Collusion through debt and managers*

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Abstract

We investigate the anticompetitive effects of debt financing and managerial incentives in a framework where managers incur personal bankruptcy costs. We characterize the strategic value for firms' shareholders of resorting to debt and managerial incentives as complementary devices to sustain collusion in the product market, provided that managerial bankruptcy costs are sufficiently responsive to the severity of financial distress. Shareholders' limited commitment to debt and managerial contracts exacerbates the reliance on the amount of debt and managerial incentives for anticompetitive purposes. These results square with the well-documented features of firms' debt structure and corporate governance in sectors plagued by collusion.

KEYWORDS: Bankruptcy, collusion, commitment, common lending, corporate governance, debt financing, managerial incentives.

JEL CLASSIFICATION: D21, G32, G33, G34, L13, L22, L41.

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1 Introduction

A recent influential strand of empirical studies has brought back attention to firms' debt structure and corporate governance as drivers for collusion in product markets, with a focus on common lending and common ownership (e.g., Antón et al., 2022; Azar et al., 2018; Dasgupta and Žaldokas, 2019; Ha et al., 2021; Saidi and Streitz, 2021). The relationship between firms' debt structure, corporate governance, and product market competition has been investigated from different angles (e.g., Allen and Gale, 2000; Buccirossi and Spagnolo, 2008; MacKay and Phillips, 2005). However, relatively little theoretical research has been conducted on the interactions between firms' debt structure and corporate governance for the sustainability of collusion. This gap is somewhat surprising in light of the consolidated empirical evidence on the anticompetitive effects of debt financing (e.g., Chevalier, 1995a, 1995b; Chevalier and Sharfstein, 1996; Kovenock and Phillips, 1995, 1997; Phillips, 1995) and managerial incentive schemes (e.g., Ha et al., 2021; Joh, 1999).

Collusion is a widespread phenomenon in product markets. The overcharges imposed by anticompetitive agreements have been estimated to be sizeable on both sides of the Atlantic (e.g., Boyer and Kotchoni, 2015; Smuda, 2014; Symeonidis, 2018). Most worryingly, in several prominent industries, firms engage in sophisticated anticompetitive tactics that challenge antitrust authorities' deterrence and detection capabilities (e.g., Asker, 2010; Marshall and Marx, 2012; Miller, 2009). Identifying the factors that may facilitate or hinder collusion has become critical to designing institutional architectures that effectively prevent collusive practices. Previous studies have significantly improved the understanding of these factors — such as monitoring of sales and communication (e.g., Harrington and Skrzypacz, 2007, 2011) as well as the use of algorithms, machine learning, and artificial intelligence (e.g., Calvano et al., 2020; Miklós-Thal and Tucker, 2019).

We aim to contribute to this literature and enrich the current policy debate by exploring the anticompetitive effects of debt financing and managerial incentives. Specifically, we characterize the strategic value for firms' shareholders of resorting to debt and managerial incentives as complementary devices to collude in the product market. Our analysis starts by considering a market for a homogeneous good where firms compete à la Bertrand by setting prices over an infinite time horizon. A firm's shareholders choose the debt structure and delegate pricing decisions to a self-interested manager, whose remuneration is contingent on the firm's profit.[1] When a firm is unable to repay its debt, bankruptcy occurs. Shareholders and managers of insolvent firms are protected by limited liability. However, managers face personal bankruptcy costs. Systematic empirical evidence indicates that defaulting managers incur bankruptcy costs, which usually take the form of reputation and stigma costs along with job losses or drastic wage cuts.[2] Lenders often explicitly ask shareholders to hire top managers that, given

[1] We refer to Section 3 for details about managerial incentive schemes. Section 6 provides microfoundations for such schemes.

[2] Sutton and Callahan (1987) document the significant stigma costs faced by top managers after bankruptcy because of discredit, loss of status, spoiled image, and embarassement. As shown by Gilson (1989) and Gilson and Vetsuypens (1993), about half of the managers of firms facing financial distress are replaced without being rehired by comparable (exchange-listed) firms during the following three years and the managers retained by their firms bear large reductions in salary and bonus. Along these lines, Eckbo and Thorburn (2003) find a median CEO income decline of 47% after bankruptcy. Eckbo et al. (2016) report that CEOs leaving the executive labor

their solid reputation for 'prudent behavior', significantly suffer from bankruptcy.[3] In line with some relevant empirical studies, we allow for managerial bankruptcy costs increasing with the severity of financial distress, which is related to the amount of unrepaid debt.[4]

In this framework, we can address a range of stimulating questions. How does debt financing affect collusion in the product market? What is the role of managers? How can managerial remuneration schemes be designed to facilitate cooperation in the product market?

We find that two opposite forces shape the impact of an expansion of debt on the sustainability of collusion, as measured by the critical discount factor above which firms can achieve the collusive monopoly outcome. On the one hand, a higher level of debt makes managers more eager to deviate by undercutting the collusive price because the reversion to the competitive equilibrium in the punishment phase leads to bankruptcy, which cancels the residual debt due to limited liability. On the other hand, higher debt inflates the bankruptcy costs that managers incur. As a result of the trade-off between these two opposite forces, a higher level of debt facilitates collusion when managerial bankruptcy costs are sufficiently responsive to the severity of financial distress. We also find that managers receiving a higher profit share are more tempted to undercut the collusive price because they can grab a larger portion of the gain from deviation. Hence, higher-powered managerial incentives hinder collusion for a given amount of debt.

Building on these results, we identify the level of debt and managerial incentives chosen by firms' shareholders to maximize collusive profits. The endogenization of these instruments leads to additional challenging questions. Under what conditions can debt and managerial incentives facilitate collusion in the product market? What is the interaction between these instruments? How does their adoption depend on the market features?

We show that, for intermediate values of the discount factor, collusion is only sustainable through an appropriate combination of debt and managerial incentives. In light of the trade-off previously discussed, managerial bankruptcy costs must be sufficiently responsive to the severity of financial distress. We know from the foregoing analysis that, for a given level of debt, higher-powered managerial incentives hamper collusion. However, endogenizing these instruments, we find that debt and managerial incentives constitute complementary strategic devices to sustain collusion in otherwise competitive industries. A higher level of debt facilitates collusion by inflating managerial bankruptcy costs but curbs firms' profits and thus induces managers to require higher-powered incentives to ensure their participation.

The combination of debt and managerial incentives that supports the collusive outcome varies with the characteristics of the market at hand. As higher debt increases managers'

market after bankruptcy experience a median estimated compensation loss equal to five times the pre-departure income. Kaplan (1994a, 1994b) provides empirical evidence about the negative relationship between top executive turnover and firm's performance. Jenter and Kanaan (2015) document that CEOs are significantly more likely to be dismissed from their jobs after bad industry performance.

[3]Gilson (1989) finds that a significant number of changes in management are initiated by creditors, especially during debt restructuring. As documented by Nini et al. (2012), creditors play an active role in corporate governance by exerting informal influence as well.

[4]Ozelge and Saunders (2012) document that the underperforming firms' likelihood of CEO forced replacement rises with the intensity of their reliance on bank loans, especially when loan covenants are violated. We refer to Eckbo et al. (2016) for an estimation of the positive impact of leverage on the probability of CEO forced turnover around bankruptcy.

bankruptcy costs, firms' shareholders expand the amount of debt whenever their managers are more tempted to deviate from the collusive agreement. This occurs in less concentrated markets, where managers can obtain (a share of) higher profits by undercutting the collusive price, or in the presence of better outside options, which allow managers to command a higher profit share and make deviations more attractive. Conversely, a reduction in the managers' temptation to deviate, stemming from more costly bankruptcy, higher collusive profit, or a larger discount factor, leads firms' shareholders to curb their debt. The complementarity between debt and managerial incentives implies that the profit share granted to managers moves in the same direction as the amount of debt.

Our analysis is robust to several extensions. In the baseline model, firms' shareholders are able to commit to contracts that specify the debt structure and managerial compensation schemes. As we argue, shareholders are able to commit to such contractual terms under many plausible circumstances. Yet, to broaden the scope of our study, we relax this hypothesis by incorporating different degrees of limited commitment into our model. Specifically, we first allow for managerial contracts to be secretly renegotiated at any time. Then, we turn to a more general setting where shareholders can also secretly renegotiate their financial obligations. Our analysis is further extended to a more extreme scenario where shareholders cannot commit at all to managerial contracts, which can be reneged upon tout court, and may also have limited commitment power on the financial structure. We show that debt and managerial incentives can still facilitate collusion as long as each lender serves (at least) two rivals in the product market — i.e., under common lending relationships, a well-known phenomenon typically associated with bank specialization. Interestingly, as shareholders with weaker commitment power find it more difficult to cooperate, limited commitment exacerbates the reliance on the amount of debt and managerial incentives for anticompetitive purposes.

As shown in the Supplementary Appendix, our results also carry over to various forms of market structure. In industries where competition is relatively soft even in the absence of collusion (e.g., because of product differentiation) or where demand is somewhat elastic, firms' profits outside the collusive phase can be significant. This makes managers more inclined to deviate and thus leads shareholders to expand the debt level and strengthen managerial incentives to stabilize collusion. Notably, our model neither resorts to unduly restrictive assumptions on functional forms nor requires direct communication across firms at any stage of the product market game, enabling us to explore the sustainability of tacit collusion.

In a nutshell, our analysis unveils a novel channel that connects firms' financial structure, corporate governance, and product market competition, thereby throwing light upon the anticompetitive effects of debt and managerial incentives. Our work can contribute to a more comprehensive understanding of the relationship between financial and product markets by complementing various well-established contributions — such as the seminal pioneering studies of Brander and Lewis (1986) and Maksimovic (1988) — according to which higher debt makes firms more aggressive in the product market. The different predictions of our model are more suitable for markets where professional managers run companies, and thus managerial incentives turn out to be relevant. Our results provide theoretical support for the aforementioned empirical evidence on the anticompetitive effects of debt financing and managerial incentive

schemes. Notably, Dasgupta and Žaldokas (2019) identify a drop in the level of debt after the breakdown of collusive activities due to the introduction of a leniency program. Establishing a causal relationship between credit concentration and industry markup, Saidi and Streitz (2021) show that common lenders serving multiple firms reduce the cost of debt and soften product market competition. This is especially the case when a higher cost of debt is more likely to drive firms into dire straits. As thoroughly discussed in Section 6, the coordination function of a common lender can also be related to the recent empirical literature about the anticompetitive effects of common ownership — i.e., the role of common lending is, de facto, equivalent to the role of common ownership. We find that common lending is more relevant to collusion in markets where firms' shareholders have limited commitment power on debt contracts.

The classical empirical observations on the relationship between debt financing and market outcomes are rather heterogeneous. In some studies, more indebted firms charge lower prices (e.g., Busse, 2002; Zingales, 1998). However, different studies indicate that, primarily in concentrated industries, higher debt is associated with softer competition, which takes the form of higher prices, lower output, and more passive investment behavior (e.g., Chevalier, 1995a, 1995b; Chevalier and Sharfstein, 1996; Kovenock and Phillips, 1995, 1997; Phillips, 1995). Our analysis can explain the anticompetitive effects of debt, especially in concentrated industries with large companies run by managers. Our findings are also consistent with other traditional empirical regularities about debt, such as stock price rises in response to debt-increasing transactions (e.g., Harris and Raviv, 1991; James, 1987).

Our work provides novel insights into the role of managers within firms, suggesting that some debt can be a firm's deliberate choice to mitigate overly aggressive managerial behavior in the product market rather than the outcome of poor managerial performance. From a normative angle, we unveil a dark side of information sharing that has been overlooked so far. The exchange of information regarding firms' financial situation and corporate governance strengthens shareholders' commitment power on debt and managerial contracts. This renders debt and managerial incentives more effective in sustaining collusion at the expense of final consumers. Our results recommend careful consideration of these aspects in the design of disclosure rules that shape financial and corporate governance regulations.

The rest of the paper unfolds as follows. Section 2 describes the related literature. Section 3 sets out the formal model. Section 4 characterizes the combination of debt and managerial incentives to sustain collusion and examines the underlying factors. Section 5 investigates the robustness of our results by considering different degrees of limited commitment to debt and managerial contracts. Section 6 provides microfoundations for managerial incentive schemes based on profit sharing. Section 7 discusses some managerial and policy implications. Section 8 concludes the analysis. Formal proofs are collected in the Appendix. In the Supplementary Appendix we extend our model to various forms of market structure.

2 Related literature

Our paper belongs to the extensive literature on the interaction between firms' financial structure and product market competition. The two traditional approaches to this issue suggest

that firms better endowed with financial assets compete more fiercely in the product market. According to the 'long purse' or 'deep pocket' theory of predatory pricing à la Telser (1966), a financially strong firm cuts its prices to drive the competitors out of the market or preempt entry. This argument has been rigorously established in various models of predation (e.g., Benoit, 1984; Bolton and Scharfstein, 1990; Fudenberg and Tirole, 1986). The 'limited liability' theory, stemming from Brander and Lewis (1986), argues that higher debt can help firms to commit to behave more aggressively in the product market. Limited liability provisions allow shareholders to ignore bad market states, creating a conflict of interests between debt holders and equity holders, as described by Jensen and Meckling (1976). Along these lines, Maksimovic (1988) identifies the negative impact of debt on the firms' ability to sustain tacit collusion in an infinite horizon model and characterizes the highest level of debt that prevents any deviation. Stenbacka (1994) provides further insights into the procompetitive effects of debt in a market for a homogeneous good where firms engage in infinitely repeated Bertrand competition and demand randomly varies over time. Poitevin (1989) shows in a model à la Brander and Lewis (1986) that a common lender may allow firms to mitigate their overly aggressive behavior in the product market through a suitable choice of the interest rate. Debt remains procompetitive and firms would be better off if they could commit not to use debt. Contrary to Poitevin (1989), we find that debt relaxes competition and a common lender helps collusion only when firms' shareholders have limited commitment power.

Extending Maksimovic's (1988) framework, Hege (1998) shows that an indebted firm can achieve the highest collusive profit by repaying its debt as quickly as possible, and bank-financed industries can sustain more collusion compared to the case of publicly traded debt. In an infinitely repeated version of the Brander and Lewis (1986) model, Damania (1997) argues that, under certain circumstances, debt holdings facilitate collusion. A major difference with Maksimovic (1988) is that demand is unknown prior to production decisions and fluctuates over time. In our paper, we establish the joint role of debt and managerial incentives in sustaining collusion, which does not rely on unknown demand fluctuations or the observability and inflexibility of the debt structure. Hirshleifer and Thakor (1992) find that managerial concern for reputation aligns managers' interests with those of bondholders. Zwiebel (1996) shows that debt can be used to credibly restrict managerial empire-building ambitions. Our work indicates that the disciplinary role of debt in shaping managerial behavior extends to the sustainability of collusion, provided that managers obtain a suitable stake of collusive profits.

Other theories have been proposed to explain the link between firms' financial structure and product market decisions. In a two-period version of the Brander and Lewis (1986) model, Glazer (1994) finds that long-term debt softens product market competition. Showalter (1995) shows that debt allows Bertrand competitors to raise prices when demand conditions are uncertain. Faure-Grimaud (2000) identifies the anticompetitive role of debt under asymmetric information between lenders and borrowing firms. Aghion et al. (2000) develop a model where the firm's management becomes softer or tougher in response to a higher need for outside finance, according to whether the initial level of outside finance is low or high. Povel and Raith (2004) find a U-shaped relationship between the output of a financially constrained firm and the level of its internal funds in a duopoly market where the constrained firm faces an uncon-

strained rival. In the presence of information asymmetries between the parties to a financial contract, Campello (2006) characterizes a non-monotonic relationship between debt and sales performance such that, beyond a certain threshold, more debt generates market share losses, providing empirical evidence for these results. In a general equilibrium model, Dellas and Fernandes (2014) find that the development of financial markets tends to lower firms' markups, despite the possible reduction in the number of firms. Lehar et al. (2020) show that upstream suppliers can offer their retailers trade credit in vertically related markets with medium concentration levels to achieve a more collusive outcome in the downstream sector compared to the case of bank financing. We refer to Sertsios (2020) for an exhaustive survey focusing on recent developments.

There exists a significant literature that investigates the effects of managerial incentives on the sustainability of collusion. Spagnolo (2000) shows that, under certain circumstances, stock-related compensation packages constitute powerful incentives for managers to support tacit collusion in repeated oligopolies. In a setting where managers prefer smooth profit streams, Spagnolo (2005) finds that firms can sustain collusive agreements at lower discount factors and collusion arises at any discount factor through capped bonus plans and incumbency rents with termination threats. Along these lines, in a dynamic oligopoly with uncertain demand, Bernhardt and Chambers (2006) show that profit sharing between firms and their employees always facilitates collusion between firms. A significant novelty of our approach with respect to the afore-discussed strands of literature dealing separately with the effects of debt financing and managerial incentives on product market competition is that we unveil the role of debt and managerial incentives as complementary strategic devices to sustain collusion. Notably, our results carry over to different forms of limited commitment to debt and managerial contracts. There also exists a vast literature in political economy that studies international cooperation between country leaders. For instance, Conconi and Sahuguet (2009) consider a model with two infinitely-lived countries run by policymakers and show that the possibility of re-election can facilitate cooperation between policymakers with finite but potentially renewable mandates relative to the case of infinitely-lived policymakers.

Our study can also contribute to the literature that considers the role of lenders as 'gatekeepers' in product markets and examines the effects of lending on market entry. The seminal paper of Bhattacharya and Chiesa (1995) shows that common lending may serve as a precommitment mechanism to share information among borrowing competitors that acquire proprietary knowledge through R&D investments, by possibly leading to a collusive outcome where only one firm enters the product market. In a 'big push' model, Da Rin and Hellmann (2002) characterize the catalytic role of dominant banks in creating new industries. Cestone and White (2003) find that a dominant investor deters market entry when its claim is sufficiently sensitive to the incumbent firm's profit — by holding equity or, equivalently, risky debt — so that entrants do not manage to obtain any funds. Differently from these contributions, in an infinite horizon model we investigate collusion among firms established in the product market and derive our results in the absence of market power in the lending sector.

Our paper is also related to the significant collection of work on collusion in vertical relationships (e.g., Jullien and Rey, 2007; Nocke and White, 2007, 2010; Piccolo and Miklós-Thal,

2012; Piccolo and Reisinger, 2011). Differently from our paper, these contributions abstract from debt financing and managerial incentives.

3 The model

Product market. We consider a market for a homogeneous good where $N \geq 2$ identical firms interact over an infinite time horizon by simultaneously setting prices in every period $\tau \in \{1, \ldots, +\infty\}$. Each firm maximizes the present discounted profit, with the common discount factor $\delta \in (0, 1)$. The per period profit accruing to each firm is $\pi > 0$ when all firms charge the collusive monopoly price. If a firm deviates from the collusive agreement by undercutting the monopoly price whereas the $N - 1$ rivals still charge it, the deviant firm (approximately) collects the industry monopoly profit $N\pi$ in the deviation period. Each firm obtains zero profit in the unique equilibrium of the stage game where the price reflects the (constant) marginal cost. Pricing decisions in each period τ become common knowledge at the beginning of period $\tau + 1$, so the game exhibits perfect monitoring.

Credit market and firms' debt structure. There exists a competitive credit market.[5] In period $\tau = 0$, before the product market game takes place, firms' shareholders can issue long-term debt instruments with infinite maturity.[6] A debt contract between firm $i \in \{1, ..., N\}$ and its lender consists of a pair $(L_i, \{b_{\tau i}\}_{\tau=1}^{+\infty})$ specifying a loan L_i received by firm i at the beginning of period $\tau = 1$ and a pledged repayment $b_{\tau i}$ by firm i in each period τ. The zero profit condition in the credit market yields $\sum_{\tau=1}^{+\infty} \delta^{\tau-1} b_{\tau i} = L_i$. Every firm employs its loan at the beginning of period $\tau = 1$ in unproductive activities, such as the distribution of dividends to shareholders or wasteful advertising.[7] In line with the literature (e.g., Brander and Lewis, 1986; Maksimovic, 1988; Stenbacka, 1994), this approach allows us to identify the strategic role of debt as a collusive device and to neutralize the well-known effects of capital investments.[8] Hence, debt repayments can only be financed through sales revenues. When a firm is unable to honor its debt, bankruptcy occurs. Firms' shareholders are protected by limited liability. Insolvent firms are sold to new owners and continue to operate in the market.[9] Without loss of generality, each firm borrows from one lender. As it will be clear, only the total size of pledged repayments matters.

Firms' organizational structure and managerial bankruptcy costs. A firm's shareholders delegate pricing decisions to a self-interested manager. In line with the empirical literature dis-

[5] Our results can be even reinforced in a credit market where the lender possesses some market power.

[6] See, e.g., Maksimovic (1988) and Stenbacka (1994). Clearly, our results still hold with a series of debt contracts, each with finite maturity, provided that a new contract is signed as soon as the previous one has expired. In practice, shareholders play a crucial role in significant matters of corporate governance, including the issue of new securities. This is especially the case in companies with large shareholders. Boubaker et al. (2017) provide compelling evidence that firms with multiple large shareholders tend to rely more heavily on bank debt financing.

[7] Including dividends into shareholders' wealth does not affect our qualitative results.

[8] The incorporation of investments into our model would complicate the analysis without providing any additional useful insights. As investments typically consist of demand-enhancing or cost-reducing activities that improve firms' profits, we refer to Section 4.3 for the impact of firms' profits on the sustainability of collusion.

[9] This approach, stemming from Maksimovic (1988) and Stenbacka (1994), is consistent with the evidence documented by Antill (2022) that 76% of defaulting large commercial and industrial companies in the United States are either reorganized or acquired. Intuitively, if firms permanently exit the market after bankruptcy, predation would occur and collusion could not be sustained.

cussed in the introduction, bankruptcy imposes personal costs on managers, despite limited liability.[10] Such costs can be also affected by the magnitude of financial distress. The bankruptcy costs of firm i's manager in period τ are given by

$$C\left(b_{\tau i}\right) \triangleq \left[k + \phi\left(b_{\tau i} - \pi_{\tau i}\right)\right] \cdot \mathbf{1}_b, \tag{1}$$

where $k \geq 0$ is a fixed component and $\phi \geq 0$ captures the responsiveness of bankruptcy costs to the severity of financial distress, measured by the amount of firm i's unrepaid debt in period τ corresponding to the difference between the pledged repayment $b_{\tau i}$ and the gross profit $\pi_{\tau i}$.[11] The formulation in (1) is sufficiently flexible to allow for different sources of managerial bankruptcy costs, which may or may not depend on the firm's indebtedness. Firm i's bankruptcy occurs in period τ if and only if the pledged repayment outweighs the gross profit, i.e., $b_{\tau i} > \pi_{\tau i}$. Thus, the indicator function $\mathbf{1}_b \in \{0, 1\}$ in (1) assumes a value of one if $b_{\tau i} > \pi_{\tau i}$ and a value of zero otherwise. Given that shareholders and managers are protected by limited liability, lenders can seize at most the firm's product market earnings in case of default.[12] A non-defaulting manager has a per period reservation utility equal to u, where $0 \leq u < \pi$. After bankruptcy has occurred, the manager's outside option declines because of reputation loss. The reservation utility of a defaulting manager is normalized to zero.[13]

Managerial incentive schemes. In every period τ the manager of firm i receives a share $\alpha_{\tau i} \in [0, 1]$ of firm i's net profit $\pi_{\tau i} - b_{\tau i}$. Thus, the total compensation of firm i's manager in period τ amounts to $\alpha_{\tau i}(\pi_{\tau i} - b_{\tau i})$. In Section 6, we provide microfoundations for the optimality of managerial incentive schemes based on profit sharing. To this aim, we extend our model to a classical moral hazard situation where shareholders induce managers to exert unobservable effort (for instance, in terms of cost-reducing or demand-enhancing activities).[14] Consistently with our formulation, managers' performance-based remuneration schemes have been systematically documented in the empirical literature (e.g., Ha et al., 2021; Joh, 1999; Kaplan, 1994a).

Collusion. Given that firms are identical, without any loss of insights, throughout the analysis we consider symmetric and stationary collusive strategies that implement the monopoly outcome. In particular, a (symmetric and stationary) collusive strategy prescribes that in period $\tau = 0$ the shareholders of each firm announce (i) a debt contract that specifies a loan L and a

[10] As argued in the introduction, bankruptcy costs can stem from reputation loss or social stigma. Bankruptcy is costly for a manager irrespective of what happens to other managers.

[11] As the loan exhibits infinite maturity, the amount of the per period default is proportional to the amount of total default. Hence, we can focus on bankruptcy costs contingent only on the severity of the firm's financial distress in the default period.

[12] We refer to Ross (1977) and Diamond (1984) for early models of bankruptcy costs in line with our formulation in (1). In Greenwald and Stiglitz (1990, 1993) bankruptcy costs increase with the firm's size, which is consistent with our approach as long as larger firms suffer from more severe financial distress. Berk et al. (2010) show that bankruptcy costs naturally arise from optimal contractual arrangements in perfectly competitive capital and labor markets.

[13] This is without loss of generality as long as the post-bankruptcy reservation utility is not too large relative to the bankruptcy fixed cost k. A higher k yields the same effect as a decline in the post-bankruptcy reservation utility.

[14] Optimal managerial schemes have been shown to exhibit linear sharing components in different moral hazard settings (e.g., Laffont and Tirole, 1986; Piccolo et al., 2014). The strategic delegation literature has traditionally adopted linear managerial schemes even in the absence of moral hazard problems (e.g., Fershtman and Judd, 1987; Sklivas, 1987).

per period pledged repayment b, where $b = (1 - \delta) L$ follows from the zero profit condition in the credit market, and (ii) a managerial contract that identifies a profit sharing rule α.[15] Clearly, a collusive strategy also dictates that each firm charges the monopoly price in period $\tau = 1$ (the outset of the product market game) and continues to do so as long as each firm charged this price in any previous period. In response to a deviation from the collusive strategy, arising either at the contractual stage in period $\tau = 0$ or at the product market stage in any period $\tau \geq 1$, firms revert to the unique equilibrium of the stage game and set the price at the marginal cost in any subsequent period. Hence, firms adopt standard grim trigger strategies to punish deviations. Given that profits drop to zero, firms cannot honor their debt contracts and go bankrupt. This behavior is rational because firms' shareholders are protected by limited liability.

Timing and equilibrium concept. The sequence of events unfolds as follows.

- In period $\tau = 0$, firms' shareholders simultaneously announce debt and managerial contracts.
- From period $\tau = 1$ onward, firms' managers engage in the product market game and contracts are executed. If a firm does not repay its debt, bankruptcy occurs.

Shareholders' announcements of debt and managerial contracts become common knowledge before the product market game commences. In the baseline model, contract announcements are binding and thus shareholders have full commitment power on both debt and managerial contracts. We look for a symmetric pure-strategy subgame perfect Nash equilibrium where the collusive monopoly outcome arises. In a number of countries, binding contract announcements can be achieved through mandatory transparency requirements, which oblige firms to disclose verifiable information about their financial structure and corporate governance.[16] In countries where transparency rules are voluntary or poorly enforced, contract announcements can still exhibit some commitment value, provided that firms are able to share confidential information. This can occur through multiple channels — such as trade associations or common intermediaries — established in a wide range of markets, which have been incidentally under close scrutiny by antitrust authorities in different countries.[17] Notably, a firm's shareholders can reinforce their commitment power on a debt contract by implementing a corporate policy that prescribes in each period the distribution of the firm's profit in excess of the per period pledged repayment in the form of dividends to shareholders. This prevents any revision of the debt contract. In Section 5, we relax the hypothesis of full commitment in different directions and show that our qualitative results persist when contract announcements are not binding and can be revised at any time.

[15] Our qualitative results carry over to different repayment obligations, which can account for the probability that the loan is not reimboursed.

[16] In Section 7, we discuss the forms of regulation that facilitate the exchange of this type of information.

[17] For instance, in 2011 the European Commission concluded that 17 steel producers (e.g., ArcelorMittal, Emesa, Global Steel Wire, voestalpine Austria Draht, and WDI) had operated between 1984 and 2002 a cartel to fix prices, share markets and exchange sensitive commercial information in all the countries that then formed the European Union except the United Kingdom, Ireland, and Greece. Details can be found at `https://ec.europa.eu/commission/presscorner/detail/en/IP_11_403` (last retrieved in March 2023).

4 Main results

4.1 A relevant benchmark

To clarify the role of managerial bankruptcy costs, we start with the benchmark case where bankruptcy does not harm managers. Consider a debt structure with a per period repayment $b \in [0, \pi]$, which cannot clearly exceed the firm's gross collusive profit π, and a managerial incentive scheme with a profit sharing rule $\alpha \in [0, 1]$. As discussed in Section 3, a firm obtains $\pi - b$ in any collusive period and $N\pi - b$ in the deviation period by undercutting the collusive monopoly price. The punishment of a deviation leads to the competitive equilibrium with the price at the marginal cost and drives the firm into bankruptcy for $b > 0$. The incentive constraint that ensures the sustainability of collusion writes as

$$\frac{\alpha}{1-\delta}(\pi - b) \geq \alpha(N\pi - b) \implies \delta \geq \frac{N-1}{N\pi - b}\pi.$$

A firm's manager abides by the collusive agreement if and only if the managerial share of the present discounted collusive profit (weakly) exceeds the managerial share of the deviation profit. In the absence of bankruptcy costs, managers' preferences are fully aligned with those of shareholders and managerial remuneration is inconsequential to the sustainability of collusion.[18] By simple inspection, we find that collusion is more difficult to support when the firm's debt increases, i.e., b goes up. A deviation from the collusive agreement triggers bankruptcy, which allows the firm to avoid the reimbursement of the residual part of the loan due to limited liability. As hinted by Maksimovic (1988), a more indebted firm is less inclined to engage in collusive activities and higher debt destabilizes collusion. Consequently, the optimal collusive strategy mandates no debt, i.e., $b = 0$. Collusion is sustainable for $\delta \geq (N-1)/N$, which corresponds to the standard condition in an infinitely repeated Bertrand game. As a loan is spent on unproductive activities and firms resort to debt only for anticompetitive purposes, we obtain that for $\delta \geq (N-1)/N$ debt cannot enlarge the scope for collusion, irrespective of the magnitude of managerial bankruptcy costs. Throughout the analysis, we consider the situation where this condition no longer holds by imposing the following assumption.

Assumption 1 $\delta < (N-1)/N$.

Assumption 1 ensures that the discount factor is not excessively large and thus collusion cannot be sustained when managers do not incur bankruptcy costs.

4.2 Collusion under managerial bankruptcy costs

The result à la Maksimovic (1988) derived in Section 4.1, according to which firms' indebtedness impairs collusion, emerges in the absence of managerial bankruptcy costs. Yet, as previously argued, managers are typically harmed by bankruptcy and thus their interests may diverge from shareholders' preferences. We first characterize the condition for the sustainability of collusion in the presence of managerial bankruptcy costs for a given level of managerial

[18] Trivially, managers are indifferent about collusion for $\alpha = 0$. We refer to Section 4.3 for a discussion of this extreme case.

profit share α and debt repayment b. Then, we study the impact of these instruments on the managers' incentives to collude. For any pair (α, b), the collusion incentive constraint becomes

$$\frac{\alpha}{1-\delta}(\pi - b) \geq \alpha(N\pi - b) - \delta C(b). \tag{2}$$

Anticipating that the punishment of a deviation will trigger bankruptcy (for $b > 0$), a firm's manager considers the bankruptcy costs $C(b)$ in (1), which are weighted by the discount factor δ because bankruptcy occurs in the period subsequent to a deviation. As described in Section 3, bankruptcy brings the managerial outside option to zero because of reputation loss, and thus a deviating manager gets zero in the continuation game. The collusion incentive constraint (2) can be rewritten as

$$\delta \geq \delta^*(\alpha, b), \tag{3}$$

where $\delta^*(\alpha, b) \in (0, 1)$ identifies the (unique) solution to the constraint (2) holding with equality.[19] Hence, $\delta^*(\alpha, b)$ denotes the critical discount factor above which collusion is sustainable. In the following lemma, we investigate how managerial incentives and debt financing affect the scope for collusion in the presence of managerial bankruptcy costs.

Lemma 1 *The critical discount factor $\delta^*(\alpha, b)$ exhibits the following features:*

(i) higher-powered managerial incentives hinder collusion — i.e., $\partial\delta^*/\partial\alpha > 0$;

(ii) a higher debt repayment facilitates collusion if and only if the responsiveness of managerial bankruptcy costs to the severity of financial distress is large enough — i.e., $\partial\delta^*/\partial b < 0$ if and only if $\phi > \widetilde{\phi}$, where $\partial\widetilde{\phi}/\partial\alpha > 0$;

(iii) higher managerial bankruptcy costs facilitate collusion — i.e., $\partial\delta^*/\partial k < 0$ and $\partial\delta^*/\partial\phi < 0$.

When managers incur bankruptcy costs, managerial incentives are no longer irrelevant to the sustainability of collusion. As bankruptcy costs make managers more willing to collude, we find from (2) and (3) that, at the critical discount factor δ^*, the profit of a deviating firm exceeds the present discounted collusive profit. A higher profit share α exacerbates the managers' temptation to deviate because they can grab a larger portion of the deviation profit. As point (i) of Lemma 1 reveals, for a given debt repayment, higher-powered managerial incentives discourage collusion.

A higher debt repayment b generates two opposite forces on the scope for collusion. On the one hand, as shown in Section 4.1, the managers of more indebted firms are more eager to deviate because this triggers bankruptcy and cancels the residual debt due to limited liability. On the other hand, insofar as bankruptcy entails managerial costs proportional to the severity of financial distress, higher debt makes managers more inclined to collude. As point (ii) of Lemma 1 reveals, the trade-off between these two opposite forces implies that debt facilitates collusion so long as the responsiveness ϕ of the managerial bankruptcy costs in (1) to the severity of financial distress is large enough. The threshold $\widetilde{\phi}$ above which debt favors collusion increases with α because a higher profit share α magnifies the managers' incentives to deviate.

[19] We refer to the proof of Lemma 1 for technical details.

Point (iii) of Lemma 1 emphasizes the effects of managerial bankruptcy costs on the sustainability of collusion. Intuitively, for any level of debt and managerial profit share, firms can more easily achieve the collusive outcome in the product market when managers bear higher bankruptcy costs.

4.3 Endogenizing debt and managerial incentives

Equipped with the results in Lemma 1, we now characterize the optimal collusive strategy that implements the monopoly outcome. At the outset of the game, the shareholders of each firm announce the managerial profit share $\alpha \in [0,1]$ and the debt repayment $b \in [0,\pi]$ that maximize the present discounted collusive profit net of managerial compensation. Shareholders' announcements of debt and managerial contracts are binding. The maximization problem of a firm's shareholders is given by

$$\max_{\{\alpha\in[0,1],b\in[0,\pi]\}} \frac{(1-\alpha)(\pi-b)}{1-\delta} \tag{4}$$

subject to the collusion incentive constraint (2) and the following participation constraint

$$\alpha(\pi-b) \geq u, \tag{5}$$

which requires that, on the equilibrium path, managers' remuneration (weakly) exceeds their reservation utility in every period. Given that a lower profit share α increases shareholders' profit in (4) and relaxes the collusion incentive constraint (2) through a reduction in the critical discount factor δ^* in (3) as established in Lemma 1, shareholders select the minimal level of $\alpha \in [0,1]$ such that the managerial participation constraint (5) is binding in equilibrium. Then, we have

$$\alpha(b) = \min\left\{1, \frac{u}{\pi-b}\right\}, \tag{6}$$

which identifies the minimal profit share $\alpha(b)$ ensuring the manager's participation for a given level of debt repayment b. In the subsequent analysis, we consider the relevant case $\alpha(b) < 1$, which occurs as long as the manager's reservation utility u is small enough.[20] Substituting (6) into the shareholders' objective function in (4) and into the collusion incentive constraint (2), the shareholders' maximization problem in (4) can be rewritten as

$$\max_{b\in[0,\pi]} \frac{\pi-b-u}{1-\delta} \tag{7}$$

subject to the following collusion incentive constraint

$$\frac{u}{1-\delta} \geq \frac{N\pi-b}{\pi-b}u - \delta C(b). \tag{8}$$

It is worth noting from (5) and (8) that a manager with a zero outside option ($u=0$) is willing to accept a contract with zero remuneration ($\alpha=0$) and to collude in the product market, ir-

[20]The extreme case $\alpha = 1$ is concisely discussed after Proposition 1. We refer to the proof of Proposition 1 for technical details.

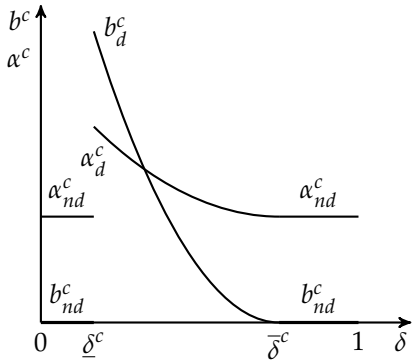

Figure 1: Debt repayment b^c and managerial profit share α^c under full commitment.

respective of the debt level and the discount factor. However, this extreme case seems rather unrealistic. Top managers have some market value, especially when they have not experienced bankruptcy, as discussed in Section 3. Hence, throughout the analysis we focus on the most plausible case of a positive outside option ($u > 0$) for non-defaulting managers so that shareholders must forego some profits to convince their managers to collude.[21] Given that the shareholders' objective function in (7) decreases with the debt repayment b, shareholders choose the lowest level of debt that ensures collusion, if it exists. In the following proposition, we derive the optimal combination of debt and managerial incentives that supports collusion when shareholders' announcements of debt and managerial contracts are binding.

Proposition 1 *Suppose that firms' shareholders are able to commit to debt and managerial contracts. Then, for a relatively small managerial reservation utility — i.e., $u < \overline{u}^c$ — there exist two thresholds $\overline{\delta}^c$ and $\underline{\delta}^c$ for the discount factor δ such that*

(i) if $\delta \geq \overline{\delta}^c$, in the collusive equilibrium, firms' shareholders issue (almost) no debt and provide managerial incentives — i.e., $b_{nd}^c \to 0$ and $\alpha_{nd}^c \in (0,1)$;

(ii) if $\delta \in \left[\underline{\delta}^c, \overline{\delta}^c\right)$, in the collusive equilibrium, firms' shareholders issue some debt and provide higher-powered managerial incentives when the responsiveness of managerial bankruptcy costs to the severity of financial distress is large enough — i.e., $b_d^c > 0$ and $\alpha_d^c \in \left(\alpha_{nd}^c, 1\right)$ for $\phi \geq \overline{\phi}^c$;

(iii) otherwise — i.e., either if $\delta < \underline{\delta}^c$ or if $\delta \in \left[\underline{\delta}^c, \overline{\delta}^c\right)$ and $\phi < \overline{\phi}^c$ — the collusive monopoly outcome is not sustainable.

The results in Proposition 1 are illustrated in Figure 1. As point (i) of Proposition 1 indicates, for a sufficiently large discount factor, collusion can be sustained through an infinitesimal amount of debt that arbitrarily converges to zero.[22] Managers receive a share of collusive profit that satisfies the managerial participation constraint (6). More relevantly, point (ii) of Proposition 1 identifies an intermediate region of values for the discount factor in which

[21] As shown in Section 6, agency conflicts within firms may also allow managers to extract some rents.

[22] This can be interpreted as the smallest level of loan that a lender is available to provide. An infinitely small amount of debt leads managers to incur the bankruptcy fixed cost k when a deviation from the collusive agreement occurs. As established in the proof of Proposition 1, it holds $\overline{\delta}^c < (N-1)/N$ and thus the interval for the discount factor δ in point (i) of Proposition 1 is non-empty under Assumption 1. The results in point (i) are only marginally different from those for $\delta \geq (N-1)/N$, where no debt is issued, as shown in Section 4.1. When k is negligible, we find that $\lim_{k \to 0^+} \overline{\delta}^c = (N-1)/N$, which implies from Assumption 1 that collusion cannot be generally sustained through an infinitely small amount of debt. We refer to the proof of Proposition 1 for technical details.

shareholders resort to some debt and provide their managers with higher-powered incentives. In light of the trade-off stemming from higher debt characterized in Lemma 1, collusion is sustainable for managerial bankruptcy costs sufficiently responsive to the severity of financial distress. As the collusion incentive constraint (2) is binding in equilibrium, the optimal level of debt equalizes the managerial share of the deviation profit net of the present discounted collusive profit with the (discounted) managerial bankruptcy costs. We know from Lemma 1 that, for a given amount of debt, higher-powered managerial incentives hinder collusion. However, endogenizing these instruments, we find that debt and managerial incentives act as complementary strategic devices to sustain collusion in the product market. To understand the rationale for this result, it is helpful to note that a higher level of debt makes managers more inclined to collude by inflating bankruptcy costs but erodes firms' profits net of debt and thus leads managers to command a higher profit share to induce their participation. Clearly, as established in point (iii) of Proposition 1, when the discount factor is relatively small or bankruptcy costs are not responsive enough to the severity of financial distress, no combination of debt and managerial incentives can achieve the collusive monopoly outcome.[23]

As argued in Section 2, previous studies have separately investigated the impact of debt and managerial incentives on product market competition. To the best of our knowledge, this paper constitutes the first attempt to identify the strategic value of combining debt and managerial incentives for anticompetitive purposes. Our analysis unveils a new channel for collusion stemming from managerial bankruptcy costs. As discussed in the introduction, this provides theoretical support for the empirically documented anticompetitive effects of debt financing and managerial incentive schemes.

We now investigate the factors that affect the optimal mix of debt and managerial incentives for the sustainability of collusion.

Proposition 2 *The debt repayment b_d^c and the profit share α_d^c characterized in Proposition 1 exhibit the following features:*

(i) *they increase with the number of firms and the managerial reservation utility — i.e., $\partial b_d^c/\partial N > 0$, $\partial b_d^c/\partial u > 0$, and $\partial \alpha_d^c/\partial N > 0$, $\partial \alpha_d^c/\partial u > 0$;*

(ii) *they decrease with the managerial bankruptcy costs, the per period collusive profit, and the discount factor — i.e., $\partial b_d^c/\partial k < 0$, $\partial b_d^c/\partial \phi < 0$, $\partial b_d^c/\partial \pi < 0$, $\partial b_d^c/\partial \delta < 0$, and $\partial \alpha_d^c/\partial k < 0$, $\partial \alpha_d^c/\partial \phi < 0$, $\partial \alpha_d^c/\partial \pi < 0$, $\partial \alpha_d^c/\partial \delta < 0$.*

Shareholders adjust the level of debt and managerial incentives according to the managers' temptation to deviate. In less concentrated markets, where the number of firms is higher, managers are more eager to deviate and capture the entire industry profit. This also occurs in the presence of more valuable managerial outside options because managers ask for a higher profit share, which magnifies their incentives to defect from the collusive agreement, as established in Lemma 1. Point (i) of Proposition 2 indicates that, under these circumstances,

[23]The focus in Proposition 1 on the case of a sufficiently small managerial reservation utility ensures the most realistic outcome where shareholders receive at least a portion of the collusive profit, i.e., $\alpha < 1$. The extreme case $\alpha = 1$ is not relevant for our analysis because shareholders forego the entire profit and become indifferent as to whether to collude or not. Even a small probability of an antitrust case would dissuade shareholders from colluding in the product market.

firms' shareholders inflate the level of debt to make bankruptcy more costly for managers and mitigate their eagerness to deviate. Given the complementarity between debt and managerial incentives, shareholders also increase the managerial profit share.

Conversely, a reduction in the managers' temptation to deviate allows shareholders to curb the magnitude of debt and managerial incentives. This occurs in a range of situations described in point (ii) of Proposition 2. Intuitively, managers are less inclined to undercut the collusive price in anticipation of more costly bankruptcy occurring in the punishment phase. In relatively more lucrative markets, where the per period collusive profit is higher, the lower wedge between deviation and collusive profits (net of debt) alleviates the managers' incentives to deviate. Finally, with a larger discount factor, managers are more patient and care to a greater extent about future collusive profits and losses from punishment relative to the spot gain from deviation. Under these circumstances, shareholders can discipline their managers through a lower level of debt and managerial incentives.

Before extending our analysis to different degrees of shareholders' limited commitment to debt and managerial contracts, we highlight some points in the following remarks.

Remark 1 It is well known that, in the absence of bankruptcy costs, for a sufficiently large discount factor such that the monopoly outcome can be sustained, an infinitely repeated Bertrand model admits a continuum of equilibria with the price ranging from the marginal cost to the monopoly level. Otherwise, there exists a unique equilibrium where the price reflects the marginal cost. This classical 'bang-bang' property no longer holds when managers incur bankruptcy costs. Despite the monopoly solution being unfeasible, firms may still manage to coordinate on supracompetitive prices. We refer to Piccolo and Miklós-Thal (2012) for insights into this direction.

Remark 2 In line with the empirical evidence discussed in the introduction, throughout the analysis we consider bankruptcy costs incurred by managers. It is worth noting that shareholders can also face bankruptcy costs, including legal fees, loss of key personnel, and bad publicity. In sectors where shareholders can directly influence the firms' pricing decisions, our results still apply by replacing managers with shareholders. As stressed in the introduction, our attention is mainly devoted to concentrated industries with large companies run by managers, whose bankruptcy costs are presumably significant. Given that managers are charged with pricing decisions, the sustainability of collusion essentially depends on their bankruptcy costs.

Remark 3 In our setting a deviation from the collusive agreement certainly leads to bankruptcy for any positive level of debt, because firms revert to the competitive equilibrium where they make zero profits. Interestingly, our qualitative findings carry over to a more sophisticated framework where bankruptcy arises only with some probability. For instance, this captures the degree of enforceability of bankruptcy rules. We can incorporate this point into our model by assigning a certain likelihood of bankruptcy costs in the collusion incentive constraint (2). In light of our results, we expect that collusion is sustainable through higher levels of debt and managerial incentives. We refer to the Supplementary Appendix for a setting where firms are still able to repay some amount of debt when collusion breaks down.

5 Limited commitment

We have assumed so far that firms' shareholders are able to commit to debt and managerial contracts. Now, we extend our analysis to various forms of limited commitment by allowing shareholders to revise in each period the contractual terms that specify the debt structure and the managerial remuneration. Can collusion still be sustained when shareholders have limited commitment power? What are the effects of limited commitment on the combination of debt and managerial incentives for anticompetitive purposes?

To suitably address these issues, we examine different degrees of limited commitment. It is well established in the literature (e.g., Dewatripont, 1988; Katz, 1991) that the commitment value of contracts vis-à-vis third parties can be significantly eroded by the agents' ability to secretly renegotiate the announced contracts. We first consider a scenario in which shareholders have the opportunity to secretly renegotiate managerial contracts, but they can still credibly announce the debt structure. Afterward, we explore secret renegotiations involving not only the managerial remuneration but also the amount of debt. Finally, we allow for a more extreme situation where shareholders cannot commit at all to managerial contracts, which can be secretly reneged upon tout court, and may also engage in secret renegotiations about debt contracts.[24] In line with the baseline model, we show that, under certain circumstances, shareholders resort to some level of debt bundled with managerial incentives to sustain collusion in the product market. As it will become apparent, the magnitude of shareholders' commitment power crucially shapes the optimal combination of debt and managerial incentives.

Under limited commitment, the timing of the game unfolds as follows.

- In period $\tau = 0$, firms' shareholders simultaneously announce debt and managerial contracts, which can be secretly modified in every period.
- From period $\tau = 1$ onward, firms' managers engage in the product market game and contracts are executed. If a firm does not repay its debt, bankruptcy occurs.

As contractual revisions are secret, the solution concept is perfect Bayesian equilibrium, with the standard refinement of 'passive beliefs' or 'market-by-market bargaining conjectures' (e.g., Hart and Tirole, 1990; McAfee and Schwartz, 1994). A firm's manager that receives an 'unexpected' (i.e., out-of-equilibrium) offer from the firm's shareholders still believes that the managers of the other firms follow their equilibrium strategies. This captures the natural idea that shareholders cannot signal to their managers information that they do not possess about the rivals, because the shareholders of each firm are independent and act simultaneously.

5.1 Renegotiations about managerial contracts

Each contract announced at the outset of the game is legally valid but in any period the contractual parties can secretly renegotiate it and stipulate another (legally valid) contract to achieve a mutually beneficial outcome. This preserves some minimal commitment value for debt and

[24] The breach of a debt contract would clearly drive the firm into bankruptcy.

managerial contracts. As argued later, our approach is reasonable in the context at hand. Essentially, firms' shareholders do not deceive the market about their contractual choices, which can, however, be subsequently amended through a secret renegotiation process. To properly identify the effects of contract renegotiations on the sustainability of collusion, we first consider the case where a firm's shareholders are able to commit to a debt structure, but in any period they can secretly renegotiate the contract with their manager at no cost.[25] The idea that debt contracts are binding turns out to be particularly compelling when lenders systematically share information on borrowers' histories, mainly in terms of their total exposure (e.g., Degryse et al., 2016). In several countries, publicly managed credit registries consolidate information on borrowers' credit worthiness, which typically includes their total indebtedness. There are also several countries, such as the United States and Italy, where different private information sharing systems — known as credit bureaus — have been voluntarily developed by financial intermediaries in response to information asymmetries between borrowers and lenders. Credit registries and bureaus often gather data about borrowers' past debts and report their total indebtedness instead of just documenting borrowers' characteristics and past delinquencies. Such information sharing activities can help firms' shareholders to confer credibility on their announcements about firms' financial situation even when they cannot refrain from secretly revising managerial contracts.

Consider a candidate equilibrium such that in period $\tau = 0$ each firm announces the pair (α, b) specifying the managerial profit share $\alpha \in [0,1]$ and the debt repayment $b \in [0,\pi]$ that support the collusive outcome. The present discounted profit accruing to a firm's shareholders in this candidate equilibrium is given by

$$V(\alpha, b) \triangleq \frac{(1-\alpha)(\pi - b)}{1-\delta}. \tag{9}$$

Furthermore, let

$$\widetilde{V}(\alpha, b) \triangleq \max_{\widetilde{\alpha}\in[0,1]} \left\{ (1-\widetilde{\alpha})(N\pi - b) \,:\, \widetilde{\alpha}(N\pi - b) - \delta C(b) \geq \frac{\alpha}{1-\delta}(\pi - b) \right\} \tag{10}$$

be the maximum profit that a firm's shareholders can obtain in a secret renegotiation process by offering the manager a new profit share $\widetilde{\alpha}$ that guarantees at least the same remuneration as in the original contract and induces the manager to deviate in the product market. The manager obtains a share $\widetilde{\alpha}$ of the deviation profit $N\pi$ net of the debt repayment b and incurs the bankruptcy costs $C(b)$ in the punishment phase.

Given that firms' shareholders are able to commit to the debt structure, it follows from (9) and (10) that any managerial contract is renegotiation proof if and only if

$$V(\alpha, b) \geq \widetilde{V}(\alpha, b). \tag{11}$$

This condition ensures that a firm's shareholders cannot benefit from secretly renegotiating the original contract with their manager at the rivals' expense. In this case, the contractual pair (α, b) is immune to renegotiations and allows shareholders to achieve the collusive out-

[25] Any positive cost of renegotiation would definitely reinforce our results.

come. In the following lemma, we reformulate the condition for the renegotiation proofness of managerial contracts in (11) and investigate whether the full commitment solution derived in Section 4.3 survives the threat of secret renegotiations.

Lemma 2 *Suppose that firms' shareholders are able to commit to debt contracts but they can secretly renegotiate managerial contracts. Then, any managerial contract is renegotiation proof if and only if*

$$\frac{\pi - b}{1 - \delta} \geq N\pi - b - \delta C(b). \tag{12}$$

The full commitment contractual pair $\left(\alpha_d^c, b_d^c\right)$ *characterized in Proposition 1 is not renegotiation proof.*

Lemma 2 shows that, when managerial contracts are potentially vulnerable to secret renegotiations, collusion is still sustainable as long as the whole firm — intended as the coalition of the firm's shareholders and manager — does not have any incentive to defect from the collusive agreement. This occurs if and only if the present discounted collusive profit (weakly) exceeds the aggregate deviation profit net of the (discounted) managerial bankruptcy costs — i.e., the renegotiation proofness constraint (12) is satisfied. Note from (2) and (12) that, differently from the case of full commitment, managers no longer affect the scope for collusion at the 'intensive margin' through the profit share α but only at the 'extensive margin' through bankruptcy costs. To gain further insights, consider a candidate equilibrium where each firm announces a contractual pair (α, b) that implements the collusive outcome. Suppose that, in a certain period, a firm's shareholders deviate from the candidate equilibrium by offering their manager a new profit share $\widetilde{\alpha}$, which induces the manager to undercut the collusive price. Anticipating that price undercutting will trigger bankruptcy, the manager commands a compensation for the associated loss. Renegotiations between the firm's shareholders and their manager are mutually beneficial when the aggregate deviation profit net of managerial bankruptcy costs more than compensates the aggregate discounted collusive profit — i.e., the renegotiation proofness constraint (12) fails to hold.

Lemma 2 also indicates that the full commitment contractual pair $\left(\alpha_d^c, b_d^c\right)$ characterized in Proposition 1, specifying the profit share α_d^c and the debt repayment b_d^c, is not robust to the threat of secret renegotiations. Recall from Lemma 1 that a larger share α tightens the collusion incentive constraint (2), which coincides with the renegotiation proofness constraint (12) if and only if $\alpha = 1$. Then, the full commitment contractual pair $\left(\alpha_d^c, b_d^c\right)$ violates the renegotiation proofness constraint (12). The full commitment solution succumbs to secret renegotiations and cannot be supported in equilibrium because it neglects the deviation incentives of the firm as a whole.

Equipped with the results in Lemma 2, we now derive the optimal design of debt and managerial incentives that satisfies the renegotiation proofness constraint (12) and thus sustains collusion under the threat of secret renegotiations about managerial contracts.

Proposition 3 *Suppose that firms' shareholders are able to commit to debt contracts but they can secretly renegotiate managerial contracts. Then, for a relatively small managerial reservation utility — i.e.,* $u < \overline{u}^{rm}$ *— there exist two thresholds* $\overline{\delta}^r$ *and* $\underline{\delta}^r$ *for the discount factor* δ *such that*

(i) if $\delta \geq \overline{\delta}^r$, in the collusive equilibrium, firms' shareholders issue (almost) no debt and provide managerial incentives — i.e., $b_{nd}^{rm} \to 0$ and $\alpha_{nd}^{rm} \in (0,1)$;

(ii) if $\delta \in \left[\underline{\delta}^r, \overline{\delta}^r\right)$, in the collusive equilibrium, firms' shareholders issue some debt and provide higher-powered managerial incentives when the responsiveness of managerial bankruptcy costs to the severity of financial distress is large enough — i.e., $b_d^{rm} > 0$ and $\alpha_d^{rm} \in \left(\alpha_{nd}^{rm}, 1\right)$ for $\phi \geq \overline{\phi}^r$;

(iii) otherwise — i.e., if $\delta < \underline{\delta}^r$ or $\delta \in \left[\underline{\delta}^r, \overline{\delta}^r\right)$ and $\phi < \overline{\phi}^r$ — the collusive monopoly outcome is not sustainable.

The debt repayment b_d^{rm} and the profit share α_d^{rm} in point (ii) are higher than in the scenario of full commitment characterized in Proposition 1 — i.e., $b_d^{rm} > b_d^c$ and $\alpha_d^{rm} > \alpha_d^c$.

Proposition 3 shows that, notwithstanding the potential vulnerability of managerial contracts to secret renegotiations, there is still scope for collusion in the product market. For intermediate values of the discount factor, shareholders can achieve the collusive outcome through an adequate mix of debt and managerial incentives, provided that managerial bankruptcy costs are sufficiently sensitive to the severity of financial distress, in line with the full commitment solution characterized in Proposition 1.[26] Note from Lemma 2 that managerial bankruptcy costs relax the renegotiation proofness constraint (12) but the profit share is inconsequential, which makes the collusive outcome more difficult to achieve. Hence, in order to insulate managerial contracts from secret renegotiations, shareholders must expand the amount of debt compared to the case of full commitment. Given the complementarity between debt and managerial incentives, a higher profit share is required by managers to ensure their participation. Comparing the thresholds for the discount factor δ between the scenarios of full commitment and renegotiations, we find that $\overline{\delta}^r > \overline{\delta}^c$ and $\underline{\delta}^r > \underline{\delta}^c$.[27] Thus, for $\delta \in \left(\overline{\delta}^c, \overline{\delta}^r\right)$ collusion can arise with an infinitesimal amount of debt under full commitment but requires some (non-negligible) level of debt under renegotiations and for $\delta \in (\underline{\delta}^c, \underline{\delta}^r)$ the collusive monopoly outcome can only be achieved under full commitment.

5.2 Renegotiations about debt and managerial contracts

In light of the results in Proposition 3, it is natural to wonder whether the sustainability of collusion crucially hinges upon the shareholders' ability to commit to debt contracts. As previously argued, debt contracts can exhibit a significant commitment value. However, the possibility that debt contracts are also susceptible to secret renegotiations definitely deserves some attention. To this aim, we now consider a scenario where shareholders' commitment power is so weak that they cannot refrain from secretly renegotiating both debt and managerial contracts. Our analysis reveals that, under certain circumstances, a combination of debt and managerial incentives can still allow collusion in the product market. To begin with, it is helpful to provide the following result.

Lemma 3 *Suppose that firms' shareholders can secretly renegotiate debt and managerial contracts. If each firm borrows from an exclusive lender, firms' shareholders issue no debt and collusion cannot be sustained.*

[26] When the discount factor is large enough, the amount of debt is so small as to converge to zero, similarly to Proposition 1. We refer to the proof of Proposition 3 for technical details.

[27] We refer to the proof of Proposition 3 for technical details.

Lemma 3 emphasizes that, under exclusive lending relationships, the collusive outcome collapses when any contract can be secretly renegotiated. An exclusive lender does not internalize the negative effects that debt renegotiations with its client impose on the other firms, which go bankrupt as a consequence of the renegotiating firm's deviation in product market. The vertical structure formed by an exclusive lender and a firm benefits from secret changes both in the debt contract, which is terminated in exchange for a transfer to the lender (at least) equal to the loan, and in the managerial contract, with a new profit share that leaves the manager (at least) as well off and induces a defection from the collusive agreement.[28]

Common lending. Things are dramatically different in the typically observed situation where a lender deals simultaneously with multiple firms — i.e., under common lending relationships. In this case, in order to accept a new proposal by one client, the lender asks for a premium in anticipation of the losses arising from bankruptcy of the other borrowing firms. Intuitively, if this premium is high enough, renegotiations about the debt contract cannot be mutually beneficial. Hence, the establishment of a financial network, with multiple firms served by the same lender, can help to sustain collusion in the product market because it allows the lender to (at least partially) internalize the negative externalities of debt renegotiations. To convey our results in the most intuitive manner, we first consider the simplest financial network where all firms borrow from a single lender. Suppose that, in a certain period (after all loans have been spent on wasteful activities), one firm makes the lender an offer that stipulates the termination of the original debt contract in exchange for a transfer from the firm to the lender. In anticipation of the firm's deviation in the product market and the consequential default of the other firms, the lender is willing to accept the deviating firm's offer for a transfer (at least) equal to $Nb/(1-\delta)$, which ensures a renegotiation premium corresponding to the present discounted value of the per period debt repayments promised by all N firms.[29] Hence, a debt contract is renegotiation proof if and only if

$$\frac{\pi - b}{1-\delta} \geq N\pi - \frac{Nb}{1-\delta} \implies b \geq \frac{(1-\delta)N - 1}{N-1}\pi, \tag{13}$$

which identifies the lowest level of debt repayment ensuring that a firm's present discounted collusive profit outweighs the gain from renegotiations given by the deviation profit net of the renegotiation premium to the lender.[30] When secret renegotiations can affect both debt and managerial contracts, the renegotiation proofness constraints (12) and (13) must be simultaneously satisfied in order to sustain collusion. The following proposition shows that, even in the case where any contract is potentially susceptible to secret renegotiations, firms' shareholders can still combine debt and managerial incentives to soften competition.

[28] Acemoglu (1998) shows that secret renegotiations within a vertical structure could be prevented by offering the manager sufficiently large rents so that the renegotiation process becomes overly expensive. Notably, Acemoglu's (1998) solution requires the manager's rents to be partially financed by the lender. This implies that the manager is simultaneously on the payroll of the lender and of the firm.

[29] As in an infinitely repeated game each subgame that commences in a certain period is identical to the original game, a contractual revision in that period corresponds to the one occurring at the outset of the game. Given the loan $L = b/(1-\delta)$ in the initial period $\tau = 1$, the present discounted value of the residual debt of each firm in any subsequent period $\tau \in \{2, ..., +\infty\}$ is $L/\delta^{\tau-1} - \sum_{t=1}^{\tau-1} b/\delta^t = b/(1-\delta)$.

[30] Note from Assumption 1 that the lowest level of debt repayment in (13) is strictly positive.

degrees of limited commitment	commitment to D renegotiations about M	renegotiations about D and M
debt level profit share	Higher than under commitment to D and M: $b_d^{rm} > b_d^c$ $\alpha_d^{rm} > \alpha_d^c$	Weakly higher than under commitment to D and renegotiations about M: $b_d^r = \max\left\{b_d^{rd}, b_d^{rm}\right\}$ $\alpha_d^r = \max\left\{\alpha_d^{rd}, \alpha_d^{rm}\right\}$
thresholds for the discount factor	Commitment vs renegotiations: $\overline{\delta}^r > \overline{\delta}^c$ and $\underline{\delta}^r > \underline{\delta}^c$	

Table 1: Debt (D) and managerial incentives (M) under limited commitment (Propositions 3 and 4).

Proposition 4 *Suppose that firms' shareholders can secretly renegotiate debt and managerial contracts. Then, for a relatively small managerial outside option — i.e., $u < \overline{u}^r$ — there exist two thresholds $\overline{\delta}^r$ and $\underline{\delta}^r$ for the discount factor δ such that*

(i) if $\delta \geq \overline{\delta}^r$, in the collusive equilibrium, firms' shareholders issue some debt and provide managerial incentives — i.e., $b_d^r = b_d^{rd} > 0$ and $\alpha_d^r = \alpha_d^{rd} \in \left(\alpha_{nd}^{rm}, 1\right)$;

(ii) if $\delta \in \left[\underline{\delta}^r, \overline{\delta}^r\right)$, in the collusive equilibrium, firms' shareholders issue some debt and provide managerial incentives when the responsiveness of managerial bankruptcy costs to the severity of financial distress is large enough — i.e., $b_d^r = \max\left\{b_d^{rd}, b_d^{rm}\right\}$ and $\alpha_d^r = \max\left\{\alpha_d^{rd}, \alpha_d^{rm}\right\}$ for $\phi \geq \overline{\phi}^r$;

(iii) otherwise — i.e., either if $\delta < \underline{\delta}^r$ or if $\delta \in \left[\underline{\delta}^r, \overline{\delta}^r\right)$ and $\phi < \overline{\phi}^r$ — the collusive monopoly outcome is not sustainable.

The debt repayment $b_d^r = \max\left\{b_d^{rd}, b_d^{rm}\right\}$ and the profit share $\alpha_d^r = \max\left\{\alpha_d^{rd}, \alpha_d^{rm}\right\}$ in point (ii) are strictly higher than in the scenario of commitment to debt contracts and secret renegotiations about managerial contracts characterized in Proposition 3 when the responsiveness of managerial bankruptcy costs to the severity of financial distress is large enough — i.e., $b_d^r = b_d^{rd} > b_d^{rm}$ and $\alpha_d^r = \alpha_d^{rd} > \alpha_d^{rm}$ for $\phi > \widetilde{\phi}^r$.

The main results in Propositions 3 and 4 are summarized in Table 1. Proposition 4 indicates that the level of debt and managerial incentives characterized in Proposition 3 in a setting where only managerial contracts can be renegotiated may not suffice to render debt contracts immune to renegotiations as well. Hence, under certain circumstances, shareholders must expand the amount of debt and strengthen managerial incentives to ensure collusion in the product market. Specifically, when the discount factor is large enough, some (non-negligible) amount of debt is required to remove any temptation to renegotiate debt contracts — i.e., the renegotiation proofness constraint (13) is binding in equilibrium. For intermediate values of the discount factor, a combination of debt and managerial incentives allows shareholders to achieve the collusive outcome, provided that managerial bankruptcy costs are sufficiently responsive to the severity of financial distress. The level of debt and managerial incentives crucially depends on the relative magnitude of the scope for renegotiations about debt vis-à-vis managerial contracts. When the responsiveness of managerial bankruptcy costs to the severity of financial distress is large enough, managers become relatively reluctant to revise their contracts and thus debt renegotiations are more attractive. In this case, the renegotiation proofness constraint (13) about debt contracts is more stringent than the renegotiation proofness constraint (12) about managerial contracts and thus it is binding in equilibrium. Other-

wise, the possibility of also renegotiating debt contracts does not alter the optimal mix of debt and managerial incentives that sustains collusion compared to the case where only managerial contracts can be renegotiated.

Multiple lenders. Our results can be generalized to multiple financial networks, where only a subset of firms borrows from the same lender. To fix ideas, consider a collusive strategy prescribing that each lender serves $M < N$ firms, which implies that there are N/M active lenders.[31] It follows from Lemma 3 that each lender must serve at least two firms in the collusive equilibrium, i.e., $M \geq 2$.[32] Following the same rationale adopted in the previous analysis, a lender is willing to accept a deviating firm's offer for a transfer (at least) equal to $Mb/(1-\delta)$, which ensures a renegotiation premium corresponding to the present discounted value of the per period debt repayments promised by the lender's M clients. Thus, a debt contract is renegotiation proof if and only if

$$\frac{\pi - b}{1-\delta} \geq N\pi - \frac{Mb}{1-\delta} \implies b \geq \frac{(1-\delta)N - 1}{M-1}\pi. \tag{14}$$

The lowest level of debt repayment in (14) that makes a debt contract immune to renegotiations decreases with the number M of firms belonging to the lender's network. Hence, the establishment of large financial networks — with a high number of firms borrowing from the same lender — reduces the amount of debt and enhances the scope for collusion. We now present our results with multiple lenders.

Proposition 5 *Suppose that firms' shareholders can secretly renegotiate debt and managerial contracts. Then, if a number $M < N$ of firms borrow from the same lender and hence there are N/M active lenders, the results characterized in Proposition 4 hold, with the debt repayment b_d^{rd} and the profit share α_d^{rd} being replaced by b_d^{rdm} and α_d^{rdm} respectively, where $b_d^{rdm} > b_d^{rd}$ and $\alpha_d^{rdm} > \alpha_d^{rd}$.*

Proposition 5 shows that firms can sustain collusion in the presence of a multiplicity of independent financial networks by increasing the level of debt and managerial incentives compared to the case of a single lender.[33] Prominent examples of such financial networks are the Japanese 'keiretsu' and the German 'house bank' system. In a similar vein, our qualitative results continue to hold when firms borrow from more than one bank (e.g., Carletti, 2004; Carletti et al., 2007; Detragiache et al., 2000), provided that each bank holds a debt share of some firms. Interestingly, our analysis can be also related to the investigation into the formation of network structures involving buyers and sellers (e.g., Kranton and Minehart, 2001).

[31] Without loss of insights, we assume that N/M is an integer number.

[32] This occurs in a wide range of plausible circumstances. As discussed in Section 7, a lender typically deals with multiple firms operating in the same market. In our setting, if a lender denies credit to a firm in order to establish an exclusive relationship with another firm, collusion would not start because the firm without credit could not engage in wasteful activities at the outset of the game in the reasonable situation where the number of potential lenders is sufficiently smaller than the number of firms. This would clearly remove any incentive to form an exclusive lending relationship. A fortiori, credit denial could be prevented through information sharing among lenders about their borrowers, which is described in Section 7.

[33] The proof of Proposition 5 is omitted. It directly follows from the proof of Proposition 4, with the only difference that the relevant renegotiation proofness constraint is (14) instead of (13). The debt repayment b_d^{rdm} and the profit share α_d^{rdm} are respectively determined by the renegotiation proofness constraint (14) and the managerial participation constraint (5), which are binding in equilibrium.

5.3 Reneging upon contracts

To substantiate the robustness of our results to more extreme forms of limited commitment, we now consider the situation where a firm's shareholders cannot commit to any contract whatsoever with their manager. Hence, shareholders can secretly renege upon the original contractual obligations and offer the manager a new contract. In other terms, the announcement of managerial contracts is cheap talk. We denote by

$$\widehat{V}(b) \triangleq \max_{\widehat{\alpha}\in[0,1]} \{(1-\widehat{\alpha})(N\pi - b) : \widehat{\alpha}(N\pi - b) - \delta C(b) \geq u\} \tag{15}$$

the maximum profit that a firm's shareholders can obtain by breaching the original contract and offering their manager a new profit share $\widehat{\alpha}$ that ensures at least the reservation utility u and induces the manager to deviate in the product market by undercutting the collusive price. The manager obtains a share $\widehat{\alpha}$ of the deviation profit $N\pi$ net of the debt repayment b and incurs the bankruptcy costs $C(b)$ in the punishment phase.[34]

As in the previous analysis, we start with the case where debt contracts can be credibly announced. Using (9) and (15), we find that a firm's shareholders cannot benefit from reneging upon any managerial contract if and only if

$$V(\alpha, b) \geq \widehat{V}(b). \tag{16}$$

Any contractual pair (α, b) that satisfies the reneging proofness constraint (16) allows shareholders to achieve the collusive outcome. We find the following results.

Proposition 6 *Suppose that firms' shareholders are able to commit to debt contracts but they can secretly renege upon managerial contracts. Then, for a relatively small managerial outside option — i.e., $u < \overline{u}^{rm\prime}$ — there exist two thresholds $\overline{\delta}^{r\prime}$ and $\underline{\delta}^{r\prime}$ for the discount factor δ such that*

(i) if $\delta \geq \overline{\delta}^{r\prime}$, in the collusive equilibrium, firms' shareholders issue (almost) no debt and provide managerial incentives — i.e., $b_{nd}^{rm\prime} \to 0$ and $\alpha_{nd}^{rm\prime} \in (0,1)$;

(ii) if $\delta \in \left[\underline{\delta}^{r\prime}, \overline{\delta}^{r\prime}\right)$, in the collusive equilibrium, firms' shareholders issue some debt and provide higher-powered managerial incentives when the responsiveness of managerial bankruptcy costs to the severity of financial distress is large enough — i.e., $b_d^{rm\prime} > 0$ and $\alpha_d^{rm\prime} \in \left(\alpha_{nd}^{rm\prime}, 1\right)$ for $\phi \geq \overline{\phi}^{r\prime}$;

(iii) otherwise — i.e., either if $\delta < \underline{\delta}^{r\prime}$ or if $\delta \in \left[\underline{\delta}^{r\prime}, \overline{\delta}^{r\prime}\right)$ and $\phi < \overline{\phi}^{r\prime}$ — the collusive monopoly outcome is not sustainable.

The debt repayment $b_d^{rm\prime}$ and the profit share $\alpha_d^{rm\prime}$ in point (ii) are strictly higher than in the scenario of commitment to debt contracts and secret renegotiations about managerial contracts characterized in Proposition 3 — i.e., $b_d^{rm\prime} > b_d^{rm}$ and $\alpha_d^{rm\prime} > \alpha_d^{rm}$.

Proposition 6 shows that the collusive role of debt and managerial incentives persists when firms' shareholders lack any commitment power on managerial contracts but they are still able to commit to the debt structure. The opportunity to secretly renege upon managerial contracts makes collusion more difficult to sustain compared to the case of secret renegotiations — i.e., the reneging proofness constraint (16) is more stringent than the renegotiation proofness

[34] Recall from Section 3 that the manager's reservation utility drops to zero after bankruptcy.

degrees of limited commitment	commitment to D reneging upon M	renegotiations about D reneging upon M
debt level profit share	Higher than under commitment to D and renegotiations about M: $b_d^{rm\prime} > b_d^{rm}$ $\alpha_d^{rm\prime} > \alpha_d^{rm}$	Weakly higher than under commitment to D and reneging upon M: $b_d^{r\prime} = \max\left\{b_d^{rd}, b_d^{rm\prime}\right\}$ $\alpha_d^{r\prime} = \max\left\{\alpha_d^{rd}, \alpha_d^{rm\prime}\right\}$
thresholds for the discount factor	Renegotiations vs reneging: $\overline{\delta}^{r\prime} > \overline{\delta}^{r}$ and $\underline{\delta}^{r\prime} > \underline{\delta}^{r}$	

Table 2: Debt (D) and managerial incentives (M) under limited commitment (Propositions 6 and 7).

constraint (11).[35] As Propositions 3 and 6 reveal, this magnifies shareholders' reliance on the amount of debt and managerial incentives for anticompetitive purposes. A higher level of debt makes managers more reluctant to deviate in the product market and reduces shareholders' benefits from breaching managerial contracts. Comparing the thresholds for the discount factor δ between the scenarios of renegotiations and reneging, we find that $\overline{\delta}^{r\prime} > \overline{\delta}^{r}$ and $\underline{\delta}^{r\prime} > \underline{\delta}^{r}$.[36] Thus, for $\delta \in \left(\overline{\delta}^{r}, \overline{\delta}^{r\prime}\right)$ collusion can arise with an infinitesimal amount of debt under renegotiations but requires some (non-negligible) level of debt under reneging and for $\delta \in \left(\underline{\delta}^{r}, \underline{\delta}^{r\prime}\right)$ the collusive monopoly outcome can only be achieved under renegotiations.

Finally, we turn to the case of secret renegotiations about debt contracts.

Proposition 7 *Suppose that firms' shareholders can secretly renegotiate debt contracts and they can secretly renege upon managerial contracts. Then, for a relatively small managerial outside option — i.e., $u < \overline{u}^{r\prime}$ — there exist two thresholds $\overline{\delta}^{r\prime}$ and $\underline{\delta}^{r\prime}$ for the discount factor δ such that*

(i) if $\delta \geq \overline{\delta}^{r\prime}$, in the collusive equilibrium, firms' shareholders issue some debt and provide managerial incentives — i.e., $b_d^{r\prime} = b_d^{rd} > 0$ and $\alpha_d^{r\prime} = \alpha_d^{rd} \in \left(\alpha_{nd}^{rm\prime}, 1\right)$;

(ii) if $\delta \in \left[\underline{\delta}^{r\prime}, \overline{\delta}^{r\prime}\right)$, in the collusive equilibrium, firms' shareholders issue some debt and provide managerial incentives when the responsiveness of managerial bankruptcy costs to the severity of financial distress is large enough — i.e., $b_d^{r\prime} = \max\left\{b_d^{rd}, b_d^{rm\prime}\right\}$ and $\alpha_d^{r\prime} = \max\left\{\alpha_d^{rd}, \alpha_d^{rm\prime}\right\}$ for $\phi \geq \overline{\phi}^{r\prime}$;

(iii) otherwise — i.e., either if $\delta < \underline{\delta}^{r\prime}$ or if $\delta \in \left[\underline{\delta}^{r\prime}, \overline{\delta}^{r\prime}\right)$ and $\phi < \overline{\phi}^{r\prime}$ — the collusive monopoly outcome is not sustainable.

The debt repayment $b_d^{r\prime} = \max\left\{b_d^{rd}, b_d^{rm\prime}\right\}$ and the profit share $\alpha_d^{r\prime} = \max\left\{\alpha_d^{rd}, \alpha_d^{rm\prime}\right\}$ in point (ii) are strictly higher than in the scenario of commitment to debt contracts and reneging upon managerial contracts characterized in Proposition 6 when the responsiveness of managerial bankruptcy costs to the severity of financial distress is large enough — i.e., $b_d^{r\prime} = b_d^{rd} > b_d^{rm\prime}$ and $\alpha_d^{r\prime} = \alpha_d^{rd} > \alpha_d^{rm\prime}$ for $\phi > \widetilde{\phi}^{r\prime}$.

The main results in Propositions 6 and 7 are summarized in Table 2. Proposition 7 indicates that the potential vulnerability of debt contracts to secret renegotiations can exacerbate the shareholders' propensity to resort to debt and managerial incentives to sustain collusion. Similarly to the case of renegotiations about debt and managerial contracts (formalized in Proposition 4), shareholders now issue some debt even for a sufficiently large discount factor — i.e.,

[35] Using (6), we find that the right-hand side of the constraint in (15) is lower than the right-hand side of the constraint in (10).

[36] We refer to the proof of Proposition 6 for technical details.

the renegotiation proofness constraint (13) is binding in equilibrium. We find from Propositions 6 and 7 that, under the possibility of reneging upon managerial contracts, the threat of secret renegotiations about debt contracts can lead shareholders to increase the level of debt and managerial incentives with respect to full commitment to debt contracts. This occurs as long as managerial bankruptcy costs are sufficiently sensitive to the severity of financial distress so that managers become relatively reluctant to deviate in the product market and debt renegotiations are more attractive. In this case, the renegotiation proofness constraint (13) is more stringent than the reneging proofness constraint (16) and thus it is binding in equilibrium.

6 Profit sharing schemes

Throughout the analysis, we consider managerial incentive schemes based on profit sharing. As discussed in Section 3, this formulation is consistent with the empirical evidence that managerial rewards are related to performance outcomes. Now, we provide microfoundations for profit sharing schemes. To fix ideas, suppose that in each period of the collusive phase a firm's performance (for a given debt repayment b) is stochastic and can be either high or low, i.e., $(\pi - b)(1 + \epsilon)$ or $(\pi - b)(1 - \epsilon)$, where $\epsilon \in (0, 1)$ is a performance gap parameter. For the sake of convenience, performance random variables are independent and identically distributed across firms and over time. A firm's manager chooses a level of effort e, which can take the form of activities affecting market demand or production costs. Such activities are non-contractible because they cannot be observed by the firm's shareholders (or verified by a court of law). This identifies a classical moral hazard setting where shareholders benefit from higher effort that improves the firm's performance but managers incur the cost of effort. The firm's performance is high with probability $\sigma(e) \in [0, 1]$ and low with complementary probability $1 - \sigma(e)$, where $\sigma(e)$ increases with e. Intuitively, higher effort makes the firm's performance more likely to be high. Without loss of insights, we assume that $\sigma(e) = e$ for any $e \in [0, 1]$. Thus, the manager's effort coincides with the probability of high performance. The cost of effort function $\psi(e)$ is increasing and convex in e, i.e., $\psi'(e) \geq 0$ (with $\psi'(e) > 0$ for $e > 0$) and $\psi''(e) > 0$, with the standard normalization $\psi(0) = 0$.[37] At the outset of the game, a firm's shareholders offer their manager a contract (α, t) that specifies a profit share $\alpha \in [0, 1]$ and a fixed transfer $t \in \mathbb{R}$ to the manager in each period. Upon accepting the shareholders' offer, the manager chooses the level of effort in each period. Then, the manager decides whether to collude or not and the firm's performance realizes. At the end of each period, payments are executed.

As the manager is protected by limited liability, the transfer t satisfies the following limited liability constraint

$$\alpha(\pi - b)(1 - \epsilon) + t \geq u, \tag{17}$$

which ensures that, on the equilibrium path, in each period the manager's remuneration in case of low performance (weakly) exceeds the reservation utility. As in each period the manager's effort affects the probability of the firm's current performance, the manager chooses the

[37] To ensure interior solutions, we impose the Inada conditions $\psi'(0) = 0$ and $\psi'(1) \to +\infty$.

level of effort that maximizes the per period expected utility. Formally, we have

$$e = \underset{\widetilde{e}\in[0,1]}{\arg\max}\, \alpha\left[\widetilde{e}\left(\pi - b\right)\left(1+\epsilon\right) + \left(1-\widetilde{e}\right)\left(\pi - b\right)\left(1-\epsilon\right)\right] + t - \psi\left(e\right).$$

Taking the first-order condition with respect to e yields the following moral hazard incentive constraint

$$2\alpha\epsilon\left(\pi - b\right) = \psi'\left(e\right), \tag{18}$$

which equalizes the manager's marginal benefit of effort with the corresponding marginal cost. Importantly, the manager decides to provide a level of effort $e > 0$ only when receiving a profit share $\alpha > 0$.

The maximization problem of a firm's shareholders can be written as

$$\max_{\{\alpha\in[0,1], t\in\mathbb{R}\}} \left(1-\alpha\right)\left[e\left(\pi - b\right)\left(1+\epsilon\right) + \left(1-e\right)\left(\pi - b\right)\left(1-\epsilon\right)\right] - t \tag{19}$$

subject to the limited liability constraint (17), which is binding in equilibrium, and the moral hazard incentive constraint (18). Replacing these constraints into the shareholders' maximization problem in (19) and taking the first-order condition with respect to e yields

$$2\epsilon\left(\pi - b\right) = \psi'\left(e\right) + e\psi''\left(e\right). \tag{20}$$

Condition (20) determines the equilibrium effort $e^{mh} > 0$ that equalizes the shareholders' marginal benefit of effort with the corresponding marginal cost inflated by the marginal cost of the managerial rent under moral hazard. Shareholders prefer to elicit some effort from their managers to achieve high performance with positive probability. Substituting $e^{mh} > 0$ into (18), we find that in equilibrium managers require the profit share $\alpha^{mh} > 0$. Hence, the optimal contract with managerial effort must include a profit sharing component and cannot specify only a fixed transfer.

A contract that provides managers only with a fixed transfer (equal to the reservation utility) allows shareholders to sustain collusion with no debt (by making managers indeed indifferent). However, this contract induces zero effort and thus brings to the firm's low performance $\pi\left(1-\epsilon\right)$ with certainty. Substituting the binding limited liability constraint (17) and the moral hazard incentive constraint (18) into the shareholders' objective function in (19), we find that a profit sharing contract is optimal if and only if

$$\left(\pi - b^{mh}\right)\left(1 - \epsilon + 2\epsilon e^{mh}\right) - e^{mh}\psi'\left(e^{mh}\right) > \pi\left(1-\epsilon\right),$$

where b^{mh} is the amount of debt that supports collusion in the moral hazard equilibrium. Using (20), we obtain after some manipulation

$$\left(e^{mh}\right)^2\psi''\left(e^{mh}\right) > b^{mh}\left(1-\epsilon\right).$$

This condition is clearly satisfied as long as the performance gap parameter ϵ is relatively high. We emphasize our main results in the following remark.

Remark 4 When firms' shareholders need to incentivize their managers to exert some effort, the optimal managerial contract involves profit sharing, provided that the moral hazard issue is significant enough. As the contractual part specifying the fixed transfer does not affect the managers' attitude towards collusion, the profit sharing schemes adopted in our framework can be derived from the shareholders' optimization problem under moral hazard.

It is worth noting that the equilibrium profit share is determined by the moral hazard incentive constraint (18) rather than by the participation constraint (5). It follows from (18) that, for a given level of effort, higher debt must be accompanied by higher-powered managerial incentives, as in our model. Introducing managerial effort creates, however, a further effect that goes in the opposite direction. We find from (20) that a higher level of debt discourages effort because it reduces the firm's net profit and thus mitigates the shareholders' benefit of effort provision.[38] Hence, in a moral hazard environment, debt and managerial incentives are still complementary strategic devices that must be jointly adopted to sustain collusion, but higher debt may not require stronger managerial incentives.

7 Managerial and policy implications

Our work delivers some implications for the role of managers within firms. Shareholders can combine debt financing and managerial incentives to discipline managers' behavior in the product market. Managers are inclined to compete too fiercely compared to what shareholders would ideally prefer. To mitigate managers' overly aggressive behavior, shareholders issue some debt, which induces managers to behave more leniently in fear of bankruptcy, and provide adequate incentives ensuring managers' participation. Thus, our findings suggest that some level of debt can be a firm's deliberate choice to soften competition rather than the outcome of poor managerial performance.

High debt has been traditionally deemed as an impediment to collusion. By identifying the anticompetitive effects of debt, our study can help policymakers with the design of compelling guidelines and protocols that assist antitrust authorities in deterring collusive activities. Notably, our results about the role of common lending in supporting collusion can be related to the literature on common ownership. A firm that owns a stake in the rivals' profits (partially) internalizes the losses imposed by a price cut on the rivals. Gilo et al. (2006) establish the conditions under which partial cross ownership facilitates collusion. As shown by Azar et al. (2018) using data from the US airline industry, common ownership generates anticompetitive effects that are more than ten times larger than what is 'presumed to be likely to enhance market power' according to the 2010 US Merger Horizontal Guidelines. We find that a common lender stabilizes collusion by internalizing the negative externalities of debt renegotiations with a firm arising from bankruptcy of the other borrowing firms. Thus, the role of common lending in sustaining collusion corresponds to the role of common ownership. Notably, in our framework, common lending only matters under limited commitment. The

[38] Applying the implict function theorem to (20) yields $\frac{\partial e^{mh}}{\partial b} = -\frac{2\epsilon}{2\psi''(e^{mh})+e^{mh}\psi'''(e^{mh})} < 0$, where the inequality follows from $-2\psi''\left(e^{mh}\right) - e^{mh}\psi'''\left(e^{mh}\right) < 0$ (second-order condition for shareholders' profit maximization).

collusive mechanism based on common lending is arguably a more subtle matter for antitrust policy than common ownership because standard arguments of bank specialization can justify the prevalence of common lenders. Antón et al. (2022) document that managerial incentives are less performance-sensitive in firms with more common ownership. The reason is that common owners (partially) internalize the reduction in the profitability of competing firms due to lower prices driven by productivity-improving managerial effort. By contrast, we find that shareholders strengthen managerial incentives for anticompetitive purposes in the absence of common ownership.

Our model also provides policy implications for the role of information sharing systems in credit markets. It has been widely recognized in the literature since Sharpe (1990) and Pagano and Jappelli (1993) that lenders can acquire monopoly power from privileged information about their own customers and information sharing among lenders may arise endogenously, which leads to an increase in the volume of lending when the problem of adverse selection is particularly severe. As pointed out by Padilla and Pagano (1997, 2000), the lenders' commitment to share information promotes the borrowers' effort to repay loans and exchanging information about borrowers' past defaults may improve their performance.

This literature has not typically considered the impact of information sharing agreements on product markets and welfare. Our study unveils a dark side of information sharing that has been hitherto neglected. The exchange of information about firms' financial structure strengthens shareholders' commitment power, which renders debt and managerial incentives more effective in sustaining collusion, especially when information involves new or outstanding debt instead of poor repayment behavior.[39] This advocates a suitable formulation of disclosure rules that shape financial and corporate governance regulations. In the United States, public companies are required to report to the Securities and Exchange Commission (SEC) certain material corporate events, including changes in financial obligations, by filing the so-called 'Form 8-K' — introduced by the Securities Exchange Act of 1934 and amended in 2004 to expand disclosure requirements — within four business days of their occurrence. Such reports are publicly available in the SEC's EDGAR database, which collects information about public companies' financial operations.[40] The well-recognized social benefits of transparency rules in terms of investors' protection and limitation of financial market manipulation can be balanced against the exacerbation of the anticompetitive effects of debt highlighted in our analysis. This point is evocative of Stigler's (1964) classical observation that transparency rules aimed at improving accountability may facilitate the formation and stability of bidding rings by allowing bidders to monitor their rivals' choices. Given that firms can be disciplined by sufficiently severe market competition and corporate governance matters exactly when competitive pressure is relatively weak (Giroud and Mueller, 2010, 2011), our results imply that transparency rules designed to improve market efficiency may be counterproductive by undermining the disciplinary role of competition.

[39] Bennardo et al. (2015) characterize a different negative aspect of information sharing in credit markets in a model where borrowers do not compete in the product market. Information sharing about borrowers' credit histories may lead the credit market to collapse insofar as it exacerbates the moral hazard problems faced by lenders competing for the same borrower.

[40] Further details are available at `https://www.sec.gov/edgar/about` (last retrieved in March 2023).

8 Concluding remarks

In this paper, we investigate the effects of debt financing and managerial incentives on product market competition when managers incur personal bankruptcy costs. Challenging the traditional view that debt hinders the firms' ability to collude, we characterize the strategic value for firms' shareholders of combining debt and managerial incentives as complementary devices to sustain collusion. The adoption of these instruments for anticompetitive purposes is more effective when shareholders are able to commit to debt and managerial contracts. Our results carry over to settings characterized by different degrees of limited commitment, where disclosure rules about firms' financial structure and corporate governance are lacking or poorly enforced. Shareholders' limited commitment expands the level of debt and managerial incentives required to support collusion in the product market. Our analysis sheds new light on the role of managers within firms and provides some policy implications for the design of antitrust institutions along with financial and corporate governance regulations that can effectively deter collusion.

As discussed in the introduction, we attempt to reconcile theory, which has traditionally emphasized the procompetitive role of debt, with a large body of empirical evidence documenting the anticompetitive effects of debt and managerial incentives. Establishing how firms' financial structure and corporate governance mechanisms can interact as determinants of collusion in product markets, our model predicts that collusion is more likely to emerge in markets where firms rely more extensively on debt financing and managers' performance-based remuneration schemes.[41] In settings with low enforcement of disclosure rules (which weakens shareholders' commitment power), firms can sustain collusion by increasing the level of debt and managerial incentives. A similar pattern is expected to arise in industries where competition is soft even in the absence of collusion (for instance, due to product differentiation) or market demand is somewhat elastic. Common lending is an additional element potentially conducive to collusion, especially when disclosure rules are poorly enforced. The anticompetitive effects of common lending share relevant similarities with those of common ownership. As such, our study delivers a range of testable predictions and new guidance on the empirical assessment of the channels through which firms' financial structure and corporate governance mechanisms affect collusion in product markets.

Appendix

This Appendix collects the proofs.

Proof of Lemma 1. Define

$$\Gamma(\alpha, b) \triangleq \frac{\alpha}{1-\delta}(\pi - b) - \alpha(N\pi - b) + \delta C(b), \tag{A1}$$

where $C(b)$ is given by (1). The collusion incentive constraint (2) is satisfied if and only if $\Gamma(\alpha, b) \geq 0$. Given that for $\alpha = 0$ it holds $\Gamma(\alpha, b) \geq 0$, we focus hereafter on $\alpha > 0$. Note

[41] A cross-country empirical investigation could exploit differences in regulations across countries that determine different managerial bankruptcy costs.

from (A1) that $\Gamma(\alpha, b) \to -\alpha(N-1)\pi < 0$ for $\delta \to 0$ and $\Gamma(\alpha, b) \to +\infty$ for $\delta \to 1$. It follows from the intermediate value theorem that there exists at least a value for $\delta \in (0,1)$ such that $\Gamma(\alpha, b) = 0$. Differentiating $\Gamma(\cdot,\cdot)$ in (A1) with respect to δ yields

$$\frac{\partial \Gamma}{\partial \delta} = \alpha \frac{\pi - b}{(1-\delta)^2} + C(b) > 0.$$

Hence, there exists a unique threshold $\delta^*(\alpha, b) \in (0,1)$ such that $\Gamma(\alpha, b) \geq 0$ if and only if $\delta \geq \delta^*(\alpha, b)$, where $\Gamma(\alpha, b) = 0$ if and only if $\delta = \delta^*(\alpha, b)$. The threshold $\delta^*(\alpha, b)$ is given by

$$\delta^*(\alpha, b) = \frac{N\alpha\pi + C(b) - \alpha b - \sqrt{[N\alpha\pi + C(b) - \alpha b]^2 - 4\alpha\pi(N-1)C(b)}}{2C(b)}. \tag{A2}$$

Applying the implicit function theorem to $\Gamma(\alpha, b) = 0$, where $\Gamma(\alpha, b)$ is defined by (A1), yields

$$\frac{\partial \delta^*}{\partial \alpha} = -\left.\frac{\partial \Gamma / \partial \alpha}{\partial \Gamma / \partial \delta}\right|_{\delta=\delta^*} = \frac{(1-\delta^*)[(1-\delta^*)(N\pi - b) - (\pi - b)]}{\alpha(\pi - b) + (1-\delta^*)^2 C(b)} > 0,$$

where the inequality follows from the positive sign of the expression in squared brackets (as $\Gamma(\alpha, b) = 0$).

Furthermore, we find that

$$\frac{\partial \delta^*}{\partial b} = -\left.\frac{\partial \Gamma / \partial b}{\partial \Gamma / \partial \delta}\right|_{\delta=\delta^*} = \frac{\delta^*(1-\delta^*)[\alpha - \phi(1-\delta^*)]}{\alpha(\pi - b) + (1-\delta^*)^2 C(b)} < 0,$$

where the inequality holds if and only if $\alpha/\phi < 1 - \delta^*$. As δ^* increases with α (see above) and decreases with ϕ (see below), we find that $\partial\delta^*/\partial b < 0$ if and only if α is small enough and ϕ is large enough. Taking the derivative of $\delta^*(\cdot,\cdot)$ in (A2) with respect to b yields

$$\begin{aligned}\frac{\partial \delta^*}{\partial b} = &-\frac{\alpha - \phi}{2C(b)} - \frac{(\alpha - \phi)[\alpha b - C(b)] - \alpha\pi[N\alpha + \phi(N-2)]}{2C(b)\sqrt{[N\alpha\pi + C(b) - \alpha b]^2 - 4\alpha\pi(N-1)C(b)}} \\ &- \phi \frac{N\alpha\pi + C(b) - \alpha b - \sqrt{[N\alpha\pi + C(b) - \alpha b]^2 - 4\alpha\pi(N-1)C(b)}}{2C(b)^2}.\end{aligned}$$

Using (1), we find that $\partial\delta^*/\partial b < 0$ if and only if $\phi > \widetilde{\phi}$, where

$$\widetilde{\phi} \triangleq \frac{N\alpha\pi - k + \sqrt{(N\alpha\pi - k)^2 + 4\alpha k\pi}}{2\pi} \text{ and } \frac{\partial \widetilde{\phi}}{\partial \alpha} = \frac{N}{2} + \frac{N^2\alpha\pi - (N-2)k}{2\sqrt{(N\alpha\pi - k)^2 + 4\alpha k\pi}} > 0.$$

Applying the implicit function theorem to $\Gamma(\alpha, b) = 0$, where $\Gamma(\alpha, b)$ is defined by (A1), we also find that

$$\frac{\partial \delta^*}{\partial k} = -\left.\frac{\partial \Gamma / \partial k}{\partial \Gamma / \partial \delta}\right|_{\delta=\delta^*} = -\frac{\delta^*(1-\delta^*)^2}{\alpha(\pi - b) + (1-\delta^*)^2 C(b)} < 0;$$

$$\frac{\partial \delta^*}{\partial \phi} = -\left.\frac{\partial \Gamma / \partial \phi}{\partial \Gamma / \partial \delta}\right|_{\delta=\delta^*} = -\frac{b\delta^*(1-\delta^*)^2}{\alpha(\pi - b) + (1-\delta^*)^2 C(b)} < 0. \quad \blacksquare$$

Proof of Proposition 1. Define

$$\Theta(b) \triangleq \frac{u}{1-\delta} - \frac{N\pi - b}{\pi - b}u + \delta C(b), \tag{A3}$$

where $C(b)$ is given by (1). The collusion incentive constraint (8) is satisfied if and only if $\Theta(b) \geq 0$. The shareholders' objective function in (7) decreases with b. Evaluating $\Theta(b)$ in (A3) at $b = 0$ (which implies $C(b) = 0$) yields

$$\Theta(b)|_{b=0} = \frac{u}{1-\delta} - Nu < 0,$$

where the inequality follows from Assumption 1. Furthermore, we have

$$\lim_{b\to 0^+} \Theta(b) = \frac{u}{1-\delta} - Nu + \delta k. \tag{A4}$$

As $\lim_{b\to 0^+} \Theta(b) = -u(N-1) < 0$ for $\delta \to 0$ and $\lim_{b\to 0^+} \Theta(b) = +\infty$ for $\delta \to 1$, it follows from the intermediate value theorem that there exists at least a value for $\delta \in (0,1)$ such that $\lim_{b\to 0^+} \Theta(b) = 0$. Given that $\lim_{b\to 0^+} \Theta(b)$ is strictly convex in δ, there exists a unique threshold $\overline{\delta}^c \in (0,1)$ such that $\lim_{b\to 0^+} \Theta(b) \geq 0$ if and only if $\delta \geq \overline{\delta}^c$, where $\lim_{b\to 0^+} \Theta(b) = 0$ if and only if $\delta = \overline{\delta}^c$. The threshold $\overline{\delta}^c$ is given by

$$\overline{\delta}^c \triangleq \frac{k + Nu - \sqrt{(k-Nu)^2 + 4ku}}{2k}, \tag{A5}$$

where $\overline{\delta}^c < (N-1)/N$ and $\lim_{k\to 0^+} \overline{\delta}^c = (N-1)/N$ (by L'Hospital's rule). Then, we find that, for $\delta \geq \overline{\delta}^c$, in the collusive equilibrium, the amount of debt is $b^c_{nd} \to 0$. Using (6), the profit share is $\alpha^c_{nd} \to u/\pi \in (0,1)$.

Now, suppose that $\delta < \overline{\delta}^c$. Note from (A4) that $\lim_{b\to 0^+} \Theta(b) < 0$ (as $\delta < \overline{\delta}^c$). Differentiating $\Theta(\cdot)$ in (A3) with respect to b yields

$$\frac{\partial \Theta}{\partial b} = \delta\phi - \frac{(N-1)\pi u}{(\pi - b)^2} \text{ and } \frac{\partial^2 \Theta}{\partial b^2} = -\frac{2(N-1)\pi u}{(\pi - b)^3} < 0,$$

which implies that $\Theta(\cdot)$ is strictly concave in b. Then, a necessary condition for $\Theta(b) \geq 0$ is

$$\lim_{b\to 0^+} \frac{\partial \Theta}{\partial b} = \delta\phi - \frac{(N-1)u}{\pi} > 0.$$

This condition holds if and only if $\delta \geq \underline{\delta}^c$, where the threshold $\underline{\delta}^c$ is given by

$$\underline{\delta}^c \triangleq \frac{(N-1)u}{\pi\phi}. \tag{A6}$$

The lowest solution to $\Theta(b) = 0$ with respect to b writes as

$$b^c_d \triangleq \frac{(1-\delta)(\pi\phi - k) - u}{2(1-\delta)\phi} - \frac{\sqrt{\delta(1-\delta)^2(k+\pi\phi)^2 + 2u(1-\delta)[\delta k + \pi\phi(2 - \delta + 2N\delta - 2N)] + \delta u^2}}{2\sqrt{\delta}(1-\delta)\phi}. \tag{A7}$$

This solution exists if and only if $\phi \geq \overline{\phi}^c$, where

$$\overline{\phi}^c \triangleq \frac{[2N(1-\delta)+\delta-2]u-\delta(1-\delta)k+2\sqrt{u(1-\delta)(N-1)[(1-\delta)(Nu-\delta k)-u]}}{\delta(1-\delta)\pi}.$$

Then, we find that, for $\delta \in \left[\underline{\delta}^c, \overline{\delta}^c\right)$ and $\phi \geq \overline{\phi}^c$, in the collusive equilibrium, the amount of debt is b_d^c in (A7), where $b_d^c > 0$, and the profit share is $\alpha_d^c = u/\left(\pi - b_d^c\right) \in \left(\alpha_{nd}^c, 1\right)$. Now, we show that $\alpha_d^c < 1$ when u is small enough. Applying the implicit function theorem to $\Theta(b) = 0$, where $\Theta(b)$ is defined by (A3), yields

$$\frac{\partial b_d^c}{\partial u} = -\left.\frac{\partial\Theta/\partial u}{\partial\Theta/\partial b}\right|_{b=b_d^c} > 0, \tag{A8}$$

where the inequality follows from $\partial\Theta/\partial u|_{b=b_d^c} = 1/(1-\delta) - \left(N\pi - b_d^c\right)/\left(\pi - b_d^c\right) < 0$ and $\partial\Theta/\partial b|_{b=b_d^c} = \delta\phi - (N-1)\pi u/\left(\pi - b_d^c\right)^2 > 0$, with b_d^c defined by (A7). This implies that $\alpha_d^c = u/\left(\pi - b_d^c\right)$ increases with u and $\alpha_d^c < 1$ if and only if $u < \overline{u}^c$, where

$$\overline{u}^c \triangleq \frac{(1-\delta)(\pi + \delta\pi\phi + \delta k - N\pi)}{\delta(\phi - \delta\phi - 1)}.$$

In the remaining region of parameters, where either $\delta < \underline{\delta}^c$ or $\delta \in \left[\underline{\delta}^c, \overline{\delta}^c\right)$ and $\phi < \overline{\phi}^c$, the collusive monopoly outcome is not sustainable. ■

Proof of Proposition 2. First, we consider the debt repayment b_d^c. Recall from the proof of Proposition 1 that $\partial\Theta/\partial b|_{b=b_d^c} > 0$, where $\Theta(b)$ is defined by (A3) and b_d^c by (A7). Furthermore, we know from (A8) that $\partial b_d^c/\partial u > 0$. Applying the implicit function theorem to $\Theta(b) = 0$ yields

$$\left.\frac{\partial\Theta}{\partial N}\right|_{b=b_d^c} = -\frac{\pi u}{\pi - b_d^c} < 0 \implies \frac{\partial b_d^c}{\partial N} = -\left.\frac{\partial\Theta/\partial N}{\partial\Theta/\partial b}\right|_{b=b_d^c} > 0;$$

$$\left.\frac{\partial\Theta}{\partial \pi}\right|_{b=b_d^c} = \frac{b_d^c u(N-1)}{\left(\pi - b_d^c\right)^2} > 0 \implies \frac{\partial b_d^c}{\partial \pi} = -\left.\frac{\partial\Theta/\partial \pi}{\partial\Theta/\partial b}\right|_{b=b_d^c} < 0;$$

$$\left.\frac{\partial\Theta}{\partial k}\right|_{b=b_d^c} = \delta > 0 \implies \frac{\partial b_d^c}{\partial k} = -\left.\frac{\partial\Theta/\partial k}{\partial\Theta/\partial b}\right|_{b=b_d^c} < 0;$$

$$\left.\frac{\partial\Theta}{\partial \phi}\right|_{b=b_d^c} = \delta b_d^c > 0 \implies \frac{\partial b_d^c}{\partial \phi} = -\left.\frac{\partial\Theta/\partial \phi}{\partial\Theta/\partial b}\right|_{b=b_d^c} < 0;$$

$$\left.\frac{\partial\Theta}{\partial \delta}\right|_{b=b_d^c} = \frac{u}{(1-\delta)^2} + k + \phi b_d^c > 0 \implies \frac{\partial b_d^c}{\partial \delta} = -\left.\frac{\partial\Theta/\partial \delta}{\partial\Theta/\partial b}\right|_{b=b_d^c} < 0.$$

Now, we turn to the profit share α_d^c. Recalling from the proof of Proposition 1 that $\alpha_d^c = u/\left(\pi - b_d^c\right)$, we find from the previous results that

$$\frac{\partial \alpha_d^c}{\partial N} = \frac{u}{\left(\pi - b_d^c\right)^2}\frac{\partial b_d^c}{\partial N} > 0;\ \frac{\partial \alpha_d^c}{\partial u} = \frac{1}{\left(\pi - b_d^c\right)^2}\left(\pi - b_d^c + u\frac{\partial b_d^c}{\partial u}\right) > 0;$$

$$\frac{\partial \alpha_d^c}{\partial \pi} = -\frac{u}{\left(\pi - b_d^c\right)^2}\left(1 - \frac{\partial b_d^c}{\partial \pi}\right) < 0;\ \frac{\partial \alpha_d^c}{\partial k} = \frac{u}{\left(\pi - b_d^c\right)^2}\frac{\partial b_d^c}{\partial k} < 0;$$

$$\frac{\partial \alpha_d^c}{\partial \phi} = \frac{u}{\left(\pi - b_d^c\right)^2} \frac{\partial b_d^c}{\partial \phi} < 0; \; \frac{\partial \alpha_d^c}{\partial \delta} = \frac{u}{\left(\pi - b_d^c\right)^2} \frac{\partial b_d^c}{\partial \delta} < 0. \quad \blacksquare$$

Proof of Lemma 2. As the shareholders' objective function in (10) decreases with $\widetilde{\alpha}$ and thus the associated constraint is binding, we find that

$$\widetilde{\alpha} = \frac{\alpha\left(\pi - b\right) + \delta\left(1 - \delta\right) C\left(b\right)}{\left(N\pi - b\right)\left(1 - \delta\right)}.$$

Substituting the expression for $\widetilde{\alpha}$ into $\widetilde{V}\left(\alpha, b\right)$ in (10) and using $V\left(\alpha, b\right)$ in (9), we obtain that the condition in (11) is satisfied if and only if

$$\frac{\pi - b}{1 - \delta} \geq N\pi - b - \delta C\left(b\right),$$

which identifies the renegotiation proofness constraint (12).

At the full commitment contractual pair $\left(\alpha_d^c, b_d^c\right)$ characterized in Proposition 1, we find from the binding collusion incentive constraint (2) that

$$V\left(\alpha_d^c, b_d^c\right) - \widetilde{V}\left(\alpha_d^c, b_d^c\right) = -\frac{\delta\left(1 - \alpha_d^c\right) C\left(b_d^c\right)}{\alpha_d^c} < 0,$$

where the inequality follows from $\alpha_d^c \in (0, 1)$ and $C\left(b_d^c\right) > 0$ (as $b_d^c > 0$). The full commitment contractual pair $\left(\alpha_d^c, b_d^c\right)$ violates the condition in (11) and thus it is not renegotiation proof. $\blacksquare$

Proof of Proposition 3. Define

$$\Lambda\left(b\right) \triangleq \frac{\pi - b}{1 - \delta} - \left(N\pi - b\right) + \delta C\left(b\right), \tag{A9}$$

where $C\left(b\right)$ is given by (1). The renegotiation proofness constraint (12) is satisfied if and only if $\Lambda\left(b\right) \geq 0$. The shareholders' objective function in (4) decreases with b. Evaluating $\Lambda\left(b\right)$ in (A9) at $b = 0$ (which implies $C\left(b\right) = 0$) yields

$$\Lambda\left(b\right)|_{b=0} = \frac{\pi}{1 - \delta} - N\pi < 0,$$

where the inequality follows from Assumption 1. Furthermore, we have

$$\lim_{b \to 0^+} \Lambda\left(b\right) = \frac{\pi}{1 - \delta} - N\pi + \delta k. \tag{A10}$$

It holds $\lim_{b \to 0^+} \Lambda\left(b\right) \geq 0$ if and only if $\delta \geq \overline{\delta}^r$, where the threshold $\overline{\delta}^r$ is given by

$$\overline{\delta}^r \triangleq \frac{N\pi + k - \sqrt{\left(N\pi - k\right)^2 + 4k\pi}}{2k}, \tag{A11}$$

where $\overline{\delta}^r \in \left(\overline{\delta}^c, \left(N - 1\right)/N\right)$ (see (A5)). Then, we find that, for $\delta \geq \overline{\delta}^r$, in the collusive equilibrium, the amount of debt is $b_{nd}^{rm} \to 0$. Using (6), the profit share is $\alpha_{nd}^{rm} \to u/\pi \in (0, 1)$.

Now, suppose that $\delta < \overline{\delta}^r$. Note from (A10) that $\lim_{b\to 0^+} \Lambda(b) < 0$ (as $\delta < \overline{\delta}^r$). Furthermore, we obtain from (A9) that $\Lambda(b)|_{b=\pi-u} = (1+\delta\phi-N)\pi + \delta k + \delta u[1-\phi(1-\delta)]/(1-\delta)$, where $b = \pi - u$ identifies the highest value for b ensuring that the shareholders' objective function in (7) is non-negative. As $\Lambda(b)|_{b=\pi-u} \to -(N-1)\pi < 0$ for $\delta \to 0$ and $\Lambda(b)|_{b=\pi-u} \to +\infty$ for $\delta \to 1$, it follows from the intermediate value theorem that there exists at least a value for $\delta \in (0,1)$ such that $\Lambda(b)|_{b=\pi-u} = 0$. Given that $\Lambda(b)|_{b=\pi-u}$ is strictly convex in δ, there exists a unique threshold $\underline{\delta}^r \in (0,1)$ such that $\Lambda(b)|_{b=\pi-u} \geq 0$ if and only if $\delta \geq \underline{\delta}^r$, where $\Lambda(b)|_{b=\pi-u} = 0$ if and only if $\delta = \underline{\delta}^r$. The threshold $\underline{\delta}^r$ is given by

$$\underline{\delta}^r \triangleq \frac{\pi(\phi+N-1)+u(1-\phi)+k}{2[k+\phi(\pi-u)]} - \frac{\sqrt{[\pi(\phi+N-1)+u(1-\phi)+k]^2 - 4\pi(N-1)[k+\phi(\pi-u)]}}{2[k+\phi(\pi-u)]}, \tag{A12}$$

where $\underline{\delta}^r > \underline{\delta}^c$ (see (A6)). Differentiating $\Lambda(\cdot)$ in (A9) with respect to b yields

$$\frac{\partial \Lambda}{\partial b} = 1 + \delta\phi - \frac{1}{1-\delta}.$$

As $\Lambda(\cdot)$ is linear in b, it follows from the intermediate value theorem that there exists a non-empty interval for b such that $\Lambda(b) \geq 0$ if and only if $\delta \geq \underline{\delta}^r$, where $\underline{\delta}^r$ is defined by (A12). In this case, $\Lambda(\cdot)$ increases with b. Deriving the unique solution to $\Lambda(b) = 0$ with respect to b, we obtain

$$b_d^{rm} \triangleq \frac{(1-\delta)(N\pi-\delta k)-\pi}{\delta[\phi(1-\delta)-1]}. \tag{A13}$$

Note from (A11) and (A12) that $\overline{\delta}^r \geq \underline{\delta}^r$ if and only if $\phi \geq \overline{\phi}^r$, where

$$\overline{\phi}^r \triangleq \frac{N\pi - k + \sqrt{(N\pi-k)^2 + 4k\pi}}{2\pi}.$$

Then, we find that, for $\delta \in \left[\underline{\delta}^r, \overline{\delta}^r\right)$ and $\phi \geq \overline{\phi}^r$, in the collusive equilibrium, the amount of debt is b_d^{rm} in (A13), where $b_d^{rm} > 0$, and the profit share is $\alpha_d^{rm} = u/\left(\pi - b_d^{rm}\right) \in \left(\alpha_{nd}^{rm}, 1\right)$. It holds $\alpha_d^{rm} < 1$ if and only if $u < \overline{u}^{rm}$, where

$$\overline{u}^{rm} \triangleq \frac{(1-\delta)[(1+\delta\phi-N)\pi+\delta k]}{\delta[\phi(1-\delta)-1]}. \tag{A14}$$

Substituting the binding collusion incentive constraint (2) into (A9) yields $\Lambda\left(b_d^c\right) < 0$. As $\Lambda(\cdot)$ increases with b, we have $b_d^{rm} > b_d^c$ and thus $\alpha_d^{rm} > \alpha_d^c$.

In the remaining region of parameters, where either $\delta < \underline{\delta}^r$ or $\delta \in \left[\underline{\delta}^r, \overline{\delta}^r\right)$ and $\phi < \overline{\phi}^r$, the renegotiation proofness constraint (12) fails to hold and thus the collusive monopoly outcome is not sustainable. ■

Proof of Lemma 3. Consider a candidate equilibrium contractual pair (α, b), with $b > 0$, announced in period $\tau = 0$ to support collusion. This yields the discounted collusive profit $V(\alpha, b)$ in (9). Now, suppose that a firm's shareholders offer the exclusive lender the reimbursement of the entire loan in period $\tau = 1$ and provide their manager with a new contract.

The shareholders' maximum profit is given by

$$V^o(\alpha, b) \triangleq \max_{\alpha^o \in [0,1]} \left\{ (1-\alpha^o) \left(N\pi - \frac{b}{1-\delta} \right) : \alpha^o \left(N\pi - \frac{b}{1-\delta} \right) \geq \frac{\alpha}{1-\delta} (\pi - b) \right\}, \quad \text{(A15)}$$

where the constraint ensures that the manager is willing to accept the new contract and deviates in the product market. As the shareholders' objective function in (A15) decreases with α^o and thus the associated constraint is binding, we find that

$$\alpha^o = \frac{\alpha(\pi - b)}{(1-\delta) N\pi - b}.$$

Substituting the expression for α^o into $V^o(\alpha, b)$ in (A15) and using $V(\alpha, b)$ in (9) yields

$$V(\alpha, b) - V^o(\alpha, b) = \frac{1 - (1-\delta) N}{1-\delta} \pi < 0,$$

where the inequality follows from Assumption 1. Then, the pair (α, b) is not renegotiation proof and cannot be supported in equilibrium. ■

Proof of Proposition 4. We know from the renegotiation proofness constraint (13) that a debt contract is renegotiation proof if and only if

$$b \geq b_d^{rd} \triangleq \frac{(1-\delta) N - 1}{N-1} \pi, \quad \text{(A16)}$$

where $b_d^{rd} > 0$ follows from Assumption 1. Recall from Lemma 2 that a managerial contract is renegotiation proof when the renegotiation proofness constraint (12) is satisfied. In light of the proof of Proposition 3, we find that, for $\delta \geq \overline{\delta}^r$, in the collusive equilibrium, the amount of debt is $b_d^{rd} > 0$. Using (6), the profit share is $\alpha_d^{rd} = u / \left(\pi - b_d^{rd} \right) \in \left(\alpha_{nd}^{rm}, 1 \right)$. It holds $\alpha_d^{rd} < 1$ if and only if $u < \overline{u}^r$, where $\overline{u}^r$ corresponds to

$$\overline{u}^{rd} \triangleq \frac{\delta \pi N}{N-1}. \quad \text{(A17)}$$

Now, suppose that $\delta < \overline{\delta}^r$. It follows from the proof of Proposition 3 that, for $\delta \in \left[\underline{\delta}^r, \overline{\delta}^r \right)$ and $\phi \geq \overline{\phi}^r$, in the collusive equilibrium, the amount of debt is $b_d^r = \max \left\{ b_d^{rd}, b_d^{rm} \right\}$ and the profit share is $\alpha_d^r = \max \left\{ \alpha_d^{rd}, \alpha_d^{rm} \right\}$. It holds $\alpha_d^r < 1$ if and only if $u < \overline{u}^r$, where $\overline{u}^r$ corresponds to $\overline{u}^{rd}$ in (A17) for $b_d^r = b_d^{rd}$ and $\overline{u}^r$ corresponds to $\overline{u}^{rm}$ in (A14) for $b_d^r = b_d^{rm}$. Using (A9), we have $\Lambda \left(b_d^{rd} \right) > 0$ if and only if

$$\phi > \widetilde{\phi}^r \triangleq \frac{\delta + N - 1}{\delta (1-\delta)} - \frac{(N-1) k}{\pi \left[(1-\delta) N - 1 \right]}.$$

As $\Lambda(\cdot)$ increases with b whenever it admits a non-empty interval for b such that $\Lambda(b) \geq 0$ (see the proof of Proposition 3), we find that, for $\delta \in \left[\underline{\delta}^r, \overline{\delta}^r \right)$ and $\phi > \widetilde{\phi}^r$, it holds $b_d^r = b_d^{rd} > b_d^{rm}$ and thus $\alpha_d^r = \alpha_d^{rd} > \alpha_d^{rm}$.

In the remaining region of parameters, where either $\delta < \underline{\delta}^r$ or $\delta \in \left[\underline{\delta}^r, \overline{\delta}^r \right)$ and $\phi < \overline{\phi}^r$, the renegotiation proofness constraint (12) fails to hold and thus the collusive monopoly outcome is not sustainable. ■

Proof of Proposition 6. As the shareholders' objective function in (15) decreases with $\widehat{\alpha}$ and thus the associated constraint is binding, we obtain that

$$\widehat{\alpha} = \frac{u + \delta C(b)}{N\pi - b}.$$

Substituting the expression for $\widehat{\alpha}$ into $\widehat{V}(b)$ in (15) and using $V(\alpha, b)$ in (9), we find that the reneging profness constraint (16) can be rewritten as

$$\frac{(1-\alpha)(\pi - b)}{1-\delta} - (N\pi - b) + u + \delta C(b) \geq 0. \tag{A18}$$

The shareholders' objective is to maximize $V(\alpha, b)$ in (9) subject to the reneging proofness constraint (A18). As a lower α increases $V(\alpha, b)$ and relaxes the reneging proofness constraint (A18), the managerial participation constraint (5) is binding in equilibrium, i.e., $\alpha = \min\{1, u/(\pi - b)\}$. In the subsequent analysis, we consider $\alpha = u/(\pi - b)$. At the end of the proof, we show that this occurs for u small enough. Define

$$\Upsilon(b) \triangleq \frac{\pi - b - u}{1-\delta} - (N\pi - b) + u + \delta C(b), \tag{A19}$$

where $C(b)$ is given by (1). As $\alpha = u/(\pi - b)$, the reneging proofness constraint (A18) holds if and only if $\Upsilon(b) \geq 0$. Note that $V(\alpha, b)$ in (9) coincides with the shareholders' objective function in (7) and $V(\alpha, b)$ decreases with b. Evaluating $\Upsilon(b)$ in (A19) at $b = 0$ (which implies $C(b) = 0$) yields

$$\Upsilon(b)|_{b=0} = \frac{\pi - \delta u}{1-\delta} - N\pi < 0,$$

where the inequality follows from Assumption 1. Furthermore, we have

$$\lim_{b \to 0^+} \Upsilon(b) = \frac{\pi - \delta u}{1-\delta} - N\pi + \delta k. \tag{A20}$$

As $\lim_{b\to 0^+} \Upsilon(b) = -\pi(N-1) < 0$ for $\delta \to 0$ and $\lim_{b\to 0^+} \Upsilon(b) = +\infty$ for $\delta \to 1$, it follows from the intermediate value theorem that there exists at least a value for $\delta \in (0,1)$ such that $\lim_{b\to 0^+} \Upsilon(b) = 0$. Given that $\lim_{b\to 0^+} \Upsilon(b)$ increases with δ, there exists a unique threshold $\overline{\delta}^{r\prime} \in (0,1)$ such that $\lim_{b\to 0^+} \Upsilon(b) \geq 0$ if and only if $\delta \geq \overline{\delta}^{r\prime}$, where $\lim_{b\to 0^+} \Upsilon(b) = 0$ if and only if $\delta = \overline{\delta}^{r\prime}$. The threshold $\overline{\delta}^{r\prime}$ is given by

$$\overline{\delta}^{r\prime} \triangleq \frac{N\pi + k - u - \sqrt{(N\pi + k - u)^2 - 4k\pi(N-1)}}{2k}, \tag{A21}$$

where $\overline{\delta}^{r\prime} \in \left(\overline{\delta}^{r}, (N-1)/N\right)$ (see (A11)). Then, we find that, for $\delta \geq \overline{\delta}^{r\prime}$, in the collusive equilibrium, the amount of debt is $b_{nd}^{rm\prime} \to 0$ and the profit share is $\alpha_{nd}^{rm\prime} \to u/\pi \in (0,1)$.

Now, suppose that $\delta < \overline{\delta}^{r\prime}$. Note from (A20) that $\lim_{b\to 0^+} \Upsilon(b) < 0$ (as $\delta < \overline{\delta}^{r\prime}$). We find from (A19) that $\Upsilon(b)|_{b=\pi-u} = \delta k - (N-1)\pi + \delta\phi(\pi - u)$, where $b = \pi - u$ is the highest value for b ensuring $V(\alpha, b) \geq 0$. It holds $\Upsilon(b)|_{b=\pi-u} \geq 0$ if and only if $\delta \geq \underline{\delta}^{r\prime}$, where the threshold $\underline{\delta}^{r\prime}$ is given by

$$\underline{\delta}^{r\prime} \triangleq \frac{(N-1)\pi}{k + \phi(\pi - u)}, \tag{A22}$$

where $\underline{\delta}^{r\prime} > \underline{\delta}^{r}$ (see (A12)). Differentiating $\mathrm{Y}(\cdot)$ in (A19) with respect to b yields

$$\frac{\partial \mathrm{Y}}{\partial b} = \delta\phi - \frac{\delta}{1-\delta}.$$

As $\mathrm{Y}(\cdot)$ is linear in b, it follows from the intermediate value theorem that there exists a non-empty interval for b such that $\mathrm{Y}(\cdot) \geq 0$ if and only if $\delta \geq \underline{\delta}^{r\prime}$, where $\underline{\delta}^{r\prime}$ is defined by (A22). In this case, $\mathrm{Y}(\cdot)$ increases with b. The unique solution to $\mathrm{Y}(b) = 0$ with respect to b writes as

$$b_d^{rm\prime} \triangleq \frac{(1-\delta)(N\pi - \delta k) - \pi + \delta u}{\delta[\phi(1-\delta) - 1]}. \tag{A23}$$

It follows from (A21) and (A22) that $\overline{\delta}^{r\prime} \geq \underline{\delta}^{r\prime}$ if and only if $\phi \geq \overline{\phi}^{r\prime}$, where

$$\overline{\phi}^{r\prime} \triangleq \frac{N\pi - k - u + \sqrt{(N\pi - k - u)^2 + 4k(\pi - u)}}{2(\pi - u)}.$$

Then, we find that, for $\delta \in \left[\underline{\delta}^{r\prime}, \overline{\delta}^{r\prime}\right)$ and $\phi \geq \overline{\phi}^{r\prime}$, in the collusive equilibrium, the amount of debt is $b_d^{rm\prime}$ in (A23), where $b_d^{rm\prime} > 0$, and the profit share is $\alpha_d^{rm\prime} = u/\left(\pi - b_d^{rm\prime}\right) \in \left(\alpha_{nd}^{rm\prime}, 1\right)$. A comparison between (A13) and (A23) yields $b_d^{rm\prime} > b_d^{rm}$. Consequently, it holds $\alpha_d^{rm\prime} > \alpha_d^{rm}$. Now, we show that $\alpha_d^{rm\prime} < 1$ when u is small enough. Applying the implicit function theorem to $\mathrm{Y}(b) = 0$, where $\mathrm{Y}(b)$ is defined by (A19), we obtain that

$$\frac{\partial b_d^{rm\prime}}{\partial u} = -\left.\frac{\partial \mathrm{Y}/\partial u}{\partial \mathrm{Y}/\partial b}\right|_{b=b_d^{rm\prime}} > 0,$$

where the inequality follows from $\partial \mathrm{Y}/\partial u|_{b=b_d^{rm\prime}} = -\delta/(1-\delta) < 0$ and $\partial \mathrm{Y}/\partial b|_{b=b_d^{rm\prime}} = \delta\phi - \delta/(1-\delta) > 0$ (as $\mathrm{Y}(\cdot)$ increases with b). This implies that $\alpha_d^{rm\prime} = u/\left(\pi - b_d^{rm\prime}\right)$ increases with u and $\alpha_d^{rm\prime} < 1$ if and only if $u < \overline{u}^{rm\prime}$, where

$$\overline{u}^{rm\prime} \triangleq \frac{\pi(1 + \delta\phi - N) + \delta k}{\delta\phi}. \tag{A24}$$

In the remaining region of parameters, where either $\delta < \underline{\delta}^{r\prime}$ or $\delta \in \left[\underline{\delta}^{r\prime}, \overline{\delta}^{r\prime}\right)$ and $\phi < \overline{\phi}^{r\prime}$, the reneging proofness constraint (A18) fails to hold and thus the collusive monopoly outcome is not sustainable. ■

Proof of Proposition 7. We know from the proof of Proposition 4 that a firm's shareholders prefer not to renegotiate a debt contract if and only if $b \geq b_d^{rd}$, where b_d^{rd} is given by (A16). Recall from the proof of Proposition 6 that a firm's shareholders cannot benefit from reneging upon a managerial contract when the reneging proofness constraint (A18) is satisfied. Then, for $\delta \geq \overline{\delta}^{r\prime}$, in the collusive equilibrium, the amount of debt is b_d^{rd} in (A16), where $b_d^{rd} > 0$, and the profit share is $\alpha_d^{rd} = u/\left(\pi - b_d^{rd}\right) \in \left(\alpha_{nd}^{rm\prime}, 1\right)$. It holds $\alpha_d^{rd} < 1$ if and only if $u < \overline{u}^{r\prime}$, where $\overline{u}^{r\prime}$ corresponds to $\overline{u}^{rd}$ in (A17).

Now, suppose that $\delta < \overline{\delta}^{r\prime}$. It follows from the proof of Proposition 6 that, for $\delta \in \left[\underline{\delta}^{r\prime}, \overline{\delta}^{r\prime}\right)$ and $\phi \geq \overline{\phi}^{r\prime}$, in the collusive equilibrium, the amount of debt is $b_d^{r\prime} = \max\left\{b_d^{rd}, b_d^{rm\prime}\right\}$ and the profit share is $\alpha_d^{r\prime} = \max\left\{\alpha_d^{rd}, \alpha_d^{rm\prime}\right\}$. It holds $\alpha_d^{r\prime} < 1$ if and only if $u < \overline{u}^{r\prime}$, where $\overline{u}^{r\prime}$ corresponds to $\overline{u}^{rd}$ in (A17) for $b_d^{r\prime} = b_d^{rd}$ and $\overline{u}^{r\prime}$ corresponds to $\overline{u}^{rm\prime}$ in (A24) for $b_d^{r\prime} = b_d^{rm\prime}$.

Using (A19), we have $\mathrm{Y}\left(b_d^{rd}\right) > 0$ if and only if

$$\phi > \widetilde{\phi}^{r\prime} \triangleq \frac{\delta + N - 1}{\delta\left(1-\delta\right)} + \frac{\left(N-1\right)\left[u - k\left(1-\delta\right)\right]}{\pi\left(1-\delta\right)\left[\left(1-\delta\right)N - 1\right]}.$$

As $\mathrm{Y}\left(\cdot\right)$ increases with b whenever it admits a non-empty interval for b such that $\mathrm{Y}\left(b\right) \geq 0$ (see the proof of Proposition 6), we find that, for $\delta \in \left[\underline{\delta}^{r\prime}, \overline{\delta}^{r\prime}\right)$ and $\phi > \widetilde{\phi}^{r\prime}$, it holds $b_d^{r\prime} = b_d^{rd} > b_d^{rm\prime}$ and thus $\alpha_d^{r\prime} = \alpha_d^{rd} > \alpha_d^{rm\prime}$.

In the remaining region of parameters, where either $\delta < \underline{\delta}^{r\prime}$ or $\delta \in \left[\underline{\delta}^{r\prime}, \overline{\delta}^{r\prime}\right)$ and $\phi < \overline{\phi}^{r\prime}$, the reneging proofness constraint (A18) fails to hold and thus the collusive monopoly outcome is not sustainable. ■

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Collusion through debt and managers
Supplementary Appendix

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1 Introduction

This Supplementary Appendix complements the paper. It extends our model to various forms of market structure and collects the associated proofs.

2 Market structure

Throughout the paper, we examine a standard Bertrand framework where firms sell a homogeneous good and compete in prices. A firm that defects from the collusive agreement is able to collect the entire industry profit, whereas in the punishment phase all firms revert to the competitive equilibrium by setting prices at the marginal cost and making zero profits, which implies that loans cannot be reimbursed and bankruptcy arises. What is the impact of the market structure on the sustainability of collusion? How do the magnitude of deviation profits and the intensity of competition affect the shareholders' propensity to use debt and managerial incentives for anticompetitive purposes?

To address these issues, in the spirit of Harrington and Chang (2009), we extend our model to a framework in which a firm's deviation profit is $\eta\pi$, where $\eta > 1$ captures the degree of demand elasticity, and a firm's per period punishment profit amounts to $\gamma\pi$, where $\gamma \in [0,1)$ is an inverse measure of the intensity of competition (for instance, driven by the degree of product differentiation).[1] It is well-known that, when firms obtain positive profits even in the absence of collusion, the strongest credible punishment could be implemented through a carrot-and-stick strategy that involves a punishment phase where firms' profits are lower than under competition, after which firms start colluding again. In our setting, however, punishment must drive firms into bankruptcy for managers to incur the bankruptcy costs and debt to sustain collusion. This removes the possibility of reverting to collusive phase and thus prevents the adoption of strategies that leads to punishment profits below the competitive level.

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[1] This setting reduces to our baseline model à la Bertrand for $\eta = N$ and $\gamma = 0$. A linear Cournot model with the collusive price at the monopoly level arises for $\eta = (N+1)^2/4N$ and $\gamma = 4N/(N+1)^2$.

Faced with a debt repayment b, a firm receives $\pi - b$ in any collusive period and $\eta\pi - b$ in the deviation period. A firm goes bankrupt in the punishment phase if and only if the debt repayment is larger than the punishment profit, i.e., $b > \gamma\pi$. This brings the manager's outside option to zero and engenders the managerial bankruptcy costs $C(b) = [k + \phi(b - \gamma\pi)] \cdot \mathbf{1}_b$, where the indicator function $\mathbf{1}_b \in \{0, 1\}$ assumes a value of one if $b > \gamma\pi$ and a value of zero otherwise. As in the baseline model, we consider a setting where a firm's shareholders are able to commit to debt and managerial contracts. The incentive constraint that ensures the sustainability of collusion writes as

$$\frac{\alpha}{1-\delta}(\pi - b) \geq \alpha\left[\eta\pi - b + \frac{\delta}{1-\delta}\max\{\gamma\pi - b, 0\}\right] - \delta C(b), \tag{S1}$$

where managers obtain zero remuneration during the punishment phase in case of bankruptcy that occurs if and only if the debt repayment exceeds the punishment profit, i.e., $b > \gamma\pi$. Without debt and bankruptcy costs, i.e., $b = C(b) = 0$, collusion is sustainable if and only if $\delta \geq (\eta - 1)/(\eta - \gamma)$. We impose the following assumption.

Assumption 2 $\delta < (\eta - 1)/(\eta - \gamma)$.

Assumption 2 is a natural extension of Assumption 1 of the paper and guarantees that the discount factor is not excessively large so that collusion cannot be sustained when managers do not incur bankruptcy costs.

To gain some insights, we provide the following result.

Lemma 1 *In the collusive equilibrium, the debt repayment must exceed the firm's punishment profit — i.e., $b > \gamma\pi$.*

Intuitively, a necessary condition for the sustainability of collusion is that managers incur bankruptcy costs after a deviation, which requires that a firm is not able to reimburse its debt in the punishment phase.[2]

In light of the result in Lemma 4, we focus hereafter on the case $b > \gamma\pi$. The collusion incentive constraint (S1) becomes

$$\frac{\alpha}{1-\delta}(\pi - b) \geq \alpha(\eta\pi - b) - \delta[k + \phi(b - \gamma\pi)]. \tag{S2}$$

As in the baseline model, the debt structure generates a trade-off in terms of collusion. A higher level of debt makes managers keener to deviate, because the consequential default in the punishment phase cancels the reimbursement of the residual debt. However, managerial bankruptcy costs increase with the amount of debt. Interestingly, the magnitude of this trade-off varies with the market structure. In particular, softer competition (i.e., a higher γ) reduces the severity of financial distress, which translates into lower managerial bankruptcy costs. Furthermore, a more elastic demand (i.e., a higher η) improves the firm's deviation profit. Both effects make the collusion incentive constraint (S2) more stringent. This suggests that in

[2]The proof of Lemma 4 is omitted. It directly follows from the observation that for $b \leq \gamma\pi$ the collusion incentive constraint (S1) fails to hold under Assumption 2.

markets with softer competition and higher elasticity of demand firms' shareholders should rely to a greater extent on the amount of debt and managerial incentives for anticompetitive purposes. The following proposition substantiates our claims.

Proposition 1 *Consider a market structure with intensity of competition $\gamma \in [0,1)$ and demand elasticity $\eta > 1$. Suppose that firms' shareholders are able to commit to debt and managerial contracts. Then, for a relatively small managerial outside option — i.e., $u < \overline{u}^{cs}$ — there exist two thresholds $\overline{\delta}^{cs}$ and $\underline{\delta}^{cs}$ for the discount factor δ such that*

(i) if $\delta \geq \overline{\delta}^{cs}$, in the collusive equilibrium, firms' shareholders issue some debt and provide managerial incentives — i.e., $b_d^{cs} \to \gamma\pi$ and $\alpha_d^{cs} \in (0,1)$;

(ii) if $\delta \in \left[\underline{\delta}^{cs}, \overline{\delta}^{cs}\right)$, in the collusive equilibrium, firms' shareholders issue a higher amount of debt and provide higher-powered managerial incentives when the responsiveness of managerial bankruptcy costs to the severity of financial distress is large enough — i.e., $b_{hd}^{cs} > \gamma\pi$ and $\alpha_{hd}^{cs} \in \left(\alpha_d^{cs}, 1\right)$ for $\phi \geq \overline{\phi}^{cs}$;

(iii) otherwise — i.e., either if $\delta < \underline{\delta}^{cs}$ or if $\delta \in \left[\underline{\delta}^{cs}, \overline{\delta}^{cs}\right)$ and $\phi < \overline{\phi}^{cs}$ — the collusive monopoly outcome is not sustainable.

Proposition 8 indicates that shareholders can still combine debt and managerial incentives to achieve collusion in product markets with structural features that differ from the standard Bertrand setting. As implied by Lemma 4, debt is now used for anticompetitive purposes even with a relatively large discount factor. Shareholders can sustain collusion for intermediate values of the discount factor by inflating debt and managerial incentives, provided that managerial bankruptcy costs are sufficiently responsive to the severity of financial distress. Based on the results in Proposition 8, we conduct the following comparative statics analysis.

Corollary 1 *The debt repayment b_d^{cs} and the profit share α_d^{cs} characterized in Proposition 8 decrease with the intensity of competition and increase with the degree of demand elasticity — i.e., $\partial b_d^{cs}/\partial\gamma > 0$, $\partial b_d^{cs}/\partial\eta > 0$, and $\partial\alpha_d^{cs}/\partial\gamma > 0$, $\partial\alpha_d^{cs}/\partial\eta > 0$.*

As the discussion after Lemma 4 suggests, softer competition (i.e., a higher γ) reduces managerial bankruptcy costs, which leads shareholders to increase the level of debt for anticompetitive purposes. Analogously, a more elastic demand (i.e., a higher η) magnifies the deviation profit and thus collusion can only be supported through an expansion of debt. This translates into a higher profit share in order to ensure managers' participation.

We now present the proofs of Proposition 8 and Corollary 1.

Proof of Proposition 8. Defining

$$\Psi(\alpha, b) \triangleq \frac{\alpha}{1-\delta}(\pi - b) - \alpha(\eta\pi - b) + \delta[k + \phi(b - \gamma\pi)] \tag{S3}$$

the collusion incentive constraint (S2) is satisfied if and only if $\Psi(\alpha, b) \geq 0$. Given that for $\alpha = 0$ it holds $\Psi(\alpha, b) \geq 0$, we focus hereafter on $\alpha > 0$. Note from (S3) that $\Psi(\alpha, b) \to -\alpha(\eta - 1)\pi < 0$ for $\delta \to 0$ and $\Psi(\alpha, b) \to +\infty$ for $\delta \to 1$. It follows from the intermediate

value theorem that there exists at least a value for $\delta \in (0,1)$ such that $\Psi(\alpha, b) = 0$. Differentiating $\Psi(\cdot,\cdot)$ in (S3) with respect to δ yields

$$\frac{\partial \Psi}{\partial \delta} = \alpha \frac{\pi - b}{(1-\delta)^2} + k + \phi(b - \gamma\pi) > 0,$$

where $b > \gamma\pi$ by Lemma 4. Then, there exists a unique threshold $\delta^{*s}(\alpha, b) \in (0,1)$ such that $\Psi(\alpha, b) \geq 0$ if and only if $\delta \geq \delta^{*s}(\alpha, b)$, where $\Psi(\alpha, b) = 0$ if and only if $\delta = \delta^{*s}(\alpha, b)$. Applying the implicit function theorem to $\Psi(\alpha, b) = 0$ yields

$$\frac{\partial \delta^{*s}}{\partial \alpha} = -\left.\frac{\partial \Psi / \partial \alpha}{\partial \Psi / \partial \delta}\right|_{\delta = \delta^{*s}} = \frac{(1-\delta^{*s})\left[(1-\delta^{*s})(\eta\pi - b) - (\pi - b)\right]}{\alpha(\pi - b) + (1-\delta^{*s})^2\left[k + \phi(b - \gamma\pi)\right]} > 0,$$

where the inequality follows from the positive sign of the expression in square brackets (as $\Psi(\alpha, b) = 0$). Then, a lower α relaxes the collusion incentive constraint (S2). As the shareholders' objective function in (4) of the paper decreases with α, the managerial participation constraint (5) of the paper is binding in equilibrium, i.e., $\alpha = \min\{1, u/(\pi - b)\}$. Suppose that $\alpha = u/(\pi - b)$. At the end of the proof, we show that this is the case for u small enough. Thus, $\Psi(\alpha, b)$ in (S3) becomes

$$\Psi(b) = \frac{u}{1-\delta} - \frac{\eta\pi - b}{\pi - b}u + \delta\left[k + \phi(b - \gamma\pi)\right]. \tag{S4}$$

Note that the shareholders' objective function in (4) of the paper decreases with b. Furthermore, it follows from Lemma 4 that in the collusive equilibrium the lowest value for b is $b \to \gamma\pi$, which yields

$$\lim_{b \to \gamma\pi} \Psi(b) = \frac{u}{1-\delta} - \frac{\eta - \gamma}{1 - \gamma}u + \delta k. \tag{S5}$$

As $\lim_{b\to\gamma\pi} \Psi(b) = -u(\eta - 1)/(1 - \gamma) < 0$ for $\delta \to 0$ and $\lim_{b\to\gamma\pi} \Psi(b) = +\infty$ for $\delta \to 1$, it follows from the intermediate value theorem that there exists at least a value for $\delta \in (0,1)$ such that $\lim_{b\to\gamma\pi} \Psi(b) = 0$. Given that $\lim_{b\to\gamma\pi} \Psi(b)$ is strictly convex in δ, there exists a unique threshold $\overline{\delta}^{cs} \in (0,1)$ such that $\lim_{b\to\gamma\pi} \Psi(b) \geq 0$ if and only if $\delta \geq \overline{\delta}^{cs}$, where $\lim_{b\to\gamma\pi} \Psi(b) = 0$ if and only if $\delta = \overline{\delta}^{cs}$. The threshold $\overline{\delta}^{cs}$ is given by

$$\overline{\delta}^{cs} \triangleq \frac{k(1-\gamma) + u(\eta - \gamma) - \sqrt{\left[k(1-\gamma) + u(\eta - \gamma)\right]^2 - 4ku(1-\gamma)(\eta - 1)}}{2k(1-\gamma)},$$

where $\overline{\delta}^{cs} < (\eta - 1)/(\eta - \gamma)$. Then, for $\delta \geq \overline{\delta}^{cs}$, in the collusive equilibrium, we find that the amount of debt is $b_d^{cs} \to \gamma\pi$ and the profit share is $\alpha_d^{cs} \to u/[(1-\gamma)\pi] \in (0,1)$, where $\alpha_d^{cs} < 1$ if and only if $u < \overline{u}^{cs} \triangleq (1-\gamma)\pi$.

Now, suppose that $\delta < \overline{\delta}^{cs}$. Note from (S5) that $\lim_{b\to\gamma\pi} \Psi(b) < 0$ (as $\delta < \overline{\delta}^{cs}$). Furthermore, differentiating $\Psi(\cdot)$ in (S4) with respect to b yields

$$\frac{\partial \Psi}{\partial b} = \delta\phi - \frac{(\eta - 1)\pi u}{(\pi - b)^2} \text{ and } \frac{\partial^2 \Psi}{\partial b^2} = -\frac{2(\eta - 1)\pi u}{(\pi - b)^3} < 0,$$

which implies that $\Psi(\cdot)$ is strictly concave in b. Hence, a necessary condition for $\Psi(\cdot) \geq 0$ is

$$\lim_{b \to \gamma\pi} \frac{\partial \Psi}{\partial b} = \delta\phi - \frac{(\eta - 1)\, u}{\pi (1-\gamma)^2} > 0.$$

This condition holds if and only if $\delta \geq \underline{\delta}^{cs}$, where the threshold $\underline{\delta}^{cs}$ is given by

$$\underline{\delta}^{cs} \triangleq \frac{(\eta - 1)\, u}{\phi\pi (1-\gamma)^2}.$$

The lowest solution to $\Psi(b) = 0$ with respect to b writes as

$$b_d^{cs} \triangleq \frac{(1-\delta)\left[\pi\phi(1+\gamma) - k\right] - u}{2(1-\delta)\phi} - \frac{\sqrt{\delta(1-\delta)^2\left[k + \pi\phi(1-\gamma)\right]^2 + 2u(1-\delta)\left[\delta k + \pi\phi(2 - \delta + 2\delta\eta - 2\eta - \gamma\delta)\right] + \delta u^2}}{2\sqrt{\delta}(1-\delta)\phi}. \tag{S6}$$

This solution exists if and only if $\phi \geq \overline{\phi}^{cs}$, where

$$\overline{\phi}^{cs} \triangleq \frac{2\eta(1-\delta) + \delta(1+\gamma) - 2}{\delta(1-\delta)(1-\gamma)^2\pi} u - \frac{k}{(1-\gamma)\pi} + \frac{2}{\delta(1-\delta)(1-\gamma)^2\pi}\sqrt{(1-\delta)(\eta-1)\, u\left[(1-\delta)\eta u + \delta\gamma u - u - \delta(1-\delta)(1-\gamma)k\right]}.$$

Then, we find that, for $\delta \in \left[\underline{\delta}^{cs}, \overline{\delta}^{cs}\right)$ and $\phi \geq \overline{\phi}^{cs}$, in the collusive equilibrium, the amount of debt is b_d^{cs} in (S6), where $b > \gamma\pi$, and the profit share is $\alpha_d^{cs} = u/\left(\pi - b_d^{cs}\right) \in \left(\alpha_{nd}^{cs}, 1\right)$. Now, we show that $\alpha_d^{cs} < 1$ when u is small enough. Applying the implicit function theorem to $\Psi(b) = 0$, where $\Psi(b)$ is given by (S4), yields

$$\frac{\partial b_d^{cs}}{\partial u} = -\left.\frac{\partial \Psi / \partial u}{\partial \Psi / \partial b}\right|_{b = b_d^{cs}} > 0,$$

where the inequality follows from $\partial\Psi/\partial u|_{b=b_d^{cs}} = 1/(1-\delta) - \left(\eta\pi - b_d^{cs}\right)/\left(\pi - b_d^{cs}\right) < 0$ and $\partial\Psi/\partial b|_{b=b_d^{cs}} = \delta\phi - (\eta - 1)\pi u/\left(\pi - b_d^{cs}\right)^2 > 0$, with b_d^{cs} defined by (S6). This implies that $\alpha_d^{cs} = u/\left(\pi - b_d^{cs}\right)$ increases with u and $\alpha_d^{cs} < 1$ if and only if $u < \overline{u}^{cs}$, where

$$\overline{u}^{cs} \triangleq \frac{(1-\delta)(\pi + \delta\phi\pi + \delta k - \eta\pi - \delta\phi\gamma\pi)}{\delta(\phi - \delta\phi - 1)}.$$

In the remaining region of parameters, where either $\delta < \underline{\delta}^{cs}$ or $\delta \in \left[\underline{\delta}^{cs}, \overline{\delta}^{cs}\right)$ and $\phi < \overline{\phi}^{cs}$, the collusive monopoly outcome is not sustainable. ■

Proof of Corollary 1. First, we consider the debt repayment b_d^{cs}. Recall from the proof of Proposition 8 that $\partial\Psi/\partial b|_{b=b_d^{cs}} > 0$, where $\Psi(b)$ is defined by (S4) and b_d^{cs} by (S6). Applying

the implicit function theorem to $\Psi(b) = 0$, we find that

$$\left.\frac{\partial \Psi}{\partial \gamma}\right|_{b=b_d^{cs}} = -\delta\phi\pi < 0 \implies \frac{\partial b_d^{cs}}{\partial \gamma} = -\left.\frac{\partial \Psi/\partial \gamma}{\partial \Psi/\partial b}\right|_{b=b_d^{cs}} > 0;$$

$$\left.\frac{\partial \Psi}{\partial \eta}\right|_{b=b_d^{cs}} = -\frac{\pi u}{\pi - b_d^{cs}} < 0 \implies \frac{\partial b_d^{cs}}{\partial \eta} = -\left.\frac{\partial \Psi/\partial \eta}{\partial \Psi/\partial b}\right|_{b=b_d^{cs}} > 0.$$

Now, we turn to the profit share α_d^{cs}. Recalling from the proof of Proposition 8 that $\alpha_d^{cs} = u/\left(\pi - b_d^{cs}\right)$, we find from the previous results that

$$\frac{\partial \alpha_d^{cs}}{\partial \gamma} = \frac{u}{\left(\pi - b_d^{cs}\right)^2}\frac{\partial b_d^{cs}}{\partial \gamma} > 0; \ \frac{\partial \alpha_d^{cs}}{\partial \eta} = \frac{u}{\left(\pi - b_d^{cs}\right)^2}\frac{\partial b_d^{cs}}{\partial \eta} > 0. \quad \blacksquare$$