Intergenerational social mobility, inequality and the Carnegie effect

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Abstract

Using US data, we show that higher income inequality reduces upward intergenerational social mobility (ISM) at the bottom of the income distribution, increases downward ISM at the top, and reduces future income. We explain these findings in a life-cycle model in which individuals are altruistic and suffer disutility of effort. Investment in education and effort increase labor earnings. Two mechanisms explain the different effects of inequality on ISM. First, due to a credit constraint, the investment in education and the future earnings of children born in low-income families are limited by parental wealth, which explains why higher inequality reduces upward ISM at the bottom of the distribution. Second, children born in affluent families exert less effort and obtain lower labor earnings when they receive a larger inheritance, which explains why higher inequality increases downward ISM at the top. These two mechanisms also determine the effect of inequality on future income.

JEL classification: D64, E21, E71, I24.

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1 Introduction

Intergenerational social mobility (ISM) measures the relationship between the economic status of parents and that of their children. Corak (2013), using cross-country data, and Chetty et al. (2014a, 2014b), using within-country data, show that cross-section income inequality reduces ISM.[1] The mechanism explaining this relation is based on classical papers on parental investment in education by Becker and Tomes (1979, 1986) and, more recently, Galor and Zeira (1993), and Alonso-Carrera, et al. (2012), among many others. In these papers, financial market imperfections limit investment in education in low-income families, which explains why these families stay poor. Since the number of low-income families increases with inequality, these imperfections explain the negative relationship between income inequality and ISM that the literature calls the Great Gatsby curve (Durlauf, 2022).

The literature studying the relationship between income inequality and ISM has focussed on poverty and its persistence across generations (see, for instance, Jarrim and Macmillan, 2015; Halter, 2015; and Caucutt and Lochner, 2020). This is obviously important for equal opportunity rights, and it is also highly relevant for efficiency. Since the persistence of poverty is explained by financial market imperfections and limited access to education, it implies that individuals born in low-income families cannot develop their talents. As a result, financial market imperfections lead to a misallocation of talent and a loss of efficiency that reduces future income. Therefore, higher inequality, by increasing poverty, reduces future income.

In this paper, we study the effect of higher inequality on ISM and future income of individuals born in families whose income is not only at the bottom of the income distribution, but also at the top. We thus contribute to the literature by studying the effect of a higher inequality on the future labor earnings of individuals born into families whose income falls at different parts of the income distribution. To this end, we use data in Chetty et al. (2014a) and we study the relationship between income inequality and ISM, for the US economy and for the cohort born in the period 1980-1982. We show that larger parental income inequality reduces upward ISM for

[1] Other empirical papers that study ISM are Blanden (2013), Björklund and Jantti (2019), Cervini-Pla (2015), Corak (2006), d'Addio (2007), Isaacs (2007), Jantti, et al. (2006), and Solon (2002).

individuals born in families whose income falls in the lower part of the income distribution (first two quintiles), has a negligible effect for individuals born in families that fall in the middle part (third quintile), and increases downward ISM for individuals born in families that fall in the upper part (last two quintiles). While the effect of income inequality on ISM at the bottom of the income distribution confirms the results previously obtained by the literature, the effect on the top of the distribution is a contribution of this paper.

We explain the different effects of income inequality on ISM in a life-cycle model in which labor earnings depend positively on education and effort. Individuals are altruistic towards their descendents. In particular, we assume that the utility depends on the bequest given to descendents. Therefore, we assume a form of warm glow altruism. The two crucial assumptions driving the relationship between income inequality and ISM are a credit constraint and disutility of effort. The credit constraint is a form of financial market imperfection that directly limits investment in education of those individuals born into low-income families and thus limits the future income that these individuals will earn as adults. We use the model to show that the credit constraint explains why an increase in income inequality reduces ISM at the bottom of the income distribution. The disutility of effort implies that individuals from affluent families exert less effort and obtain lower earnings as adults when they receive a larger inheritance. This effect of inheritances on children's effort is known as the Carnegie effect (see Degan and Thibault, 2016 and Alonso-Carrera, et al. 2020) and we show that it explains the increase in ISM at the top of the income distribution when inequality increases.[2]

An interesting implication of the analysis is that income inequality reduces future income of individuals born into both low-income and affluent families. On the one hand, greater inequality increases the amount of individuals who are financially constrained, which reduces investment in education and future income. On the other hand, it also increases the wealth of affluent families, whose children will reduce effort as adults. This also reduces future income. We provide empirical support to these

[2] Holtz-Eakin, et al. (1993), Elinder, et al. (2012) and Brown, et al. (2010) have shown a negative effect of inheritances on the labor supply.

results by showing that greater income inequality reduces the average future income of individuals born into families whose incomes are at the 25th and 75th percentiles of the income distribution.

We conclude that greater income inequality hurts future income through a reduction in education of the low-income individuals and in the effort exerted by affluent individuals.[3] An obvious question is then to measure the contribution of each mechanism in explaining the fall in future average income. To address this question, we calibrate the model to match several targets of the US economy and then we use the calibration to simulate and compare commuting zones with different income inequality. We show that 72% of the reduction in future income due to a higher income inequality is explained by lower investment in education and the rest by reduced effort.

The rest of the paper is organized as follows. Section 2 provides the empirical evidence. Section 3 explains the two mechanisms that govern individual decisions. Section 4 introduces the model and studies the effects of a higher income inequality. Section 5 describes the numerical analysis. Section 6 concludes the paper. Some technical details are relegated to an appendix.

2 Empirical evidence

In this section, we study the effect of a larger parental income inequality on ISM and on the average income that children obtain as adults. We identify this income as the future income per capita. To perform this analysis, we use data from Chetty et al. (2014a), who provides transition matrices and the estimated coefficients of Rank-Rank regressions.[4]

2.1 Transition matrices

Chetty et al. (2014a) provide transition matrices for 707 commuting zones of the US, in which parents and children are grouped by quintiles of income. They consider

[3] Brueckner and Lederman (2018) show that higher income inequality reduces human capital and future income in developed countries.

[4] See Dahl and DeLeire (2008) for an explanation of Rank-Rank regressions.

children born in the period 1980-82, parents' income is the mean of family income in the period 1996-2000 and children's income is the mean family income in the period 2011-2012. The elements of these matrices are the conditional probabilities of the children's income falling into a quintile of the national income distribution given the parents' position in this distribution. Table 1 shows two transition matrices. One is obtained as the average of the social matrices of commuting zones with a Gini index below the median of the Gini indexes and the other as the average of the matrices of commuting zones with a Gini index above the median.[5] We observe upward and downward ISM. The former implies that individuals born in low-income families move into higher income quintiles as adults and the latter implies that individuals born in affluent families move into lower income quintiles as adults. However, despite this evidence of ISM, there is large intergenerational income persistence, especially in the lower and higher quintiles. The persistence at the bottom of the income distribution is shown by the fact that the probabilities that individuals born in families in the first two quintiles stay in these two quintiles are much larger than 20%. Similarly, persistence at the top is shown by the fact that the probabilities that individuals born in affluent families stay in the higher quintiles is also much larger than 20%.

Table 1. Transition matrices

	Children						Children				
Parents	Q1	Q2	Q3	Q4	Q5	Parents	Q1	Q2	Q3	Q4	Q5
Q1	0.33	0.28	0.18	0.13	0.08	Q1	0.28	0.23	0.20	0.17	0.12
Q2	0.24	0.25	0.21	0.17	0.12	Q2	0.19	0.20	0.22	0.22	0.18
Q3	0.17	0.20	0.22	0.22	0.19	Q3	0.13	0.17	0.21	0.25	0.24
Q4	0.13	0.16	0.20	0.24	0.26	Q4	0.10	0.14	0.20	0.26	0.30
Q5	0.12	0.13	0.17	0.24	034	Q5	0.09	0.12	0.17	0.25	0.37
Average Gini: 0.47. Mobility: 0.70						Average Gini: 0.34. Mobility: 0.74					

Source. Data is from Chetty et al. (2014a). Mobility is defined as one minus the second highest eigenvalue.

We proceed to compare the two transition matrices to show the effect of income inequality on ISM. To this end, we need a measure of the ISM implied by each matrix.

[5] Gini indexes are obtained on disposable income in the period 1996-2000.

A usual measure is one minus the second highest eigenvalue (see Caballé, 2016). This measure equals zero when there is full intergenerational persistence and one when there is perfect mobility. We observe that this measure of mobility is 74% in low income inequality commuting zones and 70% in the more unequal commuting zones. This confirms the Great Gatsby curve, according to which more inequality reduces mobility (Corak, 2013 and Chetty et al., 2014a).

From the comparison between the two matrices in Table 1, we observe that in those commuting zones with a higher income inequality the probabilities of falling into the first two quintiles given that the individual was born in a family whose income is in these quintiles are larger and, in contrast, the probabilities of falling into the last two quintiles given that the family's income is in these quintiles are smaller. This shows that in commuting zones with higher inequality there is more persistence at the bottom of the distribution and there is more mobility at the top.

Table 2. OLS coefficients

	Descendents				
Parents	Q1	Q2	Q3	Q4	Q5
Q1	0.35	0.39	-0.09	-0.30	-0.34
Q2	0.36	0.35	-0.02	-0.30	-0.39
Q3	0.29	0.24	0.05	-0.23	-0.35
Q4	0.22	0.18	0.05	-0.15	-0.31
Q5	0.16	0.07	0.04	-0.08	-0.20

Note. OLS coefficients are obtained from regressing each element of the transition matrices against a constant and the Gini index. Data is from Chetty et al. (2014a).

To confirm these findings, we pool the transition matrices of the 707 commuting zones to regress each element of the transition matrices against a constant and the Gini index in each commuting zone. Table 2 provides the ordinary least square coefficient of these regressions. All coefficients are significantly different from zero at 1%. These coefficients inform about the effect that a larger parental income inequality has on each element of the transition matrix and, therefore, they inform about the effect that income inequality has on ISM. The coefficients in the main diagonal are especially informative of these effects. We observe that larger inequality increases the probability that children from low income parents (the two lower quintiles) remain in the lower quintiles and, therefore, it implies that more income inequality increases persistence for low income individuals. In the third quintile, we observe a very small positive effect, which shows that the effect of higher inequality is small in the middle part of the income distribution. Finally, for the two higher quintiles, a larger income inequality reduces the probability and, therefore, it increases downward ISM.

Table 3. Income inequality and future income

	Gini index	Children income	Parents income
Gini below the median	0.34	50,325	67,748
Gini above the median	0.47	42,641	68,551

Note: The table provides the average values for the commuting zones with a Gini index below and above the median Gini index.

We next show the effect of income inequality on future average income of the children. Table 3 splits commuting zones in two groups: one with Gini indexes below the median value and the other with Gini indexes above. The table provides average values and shows that although parents income is slightly larger in more unequal commuting zones, children income is substantially smaller. This shows that inequality has a strong negative effect on future income. This result is confirmed in Table 4. The second column of this table provides the estimated coefficients obtained from the regression of children's income against a constant, the Gini index and parents' income. The positive coefficient on parents income indicates that commuting zones with larger income in the period 1996-2000 are also commuting zones with larger income in the period 2011-2012. The negative and large coefficient associated to the Gini index confirms that a larger income inequality reduces future income. Columns 3 and 4 show that this reduction occurs both in the first quartile of the income distribution and also in the last quartile.

Table 4. Income inequality and future income. Regression

	Children income	Children P25	Child P99-P75
Constant	6.91*** (0.20)	3.91*** (0.43)	8.96*** (0.16)
Gini index	-1.21*** (0.05)	-1.24*** (0.14)	-0.36*** (0.08)
Parents income	0.39*** (0.01)	-.-	-.-
Parents P25	-.-	0.61*** (0.04)	-.-
Parents P99-P75	-.-	-.-	0.24*** (0.001)
Observations	707	707	707
R^2	0.55	0.47	0.29

Note: *** indicates p-value $<$ 0.01. Standard errors in parenthesis. Children (Parental) income is the natural logarithm of average income of children (parents) in each commuting zone. Children P25 (Parents P25) indicate the natural logarithm of income at the percentile 25th and Children P99-P75 (Parents P99-P75) indicate the natural logarithm of the difference between income at the percentiles 99th and 75th. We use the first variable as a measure of income at the first quartile and the second as a measure of the increase in income in the last quartile.

To summarize, a higher income inequality increases intergenerational persistence at the bottom of the distribution and increase intergenerational downward social mobility at the top of the distribution. In addition, higher income inequality reduces future income and this reduction affects individuals that are born in families whose income falls either at the bottom or at the top of the income distribution. In the following section, we show that these results are also obtained using the estimated coefficients of Rank-Rank regressions.

2.2 Rank-Rank regressions

Chetty et. al (2014a) provide the coefficients obtained in the Rank-Rank regressions of 2,768 counties in the US. They regress children average percentile rank in the national income distribution against a constant and parents percentile rank in the national income distribution. The slope-coefficient measures the relative income mobility, with a smaller coefficient implying more mobility. The predicted children percentile rank, which is equal to the sum of the constant and the slope-coefficient times the parents percentile rank, measures the absolute social mobility at the parents' percentile rank.

There is upward social mobility when the expected percentile rank of the children is higher than the percentile rank of the parents and downward social mobility otherwise.

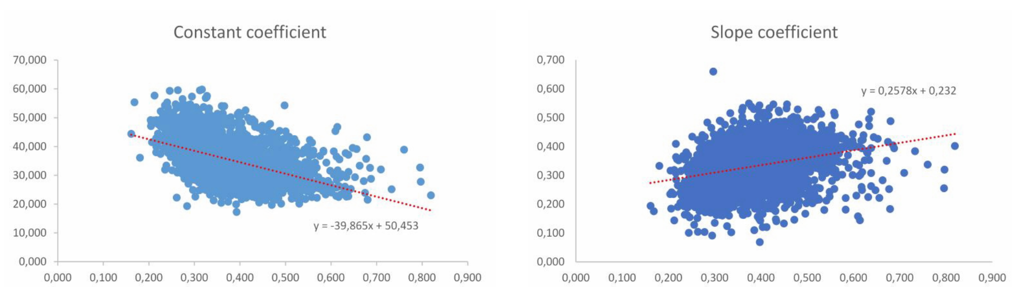
Chetty et al. (2014a) show that in all counties there is upward social mobility for individuals born in families belonging to the low percentiles of the income distribution and downward for those other individuals born in affluent families. This happens because the constant coefficient is positive and the slope is positive and less than one for all counties. In fact, using the constant and the slope coefficients, we can easily obtain the threshold of the percentile rank of the parents above which there is downward ISM and below which there is upward ISM.[6]

Table 5. Coefficients of Rank-Rank regressions

Gini	Constant	Slope	Threshold
0.32	38.1	0.31	55.2
0.45	32.4	0.35	49.6

Source. Data is from Chetty et al. (2014a).

Figure1. Coefficients of Rank-Rank regressions and Gini index



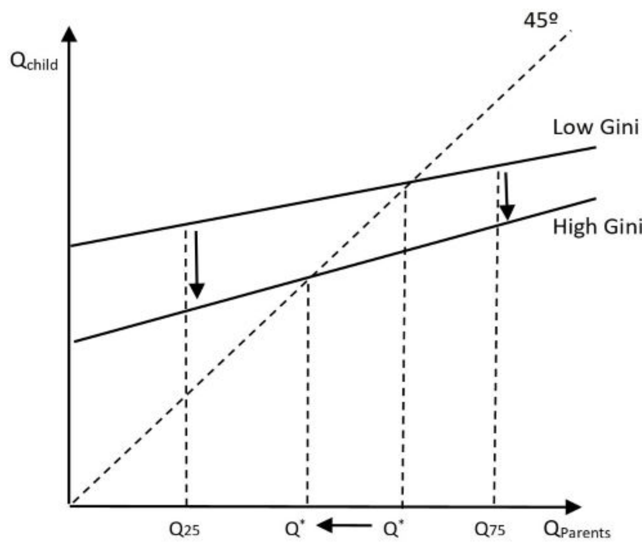
Source. Data is from Chetty et al. (2014a).

Table 5 provides the average value of the Gini index, the two estimated coefficients and the threshold of parents' percentile rank of two groups of counties: one group consisting of those counties with a Gini index below the median of the Gini indices and another group consisting of those other counties with a Gini index above the median. We observe that in those counties with larger inequality the constant is substantially smaller and the slope is slightly larger. Figure 1 shows the relation between the two

[6]This threshold is obtained as the constant divided by one minus the slope.

estimated coefficients (the constant and the slope) and the Gini index in the counties of the US. This figure confirms that a larger inequality substantially decreases the constant and it has a small and positive effect on the slope. As a result of these changes in the slope and in the constant, a higher income inequality reduces the threshold in more than 5 points, as shown in Table 5. This reduction implies that a larger inequality moves from 55% to only 50% the fraction of families whose children are in a higher position than parents in the income distribution.

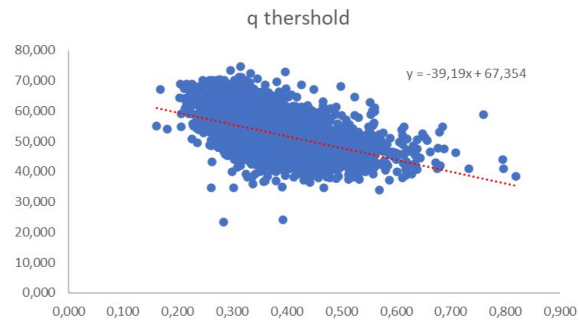
Figure 2. Children predicted percentile rank



Note: Q* indicates the thershold of the percentile rank of parents.

We take into account the results shown in Table 5 to plot Figure 2. This figure shows the predicted children percentile rank as a function of parents' percentile rank of two counties with different inequality. The county with larger inequality has a smaller constant and slightly larger slope. This figure shows the consequences for ISM of a higher inequality. First, higher inequality reduces upward ISM for the low-income and increases persistence, since descendents are closer to the percentile rank of the parents. Second, more inequality increases downward ISM for the high-income and, hence, reduces intergenerational persistence for this group of individuals. Finally, inequality reduces the threshold value of the percentile rank.

Figure 3. Threshold of percentile rank and Gini index.



To confirm these three findings, we perform two exercises. First, using the estimated coefficients, we compute the thresholds for each county and obtain that they are between 25th and 75th percentiles. We regress these thresholds against a constant and the Gini index. The results of this regression are shown in Figure 3 and confirm that more inequality reduces the threshold. Second, we regress the predicted percentile rank of the children born in families that are in the 25th and 75th percentiles against a constant and the Gini index. The results of these regressions are shown in Table 6. We obtain that higher inequality reduces these predicted percentile ranks. This confirms that upward mobility is reduced in the 25th percentile and downward mobility increases in the 75th percentile.

Table 6. Predicted percentile ranks of children

	Percentile 25	Percentile 75
Constant	55.78*** (0.407)	67.64*** (0.334)
Gini index	-32.16*** (1.036)	-19.96*** (0.850)
Observations	2768	2768
R^2	0.26	0.17

Note: *** indicates p-value < 0.01. Standard errors in parenthesis.

In Tables 7 and 8 we use data on income by county to show that a higher income inequality reduces future income, which we identify with the average income of the children. Results are in line with those obtained in Tables 3 and 4, where the analysis is based on income by commuting zone. In Table 7, we group counties according to their Gini index in two groups: high and low income inequality. The table shows that the average income of children is substantially smaller in the group of counties with more income inequality.

Table 7. Income inequality and future income

	Gini index	Child income	Parents income
Gini below the median	0.32	48,816	66,979
Gini above the median	0.45	42,784	67,860

Note: The table provides the average values for the counties with a Gini index below and above the median Gini index.

In Table 8, we confirm that the income of children declines with income inequality. We show that this effect occurs both in the first quartile of the income distribution and also in the last quartile.

Table 8. Income inequality and future income. Regression

	Children income	Children P25	Child P99-P75
Constant	6.09*** (0.09)	3.56*** (0.16)	8.75*** (0.085)
Gini index	-0.94*** (0.03)	-0.88*** (0.06)	-0.33*** (0.05)
Parents income	0.45*** (0.008)	-.-	-.-
Parents P25	-.-	0.62*** (0.01)	-.-
Parents P99-P75	-.-	-.-	0.25*** (0.007)
Observations	2768	2768	2768
R^2	0.60	0.55	0.31

Note: *** indicates p-value < 0.01. Standard errors in parenthesis. Children (Parental) income is the natural logarithm of average income of children (parents) in each commuting zone. Children P25 (Parents P25) indicate the natural logarithm of income at the percentile 25th and Children P99-P75 (Parents P99-P75) indicate the natural logarithm of the difference between income at the percentiles 99th and 75th. We use the first variable as a measure of income at the first quartile and the second as a measure of the increase in income in the last quartile.

We conclude that higher inequality increases persistence of individuals born in families whose income falls at the bottom of the distribution and increases mobility of individuals born in families whose income falls at the top. In addition, it reduces the fraction of individuals that benefit from upward social mobility and decreases future income. The reduction in income occurs both at the top and at the bottom of the income distribution.

3 Model

We proceed to build an analytically tractable model to explain the evidence discussed in the previous section. To keep the model tractable, we consider a small open economy populated by a constant number of young individuals that live for two periods. Therefore, every adult individual has a unique descendent. We also assume that a young individual i receives an inheritance, b_i, and has an innate ability, a_i. This ability is an idiosyncratic productivity shock. Inheritances and abilities introduce heterogeneity among individuals.

The inheritance is a transfer from parents that individuals receive in the first period of life. Individuals decide optimally between investing this transfer in education or saving it in financial assets.[7] The return of savings is the exogenous interest factor, R, that is set in world financial markets and the return of investment in education is a larger next period wage. More precisely, we define by $\mu(h_i)$ the investment in education necessary to obtain an education h_i. We assume that education is a continuous variable defined in the interval $h_i \in (0, \infty)$ and the function $\mu(h_i)$ is continuous, increasing and convex and, hence, $\mu_h > 0$ and $\mu_{hh} \geq 0$, where the subindex in the function indicates the argument of the partial derivative.

The ability a_i obtained, the education h_i achieved, and the effort e_i exerted determine the wage w_i of an individual i, according to the function $w_i \equiv w(a_{i,} e_i, h_i)$. We assume that this function is increasing in all arguments, jointly concave in edu-

[7] By assuming that individuals optimally decide to invest the inheritance in savings (financial assets) or in education (productive investment), we avoid the issues of overeducation or of strategic interaction between parents decision on education and children decisions on effort (Alonso-Carrera et al., 2018), which are not relevant for the purposes of this paper.

cation and effort and it introduces a complementarity between the three arguments. Therefore, we assume that $w_e > 0$, $w_h > 0$, $w_{ee} < 0$, $w_{hh} < 0$, $w_{e_ie_i}w_{hh} > (w_{e_ih})^2$, $w_{eh} > 0$, $w_{ae} > 0$ and $w_{ah} > 0$.

In the second period of life, an individual exerts effort, earns the wage and uses the wage and the return from savings to consume c_i and provide a bequests to his child, b_i'. It follows that the budget constraints in both periods of life are

$$\mu(h_i) + s_i = b_i, \tag{1}$$

$$c_i + b_i' = w(a_{i,} e_i, h_i) + s_i R. \tag{2}$$

Preferences satisfy

$$u_i = \ln c_i + \beta \ln b_i' - \phi(e_i),$$

where $\beta > 0$ is the altruism parameter and $\phi(e_i)$ is the disutility of effort. We assume that the disutility is an increasing and convex function of effort and, hence, $\phi_e > 0$ and $\phi_{ee} \geq 0$.

Finally, we assume that individuals face the following credit constraints, $s_i \geq 0$. Therefore, we introduce an imperfection in the financial markets that limits the investment in education for those individuals that receive a small inheritance.

To keep the notation simple, in what follows we eliminate the subindex i.

3.1 Decisions of the individuals

Individuals choose education, effort, consumption and bequests to maximize utility subject to the budget constraints (1) and (2). From the first order conditions, we obtain

$$b' = \beta c, \tag{3}$$

$$c = \frac{w + R(b - \mu)}{1 + \beta}, \tag{4}$$

$$w_h = R\mu_h \text{ if } b \geq \mu \text{ and } w_h > R\mu_h \text{ if } b < \mu, \tag{5}$$

$$\phi'(e) = \frac{w_e}{c} = w_e \left(\frac{1 + \beta}{w + R(b - \mu)} \right). \tag{6}$$

Given b, equations (3)-(6) determine the values of h, e, c, and b'. Equation (3) describes the intergenerational optimal allocation between consumption and bequests and (4) is the optimal consumption given this intergenerational optimal allocation. The other two equations, (5) and (6), respectively determine education and effort decisions. These two decisions deserve some detailed explanation as they determine income.

3.1.1 Educational decision

The educational decision depends on the position of individuals in the asset market. When they are credit unconstrained lenders, $b \geq \mu$, the educational decision in (5) equalizes the marginal increase in the wage due to a larger investment in education with the marginal cost of this investment. It implicitly defines the following function $h = H^u(e, a)$, with

$$\left.\frac{dh}{de}\right|_{H^u(e,a)} = -\frac{w_{he}}{w_{hh} - R\mu_h} > 0,$$
$$\left.\frac{dh}{da}\right|_{H^u(e,a)} = -\frac{w_{ha}}{w_{hh} - R\mu_h} > 0.$$

These positive derivatives show that individuals that exert more effort or obtain larger abilities invest more in education. Note that these positive relations arise because of the complementarities introduced by the wage function and that imply $w_{he} > 0$ and $w_{ha} > 0$.

When individuals are credit constrained, investment in education is limited by the inheritance. Education then satisfies the following equation: $\mu(h) = b$. This equation implicitly defines the function $h = H^c(b)$, with

$$\left.\frac{dh}{db}\right|_{H^c(b)} = \frac{1}{\mu_h} > 0.$$

A larger inheritance increases investment in education for credit constrained individuals. In contrast, investment in education does not depend on abilities or effort.

We conclude that the positive effect of inheritances on education is the consequence of the imperfection in the financial markets and it only affects credit constrained individuals. We denote this effect of bequest on education as the imperfect financial market effect (IM). This effect has been introduced in models of parental investment to explain intergenerational mobility since the seminal paper by Becker and Tomes (1976).

3.1.2 Effort decision

The effort decision in (6) equalizes the marginal cost in terms of utility of increasing effort with the marginal benefit. This benefit is obtained as the product between the marginal increase in the wage and the marginal utility of consumption. Using (4), we obtain that (6) defines a function $e = E^{u}(h, b, a)$ for unconstrained individuals and $e = E^{c}(h, a)$ for constrained individuals, with

$$\left.\frac{dh}{de}\right|_{E^{u}(h,b,a)} = \frac{\frac{\phi_{ee}}{\phi_e} - \frac{w_{ee}}{w_e} + \frac{\phi_e}{1+\beta}}{\frac{w_{eh}}{w_e}} > 0,$$
$$\left.\frac{dh}{de}\right|_{E^{c}(h,a)} = \frac{\frac{\phi_{ee}}{\phi_e} - \frac{w_{ee}}{w_e} + \frac{\phi_e}{1+\beta}}{\frac{w_{eh}}{w_e} - \frac{w_h}{w}} > 0,$$

where the last inequality holds when $\frac{w_{eh}}{w_e} \geq \frac{w_h}{w}$. This condition is satisfied when the elasticity of substitution between h and e in the wage function is smaller than one. From now on, we assume that this elasticity is smaller or equal to one. In other words, we assume complementarity between education and effort.

To study the effect of bequest on effort decisions, we consider the position of individuals in the financial market. For unconstrained individuals, the effect of a larger inheritance on effort is obtained from the following derivative:

$$\left.\frac{de}{db}\right|_{E^{u}(h,b,a)} = -\frac{\frac{R}{1+\beta}\frac{\phi_e}{w_e}}{\frac{\phi_{ee}}{\phi_e} - \frac{w_{ee}}{w_e} + \frac{\phi_e}{1+\beta}} < 0.$$

Therefore, a larger inheritance reduces effort for unconstrained individuals. As follows from (4), a larger inheritance increases consumption and, hence, reduces the marginal utility of consumption. As a result, the marginal benefit from effort decreases and (6) indicates that the marginal disutility of effort must also decrease. This explains the negative effect of inheritances on effort. This effect, know as the Carnegie effect (CE), has been considered in the analysis of intergenerational mobility by Degan and Thibault (2016) and Alonso-Carrera, et al. (2020).

For constrained individuals, the investment in education equals the inheritance. As a result, any increase in inheritances is used to increase education and does not cause any effect on consumption, nor on the marginal benefit of effort. This implies that, for credit constrained individuals, the effort decision does not depend on the inheritance received . In other words, these individuals are not affected by the CE.

Finally, the effect of abilities on effort for both groups of individuals is given by the following derivative:

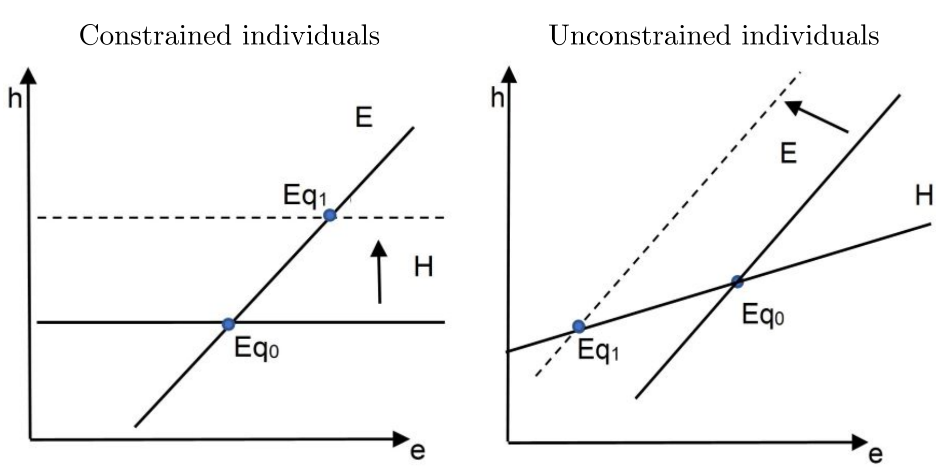
$$\left.\frac{de}{da}\right|_{E^u(h,b,a)} = \left.\frac{de}{da}\right|_{E^c(h,a)} = \frac{\frac{w_{ea}}{w_e} - \frac{\phi_e}{1+\beta}\frac{w_a}{w_e}}{\frac{\phi_{ee}}{\phi_e} - \frac{w_{ee}}{w_e} + \frac{\phi_e}{1+\beta}}.$$

The sign of this derivative is ambiguous due to substitution and income effects. The former arises because a larger ability increases the return from effort. The later is the consequence that wages are larger when abilities are larger, which increases consumption and the disutility of effort.

3.2 The effect of inheritances on individual decisions

We next use the functions E and H to study the effect of a larger inheritance on effort and education. Figure 4 plots these two functions and distinguishes between constrained and unconstrained individuals. For the later, both functions are positively sloped. Therefore, a first necessary step is to determine which function is steeper at the point where they cross. In the appendix, it is shown that concavity of the wage function implies that the function E is steeper than the function H. Taking this into account, Figure 4 shows the effect on individuals decisions of an increase in inheritances.

Figure 4. The effect of a larger inheritance on education and effort



When individuals are credit constrained, the inheritance only affects the decisions on education through the IM. A larger inheritance increases investment in education and, since the elasticity of substitution between education and effort is smaller than one, effort also increases.[8] Graphically, this is shown by the shift upwards in the function H that shifts the equilibrium from Eq0 to Eq1. In contrast, when individuals are unconstrained, the inheritance only affects effort decisions through the CE. For these individuals, a larger inheritance reduces effort and, because of the complementarity between effort and education, investment in education also declines. Graphically, this is shown by the shift to the left in the function E.

We conclude that the effect of a larger inheritance on individuals decisions depends on the position in the asset market. A larger inheritance reduces effort and education when individuals are unconstrained, but has the opposite effects when individuals are constrained. In the following section, we use particular functional forms to show that the position in the asset market depends on the inheritance received from parents.

Before moving to the next section, we briefly discuss the effect of a larger ability on individual decisions. For those individuals that are credit constrained, abilities only affect effort decisions. Therefore, when the income effect dominates, effort decreases with abilities and it increases when the substitution effect dominates. Education is independent of abilities for these individuals. For those other individuals that are not credit constrained, abilities affect both decisions. Since abilities have a positive effect on the education decision, a larger ability increases both effort and education when the substitution effect dominates and has an ambiguous effect on both decisions when the income effect dominates.

4 Inequality, income and ISM

In this section, we complete the analysis of the model described in the previous section. More precisely, we first characterize the decisions of individuals as a function of the inheritance. In so doing, we follow the analysis of the previous section and

[8] In the following section, we assume that the wage function is Cobb-Douglas. In this case, the elasticity of substitution equals one and the function J is vertical for constrained individuals. As a consequence, effort is constant and a larger inheritance only increases education.

we distinguish between unconstrained and constrained individuals. We then use the individual decisions to study the transitional dynamics implied by the model. Finally, we analyze the effect of a higher inequality on future income and ISM. To perform these analyses, we assume the following functional forms:

$$\begin{aligned}\mu &= Bh,\ B>0,\\ \phi &= De,\ D>0,\\ w &= Ah^{\alpha}e^{1-\alpha},\ A>0.\end{aligned}$$

The parameter B measures the cost of education, D measures the disutility of effort and A measures the efficiency of technology. To keep the model simple, in this section, we assume that these parameters are identical for all individuals. In the following section, we will introduce abilities by assuming that A is an idiosyncratic productivity shock.

4.1 Unconstrained individuals

Using (5), we obtain that the function H^u simplifies as follows $h_u = \Gamma e_u$, with $\Gamma = \left(\frac{\alpha A}{RB}\right)^{\frac{1}{1-\alpha}}$. The subindex u identifies optimal decisions of unconstrained individuals. We also deduce that the wage is $w_u = A\Gamma^{\alpha} e_u$.

Using (6), we obtain

$$\frac{(1-\alpha)\,w_u}{De_u} = \frac{w_u + Rb - RBh_u}{1+\beta},$$

and using the function E^u and the expression for wages, we deduce that effort satisfies

$$e_u = \frac{1+\beta}{D} - \frac{Rb}{(1-\alpha)\,A\Gamma^{\alpha}}.$$

The negative effect of the inheritance on effort is the Carnegie effect. Note that for a sufficiently large b, effort is negative. To avoid negative values of effort, we constrain the support of the distribution of inheritances by assuming that

$$b < \widetilde{b} \equiv \frac{(1+\beta)\,(1-\alpha)\,A\Gamma^{\alpha}}{DR}.$$

We use (4) to obtain consumption

$$c_u = \frac{(1-\alpha)\,A\Gamma^{\alpha}}{D},$$

and we use (3) to obtain the bequests given to offspring

$$b_u' = \beta \frac{(1-\alpha) A\Gamma^{\alpha}}{D}. \tag{7}$$

We now use the former equations to characterize the effect of the inheritance on the future income of unconstrained individuals. We define income as $I \equiv w + rb$, where $r = R - 1$ is the interest rate. Using the expression of the wage, we obtain that income, for unconstrained individuals, satisfies

$$I_u = w_u + rb = \frac{1+\beta}{D} A\Gamma^{\alpha} - \frac{\alpha}{1-\alpha} Rb - b. \tag{8}$$

Note that a larger inheritance reduces future income. This is a consequence of the Carnegie effect that reduces effort. While the negative effect on labor income is the obvious consequence of the Carnegie effect, the reduction in total income (capital and labor income) is an extreme result that arises because the disutility of effort is linear. In the Appendix, we show that if the disutility of effort is strictly convex then total income does not necessary decrease with the inheritance. The convexity of the disutility of effort implies that the reduction in effort due to a larger inheritance is smaller for wealthier individuals. As a result, effort is always strictly positive and total income may increase with the inheritance.

From (7), we observe that, regardless of the inheritance received from parents, all unconstrained individuals give the same bequests to descendents. It follows that families of unconstrained individuals attain the steady state in only one generation. At this steady state, the bequests is given by (7) and effort, education and income satisfy:

$$\begin{aligned}
b_u^* &= \beta \frac{(1-\alpha) A\Gamma^{\alpha}}{D}, \\
e_u^* &= \frac{1+\beta (1-R)}{D}, \\
h_u^* &= \Gamma \frac{1+\beta (1-R)}{D}, \\
I_u^* &= \frac{A\Gamma^{\alpha}}{D} \left[1 + \beta\alpha (1-R)\right].
\end{aligned}$$

Note that this steady state is well-defined if $e_u^* > 0$ and $I_u^* > 0$, which requires that $R < (1+\beta)/\beta$. We assume that this condition is always satisfied.

Finally, we obtain the threshold of inheritances, $\overline{b}$, that separates unconstrained from constrained individuals. This threshold is such that $\overline{b} = \mu_u$. We solve this equation to obtain

$$\overline{b} = B(1-\alpha)\Gamma\left(\frac{1+\beta}{D}\right).$$

Therefore, individuals that receive an inheritance $b > \overline{b}$ will be unconstrained lenders. The descendents will be unconstrained individuals that attained the steady state if and only if $b_u^* > \overline{b}$, which happens when

$$R > \frac{\alpha(1+\beta)}{\beta}.$$

When this condition is not satisfied, the descendents of unconstrained individuals will receive an inheritance below the threshold $\overline{b}$ and, therefore, they will be credit constrained individuals.

4.2 Credit constrained individuals

Credit constrained individuals satisfy $\mu_c = b$, where the subindex c indicates the optimal decisions of a credit constrained individual. This equation implies that education satisfies $h_c = b/B$. Using (6), we deduce that

$$e_c = \frac{(1-\alpha)(1+\beta)}{D}.$$

Note that effort is independent of inheritances, which implies that the CE does not affect effort decisions. Instead, education increases with b as a consequence of the imperfections in the credit market.

We next use the wage function to obtain

$$w_c = A\left(\frac{b}{B}\right)^{\alpha} e_c^{1-\alpha}.$$

Using (4), we deduce that

$$c_c = \frac{A}{1+\beta}\left(\frac{b}{B}\right)^{\alpha} e_c^{1-\alpha}$$

and, using (3), we get

$$b_c' = \beta\frac{A}{1+\beta}\left(\frac{b}{B}\right)^{\alpha} e_c^{1-\alpha}. \tag{9}$$

We use the expression of the wages to obtain income as the following increasing and concave function of the inheritance

$$I_c = w_c + rb = A\left(\frac{b}{B}\right)^{\alpha} e_B^{1-\alpha} + rb. \tag{10}$$

Equation (9) describes the transitional dynamics of families that initially receive $b < \bar{b}$. In these families, bequests exhibit a monotonic transition towards the following steady state:

$$\begin{aligned}
b_c^* &= \left(\frac{\beta A}{(1+\beta)\, B^{\alpha}}\right)^{\frac{1}{1-\alpha}} ec, \\
e_c^* &= e_c, \\
h_c^* &= \left(\frac{\beta A}{(1+\beta)\, B}\right)^{\frac{1}{1-\alpha}} e_c, \\
I_c^* &= \left(\frac{1}{\beta} + R\right)\left(\frac{A\beta}{(1+\beta)\, B^{\alpha}}\right)^{\frac{1}{1-\alpha}} e_c.
\end{aligned}$$

The steady state for constrained individuals exists if $b_c^* < \bar{b}$, which happens when $R < \alpha(1+\beta)/\beta$. In addition, we can can show that $b_c^* < b_u^*$ if and only if $R < \alpha(1+\beta)/\beta$.

4.3 Transitional dynamics

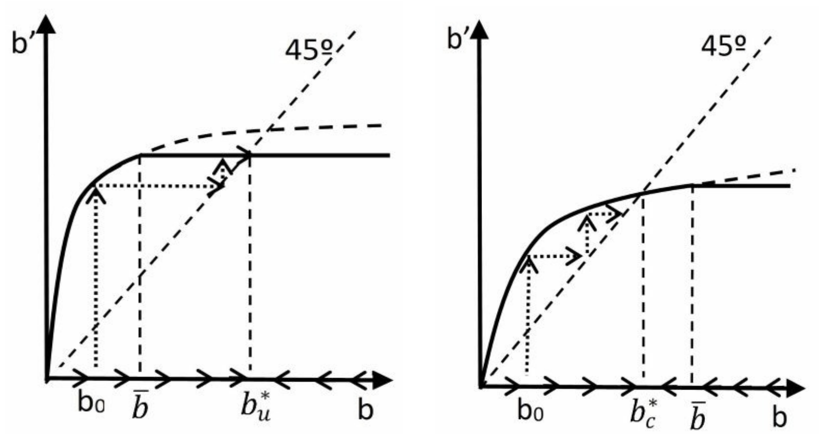
We proceed to characterize the transitional dynamics between the inheritance received by parents and the bequests given to descendents. Therefore, we study the transitional dynamics of inheritances between consecutive generations of individuals of the same family that are linked by altruism. To analyze this transition, we use (7) and (9) to define

$$b' = \begin{cases} b'_c = \beta\frac{A}{1+\beta}\left(\frac{b}{B}\right)^{\alpha} e_c^{1-\alpha} \text{ if } b < \bar{b} \\ b'_u = \beta\frac{(1-\alpha)A\Gamma^{\alpha}}{D} \text{ if } b \geq \bar{b} \end{cases}. \tag{11}$$

This function is continuous since $b'_c = b'_u$ when $b = \bar{b}$. Figure 5 plots this function and distinguishes between two different parametric cases that depend on the value of R.

Figure 5. Dynamics of bequests

$R > \alpha (1+\beta)/\beta$ $\quad$ $R < \alpha (1+\beta)/\beta$



When $R > \alpha (1+\beta)/\beta$, we have that $b_c^* > b_u^* > \bar{b}$. In this case, individuals that receive an inheritance above $\bar{b}$ have descendents that immediately jump to the steady state b_u^*. In contrast, individuals that receive an inheritance below $\bar{b}$ are credit constrained. Their descendents will be credit constrained during some generations until they inherit a quantity above $\bar{b}$ and become unconstrained lenders. In the long run, all individuals will be unconstrained lenders.

When $R < \alpha (1+\beta)/\beta$, we have $b_c^* < b_u^* < \bar{b}$. In this case, individuals that receive an inheritance below $\bar{b}$ are credit constrained. Their descendents after some generations will achieve the steady state b_c^*. In contrast, individuals that receive inheritances above $\bar{b}$ are unconstrained and have descendents that receive b_u^*. Since $b_u^* < \bar{b}$, these descendents are credit constrained. All individuals in the long run are credit constrained.

We can distinguish a third case in which $R^* = \alpha (1+\beta)/\beta$. In this limiting case, we have that $b_c^* = b_u^* = \bar{b}$. Individuals that receive an inheritances above $\bar{b}$ are unconstrained lenders and their descendents receive b_u^*. Individuals that receive an inheritance below $\bar{b}$ are credit constrained and, after some generations, their descendents will inherit b_c^*. Since $b_c^* = b_u^* = \bar{b}$, all individuals converge to a steady state in which they are unconstrained and their optimal savings are zero.

4.4 Inequality, future income and ISM

We proceed to analyze the effects of a higher inequality. Since individuals are heterogeneous only in the inheritance received, we will consider the effect of a more unequal distribution of inheritances. Therefore, in this section, a larger inequality means a more unequal distribution of inheritances.

We perform two different analysis. First, we consider the effect of a larger inequality on future income. Second, we consider the effect of a larger inequality on ISM. The ISM can be measured either by wealth (the inheritance received) or by income. Both measures are relevant, as wealth informs on utility and empirical evidence is based on income. Therefore, we consider both measures. We first analyze the effect of larger inequality on the ISM when it is measured by the difference between the inheritance of the parents and the bequest given to the children and then we perform the same analysis when the ISM is measured by the difference between parental income and the income that children will earn as adults.

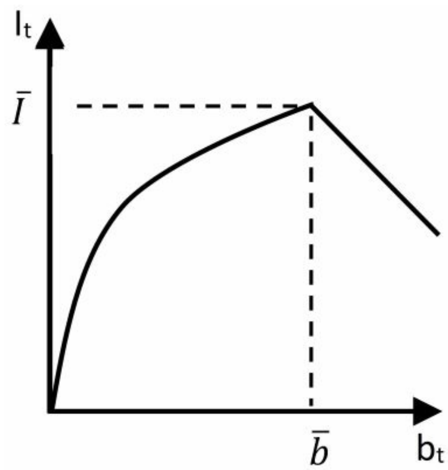
4.4.1 Inequality and future income

We use (8) and (10) to obtain the following function relating income to inheritance:

$$I = \begin{cases} I_c = A\left(\frac{b}{B}\right)^{\alpha} e_c^{1-\alpha} + Rb - b \text{ if } b < \bar{b} \\ I_u = \frac{1+\beta}{D} A\Gamma^{\alpha} - \frac{\alpha}{1-\alpha} Rb - b \text{ if } b \geq \bar{b} \end{cases} .$$

This function is continuous, since $I_u = I_c$ at $b = \bar{b}$. When $b < \bar{b}$, individuals are credit constrained. For these individuals, a larger inheritance increases investment in education and, hence, future income increases. In contrast, when $b \geq \bar{b}$, individuals are unconstrained lenders. For these individuals, a larger inheritance reduces the effort exerted, which explains the negative effect on income of a larger inheritance. Figure 6 depicts this function relating income to inheritance.

Figure 6. Income as a function of inheritance



Using Figure 6, we deduce the effect on future income of a more unequal distribution of inheritances (wealth). To this end, we assume that a more unequal distribution implies that the inheritances obtained by constrained individuals declines, whereas the inheritance received by unconstrained individuals increases. The figure then indicates that a more unequal distribution reduces income of both groups of individuals.

We conclude that this model explains the findings in Tables 4 and 8, which show that higher inequality reduces income both at the lower and at the higher quartiles of the income distribution. There is an extensive literature that analyses the effect of inequality on future income. This literature has focussed on poverty and the reduction in educational investment when individuals are credit constrained and inequality increases. The novelty of our research is to show that higher inequality also harms future income through the reduction of labor earnings of individuals born in affluent families, that we explain using the Carnegie effect. In the following section, we perform a numerical exercise to measure the importance of the two mechanisms driving the negative effect of inequality on income: the IM and the CE.

At this point, a few caveats are in order. First, an increase in inequality does not necessary imply that all unconstrained individuals benefit from a larger inheritance and all constrained individuals suffer from a lower inheritance. In fact, a more unequal distribution could imply that richer constrained individuals also benefit from a larger inheritance. This would imply that income of these group of individuals will increase. However, concavity of the income function implies that a mean preserving spread of the distribution of inheritances received by constrained individuals will reduce the average income of these individuals. As a result, aggregate income declines even if a more unequal distribution of inheritances rises the income of rich constrained individuals. We attain a similar conclusion if we consider that a more unequal distribution reduces the inheritance received by poor unconstrained individuals. This reduction will increase future income of these group of unconstrained individuals, but will not modify the general conclusion that inequality reduces future average income.

Second, the reduction of the income of unconstrained individuals due to an increase in the inheritance is the consequence of simplifying assumptions. In the appendix, we show that if we consider that effort disutility is strictly convex, then income of unconstrained individuals may increase with inheritances. In this case, even if total individual income does not fall with inheritances, labor income will fall due to the Carnegie effect. Therefore, it is still true that future income would be larger with a more equal distribution of inheritances.

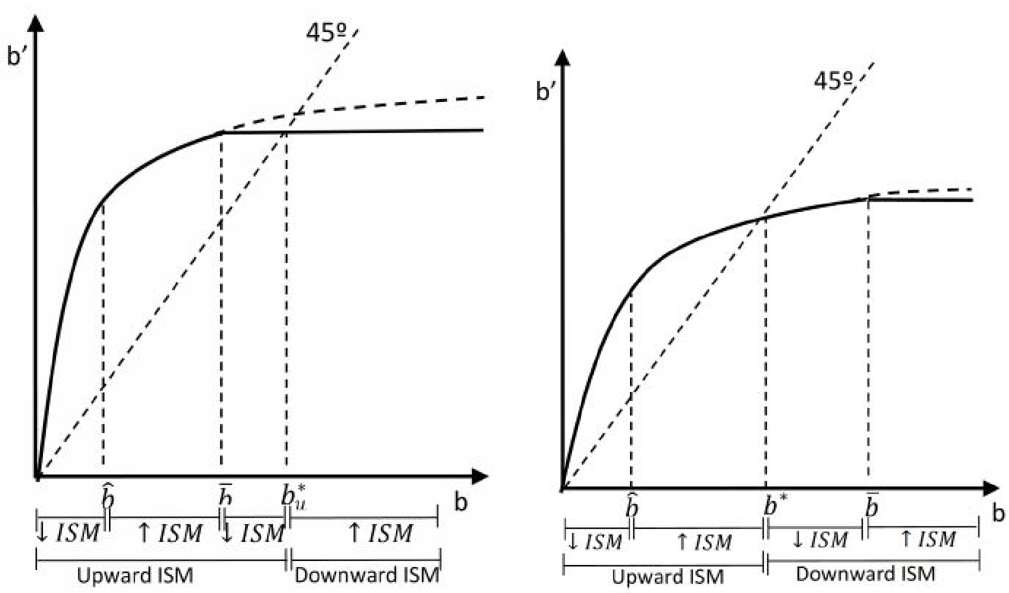
Third, the analysis in this section shows that income declines when inequality in inheritances increases, whereas the evidence discussed in Section 2 considers income inequality. In the numerical exercise of the following section, we show that a more unequal distribution of inheritances increases income inequality and reduces future income. Therefore, in the numerical exercise, we show that the model generates the negative effect of income inequality on future income shown in the evidence discussed in Section 2.

4.4.2 Inequality and ISM

We next analyze the effect on ISM of a more unequal distribution of inheritances. As a first approach, we consider that the variable determining ISM is inheritance. We claim that there is upward ISM when the inheritance received is smaller than the bequest given to children and there is downward ISM when it is larger. The relationship between the bequests given to children and the inheritance received is obtained from using the function (11), which is plotted in Figure 7.

Figure 7. ISM in inheritances

$R > \alpha\,(1+\beta)\,/\beta$ $\qquad$ $R < \alpha\,(1+\beta)\,/\beta$



Note: We assume that a higher inequality implies an increase in b if individual is unconstrained and a reduction if he is constrained.

When $R > \alpha\,(1+\beta)\,/\beta$, we observe that there is upward ISM when $b < b_u^*$ and downward otherwise. As a result, the bequest given is larger than the inheritance received for all constrained individuals and also for unconstrained individuals with $b < b_u^*$. When $R < \alpha\,(1+\beta)\,/\beta$, there is upward ISM when $b < b_c^*$. In this case, only constrained individuals with $b < b_c^*$ leave a bequest larger than the inheritance received.

Using Figure 7, we can deduce the effect of inequality on ISM. To perform this analysis, we must introduce two clarifications. First, we assume that a more unequal distribution of inheritances reduces the inheritance of constrained individuals and increases the inheritance of unconstrained individuals. Second, there is more mobility (either upward or downward) when the bequest function separates from the 45^o line. Taking these clarifications into account, we can proceed to deduce the effect of inequality on ISM in the two parametric cases.

When $R > \alpha\,(1+\beta)\,/\beta$, we observe that a larger inequality reduces upward ISM for unconstrained individuals with $b < b_u^*$, whereas it increases downward social mobility for those high-income unconstrained individuals with $b > b_u^*$. The intuition is quite immediate. We interpret a larger inequality as an increase in the inheritance received by unconstrained individuals. However, this does not translate into a larger bequest given to the descendents, since unconstrained individuals give the same bequest regardless of the inheritance received. As a result, when inequality rises, the bequests given to descendents relative to the inheritance received declines, which explains that ISM declines for families of unconstrained individuals that benefit from upward ISM and increases for families of unconstrained individuals that suffer from downward ISM.

As for the constrained individuals, a larger inequality reduces upward ISM for poor constrained individuals with $b < \widehat{b}$, whereas it increases upward ISM for those constrained individuals with $b > \widehat{b}$. The threshold $\widehat{b}$ is the value of the inheritance for which a reduction in the inheritance received is translated into a reduction in the bequest given of the same amount. Due to decreasing returns to education $(\alpha < 1)$, below this threshold, a reduction in the inheritance received causes a larger reduction in income that is translated into a larger reduction in the bequest given. On the contrary, above this threshold, the reduction in inheritances causes a smaller effect on income and, therefore, the reduction in the bequest given is smaller. The threshold $\widehat{b}$, that is obtained by equalizing the slope of the function (9) to one, is equal to

$$\widehat{b} = \left(\frac{\alpha \beta A}{(1+\beta) B^{\alpha}} \right)^{\frac{1}{1-\alpha}} e_c.$$

The condition $R < (1+\beta)/\beta$ implies that $\widehat{b} < \overline{b}$ and, as a result, the two groups of constrained individuals always exist in this economy.

When $R < \alpha(1+\beta)/\beta$, we observe some interesting differences. In this case, all families of unconstrained individuals suffer from downward ISM and, as in the previous case, a larger inequality increases downward ISM. As for the constrained individuals, we distinguish three groups. The first two groups are alike those of the previous case. In particular, inequality reduces upward ISM for constrained individuals that receive an inheritance $b < \widehat{b}$ and increases ISM for those constrained individuals that receive $b \in \left(\widehat{b}, b_c^*\right)$. The intuition is as in the previous case. In addition, there is a third group of constrained individuals that receive an inheritance $b \in \left(b_c^*, \overline{b}\right)$. The families of these constrained individuals suffer downward social mobility. A larger inequality, that reduces the inheritance obtained by these constrained individuals, reduces the bequest given to their descendents in a smaller amount. As a consequence, a larger inequality reduces downward ISM for these group of constrained individuals.

To summarize, Figure 7 shows that for families of individuals that receive low inheritance $\left(b < \widehat{b}\right)$, a more unequal distribution reduces ISM. It also shows that for families of individuals that receive large inheritance $\left(b > \max\left\{\widehat{b}, b_u^*\right\}\right)$, a higher inequality increases downward ISM. Finally, for families in the middle of the distribution, we observe different effects of inequality on ISM.

These results are based on a very particular assumption regarding the meaning of a more unequal distribution of inheritances. More unequal means an increase in the inheritance of unconstrained individuals and a reduction in the inheritance of constrained individuals. Obviously, a more unequal distribution could imply that for sufficiently rich constrained individuals inheritance increases or, on the contrary, for low-income unconstrained individuals inheritance decreases. In these cases, a larger inequality makes some individuals shift between being unconstrained and constrained. As a result, conclusions regarding the effect of a higher inequality on the middle social class will depend crucially on the interpretation of a more unequal distribution. However, the focus of our analysis is on the effect of inequality on ISM of families whose income is at the bottom and at the top of the distribution. For these families, the effect of higher inequality remains when we consider alternative interpretations of a more unequal distribution.

In what follows, we analyze the effect of a higher inequality on ISM when it is measured in terms of income. As a first step, we obtain descendents income as a function of parental income. To keep the analysis simple, we will consider that parents and their children hold the same position in the asset market. We first consider that parents and children are unconstrained individuals. Parental income of unconstrained individuals is a decreasing function of inheritance, $I_u(b)$, given by (8). Since unconstrained individuals receive an inheritance $b \geq \overline{b}$, parental income satisfies that $I_u(b) \leq I_u\left(\overline{b}\right)$, with

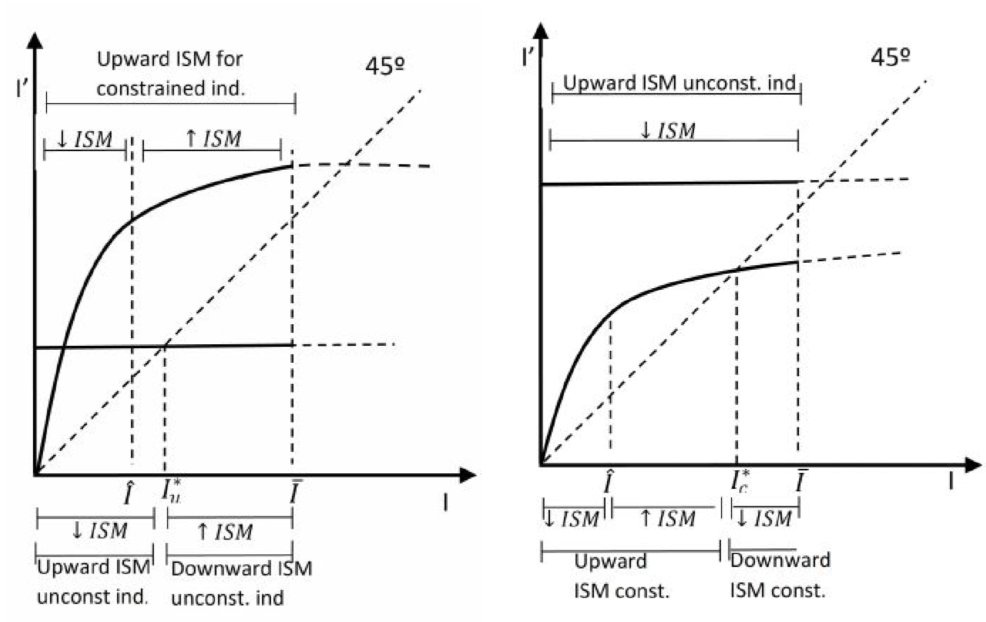
$$I_u\left(\overline{b}\right) = \left(1 + \alpha - \frac{\alpha}{R}\right) \frac{(1+\beta)(1-\alpha) A\Gamma^{\alpha}}{D}.$$

Descendents income is independent of inheritances and it is equal to $I'_u = I^*_u$.

We next consider that parents and children are constrained individuals. Parental income is an increasing function of inheritance, $I_c(b)$, given by (10). Since constrained individuals receive an inheritance $b < \bar{b}$, parental income satisfies that $I_c(b) < I_c(\bar{b})$. It is immediate to show that $I_c(\bar{b}) = I_u(\bar{b}) \equiv \bar{I}$. Therefore, parental income of both unconstrained individuals and constrained individuals satisfies $I \leq \bar{I}$. The income of the children of constrained individuals is determined by the composite function $I'_c = I_c(b'(b))$, which is obtained from (9) and (10). This composite function relates the income of children with parental inheritance. To related with parental income, we define $b(I)$ as the inverse function of $I_c(b)$. Using this inverse function, we obtain the income of children as the following function of parental income: $I'_c = I_c(b'(b(I))) \equiv g(I)$. This function satisfies the following properties. First, using (9) and (10), we obtain that $I_c(0) = 0$ and $b'(0) = 0$. This implies that $g(0) = 0$. Second, since both $I_c(b)$ and $b'(b)$ are increasing functions, $g(I)$ is also an increasing function. Third, in the appendix it is shown that if $R > \alpha(1+\beta)/\beta$ then $I^*_c > \bar{I} > I^*_u$ and $g(\bar{I}) > I^*_u$ and if $R < \alpha(1+\beta)/\beta$ then $I^*_c < \bar{I} < I^*_u$ and $g(\bar{I}) < I^*_u$. Finally, in the appendix it is also shown that the function $g(I)$ is concave. These properties imply that the relationship between the income of children and parental income is as the one shown in Figure 8.

Figure 8. ISM in income

$R > \alpha (1+\beta)/\beta$ $\quad$ $R < \alpha (1+\beta)/\beta$



Note: We assume that a higher inequality implies that I of unconstrained individuals increases and I of constrained indviduals decreases.

We use Figure 8 to analyze ISM in terms of income. First, note that the income of both constrained individuals and unconstrained individuals is distributed in the interval $\left(0, \overline{I}\right)$. When $R > \alpha (1+\beta)/\beta$, we find results similar to those obtained in Figure 7. In particular, constrained individuals always exhibit upward ISM, whereas unconstrained individuals with $I < I_u^*$ exhibit upward social mobility and those with with $I \geq I_u^*$ exhibit downward social mobility. As for the effects of a larger income inequality, it is important to clarify first the meaning of a more unequal distribution of income. Following the analysis in Figure 7, we first assume that more unequal means that the income of the constrained individuals decreases and that of the unconstrained individuals increases. Under this interpretation, we observe that families of unconstrained individuals reduce upward social mobility when $I < I_u^*$ and increase downward social mobility otherwise. For constrained individuals, upward social mobility declines when $I > \widehat{I}$ and increases otherwise. The existence of $\widehat{I}$ is shown in the appendix.

These results are in line with those discussed in the analysis of Figure 7 and consistent with the empirical evidence introduced in Section 2, since they imply that higher income inequality increases mobility at the top of the income distribution and decreases mobility at the bottom. However, since the income of unconstrained individuals is not necessary larger than that of constrained individuals, we should consider other interpretations of a larger inequality. For instance, assume that rich individuals are both constrained individuals with $I > \widehat{I}$ and unconstrained with $I > I_u^*$. Assume also that higher inequality increases the income of rich individuals and decreases the income of the rest of individuals. In this case, we observe that for constrained individuals more inequality reduces upward ISM. In contrast, for unconstrained individuals more inequality may increase ISM both upward (for poor unconstrained individuals) and downward (for rich unconstrained individuals). Provided the fraction of unconstrained individuals among rich individuals is larger, these results also explain that more inequality reduces ISM at the bottom of the distribution and increases ISM at the top.

When $R < \alpha(1+\beta)/\beta$, we find results quite different from those in Figure 7. In particular, unconstrained individuals now exhibit upward ISM, whereas constrained individuals with $I < I_c^*$ exhibit upward social mobility and those with with $I > I_c^*$ exhibit downward social mobility. Regarding the effects of a larger inequality on ISM, they crucially depend on the interpretation of a larger inequality. If it implies an increase in the income of unconstrained individuals and a reduction in the income of constrained individuals, then, as shown in the figure, it implies a reduction in ISM except for constrained individuals with $I \in \left(\widehat{I}, I_c^*\right)$. In contrast, if a larger inequality implies that richer individuals obtain more income and poor obtain less then for unconstrained individuals ISM increases if they are low-income and decreases if they are high-income. For constrained individuals, there is a reduction in ISM for poor and an increase of ISM for high-income constrained individuals.

We conclude that when we consider the effects of an increase in inequality on ISM then the results are in line with the evidence discussed in Section 2 when $R > \alpha(1+\beta)/\beta$, whereas they crucially depend on the meaning of a larger inequality when $R < \alpha(1+\beta)/\beta$. To provide a more precise analysis, we perform a numerical exercise in the following section.

5 Numerical exercise

The numerical exercise is based on the model of the previous section with two important differences. First, we assume a quadratic effort disutility, i.e. $\phi = De^2$. In Appendix C, we solve this version of the model and we show that the effects of a higher income inequality are in line with those obtained when the disutility of effort is linear.[9] However, in contrast to the model with a linear disutility, income of unconstrained individuals increases with inheritances. Therefore, the income of unconstrained individuals is larger than that of constrained individuals, which is consistent with the fact that constrained individuals tend to be concentrated in the first quintiles of the income distribution.

The second difference is the introduction of abilities as an idiosyncratic productivity shock that determines the efficiency parameter in the wage function, according to the following function: $A_t^i = \exp(a_t^i)$. Abilities are an additional source of heterogeneity among individuals that is necessary to generate the patterns of social mobility observed in the transition matrices. The intergenerational transmission of these abilities is set according to the following process:

$$a_t^i = \gamma + \rho a_{t-1}^i + \varepsilon_t,\ \ \rho \in (0,1)\,,$$

where ε_t is a normally distributed random variable with zero mean and variance σ^2. The parameter ρ determines the intergenerational correlation between abilities. The expected value of A_t^i at t satisfies

$$\mathrm{E}_t(A_{t+j}^i) = a_t^{\rho^j} \exp\left(\frac{\gamma(1-\rho^j)}{1-\rho} + \frac{\sigma^2(1-\rho^{2j})}{2(1-\rho^2)}\right)$$

and the unconditional expected value satisfies

$$\mathrm{E}(A_t^i) = \exp\left(\frac{\gamma}{1-\rho} + \frac{\sigma^2}{2(1-\rho^2)}\right).$$

We organize this section as follows. First, we explain the calibration of the parameters. Second, we simulate the calibrated economy to show individual decisions,

[9] In Appendix C, we analyze a general strictly convex effort disutility with $\phi = De^\theta$, $\theta > 1$. In the numerical exercise, we consider the quadratic case with $\theta = 2$.

the time path of the main variables and the importance of the two mechanisms: CE and IM. Finally, we analyze the effects on average income and ISM of a higher income inequality.

5.1 Calibration

We calibrate the parameters to match several targets of the US economy in the period 1996-2000. Table 9 provides the value of parameters and targets.

Table 9. Calibration

Parameter	Value	Target	Data	Model
Initial distribution of inheritances and abilities				
$\widetilde{\rho}$	0.15	Gini index 1996-2000[a]	41%	41%
A_1	8.9	Third to first quartile of labor earnings[b]	2.3	2.4
B_1	0.1124	Mean of parents abilities	-.-	1
A_2	0.20	Wealth to income ratio 1996-2000[c]	5.7	5.7
B_2	0.63	Fraction of wealth among the top 10%[d]	62%	64%
Exogenous process for abilities				
ρ	0.01	Intergenerational mobility[a]	72%	72%
σ	0.91	Intergenerational persistence at the top[a]	28%	27%
γ	-0.41	Mean of descendents abilities	-.-	1
Parameters of preferences and technology				
R	5.5427	Labor income share[e]	62%	62%
β	0.112	Wealth to income ratio 2018-2022[c]	7.2	7.2
α	0.125	Intergenerational persistence at the bottom[a]	25%	27%
B	0.0226	Average steady state value of education	-.-	1.03
D	0.2453	Average steady state value of effort	-.-	0.81

Source:
[a] Chetty et al. (2014a).
[b] US Bureau of labor statistics.
[c] FRED, St. Louis Fed.
[d] Mean value of household and non-profit organization net worth as percentage of disposable personal income, from FRED, st. Louis Fed.
[e] Penn World Table.

Two variables determine initial heterogeneity among individuals: abilities and inheritances. We obtain the initial distribution of these two variables from two Gamma distributions that are calibrated as follows:

1. The number of individuals is set to 200.000.

2. The correlation between the two distributions, $\widetilde{\rho}$, is set to match the average value of the Gini index in disposable income post taxes and transfers in the period 1996-2000. We obtain this number from Chetty et al. (2014a) as the average value of the Gini indexes in the commuting zones.

3. The mean of abilities is normalized to one and the mean of inheritances is set to match the average wealth to income ratio in the period 1996-2000, which equals 5.74.[10]

4. The variance of abilities is set to match the value in the US in the period 1996-2000 of the ratio between average labor earnings in the third and first quartiles of the labor earnings distribution. Data on the distribution of labor earnings is from the US Bureau of Labor Statistics.

5. The variance of inheritances is set to match the share of net worth held by the top 10 percentile in the period 1996-2000, which is 61.8%. Data on the distribution of wealth is from the St. Louis Fed.

Three parameters, γ, ρ and σ, determine the distribution of future abilities and, therefore, will be key to determine ISM. First, we set $\gamma = -\sigma^2/2$ to keep the mean of descendents abilities equal to one. This eliminates exogenous growth of wages. Second, ρ and σ, together with α are jointly set to match three measures of ISM that are calculated from the social matrix obtained as the average of the social matrices

[10] Data on social mobility is from Chetty et al. (2014a). They consider cohorts born in the period 1980-1982. They measure parent's income as the mean of family income in the period 1996-2000, when children were between 15 and 20 years old, and they measure children's income as the mean of family income in period 2011-2012, when they were 30 years old. Accordingly, we consider the initial period as the period 1996-2000. The initial wealth to income ratio is the mean value of household and non-profit organization net worth as percentage of disposable personal income in the period 1996-2000. This variable is obtained from FRED.

of the different commuting zones. We call the first measure intergenerational mobility and it determines ISM for the entire distribution. It is defined as 1 minus the second highest eigenvalue. There is full mobility when it is equal to one and full-intergenerational persistence when it is equal to zero. The second and third measures determine, respectively, intergenerational persistence at the bottom and at the top of the income distribution. One is obtained as the probability that an individual born in a family whose income falls in the first two quintiles will earn as an adult an income that is also in these two quintiles and the second is obtained as the probability that an individual born in a family whose income falls in the last two quintiles stays in these quintiles as an adult. More precisely, let us define by π_{ij} the probability that an individual born in a family that belongs to quintile i moves as an adult to quintile j. Then, we define persistence at the bottom as

$$PersB = \frac{\pi_{11} + \pi_{12} + \pi_{21} + \pi_{22}}{4},$$

and we define persistence at the top as

$$PersT = \frac{\pi_{44} + \pi_{45} + \pi_{54} + \pi_{55}}{4}.$$

These measures imply persistence when they are larger than 20% and a larger value implies more intergenerational persistence for that group of individuals. A larger ρ reduces ISM in all quantiles of the transition matrix, whereas a larger σ increases mobility in all quantiles of the transition matrix. Finally, the parameter α measures the return of education and, therefore, it has a large effect on mobility at the bottom. By setting the value of these three parameters, the model matches the target of intergenerational mobility and generates intergenerational persistence at the top (27% instead of 28% in the data) and at the bottom (27% instead of 25% in the data).

We set $R = 5.54$, which is the value of R for which the labor income share equals its average value in the period 1996-2000 (62%, see the Penn World Tables) when the wealth to income ratio equals 5.74. This number implies and annual interest rate equal to 7.1%, when we assume that a period is 25 years. This value is very close to the internal rate of return from Penn World Table (PWT) whose average value is 7.8%.

Finally, three parameters, β, B and D, are set to match steady state targets. Given the autoregressive process assumed for abilities, the steady state informs about long run mean values of the economy.

1. We will set β to match the steady state value of the wealth to income ratio. We approximate this value with the mean value of household and non-profit organization net worth as percentage of disposable personal income in the period 2018-2022, that we obtain from the St. Louis Fed. This value equals 7.19. To obtain the value of β, we must take into account three important aspects. First, we define wealth as Rb, to take into account that consumption is done at the end of the second period. Second, income in the data is annual, whereas in the model is life-time income generated in a 25-years period. Assuming a constant annual income, we obtain the following relation between individual annual income (I_a^i) and life-time income (I^i):

$$I^i = I_a^i \sum_{t=0}^{25} \frac{1}{R_a^t} = I_a^i \frac{\frac{1}{R_a^{25}} - 1}{\frac{1}{R_a} - 1} = 12.38 I_a^i,$$

where $R_a = 1.071$ is the annual interest factor. This implies that the capital to income ratio in the model must equal $7.19/12.38 = 0.58$. Finally, the value of β will depend on the position of individuals in the asset market at the steady state. If $R < \alpha (1+\beta)/\beta$ then individuals are constrained at the steady state and the wealth to income ratio equals[11]

$$\frac{Rb^*}{I^*} = \frac{R}{\frac{1}{\beta} + R}.$$

The value of β that makes $\frac{Rb^*}{I^*} = 0.58$ is 0.25. This value implies that $R > (1+\beta)/\beta > \alpha (1+\beta)/\beta$. Therefore, we cannot explain the large accumulation of wealth in the US economy when individuals are constrained. We consider next that $R > \alpha (1+\beta)/\beta$. Individuals are unconstrained and

$$\frac{Rb^*}{I^*} = \frac{R\beta (1-\alpha)}{1 - \alpha\beta r} = 0.58.$$

[11] The expression of Rb^*/I^* is obtained from the steady state equations. This ratio does not change when we consider a convex disutility of effort.

We solve this equation and obtain

$$\beta = \frac{0.58}{\left[1-\alpha\left(1-0.58\right)\right]R-0.58\alpha}.$$

Note that this function implies that β is an increasing function of α for which $R \in \left(\alpha\frac{(1+\beta)}{\beta}, \frac{(1+\beta)}{\beta}\right)$ for any $\alpha \in (0,1)$.[12] Therefore, the steady state with unconstrained individuals can explain the accumulation of capital in the US. In the calibration, we set β according to the previous function and at the steady state individuals are unconstrained.

2. The parameters B and D are scale parameters that determine the steady state value of e and h. We set them to have $e^* = h^* = 1$. They are equal to

$$\begin{aligned} D &= \frac{1+\beta-\beta R}{2}, \\ B &= \frac{\alpha \mathrm{E}(A_t^i)}{R} = \frac{\alpha}{R}. \end{aligned}$$

5.2 Results of the simulation

In this section, we first provide scattered plots showing individuals decisions and the patterns of social mobility for each individual. Next, we show the time path of the average value of the main variables. Finally, we simulate counterfactual economies to discuss the effect of the CE and IM mechanisms.

5.2.1 Individual decisions

Figure 9 shows that credit constrained individuals are characterized by low inheritance and large ability. The complementarity between ability and education implies that individuals with large ability face the borrowing constraint, since they obtain a larger return from education and, therefore, they would like to borrow to invest in education.

[12] Since in the data $Rb^*/I^* < 1$, we have that $R < (1+\beta)/\beta$. Since in the data $R > 1$, we have that $R > \alpha(1+\beta)/\beta$ for any $\alpha \in (0,1)$.

Figure 9. Inheritance and abilities

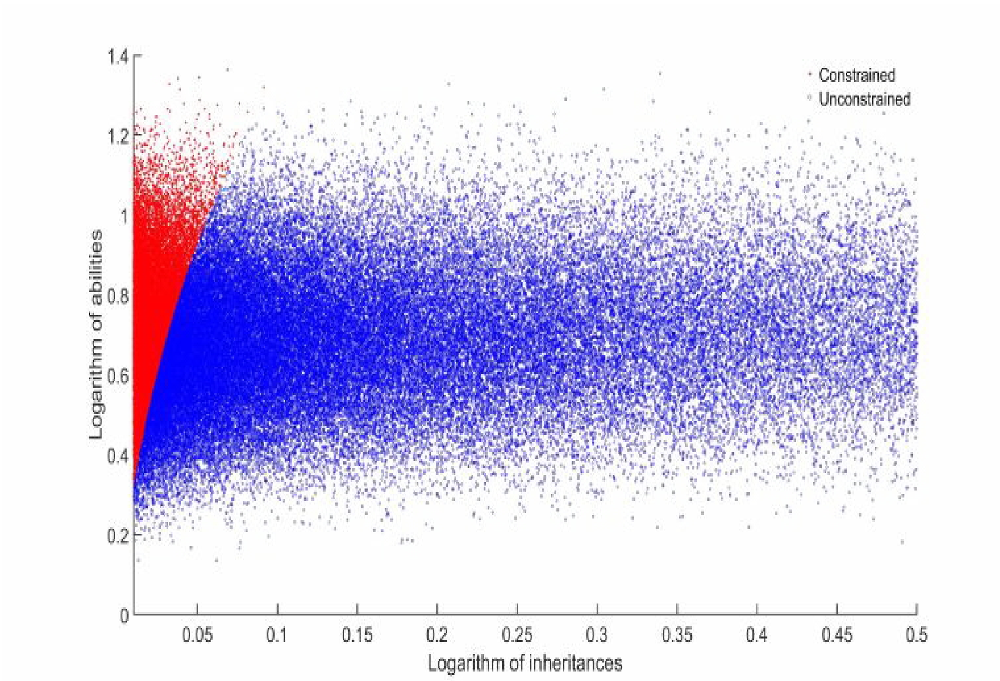
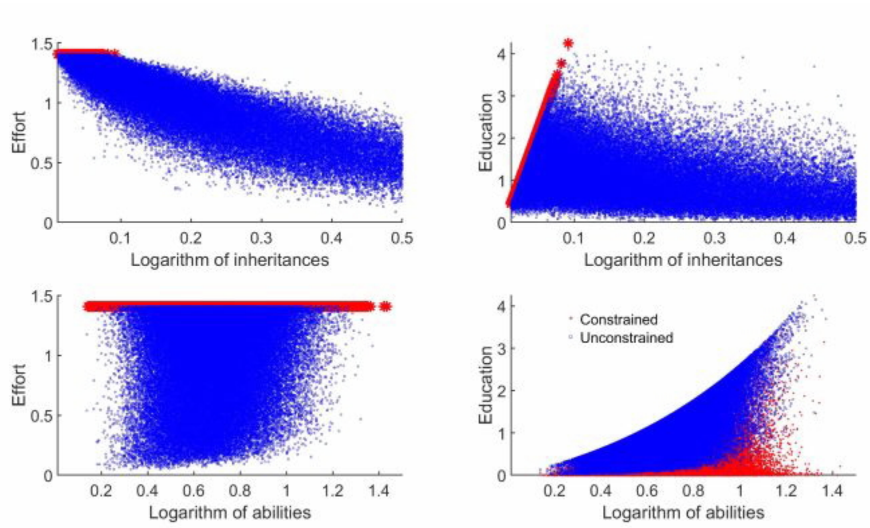


Figure 10 shows that effort is constant and large for credit constrained individuals, whereas for unconstrained individuals effort is smaller, it declines with inheritance and has not a clear relation with abilities. It also shows that education increases with bequests and abilities for credit constrained individuals. In contrast, for unconstrained individuals, it declines with inheritance and increases with abilities. These findings confirm the results obtained in Section 3.

Figure 10. Individuals decisions



In Figure 11, we show the effect of a larger inheritance on wages and income. For constrained individuals, a lager inheritance increases education and does not reduce effort. As a result, wages and income increase. For unconstrained individuals, a larger inheritance reduces both effort and education. As a result, wages decline with the inheritance. Even if wages decrease, income increases, but the effect of inheritances on income is smaller than that of constrained individuals.

Figure 11. Inheritance and income

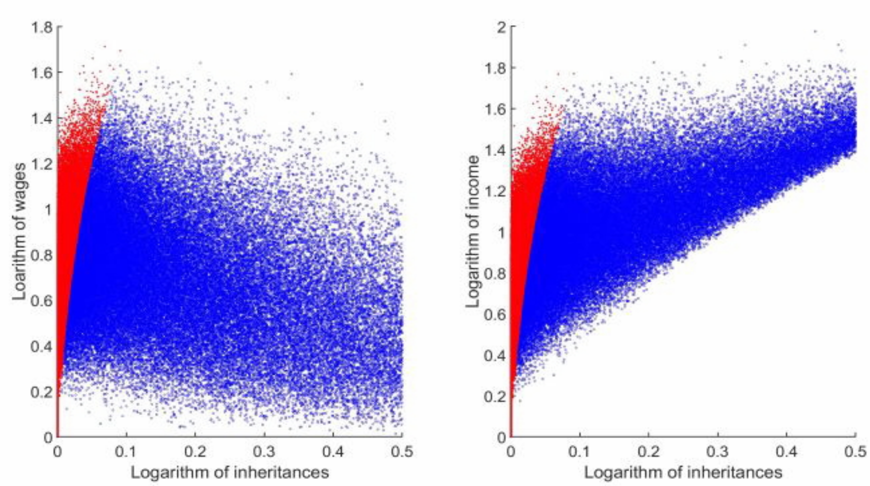
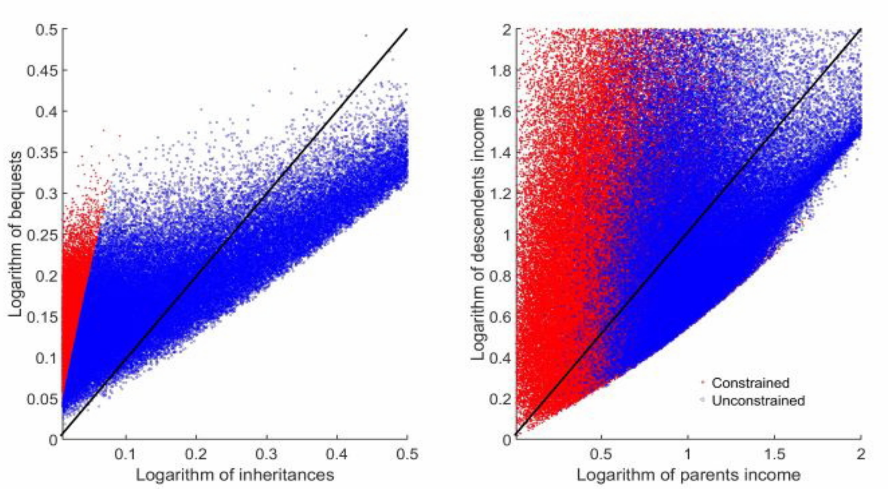


Figure 12 shows intergenerational social mobility. The first panel shows the relation between inheritance and bequests and the second the relation between parents and descendents income. Both panels show that the vast majority of constrained individuals exhibit upward ISM in wealth and also in income, while only low-income unconstrained individuals exhibit upward ISM. These findings confirm the results obtained in Section 4.

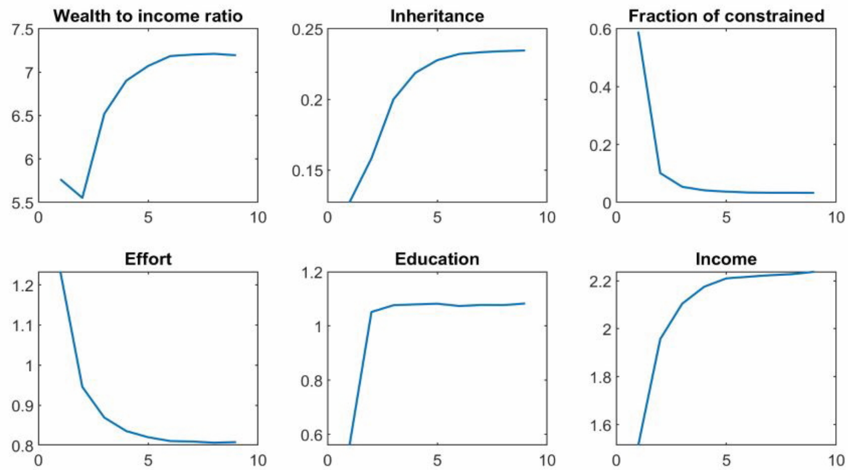
Figure 12. Intergenerational social mobility



5.2.2 Time paths

Figure 13 displays the mean values of the variables along the transition. Remember that a period is 25 years and, therefore, the transition occurs over a long period of time. As follows from the calibration in Table 9, we assume that the wealth to income ratio is initially below its steady state value. This is the driver of the transition that directly explains the increase in the mean value of inheritances. As inheritances increase, the number of credit constrained individuals declines, which also explains the reduction in effort and the increase in education. Finally, income increases along the transition as a consequence of the larger wealth and education.

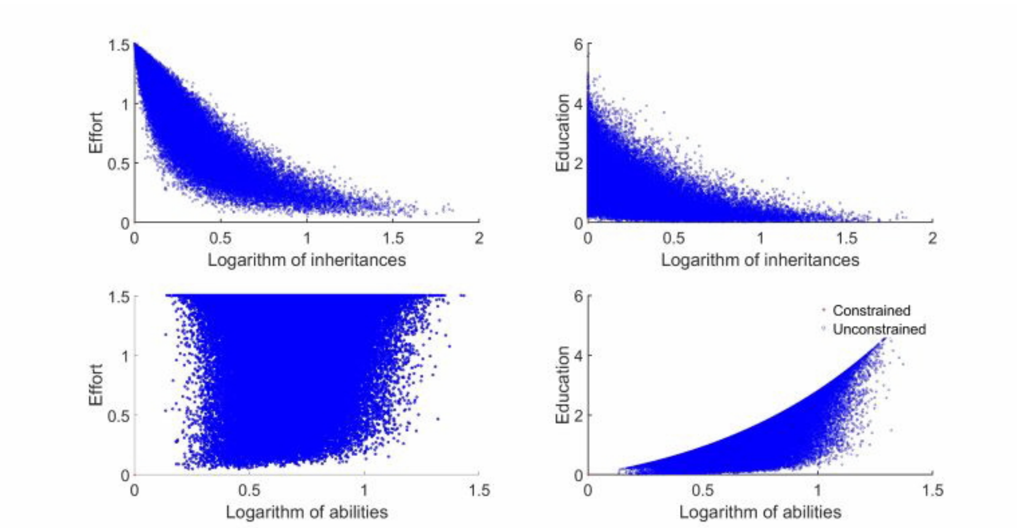
Figure 13. Time paths



5.2.3 CE and IM

We next discuss the effect of CE and IM for ISM. To this end, we simulate two counterfactual economies in which we remove these two mechanisms. We remove the CE by assuming that non-constrained individuals exert the same effort than constrained individuals and we remove the IM by assuming that the credit market is perfect. Figures 14 and 15 depict the scattered plots of effort and education decisions when the IM and CE effects are removed.

Figure 14. Individuals decisions without IM



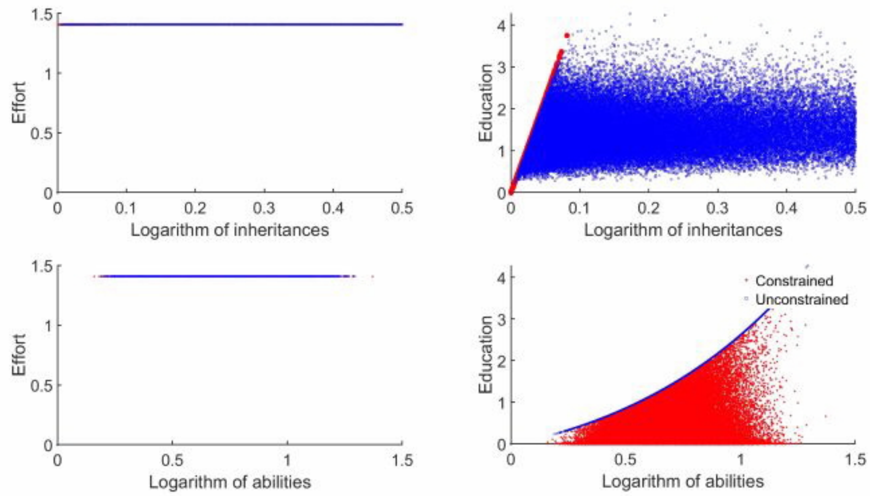
When the credit market is perfect, individuals are not credit constrained. As a result, all individuals are affected by the Carnegie effect and, hence, effort declines with the value of the inheritance. This also explains the negative effect of inheritances on education. The intuition is as follows. With perfect credit markets, the investment in education depends only on the return of education, which increases with effort. Since a larger inheritance reduces effort, it reduces the return of education and, as a result, investment in education declines. Therefore, removing imperfections in the credit market increases the negative effect that inheritances have on income. Table 10 shows that removing this imperfection reduces substantially inequality, whereas it has a small positive effect on ISM and also on persistence both at the top and at the bottom of the distribution. Two different mechanisms explain this small effect on ISM. First, individuals at the top of the distribution are not affected by imperfect credit markets. Second, individuals at the bottom benefit from the elimination of the imperfection, since they can borrow to invest in education, which increases their income and explains the large reduction in income inequality. However, these individuals must pay back the credit, which limits the increase in the bequest given to children. This explains that removing imperfections in the credit market has only a moderate effect on ISM.

Table 10. Inequality and ISM

	US economy	No IM	No CE
Income	100%	124%	111%
Gini	0.4150	0.2813	0.4327
Mobility	0.72	0.77	0.5946
Persistence bottom	0.2710	0.2664	0.2955
Persistence top	0.2655	0.2586	0.2941
π_{11}	0.3947	0.3918	0.4468
π_{55}	0.3442	0.3267	0.4303

Figure 15 shows the individual decisions when we do not consider the CE. In this case, effort is constant for both credit constrained and unconstrained individuals. This has some interesting implications for unconstrained individuals. These individuals choose education, given the abilities, to maximize income. Therefore, abilities determine education. As a result, education is uncorrelated with inheritances and it is strongly correlated with abilities. Removing the CE increases income of unconstrained individuals, which causes an increase in income inequality and a reduction in ISM, specially at the top of the distribution. Table 10 confirms these intuitions. It shows that removing the CE increases the Gini index in 3 percentage points and causes a large reduction in mobility, mainly driven by the large increase in persistence at the top of the distribution.

Figure 15. Individuals decisions without CE



5.3 A tale of two commuting zones

Chetty et al. (2014a) report huge differences in income inequality among US commuting zones. As an example, we can compare Soda Springs and New York. The first commuting zone has a very low Gini index of 0.225, whereas the Gini index of New York equals 0.684, which is one of the highest in the US. These large differences in income inequality translate into differences in ISM and future income that are in line with the empirical evidence shown in Section 2. On the one hand, mobility is much higher in Soda Springs. The expected rank of children whose parents are at the 25th percentile of the national income distribution is 55% in Soda Springs and only 41% in New York. This is a huge difference of 14 percentage points in mobility between these two commuting zones. On the other hand, income inequality reduces future income. To see this effect, we use Chetty (2114a) data on average parents' income in the period 1996-2000 and average children's income in the period 2011-2012. We observe that in Soda Springs children's income is 77% of parents income, whereas in New York it is only 52%. Clearly, future income is smaller when inequality is larger.

We next use the model to analyze the effect of income inequality on ISM and future income and to measure the role of the two mechanisms: the Carnegie effect and the credit constraint. To generate differences in income inequality, we assume that the only difference among commuting zones is in the variance of the distribution of inheritances. Therefore, all parameters are taken from Table 9 except those characterizing the distribution of inheritances, which are set to generate a mean preserving spread of the distribution shown in Table 9. More precisely, we change the value of the parameters A_2 and B_2 to keep constant the value of the mean of inheritances and to match the value of the Gini index in each commuting zone.

5.3.1 The effect of income inequality on ISM

Table 11 shows the value of the parameters characterizing the distribution of inheritances, moments of this distribution and the results of the simulation for three economies that differ only in the variance of inheritances. One economy targets the Gini index of the average of the commuting zones in the US and the other two target the Gini index of Soda Springs and New York.

Table 11. The effect of inequality on ISM

	Soda Springs	US	New York
Parameters			
A_2	0.9	0.2	0.057
B_2	0.14	0.63	2.21
Moments of the distribution			
Mean of inheritances	0.12	0.12	0.12
Variance of inheritances	0.02	0.08	0.28
Results. All mechanisms			
Gini	22.5%	41%	69%
Mobility	72%	71%	56%
Expected rank 25th	50%	40%	31%
Expected rank 75th	61%	59%	50%
Results. No IM			
Gini	21%	28%	35%
Mobility	80%	77%	77%
Expected rank 25th	51%	49%	48%
Expected rank 75th	62%	61%	60%
Results. No CE			
Gini	25%	44%	71%
Mobility	67%	58%	48%
Expected rank 25th	52%	37%	30%
Expected rank 75th	67%	64%	51%
Expected rank 90th	70%	71%	69%

Note. Expected rank 25th (75th) measures the expected rank of children whose parents are at the 25th (75th) percentile of the national income distribution.

Table 11 provides the Gini index and three measures of ISM: mobility, the expected rank 25th and the expected rank 75th. The distribution of inheritances is set so that the simulated Gini index equals the Gini index in the data in the period 1996-2000. Mobility is one minus the second highest eigenvalue of the transition matrix. Therefore, it is a measure of social mobility for the entire distribution. It shows that higher inequality reduces mobility, which is consistent with the Great Gatsby curve. Expected rank 25th is the expected rank of children born in families whose income is in the 25th percentile of the national income distribution. Note that it is larger than 25% in the three economies, implying that at this percentile there is upward social mobility. Therefore, this expected rank is a measure of upward social mobility for individuals born in families whose income falls in the bottom of the distribution. We observe that a higher inequality reduces substantially upward social mobility, from 50% in Soda Springs to only 31% in New York. The last measure of social mobility is the expect rank of children born in families whose income falls in the 75th percentile of the national income distribution. In all economies, it is lower than 75%, which implies that at this percentile we observe downward social mobility. It is a measure of social mobility for individuals born in families whose income falls in the top of the distribution. We observe that a higher inequality increases downward social mobility from 61% in Soda Springs to 50% in New York. Therefore, the results of the simulation are consistent with the evidence introduced in Section 2 and show that higher inequality reduces ISM at the bottom of the distribution, whereas increases mobility at the top. This has relevant implications for future inequality as the difference between the expected percentile ranks of children born in families at the 75th percentile and of children born in families at the 25th percentile is only 9 percentage points in Soda Springs, while it is 19 percentage points in New York.

To understand the mechanisms driving the effect of inequality on social mobility, Table 11 also provides the simulation of counterfactual economies in which either the IM or the CE are removed. When the credit constraint is removed, income inequality decreases and mobility increases. We also observe that in Soda Springs removing the credit constraint has almost no effect in the expected rank at the 25th percentile, nor at the 75th, since the number of credit constrained individuals in these percentiles is very small in this commuting zone. In contrast, in the US we observe a large effect in the 25th percentile and small in the 75th. This is explained by the fact that the fraction of credit constrained individuals is large in the 25th percentile and very small in the 75th percentile. Finally, in the very unequal New York we have credit constrained individuals both in the 25th percentile and also in the 75th percentile, which explains that removing the credit constraint increases the expected rank at both percentiles. Clearly, removing the credit constraint has a larger effect on ISM in more unequal economies, where the number of credit constrained individuals is larger.

Finally, removing the Carnegie effect increases inequality and reduces mobility. Since the Carnegie effect does not affect credit constrained individuals, it has a very mild effect on the expected rank at the 25th percentile. In contrast, it increases the expected rank at the 75th percentile, which explains the reduction in ISM. Observe also that the effect on the expected rank is larger in more equal economies. Again this is the consequence that in these economies the fraction of credit constrained individuals is smaller. Indeed, if there were no credit constrained individuals, inequality will not drive differences in the expected rank in the absence of the Carnegie effect. To see this, Table 11 shows the expected rank for individuals born in families whose income is in the 90th percentile. Among these families the fraction of credit constrained individuals is negligible and we do not observe a clear effect of income inequality on the expected rank.

We conclude that both the IM and the CE explain that inequality increases downward social mobility in affluent families and reduces upward social mobility in poor families.

5.3.2 The effect of income inequality on future income

We use the three economies described in Table 11 to analyze the effect on income of a more unequal distribution of inheritances. The results of this analysis are summarized in Table 12, which provides the mean values of several variables for the three economies. For each variable, it is shown the mean value of the variable for constrained and unconstrained individuals.

Table 12 shows that a more unequal distribution of inheritances that increases the Gini index causes a large reduction on income per capita. This result is consistent with the evidence discussed in Section 2. The effect of inequality on income is explained entirely by the effect on the average wage, which is larger in more egalitarian commuting zones for both constrained and unconstrained individuals. The average inheritance of credit constrained individuals is larger in more egalitarian commuting zones. As a result, they invest more in education, which explains the larger wage of constrained individuals. Unconstrained individuals receive a smaller average inheritance in more egalitarian commuting zones, which explains that both effort and the average wage are larger in these commuting zones. Therefore, both the Carnegie effect and the credit constraint explain the negative effect of inequality on income.

Table 12. The effect of inequality on income

	Soda Springs	US	New York
Gini	0.22	0.41	0.69
CC	0.24	0.59	0.81
Mean b	0.12	0.12	0.12
Mean b_C	0.01	0.005	0.002
Mean b_U	0.16	0.30	0.65
Mean I	1.74	1.51	1.12
Mean I_C	1.37	0.90	0.46
Mean I_U	1.85	2.39	3.87
Mean w	1.16	0.93	0.54
Mean w_C	1.30	0.88	0.46
Mean w_U	1.12	1.02	0.90
Mean e	1.17	1.23	1.29
Mean e_C	1.41	1.41	1.41
Mean e_U	1.10	0.98	0.84
Mean h	1.02	0.56	0.24
Meanh_C	0.69	0.24	0.08
Mean h_U	1.12	1.02	0.89

Note. CC measures the fraction of credit constrained individuals. The subindex c (u) indicate that the mean of the variables is computed over credit constrained (unconstrained) individuals.

The larger average wage in more egalitarian commuting zones is the consequence of three effects: the smaller fraction of credit constrained individuals, the larger average education of credit constrained individuals (IM effect) and the larger average effort exerted by unconstrained individuals (CE effect). To illustrate the importance of each mechanism, we decompose the wage gap between Soda Springs and New York. We obtain that 40% of the gap is explained by the reduction in the number of credit constrained individuals, 32% is explained by the increase in the education of constrained individuals and the remaining 28% is explained by the increase in the effort of unconstrained individuals.

6 Conclusions

We show that in the US higher income inequality reduces upward ISM at the bottom of the income distribution, but increases downward ISM at the top. We also show that higher income inequality reduces future income and this reduction happens both at the bottom and also at the top of the income distribution. We explain these findings combining two different mechanisms. One mechanism is based on a credit constraint that explains the effect of a higher income inequality on individuals that are at the bottom of the income distribution, where credit constrained individuals concentrate. A higher inequality implies that individuals at the lower percentiles of the income distribution obtain a smaller inheritance and, since they are credit constrained, they invest less in education. This explains that as inequality increases more individuals stay poor and future income declines. The second mechanism is based on endogenous effort decisions that only affect affluent unconstrained individuals and, therefore, it explains the effect of inequality at the top of the distribution. A higher inequality implies that individuals at the top percentiles of the income distribution obtain higher inheritances and, accordingly, they exert less effort, which reduces their labor earnings. This effect, known as the Carnegie effect, explains that as inequality increases there is a reduction in future income and more downward ISM.

We introduce the two mechanisms in an overlapping generations model with heterogeneous individuals. First, we assume that individuals are heterogeneous only in inheritances (wealth) and show that the model explains the empirical findings on the effect of higher inequality on future income and ISM. We then introduce innate abilities as another source of heterogeneity. We solve numerically the version of the model with innate abilities to measure the effects of an increase in income inequality when this increase is driven by a more unequal distribution of inheritances. We show that when we increase the Gini index from 22% (the lower level of inequality among commuting zones) to 69% (the highest levels of inequality), mobility falls in 14 percentage points and the expected rank of children born in families whose income is at the 25th percentile of the income distribution falls in 19 percentage points and that of children born in families whose income is at the 75th percentile falls in 11 percentage points. These are large changes in ISM that are explained by the introduction of the two mechanisms. More precisely, we show that if we remove the credit constraint then, as inequality increases, the model does not generate the reduction in the expected rank of children born in families whose income is at the 25th percentile. We also show that if we remove the Carnegie effect then a higher income inequality does not generate any significant reduction in the expected rank of children born in families whose income is at high percentiles of the income distribution. We finally show that increasing the Gini index from 22% to 69% leads to a 53% decrease in the average future labor earnings. This is a huge reduction that we explain as the consequence of three different effects. First, the number of constrained individuals increases as inequality increases, which reduces investment in education. This causes an income loss that accounts for 40% of the total reduction in labor earnings. Second, a higher inequality reduces the inheritance of constrained individuals that then reduce investment in education. This causes an additional income loss that accounts for 32% of the total reduction. Finally, a higher inequality increases the inheritance of unconstrained individuals than then reduce effort. This accounts for 28% of the total reduction.

We conclude that a higher income inequality harms future income of both low and high income individuals. Given the large effects of a higher inequality obtained in our analysis, the design of policies oriented to limit these negative effects is relevant. To design these policies, we must take into account the differences in the mechanisms relating income inequality and future income. For low-income individuals, the mechanism is based on imperfections in the credit market. Removing these imperfections is generally a Pareto improvement. In addition, removing these imperfections will reduce inequality and will increase income. In contrast, the mechanism affecting affluent individuals is the Carnegie effect, which is not based on any inefficiency. As a result, policy interventions that aim to reduce this effect will not be Pareto improving. Moreover, we have shown that removing the Carnegie effect increases inequality and reduces ISM. This discussion suggests that the design of government policies is particularly interesting in the context of the model introduced in this paper, since they may have contrasting effects on welfare and income inequality. The analysis of these policies is the aim of future research.

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A Slopes of the functions $\mathbf{E}^u$ and $\mathbf{J}^u$

The function $E^u(h,b,a)$ is steeper than $H^u(e,a)$ if the following inequality is satisfied:

$$\frac{\frac{\phi_{ee}}{\phi_e}-\frac{w_{ee}}{w_e}+\frac{\phi_e}{1+\beta}}{\frac{w_{eh}}{w_e}} > \frac{w_{he}}{R\mu_h - w_{hh}}.$$

After some simple manipulation, we deduce that

$$\left(\frac{\phi_{ee}}{\phi_e}w_e - w_{ee} + \frac{\phi_e}{1+\beta}w_e\right)R\mu_h - \left(\frac{\phi_{ee}}{\phi_e}+\frac{\phi_e}{1+\beta}\right)w_e w_{hh} > (w_{eh})^2 - w_{hh}w_{ee}.$$

The left hand side is positive, whereas concavity of the wage function implies that the right side is negative. This proves that the inequality is satisfied and, hence, the function $E^u(h,b,a)$ is stepper than the function $H^u(e,a)$.

B Characterization of the function $g(I)$

First, we remember that $g(I) = I_c(b'(b(I)))$ where

$$\begin{aligned} I_c(b') &= A\left(\frac{b'}{B}\right)^\alpha e_c^{1-\alpha} + rb', \\ b'(b) &= \beta\frac{A}{1+\beta}\left(\frac{b}{B}\right)^\alpha e_c^{1-\alpha}, \end{aligned}$$

and $b(I)$ is the inverse function of

$$I = A\left(\frac{b}{B}\right)^\alpha e_c^{1-\alpha} + rb.$$

We next obtain

$$b_c'(\bar{b}) = \frac{\beta A}{1+\beta}\Gamma^\alpha e_c$$

and we use it to get $g(\bar{I}) = I_c(b'(\bar{b}))$ that satisfies

$$g(\bar{I}) = I_c(b'(\bar{b})) = \left[\left(\frac{\beta}{1+\beta}\right)^{\alpha-1}\left(\frac{R}{\alpha}\right)^\alpha + R - 1\right]\frac{\beta A}{1+\beta}\Gamma^\alpha e_c.$$

We have that $g(\bar{I}) > I_u^*$ when

$$\left[\left(\frac{\beta}{1+\beta}\right)^{\alpha-1}\left(\frac{R}{\alpha}\right)^\alpha + R - 1\right]\beta(1-\alpha) > 1 + \alpha\beta(1-R).$$

Note that the left hand side of the previous inequality increases with R, the right hand side decreases with R and it holds with strict equality when $R = \alpha(1+\beta)/\beta$. This proves that $g(\bar{I}) > I_u^*$ if and only if $R > \alpha(1+\beta)/\beta$.

Using the definitions $\overline{I}$ and I_u^*, we obtain that $\overline{I} > I_u^*$ when

$$(1+\beta)(1-\alpha)\left(1+\alpha-\frac{\alpha}{R}\right)-1-\alpha\beta(1-R)>0.$$

The left hand side is increasing in R and the equation holds with strict equality when $R=\alpha(1+\beta)/\beta$. This proves that $\overline{I} > I_u^*$ if and only if $R>\alpha(1+\beta)/\beta$. We next use the definitions of $\overline{I}$ and I_c^* to show that $\overline{I} < I_c^*$ if

$$\left(1+\alpha-\frac{\alpha}{R}\right)\left(\frac{\alpha}{R}\right)^{\frac{\alpha}{1-\alpha}}<\left(\frac{1}{\beta}+R\right)\left(\frac{\beta}{1+\beta}\right)^{\frac{1}{1-\alpha}}.$$

Note that the left hand side of the previous inequality decreases with R, the right hand side increases with R and it holds with stick equality when $R=\alpha(1+\beta)/\beta$. This proves that $\overline{I} < I_c^*$ if and only if $R>\alpha(1+\beta)/\beta$. We next use the definitions of I_u^* and I_c^* to show that $I_u^* < I_c^*$ if

$$\left(\frac{\alpha}{R}\right)^{\frac{\alpha}{1-\alpha}}\left[1+\alpha\beta(1-R)\right]<\left(\frac{1}{\beta}+R\right)\left(\frac{\beta}{1+\beta}\right)^{\frac{1}{1-\alpha}}(1-\alpha)(1+\beta).$$

Note that the left hand side of the previous inequality decreases with R, the right hand side increases with R and it holds with strict equality when $R=\alpha(1+\beta)/\beta$. This proves that $I_u^* < I_c^*$ if and only if $R>\alpha(1+\beta)/\beta$.

We proceed to show concavity of the function $I^{'}=g(b(I))$ where

$$\begin{aligned} g(b) &= \frac{Ae_c^{1-\alpha}}{B^\alpha}\left(\underbrace{\frac{A\beta e_B^{1-\alpha}}{B^\alpha(1+\beta)}}_{\eta}\right)^{\alpha} b^{\alpha^2}+(R-1)\frac{A\beta e_B^{1-\alpha}}{B^\alpha(1+\beta)}b^\alpha \\ &= \frac{1+\beta}{\beta}\eta^{\alpha+1}b^{\alpha^2}+(R-1)\eta b^\alpha, \end{aligned}$$

and

$$I(b)=A\left(\frac{b}{B}\right)^{\alpha}e_c^{1-\alpha}+(R-1)b=\frac{1+\beta}{\beta}\eta b^\alpha+(R-1)b.$$

We have that

$$\frac{\partial g}{\partial I}=\frac{\partial g}{\partial b}\frac{\partial b}{\partial I}=\frac{\frac{\partial g}{\partial b}}{\frac{\partial I}{\partial b}},$$

and

$$\frac{\partial^2 g}{\partial I^2}=\frac{\frac{\partial^2 g}{\partial b^2}\frac{\partial I}{\partial b}-\frac{\partial g}{\partial b}\frac{\partial^2 I}{\partial b^2}}{\left(\frac{\partial I}{\partial b}\right)^2}\frac{\partial b}{\partial I}<0$$

if $\frac{\partial^2 g}{\partial b^2}\frac{\partial I}{\partial b} - \frac{\partial g}{\partial b}\frac{\partial^2 I}{\partial b^2} < 0$. It follows that when this inequality is satisfied the function g is concave. We obtain that

$$\begin{aligned}
\frac{\partial g}{\partial b} &= \alpha^2 \frac{1+\beta}{\beta}\eta^{\alpha+1} b^{\alpha^2-1} + \alpha (R-1)\eta b^{\alpha-1} > 0, \\
\frac{\partial^2 g}{\partial b^2} &= \alpha^2 \left(\alpha^2 - 1\right) \frac{1+\beta}{\beta}\eta^{\alpha+1} b^{\alpha^2-2} + \alpha(\alpha-1)(R-1)\eta b^{\alpha-2} < 0, \\
\frac{\partial I}{\partial b} &= \alpha \frac{1+\beta}{\beta}\eta b^{\alpha-1} + (R-1) > 0, \\
\frac{\partial^2 I}{\partial b^2} &= \alpha(\alpha-1)\frac{1+\beta}{\beta}\eta b^{\alpha-2} < 0.
\end{aligned}$$

Therefore, we obtain that $\frac{\partial^2 g}{\partial b^2}\frac{\partial I}{\partial b} - \frac{\partial g}{\partial b}\frac{\partial^2 I}{\partial b^2} < 0$ when

$$\begin{aligned}
& \left[\alpha^2 \left(\alpha^2 - 1\right) \frac{1+\beta}{\beta}\eta^{\alpha+1} b^{\alpha^2-2} + \alpha(\alpha-1)(R-1)\eta b^{\alpha-2}\right] \left[\alpha \frac{1+\beta}{\beta}\eta b^{\alpha-1} + R - 1\right] \\
< \; & \left[\alpha^2 \frac{1+\beta}{\beta}\eta^{\alpha+1} b^{\alpha^2-1} + \alpha(R-1)\eta b^{\alpha-1}\right] \alpha(\alpha-1)\frac{1+\beta}{\beta}\eta b^{\alpha-2},
\end{aligned}$$

which simplifies as

$$\begin{aligned}
& \left[\alpha^2 \left(\alpha^2 - 1\right) \frac{1+\beta}{\beta}\eta^{\alpha+1} b^{\alpha(\alpha-1)} + \alpha(\alpha-1)(R-1)\eta\right] \left[\alpha \frac{1+\beta}{\beta}\eta b^{\alpha-1} + R - 1\right] \\
< \; & \left[(\alpha-1)\alpha^2 \frac{1+\beta}{\beta}\eta^{\alpha+1} b^{\alpha(\alpha-1)} + (\alpha-1)\alpha(R-1)\eta\right] \alpha\frac{1+\beta}{\beta}\eta b^{\alpha-1}.
\end{aligned}$$

After some manipulations, we obtain

$$\begin{aligned}
& \left[\alpha^2 \left(\alpha^2 - 1\right) \frac{1+\beta}{\beta}\eta^{\alpha+1} b^{\alpha(\alpha-1)} + \alpha(\alpha-1)(R-1)\eta\right](R-1) \\
< \; & (1-\alpha)\alpha^3 \frac{1+\beta}{\beta}\eta^{\alpha+1} b^{\alpha(\alpha-1)} \alpha \frac{1+\beta}{\beta}\eta b^{\alpha-1}.
\end{aligned}$$

This inequality is always satisfied, since the left hand side is negative and the right hand side is positive. This proves concavity.

We finally define $\widehat{I}$, which is the value of I such that $\frac{\partial g}{\partial I} = 1$. This value satisfies $\widehat{I} < \overline{I}$ if

$$\left.\frac{\partial g}{\partial I}\right|_{I=\overline{I}} = \frac{\alpha^2 \frac{1+\beta}{\beta}\eta^{\alpha+1}\left(\overline{b}\right)^{\alpha^2-1} + \alpha(R-1)\eta\left(\overline{b}\right)^{\alpha-1}}{\alpha\frac{1+\beta}{\beta}\eta\left(\overline{b}\right)^{\alpha-1} + (R-1)} < 1,$$

which holds when

$$\alpha^2 \frac{1+\beta}{\beta}\eta^{\alpha+1}\left(\overline{b}\right)^{\alpha^2-1} < \left(\frac{1+\beta}{\beta} - (R-1)\right)\alpha\eta\left(\overline{b}\right)^{\alpha-1} + (R-1).$$

We evaluate this inequality when $b = \bar{b} = B(1-\alpha)\Gamma\left(\frac{1+\beta}{D}\right)$ and obtain

$$\begin{aligned} & \alpha^2 \frac{1+\beta}{\beta} \eta^{\alpha+1} \left[B(1-\alpha)\Gamma\left(\frac{1+\beta}{D}\right) \right]^{(\alpha+1)(\alpha-1)} \\ < & \left(\frac{1+\beta}{\beta} - (R-1) \right) \alpha\eta \left[B(1-\alpha)\Gamma\left(\frac{1+\beta}{D}\right) \right]^{\alpha-1} + R - 1. \end{aligned}$$

Using the expression of η, we obtain

$$\begin{aligned} & \alpha^2 \frac{1+\beta}{\beta} \left(\frac{A\beta}{B^\alpha (1+\beta)} \right)^{\alpha+1} (B\Gamma)^{(\alpha+1)(\alpha-1)} \\ < & \left(\frac{1+\beta}{\beta} - (R-1) \right) \alpha \frac{A\beta}{B^\alpha (1+\beta)} (B\Gamma)^{\alpha-1} + R - 1, \end{aligned}$$

which simplifies as

$$\Psi = \alpha^{1-\alpha} \left(\frac{\beta}{1+\beta} \right)^\alpha R^{1+\alpha} - \left(\frac{1+\beta}{\beta} - (R-1) \right) \frac{\beta}{1+\beta} R - (R-1) < 0.$$

Since $R < (1+\beta)/\beta$ and $R > 1$, we obtain that

$$\Psi = \alpha^{1-\alpha} \left(\frac{\beta R}{1+\beta} \right)^\alpha R - R + (R-1) \left(\frac{\beta}{1+\beta} R - 1 \right) < \left(\alpha^{1-\alpha} - 1 \right) R < 0.$$

This proves that $\widehat{I} < \overline{I}$.

C Convex disutility of effort

In the simple example of Section 4, the income of unconstrained individuals decreases with the inheritance, due to the Carnegie effect. This is an extreme feature of the model that arises because the disutility of effort is linear. In this appendix, we show that if the effort function is strictly convex then income of unconstrained individuals may not decline with inheritances and the results regarding the effects of higher inequality obtained in Section 4 still hold. Therefore, in this appendix, we modify the model of Section 4 by assuming that the effort cost function is $\phi = De^\theta$, with $\theta > 1$.

We organize the appendix following the structure of Section 4. We first analyze individual decisions of both unconstrained and constrained individuals. We then characterize the transitional dynamics and, finally, we analyze the effect of inequality on future income and on ISM.

C.1 Unconstrained individuals

Using (5), we obtain that $h_u = \Gamma e_u$, $w_u = A\Gamma^\alpha e_u$ and $\mu_u = B\Gamma e_u$. Using (6), we obtain $e_u = e(b)$ that solves

$$\left(\frac{1+\beta}{\theta D e^\theta} - 1\right)(1-\alpha) A\Gamma^\alpha e = Rb. \tag{12}$$

Note that $e(b) > 0$, $e'(b) < 0$, $e''(b) > 0$ and

$$e(0) = \left(\frac{1+\beta}{\theta D}\right)^{\frac{1}{\theta}} \equiv \widehat{e}.$$

Therefore, effort satisfies that $e_u \in (0, \widehat{e})$. Note that effort is always positive when $\theta > 1$. The negative effect of inheritances on effort is the Carnegie effect. This implies that labor earnings decline with the inheritance due to the Carnegie effect. However, total income also includes capital income and it is equal to

$$I_u = w_u + rb = \left[\frac{(R-1)(1+\beta)(1-\alpha)}{\theta D e_u^\theta} + (1-\alpha) + \alpha R\right] \frac{A\Gamma^\alpha}{R} e_u. \tag{13}$$

We can write income as a function of effort and we can compute

$$\frac{\partial I_u}{\partial e_u} = \left[\frac{(R-1)(1+\beta)(1-\alpha)(1-\theta)}{\theta D e_u^\theta} + (1-\alpha) + \alpha R\right] \frac{A\Gamma^\alpha}{R}.$$

Note that income increases with effort if and only if $e > \widetilde{e}$ where

$$\widetilde{e} = \left(\frac{(R-1)(1+\beta)(1-\alpha)(\theta-1)}{[1-\alpha+\alpha R]\,\theta D}\right)^{\frac{1}{\theta}} = \left(\frac{r(1+\beta)(1-\alpha)(\theta-1)}{(1+\alpha r)\,\theta D}\right)^{\frac{1}{\theta}}.$$

At $e_u = \widetilde{e}$, income of unconstrained individuals takes a minimum value. Using (12) and $\widetilde{e}$, we obtain that the associated value of inheritances is

$$\widetilde{b} = \left(\frac{1 + [1-(1-\alpha)\theta]\, r}{rR(\theta-1)}\right) A\Gamma^\alpha \widetilde{e}.$$

Thus, when $b = \widetilde{b}$ income takes a minimum value. It follows that if $b < \widetilde{b}$ then income decreases and if $b > \widetilde{b}$ then income increases with bequests. The intuition is as follows. For the poor unconstrained individuals that still exert large effort, an additional inheritance causes a large reduction in effort and labor earnings, whereas for rich unconstrained individuals this negative effect is small. Thus, the Carnegie effect dominates only for the poor unconstrained individuals. Also note that if $\theta = 1$

then $\widetilde{e} = 0$ and $\widetilde{e}$ increases as θ increases. Therefore, the parameter θ can be used to parametrize the intensity of the Carnegie effect.

Unconstrained individuals satisfy $b > \mu_u = B\Gamma e_u$. This implies that $Rb > BR\Gamma e_u$. Using (12), we obtain that unconstrained individuals satisfy

$$e_u < \overline{e} = \left(\frac{(1+\beta)(1-\alpha)}{\theta D} \right)^{\frac{1}{\theta}}.$$

Since e_u decreases with the inheritance, unconstrained individuals are those individuals that receive $b > \overline{b}$ and exert effort $e_u < \overline{e}$, where

$$\overline{b} = \frac{\alpha A \Gamma^\alpha}{R} \overline{e}.$$

Using (3) and (4), we obtain that

$$\begin{aligned} c_u &= \frac{(1-\alpha)}{\theta D} A \Gamma^\alpha e_u^{1-\theta}, \\ b'_u &= \beta \frac{(1-\alpha)}{\theta D} A \Gamma^\alpha e_u^{1-\theta}. \end{aligned} \tag{14}$$

Note that the inheritances of unconstrained individuals exhibit transitional dynamics when $\theta > 1$. To analyze the transitional dynamics, in Appendix C6 we characterize the function $b'_u(b)$ that is obtained from (12) and (14). We show that the function is increasing and convex with a slope smaller than one and $b'_u(b) > 0$ for all b. We also show that $b'_u\left(\overline{b}\right) < \overline{b}$ if and only if $R < \alpha(1+\beta)/\beta$. These conditions imply that we distinguish two different transitional dynamics. If $R < \alpha(1+\beta)/\beta$ then the inheritance of families of unconstrained individuals decline from one generation to the next (downward social mobility) until $b < \overline{b}$ and individuals become constrained individuals. If $R > \alpha(1+\beta)/\beta$ then inheritances of families of unconstrained individuals monotonically converge towards the following steady state:

$$\begin{aligned} e_u^* &= \left(\frac{1+\beta-\beta R}{\theta D} \right)^{\frac{1}{\theta}}, \\ h_u^* &= \Gamma e_u^*, \\ b_u^* &= \frac{\beta(1-\alpha) A \Gamma^\alpha}{\theta D} e_u^{*1-\theta}, \\ I_u^* &= \left(\frac{1-\alpha}{1+\beta-\beta R} + \alpha \right) A \Gamma^\alpha e_u^*. \end{aligned}$$

Note that the steady state for unconstrained individuals is well defined only if $R < (1+\beta)/\beta$, since otherwise effort would be negative. We assume that this condition is always satisfied.

As a final remark, we note that $e_u^* < \bar{e}$ and $b_u^* > \bar{b}$ if and only if $R > \alpha(1+\beta)/\beta$. This confirms that a steady state for unconstrained individuals exists if and only if $R > \alpha(1+\beta)/\beta$.

C.2 Constrained individuals

Credit constrained individuals satisfy $\mu_c = b$. From this equation, we obtain that $h_c = b/B$. Using (6), we obtain

$$e_c = \left(\frac{(1-\alpha)(1+\beta)}{D\theta}\right)^{\frac{1}{\theta}} = \bar{e}.$$

The wage satisfies

$$w_c = A\left(\frac{b}{B}\right)^{\alpha} \bar{e}^{1-\alpha}.$$

Using (4), we deduce that

$$c_c = \frac{A}{1+\beta}\left(\frac{b}{B}\right)^{\alpha} \bar{e}^{1-\alpha}$$

and, using (3), we get

$$b_c' = \frac{\beta A}{1+\beta}\left(\frac{b}{B}\right)^{\alpha} \bar{e}^{1-\alpha}. \tag{15}$$

We can now deduce the following results. First, we use the expression of the wages to obtain that income is the following increasing and concave function of the inheritance:

$$I_c = w_c + rb = A\left(\frac{b}{B}\right)^{\alpha} \bar{e}^{1-\alpha} + rb. \tag{16}$$

Second, using (15), we deduce that bequests of constrained individuals exhibit a monotonic transition towards the following steady state:

$$\begin{aligned}
b_c^* &= \left(\frac{\beta A}{(1+\beta)B^{\alpha}}\right)^{\frac{1}{1-\alpha}} \bar{e}, \\
e_c^* &= \bar{e}, \\
h_c^* &= \left(\frac{\beta A}{(1+\beta)B}\right)^{\frac{1}{1-\alpha}} \bar{e}, \\
I_c^* &= \left(\frac{1}{\beta}+R\right)\left(\frac{\beta A}{(1+\beta)B^{\alpha}}\right)^{\frac{1}{1-\alpha}} \bar{e}.
\end{aligned}$$

This steady state is well-defined if $b_c^* < \bar{b}$. This inequality implies

$$\left(\frac{\beta A}{(1+\beta)B^{\alpha}}\right)^{\frac{1}{1-\alpha}} \bar{e} < \frac{\alpha A \Gamma^{\alpha}}{R}\bar{e},$$

which is satisfied if and only if $R < \alpha(1+\beta)/\beta$.

C.3 Transitional dynamics

The transitional dynamics of this economy is characterized by the function $b'(b)$ that relates bequest given with inheritance received. This function is defined by parts as follows

$$b'(b) = \begin{cases} b'_c(b) & \text{if } b \leq \bar{b} \\ b'_u(b) & \text{if } b > \bar{b} \end{cases}.$$

In addition, we have

$$\begin{aligned} b'_c\left(\bar{b}\right) &= \frac{\beta A}{1+\beta}\left(\frac{\alpha A \Gamma^{\alpha}}{BR}\right)^{\alpha}\bar{e}, \\ b'_u\left(\bar{b}\right) &= \beta\frac{(1-\alpha)}{\theta D}A\Gamma^{\alpha}\bar{e}^{1-\theta}. \end{aligned}$$

It is immediate to see that $b'_c\left(\bar{b}\right) = b'_u\left(\bar{b}\right)$, which implies that the function $b'(b)$ is continuous.

Figure 16. Transitional dynamics

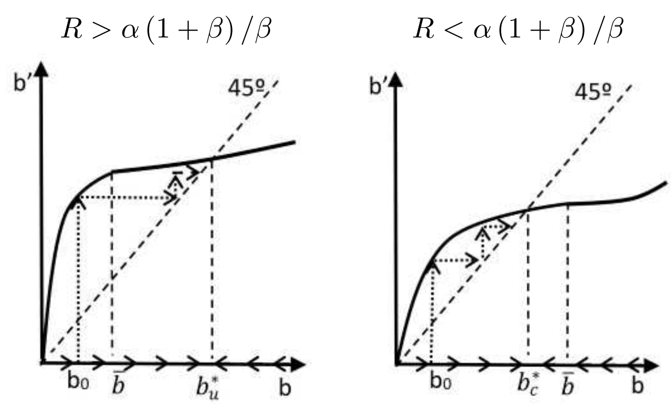


Figure 16 plots this function. We distinguish between two cases. When $R < \alpha(1+\beta)/\beta$, we have seen that there is only a steady state for constrained individuals that satisfies $b_c^* < \bar{b}$. Families of unconstrained individuals will reduce inheritances from one generation to the other until $b < \bar{b}$ and they become constrained individuals. When $R > \alpha(1+\beta)/\beta$, there is only a steady state for unconstrained individuals that satisfies $b_u^* > \bar{b}$. In this case, families of constrained individuals will increase the inheritance from one generation to the next until $b > \bar{b}$ and they become unconstrained individuals.

C.4 Inequality and future income

We analyze how inequality in wealth affects future income. As a first step, we characterize the function $I(b)$ relating income with inheritances. This function is defined as follows

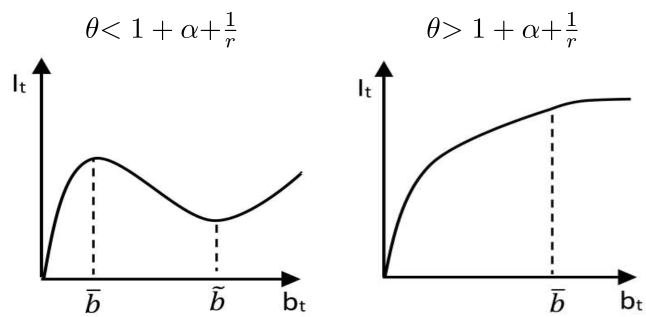
$$I(b) = \begin{cases} I_c(b) = A\left(\frac{b}{B}\right)^{\alpha} \overline{e}^{1-\alpha} + rb \text{ if } b \leq \overline{b} \\ I_u(b) = \left[\frac{(R-1)(1+\beta)(1-\alpha)}{\theta D(e(b))^{\theta}} + (1-\alpha) + \alpha R\right] \frac{A\Gamma^{\alpha}}{R} e(b) \text{ if } b > \overline{b} \end{cases}$$

It is easy to show that $I_c\left(\overline{b}\right) = I_u\left(\overline{b}\right)$, which implies that the function is continuous. In addition, $I_c(b)$ increases with b and $I_u(b)$ increases with b if and only if $b > \widetilde{b}$, where $\widetilde{b}$ is defined in Appendix C1.

To complete the characterization of the effect of bequests on income, note that $\overline{e} > \widetilde{e}$ (or equivalently $\overline{b} < \widetilde{b}$) if and only if $\theta < 1 + \alpha + 1/r$.

Figure 17 plots the function $I(b)$ in two different cases. When $\theta > \alpha+1+1/r$, then $\overline{b} > \widetilde{b}$. In this case, if $b < \overline{b}$ individuals are constrained and if $b > \overline{b}$ then individuals are unconstrained. Since $b > \overline{b} > \widetilde{b}$ then $I_u(b)$ increases with b for all $b > \overline{b}$. In contrast, when $\theta < \alpha + 1 + 1/r$, then $\overline{b} < \widetilde{b}$. In this case, if $b < \overline{b}$ individuals are constrained, if $b \in \left(\overline{b}, \widetilde{b}\right)$ then individuals are unconstrained and $I_u(b)$ decreases with b and if $b > \widetilde{b}$ then individuals are unconstrained and $I_u(b)$ increases with b.

Figure 17. Income as a function of inheritance



An increase in inequality that reduces wealth of constrained individuals and increases wealth of unconstrained individuals reduces future income even if $I_u(b)$ increases with b.This is due to the Carnegie effect that makes the slope of $I_u(b)$ be smaller than the slope of $I_c(b)$. To see this, we calculate

$$\begin{aligned}\frac{\partial I_c}{\partial b} &= \alpha A\left(\frac{1}{B}\right)^{\alpha}\overline{e}^{1-\alpha}b^{\alpha-1}+r>r,\\ \frac{\partial I_u}{\partial b} &= \frac{R}{\frac{(1+\beta)(1-\alpha)(1-\theta)}{\theta D}e_u^{-\theta}-(1-\alpha)}+r<r.\end{aligned}$$

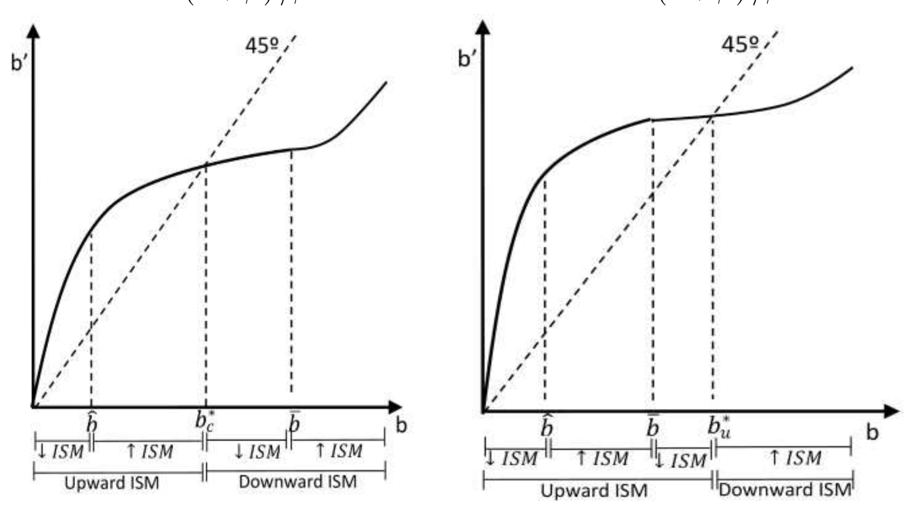
We observe that $\frac{\partial I_c}{\partial b} > \frac{\partial I_u}{\partial b}$. The intuition on this result is as follows. An additional unit of inheritance increases capital income, for both unconstrained and constrained individuals, in r. However, the effect on labor income is distinct. For constrained individuals, a larger inheritance increases education and labor earning, whereas it reduces effort and labor earnings for unconstrained individuals. This explains that a higher inequality reduces future income.

C.5 Inequality and ISM

We consider first the relation between inequality and ISM when it is measured by inheritances (wealth). We assume that a more unequal distribution of inheritances implies that inheritances increase for unconstrained individuals and decrease for constrained individuals. Under this assumption, Figure 18 shows the effects of inequality on ISM.

Figure 18. ISM in inheritances

$R<\alpha(1+\beta)/\beta$ $\qquad$ $R>\alpha(1+\beta)/\beta$



Note: We assume that a higher inequality implies an increase in the inheritance received by unconstraned individuals and a reduction of the inheritance received by constrained individuals.

When $R < \alpha(1+\beta)/\beta$, constrained individuals with $b < b_c^*$ exhibit upward ISM. Constrained individuals with $b > b_c^*$ and all unconstrained individuals exhibit downward ISM. It is easy to show that there exists a value of b, $\widehat{b}$, satisfying $\widehat{b} \in (0, b_c^*)$, such that constrained individuals with $b < \widehat{b}$ suffer a reduction in ISM when inequality increases, whereas ISM increases for constrained individuals with $b \in \left(\widehat{b}, b_c^*\right)$ and decreases for constrained individuals with $b \in \left(b_c^*, \overline{b}\right)$. For unconstrained individuals, the effect of a larger inequality on ISM follows from the fact that the slope of the function $b_u(b)$ is smaller than one, as shown in Appendix C6. A larger inequality increases ISM for these individuals.

When $R > \alpha(1+\beta)/\beta$, we observe that all constrained individuals and unconstrained individuals with $b < b_u^*$ exhibit upward ISM. Unconstrained individuals with $b > b_u^*$ exhibit downward ISM. In this case, there also exists a value of b, $\widehat{b}$, satisfying $\widehat{b} \in \left(0, \overline{b}\right)$, such that constrained individuals with $b < \widehat{b}$ suffer a reduction in ISM when inequality increases. Inequality increases ISM for constrained individuals with $b \in \left(\widehat{b}, \overline{b}\right)$ and for unconstrained individuals with $b > b_u^*$. Finally, inequality decreases ISM for poor unconstrained individuals with $b \in \left(\overline{b}, b_u^*\right)$.

We have seen that the effects of inequality on ISM are identical to those obtained in Section 4 when $\theta = 1$. They imply that a more unequal distribution increases ISM at the top of the income distribution and decreases ISM at the bottom.

We next show how a larger inequality in income affects ISM when it is measured in income. To this end, we must characterize the function $I' = g(I)$ relating parental income with descendents income. This function is defined by parts as follows

$$g(I) = \begin{cases} g_c(I) & \text{if } b \leq \overline{b} \\ g_u(I) & \text{if } b > \overline{b} \end{cases}.$$

For constrained individuals, the function $g_c(I)$ is obtained from (15) and (16). The analysis of the function $g_c(I)$ is independent of θ. Therefore, the results in Appendix B, where the properties of the function $g_c(I)$ are studied when $\theta = 1$, still hold. In this appendix, we show that this function is increasing, concave and satisfies $g_c(0) = 0$. We use (16) to obtain that

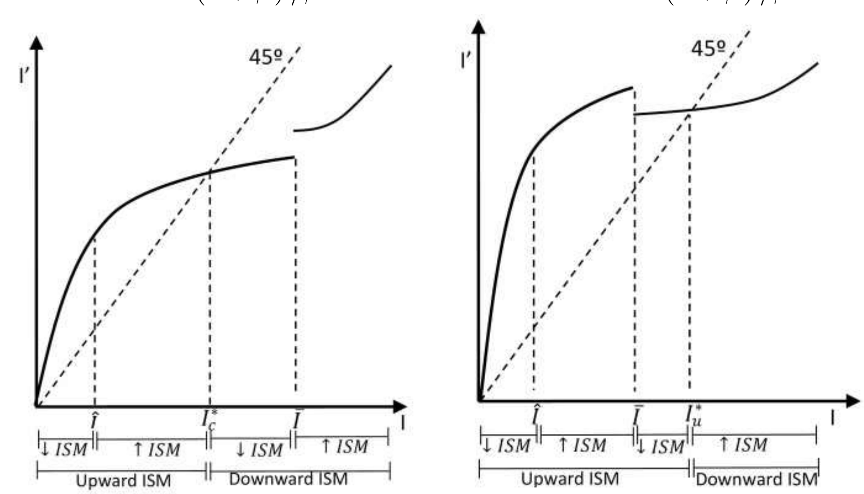
$$\overline{I} \equiv I_c\left(\overline{b}\right) = \left(\frac{1 + (1+\alpha)r}{R}\right) A\Gamma^{\alpha}\overline{e}.$$

In Appendix B, we also show that $I_c^* < \overline{I}$ if and only if $R < \alpha(1+\beta)/\beta$.

For unconstrained individuals, we assume that $\theta > 1+\alpha+1/r$.[13] This assumption implies that $g_u(I)$ is increasing. To see this, we remember that income increases with inheritances when $\theta > 1+\alpha+1/r$ and the bequest given to descendents increases with the inheritance received by parents. As a result, a larger inheritance increases both the income of parents and of the children, which implies that the slope of $g_u(I)$ is positive. The function $g_u(I)$ is obtained from (12), (13) and (14). Using these equations, we first obtain that $I_u(\overline{b}) = I_c(\overline{b}) = \overline{I}$. Second, in Appendix C7 we show that the function $g(I)$ is not continuous with $g_u(I_u(\overline{b})) > g_c(I_c(\overline{b}))$ if and only if $R < \alpha(1+\beta)/\beta$. We also show that $g_u(I_u(\overline{b})) < I_u(\overline{b})$ if and only if $R < \alpha(1+\beta)/\beta$. Finally, in the same appendix, we show that the slope of g_u is smaller than one.

Figure 19. Inequality and ISM in income

$R < \alpha(1+\beta)/\beta$ $\qquad\qquad$ $R > \alpha(1+\beta)/\beta$



Note: We assume that a higher inequality implies an increase in the income of unconstraned individuals and a reduction in the income of constrained individuals.

Figure 19 plots the function g. Note that the relation between parental income and descendents income mimics that of bequests displayed in Figure 11. This is due to the fact that the bequests given to descendents increases with the inheritance received by parents and income increases with inheritance. Therefore, $I' > I$ if and

[13] When $\theta < 1+\alpha+1/r$, the effect of a higher inheritance on income depends on b being smaller or larger than $\widetilde{b}$. This complicates the analysis of this case. In Figure 19 we assume that $\theta > 1+\alpha+1/r$.

only if $b' > b$. This implies that the intergenerational social mobility patterns for income are identical to those described for inheritances. Note that a larger income inequality increases downward social mobility at the top of the income distribution and decreases upward social mobility at the bottom.

We conclude that the main results regarding the effects of income inequality obtained in Section 4 with a linear disutility of effort still hold when the disutility of effort is strictly convex.

C.6 Transitional dynamics of unconstrained individuals when $\theta > 1$

We use (12) and (14) to characterize the function $b'_u(e(b))$. First, we obtain the following derivatives

$$\frac{\partial b'_u}{\partial b} = \frac{\partial b'_u}{\partial e}\frac{\partial e}{\partial b} = \beta(1-\theta)\frac{(1-\alpha)}{\theta D}A\Gamma^{\alpha}e_u^{-\theta}\frac{\partial e}{\partial b},$$

with

$$\frac{\partial e}{\partial b} = \frac{1}{\left(\frac{(1+\beta)(1-\theta)}{\theta D}e_u^{-\theta} - 1\right)\frac{(1-\alpha)A\Gamma^{\alpha}}{R}}.$$

Therefore, we obtain that

$$\frac{\partial b'_u}{\partial b} = \frac{\beta(1-\theta)\frac{(1-\alpha)}{\theta D}A\Gamma^{\alpha}e_u^{-\theta}}{\left(\frac{(1+\beta)(1-\theta)}{\theta D}e_u^{-\theta} - 1\right)\frac{(1-\alpha)A\Gamma^{\alpha}}{R}} = \frac{\beta(\theta-1)\frac{1}{\theta D}e_u^{-\theta}R}{\frac{(1+\beta)(\theta-1)}{\theta D}e_u^{-\theta} + 1}.$$

Notice that $\frac{\partial b'_u}{\partial b} > 0$ and, since $R < (1+\beta)/\beta$, we can easily deduce that $\frac{\partial b'_u}{\partial b} < 1$.

Second, to show convexity of the function $b'_u(e(b))$ we calculate the following derivatives:

$$\begin{aligned}\frac{\partial^2 b'_u}{\partial b^2} &= \frac{\partial^2 b'_u}{\partial e^2}\left(\frac{\partial e}{\partial b}\right)^2 + \frac{\partial b'_u}{\partial e}\frac{\partial^2 e}{\partial b^2} \\ &= \frac{\beta(1-\theta)(1-\alpha)e_u^{-\theta-1}A\Gamma^{\alpha}}{\theta D}\left[-\theta\left(\frac{\partial e}{\partial b}\right)^2 + e_u\frac{\partial^2 e}{\partial b^2}\right],\end{aligned}$$

with

$$\frac{\partial^2 e}{\partial b^2} = \frac{\frac{(1+\beta)(1-\theta)}{D}e_u^{-\theta-1}\frac{(1-\alpha)A\Gamma^{\alpha}}{R}}{\left(\left(\frac{(1+\beta)(1-\theta)}{\theta D}e_u^{-\theta} - 1\right)\frac{(1-\alpha)A\Gamma^{\alpha}}{R}\right)^2}\frac{\partial e}{\partial b} = \frac{(1+\beta)(1-\theta)}{D}e_u^{-\theta-1}\frac{(1-\alpha)A\Gamma^{\alpha}}{R}\left(\frac{\partial e}{\partial b}\right)^3.$$

Therefore, we obtain

$$\begin{aligned}
\frac{\partial^2 b_u'}{\partial b^2} &= \frac{\beta\left(1-\theta\right)\left(1-\alpha\right)e_u^{-\theta-1}A\Gamma^\alpha\left[-\theta\left(\frac{\partial e}{\partial b}\right)^2+e_u\frac{(1+\beta)(1-\theta)}{D}e_u^{-\theta-1}\frac{(1-\alpha)A\Gamma^\alpha}{R}\left(\frac{\partial e}{\partial b}\right)^3\right]}{\theta D} \\
&= \frac{\beta\left(1-\theta\right)\left(1-\alpha\right)e_u^{-\theta-1}A\Gamma^\alpha}{\theta D}\left(\frac{\partial e}{\partial b}\right)^3\frac{(1-\alpha)A\Gamma^\alpha\left[-\theta\left(\frac{(1+\beta)(1-\theta)e_u^{-\theta}}{\theta D}-1\right)+\frac{(1+\beta)(1-\theta)e_u^{-\theta}}{D}\right]}{R} \\
&= \frac{\beta\left(1-\theta\right)\left(1-\alpha\right)e_u^{-\theta-1}A\Gamma^\alpha}{D}\left(\frac{\partial e}{\partial b}\right)^3\frac{\left(1-\alpha\right)A\Gamma^\alpha}{R}>0,
\end{aligned}$$

which is positive since $\theta>1$ and $\frac{\partial e}{\partial b}<0$.

Third, we evaluate b_u' at $e=\overline{e}$ to obtain $b_u'\left(\overline{b}\right)$, which satisfies

$$b_u'\left(\overline{b}\right)=\beta\frac{(1-\alpha)}{\theta D}A\Gamma^\alpha\left(\frac{(1+\beta)\left(1-\alpha\right)}{\theta D}\right)^{\frac{1-\theta}{\theta}}.$$

It is immediate to show that $b_u'\left(\overline{b}\right)<\overline{b}$ if and only if $R<\alpha\left(1+\beta\right)/\beta$.

These conditions imply that if $R>\alpha\left(1+\beta\right)/\beta$ then there is a steady state for unconstrained individuals. To obtain the steady state, we first remember that $b'=b$ at the steady state and we rewrite (14) as

$$Rb=\beta R\frac{(1-\alpha)}{\theta D}A\Gamma^\alpha e_u^{1-\theta}$$

and, using (12), we obtain

$$\left(\frac{1+\beta}{\theta D e_u^\theta}-1\right)\left(1-\alpha\right)A\Gamma^\alpha e_u=\beta R\frac{(1-\alpha)}{\theta D}A\Gamma^\alpha e_u^{1-\theta}.$$

From this equation, we obtain the steady state value of effort

$$e_u^*=\left(\frac{1+\beta-\beta R}{\theta D}\right)^{\frac{1}{\theta}}.$$

We use (14) to obtain the steady state value of bequests

$$b_u^*=\frac{\beta\left(1-\alpha\right)A\Gamma^\alpha}{\theta D}e_u^{*\frac{1-\theta}{\theta}},$$

and the steady state value of income

$$I_u^*=\left(\frac{(1+\beta)\left(1-\alpha\right)}{1+\beta-\beta R}+\alpha\right)A\Gamma^\alpha e_u^*.$$

C.7 Characterization of the function g for unconstrained individuals when $\theta > 1$

In this appendix, we deduce three properties of the function g : continuity of the function g, the relation between $I_u'\left(b_c'\left(\overline{b}\right)\right) > I_u\left(\overline{b}\right)$ and the slope of the function g.

To show continuity of the function g, we first obtain $I_u'\left(b_u'\left(\overline{b}\right)\right)$ and $I_c'\left(b_c'\left(\overline{b}\right)\right)$. To this end, we note that

$$b_u'\left(\overline{b}\right) = \beta\frac{(1-\alpha)}{\theta D}A\Gamma^{\alpha}\overline{e}^{1-\theta} = \frac{\beta}{1+\beta}A\Gamma^{\alpha}\overline{e} = b_c'\left(\overline{b}\right).$$

Using (12), we obtain that $e^s = e\left(\overline{e}\right)$ is a decreasing function that solves

$$\left(\frac{1+\beta}{\theta D\left(e^s\right)^{\theta}} - 1\right)(1-\alpha)A\Gamma^{\alpha}e^s = R\frac{\beta}{1+\beta}A\Gamma^{\alpha}\overline{e} \tag{17}$$

This equation can be rewritten as

$$\frac{\beta}{1+\beta}rA\Gamma^{\alpha}\overline{e} + A\Gamma^{\alpha}e^s = \left(r\frac{(1+\beta)(1-\alpha)}{\theta D\left(e^s\right)^{\theta}} + R\alpha + 1 - \alpha\right)\frac{A\Gamma^{\alpha}e^s}{R} = I_u'\left(b_c'\left(\overline{b}\right)\right).$$

We use (15) and (16) to obtain

$$I_c'\left(b_c'\left(\overline{b}\right)\right) = \left[\left(\frac{\beta}{1+\beta}\frac{R}{\alpha}\right)^{\alpha-1} + \alpha\frac{r}{R}\right]\frac{R}{\alpha}\frac{\beta}{1+\beta}A\Gamma^{\alpha}\overline{e}.$$

Next, we compare $I_u'\left(b_u'\left(\overline{b}\right)\right)$ with $I_c'\left(b_c'\left(\overline{b}\right)\right)$ to determine the continuity of the function $g\left(I\right)$. We have that $I_u'\left(b_u'\left(\overline{b}\right)\right) > I_c'\left(b_c'\left(\overline{b}\right)\right)$ if and only if

$$r\frac{\beta}{1+\beta}A\Gamma^{\alpha}\overline{e} + A\Gamma^{\alpha}e^s > \left[\left(\frac{\beta}{1+\beta}\frac{R}{\alpha}\right)^{\alpha-1} + \alpha\frac{r}{R}\right]\frac{R}{\alpha}\frac{\beta}{1+\beta}A\Gamma^{\alpha}\overline{e}$$

which simplifies as follows

$$e^s > \left(\frac{\beta}{1+\beta}\frac{R}{\alpha}\right)^{\alpha}\overline{e}.$$

To see if this inequality holds, we substitute in $e^s = \left(\frac{\beta}{1+\beta}\frac{R}{\alpha}\right)^{\alpha}\overline{e}$ in (17) to obtain

$$\Sigma_1 = \left(\frac{\beta}{1+\beta}\frac{R}{\alpha}\right)^{\alpha(1-\theta)} - \alpha\left(\frac{R}{\alpha}\frac{\beta}{1+\beta}\right) - (1-\alpha)\left(\frac{\beta}{1+\beta}\frac{R}{\alpha}\right)^{\alpha}.$$

Note that $\Sigma_1 = 0$ if $\frac{\beta}{1+\beta}\frac{R}{\alpha} = 1$ and it is positive when $\frac{\beta}{1+\beta}\frac{R}{\alpha} < 1$. If $\Sigma_1 > 0$ then $e^s > \left(\frac{\beta}{1+\beta}\frac{R}{\alpha}\right)^{\alpha}\overline{e}$ and $I_u'\left(b_c'\left(\overline{b}\right)\right) > I_c'\left(b_c'\left(\overline{b}\right)\right)$. Therefore, we have that $I_u'\left(b_u'\left(\overline{b}\right)\right) > I_c'\left(b_c'\left(\overline{b}\right)\right)$ if and only if $\frac{\beta}{1+\beta}\frac{R}{\alpha} < 1$.

We proceed to show the relationship between $I'_u\left(b'_c\left(\overline{b}\right)\right) > I_u\left(\overline{b}\right)$. Note that $I'_u\left(b'_c\left(\overline{b}\right)\right) > I_u\left(\overline{b}\right)$ when

$$r\frac{\beta}{1+\beta}A\Gamma^{\alpha}\overline{e}+A\Gamma^{\alpha}e^{s} > \left(\frac{1+(1+\alpha)\,r}{R}\right)A\Gamma^{\alpha}\overline{e},$$

which simplifies as follows

$$e^{s} > \left(\frac{1+(1+\alpha)\,r}{R}-r\frac{\beta}{1+\beta}\right)\overline{e}$$

To see if this inequality holds, we substitute $e^{s} = \left(\frac{1+(1+\alpha)r}{R}-r\frac{\beta}{1+\beta}\right)\overline{e}$ in (17) to obtain

$$\Sigma_2 = \left(\frac{1+(1+\alpha)\,r}{R}-r\frac{\beta}{1+\beta}\right)^{1-\theta}-(1-\alpha)\left(\frac{1+(1+\alpha)\,r}{R}-r\frac{\beta}{1+\beta}\right)-R\frac{\beta}{1+\beta}.$$

It can be shown that if $\frac{\beta}{1+\beta}R=\alpha$ and $\theta > 1+\alpha+1/r$ then $\Sigma_2 = 0$ and $\frac{\partial\Sigma_2}{\partial R} > 0$. This implies that $g\left(I_u\left(\overline{b}\right)\right) > I_u\left(\overline{b}\right)$ if and only if $R > \alpha\frac{1+\beta}{\beta}$.

We finally show that the slope of the function $I'_u = g_u\left(I_u\right)$ is smaller than one at the steady state. To this end, we remember that in Appendix C.4 we have shown I_u is a function of inheritance with the following slope.

$$\frac{\partial I_u}{\partial b} = \frac{R}{\frac{(1+\beta)(1-\alpha)(1-\theta)}{\theta D}e_u^{-\theta}-(1-\alpha)}+r.$$

Then,

$$\frac{\partial I'_u}{\partial I_u} = \frac{\partial I'_u}{\partial b'_u}\frac{\partial b'_u}{\partial b}\frac{\partial b}{\partial I_u} = \frac{\frac{\partial I'_u}{\partial b'}}{\frac{\partial I_u}{\partial b}}\frac{\partial b'_u}{\partial b},$$

where

$$\frac{\partial b'_u}{\partial b} = \frac{\beta\,(\theta-1)\,\frac{1}{\theta D}e_u^{-\theta}R}{\left(\frac{(1+\beta)(\theta-1)}{\theta D}e_u^{-\theta}+1\right)} < 1.$$

In Appendix C.6, we have shown that $\frac{\partial b'_u}{\partial b} \in (0,1)$. At the steady state, $e'_u = e_u$ and then $\frac{\partial I'_u}{\partial b'} = \frac{\partial I_u}{\partial b}$. It follows that at the steady state $\frac{\partial I'_u}{\partial I_u} < 1$.