# On Excessive Entry in Bayes-Cournot Oligopoly*

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**Abstract**

In a Cournot industry where firms are privately informed about their marginal costs and must sink a setup cost to enter the market, entry restrictions increase expected output, entrants' profits, total welfare and, in some cases, also benefit consumers. Under Bayes-Cournot competition, firms react to the expectation (conditional on entry) of rivals' costs rather than to their actual costs. This creates scope for entry by relatively inefficient types. Entry restrictions, preventing these high-cost types from entering, increase inframarginal (lower-cost) types' and rivals' expected output. As a result, they increase profits and, unless reduce output variability too much, also consumer surplus.

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# 1 Introduction

The relationship between entry barriers in an industry, competition, and consumer and social welfare has been much debated. Bain (1956) summarized the post-war presumption that such barriers were harmful because they distorted competition. This view was challenged by von Weizsäcker (1980), who argued that entry barriers could be beneficial in the presence of economies of scale or incomplete information about product quality. Mankiw and Whinston (1986) examined systematically the conditions under which there can be excessive entry in oligopoly markets. They showed that, in Cournot games with setup costs, a free entry equilibrium results in excessive entry compared to the social optimum.

In the Mankiw-Whinston's setting, entry is excessive because of a business-stealing effect: the entry of an additional firm reduces the profitability of incumbents. Therefore, while entry barriers can increase total welfare, they always reduce consumer surplus. In this paper, we develop the Mankiw-Whinston approach to examine the impact of entry restrictions on output, consumer surplus, and total welfare in environments where firms have incomplete information on their rivals' costs. There are good reasons to think that assuming private information about costs will often be more plausible than assuming complete information, including the high demand for benchmarking of costs in many industries (Vives, 2002) and antitrust authorities' prohibitions against information exchange between firms.[1] The existence and significance of entrants' private information has also been confirmed by recent empirical work on the estimation of entry games (Grieco, 2014; Magnolfi and Roncoroni, 2022).[2]

We consider a linear-quadratic Bayes-Cournot game where firms are privately informed about their random marginal costs of production and must sink a setup cost to enter the market. In this setting, we outline a new mechanism through which entry barriers affect welfare. With incomplete information, low entry barriers lead to an adverse selection of firms entering the market: some high-cost firms enter that would not enter in a complete information world. The possibility of entry by such high-cost firms can reduce expected output, and thus harm consumer and total welfare.

Building on this simple mechanism, we show the following results. First, entry restrictions are required to maximize entrants' and industry expected output. Second, in expectation entrants themselves prefer to face regulations mandating some entry restrictions. Third, consumer surplus maximization may require entry restrictions. Therefore, the presumption that entry restrictions enhance total welfare only at the expense of lower consumer surplus is not necessarily true in the Bayes-Cournot model as opposed to the standard Cournot game.

The trade-offs shaping these results are easily illustrated in the simplest scenario where only

[1] A large literature examines the competitive and welfare effects of cost-related information sharing in oligopoly: see, e.g., Li (1985) and Shapiro (1986).

[2] Grieco (2014) shows that part of firms' observed heterogeneity comes from private information about marginal costs, contracts with suppliers and managerial ability, which they possess when making their entry decisions. Omitting private information from the model can lead to qualitatively different results. Magnolfi and Roncoroni (2022) propose a method to estimate static discrete games with weak assumptions on the information available to players. They estimate a static entry game, showing that the more restrictive complete information model may lead to underestimation of how much players' profits are affected by the presence of competitors.

one entrant faces competition from one or more incumbents that have already sunk their setup costs. In this game, facilitating entry has two effects on the entrant's expected output. On the one hand, it increases the probability of entry, thus increasing the entrant's expected output. On the other hand, reducing the setup cost raises the incumbents' (conditional) expectation of the entrant's marginal cost of production. This leads incumbents to produce a higher quantity if they face entry, and so leads the entrant to produce less.

The optimal setup cost that maximizes the entrant's expected output balances these two effects. Maximizing the entrant's expected output thus requires a positive level of setup cost, and so some restrictions on entry. The reason why the first effect does not always dominate the second can be seen by considering the limiting case where entry requires no setup costs. In this scenario, the marginal entrant type produces zero, so its contribution to expected output is nil. Yet, its presence in the market imposes a negative externality on the production of inframarginal (lower-cost) entrant types. This is because, in our Bayes-Cournot setting, the incumbents' aggregate output falls as entry barriers increase, as it does not depend on the entrant's actual cost but on its conditional expectation. As a result, increasing the setup cost slightly above zero has a positive first-order effect on the entrant's expected output. It reduces the incumbents' output for all inframarginal types, thereby increasing the production of these types while losing the marginal type that does not produce.

By a similar logic, entry restrictions are also required to maximize expected industry output. In the Bayes-Cournot model, aggregate output falls post-entry if the entrant is relatively inefficient. In that case, it does not produce much, but its entry causes a disproportionate drop in rivals' aggregate output, given that they respond to the entrant's expected cost and not to its actual cost. This reaction is absent in the standard Cournot analysis and is the main difference between our setting and Mankiw-Whinston's analysis.

The standard business-stealing logic implies that incumbents are always in favor of entry restrictions. Yet, surprisingly, the entrant also benefits in expectation from market access restrictions. Although reducing the probability of entry, and making entry more costly, a restrictive entry policy is de facto equivalent to an efficiency commitment, thereby inducing more accommodating behavior by the incumbents upon entry.

As for consumer surplus, consumers are risk lovers: their expected utility is convex in aggregate output (Waugh, 1944). Therefore, in addition to valuing expected industry output, they also value output volatility, which increases as the range of potential entrant's costs rises. Hence, entry restrictions have two types of impact on consumer surplus: a positive effect by increasing expected industry output; and a negative effect by reducing expected output volatility. On the net, consumer welfare maximization does require entry barriers so long as, in expected terms, incumbents are relatively efficient compared with the entrant. Conversely, total welfare is decreasing in the variance of industry output. Hence, net of setup costs, total welfare maximization requires restricting entry to increase expected output and reduce variance. Since firms are asymmetric, minimization of production costs requires more efficient firms to sell more, which again calls for entry restrictions.

These results remain qualitatively true with product differentiation and when entry may trigger exit by an incumbent (creative destruction). Moreover, they extend to the more general case, closer to Mankiw and Whinston (1986), where all firms are potential entrants: entry restrictions can still increase expected output, total welfare, and, in a narrower range of parameters, consumer surplus.

To sum up, our analysis provides the first attempt to examine entry regulation in a Bayes-Cournot model. Its novel message is that the presumption that entry barriers reduce expected output and consumer surplus is unwarranted when firms are privately informed about their costs. Although derived in a quantity-setting oligopoly game, our study has broader implications. It applies to all instances of strategic interactions where players must sink a setup cost to enter a game of strategic substitutes, and the entry decision signals relevant private information. This could include settings as varied as patent races (Gans and Stern, 2000), political protests (Cantoni et al., 2019), and local tax competition (Parchet, 2019).

The paper is organized as follows. After reviewing the related literature, we set up the model and derive the Bayes-Cournot equilibrium (for given entry decisions) in Section 2. In Section 3 we consider only one potential entrant. Section 4 extends the analysis to the case where the entry decision is endogenous for all firms. Section 5 concludes. All proofs and additional material are in the Appendix.

**Contribution to the literature.** There is a range of contrasting results in the literature on the relationship between entry restrictions and total welfare. In the presence of product differentiation, free entry can result in a socially inadequate number of firms in Spence (1976) and Dixit and Stiglitz (1977). In contrast, it always entails excessive entry in Salop (1979).[3]

Mankiw and Whinston (1986) find an excessive entry result for any positive setup cost. Entry by any firm induces a business-stealing effect on its rivals, which is not internalized in equilibrium. This implies that firms always have an excessive incentive to entry from the perspective of total welfare. Raising entry barriers can increase welfare, though at the expense of consumer surplus. In a similar spirit, von Weizsäcker (1980) shows that, in the presence of economies of scale or product differentiation, welfare could be improved by increasing the protection of incumbents (see also Perry, 1984, and Suzumura and Kiyono, 1987). Follow-up investigations mostly confirm this prediction's robustness: e.g., for comprehensive surveys, see Etro (2014) and Polo (2018).[4]

There has been some examination of the robustness of the excess-entry result under various sources of uncertainty. Creane (2007) considers an exogenous 'failure' probability with which a firm, upon sinking the entry cost, cannot produce: insufficient entry may occur if this probability is high enough. Silvers (2018) supposes that entrants only observe a signal of their actual entry cost and finds that the expected number of excess entrants is minimal if this signal is uninformative. In Deo and Corbett (2009), the production process is characterized by yield

[3]However, Gu and Wenzel (2009) find that the excess-entry theorem in Salop (1979) only holds when demand elasticity is sufficiently small.

[4]However, Ghosh and Morita (2007) find that the excess-entry result can be overturned in models of vertical relations. The increase in total surplus due to upstream entry is in part captured in the profits of the downstream sector. This "business-creation effect" to the downstream sector is not internalized in the upstream entry decision, and may outweigh the standard business-stealing effect in the upstream sector, leading to insufficient entry.

uncertainty, and there is insufficient entry if output variance is large enough. Uncertain production costs, which are unknown to everyone at the entry stage but common knowledge before production choices, exacerbate the excess-entry distortion in De Pinto and Goerke (2021). In all these papers, excessive or insufficient entry is always defined from a total welfare standpoint and, in contrast to our results, consumers unambiguously benefit from more entry.

Espín-Sánchez and Parra (2018) provide a sufficient condition for uniqueness of equilibrium in a general class of entry games with entrants' private information.[5] Their model allows for equilibrium outcomes in which firms self-select when entering the market, but entrants' information becomes public before firms compete. Thus, none of these papers feature private information for firms at the market interaction stage; the entry decision has no signaling value for competitors.

The selection effect of entry costs and their competitive implications have been widely analyzed in auction theory. In Samuelson (1985), bidders make their (costly) entry decisions after learning their valuations. In contrast to the case with no entry costs, expected procurement cost (resp. auctioneer's revenue) need not decline (resp. increase) with the number of potential bidders. Thus, in line with our results, policies to limit the number of bidders may be welfare-improving. In a similar setting, Menezes and Monteiro (2000) show that it is optimal for the seller to charge an entry fee and characterize the revenue-maximizing auction. More recent works by Gentry and Li (2014) and Bhattacharya et al. (2014) analyze auctions with selective entry where bidders have imperfect information on their values before entry and provide a nonparametric identification framework and an empirical application, respectively. In this literature, the objective function is the auctioneer's revenue rather than the bidders' surplus.

## 2 Model

**Setup.** Consider a set of $N \geq 2$ potential entrants (denoted by $i = 1, \ldots, N$) in an industry with Cournot competition and characterized by linear inverse demand[6]

$$P(Q) \triangleq \max\{0, 1 - Q\},$$

with $Q \triangleq \sum_{i=1}^{N} q_i$ denoting industry output.[7] Conditional on entering the market, each firm $i$ produces at a (constant) marginal cost $c_i$. Costs are ex-ante uncertain: each $c_i$ is drawn from a bounded support $[0, \bar{c}_i]$ according to a smooth cdf $F_i(c_i)$ with density $f_i(c_i)$, and mean $\mathbb{E}[c_i] \triangleq \tilde{c}_i$. Cost realizations are independent across firms. Each potential entrant (say firm $i$) privately observes its cost realization $c_i$ before deciding whether to sink a setup cost $k_i \geq 0$ to enter the market. Formally, firm $i$'s entry decision can be represented by a binary choice variable $x_i(c_i) \in \{0, 1\}$, taking value 1 if firm $i$ enters, and 0 otherwise. The level of setup costs is common knowledge, whereas private information about production costs is still present at the

[5] Other contributions examine how entrants' private information affects their location choices (Jovanovic, 1981), or the timing of entry under signaling concerns (Kolb, 2015).

[6] Following the literature on asymmetric information in oligopoly (e.g., Vives, 1999), we focus on a linear-quadratic framework, which allows us to obtain closed-form solutions in the Bayes-Cournot game.

[7] We restrict attention throughout to strictly positive prices: a sufficient condition for $Q \in (0, 1)$ in equilibrium is stated in Appendix A.

market competition stage — i.e., when firms set quantities, they are still uncertain about their rivals' marginal costs of production.

The timing of the game is as follows:

$t = 1$ Upon observing the realization of $c_i$, each firm $i$ decides whether to sink the setup cost $k_i \geq 0$ to enter the market;

$t = 2$ Entry decisions are observed, and firms active in the market engage in Bayes-Cournot competition.

The solution concept is Perfect Bayesian Equilibrium (PBE). A PBE of this game is a collection of: $(i)$ a tuple of entry and output strategies $\{(x_i(c_i))_{i=1,..,N}, (q_i(c_i, x_{-i}))_{i=1,..,N}\}$, where $x_{-i} \triangleq (x_1, ..., x_{i-1}, x_{i+1}, ..., x_N)$ is the vector of the entry decisions of firm $i$'s rivals, with the convention that $q_i(c_i, 0) = 0$; $(ii)$ for each entrant $i$, the rivals' beliefs on its cost, given its entry decision, summarized by a posterior cdf $F_i(c_i|x_i)$.[8] The strategy profile must be sequentially rational given beliefs, and beliefs must be compatible with the strategy profile — i.e., obtained by Bayes rule whenever possible. That is, each entrant optimally chooses output given the beliefs on rivals' costs, and these beliefs are obtained using Bayes rule given their entry strategies.[9]

All firms are risk neutral (i.e., maximize expected profits). As a tie-breaking condition, we posit that each firm decides not to enter the market in case of indifference.[10]

**Equilibrium.** We solve the model by backward induction. To start with, we characterize the Bayes-Cournot equilibrium for given entry decisions. Every entrant (say firm $i$, with $x_i = 1$) solves

$$\max_{q_i \geq 0} \left[ P\left( q_i + \sum_{j \neq i} q_j^e(x_j) \right) - c_i \right] q_i,$$

with $q_j^e(x_j) \triangleq \mathbb{E}[q_j(c_j, x_j)|x_j]$ being the quantity that firm $i$ expects firm $j$ to produce conditional on its entry decision $x_j \in \{0, 1\}$. Differentiating with respect to $q_i$, the first-order condition solving the above maximization problem immediately yields firm $i$'s best-response function

$$q_i^*(c_i, x_{-i}) \triangleq \frac{1}{2} \left[ 1 - c_i - \sum_{j \neq i} q_j^e(x_j) \right]. \tag{1}$$

Denoting by $n$ the number of entrants, given that $q_j^e(0) = 0$, in equilibrium we have

$$\sum_{j \neq i} q_j^e(x_j) = \frac{(n-1)(1 + \hat{c}_i) - 2 \sum_{j \neq i : x_j = 1} \hat{c}_j}{n+1}, \tag{2}$$

[8] Since firms' costs are independent, all firm $i$'s rivals have the same posterior beliefs on its cost (see Fudenberg and Tirole, 1991).

[9] We consider throughout the interesting cases where any firm enters with positive probability. Hence, there are no relevant off-path events in the game, and so no need to specify off-path beliefs.

[10] This assumption rules out the unrealistic scenario in which, if entry is unrestricted (i.e., entry costs are zero), high-cost firms enter but produce zero quantity.

where

$$\hat{c}_i \triangleq \mathbb{E}[c_i | x_i = 1] = \int_{\{c_i : x_i(c_i)=1\}} c_i dF(c_i | x_i = 1)$$

denotes the conditional expectation of firm $i$'s cost from its rivals' standpoint, conditional on the fact that this firm decided to enter the market (the same definition applies to $\hat{c}_j$ for every other active firm $j$).

From (1) we then have that, for given rivals' entry decisions, firm $i$'s expected profit after entry is

$$\pi_i^*(c_i, x_{-i}) \triangleq \left[ P\left( q_i^*(c_i, x_{-i}) + \sum_{j \neq i} q_j^e(x_j) \right) - c_i \right] q_i^*(c_i, x_{-i}) = q_i^*(c_i, x_{-i})^2. \tag{3}$$

Each firm $i$ enters the market if and only if its expected profit exceeds the setup cost. Formally,

$$\mathbb{E}[\pi_i^*(c_i, x_{-i})] > k,$$

where the expectation is taken with respect to rivals' entry decisions (i.e., realizations of the vector $x_{-i}$, given the rivals' entry strategies).

The equilibrium of the entry stage is the solution of a fixed-point problem. On the one hand, each firm $i$'s entry strategy depends on its rivals' belief $\hat{c}_i$ because its expected profit is a function of its output $q_i^*(\cdot)$, which, from (2), depends on $\hat{c}_i$. On the other hand, this rivals' belief must be compatible with firm $i$'s entry strategy.

However, as $q_i^*(\cdot)$ is decreasing in $c_i$ for all $x_{-i}$, also the expected profit falls with $c_i$. This implies that, in any candidate PBE of the game, there are cutoffs $(c_i^*)_{i=1,\ldots,N}$ such that $x_i^*(c_i) = 1$ if and only if $c_i < c_i^*$. Hence, using Bayes rule, rivals' beliefs on firm $i$'s costs, conditional on its entry, are such that

$$\hat{c}_i\left(c_i^*\right) \triangleq \mathbb{E}[c_i | c_i < c_i^*], \tag{4}$$

where, with a slight abuse of notation, we introduced the notation $\hat{c}_i(c_i^*)$ to make explicit the link between the beliefs $\hat{c}_i$ and the equilibrium cutoff $c_i^*$: firm $i$'s rivals anticipate that this firm enters if and only if it is relatively efficient, and so accordingly update their beliefs about its average cost upon observing firm $i$'s entry. The entry decision thus has a signaling component, which will be key for our results. This is because firm $i$'s rivals respond to its expected (conditional on entry) cost, not to its actual cost, which remains private information.

In the remainder of the paper we analyze the impact of restricting entry by raising firms' setup costs. In particular, we shall consider how increasing setup costs changes (expected) output, profits, consumer surplus and total welfare. In Section 3, we consider a simplified version of the model (baseline) in which there is only one entrant and $N - 1$ incumbents. In Section 4 we turn to the more general case in which all $N$ firms decide whether or not to enter.

## 3 The game with only one potential entrant

Suppose that $N-1$ firms (hereafter, incumbents) are already in the market — i.e., they either face no entry costs, or their entry costs are sunk — and that each always produces strictly positive output.[11] Hence, the entry decision is endogenous for one firm only (say firm $i$). Assume that this decision is influenced by a regulator through the choice of firm $i$'s entry cost $k_i$. To make the entry problem interesting for all $k_i \geq 0$, we assume that the support of firm $i$'s cost distribution is sufficiently wide that it may choose not to enter the market even if its setup cost $k_i$ is zero.[12]

Under these assumptions, for all incumbents $j \neq i$, their expected costs are equal to the unconditional means: $\hat{c}_j = \tilde{c}_j$. In other words, as the incumbents always operate in the market, their presence does not signal anything about their (privately observed) costs.

### 3.1 Entry decision and equilibrium

As entry requires a setup cost $k_i \geq 0$, and firm $i$ always faces competition by its $N-1$ rivals, it enters the market if and only if $\pi_i^*(c_i, \mathbf{1}) > k_i$, with $\pi_i^*(\cdot)$ being the expected profit (conditional on $c_i$) defined in (3) for $x_{-i} = (1, \ldots, 1) \triangleq \mathbf{1}$. The equilibrium cutoff $c_i^*$ is then obtained as a solution of

$$q_i^*(c_i^*, \mathbf{1}) = \sqrt{k_i}, \tag{5}$$

where the rivals' common belief $\hat{c}_i(c_i^*)$ about firm $i$'s cost is given by (4).

The link between $k_i$ and $c_i^*$ and, therefore, the link with the belief $\hat{c}_i(c_i^*)$ held by firm $i$'s rivals, hinges on the fact that the setup cost $k_i$ is common knowledge. Hence, since in equilibrium there is a one-to-one mapping between $k_i$ and $c_i^*$ (see below), any change in $k_i$ also affects $\hat{c}_i(\cdot)$ — i.e., the expectation of firm $i$'s rivals about its marginal cost — through the impact of the setup cost on firm $i$'s equilibrium strategy.

Since the entry decision is endogenous for firm $i$ only, to ease notation we omit the subscript $i$ and simply write $c_i^* \triangleq c^*$, $\hat{c}_i(c_i^*) \triangleq \hat{c}(c^*)$ and $k_i \triangleq k$. From the above analysis, the equilibrium characterization of this game can be summarized as follows:[13]

**Lemma 1.** *Let $c^*$ be the unique solution of*

$$\frac{2(1 + \sum_{j \neq i} \tilde{c}_j) - (N-1)\hat{c}(c^*)}{N+1} - c^* = 2\sqrt{k}, \tag{6}$$

*with $c^*$ being decreasing in $k$. Then, for any given $k$, the equilibrium of the game is as follows:*

[11] Appendix A details the restrictions on cost distributions that are necessary to guarantee that the $N-1$ incumbents produce positive output.

[12] That is, there exists a value $c_i' \in (0, \bar{c}_i)$ such that $q_i^*(c_i', \mathbf{1}) = 0$. Note that, as firm $i$ would never enter the market if it would then optimally produce zero output, no further restrictions on its cost distribution need to be imposed to guarantee that, conditional on entry, its output is strictly positive — i.e., $q_i(\cdot) > 0$.

[13] Since from the entrant's viewpoint $x_{-i} = \mathbf{1}$ with probability one, in what follows the vector of rivals' entry decisions is omitted from the expressions of firms' output strategies, and firm $i$'s entry cutoff $c^*$ appears as the second argument of all firms' optimal quantities conditional on firm $i$'s entry.

- *For every $c_i < c^*$, firm $i$ enters the market — i.e., $x^*(c_i) = 1$ — and produces*

$$q_i^*(c_i, c^*) = \frac{1}{2}\left[1 - c_i - \frac{(N-1)(1+\hat{c}(c^*)) - 2\sum_{j\neq i}\tilde{c}_j}{N+1}\right]. \tag{7}$$

*Every incumbent $j$ with cost $c_j$ produces*

$$q_j^*(c_j, c^*) = \frac{1}{2}\left[1 - c_j - \frac{(N-1)(1+\tilde{c}_j) - 2\left(\hat{c}(c^*) + \sum_{h\neq\{i,j\}}\tilde{c}_h\right)}{N+1}\right]. \tag{8}$$

- *For every $c_i \geq c^*$, firm $i$ does not enter — i.e., $x^*(c_i) = 0$ — and every incumbent $j$ with cost $c_j$ produces*

$$q_j^*(c_j) = \frac{1}{2}\left[1 - c_j - \frac{(N-2)(1+\tilde{c}_j) - 2\sum_{h\neq j}\tilde{c}_h}{N}\right]. \tag{9}$$

The setup cost affects the equilibrium outcome of the game only through its impact on the entrant's cutoff $c^*$. As this cost increases, firm $i$ is less likely to enter — i.e., $c^*$ falls with $k$. However, the cutoff $c^*$ also affects equilibrium outputs conditional on entry, through its effect on the rivals' expectation $\hat{c}(c^*)$. Specifically, from equation (4) we have

$$\frac{\partial \hat{c}(c^*)}{\partial c^*} = \frac{f_i(c^*)}{F_i(c^*)}[c^* - \hat{c}(c^*)] > 0. \tag{10}$$

In words, if entry becomes relatively harder — i.e., $k$ increases, and hence $c^*$ drops — the entrant is, on average, more efficient. The incumbents anticipate, therefore, that the entrant will produce relatively more, and react by reducing their outputs. By strategic substitutability, this in turn allows the entrant (for any given true cost $c_i$) to expand its output. Formally, we have

$$\frac{\partial q_i^*(\cdot)}{\partial \hat{c}} = -\frac{N-1}{2(N+1)} < 0 < \frac{\partial q_j^*(\cdot)}{\partial \hat{c}} = \frac{1}{N+1}.$$

Aggregating over the incumbents, it follows that

$$(N-1)\frac{\partial q_j(\cdot)}{\partial \hat{c}} > \left|\frac{\partial q_i(\cdot)}{\partial \hat{c}}\right|.$$

This inequality implies that the aggregate response by the incumbents is larger in magnitude than that of the entrant. At first glance, this may suggest that restricting entry by increasing $k$, and hence reducing $c^*$, cannot increase aggregate output, as it reduces aggregate output conditional on entry and also reduces the probability of entry. This conclusion is unwarranted, though. The reason is that, as we shall see below, the aggregate quantity may fall post-entry if the entrant is relatively inefficient.

### 3.2 Expected output and profits

In this section, we examine how changes in the set-up cost $k$, or equivalently changes of the cutoff $c^*$, affect firm $i$'s expected output and aggregate output. We show that both outputs are maximized at a strictly positive set-up cost: equivalently, the cutoff $c^*$ that maximizes expected aggregate output is lower than the cutoff under unrestricted entry — i.e., the cutoff $c_0^*$ that solves (6) for $k = 0$, or equivalently such that the entrant's quantity in (7) equals zero: $q_i^*(c_0^*, c_0^*) = 0$.

**Expected entrant's output.** We consider first the impact of restricting entry on the entrant's expected output. Since there is a monotonic relationship between $c^*$ and $k$, we can simply maximize firm $i$'s individual output with respect to $c^*$, and then recover $k$ from the equilibrium entry condition (6).

From (7) we have that, for given entry cutoff $c^*$, firm $i$'s expected output is

$$q_i^*(c^*) \triangleq F_i(c^*)\mathbb{E}[q_i^*(c_i, c^*) | c_i < c^*] = F_i(c^*) \frac{1 - N\hat{c}(c^*) + \sum_{j \neq i} \tilde{c}_j}{N+1}.$$

Differentiating $q_i^*(c^*)$ with respect to $c^*$, the first-order condition for an optimum requiring entry to be feasible and restricted — i.e., $c^* \in (0, c_0^*)$ — is

$$\underbrace{f_i(c^*) \frac{1 - N\hat{c}(c^*) + \sum_{j \neq i} \tilde{c}_j}{N+1}}_{\text{Output enhancing effect}} - \underbrace{F_i(c^*) \frac{N}{N+1} \frac{\partial \hat{c}(c^*)}{\partial c^*}}_{\text{Selection effect}} = 0. \tag{11}$$

Equation (11) shows that facilitating entry — i.e., increasing $c^*$ — has two effects on the entrant's expected output. First, it increases the probability that firm $i$ enters, and therefore increases the likelihood of producing: a standard output-enhancing effect. Second, a higher cutoff $c^*$ increases the incumbents' conditional expectation on the entrant's cost — i.e., $\hat{c}(c^*)$ raises with $c^*$ as seen in (10) — thereby making their output contraction post-entry relatively lower, which in turn reduces the entrant's output: a selection effect.

Clearly, whether restricting entry — i.e., setting a $c^* < c_0^*$ — maximizes firm $i$'s expected output depends on the sign of the above first-order condition at $c^* = c_0^*$. We can show the following:

**Proposition 1.** *Restricting entry increases the entrant's expected output. The cost cutoff that maximizes $q_i^*(c^*)$ is*

$$c^* = p^\dagger \triangleq \frac{1 + \sum_{j \neq i} \tilde{c}_j}{N} \in (0, c_0^*),$$

*with $p^\dagger$ being the expected price prevailing when only the $N-1$ incumbents operate in the market (i.e., in the absence of entry). The level of setup cost $k^*$ that implements the cutoff $c^* = p^\dagger$ is such that*

$$\sqrt{k^*} = \frac{(N-1)(p^\dagger - \hat{c}(p^\dagger))}{2(N+1)} > 0.$$

This proposition shows that restricting entry is necessary to maximize firm $i$'s expected output — i.e., the cutoff $c^* = p^\dagger$ that maximizes $q_i^*(c^*)$ is lower than the marginal type $c_0^*$ when entry is unrestricted. The intuition is as follows. Allowing type $c_0^*$ to enter the market, as implied by $k = 0$, does not contribute to firm $i$'s expected output since $q_i^*(c_0^*, c_0^*) = 0$. Yet, the presence of this knife-edge type imposes a negative externality on the inframarginal types — i.e., all the more efficient types $c_i < c_0^*$. This is because the incumbents' output does not depend on firm $i$'s actual cost but on their conditional expectation $\hat{c}(c^*)$. This expectation is maximal under unrestricted entry (i.e., at $c^* = c_0^*$). As a result, increasing $k$ slightly above zero (equivalently, setting $c^*$ slightly below $c_0^*$) has a first-order positive effect on the entrant's expected output because, quantities being strategic substitutes, it contracts the incumbents' output for all inframarginal entrant types, thereby increasing the output produced by these types.

**Remark (R&D investments).** The result that raising entry barriers increases the entrant's expected output has important implications for its incentives to invest in cost-reducing (or, equivalently, demand-enhancing) innovations. This is because in Cournot models the marginal revenue from these investments is proportional to the firm's output. To see this formally, suppose that, before observing the cost realization, firm $i$ can invest in R&D activities to shift downward the distribution of its marginal costs. These investments are observed by its rivals (so that all firms still hold a common prior on the entrant's cost). For simplicity, say that the investment $I$ is such that $\frac{\partial c_i}{\partial I} = -1$, and hence $\frac{\partial \hat{c}(c^*)}{\partial I} = -1$. Then, (7) implies

$$\frac{\partial q_i^*(c_i, c^*)}{\partial I} = \frac{1}{2}\left[1 + \frac{N-1}{N+1}\right] = \frac{N}{N+1}.$$

In words, the direct effect of an innovation is reinforced by a strategic effect of a fraction $(N-1)/(N+1)$.

This condition immediately delivers the familiar result that the incentive to innovate via a cost-reducing process innovation is proportional to output. Formally, differentiating firm $i$'s ex-ante expected profit with respect to $I$, given that investments do not affect the entry cutoff, yields

$$\frac{\partial F(c^*)\mathbb{E}[\pi_i^*(c_i, N)|c_i < c^*]}{\partial I} = F(c^*)\mathbb{E}\left[\frac{\partial q_i^*(c_i, c^*)^2}{\partial I}\middle| c_i < c^*\right] = \frac{2N}{N+1} q_i^*(c^*).$$

Setting a strictly positive entry cost such that $c^* = p^\dagger$, by maximizing the entrant's ex-ante expected output $q_i^*(c^*)$, also maximizes the amount of investments undertaken by the entrant.

**Expected industry output.** We now consider the impact of restricting entry on aggregate output. In addition to affecting the extent of rivalry (i.e., the number of firms in the industry) and the entrant's expected output, restricting entry also impacts the incumbents' aggregate output via their expectations on the entrant's output.

The expected aggregate output is

$$Q^*(c^*) \triangleq F_i(c^*)Q_N^*(c^*) + (1 - F_i(c^*))Q_{N-1}^*,$$

where

$$Q_N^*(c^*) \triangleq \frac{N - \hat{c}(c^*) - \sum_{j \neq i} \tilde{c}_j}{N+1}$$

and

$$Q_{N-1}^* \triangleq \frac{N - 1 - \sum_{j \neq i} \tilde{c}_j}{N},$$

are the expected values of industry output, conditional on firm $i$'s entry decision, obtained by aggregating the individual outputs derived in Lemma 1. Clearly, the lower the expectation $\hat{c}(c^*)$, the larger expected aggregate output, because the entrant is more efficient — i.e., the industry becomes, on average, more efficient.

By the same logic developed above, differentiating $Q^*(c^*)$ with respect to $c^*$ we have the following first-order condition for an interior optimum:

$$\underbrace{f_i(c^*)\left(Q_N^*(c^*) - Q_{N-1}^*\right)}_{\text{Entry enhancing effect}} - \underbrace{\frac{F_i(c^*)}{N+1}\frac{\partial \hat{c}(c^*)}{\partial c^*}}_{\text{Selection effect}} = 0. \tag{12}$$

The two effects highlighted in the above first-order condition are rather intuitive. First, facilitating entry leads to a change in aggregate output purely due to the fact that there are more states of nature in which firm $i$ enters — i.e., there is a change in rivalry as entry becomes more likely. Second, the selection effect implies that aggregate output conditional on entry falls in expected terms as the entrant becomes on average less efficient.

Notice, however, that the entry enhancing effect does not necessarily lead to an increase in aggregate output. That is, the difference

$$Q_N^*(c^*) - Q_{N-1}^* = \frac{p^\dagger - \hat{c}(c^*)}{N+1}$$

is not necessarily positive, as its sign depends on the difference between the price $p^\dagger$ that would prevail in the absence of entry and the entrant's average cost conditional on entry.

Substituting (10) into (12), it is easy to obtain that

$$\frac{\partial Q^*(c^*)}{\partial c^*} = \frac{1}{N}\frac{\partial q_i^*(c^*)}{\partial c^*},$$

which directly implies that the above first-order condition has a unique solution at $c^* = p^\dagger$. We can thus state the following.

**Proposition 2.** *Maximizing expected industry output is equivalent to maximizing the entrant's expected output. Hence, the cutoff that maximizes expected industry output is* $c^* = p^\dagger < c_0^*$.

Under unrestricted entry ($k = 0$), firm $i$ would enter as long as it produces positive, even negligible, output. Hence, the marginal entrant type $c_0^*$ does not contribute to industry output. However, under Bayes-Cournot competition, its entry causes a drop in rivals' output, given that quantities are strategic substitutes and incumbents respond to the entrant's average cost $\hat{c}(c_0^*) < c_0^*$, rather than to its actual cost. Thus, unlike in a Cournot model under complete information, here restricting entry by setting $k^* > 0$ increases expected industry output.

**Remark (linear demand).** The mechanism whereby incumbents contract output based on their expectations of the entrant's cost, which implies that restricting entry raises both the entrant's and aggregate expected outputs, is rather general. Yet, the result that the cutoff that maximizes the entrant's expected quantity is equal to the level that maximizes industry output, and coincides with the (expected) price that would prevail in the absence of entry, is driven by the linear demand specification.

The reason is that, under linear demand, the entrant's expected output as well as aggregate output under Bayes-Cournot are the same as the corresponding expected outputs in a model where costs become common knowledge after entry (i.e., the standard Cournot equilibrium is played). In this scenario, entry always increases aggregate output and, absent entry restrictions, the entrant participates so long as it produces a positive quantity, which requires its cost to be lower than the pre-entry price: $c_i < p^\dagger$. While $c^* = p^\dagger$ is, accordingly, the cutoff that maximizes expected (individual and aggregate) output regardless of whether or not firms' costs are common knowledge at the production stage, implementing this cutoff requires raising entry barriers in the Bayes-Cournot model. This is because, under private information, firm $i$'s rivals react to the average type $\hat{c}(p^\dagger) < p^\dagger$, and hence produce less than under complete information. This allows type $c_i = p^\dagger$ to make a strictly positive profit in the Bayes-Cournot model, which must be offset through the choice of a positive setup cost.

Figure 1 provides an illustration for the case with only one incumbent ($N = 2$). Absent entry, the incumbent produces the monopoly quantity $q^m$, with associated equilibrium price $p^\dagger$. But, when it faces an entrant with expected reaction function $q_i^e(c^*)$, it restricts its output to $q_j'$ instead. Anticipating that the incumbent will reduce its output even when faced with an entrant that is relatively inefficient, a type $c_i = p^\dagger$ will enter and play along the reaction function $q_i^*(p^\dagger, c^*)$. The entrant will therefore produce $q_i'$. In this case, $q_i' + q_j' = q^m$. That is, aggregate industry output with an entrant type with cost $c_i = p^\dagger$ is identical to industry output with a monopolist without threat of entry. Aggregate industry output is lower for any higher-cost entrant types: $q_i + q_j' < q^m$ for any $c_i > p^\dagger$.

**Expected profits.** We now examine the link between entry restrictions and firms' profits. The results are intuitive on the basis of the effects described above.

**Proposition 3.** *Incumbents are always harmed by entry. Firm i's expected profit net of the setup cost is maximized at a cutoff $c_\pi^* < c_0^*$ — i.e., the entrant benefits from facing entry restrictions.*

A simple business-stealing logic explains why the incumbents benefit from fully restricting entry — i.e., from setting $c^* = 0$. More surprisingly, the entrant itself benefits from the imposition of entry restrictions — i.e., setup costs that foreclose entry of relatively inefficient types. Although reducing the probability of entry, by making it more expensive, a restrictive entry policy is de facto equivalent to a commitment for the entrant to be efficient, thereby inducing more accommodating behavior by the incumbents upon entry.

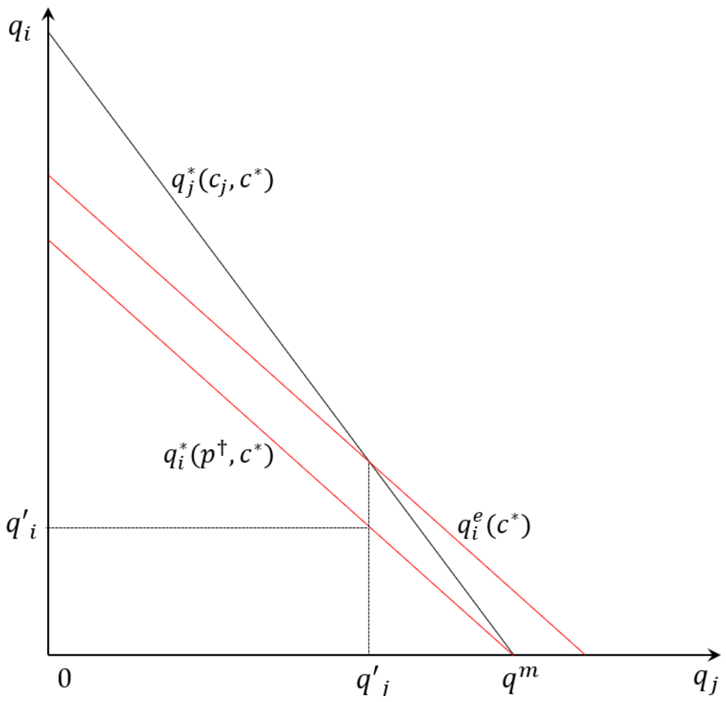



Figure 1: Best-response functions (with one incumbent).

### 3.3 Welfare standards

The result that positive setup costs increase aggregate output suggests a novel rationale for public policies that restrict entry or favor the emergence of entry barriers. This section develops a fully-fledged welfare analysis, deriving the entry cutoffs that maximize consumer surplus and total welfare.

In what follows, we assume that the distributions of firms' costs are such that both welfare objectives are single peaked in $c^*$ (in Appendix D we develop an example where this holds).

**Consumer surplus standard.** In our linear demand environment, consumer surplus is

$$S\left(Q\right) \triangleq \frac{Q^2}{2}.$$

Hence, expected consumer surplus is

$$\mathbb{E}[S\left(Q\right)] = \frac{\mathbb{E}[Q]^2 + \mathbb{V}[Q]}{2},$$

with $\mathbb{V}[Q]$ denoting the variance of aggregate output.

Thus, consumers benefit not only from higher expected aggregate output, but also from higher output variability — i.e., as is well known in the literature (e.g., Myatt and Wallace, 2015), they are *risk loving*. We have seen above that expected aggregate output is maximized at $c^* = p^\dagger$. Yet, entry by firm $i$ unambiguously increases output variability. The reason is that, both with and without entry, the output variance of each incumbent $j$ is

$$\mathbb{V}[q_j^*(\cdot)] = \frac{1}{4}\mathbb{V}[c_j],$$

which follows from (8)-(9). As cost realizations are independent across firms, for any given cutoff $c^*$ we have

$$\mathbb{V}[Q_N^*(c_i, c_{-i})] = \frac{1}{4}\sum_{j \neq i}\mathbb{V}[c_j] + \frac{1}{4}\mathbb{V}\left[c_i | c_i < \hat{c}(c^*)\right] > \frac{1}{4}\sum_{j \neq i}\mathbb{V}[c_j] = \mathbb{V}[Q_{N-1}^*(c_{-i})], \qquad (13)$$

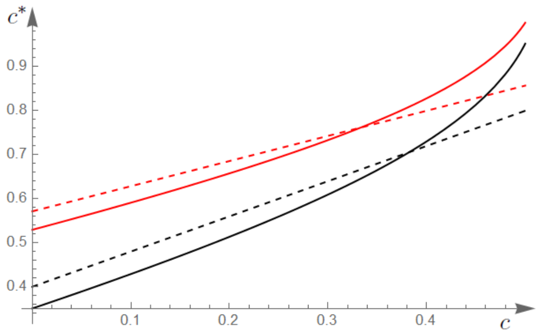
where $Q_N^*(c_i, c_{-i})$ and $Q_{N-1}^*(c_{-i})$ denote industry output with and without entry, respectively, with $c_{-i} \triangleq (c_1, ..., c_{i-1}, c_{i+1}, ..., c_N)$ being the vector of incumbents' costs.

From this argument it follows that the cutoff $c^*$ that maximizes expected consumer surplus is larger than the one that maximizes expected aggregate output: holding fixed the expectation of output, facilitating entry benefits consumers through increased output variability.

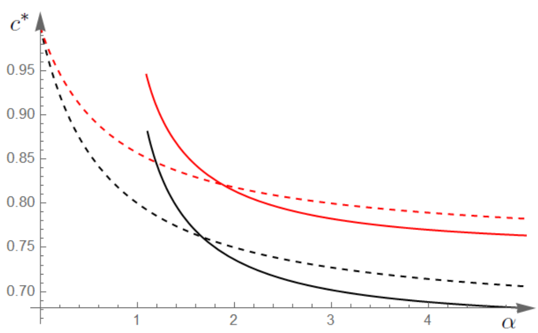
Whether raising some entry barriers (i.e., setting $k^* > 0$) is needed to maximize consumer surplus depends on the underlying parameters of the model. We can, therefore, state the following.

**Proposition 4.** *Expected consumer surplus maximization is not necessarily incompatible with the presence of entry barriers, although it requires lower barriers to entry than those mandated by expected output maximization — i.e.,* $c_S^* > p^\dagger$.

This possibility result can be established by considering a game where incumbents have the same non-random cost $c \in [0, 1/2]$,[14] and the entrant's cost follows a Beta distribution with cdf $F(c_i) = c_i^\alpha$ with $\alpha > 0$. As shown in Figure 2, which considers the case with only one incumbent ($N = 2$), consumer welfare maximization does require entry barriers so long as the incumbents' cost $c$ is sufficiently low, or the entrant's cost is likely to be large (i.e., $\alpha$ is sufficiently large).




(a) Cutoffs $c_S^*$ (continuous lines) and $c_0^*$ (dashed lines) as functions of $c$.




(b) Cutoffs $c_S^*$ (continuous lines) and $c_0^*$ (dashed lines) as functions of $\alpha$.

Figure 2: Red lines: $N = 2$; Black lines: $N = 3$.

**Total welfare standard.** We can finally turn to examine how entry restrictions affect total welfare — i.e., the sum of firms' profits and consumer surplus. We intentionally exclude the

[14]The restriction $c \leq 1/2$ ensures positive incumbents' output for all $c_i \geq 0$.

entrant's setup cost since this cost would naturally make a case for entry restrictions, as shown by Mankiw and Whinston (1986). Hence, in our linear-quadratic environment, we have

$$W(q_1, ..., q_N) \triangleq Q - \frac{1}{2}Q^2 - \sum_{h=1}^{N} c_h q_h,$$

with the convention that $q_i = 0$ if firm $i$ does not enter. Taking the expectation of the above equation, we have

$$\mathbb{E}[W(\cdot)] = \mathbb{E}[Q] - \frac{1}{2}\mathbb{E}[Q]^2 - \frac{1}{2}\mathbb{V}[Q] - \sum_{h=1}^{N} \mathbb{E}[c_h q_h].$$

The first two terms in the above expression are increasing in aggregate expected output (since by construction $Q \in [0, 1]$) and are, therefore, maximized at $c^* = p^\dagger$. Yet, unlike consumer surplus, total welfare is decreasing in output variability (as, e.g., in Myatt and Wallace, 2015). As seen above, entry unambiguously raises the variance of aggregate output, which calls for further restricting entry. Finally, as intuition suggests, minimization of production costs requires more efficient firms to produce more. Entry by a relatively inefficient firm may cause misallocation of production. This is because incumbents reduce their output expecting to face an entrant with average cost $\hat{c}(c^*) < c^*$. By strategic substitutability, a relatively inefficient entrant can optimally expand production, which implies that, holding total output fixed, average production costs are larger if entry barriers are too low.

Assuming for tractability that incumbents are ex-ante symmetric (i.e., their production costs are i.i.d.), we can show the following proposition:

**Proposition 5.** *Total welfare is maximized at a cutoff $c_W^* < p^\dagger$. Hence, maximizing total welfare requires raising entry barriers above the level needed to maximize expected output.*

This result is in line with Mankiw and Whinston (1986), although here it hinges on the effect of entry on output volatility and on the finding that expected output maximization requires entry barriers.

### 3.4 Robustness

In this section, we discuss two extensions of the baseline model that account for the possibility of exit by incumbents after entry and product differentiation.

**Creative destruction.** In the baseline model, we assumed that all firms produce a strictly positive quantity irrespective of the entrant's presence in the market. However, the incumbents' expected profit falls with the cutoff $c^*$. This is because a more selective entry policy makes the entrant on average more efficient and, therefore, it lowers the incumbents' output and profit. As a result, restricting entry could potentially marginalize the incumbents to the point that they do not produce.

Can entry restrictions still raise expected output when accounting for the possibility that some incumbents are inactive after entry? Can a restrictive entry policy still improve consumer surplus and total welfare?

To answer these questions, in Appendix C we develop an example with one incumbent only, say firm $j$ with random cost $c_j$, and assume that, for every admissible entrant's cutoff $c_i^*$, there is a non-degenerate probability with which the incumbent produces zero. Formally, for all $c_i^* \in [0, c_0^*]$, there exists a cutoff $c_j^* \in (0, \bar{c}_j)$ such that $q_j^*(c_j, c_i^*) = 0$ for $c_j > c_j^*$.

In Appendix C we show that $c_i^*$ and $c_j^*$ are positively linked: the more inefficient the entrant in expected terms, the larger the probability that the incumbent produces. Moreover, while the incumbent's expected output is always maximized when entry is fully restricted — i.e., $c_i^* = 0$ — and entry restrictions are required to maximize the entrant's expected output as well as aggregate output — i.e., $c_i^* < c_0^*$ — it is easy to find closed-form examples where, in contrast to the baseline model, the cutoff $c_i^*$ that maximizes aggregate output is lower than the cutoff that maximizes the entrant's expected output. Hence, less severe entry barriers are required to maximize the entrant's expected output than aggregate output.

The reason is that maximizing aggregate output requires internalizing the negative impact of a relatively more efficient entrant on the likelihood of exit by the incumbent, which is not considered when maximizing the entrant's expected output. As in the baseline model, it is also possible to show that restricting entry increases consumer surplus and total welfare, provided that the entrant's cost distribution is not too biased towards inefficient types compared to the incumbent's cost distribution.

**Product differentiation.** Consider the following system of demand functions

$$P_i(\cdot) \triangleq \max \left\{ 0, 1 - q_i - b \sum_{j \neq i} q_j \right\}, \quad \forall i = 1, ..., N, \tag{14}$$

with $b \in [0, 1]$ being an inverse measure of horizontal product differentiation ($b = 0$ for independent goods, and $b = 1$ in the case of perfectly homogeneous goods considered in the baseline analysis). How do the conclusions obtained above change when the differentiation parameter $b$ varies?

In Appendix C, we show that the results of the baseline model remain qualitatively unaltered under such a different specification. However, the necessity of imposing entry restrictions to maximize output, consumer surplus, and total welfare falls with the degree of product differentiation — i.e., the region of parameters in which entry restrictions enhance output, consumer surplus, and total welfare is narrower than in the baseline model. The reason is simple: as $b$ decreases, products become relatively more differentiated, and the cost of limiting entry rises because consumers like variety.[15]

# 4 Generalization to the all entrants game

In the baseline model, we considered only one potential entrant. In this section, we generalize the analysis to the case where all $N \geq 2$ firms must sink a (symmetric) setup cost $k \geq 0$ to

[15]Naturally, if $b$ is allowed to be negative, such that the firms' products are complements, quantities become strategic complements, and there is too little entry in equilibrium even if entry costs are zero.

enter the market. For the sake of tractability, we posit that their marginal costs of production are i.i.d. over the support $[0, \bar{c}]$, with cdf $F(c_i)$, density $f(c_i)$, and mean $\tilde{c}$.

We focus on a symmetric equilibrium in which every firm $i$ enters if and only if $c_i < c^*$, with $c^*$ being a decreasing function of $k$.[16] Moreover, we assume that, conditional on entry, firms always sell a strictly positive output, irrespective of the number of competitors. To guarantee this, we posit that the cost support is bounded from above[17] — i.e.,

$$\bar{c} \leq \check{c} \triangleq \frac{2 + (N-1)\tilde{c}}{N+1} \in (p^{\dagger}, 1). \tag{15}$$

This implies that, when entry is unrestricted (i.e., $k = 0$), all $N$ firms enter, even those with cost $\bar{c}$.

We show that, even with $N$ entrants who are always efficient enough to sell positive output in a $N$-firm oligopoly, restricting entry by setting $k^* > 0$, so that the implied cutoff is $c^* < \bar{c}$, may increase expected aggregate output, consumer surplus and total welfare.

## 4.1 Expected output

In a symmetric equilibrium, each firm enters with probability $F(c^*)$ and, for all entrants $i$,

$$\hat{c}_i = \mathbb{E}[c | c < c^*] \triangleq \hat{c}(c^*).$$

The expected aggregate output is

$$\mathbb{E}[Q_n^*(c^*)] = \sum_{n=0}^{N} \binom{N}{n} F(c^*)^n [1 - F(c^*)]^{N-n} Q_n^*(c^*),$$

where we defined by

$$Q_n^*(c^*) \triangleq \frac{n(1 - \hat{c}(c^*))}{n+1}$$

the expected aggregate output when $n$ firms are in the market (see Appendix A for the derivations).[18]

Differentiating the expected aggregate output with respect to $c^*$ and rearranging, we find that restricting entry, such that $c^* < \bar{c}$, increases expected industry output if and only if

$$\lim_{c^* \to \bar{c}} \frac{\partial \mathbb{E}[Q_n^*(c^*)]}{\partial c^*} < 0 \qquad \Longleftrightarrow \qquad \bar{c} > \frac{1 + (N-1)\tilde{c}}{N} = p^{\dagger}.$$

Hence, we can state the following:

[16] Although being intuitive (a higher setup cost tends to make each firm relatively less likely to enter), the existence and uniqueness, for any $k \geq 0$, of a symmetric equilibrium cutoff $c^*$, decreasing in $k$, may not be always technically guaranteed. In Appendix D we provide an example (with uniform costs' distribution) where this property holds.

[17] Note that the threshold $\check{c}$, as well as the other thresholds on $\bar{c}$ we define in what follows, depend on $\tilde{c}$, which in turn contains $\bar{c}$. In Appendix D we show that all inequalities give rise to proper bounds on $\bar{c}$ under the assumption of uniform distribution $c \sim \mathcal{U}[0, \bar{c}]$.

[18] As firms are ex-ante symmetric, each entrant's output choice only depends on the number $n - 1$ of rivals in the market, and not on the rivals' identity (i.e., the whole vector $x_{-i}$).

**Proposition 6.** *Expected aggregate output is maximized at a cutoff* $c_Q^* < \overline{c}$ *if and only if* $\overline{c} \in (p^\dagger, \check{c}]$. *Otherwise (i.e., for* $\overline{c} \leq p^\dagger$*), it is optimal to let all firms enter the market irrespective of their marginal costs.*

The intuition follows the logic of the baseline model. To see why, suppose that all firms are always enabled to enter — i.e., $c^* = \overline{c} = \check{c}$. This means that the marginal entrant type produces a negligible output. However, due to the selection effect described in the baseline model, the presence of this entrant generates an unnecessary contraction of its more efficient rivals' output, whose expectation, upon which they base output decisions, is $\tilde{c} < \overline{c}$. Under linear demand, the entry of the highest cost type reduces expected aggregate output whenever this type could not produce if its actual cost were observed by rivals, given that all other firms enter — i.e., for all $\overline{c} > p^\dagger$. Hence, restricting entry to increase expected aggregate output might be optimal also with $N$ entrants to mitigate the adverse impact of the selection effect.

## 4.2 Welfare standards

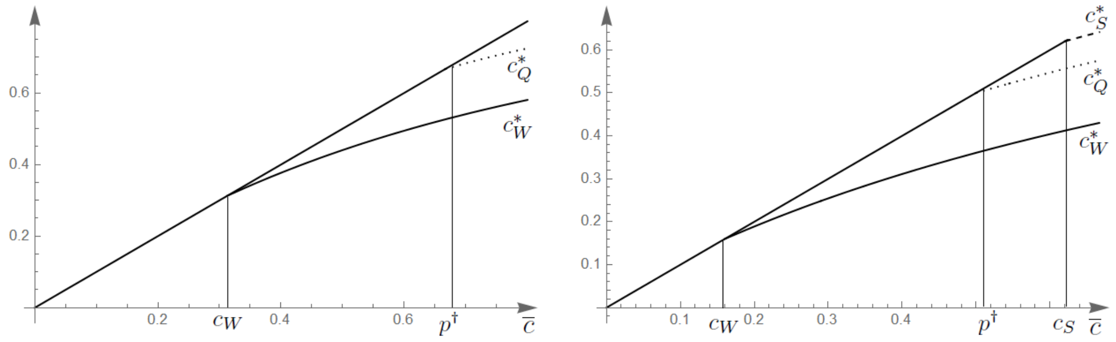
Finally, we can examine the welfare impact of entry restrictions on consumer surplus and total welfare:

**Proposition 7.** *There exist two thresholds,* $c_W < c_S$*, such that restricting entry:*

- *Increases expected total welfare if and only if* $\overline{c} > c_W$*;*
- *Increases expected consumer surplus if and only if* $N \geq 3$ *and* $\overline{c} > c_S$*.*

Entry of more firms increases output variance, which benefits consumers and harms total welfare. Moreover, entry of relatively inefficient types generates output misallocation, which further reduces total welfare. Hence, the region of parameters where restricting entry of all firms increases consumer (resp. total) welfare shrinks (resp. expands) compared to the one in which it increases aggregate output — i.e., $c_W < p^\dagger < c_S$. As intuition suggests, entry restrictions are never optimal from consumers' standpoint when there are only two potential entrants ($N = 2$). In this case, restricting entry is problematic for two reasons: ($i$) there is a higher chance that the market collapses (i.e., none of the firms enters); and ($ii$) when one firm stays out, the entrant monopolizes the market.

The conclusions drawn from the baseline model are robust in the sense that restricting entry is more likely to improve social welfare than consumer surplus. Note that the results of Proposition 7 do not imply that, in general, the maxima of the welfare objectives are such that $c_W^* \leq c_S^*$. With a generic number of firms, given that the number of entrants is itself a binomial random variable, obtaining closed-form solutions of the first-order conditions is hard. However, these results can be obtained for a duopoly ($N = 2$) and a triopoly ($N = 3$). Figure 3 illustrates the results for the case $c_i \sim \mathcal{U}[0, \overline{c}]$ (all analytical details are in Appendix D).




(a) $N = 2$. (b) $N = 3$.

Figure 3: Optimal cutoffs as functions of $\overline{c}$.

# 5 Conclusion

Our analysis has offered a first attempt to examine entry regulation in the Bayes-Cournot model. The paper's key lesson is that the presumption that entry barriers reduce output and consumer surplus is unwarranted when firms are privately informed about their production costs. These results provide an implicit justification for policies inspired by the excessive competition principle that goes beyond the insights of Mankiw and Whinston (1986), where total welfare maximization comes at the expense of consumer surplus.

The mechanism underlying these conclusions is that, in Bayes-Cournot, any active firm reacts to the conditional expectation on its rivals' costs rather than to their actual costs — i.e., the expectation of these costs conditional on rivals' entry. Regulations that influence setup costs, therefore, impact these expectations. The less (resp. more) costly entry, the less (resp. more) efficient entrants are perceived by their rivals in expected terms; therefore, the lower (resp. larger) the contraction of their output, by strategic substitutability, in response to new entry in the market. The stronger the selection effect of entry restrictions, the more intense competition upon entry is.

Although derived in a quantity-setting oligopoly game, our study has broader implications. It applies to all instances of strategic interactions where players must sink a setup cost to enter a game of strategic substitutes, and the entry decision signals relevant private information, be it political economy, international trade, patent races, etc.

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# Appendix

## A Bayes-Cournot equilibrium and technical assumptions

Suppose that $n$ firms, denoted by $i = 1, ..., n$ without loss of generality, have entered the market. From (1) it follows that each entrant $i$'s best-reply function is

$$q_i^e(\cdot) = \frac{1}{2}\left(1 - \hat{c}_i - \sum_{j \neq i} q_j^e\right), \tag{16}$$

with $j = 1, ..., n$ being an active rival ($x_j = 1$). Summing up, (16) for all entrants yields

$$\sum_{i=1}^{n} q_i^e = \frac{n - \sum_{i=1}^{n} \hat{c}_i}{n+1}. \tag{17}$$

Since $\sum_{j \neq i} q_j^e = \sum_{i=1}^{n} q_i^e - q_i^e$, substituting (17) into (16) and solving for $q_i^e$ yields

$$q_i^e = \frac{1}{n+1}\left[1 - \hat{c}_i + \sum_{j \neq i}(\hat{c}_j - \hat{c}_i)\right].$$

From $\sum_{j \neq i} q_j^e = \sum_{i=1}^{n} q_i^e - q_i^e$, we obtain the expression for the sum of the active rivals' expected output in (2).

We assume throughout that prices are always positive in all circumstances. That is, even if $c_i = 0$ (so that all firms enter) and $\hat{c}_i = \tilde{c}_i$ for all entrants $i = 1, ..., N$,

$$\sum_{i=1}^{N} \tilde{c}_i > \frac{2(N-2)}{N-1},$$

which is always true for $N = 2$.

Moreover, in the baseline model of Section 3, we assume that all firms $j \neq i$ face no entry costs ($k_j = 0$), or they already sunk these costs, and that

$$\bar{c}_j < \frac{2\left(1 + \sum_{h \neq \{i,j\}} \tilde{c}_h\right) - (N-1)\tilde{c}_j}{N+1} \quad \forall j \neq i.$$

This guarantees $q_j^*(\cdot) > 0$ in all circumstances — i.e., even if $c_j = \bar{c}_j$ and $c^* = 0$. In the all entrants' game of Section 4, as firms' costs are i.i.d., this condition boils down to (15).

## B Proofs

**Proof of Lemma 1**. The expressions for the equilibrium quantities in Lemma 1 follow from (1)-(2), given that $\hat{c}_j = \tilde{c}_j$ for all $j \neq i$, $n = N$ and $\hat{c}_i = \hat{c}(c^*)$ with entry of firm $i$, and $n = N - 1$ when the entrant stays out.

Equation (6) for the entry cutoff $c^*(\cdot)$ is derived from the indifference condition (5), imposing $\hat{c}_i = \hat{c}(c^*)$. The uniqueness of the solution to (6), provided $k$ is not too large, follows from the observation that its left-hand side is decreasing in $c^*$, as it depends negatively both on $c^*$ and $\hat{c}(c^*)$, and $\frac{\partial \hat{c}(\cdot)}{\partial c^*} > 0$ as shown in (10). This also implies that $c^*(\cdot)$ is decreasing in $k$. $\square$

**Proof of Proposition 1.** Substituting (10) into the left-hand side of (11) yields

$$\frac{\partial q_i^*(c^*)}{\partial c^*} = \frac{N}{N+1} f_i(c^*) \left(p^\dagger - c^*\right) = 0 \quad \Longleftrightarrow \quad c^* = p^\dagger.$$

Under unrestricted entry, the marginal entrant type is such that $q_i^*(c^*, c^*) = 0$, which gives

$$\frac{2\left(1 + \sum_{j\neq i} \tilde{c}_j\right) - (N-1)\hat{c}(c^*)}{N+1} - c^* = 0,$$

with the left-hand side being decreasing in $c^*$. Hence, $c_0^* > p^\dagger$ if and only if

$$\begin{aligned} \frac{2\left(1 + \sum_{j\neq i} \tilde{c}_j\right) - (N-1)\hat{c}(p^\dagger)}{N+1} - p^\dagger < 0 \quad &\Longleftrightarrow \\ \frac{2Np^\dagger - (N-1)\hat{c}(p^\dagger)}{N+1} - p^\dagger < 0 \quad &\Longleftrightarrow \\ p^\dagger > \hat{c}(p^\dagger), \quad & \end{aligned}$$

which is always true. Finally, substituting $c^* = p^\dagger$ into (6), we find the entry cost that maximizes the entrant's expected output. $\square$

**Proof of Proposition 2.** Differentiating the expected aggregate output w.r.t. $c^*$ gives

$$\frac{\partial Q^*(c^*)}{\partial c^*} = f_i(c_i^*)[Q_N^*(c^*) - Q_{N-1}^*] - F_i(c^*)\frac{1}{N+1}\frac{\partial \hat{c}(c^*)}{\partial c^*} = \frac{1}{N}\underbrace{\frac{f_i(c^*)}{N+1}\left[1 + \sum_{j\neq i}\tilde{c}_j - Nc^*\right]}_{=\frac{\partial q_i^*(c^*)}{\partial c^*}}.$$

Hence, $Q^*(c^*)$ is inverted U-shaped in $c^*$ and maximized at $c^* = p^\dagger$. $\square$

**Proof of Proposition 3.** The entrant's expected profit is

$$\begin{aligned} \pi_i^*(c^*) &\triangleq F_i(c^*)\mathbb{E}[(q_i^*(c_i, c^*))^2 | c_i < c^*] - k(c^*) \\ &= F_i(c^*)\left[(\mathbb{E}[q_i^*(c_i, c^*)|c_i < c^*])^2 + \mathbb{V}[q_i^*(c_i, c^*)|c_i < c^*]\right] - k(c^*) \\ &= F_i(c^*)\left[\left(\frac{1 - N\hat{c}(c^*) + \sum_{j\neq i}\tilde{c}_j}{N+1}\right)^2 + \frac{\hat{\sigma}(c^*)}{4}\right] - k(c^*), \end{aligned}$$

where $k(c^*)$ is the entry cost that implements $c^*$, obtained from (6), and

$$\hat{\sigma}(c^*) \triangleq \mathbb{V}\left[c_i | c_i < \hat{c}\right] = \frac{1}{F_i(c^*)}\int_0^{c^*} (c_i - \hat{c}(c^*))^2 \, dF_i(c_i),$$

is the conditional variance of the entrant's cost, with

$$\frac{\partial\hat{\sigma}(c^*)}{\partial c^*} = \frac{f_i(c^*)}{F_i(c^*)}\left[(c^* - \hat{c}(c^*))^2 - \hat{\sigma}(c^*)\right].$$

Differentiating the expected profit with respect to $c^*$ yields

$$\frac{\partial\pi_i^*(c^*)}{\partial c^*} = f_i(c^*)\left[(\mathbb{E}[q_i^*(c_i,c^*)|c_i < c^*])^2 + \frac{1}{4}\hat{\sigma}(c^*) - k(c^*)\right] +$$

$$-F_i(c^*)\left[\frac{2N}{N+1}\mathbb{E}[q_i^*(c_i,c^*)|c_i < c^*]\frac{\partial\hat{c}(c^*)}{\partial c^*} - \frac{1}{4}\frac{\partial\hat{\sigma}(c^*)}{\partial c^*} + \frac{\partial k(c^*)}{\partial c^*}\right].$$

By construction, at $c^* = c_0^*$, $k(c_0^*) = 0$, and also $\frac{\partial k(c^*)}{\partial c^*}|_{c^*=c_0^*} = 0$. The derivative above simplifies as

$$\left.\frac{\partial\pi_i^*(c^*)}{\partial c^*}\right|_{c^*=c_0^*} = -f_i(c^*)\frac{2N^2(N-1)(\hat{c}(c_0^*) - p^\dagger)^2}{(1+N)^3} < 0,$$

which implies that the entrant's expected profit is maximized at $c_\pi^* < c_0^*$.

Similarly, each incumbent $j$'s expected profit writes as

$$\pi_j^*(c^*) \triangleq F_i(c^*)\mathbb{E}[(q_j^*(c_j,c^*))^2] + [1 - F_i(c^*)]\mathbb{E}[(q_j^*(c_j))^2] =$$

$$= F_i(c^*)\left[\mathbb{E}[q_j^*(c_j,c^*)]^2 + \mathbb{V}[q_j^*(c_j,c^*)]\right] + [1 - F_i(c^*)]\left[\mathbb{E}[q_j^*(c_j)]^2 + \mathbb{V}[q_j^*(c_j)]\right].$$

Since $\mathbb{V}[q_j^*(c_j,c^*)] = \mathbb{V}[q_j^*(c_j)] = \frac{1}{4}\sigma_j$, with $\sigma_j \triangleq \mathbb{V}[c_j]$, we have

$$\pi_j^*(c^*) = \mathbb{E}[q_j^*(c_j)]^2 + \frac{1}{4}\sigma_j - F_i(c^*)\left[\mathbb{E}[q_j^*(c_j)]^2 - \mathbb{E}[q_j^*(c_j,c^*)]^2\right].$$

Moreover, as

$$\mathbb{E}[q_j^*(c_j)] - \mathbb{E}[q_j^*(c_j,c^*)] = \frac{1 + \sum_{j\neq i}\tilde{c}_j - N\hat{c}(c^*)}{N(N+1)} > 0, \quad \forall c^* \leq c_0^*,$$

each incumbent $j$'s expected profit is maximized at $F_i(c^*) = 0$, or $c^* = 0$. □

**Proof of Proposition 4.** In equilibrium,

$$\mathbb{E}[Q^2] = F_i(c^*)\left[Q_N^*(c^*)^2 + \mathbb{V}\left[Q_N^*(c_i,c_{-i})\right]\right] + (1 - F_i(c^*))\left[(Q_{N-1}^*)^2 + \mathbb{V}\left[Q_{N-1}^*(c_{-i})\right]\right],$$

where variances are given in (13). Differentiating expected consumer surplus with respect to $c^*$ and rearranging we obtain

$$\frac{\partial\mathbb{E}[S(\cdot)]}{\partial c^*} = \frac{f(c^*)}{2}\left[Q_N^*(c^*)^2 - (Q_{N-1}^*)^2 - \frac{2}{N+1}Q_N^*(c^*)(c^* - \hat{c}(c^*)) + \frac{(c^* - \hat{c}(c^*))^2}{4}\right].$$

Evaluating this derivative at $c^* = p^\dagger$ yields

$$\left.\frac{\partial\mathbb{E}[S(\cdot)]}{\partial c^*}\right|_{c^*=p^\dagger} = \frac{(3+N)(N-1)\left(1 - N\hat{c}(c^*) + \sum_{j\neq i}\tilde{c}_j\right)^2}{8N^2(N+1)^2} > 0.$$

Assuming that $\mathbb{E}[S(\cdot)]$ is single-peaked w.r.t. $c^*$, the above inequality shows that consumer surplus maximization requires $c_S^* > p^\dagger$. Figure 2 shows that expected consumer surplus can be maximized at $c_S^* < c_0^*$. The analytical steps that are necessary to obtain the figure are relegated to Appendix D. □

**Proof of Proposition 5.** The first-order condition for total welfare maximization can be written as

$$\frac{\partial \mathbb{E}[W(\cdot)]}{\partial c^*} = \frac{\partial Q^*(c^*)}{\partial c^*} - \frac{1}{2}\frac{\partial \mathbb{E}[Q^2]}{\partial c^*} - \frac{\partial}{\partial c^*}\mathbb{E}\left[\sum_h c_h q_h\right] = 0,$$

where the first two terms have been computed in the proofs of the previous results.

As for the last term, without entry, for all incumbents $j$, it must be

$$\mathbb{E}[c_j q_j^*(c_j)] = \frac{1}{2}\left[\tilde{c}(1-(N-2)q^*) - (\tilde{c}^2+\sigma)\right],$$

with $q^* \triangleq \mathbb{E}[q_j^*(c_j)]$ for all $j$, by symmetry (under the assumption of i.i.d. costs), and similarly $\tilde{c} \triangleq \tilde{c}_j$ and $\sigma \triangleq \mathbb{V}[c_j]$ for all $j$. We then have

$$\sum_{j\neq i}\mathbb{E}[c_j q_j^*(c_j)] = (N-1)\left(\frac{\tilde{c}(1-(N-1)\tilde{c})}{N} - \frac{\sigma}{2}\right).$$

If, instead, firm $i$ enters,

$$\mathbb{E}[c_j q_j^*(c_j,c^*)] = \frac{1}{2}\left[\tilde{c}(1-(N-2)q^*(c^*) - \mathbb{E}[q_i^*(\cdot)|c_i<c^*]) - (\tilde{c}^2+\sigma)\right],$$

with $q^*(c^*) \triangleq \mathbb{E}[q_j^*(c_j,c^*)]$ for all incumbents $j$, so that

$$\sum_{j\neq i}\mathbb{E}[c_j q_j^*(c_j,c^*)] = (N-1)\left(\frac{\tilde{c}(1+\hat{c}(c^*)-2\tilde{c})}{N+1} - \frac{1}{2}\sigma\right).$$

Turning to the entrant, we have

$$\mathbb{E}[c_i q_i^*(c_i,c^*)] = \frac{\hat{c}(c^*)(1+(N-1)\tilde{c}-N\hat{c}(c^*))}{N+1} - \frac{1}{2}\hat{\sigma}(c^*).$$

Overall, the derivative of total welfare w.r.t. $c^*$ simplifies to

$$\frac{\partial \mathbb{E}[W(\cdot)]}{\partial c^*} = \frac{3(c^*)^2}{8} - \frac{4(2+N+\tilde{c}(2N^2+N-3)) - (N-1)(5N+7)\hat{c}(c^*)}{4(N+1)^2}c^* +$$

$$+\frac{4(2N+1)-(N-1)((\hat{c}(c^*))^2N^2(5N+7) - 8(1+N(N+3)\tilde{c}) + 4(1-5N-14N^2-2N^3+4N^4)\tilde{c})}{8N^2(N+1)^2}.$$

Evaluating this derivative at $c^* = p^\dagger$ yields

$$\left.\frac{\partial \mathbb{E}[W(\cdot)]}{\partial c^*}\right|_{c^*=p^\dagger} = -\frac{1}{8N^2(1+N)^2}[(N-1)(2\tilde{c}(N-1)(5N+7)(1-N\hat{c}(p^\dagger))+$$

$$+(5N+7)(1-N\hat{c}(p^\dagger))^2 + (16N^4+5N^3-51N^2-41N+7)\tilde{c}^2)] < 0.$$

Under the assumption that $\mathbb{E}[W(\cdot)]$ is single-peaked w.r.t. $c^*$, this shows that total welfare maximization requires $c^*_W < p^\dagger$. From (6), this implies that the entry cost that implements $c^*_W$ is larger than the one that implements $c^* = p^\dagger$. □

**Proof of Proposition 6.** Differentiating expected aggregate output with respect to $c^*$ and rearranging, we have

$$\frac{\partial \mathbb{E}[Q_n^*(c^*)]}{\partial c^*} = f(c^*) \sum_{n=0}^{N} \frac{n}{n+1} \binom{N}{n} F(c^*)^{n-1} \Gamma_Q(c^*, n),$$

with

$$\begin{aligned}\Gamma_Q(c^*, n) \triangleq{} & [1 - F(c^*)]^{N-n} [n(1 - \hat{c}(c^*)) - (c^* - \hat{c}(c^*))] + \\ & - F(c^*)[1 - F(c^*)]^{N-n-1} (N - n)(1 - \hat{c}(c^*)).\end{aligned}$$

Restricting entry increases expected aggregate output if and only if $\frac{\partial \mathbb{E}[Q_n^*(c^*)]}{\partial c^*}|_{c^* \to \bar{c}} < 0$. Note that, since $F(c^*) \to 1$ for for $c^* \to \bar{c}$, all the terms in the above derivative vanish, except those for $n \in \{N-1, N\}$. We then have

$$\begin{aligned}\left.\frac{\partial \mathbb{E}[Q_n^*(c^*)]}{\partial c^*}\right|_{c^* \to \bar{c}} &= -\frac{N-1}{N} \binom{N}{N-1} (1 - \tilde{c}) + \frac{N}{N+1} \binom{N}{N} [N(1 - \tilde{c}) - (\bar{c} - \tilde{c})] < 0 \\ \iff \quad & \bar{c} > \frac{1 + (N-1)\tilde{c}}{N} = p^\dagger,\end{aligned}$$

which establishes the result. □

**Proof of Proposition 7.** Expected consumer surplus is

$$\mathbb{E}[S(Q)] \triangleq \frac{1}{2} \sum_{n=0}^{N} \binom{N}{n} F(c^*)^n [1 - F(c^*)]^{N-n} \left[ [Q_n^*(c^*)]^2 + \frac{n}{4} \hat{\sigma}(c^*) \right].$$

Differentiating w.r.t. $c^*$ and rearranging, we have

$$\frac{\partial \mathbb{E}[S(\cdot)]}{\partial c^*} = \frac{f(c^*)}{2} \sum_{n=0}^{N} n \binom{N}{n} F(c^*)^{n-1} \Gamma_S(c^*, n),$$

with

$$\begin{aligned}\Gamma_S(c^*, n) \triangleq{} & [1 - F(c^*)]^{N-n-1} \left[ \frac{1}{4}(c^* - \hat{c}(c^*))^2 - \frac{2(1 - \hat{c}(c^*))(c^* - \hat{c}(c^*))n}{(n+1)^2} + \frac{(1 - \hat{c}(c^*))^2 n^2}{(n+1)^2} \right] + \\ & - F(c^*)[1 - F(c^*)]^{N-n-1} (N - n) \left[ n \frac{(1 - \hat{c}(c^*))^2}{(n+1)^2} + \frac{1}{4} \hat{\sigma}(c^*) \right].\end{aligned}$$

As above, for $c^* \to \bar{c}$, all the terms in the above derivative vanish, except those for $n \in \{N-1, N\}$. We then have

$$\left.\frac{\partial \mathbb{E}[S(\cdot)]}{\partial c^*}\right|_{c^* \to \bar{c}} = \frac{f(c^*)}{2} \left[ \frac{N\bar{c}^2}{4} - \frac{N(4N + (N-1)^2 \tilde{c})}{2(N+1)^2} \bar{c} + \right.$$

$$+\frac{4(2N^2-1)+8(N^2(N-2)+1)\tilde{c}+(N^2(N-3)^2-4)\tilde{c}^2}{4N(N+1)^2}\Bigg] < 0 \iff \bar{c} > c_S,$$

with

$$c_S = \frac{2N^2(2-\tilde{c})+N(N^2+1)\tilde{c}-2(1-\tilde{c})\sqrt{N(N-2)(2N^2-1)+1}}{N(N+1)^2},$$

being lower than $\check{c}$ if and only if $N > 1+\sqrt{2}$ — i.e., for all $N \geq 3$. Consumer surplus maximization is thus always achieved under unrestricted entry (i.e., $c_S^* = \bar{c}$) for $N = 2$, whereas, for all $N \geq 3$, it requires restricting entry (i.e., $c_S^* < \bar{c}$) whenever $\bar{c} > c_S$.

The first-order condition for total welfare maximization can be written as

$$\frac{\partial \mathbb{E}[W(\cdot)]}{\partial c^*} = \frac{\partial \mathbb{E}[Q_n^*(c^*)]}{\partial c^*} - \frac{1}{2}\frac{\partial \mathbb{E}[(Q_n^*(c^*))^2]}{\partial c^*} - \frac{\partial}{\partial c^*}\mathbb{E}\left[\sum_i c_i q_n^*(c_i, c^*)\right] = 0,$$

with the convention $q_n^*(c_i, c^*) = 0$ for $c_i \geq c^*$ (i.e., for firms who do not enter). The first two terms have been computed in the proofs of the previous results. As for the last term, conditional on $n$ firms entering the market, we have

$$\mathbb{E}[c_i q_n^*(c_i, c^*)] = \hat{c}(c^*)\frac{2+(n-1)\hat{c}(c^*)}{2(n+1)} - \frac{1}{2}\left((\hat{c}(c^*))^2 + \hat{\sigma}(c^*)\right),$$

and so

$$\mathbb{E}[c_i q_n^*(c_i, c^*)] = \sum_{n=0}^{N} n\binom{N}{n} F(c^*)^n[1-F(c^*)]^{N-n}\left[\frac{\hat{c}(c^*)(1-\hat{c}(c^*))}{n+1} - \frac{1}{2}\hat{\sigma}(c^*)\right].$$

Differentiating this expected value w.r.t. $c^*$ and rearranging, we have

$$\frac{\partial \mathbb{E}[c_i q_n^*(c_i, c^*)]}{\partial c^*} = f(c^*)\sum_{n=0}^{N} n\binom{N}{n} F(c^*)^{n-1}\Gamma_C(c^*, n),$$

with

$$\Gamma_C(c^*, n) \triangleq [1-F(c^*)]^{N-n}\left[\frac{c^* - \hat{c}(c^*)(c^* - \hat{c}(c^*))}{n+1} - \frac{1}{2}(c^* - \hat{c}(c^*))^2\right] +$$

$$-F(c^*)[1-F(c^*)]^{N-n-1}(N-n)\left[\frac{\hat{c}(c^*)(1-\hat{c}(c^*))}{n+1} - \frac{1}{2}\hat{\sigma}(c^*)\right].$$

Once again, for $c^* \to \bar{c}$, all the terms in the above derivative vanish, except those for $n \in \{N-1, N\}$. We then have

$$\left.\frac{\partial \mathbb{E}[W(\cdot)]}{\partial c^*}\right|_{c^* \to \bar{c}} = \frac{3N\bar{c}^2}{8} - \frac{8-5\tilde{c}+N(4+(3N+2)\tilde{c})}{4(N+1)^2}N\bar{c} +$$

$$+\frac{4(2N+1)+8(N-1)(N^2+3N+1)\tilde{c}+(N-2)(N-1)(N+2)(3N+1)\tilde{c}^2}{8N(N+1)^2},$$

which, under the constraint $\bar{c} < \check{c}$, is negative if and only if $\bar{c} > c_W$, with

$$c_W = \frac{N(8-5\tilde{c}+N(4+(3N+2)\tilde{c}))-2(1-\tilde{c})\sqrt{(N-1)(4N^3+14N^2+15N+3)}}{3N(N+1)^2} < p^\dagger.$$

Hence, total welfare maximization requires restricting entry (i.e., $c^*_W < \overline{c}$) whenever $\overline{c} > c_W$. $\square$

# C Extensions

## C.1 Creative destruction

Consider a model with only one incumbent (firm $j$), and suppose that, for all entry cutoffs $c_i^*$, the incumbent is not active with some probability.[19] Formally, we take a distribution $F_j(c_j)$ such that $\Pr[q_j^*(c_j, c_i^*) = 0] \in (0,1)$ for all $c_i^* \in [0, c_0^*]$. Moreover, we assume that, when it finds it optimal not to produce, the incumbent does not take an exit decision observed by the entrant before quantity competition takes place: otherwise, this exit decision would entail a similar signaling effect as firm $i$'s entry decision, which is not the focus of our paper. Let $c_j^*$ be the value of $c_j$ above which the incumbent is not active: $q_j^*(c_j, c_i^*) = 0$ for $c_j \geq c_j^*$. Then, conditional on entry,

$$q_i^*(c_i) = \frac{1}{2}(1 - c_i - q_j^e) > 0,$$

this quantity being positive in equilibrium as otherwise firm $i$ would not enter, and

$$q_j^*(c_j) = \max\left[\frac{1}{2}(1 - c_i - q_i^e), 0\right].$$

Hence, expected outputs are

$$q_i^e = \int_0^{c_i^*} \frac{1}{2}[1 - c_i - q_j^e] dF_i(c_i | c_i < c^*) = \frac{1}{2}[1 - \hat{c}_i(c_i^*) - q_j^e],$$

$$q_j^e = \int_0^{c_j^*} \frac{1}{2}[1 - c_j - q_i^e] dF_j(c_j) = F_j(c_j^*)\frac{1}{2}[1 - \hat{c}_j(c_j^*) - q_i^e].$$

Solving these two equations for $(q_i^e, q_j^e)$ gives

$$q_i^e = \frac{2(1 - \hat{c}_i(c_i^*)) - F_j(c_j^*)(1 - \hat{c}_j(c_j^*))}{4 - F_j(c_j^*)},$$
$$q_j^e = \frac{(1 + \hat{c}_i(c_i^*) - 2\hat{c}_j(c_j^*))F_j(c_j^*)}{4 - F_j(c_j^*)}.$$

Then, $c_j^*$ is obtained as the lowest value of $c_j$ for which $q_j^*(\cdot) = 0$ — i.e., it solves

$$1 - c_j^* - \frac{2(1 - \hat{c}_i(c_i^*)) - F_j(c_j^*)(1 - \hat{c}_j(c_j^*))}{4 - F_j(c_j^*)} = 0, \tag{18}$$

from which it follows that $c_j^*$ is increasing in $\hat{c}_i(c_i^*)$, and so in $c_i^*$. As in the baseline model, under

[19] The analysis under the polar opposite assumption of that in the base model, namely that the incumbent exits whenever there is entry, is trivial. The entrant's expected output is maximized at $c^* = 1$, which coincides with the cutoff under unrestricted entry (since, being a monopolist, the entrant can produce positive output and make positive profits for all $c_i < 1$). However, maximizing expected industry output still requires raising entry barriers: letting an entrant of cost $c_i$ replace an incumbent monopolist having (say, for simplicity, non-random) cost $c$ increases expected industry output if and only if the entrant is more efficient than the monopolist ($c_i \leq c$): hence, $c^* = c < c_0^*$. This coincides with the cutoff that maximizes both expected consumer surplus and total welfare. Details are omitted for brevity and available upon request.

unrestricted entry, $c_i^*$ is pinned down by $q_i^*(c_i^*) = 0$:

$$1 - c_i^* = \frac{(1 + \hat{c}_i(c_i^*) - 2\hat{c}_j(c_j^*))F_j(c_j^*)}{4 - F_j(c_j^*)}.$$

As in the baseline model, maximization of both the entrant's and aggregate expected outputs require raising entry barriers — i.e., $c_i^* < c_0^*$. However, unlike in the baseline model, the two objectives are maximized at different cutoffs $c_i^*$. To see this, suppose $c_j \sim \mathcal{U}[0,1]$ and $c_i \in [0,1]$ is distributed according to $F_i(c_i) = c_i^\alpha$, with $\alpha \in (0,3)$.[20] Then, solving (18) yields

$$c_j^*(c_i^*) = 4 - 2\sqrt{3 - \hat{c}_i(c_i^*)} \in (0,1), \tag{19}$$

with $\hat{c}_i(c_i^*) = \frac{\alpha}{\alpha+1} c_i^*$, and

$$c_0^* = 4(1+\alpha)\sqrt{3 + 6\alpha + 4\alpha^2} - 6 - 2\alpha(7 + 4\alpha) \in (0,1).$$

The expected entrant's output is

$$q_i^*(c_i^*) \triangleq F_i(c_i^*)\frac{1}{2}\left[1 - \hat{c}_i(c_i^*) - \frac{(1 + \hat{c}_i(c_i^*) - 2\hat{c}_j(c_j^*))F_j(c_j^*)}{4 - F_j(c_j^*)}\right].$$

Substituting for $c_j^*(c_i^*)$ in (19), in the example at hand this yields

$$q_i^*(c_i^*) = (c_i^*)^\alpha \left(2\sqrt{3 - \frac{\alpha}{1+\alpha} c_i^*} - 3\right).$$

This quantity is maximized at

$$c_q^* = \frac{3(4 - (1+\alpha)\sqrt{3(2+\alpha)(2+3\alpha)} + \alpha(9 + 5\alpha))}{2(1+2\alpha)^2}.$$

Absent entry, the incumbent produces the monopoly output (which is assumed always positive, as otherwise firm $j$ could not be considered an incumbent). Hence, the expected aggregate output is

$$Q^*(c_i^*) \triangleq F_i(c_i^*)\frac{1}{2}\left[1 - \hat{c}_i(c_i^*) + \frac{(1 + \hat{c}_i(c_i^*) - 2\hat{c}_j(c_j^*))F_j(c_j^*)}{4 - F_j(c_j^*)}\right] + [1 - F_i(c_i^*)]\frac{1}{2}(1 - \tilde{c}_j),$$

which, in our example, simplifies to

$$Q^*(c_i^*) = \frac{1}{4}\left(1 + (c_i^*)^\alpha \left(15 - \frac{4\alpha c_i^*}{1+\alpha} - 8\sqrt{3 - \frac{\alpha}{\alpha+1} c_i^*}\right)\right).$$

[20]The restriction $\alpha < 3$ guarantees that the incumbent is inactive with positive probability for all $c_i^* \in [0,1]$.

Its maximizer, denoted by $c_Q^*$, solves

$$\frac{c_i^*}{1+\alpha}\left(\frac{1}{\sqrt{3-\frac{\alpha}{\alpha+1}c_i^*}}-1\right)+\frac{1}{4}\left(15-4\frac{\alpha}{\alpha+1}c_i^*-8\sqrt{3-\frac{\alpha}{\alpha+1}c_i^*}\right)=0,$$

with $c_Q^* < c_q^* < c_0^*$, as illustrated in Figure 4.

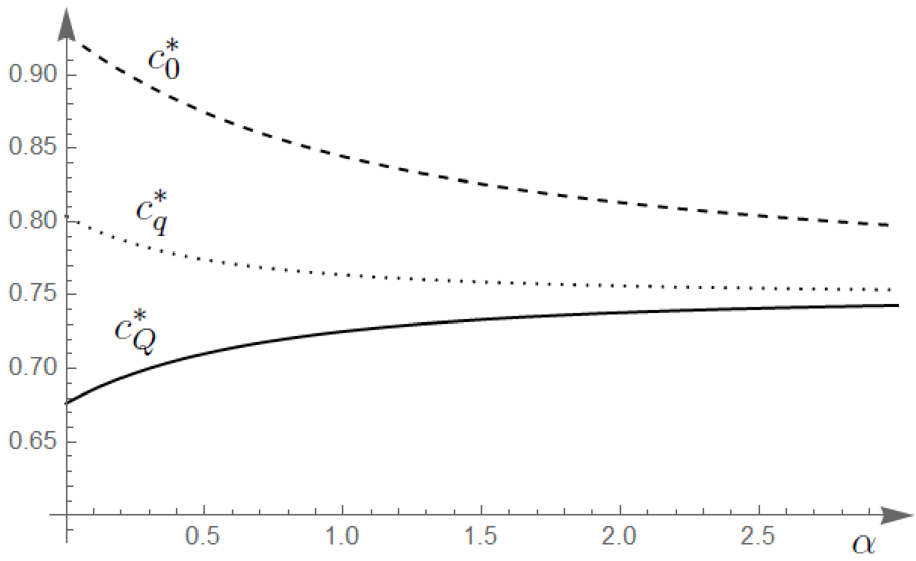


Figure 4: Optimal cutoffs as functions of $\alpha \in (0, 3)$.

Finally, restricting entry also increases consumer surplus and total welfare, provided $\alpha$ is not too large: see Figure 5 below.[21]

## C.2 Product differentiation

This appendix extends the analysis to account for horizontal product differentiation.

**Preliminaries.** Consider a representative consumer with preferences represented by the following utility function

$$U(\cdot) \triangleq \sum_h q_h - \frac{1}{2}\sum_h q_h^2 - \frac{b}{2}\sum_{h,j=1,\ldots,N,j\neq h} q_h q_j - \sum_h p_h q_h,$$

with the convention that $q_i = 0$ if firm $i$ does not enter, where $b \in [0, 1]$ is an inverse measure of product differentiation ($b = 0$ for independent goods, and $b = 1$ in the case of perfect homogeneous goods considered in the main analysis). Differentiating with respect to outputs, we immediately have the system of demand functions given in (14).

[21] The expressions of the welfare objectives are very cumbersome, and the first-order conditions cannot be solved analytically. Mathematica files are available upon request.

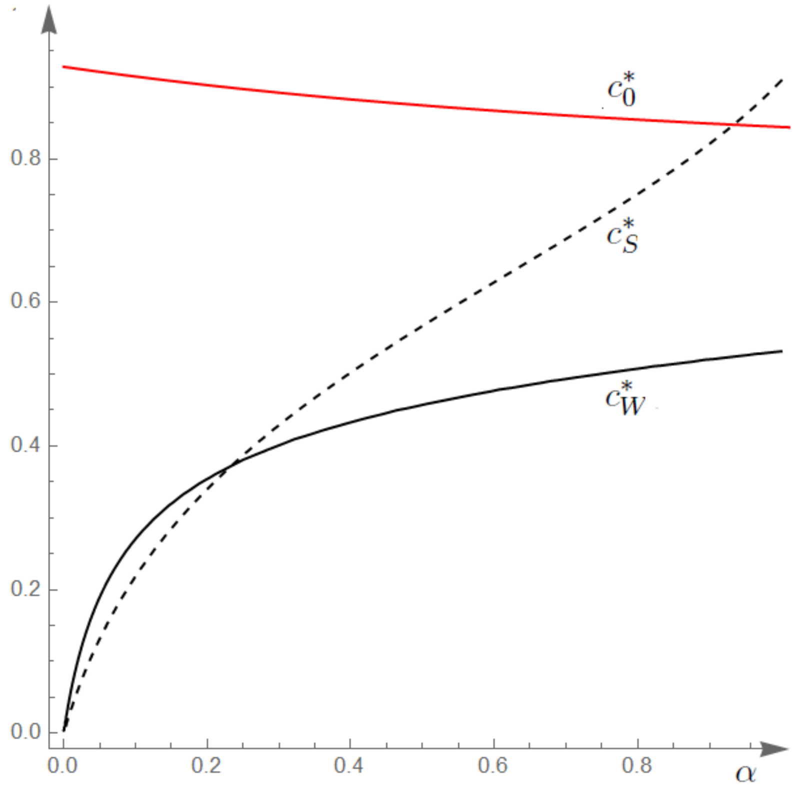


Figure 5: Optimal cutoffs as functions of $\alpha \in (0, 1)$.

Every active entrant (say firm $i$) solves

$$\max_{q_i \geq 0} \left[ 1 - q_i - b \sum_{j \neq i} q_j^e(x_j) - c_i \right] q_i,$$

with $q_j^e(x_j)$ being the quantity that firm $i$ expects to be produced by firm $j$ given its entry decision $x_j \in \{0, 1\}$.

Taking the first-order condition immediately yields firm $i$'s best-response function

$$q_i^*(c_i, x_{-i}) = \frac{1}{2} \left[ 1 - c_i - b \sum_{j \neq i} q_j^e(x_j) \right].$$

where, by the same steps carried out for the baseline model (and using the same notation), in equilibrium we have

$$\sum_{j \neq i} q_j^e(x_j) = \frac{(n-1)[2 - b(1 - \hat{c}_i)] - 2 \sum_{j \neq i : x_j = 1} \hat{c}_j}{(2 - b)(2 + b(n-1))}.$$

In what follows we consider the case where $N - 1$ firms are incumbents and only one firm (indexed by $i$) is a potential entrant. It is easy to show that, for all entry cost $k_i \triangleq k \geq 0$, there is a cutoff $c^*$, decreasing in $k$, such that firm $i$ enters if and only if $c_i < c^*$.

**Output and profits.** Firm $i$'s expected output from an ex-ante standpoint is

$$\begin{aligned} q_i^*(c^*) &\triangleq F(c_i^*)\mathbb{E}[q_i^*(c_i, c^*)|c_i < c^*] \\ &= F_i(c^*)\frac{1}{2}\left[1 - \hat{c}(c^*) - b\frac{(N-1)[2-b(1-\hat{c}(c^*))] - 2\sum_{j\neq i}\tilde{c}_j}{(2-b)(2+b(N-1))}\right], \end{aligned}$$

and it is maximized at

$$c^* = p^\dagger \triangleq \frac{2 + b\left(\sum_{j\neq i}\tilde{c}_j - 1\right)}{2 + b(N-2)},$$

where, as in the baseline model, $p^\dagger$ is the choke price from the entrant's standpoint, that is the maximum price at which it can sell a positive quantity if incumbents produce the pre-entry equilibrium outputs.[22]

Absent entry barriers (i.e., for $k = 0$), firm $i$ would enter for all $c_i < c_0^*$, with

$$c_0^* = \frac{4 - b(2 + b\hat{c}(c_0^*)(N-1) - 2\sum_{j\neq i}\tilde{c}_j)}{(2-b)(2+b(N-1))} = p^\dagger + \frac{b^2(N-1)(p^\dagger - \hat{c}(c_0^*))}{(2-b)(2+b(N-1))} > p^\dagger,$$

since $p^\dagger > \hat{c}(c_0^*)$ whenever $q_i^*(\cdot) > 0$. Hence, maximizing the expected entrant's output requires raising entry barriers.[23]

It is straightforward to see that the incumbents are always harmed by entry — i.e., each incumbent's expected profit is maximized at $c^* = 0$.

The entrant's expected profit is given by

$$\begin{aligned} \pi_i^*(c^*) &\triangleq F_i(c^*)\mathbb{E}[(q_i^*(c_i, c^*))^2|c_i < c^*] - k(c^*) \\ &= F_i(c^*)\left[(\mathbb{E}[q_i^*(c_i, c^*)|c_i < c^*])^2 + \mathbb{V}[q_i^*(c_i, c^*)|c_i < c^*]\right] - k(c^*), \end{aligned}$$

where $k(c^*)$ is the entry cost that implements $c^*$. At $c^* = c_0^*$: $k(c_0^*) = \frac{\partial k(c^*)}{\partial c^*}|_{c^*=c_0^*} = 0$, and we find

$$\left.\frac{\partial \pi_i^*(c^*)}{\partial c^*}\right|_{c^*=c_0^*} = -\frac{2b^2 f(c_0^*)(N-1)(b - 2(1-\hat{c}(c_0^*)) + b(N-2)\hat{c}(c_0^*) - b\sum_{j\neq i}\tilde{c}_j)^2}{(2-b)^3(2+b(N-1))^3} < 0,$$

from which we can conclude that restricting entry also increases the entrant's (ex-ante) expected profit.

**Welfare.** Consumer surplus is given by

$$S(q_1, ..., q_N) \triangleq \frac{1}{2}\sum_h q_h^2 + \frac{b}{2}\sum_{h,j=1,...,Nj\neq h} q_h q_j.$$

[22] Formally, absent entry, the expected aggregate quantity produced by incumbents is

$$\sum_j \mathbb{E}[q_j^*(c_j)] = \frac{N - 1 - \sum_j \tilde{c}_j}{2 + (N-2)b},$$

and so $p^\dagger = 1 - b\sum_j \mathbb{E}[q_j^*(c_j)]$.

[23] It is easy to check that, as in the base model, also expected industry output is maximized at $c^* = p^\dagger$, though, products being differentiated, in this model aggregate output is a variable of more limited interest.

Taking expectations, as $\mathbb{E}[q_i^2] = (\mathbb{E}[q_i])^2 + \mathbb{V}[q_i]$, we can see that consumers are risk-lover. Despite entry unambiguously increases the output variance, restricting entry may still benefit consumers, as we show in the following example.

Consider $N = 2$ and a deterministic incumbent's cost $c \in [0, 1/2]$. The entrant's cost is uniformly distributed over $[0, 1]$ — i.e., $F_i(c_i) = c_i$. Then, consumer surplus is maximized at

$$c_S^* = \frac{64 - 16b^2(3 - b(1-c)) - 2b\sqrt{\Delta_S}}{64 - 44b^2 + b^4},$$

where

$$\Delta_S \triangleq b^6(1-c)^2 - 8b^5(1-c) + 12b^4(3 - 2c(2-c)) + \\ - 32b^3(1-c) - 16b^2(5 - 7c(2-c)) + 64(3 - 4c(2-c)) > 0.$$

This cutoff can be lower than the one under unrestricted entry,

$$c_0^* = \frac{4(2 - b(1-c))}{8 - b^2} \in (0, 1),$$

as shown in Figure 6 below. In particular, $c_S^* < c_0^*$ provided $c$ is not too large and $b$ is large enough.

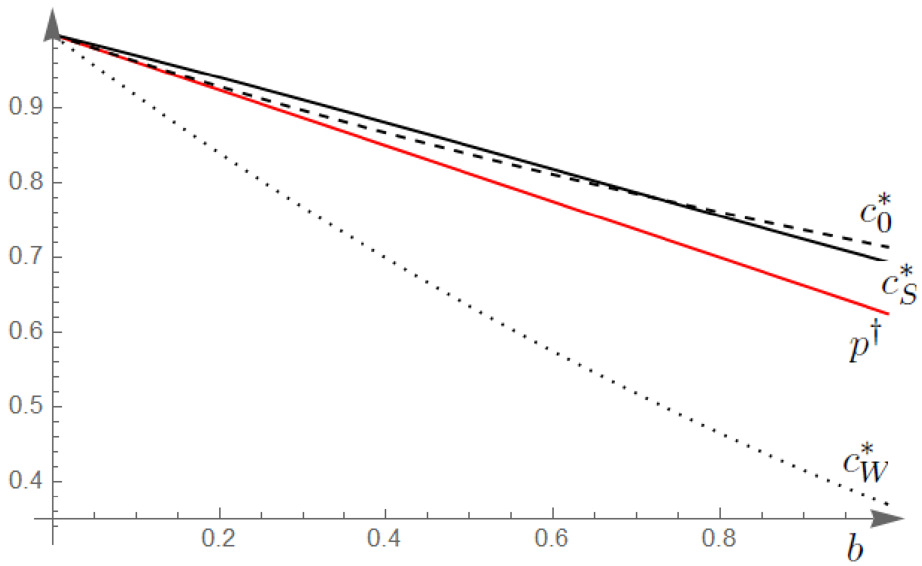


Figure 6: Optimal cutoffs as functions of $b \in [0, 1]$, for $c = 1/4$.

This figure also shows that the result $c_W^* < p^\dagger < c_S^*$ holds for all $b \in [0, 1]$. Total welfare is

$$W(q_1, ..., q_N) \triangleq \sum_h q_h - \frac{1}{2}\sum_h q_h^2 - \frac{b}{2}\sum_{h,j=1,...N, j\neq h} q_h q_j - \sum_h c_h q_h,$$

and, in this example, it is maximized at

$$c_W^* = \frac{2(96 - 8b(8 + b - b^2(1-c) - 8c) - b\sqrt{\Delta_W})}{3(64 - 12b^2 + b^4)} < p^\dagger,$$

where

$$\Delta_W \triangleq 9b^6(1-c)^2 - 24b^5(1-c) - 4b^4(23 - 26c(2-c)) + 352b^3(1-c)+$$
$$+16b^2(3 - 17c(2-c)) - 1280b(1-c) + 64(19 - 4(2-c)c) > 0.$$

# D Further material

## D.1 The example with a Beta distribution

Suppose $c_i \in [0,1]$ is distributed according to a cdf $F_i(c_i) = c_i^\alpha$ with $\alpha > 0$, so that $\hat{c}(c^*) = \frac{\alpha}{1+\alpha}c^*$. We show that consumer surplus and total welfare are single peaked w.r.t. $c^*$.

We have:

$$\frac{\partial \mathbb{E}[Q^2]}{\partial c^*} = \frac{4\alpha(2+\alpha) + (1+N)^2}{4(1+\alpha)^2(1+N)^2}(c^*)^2 + \frac{2(N - \sum_{j\neq i}\tilde{c}_j)}{(N+1)^2}c^* +$$
$$+\frac{(1+\sum_{j\neq i}\tilde{c}_j)[2N(N - \sum_{j\neq i}\tilde{c}_j) - \sum_{j\neq i}\tilde{c}_j - 1]}{N^2(N+1)^2}.$$

Since this derivative is U-shaped in $c^*$, and positive at $c_0^* = 0$, there are two possible cases: *(i)* the first-order condition $\frac{\partial \mathbb{E}[Q^2]}{\partial c^*} = 0$ has no solutions (i.e., the function $\mathbb{E}[Q^2]$ has no stationary points): in this case, $\frac{\partial \mathbb{E}[Q^2]}{\partial c^*} > 0$ for all $c^* \in [0,1]$, and so expected consumer surplus is maximized under unrestricted entry; or *(ii)* the first-order condition admits two roots: in this case, only the first root satisfies the second-order condition and is the global maximum point.

Assuming that all $N-1$ incumbents have the same (non-random) cost $c \in [0, 1/2]$, the smallest solution to the first-order condition is

$$c_S^* = \frac{2(1+\alpha)^2\left[2c + 2(1-c)N - \sqrt{4(N - c(N-1))^2 + \frac{(1+c(N-1))(4\alpha(2+\alpha)+(1+N)^2)(1-2N^2+c(N-1)(1+2N))}{(1+\alpha)^2N^2}}\right]}{4\alpha(2+\alpha) + (1+N)^2}$$

Implementing $c_S^*$ is feasible and requires a positive entry cost if and only if $c_S^* < c_0^*$, where, under this model specification, it is easy to find

$$c_0^* = \frac{2(1+\alpha)(1+(N-1)c)}{1+(1+2\alpha)N}.$$

Figure 2 shows that $c_S^* < c_0^*$ if $c$ is sufficiently small and/or $\alpha$ is large enough.

Turning to total welfare, assuming that the incumbents have i.i.d. costs, with average cost $\tilde{c}$, we also get

$$\frac{\partial \mathbb{E}[W(\cdot)]}{\partial c^*} = \frac{3 - 4\alpha(2+\alpha) + 6N + 8\alpha(2+\alpha)N + (3 + 8\alpha(2+\alpha))N^2}{8(1+\alpha)^2(1+N)^2}[c^*]^2 - \frac{2 + N + (2N^2 + N - 3)\tilde{c}}{(N+1)^2}c^* +$$
$$+\frac{1 + 2N - \tilde{c}(N-1)(\tilde{c} - 2(N^2 + 3N + 1) + \tilde{c}_j N(4N^3 - 2N^2 - 14N - 5)}{2N^2(N+1)^2}.$$

This derivative is U-shaped in $c^*$, and positive at $c_0^* = 0$. By the same arguments explained for consumer surplus, this establishes that also $\mathbb{E}[W]$ is single-peaked w.r.t. $c^*$.

## D.2 All entrants game

In this section we provide the analytical details for the material that has been used but not shown in the all entrants game.

### D.2.1 Uniform distribution

Suppose that $c \sim \mathcal{U}[0, \bar{c}]$. Then,

$$\check{c} = \frac{4}{3+N} > p^{\dagger} = \frac{2}{1+N},$$

$$c_W = \frac{8(1+N)(1+N+N^2) - 4N\sqrt{(N-1)(3+15N+2N^2(7+2N))}}{(2+N)(2+3N+8N^2+3N^3)} < p^{\dagger},$$

and

$$c_S = \frac{8(N^2(2+N)-1) - 4N\sqrt{1+N(N-2)(2N^2-1)}}{N^2(1+N)(9+N)-4} > p^{\dagger},$$

with $c_S < \check{c}$ for all $N \geq 3$.

### D.2.2 Two potential entrants

We show that, for $N = 2$, the maximum points of the welfare objectives are such that $c_W^* \leq c_Q^*$, the inequality being strict whenever $\bar{c} > p^{\dagger} = \frac{1+\check{c}}{2}$.[24]

The first-order condition for expected output maximization simplifies to

$$\frac{\partial \mathbb{E}[Q_n^*(c^*)]}{\partial c^*} = f(c^*)\left[\left(1 - \frac{2}{3}F(c^*)\right)(1 - \hat{c}(c^*)) - \left(1 - \frac{1}{3}F(c^*)\right)(c^* - \hat{c}(c^*))\right], \tag{20}$$

whereas the first-order condition for expected total welfare maximization can be written as

$$\frac{\partial \mathbb{E}[W(c^*)]}{\partial c^*} = \frac{f(c^*)}{36}(1 - c^*)\left[27(1 - c^*) - 22[1 - \hat{c}(c^*)]F(c^*)\right]. \tag{21}$$

Suppose that both $\mathbb{E}[Q_n^*(\cdot)]$ and $\mathbb{E}[W(\cdot)]$ are single peaked w.r.t. $c^*$, and maximized at $c_Q^*$ and $c_W^*$, respectively. Then, for all $\bar{c} > p^{\dagger}$, both expected output and total welfare admit an interior maximum, and, using (20) into (21), $c_W^* < c_Q^*$ if and only if

$$\left.\frac{\partial \mathbb{E}[W(\cdot)]}{\partial c^*}\right|_{c^*=c_Q^*} < 0 \quad \Longleftrightarrow \quad 9F(c^*)[2 - \hat{c}(c^*) - c^*] - 22[1 - \hat{c}(c^*)]F(c^*) < 0$$

$$\Longleftrightarrow \quad 4 + 9c^* - 13\hat{c}(c^*) > 0,$$

which is always true.

**Example 1.** Suppose that $c_i \sim \mathcal{U}[0, \bar{c}]$ for $i = 1, 2$. In this environment, we can show that there is a one-to-one relation between the entry cost $k$ and the equilibrium cutoff $c^*$, and that expected output and total welfare are single-peaked in $c^*$.

Anticipating that its rival (firm $j$) enters if and only if $c_j < c_j^*$, firm $i$'s indifference condition between entering or not the market, given the entry cost $k$, is as follows:

$$[1 - F(c_j^*)]\left[\frac{1 - c_i^*}{2}\right]^2 + F(c_j^*)\left[\frac{1}{2}\left(1 - c_i^* - \frac{1 + \hat{c}_i(c_i^*) - 2\hat{c}_j(c_j^*)}{3}\right)\right]^2 = k.$$

Imposing symmetry — i.e., $c_i^* = c_j^* = c^*$so that $\hat{c}_i(c_i^*) = \hat{c}_j(c_j^*) = \hat{c}(c^*) = c^*/2$ — this condition

[24] Recall that for $N = 2$ expected consumer surplus is maximized at $c_S^* = \bar{c}$ for all $\bar{c} \leq \check{c}$.

boils down to

$$\frac{1}{4} - \frac{(2-c^*)(10-11c^*+36\bar{c})c^*}{144\bar{c}} = k.$$

It can be easily seen that the left-hand side is decreasing in $c^*$ for all $c^* \in [0, \bar{c}]$ and $\bar{c} < \check{c} = \frac{4}{5}$. This shows that, for any entry cost $k \geq 0$, there is a corresponding equilibrium cutoff $c^*(k)$, decreasing in $k$. For all $\bar{c} > p^\dagger = \frac{2}{3}$, substituting $c_Q^* < \bar{c}$ into the left-hand side of this equilibrium condition yields the corresponding value of $k > 0$ that implements $c_Q^*$ (same applies to $c_W^*$, whenever $\bar{c} > c_W = \frac{5}{16}$).

The expected output is such that

$$\frac{\partial \mathbb{E}[Q_n^*(c^*)]}{\partial c^*} \propto \frac{(c^*)^2}{2\bar{c}} - \left(1 + \frac{2}{3\bar{c}}\right) c^* + 1,$$

hence (by the arguments of the previous section) it is single peaked w.r.t. $c^*$, and maximized at

$$c_Q^* = \frac{2}{3} + \bar{c} - \frac{1}{3}\sqrt{4 - 6\bar{c} + 9\bar{c}^2},$$

for all $\bar{c} > \frac{2}{3}$.

Expected total welfare is also single-peaked in $c^*$, as

$$\frac{\partial \mathbb{E}[W(\cdot)]}{\partial c^*} \propto 11(c^*)^2 - (27\bar{c} + 22)c^* + 27\bar{c},$$

and it is maximized at

$$c_W^* = \frac{1}{22}\left(22 + 27\bar{c} - \sqrt{484 + 729\bar{c}^2}\right),$$

for all $\bar{c} > \frac{5}{16}$.

### D.2.3 Three potential entrants

We show that, for $N = 3$, the maximum points of the welfare objectives are such that $c_W^* \leq c_Q^* \leq c_S^*$, the first inequality (resp. both inequalities) being strict whenever $\bar{c} > p^\dagger$ (resp. $\bar{c} > c_S$).

Taking the first-order condition for expected output maximization yields

$$c_Q^* = \frac{6 - F(c_Q^*)[8 - 2\hat{c}(c_Q^*)(2 - F(c_Q^*)) - 3F(c_Q^*)]}{6 - (4 - F(c_Q^*))F(c_Q^*)},$$

with $c_Q^* < \bar{c}$ if and only if $\bar{c} > p^\dagger = \frac{1+2\tilde{c}}{3}$.

Differentiating expected consumer surplus w.r.t. $c^*$, we find

$$\frac{\partial \mathbb{E}[S(\cdot)]}{\partial c^*} \propto \frac{3}{4}(1-c^*)^2 - \frac{1}{3}(1-\hat{c}(c^*))(1-c^*)F(c^*) - \frac{1}{48}(1-\hat{c}(c^*))(3-\hat{c}(c^*)-2c^*)F(c^*)^2.$$

Evaluating this derivative at $c^* = c_Q^*$ derived above gives

$$\left.\frac{\partial \mathbb{E}[S(\cdot)]}{\partial c^*}\right|_{c^*=c_Q^*} \propto \frac{(1-\hat{c}(c^*))^2 F(c^*)^2 (156 - 128F(c^*) - 20F(c^*)^2 + 16F(c^*)^3 + 3F(c^*)^4)}{48(6-(4-F(c^*))F(c^*))^2} > 0.$$

This shows that, if both $\mathbb{E}[Q_n^*(c^*)]$ and $\mathbb{E}[S(\cdot)]$ are single-peaked w.r.t. $c^*$, then $c_Q^* < c_S^*$ for all $\bar{c} > c_S$ (i.e., whenever both objectives admit an interior maximum).

Finally, turning to total welfare, we have

$$\frac{\partial \mathbb{E}[W(\cdot)]}{\partial c^*} \propto \frac{9}{8}(1-c^*)^2 - \frac{11}{6}(1-\hat{c}(c^*))(1-c^*)F(c^*) + \frac{25}{96}(1-\hat{c}(c^*))(3-\hat{c}(c^*)-2c^*)[F(c^*)]^2.$$

Evaluating this derivative at $c^* = c^*_Q$ gives

$$\left.\frac{\partial \mathbb{E}[W(\cdot)]}{\partial c^*}\right|_{c^*=c^*(Q)} = -\frac{((1-\hat{c}(c^*))^2 F(c^*)^2(1596-3200F(c^*)+2380F(c^*)^2-752F(c^*)^3+75F(c^*)^4)}{96(6-(4-F(c^*))F(c^*))^2} < 0.$$

If also $\mathbb{E}[W(\cdot)]$ is single-peaked w.r.t. $c^*$, this result implies that $c^*_Q > c^*_W$ whenever both $\mathbb{E}[Q^*(c^*)]$ and $\mathbb{E}[W]$ admit an interior maximum — i.e., for all $\bar{c} > p^\dagger$.

**Example 2.** In a symmetric equilibrium where each firm $i$ enters if and only if $c_i < c^*$, for any entry cost $k$, the cutoff $c^*$ solves the indifference condition

$$[1-F(c^*)]^2\left[\frac{1-c^*}{2}\right]^2 + 2F(c^*)[1-F(c^*)]\left[\frac{2+\hat{c}(c^*)}{6} - \frac{1}{2}c^*\right]^2 + \\ +[F(c^*)]^2\left[\frac{1+\hat{c}(c^*)}{4} - \frac{1}{2}c^*\right]^2 = k.$$

Considering the uniform distribution $c \sim \mathcal{U}[0,\bar{c}]$, this condition boils down to

$$\frac{(2-c^*)(c^*)^2(26-25c^*) - 8\bar{c}(2-c^*)(10-11c^*)c^* + 144\bar{c}^2(1-c^*)^2}{576\bar{c}^2} = k.$$

It can be easily seen that the left-hand side of this condition is decreasing in $c^*$ for all $c^* \in [0,\bar{c}]$ and $\bar{c} < \check{c} = \frac{2}{3}$. This shows that, for any entry cost $k \geq 0$, there is a corresponding equilibrium cutoff $c^*(k)$, decreasing in $k$.

We then have

$$\frac{\partial \mathbb{E}[Q^*_n(c^*)]}{\partial c^*} \propto -\frac{(c^*)^3}{2\bar{c}^2} + \frac{3(1+2\bar{c})}{4\bar{c}^2}(c^*)^2 - \left(\frac{3}{2}+\frac{2}{\bar{c}}\right)c^* + \frac{3}{2},$$

$$\frac{\partial \mathbb{E}[S]}{\partial c^*} \propto -\frac{5}{192\bar{c}^2}(c^*)^4 + \frac{1-2\bar{c}}{12\bar{c}^2}(c^*)^3 + \frac{4\bar{c}(2+3\bar{c})-1}{16\bar{c}^2}(c^*)^2 - \frac{2+9\bar{c}}{6\bar{c}}c^* + \frac{3}{4},$$

$$\frac{\partial \mathbb{E}[W(\cdot)]}{\partial c^*} \propto \frac{125}{384\bar{c}^2}(c^*)^4 - \frac{25+22\bar{c}}{24\bar{c}^2}(c^*)^3 + \frac{25+4\bar{c}(22+9\bar{c})}{32\bar{c}^2}(c^*)^2 - \left(\frac{9}{4}+\frac{11}{6\bar{c}}\right)c^* + \frac{9}{8}.$$

The three objectives are single-peaked in $c^*$ for all $\bar{c} < \frac{2}{3}$. The expressions of the maximum points, which are shown in Figure 3, are very cumbersome, hence omitted here.