Summary statistics of variables used in the analyses *in changes*.

1999-200	Mean	St.Dev.	Median	Min	Max	N
μ_{it-1}	1.2	0.2	1.1	1.0	2.3	29,074
$log(\boldsymbol{x}_{it-1})$	9.1	1.0	8.9	3.5	16.1	29,814
μ_{it}	1.2	0.2	1.2	1.0	2.3	29,094
$log(\boldsymbol{x}_{it})$	9.1	1.0	9.0	0.8	16.2	29,814
$\Delta_{t,t-1}log(r_i)$	0.2	0.5	0.1	-0.9	4.6	29,814
$\Delta_{t,t-1}log(l_i)$	0.1	0.4	0.0	-1.0	5.0	29,814
$\Delta_{t,t-1}log(\boldsymbol{x}_i)$	0.1	0.4	0.1	-1.0	5.0	29,814
% firms increasing r_i and μ_i	36%					29,814

C.2 Cost-share *vs.* production function approach.

I estimate the output elasticities with the cost-share approach mainly because the assumptions for the production function approach appear too restrictive in the settings considered in my paper. First, because I am explicitly considering imperfect competitive settings where the derived factor demand is not necessarily monotonic in (scalar Hicks-neutral) productivity shocks. While this is the key assumption for the validity of the control function approach to estimate production functions, the factor share approach is valid under any market structure configuration and output demand. Second, because in the empirical analysis I want to allow for additional sources of heterogeneities from the demand side, on top of productivity. Again, the cost-share approach can accommodate this, but the presence of demand shifters raises a fundamental challenge for the application of the production function approach since it requires including all relevant shifters (coming from either cost, demand, or market structure) in the control function itself. Third, because I am using standard firm-level where output is expressed in monetary values (i.e. revenues) rather than physical quantities. As I consider generic demand functions (without committing to a specific functional form) in which price elasticity of demand can vary with output, the approach proposed Klette and Griliches (1996) and De Loecker (2011) for CES demand is not applicable. Again, the cost-share approach is not affected by the unavailability of output data.

Of course, the cost-share approach is not assumption-free either.[40] As mentioned before, the cost shares can be computed with the available data assuming that (a) each producer statically minimizes cost each period, that (b) all inputs are flexibly adjustable, and (c) technology is characterized by constant returns to scale. Overall, in my paper, the balance of (implicit and explicit) assumptions between the two approaches clearly tilts in favor of the cost-share approach, albeit at the expense of having to assume constant returns to scale. In fact, assumptions (a) and (b) are somehow in line with my theoretical framework where firms are assumed to maximize their profit (hence minimize their costs) in choosing their input levels at every period.[41]

[40] For a more detailed discussion, De Loecker and Syverson (2021) provide a thorough discussion on the scope of application and the maintained assumptions of these two approaches.

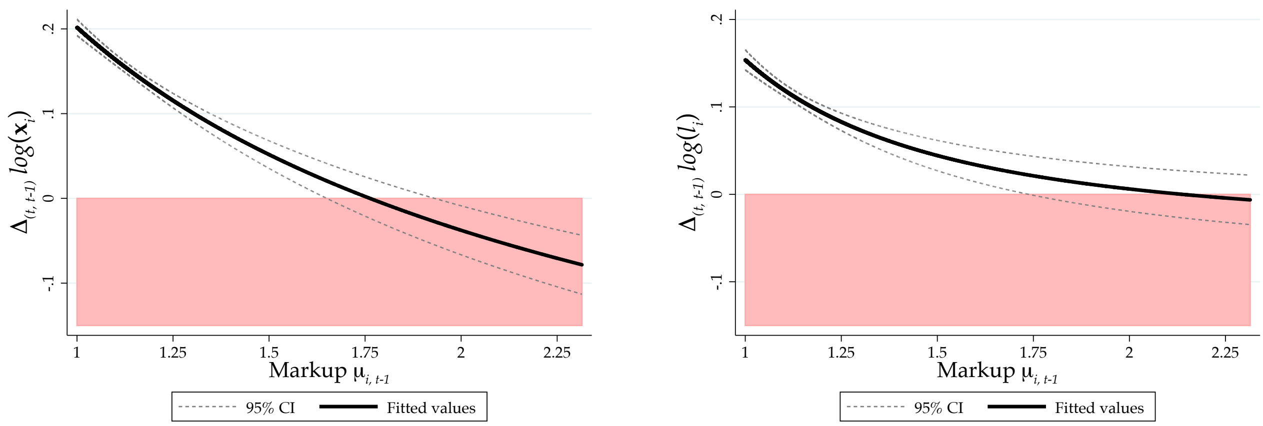
[41] In future work, I am planning to relax assumption (c) on constant returns to scale by using the methodology developed by Syverson (2004).

C.3 Empirical results.

To complement the results reported in Figure A13, I plot below the relationship between input changes and markups estimated with a more flexible fractional polynomial fit.

Figure A13. Input changes over the markup distribution of Chinese firms.

(a) Changes in composite input. **(b)** Changes in employment.

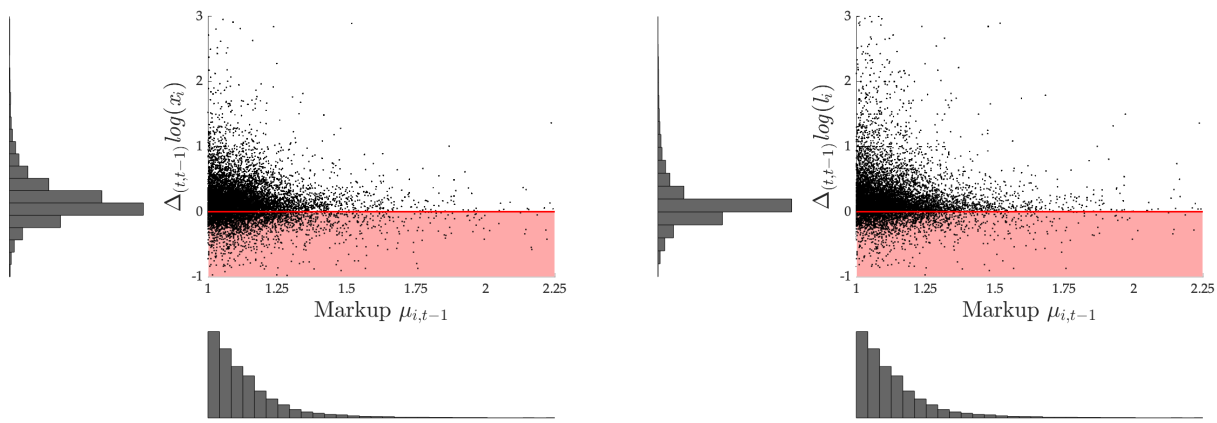


Notes: fitted values are based on a fractional polynomial of degree 2 estimated with the `fpfitci` package in Stata.

In Figure A14 I show the data underlying all my analyses in changes, together with the univariate distribution of each variable.

Figure A14. Overview of the underlying data.

(a) Changes in composite input. **(b)** Changes in employment.



In the long Table A5 that is spread over the following pages, I report for each narrowly-defined industry the estimates of the output elasticities, the number of firms used in the various analyses, and the results of the analyses in levels and in changes.

Table A5. Overview of results by narrowly-defined industry.

Industry CIC code and description	$\hat{\beta}_l$	$\hat{\beta}_m$	$\hat{\beta}_k$	$\boldsymbol{N_t}$	$\boldsymbol{N_\Delta}$	$\boldsymbol{N_{P2}}$	P1 $\boldsymbol{x}_i$	P1 l_i	P2 $\Delta\boldsymbol{x}_i$	P2 Δl_i
0600 - Coal mining and washing	0.22	0.64	0.09	1065	726	369				
0700 - Oil and gas extraction	0.07	0.56	0.37	23	14	4			-	-
0810 - Iron ore mining and dressing	0.07	0.87	0.05	216	125	60			✓	✓
0890 - Mining and dressing of other ferrous metals	0.13	0.74	0.12	23	13	8	✓	✓		
0911 - Copper mining and dressing	0.13	0.69	0.11	40	17	5				
0912 - Lead-zinc mining	0.06	0.87	0.05	135	131	6			✓	✓
0914 - Tin mining and dressing	0.10	0.79	0.08	17	5	1			-	-
0921 - Gold mining and dressing	0.05	0.89	0.05	293	219	125			✓	✓
0931 - Tungsten and molybdenum ore mining and dressing	0.09	0.80	0.09	43	28	1			-	-
1011 - Limestone and gypsum mining	0.07	0.88	0.04	136	89	29			✓	✓
1012 - Exploitation of stone for architectural decoration	0.06	0.91	0.02	147	125	83		✓		
1013 - Refractory earth-rock mining	0.05	0.91	0.03	20	13	6				
1019 - Exploitation of clay and other soil and gravel	0.05	0.91	0.02	162	157	17			✓	✓
1020 - Chemical ore mining and dressing	0.11	0.80	0.07	40	34	25	✓			✓
1030 - Salt mining	0.17	0.62	0.18	70	17	8				✓
1091 - Mining and dressing of asbestos and mica ore	0.13	0.79	0.07	12	2	1			-	-
1092 - Selection of graphite and talc	0.06	0.89	0.03	36	14	11				
1099 - Mining and dressing of other nonmetallic mines	0.06	0.88	0.04	73	33	19				✓
1310 - Grain grinding	0.03	0.93	0.03	752	727	416				✓
1320 - Feed processing	0.03	0.93	0.03	739	717	476				✓
1331 - Processing of edible vegetable oil	0.03	0.91	0.05	451	292	77			✓	✓
1332 - Non-edible vegetable oil processing	0.03	0.94	0.03	11	10	5			✓	
1340 - Sugar refinery	0.09	0.76	0.15	91	27	1			-	-
1351 - Livestock and poultry slaughter	0.03	0.93	0.03	187	187	88			✓	✓
1352 - Processing of meat pdts and by-pdts	0.03	0.93	0.03	418	407	78			✓	
1361 - Processing of frozen aquatic pdts	0.02	0.95	0.02	366	84	42			✓	
1362 - Surimi and dry-cured aquatic pdts processing	0.04	0.92	0.03	88	72	9				
1363 - Aquatic feed manuf.	0.03	0.93	0.04	40	37	8				✓
1364 - Extraction of Fish Oil	0.04	0.88	0.07	13	6	3			-	-
1391 - Manuf. of starch and starch pdts	0.04	0.91	0.04	178	175	13			✓	
1392 - Manuf. of Soybeans pdts	0.10	0.81	0.06	47	11	3			-	-
1393 - Egg processing	0.04	0.94	0.02	16	14	9				
1399 - Other unlisted agricultural and sideline food processing	0.05	0.90	0.04	296	180	93				✓
1411 - Manuf. Of Pastry and bread	0.09	0.82	0.06	80	40	13			✓	
1419 - Manuf. of biscuits and other baked goods	0.06	0.87	0.04	97	77	10		✓	✓	✓
1421 - Manuf. of candy and chocolate	0.07	0.88	0.05	69	37	10			✓	✓
1422 - Manuf. of Candied fruit	0.04	0.91	0.03	66	61	38				
1431 - Manuf. of rice and flour pdts	0.06	0.90	0.04	82	77	51				✓
1432 - Manuf. of frozen food	0.06	0.85	0.07	115	69	34				
1440 - Manuf. of liquid milk and dairy pdts	0.05	0.88	0.06	108	81	51				
1453 - Manuf. of canned vegetables and fruits	0.06	0.88	0.04	113	50	4			-	-

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[Continued from previous page]

Industry CIC code and description	$\hat{\beta}_l$	$\hat{\beta}_m$	$\hat{\beta}_k$	N_t	N_Δ	N_{P2}	P1 x_i	P1 l_i	P2 Δx_i	P2 Δl_i
1459 - Manuf. of other canned food	0.04	0.91	0.03	14	9	9				
1461 - Manuf. of monosodium glutamate	0.07	0.85	0.06	44	29	20				
1462 - Manuf. of soy sauce, vinegar and similar pdts	0.09	0.85	0.06	57	30	2			-	-
1469 - Manuf. of other condiments and fermented pdts	0.05	0.88	0.06	124	86	32				
1492 - Manuf. of frozen drinks and edible ice	0.08	0.82	0.10	73	37	16			✓	✓
1493 - Salt processing	0.11	0.69	0.16	13	2	2			-	-
1494 - Manuf. of food and feed additives	0.05	0.90	0.05	56	49	17			✓	✓
1499 - Manuf. of other unlisted food	0.04	0.91	0.04	125	74	33			✓	
1511 - Manuf. of alcohol	0.06	0.84	0.08	38	9	5		✓		✓
1521 - Manuf. of liquor	0.06	0.87	0.06	418	297	21			✓	✓
1522 - Manuf. of beer	0.09	0.73	0.16	240	91	29			✓	✓
1523 - Manuf. of yellow rice wine	0.08	0.83	0.08	23	11	0		✓	-	-
1524 - Wine production	0.05	0.88	0.05	30	8	7			✓	
1529 - Other wine manuf.	0.04	0.91	0.05	11	4	1			-	-
1531 - Manuf. of carbonated beverage	0.05	0.87	0.08	53	40	9				✓
1532 - Manuf. of bottled (canned) drinking water	0.05	0.84	0.08	46	25	18				
1533 - Manuf. of fruit juice and fruit juice beverage	0.04	0.85	0.10	71	2	0			-	-
1534 - Manuf. of milk beverage and vegetable protein beverage	0.05	0.88	0.06	74	38	9	✓	✓		
1535 - Manuf. of solid beverage	0.05	0.87	0.06	18	13	0			-	-
1540 - Refined tea processing	0.04	0.92	0.03	102	71	7			✓	✓
1610 - Tobacco redrying	0.06	0.84	0.10	55	50	11				
1620 - Manuf. of cigarettes	0.08	0.77	0.13	116	90	11				
1711 - Textile processing of cotton and chemical fiber	0.08	0.86	0.05	1102	453	28			✓	✓
1712 - Finishing printing and dyeing of cotton and chemical fiber	0.06	0.86	0.07	408	142	43				
1721 - Wool processing	0.04	0.92	0.04	24	12	0			-	-
1722 - Wool textile	0.07	0.86	0.05	376	95	12			✓	✓
1723 - Wool dyeing and finishing	0.05	0.89	0.05	37	19	5	✓	✓	✓	
1730 - Linen textile	0.09	0.85	0.05	46	5	1		✓	-	-
1741 - Silk reeling process	0.11	0.82	0.06	190	106	4		✓	-	-
1742 - Silk spinning and silk weaving processing	0.06	0.87	0.06	399	14	0			-	-
1743 - Silk-screen dyeing and finishing	0.08	0.81	0.10	67	18	4			-	-
1751 - Manuf. of cotton and chemical fiber pdts	0.09	0.85	0.04	232	123	21			✓	
1752 - Manuf. of wool pdts	0.08	0.79	0.09	13	8	6				
1753 - Manuf. of hemp pdts	0.09	0.76	0.08	14	10	3	✓	✓	-	-
1754 - Manuf. of silk pdts	0.06	0.87	0.04	37	15	10				✓
1755 - Manuf. of ropes, cables and cables	0.05	0.88	0.05	31	22	3			-	-
1756 - Manuf. of textile belt and cord fabric	0.06	0.88	0.04	69	45	25				
1759 - Manuf. of other textile pdts	0.07	0.87	0.04	314	186	46		✓	✓	
1760 - Manuf. of knitwear, woven goods and accessories	0.08	0.86	0.04	848	336	91				
1810 - Manuf. of textile and garment	0.11	0.85	0.03	3108	1333	519			✓	✓
1820 - Manuf. of textile fabric shoes	0.11	0.85	0.03	214	107	56				✓
1830 - Manuf. of hats	0.14	0.81	0.03	77	36	3			-	-

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Industry CIC code and description	$\hat{\beta}_l$	$\hat{\beta}_m$	$\hat{\beta}_k$	$\boldsymbol{N}_t$	$\boldsymbol{N}_\Delta$	$\boldsymbol{N}_{P2}$	P1 $\boldsymbol{x}_i$	P1 l_i	P2 $\Delta\boldsymbol{x}_i$	P2 Δl_i
1910 - Leather tanning processing	0.03	0.93	0.03	238	122	31		✓	✓	✓
1921 - Manuf. of leather shoes	0.12	0.84	0.03	497	262	39			✓	
1922 - Manuf. of leather garment	0.09	0.87	0.02	142	58	16				
1923 - Manuf. of suitcase and bag	0.11	0.86	0.02	244	161	32			✓	✓
1929 - Manuf. of other leather pdts	0.11	0.86	0.03	129	96	23			✓	
1931 - Fur tanning processing	0.04	0.93	0.03	39	38	2			-	-
1932 - Fur garment processing	0.06	0.91	0.03	25	17	7				
1939 - Processing of other fur pdts	0.04	0.93	0.03	23	18	9				
1941 - Feather (down) processing	0.02	0.96	0.02	48	42	24				✓
1942 - Processing of feather (down) pdts	0.06	0.90	0.03	73	16	6			✓	✓
2011 - Sawn timber processing	0.06	0.91	0.03	83	79	3			-	-
2012 - Wood chip processing	0.04	0.93	0.03	37	29	0			-	-
2021 - Manuf. of plywood	0.06	0.89	0.04	234	112	70				✓
2022 - Manuf. of fibreboard	0.07	0.78	0.13	67	22	1			-	-
2023 - Manuf. of particleboard	0.08	0.82	0.09	35	19	9				✓
2029 - Manuf. of other wood-based panels and timber	0.05	0.91	0.03	90	77	40				✓
2030 - Manuf. of Wood pdts	0.07	0.89	0.03	256	171	60				
2040 - Manuf. of bamboo, rattan, palm and grass pdts	0.07	0.89	0.03	100	97	1			-	-
2110 - Manuf. of Wood furniture	0.07	0.88	0.04	433	304	137			✓	✓
2130 - Metal furniture manuf.	0.07	0.88	0.04	99	91	57				
2190 - Manuf. of other furniture	0.07	0.88	0.03	63	31	2			-	-
2210 - Manuf. of pulp	0.06	0.86	0.06	22	10	3			-	-
2221 - Manuf. of machine-made paper and paperboard	0.07	0.86	0.06	1009	507	87				
2223 - Manuf. of processed paper	0.06	0.89	0.05	76	50	4		✓	-	-
2230 - Manuf. of paper pdts	0.07	0.87	0.05	928	377	136			✓	✓
2311 - Printing of books, newspapers and periodicals	0.14	0.74	0.11	270	138	29				
2312 - Printing industry and reproduction of recording media	0.11	0.82	0.05	23	11	0	✓	✓	-	-
2319 - Packaging, decoration and other printing	0.08	0.83	0.08	374	173	49	✓		✓	✓
2320 - Binding and other printing service activities	0.09	0.83	0.07	206	86	36			✓	✓
2330 - Reproduction of recording medium	0.08	0.70	0.22	18	7	1			-	-
2411 - Manuf. of stationery	0.09	0.86	0.03	59	43	10			✓	✓
2412 - Manuf. of pens	0.10	0.87	0.03	61	8	3			-	-
2421 - Manuf. of balls	0.14	0.83	0.04	37	18	8			✓	✓
2422 - Manuf. of sports equip. and accessories	0.08	0.87	0.04	37	28	7			✓	✓
2423 - Manuf. of training and fitness equip.	0.08	0.88	0.03	69	49	28		✓	✓	✓
2432 - Manuf. of Xi musical instruments	0.12	0.80	0.04	34	5	2			-	-
2440 - Manuf. of toys	0.11	0.85	0.03	463	147	50				
2452 - Manuf. of indoor entertainment equip.	0.09	0.86	0.04	42	21	11				
2511 - Crude oil processing and petroleum pdts manuf.	0.03	0.90	0.05	230	83	42				
2520 - Petroleum coke	0.07	0.83	0.09	153	77	36				
2611 - Manuf. of inorganic acid	0.06	0.86	0.06	83	68	6			✓	✓
2612 - Manuf. of inorganic base	0.07	0.87	0.05	44	14	5				

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Industry CIC code and description	$\hat{\beta}_l$	$\hat{\beta}_m$	$\hat{\beta}_k$	N_t	N_Δ	N_{P2}	P1 x_i	P1 l_i	P2 Δx_i	P2 Δl_i
2613 - Manuf. of inorganic salt	0.06	0.87	0.05	204	128	47				✓
2614 - Manuf. of organic chemical raw materials	0.05	0.90	0.05	218	115	10				✓
2621 - Manuf. of nitrogen fertilizer	0.11	0.73	0.15	105	21	5			✓	✓
2622 - Manuf. of phosphate fertilizer	0.09	0.84	0.06	81	50	24				✓
2623 - Manuf. of potassium fertilizer	0.03	0.92	0.05	14	2	0	✓	✓	-	-
2624 - Manuf. of compound fertilizer	0.04	0.92	0.03	169	163	71				✓
2625 - Manuf. of organic fertilizer and microbial fertilizer	0.04	0.92	0.03	53	46	16				✓
2630 - Manuf. of pesticide	0.04	0.91	0.04	164	160	64				
2641 - Manuf. of paint	0.05	0.90	0.03	349	144	55				
2642 - Manuf. of ink and similar pdts	0.05	0.91	0.04	49	26	10				✓
2643 - Manuf. of pigments	0.06	0.89	0.05	94	51	8	✓	✓	✓	
2644 - Manuf. of dye	0.05	0.91	0.03	137	53	22			✓	✓
2645 - Manuf. of packing and similar pdts for sealing	0.05	0.90	0.05	126	70	34			✓	✓
2651 - Manuf. of plastic and synthetic resin in primary form	0.04	0.91	0.04	155	118	73				
2652 - Manuf. of synthetic rubber	0.04	0.91	0.03	23	22	0			-	-
2653 - Manuf. of single (polymer) body of synthetic fiber	0.04	0.87	0.08	12	6	0			-	-
2659 - Manuf. of other synthetic materials	0.03	0.94	0.03	45	33	16				✓
2661 - Manuf. of chemical reagents and auxiliaries	0.05	0.91	0.04	299	285	83		✓		
2662 - Manuf. of special chemical pdts	0.05	0.90	0.04	248	184	96		✓	✓	✓
2663 - Manuf. of forest chemical pdts	0.05	0.91	0.04	52	52	26			✓	
2664 - Manuf. of explosives and pyrotechnic pdts	0.15	0.74	0.09	81	38	3		✓	-	-
2665 - Manuf. of electronic chemicals	0.07	0.83	0.08	42	26	8				
2666 - Manuf. of pharma. materials for environmental pollution treat.	0.06	0.86	0.06	266	205	104			✓	
2667 - Manuf. of animal glue	0.05	0.91	0.05	17	12	7				
2672 - Manuf. of cosmetic pdts	0.06	0.89	0.05	81	37	17				
2674 - Manuf. of spices and essences	0.03	0.94	0.03	62	40	15		✓		
2679 - Manuf. of other daily chemical pdts	0.08	0.87	0.03	73	56	34				
2710 - Manuf. of chemical raw materials	0.06	0.89	0.05	175	114	56			✓	✓
2720 - Manuf. of chemical preparations	0.08	0.84	0.06	181	67	32			✓	
2730 - Traditional Chinese medicine processing	0.07	0.85	0.06	260	146	26			✓	✓
2750 - Manuf. of veterinary drugs	0.06	0.88	0.05	44	30	17				
2760 - Manuf. of biological and biochemical pdts	0.08	0.85	0.06	57	39	16			✓	
2770 - Manuf. of sanitary materials and medical supplies	0.09	0.88	0.03	72	50	28				✓
2812 - Manuf. of artificial fiber (cellulose fiber)	0.06	0.87	0.06	35	21	12	✓	✓	✓	
2821 - Manuf. of nylon fiber	0.05	0.87	0.07	25	10	1			-	-
2822 - Manuf. of polyester fiber	0.05	0.86	0.09	110	24	8				✓
2829 - Manuf. of other synthetic fibers	0.06	0.86	0.08	56	21	4			-	-
2910 - Manuf. of tire	0.07	0.87	0.06	115	74	55				
2920 - Manuf. of rubber sheet, pipe and belt	0.06	0.89	0.04	99	39	27				✓
2930 - Manuf. of rubber parts	0.09	0.84	0.06	88	48	3			-	-
2940 - Manuf. of recycled rubber	0.06	0.88	0.05	43	23	9			✓	✓
2950 - Manuf. of rubber pdts for daily use and medical use	0.07	0.88	0.03	48	37	23			✓	✓

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Industry CIC code and description	$\hat{\beta}_l$	$\hat{\beta}_m$	$\hat{\beta}_k$	$\boldsymbol{N}_t$	$\boldsymbol{N}_\Delta$	$\boldsymbol{N}_{P2}$	P1 $\boldsymbol{x}_i$	P1 l_i	P2 $\Delta\boldsymbol{x}_i$	P2 Δl_i
2960 - Manuf. of rubber boots and shoes	0.11	0.85	0.03	208	162	9			✓	✓
2990 - Manuf. of other rubber pdts	0.06	0.86	0.05	96	55	23				✓
3010 - Manuf. of plastic film	0.05	0.88	0.06	233	90	57			✓	✓
3020 - Manuf. of plastic plates, pipes and profiles	0.05	0.89	0.05	288	138	72				
3030 - Manuf. of plastic silk, rope and woven goods	0.08	0.87	0.04	388	268	37			✓	✓
3040 - Manuf. of foam plastics	0.05	0.89	0.05	295	177	76				
3060 - Manuf. of plastic packing boxes and containers	0.06	0.86	0.07	185	122	8			✓	✓
3070 - Manuf. of plastic parts	0.08	0.86	0.05	159	81	31				✓
3081 - Manuf. of plastic shoes	0.10	0.86	0.03	136	113	43				
3082 - Manuf. of plastic sundries for daily use	0.07	0.86	0.05	162	106	8			✓	
3090 - Manuf. of other plastic pdts	0.07	0.87	0.05	588	310	154			✓	✓
3111 - Manuf. of cement	0.11	0.73	0.14	2782	179	65			✓	
3112 - Manuf. of lime and gypsum	0.09	0.87	0.04	48	23	10				
3121 - Manuf. of cement pdts	0.07	0.84	0.07	396	178	85				✓
3122 - Manuf. of concrete structural members	0.06	0.90	0.04	171	157	83				
3123 - Manuf. of asbestos cement pdts	0.09	0.87	0.04	33	31	19				
3124 - Manuf. of light building materials	0.05	0.91	0.03	73	46	26				
3131 - Manuf. of clay bricks and building blocks	0.11	0.84	0.04	411	285	107			✓	✓
3133 - Processing of building stone	0.05	0.90	0.03	254	151	95				
3134 - Manuf. of waterproof building materials	0.06	0.92	0.02	69	67	21			✓	
3135 - Manuf. of heat and sound insulation materials	0.06	0.90	0.05	50	17	10				
3139 - Manuf. of other building materials	0.08	0.85	0.07	39	20	0			-	-
3141 - Manuf. of flat glass	0.08	0.79	0.10	129	13	7				
3143 - Manuf. of optical glass	0.09	0.85	0.06	28	21	12				
3145 - Manuf. of glass pdts and packaging containers for daily use	0.13	0.80	0.06	230	31	16			✓	✓
3146 - Manuf. of glass insulation container	0.14	0.80	0.06	24	23	4			-	-
3147 - Manuf. of glass fiber and pdts	0.08	0.87	0.04	48	34	8				
3148 - Manuf. of glass fiber reinforced plastic pdts	0.06	0.89	0.03	110	66	34				
3149 - Manuf. of other glass pdts	0.07	0.87	0.04	63	51	1			-	-
3151 - Manuf. of sanitary ceramic pdts	0.08	0.77	0.12	386	62	13				✓
3152 - Manuf. of special ceramic pdts	0.11	0.83	0.05	47	16	4		✓	-	-
3153 - Manuf. of ceramic pdts for daily use	0.17	0.75	0.06	295	103	24				
3159 - Manuf. of garden, display art and other ceramic pdts	0.14	0.78	0.06	25	10	2			-	-
3161 - Manuf. of asbestos pdts	0.03	0.94	0.02	27	12	7				
3162 - Manuf. of mica pdts	0.07	0.85	0.06	14	4	1			-	-
3169 - Manuf. of refractory ceramic pdts	0.06	0.88	0.05	284	277	34				✓
3191 - Manuf. of graphite and carbon pdts	0.07	0.85	0.07	91	46	32		✓		
3199 - Manuf. of other nonmetallic mineral pdts	0.06	0.89	0.04	273	218	87			✓	✓
3210 - Ironmaking	0.06	0.88	0.05	362	217	56				✓
3220 - Steelmaking	0.04	0.90	0.04	84	74	13			✓	✓
3230 - Rolling processing of steel	0.04	0.92	0.03	656	324	130			✓	
3240 - Ferroalloy smelting	0.07	0.87	0.06	210	190	99				

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3311 - Copper smelting	0.04	0.93	0.03	62	61	30		✓		✓
3312 - Lead and zinc smelting	0.03	0.94	0.03	94	84	34				✓
3315 - Antimony smelting	0.06	0.88	0.05	14	10	2	✓	✓	-	-
3316 - Aluminum smelting	0.03	0.91	0.05	108	26	15			✓	
3317 - Magnesium smelting	0.07	0.86	0.05	47	34	26		✓		
3319 - Smelting of other commonly used nonferrous metals	0.06	0.89	0.04	45	26	5			✓	✓
3321 - Gold smelting	0.09	0.79	0.08	36	25	16				
3329 - Smelting of other precious metals	0.05	0.91	0.04	43	31	9				✓
3331 - Tungsten and molybdenum smelting	0.03	0.93	0.04	21	19	4		✓	-	-
3340 - Manuf. of nonferrous metal alloy	0.03	0.93	0.02	28	23	12				
3351 - Rolling processing of common non-ferrous metals	0.04	0.92	0.04	387	129	66				
3411 - Manuf. of metal structures	0.07	0.88	0.04	294	155	73			✓	
3412 - Manuf. of metal doors and windows	0.07	0.88	0.04	146	93	18			✓	
3421 - Manuf. of cutting tools	0.09	0.85	0.04	85	36	24				
3422 - Manuf. of hand tool	0.09	0.85	0.04	107	33	16			✓	✓
3423 - Manuf. of metal tools for agriculture and gardens	0.07	0.89	0.03	34	25	12			✓	✓
3424 - Manuf. of knives, scissors and similar daily metal tools	0.09	0.85	0.03	52	47	14				
3429 - Manuf. of other metal tools	0.08	0.88	0.03	24	9	5				
3431 - Manuf. of container	0.05	0.91	0.04	28	17	6				✓
3432 - Manuf. of metal pressure vessel	0.06	0.87	0.05	271	123	50			✓	✓
3440 - Manuf. of metal wire rope and its pdts	0.05	0.91	0.03	309	232	93				✓
3451 - Manuf. of metal fittings for buildings and furniture	0.09	0.87	0.03	152	79	31				
3452 - Manuf. of building decoration and plumbing pipe parts	0.08	0.87	0.03	164	96	33				
3453 - Manuf. of metal pdts for safety and fire protection	0.07	0.88	0.04	235	139	65				
3459 - Manuf. of metal pdts for other buildings and safety	0.05	0.90	0.04	59	19	11				
3460 - Metal surface treatment and heat treatment	0.07	0.88	0.04	219	174	62				
3470 - Manuf. of enamel pdts	0.10	0.83	0.04	46	21	6				
3481 - Manuf. of metal kitchen conditioning and sanitary ware	0.06	0.89	0.04	382	272	124				✓
3489 - Manuf. of other daily metal pdts	0.08	0.87	0.04	151	66	18				
3511 - Manuf. of boiler and auxiliary equip.	0.09	0.84	0.06	86	36	19				✓
3512 - Manuf. of internal combustion engine and accessories	0.10	0.83	0.06	132	68	23				✓
3519 - Manuf. of other prime mover	0.13	0.80	0.05	14	8	2			-	-
3521 - Manuf. of metal cutting machine tools	0.12	0.80	0.07	81	15	10			✓	✓
3522 - Metal forming machine tool manuf.	0.11	0.81	0.07	48	8	3			-	-
3523 - Manuf. of foundry machinery	0.09	0.85	0.04	31	23	0			-	-
3524 - Manuf. of metal cutting and welding equip.	0.09	0.86	0.06	20	9	4			-	-
3525 - Manuf. of machine tool accessories	0.10	0.82	0.06	29	13	2			-	-
3529 - Manuf. of other metal processing machinery	0.08	0.86	0.05	81	24	1			-	-
3530 - Manuf. of lifting and transportation equip.	0.09	0.85	0.06	143	57	5				
3541 - Manuf. of pumps and vacuum equip.	0.09	0.86	0.05	138	64	41			✓	
3542 - Manuf. of gas compression machinery	0.09	0.87	0.05	26	14	5				
3543 - Manuf. of valves and cocks	0.09	0.86	0.04	193	90	46				

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3544 - Manuf. of hydraulic and pneumatic power machinery	0.09	0.85	0.04	106	80	39			✓	✓
3551 - Manuf. of bearing	0.11	0.82	0.06	245	95	45			✓	✓
3552 - Manuf. of gears, transmission and driving parts	0.10	0.84	0.05	105	45	21			✓	
3571 - Manuf. of fans	0.11	0.81	0.07	53	11	6				
3573 - Manuf. of refrigeration and air-conditioning equip.	0.09	0.86	0.04	58	23	10				
3574 - Manuf. of pneumatic and electric tools	0.06	0.91	0.03	88	62	27			✓	✓
3575 - Manuf. of spray guns and similar appliances	0.09	0.84	0.05	152	78	47			✓	✓
3576 - Manuf. of special packaging equip.	0.09	0.84	0.07	19	3	1			-	-
3577 - Manuf. of weighing instruments	0.09	0.85	0.06	23	16	7				
3581 - Manuf. of metal seals	0.10	0.84	0.05	33	23	15				✓
3582 - Manuf. of fasteners and springs	0.08	0.87	0.04	217	111	51				✓
3583 - Mechanical parts processing and equip. repair	0.18	0.73	0.05	35	11	3			-	-
3589 - Manuf. of other general parts	0.08	0.87	0.04	123	98	32			✓	✓
3591 - Manuf. of steel castings	0.08	0.88	0.03	646	437	185				✓
3592 - Manuf. of forgings and powder metallurgy pdts	0.08	0.87	0.05	165	55	25				
3611 - Manuf. of mining and quarrying equip.	0.10	0.83	0.06	67	40	23				
3612 - Manuf. of special equip. for oil drilling and production	0.08	0.86	0.05	100	57	21			✓	
3615 - Manuf. of metallurgical special equip.	0.11	0.82	0.06	29	16	9				✓
3621 - Manuf. of special equip. for oil refining and chemical prod.	0.08	0.86	0.05	53	28	12			✓	✓
3622 - Manuf. of special equip. for rubber processing	0.14	0.80	0.05	16	6	2			-	-
3623 - Manuf. of special equip. for plastic processing	0.07	0.88	0.04	59	18	13				✓
3625 - Die making	0.10	0.82	0.07	115	80	48			✓	
3629 - Manuf. of other special equip. for nonmetallic processing	0.09	0.85	0.07	47	42	2			-	-
3631 - Manuf. of special equip. for food, beverage and tobacco ind.	0.09	0.86	0.05	50	18	6				
3632 - Manuf. of special equip. for agricultural processing	0.10	0.85	0.06	54	24	7			✓	
3641 - Manuf. of special equip. for pulping and papermaking	0.08	0.87	0.05	31	11	9				✓
3642 - Manuf. of special printing equip.	0.10	0.83	0.06	27	16	10				✓
3644 - Manuf. of pharmaceutical special equip.	0.08	0.88	0.05	14	4	1		✓	-	-
3645 - Manuf. of special equip. for lighting appliance production	0.08	0.86	0.05	11	6	0	✓		-	-
3651 - Manuf. of textile special equip.	0.09	0.86	0.04	146	75	24				✓
3662 - Manuf. of special equip. for electronic industry	0.10	0.81	0.08	23	9	5				✓
3669 - Manuf. of aviation, aerospace and other special equip.	0.06	0.88	0.05	26	14	9			✓	✓
3671 - Manuf. of tractor	0.07	0.87	0.05	36	6	2			-	-
3672 - Manuf. of mechanized agricultural and horticultural machinery	0.07	0.87	0.04	76	36	8			✓	✓
3673 - Manuf. of silviculture and bamboo harvesting machinery	0.05	0.91	0.03	25	8	4			-	-
3676 - Manuf. of AFAhF machinery parts	0.09	0.87	0.04	106	48	1			-	-
3679 - Manuf. of other AFAhF machinery and machinery repair	0.06	0.91	0.02	92	36	6				
3681 - Manuf. of medical diagnosis, monitoring and treatment equip.	0.11	0.84	0.05	31	17	4			-	-
3684 - Manuf. of medical, surgical and veterinary instruments	0.11	0.85	0.05	31	13	9			✓	
3691 - Manuf. of special equip. for environ. pollution prevention	0.07	0.89	0.04	32	11	1			-	-
3695 - Manuf. of social public safety equip. and equip.	0.09	0.87	0.04	58	50	4			-	-
3699 - Manuf. of other special equip.	0.09	0.87	0.04	53	37	11	✓	✓		✓

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Industry CIC code and description	$\hat{\beta}_l$	$\hat{\beta}_m$	$\hat{\beta}_k$	$\boldsymbol{N}_t$	$\boldsymbol{N}_\Delta$	$\boldsymbol{N}_{P2}$	P1 $\boldsymbol{x}_i$	P1 l_i	P2 $\Delta\boldsymbol{x}_i$	P2 Δl_i
3713 - Manuf. of railway locomotive and rolling stock parts	0.19	0.73	0.06	43	19	7				
3714 - Manuf. of special railway equip., equip. and accessories	0.18	0.77	0.06	19	9	3			-	-
3719 - Other railway equip. manuf. and equip. repair	0.26	0.70	0.04	20	12	3			-	-
3721 - Manuf. of automobile whole vehicles	0.06	0.89	0.05	38	13	8		✓		✓
3722 - Manuf. of refit automobile	0.07	0.89	0.04	68	51	2			-	-
3724 - Manuf. of automobile body and trailer	0.08	0.88	0.03	18	13	4			-	-
3725 - Automobile parts and accessories manuf.	0.09	0.84	0.06	798	288	138			✓	
3726 - Vehicle repair	0.11	0.80	0.07	269	192	38	✓	✓	✓	
3731 - Manuf. of motorcycle whole vehicle	0.03	0.95	0.03	73	47	2			-	-
3732 - Manuf. of motorcycle parts and accessories	0.07	0.88	0.04	283	180	60				
3740 - Manuf. of bicycle	0.06	0.90	0.03	140	68	5				
3751 - Metal shipbuilding	0.08	0.88	0.04	76	28	6				
3754 - Manuf. of marine supporting equip.	0.10	0.78	0.08	15	1	0			-	-
3755 - Ship repair and ship breaking	0.13	0.72	0.07	66	23	5				
3761 - Aircraft manuf. and repair	0.16	0.70	0.14	12	4	1			-	-
3792 - Manuf. of metal signs and facilities for traffic management	0.06	0.91	0.03	13	8	6				
3799 - Manuf. of other transportation equip.	0.10	0.83	0.07	13	8	3	✓	✓	-	-
3911 - Manuf. of generators and generator sets	0.08	0.87	0.04	41	14	6				
3912 - Manuf. of electric motor	0.11	0.83	0.05	114	44	32				
3919 - Manuf. of micro-motors and other motors	0.09	0.87	0.04	94	25	18				
3921 - Manuf. of transformers, rectifiers and inductors	0.09	0.85	0.05	184	97	40			✓	✓
3922 - Manuf. of capacitor and its supporting equip.	0.09	0.84	0.07	42	20	12				✓
3923 - Manuf. of distribution switch control equip.	0.08	0.87	0.04	207	98	33		✓		
3924 - Manuf. of power electronic components	0.08	0.85	0.05	95	88	48			✓	✓
3929 - Manuf. of other power transmission and control equip.	0.07	0.88	0.04	124	46	6				
3931 - Manuf. of wire and cable	0.04	0.92	0.04	722	514	189			✓	✓
3933 - Manuf. of insulation pdts	0.05	0.90	0.05	65	32	23				
3939 - Manuf. of other electrical equip.	0.05	0.90	0.04	73	44	9				
3940 - Manuf. of batteries	0.09	0.85	0.05	152	69	33				
3951 - Manuf. of household refrigeration appliances	0.07	0.86	0.05	25	10	4			-	-
3952 - Manuf. of domestic air conditioner	0.05	0.90	0.04	53	22	10				✓
3953 - Manuf. of household ventilation electrical appliances	0.06	0.89	0.04	51	19	6				
3955 - Manuf. of household cleaning and sanitary electr. appliances	0.07	0.88	0.04	35	18	5		✓		
3956 - Manuf. of household beauty and health care appliances	0.07	0.88	0.03	150	68	24			✓	
3961 - Manuf. of gas, solar energy and similar energy appliances	0.06	0.90	0.03	42	22	1			-	-
3969 - Manuf. of other non-electric household appliances	0.09	0.87	0.03	43	19	13				✓
3971 - Manuf. of electric light source	0.10	0.83	0.04	101	42	17			✓	✓
3972 - Manuf. of lighting fixtures	0.08	0.88	0.03	170	104	49				✓
3979 - Manuf. of lamp accessories and other lighting appliances	0.09	0.87	0.03	103	74	18				✓
3990 - Manuf. of other electrical machinery and equip.	0.10	0.85	0.04	40	22	13				✓
4011 - Manuf. of communication transmission equip.	0.08	0.88	0.03	177	110	46				✓
4013 - Manuf. of communication terminal equip.	0.10	0.85	0.04	30	14	4			-	-

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Industry CIC code and description	$\hat{\beta}_l$	$\hat{\beta}_m$	$\hat{\beta}_k$	$\boldsymbol{N}_t$	$\boldsymbol{N}_\Delta$	$\boldsymbol{N}_{P2}$	P1 $\boldsymbol{x}_i$	P1 l_i	P2 $\Delta\boldsymbol{x}_i$	P2 Δl_i
4031 - Manuf. of radio and TV production and transmission equip.	0.11	0.85	0.03	20	16	0			-	-
4041 - Manuf. of complete electronic computer	0.04	0.93	0.02	26	21	10				
4042 - Manuf. of computer network equip.	0.06	0.88	0.04	90	42	22				
4051 - Manuf. of electronic vacuum device	0.05	0.86	0.08	33	16	9				
4052 - Manuf. of semiconductor discrete devices	0.09	0.82	0.06	59	28	2			-	-
4053 - Manuf. of integrated circuits	0.08	0.86	0.05	71	32	3			-	-
4060 - Manuf. of electronic parts	0.10	0.83	0.05	562	482	24			✓	
4071 - Manuf. of household video equip.	0.04	0.93	0.02	45	28	13				
4072 - Manuf. of household audio equip.	0.09	0.87	0.03	87	63	15			✓	
4090 - Manuf. of other electronic equip.	0.08	0.86	0.03	125	82	24				
4111 - Manuf. of industrial automatic control system device	0.09	0.86	0.05	43	39	24				
4112 - Manuf. of electrical instruments	0.07	0.88	0.04	36	34	2			-	-
4113 - Manuf. of drawing, calculating and measuring instruments	0.11	0.85	0.02	14	5	3			-	-
4119 - Manuf. of supply instruments and other general instruments	0.08	0.87	0.03	37	18	12				
4128 - Manuf. of electronic measuring instruments	0.09	0.84	0.04	11	5	4			-	-
4129 - Manuf. of other special instruments	0.06	0.86	0.05	14	5	0			-	-
4130 - Manuf. of clocks and timing instruments	0.11	0.84	0.04	106	63	5				
4141 - Manuf. of optical instruments	0.08	0.89	0.03	39	37	22				
4142 - Manuf. of glasses	0.12	0.82	0.05	92	49	33				✓
4153 - Manuf. of cameras and equip.	0.10	0.85	0.03	25	9	2			-	-
4154 - Manuf. of copying and offset printing equip.	0.04	0.88	0.05	12	10	7				
4155 - Manuf. of special equip. for calculator and currency	0.08	0.86	0.05	16	7	2	✓		-	-
4159 - Manuf. of other cultures, office machinery	0.06	0.84	0.06	11	5	0			-	-
4190 - Manuf. and repair of other instruments and meters	0.12	0.86	0.03	20	12	8			✓	✓
4211 - Manuf. of sculpture handicraft	0.13	0.82	0.03	175	132	8			✓	✓
4212 - Manuf. of metal handicraft	0.13	0.82	0.03	41	25	5				✓
4214 - Manuf. of huahua artware	0.10	0.86	0.03	70	42	13				
4215 - Manuf. of natural plant fiber woven handicrafts	0.08	0.89	0.02	121	118	40			✓	
4216 - Manuf. of embroidery handicraft	0.08	0.87	0.03	137	62	19			✓	✓
4217 - Manuf. of carpets and tapestries	0.09	0.86	0.04	131	55	14			✓	
4218 - Manuf. of jewelry and related articles	0.05	0.92	0.02	98	89	34			✓	✓
4219 - Manuf. of other arts and crafts	0.10	0.86	0.02	356	328	96			✓	
4222 - Manuf. of mane processing, brush making and cleaning tools	0.07	0.90	0.02	69	30	17				
4229 - Manuf. of other daily sundries	0.09	0.88	0.02	80	42	7				
4290 - Other manuf. industries not listed	0.06	0.89	0.04	233	191	97			✓	✓

Notes: description based on the Chinese Industry Classification (CIC) adjusted by Brandt et al. (2014) and translated from Chinese to English with GoogleTranslate. AFAhF stands for "Agriculture, Forestry, Animal husbandry and Fishery". $\hat{\beta}_l$, $\hat{\beta}_m$ and $\hat{\beta}_k$ are the estimated output elasticities for each industry. $\boldsymbol{N}_t$ denotes the total number of firms in my sample, while $\boldsymbol{N}_\Delta$ the number of firms with markups greater than 1 and complete data in changes. Finally, $\boldsymbol{N}_{P2}$ reports the number of firms that grew in revenue and markups between 1999-2000 which I use in the analysis based on Prediction 2. The columns P1 $\boldsymbol{x}_i$ and P1 $l_i$report whether in the industry I find a non-monotonic relationship between the input (composite or employment) and revenue. This is indicated by the symbol ✓, while "-" means that the industry has not been tested at all (too few observations). The last two columns report whether predicted input changes are found negative at the highest 5ile of lagged markups.

C.3.1 Robustness checks.

In this section, I consider the robustness of the main results reported in Section 4.3. In particular, the results of the logit regressions which are shown in columns (1) and (4) of Table 3 and the results of the industry by industry in Table 4. I report the outcome of these additional analyses in Table A6. To improve readability, I report the results as % of total industries tested since the total slightly varies across the different checks.

While the baseline results are based on a linear fit, I re-estimate the relationship between the input changes and lagged markups industry by industry with *(a)* a fractional polynomial fit and *(b)* a lowess (locally weighted scatterplot smoothing) fit. Both of them provide very similar results. Then, I check whether the level at which the cost shares are aggregated and smoothed-out influences the results since this may affect my estimates of the output elasticities, and thus the composite and markups. In *(c)* I take the median cost share at the industry-province level to account for potential geographic differences in input prices. In *(d)*, I group the firms by ownership category (state owned, hybrid or collective, private, and two types of foreign firms) so that output elasticities can vary by industry and by ownership type.[42] In (e) I bring this logic to the extreme and smooth out the cost shares over time at the firm level so that the output elasticities reflect the mean cost shares over the two years. Overall, all these alternative ways of estimating the output elasticities do not affect significantly neither the estimated average effect of lag markups on input reductions nor the % of industries where I find evidence of negative predicted input changes at the highest quintile of markups. Then, I consider the role State-Owned Enterprise (SOE) in the Chinese economy. While Chen, Igami, Sawada, and Xiao (2021) provide a detailed discussion and nuanced defense of the profit maximization assumption for Chinese SOEs, they may optimize a different objective function. For this reason, in *(f)* I drop all SOEs firms (14% of firms in my sample) to check whether they are driving somehow the results. To rule out possible effects of SOEs privatization that occurred in the same period, in *(g)* I drop SOEs that were privatized between 1999 and 2000 (3.9% of firms in my sample). Again, this does not change much the results. Finally, in *(h)* I include firms with $\mu_i < 1$. In the main analysis, I do not consider those firms because a markup lower than one can not be rationalized with profit maximization. However, including them does not affect the main results.

[42] The ownership can be identified using the variable (*qiye dengji zhuce leixing*) which provides information about the firm's registered type. It distinguishes 23 exhaustive ownership types, which include joint ventures between different types of owners. I follow the classification proposed by Brandt et al. (2012) to classify each of these types into the following five basic groups: state-owned, hybrid or collective, private, and two types of foreign firms, those from Hong-Kong, Macau, and Taiwan and those from all other countries.

Table A6. Results of robustness checks.

	Estimated average M.E. Logit		At highest 5ile of μ_{it-1}	
	$Prob(\Delta \boldsymbol{x}_i < 0)$	$Prob(\Delta l_i < 0)$	$\Delta \boldsymbol{x}_i < 0$	$\Delta l_i < 0$
Baseline	**0.28*** **	**0.12*** **	**38%**	**45%**
(a) Fractional polynomial fit	-	-	40%	45%
(b) Lowess fit	-	-	41%	45%
(c) Median at (industry-province) level	0.27***	0.14***	35%	45%
(d) Median at (industry-ownership) level	0.26***	0.11***	35%	44%
(e) Mean at firm-level	0.18***	0.25***	22%	36%
(f) Without any SOEs	0.25***	0.13***	35%	42%
(g) Without SOEs privatized over 1999-2000	0.29***	0.10***	37%	41%
(h) Also with firms with $\mu_i < 1$	0.25***	0.05***	40%	43%