On Bertrand Supergames

Stefano Antonio Bolatto,[§] Luca Lambertini[§] and Giuseppe Pignataro[#]

Department of Economics, University of Bologna

§ Via San Giacomo 3, 40126 Bologna, Italy

Piazza Scaravilli 2, 40126 Bologna

stefano.bolatto@unibo.it; luca.lambertini@unibo.it; giuseppe.pignataro@unibo.it

March 15, 2023

Abstract

This paper provides a complete characterization –across the entire space of product differentiation– of the optimal one-shot punishment strategy that can sustain collusion at the profit frontier in infinitely repeated Bertrand games with discounting. Specifically, we consider the optimal design of the stick-and-carrot punishment à la Abreu (1986) that can credibly sustain a collusive phase at any level of product differentiation, given the number of cartel members. We then identify the lower envelop of the critical discount factor as well as the conditions on the marginal production cost under which the set of punishment prices is admissible. Our analysis spotlights a fundamental tradeoff, as any increase in the number of cartel members raises the efficiency of the punishment, but reduces the stability of the cartel.

JEL Codes: C73; D43; L13

Keywords: repeated games; price collusion; product differentiation; cartel stability; optimal punishment

1 Introduction

While economic research has long established the conditions for the stability of collusive agreements in markets for homogeneous goods, much remains to be said about the overall sustainability of cartels involving producers of differentiated goods. This holds true for both cases of Cournot (quantity) and Bertrand (price) competition, although the theory of supergames has contributed greatly to our understanding of the formation and stability of collusive phases in different situations of strategic oligopolistic interaction.[1] Since Friedman (1971), we know that when firms engage in a supergame, i.e., play the same game repetitively, any feasible payoff that Pareto dominates a Nash equilibrium of the constituent stage game qualifies as a subgame perfect equilibrium (SPE) of the infinitely repeated game. The only condition is that players are sufficiently patient, implying that a critical level of the discount factor exists, above which the threat of a simple grim trigger strategy (i.e., the ever-lasting reversion to the Nash equilibrium of the stage game) is sufficient to stabilize the collusive agreement.

The seminal work of Abreu (1986) has further extended our understanding of the dynamics of collusion by identifying a class of (pure) strategies in repeated games with discounting that outperforms the grim trigger strategy, leading to a lower critical discount factor able to ensure cartel stability. The punishment proposed by Abreu (1986), commonly known as 'stick-and-carrot', takes the form of a two-stage punishment, such that immediately after any individual defection from the cartel, all firms engage in a one-off price war aimed at inflicting mutual losses, so large that right after they have the incentive to resume and then stick to their tacit cooperative strategies over all subsequent periods. This leads to identifying the optimal punishment

[1] For an introduction to the theory of supergames, and a few relevant applications, see Aumann (1981), Funderberg and Maskin (1986), Segerstrom (1988) or Lambertini (1997), among others.

as the one that induces the entire industry to return to the most collusive sustainable configuration after only a single period (Shapiro, 1989).

Limited to the case of an infinitely repeated Cournot game, Abreu (1986) has shown that the stick-and-carrot can achieve the same level of efficiency as the *minmax* strategy outlined by von Neumann (1928). When players minmax each other upon the initial unilateral deviation from the cartel, they receive a zero payoff in continuation equilibria. This penal code is notoriously not globally optimal due to the lack of subgame perfection. Nonetheless, it is a valuable benchmark, as it induces the critical discount factor at its lowest attainable level, provided that a less-than-zero continuation payoff is not admitted. At least for the case of homogeneous good markets, the stick-and-carrot may not only attain this threshold, since it can also induce a zero payoff in the continuation game, but also proves to satisfy the additional requirement of subgame perfection.

Similar considerations also hold under Bertrand competition, although the existing literature has never reached the point of proposing a comprehensive analysis of the optimal punishment in a generalised framework with both product differentiation and an arbitrary number of players. Several authors (e.g. Lambson, 1995) have examined the feasibility of the stick-and-carrot in repeated Bertrand games with discounting, in which cartel members produce a homogeneous good. Others have extended this analysis to the case of product differentiation, but restricted to duopoly markets (see Ross, 1992; Häckner, 1996; Lambertini and Sasaki, 1999, 2002; Lambertini et al., 2002, 2003). Finally, some other papers (e.g. Majerus, 1988) have proposed a study of the sustainability of collusive agreements among a variable number of firms and over the entire space of production differentiation, but limited to the adoption of the grim trigger strategy of Friedman (1971) in place of the penal code advocated by Abreu (1986).

This paper aims at filling this gap in the literature. We provide here an exhaustive analysis of the conditions for cartel stability in Bertrand supergames, when the collusive phase is maintained by the threat of a punishment strategy consistent with Abreu (1986). Specifically, we consider an infinite repetition of a stage game in which a generic number of possibly differentiated good producers must decide whether to cooperate or compete on the basis of their pricing strategy. We restrict our attention to the case of collusion at the profit frontier ('full' collusion), i.e., we assume that the symmetric cooperative strategy requires all firms to set the monopoly price. Products are horizontally differentiated, and the (linear) demand system is the one firstly introduced by Bowley (1924) and then extended by Dixit (1979) and Singh and Vives (1984).[2] The set of model parameters, therefore, includes the market size, the degree of product differentiation, the number of active firms, and the symmetric and constant marginal cost of production.

We consider four possible configurations of a punishment scheme consistent with the penal code described in Abreu (1986), thereby identifying four different pairs of critical discount factors and symmetric punishment prices, which simultaneously satisfy the required cartel stability, participation, and incentive compatibility constraints. These four alternative scenarios are identified as follows. First, the deviation strategy of the initial defector from the cartel may leave all other loyal members either with a positive or nil residual demand. It has been well-known since Deneckere (1984) and Majerus (1988) that unilateral deviations from full collusion may take one of these two forms, based on the degree of product differentiation and the number of firms. Second, in the symmetric price war that follows the initial defection,

[2]This differentiates our approach from a large bulk of literature, in which product differentiation is shaped through a standard Hotelling model with two firms only (e.g. Chang 1991, 1992; or Friedman and Thisse, 1993). As for the type of product differentiation, we refer the reader to Häckner (1994) for an investigation of collusion under vertical differentiation, which we do not cover in this paper.

firms may dispose or not of a credible strategy to deviate from the punishment price they are supposed to set, yielding a strictly positive payoff in the stage game.[3]

By combining the deviation regime from collusion with the existence or not of a profitable deviation from the punishment, we fully characterize the strategies among which a potential defector may choose the one to pursue according to individual rationality, and given the extent of product differentiation and the numbers of firms operating in the marketplace. For each strategy, we pin down the retaliation price prescribed by Abreu (1986) to sustain collusion, and the associated critical discount factor. Correspondingly, we assess the conditions on firms' marginal cost required for such prices to be feasible. We therefore derive the lower envelope of the marginal cost over the entire space of product differentiation, above which a punishment scheme complying with the penal code of Abreu (1986) can be used to maintain collusion at the profit frontier.[4]

Assuming the marginal cost requirement is met, we assess the stability of the cartel under the stick-and-carrot by mapping the corresponding thresholds of the discount factor, i.e., the critical levels associated with each of the scenarios presented above. We show that duopoly and triopoly markets are special cases, as the associated lower envelopes differentiate themselves from

[3] As pointed out in Abreu (1988), "optimality [of the punishment scheme] might require that a deviant "cooperate" (in a one-period sense) in his own punishment". When a profitable deviation from the retailation price exists, a zero continuation payoff cannot be granted, implying that the the stick-and-carrot is unable to attain the lowest feasible level of the critical discount factor, identified by the minmax strategy.

[4] Below this threshold, full collusion can be sustained only by a less efficient penal code, such as the grim trigger strategy of Friedman (1971, 1986). Alternatively, firms may resort to 'partial collusion', thereby using the stick-and-carrot to sustain a cooperative strategy in which the collusive price is set at the highest possible level above the Nash Equilibrium price (but below the monopoly price) that is allowed by their time preferences, as summarized by their relevant discount factor.

the one applying in the general case with four or even more players. The reason is that, moving from the homogeneous good case to increasing levels of product differentiation, the sequence of the relevant scenarios for the construction of the optimal punishments changes as we first switch from two to three cartel members, and then from three to four or even more.

The most striking result of our analysis is that fundamental trade-off exists whenever increasing the number of firms involved in a collusive agreement. On the one hand, it increases the space of product differentiation over which the stick-and-carrot is able to ensure a zero continuation payoff by preventing any profitable deviation from the punishment, and thereby attains the lowest feasible threshold of the discount factor, i.e., the same as under the min-max strategy. On the other hand, the increased efficiency of the punishment scheme comes at a cost, here in the form of a higher level of the critical discount factor typically induced by any enlargement of the cartel size, regardless of the specific penal code in use.

The paper is organized as follows. In Section 2 we revise the conditions for cartel stability in repeated Bertrand games with discounting, and we introduce a liner demand system in the presence of possibly differentiated goods. In Section 3 we describe the deviation strategies from the collusive path, we assess cartel stability under the grim trigger strategy, and we finally characterize the optimal punishments as a function of the model parameters. Section 4 reports some conclusive remarks.

2 Preliminaries

The building blocks of the model can be laid out in the following terms. Consider a non-cooperative one-shot game involving $n \geq 2$ symmetric firms each selling a single variety of a differentiated good, and choosing its price $p_i \in [0, \overline{p}]$. The profit function of firm i is $\pi_i(p_i, \mathbf{p}_{-i})$, in which $\mathbf{p}_{-i}$ is the vector of the $n-1$ rivals' prices. $\pi_i(p_i, \mathbf{p}_{-i})$ is assumed to be continuous, twice differentiable, strictly concave and single-peaked in p_i, for any admissible $\mathbf{p}_{-i}$.

Now, define as $\pi(\mathfrak{p})$ the profits accruing to every firm in correspondence of the symmetric outcome in which all firms play the same price $\mathfrak{p}$. Then, assume $p^M = \arg\max_{\mathfrak{p}} \pi(\mathfrak{p})$ is a singleton, and denote the corresponding individual (industry) profits as π^M $(\Pi^M = n\pi^M)$. Moreover, let (i) $\pi^m \equiv \sup \pi_i(x_i)|_{n=1}$ be the pure monopoly profit accruing to firm i when all of the $n-1$ rivals either shut production down, do not sell or quit the market, so that firm i may indeed set its own price undisturbed.

Additionally, suppose the game produces a unique Nash equilibrium in pure strategies, in correspondence of which all firms play the price $p^N < p^M$, yielding an individual profit equal to $\pi^N < \pi^M$, so that the Nash equilibrium outcome lies inside the frontier of industry profits. As a last step, assume that there exists a unique level of the price, $\mathsf{p} \in [0, \overline{p}]$ such that $\pi(\mathsf{p}) = 0$. At this price, if adopted by all firms, individual and industry profits are driven to zero or, equivalently, by adopting p all firms minmax each other.

This structure can be the constituent stage game of the infinitely repeated game over discrete time $t = 0, 1, 2, ...\infty$, in which firms adopt the common and time invariant factor $\delta \in [0, 1]$ to discount future profits. In this regard, we assume the existence of a price $\underline{p} \in [0, \overline{p}]$ such that

$$\pi_i(\underline{p}) + \frac{\delta \pi^M}{1-\delta} \leq 0 \qquad (1)$$

where $\pi_i(\underline{p})$ measures firm i's profits when all firms play the price $\underline{p}$. This amounts to saying that the industry-wide adoption of $\underline{p}$ at any time t annihilates the discounted profit flow associated with the truncated portion of the supergame starting at t. Since $\pi^M > 0$, then necessarily $\pi_i(\underline{p})\big|_{n=1} < 0$ and, if $\partial^2 \pi_i / \partial p_i \partial p_j > 0$, whereby prices behave as strategic complements (Bulow *et al.*, 1985), we may expect $\underline{p}$ to lie below marginal production cost, but not below zero. Indeed, in the remainder of the paper, we will come back to the possibility that $\underline{p} \notin (0, \overline{p}]$ and discuss the consequences of having a corner at $\underline{p} = 0$ on collusion.

Firms want to stabilise tacit collusion delivering per-period individual profits $\pi^C \in \left(\pi^N, \pi^M\right]$, through the adoption of a price $p^C \in \left(p^N, p^M\right]$. Let $\pi^D\left(p^C\right) > \pi^C$ be the profit delivered by the unilateral deviation from the collusive path, which, as we know from Deneckere (1983, 1984)[5] is piecewise linear in the cheated firms' collusive price as it is sensitive to the degree of product differentiation.

In general, the stability of collusion requires:

$$\frac{\pi^C}{1-\delta} \geq \pi^D(p^C) + \Pi^P \tag{2}$$

where Π^P is a compact representation of the continuation payoff. The exact nature and structure of this continuation payoff depends on the type of punishment being envisaged to deter deviations from the collusive path.

The most commonly used is the infinite reversion to the Nash equilibrium of the constituent stage game, as in Friedman (1971), whereby the discounted continuation payoff is $\Pi^P = \delta\pi^N((1-\delta)$. The adoption of the Nash threat ensures subgame perfection and allows to identify the following threshold of the discount factor, above which collusion is perpetually stable:

$$\frac{\pi^C}{1-\delta} \geq \pi^D(p^C) + \frac{\delta\pi^N}{1-\delta} \Longleftrightarrow \delta \geq \frac{\pi^D(p^C) - \pi^C}{\pi^D(p^C) - \pi^N} \equiv \delta_F \tag{3}$$

where subscript F stands for *Friedman*. It is worth noting that, if firms sell a homogeneous good produced at the same common and constant marginal cost $c \in [0, a)$, then

$$\delta_F|_{\pi^N=0} = \frac{\pi^D(p^C) - \pi^C}{\pi^D(p^C)} \tag{4}$$

because $p^N = c$ and therefore $\pi^N = 0$. In this case, the Nash reversion replicates the performance associated with the adoption of minmax strategies

[5]See also Majerus (1988), Ross (1992), Rothschild (1992) and Albæk and Lambertini (1998), among others.

à la von Neumann, whereby $\Pi^P = 0$ and the critical threshold δ_{vN} coincides with (4).

Observe that $\delta_F|_{\pi^N=0} = \delta_{vN}$ identifies the lowest possible level of $\delta \in [0,1]$ at which firms may sustain forever a given collusion level at π^C. The reason is that $\Pi^P = 0$ is the lowest admissible value of the continuation payoff compatible with the generic firm's individual rationality constraint: should Π^P fall even slightly below zero, firms would quit the supergame.

Yet, with the exception represented by the aforementioned special case, the Nash reversion is unable to reproduce δ_{vN}. That is, in general, the Nash reversion is an inefficient punishment. However, one cannot literally invoke minmax strategies as, with the same exception, the minmax equilibrium does not coincide with the Nash equilibrium in a variable-sum game and therefore is not subgame perfect.

Efficiency and subgame perfection go hand in hand in the design of the stick-and-carrot punishment (Abreu, 1986, 1988) meeting the following set of constraints:

$$\pi^D(p^C) - \pi^C \leq \delta\left(\pi^C - \pi^P\right) \tag{5}$$

$$\pi^D(p^P) - \pi^P \leq \delta\left(\pi^C - \pi^P\right) \tag{6}$$

$$\pi^P + \frac{\delta\pi^C}{1-\delta} \geq 0 \tag{7}$$

where π^P is the one-shot punishment payoff and $\pi^D(p^P)$ is the payoff generated by the unilateral optimal deviation from the punishment price p^P. Inequality (5) must hold in order for the collusive path to be stable. Inequality (6) must be met for firms to implement the optimal punishment p^P. Constraint (7) is the participation constraint whereby the discounted continuation payoff cannot be negative. Assumption (1) ensures the existence of a price p^P such that participation constraint (7) can be satisfied (at least) at the margin.

The above representation of constraints (5-7) can be found in Abreu (1986, Lemma 17, p. 204) and reflects the scenario in which $\pi^D(p^P) > 0$, i.e., the unilateral deviation from the punishment price is profitable. However, from theorems 18-19 in Abreu (1986, p. 205), we know that the critical threshold for the stability of collusion is minimised when $\pi^D(p^P) = 0$, in which case (6) and (7) coincide except for a constant, so that one can solve either (5-6) or (5-7) w.r.t. δ and π^P to obtain

$$\begin{gathered}\delta \geq \frac{\pi^D(p^C) - \pi^C}{\pi^D(p^C)} \equiv \delta_A = \delta_{vN} = \delta_F|_{\pi^N=0} \\ \pi^P \leq \pi^C - \pi^D(p^C) < 0\end{gathered} \tag{8}$$

where subscript A stands for Abreu. In (8), we see that (i) the critical threshold of δ is indeed minimised, and (ii) the one-off punishment involves a negative profit whose amount, in absolute value, is at least as large as the net instantaneous incentive to unilaterally abandon collusion.

If instead $\pi^D(p^P) > 0$, one must solve (5-6) to obtain

$$\begin{gathered}\delta \geq \frac{\pi^D\left(p^C\right) - \pi^C}{\pi^D\left(p^C\right) - \pi^D\left(p^P\right)} \equiv \widehat{\delta}_A \\ \pi^P \leq \pi^C - \pi^D\left(p^C\right) + \pi^D\left(p^P\right) < 0\end{gathered} \tag{9}$$

with $\widehat{\delta}_A \in (\delta_A, \delta_F)$, and then check (7) to see that

$$\frac{\left[\pi^D\left(p^C\right) - \pi^D\left(p^P\right)\right]\pi^D\left(p^P\right)}{\pi^C - \pi^D\left(p^P\right)} > 0 \tag{10}$$

The procedure yielding the above solutions relies on taking the degree of collusion π^C as given, whereby the two unknowns to be determined are the critical level of the discount factor δ and the intensity of the punishment π^P. As soon as one specifies the functional forms of demand and cost conditions, one can alternatively (i) suppose firms collude along the frontier of industry profits, and solve the relevant system of inequalities w.r.t. δ and p^P, or (ii) suppose firms activate partial collusion at some $p^C \in \left(p^N, p^M\right)$ and solve the relevant system of inequalities w.r.t. p^C and p^P, thereby obtaining cartel and punishment price parametric in the discount factor.

One aspect deserving some additional attention before delving into the details of the ensuing models is the fact that $\pi^P < 0$ strictly requires the punishment price to lie below marginal cost, and this is true for homogeneous and differentiated goods alike. The critical side of the matter is that p^P cannot go below zero as this would invalidate the demand system. Therefore, the size of the admissible interval $[0, c)$ is bound to play a relevant role insofar as it puts an upper bound to the potential deterrence embodied in the stick p^P. More explicitly, any natural or artificial increase in marginal cost may have pro-collusive effects as it expands the admissible range of the punishment price (Lambertini and Sasaki, 2001). We will come back explicitly on this issue in the remainder of the paper.

2.1 Demand and technology

In order to make the model explicitly solvable, we assume that in each period the demand function for each variety or firm i has the same form as in Bowley (1924) and Singh and Vives (1984):

$$q_i = \frac{a(1-s) - p_i\left[1 + s(n-2)\right] + s\sum_{j \neq i}^{n-1} p_j}{(1-s)\left[1 + s(n-1)\right]} \quad \forall i = 1, 2, ...n \tag{11}$$

in which $a > 0$ is the choke price common to all varieties, and p_i and p_j are, respectively, the price of firm i and that of one of its $n-1$ rivals. Parameter $s \in [-1, 1]$ measures the degree of complementarity (if negative) or substitutability (if positive), between any two varieties. In the latter case, s is an inverse measure of product differentiation. The special case $s = 0$ portrays a situation in which firms' products are not interacting at all and each firm is a pure single-product monopolist on a separate market. Indeed, in what follows, we will confine our attention to $s \in (0,1]$ so as to discuss price collusion among substitutes.

Firms are endowed with the same production technology characterised by the common marginal cost $c \in (0,a)$. Later, we will investigate the scenario in which vertical separation prevails and downstream firms source a homogeneous intermediate good at a wholesale price $w \in (c,a)$ on the input market. For the moment, we stipulate that firms do not outsource anything, i.e., they are vertically integrated.

3 Full collusion

The first step consists in offering a brief reconstruction of the knowledge we have inherited from the initial debate investigating price collusion on the basis of Friedman's (1971) grim trigger strategies, in particular Deneckere (1983, 1984) Majerus (1988).

Suppose all firms are member of a cartel aiming at maximising collusive profits. The collusive price p^C is therefore equal to the monopoly price $p^M = (a+c)/2$, while individual quantities and profits are

$$q^C = \frac{(a-c)}{2\left[1+s(n-1)\right]} = q^M = \frac{Q^M}{n}\ ;\ \pi^C = \frac{(a-c)^2}{4\left[1+s(n-1)\right]} = \pi^M \qquad (12)$$

In particular, the profit π^C in eq. (12) corresponds to the fraction $1/n$ of the joint profit Π^M generated by the cartel as a whole.

In the Nash equilibrium of the constituent game, all firms price at

$$p^N = \frac{a(1-s)+c(1+s(n-2))}{2+s(n-3)} \in [c,a)\ \ \forall s \in (0,1],\ n \geq 2 \qquad (13)$$

The Nash equilibrium individual quantity and profits are

$$\begin{aligned} q^N &= \frac{(a-c)\left[1+s(n-2)\right]}{\left[2+s(n-3)\right]\left[1+s(n-1)\right]} \\ \pi^N &= \frac{(a-c)^2\left[1+s(n-2)\right](1-s)}{\left[2+s(n-3)\right]^2\left[1+s(n-1)\right]} \end{aligned} \tag{14}$$

Clearly, we have $p^M > p^N \geq c$, $q^N > q^M > 0$ and $\pi^M > \pi^N \geq 0$ for all $s \in (0,1]$ and $n \geq 2$.

3.1 Deviations from the collusive path

As shown by Deneckere (1984) and Majerus (1988), the unilateral deviation from a price-setting cartel locating itself on the frontier of industry profits can take two different forms depending on the degree of product differentiation. In particular, Majerus (1988, p. 296) shows that for any

$$s \in (0, \hat{s}) \,,\ \hat{s}(n) = \frac{n-3+\sqrt{n^2-1}}{3n-5} \in \left(2/3, \sqrt{3}-1\right] \ \forall n \geq 2 \tag{15}$$

the defecting firm adopts the best response strategy to deviate from the monopoly price p^M charged by the other $n-1$ cartel members:

$$p^D(p^M) = \frac{a\left[2+s(n-3)\right]+c(\left[2+s(3n-5)\right]}{4\left[1+s(n-2)\right]} \in (c,a) \tag{16}$$

The above price delivers the following deviation quantity and profits:

$$\begin{aligned} q^D(p^M) &= \frac{(a-c)(2+s(n-3))}{4(1-s)\left[1+s(n-1)\right]} \\ \pi^D(p^M) &= \frac{(a-c)^2\left[2+s(n-3)\right]^2}{16(1-s)\left[1+s(n-1)\right]\left[1+s(n-2)\right]} \end{aligned} \tag{17}$$

which are both positive for all $s \in (0, \hat{s})$ and $n \geq 2$. In turn, each of the $n-1$ cheated firms sells

$$q^{\ell} = \frac{(a-c)[2 - s^2(3n-5) + 2s(n-3)]}{4(1-s)\,[1+s(n-1)]\,[1+s(n-2)]} \tag{18}$$

which is also positive for all $s \in (0, \hat{s})$. Superscript ℓ stands for *loyal*.

For all $s \in [\hat{s}, 1]$, firm i's deviation price $p^D(p^M)$ would drive the loyal firms' outputs below zero, i.e.,

$$q^{\ell} = \frac{a(1-s) - p^M\,[1+s(n-2)] + s\,\left[(n-2)p^M + p^D(p^M)\right]}{(1-s)\,[1+s(n-1)]} \leq 0 \,\forall\, s \in [\hat{s}, 1] \tag{19}$$

Hence, if indeed $s \in [\hat{s}, 1]$ - or, if substitutability is high enough - the defecting firm is aware that deviating along its own best reply is unfeasible and that it must design an alternative deviation price accounting for (19). The resulting deviation price captures the entire demand existing at that price level while driving the opponents' residual demand down to zero:

$$p^{D\prime}(p^M) = \frac{a(2s-1)+c}{2s} \in (c, a)\ \forall\, s \in [\hat{s}, 1] \tag{20}$$

The adoption of $p^{D\prime}(p^M)$ makes the defector a spurious monopolist for all $s \in [\hat{s}, 1)$ since the resulting quantity and profits are strictly lower than pure monopoly profits (which, under constant returns to scale, are simultaneously equal to Π^M and π^m) except in the special case of product homogeneity, in which the present model collapses to the textbook Bertrand game where $p^N = c$. Also note that $p^{D\prime}(p^M) = p^M$ at $s = 1$, as in this case the defecting firm becomes a monopolist via an arbitrarily small discount $\varepsilon > 0$, which can be disregarded, on the pure monopoly price.

This deviation strategy delivers profits

$$\pi^{D\prime}(p^M) = \frac{(2s-1)(a-c)^2}{4s^2} \tag{21}$$

and a quantity $q^{D\prime} = (a-c)/2s$. It is easily proved that deviation prices (16) and (20) intersect at $s = \hat{s}$, with $p^{D\prime}(p^M) < p^D(p^M)$ for all $s \in (0, \hat{s})$ and $p^{D\prime}(p^M) > p^D(p^M)$ for all $s \in (\hat{s}, 1]$.

3.2 Cartel stability with grim trigger strategies

Suppose firms adopt the infinite reversion to the Bertrand-Nash equilibrium of the constituent, stage-game to deter deviations, as in Friedman (1971). Depending on the value of s, the critical threshold of the discount factor above which full collusion is stable forever can be written as

$$\delta_F = \frac{\pi^D(p^M) - \pi^M}{\pi^D(p^M) - \pi^N} \,\forall\, s \in (0, \hat{s},) \qquad \delta_F' = \frac{\pi^{D'}(p^M) - \pi^M}{\pi^D(p^M) - \pi^N} \,\forall\, s \in [\hat{s}, 1] \tag{22}$$

and, using the appropriate expressions, one may replicate the result originally derived by Majerus (1988):

$$\delta_F = \frac{[2 + s(n-3)]^2}{s^2 [17 + (n-10)n] + 8 [1 + s(n-3)]}, \;\forall\, s \in (0, \hat{s},)$$

$$\delta_F' = \frac{[2 + s(n-3)]^2 [s (s(2n-3) - n + 3) - 1]}{s^4 (n-1)^2 + [2 + s(n-3)]^2 [s (s(2n-3) - n + 3) - 1]}, \forall\, s \in [\hat{s}, 1] \tag{23}$$

While $\partial \delta_F / \partial s > 0$ for all $s \in (0, \hat{s})$, for $n = 2, 3, 4$, δ_F' reaches a peak at

$$s_F^{'} = \frac{2 \left[2(n-3) + \sqrt{2 [n(n+2) - 3]} \right]}{7(2n-3) - n^2} \tag{24}$$

which is equal to one at $n = 5$. This means that the lower envelope of δ_F and δ_F' is single-peaked in s for all $n = 2, 3, 4$, while it is monotone in the degree of product substitutability for all $n \geq 5$.

Finally, it is easily checked that $\lim_{s\rightarrow 1}\delta_F' = (n-1)/n$ and then also that the latter expression becomes equal to $1/2$ if $n=2$. This completes the reconstruction of full collusion with n firms and imperfect substitute goods as we know it from Majerus (1988), which boils down to the following

Proposition 1 *(Majerus, 1988) Full collusion in prices in a differentiated oligopoly is sustainable iff* $\delta \geq \delta_F^*$, *with* $\delta_F^* = \delta_F \, \forall \, s \in (0, \hat{s},)$ *and* $\delta_F^* = \delta_F' \, \forall \, s \in [\hat{s}, 1]$. *If the cartel is small enough,* δ_F^* *has a peak at* $s = s_F^{'}$; *otherwise, the critical threshold of the discount factor is monotonically increasing in the degree of product substitutability.*

3.3 Optimal punishments

If instead firms adopt the stick-and-carrot punishment as in Abreu (1986; see also Segerstrom, 1988), the relevant system of constraints is (5-7), with either $\pi^{DP}(p^P) = 0$ or $\pi^{DP}(p^P) > 0$. In what follows, profits from collusion and those generated by the unilateral deviation from the cartel path are the same as above, i.e., either $\pi^D(p^M)$ or $\pi^{D'}(p^M)$ depending on whether the cheating firm deviates along its best reply or to spurious monopoly. We shall consider the four possible scenarios, in the following sequence:

- $\pi^{DP}(p^P) = 0$, with the deviation from the cartel path either to monopoly or along the best reply (if so, leaving a positive market shares to loyal firms. Note that $\pi^{DP}(p^P) = 0$ implies that the adoption of the optimal punishment replicates the critical threshold of δ associated with minmax strategies, by driving to zero the continuation payoff.

- $\pi^D(p^P) > 0$, coupled with the same alternative deviations from collusion.

This gives rise to four different critical thresholds of the discount factor, and our task is to correctly identify their respective domains in the range of product substitutability, i.e., over $s \in (0,1]$, so as to identify their envelope for any value of s. In the four cases, one must also check the admissibility of the resulting punishments prices p^P, which shall not fall below zero (although, of course, they may indeed locate below marginal cost c).

3.3.1 Stick-and-carrot with zero continuation payoff

If $\pi^{DP}(p^P) = 0$ in (6), the system (5-7) can be written as follows. The first condition is related to cartel stability, and takes the form of either

$$\pi^D(p^M) - \pi^M \leq \delta\left(\pi^M - \pi^P\right) \tag{25}$$

or

$$\pi^{D'}(p^M) - \pi^M \leq \delta\left(\pi^M - \pi^P\right) \tag{26}$$

depending on the nature of the deviation from the cartel price. The two additional conditions are

$$-\pi^P \leq \delta\left(\pi^M - \pi^P\right) \tag{27}$$

$$\pi^P + \frac{\delta\pi^M}{1-\delta} \geq 0 \tag{28}$$

with the second and third constraints coinciding up to scalar, so that any punishment price p^P satisfying either inequality (27) or (28) will also satisfy the other. Given $\pi^P = (a - p^P)(p^P - c)/(1 + s(n-1))$, the above system therefore reduces to two inequalities in two unknowns, δ and p^P. The solution for δ is $\delta_A = \delta_{vN} = \delta_F|_{\pi^N=0}$ as in (8): with $\pi^{DP}(p^P) = 0$, the *stick-and-carrot* penal code indeed yields a continuation payoff which is endogenously nil, and the system reduces to

$$\pi^P \leq \pi^M - \pi^{\mathcal{D}}(p^M) < 0;\ \delta \geq \frac{\pi^{\mathcal{D}}(p^M) - \pi^M}{\pi^D(p^M)} \equiv \delta_{vN};\ \mathcal{D} = D, D' \tag{29}$$

while the associated critical threshold of the discount factor coincides with the one resulting from the infinite reversion to minmax strategies.

Now suppose the unilateral deviation from the collusive path takes place along the best reply, in such a way that the relevant system consists of (25) and (27). This delivers

$$\begin{aligned} \delta_{A0} &= \frac{s^2(n-1)^2}{[2+s(n-3)]^2} \\ p_0^P &\leq \frac{a+c}{2} - \frac{(a-c)\,[2+s(n-3)]\sqrt{(1-s)\,[1+s\,(n-2)]}}{4\,(1+s)\,[1+s\,(n-2)]} \end{aligned} \tag{30}$$

with $\delta_{A0} \in (0,1)$ and $p_0^P \in (0,c)$ for all

$$\begin{aligned} c > c_0^P &= \max\left\{0, \frac{a\left[2\left(\sqrt{\Phi}-1\right)\right] - s\left(2\,(1+s\,(n-2)) + (n-3)\sqrt{\Phi}\right)}{2\left(1+\sqrt{\Phi}\right) + s\left[2\,(1+s\,(n-2)) + (n-3)\sqrt{\Phi}\right]}\right\}; \\ \Phi &\equiv (1-s)\,[1+s\,(n-2)] > 0 \end{aligned} \tag{31}$$

as well as all $s \in (0, \hat{s},)$ and $n \geq 2$. Moreover, δ_{A0} is increasing and convex in s for all $s \in (0, \hat{s},)$ and all $n \geq 2$. The appearance of a zero in the subscript mnemonics for the fact that deviation from the punishment involves zero profits as it is convenient for the defector to shut down production.

If instead the deviation grants spurious monopoly to the defecting firm, the relevant set of conditions is composed by (26) and (27), whereby the critical threshold of δ above which full collusion is stable and the maximum punishment price sustaining it are

$$\begin{aligned} \delta'_{A0} &= \frac{s\,[3\,(1-s) + n(2s-1)] - 1}{(2s-1)\,[1+s\,(n-1)]} \\ p_0^{P'} &\leq \frac{a+c}{2} - \frac{(a-c)\sqrt{(2s-1)\,[1+s\,(n-1)]}}{2s} \end{aligned} \tag{32}$$

with $p_0^{P'} > 0$ for all

$$c > c_0^{P'} = \max\left\{0, \frac{a\left(\sqrt{(2s-1)\left[1+s\left(n-1\right)\right]} - 2s\right)}{\sqrt{(2s-1)\left[1+s\left(n-1\right)\right]} + 2s}\right\} \quad (33)$$

It is also worth noting that

$$\frac{\partial \delta'_{A0}}{\partial n} > 0; \; \frac{\partial^2 \delta'_{A0}}{\partial n^2} < 0; \; \frac{\partial \delta'_{A0}}{\partial s} > 0; \; \frac{\partial^2 \delta'_{A0}}{\partial s^2} < 0 \, \forall \, s \in [\hat{s}, 1] \text{ and } n \geq 2 \quad (34)$$

and

$$\lim_{n\to\infty} \delta'_{A0} = 1\,; \; \lim_{s\to 1} \delta'_{A0} = \frac{n-1}{n} \quad (35)$$

which amounts to saying that (i) δ'_{A0} is increasing and concave in the number of firms and the level of substitutability, as intuition would a priory suggests; (ii) if the cartel becomes infinitely large, firms become altogether unable to collude irrespective of the exact level of product substitutability, which becomes immaterial in this respect; and (iii) if differentiation vanishes completely, then the critical level of the discount factor retrieves the functional form we are accustomed with from the IO textbooks (and of course its limit is one if cartel size becomes infinitely large).

A cautionary note is in order. The analysis carried out in this subsection suggests that we may expect a switch from δ_{A0} to δ'_{A0} as s decreases from 1 to $\hat{s}$. Yet, this is the case provided firms may consistently drive to zero the continuation payoff (or, equivalently, if $\pi^{DP}(p^P) = 0$) for all $s \in (0, 1]$. As we shall see below, this is not the case, the reason being that, as soon as $\pi^{DP}(p^P)$ becomes positive, firms' profit incentives drive them off von Neumann's track.

3.3.2 Stick-and-carrot with profitable deviation from the punishment

Here we treat the alternative scenario in which there exists a profitable deviation from the one-shot punishment price. If this is the case, the relevant set of conditions is made up by either (25) or (26), coupled with

$$\pi^D(p^P) - \pi^P \leq \delta\left(\pi^M - \pi^P\right) \tag{36}$$

in which $\pi^D(p^P) > 0$ is the profit engendered by the optimal deviation against the punishment price p^P along the defecting firm's best reply. In both cases, (28) is loose.

For all for all $s \in (0, \hat{s})$, any defection from the cartel grants the loyal firm a positive market share, and the relevant system includes (25) and (36). This delivers

$$\begin{aligned} \delta_{A+} &= \frac{[2+s(n-3)]^2}{16\,(1-s)\,[1+s(n-2)]} \\ p_+^P &\leq \frac{a\,[2-s\,(n+1)] + c\,[2+s\,(3n-5)]}{4+2s\,(n-3)} \end{aligned} \tag{37}$$

with $p_+^P > 0$ for all

$$c > c_+^P = \max\left\{0, \frac{a\,[s\,(n+1)-2]}{2+s\,(3n-5)}\right\} \tag{38}$$

Here, the appearance of a plus in the subscript signals that the optimal deviation from the punishment along the best reply may grant the defecting firm a positive profit (see below).

If $s \in [\hat{s}, 1]$, the deviation from the cartel turns the defector into a monopolist, and therefore one has to solve the system formed by (26) and (36), obtaining

$$\begin{aligned} \delta'_{A+} &= \frac{[2+s(n-3)]^2\,[s\,(3\,(1-s)+n\,(2s-1))-1]}{\left[s^2(n-1)+2\sqrt{\Psi}\right]^2} \\ p_+^{P'} &\leq \frac{a\left[s\,(1-s)-\sqrt{\Psi}\right] + c\left[s\,(1+s\,(n-2))+\sqrt{\Psi}\right]}{s\,[2+s\,(n-3)]} \end{aligned} \tag{39}$$

with

$$\Psi \equiv (1-s)\left[1+s(n-2)\right]\left[s^2(2n-3)-s(n-3)-1\right] \geq 0$$
$$\forall s \in \left[\underline{s}, 1\right], \; \underline{s} \equiv \frac{n-3+\sqrt{n(n+2)-3}}{2(2n-3)} \tag{40}$$

and $p_+^{P'} > 0$ for all

$$c > c_+^{P'} = \max\left\{0, \frac{a\left[\sqrt{\Psi}-s(1-s)\right]}{s\left[1+s(n-2)\right]+\sqrt{\Psi}}\right\} \tag{41}$$

Concerning the deviation from the punishment price, this yields the following production levels, in the two relevant ranges of substitutability. For all $s \in (0, \hat{s})$, defecting along one's own reaction function involves selling the quantity

$$q^D\left(p_+^P\right) = \frac{(a-c)\left[4-s\left(12-7s+n\left(s(n-2)-4\right)\right)\right]}{4(1-s)\left[2+s(n-3)\right]\left[1+s(n-1)\right]} \geq 0$$
$$\forall s \in \left[\widetilde{s}, 1\right], \; \widetilde{s} \equiv \frac{2\left[n+\sqrt{2}(n-1)-3\right]}{n(n+2)-7} \tag{42}$$

Otherwise, if $s \in [\hat{s}, 1]$, the optimal defection from the punishment allows the deviating firm to sell

$$q^D\left(p_+^{P'}\right) = \frac{(a-c)\left[2(1-s)\left(1+s(n-2)\right)-(n-1)\sqrt{(1-s)\left(1+s(n-2)\right)\Lambda}\right]}{4(1-s)\left[2+s(n-3)\right]\left[1+s(n-1)\right]} \geq 0$$

$$\forall s \in \left[\widetilde{s}', 1\right], \; \widetilde{s}' \equiv \frac{n\left[n(n-5)+11\right]-15+\left(n^2-1\right)\sqrt{n(n+2)-5}}{2\left[n\left(n(2n-7)+12\right)-11\right]} \tag{43}$$

where $\Lambda \equiv s^2(2n-3)-s(n-3)-1 > 0$ for all $s \in [\underline{s}, 1]$. In view of the above considerations, we must assess the sign of the critical levels of s at which adeviation regime switch takes place, in order to correctly identify the shape of the critical threshold of the discount factor.

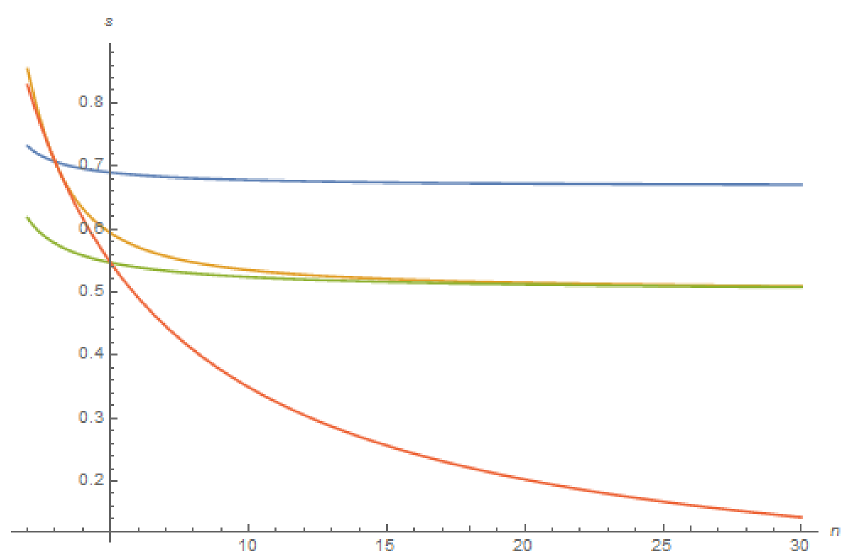
The discussion carried out thus far delivers the following

Remark 2 *The set of optimal punishment prices* $\mathbb{P}^P = \left(p_0^P, p_0^{P'}, p_+^P, p_+^{P'}\right)$ *is admissible iff* $c > \max\left\{c_0^P, c_0^{P'}, c_+^P, c_+^{P'}\right\}$ *for all* $s \in (0,1]$ *and* $n \geq 2$.

3.3.3 Assessing cartel stability

Now, provided the requirement stated in the Remark above is met, we are in a position to evaluate cartel stability under stick-and-carrot punishments by mapping the above critical levels of the discount factor, i.e., the elements of the set $\left\{\delta_{A0}, \delta'_{A0}, \delta_{A+}, \delta'_{A+}\right\}$. However, before doing so, it is useful to observe the map of the critical levels of s, i.e., $\{\hat{s}, \underline{s}, \widetilde{s}', \widetilde{s}\}$, appearing in Figure 1.

Figure 1: The critical levels of product substitutability.



Notes: Colour codes: $\widetilde{s}'$=yellow; $\widetilde{s}$ =red; $\widehat{s}$ =blue; $\underline{s}$ =green.

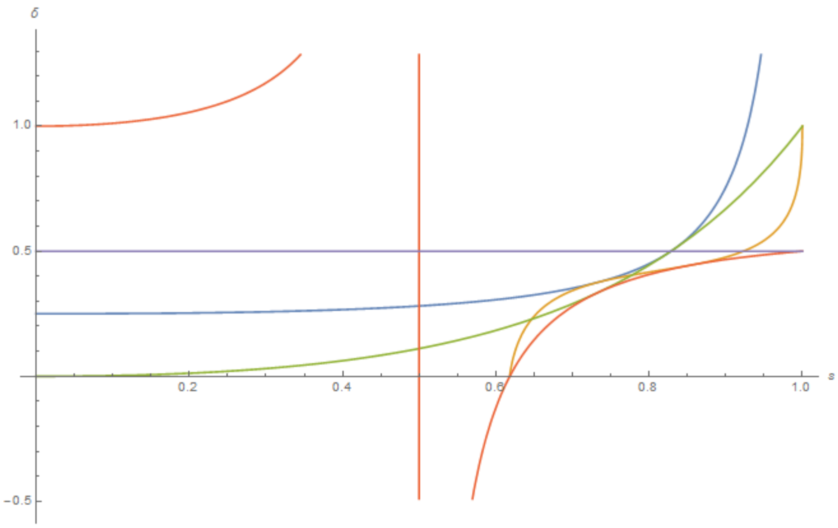
In the duopoly case, $\widetilde{s}' = \left(3\sqrt{5}-5\right)/2 > \widetilde{s} = 2\left(\sqrt{2}-1\right) > \widehat{s} = \sqrt{3}-1 > \underline{s} = \left(\sqrt{5}-1\right)/2$. Now note that $\hat{s} = \underline{s} = \widetilde{s}' = \sqrt{2}/2 \simeq 0.707$ at $n = 3$, as can be easily ascertained from the relevant expressions appearing in (15), (40) and (42). If (i) $n = 4$, $\hat{s} > \widetilde{s}' > \widetilde{s} > \underline{s}$; (ii), $n = 5$, $\hat{s} > \widetilde{s}' > \widetilde{s} = \underline{s}$; then, for all $n \geq 6$, $\hat{s} > \widetilde{s}' > \underline{s} > \widetilde{s}$. Taking into account the integer constraint, this implies that, as far as cartel stability is concerned, $n = 2$ and $n = 3$ are bound to be special cases, while the residual (and infinitely many) cases arising for $n \geq 4$ can be easily predicted to deliver the same qualitative picture. First we treat the two initial special cases, and then the properties of the general case arising for any $n \geq 4$.

Lemma 3 *Suppose $n = 2$, and therefore $\widetilde{s}' > \widetilde{s} > \widehat{s} > \underline{s}$, and consider s decreasing from 1 to 0. For all $s \in [\widetilde{s}', 1]$, the relevant threshold of the discount factor is δ'_{A0}; for all $s \in [\hat{s}, \widetilde{s}')$, it is δ'_{A+}; finally, for all $s \in (0, \hat{s})$, it is δ_{A+}.*

Proof We may consider first the duopoly model, for which the relevant envelope of critical discount factor levels can be deduced from Figure 2.

Now examine the graph as substitutability progressively decreases from $s = 1$. There, firms sell the same homogeneous good and the Bertrand-Nash equilibrium price equal marginal cost c, with the critical threshold for full collusion being $\delta'_{A0} = 1/2$. Then, as soon as $s = 0.\overline{9}$, stabilising collusion along the frontier of industry profits requires $\delta \geq \delta'_{A0}$ until the tangency point between the red and yellow curves is reached at $\widetilde{s}' \simeq 0.854$. From this value of s to $\hat{s} = \sqrt{3} - 1 \simeq 0.732$, the relevant threshold is δ'_{A+}. Then, for any $s \in \left(0, \sqrt{3}-1\right)$, full collusion is stable iff $\delta \geq \delta_{A+}$. All of this implies that deviation from the punishment price plays a role in determining cartel stability for all $s \in (0, \widetilde{s}')$, and for $n = 2$ this means more than 85% of the substitutability range.■

Figure 2: The envelop of the critical discount factor, $n = 2$.



Notes: Colour codes: blue=δ_{A+}; yellow=δ'_{A+}; green=δ_{A0}; red=δ'_{A0}.

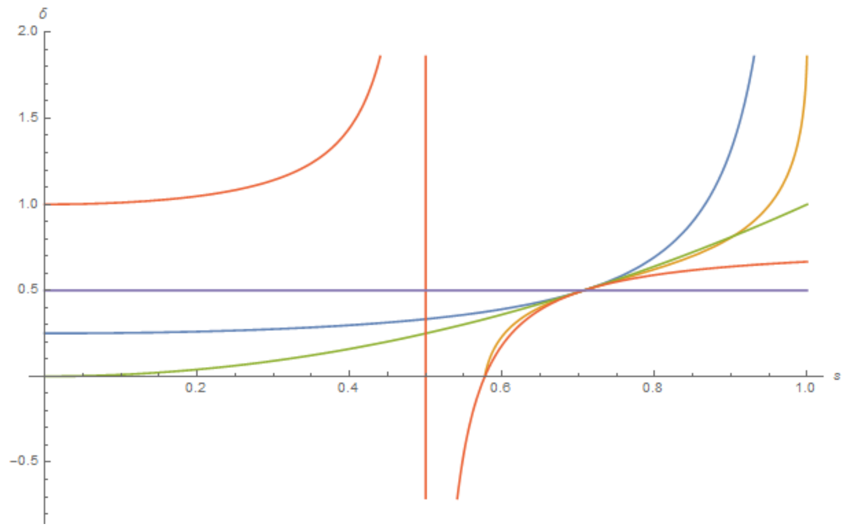
In triopoly, the following holds:

Lemma 4 *Suppose $n = 3$, and therefore $\widetilde{s}' = \widetilde{s} = \widehat{s} > \underline{s}$, and consider s decreasing from 1 to 0. For all $s \in [\widetilde{s}' = \widetilde{s} = \widehat{s}, 1]$, the relevant threshold of the discount factor is δ'_{A0}; then, for all $s \in (0, \widetilde{s}' = \widetilde{s} = \widehat{s})$, it is δ_{A+}.*

Proof If $n = 3$, the relevant envelope of the discount factors identifying the threshold for cartel stability emerges from the examination of Figure 3.

In this case, δ_{A0}, δ'_{A0}, δ_{A+} and δ'_{A+} are tangent to each other at $\hat{s} = \underline{s} = \widetilde{s}' = \sqrt{2}/2$. Therefore, for all $s \in [\hat{s} = \underline{s} = \widetilde{s}', 1]$, the relevant threshold is δ'_{A0}, while for all $s \in (0, \hat{s} = \underline{s} = \widetilde{s}')$ it is necessarily δ_{A0}, because in this range $q^D\left(p^P_+\right) = 0$, and consequently the deviation from the punishment necessarily involves shutting down production.■

Figure 3: The envelop of the critical discount factor, $n = 3$.



Notes: Colour codes: blue=δ_{A+}; yellow=δ'_{A+}; green=δ_{A0}; red=δ'_{A0}.

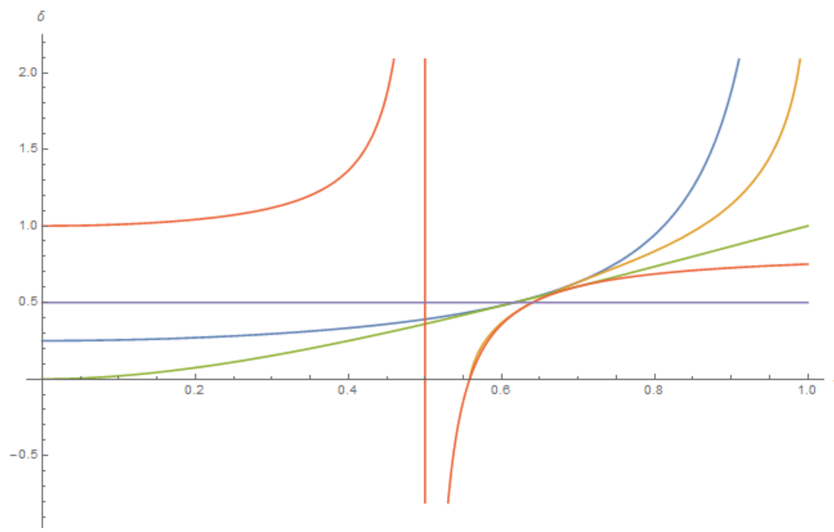
For any larger cartel, we may formulate the following:

Lemma 5 *Suppose $n \geq 4$, and therefore $\widehat{s} > \widetilde{s}' > \max\{\widetilde{s}, \underline{s}\}$, and consider s decreasing from 1 to 0. For all $s \in [\widehat{s}, 1]$, the relevant threshold of the discount factor is δ'_{A0}; then, for all $s \in [\widetilde{s}', \widehat{s})$, it is δ_{A0}; finally, for all $s \in (0, \widetilde{s}')$, it is δ_{A+}.*

Proof The last scenario applies to any $n \geq 4$, since $\hat{s} > \widetilde{s}'$. To validate the above claim, we examine the cases of $n = 4, 5$ which suffice to prove the result.

As illustrated in Figure 4 (where the plot indeed refers to $n = 4$), if the number of firms is sufficiently large, and proceeding as usual from $s = 1$ towards $s = 0$, the relevant threshold is δ'_{A0}, followed by δ_{A0} at the tangency point between these two, and then by δ_{A+} (because the switch from δ'_{A+} to δ_{A+} takes place in correspondence of $\hat{s}$).

Figure 4: The envelop of the critical discount factor, $n = 4$.



Notes: Colour codes: blue=δ_{A+}; yellow=δ'_{A+}; green=δ_{A0}; red=δ'_{A0}.

This becomes more evident for a larger number of firms. In fact $n = 5$ suffices to visualise the situation emerging in general for sufficiently large cartels, as illustrated in Figure 5, where the transition (right to left) from the red curve (δ'_{A0}) to the green (δ_{A0}) and finally to the blue one (δ_{A+}) is easier to detect.

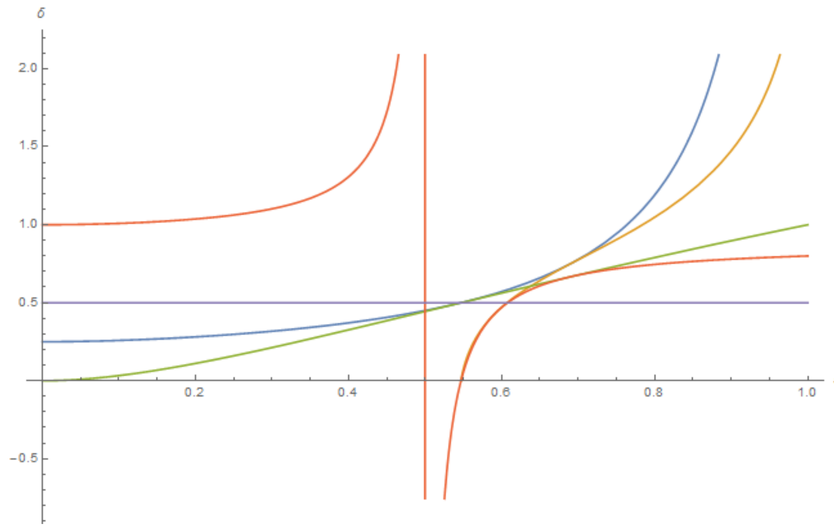
Analogous considerations hold for any larger cartel size.■

As a straightforward consequence of Lemmata 3, 4 and 5, we may draw the following implication:

Proposition 6 *As the number of cartel members increases, the subset of s in which the continuation payoff is nil expands.*

In plain words, this means that as the cartel size grows larger, the firms involved in it find it easier to follow von Neumann's track though the adoption of optimal punishments minimising the critical discount factor for cartel

Figure 5: The envelop of the critical discount factor, $n = 5$.



Notes: Colour codes: blue=δ_{A+}; yellow=δ'_{A+}; green=δ_{A0}; red=δ'_{A0}.

stability along the frontier of industry profits. Yet, this is no free lunch, as the expansion of the cartel size bring about the usual effect of rising this threshold monotonically as n increases. This yields our final result:

Corollary 7 *Increasing cartel size involves a tradeoff between the efficiency of the punishment and the stability of collusion.*

The last step consists in assessing the critical threshold engendered by Friedman's (1971) grim trigger strategies with the ones delivered by Abreu's (1986) optimal punishments. This exercise yields the following

Proposition 8 *For all $n \geq 2$, the lower envelope of δ_F and δ'_F lies above the lower envelope of δ_{A0}, δ'_{A0}, δ_{A+} and δ'_{A+}, except in the special case of full substitutability, where $\delta'_F = \delta'_{A0} = (n-1)/n$.*

The full proof is omitted as it can be easily reproduced using the corresponding expressions listed along the foregoing analysis, and it may also be intuitively grasped by plotting the critical discount factors. This result can be reformulated by saying that in presence of optimal punishments, provided the latter are feasible - which means that the condition appearing in Remark 2 is met - systematically facilitates collusion along the frontier of industry profits, even if the deviation from the punishment is profitable in some portion of the range of product differentiation.

4 Concluding remarks

Our analysis has unveiled the conditions under which a collusive path at the profit frontier can be sustained by the threat of the optimal punishments of Abreu (1986) when producers of possibly differentiated goods repeatedly engage in a Bertrand game with discounting. For each degree of product differentiation and number of cartel members, a critical level of the symmetric marginal costs exists, above which firms may credibly enforce a one-shot punishment strategy in case of unilateral defections from the cartel, such that cooperation is resumed after one period only. Accordingly, we have defined the level of the discount factor that is relevant to implement the optimal punishment at every level of product differentiation. Interestingly, we have noted that the patterns arising in the duopoly and triopoly cases cannot be generalized to the other oligopoly cases with larger cartel size, as the feasibility of a punishment strategy able to induce a zero continuation payoff is not only more limited over the space of product differentiation, but also differently related to the deviation strategy adopted by the potential defector to cease the collusive phase.

A novel and interesting trade-off has emerged. When increasing the number of active firms on the marketplace, a larger space of product differentiation becomes compatible with a stick-and-carrot punishment strategy able to reproduce the same efficiency of the minmax, although the critical discount factor required to stabilize the cartel increases. In other words, in Bertrand supergames with differentiated goods, the penal code of Abreu (1986) entails a trade-off between the efficiency of the punishment scheme and the stability of the cartel (i.e., the requested level of patience of their members), whenever the number of firms involved in the collusive agreement expands.

A few relevant aspects have however remained open and call for future in-dept analysis. First, when marginal costs are too low, such that the optimal punishment prices cannot be admitted, firms may still sustain a collusive path under the threat of the stick-and-carrot, yet on the condition that the cooperative price is set below the monopoly price, thus resulting in collusive profits beneath the industry frontier. Shedding light on the sustainability of 'partial' collusion thus appears to us as a promising avenue for follow-up research, which could also help reconnect our analysis to the strand of literature on collusive pricing and cartel stability (see, for instance, Harrington and Chen, 2006; and Harrington 2004, 2005, for an investigation of optimal cartel pricing when buyers and antitrust authorities may detect and/or sanction the formation of collusive agreements).

Second, much is still largely unexplored in regard to both feasibility and implications of the optimal punishments when product differentiation combines with firm heterogeneity (see Harrington, 1989, 2017) and, more specifically, with cost asymmetry (in the wake of Bae, 1987). The extant literature has indeed so far investigated this issue mainly by focusing on the grim trigger strategy or the minmax strategy or it has adopted the optimal punishments yet in the context of homogeneous good markets. Future research on this matter is then highly desirable.

References

[1] Abreu, D.J. (1986), "Extremal Equilibria of Oligopolistic Supergames", *Journal of Economic Theory*, **39**, 191-225.

[2] Abreu, D.J. (1988), "On the Theory of Infinitely Repeated Games with Discounting", *Econometrica*, **56**, 383-96.

[3] Albæk, S. and L. Lambertini (1998), "Collusion in Differentiated Duopolies Revisited", *Economics Letters*, **59**, 305-8.

[4] Aumann, R.J. (1981), "Survey of Repeated Games", in R. J. Aumann *et al.* (eds), *Essays in Game Theory*, Mannheim, Bibliographisches Institut.

[5] Bae, H. (1987), "A Price-setting Supergame between Two Heterogeneous Firms", *European Economic Review*, **31**, 1159-1171.

[6] Bowley, A.L. (1924), "The Mathematical Groundwork of Economics", Oxford University Press, Oxford.

[7] Bulow, J., J. Geanakoplos and P. Klemperer (1985), "Multimarket Oligopoly: Strategic Substitutes and Complements", *Journal of Political Economy*, **93**, 488-511.

[8] Chang, M.H. (1991), "The Effects of Product Differentiation on Collusive Pricing", *International Journal of Industrial Organization*, **9**, 453-69.

[9] Chang, M.H. (1992), "Intertemporal Product Choice and Its Effects on Collusive Firm Behavior", *International Economic Review*, **33**, 773-93.

[10] Deneckere, R. (1983), "Duopoly Supergames with Product Differentiation", *Economics Letters*, **11**, 37-42.

[11] Deneckere, R. (1984), "Corrigenda", *Economics Letters*, **15**, 385-387.

[12] Dixit, A. (1979), "A Model of Duopoly Suggesting a Theory of Entry Barriers", *Bell Journal of Economics*, **10**, 20-32.

[13] Friedman, J.W. (1971), "A Non-Cooperative Equilibrium for Supergames", *Review of Economic Studies*, **28**, 1-12.

[14] Friedman, J.W. (1986), *Game Theory with Applications to Economics*, Oxford, Oxford University Press.

[15] Friedman, J.W. and J.F. Thisse (1993), "Partial Collusion Fosters Minimum Product Differentiation", *Rand Journal of Economics*, **24**, 631-645.

[16] Fudenberg, D. and e. Maskin (1986), "The Folk Theorem in Repeated Games with Discounting or with Incomplete Information", *Econometrica*, **54**, 533-54.

[17] Häckner, J. (1994), "Collusive Pricing in Markets for Vertically Differentiated Products", *International Journal of Industrial Organization*, **12**, 155-77.

[18] Häckner, J. (1996), "Optimal Symmetric Punishments in a Bertrand Differentiated Product Duopoly", *International Journal of Industrial Organization*, **14**, 611-30.

[19] Harrington, J. (1989), "Collusion among Asymmetric Firms: the Case of Different Discount Factors", *International Journal of Industrial Organization*, **7**, 289-307.

[20] Harrington, J. (2004), "Cartel Pricing Dynamics in the Presence of an Antitrust Authority", *The RAND Journal of Economics*, **35**, 651-673.

[21] Harrington, J. (2005), "Optimal Cartel Pricing in the Presence of an Antitrust Authority", *International Economic Review*, **46**, 145-169.

[22] Harrington, J. (2016), "Heterogeneous Firms can Always Collude on a Minimum Price", *Economic Letters*, **138**, 46–49.

[23] Harrington, J. (2017), "The Deterrence of Collusion by a Structural Remedy", *Economic Letters*, **160**, 78-81.

[24] Harrington, J. and J. Chen (2006) "Cartel Pricing Dynamics with Cost Variability and Endogenous Buyer Detection", *International Journal of Industrial Organization*, **24**, 1185-1212.

[25] Lambertini, L. (1997), "Prisoners' Dilemma in Duopoly (Super)Games", *Journal of Economic Theory*, **77**, 181-91.

[26] Lambertini, L. and D. Sasaki (1999), "Optimal Punishments in Linear Duopoly Supergames with Product Differentiation", Journal of Economics, **69**, 173-88.

[27] Lambertini, L. and D. Sasaki (2001), "Marginal Costs and Collusive Sustainability", *Economics Letters*, **72**, 117-20.

[28] Lambertini, L. and D. Sasaki (2002), "Non-Negative Quantity Constraints and the Duration of Punishment", *Japanese Economic Review*, **53**, 77-93.

[29] Lambertini, L., S. Poddar and D. Sasaki (2002), "Research Joint Ventures, Product Differentiation and Price Collusion", *International Journal of Industrial Organization*, **20**, 829-54.

[30] Lambertini, L., S. Poddar and D. Sasaki (2003), "RJVs in Product Innovation and Cartel Stability", *Review of Economic Design*, **7**, 465-77.

[31] Lambson, V. (1995), "Optimal Penal Codes in Nearly Symmetric Bertrand Supergames with Capacity Constraints", *Journal of Mathematical Economics*, **24**, 1-22.

[32] Luce, R. D. and H. Raiffa (1957), *Games and Decisions*, New York, Wiley.

[33] Majerus, D. (1988), "Price vs Quantity Competition in Oligopoly Supergames", *Economics Letters*, **27**, 293-7.

[34] von Neumann, J. (1928), "Zur theorie der gesellschaftsspiele", *Mathematische Annalen*, **100**, 295-320.

[35] Ross, T.W. (1992), "Cartel Stability and Product Differentiation", *International Journal of Industrial Organization*, **10**, 1-13.

[36] Rothschild, R. (1992), "On the Sustainability of Collusion in Differentiated Duopolies", *Economics Letters*, **40**, 33-7.

[37] Shapiro, C. (1989), "Theories of Oligopoly Behavior", in R. Schmalensee and R. Willig (eds), *Handbook of Industrial Organization*, vol. I, Amsterdam, North-Holland.

[38] Segerstrom, P. (1988), "Demons and Repentance", *Journal of Economic Theory*, **45**, 32-52.

[39] Singh, N. and X. Vives (1984), "Price and Quantity Competition in a Differentiated Duopoly", *RAND Journal of Economics*, **15**, 546-54.