Common ownership unpacked*

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Abstract

In this paper we study the market effects of common ownership in a setting where any ownership structure and any shareholder size is allowed. We depart from the standard reduced form approach of assuming that firms maximize a weighted average of shareholders' portfolios, and instead study the collective choice problem of shareholders head-on. In our model shareholder meetings elect firm managers by one-share one-vote majority rule. Managers differ in their degree of aversion to the negative externality of production. Voting for socially concerned managers therefore provides a mechanism for common owners to direct away the firm from own profit towards industry profit maximization. We show that allowing shareholders of any size to freely diversify their portfolio leads to monopolistic outcomes. Our results have the novel policy implication that the anticompetitive effects of common ownership can emerge even when blockholders are undiversified, but the majority of shares belongs to small diversified shareholders, indicating that small diversified portfolios may also be a threat.

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1 Introduction

Common ownership—investors simultaneously holding stocks of multiple firms which compete in the same industry—has been of interest for competition economists since the formulation of the "common ownership hypothesis" (see, e.g., Rotemberg, 1984, Bresnahan & Salop, 1986). The latter suggests that if firms maximize shareholder value and if shareholders also hold stakes in firms' rivals, firms may to some extent take into account the profits of their competitors in their objective function, which would result in softer competition. The hypothesis has gained a large deal of attention from both competition scholars and authorities worldwide in relation to the growth of institutional investors (e.g., index funds and exchange-traded funds). This is so as for the case of highly diversified institutional investors, holding an industry-wide portfolio of publicly-traded firms, the hypothesis would predict a particularly concerning outcome, namely that common ownership might lead to an industry-wide monopoly, with potentially enormous harm for consumers (see, e.g., Elhauge, 2016, Posner, Scott Morton, & Weyl, 2017, Scott Morton & Hovenkamp, 2018, Backus, Conlon, & Sinkinson, 2019, Hemphill & Kahan, 2020).[1]

Recently, a rich theoretical and empirical literature has developed investigating such anticompetitive effects, with Azar, Schmalz, and Tecu (2018), Azar and Vives (2021), and Antón, Ederer, Giné, and Schmalz (2023) being, arguably, among the most salient contributions (see, also, the literature reviews by Backus et al., 2019; Schmalz, 2018, 2021). However, despite these significant advances, the specific way in which common ownership shapes the firm's objective function in a setting where shareholders are heterogeneously diversified across firms remains, largely, unexplored.[2] For example, how can institutional investors influence corporate behavior despite being typically non controlling in terms of stock size and therefore unable to cast a decisive vote on any issue? Importantly, as diversified owners, such shareholders might want the manager of a firm to maximize the collective profits of the industry, while undiversified owners that only hold shares in that firm will instead want the manager to maximize the firm's own profits only. As different shareholders who are unevenly diversified across firms have different preferences, the determination of the firm's objective constitutes a collective choice problem.

In this paper we study the collective choice problem of shareholders and show that what matters for the anticompetitive effects of common ownership to emerge is not the size of diversified shareholders, but whether they collectively have control of the firm. This is important, as (institutional) investors might be *individually small but collectively large* in terms of stock size.[3]

[1]On the other hand, cross-industry common ownership might have pro-competitive effects (see, e.g., Azar & Vives, 2021 and Azar & Vives, 2022.)

[2]This lack of clarity is often put forward as the reason preventing a full-fledged regulatory action (see, e.g., OECD, 2017, Federal Trade Commission, 2018, European Commission, 2020, European Parliament, 2020.)

[3]Schmalz (2018) reports that institutional investors including the "big three" (BlackRock, Vanguard,

The literature on common ownership has so far acknowledged but largely abstracted away from this collective choice problem and considered the ad hoc assumption that a firm manager maximizes a control-weighted sum of the portfolios of firm's shareholders.[4] The control weights are increasing in the shareholder's stake in the firm, and they are quite disproportional in favor of large shareholders. Strikingly, this leads to the prediction that when ownership by diversified (resp. undiversified) shareholders gets sufficiently dispersed, the manager's objective function fully reflects the interests of the less dispersed undiversified (resp. diversified) blockholders, *even if the former group collectively owns an overwhelming majority of shares.*[5]

As recognized by a number of authors, this feature of the dominant formulation is often at odds with standard majoritarian corporate voting mechanisms (Gramlich & Grundl, 2017, O'Brien & Waehrer, 2017, Brito et al., 2023, Vravosinos, 2023). While it is plausible that blockholders have disproportionately more voting power than small shareholders, e.g., because they have larger stakes and therefore are more engaged in the election process, assuming that small shareholders have a cumulatively negligible effect on outcomes—especially when they hold a large majority of shares—seems to be a stretch and it is not supported by real world observations. For example, multiple cases are reported where individual activist shareholders were able to secure large percentages of votes in favor of their own governance proposals (Gordon & Pound, 1993) or encourage other shareholders to withhold votes toward a director's election to express discontent with management (Del Guercio, Seery, & Woidtke, 2008). In addition, retail investors have been successful in increasing their influence on governance decisions by forming coalitions to coordinate their voting (Strickland, Wiles, & Zenner, 1996, Gillan & Starks, 2000). In some instances the management itself has encouraged turnout of retail shareholders for corporate decisions where it needed more voting support on (Lee & Souther, 2020). There is also evidence that retail shareholders participate in voting even when their holding is so small that their vote would have a negligible impact on the outcome,

State Street) are among the top ten shareholders of all the largest US airlines. They typically hold less than 15% of the stock in each carrier individually but collectively hold between 39% and 55%. Similar patterns can be observed for US banks and supermarkets.

[4]This control-weighted formulation was developed by O'Brien and Salop (2000) based on previous work by Rotemberg (1984) and Bresnahan and Salop (1986) and has been used, explicitly or implicitly, by the bulk of literature including recent contributions by Azar et al. (2018), Newham, Seldeslachts, and Banal-Estañol (2018), López and Vives (2019), Backus, Conlon, and Sinkinson (2021a), Backus, Conlon, and Sinkinson (2021b), Azar, Raina, and Schmalz (2022), Azar and Ribeiro (2022), Banal-Estañol, Seldeslachts, and Vives (2022), and Antón et al. (2023). See Schmalz (2018) and Schmalz (2021) for a comprehensive list of older contributions.

[5]For example, when a large number of sufficiently small identical diversified shareholders collectively owns 95% of a firm while the remaining 5% of shares are owned by a single undiversified blockholder, the objective function of the firm will only reflect the interests of the blockholder despite an overwhelming majority of diversified shareholders. Analogously, when a large number of sufficiently small identical undiversified shareholders collectively owns 95% of a firm, while the remaining 5% of shares are owned by a single diversified blockholder, only the interests of the diversified blockholder will be reflected (see, e.g., Brito, Elhauge, Ribeiro, & Vasconcelos, 2023).

which is consistent with the presence of non-monetary benefits from voting (Brav, Cain, & Zytnick, 2022).

In addition to being questioned by existing evidence, the assertion of the standard approach that, in the presence of blockholders, the diversity of the portfolios of small shareholders does not matter, may also induce scholars and authorities to focus only on large diversified shareholders, while disregarding a majority of small diversified shareholders as a potential concern. This can lead to misguided policy prescriptions, for example, that breaking up in multiple small parts the large diversified investors or imposing a limit on their holdings in a given industry, would be sufficient for the anticompetitive forces of common ownership to fade away (see, e.g., Posner et al., 2017).

In this paper we address this issue and we relax the dominant assumption. We study the collective choice problem of shareholders in a standard majoritarian voting setting and allow for a wide dispersion or concentration of shares across individual shareholders. This permits us to study the market effects of common ownership in a setting where any ownership structure and any size of shareholder is allowed, and to establish that what matters for the monopoly-generating forces of common ownership to appear is the collective, rather than the individual, size of shareholders with common interests: a majority of infinitesimally small diversified owners will lead to anticompetitive outcomes.

In our model we consider a duopolistic industry with publicly traded firms and a large number of shareholders who are allowed to own different shares of each firm. Shareholders are represented by agents that participate and vote in shareholder meetings to elect firm managers by one-share one-vote majority rule. While shareholders only care about the value of their portfolio, managers differ in their alternative values, which we understand here, simply, as different degrees of aversion to the negative externalities induced to the society by firm production. For example, this can be interpreted as managers having different social responsibility concerns.[6] This feature is modeled as the manager bearing an individual private cost, which is increasing in the level of production, implying that the more socially concerned the manager is, the farther she will want to direct the firm away from the profit-maximizing output level. The characteristics of the managers are publicly observable (e.g., because of manager reputation). Once authority is delegated to a manager, her objective is to engage in quantity competition à la Cournot.

We show that shareholders have single-peaked preferences over manager types, which implies that a unique manager type will collect the majority of votes and emerge as

[6]There is evidence that management can have alternative values that influence its decisions, beyond the goal of maximizing firm profits (see, e.g., Cespa & Cestone, 2007, Bénabou & Tirole, 2010). Indeed, as noted by Goran Lindahl, ABB's group executive vice president, *"In the end, managers are loyal not to a particular boss or even to a company but to a set of values they believe in and find satisfying."* (Bartlett & Ghoshal, 1994). In reality, shareholders might also have values beyond portfolio value maximization (see, e.g., Hart & Zingales, 2017, Broccardo, Hart, & Zingales, 2022, Oehmke & Opp, 2023), which could be reflected in managements' choices. Assuming that shareholders only care about profits is a conservative modelling choice, since additional concerns might give rise to anticompetitive forces in rather direct ways.

the (Condorcet) winner in each firm. In addition, shareholders' ideal manager type is increasing in the degree of diversification of their portfolio. Therefore, if the majority of votes is held by (sufficiently) diversified shareholders, in equilibrium managers with positive social concerns will be elected. The elected managers will therefore unilaterally choose lower output levels than the profit-maximizing ones, thereby internalizing the negative externality caused by one firm to another, which boosts industry-level profits.

We also show that common ownership arises in a competitive equilibrium, if we introduce a pre-stage in which initially undiversified shareholders are allowed to trade shares and, thus, endogenously determine the diversification of their portfolios. In equilibrium, shareholders choose to acquire equal interests in both firms, leading to symmetric full diversification, which implies that managers with high enough social concerns will be elected, resulting in the monopoly outcome.

Our results therefore provide a novel and relevant policy implication. Indeed, authorities traditionally fear that competition will be hindered by single agents controlling substantial shares in multiple companies of the same industry, while they do not perceive small shareholders as a threat. As we demonstrate in this paper, a majority of (infinitesimally) small diversified shareholders is sufficient to generate anti-competitive forces, even in presence of undiversified blockholders. For example, consider the different implications that our results would have relative to the standard approach in the context of the recent US reform on "pass-through" voting. This reform, in the attempt of reducing concentration of voting power among a few asset managers, allows investors of funds to vote directly on their shares rather than delegating their voting rights to fund managers (see, e.g., Malenko & Malenko, 2023). While the standard approach would predict that, indeed, the voting power of diversified shareholders would fade away because of dispersion, our setup predicts that a diluted majority of diversified investors will still be relevant.

We contribute to the literature on common ownership in several ways. Our main contribution is to depart from the reduced form approach and instead fully investigate the interconnected collective choice problems of the firms. Indeed, several studies (Azar, 2012; Azar, 2017; Brito, Elhauge, Ribeiro, & Vasconcelos, 2018; Moskalev, 2019; Brito et al., 2023) microfounded the standard formulation through probabilistic voting models. In these models candidates choose strategy proposals to maximize their vote share or their expected utility from corporate office and shareholders' utility depends not only on expected portfolio returns from proposed strategies but also from characteristics of the candidates. These assumptions—which were borrowed from political electoral competition models—not only essentially impose the existence of an equilibrium in the candidates' strategy formation stage but are not always in line with corporate governance mechanisms which rely on the majoritarian principle.[7] On the other hand, we model the collective

[7] Brito et al. (2023) also stress the limitations of the dominant formulation and make significant progress relative to the literature. They formally argue that if candidates' fixed characteristics are

choice problem of shareholders in a canonical way, i.e., instead of seeking a model to justify a given objective function, we start with standard underlying assumptions: we consider that shareholders simply care to maximize the value of their portfolio, and then make a collective choice following one-share-one-vote majority rule, which is the standard voting approach adopted in corporations to elect managers. We show that the problems of shareholders are well-behaved, i.e., that objective functions are quasi-concave in the decision variable (i.e., the manager type). This implies that a Condorcet winner manager type always exists and it is the one most preferred by the "median"—in terms of diversification of interests—shareholder. The resulting objective function of the firm will therefore reflect the interests of the majority.[8]

The second contribution of our analysis is to provide a novel mechanism through which common ownership arises and results in anticompetitive outcomes without the need for the manager to internalize portfolio considerations in its objective function. This is relevant, as the extent to which managers are willing or able to take into account shareholder portfolio interests in their choices might be importantly constrained by agency problems (e.g., managerial entrenchment) or legal constraints (e.g., fiduciary duty). The only paper we are aware of that acknowledges the importance of these constraints is Antón et al. (2023). The paper studies performance-sensitive managerial compensation schemes as the main channel and shows that, by designing low-powered incentives (or rather more passively, choosing not to design high-powered ones), shareholders induce managers (that are only responsive to their compensation scheme) to exert low levels of productivity-improving effort, which softens output competition. We propose an alternative mechanism based on voting for manager types, and we show that it is sufficient for the management to have alternative values in addition to profits (and these preferences being observable by the owners) for anticompetitive effects to emerge.[9]

Third, while some recent papers in the literature have studied endogenous portfolio

profit-relevant, rather than profit-irrelevant as assumed by the previous studies, then the probabilistic voting model can avoid the unintuitive result that infinitesimal shareholders have zero weight even when they collectively have the majority. However, any approach that relies on a probabilistic voting model is bound to clash with majoritarian principles, in the sense that the equilibrium objective function will assign a positive weight to non-infinitesimal diversified (resp. undiversified) shareholders even when non-infinitesimal undiversified (resp. diversified) shareholders control an overwhelming majority of shares. Vravosinos (2023) provides an alternative model of corporate control based on Nash bargaining rather than shareholder voting, which is shown to accommodate better the issue of ownership dispersion compared to the standard approach.

[8]Notice that in the example where 95% of shares are held by small undiversified owners and 5% of shares are held by a diversified blockholder, majority voting excludes that the objective function of the firm amounts to something other than own profits. This is because the median shareholder—the one that has at least 50% of votes on her left and on her right when shareholders are ordered according to their degree of diversification—would necessarily be an undiversified one. She would prefer a socially unconcerned manager which would maximize own-firm profits only.

[9]Antón et al. (2023) assume that in each firm there is a majority owner who designs the incentive contract for the manager, thereby avoiding the unintuitive results of the dominant formulation but yet abstracting from the collective choice problem as the rest of the literature.

choices and found that this leads to a worsening of anticompetitive effects, they have all done so employing the standard control weighted formulation. We extend this result, by showing that common ownership—and hence monopoly outcomes—may arise endogenously also when the firm's objective function is decided through a standard collective choice procedure.[10]

Finally, we contribute to the literature relating common ownership to corporate social responsibility (CSR) or environmental, social and governance (ESG) issues. A number of papers in the corporate law and finance literature studied the effects of common ownership on CSR and in turn the market effects.[11] However, papers that formalize these mechanisms are generally missing.[12] As far as we know the only attempt in this sense was made by Dai and Qiu (2020) which however still builds on the standard control-weighted approach, and considers a very different mechanism, where CSR investments are modeled as a weight on consumer surplus and used as a commitment device to expand output aggressively in the future. We provide a mechanism that explains what motivates profit-seeking, self-interested firms to embrace CSR goals. We show that higher portfolio diversification, i.e., a higher level of common ownership, renders CSR compliance more advantageous for shareholders and thus motivates them to incorporate CSR into firm's decision making. This suggests that common ownership may function as a self-regulating mechanism in favor of CSR. In this sense we contribute to the literature on the "bright side" of common ownership and on green antitrust.[13]

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 characterizes the equilibrium outcome for a given ownership structure. Section 4 endogenizes the choice of shareholdings. Section 5 provides a numerical example to

[10]Piccolo and Schneemeier (2021) and Hemphill and Kahan (2021) show that crowding out of undiversified investors occurs in equilibrium. In a setting with Bertrand competition with homogeneous products Bayona, López, and Manganelli (2022) study ownership configurations that can sustain monopoly pricing in equilibrium, and show that these structures can emerge as the solution of network formation or bargaining games among investors. Moreno and Petrakis (2022) focus on large investors and find equilibria with symmetric portofolios leading to monopolistic outcomes. In a simple Cournot setting, Papadopoulos (2022) shows that cross-ownership schemes can imitate and outperform any partial merger for gaining market power.

[11]For example, Condon (2020) attributes investor climate activism to the rise of common ownership by portfolio firms. In the fossil fuel industry, it has led to commitment to emissions reduction targets, discontinuance of political lobbying against greenhouse gas regulations and more disclosure of climate risk. Coffee Jr (2021) reports a higher demand for ESG disclosure due to common ownership. Choice of managers based on their CSR concerns in contexts without common ownership has been studied by Manasakis, Mitrokostas, and Petrakis (2014).

[12]Empirical contributions have been provided by Dyck, Lins, Roth, and Wagner (2019), Chen, Dong, and Lin (2020), Cheng, (Helen) Wang, and Wang (2022), and DesJardine, Grewal, and Viswanathan (2022).

[13]For example, Bayona & López, 2018, López & Vives, 2019, Anton, Ederer, Gine, & Schmalz, 2021 find that common ownership can stimulate innovation via spillover effects. However, as noticed by Schmalz (2018), this positive effect needs to be balanced against the negative anticompetive impact, so the net effect on social welfare is ambiguous. Schinkel and Spiegel (2017) and Schinkel, Spiegel, and Treuren (2022) find that while allowing a cartel on production decisions can increase CSR efforts, the welfare net effects can be negative. Assessing net welfare effects is beyond the scope of this paper.

illustrate the main results. Section 6 provides an extension for the case of more than two firms. Section 7 concludes.

2 The Model

Consider an industry with two publicly traded firms, indexed by $i = 1, 2$ that sell an homogeneous good and face inverse demand $p = a - \sum_i q_i$, where q_i is the quantity sold by firm i and p is the price of the good.[14] For simplicity, we normalize production costs to zero, so the profit of firm i is $\Pi_i(q_1, q_2) = (a - q_i - q_j)q_i, i, j = 1, 2, j \neq i$.

2.1 Shareholders

Let K be the set of shareholders, indexed by $k = 1, 2, .., |K|$. A shareholder $k \in K$ is characterized by a portfolio of shares $s^k = (s_1^k, s_2^k)$, where $s_i^k \in [0, 1]$, is a percentage of the total shares of firm $i = 1, 2$ that k owns, with $s_i^k > 0$ for at least one $i = 1, 2$.

Shareholders are only interested in maximizing their wealth, i.e., the value of their portfolio. For shareholder k this is formally defined as

$$V^k = s_1^k \Pi_1(q_1, q_2) + s_2^k \Pi_2(q_1, q_2) \tag{1}$$

The following definition introduces an indicator that captures the degree of diversification of shareholder k's portfolio.

Definition 1. *The relative interest of shareholder k in firm $i = 1$ is*

$$\sigma^k = \frac{s_1^k}{s_1^k + s_2^k}. \tag{2}$$

Therefore the relative interest of shareholder k in firm $i = 2$ is $1 - \sigma^k$. By construction, $\sigma^k \in [0, 1]$, for every $k \in K$ and each shareholder k is characterized by a unique σ^k.

It will be useful for the rest of the analysis to arrange all shareholders on the $[0, 1]$ line by defining an order on K and ranking them with respect to their relative interest in firm 1. Therefore, without loss of generality, let us assume $\sigma^1 \leq \sigma^2 \leq ... \leq \sigma^k \leq ... \leq \sigma^{|K|}$. The order is increasing from left to right, i.e., shareholders that are closer to 0 have less (resp. more) relative interest in firm 1 (resp. firm 2) than shareholders located closer to 1 and obviously if a shareholder has no interest in firm 1 (resp. firm 2), then she is located on the extreme point 0 (resp. 1).

[14] Various papers in the literature focus on Cournot competition with homogeneous goods (see, e.g., Azar & Vives, 2021, Vravosinos, 2023, and Vives & Vravosinos, 2023). Other papers focus on Bertrand competition with homogeneous goods (see, e.g., Bayona et al., 2022) or product differentiation (see, e.g., López & Vives, 2019, Antón et al., 2023.)

2.2 Managers

There is a continuum of manager types $m \in [0, M] \subset \Re_+$ that are available for hiring in the industry. The manager type represents their *social concern*, which we define as the manager's degree of aversion to externalities such as pollution, climate change, or inequality in the workplace that may be inevitably induced by production.[15] The manager types range from 0 that denotes "no concern" to M that denotes "maximum concern". Manager types are common knowledge to shareholders.[16] It is also common knowledge that if manager m_i is appointed to run firm i, she will choose output so that it maximizes her own utility function,

$$\max_{q_i} U^{m_i}(q_i) = \Pi_i(q_1, q_2) - \frac{m_i}{2} q_i^2. \tag{3}$$

The manager's objective function implies that she cares only about maximizing the product market profit of the firm she runs,[17] nevertheless she bears an individual cost $(m_i/2)q_i^2$ because of her social concern, which is not internalized by the firm. Notice that while the type of the manager could be negative, because the manager might value the positive externality of production (e.g., in terms of higher consumer surplus) more than its negative externality (e.g., in terms of pollution), incorporating negative manager types in the analysis offers very little additional insights. As it will soon be clear, when shareholders only care about the value of their portfolio, they will typically want the firm to produce either the profit maximizing level of output or a lower level, but not a higher level—so that in equilibrium managers with a negative type will not be chosen.[18]

2.3 Corporate Governance and Output Decisions

The output decision in each firm is taken by a manager who is elected by shareholders.[19]

[15]Therefore, in our model CSR efforts only amount to reducing the negative externality of production via a reduction of the output level. In other related works CSR is modelled as an additional choice variable for the firm, typically an investment that is valued by consumers (see e.g., Schinkel and Spiegel (2017), Dai and Qiu (2020), Schinkel et al. (2022)).

[16]We may think of the set of manager types as different persons that have developed a certain reputation or have made public announcements on CSR or ESG issues. Alternatively, a firm may build its own preferred manager profile and look it up in the market for managers. We assume that the market for managers is quite rich and any conceivable type is available for hiring.

[17]Unlike what is standard in the literature, we do not assume that a manager takes into account shareholders' portfolios and hence their interests in other firms according to a control-weighted objective function, or the interest of any dominant or majority shareholder. As mentioned, such dominant approach abstracts away from important aspects in corporate governance, such as agency problems, legal constraints, and collective choice.

[18]An alternative interpretation of manager type is in terms of manager efficiency, i.e., cost or benefit of managerial effort to increase production (similar to Antón et al., 2023). More generally, the type can represent any alternative concern that the manager may have beyond profits.

[19]While Antón et al. (2023) work under the assumption that shareholders decide compensation schemes, voting on managers' types seems to be also a plausible mechanism through which shareholders can exert influence on the firm's objective function. Indeed, Shekita (2021) includes manager appoint-

We assume that shareholders do not participate themselves in the general meetings of shareholders but instead delegate proxies, i.e., agents who vote on their behalf on a given proposal. *Representation by proxies* ensures that no common owner can exert any form of simultaneous influence over the competing firms, and, hence, eliminates anti-competitive forces originating from coordination. Therefore, any potential effect of common ownership on outcomes will be purely driven by the richer incentive structure that common ownership generates, and not by the fact that a common owner can facilitate any kind of collusion among firms.[20] Let k_1 be the proxy that represents shareholder k in firm 1 and k_2 the proxy that represents k in firm 2. In the respective shareholders' meetings, the proxies of k vote independently, without any communication between themselves, to elect manager m_i in firm i according to one share - one vote majority rule. Given the choice of manager in firm j, m_j, for proxy k_i the most favorable manager to run firm i is candidate m_i that maximizes the portfolio value of shareholder k,

$$V_i^k(m_{1,}m_2) = s_1^k \Pi_1(m_{1,}m_2) + s_2^k \Pi_2(m_{1,}m_2) \tag{4}$$

where $\Pi_i(m_{1,}m_2)$ are the equilibrium profits from the production subgame with m_1 and m_2 as manager types.[21] We may express the portfolio value of shareholder k from the viewpoint of proxy k_i, as a function of her relative interest in firm i by using the following monotonic transformation of $V_i^k(m_{1,}m_2)$:[22]

$$\tilde{V}_i^k(m_1, m_2) = V_i^k(m_{1,}m_2)/(s_1^k + s_2^k) = \sigma^k \Pi_1(m_1, m_2) + (1 - \sigma^k)\Pi_2(m_1, m_2). \tag{5}$$

We build a two-stage game to analyze the choice of managers by shareholders and the subsequent choice of outputs by each elected manager.

At $t = 1$, the voting stage, the proxies in each firm $i = 1, 2$ elect simultaneously the type of the firm's manager, m_i that will be in charge of the production decision in the next stage. The election process takes place in two sub-stages. In sub-stage $t = 1_a$ the

ment in its compilation of observed cases of different influence mechanisms. Antón et al. (2023) mentions that Virgin, who was controlled by less diversified shareholders than other publicly-listed US airlines, was associated with higher corporate quality and more aggressive pricing. Our work validates this suggestive evidence, and highlights that alternative mechanisms of shareholder influence over the firm's objective functions can be employed, essentially, to the same effect.

[20] This assumption is standard in the literature studying shareholder voting under common ownership. It is usually integrated in a property called *conditional sincerity*, according to which shareholders are assumed to vote sincerely in the shareholder meeting of a given firm, taking as given the decisions of the other firms. See, for instance, Azar (2012); Azar (2017); Brito et al. (2018); Brito et al. (2023). In this paper, we only keep the latter part of the assumption, and do not impose the sincerity constraint.

[21] To avoid any confusion, V_1^k is the portfolio value of shareholder k, from the viewpoint of proxy 1. That is, this proxy takes as given m_2 when she computes the portfolio value of shareholder k for each alternative m_1. Proxy 2 operates in a symmetric manner. Of course, ex-post, at the equilibrium pair of manager types, $V_1^k = V_2^k = V^k$.

[22] Evidently, $\partial\tilde{V}_i^k(m_1, m_2)/\partial m_i > 0$ (< 0) if and only if $\partial V_i^k(m_1, m_2)/\partial m_i > 0$ (< 0). Therefore, it suffices to study $\tilde{V}_i^k(m_1, m_2)$ in order to infer the preferences of agent k_i over manager types m_i, for any given m_j.

proxy of each shareholder proposes a manager type, and then, in sub-stage $t = 1_b$ all proxies vote using majority rule (one-share-one-vote) among the shareholders' proposals plus an exogenous proposal (e.g., the type of the incumbent manager).

According to this rule, the proposed manager type that collects a majority of votes over each other proposed manager type is the winner of the procedure. If the procedure cannot pin down a unique winner, then the exogenous proposal is implemented.[23]

At $t = 2$, the production stage, the elected managers choose simultaneously the level of production q_i (Cournot competition).

We focus on "sincere" subgame perfect equilibria. That is, we focus on sincere voting behavior at sub-stage $t = 1_b$, whenever such behavior constitutes an equilibrium behavior.

As it will soon become evident, whenever a Condorcet winner manager type exists, then: a) sincere voting is an equilibrium behavior in the voting stage for any possible set of proposals,[24] and b) the posited election procedure identifies the unique stable winning type, in the sense that no shareholder can ever succeed in building a majority coalition that is willing to replace the winner with an alternative candidate. That is, our election procedure captures in an, arguably, effective manner the main features of real world manager election procedures, according to which certain (active) shareholders propose to replace the incumbent with a particular challenger. Therefore, one of the main aspirations of the subsequent formal analysis is to establish that, indeed, in our setting, a Condorcet winner manager type exists generically.

3 Equilibrium

To identify the equilibria of the game, we use backward induction.

At $t = 2$, the manager of each firm $i = 1, 2$ chooses q_i to solve (3) or

$$q_i(q_j) = \frac{a - q_j}{2 + m_i}. \tag{6}$$

Solving the system of reactions functions above we obtain the equilibrium quantities and

[23]One could employ alternative structures of the election stage, leading essentially to the same outcome. Indeed, it could be the case that managers decided to apply for the job taking some minimum cost (e.g., to send a CV), in the fashion of citizen candidates' models (see, e.g., Osborne & Slivinski, 1996 and Besley & Coate, 1997). It could also be the case that proxies decide a compensation scheme like in Antón et al. (2023) or even directly the policy of the firm, as in the probabilistic voting models (e.g., Azar, 2012, Azar, 2017, Brito et al., 2018, Moskalev, 2019, Brito et al., 2023). As it will be evident in the subsequent sections, the analysis would still lead to similar conclusions since the existence of a majority winner is guaranteed under several alternative specifications.

[24]A Condorcet winner in this context exists if, given common beliefs regarding the manager type of the competing firm, the ideal manager type of a certain shareholder is preferred by a majority (i.e., a subset of shareholders owning a majority of shares) over any other manager's type.

profits as functions of manager types:

$$q_i(m_i, m_j) = \frac{a(1+m_j)}{3+2m_j+m_i(2+m_j)}, i, j = 1, 2, i \neq j, \tag{7}$$

$$\Pi_i(m_i, m_j) = \frac{a^2(1+m_i)(1+m_j)^2}{[3+2m_j+m_i(2+m_j)]^2}, i, j = 1, 2, i \neq j. \tag{8}$$

That is, each subgame admits a unique Nash equilibrium.

Notice that, when managers are socially unconcerned the outcome reduces to the standard Cournot duopoly one: $q_1(0,0) = q_2(0,0) = a/3$ and $\Pi_1(0,0) = \Pi_2(0,0) = a^2/9$. Each firm's quantity and profit is decreasing in its own manager type and increasing in the one of the other firm. This is intuitive: on one hand, the more socially concerned the manager is, the more she is willing to decrease the output level relative to the profit maximizing one. This results in lower profits. On the other hand, the more socially concerned the manager of the other firm is (therefore the lower the optimal level of output of the other firm), the higher the optimal output level prescribed by the reaction function, which results in higher profits. The effect of social concerns of managers on product market outcomes is therefore analogous to the one of asymmetric production costs in the standard Cournot model: the higher the social concern in a firm, the more the firm reduces the externality imposed on the rival and therefore the higher the benefit for the latter. This results in a softening of competition, which moves the market outcome away from Cournot and towards monopoly.[25]

A similar effect is reached in the symmetric case where both firms elect manager types with the same level of social concern. By reducing their own output levels, both firms reduce the externality they impose on each other, thereby boosting industry profits. For example, when both managers are of unitary type, the monopoly outcome is reached, with each firm producing half of the monopoly output and getting half of the monopoly profits: $q_1(1,1) = q_2(1,1) = a/4$, and $\Pi_1(1,1) = \Pi_2(1,1) = a^2/8$.

As we see next, this is the mechanism that shareholders lever to aim at portfolio maximization.

At $t = 1$, the proxies in both firms choose the types of their managers simultaneously, following majority rule. Given m_j, the manager type chosen by firm j, proxy k_i will vote for own manager type m_i that maximizes the portfolio value of shareholder k given by (5) taking into consideration that the elected manager will choose output according to (7) resulting in profit (8). In particular, proxy k_i takes into account that for any m_j, the output and profit of firm i are negatively related to own manager type, m_i. Through simple majority voting, firm i will then choose manager type m_i if, given m_j, type m_i is

[25] An increase in own manager's social concern reduces output and profit for the firm and increases output and profits for the rival. Overall, total output decreases and total profits increase. For a high enough social concern of own manager type, the firm allows the rival to behave almost as a monopolist. Total output and total profits are therefore bounded at the monopoly levels.

preferred by a majority over any other manager type. We consider that each proxy has, in the relevant firm, a percentage of votes equal to the percentage of shares held by the shareholder that she represents.

The outcome of the voting procedure in the first stage may not be well-defined if there is a tie or if social preferences turn out to be intransitive. Therefore we will start by demonstrating that each proxy of shareholder k has single-peaked preferences over manager types, for any given manager type (expected to be) selected by the other firm.

Lemma 1. *Given m_2, the preferences of agent k_1 over manager types for firm 1 are single-peaked. The ideal manager type of agent k_1, given m_2, denoted $m_1^{k_1}(m_2)$, is weakly decreasing in the relative interest σ^k, and it is equal to:*

$$m_1^{k_1}(m_2) = \begin{cases} M, & \text{if } \sigma^k \le \frac{2}{4+m_2} \\ \min\{\frac{2-3\sigma^k}{4\sigma^k+\sigma^k m_2-2}, M\}, & \text{if } \frac{2}{4+m_2} < \sigma^k < \frac{2}{3} \\ 0, & \text{if } \sigma^k \ge \frac{2}{3}. \end{cases} \tag{9}$$

Similarly, given m_1 the preferences of proxy k_2 over manager types for firm 2 are single-peaked, and $m_2^{k_2}(m_1)$ is weakly increasing in σ^k.

Proof. Assume that firm 2 is expected to appoint a manager of type m_2. Then the proxy k_1 representing shareholder k in firm 1, participates in the decision of m_1 and wants to maximize the wealth of shareholder k which is given by (5).

It suffices to study $\tilde{V}_1^k(m_1, m_2)$ in order to infer the preferences of agent k_1 over manager types m_1, for any given m_2. The latter are single peaked if, for every fixed m_2, $\tilde{V}_1^k(m_1, m_2) : [0, M] \mapsto \Re$ is quasi-concave.

First, we observe that, for each admissible m_2, $\partial\tilde{V}_1^k(m_1, m_2)/\partial m_1 < 0$ for every positive m_1 if and only if $\sigma^k \ge \frac{2}{3}$. That is, if $\sigma^k \ge \frac{2}{3}$ the agent has single-peaked preferences on $[0, M]$, with a peak at zero. Then, we notice that, for each admissible m_2, $\partial\tilde{V}_1^k(m_1, m_2)/\partial m_1 > 0$ for every positive m_1 if and only if $\sigma^k \le \frac{2}{4+m_2}$. That is, if $\sigma^k \le \frac{2}{4+m_2}$ the agent has single-peaked preferences on $[0, M]$, with a peak at M.

Finally, we have that $\partial\tilde{V}_1^k(m_1, m_2)/\partial m_1 = 0$ if and only if $m_1 = \frac{2-3\sigma^k}{4\sigma^k+\sigma^k m_2-2}$. This is a positive number if and only if $\frac{2}{4+m_2} < \sigma^k < \frac{2}{3}$. Moreover, $\partial\tilde{V}_1^k(m_1, m_2)/\partial m_1|_{m_1=0} = \frac{a^2(1+m_2)^2(2-3\sigma^k)}{(3+2m_2)} > 0$ for every $\sigma^k < \frac{2}{3}$. That is, for each admissible m_2, $\tilde{V}_1^k(m_1, m_2)$ is quasi-concave with respect to $m_1 \in [0, M]$, establishing that all agents' preferences are single-peaked on $[0, M]$. Indeed, if $\frac{2}{4+m_2} < \sigma^k < \frac{2}{3}$ and $\frac{2-3\sigma^k}{4\sigma^k+\sigma^k m_2-2} < M$, then the agent's peak is equal to $\frac{2-3\sigma^k}{4\sigma^k+\sigma^k m_2-2}$, and if $\frac{2}{4+m_2} < \sigma^k < \frac{2}{3}$ and $\frac{2-3\sigma^k}{4\sigma^k+\sigma^k m_2-2} \ge M$, then the agent's peak is equal to M.

We also notice that the derivative of $\frac{2-3\sigma^k}{4\sigma^k+\sigma^k m_2-2}$ with respect to σ^k is equal to $-\frac{2m_2+2}{((m_2+4)\sigma^k-2)^2} < 0$ for every $\sigma^k > \frac{2}{4+m_2}$ i.e., the peak of each proxy is monotonic and decreasing in the relative interest. That is, for every m_2, agent k_1 representing shareholder k in firm 1 has single-peaked preferences over m_1 with ideal manager type $m_1^{k_1}(m_2)$ weakly decreasing in σ^k.

Similarly one can establish that given m_1 the preferences of agent k_2 over manager types for firm 2 are single-peaked, and $m_2^{k_2}(m_1)$, is weakly increasing in σ^k, and is equal to:

$$m_2^{k_2}(m_1) = \begin{cases} 0, \quad \text{if } \sigma^k \leq \frac{1}{3} \\ \min\{\frac{1-3\sigma^k}{4\sigma^k+\sigma^k m_1 - m_1 - 2}, M\}, \quad \text{if } \frac{1}{3} < \sigma^k < \frac{2+m_1}{4+m_1} \\ M, \quad \text{if } \sigma^k \geq \frac{2+m_1}{4+m_1} \end{cases} \tag{10}$$

□

Lemma 1 indicates that for a given manager type chosen by the other firm, the larger the relative interest of a shareholder in a firm, the lower the ideal manager type of her proxy in that firm. This is so as the higher the relative interest in a firm the closer her preferences will be to that firm's own profit maximization. Indeed, for the proxy to prefer a strictly positive manager type, the relative interest in the firm needs to be low enough. Additionally, the larger the expected manager type of the other firm, the lower the ideal type (as the optimal level of output is higher).

Given that the ideal manager of each proxy of shareholder $k = 1...|K|$ in firm i is monotonic with respect to the shareholder's relative interest in firm i, the order of the ideal managers can be derived from the order of relative interests. In particular, the order of ideal managers $m_1^{k_1}(m_2)$ of voters in firm 1 on the $[0, M]$ interval will be opposite to the order of the relative interests σ^k on the $[0, 1]$ interval: the higher the relative interest in firm 1, the closer to 0 is the ideal manager. On the other hand, the order of ideal managers $m_2^{k_2}(m_1)$ in firm 2 will be the same as the order of relative interests σ^k (see numerical example in Section 5).

Since the preferences of shareholders' proxies over m_1, given m_2, are single-peaked, we know by the median voter theorem that there exists an unique ideal manager type that will be preferred by a majority of shareholders' votes over any other ideal manager type (i.e., a Condorcet winner). Moreover, that manager type is the ideal manager type of the median voter of firm 1 (see, e.g., Persson & Tabellini, 2002). The median voter in our context can be defined as follows.

Definition 2. *The median voter of firm i, denoted by μ_i, is the proxy representing median shareholder $k(\mu_i)$, such that $\sum_{k \leq k(\mu_i)} s_i^k \geq \frac{1}{2}$ and $\sum_{k \geq k(\mu_i)} s_i^k \geq \frac{1}{2}$.*

The median shareholder $k(\mu_i)$ has more that 50% of votes or shares on her left (including her) and on her right (including her), when other shareholders are ordered according to their relative interest (or according the ideal managers of their proxies). A median voter of firm i always exists and, except some non-generic cases in which $\sum_{k' \leq k} s_i^{k'} = \frac{1}{2}$

for some k', the median voter of firm i is unique. Our analysis will focus only on the generic scenarios with a unique median voter in each firm, μ_1 and μ_2.

Proxies that represent shareholders with non-identical portfolios will in general disagree on which manager to select. A shareholder with a more diversified portfolio will favor a manager that is expected to lead to higher industry profits whereas a undiversified shareholder will favor a manager that cares more about profits at the firm level. Despite the disagreement, due to single-peaked preferences a winner manager type exists out of any collection of proposals. That is, in every equilibrium that satisfies our refinement the Condorcet winner proposal will be supported by a majority. Therefore, in equilibrium, the median voter of the firm proposes her ideal manager type, and this type prevails over all other proposals. We summarize these observations in the following lemma.

Lemma 2. *At $t = 1$, given m_2, firm 1 will appoint a manager of type $m_1^{\mu_1}(m_2)$; and given m_1, firm 2 will appoint a manager of type $m_2^{\mu_2}(m_1)$.*

Proof. Omitted. □

This approach therefore avoids the inconsistent results obtained with the control-weighted formulation adopted by the literature such that the manager's objective function might not be the one preferred by the majority of shareholders. The majority principle ensures that the equilibrium manager's objective, as determined by the winning elected manager, will always be the one preferred by the majority of votes. For example, if the majority of votes are held by undiversified shareholders, it can never happen that the winning manager will have positive social concerns, and therefore that the objective function of the firm is something other than own profit maximization.

3.1 The Effects of Common Ownership on Market Outcomes

Given the allocation of shares across individual shareholders and their induced relative interests, due to Lemma 1 we may identify the effects of common ownership on equilibrium manager types and therefore on market outcomes by relying solely on the relative interests of the median shareholders $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)})$.

We may distinguish two polar cases that serve as points of reference: *i)* when the median shareholders are completely undiversified, i.e., $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) = (1, 0)$ and *ii)* when the median shareholders are completely diversified, i.e., $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) = (1/2, 1/2)$.

In case *i)* of no diversification, firms are run as if there are no common owners and appoint managers with no social concerns, $m_1^*(1, 0) = m_2^*(1, 0) = 0$.[26] Equilibrium quantities and profits are equivalent to the standard Cournot equilibrium with a single undiversified owner-manager in each firm, $q_1 = q_2 = a/3$ and $\Pi_1 = \Pi_2 = a^2/9$.

[26]This entails also the case where firms have no common owners at all, i.e., for each $k \in K$, either $\sigma^k = 1$ or $\sigma^k = 0$.

In case *ii)* of complete diversification, both firms are run as if they are owned by the same shareholder, or, more accurately, by two identical shareholders who have the same preferences because they hold exactly the same portfolio $\tilde{V}^{k(\mu_1)}(m_1,m_2) = \tilde{V}^{k(\mu_2)}(m_1,m_2)$. These identical shareholders will appoint identical managers with relatively high social concerns, $m_1^*(1/2, 1/2) = m_2^*(1/2, 1/2) = 1$. Equilibrium quantities and profits are equivalent to the standard monopoly equilibrium with a single owner-manager in both firms, $q_1 = q_2 = a/4$ and $\Pi_1 = \Pi_2 = a^2/8$.

Departing from these polar cases that induce "extreme" market outcomes like Cournot or monopoly, the whole range of equilibrium choices of managers for any pair of relative interests of the median shareholders $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) \in (0, 1] \times [0, 1)$ can be characterized. In this section we focus on a set of pairs of relative interests of median shareholders that lead to the appointment of non-extreme manager types for both firms, i.e., $0 < m_i^* < M, i = 1, 2$ (a complete characterization is provided in the Appendix).

Observe that if $\sigma^{k(\mu_1)} \in (1/2, 2/3)$, then it follows from Lemma 1 that at equilibrium $m_1^* > 0$ and, for M large enough, $m_1^* < M$. Similarly, if $\sigma^{k(\mu_2)} \in (1/3, 1/2)$, then at equilibrium $m_2^* > 0$ and $m_2^* < M$, if M is large enough. More formally:

Lemma 3. *Let the median shareholders of both firms be sufficiently diversified, i.e.,* $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) \in \left(\frac{1}{2}, \frac{2}{3}\right) \times \left(\frac{1}{3}, \frac{1}{2}\right)$ *and M be sufficiently large, then both firms will appoint managers with interior social concerns, i.e.,* $m_1^*, m_2^* \in (0, M)$.

Proof. In equilibrium, the beliefs of the agents participating in the manager's appointment in firm 1 (resp. 2) should have correct expectations about the appointed manager's type in firm 2 (resp. 1).

The ideal manager type of the median proxy in firm 1 is given by the maximization of (5) given m_2 or

$$\begin{aligned}\tilde{V}_1^{k(\mu_1)}(m_1,m_2) &= \sigma^{k(\mu_1)}\Pi_1(m_1,m_2) + (1 - \sigma^{k(\mu_1)})\Pi_2(m_1,m_2), \\ &= \sigma^{k(\mu_1)}(a - q_1(m_1, m_2) - q_2(m_1, m_2))q_1(m_1, m_2) \\ &\quad +(1 - \sigma^{k(\mu_1)})(a - q_1(m_1, m_2) - q_2(m_1, m_2))q_2(m_1, m_2).\end{aligned} \tag{11}$$

Solving $\partial\tilde{V}_1^{k(\mu_1)}(m_1,m_2)/\partial m_1 = 0$ for m_1, we obtain

$$m_1(m_2) = \frac{2 - 3\sigma^{k(\mu_1)}}{4\sigma^{k(\mu_1)} + \sigma^{k(\mu_1)}m_2 - 2}. \tag{12}$$

We define $\tilde{V}_2^{k(\mu_2)}(m_1, m_2)$ similarly to (11) and from $\partial\tilde{V}_2^{k(\mu_2)}(m_1,m_2)/\partial m_2 = 0$ we obtain

$$m_2(m_1) = \frac{1 - 3\sigma^{k(\mu_2)}}{m_1\sigma^{k(\mu_2)} - m_1 + 4\sigma^{k(\mu_2)} - 2} \tag{13}$$

Solving the system of equations (12) and (13) we obtain the equilibrium manager types in the voting stage at $t = 1$ as a function of the exogenously given relative interests of the median shareholders $k(\mu_1), k(\mu_2) \in K$ in firms 1 and 2 respectively.

$$\begin{aligned} m_1^*(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) &= \frac{\sigma^{k(\mu_1)}(3 - 6\sigma^{k(\mu_2)}) + 4\sigma^{k(\mu_2)} - 2}{(2\sigma^{k(\mu_1)} - 1)(\sigma^{k(\mu_2)} - 1)}, \\ m_2^*(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) &= \frac{(2\sigma^{k(\mu_1)} - 1)(3\sigma^{k(\mu_2)} - 1)}{\sigma^{k(\mu_1)} - 2\sigma^{k(\mu_1)}\sigma^{k(\mu_2)}}. \end{aligned} \tag{14}$$

The system of simultaneous inequalities $m_1^*(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) > 0$ and $m_2^*(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) > 0$ is true if $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2}) \in \left(\frac{1}{2}, \frac{2}{3}\right) \times \left(\frac{1}{3}, \frac{1}{2}\right)$ and M is large enough so that $m_1^*(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) < M$ and $m_2^*(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) < M$.

Hence, if $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2}) \in \left(\frac{1}{2}, \frac{2}{3}\right) \times \left(\frac{1}{3}, \frac{1}{2}\right)$ and M is large enough, then $0 < m_i^* < M, i = 1, 2$. □

We can now characterize the effects of a change in the degree of diversification of the portfolios of the median shareholders on the equilibrium manager types, on output and on profits.

Proposition 1. *Let* $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2}) \in \left(\frac{1}{2}, \frac{2}{3}\right) \times \left(\frac{1}{3}, \frac{1}{2}\right)$ *and* M *be sufficiently large. If the median shareholder of firm 1 becomes more diversified,* $d\sigma^{k(\mu_1)} < 0$*, then a more (resp. less) socially concerned manager will be elected in firm 1 (resp. firm 2), the production and profit of firm 1 (resp. firm 2) will decrease (resp. increase), total output will decrease and industry profits will rise.*

Proof. We differentiate (14), (7) and (8) to obtain

$$\begin{aligned} \frac{dm_1^*}{d\sigma^{k(\mu_1)}} &= \frac{1 - 2\sigma^{k(\mu_2)}}{(1 - 2\sigma^{k(\mu_1)})^2(\sigma^{k(\mu_2)} - 1)} < 0, \frac{dm_2^*}{d\sigma^{k(\mu_1)}} = \frac{1 - 3\sigma^{k(\mu_2)}}{(\sigma^{k(\mu_1)})^2(2\sigma^{k(\mu_2)} - 1)} > 0, \\ \frac{dq_1^*}{d\sigma^{k(\mu_1)}} &= \frac{a(\sigma^{k(\mu_2)} - 1)(2\sigma^{k(\mu_2)} - 1)}{2(\sigma^{k(\mu_1)} - \sigma^{k(\mu_2)})^2} > 0, \frac{dq_2^*}{d\sigma^{k(\mu_1)}} = \frac{a\sigma^{k(\mu_2)}(2\sigma^{k(\mu_2)} - 1)}{2(\sigma^{k(\mu_1)} - \sigma^{k(\mu_2)})^2} < 0, \\ \frac{d(q_1^* + q_2^*)}{d\sigma^{k(\mu_1)}} &= \frac{a(1 - 2\sigma^{k(\mu_2)})^2}{2(\sigma^{k(\mu_1)} - \sigma^{k(\mu_2)})^2} > 0, \\ \frac{d\Pi_1^*}{d\sigma^{k(\mu_1)}} &= \frac{a^2(\sigma^{k(\mu_2)} - 1)(2\sigma^{k(\mu_2)} - 1)(\sigma^{k(\mu_1)}(8\sigma^{k(\mu_2)} - 3) - 5\sigma^{k(\mu_2)} + 2)}{4(\sigma^{k(\mu_1)} - \sigma^{k(\mu_2)})^3} > 0, \\ \frac{d(\Pi_1^* + \Pi_2^*)}{d\sigma^{k(\mu_1)}} &= \frac{a^2(2\sigma^{k(\mu_2)} - 1)\left(\sigma^{k(\mu_1)}(8\sigma^{k(\mu_2)} - 5)\sigma^{k(\mu_2)} + \sigma^{k(\mu_1)} - 3(\sigma^{k(\mu_2)})^2 + \sigma^{k(\mu_2)}\right)}{4(\sigma^{k(\mu_1)} - \sigma^{k(\mu_2)})^3} < 0. \end{aligned}$$

□

Sometimes there are events that cause a joint change in the portfolio diversification of median shareholders. For instance, in the absence of money or other assets, a change of portfolio of one shareholder affects the overall ownership structure of the two firms, because a purchase of shares in a firm is only feasible by the sale of shares in the other firm and vice-versa. Therefore, a change of relative interest of a shareholder induces an opposite change in the relative interest of at least one other shareholder. This negative relation of relative interests of shareholders across different firms can have important consequences on outcomes when the involved shareholders are the median shareholders of the two firms. To study such joint opposite shifts in the relative interests of median shareholders we focus on a salient class of share distributions that satisfy a symmetry condition, as described below.

Assumption 1. *The relative interests of median shareholders satisfy "symmetry" if*

$$1 - \sigma^{k(\mu_2)} = \sigma^{k(\mu_1)} = \hat{\sigma}. \tag{15}$$

This assumption dictates that the median shareholder of firm 1 cares about the profits of firm 1 (resp. 2) as much as the median shareholder of firm 2 cares about the profits of firm 2 (resp. 1), and allows us to capture the degree of common ownership in that industry by a single parameter, $\hat{\sigma}$.

Proposition 2. *Let* $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) \in \left(\frac{1}{2}, \frac{2}{3}\right) \times \left(\frac{1}{3}, \frac{1}{2}\right)$, *$M$ be sufficiently large, and Assumption 1 hold. If the degree of common ownership increases, $d\hat{\sigma} < 0$, both firms will choose managers with higher social concerns, leading to lower production in each firm and to higher wealth for all shareholders.*

Proof. We substitute (15) in (14) and we obtain

$$m_1^* = m_2^* = m^* = \frac{2}{\hat{\sigma}} - 3. \tag{16}$$

Differentiating (16), (7), and (11) we obtain $dm^*/d\hat{\sigma} = -\frac{2}{(\hat{\sigma})^2} < 0, dq_i^*/d\hat{\sigma} = \frac{2}{(\hat{\sigma})^2} \frac{a}{(3+m)^2} > 0$ and $d\tilde{V}^{k*}/d\hat{\sigma} = \frac{1}{2a^2(1-2\hat{\sigma})} < 0$. □

The results above suggest that if shareholders are sufficiently diversified across the two firms, proxies will elect socially concerned managers in both firms. In both firms therefore managers will unilaterally deviate downward from the profit maximizing output level in order to internalize the negative externality caused by their firm. This results in firms internalizing the externalities they impose on each other, which softens competition and boosts shareholders portfolio values, close to industry-level profits. This result therefore not only confirms the anticompetitive effects found in the literature, but also provides two novel insights. First, diversified shareholders do not need to be individually big in terms of share for anticompetitive effects to emerge, they can be as small as they want as long as they collectively have the majority of votes (so that the median shareholder is a diversified one). Second, shareholders do not need to directly influence the choices of managers (i.e., managers do not need to take into account preferences of shareholders) but anticompetitive forces can emerge indirectly provided managers have alternative values in addition to profits. The next section shows that by allowing shareholders to freely choose their degree of diversification, the full monopoly outcome is reached.

4 Endogenous Choice of Shareholdings

We have shown that the manager appointed in each firm is the ideal manager of median voter μ_i that represents median shareholder $k(\mu_i)$. Strategic shareholders may be able to affect the voting outcome by changing their portfolio. This may happen by trading their shares in a competitive stock market or by bilateral or multilateral exchanges of shares out of the market. To capture stock trading, we may introduce stage $t = 0$, prior to the voting stage, where shareholders trade shares taking into account that their relative interests in the two firms will influence the manager choice and consequently the production decision.

Equilibrium in the stock market requires that stock prices (or relative prices in case of OTC trades) are such that no unilateral deviation is profitable by any shareholder.

We elaborate on a simple case where each firm is initially owned by a single shareholder who is obviously the median voter and there is no common ownership, and later we discuss the general case with multiple shareholders. Will the owner of either company have any incentive to trade shares and change her relative interests so that she influences the manager appointment in each firm and consequently the output choice?

Suppose that shareholder 1 owns 100% of firm 1 and shareholder 2 owns 100% of firm 2. Portfolios are $s^1 = (1, 0)$ and $s^2 = (0, 1)$, relative interests are $\sigma^1 = 1$ and $\sigma^2 = 0$, and the choice of managers types according to Lemma 1 would be $m_1^*(1, 0) = m_2^*(1, 0) = 0$, resulting in $q_1 = q_2 = a/3, \Pi_1 = \Pi_2 = a^2/9$. Firms and profits are symmetric, so if owners exchanged the entirety of their firms, they would be indifferent.

Now suppose that shareholders exchange a percentage x of the shares of their firms. This trade can be a barter exchange or an exchange based on stock market prices. Under zero net supply of money, this amounts to selling x of firm 1 to buy x from firm 2 through the stock market at the price of 1. Suppose $x = 1/100$. Then, portfolios become $s^1 = (99/100, 1/100)$ and $s^2 = (1/100, 99/100)$ and the relative interests in firm 1 are $\sigma^1 = 99/100$ and $\sigma^2 = 1/100$. Nevertheless, equilibrium manager types do not change, i.e., $m_1^*(99/100, 1/100) = m_2^*(99/100, 1/100) = 0$, hence the profits remain the same and owners of both firms would be indifferent to that trade. In fact, for $x \in [0, 1/3]$, $m_1^*(1 - x, x) = m_2^*(1 - x, x) = 0$ but for $x \in (1/3, 1/2)$, $m_1^*(1 - x, x) = m_2^*(1 - x, x) > 0$ and consequently output will decrease and profits will increase due to the choice of more socially concerned managers.[27] For example for $x = 40/100$, portfolios are $s^1 =$

[27]For $x \in (1/2, 1]$ control reverses, i.e., shareholder 2 (resp. 1) becomes the median shareholder of firm 1 (resp. 2), and everything is symmetric to the case currently considered. In the degenerate case of $x = 1/2$, shareholders are completely identical, and hence it does not matter who is considered to be

$(60/100, 40/100)$, $s^2 = (40/100, 60/100)$ and $\sigma^1 = 60/100$, $\sigma^2 = 40/100$. Equilibrium manager types are $m_1^*(60/100, 40/100) = m_2^*(60/100, 40/100) = 1/3$ and $q_1 = q_2 = 3a/10 < a/3, \Pi_1 = \Pi_2 = 3a^2/25 > a^2/9$. It follows that a mutually beneficial symmetric share exchange exists and therefore portfolios $s^1 = (1, 0)$ and $s^2 = (0, 1)$ cannot be equilibrium portfolios. It can be easily checked that there exists a set of Pareto improving portfolios with respect to no-common ownership of the type $s^1 = (1-x, x)$, $s^2 = (x, 1-x)$ for $x \in (1/3, 1/2)$. Moreover there is a unique Pareto optimal ownership structure, $s^{1*} = (1/2 + \epsilon, 1/2 - \epsilon)$, $s^{2*} = (1/2 - \epsilon, 1/2 + \epsilon)$ with $\epsilon \to 0$ that induces $m_1^* = m_2^* = 1$ and results in the monopoly outcome $q_1 = q_2 = a/4, \Pi_1 = \Pi_2 = a^2/8$.

The example can also be presented in the context of a competitive stock market. Let $\bar{s}^k, k \in K$ be the set of initial portfolios. We may define a market equilibrium as follows.

Definition 3. *A market equilibrium of a stock market economy is a set s^* of portfolios, one for each investor, and a set ρ^* of share prices, one for each firm, such that every investor $k \in K$ chooses s^{k*} to maximize her portfolio value V^k given her budget constraint $\sum_i \rho_i^* \bar{s}_i^k = \sum_i \rho_i^* s_i^{k*}$, and market capacity constraints $\sum_i s_i^{k*} = 1$, for every i.*[28]

Starting from an initial situation where shareholders are not diversified, i.e., the first shareholder owns all the shares of firm 1 and the second shareholder owns all the shares of firm 2, we show that prices $\rho_1^* = \rho_2^* = 1$ induce an allocation of shares that corresponds to full diversification $s^k = (1/2, 1/2), k = 1, 2$ and that this allocation and price vector constitute a competitive equilibrium.

Notice that given the initial endowments, the posited prices and the market capacity constraints, the equilibrium allocation should be symmetric (i.e., Assumption 1 should hold). By substituting the budget and the market capacity constraints in (2) and then the corresponding relative interests in (14), we identify the equilibrium manager types as a function of the shareholdings, when shareholdings induce interior equilibrium manager types, i.e., when $(s_1^1, s_1^2) \in (1/3, 2/3)^2$. Subsequently, if we substitute these manager types in (8), the shareholders' wealth, given by (4), becomes

$$V^1(s_1^1) = (1/2)a^2(1 - s_1^1)s_1^1, \tag{17}$$

$$V^2(s_1^2) = (1/2)a^2(1 - s_1^2)s_1^2. \tag{18}$$

If $(s_1^1, s_1^2) \notin (1/3, 2/3)^2$, then $m_1^* = m_2^* = 0$ and, hence each shareholder's wealth is equal to $a^2/9$. Maximizing the above for each shareholder with respect to her shareholdings,

the median shareholder of each firm.

[28]Notice that our environment has externalities, in the sense that, in order to compute their (expected) utility for any profile of shareholdings, shareholders need to know not only their shares in each firm, but also the shareholdings of the other shareholders. Arrow and Hahn (1971) defined a more general competitive equilibrium notion which accommodates for externalities (see, e.g., Casella, Llorente-Saguer, & Palfrey, 2012 for a discussion). Our definition of competitive equilibrium is compatible with this approach.

we obtain $s^{k*} = (1/2, 1/2), k = 1, 2$ and therefore shareholders will equalize their relative interests across firms, $\sigma^1 = \sigma^2 = 1/2$. The choice of managers types are $m_1^*(1/2, 1/2) = m_2^*(1/2, 1/2) = 1$ and results in the monopoly equilibrium $q_1 = q_2 = a/4, \Pi_1 = \Pi_2 = a^2/8$.

When only two shareholders are present each share not held by one shareholder should automatically belong to the other one. When multiple shareholders exist, one should first specify how trade is conducted. In what follows, we assume that each trader can decide not only which share to trade, but also the identity of the other trader (i.e., shareholder k buys one share of firm i from shareholder k'). However, we stress that our conclusions continue to hold also under alternative assumptions (e.g., with anonymous demands and rationing rules).

First, it is easy to see that full diversification, $s^{k*} = (1/|K|, 1/|K|)$ for each $k = 1, 2, ..., |K|$, and equal prices $\rho_1^* = \rho_2^* = 1$ is also an equilibrium, if all shareholders start with equally valuable endowments. Indeed, any change in shareholdings either leads to lower profits, or does not induce any change to the preferences of the median shareholder of any firm, therefore leaving outcomes invariant. Moreover, if we consider generic distributions of initial shareholdings, we can see that the median shareholders of the firms have incentives to trade with each other, to increase their diversification and ensure more favorable outcomes for both of them. In general, it is true that in this more general case, equilibria without full diversification may also exist, but the full diversification equilibrium is always present, and trade dynamics that are present in generic distributions of shareholdings also push towards diversification.

5 A Numerical Example

Let $|K| = 9$ be the number of shareholders. There are 2 firms. Suppose the number of shares in each firm is 100 and each share gives one vote. The allocation of shares across shareholders of firm 1 is given by $\{23, 12, 17, 14, 18, 0, 5, 7, 4\}$ and of firm 2 by $\{9, 6, 8, 15, 16, 13, 12, 10, 11\}$ where the first element (extreme left) in the list is the percentage of shares (or votes) for the first shareholder, the second element is the percentage for the second etc. Therefore the first shareholder has portfolio $(23, 9)$, while the second $(12, 6)$. Then the relative interests in firm 1 are

$$\left\{\frac{23}{32}, \frac{12}{18}, \frac{17}{25}, \frac{14}{29}, \frac{18}{34}, 0, \frac{5}{17}, \frac{7}{17}, \frac{4}{15}\right\},$$

where the numerator in each fraction represents the number of shares in firm 1 and the denominator represents the sum of shares of a shareholder in both firms.

Arranging shareholders in increasing order according to their relative interests in firm

1, $\sigma = \{..., \sigma^k, ...\}$, gives

$$\begin{aligned}\sigma &= \{\sigma^1, \sigma^2, \sigma^3, \sigma^4, \sigma^5, \sigma^6, \sigma^7, \sigma^8, \sigma^9\} \\ &= \left\{0, \frac{4}{15}, \frac{5}{17}, \frac{7}{17}, \frac{14}{29}, \frac{18}{34}, \frac{12}{18}, \frac{17}{25}, \frac{23}{32}\right\} \\ &= \{0, 0.26, 0.29, 0.41, 0.48, 0.53, 0.66, 0.68, 0.72\}\end{aligned}$$

The above arrangement of relative interests provides also information on the preferences of each shareholder for the ideal manager in both firms. In particular σ implies the following complete order of ideal managers of firm 1, for any admissible type of manager of firm 2

$$m_1^{1*} \geq m_1^{2*} \geq m_1^{3*} \geq m_1^{4*} \geq m_1^{5*} \geq m_1^{6*} \geq m_1^{7*} \geq m_1^{8*} \geq m_1^{9*}.$$

For example, for $m_2 = \frac{13}{2}$, the ideal manager types in firm 1 are $\{M, \frac{3}{2}, \frac{38}{37}, \frac{26}{79}, \frac{16}{89}, \frac{14}{121}, 0, 0, 0\}$.

On the other hand, $1 - \sigma$ provides a descending order of relative interests in firm 2,

$$\begin{aligned}1 - \sigma &= \{1 - \sigma^1, 1 - \sigma^2, 1 - \sigma^3, 1 - \sigma^4, 1 - \sigma^5, 1 - \sigma^6, 1 - \sigma^7, 1 - \sigma^8, 1 - \sigma^9\} \\ &= \left\{\frac{13}{13}, \frac{11}{15}, \frac{12}{17}, \frac{10}{17}, \frac{15}{29}, \frac{16}{34}, \frac{6}{18}, \frac{8}{25}, \frac{9}{32}\right\} \\ &= \{1, 0.73, 0.70, 0.58, 0.51, 0.47, 0.33, 0.32, 0.28\}.\end{aligned}$$

that induces the following complete order of ideal managers of firm 2, for any admissible manager type in firm 1

$$m_2^{1*} \leq m_2^{2*} \leq m_2^{3*} \leq m_2^{4*} \leq m_2^{5*} \leq m_2^{6*} \leq m_2^{7*} \leq m_2^{8*} \leq m_2^{9*}.$$

For example, for $m_1 = 0$, the ideal manager types for firm 2 are $\{0, 0, 0, \frac{2}{3}, \frac{13}{2}, M, M, M, M\}$.[29]

The negative monotonic relationship between σ^k and m_1^{k*} and the positive one between σ^k and m_2^{k*} is established by Lemma 1. For example, shareholder 1 on the extreme left of σ has zero shares in firm 1 and hence zero relative interest in firm 1, $\sigma^1 = 0$, and would therefore wish that firm 1 elected a manager with maximum social concern, while firm 2 elected a manager with minimum social concern, because her relative interest in firm 2 is $1 - \sigma^1 = 1$. On the other hand, on the right extreme of σ, shareholder 9 with relative interest of $\sigma^9 = 23/32$ in firm 1 has opposite preferences to shareholder 1. Shareholder 9 prefers the least socially concerned manager in firm 1, in comparison to the rest of shareholders in firm 1, while she prefers the most concerned one in firm 2, because of her relative interest in firm 2, $1 - \sigma^9 = 9/32$.

Obviously shareholders disagree on the choice of manager.

[29] The values of $m_2 = 13/2$ and $m_1 = 0$ used to provide a numerical exemplification of the ranking of manager types in, respectively, firm 1 and firm 2, are actually the equilibrium choices of m_2 and m_1, as characterized in (19).

Let us focus on firm 1. In a shareholders' meeting, the ideal manager of the median voter of firm 1 will collect the majority of votes. The median voter (the proxy of the median shareholder) in each firm is the one that has more than 50% of votes on her left (including her) and on her right (including her) on the ordered sequence of relative interests. The votes are given by the numerators in the ordered sequence σ for firm 1 $\{0, 4, 5, 7, 14, 18, 12, 17, 23\}$ and $1 - \sigma$ for firm 2, $\{13, 11, 12, 10, 15, 16, 6, 8, 9\}$. Therefore, the median voter in firm 1, μ_1, is the proxy representing median shareholder $k(\mu_1) = 7$ with 12 shares (as $0+4+5+7+14+18+12 = 60$ and $12+17+23 = 52$) and a relative interest of $\sigma^7 = 12/18$ in firm 1 (and of $1 - \sigma^7 = 6/18$ in firm 2). On the other hand, the median voter in firm 2, μ_2, is the proxy representing median shareholder $k(\mu_2) = 5$ with 15 shares (as $13 + 11 + 12 + 10 + 15 = 61$ and $15 + 16 + 6 + 8 + 9 = 54$) and a relative interest of $1 - \sigma^5 = 15/29$ in firm 2 (and of $\sigma^5 = 14/29$ in firm 1).

The ideal manager types of the median voters are those that maximize their respective shareholders' portfolio values (11) that is,[30]

$$m_1^*(\sigma^7, 1 - \sigma^5) = m_1^*(12/18, 15/29) = 0, \tag{19}$$
$$m_2^*(\sigma^7, 1 - \sigma^5) = m_2^*(12/18, 15/29) = 13/2.$$

According to (7), these manager types will choose quantities $q_1^*(0, 13/2) = 15a/32$ and $q_2^*(0, 13/2) = a/16$, obtaining, according to (8), profits $\Pi_1^*(0, 13/2) = (225a^2)/1024$ and $\Pi_2^*(0, 13/2) = (15a^2)/512$ which sum up to 112% of the benchmark Cournot profits and 99,6% of the monopoly profits.

As the median shareholder of firm 2 is highly diversified she chooses a socially concerned manager. On the other hand the median shareholder of firm 1 is not so diversified, so she chooses a manager with zero concerns. This consistently results in firm 2 (resp. firm 1) choosing a lower (resp. higher) output level and gaining lower (resp. higher) profit relative to the Cournot benchmark without social concerns. Due to such softening of competition total profits are higher than in the Cournot benchmark and approach the monopoly profits.

6 Extension to I firms

Understanding how common ownership affects outcomes in contexts with two competing firms is of paramount importance. However, it is not straightforward that the main insights provided above—in particular, the existence of a Condorcet winner manager type—extend also to settings with multiple firms. In duopolies, the identity of the decisive shareholder of each firm (i.e., the median shareholder), depended on the ordering of

[30] According to (9), $m_1^*(\sigma^7, 1 - \sigma^5) = 0$, while according to (10), when $m_1 = 0$, $m_2^*(\sigma^7, 1 - \sigma^5)$ is an interior solution given by (13).

relative interests, and not on the manager type of the other firm. Does this also hold in multi-firm contexts? If so, which is the relevant ordering of the shareholders? If not, can we still show that a Condorcet winner manager type exists although identifying it might be more complicated? Or is it instead the case that the single-peakedness property that we established in duopolies collapses altogether and stable majority outcomes are not guaranteed anymore?

To get some answers, we now extend the analysis to the case where there are I firms in the industry, with $i = 1, 2, .., I$, facing inverse demand $p = 1 - Q$, where $Q = \sum_i q_i$. The portfolio of shareholder k is a vector of shares $s^k = (s_i^k)_i^I \in \Re_+^I$.

Let $m = (m_i)_i^I \in \Re_+^I$ be a profile of managers that have been elected at the first stage of the game. At the second stage, the manager of firm i maximizes utility given by (3) with respect to q_i where $\Pi_i(q_1, ..., q_I) = (1 - Q)q_i$. From the first order condition we have that $1 - q_i - m_i q_i - Q = 0$, or $q_i = (1 - Q)/(1 + m_i)$. Setting $g_i(m_i) = 1/(1 + m_i)$ and summing over all firms we obtain $Q = (1 - Q)G(m)$, where $G(m) = \sum_i g_i(m_i)$, or $Q = G(m)/(1 + G(m))$. Then $1 - Q = 1/(1 + G(m))$. Therefore, at $t = 2$ the utility maximizing quantity that will be chosen by the manager of firm i is

$$
\begin{aligned}
q_i(m) = g_i(m_i)(1 - Q) &= \frac{g_i(m_i)}{1 + G(m)} \\
&= \left[(1 + m_i)(1 + \sum_i \frac{1}{1 + m_i})\right]^{-1}.
\end{aligned}
$$

Therefore, the equilibrium profit of firm i is

$$\Pi_i(m) = (1 - Q)q_i^* = [1 + G(m)]^{-2} g_i(m_i). \tag{20}$$

At $t = 1$, given the profile of managers chosen by all the other firms except firm i, $m_{-i} = (m_j)_j^I, j = 1, ..., I, j \neq i$, we may write the portfolio value of shareholder k as

$$
\begin{aligned}
V_i^k(m_i, m_{-i}) = \sum_i^I s_i^k \Pi_i(m_i, m_{-i}) &= \sum_i^I s_i^k [1 + G(m)]^{-2} g_i(m_i) \\
&= [1 + G(m)]^{-2} \sum_i^I s_i^k g_i(m_i) \\
= [1 + G_{-i}(m_{-i}) + g_i(m_i)]^{-2} &\left(s_i^k g_i(m_i) + \sum_{j \neq i}^I s_j^k g_j(m_j) \right),
\end{aligned} \tag{21}
$$

where $\Pi_i(m_i, m_{-i})$ are the profits corresponding to the (m_i, m_{-i}) profile of manager types and $G_{-i}(m_{-i}) = \sum_{j \neq i} g_j(m_j)$. Let $g_{-i} = (g_j)_j^I, j = 1, ..., I, j \neq i$, then the ideal manager of shareholder k in firm i is given by the the maximization of $V_i^k(g_i, g_{-i}) \equiv V_i^k(m_i, m_{-i})$

with respect to $g_i(m_i)$, which yields

$$g_i^*(m_i) = 1 + G_{-i}(m_{-i}) - \frac{2W_{-i}^k}{s_i^k} \tag{22}$$

where $W_{-i}^k = \sum_{j\neq i}^I s_j^k g_j(m_j)$, or in terms of manager types,

$$\frac{1}{1+m_i^*} = 1 + \sum_{j\neq i}^I \frac{1}{1+m_j} - \frac{2}{s_i^k}\sum_{j\neq i}^I s_j^k \frac{1}{1+m_j}. \tag{23}$$

We may interpret W_{-i}^k as a weighted average of the shareholdings of shareholder k in all firms except firm i, using as weights decreasing functions of the respective manager types.

Analogously to Lemma 1, we can demonstrate that each proxy of shareholder k in firm i has single-peaked preferences over manager types, for any profile of manager types that is expected to be selected by the other firms, m_{-i}. The key difference between the two-firm and the multi-firm case, lies in the fact that in the former the identity of the median shareholder in firm i does not depend on the type of the manager expected to be appointed in firm j, while in the latter it does.

Lemma 4. *Given a profile of managers m_{-i}, the preferences of agent k_i over manager types for firm i are single-peaked. Moreover, the ideal manager type of agent k_i, given m_{-i}, denoted by $m_i^{k_i}(m_{-i})$, is weakly decreasing in $\frac{s_i^k}{W_{-i}}$.*

Proof. Given m_{-i}, the managers that are expected to be appointed by the other firms, proxy k_i representing shareholder k in firm i, maximizes the portfolio value given by (21) with respect to g_i or[31]

$$V_i^k(g_i, g_{-i}) = [1 + G_{-i} + g_i]^{-2}(s_i^k g_i + W_{-i}^k) \tag{24}$$

We will indirectly infer the preferences of agent k_i over manager types m_i, for any given m_{-i} by inferring her preferences over g_i given $g_{-i} = (g_j)_j^I, j = 1, ..., I, j \neq i$. The latter are single-peaked if, for every fixed g_{-i}, $V_i^k(g_i, g_{-i}) : [(1+M)^{-1}, 1] \mapsto \Re$ is quasi-concave. Differentiation yields

$$\frac{\partial V_i^k(g_i, g_{-i})}{\partial g_i} = \frac{(g_i - G_{-i} - 1)s_i^k + 2W_{-i}^k}{(g_i + G_{-i} + 1)}. \tag{25}$$

[31]For notational brevity we write $g_i = g_i(m_i), g_j = g_j(m_j), G_{-i} = G_{-i}(m_{-i}), \sigma_i^k(g_{-i}) = \sigma_i^k$.

Then for every $g_i \in [(1+M)^{-1}, 1]$,

$$\frac{\partial V_i^k(g_i, g_{-i})}{\partial g_i} \quad \begin{cases} > 0, & \text{if } g_i < 1 + G_{-i} - \frac{2W_{-i}}{s_i^k}, \\ = 0, & \text{if } g_i = 1 + G_{-i} - \frac{2W_{-i}}{s_i^k}, \\ < 0, & \text{if } g_i > 1 + G_{-i} - \frac{2W_{-i}}{s_i^k}. \end{cases} \tag{26}$$

Given g_{-i} and hence G_{-i}, $V_i^k(g_i, g_{-i})$ is increasing in g_i, reaches a maximum at $g_i^* = 1+G_{-i}-\frac{2W_{-i}}{s_i^k}$ which belongs to $[(1+M)^{-1}, 1]$ if and only if $(1+M)^{-1} \leq 1+G_{-i}-\frac{2W_{-i}}{s_i^k} \leq 1$ or $\frac{2}{G_{-i}} \geq \frac{s_i^k}{W_{-i}} \geq \frac{2}{G_{-i}+\frac{M}{(1+M)}}$, and then it is decreasing in g_i.

Now, if $\partial V_i^k(g_i, g_{-i})/\partial g_i > 0$ everywhere in $[(1+M)^{-1}, 1]$ then by (26) $g_i < 1+G_{-i} - \frac{2W_{-i}}{s_i^k}$ and $\partial V_i^k(g_i, g_-)/\partial g_i|_{g_i=1} = \frac{s_i G_{-i} - 2W_{-i}}{(2+G_{-i})^3} > 0$ because for $g_i = 1$ by (26) $G_{-i} - \frac{2W_{-i}}{s_i^k} > 0$ or $\frac{s_i^k}{W_{-i}} > \frac{2}{G_{-i}}$.

Also, if $\partial V_i^k(g_i, g_{-i})/\partial g_i < 0$ everywhere in $[(1+M)^{-1}, 1]$ then by (26) $g_i > 1 + G_{-i} - \frac{2W_{-i}}{s_i^k}$ and $\partial V_i^k(g_i, g_-)/\partial g_i|_{g_i=(1+M)^{-1}} = \frac{s_i^k[1-(1+M)^{-1}+G_{-i}]-2W_{-i}}{[(1+M)^{-1}+G_{-i}+1]^3} < 0$ because for $g_i = (1+M)^{-1}$, from (26) we have $(1+M)^{-1} > 1+G_{-i} - \frac{2W_{-i}}{s_i^k} \Leftrightarrow 2W_{-i} > s_i^k[1+G_{-i} - (1+M)^{-1}] \Leftrightarrow \frac{s_i^k}{W_{-i}} \leq \frac{2}{G_{-i}+\frac{M}{(1+M)}}$.

Therefore, we have shown that for $\frac{2}{G_{-i}} \geq \frac{s_i^k}{W_{-i}} \geq \frac{2}{G_{-i}+\frac{M}{(1+M)}}$, $V_i^k(g_i, g_{-i})$ is quasi-concave and has a single peak in $[(1+M)^{-1}, 1]$ which is $g_i^* = 1 + G_{-i} - \frac{2W_{-i}}{s_i^k}$; for $\frac{s_i^k}{W_{-i}} > \frac{2}{G_{-i}}$, $V_i^k(g_i, g_{-i})$ is increasing, quasi-concave and has a single peak at 1; and for $\frac{s_i^k}{W_{-i}} < \frac{2}{G_{-i}+\frac{M}{(1+M)}}$, and, otherwise, $V_i^k(g_i, g_{-i})$ is decreasing, quasi-concave and has a single peak at $(1+M)^{-1}$. Therefore, we have established that for every m_{-i} and hence g_{-i}, agent k_i representing shareholder k in firm i has single-peaked preferences over g_i.

Finally, we observe that for any fixed G_{-i}, g_i^* is increasing in $\frac{s_i^k}{W_{-i}}$, which suggests that $m_i^{k_i}(m_{-i})$ is decreasing in $\frac{s_i^k}{W_{-i}}$, for every fixed m_{-i}.

□

We find that the order of shareholders' ideal manager types regarding firm i follows the order of $\frac{s_i^k}{W_{-i}}$, which depends both on the shareholdings and on the manager types expected to be appointed in the other firms. That is, the problem is substantially more involved than the two firm case, but still well-behaved, in the sense that for any distribution of manager types of the other firms, there is always a Condorcet winner manager type in firm i.[32]

[32]Notice that for $I = 2$, we have that $\frac{s_i^k}{W_{-i}} = \frac{s_i^k}{s_j^k g_j}$. That is, the order of shareholders' ideal manager types regarding firm i follows the order of $\frac{s_i^k}{s_j^k}$, which a) does not depend on the manager type expected to be appointed in firm j, and b) it is a monotonic transformation of the relative interest of shareholder k in firm i.

7 Conclusion

In this paper we study the market effects of common ownership in a setting where any ownership structure and any shareholder size is allowed. We show that what matters for the anticompetitive effects of common ownership to emerge is not the size of individual shareholders but whether they collectively have control of the firm.

In our analysis we relax the ad hoc assumption of a control-weighted objective function adopted by the literature and instead study the collective choice problem of shareholders from primitives. In our model we focus on a duopolistic industry where shareholders are allowed to own different shares of each firm and are only interested in maximizing the value of their portfolio. Shareholders are represented by agents that participate and vote independently in shareholders meetings to elect firm managers by one-share-one-vote majority rule. Managers differ in their degree of aversion to the externalities of firm production, and after being elected they engage in Cournot product market competition.

Our main results are as follows. First, shareholders have single-peaked preferences over manager types, which implies that a unique manager type will collect the majority of votes and emerge as the (Condorcet) winner in each firm. The winning manager type corresponds to the ideal manager type of the median shareholder of that firm. Second, a shareholder's ideal manager type is higher the more diversified is the shareholder. We show that if the majority of votes is held by sufficiently diversified shareholders (so that the median shareholder is sufficiently diversified), firms will elect managers with strictly positive social concerns. These managers types will push output levels below the profit maximizing ones, softening competition and boosting industry level profits. Last, by endogenizing the ownership structure, i.e., allowing initially undiversified shareholders to trade shares and freely diversify their portfolio, we show that they will choose to fully diversify, i.e., acquire equal interest in both firms, which will lead to elect managers with relatively high social concerns, resulting in the monopoly outcome.

Our results have therefore the novel policy implication that competition might be hindered not only by single large investors holding controlling or substantial shares in firms but also by a multitude of small shareholders that collectively have control of the firms—even in presence of undiversified blockholders. This might be especially relevant in the context of some recent policy proposals that, in the attempt to contain the potential anticompetitive effects of common ownership, recommend to fragment institutional investors or to impose a cap on their holdings in a given industry.

Our analysis can be extended in a number of directions. For example, it would be relevant to show that results are robust to the relaxation of parametric assumptions, alternative oligopolistic settings (e.g., with product differentiation), as well as to alternative specifications of the manager's objective function, including the standard formulation based on weighted shareholders' portfolios.

A Appendix

Lemma 3 focuses on a set of parametrizations that lead to the choice of equilibrium manager types with strictly positive, non extreme, social concerns i.e., $0 < m_i^* < M, m_i^* = 1, 2$. We complement Lemma 3, by fully characterizing the equilibrium manager types for each admissible parametrization, $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}, M)$.

Lemma 5. *Let $M > 0$ and $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) \in (0, 1] \times [0, 1)$ be the relative interest of the median voter of firm 1 and firm 2 respectively in firm 1. Then managers will be chosen according to the following conditions.*

$$i) \text{ if } (\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) \in \left(0, \frac{2}{3+M}\right] \times \left[\frac{1+M}{3+M}, 1\right), \text{ then } m_1^* = M, m_2^* = M,$$

$$ii) \text{ if } (\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) \in \left[\frac{2}{3}, 1\right] \times \left[0, \frac{1}{3}\right], \text{ then } m_1^* = 0, m_2^* = 0,$$

$$iii) \text{ if } (\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) \in \left[\frac{2}{3}, 1\right] \times \left(\frac{1+2M}{3+4M}, 1\right), \text{ then } m_1^* = 0, m_2^* = M,$$

$$iv) \text{ if } (\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) \in \left[\frac{2}{3}, 1\right] \times \left[\frac{1}{3}, \frac{1+2M}{3+4M}\right), \text{ then } m_1^* = 0, 0 < m_2^* < M,$$

$$v) \text{ if } (\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) \in \left(0, \frac{M(\sigma^{k(\mu_2)} - 1) + 4\sigma^{k(\mu_2)} - 2}{2M(\sigma^{k(\mu_2)} - 1) + 6\sigma^{k(\mu_2)} - 3}\right] \times \left(\frac{1}{3}, \frac{1+M}{3+M}\right), \text{ then } m_1^* = M, 0 < m_2^* < M,$$

$$vi) \text{ if } (\sigma^{k(\mu_1)}, \sigma^{k(\mu_2}) \in \left(\frac{2+2M}{3+4M}, \frac{2}{3}\right) \times \left[0, \frac{1}{3}\right], \text{ then } 0 < m_1^* < M, m_2^* = 0,$$

$$vii) \text{ if } (\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) \in \left(0, \frac{2+2M}{3+4M}\right) \times \left[0, \frac{1}{3}\right], \text{ then } m_1^* = M, m_2^* = 0,$$

$$viii) \text{ if } (\sigma^{k(\mu_1)}, \sigma^{k(\mu_2}) \in \left(\frac{2}{3+M}, \frac{2}{3},\right) \times \left(\frac{(2+M)\sigma^{k(\mu_1)} - 1}{2(3+M)\sigma^{k(\mu_1)} - 3}, 1\right), \text{ then } 0 < m_1^* < M, m_2^* = M.$$

$$ix) \text{ if } (\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) \text{ does not satisfy any of the conditions above, then } 0 < m_1^* < M, 0 < m_2^* < M.$$

Proof. From Lemma 1 we have the following possible configurations of equilibrium manager types:

Case i): $m_1^* = m_2^* = M$.

If $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2}) \in \left(0, \frac{2}{4+m_2}\right] \times \left[\frac{2+m_1}{4+m_1}, 1\right)$, then according to (9) and (10) $m_1 = m_2 = M$, and therefore $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2}) \in \left(0, \frac{2}{4+M}\right] \times \left[\frac{2+M}{4+M}, 1\right)$.

If $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2}) \in \left(0, \frac{2}{4+m_2}\right] \times \left(\frac{1}{3}, \frac{2+m_1}{4+m_1}\right)$, then $m_2 = \min\{\frac{1-3\sigma^{k(\mu_2)}}{4\sigma^{k(\mu_2)}+\sigma^{k(\mu_2)}m_1-m_1-2}, M\}$ and $m_1 = M$. If $M < \frac{1-3\sigma^{k(\mu_2)}}{4\sigma^{k(\mu_2)}+\sigma^{k(\mu_2)}M-M-2}$, then $m_2 = M$. Solving the latter inequality for $\sigma^{k(\mu_2)} \in \left(\frac{1}{3}, \frac{2+M}{4+M}\right)$, we obtain $\sigma^{k(\mu_2)} \in \left(\frac{1+M}{3+M}, \frac{2+M}{4+M}\right)$. So if $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2}) \in \left(0, \frac{2}{4+M}\right] \times \left(\frac{1+M}{3+M}, \frac{2+M}{4+M}\right)$, then $m_1^* = m_2^* = M$.

If $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) \in \left(\frac{2}{4+m_2}, \frac{2}{3}\right) \times \left[\frac{2+m_1}{4+m_1}, 1\right)$, then $m_1 = \min\left\{\frac{2-3\sigma^{k(\mu_1)}}{4\sigma^{k(\mu_1)}+\sigma^{k(\mu_1)}m_2-2}, M\right\}$ and $m_2 = M$. If $M < \frac{2-3\sigma^{k(\mu_1)}}{4\sigma^{k(\mu_1)}+\sigma^{k(\mu_1)}M-2}$, then $m_1 = M$. Solving the latter inequality for $\sigma^{k(\mu_1)} \in \left(\frac{2}{4+M}, \frac{2}{3}\right)$, we obtain $\sigma^{k(\mu_1)} \in \left(\frac{2}{4+M}, \frac{2}{3+M}\right)$. So if $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) \in \left(\frac{2}{4+M}, \frac{2}{3+M}\right) \times \left[\frac{2+M}{4+M}, 1\right)$, then $m_1^* = m_2^* = M$.

If $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) \in \left(\frac{2}{4+m_2}, \frac{2}{3}\right) \times \left(\frac{1}{3}, \frac{2+m_1}{4+m_1}\right)$, then $m_1 = \min\left\{\frac{2-3\sigma^{k(\mu_1)}}{4\sigma^{k(\mu_1)}+\sigma^{k(\mu_1)}m_2-2}, M\right\}$ and $m_2 = \min\{\frac{1-3\sigma^{k(\mu_2)}}{4\sigma^{k(\mu_2)}+\sigma^{k(\mu_2)}m_1-m_1-2}, M\}$. If $\frac{2-3\sigma^{k(\mu_1)}}{4\sigma^{k(\mu_1)}+\sigma^{k(\mu_1)}m_2-2} > M$ and $\frac{1-3\sigma^{k(\mu_2)}}{4\sigma^{k(\mu_2)}+\sigma^{k(\mu_2)}m_1-m_1-2} > M$, then $m_1 = m_2 = M$. Solving the latter system of inequalities for $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2}) \in \left(\frac{2}{4+M}, \frac{2}{3}\right) \times \left(\frac{1}{3}, \frac{2+M}{4+M}\right)$, we obtain $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2}) \in \left(\frac{2}{4+M}, \frac{2}{3+M}\right) \times \left(\frac{1+M}{3+M}, \frac{2+M}{4+M}\right)$.

Case ii): $m_1^* = m_2^* = 0$.

According to (9) and (10) this case occurs when $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2}) \in \left[\frac{2}{3}, 1\right] \times \left[0, \frac{1}{3}\right]$. Moreover for $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2}) \in \left(\frac{2}{4+m_2}, \frac{2}{3}\right) \times \left(\frac{1}{3}, \frac{2+m_1}{4+m_1}\right)$ it is not possible that both peaks are negative, i.e. $m_1 = \frac{2-3\sigma^{k(\mu_1)}}{4\sigma^{k(\mu_1)}-2} < 0$ and $m_2 = \frac{1-3\sigma^{k(\mu_2)}}{4\sigma^{k(\mu_2)}-2} < 0$ or one peak is zero and the other is negative. That is, for $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2}) \in \left(\frac{2}{4+m_2}, \frac{2}{3}\right) \times \left(0, \frac{1}{3}\right]$ it cannot be that $m_1 = \frac{2-3\sigma^{k(\mu_1)}}{4\sigma^{k(\mu_1)}-2} < 0$ and $m_2 = 0$. Also, for $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2}) \in \left[\frac{2}{3}, 1\right] \times \left(\frac{1}{3}, \frac{2+m_1}{4+m_1}\right)$ it is not possible that $m_1 = 0$ and $m_2 = \frac{1-3\sigma^{k(\mu_2)}}{4\sigma^{k(\mu_2)}-2} < 0$.

Case iii): $m_1^* = 0$ and $m_2^* = M$.

If $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) \in \left[\frac{2}{3}, 1\right] \times \left[\frac{1}{2}, 1\right])$, then from (9) and (10) we have $m_1^* = 0$ and $m_2^* = M$. Also, if $\sigma^{k(\mu_2)} \in \left(\frac{1}{3}, \frac{2+m_1}{4+m_1}\right)$ and $\frac{1-3\sigma^{k(\mu_2)}}{4\sigma^{k(\mu_2)}-2} > M$, then $m_1 = 0$ and $m_2 = M$. Solving the latter inequality for $\sigma^{k(\mu_2)} \in \left(\frac{1}{3}, \frac{1}{2}\right)$ we obtain $\sigma^{k(\mu_2)} \in \left(\frac{1+2M}{3+4M}, \frac{1}{2}\right)$. Hence, if $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) \in \left[\frac{2}{3}, 1\right] \times \left(\frac{1+2M}{3+4M}, 1\right)$, then $m_1^* = 0$ and $m_2^* = M$. Moreover, if $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2}) \in \left(\frac{2}{4+m_2}, \frac{2}{3}\right) \times \left(\frac{2+m_1}{4+m_1}, 1\right)$, then the peak $m_1 = \frac{2-3\sigma^{k(\mu_1)}}{4\sigma^{k(\mu_1)}-2}$ cannot be negative and if $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) \in \left(\frac{2}{4+m_2}, \frac{2}{3}\right) \times \left(\frac{1}{3}, \frac{2+m_1}{4+m_1}\right)$, then the peak $m_1 = \frac{2-3\sigma^{k(\mu_1)}}{4\sigma^{k(\mu_1)}-2}$ cannot be negative and the peak $m_2 = \frac{1-3\sigma^{k(\mu_2)}}{4\sigma^{k(\mu_2)}-2}$ cannot be greater than M.

Case iv): $m_1^* = 0$ and $0 < m_2^* < M$.

According to (9), if $\sigma^{k(\mu_1)} \in \left[\frac{2}{3}, 1\right]$ then $m_1^* = 0$, while according to (10), if $\sigma^{k(\mu_2)} \in \left(\frac{1}{3}, \frac{1}{2}\right)$ then $m_2^*(0) = \frac{1-3\sigma^{k(\mu_2)}}{4\sigma^{k(\mu_2)}-2} < M$ which is true if $\sigma^{k(\mu_2)} \in \left(\frac{1}{3}, \frac{1+2M}{3+4M}\right)$. Moreover the optimal manager type $\frac{2-3\sigma^{k(\mu_1)}}{4\sigma^{k(\mu_1)}+\sigma^{k(\mu_1)}m_2-2}$ cannot be negative for $\sigma^{k(\mu_1)} \in \left(\frac{2}{4+m_2}, \frac{2}{3}\right)$ and $0 \le m_2 \le M$. Therefore if $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2}) \in \left[\frac{2}{3}, 1\right] \times \left[\frac{1}{3}, \frac{1+2M}{3+4M}\right)$, then $m_1^* = 0$ and $0 < m_2^* < M$.

Case v): $m_1^* = M$ and $0 < m_2^* < M$.

If $\sigma^{k(\mu_1)} \in \left(0, \frac{2}{4+m_2}\right]$, then from (9) $m_1^* = M$. In (10), for $m_1 = M$, if $\sigma^{k(\mu_2)} \in \left(\frac{1}{3}, \frac{2+M}{4+M}\right)$, then $m_2(M) = \frac{1-3\sigma^{k(\mu_2)}}{4\sigma^{k(\mu_2)}+\sigma^{k(\mu_2)}M-M-2} < M$. Solving the latter inequality for $\sigma^{k(\mu_2)}$ we obtain $\sigma^{k(\mu_2)} \in \left(\frac{1}{3}, \frac{1+M}{3+M}\right]$. Substituting $m_2 = \frac{1-3\sigma^{k(\mu_2)}}{4\sigma^{k(\mu_2)}+\sigma^{k(\mu_2)}M-M-2}$ in $\sigma^{k(\mu_1)} \in \left(0, \frac{2}{4+m_2}\right]$ and taking into account $\sigma^{k(\mu_2)} \in \left(\frac{1}{3}, \frac{1+M}{3+M}\right)$ we obtain $\sigma^{k(\mu_1)} \in \left(0, \frac{2M(\sigma^{k(\mu_2)}-1)+8\sigma^{k(\mu_2)}-4}{4M(\sigma^{k(\mu_2)}-1)+13\sigma^{k(\mu_2)}-7}\right]$. Also, $m_1^* = M$ occurs when $\sigma^{k(\mu_1)} \in \left(\frac{2}{4+m_2}, \frac{2}{3}\right)$ and $\frac{2-3\sigma^{k(\mu_1)}}{4\sigma^{k(\mu_1)}+\sigma^{k(\mu_1)}m_2-2} > M$. $m_2(M) = \frac{1-3\sigma^{k(\mu_2)}}{4\sigma^{k(\mu_2)}+\sigma^{k(\mu_2)}M-M-2} < M$ when $\sigma^{k(\mu_2)} \in \left(\frac{1}{3}, \frac{2+M}{4+M}\right)$. The inequalities are compatible when

$\sigma^{k(\mu_2)} \in \left(\frac{1}{3}, \frac{1+M}{3+M}\right)$ and $\sigma^{k(\mu_1)} \in \left(\frac{2M(\sigma^{k(\mu_2)}-1)+8\sigma^{k(\mu_2)}-4}{4M(\sigma^{k(\mu_2)}-1)+13\sigma^{k(\mu_2)}-7}, \frac{M(\sigma^{k(\mu_2)}-1)+4\sigma^{k(\mu_2)}-2}{2M(\sigma^{k(\mu_2)}-1)+6\sigma^{k(\mu_2)}-3}\right]$. Therefore if $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) \in \left(0, \frac{M(\sigma^{k(\mu_2)}-1)+4\sigma^{k(\mu_2)}-2}{2M(\sigma^{k(\mu_2)}-1)+6\sigma^{k(\mu_2)}-3}\right] \times \left(\frac{1}{3}, \frac{1+M}{3+M}\right)$, then $m_1^* = M$ and $0 < m_2^* < M$.

Case *vi)*: $0 < m_1^* < M$ and $m_2^* = 0$.

According to (10), if $\sigma^{k(\mu_2)} \in \left[0, \frac{1}{3}\right]$ then $m_2^* = 0$, while according to (9), if $\sigma^{k(\mu_1)} \in \left(\frac{1}{2}, \frac{2}{3}\right)$ then $m_1^*(0) = \frac{2-3\sigma^{k(\mu_1)}}{4\sigma^{k(\mu_1)}-2} < M$ which is true if $\sigma^{k(\mu_1)} \in \left(\frac{2+2M}{3+4M}, \frac{2}{3}\right)$. Moreover, the optimal manager type $\frac{1-3\sigma^{k(\mu_2)}}{4\sigma^{k(\mu_2)}+\sigma^{k(\mu_2)}m_1-m_1-2}$ cannot be negative for $\sigma^{k(\mu_2)} \in \left(\frac{1}{3}, \frac{2+m_1}{4+m_1}\right)$ and $0 \le m_1 \le M$. Therefore if $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) \in \left(\frac{2+2M}{3+4M}, \frac{2}{3}\right) \times \left[0, \frac{1}{3}\right]$, then $0 < m_1^* < M$ and $m_2^* = 0$.

Case *vii)*: $m_1^* = M$ and $m_2^* = 0$.

If $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) \in \left(0, \frac{1}{2}\right] \times \left[0, \frac{1}{3}\right]$, then from (9) and (10) we have $m_1^* = M$ and $m_2^* = 0$. Also, if $\sigma^{k(\mu_1)} \in \left(\frac{2}{4+m_2}, \frac{2}{3}\right)$ and $\frac{2-3\sigma^{k(\mu_1)}}{4\sigma^{k(\mu_1)}-2} > M$, then $m_1 = M$ and $m_2 = 0$. Solving the latter inequality for $\sigma^{k(\mu_1)} \in \left(\frac{1}{2}, \frac{2}{3}\right)$ we obtain $\sigma^{k(\mu_1)} \in \left(\frac{1}{2}, \frac{2+2M}{3+4M}\right)$. Hence, if $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) \in \left(0, \frac{2+2M}{3+4M}\right) \times \left[0, \frac{1}{3}\right]$, then $m_1^* = M$ and $m_2^* = 0$. Moreover, if $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) \in \left(0, \frac{2}{4+m_2}\right] \times \left(\frac{1}{3}, \frac{2+m_1}{4+m_1}\right)$, then the peak $m_2 = \frac{1-3\sigma^{k(\mu_2)}}{4\sigma^{k(\mu_2)}-2}$ cannot be negative and if $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) \in \left(\frac{2}{4+m_2}, \frac{2}{3}\right) \times \left(\frac{1}{3}, \frac{2+m_1}{4+m_1}\right)$, then the peak $m_1 = \frac{2-3\sigma^{k(\mu_1)}}{4\sigma^{k(\mu_1)}-2}$ cannot be greater than M and the peak $m_2 = \frac{1-3\sigma^{k(\mu_2)}}{4\sigma^{k(\mu_2)}-2}$ cannot be negative.

Case *viii)*: $0 < m_1^* < M$ and $m_2^* = M$.

If $\sigma^{k(\mu_2)} \in \left[\frac{2+m_1}{4+m_1}, 1\right)$, then from (10) $m_2^* = M$. In (9), for $m_2 = M$, if $\sigma^{k(\mu_1)} \in \left(\frac{2}{4+M}, \frac{2}{3},\right)$, then $m_1(M) = \frac{2-3\sigma^{k(\mu_1)}}{4\sigma^{k(\mu_1)}+\sigma^{k(\mu_1)}M-2} < M$. Solving the latter inequality for $\sigma^{k(\mu_1)}$ we obtain $\sigma^{k(\mu_1)} \in \left(\frac{2}{3+M}, \frac{2}{3}\right]$. Substituting $m_1 = \frac{2-3\sigma^{k(\mu_1)}}{4\sigma^{k(\mu_1)}+\sigma^{k(\mu_1)}M-2}$ in $\sigma^{k(\mu_2)} \in \left[\frac{2+m_1}{4+m_1}, 1\right)$ and taking into account $\sigma^{k(\mu_1)} \in \left(\frac{2}{3+M}, \frac{2}{3}\right]$ we obtain $\sigma^{k(\mu_2)} \in \left[\frac{(5+2M)\sigma^{k(\mu_1)}-2}{(13+4M)\sigma^{k(\mu_1)}-6}, 1\right)$. Also, $m_2^* = M$ occurs when $\sigma^{k(\mu_2)} \in \left(\frac{1}{3}, \frac{2+m1}{4+m_1}\right)$ and $\frac{1-3\sigma^{k(\mu_2)}}{4\sigma^{k(\mu_2)}+\sigma^{k(\mu_1)}m_1-m_1-2} > M$. $m_1(M) = \frac{2-3\sigma^{k(\mu_1)}}{4\sigma^{k(\mu_1)}+\sigma^{k(\mu_1)}M-2} < M$ when $\sigma^{k(\mu_1)} \in \left(\frac{2}{4+M}, \frac{2}{3}\right)$. The inequalities are compatible when $\sigma^{k(\mu_1)} \in \left(\frac{2}{3+M}, \frac{2}{3},\right)$ and $\sigma^{k(\mu_2)} \in \left(\frac{(2+M)\sigma^{k(\mu_1)}-1}{2(3+M)\sigma^{k(\mu_1)}-3}, \frac{(5+2M)\sigma^{k(\mu_1)}-2}{(13+4M)\sigma^{k(\mu_1)}-6}\right)$. Therefore if $(\sigma^{k(\mu_1)}, \sigma^{k(\mu_2)}) \in \left(\frac{2}{3+M}, \frac{2}{3},\right) \times \left(\frac{(2+M)\sigma^{k(\mu_1)}-1}{2(3+M)\sigma^{k(\mu_1)}-3}, 1\right)$, then $0 < m_1^* < M$ and $m_2^* = M$.

$\square$

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