Dealing with Heterogeneous Creditors in Sovereign Bond Restructurings

[PRELIMINARY DRAFT]

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Abstract

This paper considers a simple model of sovereign debt restructuring with multiple bonds and heterogeneous creditors to analyse the optimal use of modification provisions by a government willing to conduct a bond exchange at minimum cost. In particular, in the presence of "enhanced" collective action clauses (CACs), we provide conditions under which two-limb voting dominates single-limb aggregation—thus accounting for the approach taken by Argentina and Ecuador to restructure their bonded debt in the summer of 2020. We also discuss the ongoing reform of Euro Area CACs in light of our results.

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Introduction

Since their systematic introduction in New York-law governed sovereign bonds in 2003, collective action clauses (CACs) have been viewed as a key pillar of the international debt architecture. Indeed, by enabling the implementation of a restructuring without the unanimous consent of creditors, such provisions constitute an essential element in the contractual toolkit towards the orderly resolution of sovereign debt distress.[1]

In practice, over the last two decades, CACs have appeared in various forms in sovereign bond contracts. In their latest incarnation, the so-called 'enhanced' CACs (ICMA, 2014) provide that in the context of a restructuring involving multiple bond series, the sovereign can choose among three voting procedures (or 'modification methods') to determine which series are swept into the proposed bond exchange:

- the first procedure operates *series-by-series*, allowing a supermajority of participating creditors (usually 75%) to bind a dissenting minority within a bond issue;
- the *'two-limb'* mechanism relies both on the voting outcomes within bond series and on the aggregate outcome across series. The voting thresholds in this hybrid procedure are typically set at 50% and 66 2/3%, respectively;[2]
- the *'single-limb'* aggregation procedure exclusively relies on the aggregate voting outcome, with a supermajority threshold of 75% and the additional constraint—known as *uniform applicability* condition—that all bond series receive the same exchange terms.

In view of the highly effective use of simple aggregation in the context of the Greek private restructuring of 2012 (see Zettelmeyer et al. (2013)), the presumption when 'enhanced' CACs were introduced was that the latter procedure would be the method of choice to conduct bond exchanges.[3] Yet in the summer of 2020, in the first two instances when these contractual provisions were tested in practice, the Argentine and Ecuadorian governments opted in favour of two-limb aggregation—at odds with the belief commonly held in policy circles that single-limb aggregation would be the most potent tool to facilitate restructurings.[4]

[1]For background information on institutional and legal details, see Buchheit and Gulati (2002), Buchheit et al. (2019), Gelpern and Heller (2015), and Weidemaier and Gulati (2013), as well as IMF (2014) and IMF (2020).

[2]When two-limb aggregation was first introduced (e.g., Uruguay 2003, Argentina 2005), the voting thresholds were set at 66 2/3% and 85%, respectively.

[3]See the original proposal in IMF (2014), and Gelpern and Heller (2015) and Sobel (2016) for further discussion.

[4]For a detailed account of the eventful and controversial Argentine restructurings, see among others Buchheit and Gulati (2020), Clark and Lyratzakis (2020), de la Cruz and Lagos (2021), and Setser (2020).

Motivated by these recent developments, this paper constitutes a first attempt at providing an economic analysis of enhanced CACs in sovereign debt workouts. To do so, we consider a setup with multiple bond series and heterogeneous creditors, allowing for heterogeneity both within bonds (capturing cross-investor differences in discount rates, regulatory constraints, information, or litigation costs) and across bonds (which may differ, e.g., in terms of maturity or coupon rate,[5] as well as in their size or bondholder base). Our aim is to characterize the optimal choice of aggregation procedure by a debtor government in a restructuring.

Specifically, our stylised analytical framework features two bonds held by two different continua of investors. Within each group, investors have heterogeneous reservation values—i.e., they value the payoff from holding out of the restructuring differently—and the distribution of reservation values differs across the two bonds. Relative to the single-limb procedure, two-limb aggregation brings the benefit of allowing for differentiated offers across the two bonds, but also comes with a cost arising from the additional series-by-series constraints. In a parametric example, we show that two-limb aggregation is the optimal procedure for the government when the relative notional size of the 'expensive' bond—i.e., the bond whose holders tend to have higher reservation values—is large enough. Conversely, the single-limb procedure is best when the size of the expensive bond is small—in which case the benefit from differentiated offers is dominated by the cost associated with the need to meet the individual series threshold for the expensive bond.

Our modelling environment, or extensions thereof, would be well-suited to investigate a number of related research questions such as the strategic interactions among creditors under aggregated voting, sub-aggregation and redesignation strategies, the role of large (non-atomistic) investors who may hold multiple bond series, or the endogenous sorting between bond characteristics and creditor types. Our analysis may also be extended to allow for interlocking debt stocks featuring different CACs specifications, as was the case in Argentina 2020 with the two subsets of Kirchner and Macri bonds.

Related Literature. The paper contributes to the theoretical economic literature on CACs in sovereign debt restructuring. Existing work—such as Haldane et al. (2005), Engelen and Lambsdorff (2009), Bi et al. (2016)—has looked at settings with a single bond instrument to study how strategic creditor interactions and restructuring outcomes are affected by the introduction of a supermajority rule in place of a unanimity requirement.[6] By design, these

[5] These dimensions of heterogeneity become irrelevant in circumstances where cross-default clauses are triggered and all bonds series are accelerated upon occurrence of an event of default prior to the restructuring.

[6] Also in a one-bond setting, Ghosal and Thampanishvong (2013) analyses the impact of a 'strengthening' of CACs (i.e., a decrease in the supermajority threshold) on interim vs ex ante efficiency in a setup featuring debtor moral hazard and coordination frictions due to incomplete information.

papers are silent on cross-bond heterogeneity and aggregation. Instead, we adopt a setting with multiple bonds to address questions that are specifically related to enhanced CACs.[7]

On the empirical front, Fang et al. (2021) and Asonuma et al. (2023) provide evidence on bond-level restructuring outcomes. Their restructuring sample comprises bonds without CACs as well as bonds with 'old-style' series-by-series CACs, together with Greek local-law bonds with ("retrofitted") single-limb aggregation. Fang et al. (2021) consider the combined impact of CACs and haircuts on participation rates at the bond level, and document their variation both across and within restructuring episodes. Asonuma et al. (2023) analyse bond-specific haircuts, showing that they are negatively related to the maturity of the instrument. These works are an important first step in the analysis of creditor heterogeneity in restructuring episodes, and while they do not directly speak to the predictions derived from our analysis, they provide an ideal testing ground for those in the future.

Our work is also connected to a series of empirical papers investigating empirically how the inclusion of various versions of CACs affects sovereign bond prices and yields, including early contributions by Becker et al. (2003) and Eichengreen and Mody (2004), and more recent ones by Carletti et al. (2016), Carletti et al. (2020) and Chung and Papaioannou (2020). Theoretical predictions on the fair pricing of CACs must build on, among other things, a fine understanding of how CACs are used and actually play out in restructuring times. Our work may thus inform such empirical investigations.

1 General Framework

This section introduces a general framework to analyze the government's optimal restructuring strategy in a multiple-bond setting, allowing for creditor heterogeneity both within and across bond series.

Bonds and Bondholders. There is a countable set $\mathcal{B}$ of bond series to be restructured, with $|\mathcal{B}| \geq 2$.[8] The relative size of bond series i, expressed as a fraction of the whole restructuring pool, is given by λ_i, with $\sum_{i \in \mathcal{B}} \lambda_i = 1$. Each bondholder is atomistic and assigns an

[7]Early analyses of CACs in a one-bond setup may still be relevant in practice: first, in the restructuring of bonds that only feature old-style series-by-series CACs; second, in the case of new issuer countries that have only one bond outstanding. Ecuador restructured one single bond in 2012, and so did Belize in 2013 and Mozambique in 2019.

[8]Given a total number $N \geq 2$ of outstanding bonds that may potentially be included in the restructuring, there are $2^N - (N+1)$ candidate pools that contain two bonds or more. For expositional simplicity, we ignore the endogenous determination of the restructuring pool $\mathcal{B}$. In practice, a variety of reasons may lead a government to leave one or more bond series out of the restructuring perimeter.

idiosyncratic value to holding out from the bond exchange.[9] The reservation values of holders of bond series i are distributed according to the cumulative distribution function F_i. Each CDF reflects within-series creditor heterogeneity, while differences in CDFs across series may arise from differences in bond characteristics (e.g., payment terms) and/or creditor base. A bondholder optimally accepts a restructuring offer w from the government if it is at least as high as her value of holding out. We assume that there is no uncertainty over consent shares, and that the debtor government knows all $\{F_i\}_{i \in \mathcal{B}}$. Hence the share of holders of bond series i that give their consent to an offer w is given by $F_i(w)$.

Voting Rules. We shall consider three different modification methods, as outlined in the introduction. We denote the series-by-series procedure with the subscript 0, and we use the subscripts 1 and 2 to denote the single-limb and two-limb procedures, respectively. Under series-by-series voting, an entire bond series i is restructured if the share of consent within this series is greater than or equal to a given threshold τ_0. According to the two-limb procedure, all bond series in the aggregated pool are restructured if the share of consent within each series is greater than or equal to the threshold τ_2^{s} and the share of consent over the entire pool is no smaller than $\tau_2^{\mathrm{a}} > \tau_2^{\mathrm{s}}$. Finally, under 'single-limb' voting, the uniform applicability condition requires that the same offer be made to all bond series, and CACs are triggered as long as an aggregate threshold τ_1 is reached.[10] Enhanced CACs allow the sovereign to choose among these three voting rules to implement a restructuring.[11]

1.1 Government's Problem

We now consider the problem of a government who wants to restructure all bonds series $i \in \mathcal{B}$.

The set of constraints that need to be satisfied by the exchange proposal in order to fully achieve this objective depends on the elected modification method.[12] With series-by-series

[9]We take these reservation values as exogenous, thus abstracting from explicit strategic considerations. There exists little evidence on holdout payoffs, apart from well-publicised cases such as the Argentine settlement following the 2001 default—see Cruces and Samples (2016). Schumacher et al. (2021) provide empirical evidence on the incidence of sovereign debt litigation.

[10]The rationale of the uniform applicability condition is to provide a safeguard to ensure inter-creditor equity. That is, to avoid that holders of large enough bond series effectively dictate terms that are discriminatory against smaller series (see Section B of IMF (2014)).

[11]In the standard ICMA version published in 2014-15, the threshold values are $\tau_0 = \tau_1 = 3/4$, $\tau_2^{\mathrm{s}} = 1/2$, and $\tau_2^{\mathrm{a}} = 2/3$.

[12]The government may want to partition $\mathcal{B}$ into subsets $\mathcal{B}_0$, $\mathcal{B}_1$, $\mathcal{B}_2$ with $\cup_m \mathcal{B}_m = \mathcal{B}$ in order to use different modification methods on these different subsets, and may possibly also want to resort to sub-aggregation pools $\{B_{m,p}\}$ such that $\cup_p \mathcal{B}_{m,p} = \mathcal{B}_m$ for $m = 1, 2$. For simplicity, we assume here that each procedure is applied to the entire pool $\mathcal{B}$, without resorting to sub-aggregation.

voting, the restructuring offer $\mathbf{w} = \{w_i\}_{i\in\mathcal{B}}$ must be such that

$$F_i(w_i) \geq \tau_0 \quad \text{for all } i \in \mathcal{B}. \tag{1}$$

Using the two-limb procedure over the entire pool, the offer must satisfy

$$F_i(w_i) \geq \tau_2^{\mathrm{s}} \quad \text{for all } i \in \mathcal{B}, \tag{2}$$

$$\sum_{i\in\mathcal{B}} \lambda_i F_i(w_i) \geq \tau_2^{\mathrm{a}}. \tag{3}$$

With single-limb aggregation over the entire pool, the 'uniform' offer w must be such that

$$\sum_{i\in\mathcal{B}} \lambda_i F_i(w) \geq \tau_1. \tag{4}$$

In choosing its restructuring strategy and the offer w_i made to each bond series $i \in \mathcal{B}$, the government wishes to minimise the total payout to bondholders

$$C = \boldsymbol{\lambda}.\mathbf{w} = \sum_{i\in\mathcal{B}} \lambda_i w_i. \tag{5}$$

1.2 Optimal Exchange Offer for a Given Modification Method

A preliminary step towards comparing two-limb vs single-limb aggregation consists in characterizing the optimal restructuring proposal under each procedure.

Single-Limb Offer. The optimal uniform offer $w^\star$ under single-limb voting is such that

$$\sum_{\mathcal{B}} \lambda_i F_i(w^\star) = \tau_1, \tag{6}$$

i.e., the weighted-average acceptance rate is equal to the aggregate acceptance threshold τ_1.

Two-Limb Optimization. In contrast, two-limb aggregation entails a non-degenerate constrained optimization problem, which we formulate as follows.

$$\min_{\{w_i\}} \sum_{\mathcal{B}} \lambda_i w_i$$

subject to

$$\sum_{\mathcal{B}} \lambda_i F_i(w_i) = \tau_2^{\mathrm{a}},$$

$$F_i(w_i) \geq \tau_2^{\mathrm{s}}, \qquad i \in \mathcal{B}.$$

In practice, $\tau_2^{\mathrm{s}} < \tau_2^{\mathrm{a}}$, implying that the tolerated holdout rate at the aggregate level is smaller than at individual series level. Thus high holdout rates for some series are acceptable as long as they are not too large, and that the aggregate 'average' constraint is satisfied.

1.3 Optimal Aggregation Procedure: Preliminary Considerations

It will prove useful to consider the auxiliary problem

$$S(\tau, \boldsymbol{\lambda}) = \min_{\{w_i\}} \sum_{\mathcal{B}} \lambda_i w_i \tag{7}$$

subject to

$$\sum_{\mathcal{B}} \lambda_i F_i(w_i) = \tau.$$

Using the alternative formulation in terms of holdout rates, the corresponding Lagrangian is

$$\mathcal{L} = \sum_{\mathcal{B}} \lambda_i w_i + \xi \Big(\tau^{\mathrm{a}} - \sum_{\mathcal{B}} \lambda_i F_i(w_i) \Big). \tag{8}$$

Assuming differentiability, the first-order condition requires

$$f_i(w_i) = \frac{1}{\xi} \quad \text{for all } i, \tag{9}$$

which implies that exchange offers are such that $f_i(w_i)$ is equalized across all bonds i. This is indeed optimal since a marginal increase in the offer to series i by \$1 has a cost of λ_i while it raises the aggregate approval rate by $\lambda_i f_i(w_i)$.

Single-Limb vs Two-Limb: Key Trade-Offs. The government's problem under the single-limb and two-limb procedures can be construed in light of the auxiliary problem (7), which only features an aggregate participation constraint. Specifically, under single-limb voting, the offer needs to satisfy the additional "uniform applicability" restriction $w_i = w$, which simplifies the problem into (6). Instead, under two-limb voting, the government needs to take into account the individual series-by-series participation constraints (2). Moreover the aggregate partipation thresholds may differ across procedures. Under the assumption that $\tau_2^{\mathrm{s}} < \tau_2^{\mathrm{a}} \leq \tau_1$, the relative appeal of single-limb voting can thus be summarized as follows:

- advantage: removes the individual series-by-series constraints;
- drawback: adds the uniform applicability restriction $w_i = w$, and requires higher aggregate approval rate in the case where $\tau_2^{\mathrm{a}} < \tau_1$.

These observations have immediate corollaries, establishing sufficient conditions under which a particular aggregation method is optimal.

(i) If the solution $\widehat{\mathbf{w}} = \{\widehat{w}_i\}$ to the auxiliary problem satisfies all individual participation constraints, that is $F_i(\widehat{w}_i) \geq \tau_2^{\mathrm{s}}$ for all i, then two-limb is generically optimal.

(ii) If the single-limb uniform offer, given by the unique solution $w^\star$ to (6), is such that $F_i(w^\star) \geq \tau_2^{\text{s}}$ for all i, then two-limb is generically optimal.

(iii) If the solution $\widehat{\mathbf{w}} = \{\widehat{w}_i\}$ to the auxiliary problem is such that $\widehat{w}_i = \widehat{w}$ for all i and $F_i(\widehat{w}) < \tau_2^{\text{s}}$ for at least one series, then single-limb is optimal if $\tau_1 = \tau_2^{\text{a}}$.

A voting procedure is more appealing when its unique advantage is more valuable and the additional constraints that it entails are less costly. In configuration (i), the additional two-limb constraints are costless. In configuration (ii), the unique advantage of single-limb is worthless. In configuration (iii), the additional constraints attached to single-limb are costless while its unique advantage is valuable. Going beyond these general sufficient conditions requires making specific assumptions on the environment—that is, on the reservation value distributions F_i, on the relative bond sizes λ_i, and on the various voting thresholds. To start with, we consider the simplest possible case, with only two bond series.

2 Two-Bond Setup

We now consider two bonds, H and L, with relative weights $\lambda_H = \lambda \in (0,1)$ and $\lambda_L = 1 - \lambda$. We denote by $F_i : \mathbb{R}_+ \to [0,1]$ the cumulative distribution function of reservation values for bond $i \in \{H, L\}$, which we assume to be twice differentiable. We let $\mathcal{S}_i \subseteq \mathbb{R}^+$ denote the support of the distribution of reservation values, and we denote by f_i the corresponding density function. We assume that holders of bond H tend to have higher reservation values, so that

$$F_H(w) < F_L(w) \quad \text{for all } w \in \text{Int } (\mathcal{S}_H \cup \mathcal{S}_L),$$

or equivalently

$$F_L^{-1}(p) < F_H^{-1}(p) \quad \text{for all } p \in (0,1). \tag{10}$$

This may reflect the fact that bond H has better payment terms (e.g., shorter maturity[13] and/or high coupon rate), or that its holders have better litigation skills (e.g., vulture funds). For convenience, we shall simply refer to bond H as the 'high-valuation' or 'expensive' bond. In analyzing the problem of the government who wants to restructure the two bonds at minimal cost, we proceed under the (realistic) assumption that

$$\tau_2^{\text{s}} < \tau_2^{\text{a}} \leq \tau_1 \leq \tau_0.$$

[13]Given a coupon rate c and yield y, a bond's value is decreasing in time-to-maturity—so that a short-dated bond is indeed more valuable, holding everything else constant—if (and only if) $c < y$. In practice, bonds of different maturities may also differ in terms of coupon rates and yield-to-maturities.

In particular, since $\tau_2^{\mathrm{s}} < \tau_2^{\mathrm{a}} \leq \tau_0$, it is immediate to see that two-limb aggregation dominates series-by-series voting. We can thus restrict our attention to the optimal choice of aggregation procedure.

2.1 Single-Limb Aggregation

Under single-limb voting, the government's exchange offer must satisfy the uniform applicability condition, which requires the offer to be the same across series. The cost-minimizing offer w^* is such that the aggregate consent requirement (4) holds as an equality, that is

$$\lambda F_H(w^*) + (1-\lambda) F_L(w^*) = \tau_1. \tag{11}$$

Given the stochastic ordering of F_H and F_L, it is immediate to see that

$$F_H(w^*) < \tau_1 < F_L(w^*). \tag{12}$$

Moreover, w^* is strictly increasing in the relative size λ of the expensive bond, with

$$\lim_{\lambda \downarrow 0} w^* =: w^*(0) = F_L^{-1}(\tau_1) \quad \text{and} \quad \lim_{\lambda \uparrow 1} w^* = F_H^{-1}(\tau_1). \tag{13}$$

2.2 Two-Limb Aggregation

Under two-limb aggregation, the government wishes to minimise its total spend

$$\min_{w_H,\, w_L} \ \lambda w_H + (1-\lambda) w_L$$

subject to

$$F_i(w_i) \geq \tau_2^{\mathrm{s}} \qquad i = H, L \tag{14}$$

$$\lambda F_H(w_H) + (1-\lambda) F_L(w_L) \geq \tau_2^{\mathrm{a}}. \tag{15}$$

Since $\tau_2^{\mathrm{s}} < \tau_2^{\mathrm{a}}$, the aggregate constraint (15) is binding.

To characterise the equilibrium, we can express the offer w_H made to the holders of bond H as a function of the offer w_L made to the other group of less demanding creditors:

$$w_H = F_H^{-1}\left(\frac{\tau_2^{\mathrm{a}} - (1-\lambda) F_L(w_L)}{\lambda}\right). \tag{16}$$

Note that if w_L is such that

$$\tau_2^{\mathrm{s}} \leq F_L(w_L) \leq \frac{\tau_2^{\mathrm{a}} - \lambda \tau_2^{\mathrm{s}}}{1-\lambda}, \tag{17}$$

then the series-by-series constraints in (14) are also satisfied—in particular, the second inequality guarantees that $F_H(w_H) \geq \tau_2^{\mathrm{s}}$. Naturally, the negative relationship in (16) captures the fact that if a more generous offer is made to the L-group, thus increasing the share of consent among them, a more stringent offer can be made to the other group without compromising the aggregate consent requirement.

We can thus consider the equivalent, transformed problem

$$\min_{w_L} \lambda F_H^{-1}\left(\frac{\tau_2^{\mathrm{a}} - (1-\lambda)F_L(w_L)}{\lambda}\right) + (1-\lambda)w_L \tag{18}$$

subject to constraint (17). In the unconstrained case where (17) is not binding, the first-order condition is given by

$$f_H\left(w_H(w_L)\right) = f_L(w_L) \tag{19}$$

where $w_H(w_L)$ is defined in (16).

2.3 Optimal Aggregation Procedure

In words, the advantage of single-limb aggregation is that it removes the series-by-series constraint. This is most valuable when the share λ of the expensive bond is low, in which case it possible to compensate a small consent share in the H-bond with a larger majority in the L-bond. On the other hand, the drawback of single-limb aggregation is that it involves the uniform applicability condition, which does not allow any price discrimination by the debtor, and that it possibly implies a higher aggregate threshold than the two-limb rule. Both these channels are stronger, the higher is the heterogeneity among the creditors of the two different bonds.

Mathematically, we can characterise the optimal procedure explicitly in some circumstances. First, in view of Corollary (i) from Section 1.3, if the inequalities in (17) are satisfied by the unconstrained solution of (19), then the two-limb method is optimal. Second, another sufficient condition for the two-limb procedure to be optimal, stated in Corollary (ii), is when $F_H(w^\star) \geq \tau_2^{\mathrm{s}}$, or equivalently $w^\star \geq F_H^{-1}(\tau_2^{\mathrm{s}}) =: \underline{w}_H$. We can thus distinguish between two configurations, depending on the ranking of $w^*(0) = F_L^{-1}(\tau_1)$ and $\underline{w}_H$, namely:

- if $F_L^{-1}(\tau_1) < F_H^{-1}(\tau_2^{\mathrm{s}})$, there exists $\lambda_\dagger \in (0,1)$ such that $w^\star(\lambda_\dagger) = F_H^{-1}(\tau_2^{\mathrm{s}})$, and two-limb is optimal for $\lambda \geq \lambda_\dagger$;
- if $F_L^{-1}(\tau_1) \geq F_H^{-1}(\tau_2^{\mathrm{s}})$, two-limb is optimal for every λ.

3 Parametric Example: Exponential Distributions

We now consider a special incarnation of the two-bond setup where holdout values for each bond are exponentially distributed, that is,

$$F_i(w) = 1 - e^{-\frac{w}{\phi_i}}, \qquad w \in \mathcal{S}_i = \mathbb{R}^+, \tag{20}$$

implying that

$$F_i^{-1}(\tau) = -\phi_i \log(1-\tau), \quad \text{for all } \tau \in]0,1[. \tag{21}$$

In this case, the distribution of reservation values for bond i has mean ϕ_i and variance ϕ_i^2, and a single parameter provides a sufficient statistic of the stochastic dominance ordering. We assume that $\phi_H > \phi_L$, i.e., holders of bond H tend to have higher holdout values, so that $F_H(w) < F_L(w)$ for all $w > 0$. We further assume that

$$F_L^{-1}(\tau_1) < F_H^{-1}(\tau_2^{\mathrm{s}}) \quad \Leftrightarrow \quad \frac{\phi_H}{\phi_L} > \frac{\log(1-\tau_1)}{\log(1-\tau_2^{\mathrm{s}})} > 1, \tag{22}$$

otherwise two-limb voting is optimal for all λ, as discussed in Section 2.3.

Two-Limb Aggregation. As in equation (16), we express w_H as a function of w_L using the aggregate constraint:

$$w_H = -\phi_H \log\left(\frac{\tau_2^{\mathrm{a}} - 1 - \lambda + \lambda e^{-w_L/\phi_L)}}{\lambda}\right). \tag{23}$$

With this, we can write down the unconstrained two-limb problem as per equation (18). The unconstrained solution for w_L is

$$w_L^{unc} = \phi_L \log\left(\frac{\lambda\frac{\phi_H}{\phi_L} + 1 - \lambda}{1 - \tau_2^{\mathrm{a}}}\right). \tag{24}$$

As stated previously in the general case, this is feasible if the series-by-series constraints are satisfied, that is if

$$\tau_2^{\mathrm{s}} \leq 1 - e^{-\frac{w_L^{unc}}{\phi_L}} \leq \frac{\tau_2^{\mathrm{a}} - \lambda\tau_2^{\mathrm{s}}}{1-\lambda}. \tag{25}$$

These two inequalities require

$$\lambda\left(\frac{\phi_H}{\phi_L} - 1\right) \geq \frac{\tau_2^{\mathrm{s}} - \tau_2^{\mathrm{a}}}{1 - \tau_2^{\mathrm{s}}} \tag{26}$$

$$\lambda\left(\frac{\phi_H}{\phi_L} - 1\right) \geq \frac{\phi_H}{\phi_L}\frac{1 - \tau_2^{\mathrm{a}}}{1 - \tau_2^{\mathrm{s}}} - 1. \tag{27}$$

The first inequality is always satisfied, since $\tau_2^{\mathrm{s}} < \tau_2^{\mathrm{a}}$. The second inequality (27) holds for λ larger than a cutoff value which is a function of the thresholds $\tau_2^{\mathrm{s}}, \tau_2^{\mathrm{a}}$ and of the distribution parameters ϕ_H, ϕ_L.

Optimal voting procedure. Together with corollary (i), the above results imply that two-limb is optimal

- when $\lambda \geq (1-\tau_2^{\mathrm{a}})/(1-\tau_2^{\mathrm{s}})$
- when $\frac{\phi_H}{\phi_L} \leq \frac{1-\tau_2^{\mathrm{s}}}{1-\tau_2^{\mathrm{a}}}$
- in the remainder of the parameter space where (25) is satisfied, which happens for $\frac{\phi_H}{\phi_L}$ sufficiently small and/or for λ sufficiently large.

Indeed for such parameter values, the unconstrained two-limb offer is feasible. When instead (27) is violated and the unconstrained two-limb offer is not feasible due to the binding participation constraint on bond H, the comparison between the two aggregation procedures is non-trivial. In this case, the two-limb offers are such that

$$F_H(w_H) = \tau_2^{\mathrm{s}}, \tag{28}$$

$$F_L(w_L) = \frac{\tau_2^{\mathrm{a}} - \lambda\tau_2^{\mathrm{s}}}{1-\lambda} > \tau_2^{\mathrm{a}}, \tag{29}$$

and the total spend for the government is given by

$$\begin{aligned} \mathcal{C}_2 &= \lambda F_H^{-1}(\tau_2^{\mathrm{s}}) + (1-\lambda) F_L^{-1}\left(\frac{\tau_2^{\mathrm{a}} - \lambda\tau_2^{\mathrm{s}}}{1-\lambda}\right) \\ &= \lambda\phi_H \log\left(\frac{1}{1-\tau_2^{\mathrm{s}}}\right) + (1-\lambda)\phi_L \log\left(\frac{1-\lambda}{1-\tau_2^{\mathrm{a}} - \lambda(1-\tau_2^{\mathrm{s}})}\right). \end{aligned} \tag{30}$$

Under single-limb voting, the optimal uniform offer w^* is implicitly defined by

$$F(w^*) := \lambda F_H(w^*) + (1-\lambda)F_L(w^*) = 1 - \lambda e^{-w^*/\phi_H} - (1-\lambda)e^{-w^*/\phi_L} = \tau_1, \tag{31}$$

and the total cost for the government is $\mathcal{C}_1 = w^*$. Therefore

$$\mathcal{C}_1 < \mathcal{C}_2 \quad \Leftrightarrow \quad F(\mathcal{C}_2) > \tau_1. \tag{32}$$

This gives rise to an explicit condition on parameter values. Namely, single limb is optimal for parameter values such that the two-limb unconstrained offer is not feasible and such that

$$\lambda\left(\kappa_2^{\mathrm{s}}\right)^{\lambda}\left(\frac{\kappa_2^{\mathrm{a}} - \lambda(\kappa_2^{\mathrm{s}})}{1-\lambda}\right)^{(1-\lambda)\frac{\phi_L}{\phi_H}} + (1-\lambda)\left(\kappa_2^{\mathrm{s}}\right)^{\lambda\frac{\phi_H}{\phi_L}}\left(\frac{\kappa_2^{\mathrm{a}} - \lambda(\kappa_2^{\mathrm{s}})}{1-\lambda}\right)^{(1-\lambda)} < 1 - \tau_1. \tag{33}$$

where we defined $\kappa_2^j = 1 - \tau_2^j$ for $j \in \{\mathrm{s}, \mathrm{a}\}$ in order to lighten up the notation.

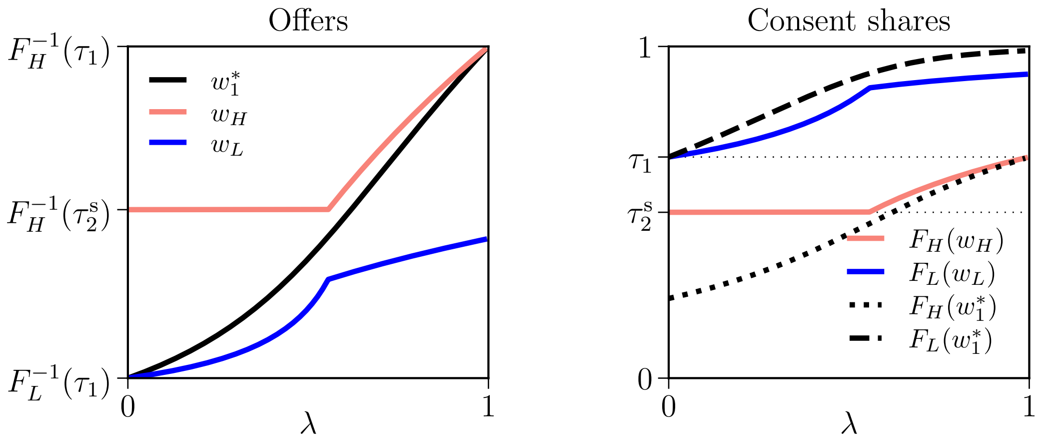

Figure 1: Optimal restructuring offers and consent shares.

Numerical example. We perform a numerical example to explore how the optimal aggregation procedure and restructuring offers depend on the relative size of the two bonds. We assume that $\tau_1 = \tau_2^a$ in our baseline calibration.[14]

Figure 1 characterises optimal offers (left panel) and consent shares (right panel) under the two different aggregation rules. Optimal offers and consent shares under two-limb aggregation are represented as blue and red solid lines for the cheap and expensive bond, respectively, whereas the optimal offer and consent shares under single-limb aggregation are represented by the black (solid or dashed) lines. Consider the two-limb rule first. When λ is large enough, the unconstrained two-limb offers are possible, both w_H and w_L are increasing in λ and consent shares are strictly larger than the series-by-series threshold, which are indicated by the horizontal dotted lines in the right panel. When instead λ is low, the series-by-series constraint for the expensive bond binds, w_H is flat and the H-bond consent share equals τ_2^s, and the government sets w_L below its unconstrained level until the aggregate threshold binds. The single-limb rule is somewhat easier to characterise: the optimal offer is a smooth function of the share of the expensive bond, and the right panel shows how the optimal consent shares of the two bonds are an almost parallel function of λ.

Figure 2 illustrates the debtor government total spend as a function of the two aggregation methods. The single-limb rule is illustrated by the solid black line, and the two-limb rule by the red-blue coloured line. To make the example clearer, we assumed that $\tau_1 = \tau_2^a$, so the total spend of the two strategies is the same when there is only one bond and therefore no creditor heterogeneity. The figure is an illustration of what we discussed earlier: the single-limb method dominates when the share of the expensive bond is low, while the two-limb method is cheaper when λ is high and the series-by-series constraint does not bind.

[14] Specifically, for the CACs thresholds, we use the typical values of $\tau_1 = 2/3, \tau_2^a = 2/3, \tau_2^s = 1/2$. For the exponential distributions we use parameters $\phi_H = 0.8, \phi_L = 0.02$.

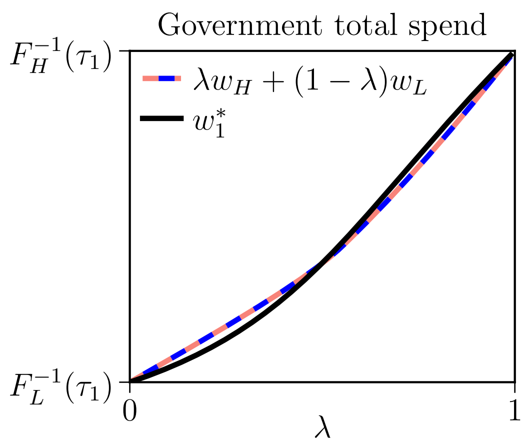

Figure 2: Optimal restructuring offers (left panel) and total government spend (right panel).

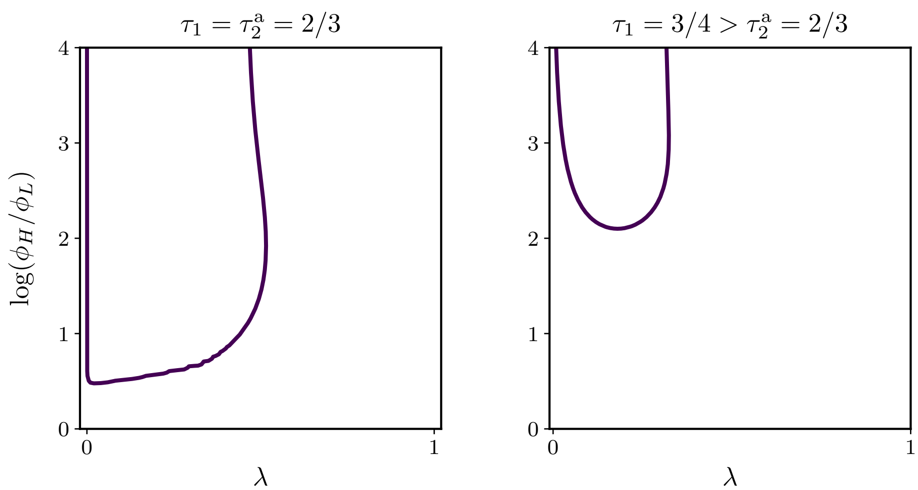

Figure 3: Optimal aggregation method as a function of bond heterogeneity ϕ_H/ϕ_L and relative bond size λ.

Finally, Figure 3 illustrates which aggregation method dominates depending on the heterogeneity between bonds, which is represented by the ratio of the exponential distribution parameters ϕ_H/ϕ_L (a higher value clearly means more heterogeneity) and the relative size λ of the expensive bond. The left panel shows the case, assumed in the previous two figures, where the aggregate thresholds coincide; the right panel instead assumes the threshold values of the ICMA CACs and shows how the region where the single-limb method dominates shrinks when its threshold increases.

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