The Deposits Channel of Monetary Policy
A Critical Review

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Abstract

Drechsler, Savov, and Schnabl (2017) claim that increases in the monetary policy rate lead to reductions in bank deposits, which account for the subsequent contraction in lending. This paper reviews their theoretical analysis, showing that the relationship between the policy rate and the equilibrium amount of deposits is either flat or upward sloping in the relevant range. Then, it constructs an alternative model, based on a simple microfoundation for the households' demand for deposits, where an increase in the policy rate always increases the equilibrium amount of deposits. These results question the theoretical underpinnings of the "deposits channel" of monetary policy transmission.

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"When the Fed funds rate rises, banks widen the interest spreads they charge on deposits, and deposits flow out of the banking system. Since banks rely heavily on deposits for their funding, these outflows induce a contraction in lending."

Drechsler, Savov, and Schnabl (2017)

1 Introduction

The paper by Drechsler, Savov, and Schnabl (2017), henceforth DSS, presents an interesting empirical analysis of the effect of changes in the policy rate on the amount of bank deposits in local markets characterized by different degrees of market power. In particular, they show in a panel regression that increases in the Federal funds rate lead to negative changes in deposits at bank branches located in concentrated counties relative to those in less concentrated counties. This result is then used to propose a novel explanation of the effect of monetary policy on bank lending, called the "deposits channel" of monetary policy. In their words: "Deposits are the main source of funding for banks. Their stability makes them particularly well suited for funding risky and illiquid assets. As a result, when banks contract deposit supply they also contract lending." The paper also constructs a theoretical model of imperfect competition in a local banking market which can account for their empirical findings.

This paper presents a critical review of DSS's theoretical model, showing that showing that the relationship between the policy rate and the equilibrium amount of deposits is either flat or upward sloping in the relevant range. Since their model does not yield simple analytical solutions, I construct an alternative model of imperfect competition in a local banking market, based on a simple microfoundation for the households' demand for deposits, which is consistent with their panel results and contradicts their deposits channel claim. In this model, increases in the policy rate always increase the equilibrium amount of deposits.

Before going into the details of the original and the alternative model, I would like to briefly comment on DSS's empirical results. Their paper starts presenting some suggestive time series evidence showing a negative correlation between changes in the Federal funds rate and changes in various types of bank deposits. But since business cycle developments may be driving all these variables, they propose an identification strategy that relies on panel data on deposit rates and deposit holdings at bank branches. Exploiting the variation in market power at the county level, they compare the effect of changes in the Federal funds rate in branches of the same bank located in different counties.

The key panel regressions have as dependent variables (i) the quarterly change in the spread s_{it} between the Federal funds rate r_t and the deposit rate of a bank's branch i, and (ii) the annual log change in the deposits D_{it} of branch i. The main explanatory variable is an interaction term between the change in the Federal funds rate r_t and the Herfindahl index HHI_i computed with the deposit market shares of the banks operating in the county where branch i is located. Thus, they estimate the following equation

$$\Delta y_{it} = \alpha_i + \gamma(\Delta r_t \times \text{HHI}_i) + \text{ Controls } + \varepsilon_{it}, \tag{1}$$

where Δy_{it} is either Δs_{it} or $\Delta \ln D_{it}$, and the controls include bank-time fixed effects which take care of time-varying differences between banks. Bank-specific characteristics (such as lending opportunities) are controlled by comparing branches of the same bank in counties with different concentrations.

The results for the *spreads equation* show that γ_s is positive and statistically significant, which means that an increase in the Federal funds rate leads to larger changes in spreads (and smaller changes in deposit rates) at branches located in high concentration counties. The results for the *deposits equation* show that γ_D is negative and statistically significant, which means that an increase in the Federal funds rate leads to smaller changes in deposits at branches located in high concentration counties.[1]

The problem arises with the particular interpretation by DSS of these empirical results. They write: "Following an increase in the Federal funds rate, the bank's branches in more concentrated counties (...) experience *larger outflows* relative to its branches in less concentrated counties" (my italics). But they could have equally written: "Following an increase in the Federal funds rate, the bank's branches in more concentrated counties (...) experience *smaller inflows* relative to its branches in less concentrated counties."

[1] However, it should be noted that Begenau and Stafford (2022) criticize these results because of widespread use of uniform deposit price setting policies among US commercial banks.

More importantly, from here they jump to the conclusion that these results imply that "when the Federal funds rate rises, banks widen the interest spreads they charge on deposits, and *deposits flow out of the banking system*" (my italics). Put it differently, the fact that coefficient γ_D is negative and statistically significant in the panel equation for deposits does not imply that increases in the Federal funds rate lead to reductions in the aggregate amount of deposits.

My claim is then that the deposits channel of monetary policy does not follow from the empirical results in DSS. In fact, it does not follow from their theoretical model either, which is the focus of this paper to which I turn now.

DSS's model features a representative household with an initial wealth that can be invested in three types of assets: cash that pays a zero interest rate, deposits of a set of n banks that pay the equilibrium deposit rate, and bonds that pay the policy rate set by the central bank. The demand for bank deposits is derived from a utility function that depends on final wealth and liquidity services provided by cash and deposits. Banks offer differentiated deposits and compete à la Bertrand by setting deposit rates, or equivalently spreads between the policy rate and the deposit rate. Equilibrium spreads are derived from the symmetric Nash equilibrium of the game played by the banks. Armed with this framework, the question is then what is the effect on equilibrium spreads and deposits of an exogenous change in the policy rate.

The model is somewhat complicated, and DSS derive results for the limit case in which the weight of liquidity services in the household's utility function goes to zero. My approach to the analysis of their model does not require this assumption, and starts with the simple case of a monopoly bank. In this case, I show that increases in the policy rate increase equilibrium spreads, but contrary to DSS's claim they have no effect on equilibrium deposits for low rates and a positive effect on equilibrium deposits for higher rates in the relevant range.

To understand the reason for these results, note that when the policy rate tends to zero, the deposit rate converges to the zero interest on cash. But since both cash and deposits yield utility, as rates tend to zero it would be optimal for the household to short the bond in order to invest more than her initial wealth in liquid assets. However, if borrowing at the policy rate is not feasible, the household would have to invest all her wealth in cash and deposits. For sufficiently low values of the policy rate, the monopoly bank would set a zero deposit rate, in which case over this range the equilibrium amount of cash and deposits would not change with the policy rate. Beyond this threshold, deposit rates would start to go up, and the household would increase her holdings of deposits (and reduce her holdings of cash). For sufficiently high rates, the household may start to invest in bonds, in which case I show that further increases in the policy rate have ambiguous effects on equilibrium deposits. Specifically, there is a negative substitution effect due to the increase in spreads and a positive income effect due to higher return to the household's initial wealth.

Next, I consider the general model with n banks, showing that the results are in line with those of the monopoly model. Moreover, since bank competition will tend to increase deposit rates, the range of values of the policy rate over which the household will not hold bonds will widen, extending the positive relationship between the policy rate and the equilibrium amount of deposits.

It should be noted that, due to the complexity of the model, many of the previous results rely on numerical solutions. To verify their robustness, I construct a simple model of Cournot competition in a local banking market for which analytical results can be derived.

The model has a continuum of heterogeneous households that differ in a utility premium associated with liquid assets. As before, households can invest their initial wealth in cash that pays a zero interest rate, bank deposits that pay the equilibrium deposit rate, and bonds that pay the policy rate set by the central bank. Assuming that cash provides higher liquidity services than deposits, I derive the households' aggregate demand for deposits as a function of the spread between the policy rate and the deposit rate, and then compute the corresponding Cournot equilibrium for a deposit market with n banks.

The analytical results of the alternative model show that, in line with the empirical results in DSS, the equilibrium spread s^* satisfies

$$\gamma_s = \frac{\partial^2 s^*}{\partial r \partial \mathrm{HHI}} > 0, \tag{2}$$

and the equilibrium amount of deposits D^* satisfies

$$\gamma_D = \frac{\partial^2 D^*}{\partial r \partial \mathrm{HHI}} < 0 \tag{3}$$

where $\mathrm{HHI} = 1/n$ is the Herfindahl index for a market with n identical banks. Moreover, it is also the case that, contrary to DSS's claim, D^* is always increasing in the policy rate r.

This paper is related to the growing literature on the transmission of monetary policy when banks have market power; see, for example, Corbae and Levine (2018), Wang et al. (2022), and Abadi et al. (2023). It is also related to papers that analyze the pass-through from policy rates to deposits and loan rates; see, for example, Eggertsson et al. (2019) and Ulate (2021). In terms of results, it is closer to the papers that emphasize the heterogeneous effects of monetary policy, such as Kashyap and Stein (2000), for large and small banks, Jiménez et al. (2012), for banks with different levels of capital, Martinez-Miera and Repullo (2020), for banks with different degrees of market power, and Heider et al. (2019) and Repullo (2020), for banks with different balance sheet structures.

The remainder of the paper is structured as follows. Section 2 presents my critical review of DSS's theoretical model, starting with the monopoly case and then analyzing the general oligopoly case. Section 3 presents the alternative Cournot model. Section 4 concludes.

2 Review of DSS's Model

Consider a representative household with *initial wealth* W_0 and preferences described by a CES utility function over *final wealth* W and *liquidity services* l such that

$$U(W, L) = \left(W^{\frac{\rho-1}{\rho}} + (\lambda L)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}, \tag{4}$$

where $\lambda > 0$ captures the utility of liquidity services relative to final wealth. Following DSS, it is assumed that final wealth and liquidity services are complements, so the elasticity of substitution satisfies $0 < \rho < 1$.

The utility function in (4) is more appealing than the one in DSS (where λ is raised to the power of 1), since it implies that when the elasticity of substitution ρ tends to zero we get a

Leontief utility function $U(W, L) = \min\{W, \lambda L\}$ in which liquidity services are a proportion $1/\lambda$ of final wealth.

Liquidity services L are derived from a CES function over *cash* M and *deposits* D such that

$$L(M, D) = \left(M^{\frac{\epsilon-1}{\epsilon}} + (\delta D)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}, \tag{5}$$

where $\delta > 0$ captures the liquidity of deposits relative to cash. Following DSS, it is assumed that cash and deposits are substitutes, so the elasticity of substitution satisfies $\epsilon > 1$.

Finally, deposits D are a composite good produced by a set of n banks according to

$$D = \left(\frac{1}{n}\sum_{i=1}^{n}(nD_i)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}, \tag{6}$$

where D_i are the deposits of bank $i = 1, 2, ..., n$. Following DSS, it is assumed that the deposits of the different banks are substitutes, so the elasticity of substitution satisfies $\eta > 1$.

The function in (6) is slightly different from the one in DSS, which is

$$D = \left(\frac{1}{n}\sum_{i=1}^{n} D_i^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}. \tag{7}$$

In this expression $D_1 = ... = D_n$ implies $D = D_1 = ... = D_n$, so D_i cannot be interpreted as the deposits of bank i. In contrast, with the function in (6), $D_1 = ... = D_n$ implies $D = D_1 + ... + D_n$.

The representative household can invest her initial wealth W_0 in three types of assets: *cash* M that pays a zero interest rate, *deposits* D_i of bank $i = 1, 2, ...n$ that pay an interest rate r_i, and *bonds* that pay an interest rate r equal to the monetary policy rate set by the central bank. Final wealth W is then given by

$$W = M + \sum_{i=1}^{n} D_i(1 + r_i) + (W_0 - M - \sum_{i=1}^{n} D_i)(1 + r). \tag{8}$$

Letting $r_i = r - s_i$, where s_i is the spread charged by bank i, final wealth simplifies to

$$W = W_0(1 + r) - \sum_{i=1}^{n} D_i s_i - Mr. \tag{9}$$

According to this expression, final wealth W equals the market return of the initial wealth $W_0(1 + r)$ minus the opportunity cost of deposit holdings $\sum_{i=1}^{n} D_i s_i$ and the opportunity cost of cash holdings Mr.

To simplify the analysis, in what follows I assume that the parameters of the liquidity services function take the values $\delta = 1$ and $\epsilon = 2$, so (5) becomes

$$L(M, D) = \left(M^{\frac{1}{2}} + D^{\frac{1}{2}}\right)^2. \tag{10}$$

To review the results of DSS's model, it is convenient to start with the case of a monopoly bank ($n = 1$).

2.1 The model with a monopoly bank

To assess the effect of a change in the monetary policy rate r on the amount of deposits D the household wants to hold, one has to determine the equilibrium spread s^* set by the monopolist, which in turn requires deriving the household's demand for deposits as a function of the policy rate r and the spread s.

To derive the demand for deposits faced by the monopoly bank, let

$$X = Mr + Ds \tag{11}$$

denote the opportunity cost of the liquidity held by the household. The optimal way to allocate X between cash M and deposits D is obtained by solving

$$\max_{M,D} \left(M^{\frac{1}{2}} + D^{\frac{1}{2}}\right)^2 \tag{12}$$

subject to (11). The first-order conditions that characterize the solution to this problem are

$$\left(M^{\frac{1}{2}} + D^{\frac{1}{2}}\right) M^{-\frac{1}{2}} = \mu r, \tag{13}$$

$$\left(M^{\frac{1}{2}} + D^{\frac{1}{2}}\right) D^{-\frac{1}{2}} = \mu s, \tag{14}$$

where μ denotes the Lagrange multiplier associated with the constraint. Dividing (13) by (14) gives

$$\left(\frac{D}{M}\right)^{\frac{1}{2}} = \frac{r}{s}. \tag{15}$$

Substituting (15) into (13) and solving for the Lagrange multiplier μ gives

$$\mu = \frac{1}{r} + \frac{1}{s}. \tag{16}$$

Solving for M in (15) substituting the result into (11) and solving for D implies

$$D = \frac{X}{\mu s^2}. \tag{17}$$

And from here it follows that

$$M = \frac{X}{\mu r^2}. \tag{18}$$

Substituting these results into the liquidity services function (10) gives

$$L = \frac{X}{\mu}\left(\frac{1}{r} + \frac{1}{s}\right)^2 = \mu X. \tag{19}$$

Now, substituting (11) into (9) final wealth becomes

$$W = W_0(1+r) - X. \tag{20}$$

Finally, substituting (19) and (20) into the household's utility function (4) yields the following maximization problem

$$\max_X \left[(W_0(1+r) - X)^{\frac{\rho-1}{\rho}} + (\lambda\mu X)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}. \tag{21}$$

The first-order condition that characterizes the solution to this problem is

$$(W_0(1+r) - X)^{-\frac{1}{\rho}} = \lambda\mu(\lambda\mu X)^{-\frac{1}{\rho}}, \tag{22}$$

which implies

$$X = \frac{W_0(1+r)}{1 + (\lambda\mu)^{1-\rho}}. \tag{23}$$

Substituting this result into (17) gives the following demand for deposits faced by the monopoly bank

$$D(s;r) = \frac{W_0(1+r)}{\mu s^2[1 + (\lambda\mu)^{1-\rho}]}, \tag{24}$$

It can be checked that $\partial D/\partial s < 0$, so the demand function is decreasing in the spread s, and that $\partial D/\partial r > 0$, so an increase in the policy rate r leads to an outward shift in the demand for deposits.

Assuming that the monopoly bank earns the policy rate r on its investments, and given that the deposit rate is $r - s$, it follows that its profits are

$$\pi(s;r) = [r - (r-s)]D(s;r) = sD(s;r). \tag{25}$$

To maximize profits for any given value of the policy rate r, the monopoly bank chooses the equilibrium spread

$$s^*(r) = \arg\max_{0\leq s\leq r}[sD(s;r)], \tag{26}$$

which implies the following equilibrium amount of deposits

$$D^*(r) = D(s^*(r);r). \tag{27}$$

However, it is important to note that if the deposit rate $r - s$ cannot be negative, the demand for deposits becomes unbounded for low values of the policy rate r. To show this, note that $0 \leq s \leq r$ implies $\lim_{r\to 0} s = 0$ and $\lim_{r\to 0}(s/r) \leq 1$, so for $0 < \rho < 1$ the denominator of (24) satisfies

$$\lim_{r\to 0}[\mu s^2[1 + (\lambda\mu)^{1-\rho}]] = \lim_{r\to 0}\left[s\left(\frac{s}{r}+1\right) + \lambda^{1-\rho}s^{\rho}\left(\frac{s}{r}+1\right)^{2-\rho}\right] = 0. \tag{28}$$

Hence, for low values of r the constraint $M + D \leq W_0$ is always violated. This is easy to explain: if cash and deposits yield valuable liquidity services and (in the limit) pay the same return $r = 0$ as bonds, the household would want to short an infinite amount of bonds to invest the proceeds in the liquid assets.

The constraint $M + D \leq W_0$ may also be violated for low values of the parameter λ that captures the utility of liquidity services relative to final wealth, and high values of the parameter ρ that captures the elasticity of substitution between final wealth and liquidity services. This follows from the fact that, by (17) and (18), the demands for deposits and cash are increasing in the opportunity cost X of the liquidity held by the household, which by (23) is decreasing in λ and increasing in ρ.[2] The intuition for the first result is that increases in λ increase the utility (relative to final wealth) of a given level of liquidity services, reducing its demand. The intuition for the second result is that, starting from the Leontief case $\rho = 0$ where liquidity services are a proportion $1/\lambda$ of final wealth, and given that the budget constraint (9) implies that the "prices" of cash r and deposits $s \leq r$ are small, increases in ρ lead to increases in the amount invested in liquid assets.

[2] The latter result requires that $\lambda\mu > 1$, which holds for plausible values of λ (greater than 1) and plausible values of the policy rate r (smaller than 100%), which by (16) imply $\mu > 1$.

Figure 1 illustrates these results by showing the relationship between the policy rate r (in the horizontal axis) and the equilibrium amounts of invested in liquid assets $\mathcal{L}^*(r) = M^*(r) + D^*(r)$ (in the vertical axis). Panel A shows that for $\rho = 0.5$ the higher the value of λ the lower $\mathcal{L}^*(r)$ for any value of r. Panel B shows that for $\lambda = 4$ the higher the value of ρ the higher $\mathcal{L}^*(r)$ for any value of r.

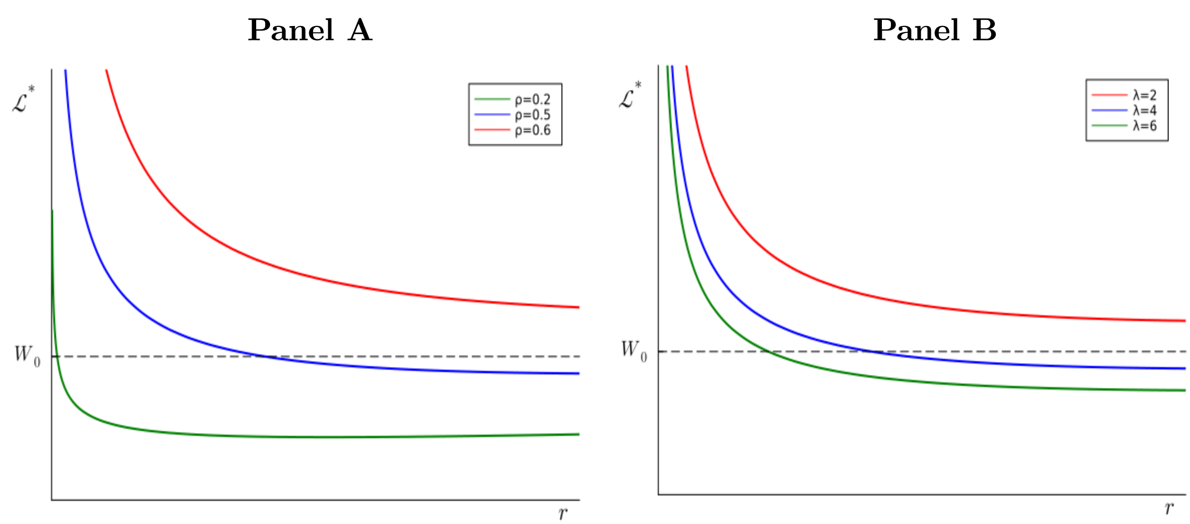

Figure 1. Equilibrium amounts invested in liquid assets

Panel A shows the relationship between the equilibrium amounts invested in liquid assets (cash and deposits) and the policy rate for lambda = 4 and several values of rho, and Panel B the relationship between the equilibrium amounts invested in liquid assets (cash and deposits) and the policy rate for rho = 0.5 and several values of lambda.

The conclusion is that for low values of the policy rate r, low values of the parameter λ that captures the utility of liquidity services relative to final wealth, and high values of the parameter ρ that captures the elasticity of substitution between final wealth and liquidity services the constraint $M + D \leq W_0$ is violated. Hence, to complete the analysis of the monopoly model I next analyze the case where the household's choice of cash and deposits is restricted to satisfy the constraint $M + D \leq W_0$.

Consider first a case in which λ and ρ are such that the constraint $M + D \leq W_0$ is always binding. Since final wealth is $W = M + D(1 + r_1) = W_0 + Dr_1$, where r_1 denotes the deposit rate set by the monopoly bank, the problem of the household may be written as

$$\max_D \left[(W_0 + Dr_1)^{\frac{\rho-1}{\rho}} + \left[\lambda \left((W_0 - D)^{\frac{1}{2}} + D^{\frac{1}{2}} \right)^2 \right]^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}. \tag{29}$$

The first-order condition is

$$(W_0 + Dr_1)^{-\frac{1}{\rho}} r_1 = \lambda^{\frac{\rho-1}{\rho}} \left(M^{\frac{1}{2}} + D^{\frac{1}{2}} \right)^{-\frac{2}{\rho}} \left[\left(\frac{D}{M} \right)^{\frac{1}{2}} - \left(\frac{M}{D} \right)^{\frac{1}{2}} \right]. \tag{30}$$

This expression implies that when $r_1 = 0$, that is when the deposit rate paid by the monopoly bank equals zero, the solution is $D = M = W_0/2$, so the household's initial wealth is equally divided between cash and deposits.[3] Similarly, for $r_1 > 0$, it must be the case that $D/M > 1$, which implies $D > W_0/2 > M$, so the household holds more deposits than cash.

The first-order condition (30) defines an implicit function $\overline{D}(r - s)$, where $s = r - r_1$. Since the monopoly bank earns the policy rate r on its investments, it follows that its profits are

$$\overline{\pi}(s; r) = s\overline{D}(r - s). \tag{31}$$

To maximize profits for any given value of the policy rate r, the monopoly bank chooses the equilibrium spread

$$s^*(r) = \arg \max_{0 \le s \le r} [s\overline{D}(r - s)], \tag{32}$$

which implies the following equilibrium amount of deposits

$$D^*(r) = \overline{D}(r - s^*(r)). \tag{33}$$

Clearly, for low values of the policy rate r, say $r \le \widetilde{r}$, the solution will be at the corner $s^*(r) = r$, where the equilibrium deposit rate is $r_1^* = 0$ and the equilibrium amount of deposits is $D^* = W_0/2$. For higher values of r the solution will be interior and characterized by a spread $s^*(r)$, so the equilibrium deposit rate will be $r_1^* = r - s^*(r)$. For interior solutions the first-order condition is

$$(r - r_1^*)\overline{D}'(r_1^*) - \overline{D}(r_1^*) = 0, \tag{34}$$

[3] It can be checked that when the parameter δ of the liquidity services function (5) that captures the liquidity of deposits relative to cash is smaller (greater) than 1, the household holds more (less) deposits than cash when the deposit rate $r_1 = 0$.

which implies

$$\overline{D}'(r_1^*) = \frac{\overline{D}(r_1^*)}{r - r_1^*} > 0. \tag{35}$$

Differentiating the first-order condition (34), and assuming that the second-order condition is satisfied, gives

$$\frac{dr_1^*}{dr} = -\frac{\overline{D}'(r_1^*)}{(r - r_1^*)\overline{D}''(r_1^*) - 2\overline{D}'(r_1^*)} > 0, \tag{36}$$

which by (35) implies

$$\frac{dD^*}{dr} = \overline{D}'(r_1^*)\frac{dr_1^*}{dr} > 0. \tag{37}$$

Thus, for $r > \widetilde{r}$ higher policy rates lead to an increase in the equilibrium amount of deposits.

Figure 2 illustrates these results for $\lambda = 2$ and $\rho = 0.5$, where as shown in Figure 1 the constraint $M + D \leq W_0$ is always binding. Panel A shows the relationship between the policy rate r (in the horizontal axis) and the equilibrium deposit rate $r_1^*(r)$ (in the vertical axis), while Panel B shows the relationship between the policy rate r (in the horizontal axis) and the equilibrium amount of deposits $D^*(r)$ (in the vertical axis). For $r \leq \widetilde{r}$ the equilibrium deposit rate is zero and the equilibrium amount of deposits is $W_0/2$, while for $r > \widetilde{r}$ both the equilibrium deposit rate and the equilibrium amount of deposits are increasing in the policy rate.

Next, consider the case in which λ and ρ are such that the constraint $M + D \leq W_0$ is binding for $r < \overline{r}$. In this case one has to take into account that the spread chosen by the monopoly bank cannot be so low that the household would want to invest part of her wealth in bonds. To formalize this constraint, let $\widehat{s}(r)$ be the value of the spread s such that

$$M(s;r) + D(s;r) = \left(\frac{1}{r^2} + \frac{1}{s^2}\right)\frac{W_0(1+r)}{\mu[1 + (\lambda\mu)^{1-\rho}]} = W_0. \tag{38}$$

Since $M(s;r) + D(s;r)$ is decreasing in the spread s for $s \leq r$ (see the Appendix), it follows that $\widehat{s}(r)$ is unique. Clearly, the monopoly bank's choice of spread s has to be smaller than or equal to $\widehat{s}(r)$, for otherwise the household would want to invest part of her wealth in bonds, which contradicts the fact that $r < \overline{r}$.

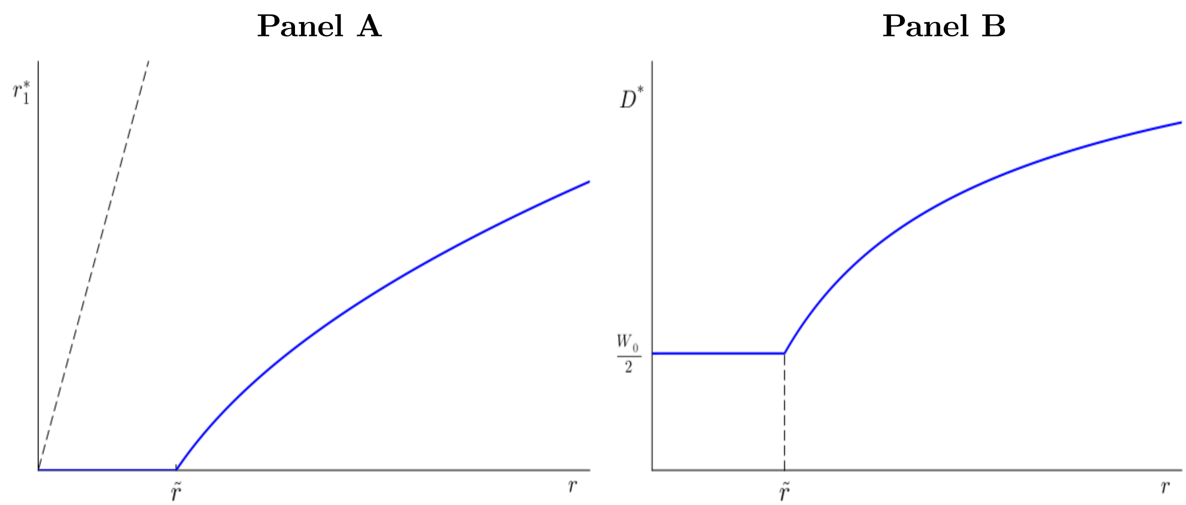

Figure 2. Equilibrium deposit rates and deposits for monopoly

Panel A shows the relationship between the equilibrium deposit rate and the policy rate and Panel B the relationship between the equilibrium amount of deposits and the policy rate for the monopoly model when lambda = 2 and rho = 0.5.

Substituting $\widehat{s}(r) = r$ into (38) and rearranging gives $r = \widetilde{r} = (2\lambda)^{\frac{\rho-1}{\rho}}$. One can show that for $r \leq \widetilde{r}$ the bank's choice of spread will be $s^*(r) = r$, so the equilibrium deposit rate will be $r_1^* = 0$ and the equilibrium amount of deposits will be $W_0/2$. For values of the policy rate r in the interval $[\widetilde{r}, \overline{r}]$ the bank's choice of spread will be at the corner $s^*(r) = \widehat{s}(r)$ that ensures that the household would not want to invest in bonds.

Figure 3 illustrates these results for $\lambda = 4$ and $\rho = 0.5$, where as shown in Figure 1 there is a range $[0, \overline{r}]$ of values of the policy rate r where the constraint $M + D \leq W_0$ is binding. Panel A shows the relationship between the policy rate r (in the horizontal axis) and the equilibrium deposit rate $r_1^*(r)$ (in the vertical axis), while Panel B shows the relationship between the policy rate r (in the horizontal axis) and the equilibrium amount of deposits $D^*(r)$ (in the vertical axis). For $r \leq \widetilde{r}$ the equilibrium deposit rate is zero and the equilibrium amount of deposits is $W_0/2$. For r in the interval $[\widetilde{r}, \overline{r}]$ both the equilibrium deposit rate and the equilibrium amount of deposits are increasing in the policy rate. Finally, for $r > \overline{r}$, where the constraint $M + D \leq W_0$ is no longer binding, the equilibrium deposit rate is increasing but the equilibrium amount of deposits is decreasing in the policy rate.

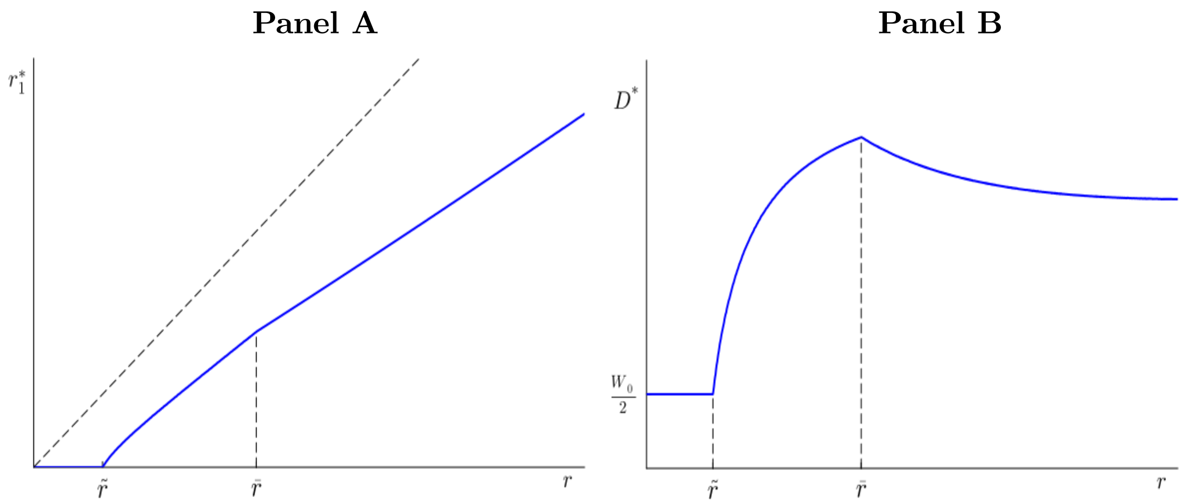

Figure 3. Equilibrium deposit rates and deposits for monopoly

Panel A shows the relationship between the equilibrium deposit rate and the policy rate and Panel B the relationship between the equilibrium amount of deposits and the policy rate for the monopoly model when lambda = 4 and rho = 0.5.

It should be noted that the equilibrium amount of deposits $D^*(r)$ is not always decreasing in the policy rate r for $r \geq \overline{r}$, where the constraint $M + D \leq W_0$ is not binding. To see this, consider the household's budget constraint (9) rewritten as

$$Mr + Ds + W = W_0(1 + r).$$

An increase in the policy rate r increases the "price" of cash r and the "price" of deposits s (since s is increasing in r), so there is a substitution effect that reduces the demand for cash and deposits. At the same time, the increase in r produces an income effect that reduces the demand for cash and deposits, so the final effect is in general ambiguous.

In fact, for low values of the elasticity of substitution ρ the substitution effect is small, so the equilibrium amount of deposits $D^*(r)$ is increasing in the policy rate r. In particular, in the Leontief case $\rho = 0$ the limit in (28) is bounded below by λ, which means that the demand for deposits (24) does not become unbounded. Moreover, solving for the first-order condition of the monopoly bank's maximization problem (26) for $\rho = 0$ gives

$$s^*(r) = \left(\frac{1}{\lambda r} + \frac{1}{r^2}\right)^{-\frac{1}{2}}, \tag{39}$$

which implies

$$D^*(r) = \frac{W_0(1+r)}{1+2\lambda\mu^*}. \tag{40}$$

But since μ^* is decreasing in r, it follows that the equilibrium amount of deposits $D^*(r)$ is increasing in r. By continuity, this implies that $D^*(r)$ will also be increasing in the policy rate r for low values of the elasticity of substitution ρ.

Summing up, I have shown that for low values of the policy rate r the household will invest her entire initial wealth in cash and deposits of the monopoly bank, in which case the equilibrium amount of deposits will be either constant or increasing in the policy rate.[4] Moreover, the range of values of the policy rate for which this result obtains is decreasing in the parameter λ that captures the utility of liquidity services relative to final wealth, and increasing in the parameter ρ that captures the elasticity of substitution between final wealth and liquidity services. Finally, the result also obtains for high values of the policy rate for which the household will start to invest in bonds as long as the elasticity of substitution is sufficiently low. The conclusion is that, contrary to the claim in DSS, for reasonable parameter values, an increase in the policy rate does not lead to a reduction in bank deposits.

2.2 The oligopoly model

I next consider DSS's oligopoly model in which n banks indexed by $i = 1, 2, ..., n$ compete in the deposit market by setting spreads s_i between the policy rate r and their deposit rate r_i.

As in the case of the monopoly model, to derive the demand for deposits of the n banks for a policy rate r and set of spreads $s_1, ..., s_n$ let

$$X = Mr + \textstyle\sum_{i=1}^{n} D_i s_i. \tag{41}$$

[4] It should be noted that these "low values" of the policy rate $r \leq \bar{r}$ need not be low. For example, in the case plotted in Figure 2, where $\lambda = 4$ and $\rho = 0.5$, $\bar{r} = 0.404$, that is a policy rate of 40.4%.

denote the opportunity cost of the liquidity held by the household. Substituting (6) into (10), it follows that the optimal way to allocate X between cash M and deposits $D_1, ..., D_n$ is obtained by solving

$$\max_{M,D_1,...,D_n} \left[M^{\frac{1}{2}} + \left(\frac{1}{n} \sum_{i=1}^{n} (nD_i)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{2(\eta-1)}} \right]^2 \tag{42}$$

subject to (41). Following the same steps as in Section 2.1, it can be shown (see the Appendix for the details) that the solution to this problem is given by

$$M = \frac{X}{\mu r^2}, \tag{43}$$

$$D_i = s_i^{-\eta} \left(\frac{1}{n} \sum_{i=1}^{n} s_i^{1-\eta} \right)^{\frac{2-\eta}{\eta-1}} \frac{X}{n\mu}, \tag{44}$$

where

$$\mu = \frac{1}{r} + \left(\frac{1}{n} \sum_{i=1}^{n} s_i^{1-\eta} \right)^{\frac{1}{\eta-1}}. \tag{45}$$

Substituting these results into (6) and rearranging then gives

$$D = \left(\frac{1}{n} \sum_{i=1}^{n} s_i^{1-\eta} \right)^{\frac{2}{\eta-1}} \frac{X}{\mu}. \tag{46}$$

which using (10), (43), and (45) implies

$$L = \left(M^{\frac{1}{2}} + D^{\frac{1}{2}} \right)^2 = \mu X. \tag{47}$$

Substituting this result into the household's utility function (4) yields the same maximization problem (21) as in the case of the monopoly bank, whose solution is given by (23). Finally, substituting this result into (44) implies the following demand for the deposits of bank i

$$D_i(s_1, ..., s_n; r) = s_i^{-\eta} \left(\frac{1}{n} \sum_{i=1}^{n} s_i^{1-\eta} \right)^{\frac{2-\eta}{\eta-1}} \frac{W_0(1+r)}{n\mu[1 + (\lambda\mu)^{1-\rho}]}. \tag{48}$$

Assuming, as before, that banks earn the policy rate r on their investments, the profits of bank i are

$$\pi_i(s_1, ..., s_n; r) = s_i D_i(s_1, ..., s_n; r). \tag{49}$$

A symmetric Nash equilibrium of the Bertrand game played by the n banks for a given policy rate r is characterized by a solution to the following equation

$$s^* = \arg\max_s \left\{ s^{1-\eta} \left(\frac{1}{n} \left[s^{1-\eta} + (n-1)(s^*)^{1-\eta} \right] \right)^{\frac{2-\eta}{\eta-1}} \frac{W_0(1+r)}{n\mu[1+(\lambda\mu)^{1-\rho}]} \right\}, \tag{50}$$

where

$$\mu = \frac{1}{r} + \left(\frac{1}{n} \left[s^{1-\eta} + (n-1)(s^*)^{1-\eta} \right] \right)^{\frac{1}{\eta-1}}. \tag{51}$$

Let $s^*(r)$ denote the equilibrium spread corresponding to the policy rate r. Substituting $s^*(r)$ into (46), and using (23), gives the equilibrium amount of deposits

$$D^*(r) = \frac{W_0(1+r)}{\mu^*[s^*(r)]^2[1+(\lambda\mu^*)^{1-\rho}]}, \tag{52}$$

where

$$\mu^* = \frac{1}{r} + \frac{1}{s^*(r)}. \tag{53}$$

It should be noted that for $n = 1$ the demand for deposits $D_1(s_1; r)$ in (48) coincides with the expression (24) obtained in Section 2.1, so the monopoly model is indeed a special case of the oligopoly model. As in the case of monopoly, one can show that for low values of the policy rate r the demand for deposits becomes unbounded. In fact, since increases in the number of banks n translate into lower equilibrium spreads and higher equilibrium deposits, the constraint $M + \sum_{i=1}^n D_i \leq W_0$ will be violated for a larger range of parameter values.

Hence, to complete the analysis of the model, I next consider the case where the household's choice of cash and deposits is restricted to satisfy constraint $M + \sum_{i=1}^n D_i \leq W_0$. For a given set of deposit rates $r_1, ..., r_n$, final wealth is $W = M + \sum_{i=1}^n D_i(1+r_i) = W_0 + \sum_{i=1}^n D_i r_i$, so the problem of the household may be written as

$$\max_{D_1,...,D_n} \left[\left(W_0 + \textstyle\sum_{i=1}^n D_i r_i\right)^{\frac{\rho-1}{\rho}} + \left[\lambda \left(\left(W_0 - \textstyle\sum_{i=1}^n D_i\right)^{\frac{1}{2}} + D^{\frac{1}{2}} \right)^2 \right]^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \tag{54}$$

where D is given by (6). The first-order conditions for $i = 1, ..., n$ are

$$\begin{aligned} \left(W_0 + \textstyle\sum_{i=1}^n D_i r_i\right)^{-\frac{1}{\rho}} r_i \quad &= \quad \lambda^{\frac{\rho-1}{\rho}} \left(\left(W_0 - \textstyle\sum_{i=1}^n D_i\right)^{\frac{1}{2}} + D^{\frac{1}{2}} \right)^{\frac{\rho-2}{\rho}} \\ &\quad \cdot \left(\left(W_0 - \textstyle\sum_{i=1}^n D_i\right)^{-\frac{1}{2}} - D^{\frac{2-\eta}{2\eta}} (nD_i)^{-\frac{1}{\eta}} \right). \end{aligned} \tag{55}$$

To characterize the symmetric Nash equilibrium of the Bertrand game played by the n banks, one has to compute the demand for deposits $\overline{D}_i(r_i, r_{-i})$ of bank i when it pays a deposit rate r_i and the other $n-1$ banks pay a deposit rate r_{-i}. This requires to solve the following system of two nonlinear equations

$$(W_0 + D_i r_i + (n-1)D_{-i}r_{-i})^{-\frac{1}{\rho}} r_i = \lambda^{\frac{\rho-1}{\rho}} \left((W_0 - D_i - (n-1)D_{-i})^{\frac{1}{2}} + D^{\frac{1}{2}}\right)^{\frac{\rho-2}{\rho}} \cdot \left((W_0 - D_i - (n-1)D_{-i})^{-\frac{1}{2}} - D^{\frac{2-\eta}{2\eta}} (nD_i)^{-\frac{1}{\eta}}\right), \quad (56)$$

$$(W_0 + D_i r_i + (n-1)D_{-i}r_{-i})^{-\frac{1}{\rho}} r_{-i} = \lambda^{\frac{\rho-1}{\rho}} \left((W_0 - D_i - (n-1)D_{-i})^{\frac{1}{2}} + D^{\frac{1}{2}}\right)^{\frac{\rho-2}{\rho}} \cdot \left((W_0 - D_i - (n-1)D_{-i})^{-\frac{1}{2}} - D^{\frac{2-\eta}{2\eta}} (nD_{-i})^{-\frac{1}{\eta}}\right), \quad (57)$$

where

$$D = \left(\frac{1}{n}(nD_i)^{\frac{\eta-1}{\eta}} + \frac{n-1}{n}(nD_{-i})^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}. \quad (58)$$

Note that dividing (56) by (57) gives

$$\frac{r_i}{r_{-i}} = \frac{(W_0 - D_i - (n-1)D_{-i})^{-\frac{1}{2}} - D^{\frac{2-\eta}{2\eta}} (nD_i)^{-\frac{1}{\eta}}}{(W_0 - D_i - (n-1)D_{-i})^{-\frac{1}{2}} - D^{\frac{2-\eta}{2\eta}} (nD_{-i})^{-\frac{1}{\eta}}}. \quad (59)$$

Hence, $r_i > r_{-i}$ implies

$$(nD_i)^{-\frac{1}{\eta}} < (nD_{-i})^{-\frac{1}{\eta}}, \quad (60)$$

which gives $D_i > D_{-i}$. Thus, if bank i pays a higher deposit rate than the other $n-1$ banks, it will get a higher amount of deposits.

To characterize the symmetric Nash equilibrium for a given policy rate r one has to compute the optimal deposit rate r_i chosen by bank i when the other $n-1$ banks pay a deposit rate r_{-i}. As before, one has to take into account that the deposit rate r_i cannot be so low that the household would want to invest part of her wealth in bonds. To formalize this constraint, let $\widehat{s}_i(r_{-i}; r)$ be the value of the spread s_i such that

$$M(s_1, ..., s_n; r) + \textstyle\sum_{i=1}^n D_i(s_1, ..., s_n; r) = W_0, \quad (61)$$

where $s_{-i} = r - r_{-i}$. Then, the range of feasible values of the deposit rate r_i set by bank i is bounded below by $\widehat{r}_i = \max\{r - \widehat{s}_i(r_{-i}; r), 0\}$. The symmetric Nash equilibrium for a given policy rate r, denoted $r^*(r)$, is a solution to the equation

$$r^* = \arg\max_{\widehat{r}_i \leq r_i \leq r} [(r - r_i)\overline{D}_i(r_i, r^*)], \tag{62}$$

which implies the following equilibrium aggregate amount of deposits

$$D^*(r) = n\overline{D}_i(r^*(r), r^*(r)). \tag{63}$$

Depending on the policy rate r, the solution to this problem may be characterized by a zero or a positive deposit rate. If $r^*(r) = 0$ one can show that $D^*(r) = W_0/2$, so the household's initial wealth is equally divided between cash and deposits. To see this note that in this case (55) simplifies to

$$(W_0 - D)^{-\frac{1}{2}} = D^{\frac{2-\eta}{2\eta}} D^{-\frac{1}{\eta}} = D^{-\frac{1}{2}}, \tag{64}$$

which implies the result.

Figure 4 illustrates these results for the same parameter values in Figure 2, that is $\lambda = 2$ and $\rho = 0.5$, and for the monopoly ($n = 1$) and duopoly ($n = 2$) case. In the latter case, the elasticity of substitution between the deposits of the two banks is $\eta = 1.2$.[5] Panel A shows the relationship between the policy rate r (in the horizontal axis) and the equilibrium deposit rate $r^*(r)$ (in the vertical axis), while Panel B shows the relationship between the policy rate r (in the horizontal axis) and the equilibrium amount of deposits $D^*(r)$ (in the vertical axis). In both panels dashed lines correspond to the case $n = 1$ and solid lines to the case $n = 2$. Going from monopoly to duopoly shrinks the range of values of the policy rate r for which the equilibrium deposit rate equals zero and increases equilibrium deposit rates and quantities outside of this range.

[5] Higher values of η would strengthen the effect of competition on equilibrium deposit rates and quantities.

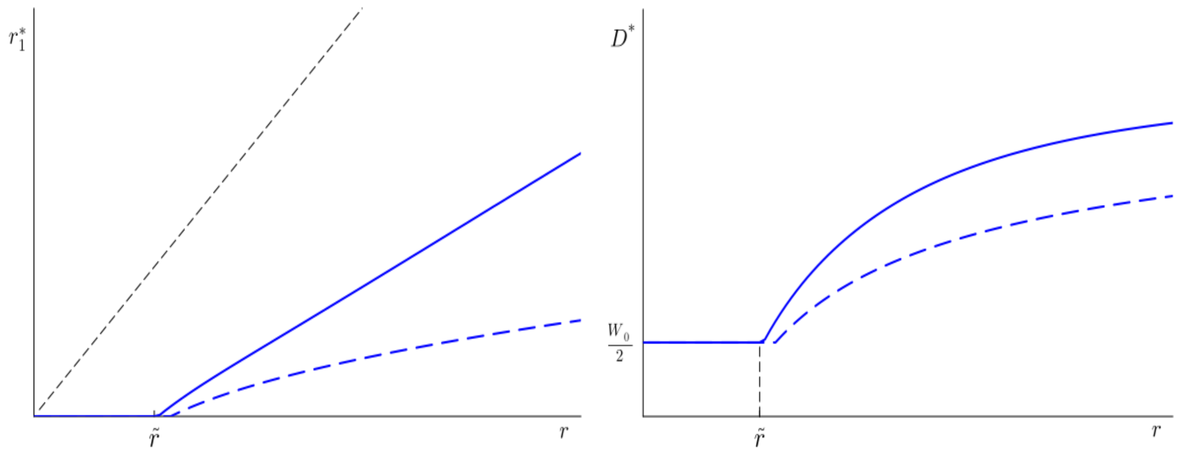

Figure 4. Equilibrium deposit rates and deposits for duopoly and monopoly

Panel A shows the relationship between the equilibrium deposit rate and the policy rate and Panel B the relationship between the equilibrium amount of deposits and the policy rate for the duopoly (solid lines) and the monopoly model (dashed lines) when lambda = 2 and rho = 0.5.

The conclusion that follows from the analysis of the oligopoly model reinforces the conclusion of the monopoly model: *Contrary to the claim in DSS, for reasonable parameter values, increases in the policy rate generally increase equilibrium deposits*, except in the case where the policy rate is close to zero and there is a zero lower bound on deposit rates, in which case they do not have any effect on equilibrium deposits.

3 An Alternative Model

This section explores the robustness of the previous results when replacing DSS's Bertrand competition model with differentiated deposits by a standard Cournot model of competition in the deposit market, based on a simple microfoundation for the households' demand for deposits. In this model an increase in the policy rate always leads to an increase in the equilibrium amount of deposits.

Consider a model with heterogeneous households that differ in a utility premium associated with liquid assets. Specifically, suppose that there is a measure one of atomistic households with unit wealth characterized by a *liquidity premium* x that is uniformly distributed in $[0,1]$. A household of type x can invest her wealth in three assets, namely cash that pays a zero interest rate, bank deposits that pay an interest rate r_D, and bonds that pay an interest rate r taken to be equal to the monetary policy rate set by the central bank. Investing in cash yields utility

$$U_C(x) = 1 + \gamma x, \tag{65}$$

investing in deposits yields utility

$$U_D(x) = 1 + r_D + x, \tag{66}$$

and investing in bonds yields utility

$$U_B(x) = 1 + r. \tag{67}$$

It is assumed that cash provides higher liquidity services than deposits, so $\gamma > 1$.[6]

There are n identical banks that compete à la Cournot in the deposit market and invest the funds raised in assets that pay the policy rate r, so their profits for unit of deposits are given by the spread $s = r - r_D$. To compute the Cournot equilibrium I next derive the households' aggregate demand for deposits $D(s)$ as a function of the spread s.

A household of type x will put all her wealth in cash if $U_C(x) > \max\{U_D(x), U_B(x)\}$, she will put all her wealth in bank deposits if $U_D(x) > \max\{U_C(x), U_B(x)\}$, and she will put all her wealth in bonds if $U_B(x) > \max\{U_C(x), U_D(x)\}$.

For a characterization of the households' investment decisions refer to Figure 5. The horizontal axis represents the liquidity premium x, while the vertical axis represents the households' utilities associated with the three assets. The horizontal red line with intercept $1 + r$ shows the utility of bond investments $U_B(x)$, the green line with intercept $1 + r_D$ and unit slope shows the utility of bank deposits $U_D(x)$, and the blue line with intercept 1 and

[6] This model of the demand for deposits builds on Martinez-Miera and Repullo (2020) by adding the possibility of investing in highly liquid cash.

slope $\gamma > 1$ shows the utility of cash $U_C(x)$. A household is indifferent between deposits and bonds when her liquidity premium x satisfies $U_D(x) = 1 + r_D + x = 1 + r = U_B(x)$, which gives the boundary point

$$\overline{x} = r - r_D = s. \tag{68}$$

A household is indifferent between cash and deposits when her liquidity premium x satisfies $U_C(x) = 1 + \gamma x = 1 + r_D + x = U_D(x)$, which gives the boundary point

$$\widehat{x} = \frac{r_D}{\gamma - 1} = \frac{r - s}{\gamma - 1}. \tag{69}$$

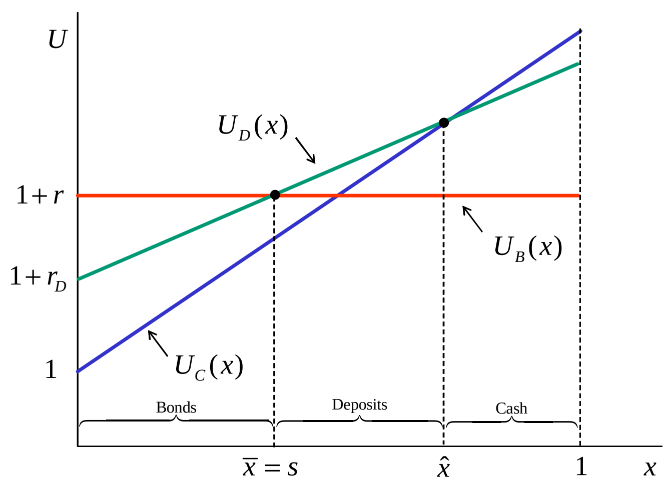

Figure 5. Utility of bonds, deposits, and cash

This figure shows the utility of investing in bonds (red line), bank deposits (green line), and cash (blue line) for the range of values of the households' liquidity premium.

I focus on the case depicted in Figure 4, where there is a positive mass of households that put their wealth in deposits.[7] Moreover, parameter γ is assumed to be sufficiently high so that there is a positive demand for cash. Since each household has a unit amount of wealth and the liquidity premium is uniformly distributed in $[0, 1]$, it follows that the (linear)

[7]Although the boundary points $\overline{x}$ and $\widehat{x}$ depend on the (endogenous) spread s, the Cournot equilibrium is characterized by a spread such that $\overline{x} < \widehat{x}$.

demand for deposits is given by

$$D(s;r) = \widehat{x} - \overline{x} = \frac{r - \gamma s}{\gamma - 1}. \quad (70)$$

To derive the Cournot equilibrium it is convenient to work with the inverse demand for deposits implied by (70), which is

$$s(D;r) = \frac{r - (\gamma - 1)D}{\gamma}. \quad (71)$$

Let d_i denote the deposits chosen by bank $i = 1, ..., n$, so the total amount of deposits is $D = \sum_{i=1}^{n} d_i$. Assuming, as before, that banks earn the policy rate r on their investments, the profits of bank i are then given by

$$\pi_i(d_1, ..., d_n; r) = (r - r_D)d_i = s(D;r)d_i. \quad (72)$$

A *symmetric Cournot equilibrium* for a given policy rate r is characterized by a solution to the equation

$$d^* = \arg\max_d [s(d + (n-1)d^*; r)d]. \quad (73)$$

Using (71), the first-order condition is

$$s(nd^*) + s'(nd^*)d^* = \frac{r - (\gamma - 1)nd^*}{\gamma} - \frac{\gamma - 1}{\gamma}d^* = 0, \quad (74)$$

which implies

$$D^*(r) = nd^*(r) = \frac{nr}{(n+1)(\gamma - 1)}. \quad (75)$$

The Cournot equilibrium has two interesting properties, namely

$$\frac{dD^*}{dr} = \frac{n}{(n+1)(\gamma - 1)} > 0, \quad (76)$$

$$\frac{\partial^2 D^*}{\partial r \partial n} = \frac{1}{(n+1)^2(\gamma - 1)} > 0. \quad (77)$$

According to (76), an increase in the policy rate r always increases the equilibrium amount of deposits $D^*(r)$. According to (77), the positive effect of the policy rate r on equilibrium deposits $D^*(r)$ is stronger when banks have low market power (high n).

To relate this latter result to the empirical results in DSS, it is convenient to rewrite (75) in terms of the Herfindahl index for a market with n identical banks, $\mathrm{HHI} = 1/n$. Solving for n in this expression, and substituting it into (75) gives

$$D^*(r) = \frac{r}{(1 + \mathrm{HHI})(\gamma - 1)}. \tag{78}$$

From here it follows that

$$\frac{dD^*}{dr} = \frac{1}{(1 + \mathrm{HHI})(\gamma - 1)} > 0, \tag{79}$$

$$\frac{\partial^2 D^*}{\partial r \partial \mathrm{HHI}} = -\frac{1}{(1 + \mathrm{HHI})^2(\gamma - 1)} < 0. \tag{80}$$

This latter result implies that an increase in the policy rate leads to smaller changes in deposits in banks located in high concentrated markets, and corresponds to the result $\gamma_D < 0$ in DDS's deposits panel regression. Moreover, (79) and (80) imply that

$$\frac{dD^*}{dr} = -(1 + \mathrm{HHI})\gamma_D. \tag{81}$$

Thus, in this model the sign of the interaction term γ_D is negative (as in DSS's panel regressions) if and only if the equilibrium amount of deposits $D^*(r)$ is increasing in the policy rate r.

4 Conclusion

This paper has reviewed the claim in Drechsler, Savov, and Schnabl (2017) that the transmission of monetary policy should be understood from the liability side of banks' balance sheets. In particular, they argue that there is a "deposits channel" whereby increases in the policy rate widen deposit rate spreads, leading to deposit outflows that reduce banks' lending capacity. I have shown that, contrary to their claim, in their theoretical model of imperfect competition in a local banking market, the relationship between the policy rate and the equilibrium amount of deposits is either flat or upward sloping in the relevant range. I have also constructed an alternative model, based on a simple microfoundation for the households' demand for deposits, which is consistent with their panel results and where increases in the policy rate always increase the equilibrium amount of deposits.

I would like to conclude with a comment on DSS's approach. They look at the effect of monetary policy on bank lending through the lens of deposit taking. In this approach, the characteristics of the loan market take a back seat. It is true that "deposits are a special source of funding for banks, one that it is not perfectly substitutable with wholesale funding." But it is also true that if the focus of the analysis is on bank lending, characteristics such as market power and risk-taking in lending should have a prominent role. For this reason, one should aim at building models that encompass both sides of banks' balance sheets.

Appendix

Demand for cash and deposits in DSS's monopoly model To prove that

$$M(s;r) + D(s;r) = \left(\frac{1}{r^2} + \frac{1}{s^2}\right) \frac{W_0(1+r)}{\mu[1+(\lambda\mu)^{1-\rho}]} \tag{82}$$

is decreasing in the spread s for $s \leq r$, note first that

$$\left(\frac{1}{r^2} + \frac{1}{s^2}\right) \frac{1}{\mu} = \frac{\left(\frac{1}{r} + \frac{1}{s}\right)^2 - \frac{2}{rs}}{\frac{1}{r} + \frac{1}{s}} = \frac{1}{r} + \frac{1}{s} - \frac{2}{r+s}, \tag{83}$$

Hence, one has to prove that

$$\frac{d}{ds}\left[\frac{\frac{1}{r} + \frac{1}{s} - \frac{2}{r+s}}{1+(\lambda\mu)^{1-\rho}}\right] = \frac{\left(\frac{2}{(r+s)^2} - \frac{1}{s^2}\right)[1+(\lambda\mu)^{1-\rho}] + \frac{\frac{1}{r}+\frac{1}{s}-\frac{2}{r+s}}{\mu s^2}(1-\rho)(\lambda\mu)^{1-\rho}}{[1+(\lambda\mu)^{1-\rho}]^2} < 0. \tag{84}$$

Since

$$\frac{2}{(r+s)^2} - \frac{1}{s^2} = \frac{s^2 - r^2 - 2sr}{(r+s)^2 s^2} < 0, \tag{85}$$

for $0 < s \leq r$, and

$$\frac{\frac{1}{r} + \frac{1}{s} - \frac{2}{r+s}}{\mu s^2} = \frac{\mu - \frac{2}{r+s}}{\mu s^2} = \left(1 - \frac{2rs}{(r+s)^2}\right)\frac{1}{s^2} = \frac{r^2+s^2}{(r+s)^2 s^2} > 0, \tag{86}$$

it follows that

$$\frac{s^2 - r^2 - 2sr}{(r+s)^2 s^2}[1+(\lambda\mu)^{1-\rho}] + \frac{r^2+s^2}{(r+s)^2 s^2}(1-\rho)(\lambda\mu)^{1-\rho} < \frac{2(s-r)(\lambda\mu)^{1-\rho}}{(r+s)^2 s} \leq 0, \tag{87}$$

which implies the result.

Demand for deposits in DSS's oligopoly model The first-order conditions that characterize the solution to (42) subject to (41) are

$$\left(M^{\frac{1}{2}} + D^{\frac{1}{2}}\right) M^{-\frac{1}{2}} = \mu r, \tag{88}$$

$$\left(M^{\frac{1}{2}} + D^{\frac{1}{2}}\right) D^{-\frac{1}{2}} D^{\frac{1}{\eta}} (nD_i)^{-\frac{1}{\eta}} = \mu s_i, \tag{89}$$

where μ denotes the Lagrange multiplier associated with the constraint. To solve for μ, first note that by (89) we have

$$D_i = D_1 \left(\frac{s_1}{s_i}\right)^{\eta}, \tag{90}$$

which by the definition (6) of D implies

$$D = \left(\frac{1}{n}\sum_{i=1}^{n}(nD_i)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} = nD_1 s_1^{\eta}\left(\frac{1}{n}\sum_{i=1}^{n} s_i^{1-\eta}\right)^{\frac{\eta}{\eta-1}}. \tag{91}$$

From here it follows that

$$D_i s_i^{\eta} = D_1 s_1^{\eta} = \frac{D}{n}\left(\frac{1}{n}\sum_{i=1}^{n} s_i^{1-\eta}\right)^{-\frac{\eta}{\eta-1}}. \tag{92}$$

Now, dividing (88) by (89) gives

$$\left(\frac{D}{M}\right)^{\frac{1}{2}}\left(\frac{nD_i}{D}\right)^{\frac{1}{\eta}} = \frac{r}{s_i}, \tag{93}$$

which using (92) implies

$$\left(\frac{D}{M}\right)^{\frac{1}{2}} = r\left(\frac{1}{n}\sum_{i=1}^{n} s_i^{1-\eta}\right)^{\frac{1}{\eta-1}}. \tag{94}$$

Using this result together with (88) gives

$$1 + \left(\frac{D}{M}\right)^{\frac{1}{2}} = 1 + r\left(\frac{1}{n}\sum_{i=1}^{n} s_i^{1-\eta}\right)^{\frac{1}{\eta-1}} = \mu r, \tag{95}$$

which implies

$$\mu = \frac{1}{r} + \left(\frac{1}{n}\sum_{i=1}^{n} s_i^{1-\eta}\right)^{\frac{1}{\eta-1}}. \tag{96}$$

To solve for D_i use (92) and (94) to get

$$D_i s_i = s_i^{1-\eta}\frac{D}{n}\left(\frac{1}{n}\sum_{i=1}^{n} s_i^{1-\eta}\right)^{-\frac{\eta}{\eta-1}} = \frac{1}{n}s_i^{1-\eta}Mr^2\left(\frac{1}{n}\sum_{i=1}^{n} s_i^{1-\eta}\right)^{\frac{2-\eta}{\eta-1}}, \tag{97}$$

which implies

$$\sum_{i=1}^{n} D_i s_i = Mr^2\left(\frac{1}{n}\sum_{i=1}^{n} s_i^{1-\eta}\right)^{\frac{1}{\eta-1}}. \tag{98}$$

Substituting this result into (41) and using (96) gives

$$X = Mr + \sum_{i=1}^{n} D_i s_i = Mr^2\mu, \tag{99}$$

which implies

$$M = \frac{X}{\mu r^2}. \tag{100}$$

Finally, substituting this result into (97) and solving for D_i gives

$$D_i = s_i^{-\eta}\left(\frac{1}{n}\sum_{i=1}^{n} s_i^{1-\eta}\right)^{\frac{2-\eta}{\eta-1}}\frac{X}{n\mu}, \tag{101}$$

as required.

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