INFORMING TO DIVERT ATTENTION *

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Abstract

I study a multidimensional Sender-Receiver game in which Receiver can acquire limited information after observing the Sender's signal. Depending on the parameters describing the conflict of interest between Sender and Receiver, I characterise optimal information disclosure and the information acquired by Receiver as a response. I show that in the case of partial conflict of interests (aligned on some dimensions and misaligned on others) Sender uses the multidimensionality of the environment to divert Receiver's attention away from the dimensions of misalignment of interests. Moreover, there is negative value of information in the sense that Receiver would be better off if she could commit not to extract private information or to have access to information of lower quality. I present applications to consumer's choice and informational lobbying.

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1 Introduction

Economic agents, whether individuals, firms or politicians, must make decisions concerning issues on which they do not possess full knowledge. In these circumstances the agents need to rely on the expertise of the more informed parties. While having access to more complete and better information these informed experts might, however, be self-interested and, thus, provide information strategically to influence the resulting decisions. Apart from affecting decisions directly by making decision-makers more informed, experts' information transmission also has an indirect effect by changing decision-makers' preference for information and, thus, by altering their own search for information.

In this paper, I study this new role for information provision - directing the decision-maker's search for information when this search is limited by some exogenous constraints. I show that, whenever the decision-maker cannot obtain information on the issues separately, the expert's best strategy might be counterintuitive: to provide some information on the issue which she wants to hide in order to divert the decision-maker's attention toward another issue. Moreover, in cases when the expert would not want the decision maker to learn anything on either of the issues (misaligned interests), it still might be optimal for her to provide some amount of information on one of the issues to divert the decision-maker's attention towards a more favorable issue.

From the perspective of the decision-maker she always benefits from the expert's information compared to the case when she makes decisions on her own. However, she might prefer to face stronger limitations on her information acquisition process (higher costs) to benefit from more information disclosure by the expert. In other words, for some types of conflict of interest, the decision-maker is facing a negative value of capacity to acquire information.

From a technical perspective, I study a multi-dimensional Sender-Receiver framework with quadratic preferences and Receiver's access to additional information after she observes Sender's signals. In the spirit of Bayesian Persuasion literature, Sender commits to a collection of linear signals before the state of the world is realized. In her turn, Receiver, upon observing the realization of Sender's signal(-s), can obtain one additional linear signal with the weights and precision of her choice. Receiver's signal is assumed to be costly, with the cost function being represented either by entropy costs as in Rational Inattention literature, or by a convex increasing precision-dependent function which is standard for many applications.

In such a framework, Receiver's choice of private information depends on the amount of attention (i.e. relative weights) given to different dimensions by Sender and the quality (i.e. precision) of Sender's information. In the case of quadratic preferences and the multiplicity of Receiver's actions, Receiver always chooses to obtain information on the most uncertain dimension. Thus, by strategically changing the relative uncertainty of dimensions, Sender affects Receiver's preference for information and, hence, her learning process.

The results show that the motives for information transmission depend on the conflict of interest. Due to the assumption of quadratic preferences, the objectives of Sender and Receiver can be described in terms of the posterior uncertainty of Receiver. Hence, the conflict of interests on each dimension can be pinned down by whether Sender benefits from Receiver's learning of this dimension or not.

In the presence of a partial conflict of interests (the case in which Sender wants to reveal one dimension but to hide another one), she faces two competing strategies. The first one is intuitive: Sender reveals the dimension on which interests are aligned and Receiver (partially) learns the other dimension. The second strategy is less intuitive: Sender partially provides information on the dimension where interests are misaligned to change Receiver's preferences for information and to make her obtain information on the dimension of alignment. Whenever Receiver has sufficiently low costs of information acquisition and is thus able to obtain a precise signal, the second strategy is preferred by Sender. Hence, Sender provides information with the goal of diverting Receiver's attention away from the dimension of misalignment of interests.

With fully misaligned interests, I show that contrary to the standard intuition, information transmission is possible and is also driven by attention diverting motives of Sender. Indeed, with fully misaligned interests the trade-off is either to reveal nothing and Receiver obtains information on the more uncertain dimension or to partially reveal this more uncertain dimension to switch Receiver's focus away from it. Depending on the relative conflict of interests on the dimensions the second type of solution might be chosen by Sender.

I extend the baseline framework in several directions. Firstly, I consider the case in which Receiver needs to make a unique decision based on the two dimensions of the state of the world. This set up is particularly important as it is highly relevant for multiple real life applications: optimal funding based on different features of a project, optimal grade, design of optimal rankings etc. I show that if Receiver can observe only one of the two dimensions, but not a mixture of them, then the main intuitions hold, that is Sender still diverts Receiver's attention under some conflicts of interest. Secondly, I extend the main framework of the paper to allow Receiver to observe multiple signals while facing a budget constraint. In this case, Sender does not divert attention anymore but chooses a more aggressive strategy - to reveal no information even when there is a partial alignment of interests in order to complicate Receiver's learning.

An important application of the results is consumer's choice in the presence of taste shocks and information acquisition constraints. Kőszegi and Matějka [2020] build a theory of mental budgeting and naive diversification and show that consumers either keep the budget unchanged and vary its share spent on different goods (mental budgeting) if the goods are substitutes, or vary the budget keeping its division between the goods (naive diversification) if the goods are complements. The results are generated by the assumption that consumers cannot learn the taste shocks for all the goods due to information costs and, thus, decide to focus either more on relative tastes for the goods or the total taste. The theoretical framework of this paper naturally extends the one by Kőszegi and Matějka [2020] by adding a Sender, i.e. an advertiser or a producer. In Section 5.1 I consider an advertiser who wants to maximize the total spendings of consumers while minimizing the difference in spendings on different products. I show that, while the naive diversification logic for complements stands, consumers do not use mental budgeting in the presence of the advertiser due to the information policy of the latter. Moreover, I show that the advertiser might find a diverting attention strategy optimal, i.e. to emphasize the difference in tastes between the two goods to make consumers question the total taste. Also, even though in expectation consumer is always better off in the presence of Sender even in the presence of a conflict of interests, she is ex-post worse off compared to the no Sender benchmark if her prior beliefs are sufficiently correct.

Another important example to which the theoretical results of the paper apply is the case of informational lobbying. Most of the time, as in my framework policy-makers face multiple decisions on different issues. Moreover, as empirical evidence suggests (see, for instance, Bertrand et al. [2014]) lobbyists tend to tailor the type of information they provide to the preference and expertise area of the policy-maker they are facing. The classic lobbying literature is mostly concerned with the question of whether informational lobbying is detrimental to the decision-making. In contrast, this paper looks at the question of optimal information provision and optimal policy-maker's access to information. While in my setting the policy-maker is always better informed in the presence of a lobbyist, she might receive less information than possible if the lobbyist is convinced the policy-maker is well informed and the interests are only partially aligned. Moreover, the theoretical results in the paper suggest that the lobbyist might take into account the subsequent information search of the policy-maker. In this scenario, she would either underprovide information on an issue with shared preference or strategically provide information on unfavorable issue. In this case, a policy-maker can benefit from ex-ante committing to the type of information she is going to obtain or from artificially decreasing the quality/quantity of information available. These theoretical intuitions provide the basis for future empirical research on the frequency of such behavior in real world settings.

The rest of the paper proceeds as follows. Section 1.1 discusses the related literature, Section 2 provides a simple example illustrating the main results, Section 3 provides the general model and extends the results, Section 4 presents various extensions and numerical illustrations, Section 5 applies the results to the consumer's choice problem a la Kőszegi and Matějka [2020] and Section 6 concludes.

1.1 Literature Review

This work contributes to several strands of the literature.

The commitment assumption imposed on Sender relates to the literature on Bayesian persuasion starting from Kamenica and Gentzkow [2011] and Rayo and Segal [2010].

In particular, there are two recent blocks of research: one focuses on multidimensional persuasion under different assumptions (without allowing Receiver access to additional information of her choice), the other considers the uni-dimensional persuasion problem with ex-post information acquisition by Receiver.

In the first of the two blocks, Tamura [2018] extends the classic Bayesian persuasion settings to the multidimensional case. Among other results it shows the optimality of the linear signals under Gaussian prior beliefs. Velicheti et al. [2023] extends the framework by introducing multiple senders with possibly different objectives under Gaussian beliefs and quadratic payoffs. Sayin and Başar [2021] provide analysis of persuasion with state-dependent quadratic payoffs for general distributions. Farokhi et al. [2016] and Sayin and Başar [2018] are other important contributions to the literature. Jain [2018] considers a two-dimensional Sender-Receiver framework in which commitment (Bayesian persuasion) is possible on one dimension while on the other dimension communication is in the form of cheap talk. Khantadze et al. [2021] study persuasion of multiple Receivers in a binary multidimensional framework with one action per dimension.

In the literature on persuasion with private information acquisition, Bizzotto et al. [2020] and Matyskova and Montes [2023] show that negative value of information may arise in a uni-dimensional setting in which Receiver has access to an additional signal afterwards. Bizzotto et al. [2020] study a binary framework with fixed precision of Receiver's signal. Matyskova and Montes [2023] fully solve the model with Shannon entropy costs of private information for Receiver and show that Receiver's equilibrium payoff is not necessarily monotonic in the level of informativeness (costs parameter).

This paper combines the two strands described above as the only way to study the diverting attention motives for Sender. Indeed, in a uni-dimensional framework there is no other dimension to divert Receiver's attention to, while in multidimensional frameworks without Receiver's own search for information, Sender does not need to take into account the effect of her information on Receiver's information strategy.

One of the information cost functions I allow for Receiver is entropy costs which relates this paper to the literature on rational inattention which starts from Sims [2003]. From the recent contributions, Kőszegi and Matějka [2020] consider a consumer's multiproduct consumption problem in the presence of taste and price shocks. Information acquisition about the shocks is costly, thus, the consumer strategically decides which of them to observe and to which extent. In the 2 goods example with the taste shocks, Kőszegi and Matějka [2020] shows that (under some restrictions on the available signals), if the goods are substitutes, consumers do not gather information on the total taste and thus keep the total spending fixed. However, in the case of complements, the consumer diversifies and varies the total spending while keeping the consumption of the 2 goods equal. In Section 5 I discuss this example in more detail and show how the introduction of Sender (for example, an advertiser, or a producer) with potentially different objectives from the consumer changes the consumption decision of the latter.

Hu [2020] considers a multidimensional Sender-Receiver framework with Rationally inattentive Receiver. While the solution to the generalized Receiver's information acquisition problem is similar to this paper, the role of Sender in Hu [2020] is different: Sender can either change the relative importance assigned to the dimensions by Receiver, or prevent Receiver from acquiring information on one of the dimensions. However, compared to the framework of the current paper, Sender does not decide on the optimal amount of information provision, hence, they exclude diverting attention motives.

From a technical point of view, there is an extensive literature which relies on Gaussian beliefs and a linear signals structure. See for example, Liang and Mu [2020], Liang et al. [2021].

This paper is closely related to the literature on informational lobbying. Cotton and Dellis [2016] study a binary two-dimensional framework, but they assume that the information on both sides is either perfect or absent. Thus, the work abstracts from the diverting attention motives which are the main focus of this work. Ellis and Groll [2020] consider a uni-dimensional setting in which a budget-constrained lobbyist can either provide information or subsidies (or both) after which a budget-constrained policy-maker can search for information herself. The policy-maker may benefit from being more budget-constrained in their setting. Cotton and Li [2018] consider a framework in which a politician may obtain information about the policy issues before lobbyists make the decision on monetary funding. They show that the policy-maker may prefer to commit to information of lower quality to induce a competition between lobbyist leading to higher monetary transfers. Other important contributions with a similar approach to informational lobbying are Dellis and Oak [2019], de Bettignies and Zabojnik [2019] and Hirsch et al. [2019]. This work contributes to the field by being the first to focus attention on the lobbyist's optimal information provision rather than whether the lobbying itself is detrimental.

Finally, the theoretical framework of the paper allows interpretation of the results in relation to many other applications. The literature has used closely related frameworks to study different issues. For political economy and media competition see, for instance, Duggan and Martinelli [2011], Perego and Yuksel [2022] and Yuksel [2022]. In these works policies are multidimensional and the learning technology for citizens or information provided by media are linear signals. The important role of multidimensionality was also pointed out in the question of bonus renumeration (Bénabou and Tirole [2016] and Fehr and Schmidt [2004]) and in a career concerns framework (Dewatripont et al. [1999]).

2 The Model

There are 2 agents - Sender and Receiver. There is a 2-dimensional state of the world $\theta = (\theta_1, \theta_2) \in \mathbb{R}^2$. Sender and Receiver have a common prior over the state of the world (prior expectations are normalized to 0):

$$\theta \overset{F}{\sim} \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} v_1 & \rho\sqrt{v_1 v_2} \\ \rho\sqrt{v_1 v_2} & v_2 \end{pmatrix} \right) = \mathcal{N}(\mu, \Sigma)$$

with $\mu = (0,0)^T$.

Throughout the paper it is assumed that $v_1 > v_2$. The special case of $v_1 = v_2$ will be discussed later when the intuition for the main results will be provided.

Information. Prior to making a decision Receiver obtains information from 2 sources sequentially.

Firstly, Sender commits ex-ante (before learning the state) to send a *set* of linear signals of the form:

$$S_S = \alpha_S \cdot \theta + \varepsilon_S$$

with $\varepsilon_S \sim \mathcal{N}(0, \sigma_S^2)$. That is, Sender chooses α_S - a matrix of $n \times 2$ for an arbitrary $n \in \mathbb{Z}_+$ and a noise σ_S^2 - a vector of length n. I denote the interim beliefs of Receiver, after the realization of S_S is observed by $(\check{\mu}, \check{\Sigma})$.

After observing the realization of Sender's signals, Receiver obtains *one* additional linear signal of her choice:

$$S_R = \alpha_R \cdot \theta + \varepsilon_R$$

with $\varepsilon_R \sim \mathcal{N}(0, \sigma_R^2)$. That is, Receiver chooses α_R - a vector of dimension 2 and a noise σ_R^2. I denote the posterior beliefs of Receiver by $(\widetilde{\mu}, \widetilde{\Sigma})$.

I impose directly the assumption of linear signals. Extensive literature studying persuasion in multidimensional settings shows the optimality of linear signals for Sender (in the absence of the additional information acquisition on Receiver's side) in case of Gaussian beliefs and quadratic preference. See, Tamura [2018], Sayin and Başar [2021], Akyol et al. [2016] for references.

Beyond technical convenience linear signals provide also a meaningful economic interpretation, namely the amount of focus given to each of the dimension. For instance, if Receiver chooses α_R such that $\alpha_{R_1} >> \alpha_{R_2}$, the signal she observes is much more informative about dimension 1 of the state than dimension 2. So, for instance, Heidhues et al. [2021] demonstrate that consumers, when searching for information on different products, tend to either focus on one product learning its characteristic in details, or browse through information on all the products without learning anything deeply. This pattern corresponds well to the predictions obtained by imposing linear signals. Moreover, in the spirit of Kőszegi and Matějka [2020] whenever the state of the world reflects a consumer's taste over two different products, it might natural to assume that the consumer might observe her taste for one or the other, or the relative taste, but not to observe both separately.

In the baseline framework I assume that Sender can provide an arbitrary number of signals while Receiver can choose only one signal later. This assumption is suitable for multiple applications. As was discussed above in the case of a consumer limitations to her search can be natural and go in line with the empirical evidence. However, the advertiser is able to provide multiple information in different form, thus, multiple signals are possible. As another example, one can think of a regulator conducting their own check of a new pharmaceutical product, in which they can run one type of experiment for an arbitrary sample size, but have no resources to design separate experiments. Another example can be job hiring process - the candidate is often free in submitting any supporting information for her portfolio, while the hiring side is often restricted in the number of interviews/tests it can conduct.

Payoffs and costs. I assume quadratic payoffs for Sender and Receiver with:

$$u_i(a,\theta) = -\left\|Q_i^\theta \theta + Q_i^a a\right\|^2 \tag{1}$$

for some arbitrary Q_i^θ and Q_i^a of size 2×2 for $i \in \{R,S\}$.

On top of that, Receiver is facing costs of information acquisition. I consider the 2 following cost specification:

– (entropy costs) $c(\check{\Sigma},\widetilde{\Sigma}) = \frac{\lambda}{2}\log\left(\frac{|\widetilde{\Sigma}|}{|\check{\Sigma}|}\right)$.

– (precision-dependent costs) $c(1/\sigma_R^2) = \lambda f(1/\sigma_R^2)$ with $f : \mathbb{R}_+ \to \mathbb{R}$ with $f'(\cdot) > 0$, $f''(\cdot) > 0$ and some $\lambda > 0$.

Thus, the total payoff of Receiver is given by:

$$\widetilde{u}_R(a,\theta,S_R) = u_i(a,\theta) - c(\cdot).$$

Timing. To summarize, the timing of the model is the following:

- $t = 1$: Sender commits to (α_S, σ_S^2);

- $t = 2$: The state is realized;

- $t = 3$: S_S are realized and observed by Receiver, who updates her beliefs and selects (α_R, σ_R^2);

- $t = 4$: S_R is realized, Receiver chooses an action $a \in \mathbb{R}^2$.

3 Illustrative Example

In this section I present and solve a simplified version of the framework which captures the main intuition for the general results.

3.1 Setting

I assume $\alpha_i^T \in \{(1,0),(0,1)\}$, that is, both Sender and Receiver are restricted to choose a unique signal which reveals (partially) one of the dimensions[1]

[1] The notation x^T refers to a transpose of vector x throughout the paper. X^T refers to the transpose of the matrix X.

I also assume the following payoff functions:

$$u_R(a,\theta) = -(a_1-\theta_1)^2-(a_2-\theta_2)^2. \tag{2}$$

$$u_S(a,\theta) = \sum_{i\in\{1,2\}} \Big(-\beta_i(a_i-\theta_i)^2-(1-\beta_i)(a_i-a_i^*)^2\Big). \tag{3}$$

To fix ideas, consider that Receiver is a policy maker who has to make decisions on two policy issues and wants them to be appropriate for the state of the world (θ). Sender, an informational lobbyist, only partially shares the interests of Receiver and would prefer the action to be "distorted" towards a_i^*. Parameters β_i capture the extent to which lobbyist's incentives are aligned with the ones of the policy maker on dimension i.

I assume Receiver faces entropy costs of information acquisition. Hence, if she chooses to observe a signal on dimension i the costs are:

$$c(\check{v}_i,\widetilde{v}_i) = -\frac{\lambda}{2}\log\left(\frac{\widetilde{v}_i}{\check{v}_i}\right).$$

Finally, assume that dimensions of the state of the world are not correlated, hence, $\rho=0$; w.l.o.g. assume $v_1>v_2$.

3.2 Optimal information provision

Given Receiver's payoff (2), the optimal action conditional on the information obtained is:

$$a_R = \widetilde{\mu}$$

Taking this into account, ex-ante expected payoffs of Receiver (2) and Sender (3) can be written as:

$$\mathbb{E}u_R(a,\theta) = -\widetilde{v}_1-\widetilde{v}_2$$

and

$$\mathbb{E}u_S(a,\theta) = \text{const}-(2\beta_1-1)\widetilde{v}_1-(2\beta_2-1)\widetilde{v}_2 \tag{4}$$

correspondingly. Note, that such representation of expected payoffs makes the conflict of interest apparent: if the coefficient in front of $\widetilde{v}_i$ (-$2\beta_i-1$) is positive then Sender benefits from Receiver's uncertainty and, thus, is not interested in disclosing any information on this dimensions. Conversely, if the coefficient is negative, Sender would ideally induce full learning. This follows directly from (4) and has an intuitive interpretation: the interests are *aligned* iff Sender puts relatively higher weight to the decision matching the true state of the world ($\beta_i>1/2$), i.e. to the Receiver's objective, and are *misaligned* otherwise ($\beta_i<1/2$). I distinguish the three following possibilities:

– Interests are *fully aligned* if $\beta_1 > 1/2$ and $\beta_2 > 1/2$;

– are *fully misaligned* if $\beta_1 < 1/2$ and $\beta_2 < 1/2$;

– are *partially aligned* if $(\beta_1 - 1/2)(\beta_2 - 1/2) < 0$.

One additional definition is needed before stating the optimal information provision in this framework.

Definition 1. *Sender diverts Receiver's attention with her signal if:*

– *Sender provides information on the dimension where interests are misaligned;*

– *Receiver would obtain information on this dimension in the absence of Sender's signal and obtains information on the other dimension after observing Sender's signal.*

I now formulate the optimal Sender's strategy, and prove the result in the next section.

Proposition 1. *Sender diverts Receiver's attention by partially disclosing dimension 1 iff*

1. *Incentives of Sender and Receiver are partially aligned with* $\beta_1 < 1/2, \beta_2 > 1/2$ *and*

$$\frac{\lambda}{2} < \frac{-(2\beta_1 - 1)}{2(\beta_2 - \beta_1)} v_2 \equiv \frac{\lambda^*}{2},$$

or

2. *Incentives of Sender and Receiver are fully misaligned* ($\beta_1 < 1/2$ *and* $\beta_2 < 1/2$) *and*

$$(v_2 - \lambda/2)(\beta_1 - \beta_2) < 0.$$

Proposition 1 pins down the set of cases in which Sender prefers to use seemingly counter intuitive strategy of diverting attention. This is driven by the effect which Sender's choice of signal has on the Receiver's focus on different dimensions when Receiver's cost of information acquisition are low. Indeed, in the cases underlined in the Proposition 1 Sender reveals the dimension she wants to hide. The intuition for this is the following: whenever Receiver is able to obtain sufficiently precise signal on her own, it is too costly for Sender to allow Receiver to learn the dimension of misalignment. Thus, using the fact that Receiver can obtain only one signal, she adjusts Receiver's uncertainty to force Receiver to learn the dimension where interests are aligned.

The argument is similar in the case of full misaligned of interests when the misalignment is higher on initially more uncertain dimension (case 2 in Proposition 1). In this case, information provision is possible for diverting attention reasons even though ideally Sender would prefer to reveal no information.

Such counter-intuitive strategic behavior of Sender has implications for Receiver's welfare as described by the next result. To illustrate that I fix some arbitrary $V_1 < 0$, $V_2 > 0$, v_1 and v_2 and alter the costs parameter λ. I slightly abuse the notation and write $\mathbb{E}[u_R(\lambda)]$ for the expected payoff of Receiver given the optimal strategies described above and costs λ.

Proposition 2. *R's utility is non-monotonic in her costs of information acquisition: there exists an interval* $(\underline{\lambda}, \lambda^*)$ *such that* $\mathbb{E}[u_R(\lambda)] < \mathbb{E}[u_R(\overline{\lambda})]$ *for all* $\lambda \in (\underline{\lambda}, \lambda^*)$.

Proof. The result follows intuitively from Proposition 1. Indeed, the payoff from the diverting attention solution at λ^* is:

$$u_R(a,\theta) - c(\cdot) = -v_2 - \lambda^*/2 + \frac{\lambda^*}{2}\log\left(\frac{\lambda^*}{2v_2}\right)$$

The payoff from the intuitive solution at λ^* is:

$$u_R(a,\theta) - c(\cdot) = -\lambda^*/2 + \frac{\lambda^*}{2}\log\left(\frac{\lambda^*}{2v_1}\right)$$

Note that at λ^* Sender is indifferent between the two solutions.

The payoff from the intuitive solution at the threshold λ^* exceeds the payoff from the diverting attention solution at this threshold if:

$$-v_2 < \lambda^*/2\log\left(\frac{v_2}{v_1}\right) \tag{5}$$

If (5) holds, then the diverting attention solution generates lower utility for Receiver also in the region of λ^* which completes the proof.

□

Note that in the interval $(\underline{\lambda}, \lambda^*)$ described in Proposition 2 Receiver's expected utility is decreasing in λ. Moreover it is decreasing for any $\lambda > \lambda^*$. However, at λ^* - the point at which Sender changes her strategy from diverting attention to intuitively revealing dimension of alignment, the expected utility is discontinuous and jumps upwards. Figure 1 illustrates this point.

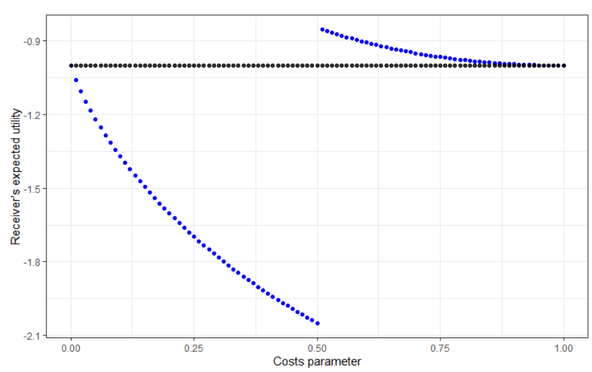


Figure 1: Receiver's expected utility (in blue) vs. no Sender benchmark (in black) as a function of the costs parameter λ. Parameters: $\beta_1 = 0.25$, $\beta_2 = 0.75$, $v_1 = 1.5$, $v_2 = 1$.

3.3 Solution

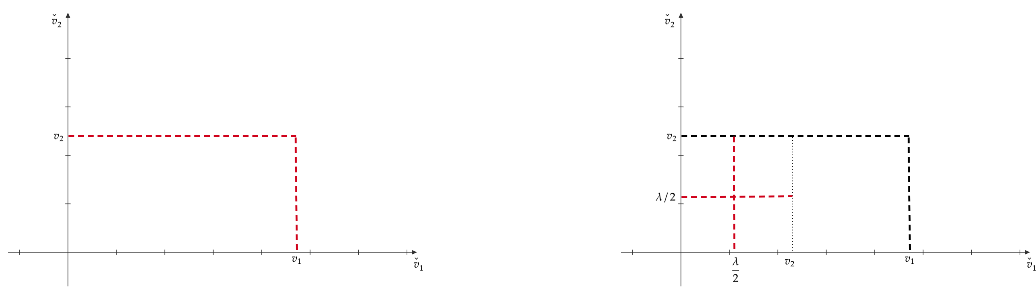
This section presents the full solution including Sender's and Receiver's strategies in the cases discussed by Proposition 1. To begin with, note that since Sender, by assumption, can provide a (possibly noisy) signal on one dimension only there are only two types of interim uncertainties she can induce on the Receiver's side:

- $(\check{v}_1, v_2)$ with $\check{v}_1 \leq v_1$ if $S_S = \theta_1 + \varepsilon_S$
- $(v_1, \check{v}_2)$ with $\check{v}_2 \leq v_2$ if $S_S = \theta_2 + \varepsilon_S$

Receiver's utility function is given by the sum of quadratic losses and she bears entropy costs of information collection. Hence, given any pair of interim uncertainties $(\check{v}_1, \check{v}_2)$, she learns the more uncertain dimension with a signal which makes the posterior variance equal to $\lambda/2$. That is the posterior beliefs are:

- $(\lambda/2, \check{v}_2)$ if $\check{v}_1 > \check{v}_2$ and $\check{v}_1 > \lambda/2$
- $(\check{v}_1, \lambda/2)$ if $\check{v}_1 < \check{v}_2$ and $\check{v}_2 > \lambda/2$
- $(\check{v}_1, \check{v}_2)$ if $\check{v}_1 < \lambda/2$ and $\check{v}_2 < \lambda/2$

The set of attainable posterior beliefs is presented in Figure 2.



(a) Set of attainable interim beliefs

(b) Set of attainable posterior beliefs (in red)

Figure 2: Attainable beliefs when Sender and Receiver have access to a single signal each

Notice that the attainable set of posterior beliefs is non-convex. For each possible alignment/misalignment of interests one of the four extreme points of the state will be the optimal solution for Sender. The information flow to reach each of these posteriors is the following:

– *Solution A:* Sender fully reveals dimension 1 ($S_S = \theta_1$) and Receiver learns dimension 2 until $\lambda/2$;

– *Solution B:* Sender doesn't reveal any information, Receiver learns dimension 1 until $\lambda/2$;

– *Solution C:* Sender reveals dimension 1 until v_2, Receiver learns dimension 2 until $\lambda/2$;

– *Solution D:* Sender reveals dimension 2 ($S_S = \theta_2$) and Receiver learns dimension 1 until $\lambda/2$.

Notice that in solution C the only goal of information provision for Sender is to divert Receiver's attention from dimension 1 by making it more certain.

There are two main take-aways from Proposition 1. Firstly, with partial alignment of interests Sender might prefer to disclose the dimension on which interests are misaligned to divert Receiver's attention instead of fully disclosing the dimension of alignment of interests. The intuition is the following: in case Sender discloses dimension on which interests are aligned, Receiver will choose a signal informative about the other dimension. In case the costs of information acquisition for Receiver are low, her chosen signal is very precise, and, thus, costly for Sender. Thus, in the case of well informed Receiver, Sender prefers to settle for only partial revelation of the dimension of alignment of interests to prevent information acquisition on the other dimension by Receiver.

Secondly, in the case of fully misaligned interests (Sender prefers Receiver not to learn any of the dimensions), the information transmission is still possible counter to the first intuition. Sender prefers to reveal some information if, in the absence of it, Receiver learns the dimension on which interests are more misaligned (lower $|\beta_i|$). In this case, information is revealed to divert Receiver's attention from the dimension of higher misalignment of interests.

An important case is the one with $\lambda = 0$, that is the information is costless for Receiver and the only restriction is that the signal must contain information about one of the two dimensions exclusively. Then whenever $\beta_1 < 1/2$ and $\beta_2 > 1/2$ (interests are misaligned on the more uncertain dimension and aligned on the other one) the unique solution for Sender for all parameters is the one based on the attention diversion.

Another important takeaway from the simple framework is the Receiver's choice of information to acquire: namely, she chooses to observe the dimension which is least known to her at the moment. This feature is in contrast to what is often obtained in the search literature (see, for instance, Gossner et al. [2021]) where once Receiver's attention was focused on one item she is relatively more likely to observe it in the future. The reason for this difference lies in the combination of the utility function of Receiver - she wants to learn the state and treats equally both dimensions, and the cost function[2].

[2] There is a big range of cost functions which allow such dynamics (given the symmetry of u_R). Any non-decreasing precision-dependent cost function leads to the learning of the most uncertain dimension.

4 General results

Now I go back to the original formulation of the problem. Section 4.1 reformulates the problem as a linear programming, Section 4.2 provides the solution to Receiver's information acquisition problem. Section 4.3 gives the main general results.

4.1 LP reformulation of the problem

From (1) it follows that the optimal action of Receiver is given by:

$$a_R = -(Q_R^{a^T} Q_R^a)^{-1} Q_R^{a^T} Q_R^\theta \widetilde{\mu}.$$

Then as is established in the literature (see, for instance, Velicheti et al. [2023] or Lemma 1 in Tamura [2018]) there exist symmetric 2×2 matrices V_R and V_S such that

$$\mathbb{E}[u_i(a^*, \theta)] = \mathbb{E}[\widetilde{\mu}^T V_i \widetilde{\mu}] + \text{const}, \tag{6}$$

for $i \in \{R, S\}$.

From (1) it is possible to link V_is to the original parameters of the model in the following way:

$$V_R = Q_R^{\theta^T} Q_R^\theta \tag{7}$$

and

$$V_S = \Lambda^T \Lambda - \Lambda^T Q_S^\theta - Q_S^{\theta^T} \Lambda$$

with $\Lambda = Q_S^a (Q_R^{a^T} Q_R^a)^{-1} Q_R^{a^T} Q_R^\theta$ (see, for instance, Tamura [2018] or Velicheti et al. [2023] for the derivation).

In the example of Section 3 the corresponding matrices V_S and V_R are:

$$V_S = \begin{pmatrix} 2\beta_1 - 1 & 0 \\ 0 & 2\beta_2 - 1 \end{pmatrix} \qquad V_R = \begin{pmatrix} 1 & 0 \\ 0 & 1. \end{pmatrix}$$

4.2 Receiver's optimal information acquisition

For the main results I focus on Receiver with the payoff such that $V_R = I$ which allows us to have the closed-form solution for both Receiver's and Sender's problems. Then (7) implies that Q_R^θ must be of the form:

$$Q_R^\theta = \begin{pmatrix} q_1 & q_2 \\ -q_2 & q_1 \end{pmatrix}$$

with $q_1^2 + q_2^2 = 1$. Note that the assumption on V_R does not impose any restriction on Q_R^a.

In Section 5.1 the relaxation of this restriction on V_R will be discussed.

Such V_R implies that uncertainty on different dimensions is equally costly for Receiver and she wants to learn the state of the world as precisely as possible. Thus, she wants to reduce the sum of the posterior uncertainties the most as stated in the following Lemma.

From now on I denote by $v(\alpha) = \alpha^T \Sigma \alpha$ the variance on dimension $\alpha \cdot \theta$.

Lemma 1. *Assume $V_R = I$. Receiver optimally obtains a signal $S_R = \alpha_R \cdot \theta + \varepsilon_R$ such that $v(\alpha_R)$ is maximized given $\|\alpha_R\| = 1$.*

Proof. **Step 1:** Given (6) the ex-ante expected payoff of Receiver can be written as:

$$\mathbb{E}u_R(a,\theta) = -\widetilde{v}_1 - \widetilde{v}_2 + \text{const.} \tag{8}$$

Assume first that there are no costs of information acquisition for Receiver on top of the restriction of a unique signal being available. Consider arbitrary interim beliefs $\check{\Sigma}$. There exists a rotation matrix U such that $\check{\Sigma}^U = U^T \check{\Sigma} U$ is diagonal[3]. That is, there exists another basis in which the dimensions are not correlated. Then the payoff in (8) can be rewritten in the following way:

$$\mathbb{E}u_R(a,\theta) = -[U^T \widetilde{\Sigma}^U U]_{11} - [U^T \widetilde{\Sigma}^U U]_{22} + \text{const} = -\widetilde{v}_1^U - \widetilde{v}_2^U + \text{const.}$$

Then in this new basis U the problem is trivial - the optimal learning strategy of Receiver is to learn a dimension $i^U = \arg\max_i v_i^U$.

Note that the dimension $i^U = \arg\max_i v_i^U$ corresponds to the eigenvector of $\check{\Sigma}$ with the highest eigenvalue. Thus, the corresponding signal indeed discloses this dimension, i.e. $S_R = \max_{\alpha:\|\alpha\|=1} \mathrm{Var}(\alpha\cdot\theta)$.

Step 2: Now I take the costs into consideration. Consider arbitrary precision dependent cost function $\lambda c(1/\sigma_R^2)$ with $c(\cdot)$ being an increasing function. Notice that for any fixed costs $\bar{c}$ (i.e. for a fixed precision of a signal) the uncertainty is reduced the most if the signal is on the dimension $\alpha_R = \arg\max_{\alpha:\|\alpha\|=1} \mathrm{Var}(\alpha\cdot\theta)$. Thus, the statement of the lemma for precision-dependent costs.

Next consider the entropy costs $c(\check{\Sigma}, \widetilde{\Sigma})$. Fix some costs c and denote $C = e^c$. Then if Receiver observes a signal S_R with α_R such that it generates costs c it must satisfy:

$$\mathrm{Var}(S_R) = \frac{v(\alpha_R)}{1-C}.$$

This follows from the fact that given entropy costs $C = \sigma^2/(v(\alpha_R) + \sigma^2)$ where σ^2 is the noise in S_R and that $\mathrm{Var}(\alpha_R) = v(\alpha_R) + \sigma^2$.

Then, for given costs the signal which maximizes the uncertainty reduction is the solution of:

[3]The rotation matrix has a form $U = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ where a is the co-sinus of the rotation angle and b is the sinus of the rotation angle.

$$\max_{\alpha_R}(1-C)\frac{\mathrm{Cov}^T(S_R,\theta)\mathrm{Cov}(S_R,\theta)}{v(\alpha_R)}$$

The solution is independent of C and coincides with $\arg\max_{\alpha_R} v(\alpha_R)$. Thus, for each targeted costs, the signal which minimizes the ex-post uncertainty is the one which uncovers the most uncertain dimension. This completes the proof.

□

I do not impose any restrictions on V_S, thus, any quadratic preference are allowed for Sender. Note, however, that there exists a rotation matrix U' such that $V_S^U = U'^T V_S U'$ is diagonal. That is, there exists a new basis such that Sender cares exclusively about the posterior uncertainties of Receiver about the associated axes, but not the correlations. Also note that since $V_R = I$ by assumption, $V_R^U = U'^T V_R U' = I$, thus in the new basis Receiver also wants to minimize the sum of residual uncertainties, i.e. to learn both dimensions as precisely as possible.

Thus, Receiver's solution satisfies Lemma 1, that is she observes the most uncertain dimension. The optimal noise in the signal can be obtained as the solution to:

$$\max_{\sigma^2} -\frac{v\sigma^2}{v+\sigma^2} - c(\cdot)$$

where $v = \max_\alpha v(\alpha)$.

I denote the resulting optimal posterior beliefs given the interim beliefs $\check{\Sigma}$ by $\widetilde{\Sigma}(\check{\Sigma})$. Notice that with entropy costs posterior beliefs on the dimension $arg \max_\alpha v(\alpha)$ are equal to $\min\{v(\alpha), \lambda/2\}$ while for the convex increasing precision-dependent costs it is an increasing continuous function of $v(\alpha)$.

To simplify notations I am going to assume directly V_S to be diagonal which is, as described above w.l.o.g. Sender's problem can be written then in the following way:

$$\begin{gathered}\max_{\Sigma'} -e^T V_S \circ \widetilde{\Sigma}(\Sigma')e \\ \text{s.t.} \\ \Sigma - \Sigma' \geq 0\end{gathered} \tag{9}$$

Note that according to (9) Sender is choosing the interim beliefs while it is standard in the literature to consider optimization over posteriors. While the problem is easily rewritten as an optimization over posteriors the formulation with interim beliefs is more convenient given the solution method to obtain the main results.

For better understanding of the results another formulation is also useful:

$$\begin{gathered}\max_{U_S,\Sigma'} -e^T V_S \circ \widetilde{\Sigma}(U_S\Sigma' U_S^T)e \\ \text{s.t.} \\ \Sigma - U_S\Sigma' U_S^T \geq 0 \\ \text{and} \\ \Sigma' \text{ is diagonal with } \Sigma'_{11} > \Sigma'_{22}.\end{gathered} \tag{10}$$

Formulation (10) states that Sender can choose a dimension of maximal uncertainty for Receiver (that is the rotation U_S of beliefs) and the interim beliefs in this basis.

Denote by $\widetilde{v}(v)$ the solution to the following unidimensional problem of Receiver:

$$\widetilde{v}(v) = \max_{v'} -v' - c(\cdot)$$

That is, if Receiver decides to obtain a signal on a dimension with uncertainty v, the optimal posterior belief is $\widetilde{v}(v)$. Note that due to the assumptions on the costs of information acquisition $\widetilde{v}(v)$ is non-decreasing.

In the benchmark framework of Section 3, notion of the conflict of interest was direct in the sense that a dimension i with $V_i > 0$ was considered a dimension of alignment and a dimension with $V_i < 0$ - a dimension of misalignment. For the general results, however, the extended notion of alignment is needed.

Definition 2. *The interests on a dimension $x\cdot\theta$ with $\|x\| = 1$ are aligned if $V_1x_1^2 + V_2x_2^2 > 0$ and misaligned otherwise.*

The definition partitions the space of the dimensions in 2 parts. The intuitive meaning of it is as follows: if Receiver would learn some arbitrary dimension $x\cdot\theta$ it would reduces her uncertainty on both original dimensions at the same time. Thus, there are 2 potential effects for Sender: the positive effect of learning on the original dimension of alignment and the negative effect of learning on the original dimension of misalignment. Depending on which effect dominates a dimension $x\cdot\theta$ is either a dimension of alignment or misalignment.

To see that such Definition comes naturally, consider the problem of Sender as in (10). In the maximizing pair $(U_S, \check{\Sigma}^{U_S})$, $\check{\Sigma}^{U_S})$ is diagonal, thus, the problem of the expected payoff of the Sender can be written as:

$$\mathbb{E}u_S(a,\theta) = -(V_1a^2 + V_2b^2)\widetilde{v}_1^{U_S} - (V_2a^2 + V_1b^2)\widetilde{v}_2^{U_S}$$

where a, b are the entrances of the rotation matrix U_S and $\widetilde{\Sigma}(\check{\Sigma}^{U_S}) = \begin{pmatrix}\widetilde{v}_1^{U_S} & 0 \\ 0 & \widetilde{v}_2^{U_S}\end{pmatrix}$.

Hence, $V_1^{U_S} \equiv (V_1a^2 + V_2b^2)$ and $V_2^{U_S} \equiv (V_2a^2 + V_1b^2)$ are the coefficients with which dimensions 1^{U_S} and 2^{U_S} enter the decision problem. Thus, as in the case of the reduced problem presented in Section 3, when $V_1^{U_S} > 0$ the interests on the dimension 1^{U_S} are aligned in the sense that in Sender's ideal scenario the posterior uncertainty of Receiver on this dimension is 0. On the contrary, when $V_1^{U_S} < 0$ the interests on the dimension 1^{U_S} are misaligned in the sense that ideally Sender prefers Receiver's posterior uncertainty on this dimension to be as high as possible. The same holds for the dimension 2^{U_S}.

4.3 Main results

The first theorem addresses the entropy costs: it provides the complete characterization of the optimal solution. The second theorem includes the statement for convex precision-dependent costs. While it does not provide the complete characterization it demonstrates the existence of the region with the diverting attention solution.

In this section I always use U for the rotation which diagonalizes prior beliefs and U_S for the rotation of interim (and thus posterior) beliefs - the choice variable of Sender according to the formulation (10). Also for any rotation U' I write $1^{U'}$ for the dimension 1 in the basis associated with U' and $2^{U'}$ for the orthogonal dimension. For any rotation U'_S I assume the interim beliefs chosen are such that $\check{v}_1^{U'_S} > \check{v}_2^{U'_S}$. Otherwise, one can switch the rotation to the orthogonal one. In other words, by choosing U_S Sender chooses the dimension of maximal uncertainty in interim beliefs.

The following notation is necessary for the results:

$$v^* = \frac{1}{2}\left(v_1 + v_2 - \sqrt{(v_1 - v_2)^2 + 4v_1 v_2 \rho^2}\right). \tag{11}$$

Intuitively, v^* is the uncertainty on the dimension of minimal uncertainty (dimension 2^U).

Theorem 1. *Assume Receiver faces entropy costs of information acquisition. Then Sender always induces diagonal $\check{\Sigma}$. Moreover, Sender diverts Receiver's attention away by providing partial information on one of the dimensions of misalignment if:*

- *The conflict of interests is partial with the misalignment on the more uncertain dimension ($V_1 < 0$, $V_2 > 0$) and*

$$\frac{\lambda}{2} \geq -\frac{-V_1}{-V_1 + V_2} v^* \tag{12}$$

- *The interests are fully misaligned ($V_1 < 0$, $V_2 < 0$) and $|V_1| > |V_2|$.*

If these conditions are not satisfied, Sender fully reveals a dimension of alignment of interests in case of the partial conflict, and provides no information in the case of fully misaligned interests.

The proof of the result is given in the Appendix. The main part of the proof demonstrates that Sender always prefers to get rid of the correlation between the original dimensions by setting $\check{\Sigma}$ to be diagonal. After this step the problem becomes identical to the one discussed in Section 3.1.

The diverting attention solution is obtained in the following way: to reveal the dimension of maximal uncertainty 1^U until the uncertainty level v^*. Since by assumption $V_1 < 0$, $V_2 > 0$ and $v_1 > v_2$, the dimension of maximal uncertainty is a dimension of misalignment. Hence, the solution is indeed diverting attention according to Definition 1. In contrast when conditions of Theorem 1 are not satisfied Sender chooses an intuitive solution: to fully reveal dimension 2.

The next result is an analog of Theorem 1 for the case of Receiver facing a convex precision-dependent costs of information acquisition.

Theorem 2. *Assume Receiver faces convex precision-dependent costs of information acquisition. Then Sender diverts Receiver's attention by providing information on one of the dimensions of misalignment for any partial conflicts of interests when Receiver is sufficiently well informed, that is λ is low enough. Moreover, when the relative conflict of interest on the dimension of misalignment is high and the dimensions are strongly correlated Sender might prefer to reveal no information.*

The formal proof of the result is left for the Appendix.

Note that Theorem 2 uses the generalized notion of alignment of interests. That is, even for non-diverting attention solution the information might be provided on both original dimensions. However, what matters is if the combined information is the one Sender overall wants to reveal or hide (hence, the generalized notion). On top of that there are qualitatively new possible solutions Sender might find optimal compared to the case of entropy costs of information acquisition.

Next Lemma describes in detail the structure of possible equilibrium strategies of Sender.

Lemma 2. *There are three strategies for Sender which can occur in equilibrium when Receiver is facing convex costs of information acquisition:*

- *(intuitive) To reveal fully one of the dimensions of alignment of interests $\alpha \cdot \theta$ such that*[4]*:*
$$\max_{\alpha}(\alpha^{\perp} \cdot V)\tilde{v}(v_1^{U_S})$$
- *(diverting attention) To reveal the dimension of maximal uncertainty until v^*;*
- *To choose U_S such that $V_1^{U_S} < 0$, $V_2^{U_S} < 0$ forcing Receiver to learn a dimension of misalignment. It is done by providing partial information either on a dimension of alignment, or on a dimension of misalignment or by providing no information if $U_S = U$.*

[4] $\alpha^{\perp}$ stands for a vector orthogonal to α.

There are several take-away(s) from Lemma 2. First, notice that the third type of solution (not intuitive or diverting attention) includes strategies such that Sender provides information on a dimension of misalignment and Receiver learns herself some other dimension of misalignment. However, since Receiver would learn on her own the dimension of highest uncertainty, these solutions do not satisfy Definition 1.

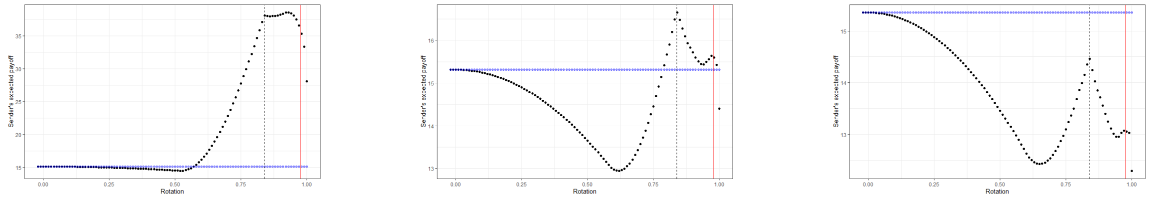
The other fact to notice is that the third type of the solution includes no information provision. This might be optimal for Sender to reveal no information to Receiver even when interests are partially aligned. This can occur if the conflict of interests on dimension 1 is relatively high ($|V_1| >> V_2$) and/or correlation between the dimensions is too strong.

An important implication of Theorem 2 is that whenever Receiver is facing low costs of information acquisition (low λ) the diverting attention solution dominates any other solution.

The Figures below illustrate the intuition above. Figure 3 illustrates the case in which the conflict of interests on dimension 1 is relatively strong and the dimensions are highly correlated. The horizontal axis represents rotation U_S given by the co-sinus of the rotation angle (b). The vertical axis presents Sender's expected payoff given by (10). In all of the figures the black dotted vertical line represents the dimension of maximal uncertainty. That is $V_1^U < 0$ and $V_2^U < 0$ in this example. The red solid line gives the threshold such that, for all U_S with b higher than this threshold (to the right from the red line), $V_2^{U_S} > 0$. The values for the U_S with b above the red line threshold are the intuitive solution from Lemma 2, the solutions on the left are of the third type from Lemma 2. The cost function used is:

$$c(1/\sigma^2) = \left(\frac{1}{\sigma^2}\right)^{10},$$

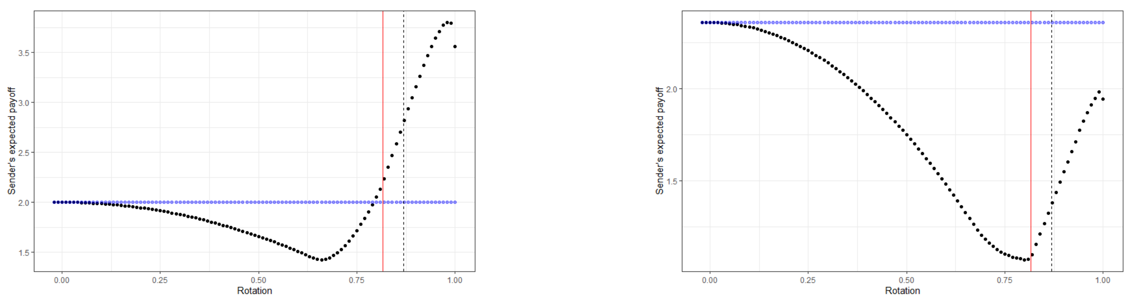
which is indeed convex. It follows then that for high costs of information acquisition for Receiver, Sender chooses the third type of solution from Lemma 2, that is the solution which is neither intuitive nor diverting attention. The intuition is the following: Sender wants to reveal some information since the interests are partially aligned, however, the correlation is too high, thus, revealing the dimension of alignment is too costly. For the intermediate level of costs she chooses to reveal nothing. This is intuitive: on the one hand the dimensions are highly correlated, thus, Sender doesn't want to reveal too much. On the other hand the costs are still high enough to prevent Sender from diverting Receiver's attention. Finally, Figure 3c shows that whenever Receiver is sufficiently well informed diverting attention solution generates the highest payoff for Sender.



(a) High cost (b) Intermediate cost (c) Low cost

Figure 3: Sender's payoff from different candidate solutions for different costs levels. $|V_1|/V_2 = 20$, $\rho = 0.8$

Figure 4 provides the same illustration for the case when the conflict of interests is relatively weak and the dimensions are less correlated with each other. In this case, as discussed above, there are only two possible equilibrium solutions: an intuitive one when costs of information acquisition are high as illustrated in Figure 4a, and a diverting attention one when the costs are low as illustrated in Figure 4b.



(a) High cost (b) Low cost

Figure 4: Sender's payoff from different candidate solutions for different costs levels. $|V_1|/V_2 = 2$, $\rho = 0.6$

The discussion of the case with the strong conflict of interests $|V_1| > V_2$ in Theorem 2 provides the intuition for the case of fully misaligned interests ($V_1 < 0$, $V_2 < 0$). The next result provides the set of possible equilibrium strategies for Sender in this case.

Corollary 1. *When the interests are fully misaligned ($V_1 < 0$, $V_2 < 0$) there are 3 types of solution which maximize Sender's payoff depending on the parameters:*

- *(no information) To provide no information;*
- *(diverting attention) To partially reveal the dimension of maximal uncertainty until v^*;*
- *(diverting attention) To partially reveal some dimension of misalignment (different from the one of maximal uncertainty).*

If one of the last 2 solutions is chosen in equilibrium, Sender diverts Receiver's attention.

This result follows directly from the case of $|V_1| > V_2$ in the proof of Theorem 2. Note that with fully misaligned interests all dimensions are dimensions of misalignment, thus, any information provision in equilibrium has diverting attention motives as its goal.

The final step in the section is to show that in the case of a partial conflict of interests Receiver might prefer to face higher costs of information acquisition. In this case Sender has more incentives to disclose information in a non-strategic way, that is to choose an intuitive solution.

Proposition 3. *Assume Receiver is facing entropy costs of information acquisition. There exists an interval $(\underline{\lambda}, \overline{\lambda})$ such that for every $\lambda \in (\underline{\lambda}, \overline{\lambda})$, Receiver's payoff net of costs is higher at $\overline{\lambda}$.*

Proof. The results is a direct consequence of Proposition 2. Indeed, note that if the result holds for the comparison of the diverting attention solution and the intuitive solution with $U_S = I$ then it holds for the optimal intuitive solution (since the costs of information for Receive increase while the gains remain the same). $\square$

The similar result holds in the case of convex precision-dependent costs of information acquisition for Receiver.

Proposition 4. *Assume a partial conflict of interests with $V_1 < 0$, $V_2 > 0$ and that Receiver is facing a convex precision-dependent costs of information acquisition. Then for some values of parameters there exists an interval $(\underline{\lambda}, \overline{\lambda})$ such that for every $\lambda \in (\underline{\lambda}, \overline{\lambda})$, Receiver's payoff net of costs is higher at $\overline{\lambda}$.*

Proof. The results is a consequence of the two following observations.

Firstly, as was already established earlier for sufficiently low costs of information acquisition Sender always chooses a diverting attention solution.

Secondly, in the region around $\rho = 0$ Receiver is facing a negative value of information for some parameters of the model. Indeed, assume $\rho = 0$. Sender chooses the "diverting attention" solution if

$$-V_1 v_2 - V_2 \widetilde{v}(v_2) \geq -V_1 \widetilde{v}(v_1).$$

Since the LHS is decreasing in λ and the RHS is increasing in λ there is a threshold $\overline{\lambda}$ such that Sender choose the diverting attention solution for all $\lambda < \overline{\lambda}$. Thus, Receiver is facing a negative value of information if:

$$-v_2 - \widetilde{v}(v_2) - \overline{\lambda} c(1/\sigma^2) < -\widetilde{v}(v_1) - \overline{\lambda} c(1/\sigma'^2),$$

where σ^2 is such that $\widetilde{v}(v_2) = v_2\sigma^2/(v_2+\sigma^2)$ and σ'^2 is such that $\widetilde{v}(v_1) = v_1\sigma'^2/(v_1+\sigma'^2)$. This holds for some costs functions and prior beliefs.

The same logic holds with the increase of the $|\rho|$. $\square$

The last remark of the section deal with the linear precision-dependent costs of information acquisition, that is $c(1/\sigma^2) = \lambda/\sigma^2$. Note that this cost function corresponds to the LLR costs introduced in Pomatto et al. [2018]. Facing such costs of information acquisition Receiver behaves in the same way as if facing entropy costs: she learns the most uncertain dimensions until $\sqrt{\lambda}$. Thus, results of Theorem 1 and Proposition 3 apply.

5 Applications

In this section I discuss some important though not exhaustive applications of the theoretical results presented above to the real life situations. Section 5.1 studies the choice of optimal consumption bundle by a rationally inattentive consumer adding an information provider to the framework presented in recent study by Kőszegi and Matějka [2020]. Section 5.2 discusses the implications of the theoretical results to informational lobbying framework.

5.1 Consumer's choice

In this section I apply my results to study the effect of the presence of advertiser or producer's information on consumer choice using the model presented in Kőszegi and Matějka [2020] (I focus on the example of Section II). First, I present the consumer's side model and then add an advertiser.

In the model of Kőszegi and Matějka [2020] a consumer needs to choose the consumption level of 2 goods facing taste shocks. Her utility of consumption of the goods is quadratic in tastes and consumption levels and takes the following form:

$$u_C(a,\theta) = (\bar{\theta} + \theta_1)a_1 + (\bar{\theta} + \theta_2)a_2 - \frac{a_1^2}{2} - \frac{a_2^2}{2} - \gamma a_1 a_2 - (a_1 + a_2)$$

where

- $\bar{\theta} > 1$ is the average taste for 2 goods;
- θ_1,θ_2 are independent random taste shocks (state of the world) distributed according to $\mathcal{N}(0, v_{\theta_i})$;
- $\gamma \in (-1, 1)$ is a substitutability parameter with the goods being substitutes when $\gamma > 0$ and complements when $\gamma < 0$ (neither for $\gamma = 0$);
- prices of both goods are normalized to 1.

The consumer can observe one of the tastes θ_1, θ_2, the relative taste $\theta_- = \theta_1 - \theta_2$ or the total taste $\theta_+ = \theta_1 + \theta_2$ but not several of these at the same time.

The consumer's problem can then be written in terms of relative and total tastes θ_- and θ_+ and solved for $a_- = a_1 - a_2$, $a_+ = a_1 + a_2$. The optimal consumption of the consumer is given by:

$$a_- = \frac{\widetilde{\mu}_-}{1-\gamma} \quad \text{and} \quad a_+ = \frac{2(\bar{\theta}-1)+\widetilde{\mu}_+}{1+\gamma}$$

where $\widetilde{\mu}_-$ and $\widetilde{\mu}_+$ are posterior expectations of the relative taste θ_- and the total taste θ_+ correspondingly. Then the consumer's expected utility rewrites as follows:

$$\mathbb{E}u_C(a,\theta) = -\frac{1}{1-\gamma}\widetilde{v}_- - \frac{1}{1+\gamma}\widetilde{v}_+$$

where $\widetilde{v}_-$ and $\widetilde{v}_+$ are posterior uncertainties about the relative and the total taste.

In such framework Kőszegi and Matějka [2020] shows that if the tastes are uncorrelated (i.e. Σ diagonal):

- and the goods are substitutes ($\gamma > 0$), consumer observes θ_- and, thus a_+ is fixed with $a_+ = \frac{2(\bar{\theta}-1)}{1+\gamma}$, however the relative consumption a_- varies depending on the signal. The phenomena is known as "mental budgeting";
- and the goods are complements ($\gamma < 0$), consumer observes θ_+ and, thus a_- is fixed with $a_- = 0$, however the total consumption a_+ varies depending on the signal. The phenomena is known as "naive diversification".

The natural second step is to ask if and how the consumer's behavior changes in the presence of an additional information provided by an advertiser or a producer prior to consumer's own information search. One relevant example is a presentation of new products, for instance, in hi-tech industry, where the information provided by the producer comes before consumer has access to any other search.

Prior to solving this question, I modify slightly the example by introducing correlations between the taste shocks (which is a natural assumption for products of the same class, or produced by the same manufacturer). The presence of $\rho \neq 0$ leads to $v_- \neq v_+$, thus, the consumer is unequally uncertain initially about the relative taste and the total taste with $v_- < v_+$ if $\rho > 0$ and $v_- > v_+$ otherwise. On top of that, assume that the consumer faces the entropy costs of information acquisition with scaling parameter λ which goes in line with the main assumption of Kőszegi and Matějka [2020]. In this case the consumer chooses to observe the relative taste θ_- if:

$$\frac{1}{1-\gamma}v_- - \frac{1}{1+\gamma}v_+ > \frac{\lambda}{2}\log\left[\frac{1+\gamma}{1-\gamma}\frac{v_-}{v_+}\right]$$

and she learns the dimension until $\lambda/2$.

I now add Sender which can be an advertiser or a producer of the goods. Assume that Sender wants to increase the total spendings of the consumer while having them heterogeneous enough across the goods. For instance, a producer of the goods enjoys high total gain but would prefer consumers to buy both rather than to leave one good on the shops shelves. Thus she would prefer to sell an equal amount of both goods. Such payoff of Sender can be written as:

$$u_S(a,\theta) = a_+^2 - a_-^2$$

It translates into the expected payoff:

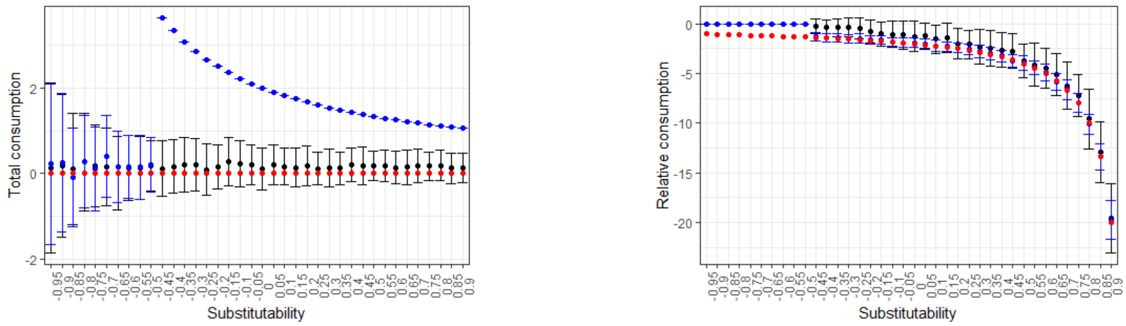
$$\mathbb{E}u_S(a,\theta) = \frac{1}{(1-\gamma)^2}\widetilde{v}_- - \frac{1}{(1+\gamma)^2}\widetilde{v}_+$$

Thus, there is a partial conflict of interests between Sender and the consumer. Given that θ_- and θ_+ are uncorrelated by assumption there are 2 possible solutions for Sender:

- To communicate perfectly the total taste for the goods θ_+ while remaining silent about the relative taste θ_-. For instance, in practice this can be done by communicating the properties and the benefits of the consumption of the goods from the category while remaining silent on the relative advantages of the two products.

- To reveal part of the information on the relative taste, to force the consumer to force on the total taste. In practice, in case of the products-substitutes the sender can provide some information on the relative benefits of one product over the other on some features (but not all of them) without focusing on the benefits of the products from this category.

Figures 5 and 7 present the consumption behavior of the consumer as a function of the substitutability of the goods in the presence of Sender. In both the black dots correspond to the case when Sender is present, blue dots to the benchmark case with no Sender. Finally, red dots show the optimal consumption level if full information would be available. Figure 5 shows the consumption patterns of an optimistic consumer compared to the realized state. One can see that the pattern for the relative consumption remains the same compared to the baseline scenario without Sender: she observes no information on the relative taste when the goods are substitutes and receives such information otherwise. However the behavior of the consumer changes with respect to the total consumption when goods are compliments. While for substitutes the pattern resembles the one of the no Sender benchmark, for the compliments the mental budgeting never occurs: consumer is always observing some information about the total taste due to diverting attention strategy of Sender. Also consumer is making better choices on average in the presence of Sender in terms of total consumption, but worse choices (further from ideal points) in terms of relative consumption. This goes in line with the diverting attention intuition - since Sender is interested in consumer's learning of total taste but wants to hide the relative taste the switch in the behavior occurs towards more attention to total taste.



(a) Total consumption (b) Relative consumption

Figure 5: Total and relative consumption of the consumer optimistic towards relative taste of good 1 and total taste

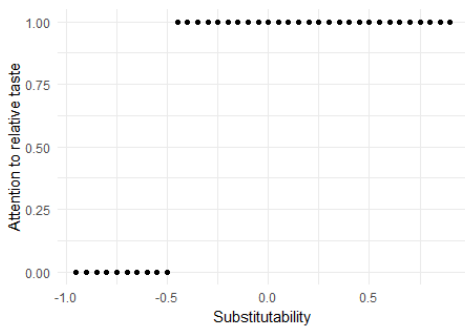
Figure 6 demonstrates the weights (attention) assigned to θ_- by the consumer and Sender. Figure 6a shows the benchmark case of no Sender: consumer learns θ_+ when the goods are complements and θ_- when the goods are substitutes (which corresponds to her benchmark consumption behavior). Figure 6b shows the attention of the consumer in the presence of Sender. She never chooses to obtain information on θ_- and focuses 100% of her attention on θ_+. This happens due to Sender's information provision strategy (Figure 6c): when the goods are substitutes and the consumer would observe the relative taste on her own, Sender chooses to partially reveal θ_- to make the consumer switch her attention to the total taste θ_+. Thus Sender chooses to divert the consumer's attention.

The same holds for a pessimistic consumer. This is the case due to the fact that Sender commits to her strategy before learning the state, and thus the uncertainties are the optimization variables for both, Sender and the consumer.

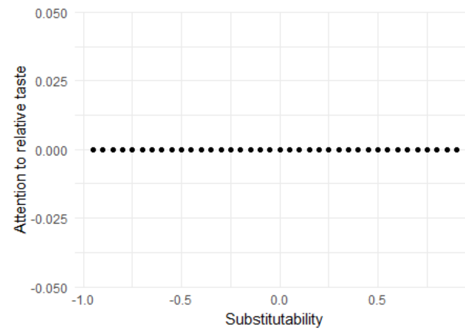
One can see that the presence of Sender, which is natural in the optimal consumption problem, can strongly affect the consumer's behavior and significantly decrease the presence of mental budgeting. Moreover, Sender's information does not necessarily reveal the information about the goods which is beneficial for Sender: whenever the goods are substitutes she provides information to change the consumer's focus rather than to reveal information in an intuitive way.

Next I illustrate implications for consumer's welfare caused by the presence of Sender in terms of realized payoffs. First I fix some value of θ_+ and vary the relative taste θ_-.

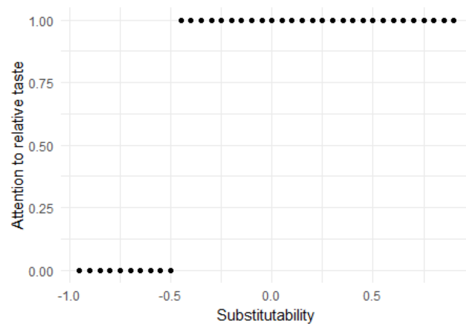
As seen in Figure 6 there are 3 qualitatively different cases: when the goods are substitutes ($\gamma > 0$), goods are complements with γ sufficiently close to 0 and complements with $\gamma << 0$. I compare consumer realized payoffs for these 3 cases which are described in Table 1. Intuitive and Diverting attention solutions differ by the cost parameter for consumer ($\lambda = 3$ versus $\lambda = 1$). The benchmark columns represent the case of no Sender (thus, the initial Kőszegi and Matějka [2020] framework): 'W' corresponds to the weight assigned to the relative taste θ_- and 'N' to the amount of



(a) Receiver's attention to θ_- in the absence of Sender



(b) Receiver's attention to θ_- in the presence of Sender



(c) Sender's attention to θ_-

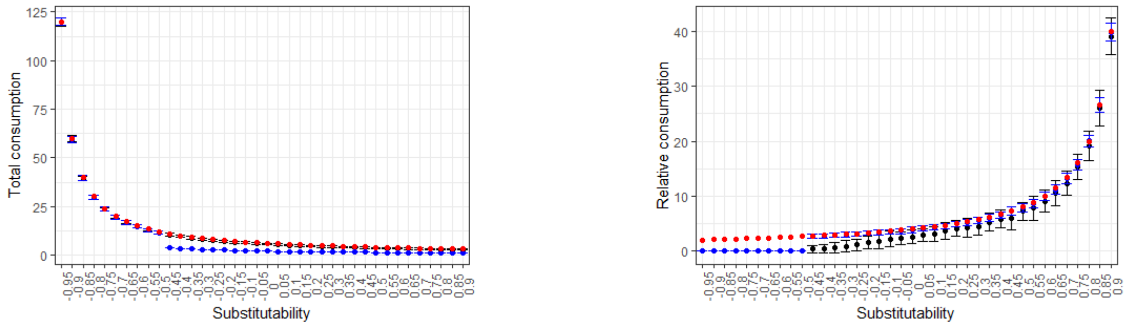
Figure 6: Total and relative consumption of the consumer pessimistic towards relative taste of good 1 and total taste

noise (σ_R^2) chosen by the consumer. The persuasion columns represent the framework with Sender. 'W S' stands for the weight assigned to the relative taste θ_- by Sender and 'N S' for the noise σ_S^2 chosen by Sender. 'W' and 'N' stand for the similar choice variables for the consumer (chosen upon observing Sender's information). Note that it is assumed directly here that Sender sends a unique signal, the previous discussion in the section shows that it is indeed optimal.

As was shown before for the case when goods are substitutes in the absence of Sender the consumer observes the relative taste θ_-, intuitively the signal is more precise when λ is lower. In the case of high costs for the consumer Sender fully reveals the total taste (intuitive solution), but in the case of low costs she provides just enough information on the relative taste to discourage consumer's further search in this direction.

Figures 8-10 represent the realized outcomes for Benchmark and Persuasion cases for the parameters described above. In all this illustrations θ_+ is fixed and θ_- varies from -10 to 10 (horizontal axis in the graphs). In each graph black lines represent the distribution (mean+standard deviation) in the Persuasion case and blue lines - in the Benchmark no Sender case.

Figure 8 (first column) illustrates the case when goods are substitutes. In Figures



(a) Total consumption (b) Relative consumption

Figure 7: Total and relative consumption of the consumer pessimistic towards relative taste of good 1 and total taste

Type of goods	γ	Type of solution	Benchmark		Persuasion			
			W	N	W S	N S	W	N
Substitutes	0.5	Intuitive	1	0.83	0	0	1	0.83
		Diverting	1	0.26	1	1.14	0	1
Complements	-0.4	Intuitive	1	2.85	0	0	1	2.85
		Diverting	1	0.77	1	55.9	0	0.33
	-0.8	Intuitive	0	0.33	0	0	1	4.08
		Diverting	0	0.1	0	7.41	0	0.1

Table 1: Cases studied for fixed θ_+ with the corresponding attention and noise in the main framework and no Sender benchmark

8a and 8b consumer has high costs of information acquisition, thus the solution chosen by Sender is intuitive (the top line of Table 1). In the case of Figure 8a, however, consumer has incorrect prior beliefs about the total taste θ_+ (that is $\mu_+ \neq \theta_+$), while in in the Figure 8b her beliefs are correct. With incorrect beliefs she benefits from the presence of Sender. Indeed, Sender reveals fully the total taste, thus, correcting mistake in the priors. In the absence of Sender, however, the consumer never learns the total taste and, thus, always sets the wrong total consumption. On the other hand, when the prior beliefs are correct the consumer is indifferent to the presence of Sender since her total consumption is always correct.

In Figures 8c and Figures 8d consumer has high costs of information acquisition, thus Sender diverts consumer's attention away from the relative taste θ_- (the second line of Table 1). Again Figure 8c shows the case when prior beliefs about the total taste are incorrect and Figure 8d case when the prior beliefs are correct. The consumer is relatively better off in the presence of Sender when beliefs are incorrect. However, in case of correct beliefs, or only a small mistake, consumer is better off without Sender. Indeed, as Table 1 shows, in this case consumer learns much less on relative taste with the addition of Sender (the noise in the signal is 1.14 compared to 0.26 in the absence of Sender). If beliefs are correct this difference is not compensated by learning more about the total taste.

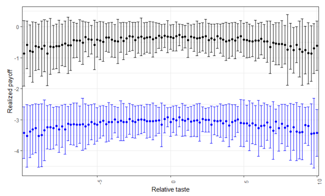
Figures 9 and 10 provide the similar analysis for the case of complements. In Figure 9 even though the goods are complements, the consumer still chooses to observe the relative taste in the absence of Sender. Thus, the patterns and interpretations are similar to the case of substitutes in Figure 8.

In the case presented in Figure 10 the consumer learns herself the total taste, thus, the diverting attention solution as defined by Definition 1 does not exist. In case of intuitive solution Sender reveals the total taste entirely. Hence, the consume benefits from the presence of Sender in case of incorrect prior beliefs (Figure 10a). On the other hand, whenever the consumer has too low costs of information acquisition Sender prefers to reveal total taste just enough to keep consumer's focus on the same question. The consumer neither benefits nor loses from the presence of Sender (Figures 10c-10d).

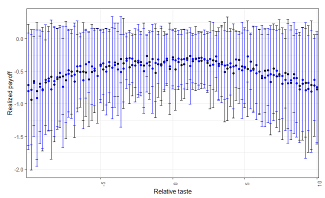
Figures 11 and 12 fix the value of relative taste θ_- and vary the total taste θ_+. Figure 11 looks at the case of goods being substitutes and Figure 12 - at the case when goods are complements with high $|\gamma|$. If the consumer learns the relative taste in the absence of the Sender (substitutes as in Figure 11 of weak complements) consumer benefits a lot from the presence of Sender unless her beliefs on total taste are correct (as was shown in Figures 8d and 9d).

However, when the goods are strong complements and consumer would learn total taste in the absence of Sender (Figure 12) she benefits less from the presence of Sender, especially she is indifferent facing low costs of information acquisition.

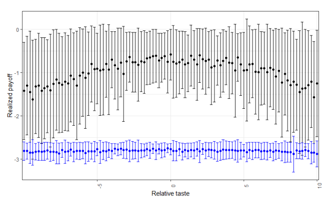
Overall, the consumer benefits from the presence of Sender when her prior beliefs are far from the realized state, but can be worse off if her initial beliefs on one of the dimensions (total taste or relative taste) are confirmed while happen to be wrong on the other. In this case she loses more if she is well-informed in the sense of lower costs of information acquisition. Note, however, that, as theoretical results show, in expectation the consumer is always weakly better off in the presence of Sender, that is in expectation the positive effects outweigh the negative ones.



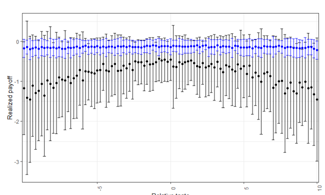
(a) High costs, incorrect prior belief



(b) High costs, correct prior belief

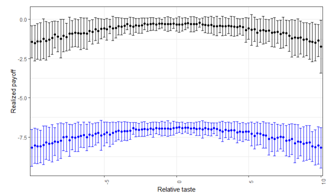


(c) Low costs, incorrect prior belief

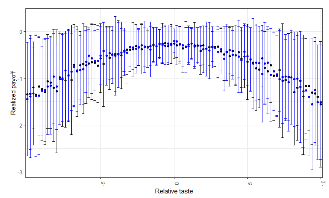


(d) Low costs, correct prior belief

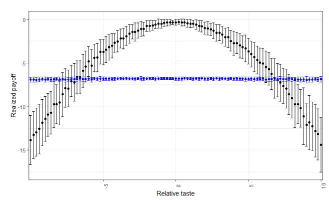
Figure 8: Consumer's realized payoffs when goods are substitutes



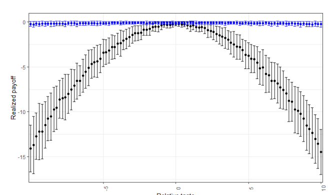
(a) High costs, incorrect prior belief



(b) High costs, correct prior belief

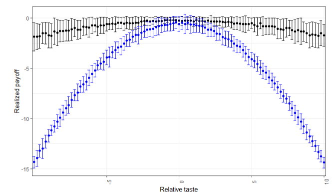


(c) Low costs, incorrect prior belief

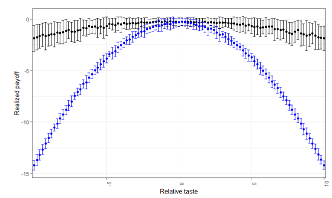


(d) Low costs, correct prior belief

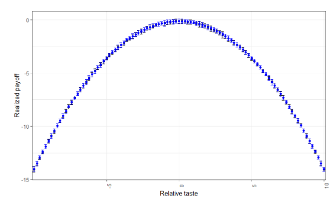
Figure 9: Consumer's realized payoffs when goods are complements, low $|\gamma|$



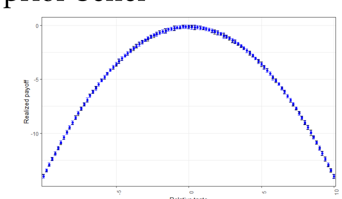
(a) High costs, incorrect prior belief



(b) High costs, correct prior belief

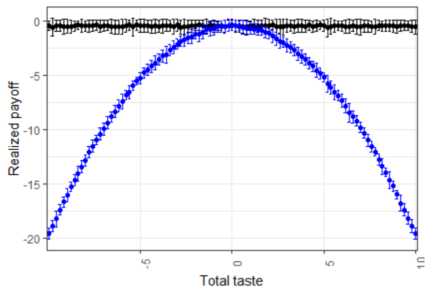


(c) Low costs, incorrect prior belief

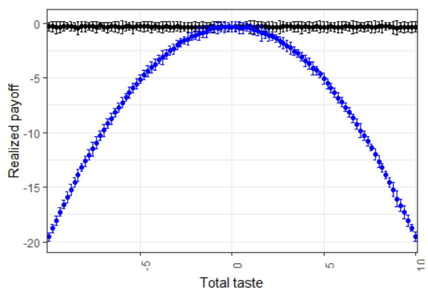


(d) Low costs, correct prior belief

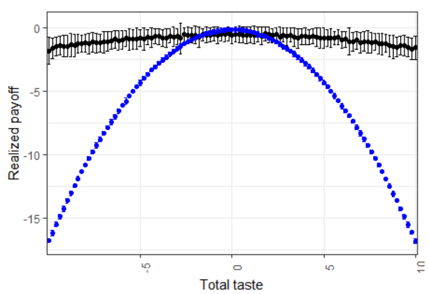
Figure 10: Consumer's realized payoffs when goods are complements, high $|\gamma|$



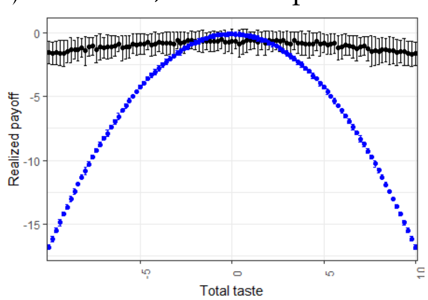
(a) High costs, incorrect prior belief



(b) High costs, correct prior belief

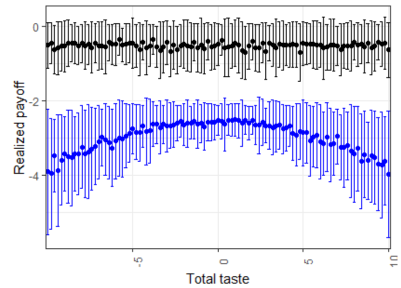


(c) Low costs, incorrect prior belief

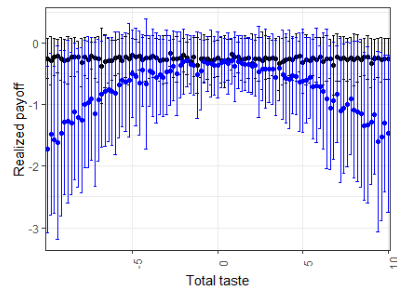


(d) Low costs, correct prior belief

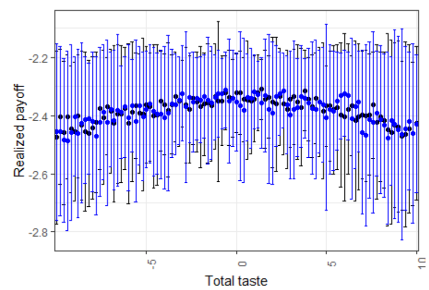
Figure 11: Consumer's realized payoffs when goods are substitutes



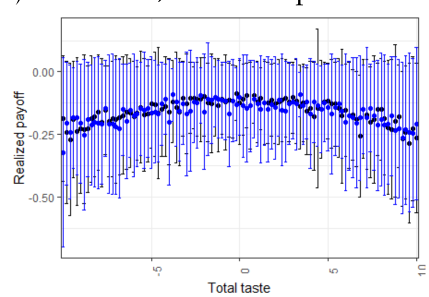
(a) High costs, incorrect prior belief



(b) High costs, correct prior belief



(c) Low costs, incorrect prior belief



(d) Low costs, correct prior belief

Figure 12: Consumer's realized payoffs when goods are complements

5.2 Implications for lobbying

As the results of the previous sections show in the case a policy-maker needs to decide several issues on some of which her interests are conflicting with the ones of a lobbyist, the latter might decide to provide some information she would not disclose ideally just to divert the policy-maker attention. Note, however, that contrary to the majority of existing literature on informational lobbying the results do not question if the presence of a lobbyist is harmful for the policy-maker. The policy-maker in my settings is always at least weakly better-off in the presence of a lobbyist even in the presence of a conflict of interests.

However, the payoff of the policy-maker depends on her costs of information acquisition in 2 ways:

- Directly, by affecting how much information the policy-maker can acquire;
- Indirectly, by affecting the choice of information of the lobbyist.

While through the first channel the policy-maker always benefits from having lower costs of information acquisition, as theoretical results demonstrate the second effect is not that straightforward.

Policy-maker has several possible solutions to overcome a negative secondary effect. One is to collect all the information she wants prior to the interaction with the lobbyist if it is possible. In this case there is no diverting attention motives for the lobbyist so she either reveals fully the issues where interests are aligned or reveals nothing if there is no such issues. Note that in this case policy-maker needs to either be able to commit to no ex-post information acquisition (by timing of the decision, for example) or to have full access to potential information without interaction with the lobbyist (so lobbyist's information doesn't change the policy-maker's abilities to process/acquire information).

However, it might be that information acquisition is not possible for the policy-maker before the interaction with the lobbyist. In this case she can commit to the direction of her private learning by specifying the type of research she will conduct/information source she will use (for instance, by a contract or a public agenda).

On the contrary, creating precise agenda for the communication with the lobbyist (that is fixing the issues/dimensions on which information will be provided) might not be effective, but kill lobbyist's incentives to provide high quality (precise) information.

6 Extensions

6.1 Single action

Until now the paper focused on the case when Receiver needs to take multiple decisions. While reasonable for some applications, in others it is more suitable to assume that Receiver needs to take one action based on the information about different dimensions of the state of the world. The examples include, for instance, an optimal investment in a project with multiple (unknown) features, an optimal bonus for an employer, an optimal consumption of a product and so on. To model these problems assume that the payoff of Sender is given in the following way:

$$u_R(a,\theta) = -(a - \gamma_R \cdot \theta)^2 \tag{13}$$

with $\|\gamma_R\| = 1$. That is Receiver believes that the optimal action is matching the state with the vector of weights γ. Assume that the objective of Sender is similar but she has a different view on the weights which should be assigned to different dimensions of the state of the world:

$$u_S(a,\theta) = -(a - \gamma_S \cdot \theta)^2$$

with $\|\gamma_S\| = 1$. In this section I assume that Σ is diagonal, i.e. the dimensions are uncorrelated.

Note that if Receiver has access to any signal which can be a linear combination of the dimensions, she always focuses on observing $\gamma_R \cdot \theta$, thus, no diverting of attention is possible. However, this logic does not hold anymore if the available signals are on one of the two dimensions but no mixtures are allowed.

Given the diagonal prior beliefs and the payoff function specified above, the pair of ex-ante expected payoffs can be written as follows:

$$\mathbb{E}u_R = -\gamma_{R_1}^2 \widetilde{v}_1 - \gamma_{R_2}^2 \widetilde{v}_2 \tag{14}$$

$$\mathbb{E}u_S = -(2\gamma_{S_1}\gamma_{R_1} - \gamma_{R_1}^2)\widetilde{v}_1 - (2\gamma_{S_2}\gamma_{R_2} - \gamma_{R_2}^2)\widetilde{v}_2.$$

I keep the assumption of $v_1 > v_2$ w.l.o.g. Then the interests of Sender and Receiver are partially aligned with the misalignment on the more uncertain dimension if Sender assigns sufficiently more weight to the dimension 2 in determining the correct action.

Proposition 5. *Under partial misalignment of interests and unique action for Receiver, Sender diverts Receiver's attention if:*

- *In the absence of Sender's information Receiver learns dimension 1, if the following condition is satisfied:*

$$\gamma_{R_1}^2\left(v_1 - \frac{\lambda}{2}\right) > \gamma_{R_2}^2\left(v_2 - \frac{\lambda}{2}\right), \tag{15}$$

- *diverting attention generates higher payoff for Sender compared to fully revealing dimension of alignment of interests.*

Sender provides no information if:

- *(15) does not hold, and*

– *no information provision generates higher payoff for Sender compared to fully revealing dimension of alignment of interests.*

In these cases Receiver faces the negative value of information.

The formal proof and the similar statement for the convex costs is left for the Appendix.

Thus diverting attention motives are present even in the case when Receiver has only one decision to make if the signal space is restricted to include only non-mixing signals. Similar result holds in the case of fully misaligned interests (presented in the Appendix).

An important case is when Sender just wants to maximize the action of Receiver, while Receiver holds the same preference as in (13). This is the case of fully misaligned interests with respect to posterior uncertainties. For instance, if Sender's payoff is given by a^2, the expected payoff is:

$$\mathbb{E}u_S = \gamma_{R_1}^2 \widetilde{v}_1 + \gamma_{R_2}^2 \widetilde{v}_2.$$

That is, the incentives of Sender and Receiver are opposing (0-sum game). In this case Sender might prefer to reveal some information if either Receiver learns dimension 1 in the absence of Sender and Receiver assigns sufficiently high relative weight to dimension 1 ($\gamma_{R_1}^2/\gamma_{R_2}^2$), or if Receiver learns dimension 2 in the absence of Sender and Receiver assigns sufficiently high relative weight to dimension 2 ($\gamma_{R_2}^2/\gamma_{R_1}^2$). Thus, information provision is possible even in the case of opposing interests. Moreover, given that the payoffs are as in a 0-sum game, Receiver would prefer no information provision from Sender.

The last example can be a good description of an interaction in a job hiring process, where the candidate provides portfolio first, and then the firm decides which abilities to test further. In this case, the firm might prefer to announce ex-ante which type of test it wants to provide, to discourage strategic information provision by the candidate.

6.2 Budget constraint on Receiver's private information acquisition

So far the paper assumed that Receiver has access to a unique signal while Sender can commit to send any number of linear signals. In this section this assumption is replaced by a budget constraint: Receiver can observe any number of costly linear signals, but the total costs of obtaining this information cannot exceed a certain exogenous threshold. For clarity we assume in this section that dimensions are not correlated, that is $\rho = 0$.

Consider entropy costs of information acquisition. The budget constraint takes then the following form: for any collection of signals $S_R = (S_{R_1}, S_{R_2}, \ldots, S_{R_k})$

$$-\sum_{i=1}^{k} \log \frac{v_k'}{v_k} \leq C \tag{16}$$

where C is some constant and v_k' and v_k are the posterior and prior uncertainties for signal k correspondingly.

Receiver problem than writes:

$$\max_{S_R} -\widetilde{v}_1 - \widetilde{v}_2$$

$$\text{s.t. } (16).$$

Consider some interim beliefs of Receiver $(\check{v}_1, \check{v}_2)$ and denote $i = arg\max_{i\in\{1,2\}} \check{v}_i$ and by j the remaining dimension. The optimal strategy for Receiver given interim beliefs $(\check{v}_1, \check{v}_2)$ is then:

– if $\log \frac{v_j}{v_i} \leq C$ Receiver chooses posterior beliefs such that $\widetilde{v}_1 = \widetilde{v}_2 = v$ with

$$\log \frac{\widetilde{v}_1}{\check{v}_1} + \log \frac{\widetilde{v}_2}{\check{v}_2} = C; \tag{17}$$

– if $\log \frac{v_j}{v_i} > C$ Receiver observes dimension i (more uncertain) with

$$\log \frac{\widetilde{v}_i}{\check{v}_i} = C$$

setting posterior beliefs to $(\widetilde{v}_i, v_j)$.

Thus Receiver equalizes the uncertainty on the dimensions if possible (making it as small as costs allow), and reduces the uncertainty on the most uncertain dimension otherwise.

Assume a partial conflict of interests, that is $V_1 < 0$, $V_2 > 0$. In this case there are 2 types of solution available for Sender:

– To reveal no information;

– To reveal fully dimension 2 so that Receiver learns dimension 1.

The first solution generates the expected payoff for Sender of $v(-V_1 - V_2)$ if $\log(v_2/v_1) \leq C$ with v given by (17) and of $-V_1\widetilde{v}_1(v_1) - V_2 v_2$ otherwise. The second solution generates the payoff of $-V_1\widetilde{v}_1(v_1)$. If $-V_1(v - \widetilde{v}(v_1)) < V_2 v$ Sender prefers to send no information to let Receiver obtain all the information on her own. This result generalizes for any convex precision-dependent cost function.

Proposition 6. *If the interests of Sender and Receiver are partially aligned with misalignment on the more uncertain dimension and*

$$-V_1(v - \widetilde{v}(v_1)) > V_2 v$$

with v given by (17), Sender provides no information to Receiver.

It follows that with a budget-constrained Receiver Sender cannot benefit from diverting attention. Instead, she prefers to provide no information even if the interests are partially aligned. The intuition for this result lies in the fact that Receiver always tries to smooth the uncertainty. Thus she is not facing a trade-off of what to observe, so diverting attention becomes impossible.

7 Conclusion

Information provision from an informed to an uninformed party is a part of almost all economic interactions. There are multiple possible reasons to reveal information: improving the quality of the decisions made, reputation concerns, etc. In this paper I uncover a new role for information provision: to divert the attention of the receiving side from unfavorable issues.

For these purposes, I consider a multidimensional Sender-Receiver framework with commitment in which Sender provides information to Receiver, and the latter may extract some additional information afterwards. In such setting information from Sender has two effects: the standard *persuasion* effect and the effect of *directing the subsequent search* for information.

I show that different reasons for information provision dominate depending on the conflict of interests and Receiver's cost of information acquisition. Whenever interests are partially aligned or fully misaligned Sender might prefer to reveal some information on the dimension where interests are misaligned (stronger misaligned) to divert Receiver's attention away from this dimension and force Receiver to search information on the other dimension. One of the main reasons for such counter intuitive strategy lies in the Receiver's learning dynamics: given any beliefs, it is optimal for her to obtain information on the dimension of maximal uncertainty. Given that, in case the interests are strongly misaligned on the initially more uncertain dimension, Sender prefers to give some information about it so that Receiver would seek information on the other dimension.

Moreover, if Receiver bears costs of information acquisition, in the case described above she faces negative value of information: whenever the costs are sufficiently low Receiver prefers to have the costs increased.

The theoretical results obtained in the paper shed new light on many economic situations. One set of questions to which the results are of a particular interest is of a consumer choice of optimal bundle. While the literature demonstrates how the consumption patterns are affected due to limited attention of consumer in the presence of taste shocks (see Kőszegi and Matějka [2020]), I show the effect which the presence of strategic Sender of information (advertiser or producer) has on these patterns. Among others I show that "mental budgeting" is much less likely to happen if Sender wants to incentivize as high spendings as possible while the "naive diversification" persists.

The framework is natural for the studies of the environments which involve expert advice. In particular, it is applicable to the case of informational lobbying as it often includes multidimensionality and the sequential information acquisition structure (see, for instance, Cotton and Dellis [2016] and Ellis and Groll [2020]) For this case my paper provides a new perspective relative to the literature: while most of the existing papers are concerned with whether informational lobbying is detrimental for the quality of the decision making, I study the optimal lobbyist's and policy-maker's behavior in the settings where the lobbying has on average a positive effect. The applicability of the results is not restricted to this particular environment. One can think of consultants for government bodies, financial advice, hiring processes, etc. This is particularly the case taking into account the extension of the framework which allows Receiver to take a single action based on the combination of the features (dimensions).

By uncovering new motives for information provision, this paper opens the door to a wide range of follow-up questions. On the technical side, an important issue is determining the conditions under which Receiver is willing to learn a more uncertain dimension which is the building bloc for the optimality of attention diversion for Sender. Another important exercise is to extend the framework beyond the continuous world and normal distributions. From the applied perspective, the use of the framework for consumer choice problem allows to study the simultaneous decision of firms on the pricing and information policy. I leave these questions for future research.

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A Proof of Theorem 1

Consider first Receiver who is facing entropy costs of information acquisition. As was shown Sender's problem can be decomposed in finding a rotation matrix U_S and beliefs $\check{\Sigma}$ such that $U_S \check{\Sigma} U_S^T$ is diagonal. For each given rotation U_S the set of feasible interim beliefs (that is such that $\Sigma - \check{\Sigma} \succeq 0$) is given by the following inequality:

$$\check{v}_2^{U_S} \leq \frac{\check{v}_1^{U_S}(v_1 b^2 - 2ab\rho\sqrt{v_1 v_2} + v_2 a^2) - v_1 v_2 (1-\rho^2)}{\check{v}_1^{U_S} - v_1 a^2 - 2ab\rho\sqrt{v_1 v_2} - v_2 b^2} = \phi^{U_S}(\check{v}_1^{U_S}) \tag{18}$$

where a, b are the components of U_S:

$$U_S = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$$

with $a^2 + b^2 = 1$. Note, that (18) takes into account the fact that there is no correlation between the dimensions of the basis $U_S^T U_S$. Note also that for each rotation the boundary on the RHS of (18) includes the point $(v_1^{U_S} = v^*, v_2^{U_S} = v^*)$ with v^* given by (11).

With respect to the basis $U_S^T U_S$ the expected payoff of Sender can be formulated as:

$$\mathbb{E}u_S = \text{const} - \widetilde{v}_1^{U_S}\left(V_1 a^2 + V_2 b^2\right) - \widetilde{v}_2^{U_S}\left(V_1 b^2 + V_2 a^2\right) = \text{const} - \widetilde{v}_1^{U_S} V_1^{U_S} - \widetilde{v}_2^{U_S} V_2^{U_S} \tag{19}$$

with $V_1^{U_S} \equiv V_1 a^2 + V_2 b^2$, $V_2^{U_S} \equiv V_1 b^2 + V_2 a^2$. Then there are 4 possible cases.

Case 1: $V_1^{U_S} > 0$, $V_2^{U_S} > 0$. The solution is then full revelation of 1^{U_S} and 2^{U_2} leading to deterministic 0 payoff.

Case 2: $V_1^{U_S} < 0$, $V_2^{U_S} > 0$. Thus, Sender wants to induce the highest possible uncertainty in dimension 1^U and the lowest possible in dimension 2^U.

There are 2 types of potential solution (in terms of $\widetilde{v}_1^{U_S}$ and $\widetilde{v}_2^{U_S}$) in such case:

- To fully reveal dimension 2^{U_S} without revealing any information on dimension 1^{U_S} so that Receiver learns dimension 1^{U_S} herself. This leads to posterior beliefs $(\widetilde{v}_1^{U_S}, \widetilde{v}_2^{U_S}) = (\lambda/2, 0)$ if $v_1^{U_S} \geq \lambda/2$ and $(\widetilde{v}_1^{U_S}, \widetilde{v}_2^{U_S}) = (v_1^{U_S}, 0)$ if $v_1^{U_S} < \lambda/2$. Then this type of solution generates the highest payoff for $U_S = I$ which satisfies the condition of $V_1^{U_S} < 0$, $V_2^{U_S} > 0$. Indeed $V_1^{U_S}$ is minimized at $U_S = I$ while $V_2^{U_S}$ is maximized at this point.

- To reveal some information on dimension 1^{U_S} to make it less uncertain and force Receiver to learn dimension 2^{U_S}. Then the posterior beliefs are $(\widetilde{v}_1^{U_S}, \widetilde{v}_2^{U_S}) = (v^*, \lambda/2)$. Thus, vector $(\widetilde{v}_1^{U_S}, \widetilde{v}_2^{U_S})$ is independent of U_S, so the rotation U_S enters Sender's utility only through the coefficients $V_1^{U_S}$ and $V_2^{U_S}$. Then the maximum is attained for $U_S = I$.

Case 3: $V_1^{U_S} > 0$, $V_2^{U_S} < 0$. Note that this case is fully symmetric to Case 2 and the same solutions apply with U_S being 90 degrees rotation matrix which corresponds to the renaming of the axis.

Case 4: $V_1^{U_S} < 0$, $V_2^{U_S} < 0$.

There are 2 candidate solutions (in terms of posterior uncertainties): $(\lambda/2, v_2^{U_S})$ and $(v_2^{U_S}, \lambda/2)$. It follows from Cases 2 and 3 that the utility of Sender from these 2 solutions is maximized when $U_S = I$ which contradicts $V_1^{U_S} < 0$, $V_2^{U_S} < 0$. Thus, there is no solution in this region.

In result, one can conclude that for entropy costs of information acquisition it is always possible for Sender to remove correlation from Receiver's belief, that is, $\check{\rho} = 0$.

Then the two pairs of equilibrium beliefs which can be achieved in equilibrium are $(\widetilde{v}_1^{U_S}, \widetilde{v}_2^{U_S}) = (\lambda/2, 0)$ and $(\widetilde{v}_1^{U_S}, \widetilde{v}_2^{U_S}) = (v^*, \lambda/2)$ if $\lambda/2 < v^*$ and only one pair $(\widetilde{v}_1^{U_S}, \widetilde{v}_2^{U_S}) = (\lambda/2, 0)$ otherwise. Then it is easy to see that Sender prefers the diverting attention solution if condition 12 is violated.

B Proof of Theorem 2, Lemma 2

Consider now a convex precision-dependent costs. First of all notice that $V_1 < 0$, $V_2 > 0$ and the definition of the rotation such that dimension $1^{U'}$ is the one with the highest uncertainty, $V_1^U < 0$.

Following the same logic as in the proof of Theorem 1 for each possible rotation U_S chosen by Sender equation (19) holds. There are then 2 different cases: $|V_1| < V_2$, that is, relative conflict of interests on dimension 1 is smaller than the agreement on dimension 2; and $|V_1| > V_2$, that is, relative conflict of interests on dimension 1 is bigger than the agreement on dimension 2.

Case 1: $|V_1| < V_2$. That means that for any U_S either $V_1^{U_S} < 0$ and $V_2^{U_S} > 0$, or $V_1^{U_S} > 0$ and $V_2^{U_S} > 0$, or $V_1^{U_S} > 0$ and $V_2^{U_S} < 0$. Figure 13 illustrates this statement.

First step is to show that in equilibrium $\operatorname{sgn} b = \operatorname{sgn} \rho$ - that is Sender chooses rotation U_S in the direction of maximal uncertainty and not away from it. In other words, Sender does not change the sign of the correlation between the dimensions for Receiver. Notice first, that optimal U_S cannot be such that $V_1^{U_S} > 0$ and $V_2^{U_S} > 0$. Indeed, the maximal payoff Sender can achieve in this case is 0 while strictly positive payoffs are attainable for other choices of U_S.

Assume now that Sender optimally sets some U_S such that $V_1^{U_S} < 0$ and $V_2^{U_S} > 0$. In this case Sender chooses one of the 2 solutions (in terms of interim beliefs induced): either $\check{v}_1^{U_S} = (\phi^{U_S})^{-1}(0)$, $v_2^{U_S} = 0$ or $v_1^{U_S} = v_2^{U_S} = v^*$. Note that v^* does not depend on the choice of the U_S, thus, the highest payoff for solution the second solution is achieved for $U_S = I$ (it follows from (19)). Thus, if Sender optimally chooses $U_S \neq I$ such that $V_1^{U_S} < 0$ and $V_2^{U_S} > 0$ she implements the posterior beliefs

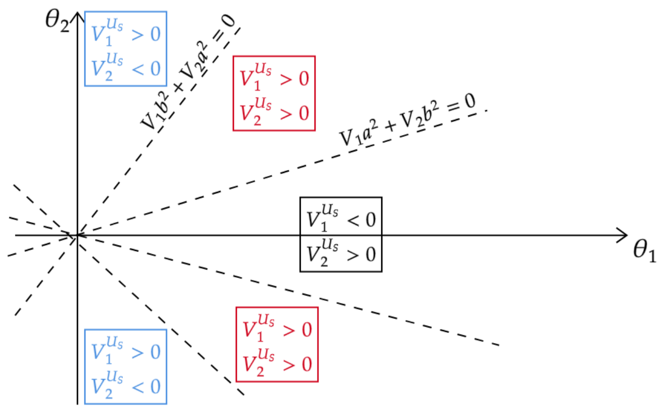


Figure 13: Coefficient sign as a function of rotation U_S

$v_1^{U_S} = (\phi^{U_S})^{-1}(0)$, $v_2^{U_S} = 0$.

Assume now that $\operatorname{sgn} b \neq \operatorname{sgn} \rho$ for the optimal U_S and the optimal interim beliefs are of the type $\check{v}_1^{U_S} = (\phi^{U_S})^{-1}(0)$, $v_2^{U_S} = 0$. Thus, there exists a symmetric U_S' with an entry $b' = -b$ such that $V_1^{U_S} = V_1^{U_S'}$ and $V_2^{U_S} = V_2^{U_S'}$. Then it follows from (18) that $(\phi^{U_S})^{-1}(0) < (\phi^{U_S'})^{-1}(0)$ leading also to a higher posterior belief $\widetilde{v}((\phi^{U_S'})^{-1}(0)) > \widetilde{v}((\phi^{U_S})^{-1}(0))$. Thus U_S does not maximize Sender's payoff contradicting the assumption that it is the optimal choice of the rotation.

For the remaining case, assume that Sender optimally sets some U_S such that $V_1^{U_S} > 0$ and $V_2^{U_S} < 0$. By definition, rotation U_S is such that dimension 1^{U_S} is the dimension of the higher uncertainty (relative to 2^{U_S}). Then there are 2 possible solutions for Sender for each fixed U_S: to induce interim beliefs $v_1^{U_S} = v_2^{U_S} = v^*$ or $v_1^{U_S} = 0$, $v_2^{U_S} = v^*$. Note that both solutions do not depend on the sign of the b, that is, on the sign of induced correlation. Moreover, both solutions generate the highest payoff whenever $b^2 = 1$ (that is U_S is 90 degree rotation).

Thus, restricting attention to the cases when $\operatorname{sgn} b = \operatorname{sgn} \rho$ is without loss of generality.

In the next step consider all possible candidate equilibrium solutions for Sender discussed above:

– **Solution 1:** U_S rotates by 90 degrees setting the interim uncertainties in the new basis to $(0, v^*)$. Generated payoff: $-V_1 \frac{v^* \sigma^2}{v^* + \sigma^2}$;

– **Solution 2:** U_S rotates by 90 degrees setting the interim uncertainties in the new basis to (v^*, v^*). Generated payoff: $-V_2 \frac{v^* \sigma^2}{v^* + \sigma^2} - V_1 v^*$;

– **Solution 3:** Setting $U_S = I$ with the interim uncertainties (v^*, v^*). Generated payoff: $-V_1 v^* - V_2 \frac{v^* \sigma^2}{v^* + \sigma^2}$;

– **Solution 4:** Setting U_S to $arg\max_{U'} -V_1^{U'}(\phi^{U'})^{-1}(0)\sigma^2/((\phi^{U'})^{-1}(0)$ setting the interim beliefs to $((\phi^{U'})^{-1}(0), 0)$.

Note that Solutions 2 and 3 are identical. Also Solution 1 is dominated by Solution 4 for Sender: Putting $U_S = I$ in Solution 4 generates a higher payoff for Sender than Solution 1. Thus, the optimal solution is given by U_S such that $V_1^{U_S} < 0$, $V_1^{U_S} > 0$.

The next step is to show that optimal U_S is strictly in between U (the dimension of maximal uncertainty) and I. By contradiction, assume that the optimal rotation is not in this region and U_S rotates beliefs of Receiver away from $U = I$. By previous argument, it is still in the region such that $V_1^{U_S} < 0$, $V_2^{U_S} > 0$ (note that it means that $V_1^{U} < 0$, $V_2^{U} > 0$). Consider now a rotation U_S' which is symmetric to U_S with respect to U. Such rotation leads to the same attainable set of pairs of $(\check{v}_1^{U_S'}, \check{v}_2^{U_S'})$ as for U_S. Thus, the optimal interim beliefs given U_S are the same - that is to induce $((\phi^{U_S'})^{-1}(0)), 0)$. Note, however, that $|V_1^{U_S'}| > |V_1^{U_S}|$ leading to a higher payoff for Sender.

Solution 3 is obtained in the following way: to reveal dimension 1^U until $v_1^U = v^*$. Thus, if $V_1^U < 0$ Sender finds it optimal to reveal the dimension where interests are misaligned according to Definition 2.

For Solution 4, however, Sender fully reveals dimension 2^{U_S} with $U_S = arg\max_{U'} -V_1^{U'}\widetilde{v}((\phi^{U'})^{-1}(0))$. As was proven before, payoff-maximizing U_S is such that $V_1^{U_S} < 0$, $V_2^{U_S} > 0$, thus, information is provided on the dimension where interests are aligned.

Case 2: Assume now $|V_1| > V_2$, that is, the disagreement on the dimension 1 is stronger than the agreement on the dimension 2. That is there are 3 zones for the U_S (illustrated on Figure 14): such that $V_1^{U_S} < 0$, $V_2^{U_S} > 0$; such that $V_1^{U_S} < 0$, $V_2^{U_S} < 0$ and that $V_1^{U_S} > 0$, $V_2^{U_S} < 0$. Again it is never strictly beneficial for Sender to change the sign of the correlation between the dimensions. For rotations U_S such that $V_1^{U_S} V_2^{U_S} < 0$ the same argument as in Case 1 applies. I now show that it also applies for the remaining case of $V_1^{U_S} < 0$, $V_2^{U_S} < 0$.

Consider some U_S such that $V_1^{U_S} < 0$, $V_2^{U_S} < 0$ and $\operatorname{sgn} b = \operatorname{sgn} \rho$ and another U_S' which is symmetric to U_S, that is $a' = a$, $b' = -b$. That leads to $V_1^{U_S'} = V_1^{U_S}$, $V_2^{U_S'} = V_2^{U_S}$. Note that by assumption $\check{v}_1^{U_S} > \check{v}_2^{U_S}$ for any U_S (thus, it is also true for U_S'). Also note that $v_1^{U_S} > v_1^{U_S'}$ (and $v_2^{U_S} < v_2^{U_S'}$). Thus for each $\check{v}_2^{U_S'} = \check{v}_2^{U_S}$ Sender can induce as an interim belief, $(\phi^{U_S'})^{-1}(\check{v}_2^{U_S'}) \leq (\phi^{U_S})^{-1}(\check{v}_2^{U_S})$. Together with the convexity of the costs function it means that the set of attainable posterior beliefs at U_S' is a subset of the corresponding set at U_S. Thus, Sender cannot obtain strictly higher payoff by setting U_S' with $\operatorname{sgn} b' \neq \operatorname{sgn} \rho$ allowing to focus on the solutions in which Sender induces interim correlation of the same sign as in the prior beliefs.

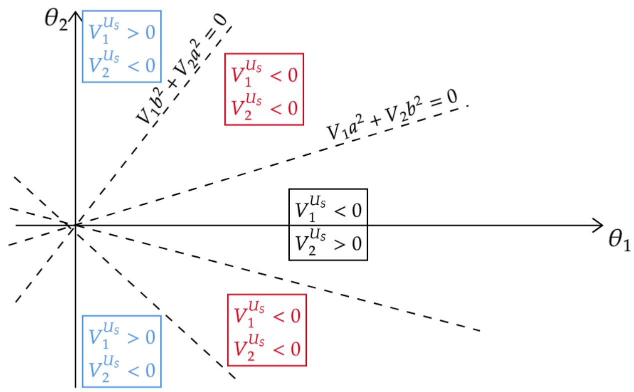
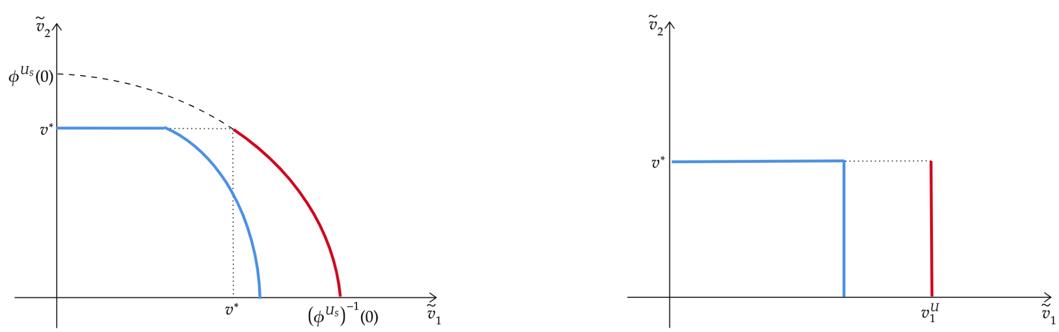


Figure 14: Coefficient sign as a function of rotation U_S, $|V_1| > V_2$

For the same reason as in Case 1, among the U_S such that $V_1^{U_S} V_2^{U_S} < 0$ there are only 2 candidate solutions: either Sender diverts attention by revealing dimension 1^U until v^*, or intuitively reveals dimension 2.

Hence, the remaining case with a potential solution(s) is to set U_S such that $V_1^{U_S} < 0$ and $V_2^{U_2} < 0$. Figures 15a-15b show the set of attainable beliefs taking into account the fact that $\check{v}_1^{U_S} > \check{v}_2^{U_S}$. Figure 15a presents the case when $U_S \neq I$, and Figure 15b - the case when $U_S = I$, that is the beliefs are not rotated.



(a) $U_S \neq I$

(b) $U_S = I$

Figure 15: Attainable posteriors for U_S such that $V_1^{U_S} < 0, V_2^{U_2} < 0$

Since in this region $V^{U_S} < 0$ and $V_2^{U_S} < 0$, the best pair of posteriors for Sender for each fixed U_S belongs to the frontier (in blue on Figure 15). Note first that for any U_S, any solution $(\check{v}_1^{U_S}, \phi(\check{v}_1^{U_S}))$ such that $\tilde{v}(\check{v}_1^{U_S}) < v^*$ is dominated by the diverting attention solution. Indeed, Sender's payoff from inducing interim beliefs $(\check{v}_1^{U_S}, \phi(\check{v}_1^{U_S}))$ generates the payoff of:

$$-V_1^{U_S}\widetilde{v}(\check{v}_1^{U_S})-V_2^{U_S}\phi(\check{v}_1^{U_S})<-V_1^{U_S}v^*-V_2^{U_S}v^*=(V_1-V_1^{U_S}-V_2^{U_S})v^*-V_1v^*= \\ -V_1v^*-V_2v^*<-V_1v^*-V_2\widetilde{v}(v^*)$$

where the last term is Sender's payoff from the diverting attention solution. The first equality uses the fact that $V_1^{U_S}+V_2^{U_S}=V_1+V_2$ for any U_S.

Moreover, Solution 3 exists only if U is such that $V_1^U<0$ and $V_2^U<0$ that is correlation between the dimensions is sufficiently high or $|V_1|>>V_2$. Indeed, consider some U_S such that $V_1^{U_S}<0$ and $V_2^{U_S}<0$ while $V_1^U<0$ and $V_2^U>0$, and assume that some pair of posteriors $(\widetilde{v}_1^{U_S},\widetilde{v}_2^{U_S})$ maximizes Sender's payoff for given U_S. Now consider some other U_S' such that $V_1^{U_S'}<0$, $V_2^{U_S'}<0$ and $V_1^{U_S'}<V_1^{U_S}$. Note that the pair of posteriors $(\widetilde{v}_1^{U_S},\widetilde{v}_2^{U_S})$ is attainable under U_S' since the set of attainable posterior beliefs grows. Indeed, $\left(\phi^{U_S'}\right)^{-1}(0)>\left(\phi^{U_S}\right)^{-1}(0)$. Note moreover that $V_1^{U_S}+V_2^{U_S}=V_1^{U_S'}+V_2^{U_S'}=V_1+V_2$. Thus, $V_1^{U_S}-V_1^{U_S'}=V_2^{U_S'}-V_2^{U_S}=\Delta>0$. Then Sender's payoff from choosing the posterior uncertainties $(\widetilde{v}_1^{U_S},\widetilde{v}_2^{U_S})$ under U_S' is:

$$-V_1^{U_S'}\widetilde{v}_1^{U_S}-V_2^{U_S'}\widetilde{v}_2^{U_S}=-(V_1^{U_S}-\Delta)\widetilde{v}_1^{U_S}-(V_2^{U_S'}+\Delta)\widetilde{v}_2^{U_S}= \\ -V_1^{U_S}\widetilde{v}_1^{U_S}-V_2^{U_S}\widetilde{v}_2^{U_S}+\Delta(\widetilde{v}_1^{U_S}-\widetilde{v}_2^{U_S}) \quad (20)$$

Note that according to the previous argument, U_S can be optimal only if $\widetilde{v}_1^{U_S}>v^*$. Thus, $\widetilde{v}_1^{U_S}-\widetilde{v}_2^{U_S}>0$. Then it follows from (20) that U_S' generates strictly higher payoff for Sender.

Moreover, the optimal U_S can only be in between U and I. For the U_S such that $V_1^{U_S}<0$ and $V_2^{U_S}<0$ and $V_1^{U_S}>V_1^U$ any solution is dominated by the non revealing one. Indeed, non-revealing solution generates payoff of:

$$-V_1^U\widetilde{v}(v_1^U)-V_2^Uv^*$$

Now consider any other solution U_S with $V_1^{U_S}>V_1^U$ and optimal posteriors $(v_1^{U_S},v_2^{U_S})$. The payoff is:

$$-(V_1^U+\Delta)\widetilde{v}(v_1^{U_S})-(V_2^U-\Delta)v_2^{U_S}\leq-V_1^U\widetilde{v}(v_1^{U_S})-V_2^Uv^*+\Delta(v_2^{U_S}-\widetilde{v}(v_1^{U_S}))< \\ -V_1^U\widetilde{v}(v_1^{U_S})-V_2^Uv^*<-V_1^U\widetilde{v}(v_1^U)-V_2^Uv^*$$

The second to last inequality comes from the fact that otherwise solution U_S is dominated by the diverting attention one if $v_2^{U_S}-\widetilde{v}(v_1^{U_S})>0$.

Then there are 3 possible solutions:

– **Solution 1:** Setting $U_S = I$ with the interim uncertainties (v^*, v^*). Generated payoff: $-V_1 v^* - V_2 \frac{v^* \sigma^2}{v^* + \sigma^2}$;

– **Solution 2:** Setting U_S to $arg\max_{U'} -V_1^{U'}(\phi^{U'})^{-1}(0)\sigma^2/((\phi^{U'})^{-1}(0)$ setting the interim beliefs to $((\phi^{U'})^{-1}(0), 0)$ (optimal U_S such that $V_1^{U_S} < 0$, $V_2^{U_S} > 0$);

– **Solution 3:** Setting U_S to $arg\max_{U',v} -V_1^{U'}(\phi^{U'})^{-1}(v)\sigma^2/((\phi^{U'})^{-1}(v) - V_2 v$ with interim beliefs $(v, \phi^{U_S})^{-1}(v))$ (optimal U_S such that $V_1^{U_S} < 0$, $V_2^{U_S} < 0$).

As was discussed for Case 1, Solution 1 is the diverting attention Solution while Solution 2 is the one in which Sender fully reveals a dimension on which interests are aligned.

Finally, note that if $\widetilde{v}(v_1^U) < v^*$ then for any $v' < \widetilde{v}(v_1^U)$, $\widetilde{v'} < v^*$ where v_1^U is the uncertainty on the dimension of maximal uncertainty. This happens if costs are sufficiently low. In this case Solution 1 dominates any other solution.

Moreover, for sufficiently low of information acquisition there Solution 2 is also dominated by the diverting attention Solution 1. Indeed the payoff from Solution 1 (assuming some optimal U_S') is $-V_1^{U_S'}\widetilde{v}(v_1^{U_S})$ and it converges to 0 as $\lambda \to 0$ (the cost parameter).

The payoff from the diverting attention solution is given by $-V_1 v^* - V_2 \widetilde{v}(v^*)$, thus, it is bounded away from 0 by $-V_1 v^*$ which completes the proof.

C Proof of Proposition 5

Notice first that since Sender can induce any pair of interim beliefs such that $\check{v}_1 \leq v_1$ and $\check{v}_2 < v_2$ she cannot benefit from inducing correlations for Receiver. Thus, it is possible to focus on diagonal interim beliefs.

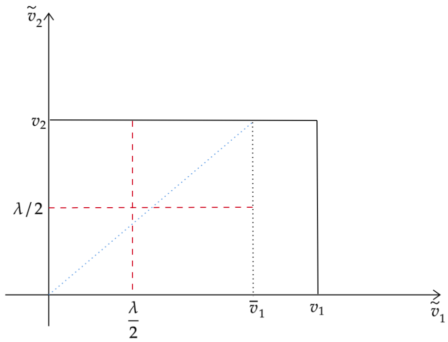
Condition (15) follows directly from the problem (14) of Receiver. Note that v_2 such that (15) is an equality is an increasing linear function of v_1.

Then, Figure 16a represents the set of attainable posteriors if condition (15) is satisfied and Figure 16b - when it is not satisfied. Blue dotted ligne shows the constraint itself. In Figure 16a $\bar{v}_1 = (v_2 - \lambda/2)\gamma_{R_2}^2/\gamma_{R_1}^2 + \lambda/2$. In Figure 16b $\bar{v}_2 = (v_1 - \lambda/2)\gamma_{R_1}^2/\gamma_{R_2}^2 + \lambda/2$

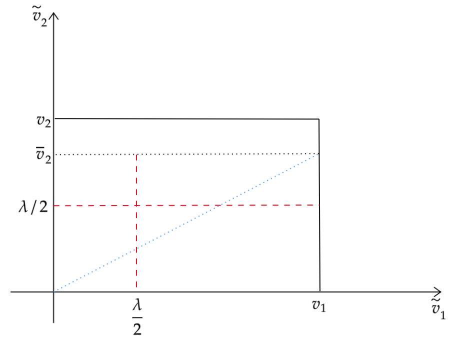
The solution then is similar to the one presented in Section 3.2. Note, however, that in the case of condition (15) being not satisfied, that is if in the absence of Sender's information Receiver learns dimension 2, there are 2 possible equilibrium solutions: intuitive disclosure of the dimension of alignment and no disclosure at all. That is, no diverting attention solution is present.

For convex precision-dependent costs, condition (15) rewrites as:

$$\gamma_{R_1}^2 (v_1 - \widetilde{v}(v_1)) > \gamma_{R_2}^2 (v_2 - \widetilde{v}(v_2))$$



(a) (15) satisfied



(b) (15) not satisfied

Figure 16: Attainable posterior beliefs, single action for Receiver

Note that due to the convexity of the costs, boundary v_2 is still an increasing function of v_1. Moreover, it is never optimal for Receiver to learn a dimension fully. Then the same argument holds.