# How do Platforms Appeal to Buyers?*

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**Abstract**

In this paper, we consider a model of platform competition to examine the mechanisms through which asymmetric platforms attract different agents. Specifically, we analyze how platforms strategically choose different attributes to appeal to the buyers. We consider a two-stage game where heterogeneous platforms simultaneously choose features on the buyers' side in the first stage and membership fees in the second stage. Our results show that the equilibrium values of attributes depend significantly on the relative strengths of cross-network effects along with the degree of heterogeneity between platforms. Buyers' decisions to join a platform therefore are influenced not only by the membership fees and cross-network effects but also by the range of functionalities offered by the platform. Furthermore, even though such attributes are offered solely on the buyers' side in our model, sellers' participation is also significantly affected by them via their interactions with the membership fees and cross-network effects.

## 1 Introduction

In recent times, there has been a significant surge in the volume of electronic commerce attributable to the widespread availability of the Internet. According to Euromonitor International's 2018 report, the proportion of retail sales conducted online accounted for 13.7% and 17% in the United States and the United Kingdom, respectively, while globally, it represented 11.5% of all retail sales. These figures translate into substantial revenue, with online retail sales reaching over $400 billion, $86 billion, and $1.7 trillion for the USA, UK, and worldwide, respectively.[1]

E-commerce typically involves buying and selling goods or services through online platforms, which is a business model connecting buyers and sellers, enabling them to engage in value-

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[1]Data is taken from Exhibit 7 of Amazon.com, 2019 Case 716-402.

creating exchanges. It is common that a dominant platform is present in this type of market, such as *Amazon* in the online retailing sector[2], *Airbnb* in lodging services[3], and *Uber* in the ride-hailing industry[4], among others. The underlying factors that contribute to these platforms' successful attraction and retention of agents have generated significant scholarly and practical interest. One potential explanation for their success is the platform's ability to serve as an intermediary between agents and actively shape the business model. It is this active involvement that may give rise to heterogeneity among platforms, and may, in turn, affect agents' incentives and valuations regarding which platform to join.

In this study, we present a framework for analysing how platforms appeal to agents, specifically on the buyers' side. Our argument is that buyers' decisions to join a platform are not only based on membership fees and cross-network effects but also on other attributes platforms offer.[5] The combination of these three elements determines which platform buyers find most appealing. Buyers are more inclined to join a platform that has built a favourable reputation and brand image over time by offering a diverse range of features. As the quality of the platform's features increases, buyers' perception of the platform's benefits improves, resulting in a stronger reputation and brand image, thereby increasing the likelihood of buyers joining the platform.

A specific example is *Amazon*, which not only works as an intermediary between buyers and sellers but also has an active function adopting a customer-centric approach to generate attributes that create value. For buyers, the platform's benefits proposition transcends beyond product pricing. It extends to the ability to appeal to and initiate a loyal customer base, enhancing their browsing experience through the provision of flexible delivery options, an extensive product assortment, swift checkout processes, and a lenient refund and return policy. On the seller side, having their products affiliated with Amazon's brand name enhances their credibility with customers and leverages the platform's Prime audience. Wells et al. (2019) observed that the majority of attributes developed by *Amazon* are primarily buyer-oriented. *Amazon* strives to attract buyers to its site by developing various attributes to meet their needs.

This research makes a twofold novel contribution to the existing literature on two-sided markets. Firstly, we introduce the platform's features as a form of vertical product differentiation on the buyers' side, shedding light on the importance of quality attributes in shaping market structure. Secondly, we analyse the intricate interactions between these quality attributes and cross-group network effects to gain insights into the resulting market configurations. By exploring these dimensions, our study expands the understanding of two-sided markets and offers valuable insights for market participants and policymakers alike.

Vertical differentiation refers to the differentiation of products or services offered by platforms based on their perceived quality, features, or attributes that cater to the distinct needs of both sides of the market. Platforms offer different levels of quality to enhance their fea-

[2] Amazon.com, 2019 Case 716-402.

[3] World's Leading Online Travel Accommodation Marketplace 2020, accessed August 2021

[4] Global Top 100 Brands 2019

[5] For simplification purposes attributes, features, and characteristics are used interchangeably throughout the paper.

tures, functionality, user experience, or service level to attract and retain users on both sides of the market. Rather than attempting to capture all possible features a platform may have, we integrate them into a single variable representing buyers' perception of the quality of the platform.

Our model builds on the framework of Armstrong (2006), where equilibrium membership fees depend on cross-group network effects, and the literature on vertical differentiation, including Mussa and Rosen (1978); Gabszewicz and Thisse (1979); Shaked and Sutton (1982, 1983), which identify consumer income as a source of differentiation. We extend Armstrong (2006) model by introducing the level of features offered on the buyers' side as a strategic variable on the vertical dimension. This allows for the existence of asymmetric platforms in equilibrium, as shown by Gabszewicz and Wauthy (2014).

The provision of attributes by platforms creates a competitive advantage in attracting agents in a two-sided market. This competitive advantage can be understood as heterogeneity within a vertically differentiated product space, where agents prefer platforms offering more attributes compared to those offering fewer attributes. The concept of vertical product differentiation space was first explored by scholars such as Mussa and Rosen (1978); Gabszewicz and Thisse (1979); Shaked and Sutton (1982, 1983). Mussa and Rosen (1978) investigated a monopoly pricing model for quality differentiated goods, and found that a monopolist cannot price discriminate in the usual way, but rather assigns a price-quality pair to customers to partially discriminate against them, thereby reducing the quality sold to customers compared to a competitive market. Gabszewicz and Thisse (1979) analysed a non-cooperative price equilibrium between firms, where consumers have different willingness to pay for quality improvements, and found that with less income disparity, the firm selling the lowest quality product will exit the market. Moreover, when consumers' tastes are less differentiated, Cournot's equilibrium price is near zero. Shaked and Sutton (1982, 1983) studied vertical differentiation in a competitive market and found that firms differentiate themselves by choosing distinct qualities to lower price competition and earn positive profits.

Subsequently, the seminal works of Economides (1989); Neven and Thisse (1990) were the first to jointly examine both horizontal and vertical product differentiation spaces. Horizontal differentiation pertains to the range of products offered, while vertical differentiation refers to the quality of the products sold in the market. Both studies yield comparable results, showing that firms maximise one dimension (variety) while minimising the other characteristic (quality) to gain a larger market share and increase profits. Building on these findings, Irmen and Thisse (1998) extended the previous models to include multiple characteristics and report similar results, indicating that firms choose to maximise differentiation in the dominant characteristic and minimise the remaining attributes to reduce price competition.

These models have undergone extensions to encompass a diverse range of sectors. Degryse (1996) explored banking services, Baake and Boom (2001) examined markets with network externalities. Inderst and Irmen (2005) focused on space and time as strategic variables in horizontal product differentiation, specifically in the retail markets, and Hansen and Nielsen (2011) investigated price as a proxy for quality in the trade between two countries. Garella and

Lambertini (2014) identifies situations in which firms select maximum differentiation in both characteristics by studying economies of scope. Finally, Barigozzi and Ma (2018) developed a general specification model that allows for general consumer preference distributions, general production cost functions (increasing and convex), and firms selecting any arbitrary number of quality characteristics.

Recent studies have explored the intersection of two-sided markets and vertical differentiation. For instance, Gabszewicz and Wauthy (2014) introduced heterogeneity among participants and found that platform competition with cross-group externalities and vertical differentiation can result in the equilibrium coexistence of asymmetric platforms. Zennyo (2016) investigated vertically differentiated two-sided markets and found that in a sequential game, both platforms charged the same per-transaction fee in equilibrium, even with quality asymmetries. Under certain conditions, a low-quality platform was found to have higher profits than a high-quality platform. Roger (2017) studied two-sided markets where platforms compete for agents on both sides of the market, and concluded that when cross-group externalities are too strong, pure-strategy equilibrium may not exist. Lastly, Etro (2021) considered the differences between device-funded and ad-funded platforms. His results showed that device-funded platforms are more aligned with consumers because they provide high-quality products and services, while ad-funded platforms offer products at competitive prices and free services.

The seminal models of Caillaud and Jullien (2003); Armstrong (2006); Rochet and Tirole (2003, 2006) analysing two-sided markets have been extended in various directions by subsequent research. Belleflamme and Toulemonde (2009); Hagiu (2009); Belleflamme and Toulemonde (2016); Belleflamme and Peitz (2019a) introduced competition among sellers and investigate how pricing equilibrium, product variety, and the optimal number of platforms are affected in the presence of a monopolistic or duopolistic platform. Their findings indicate that while consumers and producers prefer product variety, platforms prefer to minimise differentiation among them. Weyl (2010) proposed a nonlinear tariff that is conditional on the participation of agents on both sides in order to address the problem of equilibrium multiplicity. Choi (2010); Choi et al. (2017) investigated the impact of tying in a two-sided market where agents can use multiple platforms. They find that allowing multi-homing can improve welfare through tying. Gao (2018) analysed the effects of overlapping agents on both sides of a platform. Finally, Karle et al. (2020); Jeitschko and Tremblay (2020) examined how agents endogenously determine whether to singlehome or multihome.

Our model consists of two stages, where agents can join one platform (single home) only and platforms simultaneously determine the level of attributes they offer on the buyers' side in the first stage, and then determine membership fees in the second stage. We find equilibrium membership fees follow Armstrong (2006) result, but are adjusted by the differences in attributes offered by platforms on the buyers' side, and weighted by the cross-group network effect one side exercises on the other side.

Our first key finding is that the difference in attributes on the buyers' side between two competing platforms not only affects their behaviour but also has an impact on the sellers' side due to the presence of cross-group network effects on both sides of the market. We analyse two different scenarios based on the strength of these cross-group network effects. The first scenario establishes identical indirect network effects on both sides of the market. The second scenario analyses when the network effects are distinct. We find that when both cross-network effects are equal, the sellers' equilibrium membership fee remains as Armstrong (2006) stated, indicating that the difference in attributes on the buyers' side only impacts buyers' decisions.

We establish conditions for a max-min strategy to enhance profits, as demonstrated in the early works of Economides (1989) and Neven and Thisse (1990) and the generalised model of Irmen and Thisse (1998). Specifically, we identified two scenarios where such a strategy is effective: when the cross-group network effects on both sides of the market are equal, and when the cross-group network effect buyers have on sellers is greater than the impact sellers exert on buyers. In the former situation, platforms differentiate themselves as much as possible on attributes on the buyers' side (vertical dimension) and as little as possible on the product differentiation cost (horizontal dimension). In the latter setting, platforms differentiate themselves as little as possible on attributes on the buyers' side and as much as possible on the horizontal dimension to maximise profits. Furthermore, we find conditions for a max-max strategy to maximise profits, as seen in recent studies by Garella and Lambertini (2014); Barigozzi and Ma (2018). In particular, we find platforms differentiate as much as possible on both dimensions when the cross-group network effect exerted by sellers on buyers outweighs those exercised by buyers on sellers.

The rest of the paper is structured as follows. Section 2 outlines the model primitives, while section 3 presents the solution to stage 2 of the model to obtain equilibrium membership fee configurations. Section 4 provides the solution to stage 1 of the model, deriving equilibrium attribute configurations on the buyers' side. In addition, section 5 analyses and compares market structure where the cross-group network effects on both sides of the market are identical and opposite. In both cases, we express the strategic variables as a function of the model parameters and provide intuitive explanations for the results. The paper concludes in section 6.

## 2 Model

We consider a model of platform competition with cross-group external effects and attributes on the buyers' side. There are three different players: platforms, buyers and sellers. The model follows Armstrong (2006) considering two platforms that are horizontally differentiated and charge access fees to both sides of the market. Agents choose to join a single platform, i.e. agents singlehome. In this model we introduce the level of attributes $q_b$ as a strategic variable capturing various platform features on buyers' side: the higher the value of $q_b$, the more attractive the platform is for buyers, given membership fees.

Two platforms engage in competition through membership fees and attributes offered on the buyers' side. This setup is designed to facilitate interactions between a unit mass of sellers and buyers, generating positive cross-group network effects. Positioned at the extremes of a unit interval, the platforms exhibit horizontal differentiation à la Hotelling and bear a constant cost of $f_b$ and $f_s$ per additional buyer and seller, respectively. Buyers and sellers, uniformly distributed across this interval, face a cost of visiting a platform that increases linearly in distance, with rates $\tau_b$ and $\tau_s$, respectively. This cost can be interpreted as a potential mismatch with buyers' and sellers' preferences. Considering our focal point is the relationship between the cross-group network effects and the attributes a platform offers on the buyers' side, we assume, that the cost associated with visiting a platform is homogeneous across both platforms and both sides. This means that both buyers and sellers face the same disutility cost when their preferences are mismatched, and we defined it as $\tau_b = \tau_s = \tau$.

Buyers, upon joining the platform, purchase one unit of a differentiated product from each active seller on the same platform. For each trade, buyers and sellers obtain a net gain of $\upsilon$ and $\pi$, respectively. Additionally, there exists a stand-alone benefit of $R_b$ for buyers and $R_s$ for sellers when they visit the platform, a benefit uniform across all platforms. We define $\eta_b^i$ and $\eta_s^i$ as the mass of active buyers and sellers on platform $i$. The membership fees charged to buyers and sellers on platform $i$ are denoted as $p_b^i$ and $p_s^i$, respectively.

In addition, buyers receive $q_b^i$ for the attributes platform $i$ offers. Platform $i$'s for $i = 1, 2$ production cost of providing these attributes on buyers' side is set as $C^i\left(q_b^i\right) = \frac{1}{2}\alpha^i\left(q_b^i\right)^2$. The parameter $\alpha^i$ captures the efficiency of platform $i$ developing characteristics on the buyers' side. We assume $0 < \alpha^i < \alpha^j$ for $i, j = 1, 2;\ i \neq j$, meaning platform one is more efficient in developing these attributes compared to platform two. This is possible, either because platform one can produce more features with the same inputs or deliver the same level of features at a lower cost. As a result, platforms are heterogeneous in terms of both product differentiation and the characteristics they offer on the buyers' side.

A buyer and seller, respectively, obtain a surplus of visiting platform $i$ of:

$$\nu_b^i = R_b + q_b^i + \upsilon\eta_s^i - p_b^i \tag{1a}$$

$$\nu_s^i = R_s + \pi\eta_b^i - p_s^i \tag{1b}$$

The model consists of two stages. In the first stage platforms simultaneously choose characteristics on buyer's side and in the second stage choose simultaneously membership fees and then agents choose simultaneously which platform to join. We analyse different cases using the previous framework in the next sections.

**Assumptions**

Several conditions must be met by the parameters. Firstly, the second-order conditions of the platform maximisation problem at both stages of the game are crucial as they ensure a unique equilibrium where both platforms remain active. Secondly, we need to secure full participation on both sides. Specifically, indifferent buyers and sellers must attain a positive net surplus at equilibrium. We need for buyers $\nu_b^i > \frac{1}{2}\tau$ and for sellers $\nu_s^i > \frac{1}{2}\tau$, where $i = 1, 2$. Thirdly, we enforce a limitation on the number of buyers and sellers participating in the market, constraining $0 < \eta_k^i < 1$ for $k = b, s$ and $i = 1, 2$. Finally, we require positive equilibrium attributes denoted by $q_b^i > 0$.

Therefore, we gather the following set of assumptions:

**A.1.** $\tau > \frac{\pi+\upsilon}{2}$ and $\tau < \frac{2\pi+\upsilon}{3}$ if $\pi > \upsilon$ and $\tau < \frac{\pi+2\upsilon}{3}$ if $\upsilon > \pi$

**A.2.** $\nu_b^i > \frac{1}{2}\tau$ and $\nu_s^i > \frac{1}{2}\tau$, $i = 1, 2$

**A.3.** $\alpha^i > \frac{2\tau}{\Sigma}$, $i = 1, 2$ where $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$

The initial assumption, denoted as **A.1**, stipulates that the product differentiation parameter must fall within the range defined by the cross-group network effects. Meeting the lower bound of this range ensures the existence of a unique equilibrium in stage 2 of the game, during which both platforms are operational. Additionally, this condition constraints agents' market shares to a unit interval, $\eta_k^i \in (0, 1)$, where $k = b, s$.

Assumption **A.2** guarantees the full participation of agents on both sides of the market because the indifferent agent achieves a positive net surplus at equilibrium.

Assumption **A.3**, combined with the lower bound of Assumption **A.1**, ensures positive equilibrium attributes on the buyers' side. Furthermore, Assumption **A.3** alone serves as a sufficient condition to establish a unique equilibrium at stage 1 of the game, as the second-order conditions are satisfied.[6]

## 3 Equilibrium membership fees

We develop a two-stage model of two-sided markets with vertical differentiation where agents singlehome. We solve our model using backward induction. In this section, we solve the second stage of the game where platforms choose simultaneously membership fees and then agents choose simultaneously which platform to join, assuming the level of attributes on the buyers' side as given. We obtain market shares and platform profits at equilibrium. We develop some intuition for the results.

A marginal agent $k$, $k = b, s$, $b \neq s$ indifferent between joining the two platforms $i, j$, $i \neq j$ is located at $x_k$ solves: $\nu_k^i - \tau x_k = \nu_k^j - \tau(1 - x_k)$, then $x_k = \frac{1}{2} + \frac{\nu_k^i - \nu_k^j}{2\tau}$.

Agents located between 0 and $x_k$ visit platform $i$ and those located between $x_k$ and 1 visit platform $j$. It follows that $\eta_k^i = x_k$ and $\eta_k^j = (1 - x_k)$; and the total number of agents on each side is $\eta_k^i + \eta_k^j = 1$; $k = b, s$ and $b \neq s$. Then we obtain the number of buyers and sellers for each platform using expressions for $x_b$ and $x_s$ together with the gross surpluses given by equations (1a) and (1b):

$$\eta_b^i = \frac{1}{2} + \frac{\tau\left(q_b^i - q_b^j\right) + \tau\left(p_b^j - p_b^i\right) + \upsilon\left(p_s^j - p_s^i\right)}{2(\tau^2 - \pi\upsilon)} \tag{2a}$$

$$\eta_s^i = \frac{1}{2} + \frac{\pi\left(q_b^i - q_b^j\right) + \tau\left(p_s^j - p_s^i\right) + \pi\left(p_b^j - p_b^i\right)}{2(\tau^2 - \pi\upsilon)} \tag{2b}$$

We are interested in a solution where both platforms remain active. Under Assumption

[6] See Appendix A.2 and A.6 for details on the second-order conditions at stages 2 and 1 of the game, respectively. See Appendix A.8 for details on conditions on market shares.

**A.1**, we can observe that conditions $\tau^2 > \pi\upsilon$ and $0 < \eta_k^i < 1$ for $k = b, s$ hold. This implies that not only do buyers' and sellers' market shares decrease when their own side's membership fee increases, but also when the membership fee of the other side increases.[7] In other words, the market shares of both sides are influenced by changes in fees on either side of the market.[8]

**Definition 1.** *An equilibrium at stage two of the game is a pair* $p_b^i, p_s^i$ *where* $i = 1, 2$*, such that* $p_b^i$ *and* $p_s^i$ *solves the platform maximisation problem* $\max_{\{p_b^i, p_s^i\}} \Pi^i = (p_b^i - f_b)\eta_b^i(p_k^i, q_k^i) + (p_s^i - f_s)\,\eta_s^i\,(p_k^i, q_k^i) - \frac{\alpha^i (q_b^i)^2}{2}$ $\forall\ i = 1, 2$ *and* $k = b, s$*,* $b \neq s$*, where* $\eta_b^i\,(p_k^i, q_k^i)$ *and* $\eta_s^i\,(p_k^i, q_k^i)$ *are given in(2a) and (2b).*

From the first-order conditions for platform $i$'s $i, j = 1, 2$, $i \neq j$ maximisation problem, the following best responses functions are obtained:[9]

$$p_b^i = \frac{f_b + \tau + p_b^j}{2} + \frac{\left(q_b^i - q_b^j\right)}{2} - \frac{\upsilon\left(p_s^i - p_s^j\right)}{2\tau} - \frac{\pi\left(\upsilon + p_s^i - f_s\right)}{2\tau} \tag{3a}$$

$$p_s^i = \frac{f_s + \tau + p_s^j}{2} + \frac{\pi\left(q_b^i - q_b^j\right)}{2\tau} - \frac{\pi\left(p_b^i - p_b^j\right)}{2\tau} - \frac{\upsilon\left(\pi + p_b^i - f_b\right)}{2\tau} \tag{3b}$$

The best strategy for platform $i$ when the difference in characteristics $q_b^i - q_b^j$ on buyers' side is positive[10] is to increase the membership fee on *both* sides of the market. At the same time, platform $i$'s best response is to increase the membership fee on both sides when the other platform increases the fee on both sides, as well. However, platform $i$ decreases the membership fee on one side when the membership fee on the other side increases. In this context, membership fees' best responses are strategic substitutes amongst platforms but strategic complements between sides.

Although platform $i$'s best response is to increase both sides' membership fee when the difference in attributes is positive, on the sellers' side the best response is boosted when the cross-group network effect buyers exert on sellers $\pi$ increases. This behaviour is common in two-sided markets where sellers benefit as more buyers join the platform. Platform $i$ developed attributes on buyers' side appealing to more buyers because they can enjoy more features, but also appealing to more sellers given the cross-group network effect.

The next step is to solve the best response functions (3a) and (3b) to obtain the equilibrium membership fees as a function of the model parameters and the difference in attributes on buyers'

[7]Alternatively, if the cross-group network effects outweigh the opportunity cost associated with mismatched preferences on both sides of the market, i.e., $\tau^2 < \pi\upsilon$, both sides' market shares would become an increasing function of their membership fee. Consequently, both buyers and sellers would opt for the same platform, leading to a tipping point in the market.

[8]The partial derivative of equations (2a) and (2b) with respect to both membership fees are negative, as long as $\tau^2 > \pi\upsilon$. Considering that $\tau - \frac{\pi+\upsilon}{2} > 0$, then $\tau^2 - \pi\upsilon > 0$ as well. We can see this because $(\pi + \upsilon)^2 > 4\pi\upsilon \Rightarrow (\pi - \upsilon)^2 > 0$ if $\pi \neq \upsilon$.

[9]See Appendix A.1 for details.

[10]When we refer to the difference of a strategic variable: $membership\ fees$, *market-shares*, *attributes* and $platforms$' $profits$, it is always between platform $i$ and platform $j$.

side:[11]

$$p_b^i = f_b + \tau - \pi + \left[\frac{3\tau^2 - \pi(\pi + 2\upsilon)}{9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)}\right]\left(q_b^i - q_b^j\right) \tag{4a}$$

$$p_s^i = f_s + \tau - \upsilon + \left[\frac{\tau(\pi - \upsilon)}{9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)}\right]\left(q_b^i - q_b^j\right) \tag{4b}$$

First, notice the difference in attributes $q_b^i - q_b^j$ is part of the equilibrium membership fees on both sides of the market, even though they were developed only on buyers' side. Sellers' side is affected by the difference in characteristics on the other side because of the cross-group network effects one side exerts on the other side. Therefore platforms adjust sellers' membership fees taking into account the difference in features on buyers' side.

Both agents' equilibrium membership fees on platform $i$ are a function of two terms. The first term is Armstrong (2006) result, the marginal cost of an extra agent $f_k$, $k = b, s$ and $b \neq s$, the disutility for mismatch preference $\tau$, and the cross-group network effect this side exerts on the other side, $\pi$ for buyers and $\upsilon$ for sellers. The second term captures the difference in attributes developed on buyers' side $q_b^i - q_b^j$. This extra markup could be positive or negative depending on which side exerts a stronger cross-network effect on the other side.

In a one-sided market, a firm typically increases its prices as it offers more attributes to customers. However, in a two-sided market, pricing dynamics are influenced by the interplay of cross-group network effects on both sides of the market. As a result, membership fees on one side may actually decrease despite platforms offering additional features, as they can offset this decrease by charging a higher fee on the other side, using the indirect network effects present in the market.

We summarise our discussion in the next proposition:

**Proposition 1.** *For $\left(q_b^i - q_b^j\right) > 0$, whenever this difference in attributes increases, platform $i$,*

*(i) Increases buyers' and decreases sellers' equilibrium membership fees, whenever the cross-group network effect experienced by buyers is higher than the one experienced by sellers $(i.e., \upsilon > \pi)$; Whereas*

*(ii) Increases sellers' and decreases buyers' equilibrium membership fees, whenever the influence exerted on sellers by buyers outweighs the impact on buyers by sellers $(i.e., \pi > \upsilon)$.*

**Proof:** See Appendix A.3

Platform $i$ appeals to more agents by increasing the features on buyers' side, attracting more buyers directly and more sellers indirectly since the cross-group network effect. This creates a positive loop considering more agents are attracted on both sides, i.e buyers join platform $i$ given there are more features developed for them, sellers join as well because more buyers joined, then more buyers,..., and this behaviour continues.

[11] We are interested in obtaining an equilibrium where both platforms are active. Therefore, assuming condition **A.1** holds, platform $i$, $i = 1, 2$ profit function is concave and the second-order conditions of the maximisation problem are satisfied. See Appendix A.2 for more details.

Platform $i$, decides to charge a lower fee on the side that exerts a stronger cross-group network effect on the other side. On the one hand, platform $i$ decreases buyers' fee if the influence buyers exert on sellers is higher than sellers on buyers, $(\pi > \upsilon)$. On the contrary, platform $i$ decreases sellers' fee if the cross-group network effect sellers exert on buyers is higher than the impact buyers exert on sellers $(\upsilon > \pi)$.

### Equilibrium market shares and profits

At equilibrium, agents' market shares for platform $i$, $i = 1, 2$ are:[12]

$$\eta_b^i = \frac{1}{2} + \left[ \frac{3\tau}{2\left[9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)\right]} \right] \left( q_b^i - q_b^j \right) \tag{5a}$$

$$\eta_s^i = \frac{1}{2} + \left[ \frac{(\pi + 2\upsilon)}{2\left[9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)\right]} \right] \left( q_b^i - q_b^j \right) \tag{5b}$$

As with equilibrium membership fees, we find that even when platform features are exclusively developed on the buyers' side, the difference in attributes impacts both sides' market shares. Sellers are attracted to join platform $i$ even in the absence of tailored attributes for them, through the influence of cross-group network effects. Furthermore, buyers' and sellers' market shares experience an increase when there is a positive difference in attributes developed on the buyers' side $(q_b^i - q_b^j)$, regardless of which side places a higher value on interaction with the other side.[13]

Platform $i$ can increase its position in the market by developing more attributes on buyers' side. Buyers and sellers will be drawn to join platform $i$, buyers will join to enjoy more features developed for them and sellers will join because they can interact with more buyers (cross-group network effects).

As we already have the equilibrium membership fees and market shares on both sides of the market, we can compute equilibrium profits for platform $i$, $i = 1, 2$ as:

$$\Pi^i = \tau - \frac{(\pi + \upsilon)}{2} + \frac{\tau \left( q_b^i - q_b^j \right)^2 + X \left( q_b^i - q_b^j \right)}{2\left[9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)\right]} - \frac{\alpha^i \left( q_b^i \right)^2}{2} \tag{6}$$

where $X \equiv 6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)$.

Equilibrium profits of platform $i$ are equal to the degree of product differentiation on both sides of the market adjusted downwards by the cross-group network effects, $\pi$ and $\upsilon$. This result is as Armstrong (2006). Furthermore, profits are adjusted by two additional terms. The first is an extra markup associated with the difference in attributes on buyers' side and the second is the cost of developing these attributes.

[12] We limit our results to buyers' and sellers' market shares constrained on the unit interval, $\eta_k$, $k = b, s$ and $b \neq s$ between zero and one. Form more details see Appendix A.8.

[13] Partially derive equilibrium market-shares on equations (5a) and (5b) respect the difference in attributes on buyers' side. The denominator $9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$ is positive if Assumption **A.1** holds.

We can see from equation (6) that platform $i$'s equilibrium profits increase when additional attributes on buyers' side are developed.[14] When platforms offer new and innovative features, they can appeal to more agents (buyers and sellers) and increase customer satisfaction and loyalty leading to increase profits.

## 4 Equilibrium values of attributes

In this section, we find the equilibrium values of attributes on buyers' side at stage 1 of the game. Platform $i$ differentiates by the number of features offered on buyers' side, measured by $q_b^i$. There is a cost of providing $q_b^i$ of $C\left(q_b^i\right) = \frac{1}{2}\alpha^i\left(q_b^i\right)^2$, where $\alpha^j > \alpha^i > 0$, $i \neq j$. The parameter $\alpha^i$ measures the efficiency platform $i$ has in developing attributes on buyers' side. The fact that platform $i$ is more efficient in developing attributes can be related to specialisation in certain technology, experience in having a better understanding of buyers' needs, or innovation by investing more in research and development.

In stage 1 platforms simultaneously choose the characteristics' levels on buyers' side $q_b^i$, $i = 1, 2$. We can state the next definition:[15]

**Definition 2.** *An equilibrium $q_b^i$, $i = 1, 2$, is such that $q_b^i$ solves the platform maximisation problem* $\max_{\{q_b^i\}} \Pi^i \equiv \left(p_b^i - f_b\right)\eta_b^i + \left(p_s^i - f_s\right)\eta_s^i - \frac{\alpha^i\left(q_b^i\right)^2}{2}$, *where $p_b^i$ and $p_s^i$ are given by equations (4a) and (4b) and $\eta_b^i$ and $\eta_s^i$ are given by equations (5a) and (5b).*

From the first-order conditions for platform $i$'s maximisation problem, we obtained the following best response functions:

$$q_b^i = \frac{-\tau q_b^j}{\left(\alpha^i\Sigma - \tau\right)} + \frac{6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)}{2\left(\alpha^i\Sigma - \tau\right)} \quad \forall i, j = 1, 2 \ i \neq j \tag{7a}$$

$$q_b^j = \frac{-\tau q_b^i}{\left(\alpha^j\Sigma - \tau\right)} + \frac{6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)}{2\left(\alpha^j\Sigma - \tau\right)} \quad \forall i, j = 1, 2 \ i \neq j \tag{7b}$$

where $\Sigma \equiv 9\tau^2 - (\pi + 2\upsilon)(2\pi + \upsilon)$

We note attributes are strategic substitutes: considering the best response functions in equations (7a) and (7b). An attribute increase is a profit-maximising strategy platform $i$ follows to platform $j$'s attributes reduction.[16]

Solving the system of best response functions we obtain the equilibrium attributes on buyers'

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[14] This can be seen by partially derive equation (6) with respect to $q_b^i$.That is $\frac{\partial \Pi^i}{\partial q_b^i} = \frac{2q_b^i\left(\tau - \alpha^i\Sigma\right) - 2\tau q_b^j + X}{2\Sigma} > 0$ if $q_b^i > \frac{2\tau q_b^j - X}{2\left(\tau - \alpha^i\Sigma\right)}$. The previous condition is satisfied if assumption **A.3** holds. For more details see Appendix A.4.

[15] See Appendix A.5 for details.

[16] Partially derived equations (7a) and (7b) respect to $q_b^j$ and $q_b^i$, respectively.

side as a function of the model parameters for platform $i, j$, $\forall\ i, j = 1, 2$, and $i \neq j$, that is:[17]

$$q_b^i = \frac{\left(\alpha^j \Sigma - 2\tau\right)\left[6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)\right]}{2\Sigma\left[\alpha^i \alpha^j \Sigma - (\alpha^i + \alpha^j)\tau\right]} \tag{8a}$$

$$q_b^j = \frac{\left(\alpha^i \Sigma - 2\tau\right)\left[6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)\right]}{2\Sigma\left[\alpha^i \alpha^j \Sigma - (\alpha^i + \alpha^j)\tau\right]} \tag{8b}$$

where $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$.

From equations (8a) and (8b) we notice that the efficiency parameter is what differentiates equilibrium attributes on buyers' side. Platform $i$ increases attributes when platform $j$ is less efficient in developing characteristics on buyers' side (bigger $\alpha^j$), as long as Assumptions **A.1** and **A.3** hold[18]. Platform $i$ enhances attributes offered on the buyers' side to build its reputation as a high-quality intermediary to become a trusted and respected leader in the industry.

Finally, we note that the difference in attributes on buyers' side, using equations (8a) and (8b) is:

$$\left(\Delta q_b^i\right) \equiv (q_b^i) - (q_b^j) = \frac{\left(\alpha^j - \alpha^i\right)\left[6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)\right]}{2\left[\alpha^i \alpha^j \Sigma - (\alpha^i + \alpha^j)\tau\right]} \tag{9}$$

where $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$.

We observe from equation (9) that the equilibrium difference in attributes on buyers' side is positive when platform $i$ demonstrates greater efficiency in features development compared to platform $j$ ($\alpha^j > \alpha^i$), and when Assumptions **A.1** and **A.3** hold. Platform $i$ strives to differentiate itself as much as possible from platform $j$ based on the interplay of the cross-group network effects exerted on each other.[19]

Therefore, recognising the significance of the impacts that the difference in cross-group network effects has on platform attributes and overall market equilibrium (fees, market shares and profits), our focus now shifts towards a comprehensive analysis of these effects in the subsequent section.

## 5 Analysis of cross-group network effects on market configurations

In this section, we study how cross-group network effects shape the structure and dynamics of the market. We explore two distinct scenarios to gain insights into the interactions between platform's attributes and cross-group network effects. Firstly, we consider a benchmark case where cross-group network effects are identical on both sides of the market. Secondly, we explore a scenario where the cross-side network impacts are allowed to differ.

[17]Assuming condition **A.3** holds, platform $i$, $i = 1, 2$ profit function is concave and the second order conditions of the maximisation problem at stage 1 of the model, $\alpha^i > \frac{\tau}{\Sigma}$ and $\alpha^i > \frac{\alpha^j \tau}{\alpha^j \Sigma - \tau}$, are met. We can also see the latter condition is more stringent than the former. See Appendix A.6 for more details.

[18]Partially derive $q_b^i$ on equation (8a) respect to $\alpha^j$. $\frac{\partial q_b^i}{\partial \alpha^j} = \frac{X\tau}{2\Sigma A^2}\left(\alpha^i \Sigma - 2\tau\right) > 0$, where $X \equiv 6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)$, $\Sigma = 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$ and $A \equiv \left(\alpha^i \alpha^j \Sigma - \left(\alpha^i + \alpha^j\right)\tau\right)$

[19]For further details, refer to Appendix A.7.

### 5.1 Benchmark scenario: Identical cross-group network effects, $\pi = \upsilon$

In this section, we develop a special scenario where the cross-group network effects are identical on both sides of the market, ($\pi = \upsilon$). We use the game's solution at stage 1 to obtain the strategic variables as a function of the model's parameters and derive some intuition for the results that is going to help us to examine asymmetric network effects in the next section.

Using equations (8a) and (8b) and the fact that $\pi = \upsilon$, equilibrium attributes on buyers' side are:[20]

$$\left(q_b^i\right)^{sc} = \frac{9\alpha^j\left(\tau^2 - \pi^2\right) - 2\tau}{3\left[9\alpha^i\alpha^j\left(\tau^2 - \pi^2\right) - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{10a}$$

$$\left(q_b^j\right)^{sc} = \frac{9\alpha^i\left(\tau^2 - \pi^2\right) - 2\tau}{3\left[9\alpha^i\alpha^j\left(\tau^2 - \pi^2\right) - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{10b}$$

Equilibrium attributes on buyers' side on equations (10a) and (10b) are positive as long as Assumption **A.1** and **A.3** holds[21], and considering identical cross-group network effects on both sides of the market we can state the next proposition:

**Proposition 2.** *Equilibrium attributes on buyers' side decrease in the product differentiation parameter $\tau$ and increase in the cross-group network effect ($\pi = \upsilon$). Moreover, an increase in the cross-group network effect is stronger in platform $i$ than in platform $j$.*

**Proof:** See Appendix B.1

Based on Proposition 2, when platforms prioritise maximising their product differentiation parameter in the horizontal dimension, they tend to minimise the heterogeneity of attributes on buyers' side in the vertical dimension. As platform $i$ increases its product differentiation parameter across both sides of the market, it no longer has incentives to further enhance attributes on buyers' side. This is due to the cost associated with simultaneously differentiating on both the horizontal and vertical dimensions.

Instead, to gain a competitive advantage, platform $i$ opts for a broader degree of product differentiation, catering to a wide range of preferences from both buyers and sellers. Rather than focusing on increasing the level of features on buyers' side for a specific set of preferences, platform $i$ engages in less intense competition for the same pool of agents as the degree of product differentiation expands. Consequently, agents become more captive and there is reduced pressure to develop additional attributes on the buyers' side.

We notice also from Proposition 2 that platform $i$ increases the attributes on buyers' side whenever the cross-group network effects increase because this attracts directly more buyers and more sellers, given the cross-side network effects. This creates a positive loop where the more agents use platform $i$, the more valuable it becomes to buyers and sellers, which in turn attracts even more agents.

**Corollary 1.** *The difference in attributes on buyers' side decreases when there is a higher*

[20] The second order conditions at stage 1 of the game turn to $\alpha^i > \frac{\tau}{9\sigma}$ and $\alpha^i > \frac{\alpha^j\tau}{9\alpha^j\sigma - \tau}$ where $\sigma = \tau^2 - \pi^2$ and both are satisfied if condition **A.3** holds.

[21] When $\upsilon = \pi$ Assumption **A.1** turns to $\tau > \pi$ and Assumption **A.3** turns to $\alpha^i > \frac{2\tau}{9\sigma}$, where $\sigma \equiv \tau^2 - \pi^2$.

*product differentiation on both sides of the market and increases when the cross-group network effects become stronger.*

**Proof:** See Appendix B.1

Corollary 1 extends the proven arguments on Proposition 2 to the difference in attributes on buyers' side. For this reason, the intuition is the same as in Proposition 2.

**Equilibrium membership fees**

We now obtain equilibrium membership fees, market shares and platforms profits as a function of the model parameters.

For the equilibrium membership fees we have:

$$\left(p_b^i\right)^{sc} = f_b + \tau - \pi + \frac{\left(\alpha^j - \alpha^i\right)\sigma}{9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau} \quad = \quad f_b + \tau - \pi + \frac{1}{3}\left(\Delta q_b^i\right)^{sc} \tag{11a}$$

$$\left(p_s^i\right)^{sc} = f_s + \tau - \upsilon; \quad \upsilon = \pi \tag{11b}$$

When the cross-group network effects are identical on both sides of the market $\pi = \upsilon$, platforms charge symmetric fees on the sellers' side. This is a consequence that the difference in attributes on buyers' side does not influence sellers' fees when the cross-network effects are the same. Platform $i$ charges sellers the same fee as in Armstrong (2006) seminal model.

However, buyers' equilibrium membership fee is higher than it would have been without the development of specific features for them. This is due to the positive extra markup denoted by $\frac{1}{3}\left(\Delta q_b^i\right)^{sc}$. Consequently, platform $i$ lacks the option to discern which side values interaction more, and thus, cannot adjust the fee accordingly when the cross-group network effects are identical on both sides of the market.

We examine the effects of the model parameters on the difference in equilibrium fees between the two platforms, under the assumption that platform $i$ is more efficient in developing attributes compared to platform $j$. Hence, we set the following:

**Proposition 3.** *The difference in equilibrium fees buyers pay decreases when there is a greater heterogeneity between platforms (higher $\tau$) and increases when platforms become more valuable for both groups (stronger $\pi = \upsilon$). In addition, buyers' fee is more expensive in platform $i$ than $j$ whenever the cross-group network effect is stronger.*

**Proof:** See Appendix B.2

Proposition 3 reveals that as the product differentiation parameter increases, platform $i$ reduces buyers' fees because the difference in attributes between platforms decreases. This fee reduction serves as an incentive to attract more buyers to platform $i$. Platform $i$ raises sellers' fees, as indicated in equation (11b), to compensate for the decrease in buyers' fees. Conversely, when the cross-group network effect ($\pi = \upsilon$) increases, platform $i$ raises buyers' fees as it has developed more attributes to enhance their experience. Simultaneously, platform $i$ lowers sellers' fees to encourage greater participation from sellers, as observed in equation (11b).[22]

An increase in the cross-group network effect has a greater impact on buyers' equilibrium membership fee in platform $i$. This is because platform $i$ is more proficient in developing features, which attracts a larger number of buyers. Consequently, platform $i$ exploits this by charging buyers a higher fee, allowing it to extract a greater portion of buyers' surplus.

### Equilibrium market shares and profits

Using equations (5a), (5b) and (10a), (10b) we obtain the following equilibrium market shares:[23]

$$\left(\eta_b^i\right)^{sc} = \frac{1}{2} + \frac{\left(\alpha^j - \alpha^i\right)\tau}{2\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{12a}$$

$$\left(\eta_s^i\right)^{sc} = \frac{1}{2} + \frac{\left(\alpha^j - \alpha^i\right)\pi}{2\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{12b}$$

where $\sigma \equiv \tau^2 - \pi\upsilon$

Platform $i$ gains a larger market share among both buyers and sellers considering is more efficient in developing attributes on buyers' side. Equilibrium market shares on both sides increase when the cross-group network effect is stronger ($\pi = \upsilon$). Platform $i$ becomes more valuable to both buyers and sellers as the cross-group network effects strengthen, resulting in the development of more attributes for buyers. This positive feedback loop contributes to a rapid expansion of platform $i$'s market share, potentially leading to its dominance in the market.[24]

Using equilibrium membership fees (11a) and (11b) and equilibrium market shares (12a) and (12a) we obtain platform $i$, $i = 1, 2$ equilibrium profits as a function of the equilibrium features configurations:

$$\left(\Pi^i\right)^{sc} = \tau - \pi + \frac{9\sigma\left(\alpha^j - \alpha^i\right)\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau\right] - \alpha^i\left(9\alpha^j\sigma - 2\tau\right)\left(9\alpha^i\sigma - 2\tau\right)}{18\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau_s\right]^2} \tag{13}$$

where $\sigma \equiv \tau^2 - \pi^2$.

Platform $i$ equilibrium profits are a function of two terms. The first term is similar to Armstrong (2006) having product differentiation on both sides of the market ($\tau_b = \tau_s = \tau$) and cross-group network effects ($\pi = \upsilon$). The second term is an extra markup related to the difference in attributes on buyers' side between both platforms, which is positive assuming condition **A.3** holds.[25]

[22] The proof for Proposition 3 is straightforward, partially derive equation (11a) with respect to the model parameters. For details see Appendix B.2.

[23] Conditions for agents market shares distributed in the unit interval given all agents participate are: $\eta_b^i \in [0, 1]$ if $\alpha^i > \frac{2\tau}{9\sigma}$ for $i = 1, 2$ and $\eta_s^i \in [0, 1]$ if $\alpha^i > \frac{\alpha^j(\tau-\pi)}{9\alpha^j\sigma-(\tau-\pi)}$ for $i, j = 1, 2$ and $i \neq j$, both conditions are satisfied assuming condition **A.3** holds. For details see Appendix B.3.

[24] Partially derive (12a) and (12b) respect the model parameters. For details see Appendix B.4.

[25] See Appendix B.5 for details.

We obtain some insights into platforms' strategy to maximise profits in the following proposition:

**Proposition 4.** *The difference in equilibrium profits between platform $i$ and $j$ decreases as platform $i$ intensifies its differentiation from platform $j$ (higher $\tau$) and increases the more valuable it becomes for both agents inasmuch as the cross-group network effect ($\pi = \upsilon$) turns stronger.*

**Proof:** See Appendix B.6

Proposition 4 contrasts with Armstrong (2006) where equilibrium platforms' profits are increasing on product differentiation and decreasing on cross-group network effects. In our benchmark scenario, the effects in equilibrium profits are the opposite.

As both platforms become more differentiated, there is a decrease in the development of attributes on buyers' side. Consequently, the number of buyers joining the platform decreases, along with the number of sellers, considering the cross-group network effect. As a result, platform $i$ has a smaller pool of agents to charge additional fees to, leading to a decline in the difference in equilibrium profits.

Conversely, an increase in cross-group network effects leads to offering more attributes on the buyers' side. This attracts a larger number of buyers and sellers, taking into account the cross-effect of the networks. In response, platform $i$ charges a higher fee on the buyers' side and a lower fee on the sellers' side, as indicated in Proposition 3. Accordingly, platform $i$ charges an additional fee per additional agent, resulting in increased profits.

These findings align with the early work conducted by Economides (1989) and Neven and Thisse (1990) and the generalised model by Irmen and Thisse (1998). These studies suggest that platforms' profit-maximising strategy involves maximising differentiation on one dimension while minimising differentiation on the other dimension. In the current scenario, platform $i$ increases the vertical dimension by developing attributes on the buyers' side when the horizontal dimension, representing the product differentiation parameter on both sides of the market, decreases.

### 5.2 Non-Identical cross-group network effects, $\pi \neq \upsilon$

In this section, our objective is to analyse the presence of asymmetric cross-group network effects. To ensure that the analysis remains tractable without sacrificing its essence, we simplify the model by setting the side that exerts a weaker network effect on the other side to zero.[26]

The first case we consider is when buyers value interactions more than sellers ($\upsilon > \pi$). To keep our analysis tractable, without any loss of generality, we normalise the value of $\pi$ to be approximately zero. The second case we examine is when sellers value interaction more than buyers ($\pi > \upsilon$). Again, for simplicity, we normalise the value of $\upsilon$ to be approximately zero. By using the game's solution at stage 1, we obtain the strategic variables as functions of the model's parameters and derive insights into the results.

[26] Ideally, what we mean is that the network effect exerted by this side is negligible compared to the magnitude of the network effect originating from the other side.

### Equilibrium attribute values

Using equations (8a) and (8b) we obtain platform $i$'s equilibrium attributes on buyers' side:[27]

$$\text{When } \upsilon > \pi \ (\pi = 0), \ \Rightarrow \ \left(q_b^i\right)\Big|_{\upsilon>\pi} = \frac{\left(\alpha^j\sigma_\upsilon - 2\tau\right)\left[2\left(3\tau^2 - \upsilon^2\right) + \tau\upsilon\right]}{2\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{14a}$$

$$\text{When } \pi > \upsilon \ (\upsilon = 0), \ \Rightarrow \left(q_b^i\right)\Big|_{\pi>\upsilon} = \frac{\left(\alpha^j\sigma_\pi - 2\tau\right)\left[6\tau^2 - \pi^2 - \tau\pi\right]}{2\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{14b}$$

where $\sigma_\upsilon \equiv 9\tau^2 - 2\upsilon^2$ and $\sigma_\pi \equiv 9\tau^2 - 2\pi^2$ .

We can observe on equation (14a) and (14b) that equilibrium attributes on buyers' side $\left(q_b^i\right)\Big|_{\upsilon>\pi}$ and $\left(q_b^i\right)\Big|_{\pi>\upsilon}$ are positive if Assumptions **A.1** and **A.3** holds. Then we can state the next proposition:

**Proposition 5.** *Equilibrium attributes on the buyers' side decrease as the degree of product differentiation grows and increase with a stronger cross-group network effect.*

**Proof:** See Appendix C.1

Proposition 2 and Proposition 5 provide similar insights regarding equilibrium attributes on buyers' side. Regardless of whether the cross-group network effects are identical or if one side exerts a stronger network effect on the other, these propositions establish that equilibrium attributes on the buyers' side unambiguously decrease with a higher degree of product differentiation and increase with stronger cross-group network effects.

Proposition 5 is based on the observation that as the degree of product differentiation ($\tau$) increases, platform $i$ engages in less aggressive competition for both agents. This is because the unique and distinct nature of its services reduces the need to develop additional attributes on buyers' side to attract them. Conversely, when there is a stronger relationship between the two groups, characterised by increased features on the buyers' side, platform $i$ becomes more valuable to both agents. The growth of one group enhances the value of the other group, resulting in mutual growth.[28]

[27] The second order conditions at stage 1 of the game when $\upsilon > \pi$, $\pi = 0$ are $\alpha^i > \frac{\tau}{\sigma_\upsilon}$ and $\alpha^i > \frac{\alpha^j\tau}{\alpha^j\sigma_\upsilon - \tau}$ and when $\pi > \upsilon$, $\upsilon = 0$ are $\alpha^i > \frac{\tau}{\sigma_\pi}$ and $\alpha^i > \frac{\alpha^j\tau}{\alpha^j\sigma_\pi - \tau}$, where $\sigma_\upsilon \equiv 9\tau^2 - 2\upsilon^2$ and $\sigma_\pi \equiv 9\tau^2 - 2\pi^2$ and both are satisfied if condition **A.3** holds.

[28] Conditions $\tau > \frac{\upsilon}{2}$ and $\tau > \frac{\pi}{2}$ are satisfied considering Assumption **A.1** holds.

**Equilibrium market shares**

We proceed to obtain the equilibrium market shares on both sides of the market using equations (5a), (5b) and (14a), (14b)[29]

$$\eta_b^i = \frac{1}{2} + \frac{\left(\alpha^j - \alpha^i\right) 3\tau \left[6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)\right]}{4\Sigma\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{15a}$$

$$\eta_s^i = \frac{1}{2} + \frac{\left(\alpha^j - \alpha^i\right)(\pi + 2\upsilon)\left[6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)\right]}{4\Sigma\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{15b}$$

where $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$.

Based on the equilibrium market shares (15a) and (15b), we can conclude that platform $i$ gains a competitive advantage over its rival by being more efficient in developing attributes on buyers' side. This advantage holds regardless of whether the cross-group network effects are identical ($\pi = \upsilon$), as mentioned in Section 5.1, or if the indirect network effect exerted by sellers on buyers is larger ($\upsilon > \pi,\ \pi = 0$), or if the cross-group network effect exerted by buyers on sellers is stronger ($\pi > \upsilon,\ \upsilon = 0$) in Section 5.2. Platform $i$ outperforms platform $j$ because it is capable of producing superior features on the buyers' side with fewer resources and in less time.[30]

The following claim captures the impact of model parameters $\tau$ and $\pi$, $\upsilon$, on buyers' and sellers' equilibrium market shares.[31]

**Claim 1.** *Buyers' and sellers' equilibrium market shares decrease when platform $i$ is more heterogeneous (higher $\tau$) and increase when the cross-group network effects become stronger (higher $\upsilon$, $\pi$).*

**Proof:** See Appendix C.3

The claim states that as platform $i$ becomes more heterogeneous, the number of attributes on buyers' side decreases, resulting in diminished incentives for agents to join platform $i$. Conversely, as the cross-group network effects increase, platform $i$ becomes more valuable, leading to more agents joining the platform on both sides of the market.[32]

The next step is to obtain platform $i$, $i = 1, 2$ equilibrium profits as a function of the equilibrium features on equation (8a) and (8b):

$$\Pi^i = \tau - \frac{\pi + \upsilon}{2} + \left[\frac{\left(\alpha^j - \alpha^i\right)\Sigma\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i + \alpha^j\right)\tau\right] - \alpha^i\left(\alpha^j\Sigma - 2\tau\right)\left(\alpha^i\Sigma - 2\tau\right)}{8\Sigma^2\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\right] X^2 \tag{16}$$

[29] Considering all agents participate, buyers' and sellers' market shares are distributed in the unit interval as long as Assumptions **A.1** and **A.3** hold. When $\upsilon > \pi$ and without loss of generality $\pi = 0$, $\frac{\upsilon}{2} < \tau < \frac{2\upsilon}{3}$ and $\alpha^i > \frac{2\tau}{\sigma_\upsilon}$, $\sigma_\upsilon \equiv 9\tau^2 - 2\upsilon^2$. When $\pi > \upsilon$ and without loss of generality $\upsilon = 0$, $\frac{\pi}{2} < \tau < \frac{2\pi}{3}$ and $\alpha^i > \frac{2\tau}{\sigma_\pi}$, $\sigma_\upsilon \equiv 9\tau^2 - 2\pi^2$. For details see Appendix A.8.

[30] As discussed in Section 4, it is observed that the expressions $6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)$ and $9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$ are positive, provided that Assumptions **A.1** and **A.3** hold.

[31] As we observe equilibrium market shares on buyers' side is $\eta_b^i = \frac{1}{2} + \frac{3\tau}{2\Sigma}\Delta q_b i$ and on sellers' side is $\eta_s^i = \frac{1}{2} + \frac{(\pi+2\upsilon)}{2\Sigma}\Delta q_b i$

[32] The detailed derivation of these results can be found in Appendix C.3, where equations (15a) and (15b) are partially derived with respect to the model parameters.

where $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$ and $X \equiv 6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)$.

Platform $i$'s equilibrium profits are a function of two terms. The first term is similar to Armstrong (2006), product differentiation cost and cross-side network effects on both sides of the market. The second term is a markup related to the difference in attributes on buyers' side, which is positive assuming condition **A.3** holds.[33]

### Equilibrium membership fees

#### Case 1: When sellers exert a stronger influence on buyers: $\upsilon > \pi \, (\pi = 0)$.

In this case, we have:

$$\left(p_b^i\right)\Big|_{\upsilon>\pi} = f_b + \tau + \frac{3\left(\alpha^j - \alpha^i\right)\tau^2\left[2\left(3\tau^2 - \upsilon^2\right) + \tau\upsilon\right]}{2\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{17a}$$

$$\left(p_s^i\right)\Big|_{\upsilon>\pi} = f_s + \tau - \upsilon - \frac{\left(\alpha^j - \alpha^i\right)\tau\upsilon\left[2\left(3\tau^2 - \upsilon^2\right) + \tau_s\upsilon\right]}{2\sigma_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{17b}$$

where $\sigma_\upsilon \equiv 9\tau^2 - 2\upsilon^2$.

Note that the extra markup on equations (17a) and (17b) is positive considering Assumptions **A.1** and **A.3** hold. Therefore, when the cross-group network effect sellers exert on buyers outweighs the effect buyers exert on sellers, platform $i$ implements a pricing strategy that deviates from the seminal results by Armstrong (2006). Specifically, platform $i$ charges on the buyers' side an additional markup while reducing sellers' subscription fees. That is $\left(p_b^i\right)\Big|_{\upsilon>\pi} > \left(p_b^i\right)^{Armstrong}$ and $\left(p_s^i\right)\Big|_{\upsilon>\pi} < \left(p_s^i\right)^{Armstrong}$.

Next, we characterise the impacts on equilibrium fees considering platform $i$ pricing strategy on both market sides.

**Proposition 6.a.** *For $\upsilon > \pi \, (\pi = 0)$, the difference in equilibrium membership fees*

*(i) On buyers' side decreases and on sellers' side increases when $\tau$ increases.*

*(ii) On buyers' side increases and on sellers' side decreases as the cross-group network effect becomes stronger (i.e., when $\upsilon$ increases).*

**Proof:** See Appendix C.2.

According to Proposition 6.a, as the degree of product differentiation increases, there is no need for platform $i$ to develop additional attributes on the buyers' side. This is because platform $i$ is perceived as offering unique and distinct services compared to other platforms. As a result, the features on buyers' side decrease, which can discourage buyers from joining platform $i$.

To counteract this potential decrease in buyer participation, platform $i$ adjusts its pricing strategy by charging a lower fee on the buyers' side. This lower fee is aimed at attracting and retaining buyers. To compensate for the revenue loss from lower buyer fees, platform $i$ charges

[33] See Appendix C.4 for details.

a higher fee on the sellers' side. The higher fee is justified by the increased participation of sellers due to the positive cross-group network effect.

This finding contrasts with the results of Armstrong (2006), where membership fees on both sides of the market increase as the degree of product differentiation increases. The difference arises from the fact that in our model, platforms adjust their pricing strategies indirectly by manipulating the features developed on the buyers' side, rather than directly adjusting the membership fees.

Furthermore, when the cross-group network effect exerted by sellers on buyers is stronger, platform $i$ increases the attributes on buyers' side. This strategy aims to appeal to more buyers and incentivize their participation in the platform. Consequently, platform $i$ charges a higher fee to buyers, reflecting the additional value provided through the developed attributes. Additionally, the stronger cross-group network effect encourages more sellers to join the platform, as they benefit from the increased buyer participation. To attract and retain sellers, platform $i$ charges them a lower fee.

This result aligns with existing findings in the literature on two-sided markets, where platforms often adjust their pricing strategies by charging a lower subscription fee on the side that exerts a more substantial influence on the other side. In this particular scenario, sellers have a more prominent effect on buyers. By charging a lower fee to sellers, platform $i$ promotes their participation, which, in turn, attracts more agents on both sides of the market.

**Case 2: When buyers exert a stronger influence on sellers, $\pi > \upsilon\,(\upsilon = 0)$.**

In this case, we have:

$$\left(p_b^i\right)\Big|_{\pi>\upsilon} = f_b + \tau - \pi + \frac{\left(\alpha^j - \alpha^i\right)\left(3\tau^2 - \pi^2\right)\left[6\tau^2 - \pi^2 - \tau\pi\right]}{2\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{18a}$$

$$\left(p_s^i\right)\Big|_{\pi>\upsilon} = f_s + \tau + \frac{\left(\alpha^j - \alpha^i\right)\tau\pi\left[6\tau^2 - \pi^2 - \tau\pi\right]}{2\sigma_\pi\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i + \alpha^j\right)\tau\right]} \tag{18b}$$

where $\sigma_\pi \equiv 9\tau^2 - 2\pi^2$.

When the cross-group network effect exerted by buyers on sellers is stronger than the effect sellers have on buyers, platform $i$ also adopts a pricing strategy that deviates from the seminal results presented in Armstrong (2006) as in **case 1**. Specifically, platform $i$ charges a lower subscription fee for buyers, $\left(p_b^i\right)\Big|_{\pi>\upsilon} < \left(p_b^i\right)^{Armstrong}$. Additionally, platform $i$ applies an extra markup on sellers' side, $\left(p_s^i\right)\Big|_{\pi>\upsilon} > \left(p_s^i\right)^{Armstrong}$.[34]

**Proposition 6.b.** *For $\pi > \upsilon\,(\upsilon = 0)$, the difference in equilibrium membership fees*

*(i) On buyers' side increases and on sellers' side decreases when $\tau$ increases.*

*(ii) On buyers' side decreases and on sellers' side increases as the cross-group network effect becomes stronger (i.e., when $\pi$ increases).*

[34]Conditions are satisfied considering Assumptions **A.1** and **A.3** hold.

**Proof:** See Appendix C.2.

It is noteworthy that platform $i$'s pricing strategy in Proposition 6.a is the opposite of Proposition 6.b.

According to Proposition 6.b, platform $i$ adjusts its pricing strategy by lowering the equilibrium fee for sellers, recognising their higher valuation of interaction with the other side of the market. In response to a reduction of features on buyers' side, given an increase in the degree of product differentiation.

On the one hand, this strategy discourages buyers from joining the platform, and as a result, it also affects the sellers' participation due to the cross-group network effect. On the other hand, sellers' fee reduction attracts more of them and, in turn, encourages buyers to join the platform due to the positive cross-group network effect. However, to compensate for the fee decrease on the sellers' side, platform $i$ charges a higher fee to buyers.

Furthermore, when the cross-group network effect exerted by buyers on sellers is stronger, platform $i$ develops more attributes on buyers' side to appeal to a larger number of participants. This increased attractiveness of the platform to sellers, who value interaction more, leads to a higher equilibrium fee charged to them. At the same time, platform $i$ adopts a pricing policy of lowering buyers' subscription fees. This strategy creates a positive feedback loop, as the lower fees attract more buyers, which in turn further enhances the benefits of platform $i$.

### Equilibrium Platform Profits

#### Case 1: When sellers exert a stronger influence on buyers: $\upsilon > \pi \, (\pi = 0)$.

**Proposition 7.a.** *For $\upsilon > \pi$ (i.e., sellers exert a stronger cross-group network effect on buyers' side) the difference in equilibrium profits increases as the degree of product differentiation and the indirect network effect grow.*

**Proof:** See Appendix C.5

Proposition 7.a shows that when the cross-group network effect exerted by sellers on buyers is stronger, $\upsilon > \pi, (\pi = 0)$, the difference in equilibrium profits increases. This is because as platforms become more valuable to buyers (indicated by higher $\upsilon$), the profit-increasing strategy involves developing additional attributes on their side. This prompts agents from both sides of the market to join, resulting in an additional fee per agent and ultimately leading to an increase in the platform's profits.

The result on Proposition 7.a aligns with more recent research by Garella and Lambertini (2014) and Barigozzi and Ma (2018), which suggests that platforms strive to differentiate themselves on both dimensions to maximise profits. Specifically, platforms aim to increase the degree of product differentiation in the horizontal dimension by becoming more heterogeneous, and in the vertical dimension by enhancing features on the buyers' side, as buyers are highly valued by platforms. By pursuing these strategies, platforms can effectively increase their profits in the market.

**Case 2: When buyers exert a stronger influence on sellers, $\pi > \upsilon\,(\upsilon = 0)$.**

**Proposition 7.b.** *For $\pi > \upsilon$ (i.e., buyers exert a stronger cross-group network effect on sellers' side) the difference in equilibrium profits increases as the degree of product differentiation grows and decreases as the cross-group network effect rises.*

**Proof:** See Appendix C.5

It is important to notice that contrary to the previous scenario where the cross-group network effects on both sides are identical when the indirect network effects on both sides of the market are different, the difference in equilibrium profits increase in the product differentiation cost $\tau$ as in the seminal model of Armstrong (2006).

Proposition 7.a and 7.b specify that when platforms are more heterogeneous the difference in equilibrium profits increases whether one side influences the other more or vice versa. The mechanism is as follows:

- Platform $i$ offers unique and differentiated services compared to other platforms, there is no necessity to develop additional attributes on buyers' side. Consequently, the features available to buyers decrease, which can lead to a decrease in their motivation to continue using or join platform $i$ on both sides of the market.
- If buyers value interaction more than sellers, platform $i$ charges them a lower fee. To balance this, charges a higher fee on the sellers' side, as more sellers are expected to join due to the cross-group network effect. This combination of pricing strategies leads to an increase in the difference in equilibrium profits.
- Conversely, when the cross-group network effect exerted by buyers on sellers is stronger, platform $i$ adjusts its pricing strategy by lowering sellers' equilibrium fee. This strategy encourages more buyers to join, driven by the cross-group network effect. To offset the fee decrease on the sellers' side, platform $i$ charges buyers a higher fee.

On the contrary, as seen in Proposition 7.b the result driven from the cross-group network effect may seem counterintuitive because features developed on buyers' side increase as platform $i$ becomes more valuable for both agents (higher $\pi$), attracting more agents to join and generating additional fees per agent. However, the increase in sellers' cross-group network effect enhances their value, leading platforms to engage in more intense competition to attract sellers. This intensified competition prompts platforms to develop more attributes on buyers' side, further escalating competition and as mentioned in Proposition 6.a and 6.b a decrease in the difference in equilibrium profits.

As in the scenario where the cross-group network effects on both sides of the market are identical, Proposition 7.b aligns with the earlier work of Economides (1989) and Neven and Thisse (1990), as well as the generalised model of Irmen and Thisse (1998). This result suggests that platforms strive to maximise their differentiation on one dimension while minimising it on the other to increase profits. Specifically, platforms focus on increasing differentiation in the horizontal dimension by becoming more heterogeneous, while reducing differentiation in the vertical dimension by developing fewer features on buyers' side when the cross-group network effect exerted by sellers decreases.

## 6 Conclusion

In this paper, we have considered a two-stage model for a two-sided market that incorporates the concept of vertical differentiation. By analysing the intricate interplay between quality attributes and cross-group network effects, our research provides valuable insights into various market configurations. Our analysis enables us to explore the relations between price competition, cross-group network effects, and the platform's choice of quality in two-sided markets that are differentiated both horizontally and vertically, thus extending the seminal findings of Armstrong (2006).

We introduced platform attributes on the buyers' side to account for the vertical dimension. In the first stage of the model, platforms selected the level of attributes they offer to buyers simultaneously. In the second stage, platforms simultaneously chose membership fees. The equilibrium membership fees, market shares, and profits were determined by the difference in attributes on the buyers' side. Although the features were developed only on the buyers' side, they also influenced decisions on the sellers' side. As a result, we demonstrate that vertical differentiation allows for the existence of asymmetric platforms in equilibrium. Overall, our contribution is to provide a comprehensive model that captures the dynamics of competition in two-sided markets with vertical differentiation.

Our study examines two scenarios depending on the strength of the cross-group network effects. Specifically, we consider the following scenarios: Firstly, we explore a case where the indirect network effects on both sides of the market are identical. Secondly, we centre our attention where sellers' cross-group network effect on buyers is stronger than buyers' impact on sellers, normalising sellers' network effect to zero. Then, we analyse where buyers' cross-group network effect on sellers is stronger than sellers' impact on buyers, normalising buyers' network effect to zero. By examining these scenarios, we contribute to the existing literature on two-sided markets by offering insights into the influence of cross-group network effects and attributes as a vertical differentiation variable on platform competition. This knowledge can be leveraged to devise effective strategies that enhance platform performance and support overall market welfare.

Our analysis shows platforms use attributes on buyers' side as the main trigger to adjust their strategies to appeal to agents and boost profits. We find that the more heterogeneous platforms are, the fewer attributes they develop on the buyers' side. Whereas the more valuable platforms become given a stronger cross-group network effect, the more attributes are offered on the buyers' side. This mechanism drives platforms to adjust equilibrium membership fees and profits. Our analysis also uncovers interesting insights into the impact of model parameters on equilibrium membership fees, which are contingent on the relative strength of cross-group network effects between the two sides of the market. By providing such granular insights, platforms design optimal pricing strategies in two-sided markets with attributes on the buyers' side.

We also identify the optimal conditions for platforms to maximise their profits by strategically balancing the degree of product differentiation on the horizontal dimension and attributes on the buyers' side on the vertical dimension. This finding aligns with previous research conducted by Garella and Lambertini (2014) and Barigozzi and Ma (2018). Specifically, we observe that this optimal strategy occurs when the cross-group network effect exerted by sellers on buyers is stronger than the impact buyers have on sellers. Moreover, we establish the conditions under which it is optimal to maximise one dimension while minimising the other dimension to enhance profitability. This pattern is consistent with earlier studies, including Economides (1989) and Neven and Thisse (1990), as well as the generalised model proposed by Irmen and Thisse (1998).

Our findings shed light on the strategic trade-offs platforms face in two-sided markets with vertical differentiation seen as attributes on the buyers' side and provide important insights for platform managers and policymakers seeking to optimise their pricing strategies. By understanding the optimal conditions for maximising profits, platforms can enhance their performance and contribute to the overall welfare of the market.

One potential extension of the study involves incorporating features on the sellers' side, which would contribute to a more comprehensive model that better reflects real-world dynamics. Additionally, enabling both buyers and sellers to engage in multihoming would provide valuable insights into how platforms define their pricing strategies. By including these two additional features, a more thorough understanding of the platform's decision-making processes can be attained.

# A Appendix 1

## A.1 Maximisation Problem

Platforms maximise the next expression with respect to both sides membership fees to have:

$$\max_{\{p_b^i, p_s^i\}} \Pi^i \equiv \left(p_b^i - f_b\right)\eta_b^i\left(p_k^i, q_k^i\right) + \left(p_s^i - f_s\right)\eta_s^i\left(p_k^i, q_k^i\right) - \frac{\alpha^i\left(q_b^i\right)^2}{2}, \ i = 1,2 \text{ and } k \in \{b,s\}$$

First-order conditions:

$$\frac{\partial \Pi^i}{\partial p_b^i} \Rightarrow 2\tau p_b^i + (\pi+\upsilon)\,p_s^i - \tau p_b^j - \upsilon p_s^j = \tau f_b + \pi f_s + \left(\tau^2 - \pi\upsilon\right) + \tau\left(q_b^i - q_b^j\right) \quad \text{(a1)}$$

$$\frac{\partial \Pi^i}{\partial p_s^i} \Rightarrow (\pi+\upsilon)\,p_b^i + 2\tau p_s^i - \pi p_b^j - \tau p_s^j = \tau f_s + \upsilon f_b + \left(\tau^2 - \pi\upsilon\right) + \pi\left(q_b^i - q_b^j\right) \quad \text{(a2)}$$

$$\frac{\partial \Pi^j}{\partial p_b^j} \Rightarrow -\tau p_b^i - \upsilon p_s^i + 2\tau p_b^j + (\pi+\upsilon)\,p_s^j = \tau f_b + \pi f_s + \left(\tau^2 - \pi\upsilon\right) + \tau\left(q_b^j - q_b^i\right) \quad \text{(a3)}$$

$$\frac{\partial \Pi^j}{\partial p_s^j} \Rightarrow -\pi p_b^i - \tau p_s^i + (\pi+\upsilon)\,p_b^j + 2\tau p_s^j = \tau f_s + \upsilon f_b + \left(\tau^2 - \pi\upsilon\right) + \pi\left(q_b^j - q_b^i\right) \quad \text{(a4)}$$

Solve $p_s^j$ from equation $(a3)$ and substitute it into equations $(a1)$, $(a2)$ and $(a4)$ to obtain:

$$\tau(2\pi+\upsilon)\,p_b^i + \pi(\pi+2\upsilon)\,p_s^i + \tau(\upsilon-\pi)\,p_b^j = \tau(\pi+2\upsilon)\,f_b + \pi(\pi+2\upsilon)\,f_s + \left(\tau^2-\pi\upsilon\right)(\pi+2\upsilon) + \tau\pi\left(q_b^i - q_b^j\right) \quad \text{(a5)}$$

$$-\left[\tau^2 - (\pi+\upsilon)^2\right]p_b^i + \tau(2\pi+\upsilon)\,p_s^i + \left[2\tau^2 - \pi(\pi+\upsilon)\right]p_b^j = \left[\tau^2 + \upsilon(\pi+\upsilon)\right]f_b + \tau(2\pi+\upsilon)\,f_s + (\tau+(\pi+\upsilon))\left(\tau^2-\pi\upsilon\right) - \left(\tau^2 - \pi(\pi+\upsilon)\right)\left(q_b^i - q_b^j\right) \quad \text{(a6)}$$

$$\left[2\tau^2 - \pi(\pi+\upsilon)\right]p_b^i + \tau(\upsilon-\pi)\,p_s^i - \left[4\tau^2 - (\pi+\upsilon)^2\right]p_b^j = -\left[2\tau^2 - \upsilon(\pi+\upsilon)\right]f_b + \tau(\upsilon-\pi)\,f_s - (2\tau-(\pi+\upsilon))\left(\tau^2-\pi\upsilon\right) + \left[2\tau^2 - \pi(\pi+\upsilon)\right]\left(q_b^i - q_b^j\right) \quad \text{(a7)}$$

Solve $p_b^j$ from equation (a7) and substitute it into equation (a5) and (a6) to obtain:

$$\tau\left[6\tau^2 - (\pi+\upsilon)^2 - 2\pi\upsilon\right]p_b^i + \left[\tau^2(5\pi+\upsilon) - \pi(\pi+\upsilon)(\pi+2\upsilon)\right]p_s^i = \tau\left[6\tau^2 - (\pi+\upsilon)^2 - 2\pi\upsilon\right]f_b + \left[\tau^2(5\pi+\upsilon) - \pi(\pi+\upsilon)(\pi+2\upsilon)\right]f_s + \left[6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon)\right]\left(\tau^2-\pi\upsilon\right) + \tau(\upsilon-\pi)\left(\tau^2-\pi\upsilon\right) + 2\tau\left(\tau^2-\pi\upsilon\right)\left(q_b^i - q_b^j\right) \quad \text{(a8)}$$

$$\left[\tau^2(\pi+5\upsilon)-\upsilon(\pi+\upsilon)(2\pi+\upsilon)\right]p_b^i+\tau\left[6\tau^2-(\pi+\upsilon)^2-2\pi\upsilon\right]p_s^j=\left[\tau^2(\pi+5\upsilon)\right.$$
$$\left.-\upsilon(\pi+\upsilon)(2\pi+\upsilon)\right]f_b+\tau\left[6\tau^2-(\pi+\upsilon)^2-2\pi\upsilon\right]f_s+\left[6\tau^2-(\pi+\upsilon)(2\pi+\upsilon)\right]\left(\tau^2-\pi\upsilon\right)$$
$$+\tau(\pi-\upsilon)\left(\tau^2-\pi\upsilon\right)+(\pi+\upsilon)\left(\tau^2-\pi\upsilon\right)\left(q_b^i-q_b^j\right)\quad\text{(a9)}$$

The next step is to solve the system of equations (a8) and (a9) to obtain:

$$p_b^i=f_b+\tau-\pi+\left[\frac{3\tau^2-\pi(\pi+2\upsilon)}{9\tau^2-(2\pi+\upsilon)(\pi+2\upsilon)}\right]\left(q_b^i-q_b^j\right)\quad\forall\ i,j=1,2\text{and}i\neq j$$
$$p_s^i=f_s+\tau-\upsilon-\left[\frac{\tau(\upsilon-\pi)}{9\tau^2-(2\pi+\upsilon)(\pi+2\upsilon)}\right]\left(q_b^i-q_b^j\right)\quad\forall\ i,j=1,2\text{and}i\neq j$$

## A.2 Second-order conditions at stage 2

We obtain the following second-order conditions of the profit maximisation at stage 2 of the game using the first-order conditions a1 and a2 of the previous section: $\frac{\partial^2\Pi^i}{\partial(p_b^i)^2}=-\frac{2\tau}{2(\tau^2-\pi\upsilon)}0$, $\frac{\partial^2\Pi^i}{\partial p_b^i\partial p_s^i}=-\frac{(\pi+\upsilon)}{2(\tau^2-\pi\upsilon)}$, $\frac{\partial^2\Pi^i}{\partial(p_s^i)^2}=-\frac{2\tau}{2(\tau^2-\pi\upsilon)}$ and $\frac{\partial^2\Pi^i}{\partial p_s^i\partial p_b^i}=-\frac{(\pi+\upsilon)}{2(\tau^2-\pi\upsilon)}$.

We can define the Hessian matrix as:

$$\mathrm{H}=\begin{pmatrix}\Pi^i_{p_b^ip_b^i}\equiv\frac{\partial^2\Pi^i}{\partial(p_b^i)^2}=-\frac{\tau}{(\tau^2-\pi\upsilon)} & \Pi^i_{p_b^ip_s^i}\equiv\frac{\partial^2\Pi^i}{\partial p_b^i\partial p_s^i}=-\frac{(\pi+\upsilon)}{2(\tau^2-\pi\upsilon)}\\ \Pi^i_{p_s^ip_b^i}\equiv\frac{\partial^2\Pi^i}{\partial p_s^i\partial p_b^i}=-\frac{(\pi+\upsilon)}{2(\tau^2-\pi\upsilon)} & \Pi^i_{p_s^ip_s^i}\equiv\frac{\partial^2\Pi^i}{\partial(p_s^i)^2}=-\frac{\tau}{(\tau^2-\pi\upsilon)}\end{pmatrix}$$

$H$ must be negative definite, that is $\det|H|>0$ and $\Pi^i_{p_bp_b}<0$. For $\Pi^i_{p_bp_b}$ to be negative, the denominator $\tau^2-\pi\upsilon$ has to be positive because the numerator is always positive, then: $\boldsymbol{\tau^2>\pi\upsilon}$ and $\det|H|>0\Rightarrow\det|H|=\Pi_{p_bp_b}\times\Pi_{p_sp_s}-\Pi_{p_sp_b}\times\Pi_{p_bp_s}>0\Rightarrow\frac{\tau^2}{(\tau^2-\pi\upsilon)^2}-\frac{(\pi+\upsilon)^2}{4(\tau^2-\pi\upsilon)^2}>0\Leftrightarrow$ $\boldsymbol{4\tau^2>(\pi+\upsilon)^2}$.

The second order conditions of the profit maximisation are $\tau^2>\pi\upsilon$ and $4\tau^2>(\pi+\upsilon)^2$, where condition $\tau^2>\pi\upsilon$ is less stringent than $4\tau^2>(\pi+\upsilon)^2$. Therefore if $4\tau^2>(\pi+\upsilon)^2$ is satisfied, condition $\tau^2>\pi\upsilon$ is satisfied as well. We can expressed condition $4\tau^2>(\pi+\upsilon)^2$ as $\tau>\frac{(\pi+\upsilon)}{2}$ which is assumption **A.1**.

## A.3 Proof of Proposition 1

*Proof.* For platform $i$, partially derived equilibrium membership fees with respect to the difference in attributes on buyers' side. We define $\Delta q_b^i\equiv q_b^i-q_b^j$. For buyers' equilibrium membership fee we have $\frac{\partial p_b^i}{\partial\Delta q_b^i}=\frac{3\tau^2-\pi(\pi+2\upsilon)}{9\tau^2-(2\pi+\upsilon)(\pi+2\upsilon)}$. To see if the previous expression is positive we see first if the denominator is positive and then if the numerator is positive as well. The denominator is positive if the lower bound of condition **A.1** holds. That is $\frac{(\pi+\upsilon)^2}{4}>\frac{(2\pi+\upsilon)(\pi+2\upsilon)}{9}\Leftrightarrow$ $(\pi-\upsilon)^2>0$ if $\pi\neq\upsilon$. To see if the numerator is positive we use the same steps as before, that is $\frac{(\pi+\upsilon)^2}{4}>\frac{\pi(\pi+2\upsilon)}{3}\Leftrightarrow(3\upsilon+\pi)(\upsilon-\pi)>0$ if $\upsilon>\pi$. Therefore $\partial p_b^i/\partial\Delta q_b^i>0$ if $\upsilon>\pi$

and $\pi \neq \upsilon$. For sellers' equilibrium membership fee we have $\frac{\partial p_s^i}{\partial \Delta q_b^i} = \frac{\tau_s(\pi-\upsilon)}{9\tau^2-(2\pi+\upsilon)(\pi+2\upsilon)}$ which is positive if $\pi > \upsilon$ and $\pi \neq \upsilon$. Therefore $\partial p_s^i / \partial \Delta q_b^i > 0$ if $\pi > \upsilon$ and $\pi \neq \upsilon$. □

### A.4 Impact on equilibrium profits

Partially derived equilibrium profits of platform $i$ on equation 6, $\frac{\partial \Pi^i}{\partial q_b^i} = \frac{2q_b^i(\tau-\alpha^i\Sigma)-2\tau q_b^j+X}{2\Sigma} > 0$ if $q_b^i > \frac{2\tau q_b^j - X}{2(\tau-\alpha^i\Sigma)}$. The previous expression is positive if both numerator and denominator are positive or negative. For the denominator to be positive $2\left(\tau - \alpha^i\Sigma\right) > 0$ if $\alpha^i < \frac{\tau}{\Sigma}$. However, if assumption **A.3** holds, the previous condition is not satisfied. Therefore, the denominator is negative. That is $2\left(\tau - \alpha^i\Sigma\right) < 0$ if $\alpha^i > \frac{\tau}{\Sigma}$, and the previous condition is satisfied if assumption **A.3** holds. The numerator is negative if $q_b^j < \frac{X}{2\tau}$. Substituting $q_b^j$ from equation 8b we have $\frac{\left(\alpha^i\Sigma-2\tau\right)X}{2\Sigma[\alpha^i\alpha^j\Sigma-(\alpha^i+\alpha^j)\tau]} < \frac{X}{2\tau} \Rightarrow \Sigma\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i+\alpha^j\right)\tau\right] > \tau\left(\alpha^i\Sigma - 2\tau\right)$ $\Rightarrow \alpha^i\left(\alpha^j\Sigma - 2\tau\right) > \tau\left(\alpha^j\Sigma - 2\tau\right) \Rightarrow \alpha^i > \frac{\tau}{\Sigma}$ if $\alpha^i > \frac{2\tau}{\Sigma}$. The previous condition is satisfied if assumption **A.3** holds.

### A.5 Attributes Maximisation Problem

We obtain buyers' equilibrium attributes for platform $i,j$, where $i,j = 1,2$ and $i \neq j$, maximising equation 6:
First-order conditions:

$$\frac{\partial \Pi^i}{\partial q_b^i} \Rightarrow 2\left[\alpha^i\left[9\tau^2 - (2\pi+\upsilon)(\pi+2\upsilon)\right] - \tau\right] q_b^i = -2\tau q_b^j + 6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon) + \tau(\upsilon-\pi) \tag{a10}$$

$$\frac{\partial \Pi^j}{\partial q_b^j} \Rightarrow 2\left[\alpha^j\left[9\tau^2 - (2\pi+\upsilon)(\pi+2\upsilon)\right] - \tau\right] q_b^j = -2\tau q_b^i + 6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon) + \tau(\upsilon-\pi) \tag{a11}$$

We solve the previous system of equations to obtain:

$$q_b^i = \frac{\left(\alpha^j\Sigma - 2\tau\right)\left[6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon) + \tau(\upsilon-\pi)\right]}{2\Sigma\left[\alpha^i\alpha^j\Sigma - (\alpha^i+\alpha^j)\tau\right]}, \quad \forall\, i,j = 1,2 \text{ and } i \neq j$$

where $\Sigma \equiv 9\tau^2 - (2\pi+\upsilon)(\pi+2\upsilon)$

### A.6 Second-order conditions at stage 1

We obtain the following second-order conditions of the profit maximisation at stage 1 of the game using the first-order conditions a10 and a11: $\frac{\partial^2\Pi^i}{\partial (q_b^i)^2} = \frac{\tau-\alpha^i\Sigma}{\Sigma}$, $\frac{\partial^2\Pi^i}{\partial q_b^i\partial q_b^j} = -\frac{\tau}{\Sigma}$, $\frac{\partial^2\Pi^i}{\partial (q_b^j)^2} = \frac{\tau-\alpha^j\Sigma}{\Sigma}$ and $\frac{\partial^2\Pi^i}{\partial q_b^j\partial q_b^i} = -\frac{\tau}{\Sigma}$. Where $\Sigma \equiv 9\tau^2 - (2\pi+\upsilon)(\pi+2\upsilon)$. We define the Hessian matrix as:

$$\mathrm{H} = \begin{pmatrix} \Pi^i_{q_b^i q_b^i} \equiv \frac{\partial^2\Pi^i}{\partial (q_b^i)^2} = \frac{\tau-\alpha^i\Sigma}{\Sigma} & \Pi^i_{q_b^i q_b^j} \equiv \frac{\partial^2\Pi^i}{\partial q_b^i\partial q_b^j} = -\frac{\tau}{\Sigma} \\ \Pi^i_{q_b^j q_b^i} \equiv \frac{\partial^2\Pi^i}{\partial q_b^j\partial q_b^i} = -\frac{\tau}{\Sigma} & \Pi^i_{q_b^j q_b^j} \equiv \frac{\partial^2\Pi^i}{\partial (q_b^j)^2} = \frac{\tau-\alpha^j\Sigma}{\Sigma} \end{pmatrix}$$

$H$ has to be negative definite, that is det $|H| > 0$ and $\Pi^i_{q^i_b q^i_b} < 0$. For $\Pi^i_{q^i_b q^i_b}$ to be negative, the numerator has to be negative because the denominator is positive. The denominator is positive if condition **A.1** holds. That is $\frac{(\pi+\upsilon)^2}{4} > \frac{(2\pi+\upsilon)(\pi+2\upsilon)}{9} \Leftrightarrow (\pi-\upsilon)^2 > 0$ if $\pi \neq \upsilon$. Now, the numerator $\left(\tau - \alpha^i\Sigma\right)$ is negative if $\boldsymbol{\alpha^i > \frac{\tau}{\Sigma}}$. For det $|H| > 0 \Rightarrow$ det $|H| = \Pi^i_{q^i_b q^i_b} \times \Pi^i_{q^i_s q^i_s} - \Pi^i_{q^i_s q^i_b} \times \Pi^i_{q^i_b q^i_s} > 0$. We have $\frac{\left(\tau-\alpha^i\Sigma\right)\left(\tau-\alpha^j\Sigma\right)}{\Sigma^2} - \frac{\tau^2}{\Sigma^2} > 0$. Then $\Sigma\left(\alpha^i\alpha^j\Sigma - \left(\alpha^i+\alpha^j\right)\tau\right) > 0$ if $\boldsymbol{\alpha^i > \frac{\alpha^j\tau}{\alpha^j\Sigma-\tau}}$.

The second-order conditions of profit maximisation are $\alpha^i > \frac{\tau}{\Sigma}$ and $\alpha^i > \frac{\alpha^j\tau}{\alpha^j\Sigma-\tau}$. Where the former condition is less stringent than the latter. Consequently, if $\alpha^i > \frac{\alpha^j\tau}{\alpha^j\Sigma-\tau}$ is satisfied, $\alpha^i > \frac{\tau}{\Sigma}$ is satisfied as well.

## A.7 Positive Equilibrium Attributes

For $q^i_b > 0\ \forall i = 1,2$ we have $\frac{\left(\alpha^j\Sigma-2\tau\right)\left[6\tau^2-(\pi+\upsilon)(\pi+2\upsilon)+\tau(\upsilon-\pi)\right]}{2\Sigma\left[\alpha^i\alpha^j\Sigma-(\alpha^i+\alpha^j)\tau\right]} > 0$. Let us define $X \equiv 6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon) + \tau(\upsilon-\pi)$. First, the denominator is positive if condition $\alpha^i > \frac{\alpha^j\tau_s}{\alpha^j\Sigma-\tau}$ holds, which satisfied the SOC at stage 1 of the game and if condition **A.1** holds. Next, the numerator $\left(\alpha^j\Sigma - 2\tau\right)X$ is positive if both parts $\alpha^j\Sigma - 2\tau$ and $X$ are positive or negative. When both parts are positive $\alpha^j\Sigma - 2\tau > 0$ if $\alpha^j > \frac{2\tau}{\Sigma}$ and $X \equiv 6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon) + \tau(\upsilon-\pi) > 0$. We can rearrange $X$ to have a polynomial of degree two in $\tau$, that is $6\tau^2 - (\pi-\upsilon)\tau - (\pi+\upsilon)(\pi+2\upsilon)$ Using the quadratic formula we have: $\tau > \frac{(\pi-\upsilon)\pm\sqrt{(\pi-\upsilon)^2+24(\pi+\upsilon)(\pi+2\upsilon)}}{12} \Rightarrow \tau > \frac{(\pi-\upsilon)\pm(5\pi+7\upsilon)}{12}$. $\Rightarrow \tau_1 > \frac{\pi+\upsilon}{2}$ and $\tau_2 > \frac{-(\pi+2\upsilon)}{3}$, the second root doesn't satisfy a positive product differentiation cost. Therefore $X > 0$ if $\tau > \frac{\pi+\upsilon}{2}$ which is the same assumption as **A.1**.

Now, when both parts of the numerator of $q^i_b$ are negative we have $X < 0$ and $\alpha^j\Sigma - 2\tau_s < 0$ if $\alpha^j < \frac{2\tau}{\Sigma}$ for attributes configurations to be positive. Then we have a range over $\alpha^i$, $\frac{\alpha^j\tau}{\alpha^j\Sigma-\tau} < \alpha^i < \frac{2\tau}{\Sigma}$, where the lower bound is from the S.O.C of the maximisation problem at stage 1. However, to establish the previous condition we need to have $\frac{2\tau}{\Sigma} > \frac{\alpha^j\tau}{\alpha^j\Sigma-\tau}$ which holds only if $\alpha^j > \frac{2\tau}{\Sigma}$ which is a contradiction of the initial condition.

Now we establish which condition over $\alpha^i$ for all $i = 1,2$ is more stringent. We know from the previous section A.6 if conditions $\alpha^i > \frac{\tau}{\Sigma}$ and $\alpha^i > \frac{\alpha^j\tau}{\alpha^j\Sigma-\tau}$ holds, the S.O.C are satisfied. Furthermore we know condition $\alpha^i > \frac{\alpha^j\tau}{\alpha^j\Sigma-\tau}$ is more stringent than $\alpha^i > \frac{\tau}{\Sigma}$. Now if condition $\alpha^i > \frac{2\tau}{\Sigma}$ holds, we have positive equilibrium attributes. Then we can see that $\frac{2\tau}{\Sigma} > \frac{\alpha^j\tau}{\alpha^j\Sigma-\tau}$ if $\alpha^j > \frac{2\tau}{\Sigma}$ which is the initial condition. Therefore if condition $\alpha^i > \frac{2\tau}{\Sigma}\ \forall\ i = 1,2$ holds, the S.O.C at stage 1 of the game are satisfied.

Therefore the condition for a positive characteristics configuration $q^i_b > 0$ is $\alpha^i > \frac{2\tau}{\Sigma}\ \forall i = 1,2$ which is assumption **A.3** as long as condition **A.1** holds.

## A.8 Market-shares conditions

For $0 < \eta^i_b < 1$. For $\eta^i_b > 0 \Rightarrow \frac{1}{2} + \frac{3\tau X\left(\alpha^j-\alpha^i\right)}{4\Sigma\left[\alpha^i\alpha^j\Sigma-(\alpha^i+\alpha^j)\tau\right]} > 0 \Rightarrow 3\tau X\left(\alpha^j - \alpha^i\right) + 2\Sigma\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i+\alpha^j\right)\tau\right] > 0$ and is positive if $\alpha^i > \frac{\alpha^j\tau(2\Sigma-3X_\upsilon)}{2\Sigma(\alpha^j\Sigma-\tau)-3\tau X}$. If condition **A.3** holds the previous condition is satisfied. That is $\frac{2\tau}{\Sigma} > \frac{\alpha^j\tau(2\Sigma-3X)}{2\Sigma(\alpha^j\Sigma-\tau)-3\tau X} \Rightarrow 4\Sigma\left(\alpha^j\Sigma - \tau\right) - 6\tau X > \alpha^j\Sigma\left(2\Sigma - 3X\right)$ $\Rightarrow \left(\alpha^j\Sigma - 2\tau\right)\left(2\Sigma + 3X\right) > 0$. The previous expression is satisfied if condition **A.3** holds and

$(2\Sigma + 3X) > 0$. For $2\Sigma + 3X > 0 \Rightarrow 2\left[9t^2 - (2\pi + \upsilon)(\pi + 2\upsilon)\right] + 3\left[6t^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)\right] \Rightarrow \left[36t^2 - (\pi + 2\upsilon)(7\pi + 5\upsilon) + 3\tau(\upsilon - \pi)\right] > 0$ The previous expression is positive if condition **A.1** holds. That is $\frac{(\pi+\upsilon)^2}{4} > \frac{(\pi+2\upsilon)(7\pi+5\upsilon)+3\tau(\pi-\upsilon)}{36}$ if $\tau < \frac{2\pi+\upsilon}{3}$ and $\pi > \upsilon$ or $\tau > \frac{2\pi+\upsilon}{3}$ and $\upsilon > \pi$.

For $\eta_b^i < 1 \Rightarrow \frac{1}{2} + \frac{3\tau X\left(\alpha^j - \alpha^i\right)}{4\Sigma[\alpha^i\alpha^j\Sigma - (\alpha^i+\alpha^j)\tau]} < 1 \Rightarrow 2\alpha^j\Sigma\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i + \alpha^j\right)\tau\right] - 3\tau X\left(\alpha^j - \alpha^i\right) > 0$ and is positive if $\alpha^i > \frac{\alpha^j\tau(3X+2\Sigma)}{2\Sigma(\alpha^j\Sigma-\tau)+3\tau X}$. If condition **A.3** holds the previous condition is satisfied. That is $\frac{2\tau}{\Sigma} > \frac{\alpha^j\tau(3X+2\Sigma)}{2\Sigma(\alpha^j\Sigma-\tau)+3\tau X} \Rightarrow 4\Sigma\left(\alpha^j\Sigma - \tau\right) + 6\tau X > \alpha^j\Sigma(3X + 2\Sigma)$ $\Rightarrow (2\Sigma - 3X)\left(\alpha^j\Sigma - 2\tau\right) > 0$. The previous expression is satisfied if condition **A.3** holds and $(2\Sigma - 3X) > 0 \Rightarrow 2\left[9t^2 - (2\pi + \upsilon)(\pi + 2\upsilon)\right] - 3\left[6t^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)\right] \Rightarrow$ $(\pi + 2\upsilon)(\upsilon - \pi) - 3\tau(\upsilon - \pi) > 0$ if $\tau < \frac{\pi+2\upsilon}{3}$ and $\upsilon > \pi$ or $\tau > \frac{\pi+2\upsilon}{3}$ and $\pi > \upsilon$.

For $0 < \eta_s^i < 1$. For $\eta_s^i > 0 \Rightarrow \frac{1}{2} + \frac{(\pi+2\upsilon)\left(\alpha^j-\alpha^i\right)X}{4\Sigma[\alpha^i\alpha^j\Sigma-(\alpha^i+\alpha^j)\tau]} > 0 \Rightarrow (\pi + 2\upsilon)\left(\alpha^j - \alpha^i\right)X + 2\Sigma\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i + \alpha^j\right)\tau\right] > 0$ and is positive if $\alpha^i > \frac{\alpha^j(2\Sigma\tau-(\pi+2\upsilon)X)}{2\Sigma(\alpha^j\Sigma-\tau)-(\pi+2\upsilon)X}$. To see if the previous condition is satisfied, we can use the S.O.C of stage 1 of the game $\alpha^i > \frac{\alpha^j\tau}{\alpha^j\Sigma-\tau}$. That is $\frac{\alpha^j\tau}{\alpha^j\Sigma-\tau} > \frac{\alpha^j(2\Sigma\tau-(\pi+2\upsilon)X)}{2\Sigma(\alpha^j\Sigma-\tau)-(\pi+2\upsilon)X} \Rightarrow (\pi + 2\upsilon)X\left(\alpha^j\Sigma - 2\tau\right) > 0$ The previous expression is positive if condition **A.1** and **A.3** holds.

For $\eta_s^i < 1 \Rightarrow \frac{1}{2} + \frac{(\pi+2\upsilon)\left(\alpha^j-\alpha^i\right)X}{4\Sigma[\alpha^i\alpha^j\Sigma-(\alpha^i+\alpha^j)\tau]} < 1 \Rightarrow 2\Sigma\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i + \alpha^j\right)\tau\right] - (\pi + 2\upsilon)\left(\alpha^j - \alpha^i\right)$ $X > 0$ and is positive if $\alpha^i > \frac{\alpha^j[2\Sigma\tau+(\pi+2\upsilon)X]}{2\Sigma(\alpha^j\Sigma-\tau)+(\pi+2\upsilon)X}$. The previous condition is satisfied if assumption **A.3** holds. That is $\frac{2\tau}{\Sigma} > \frac{\alpha^j[2\Sigma\tau+(\pi+2\upsilon)X]}{2\Sigma(\alpha^j\Sigma-\tau)+(\pi+2\upsilon)X} \Rightarrow \left(\alpha^j\Sigma - 2\tau\right)(2\Sigma\tau - (\pi + 2\upsilon)X) > 0$. Expression $2\Sigma\tau - (\pi + 2\upsilon)X$ can be simplify to $\Rightarrow 2\tau\left[9t^2 - (2\pi + \upsilon)(\pi + 2\upsilon)\right] - (\pi + 2\upsilon)\left[6t^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)\right] \Rightarrow 6\tau^2\left[3\tau - (\pi + 2\upsilon)\right] - 3\tau(\pi + 2\upsilon)(\pi + \upsilon) + (\pi + \upsilon)(\pi + 2\upsilon)^2 \Rightarrow$ $\left[6t^2 - (\pi + \upsilon)(\pi + 2\upsilon)\right]\left[3\tau - (\pi + 2\upsilon)\right]$. Then $\eta_s^i < 1$ if $\left(\alpha^j\Sigma - 2\tau\right)\left[6t^2 - (\pi + \upsilon)(\pi + 2\upsilon)\right]\left[3\tau - (\pi + 2\upsilon)\right]$. The previous expression is satisfied if condition **A.3** holds and $\left[6t^2 - (\pi + \upsilon)(\pi + 2\upsilon)\right] > 0$ and $3\tau - (\pi + 2\upsilon) > 0$. The first part is satisfied if assumption **A.1** holds. That is $\frac{(\pi+\upsilon)^2}{4} > \frac{(\pi+\upsilon)(\pi+2\upsilon)}{6}$ if $\pi > \upsilon$. The second part is positive if $\tau > \frac{(\pi+2\upsilon)}{3}$.

In summary condition $0 < \eta_b^i < 1$, $i = 1, 2$ is satisfied if assumption **A.1** and **A.3** holds and $\tau < \frac{2\pi+\upsilon}{3}$ if $\pi > \upsilon$ or $\tau > \frac{2\pi+\upsilon}{3}$ if $\upsilon > \pi$, and $\tau < \frac{\pi+2\upsilon}{3}$ and $\upsilon > \pi$ or $\tau > \frac{\pi+2\upsilon}{3}$ and $\pi > \upsilon$. Additionally, condition $0 < \eta_s^i < 1$, $i = 1, 2$ is satisfied if assumption **A.1** and **A.3** holds and $\tau > \frac{\pi+\upsilon}{2}$ if $\pi > \upsilon$ and $\tau > \frac{(\pi+2\upsilon)}{3}$.

Therefore, condition $0 < \eta_k^i < 1$, $i = 1, 2$ and $k = b, s$, $b \neq s$ is satisfied if assumption **A.3** holds and $\frac{\pi+\upsilon}{2} < \tau < \frac{2\pi+\upsilon}{3}$ if $\pi > \upsilon$ or $\frac{\pi+\upsilon}{2} < \tau < \frac{\pi+2\upsilon}{3}$ if $\upsilon > \pi$, which is assumption **A.3**.

# B Benchmark Scenario: $\pi = \upsilon$

## B.1 Proof of Proposition 2

*Proof.* To prove Proposition 2 partially derive equations 10a with respect to $\tau$ and $\pi$. Considering we assumed platform $j$ is less efficient in developing features on buyers' side than platform $i$, we have $\frac{\partial\left(q_b^i\right)^{sc}}{\partial\tau} = \frac{-3\alpha^j\left(\alpha^j-\alpha^i\right)\left(\tau+\pi^2\right)}{[9\alpha^i\alpha^j\sigma-(\alpha^i+\alpha^j)\tau]^2} < 0$ and $\frac{\partial\left(q_b^i\right)^{sc}}{\partial\pi} = \frac{6\alpha^j\left(\alpha^j-\alpha^i\right)\tau\pi}{[9\alpha^i\alpha^j\sigma-(\alpha^i+\alpha^j)\tau]^2} > 0$. Furthermore, $\frac{\partial\left(q_b^i\right)^{sc}}{\partial\pi} - \frac{\partial\left(q_b^j\right)^{sc}}{\partial\pi} > 0 \Rightarrow \frac{6\left(\alpha^i+\alpha^j\right)\left(\alpha^j-\alpha^i\right)\tau\pi}{[9\alpha^i\alpha^j\sigma-(\alpha^i+\alpha^j)\tau]^2} > 0$ if $\alpha^j > \alpha^i$.

$$\frac{\partial\left(\Delta q_b^i\right)^{sc}}{\partial\tau}=\frac{-3\left(\alpha^i+\alpha^j\right)\left(\alpha^j-\alpha^i\right)\left(\tau+\pi^2\right)}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2}<0 \text{ and } \frac{\partial\left(\Delta q_b^i\right)^{sc}}{\partial\pi}=\frac{6\left(\alpha^i+\alpha^j\right)\left(\alpha^j-\alpha^i\right)\tau\pi}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2}>0 \qquad \square$$

### B.2 Proof of Proposition 3

*Proof.* To prove Proposition 3 we partially derive the difference in equilibrium membership fees on buyers' side, from equations 11a 11b with respect to $\tau$ and $\pi$. Considering we assumed platform $j$ is less efficient in developing features on buyers' side than platform $i$, $\left(\alpha^j>\alpha^i\right)$ we have $\frac{\partial\left(\Delta p_b^i\right)^{sc}}{\partial\tau}=\frac{-2\left(\alpha^i+\alpha^j\right)\left(\alpha^j-\alpha^i\right)\left(\tau+\pi^2\right)}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2}<0$ and $\frac{\partial\left(\Delta p_b^i\right)^{sc}}{\partial\pi}=\frac{4\left(\alpha^i+\alpha^j\right)\left(\alpha^j-\alpha^i\right)\tau\pi}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2}>0$.

On the other hand, $\frac{\partial\left(p_b^i\right)^{sc}}{\partial\pi}>\frac{\partial\left(p_b^j\right)^{sc}}{\partial\pi}\Rightarrow-1+\frac{6\left(\alpha^i+\alpha^j\right)\left(\alpha^j-\alpha^i\right)\tau\pi}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2}>-1+\frac{6\left(\alpha^i+\alpha^j\right)\left(\alpha^i-\alpha^j\right)\tau\pi}{\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2}$ $\Rightarrow 2\left(\alpha^j-\alpha^i\right)>0$ $\square$

### B.3 Market-shares conditions

For $0<\left(\eta_b^i\right)^{sc}<1$. For $\left(\eta_b^i\right)^{sc}>0\Rightarrow\frac{1}{2}+\frac{\left(\alpha^j-\alpha^i\right)\tau}{2\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]}>0\Rightarrow\left(\alpha^j-\alpha^i\right)\tau> -9\alpha^i\alpha^j\sigma+\left(\alpha^i+\alpha^j\right)\tau\Rightarrow 9\alpha^j\sigma>2\tau\Leftrightarrow\alpha^j>\frac{2\tau}{9\sigma}$. For $\left(\eta_b^i\right)^{sc}<1\Rightarrow\frac{1}{2}+\frac{\left(\alpha^j-\alpha^i\right)\tau}{2\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]}<0$ $\Rightarrow\left(\alpha^j-\alpha^i\right)\tau<9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\Rightarrow 2\tau<9\alpha^i\sigma\Leftrightarrow\alpha^i>\frac{2\tau}{9\sigma}$.

For $0<\left(\eta_s^i\right)^{sc}<1$. For $\left(\eta_s^i\right)^{sc}>0\Rightarrow\frac{1}{2}+\frac{\left(\alpha^j-\alpha^i\right)\pi}{2\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]}>0\Rightarrow\left(\alpha^j-\alpha^i\right)\pi> -9\alpha^i\alpha^j\sigma+\left(\alpha^i+\alpha^j\right)\tau\Rightarrow 9\alpha^i\alpha^j\sigma-\alpha^i\left(\tau+\pi\right)-\alpha^j\left(\tau-\pi\right)>0\Leftrightarrow\alpha^i>\frac{\alpha^j\left(\tau-\pi\right)}{9\alpha^j\sigma-\left(\tau+\pi\right)}$. If condition $\alpha^i>\frac{2\tau}{9\sigma}$ holds, the previous term is satisfied. That is: $\frac{2\tau}{9\sigma}>\frac{\alpha^j\left(\tau-\pi\right)}{9\alpha^j\sigma-\left(\tau+\pi\right)}\Rightarrow 18\alpha^j\sigma\tau-2\tau\left(\tau+\pi\right)> 9\alpha^j\sigma\left(\tau-\pi\right)\Rightarrow 9\alpha^j\sigma\left(\tau+\pi\right)>2\tau\left(\tau+\pi\right)\Leftrightarrow\alpha^j>\frac{2\tau}{9\sigma}$. For $\left(\eta_s^i\right)^{sc}<1\Rightarrow\frac{1}{2}+\frac{\left(\alpha^j-\alpha^i\right)\pi}{2\left[9\alpha^i\alpha^j\sigma-\left(\alpha^i+\alpha^j\right)\tau\right]}< 1\Rightarrow 9\alpha^i\alpha^j\sigma-\alpha^j\left(\tau+\pi\right)-\alpha^i\left(\tau-\pi\right)>0\Leftrightarrow\alpha^j>\frac{\alpha^i\left(\tau-\pi\right)}{9\alpha^i\sigma-\left(\tau+\pi\right)}$. If condition $\alpha^i>\frac{2\tau}{9\sigma}$ holds, the previous term is satisfied. That is: $\frac{2\tau}{9\sigma}>\frac{\alpha^i\left(\tau-\pi\right)}{9\alpha^i\sigma-\left(\tau+\pi\right)}\Rightarrow 18\alpha^i\sigma\tau-2\tau_s\left(\tau+\pi\right)>9\alpha^i\sigma\left(\tau-\pi\right)\Rightarrow 9\alpha^i\sigma\left(\tau+\pi\right)>2\tau\left(\tau+\pi\right)\Leftrightarrow\alpha^i>\frac{2\tau}{9\sigma}$.

$\therefore$ Condition $0<\left(\eta_k^i\right)^{sc}<1$ for $i,j=1,2,\ i\neq j$ and $k=b,s$ and $b\neq s$ is satisfied if condition $\alpha^i>\frac{2\tau}{9\sigma}$ holds.

### B.4 Impacts on Equilibrium Market-shares

| **Impacts on equilibrium market shares on buyers' side** | **Impacts on equilibrium market shares on sellers' side** |
|---|---|
| $\frac{\partial(\eta_b^i)^{sc}}{\partial\tau} = \frac{-9\alpha^i\alpha^j(\alpha^j-\alpha^i)(\tau^2+\pi^2)}{2[9\alpha^i\alpha^j\sigma-(\alpha^i+\alpha^j)\tau]^2} < 0$ | $\frac{\partial(\eta_s^i)^{sc}}{\partial\tau} = \frac{-2\pi(\alpha^j-\alpha^i)[18\alpha^i\alpha^j\tau-(\alpha^i+\alpha^j)]}{2[9\alpha^i\alpha^j\sigma-(\alpha^i+\alpha^j)\tau]^2} < 0$ |
| $\frac{\partial(\eta_b^i)^{sc}}{\partial\pi} = \frac{9\alpha^i\alpha^j(\alpha^j-\alpha^i)\tau\pi}{[9\alpha^i\alpha^j\sigma-(\alpha^i+\alpha^j)\tau]^2} > 0$ | $\frac{\partial(\eta_s^i)^{sc}}{\partial\pi} = \frac{(\alpha^j-\alpha^i)[9\alpha^i\alpha^j(\tau+\pi^2)-(\alpha^i+\alpha^j)\tau]}{[9\alpha^i\alpha^j\sigma-(\alpha^i+\alpha^j)\tau]^2} > 0$ |

For the impacts on sellers' equilibrium market shares, we have that $18\alpha^i\alpha^j\tau - (\alpha^i + \alpha^j)$ is positive if $\alpha^i > \frac{\alpha^j}{18\alpha^j\tau-1}$. The previous condition is met as long as Assumption **A.3** holds. That is $\frac{2\tau}{9\sigma} > \frac{\alpha^j}{18\alpha^j\tau-1}$ if $\alpha^j > \frac{2\tau}{9(4\tau^2-\sigma)}$ and the previous condition is met as long as Assumption **A.3** holds. That is $\frac{2\tau}{9\sigma} > \frac{2\tau}{9(4\tau^2-\sigma)}$ if $2(\tau^2 + \pi^2) > 0$.

We have $9\alpha^i\alpha^j(\tau + \pi^2) - (\alpha^i + \alpha^j)\tau$ is positive if $\alpha^i > \frac{\alpha^j\tau}{9\alpha^j(\tau^2+\pi^2)-\tau}$. The previous condition is met as long as Assumption **A.3** holds. That is $\frac{2\tau}{9\sigma} > \frac{\alpha^j\tau}{9\alpha^j(\tau^2+\pi^2)-\tau}$ if $\alpha^j > \frac{2\tau}{9(\tau^2+3\pi^2)}$ and the previous condition is met as long as Assumption **A.3** holds. That is $\frac{2\tau}{9\sigma} > \frac{2\tau}{9(\tau^2+3\pi^2)}$ if $4\pi^2 > 0$

### B.5 Positive Equilibrium Profits

To see if the second term of equilibrium profits for platform $i$, equation 13, is positive we do the following:

$$(\Pi^i)^{sc} = \tau - \frac{2\pi}{2} + \underbrace{\frac{9\sigma(\alpha^j - \alpha^i)\left[9\alpha^i\alpha^j\sigma - (\alpha^i + \alpha^j)\tau\right] - \alpha^i(9\alpha^j\sigma - 2\tau)(9\alpha^i\sigma - 2\tau)}{18\left[9\alpha^i\alpha^j\sigma - (\alpha^i + \alpha^j)\tau\right]^2}}_{\theta}$$

If condition $\alpha^i > \frac{2\tau}{9\sigma}$ holds, the denominator of $\theta$ is positive, then we need to verify the sign of the numerator $9\sigma(\alpha^j - \alpha^i)\left[9\alpha^i\alpha^j\sigma - (\alpha^i + \alpha^j)\tau\right] - \alpha^i(9\alpha^j\sigma - 2\tau)(9\alpha^i\sigma - 2\tau)$. We can rearrange it to have a polynomial of degree two in $\alpha^i$, that is $-27\sigma\left[6\alpha^j\sigma - \tau\right](\alpha^i)^2 + \left[9^2(\alpha^j)^2\sigma^2 + 2\tau(9\alpha^j\sigma - 2\tau)\right]\alpha^i - 9(\alpha^j)^2\sigma\tau$. Using the quadratic formula we have:

$$\alpha^i > -\frac{-\left[9^2(\alpha^j)^2\sigma^2 + 2\tau(9\alpha^j\sigma - 2\tau)\right]}{54\sigma(6\alpha^j\Sigma - \tau)} \pm \frac{\sqrt{\left[9^2(\alpha^j)^2\sigma^2 + 2\tau(9\alpha^j\sigma - 2\tau)\right]^2 - 12 * 9^2(\alpha^j)^2\sigma^2\tau(6\alpha^j\sigma - \tau)}}{54\sigma(6\alpha^j\Sigma - \tau)}$$

$$\alpha^i > -\frac{-\left[9^2(\alpha^j)^2\sigma^2 + 2\tau(9\alpha^j\sigma - 2\tau)\right]}{54\sigma(6\alpha^j\Sigma - \tau)}$$

$$\pm \frac{\sqrt{9^4 \left(\alpha^j\right)^4 \sigma^4 - 4 * 9^2 \left(\alpha^j\right)^2 \sigma^2 \tau \left(9\alpha^j \sigma - \tau\right) + 4\tau_s \left(9\alpha^j \sigma - 2\tau\right)}}{54\sigma \left(6\alpha^j \Sigma - \tau\right)}$$

$$\alpha^i > -\frac{-\left[9^2 \left(\alpha^j\right)^2 \sigma^2 + 2\tau \left(9\alpha^j \sigma - 2\tau\right)\right] \pm \sqrt{\left(9\alpha^j \sigma - 2\tau\right)^2 \left[9^2 \left(\alpha^j\right)^2 \sigma^2 + 4\tau^2\right]}}{54\sigma \left(6\alpha^j \Sigma - \tau\right)}$$

$$\alpha^i > -\frac{-\left[9^2 \left(\alpha^j\right)^2 \sigma^2 + 2\tau \left(9\alpha^j \sigma - 2\tau\right)\right] \pm \left(9\alpha^j \sigma - 2\tau\right) \sqrt{9^2 \left(\alpha^j\right)^2 \sigma^2 + 4\tau^2}}{54\sigma \left(6\alpha^j \Sigma - \tau\right)} \equiv \alpha^i_{ry} \; y = 1, 2$$

We verify if the first root of the polynomial $\alpha^i_{r1}$ is satisfied. If condition $\alpha^i > \frac{2\tau}{9\sigma}$ holds, the previous expression is satisfied. That is $\left(\alpha^i\right)^{sc} > \alpha^i_{r1} \Leftrightarrow \frac{2\tau}{9\sigma} > \frac{\left[9^2\left(\alpha^j\right)^2\sigma^2 + 2\tau\left(9\alpha^j\sigma - 2\tau\right)\right] - \left(9\alpha^j\sigma - 2\tau\right)\sqrt{9^2\left(\alpha^j\right)^2\sigma^2 + 4\tau^2}}{54\sigma\left(6\alpha^j\Sigma - \tau\right)} \Rightarrow 12\tau\left(6\alpha^j\sigma - \tau\right) > 9^2\left(\alpha^j\right)^2\sigma^2 + 2\tau\big(9\alpha^j\sigma - 2\tau\big) - \left(9\alpha^j\sigma - 2\tau\right)\sqrt{\left(9^2\alpha^j\right)^2\sigma^2 + 4\tau^2} \Rightarrow \left(9\alpha^j\sigma - 2\tau\right)\sqrt{\left(9^2\alpha^j\right)^2\sigma^2 + 4\tau^2} > \left(9\alpha^j\sigma - 2\tau\right)\left(9\alpha^j\sigma - 4\tau\right) \Rightarrow 3\tau\left(6\alpha^j\sigma - \tau\right) > 0$ if $\alpha^j > \frac{\tau}{6\sigma}$. Then the previous expression is satisfied if condition $\alpha^i > \frac{2\tau}{9\sigma}$ holds. That is $\frac{2\tau}{9\sigma} > \frac{\tau}{6\sigma} \Leftrightarrow 4 > 3$.

We follow the same logic to verify if the second root of the polynomial $\alpha^i_{r2}$ is satisfied. That is $\left(\alpha^i\right)^{sc} > \alpha^i_{r2}$. That is $\frac{2\tau}{\sigma} > \frac{\left[9\left(\alpha^j\right)^2\sigma^2 + 2\tau\left(9\alpha^j\sigma - 2\tau\right)\right] + \left(9\alpha^j\sigma - 2\tau\right)\sqrt{9\left(\alpha^j\right)^2\sigma^2 + 4\tau^2}}{54\sigma\left(6\alpha^j\sigma - \tau\right)} \Rightarrow 12\tau\left(6\alpha^j\sigma - \tau\right) > 9^2\left(\alpha^j\right)^2\sigma^2 + 2\tau\left(9\alpha^j\sigma - 2\tau\right) + \left(9\alpha^j\sigma - 2\tau\right)\sqrt{\left(9^2\alpha^j\right)^2\sigma^2 + 4\tau^2} \Rightarrow \left(9\alpha^j\sigma - 2\tau\right)\left(9\alpha^j\sigma - 4\tau\right) + \left(9\alpha^j\sigma - 2\tau\right)\sqrt{9^2\left(\alpha^j\right)^2\sigma^2 + 4\tau^2} < 0 \Rightarrow 2\left[9^2\sigma^2\left(\alpha^j\right)^2 - 36\sigma\tau_s\alpha^j + 10\tau\right] < 0$. To verify if the previous expression is negative we can use the quadratic formula to compute both roots. That is $\alpha^j < \frac{36\sigma\tau \pm \sqrt{16 * 9^2\sigma^2\tau^2 - 40 * 9^2\sigma^2\tau^2}}{2 * 9^2\sigma^2} \Rightarrow \alpha^j < \frac{2\tau \pm \tau\sqrt{-6}}{9\sigma}$. Then, the second root $\alpha^i_{r2}$ does not satisfy the condition because it does not have real roots. Therefore, $9\sigma\left(\alpha^j - \alpha^i\right)\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau\right] - \alpha^i\left(9\alpha^j\sigma - 2\tau\right)\left(9\alpha^i\sigma - 2\tau\right) > 0$ is satisfied if Assumption **A.3** $\alpha^i > \frac{2\tau}{9\sigma}$ holds.

### B.6 Proof of Proposition 4

*Proof.* Partially derive the difference in equilibrium profits between platform $i$ and $j$ with respect to product differentiation cost $\tau$ and the cross-group network effect $\pi = \upsilon$ and considering we assumed platform $i$ is more efficient developing attributes on buyers' side than platform $j$, we have:

$$\frac{\partial\left(\Delta\Pi^i\right)^{sc}}{\partial\tau} = \frac{-\left(\alpha^j - \alpha^i\right)\left(\tau^2 + \pi^2\right)\left[\left(\alpha^i\right)^2\left(9\alpha^j\sigma - 2\tau\right) + \left(\alpha^j\right)^2\left(9\alpha^i\sigma - 2\tau\right)\right]}{\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau\right]^3} < 0$$

$$\frac{\partial\left(\Delta\Pi^i\right)^{sc}}{\partial\pi} = \frac{2\left(\alpha^j - \alpha^i\right)\pi\left[\left(\alpha^i\right)^2\left(9\alpha^j\sigma - 2\tau\right) + \left(\alpha^j\right)^2\left(9\alpha^i\sigma - 2\tau\right)\right]}{\left[9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau\right]^3} > 0$$

where $\sigma \equiv \tau^2 - \pi^2$.

As it can be observed expression $9\alpha^i\sigma - 2\tau$, $i = 1, 2$ and $9\alpha^i\alpha^j\sigma - \left(\alpha^i + \alpha^j\right)\tau$ are positive as long as Assumption **A.3** holds. □

## C Scenario: $\upsilon \neq \pi$

### C.1 Proof of Proposition 5

*Proof.* Partially derive equilibrium attributes on buyers' side with respect to the product differentiation cost $\tau$ and cross-group network effects $\upsilon$ and $\pi$.

$$\frac{\partial\left(q_b^i\right)^\upsilon}{\partial\tau}=\frac{-\left[\alpha^iH_\tau^\upsilon-\alpha^jW_\tau^\upsilon\right]}{2\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]^2}<0$$

where $\sigma_\upsilon\equiv 9\tau^2-2\upsilon^2$, $X_\upsilon\equiv 2\left(3\tau^2-\upsilon^2\right)+\tau\upsilon$, $H_\tau^\upsilon\equiv-\sigma_\upsilon X_\upsilon\left(\alpha^j\sigma_\upsilon-\tau\right)\left(18\alpha^j\tau-2\right)+\left[18\tau X_\upsilon-\sigma_\upsilon\left(12\tau+\upsilon\right)\right]\left(\alpha^j\sigma_\upsilon-\tau\right)\left(\alpha^j\sigma_\upsilon-2\tau\right)+18\alpha^j\sigma_\upsilon\tau X_\upsilon\left(\alpha^j\sigma_\upsilon-2\tau\right)-\sigma_\upsilon X_\upsilon\left(\alpha^j\sigma_\upsilon-2\tau\right)$ and $W_\tau^\upsilon\equiv\sigma_\upsilon\tau X_\upsilon\left(18\alpha^j\tau-2\right)+\tau\left[18\tau X_\upsilon-\sigma_\upsilon\left(12\tau+\upsilon\right)\right]\left(\alpha^j\sigma_\upsilon-2\tau\right)+\sigma_\upsilon X_\upsilon\left(\alpha^j\sigma_\upsilon-2\tau\right)$.

We can see expression $\alpha^iH_\tau^\upsilon-\alpha^jW_\tau^\upsilon$ is positive if $\alpha^j>\frac{\alpha^jW_\tau^\upsilon}{H_\tau^\upsilon}$ and is satisfied if condition **A.3** when $\upsilon>\pi$, $\pi=0$ $\boldsymbol{\alpha^i>\frac{2\tau}{\sigma_\upsilon}}$ for $i=1,2$ holds. That is $\frac{2\tau}{\sigma_\upsilon}>\frac{\alpha^jW_\tau^\upsilon}{H_\tau^\upsilon}\Rightarrow\left(\alpha^j\sigma_\upsilon-2\tau\right)\Big[-\tau X_\upsilon\sigma_\upsilon\left(18\alpha^j\tau-2\right)-\tau\sigma_\upsilon\left(12\tau+\upsilon\right)\left(\alpha^j\sigma_\upsilon-2\tau\right)+18\tau^2X_\upsilon\left(\alpha^j\sigma_\upsilon-2\tau\right)+36\alpha^j\tau^2\sigma_\upsilon X_\upsilon-\sigma_\upsilon X_\upsilon\big(\alpha^j\sigma_\upsilon+2\tau\big)\Big]>0\Rightarrow\alpha^j\sigma_\upsilon\left[36\tau^2X_\upsilon-\tau\sigma_\upsilon\left(12\tau+\upsilon\right)-\sigma_\upsilon X_\upsilon\right]-2\tau^2\left[18\tau X_\upsilon-\sigma_\upsilon\left(12\tau+\upsilon\right)\right]>0$.

The previous expression is positive if $\alpha^j>\frac{2\tau_s^2\left[18\tau X_\upsilon-\sigma_\upsilon(12\tau+\upsilon)\right]}{\sigma_\upsilon\left[36\tau^2X_\upsilon-\tau\sigma_\upsilon(12\tau+\upsilon)-\sigma_\upsilon X_\upsilon\right]}$ and is satisfied if condition $\boldsymbol{\alpha^i>\frac{2\tau}{\sigma_\upsilon}}$ for $i=1,2$ holds. That is $\frac{2\tau}{\sigma_\upsilon}>\frac{2\tau^2\left[18\tau X_\upsilon-\sigma_\upsilon(12\tau+\upsilon)\right]}{\sigma_\upsilon\left[36\tau^2X_\upsilon-\tau\sigma_\upsilon(12\tau+\upsilon)-\sigma_\upsilon X_\upsilon\right]}\Rightarrow\left(18\tau^2-\sigma_\upsilon\right)X_\upsilon>0\Leftrightarrow\left(9\tau^2+2\upsilon^2\right)X_\upsilon>0$ if $\boldsymbol{\tau>\frac{\upsilon}{2}}$, which is satisfied if Assumption **A.1** holds.

$$\frac{\partial\left(q_b^i\right)^\upsilon}{\partial\upsilon}=\frac{\left[\alpha^iH_\upsilon^\upsilon-\alpha^j\tau W_\upsilon^\upsilon\right]}{4\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]^2}>0$$

Where $\sigma_\upsilon\equiv 9\tau^2-2\upsilon^2$, $X_\upsilon\equiv 2\left(3\tau^2-\upsilon^2\right)+\tau\upsilon$, $H_\upsilon^\upsilon\equiv\left(\alpha^j\sigma_\upsilon-2\tau\right)\left(\alpha^j\sigma_\upsilon-\tau\right)\left[\sigma_\upsilon\left(\tau-4\upsilon\right)+4\upsilon X_\upsilon\right]-4\alpha^j\upsilon X_\upsilon\sigma_\upsilon\tau$ and $W_\upsilon^\upsilon\equiv-4\alpha^j\upsilon X_\upsilon\sigma_\upsilon+\left(\alpha^j\sigma_\upsilon-2\tau\right)\left[\sigma_\upsilon\left(\tau-4\upsilon\right)+4\upsilon X_\upsilon\right]$.

We can see expression $\alpha^iH_\upsilon^\upsilon-\alpha^j\tau W_\upsilon^\upsilon$ is positive if $\alpha^i>\frac{\alpha^j\tau W_\upsilon^\upsilon}{H_\upsilon^\upsilon}$ and is satisfied if condition $\boldsymbol{\alpha^i>\frac{2\tau}{\sigma_\upsilon}}$ for $i=1,2$ holds. That is $\frac{2\tau}{\sigma_\upsilon}>\frac{\alpha^j\tau_sW_\upsilon^\upsilon}{H_\upsilon^\upsilon}\Rightarrow\left(\alpha^j\sigma_\upsilon-2\tau\right)\Big[\left(\alpha^j\sigma_\upsilon-2\tau\right)\left[\sigma_\upsilon\left(\tau-4\upsilon\right)+4\upsilon X_\upsilon\right]+4\alpha^j\sigma_\upsilon\upsilon X_\upsilon\Big]>0$. The previous expression is positive if $\alpha^j>\frac{2\tau[\sigma_\upsilon(\tau-4\upsilon)+4\upsilon X_\upsilon]}{\sigma_\upsilon[\sigma_\upsilon(\tau-4\upsilon)+8\upsilon X_\upsilon]}$ and is satisfied if condition $\boldsymbol{\alpha^i>\frac{2\tau}{\sigma_\upsilon}}$ for $i=1,2$ holds. That is $\frac{2\tau}{\sigma_\upsilon}>\frac{2\tau[\sigma_\upsilon(\tau-4\upsilon)+4\upsilon X_\upsilon]}{\sigma_\upsilon[\sigma_\upsilon(\tau-4\upsilon)+8\upsilon X_\upsilon]}\Leftrightarrow 4\upsilon X_\upsilon>0$ if $\boldsymbol{\tau>\frac{\upsilon}{2}}$, which is satisfied if Assumption **A.1** holds.

$$\frac{\partial\left(q_b^i\right)^\pi}{\partial\tau}=\frac{-\left[\alpha^iH_\tau^\pi-\alpha^jW_\tau^\pi\right]}{2\sigma_\pi^2\left[\alpha^i\alpha^j\sigma_\pi-\left(\alpha^i+\alpha^j\right)\tau_s\right]^2}<0$$

where $\sigma_\pi\equiv 9\tau^2-2\pi^2$, $X_\upsilon\equiv 6\tau^2-\pi^2-\tau\pi$, $H_\tau^\pi\equiv-\sigma_\pi X_\pi\left(\alpha^j\sigma_\pi-\tau\right)\left(18\alpha^j\tau-2\right)+\big[18\tau X_\pi-\sigma_\pi\left(12\tau-\pi\right)\big]\left(\alpha^j\sigma_\pi-\tau\right)\left(\alpha^j\sigma_\pi-2\tau\right)+18\alpha^j\sigma_\pi\tau X_\pi\left(\alpha^j\sigma_\pi-2\tau\right)-\sigma_\pi X_\pi\left(\alpha^j\sigma_\pi-2\tau\right)$ and $W_\tau^\pi\equiv\sigma_\pi\tau X_\pi\left(18\alpha^j\tau-2\right)+\tau\left[18\tau X_\pi-\sigma_\pi\left(12\tau-\pi\right)\right]\left(\alpha^j\sigma_\pi-2\tau\right)+\sigma_\pi X_\pi\left(\alpha^j\sigma_\pi-2\tau\right)$.

We can see expression $\alpha^iH_\tau^\pi-\alpha^jW_\tau^\pi$ is positive if $\alpha^j>\frac{\alpha^jW_\tau^\pi}{H_\tau^\pi}$ and is satisfied if condition **A.3** when $\pi>\upsilon$, $\upsilon=0$, $\boldsymbol{\alpha^i>\frac{2\tau}{\sigma_\pi}}$ for $i=1,2$ holds. That is $\frac{2\tau}{\sigma_\pi}>\frac{\alpha^jW_\tau^\pi}{H_\tau^\pi}\Rightarrow\left(\alpha^j\sigma_\pi-2\tau\right)\Big[-$

$\tau X_\pi \sigma_\pi \left(18\alpha^j\tau - 2\right) - \tau\sigma_\pi\left(12\tau - \pi\right)\left(\alpha^j\sigma_\pi - 2\tau\right) + 18\tau^2\tau X_\pi\left(\alpha^j\sigma_\pi - 2\tau\right) + 36\alpha^j\tau^2\sigma_\pi X_\pi - \sigma_\pi X_\upsilon\big(\alpha^j \sigma_\pi + 2\tau\big)\big] > 0 \Rightarrow \alpha^j\sigma_\pi\left[36\tau^2 X_\pi - \tau\sigma_\pi\left(12\tau - \pi\right) - \sigma_\pi X_\pi\right] - 2\tau^2\left[18\tau X_\pi - \sigma_\pi\left(12\tau - \pi\right)\right] > 0$. The previous expression is positive if $\alpha^j > \frac{2\tau\left[36\tau^2 X_\pi - \sigma_\pi(12\tau-\pi)\right]}{\sigma_\pi\left[36\tau^2 X_\pi - \tau\sigma_\pi(12\tau-\pi) - \sigma_\pi X_\pi\right]}$ and is satisfied if condition $\boldsymbol{\alpha^i > \frac{2\tau_s}{\sigma_\pi}}$ for $i = 1, 2$ holds. That is $\frac{2\tau}{\sigma_\pi} > \frac{2\tau\left[36\tau^2 X_\pi - \sigma_\pi(12\tau-\pi)\right]}{\sigma_\pi\left[36\tau^2 X_\pi - \tau\sigma_\pi(12\tau-\pi) - \sigma_\pi X_\pi\right]} \Rightarrow \sigma_\pi X_\pi > 0 \Leftrightarrow 2\pi^2 X_\pi > 0$ if $\boldsymbol{\tau > \frac{\pi}{2}}$, which is satisfied if Assumption **A.1** holds.

$$\frac{\partial\left(q_b^i\right)^\pi}{\partial\pi} = \frac{\left[\alpha^i H_\pi^\pi - \alpha^j\tau W_\pi^\pi\right]}{4\sigma_\pi^2\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i + \alpha^j\right)\tau\right]^2} > 0$$

where $\sigma_\pi \equiv 9\tau^2 - 2\pi^2$, $X_\pi \equiv 6\tau^2 - \pi^2 - \tau\pi$, $H_\pi^\pi \equiv \left(\alpha^j\sigma_\pi - 2\tau\right)\left(\alpha^j\sigma_\pi - \tau\right)\left[4\pi X_\pi - \sigma_\pi\left(\tau + 2\pi\right)\right] - 4\alpha^j\pi X_\pi\sigma_\pi\tau$ and $W_\pi^\pi \equiv -4\alpha^j\pi X_\pi\sigma_\pi + \left(\alpha^j\sigma_\pi - 2\tau\right)\left[4\pi X_\pi - \sigma_\pi\left(\tau + 4\pi\right)\right]$.

We can see expression $\alpha^i H_\pi^\pi - \alpha^j\tau W_\pi^\pi$ is positive if $\alpha^i > \frac{\alpha^j\tau W_\pi^\pi}{H_\pi^\pi}$ and is satisfied if conditions $\boldsymbol{\alpha^i > \frac{2\tau}{\sigma_\pi}}$ for $i = 1, 2$ holds. That is $\frac{2\tau}{\sigma_\pi} > \frac{\alpha^j\tau W_\pi^\pi}{H_\pi^\pi} \Rightarrow \left(\alpha^j\sigma_\pi - 2\tau\right)\big[\left(\alpha^j\sigma_\pi - 2\tau\right)\left[4\pi X_\pi - \sigma_\pi\left(\tau + 2\pi\right)\right] + 4\alpha^j\sigma_\pi\pi X_\pi\big] > 0$. The previous expression is positive if $\alpha^j > \frac{2\tau\left[4\pi X_\pi - \sigma_\pi(\tau+2\pi)\right]}{\sigma_\pi\left[8\pi X_\pi - \sigma_\pi(\tau+2\pi)\right]}$ and is satisfied if conditions $\boldsymbol{\alpha^i > \frac{2\tau}{\sigma_\pi}}$ for $i = 1, 2$ holds. That is $\frac{2\tau}{\sigma_\pi} > \frac{2\tau\left[4\pi X_\pi - \sigma_\pi(\tau+2\pi)\right]}{\sigma_\pi\left[8\pi X_\pi - \sigma_\pi(\tau+2\pi)\right]} \Leftrightarrow 4\pi X_\pi > 0$ if $\boldsymbol{\tau > \frac{\pi}{2}}$, which is satisfied if Assumption **A.1** holds. □

### C.2 Proof of Proposition 6.a and 6.b

*Proof.* Partially derive buyers and sellers equilibrium membership fees with respect to the product differentiation cost $\tau$ and cross-group network effects $\upsilon$ and $\pi$.

***I.*** When the cross-group network effect sellers exert on buyers is stronger than buyers on sellers, $\boldsymbol{\upsilon > \pi}$, and without loss of generality we assumed $\boldsymbol{\pi = 0}$.

The impact of product differentiation cost $\tau$ on buyers' equilibrium membership fee is done in the difference between platform $i$ and $j$, that is $\left(\Delta p_b^i\right)^\upsilon \equiv \left(p_b^i\right)^\upsilon - \left(p_b^j\right)^\upsilon$

$$\frac{\partial\left(\Delta p_b^i\right)^\upsilon}{\partial\tau} = \frac{-3\tau\left(\alpha^j - \alpha^i\right)\left[\alpha^i (G_b)_\tau^\upsilon - \alpha^j\tau(P_b)_\tau^\upsilon\right]}{\sigma_\upsilon^2\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i + \alpha^j\right)\tau\right]^2} < 0$$

where $\sigma_\upsilon \equiv 9\tau^2 - 2\upsilon^2$, $X_\upsilon \equiv 2\left(3\tau^2 - \upsilon^2\right) + \tau\upsilon$, $(G_b)_\tau^\upsilon \equiv \left(\alpha^j\sigma_\upsilon - \tau\right)\left[-\sigma_\upsilon\left[2X_\upsilon + \tau\left(12\tau + \upsilon\right)\right] + 18\tau^2 X_\upsilon\right] + 18\alpha^j X_\upsilon\sigma_\upsilon\tau^2 - X_\upsilon\sigma_\upsilon\tau$ and $(P_b)_\tau^\upsilon \equiv -\sigma_\upsilon\left[2X_\upsilon + \tau\left(12\tau + \upsilon\right)\right] + 18\tau^2 X_\upsilon + X_\upsilon\sigma_\upsilon$.

We can see expression $\alpha^i (G_b)_\tau^\upsilon - \alpha^j\tau(P_b)_\tau^\upsilon$ is positive if $\alpha^i > \frac{\alpha^j\tau(P_b)_\tau^\upsilon}{(G_b)_\tau^\upsilon}$ and is satisfied if Assumption **A.3** when $\upsilon > \pi$ and $\pi = 0$ $\boldsymbol{\alpha^i > \frac{2\tau}{\sigma_\upsilon}}$ for $i = 1, 2$ holds. That is $\frac{2\tau}{\sigma_\upsilon} > \frac{\alpha^j\tau(P_b)_\tau^\upsilon}{(G_b)_\tau^\upsilon} \Rightarrow 2(G_b)_\tau^\upsilon - \alpha^j\sigma_\upsilon(P_b)_\tau^\upsilon > 0 \Rightarrow 2\left(\alpha^j\sigma_\upsilon - \tau\right)\big[-\sigma_\upsilon\left[2X_\upsilon + \tau\left(12\tau + \upsilon\right)\right] + 18\tau^2 X_\upsilon\big] + 36\alpha^j X_\upsilon\sigma_\upsilon\tau^2 - 2X_\upsilon\sigma_\upsilon\tau > \alpha^j\sigma_\upsilon\left[\sigma_\upsilon\left[2X_\upsilon + \tau\left(12\tau + \upsilon\right)\right] + 18\tau^2 X_\upsilon\right] + \alpha^j\sigma_\upsilon^2 X_\upsilon$ and is positive if $\alpha^j > \frac{2\tau\left[-\sigma_\upsilon\left[2X_\upsilon + \tau_\upsilon(12\tau+\upsilon)\right] + 18\tau^2 X_\upsilon + X_\upsilon\sigma_\upsilon\right]}{\sigma_\upsilon\left[-\sigma_\upsilon\left[2X_\upsilon + \tau(12\tau+\upsilon)\right] + 18\tau^2 X_\upsilon + 36X_\upsilon\tau^2 - \sigma_\upsilon X_\upsilon\right]}$ and is satisfied if condition $\boldsymbol{\alpha^i > \frac{2\tau}{\sigma_\upsilon}}$ for $i = 1, 2$ holds. That is $\frac{2\tau}{\sigma_\upsilon} > \frac{2\tau\left[-\sigma_\upsilon\left[2X_\upsilon + \tau_\upsilon(12\tau+\upsilon)\right] + 18\tau^2 X_\upsilon + X_\upsilon\sigma_\upsilon\right]}{\sigma_\upsilon\left[-\sigma_\upsilon\left[2X_\upsilon + \tau(12\tau+\upsilon)\right] + 18\tau^2 X_\upsilon + 36X_\upsilon\tau^2 - \sigma_\upsilon X_\upsilon\right]} \Rightarrow 2X_\upsilon\left(18\tau^2 - \sigma_\upsilon\right) > 0$ if $\tau > \frac{\upsilon}{2}$.

For $\frac{\partial\left(p_b^i\right)^\upsilon}{\partial\upsilon}$ we transform buyers' equilibrium membership fee on equation 17a into $\left(p_b^i\right)^\upsilon =$

$f_b + \tau + \frac{3\tau^2}{\sigma_\upsilon}\Delta q_b^i$. Then $\frac{\partial\left(p_b^i\right)^\upsilon}{\partial\upsilon} = \frac{12\tau^2}{\sigma_\upsilon^2}\left(\Delta q_b^i\right)^\upsilon + \frac{3\tau^2}{\sigma_\upsilon}\frac{\partial\left(\Delta q_b^i\right)^\upsilon}{\partial\upsilon}$. We know $\left(\Delta q_b^i\right)^\upsilon > 0$ and $\sigma_\upsilon > 0$ if Assumptions **A.3** and **A.1** hold respectively, when $\upsilon > \pi$, $\pi = 0$. Then we need to establish the sign for $\frac{\partial\left(\Delta q_b^i\right)^\upsilon}{\partial\upsilon}$. That is $\frac{\partial\left(\Delta q_b^i\right)^\upsilon}{\partial\upsilon} = \frac{\left(\alpha^j-\alpha^i\right)\left[\alpha^i\alpha^j[\sigma_\upsilon(\tau-4\upsilon)+4\upsilon X_\upsilon]-\left(\alpha^i+\alpha^j\right)\tau(\tau-4\upsilon)\right]}{2[\alpha^i\alpha^j\sigma_\upsilon-(\alpha^i+\alpha^j)\tau]^2} > 0$. As we can observe that $\alpha^i > \frac{\alpha^j\tau(\tau-4\upsilon)}{\alpha^j[\sigma_\upsilon(\tau-4\upsilon)+4\upsilon X_\upsilon]-\tau(\tau-4\upsilon)}$ and is met if $\alpha^i > \frac{2\tau}{\sigma_\upsilon}$ holds. That is $\frac{2\tau}{\sigma_\upsilon} > \frac{\alpha^j\tau(\tau-4\upsilon)}{\alpha^j[\sigma_\upsilon(\tau-4\upsilon)+4\upsilon X_\upsilon]-\tau(\tau-4\upsilon)}$ if $\alpha^j > \frac{2\tau(\tau-4\upsilon)}{\sigma_\upsilon(\tau-4\upsilon)+8\upsilon X_\upsilon}$, using the same procedure as before, $\frac{2\tau}{\sigma_\upsilon} > \frac{2\tau(\tau-4\upsilon)}{\sigma_\upsilon(\tau-4\upsilon)+8\upsilon X_\upsilon} \Rightarrow 8\upsilon X_\upsilon > 0$ if $\tau > \frac{\upsilon}{2}$, which is the lower bound of Assumption **A.1**. Then $\frac{\partial\left(p_b^i\right)^\upsilon}{\partial\upsilon} > 0$.

For $\frac{\partial\left(p_s^i\right)^\upsilon}{\partial\tau}$ we transform sellers' equilibrium membership fee on equation 17b into $\left(p_s^i\right)^\upsilon = f_s + \tau - \upsilon - \frac{\tau}{\sigma_\upsilon}\left(\Delta q_b^i\right)^\upsilon$. Then $\frac{\partial\left(p_s^i\right)^\upsilon}{\partial\tau} = 1 - \left[\frac{-\upsilon\left(9\tau^2+2\upsilon^2\right)}{\sigma_\upsilon^2}\left(\Delta q_b^i\right)^\upsilon + \frac{\tau}{\sigma_\upsilon}\frac{\partial\left(\Delta q_b^i\right)^\upsilon}{\partial\tau}\right]$. As was mentioned previously $\left(\Delta q_b^i\right)^\upsilon > 0$ and $\sigma_\upsilon > 0$ if Assumptions **A.3** and **A.1** hold respectively, when $\upsilon > \pi$, $\pi = 0$. Then we need to establish the sign for $\frac{\partial\left(\Delta q_b^i\right)^\upsilon}{\partial\tau}$.
That is $\frac{\partial\left(\Delta q_b^i\right)^\upsilon}{\partial\tau} = \frac{-\left(\alpha^j-\alpha^i\right)\left[\alpha^i\alpha^j[18\tau X_\upsilon-\sigma_\upsilon(12\tau+\upsilon)]-\left(\alpha^i+\alpha^j\right)[X_\upsilon-\tau(12\tau+\upsilon)]\right]}{2[\alpha^i\alpha^j\sigma_\upsilon-(\alpha^i+\alpha^j)\tau]^2} < 0$. As we can observe that $\alpha^i > \frac{\alpha^j[X_\upsilon-\tau(12\tau+\upsilon)]}{\alpha^j[18\tau X_\upsilon-\sigma_\upsilon(12\tau+\upsilon)]-[X_\upsilon-\tau(12\tau+\upsilon)]}$ and is met if $\alpha^i > \frac{2\tau}{\sigma_\upsilon}$ holds. That is $\frac{2\tau}{\sigma_\upsilon} > \frac{\alpha^j[X_\upsilon-\tau(12\tau+\upsilon)]}{\alpha^j[18\tau X_\upsilon-\sigma_\upsilon(12\tau+\upsilon)]-[X_\upsilon-\tau(12\tau+\upsilon)]}$ if $\alpha^j > \frac{2\tau[X_\upsilon-\tau(12\tau+\upsilon)]}{36\tau^2X_\upsilon-\sigma_\upsilon\tau(12\tau+\upsilon)}$, using the same procedure as before, $\frac{2\tau}{\sigma_\upsilon} > \frac{2\tau[X_\upsilon-\tau(12\tau+\upsilon)]}{36\tau^2X_\upsilon-\sigma_\upsilon\tau(12\tau+\upsilon)} \Rightarrow 2X_\upsilon\left(9\tau^2+2\upsilon^2\right) > 0$ if $\tau > \frac{\upsilon}{2}$, which is the lower bound of Assumption **A.1** when $\upsilon > \pi$, $\pi = 0$. Then $\frac{\partial\left(p_s^i\right)^\upsilon}{\partial\tau} > 0$.

For $\frac{\partial\left(p_s^i\right)^\upsilon}{\partial\upsilon}$ we use the same expression as before $\left(p_s^i\right)^\upsilon = f_s + \tau - \upsilon - \frac{\tau}{\sigma_\upsilon}\left(\Delta q_b^i\right)^\upsilon$. Then $\frac{\partial\left(p_s^i\right)^\upsilon}{\partial\upsilon} = -1 - \left[\frac{-\tau\left(9\tau^2+2\upsilon^2\right)}{\sigma_\upsilon^2}\left(\Delta q_b^i\right)^\upsilon + \frac{\tau}{\sigma_\upsilon}\frac{\partial\left(\Delta q_b^i\right)^\upsilon}{\partial\upsilon}\right]$. As was noted before $\left(\Delta q_b^i\right)^\upsilon > 0$ and $\sigma_\upsilon > 0$ if Assumptions **A.3** and **A.1** hold respectively and $\frac{\partial\left(\Delta q_b^i\right)^\upsilon}{\partial\upsilon} > 0$ if Assumptions **A.3** and the lower bound of **A.1**, $\tau > \frac{\upsilon}{2}$ hold. Then $\frac{\partial\left(p_s^i\right)^\upsilon}{\partial\upsilon} < 0$.

***II.*** When the cross-group network effect buyers exert on sellers is stronger than sellers on buyers, $\boldsymbol{\pi > \upsilon}$, and without loss of generality, we assumed $\boldsymbol{\upsilon = 0}$.

For $\frac{\partial\left(p_b^i\right)^\pi}{\partial\tau}$ we transform buyers' equilibrium membership fee on equation 18a into $\left(p_b^i\right)^\pi = f_b + \tau - \pi + \frac{3\tau^2-\pi^2}{\sigma_\pi}\Delta q_b^i$. Then $\frac{\partial\left(p_b^i\right)^\pi}{\partial\tau} = 1 + \left[\frac{6\tau\pi^2}{\sigma_\pi^2}\left(\Delta q_b^i\right)^\pi + \left(\frac{3\tau^2-\pi^2}{\sigma_\pi}\right)\frac{\partial\left(\Delta q_b^i\right)^\pi}{\partial\tau}\right]$. We know $\left(\Delta q_b^i\right)^\pi > 0$ and $\sigma_\pi > 0$ if Assumptions **A.3** and **A.1** hold respectively, when $\pi > \upsilon$, $\upsilon = 0$. Furthermore, we know $3\tau^2 - \pi^2$ is negative because is outside the lower bound of Assumption **A.1**. Then we need to establish the sign for $\frac{\partial\left(\Delta q_b^i\right)^\pi}{\partial\tau}$.
That is $\frac{\partial\left(\Delta q_b^i\right)^\pi}{\partial\tau} = \frac{-\left(\alpha^j-\alpha^i\right)\left[\alpha^i\alpha^j[18\tau X_\pi-\sigma_\pi(12\tau-\pi)]\right]-\left(\alpha^i+\alpha^j\right)[X_\pi-\tau(12\tau-\pi)]}{2[\alpha^i\alpha^j\sigma_\pi-(\alpha^i+\alpha^j)\tau]^2} < 0$. As we can observe that $\alpha^i > \frac{\alpha^j[X_\pi-\tau(12\tau-\pi)]}{\alpha^j[18\tau X_\pi-\sigma_\pi(12\tau-\pi)]-[X_\pi-\tau(12\tau-\pi)]}$ and is met if $\alpha^i > \frac{2\tau}{\sigma_\pi}$ holds. That is $\frac{2\tau}{\sigma_\pi} > \frac{\alpha^j[X_\pi-\tau(12\tau-\pi)]}{\alpha^j[18\tau X_\pi-\sigma_\pi(12\tau-\pi)]-[X_\pi-\tau(12\tau-\pi)]}$ if $\alpha^j > \frac{2\tau[X_\pi-\tau(12\tau-\pi)]}{36\tau^2X_\pi-\sigma_\pi\tau(12\tau-\pi)}$, using the same procedure as before, $\frac{2\tau}{\sigma_\pi} > \frac{2\tau[X_\pi-\tau(12\tau-\pi)]}{36\tau^2X_\pi-\sigma_\pi\tau(12\tau-\pi)} \Rightarrow 2X_\pi\left(9\tau^2+2\pi^2\right) > 0$ if $\tau > \frac{\pi}{2}$, which is the lower bound of Assumption **A.1** when $\pi > \upsilon$, $\upsilon = 0$. Then $\frac{\partial\left(p_b^i\right)^\pi}{\partial\tau} > 0$.

For $\frac{\partial\left(p_b^i\right)^\pi}{\partial\pi}$ we use the same expression as before $\left(p_b^i\right)^\pi = f_b + \tau - \pi + \frac{3\tau^2-\pi^2}{\sigma_\pi}\left(\Delta q_b^i\right)^\pi$. Then $\frac{\partial\left(p_b^i\right)^\pi}{\partial\tau} = -1 - \frac{6\tau^2\pi}{\sigma_\pi^2}\left(\Delta q_b^i\right)^\pi + \left(\frac{3\tau^2-\pi^2}{\sigma_\pi}\right)\frac{\partial\left(\Delta q_b^i\right)^\pi}{\partial\pi}$. As was noted before $\left(\Delta q_b^i\right)^\pi > 0$, $\sigma_\pi > 0$ and

$3\tau^2-\pi^2<0$ if Assumptions **A.3** and **A.1** hold respectively, when $\pi>\upsilon$, $\upsilon=0$. Then we need to establish the sign for $\frac{\partial\left(\Delta q_b^i\right)^\pi}{\partial\pi}$. That is $\frac{\partial\left(\Delta q_b^i\right)^\pi}{\partial\pi}=\frac{\left(\alpha^j-\alpha^i\right)\left[\alpha^i\alpha^j\left[4\pi X_\pi-\sigma_\pi(\tau+2\pi)\right]+\left(\alpha^i+\alpha^j\right)\tau(\tau+2\pi)\right]}{2\left[\alpha^i\alpha^j\sigma_\pi-\left(\alpha^i+\alpha^j\right)\tau\right]^2}>0$ if $\alpha^i>\frac{-\alpha^j\tau(\tau+2\pi)}{\alpha^j\left[4\pi X_\pi-\sigma_\pi(\tau+2\pi)\right]+\tau(\tau+2\pi)}$. The previous expression is met if Assumptions **A.3** holds. That is $\frac{2\tau}{\sigma_\pi}>\frac{-\alpha^j\tau(\tau+2\pi)}{\alpha^j\left[4\pi X_\pi-\sigma_\pi(\tau+2\pi)\right]+\tau(\tau+2\pi)}$ if $\alpha^j>\frac{-2\tau(\tau+2\pi)}{8\pi X_\pi-\sigma_\pi(\tau+2\pi)}$ and using the same procedure as before, $\frac{2\tau}{\sigma_\pi}>\frac{-2\tau}{8\pi X_\pi-\sigma_\pi(\tau+2\pi)}\Rightarrow 8\pi X_\pi>0$ if $\tau>\frac{\pi}{2}$, which is the lower bound of Assumption **A.1** when $\pi>\upsilon$, $\upsilon=0$. Then $\frac{\partial\left(p_b^i\right)^\pi}{\partial\pi}<0$

The impact of product differentiation cost $\tau$ on sellers' equilibrium membership fee is done in the difference between platform $i$ and $j$, that is $\left(\Delta p_s^i\right)^\pi\equiv\left(p_s^i\right)^\pi-\left(p_s^j\right)^\pi$

$$\frac{\partial\left(\Delta p_s^i\right)^\pi}{\partial\tau}=\frac{-\pi\left(\alpha^j-\alpha^i\right)\left[\alpha^i(G_s)_\tau^\pi-\alpha^j\tau_s^2(P_s)_\tau^\pi\right]}{\sigma_\pi^2\left[\alpha^i\alpha^j\sigma_\pi-\left(\alpha^i+\alpha^j\right)\tau\right]^2}<0$$

where $\sigma_\pi\equiv 9\tau^2-2\pi^2$, $X_\pi\equiv 6\tau^2-\pi^2-\tau\pi$, $(G_s)_\tau^\pi\equiv\alpha^j\sigma_\pi\left[36\tau^2X_\pi-\sigma_\pi X_\pi-\sigma_\pi\tau\left(12\tau-\pi\right)\right]-\tau^2\left[18\tau X_\pi-\sigma_\pi\left(12\tau-\pi\right)\right]$ and $(P_s)_\tau^\pi\equiv 18\tau^2X_\pi-\sigma_\pi\tau\left(12\tau-\pi\right)$.

We can see expression $\alpha^i(G_s)_\tau^\pi-\alpha^j\tau^2(P_s)_\tau^\pi$ is positive if $\alpha^i>\frac{\alpha^j\tau^2(P_s)_\tau^\pi}{(G_s)_\tau^\pi}$ and is satisfied if condition $\boldsymbol{\alpha^i>\frac{2\tau}{\sigma_\pi}}$ for $i=1,2$ holds. That is $\frac{2\tau}{\sigma_\pi}>\frac{\alpha^j\tau^2(P_s)_\tau^\pi}{(G_s)_\tau^\pi}\Rightarrow 2(G_s)_\tau^\pi-\alpha^j\sigma_\pi\tau(P_s)_\tau^\pi>0$ $\Rightarrow\alpha^j\sigma_\pi\left[54\tau^2X_\pi-2\sigma_\pi X_\pi-\sigma_\pi\tau\left(12\tau-\pi\right)\right]-2\tau^2\left[18\tau X_\pi-\sigma_\pi\left(12\tau-\pi\right)\right]>0$ and is positive if $\alpha^j>\frac{2\tau^2\left[18\tau X_\pi-\sigma_\pi(12\tau-\pi)\right]}{\sigma_\pi\left[54\tau^2X_\pi-2\sigma_\pi X_\pi-\sigma_\pi\tau(12\tau-\pi)\right]}$ and is satisfied if condition $\boldsymbol{\alpha^i>\frac{2\tau_s}{\sigma_\pi}}$ for $i=1,2$ holds. That is $\frac{2\tau_s}{\sigma_\pi}>\frac{2\tau^2\left[18\tau X_\pi-\sigma_\pi(12\tau-\pi)\right]}{\sigma_\pi\left[54\tau^2X_\pi-2\sigma_\pi X_\pi-\sigma_\pi\tau(12\tau-\pi)\right]}\Rightarrow 2X_\pi\left(9\tau^2+2\sigma_\pi^2\right)>0$ if $\tau>\frac{\pi}{2}$, which is the lower bound of Assumption **A.1** when $\pi>\upsilon$, $\upsilon=0$.

For $\frac{\partial\left(p_s^i\right)^\pi}{\partial\pi}$ we transform sellers' equilibrium membership fee on equation 18b into $\left(p_s^i\right)^\pi=f_s+\tau+\frac{\tau\pi}{\sigma_\pi}\left(\Delta q_b^i\right)^\pi$. Then $\frac{\partial\left(p_s^i\right)^\pi}{\partial\pi}=\left[\frac{\tau\left(9\tau^2+2\pi^2\right)}{\sigma_\pi^2}\right]\left(\Delta q_b^i\right)^\pi+\frac{\tau\pi}{\sigma_\pi}\frac{\partial\left(\Delta q_b^i\right)^\pi}{\partial\pi}$. As was noted before $\left(\Delta q_b^i\right)^\pi>0$, $\sigma_\pi>0$ and $\frac{\partial\left(\Delta q_b^i\right)^\pi}{\partial\pi}>0$ if Assumptions **A.3** and **A.1** hold when $\pi>\upsilon$, $\upsilon=0$. Then $\frac{\partial\left(p_s^i\right)^\pi}{\partial\pi}>0$. □

### C.3 Proof of Claim 1

*Proof.* Partially derive equilibrium market shares with respect to the product differentiation cost $\tau$ and cross-group network effects $\upsilon$ and $\pi$.

$$\frac{\partial\eta_b^i}{\partial\tau}=\frac{-3\left(\alpha^j-\alpha^i\right)\left[\alpha^i\alpha^j\Sigma\left[36\tau^2X-\Sigma\left[X+\tau\left(12\tau+\left(\upsilon-\pi\right)\right)\right]\right]\right]}{4\Sigma^2\left[\alpha^i\alpha^j\Sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2}+\frac{3\left(\alpha^j-\alpha^i\right)\left[\left(\alpha^i+\alpha^j\right)\tau\left[18\tau^2X-\Sigma\tau\left(12\tau+\left(\upsilon-\pi\right)\right)\right]\right]}{4\Sigma^2\left[\alpha^i\alpha^j\Sigma-\left(\alpha^i+\alpha^j\right)\tau\right]^2}<0$$

where $X\equiv 6\tau^2-\left(\pi+\upsilon\right)\left(\pi+2\upsilon\right)+\tau\left(\upsilon-\pi\right)$ and $\Sigma\equiv 9\tau^2-\left(2\pi+\upsilon\right)\left(\pi+2\upsilon\right)$

We observe $\frac{\partial\eta_b^i}{\partial\tau}<0$ if $\alpha^i>\frac{\alpha^j\tau B}{\alpha^j\Sigma\Theta-\tau B}$. Where $\Theta\equiv 36\tau^2X-\Sigma\left[X+\tau\left(12\tau+\left(\upsilon-\pi\right)\right)\right]$ and $B\equiv 18\tau^2X-\Sigma\tau\left(12\tau+\left(\upsilon-\pi\right)\right)$. The previous expression is met if Assumption **A.3** holds, that is $\frac{2\tau}{\Sigma}>\frac{\alpha^j\tau B}{\alpha^j\Sigma\Theta-\tau B}$ if $\alpha^j>\frac{2\tau B}{\Sigma(2\Theta-B)}$ and using the same procedure as before, we have $\frac{2\tau}{\Sigma}>\frac{2\tau B}{\Sigma(2\Theta-B)}$ if $\Theta-B>0\Rightarrow X\left(9\tau^2+\left(2\pi+\upsilon\right)\left(\pi+2\upsilon\right)\right)>0$. The previous expression is met as long as the lower bound of Assumption **A.1** holds.

$$\frac{\partial \eta_b^i}{\partial \pi} = \frac{3\left(\alpha^j - \alpha^i\right)\tau\alpha^i\alpha^j\Sigma\left[2X\left(4\pi+5\upsilon\right) - \Sigma\left(\tau+\left(2\pi+3\upsilon\right)\right)\right]}{4\Sigma^2\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i+\alpha^j\right)\tau\right]^2} - \frac{3\left(\alpha^j - \alpha^i\right)\tau\left[\left(\alpha^i+\alpha^j\right)\tau\left[X\left(4\pi+5\upsilon\right) - \Sigma\left(\tau+\left(2\pi+3\upsilon\right)\right)\right]\right]}{4\Sigma^2\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i+\alpha^j\right)\tau\right]^2} > 0$$

where $X \equiv 6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon) + \tau(\upsilon-\pi)$ and $\Sigma \equiv 9\tau^2 - (2\pi+\upsilon)(\pi+2\upsilon)$

We observe $\frac{\partial \eta_b^i}{\partial \pi} > 0$ if $\alpha^i > \frac{\alpha^j\tau B}{\alpha^j\Sigma\Theta-\tau B}$. Where $\Theta \equiv 2X(4\pi+5\upsilon) - \Sigma(\tau+(2\pi+3\upsilon))$ and $B \equiv X(4\pi+5\upsilon) - \Sigma(\tau+(2\pi+3\upsilon))$. The previous expression is met if Assumption **A.3** holds, that is $\frac{2\tau}{\Sigma} > \frac{\alpha^j\tau B}{\alpha^j\Sigma\Theta-\tau B}$ if $\alpha^j > \frac{2\tau B}{\Sigma(2\Theta-B)}$ and using the same procedure as before, we have $\frac{2\tau}{\Sigma} > \frac{2\tau B}{\Sigma(2\Theta-B)}$ if $\Theta - B > 0 \Rightarrow (4\pi+5\upsilon)X > 0$. The previous expression is met as long as the lower bound of Assumption **A.1** holds.

$$\frac{\partial \eta_b^i}{\partial \upsilon} = \frac{3\left(\alpha^j - \alpha^i\right)\tau\alpha^i\alpha^j\Sigma\left[\Sigma\left(\tau-\left(3\pi+4\upsilon\right)\right) + 2X\left(5\pi+4\upsilon\right)\right]}{4\Sigma^2\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i+\alpha^j\right)\tau\right]^2} - \frac{3\left(\alpha^j - \alpha^i\right)\tau\left(\alpha^i+\alpha^j\right)\tau\left[\Sigma\left(\tau-\left(3\pi+4\upsilon\right)\right) + X\left(5\pi+4\upsilon\right)\right]}{4\Sigma^2\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i+\alpha^j\right)\tau\right]^2} > 0$$

where $X \equiv 6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon) + \tau(\upsilon-\pi)$ and $\Sigma \equiv 9\tau^2 - (2\pi+\upsilon)(\pi+2\upsilon)$

We observe $\frac{\partial \eta_b^i}{\partial \upsilon} > 0$ if $\alpha^i > \frac{\alpha^j\tau B}{\alpha^j\Sigma\Theta-\tau B}$. Where $\Theta \equiv \Sigma(\tau-(3\pi+4\upsilon)) + 2X(5\pi+4\upsilon)$ and $B \equiv \Sigma(\tau-(3\pi+4\upsilon)) + X(5\pi+4\upsilon)$. The previous expression is met if Assumption **A.3** holds, that is $\frac{2\tau}{\Sigma} > \frac{\alpha^j\tau B}{\alpha^j\Sigma\Theta-\tau B}$ if $\alpha^j > \frac{2\tau B}{\Sigma(2\Theta-B)}$ and using the same procedure as before, we have $\frac{2\tau}{\Sigma} > \frac{2\tau B}{\Sigma(2\Theta-B)}$ if $\Theta - B > 0 \Rightarrow (5\pi+4\upsilon)X > 0$. The previous expression is met as long as the lower bound of Assumption **A.1** holds.

$$\frac{\partial \eta_s^i}{\partial \tau} = \frac{-\left(\alpha^j - \alpha^i\right)\left(\pi+2\upsilon\right)\left[\alpha^i\alpha^j\Sigma\left[36\tau X - \Sigma\left(12\tau+\left(\upsilon-\pi\right)\right)\right]\right]}{4\Sigma^2\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i+\alpha^j\right)\tau\right]^2} + \frac{\left(\alpha^j - \alpha^i\right)\left(\pi+2\upsilon\right)\left[\left(\alpha^i+\alpha^j\right)\left[X\left(18\tau^2+\Sigma\right) - \Sigma\tau\left(12\tau+\left(\upsilon-\pi\right)\right)\right]\right]}{4\Sigma^2\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i+\alpha^j\right)\tau\right]^2} < 0$$

where $X \equiv 6\tau^2 - (\pi+\upsilon)(\pi+2\upsilon) + \tau(\upsilon-\pi)$ and $\Sigma \equiv 9\tau^2 - (2\pi+\upsilon)(\pi+2\upsilon)$

We observe $\frac{\partial \eta_s^i}{\partial \tau} < 0$ if $\alpha^i > \frac{\alpha^j B}{\alpha^j\Sigma\Theta-B}$. Where $\Theta \equiv 36\tau X - \Sigma(12\tau+(\upsilon-\pi))$ and $B \equiv X\left(18\tau^2+\Sigma\right) - \Sigma\tau(12\tau+(\upsilon-\pi))$. The previous expression is met if Assumption **A.3** holds, that is $\frac{2\tau}{\Sigma} > \frac{\alpha^j B}{\alpha^j\Sigma\Theta-\tau B}$ if $\alpha^j > \frac{2\tau B}{\Sigma(2\tau\Theta-B)}$ and using the same logic as before, $\frac{2\tau}{\Sigma} > \frac{2\tau B}{\Sigma(2\tau\Theta-B)}$ if $2(\tau\Theta - B) > 0 \Rightarrow X\left(9\tau^2 + (2\pi+\upsilon)(\pi+2\upsilon)\right) > 0$. The previous expression is met as long as the lower bound of Assumption **A.1** holds.

$$\frac{\partial \eta_s^i}{\partial \pi} = \frac{\left(\alpha^j - \alpha^i\right)\alpha^i\alpha^j\Sigma\left[X\left[\Sigma+2\left(\pi+2\upsilon\right)\left(4\pi+5\upsilon\right)\right] - \Sigma\left(\pi+2\upsilon\right)\left(\tau+\left(2\pi+3\upsilon\right)\right)\right]}{4\Sigma^2\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i+\alpha^j\right)\tau\right]^2} - \frac{\left(\alpha^j - \alpha^i\right)\left(\alpha^i+\alpha^j\right)\tau\left[X\left[\Sigma+\left(\pi+2\upsilon\right)\left(4\pi+5\upsilon\right)\right] - \Sigma\left(\pi+2\upsilon\right)\left(\tau+\left(4\pi+5\upsilon\right)\right)\right]}{4\Sigma^2\left[\alpha^i\alpha^j\Sigma - \left(\alpha^i+\alpha^j\right)\tau\right]^2} > 0$$

where $X \equiv 6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)$ and $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$

We observe $\frac{\partial \eta_s^i}{\partial \pi} > 0$ if $\alpha^i > \frac{\alpha^j \tau B}{\alpha^j \Sigma \Theta - \tau B}$. Where $\Theta \equiv X\big[\Sigma + 2(\pi + 2\upsilon)(4\pi + 5\upsilon)\big] - \Sigma(\pi + 2\upsilon)\big(\tau + (2\pi + 3\upsilon)\big)$ and $B \equiv X[\Sigma + (\pi + 2\upsilon)(4\pi + 5\upsilon)] - \Sigma(\pi + 2\upsilon)\big(\tau + \big(2\pi + 3\upsilon\big)\big)$. The previous expression is met if Assumption **A.3** holds, that is $\frac{2\tau}{\Sigma} > \frac{\alpha^j \tau B}{\alpha^j \Sigma \Theta - \tau B}$ if $\alpha^j > \frac{2\tau B}{\Sigma(2\Theta - B)}$ and using the same procedure as before, we have $\frac{2\tau}{\Sigma} > \frac{2\tau B}{\Sigma(2\Theta - B)}$ if $\Theta - B > 0 \Rightarrow X(\pi + 2\upsilon)(4\pi + 5\upsilon) > 0$. The previous expression is met as long as the lower bound of Assumption **A.1** holds.

$$
\begin{aligned}
\frac{\partial \eta_s^i}{\partial \upsilon} = {} & \frac{\left(\alpha^j - \alpha^i\right)\alpha^i \alpha^j \Sigma\left[2X\left[\Sigma + (\pi + 2\upsilon)(5\pi + 4\upsilon)\right] + \Sigma(\pi + 2\upsilon)(\tau - (3\pi + 4\upsilon))\right]}{4\Sigma^2\left[\alpha^i \alpha^j \Sigma - \left(\alpha^i + \alpha^j\right)\tau\right]^2} \\
& - \frac{\left(\alpha^j - \alpha^i\right)\left(\alpha^i + \alpha^j\right)\tau\left[X\left[2\Sigma + (\pi + 2\upsilon)(5\pi + 4\upsilon)\right] + \Sigma(\pi + 2\upsilon)(\tau - (3\pi + 4\upsilon))\right]}{4\Sigma^2\left[\alpha^i \alpha^j \Sigma - \left(\alpha^i + \alpha^j\right)\tau\right]^2} > 0
\end{aligned}
$$

where $X \equiv 6\tau^2 - (\pi + \upsilon)(\pi + 2\upsilon) + \tau(\upsilon - \pi)$ and $\Sigma \equiv 9\tau^2 - (2\pi + \upsilon)(\pi + 2\upsilon)$

We observe $\frac{\partial \eta_s^i}{\partial \upsilon} > 0$ if $\alpha^i > \frac{\alpha^j \tau B}{\alpha^j \Sigma \Theta - \tau B}$. Where $\Theta \equiv 2X\big[\Sigma + (\pi + 2\upsilon)(5\pi + 4\upsilon)\big] + \Sigma(\pi + 2\upsilon)\big(\tau - (3\pi + 4\upsilon)\big)$ and $B \equiv X[2\Sigma + (\pi + 2\upsilon)(5\pi + 4\upsilon)] + \Sigma(\pi + 2\upsilon)\big(\tau - \big(3\pi + 4\upsilon\big)\big)$. The previous expression is met if Assumption **A.3** holds, that is $\frac{2\tau}{\Sigma} > \frac{\alpha^j \tau B}{\alpha^j \Sigma \Theta - \tau B}$ if $\alpha^j > \frac{2\tau B}{\Sigma(2\Theta - B)}$ and using the same procedure as before, we have $\frac{2\tau}{\Sigma} > \frac{2\tau B}{\Sigma(2\Theta - B)}$ if $\Theta - B > 0 \Rightarrow X(\pi + 2\upsilon)(5\pi + 4\upsilon) > 0$. The previous expression is met as long as the lower bound of Assumption **A.1** holds. □

### C.4 Positive Equilibrium Profits

To see if the second term of equilibrium profits for platform $i$ in equation (16), is positive we do the following:

$$
\Pi^i = \tau - \frac{\pi + \upsilon}{2} + \underbrace{\left[\frac{\left(\alpha^j - \alpha^i\right)\Sigma\left[\alpha^i \alpha^j \Sigma - \left(\alpha^i + \alpha^j\right)\tau\right] - \alpha^i\left(\alpha^j \Sigma - 2\tau\right)\left(\alpha^i \Sigma - 2\tau\right)}{8\Sigma^2\left[\alpha^i \alpha^j \Sigma - \left(\alpha^i + \alpha^j\right)\tau\right]^2}\right]}_{\theta} X^2
$$

We noticed the denominator of $\theta$ is positive, then we need to verify the sign of the numerator $\left(\alpha^j - \alpha^i\right)\Sigma\left[\alpha^i \alpha^j \Sigma - \left(\alpha^i + \alpha^j\right)\tau\right] - \alpha^i\left(\alpha^j \Sigma - 2\tau\right)\left(\alpha^i \Sigma - 2\tau\right)$. We can rearrange the expression to have a polynomial of degree two in $\alpha^i$, that is $-\Sigma\left[2\alpha^j \Sigma - 3\tau\right]\left(\alpha^i\right)^2 + \left[\,\left(\alpha^j\right)^2 \Sigma^2\right.$

$+2\tau\left(\alpha^j\Sigma-2\tau\right)\big]\alpha^i-\left(\alpha^j\right)^2\Sigma\tau$. Using the quadratic formula we have:

$$\alpha^i > -\frac{-\left[\left(\alpha^j\right)^2\Sigma^2+2\tau\left(\alpha^j\Sigma-2\tau\right)\right]}{\Sigma\left[2\alpha^j\Sigma-3\tau\right]}$$

$$\mp\frac{\sqrt{\left[\left(\alpha^j\right)^2\Sigma^2+2\tau\left(\alpha^j\Sigma-2\tau\right)\right]^2-4\left(\Sigma\right)^2\left(\alpha^j\right)^2\tau\left[2\alpha^j\Sigma-3\tau\right]}}{\Sigma\left[2\alpha^j\Sigma-3\tau\right]}$$

$$\alpha^i > -\frac{-\left[\left(\alpha^j\right)^2\Sigma^2+2\tau\left(\alpha^j\Sigma-2\tau\right)\right]\pm\sqrt{\left(\alpha^j\Sigma-2\tau\right)^2\left[\left(\alpha^j\right)^2\Sigma^2+4\tau^2\right]}}{\Sigma\left[2\alpha^j\Sigma-3\tau\right]}$$

$$\alpha^i > -\frac{-\left[\left(\alpha^j\right)^2\Sigma^2+2\tau\left(\alpha^j\Sigma-2\tau\right)\right]\pm\left(\alpha^j\Sigma-2\tau\right)\sqrt{\left(\alpha^j\right)^2\Sigma^2+4\tau^2}}{\Sigma\left[2\alpha^j\Sigma-3\tau\right]}\equiv\alpha^i_{ry}\ y=1,2$$

As long as Assumption **A.3** holds, the previous condition over $\alpha^i$ is satisfied. For the first root of the polynomial $\alpha^i_{r1}$ we have $\frac{2\tau}{\Sigma} > \frac{\left[\left(\alpha^j\right)^2\Sigma^2+2\tau\left(\alpha^j\Sigma-2\tau\right)\right]-\left(\alpha^j\Sigma-2\tau\right)\sqrt{\left(\alpha^j\right)^2\Sigma^2+4\tau^2}}{\Sigma\left[2\alpha^j\Sigma-3\tau\right]} \Rightarrow$ $4\tau\left(2\alpha^j\Sigma-3\tau\right) > \left(\alpha^j\right)^2\Sigma^2+2\tau\left(\alpha^j\Sigma-2\tau\right)-\left(\alpha^j\Sigma-2\tau\right)\sqrt{\left(\alpha^j\right)^2\Sigma^2+4\tau^2} \Rightarrow \left(\alpha^j\Sigma-2\tau\right)$ $\sqrt{\left(\alpha^j\right)^2\Sigma^2+4\tau^2} > \left(\alpha^j\Sigma-2\tau\right)\left(\alpha^j\Sigma-4\tau\right) \Rightarrow \left(\alpha^j\right)^2\Sigma^2+4\tau^2 > \left(\alpha^j\Sigma-4\tau\right)^2 \Rightarrow \tau\left(2\alpha^j\Sigma-3\tau\right)$ $> 0$ if $\alpha^j > \frac{3\tau}{2\Sigma}$. The previous condition is met as long as Assumption **A.3** holds.

For the second root of the polynomial $\alpha^i_{r2}$ we have $\frac{2\tau}{\Sigma} > \frac{\left[\left(\alpha^j\right)^2\Sigma^2+2\tau\left(\alpha^j\Sigma-2\tau\right)\right]+\left(\alpha^j\Sigma-2\tau\right)\sqrt{\left(\alpha^j\right)^2\Sigma^2+4\tau^2}}{\Sigma\left[2\alpha^j\Sigma-3\tau\right]} \Rightarrow 4\tau\left(2\alpha^j\Sigma-3\tau\right) > \left(\alpha^j\right)^2\Sigma^2+2\tau\left(\alpha^j\Sigma-2\tau\right)+\left(\alpha^j\Sigma-2\tau\right)\sqrt{\left(\alpha^j\right)^2\Sigma^2+4\tau^2} \Rightarrow -\left(\left(\alpha^j\right)^2\Sigma^2+4\tau^2\right) > \left(\alpha^j\Sigma-4\tau\right)^2 \Rightarrow \Sigma^2\left(\alpha^j\right)^2-4\Sigma\tau\alpha^j+10\tau^2 < 0$. To verify if the previous expression is negative we can use the quadratic formula to compute both roots. That is $\alpha^j < \frac{4\Sigma\tau\pm\sqrt{16\Sigma^2\tau^2-40\Sigma^2\tau^2}}{2\Sigma^2} \Rightarrow \alpha^j < \frac{2\tau\left(1\pm\sqrt{-6}\right)}{\Sigma}$. The second root does not have a real solution. Then, only the solution of the first root met the initial condition.

Therefore, $\left(\alpha^j-\alpha^i\right)\Sigma\left[\alpha^i\alpha^j\Sigma-\left(\alpha^i+\alpha^j\right)\tau\right]-\alpha^i\left(\alpha^j\Sigma-2\tau\right)\left(\alpha^i\Sigma-2\tau\right)$ is positive as long as Assumption **A.3** holds.

### C.5 Proof of Proposition 7.a and 7.b

*Proof.* Partially derive the difference in equilibrium profits between platform $i$ and $j$ with respect to product differentiation cost $\tau$ and cross-group network effects $\upsilon$ and $\pi$. The partial derivatives consist of two expressions multiplying the efficiency parameter of developing attributes for both platforms. The process to prove the sign of each expression multiplying the efficiency parameter on both platforms is the same. Therefore, we are only proving the expression multiplying $\alpha^i$. For the second expression, the one multiplying $\alpha^j$ the process is the same.

$$\frac{\partial\left(\Delta\Pi^i\right)^\upsilon}{\partial\tau}=\frac{\left(\alpha^j-\alpha^i\right)X_\upsilon\left[\alpha^i\left[\alpha^i\sigma_\upsilon(\theta^\upsilon_\tau)^i+2\tau(\omega^\upsilon_\tau)^i\right]+\alpha^j\left[\alpha^j\sigma_\upsilon(\theta^\upsilon_\tau)^j+2\tau(\omega^\upsilon_\tau)^j\right]\right]}{4\sigma^3_\upsilon\left[\alpha^i\alpha^j\sigma_\upsilon-\left(\alpha^i+\alpha^j\right)\tau\right]^3}>0$$

where $\sigma_\upsilon \equiv 9\tau^2 - 2\upsilon^2$, $X_\upsilon \equiv 6\tau^2 - 2\upsilon^2 + \tau\upsilon$, $(\theta_\tau^\upsilon)^i \equiv \alpha^j\sigma_\upsilon\big[90\tau^2 X_\upsilon + X_\upsilon\left(18\tau^2+\sigma_\upsilon\right) - 7\sigma_\upsilon\tau\left(12\tau+\upsilon\right) - \frac{3}{2}\alpha^j\sigma_\upsilon\left(18\tau X_\upsilon - \sigma_\upsilon\left(12\tau+\upsilon\right)\right)\big] + 2\tau\left[2\sigma_\upsilon\tau\left(12\tau+\upsilon\right) - X_\upsilon\left(18\tau^2+\sigma_\upsilon\right)\right]$ and $(\omega_\tau^\upsilon)^i \equiv \alpha^j\sigma_\upsilon\left[3\sigma_\upsilon\tau\left(12\tau+\upsilon\right) - X_\upsilon\left(18\tau^2+\sigma_\upsilon\right) + X_\upsilon\left(\sigma_\upsilon - 36\tau^2\right)\right] + 2\tau\big[X_\upsilon\left(18\tau^2+\sigma_\upsilon\right) - \sigma_\upsilon\tau\left(12\tau+\upsilon\right) - \sigma_\upsilon X_\upsilon\big]$ for $i,j=1,2$ and $i\neq j$.

We can see expression $\alpha^i\sigma_\upsilon(\theta_\tau^\upsilon)^i + 2\tau(\omega_\tau^\upsilon)^i$ is positive if $\alpha^i > \frac{-2\tau(\omega_\tau^\upsilon)^i}{\sigma_\upsilon(\theta_\tau^\upsilon)^i}$, for $i,j=1,2$ and $i\neq j$ and is satisfied if Assumption **A.3** when $\upsilon > \pi$, $\pi = 0$ $\boldsymbol{\alpha^i > \frac{2\tau}{\sigma_\upsilon}}$ for $i=1,2$ holds. That is $\frac{2\tau}{\sigma_\upsilon} > \frac{-2\tau(\omega_\tau^\upsilon)^i}{\sigma_\upsilon(\theta_\tau^\upsilon)^i} \Rightarrow (\theta_\tau^\upsilon)^i + (\omega_\tau^\upsilon)^i > 0$. We can see the previous expression is positive. That is $\alpha^j\sigma_\upsilon\big[90\tau^2X_\upsilon + X_\upsilon\left(18\tau^2+\sigma_\upsilon\right) - 7\sigma_\upsilon\tau\left(12\tau+\upsilon\right) - \frac{3}{2}\alpha^j\sigma_\upsilon\left(18\tau X_\upsilon - \sigma_\upsilon\left(12\tau+\upsilon\right)\right)\big] + 2\tau\left[2\sigma_\upsilon\tau\left(12\tau+\upsilon\right) - X_\upsilon\left(18\tau^2+\sigma_\upsilon\right)\right] + \alpha^j\sigma_\upsilon\left[3\sigma_\upsilon\tau\left(12\tau+\upsilon\right) - X_\upsilon\left(18\tau^2+\sigma_\upsilon\right) + X_\upsilon\left(\sigma_\upsilon - 36\tau^2\right)\right]$ $+2\tau\big[X_\upsilon\left(18\tau^2+\sigma_\upsilon\right) - \sigma_\upsilon\tau\left(12\tau+\upsilon\right) - \sigma_\upsilon X_\upsilon\big] > 0$

$\Rightarrow \alpha^j\sigma_\upsilon\left[54\tau^2X_\upsilon - 4\sigma_\upsilon\tau\left(12\tau+\upsilon\right) + \sigma_\upsilon X_\upsilon\right] + 2\tau\left[\sigma_\upsilon\tau\left(12\tau+\upsilon\right) - \sigma_\upsilon X_\upsilon\right] - \frac{3}{2}\left(\alpha^j\right)^2\sigma_\upsilon^2\left[18\tau X_\upsilon - \sigma_\upsilon\left(12\tau+\upsilon\right)\right] > 0 \Rightarrow \left(\alpha^j\sigma_\upsilon - 2\tau\right)\left[\frac{3}{2}\alpha^j\sigma_\upsilon\left(\sigma_\upsilon\left(12\tau+\upsilon\right) - 18\tau X_\upsilon\right) - \left(\sigma_\upsilon\tau\left(12\tau+\upsilon\right) - \sigma_\upsilon X_\upsilon\right)\right] > 0$. The previous expression is positive if $\alpha^j > \frac{2[\sigma_\upsilon\tau(12\tau+\upsilon) - \sigma_\upsilon X_\upsilon]}{3\sigma_\upsilon[\sigma_\upsilon(12\tau+\upsilon) - 18\tau X_\upsilon]}$ and is satisfied if Assumption **A.3** when $\pi > \upsilon$, $\upsilon = 0$, $\boldsymbol{\alpha^i > \frac{2\tau}{\sigma_\upsilon}}$ for $i=1,2$ holds. That is $\frac{2\tau}{\sigma_\upsilon} > \frac{2[\sigma_\upsilon\tau(12\tau+\upsilon) - \sigma_\upsilon X_\upsilon]}{3\sigma_\upsilon[\sigma_\upsilon(12\tau+\upsilon) - 18\tau X_\upsilon]} \Rightarrow 3\tau\big[\sigma_\upsilon\left(12\tau+\upsilon\right) - 18\tau X_\upsilon\big] > \sigma_\upsilon\tau\left(12\tau+\upsilon\right) - \sigma_\upsilon X_\upsilon \Rightarrow 2\left(-27\tau^4 + 2\upsilon^4\right) + 3\tau\upsilon\left(-9\tau^2 + 10\tau\upsilon - 2\upsilon^2\right) > 0$. The previous expression is positive if $\tau < \left(\frac{2}{\sqrt[4]{27}}\right)\upsilon$ and $\tau > \left(\frac{5-\sqrt{7}}{9}\right)\upsilon$, and both conditions are satisfied as long as Assumption **A.1** holds.

$$\frac{\partial\left(\Delta\Pi^i\right)^\upsilon}{\partial\upsilon} = \frac{\left(\alpha^j - \alpha^i\right)X_\upsilon\left[\alpha^i\left[\alpha^i\sigma_\upsilon(\theta_\upsilon^\upsilon)^i + 2\tau^2(\omega_\upsilon^\upsilon)^i\right] + \alpha^j\left[\alpha^j\sigma_\upsilon(\theta_\upsilon^\upsilon)^j + 2\tau^2(\omega_\upsilon^\upsilon)^j\right]\right]}{4\sigma_\upsilon^3\left[\alpha^i\alpha^j\sigma_\upsilon - \left(\alpha^i+\alpha^j\right)\tau\right]^3} > 0$$

where $\sigma_\upsilon \equiv 9\tau^2 - 2\upsilon^2$, $X_\upsilon \equiv 6\tau^2 - 2\upsilon^2 + \tau\upsilon$, $(\theta_\upsilon^\upsilon)^i \equiv \frac{3}{2}\alpha^j\sigma_\upsilon\left(\alpha^j\sigma_\upsilon - 2\tau\right)\left[\sigma_\upsilon\left(\tau - 4\upsilon\right) + 4\upsilon X_\upsilon\right] + 4\tau\upsilon X_\upsilon\left(\alpha^j\sigma_\upsilon - 2\tau\right) - \left[\sigma_\upsilon\left(\tau-4\upsilon\right) + 4\upsilon X_\upsilon\right]\left(4\alpha^j\sigma_\upsilon\tau - 4\tau^2\right)$ and $(\omega_\upsilon^\upsilon)^i \equiv \left(\alpha^j\sigma_\upsilon - 2\tau_s\right)\left[\sigma_\upsilon\left(\tau - 4\upsilon\right) + 4\upsilon X_\upsilon\right] + 2\alpha^j\sigma_\upsilon\left[\sigma_\upsilon\left(\tau-4\upsilon\right) + 4\upsilon X_\upsilon\right]$ for $i,j=1,2$ and $i\neq j$.

We can see expression $\alpha^i\sigma_\upsilon(\theta_\upsilon^\upsilon)^i + 2\tau^2(\omega_\upsilon^\upsilon)^i$ is positive if $\alpha^i > \frac{-2\tau^2(\omega_\upsilon^\upsilon)^i}{\sigma_\upsilon(\theta_\upsilon^\upsilon)^i}$, for $i,j=1,2$ and $i\neq j$ and is satisfied if Assumption **A.3** for $\upsilon > \pi$, $\pi = 0$ $\boldsymbol{\alpha^i > \frac{2\tau}{\sigma_\upsilon}}$ for $i=1,2$ holds. That is $\frac{2\tau}{\sigma_\upsilon} > \frac{-2\tau^2(\omega_\upsilon^\upsilon)^i}{\sigma_\upsilon(\theta_\upsilon^\upsilon)^i} \Leftrightarrow (\theta_\tau^\upsilon)^i + \tau(\omega_\upsilon^\upsilon)^i > 0$. We can see the previous expression is positive. That is $\frac{3}{2}\alpha^j\sigma_\upsilon\left(\alpha^j\sigma_\upsilon - 2\tau\right)\left[\sigma_\upsilon\left(\tau-4\upsilon\right) + 4\upsilon X_\upsilon\right] + 4\tau\upsilon X_\upsilon\left(\alpha^j\sigma_\upsilon - 2\tau\right) - \left[\sigma_\upsilon\left(\tau-4\upsilon\right) + 4\upsilon X_\upsilon\right]\left(4\alpha^j\sigma_\upsilon\tau - 4\tau^2\right) + \tau\left(\alpha^j\sigma_\upsilon - 2\tau\right)\left[\sigma_\upsilon\left(\tau-4\upsilon\right) + 4\upsilon X_\upsilon\right] + 2\alpha^j\sigma_\upsilon\tau\left[\sigma_\upsilon\left(\tau-4\upsilon\right) + 4\upsilon X_\upsilon\right] > 0 \Rightarrow \left(\alpha^j\sigma_\upsilon - 2\tau\right)\left[\left(\frac{3}{2}\alpha^j\sigma_\upsilon - \tau\right)\left[\sigma_\upsilon\left(\tau-4\upsilon\right) + 4\upsilon X_\upsilon\right] + 4\tau\upsilon X_\upsilon\right] > 0$. We can see the previous expression is positive if $\alpha^j > \frac{2\sigma_\upsilon\tau(\tau-4\upsilon)}{3\sigma_\upsilon[\sigma_\upsilon(\tau-4\upsilon) + 4\upsilon X_\upsilon]}$ and is satisfied if condition $\boldsymbol{\alpha^i > \frac{2\tau}{\sigma_\upsilon}}$ for $i=1,2$ holds. That is $\frac{2\tau}{\sigma_\upsilon} > \frac{2\sigma_\upsilon\tau(\tau-4\upsilon)}{3\sigma_\upsilon[\sigma_\upsilon(\tau-4\upsilon) + 4\upsilon X_\upsilon]} \Rightarrow 3\left[\sigma_\upsilon\left(\tau-4\upsilon\right) + 4\upsilon X_\upsilon\right] > \sigma_\upsilon\left(\tau-4\upsilon\right) \Rightarrow 9\tau^3 + 4\tau\upsilon^2 - 4\upsilon^3 > 0$ if $\tau > \frac{\sqrt[3]{4}}{3}\upsilon$, which is satisfied as long as Assumption **A.1** holds.

$$\frac{\partial\left(\Delta\Pi^i\right)^\pi}{\partial\tau} = \frac{\left(\alpha^j - \alpha^i\right)X_\pi\left[\alpha^i\left[\alpha^i\sigma_\pi(\theta_\tau^\pi)^i + 2\tau(\omega_\tau^\pi)^i\right] + \alpha^j\left[\alpha^j\sigma_\pi(\theta_\tau^\pi)^j + 2\tau(\omega_\tau^\pi)^j\right]\right]}{4\sigma_\pi^3\left[\alpha^i\alpha^j\sigma_\pi - \left(\alpha^i+\alpha^j\right)\tau\right]^3} > 0$$

where $\sigma_\pi \equiv 9\tau^2 - 2\pi^2$, $X_\pi \equiv 6\tau^2 - \pi^2 - \tau\pi$, $(\theta_\tau^\pi)^i \equiv \alpha^j\sigma_\pi\big[90\tau^2X_\pi + X_\pi\left(18\tau^2+\sigma_\pi\right) - 7\sigma_\pi\tau\left(12\tau-\pi\right) - \frac{3}{2}\alpha^j\sigma_\pi\left(18\tau X_\pi - \sigma_\pi\left(12\tau-\pi\right)\right)\big] + 2\tau\left[2\sigma_\pi\tau\left(12\tau-\pi\right) - X_\pi\left(18\tau^2+\sigma_\pi\right)\right]$ $(\omega_\tau^\pi)^i$

$\equiv \alpha^j\sigma_\pi \left[3\sigma_\pi\tau\left(12\tau-\pi\right)-X_\pi\left(18\tau^2+\sigma_\pi\right)+X_\pi\left(\sigma_\pi-36\tau^2\right)\right]+2\tau\left[X_\pi\left(18\tau^2+\sigma_\pi\right)-\sigma_\pi\tau\left(12\tau-\pi\right)-\sigma_\pi X_\pi\right]$ for $i,j=1,2$ and $i\neq j$.

We can see expression $\alpha^i\sigma_\pi(\theta^\pi_\tau)^i+2\tau(\omega^\pi_\tau)^i$ is positive if $\alpha^i>\frac{-2\tau(\omega^\pi_\tau)^i}{\sigma_\pi(\theta^\pi_\tau)^i}$, for $i,j=1,2$ and $i\neq j$ and is satisfied if if Assumption **A.3** for $\pi>\upsilon$, $\upsilon=0$, $\boldsymbol{\alpha^i>\frac{2\tau}{\sigma_\pi}}$ for $i=1,2$ holds. That is $\frac{2\tau}{\sigma_\pi}>\frac{-2\tau(\omega^\pi_\tau)^i}{\sigma_\pi(\theta^\pi_\tau)^i}\Leftrightarrow(\theta^\pi_\tau)^i+(\omega^\pi_\tau)^i>0$. We can see the previous expression is positive. That is $\alpha^j\sigma_\pi\left[90\tau^2X_\pi+X_\pi\left(18\tau^2+\sigma_\pi\right)-7\sigma_\pi\tau\left(12\tau-\pi\right)-\frac{3}{2}\alpha^j\sigma_\pi\left(18\tau X_\pi-\sigma_\pi\left(12\tau-\pi\right)\right)\right]+2\tau\left[2\sigma_\pi\tau\left(12\tau-\pi\right)-X_pi\left(18\tau^2+\sigma_\pi\right)\right]+\alpha^j\sigma_\pi\left[3\sigma_\pi\tau\left(12\tau-\pi\right)-X_\pi\left(18\tau^2+\sigma_\pi\right)+X_\pi\left(\sigma_\pi-36\tau^2\right)\right]+2\tau\left[X_\pi\left(18\tau^2+\sigma_\pi\right)-\sigma_\pi\tau\left(12\tau-\pi\right)-\sigma_\pi X_\pi\right]>0\Rightarrow\alpha^j\sigma_\pi\left[54\tau^2X_\pi-4\sigma_\pi\tau\left(12\tau-\pi\right)+\sigma_\pi X_\pi\right]+2\tau\left[\sigma_\pi\tau\left(12\tau-\pi\right)-\sigma_\pi X_\pi\right]-\frac{3}{2}\left(\alpha^j\right)^2\sigma_\pi^2\left[18\tau X_\pi-\sigma_\pi\left(12\tau-\pi\right)\right]>0\Rightarrow\left(\alpha^j\sigma_\pi-2\tau\right)\left[\frac{3}{2}\alpha^j\sigma_\pi\left(\sigma_\pi\left(12\tau-\pi\right)-18\tau X_\pi\right)-\left(\sigma_\pi\tau\left(12\tau-\pi\right)-\sigma_\pi X_\pi\right)\right]>0$. The previous expression is positive if $\alpha^j>\frac{2[\sigma_\pi\tau(12\tau-\pi)-\sigma_\pi X_\pi]}{3\sigma_\pi[\sigma_\pi(12\tau-\pi)-18\tau X_\pi]}$ and is satisfied if condition $\boldsymbol{\alpha^i>\frac{2\tau}{\sigma_\pi}}$ for $i=1,2$ holds. That is $\frac{2\tau}{\sigma_\pi}>\frac{2[\sigma_\pi\tau(12\tau-\pi)-\sigma_\pi X_\pi]}{3\sigma_\pi[\sigma_\pi(12\tau-\pi)-18\tau X_\pi]}\Rightarrow 3\tau\left[\sigma_\pi\left(12\tau-\pi\right)-18\tau X_\pi\right]>\sigma_\pi\tau\left(12\tau-\pi\right)-\sigma_\pi X_\pi\Rightarrow 2\left(-27\tau^4+\pi^4\right)+3\tau\pi\left(9\tau^2-10\tau\pi+2\pi^2\right)>0$. The previous expression is positive if $\tau<\left(\frac{1}{\sqrt[4]{27}}\right)\pi$ and $\tau>\left(\frac{5-\sqrt{7}}{9}\right)\pi$, and both conditions are satisfied as long as Assumption **A.1** holds.

$$\frac{\partial\left(\Delta\Pi^i\right)^\pi}{\partial\pi}=\frac{-\left(\alpha^j-\alpha^i\right)X_\pi\left[\alpha^i\left[\alpha^i\sigma_\pi(\theta^\pi_\pi)^i+2\tau^2(\omega^\pi_\pi)^i\right]+\alpha^j\left[\alpha^j\sigma_\pi(\theta^\pi_\pi)^j+2\tau^2(\omega^\pi_\pi)^j\right]\right]}{4\sigma_\pi^3\left[\alpha^i\alpha^j\sigma_\pi-\left(\alpha^i+\alpha^j\right)\tau\right]^3}<0$$

where $\sigma_\pi\equiv 9\tau^2-2\pi^2$, $X_\pi\equiv 6\tau^2-\pi^2-\tau\pi$, $(\theta^\pi_\pi)^i\equiv\frac{3}{2}\alpha^j\sigma_\pi\left(\alpha^j\sigma_\pi-2\tau\right)\left[\sigma_\pi\left(\tau+2\pi\right)-4\pi X_\pi\right]-4\tau\pi X_\pi\left(\alpha^j\sigma_\pi-2\tau\right)-\left[\sigma_\pi\left(\tau+2\pi\right)-4\pi X_\pi\right]\left(4\alpha^j\sigma_\pi\tau-4\tau^2\right)$ and $(\omega^\pi_\pi)^i\equiv\left(\alpha^j\sigma_\pi-2\tau\right)\left[\sigma_\pi\left(\tau+2\pi\right)-4\pi X_\pi\right]+2\alpha^j\sigma_\pi\left[\sigma_\pi\left(\tau+2\pi\right)-4\pi X_\pi\right]$ for $i,j=1,2$ and $i\neq j$.

We can see expression $\alpha^i\sigma_\pi(\theta^\pi_\pi)^i+2\tau^2(\omega^\pi_\pi)^i$ is positive if $\alpha^i>\frac{-2\tau^2(\omega^\pi_\pi)^i}{\sigma_\pi(\theta^\pi_\pi)^i}$, for $i,j=1,2$ and $i\neq j$ and is satisfied if Assumption **A.3** for $\pi>\upsilon$ , $\upsilon=0$, $\boldsymbol{\alpha^i>\frac{2\tau}{\sigma_\pi}}$ for $i=1,2$ holds. That is $\frac{2\tau}{\sigma_\pi}>\frac{-2\tau^2(\omega^\pi_\pi)^i}{\sigma_\pi(\theta^\pi_\pi)^i}\Leftrightarrow(\theta^\pi_\pi)^i+\tau(\omega^\pi_\pi)^i>0$. We can see the previous expression is positive. That is $\frac{3}{2}\alpha^j\sigma_\pi\left(\alpha^j\sigma_\pi-2\tau\right)\left[\sigma_\pi\left(\tau+2\pi\right)-4\pi X_\pi\right]+4\tau\pi X_\pi\left(\alpha^j\sigma_\pi-2\tau\right)-\left[\sigma_\pi\left(\tau+2\pi\right)-4\pi X_\pi\right]\left(4\alpha^j\sigma_\pi\tau-4\tau^2\right)+\tau\left(\alpha^j\sigma_\pi-2\tau\right)\left[\sigma_\pi\left(\tau+2\pi\right)-4\pi X_\pi\right]+2\alpha^j\sigma_\pi\tau\left[\sigma_\pi\left(\tau+2\pi\right)-4\pi X_\pi\right]>0\Rightarrow\left(\alpha^j\sigma_\pi-2\tau\right)\left[\left(\frac{3}{2}\alpha^j\sigma_\pi-\tau\right)\left[\sigma_\pi\left(\tau+2\pi\right)-4\pi X_\pi\right]-4\tau\pi X_\pi\right]>0$. We can see the previous expression is positive if $\alpha^j>\frac{2\tau(\tau+2\pi)}{3\left[\sigma_\pi\left(\tau+2\pi\right)-4\pi X_\pi\right]}$ and is satisfied if condition $\boldsymbol{\alpha^i>\frac{2\tau}{\sigma_\pi}}$ for $i=1,2$ holds. That is $\frac{2\tau}{\sigma_\pi}>\frac{2\tau(\tau+2\pi)}{3\left[\sigma_\pi\left(\tau+2\pi\right)-4\pi X_\pi\right]}\Rightarrow 3\left[\sigma_\pi\left(\tau+2\pi\right)-4\pi X_\pi\right]>\sigma_\pi\left(\tau+2\pi\right)\Rightarrow\tau\sigma_\pi+2\pi\left[-9\tau^2+\pi^2+3\tau\pi\right]>0$ if $-9\tau^2+\pi^2+3\tau\pi$ is positive. We can rearrange the previous expression to have a polynomial of degree two in $\tau$, that is $-9\tau^2+3\pi\tau+\pi^2>0$ and using the quadratic formula to obtain both roots on $\tau$ we have $\tau_1>\left(\frac{1-\sqrt{5}}{6}\right)\pi$ and $\tau_2>\left(\frac{1+\sqrt{5}}{6}\right)\pi$. The first root $\tau_1$ is not satisfied if Assumption **A.1** holds since we have a square root of a negative number. Therefore only the second root is satisfied as long as Assumption **A.1** when $\pi>\upsilon$, $\upsilon=0$ holds. □

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