Price and non-price strategies for streaming media platforms in monopoly and competitive environments

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This paper presents a theoretical model for analyzing the optimal decisions for a streaming media platform, which offers two versions of the service to users: an advertising-based version and a premium version. In the baseline model, I find conditions for optimality of running both services or just one as well as the optimal subscription price for the premium version, which is higher the higher the average nuisance cost of advertising and the higher the attractiveness of users for advertisers.

The model is extended by including additional features of real-life platforms not analyzed in previous works to examine the effects on subscription fee and on premium version's demand. When the platform advertises its premium version in the free version both the subscription fee and the number of subscribers increase. When, in addition to advertising nuisance, the subscription decision depends on user's disposable income, the subscription fee is reduced, but this effect is mitigated if high income consumers are more appealing for advertisers, as the platform tries to turn some high-income subscribers into free users in order to increase advertising revenue.

Finally, competition among platforms is analyzed. When users singlehome, platforms avoid differentiation and choose the same media content but this product homogeneity does not prevent them from charging the monopoly price due to user heterogeneity in the nuisance suffered from advertising. When users are allowed to multihome (use both platforms) platforms reduce subscription fees and differentiate from each other to incentivize more users to use both platforms. In equilibrium, only free-version users multihome while premium-version users subscribe to one single platform.

Keywords: Streaming media, two-sided markets, network effects, pricing, freemium, monopoly, duopoly, multihoming.

JEL codes: D42, D43, L12, L13.

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1. Introduction

Streaming services are nowadays used by millions of users all over the world for watching videos and listening to music (among other uses) and is still a growing business. What defines streaming services (unlike downloads or digital purchases) is that the user has the right to use the content (depending on the subscription plan) but she never owns the files and, once the subscription is interrupted, the content may not be used any more.

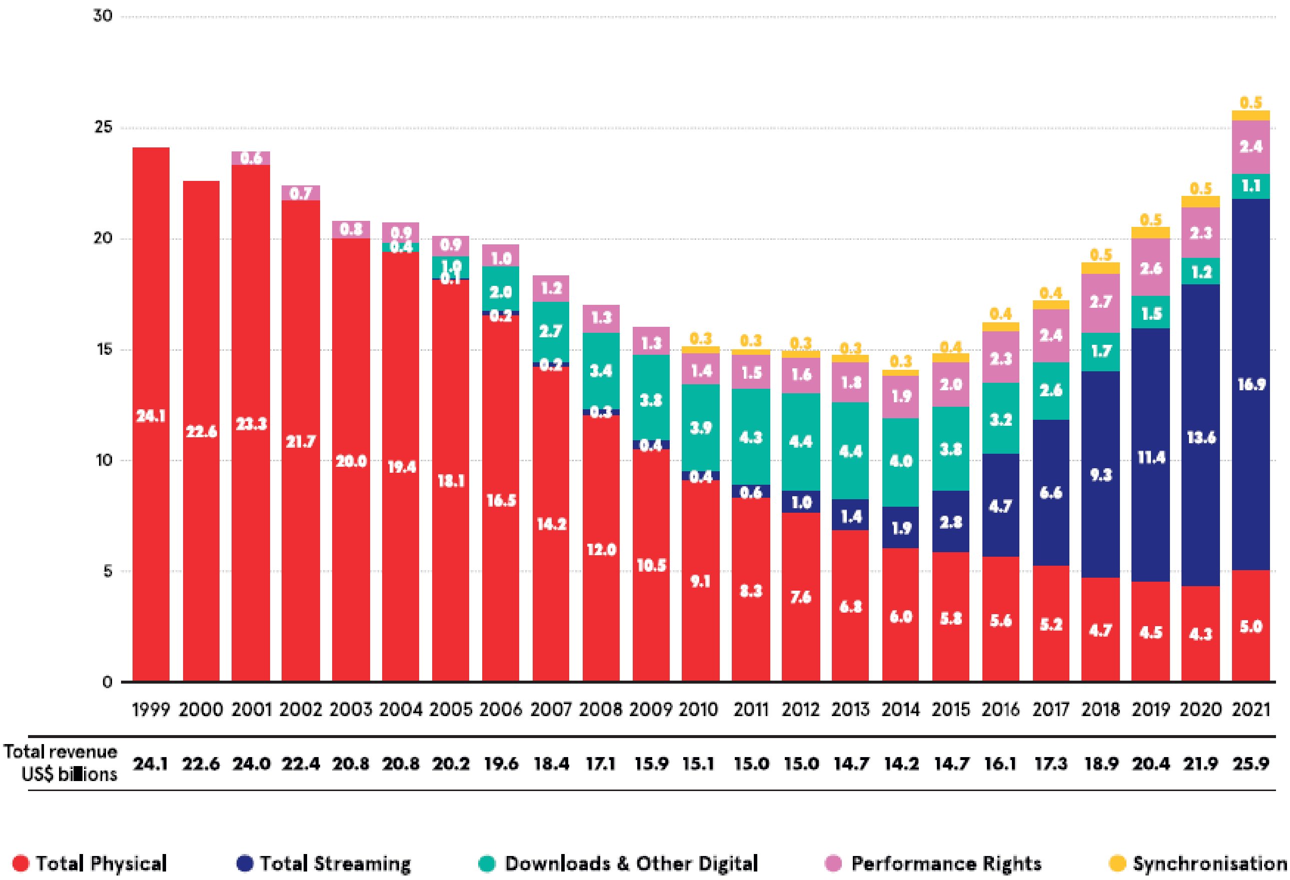

Figure 1: Overall recorded music industry revenue, by format. Period 1999-2021. Source: 2022 Global Music Report. International Federation of the Phonographic Industry (IFPI).

Figure 1, extracted from 2022 Global Music Report published by the International Federation of the Phonographic Industry (IFPI), shows the evolution of worldwide music industry revenues by format.[2] We can observe that music streaming became the

[2] Revenue from performance rights corresponds to the one which is obtained from the use of recorded music by broadcasters and public venues while the revenue from synchronization is obtained from the use of recorded music in advertising, film, games and TV (International Federation of the Phonographic Industry (2022a)).

dominant source of revenue for the industry in 2017 and its revenue is still increasing, leading to a recovery in the overall industry revenue, which had a falling trend for more than a decade.

Some of the music streaming services (like Spotify or Deezer) have a free version, which is fully financed by selling advertising content to firms, and a premium version, accessible only to those users who subscribe by paying a fee. This business model is typically called the ‘freemium’ business model (this term was coined by Anderson (2009)). Apart from not including advertising content, which most often reduces the utility of users, the premium version may also offer some extra features not available to free users related to recommendations, additional content, accessibility off-line, quality of sound or the possibility of connecting several devices simultaneously.

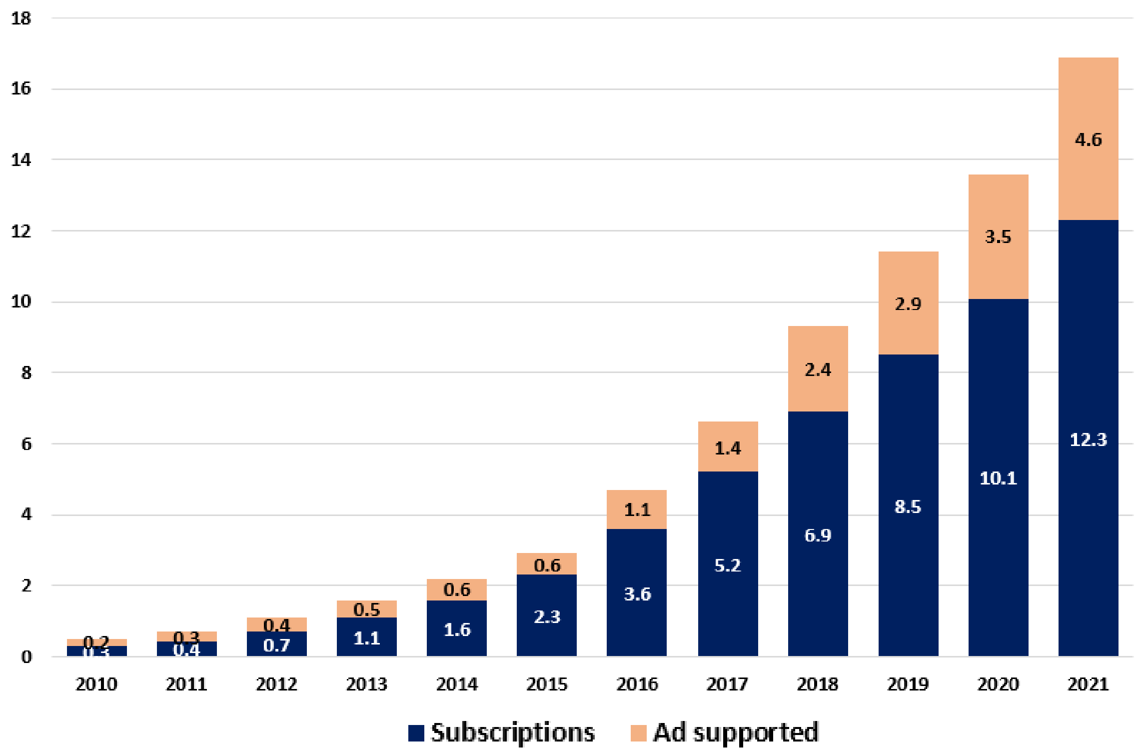

Figure 2: Worldwide music streaming revenue, by format. Period 2010-2021. Source: Statista, MIDiA Research and International Federation of the Phonographic Industry (IFPI).

As we can observe in Figure 2, there has been a constant growth in the last decade in both music streaming revenue from subscriptions and from advertising, with revenue from subscriptions representing around 75% of the overall streaming music revenue.

Streaming services fit the analysis of two-sided markets (Rochet and Tirole (2003, 2006), Caillud and Jullien (2003), Armstrong (2006) and Hagiu (2006)). This strand of literature typically analyzes the optimal strategies of platforms dealing with two groups of customers which exert cross-group network externalities on each other. There are some works which analyze the pricing decisions of streaming platforms (and the freemium business model) which mainly focus on either monopoly platforms or competing platforms with singlehoming users (Tag (2009), Thomes (2013), Sato (2019), Zennyo (2020)). While including the possibility of multihoming users complicate the analysis, it includes realism in terms of closeness to the real behavior of streaming music users. Figure 3 provides some evidence of the multihoming behavior of users by reporting values of share of users of a streaming platform who also use another platform.

	who also use	Spotify	Soundcloud	Apple Music
Share of users of	Spotify		42%	31%
	Soundcloud	29%		30%
	Apple Music	18%	20%	

Figure 3: Share of users of selected streaming music services who also use another selected service. Data corresponds to worldwide users in 2021. Source: GlobalWorldIndex.

In the present paper, I analyze, through a theoretical two-sided market model, the optimal decisions of a streaming platform. The focus is on both price and non-price strategies. Regarding price, I find conditions for the optimal subscription fee for the premium version. Additionally, the optimality of non-price strategies, such as the introduction of extra features to the premium version or the inclusion of advertisements of the premium version in the free version, is also analyzed. Advertising the premium version in the free version, a frequent practice in streaming media markets, may have a double purpose: first, to inform about the existence and features of the premium version; and also, to introduce additional nuisance in the free version in order to induce more subscriptions. These analysis are performed in the context of a monopoly platform, where I also analyze the case where the decision to subscribe depends on user's disposable income, in addition to advertising nuisance, an assumption which increases the closeness to reality. The results show that, when user's income is an issue in the decision to subscribe, the subscription fee is lower. In this context of user's income heterogeneity, I also analyze the case where the externality of users on advertisers depends on users' disposable income and I find that subscription fee is higher if high-income consumers are more appealing for advertisers than low-income users, mitigating the mentioned decrease in price caused by the influence of user's income on subscription decisions.

The present paper is the first one which analyzes the motivation and implications of self-advertising: the common practice in some of these platforms of advertising the premium version in the free version. According to International Federation of the Phonographic Industry (2022b), the most important reason for user's subscription decision is to avoid advertisements interrupting the music, followed by the "access to millions of songs" and the ability to "listen to what I want when I want". As mentioned above, self-advertising has a double purpose: first, to inform about the existence and features of the premium version and, also, to add an extra nuisance, which is most probably the most relevant driver of subscriptions and profit for the platform, as it is backed by the model's results, in detriment of both consumer surplus and social welfare.

Duopoly competition is also analyzed in this paper with two main innovations with respect to previous literature: first, I take as endogenous the platform's decision of streaming content by considering endogenous locations of platforms in the product characteristics' space; additionally, I analyze the model in two alternatives cases: with singlehoming users and with multihoming users, as it has been already considered by DeValve and Pekeč (2022) . The analysis in the present paper complements the one by DeValve and Pekeč by analyzing the platform's decision of differentiation from their rival, which is assumed as endogenous, and also includes a higher degree of consumer heterogeneity, by consider advertising nuisance following a continuous distribution (while it may only take two values in DeValve and Pekeč's analysis).

The equilibrium in the case of singlehoming users (which is solved for the symmetric case, with users of both platforms exerting the same degree of network externalities to advertisers) shows minimum differentiation, as platforms choose the same location in the streaming content space. Despite this lack of differentiation, user heterogeneity allows the platforms to charge the monopoly price, so each duopolist earns a profit equal to half the platform's profit in the monopoly case. When users may multihome, platforms differentiate from each other and they even choose maximum differentiation for many ranges of model's parameters. Multihoming takes place in the free version only while premium version users subscribe to one single platform.

The reminder of this paper is organized as follows: In Section 2; I present and discuss the related literature. In Section 3, I present the monopoly model, which is solved for two alternative scenarios: first, when the monopoly platform only runs the free version; and, alternatively, when, in addition to that version, it also runs a premium (advertising-free) version. In section 4, I consider some extensions to the base model related to supply (first three subsections) and demand (last two) sides. In Section 5, I analyze the duopoly model both in the case of singlehoming and multihoming users. Finally, Section 6 concludes and provides some discussion about.

2. Related literature

Streaming services fit the analysis of two-sided markets (Rochet and Tirole (2003, 2006), Caillud and Jullien (2003), Armstrong (2006) and Hagiu (2006)). This strand of literature typically analyzes the optimal strategies of platforms dealing with two groups of customers which exert cross-group network externalities on each other. As described by Armstrong (2006), a platform targets one group more aggressively than the other if that group "causes larger benefits to the other group than vice versa". When there is competition among platforms, the pricing structure is also affected by whether a group singlehomes or multihomes (i.e. whether a group's users may custom a single platform or more than one) and by the degree of differentiation across platforms. These issues of multihoming and platform differentiation and the effects on groups' decisions and surplus was formalized in Armstrong and Wright (2007), who analyzed the role of exclusive contracts in preventing multihoming. According to Hagiu (2009), the multihoming issue may induce further subsidization of one group in order for the platform to become dominant so it can induce members of the other group to leave the rival platform and singlehome on it.

In the case of streaming services, there are negative network externalities on the buyer side (as analyzed by Gal-Or and Dukes (2003), Anderson and Coate (2005), Peitz and Valletti (2008) and Reisinger (2012), among many others) as the user typically suffers a nuisance from the advertising included in the platform. This feature often leads to serving the users for free (or even at a negative price, if that is possible (Reisinger (2012)) obtaining all the profit from the seller side. The user is considered a loss leader, who needs to be attracted to the platform in order to gain advertising income.

Regarding the streaming media market and the freemium business model, there is a flourishing literature analyzing the conditions under which a platform increases profits by running both versions instead of one, the optimal price for each service and the effect of the business model on the level of advertising and welfare. Prasad et al. (2003) first pointed out that advertising may have two uses: first, as a source of revenues, and also, as a tool to segment the market between users with different degrees of ad aversion. These groups of users may, in turn, be of different value for advertisers, and this feature allows the platform to obtain a higher advertising income. The first work which analyzed the option to pay to reduce advertising is the one by Tag (2009), who defined the conditions for a platform to optimally introduce a fee-based version in addition to an advertising-based version. Those conditions were analyzed by Thomes (2013). In his work, the key variable for the optimal business model is the nuisance cost coefficient which, for low values, may lead the platform to offer the free service alone. Thomes' results point out that the socially optimal outcome does not correspond to freemium but to one single service (with either low or high quality) offered for free.

Sato (2019) and Lin (2020) analyze the freemium business model as a practice of second-degree price discrimination by platforms. In Sato (2019) the freemium business model is not imposed ex-ante but turns out to be optimal when the benefit of advertisers from their transaction with users is sufficiently large relative to the intrinsic value of platform's service.[3] Sato analyzed competition between streaming platforms for first time finding that platforms' equilibrium menu is freemium if the willingness to pay of advertisers is sufficiently large. Zennyo (2020) formalizes the model of platform competition finding three alternative equilibria (both platforms running a free version only, both platforms running both versions and an asymmetric equilibrium where only one platform introduces the premium version) with the eventual equilibrium depending primarily on the value of the fixed cost of the premium version. DeValve and Pekeč (2022) also find alternative equilibria in a model of competition, introducing, for first time, the possibility of users' multihoming. When platforms set one single menu of subscription price and advertising amount there may be an asymmetric equilibrium, with some consumers multihoming and some others either singlehoming or abstaining. By contrast, when platforms set two menus (which corresponds to freemium business for some parameter regions) all agents multihome. A key parameter in their analysis is the degree of platform differentiation which, in their model, is analyzed through an exogenous parameter.

Carroni and Paolini (2020) and Chi et al. (2021) introduce content providers' decision of accepting their content to be streamed through the platform in exchange of a royalty. Both works analyze the monopoly case only. Carroni and Paolini consider a market size parameter which, joint with the quality of the premium version, plays a key role in determining whether the platform offers only the basic service, the premium service or both.

On a more empirical ground, Waldfogel (2020) and Colbjornsen et al. (2021) report data on prices charged by music streaming platforms (and also by video platforms in the case of Colbjornsen et al. (2021)) in different countries.[4] Their evidence points out that music streaming markets tend to be symmetric in the sense that different platforms tend to

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[3] Another work with a similar focus is the one by Jeon et al (2022) where authors analyze the conditions for a monopoly platform to set a menu of price-advertising schedules or a single one (which implies *pooling*) considering two cases regarding cross-group network effects: when there is congruence (with users who obtain a higher utility from the platform exerting a higher externality on advertisers) or conflict (when the reverse happens) between the sides.

[4] Additionaly, Waldfogel (2020) estimates the level of consumer surplus and welfare in countries where music streaming may be modeled as a monopoly (served by Spotify) and as a duopoly (where, in addition to Spotify, Apple Music is also active).

stream a very similar portfolio of content and charge identical prices where video streaming is characterized by differentiation across platforms and different prices, with Netflix typically charging a higher subscription fee due to vertical product differentiation.

3. Monopoly model

In this section, I present the features of the model under the assumption that there is a single platform offering streaming services. The section is divided in two subsections. In the first subsection, the platform offers a free version only, financed by selling advertising; and, in the second subsection, it offers a premium version, in addition to the free one, which allows users to enjoy the same content, free from advertising, after paying a subscription fee.

3.1. Free version only

Let us consider a streaming service i, which is free for users and is financed by selling advertising, whose multimedia content has an identical value v for all potential users. There is a mass 1 of users who differ in the nuisance (disutility) they obtain from the advertisements, which are screened (or aired) through the service. The value of the nuisance suffered by users is assumed to be uniformly distributed in $[0,1]$ with a nuisance cost coefficient equal to t, which is assumed to reduce linearly the utility enjoyed by the user from the consumption of the streaming service. Therefore, the utility obtained by a user of the free version of service i, with a nuisance value $\tilde{x}$ ($0 \leq \tilde{x} \leq 1$), is $u_i^f(\tilde{x}) = v - t\tilde{x}$. We assume that $v < t$ in order to guarantee that some potential users do not use the service in the absence of the premium version. Without loss of generality, I assume that both marginal and fixed costs are zero.

In order to simplify the analysis I assume that there is one single seller advertising her products in platform i, obtaining a benefit $\alpha_i \geq 0$ from each user of the streaming service. Given a number of users n_i^f of the free version of service i, the seller's utility from hiring advertising content on that platform is $u_i^a = \alpha_i n_i^f - p_i^a$, where p_i^a is the price of advertising in the free version of platform i. To simplify the analysis, it is assumed that the platform has the whole bargaining power when dealing with the advertiser and is thus able to extract all the surplus from the advertising channeled through platform i. Therefore, the price charged by platform i to its advertiser is $p_i^a = \alpha_i n_i^f$.

Let us now solve for the value of the users' demand for the free version of streaming service i. The users who demand the service are those who obtain a nonnegative utility from using the platform will demand the service. These are all users with a nuisance value x such that $v - tx \geq 0$, which corresponds to those with a value $x \leq \frac{v}{t}$. As the distribution of the nuisance value is uniform in the set $[0,1]$, the mass of users of the free version of platform i is

$$n_i^f = \frac{v}{t}. \quad (1)$$

The price paid by the advertiser on platform i is therefore

$$p_i^a = \frac{\alpha_i v}{t}. \quad (2)$$

The profit of platform i when the latter runs the free version only is

$$\pi_i = \frac{\alpha_i v}{t}. \quad (3)$$

With the assumptions used in the model, the results show that a fraction $\frac{v}{t}$ of all potential customers use the free version of the service while the rest, $1 - \frac{v}{t}$, do not use the streaming service in the absence of a premium version. The profit of the firm comes only from the sales of advertisements and is equal to the whole value of the externality from users to advertisers.

3.2. Free version and premium version

Let us now assume that, in addition to the free version of streaming service i, described above, the platform offers a premium version with the same features of the free version without any advertising content. Users can gain access to the premium version of the service by paying a subscription fee p_i. Therefore, the utility enjoyed by any user of the premium version of streaming service i is $u_i^p(x) = v - p_i$ for all x. A consumer will prefer the free version rather than the premium version if $u_i^f(x) = v - tx \geq u_i^p(x) = v - p_i$. This implies a demand for the free version of $n_i^f = \frac{p_i}{t}$ and a demand for the premium version of $n_i^p = 1 - \left(\frac{p_i}{t}\right)$.

The advertising price in this case is $p_i^a = \frac{\alpha_i p_i}{t}$, yielding a value for the profit of platform i, which is the sum of the income obtained from the free version (through advertising revenues) plus the income obtained from the premium version (through the subscription fees paid by the premium users):

$$\pi_i = p_i^a + p_i n_i^p \ .$$

Substituting each term as a function of the subscription fee p_i, the problem of the firm can be written as

$$max_{p_i} \pi_i = \frac{\alpha_i p_i}{t} + p_i \left(1 - \frac{p_i}{t}\right).$$

The first-order condition of that problem yields an optimal value for the subscription price of the premium version

$$p_i = \frac{t+\alpha_i}{2}, \qquad (4)$$

yielding a mass of subscribers to the premium version

$$n_i^p = \frac{1}{2} - \frac{\alpha_i}{2t}, \qquad (5)$$

a mass of users of the free version

$$n_i^f = \frac{1}{2} + \frac{\alpha_i}{2t}, \qquad (6)$$

and an advertising price

$$p_i^a = \alpha_i \frac{t+\alpha_i}{2t}. \qquad (7)$$

The equilibrium profit of platform i is

$$\pi_i = \frac{(t+\alpha_i)^2}{4t}. \qquad (8)$$

Let us now comment on the results just obtained. First, by running a premium version, the platform reaches all the potential users, as all potential customers end up using either the free or the premium version of the streaming service when the latter is offered. Second, the subscription fee increases with both t and α_i (as we observe in (4)). The higher t, the higher the nuisance caused by advertising on users of the free version and thus the higher the willingness to pay for the premium version. The higher α_i, the higher the externality caused by users on advertisers and thus the ability of the platform to extract income from the latter. The opportunity cost of running the premium version is therefore higher so the platform needs to increase the subscription fee in order for the premium version to be profitable. Third, there are more subscribers of the free version than subscribers (by comparing (5) and (6)). Fourth, there are more subscribers the higher the disutility of advertisers on users, t, as they try to avoid that disutility, and the lower the externality of users to advertisers, α_i, as the platform increases the subscription fee when that externality increases (as we observe in (5)). Fifth, the subscribers of the premium version are those users with a higher nuisance value and correspond to those who were not using the platform in the absence of a premium version plus a mass $\frac{(2v-t-\alpha_i)}{2t}$ of users who switch from the free to the premium version. As the number of users of the free version is reduced, the advertising price is reduced, so it is lower in (7) than in (2).

By comparing (3) and (8) we can define the increase in profit due to the running of the premium version as $B = \frac{(t+\alpha_i)^2}{4t} - \frac{\alpha_i v}{t}$. Let us comment on the features of this value. First, $B > 0$ if $v < \tilde{v} = \frac{(t+\alpha_i)^2}{4\alpha_i}$, so the firm is better off by running a premium version of the streaming service when the value of the multimedia content for the user, net of nuisance and subscription fee, is not too high, specifically, when it is lower than $\tilde{v}$. Otherwise, it is more profitable to run a free version only. Second, $\frac{\partial B}{\partial \alpha_i} = \frac{1}{2} + \frac{\alpha_i}{2t} - \frac{v}{t} < 0$, so the extra benefit from running the premium version is smaller the higher the externality obtained by the advertiser from each user of the platform. A higher externality increases both the ability to obtain more income from advertisers and the subscription fee but the former effect dominates. Finally, $\frac{\partial B}{\partial t} = \frac{1}{4} - \left(\frac{\alpha_i}{2t}\right)^2$ is positive if and only if $\alpha_i < t$. A higher t increases the willingness of users to pay for the premium version and a higher α_i increases the willingness of advertisers to pay for the advertising in the premium version. When the former dominates, the profitability of the premium version increases with the nuisance cost.

4. Extensions of the monopoly model

Let us now consider some extensions to the monopoly model developed in Section 2.

4.1. Adding extra content to the premium version

In addition to the nonexistence of advertising, some platforms add some extra content to their premium versions, which are not accessible for the users of the free version (some of the most common features of premium services are detailed in Carroni and Paolini (2020, p. 1-2)). Let us now extend the base model just described by considering that the firm can add extra content, which may exclusively be enjoyed by the subscribers of the premium version. That extra content increases the utility of the users in a constant value ϑ. The marginal cost of those extra features is c per user of the premium version and the fixed cost of including that content is F.[5]

In order to obtain the equilibrium prices and profit, we proceed in an analogous way as in the base model by comparing the utility enjoyed by a user of the free version $u_i^f(x) = v - tx$ with the utility enjoyed by a user of the premium version, which is now $u_i^p(x) = v + \vartheta - p_i$. The demand for the free and premium versions are $n_i^f = \frac{(p_i - \vartheta)}{t}$ and $n_i^p = 1 - \left[\frac{(p_i - \vartheta)}{t}\right]$ respectively.

The advertising price is $p_i^a = \frac{\alpha_i(p_i - \vartheta)}{t}$, so we can rewrite the problem of the firm as a function of the subscription fee p_i in the following way:

[5] A positive fixed cost for the premium version is also present in Zennyo (2020), even though in his model the premium version is assumed to have the same properties for the user as the free version except the absence of advertising.

$$max_{p_i}\pi_i = \alpha_i \frac{p_i - \vartheta}{t} + (p_i - c)\left(1 - \frac{p_i - \vartheta}{t}\right) - F$$

The equilibrium price for the premium version obtained from the first order condition is

$$p_i = \frac{t+\alpha_i+\vartheta+c}{2}, \tag{9}$$

the number of subscribers of the premium version is

$$n_i^p = \frac{1}{2} - \frac{\alpha_i+c-\vartheta}{2t}, \tag{10}$$

the number of users of the free version is

$$n_i^f = \frac{1}{2} + \frac{\alpha_i+c-\vartheta}{2t}, \tag{11}$$

and the advertising price is

$$p_i^a = \alpha_i \frac{t+\alpha_i+c-\vartheta}{2t}. \tag{12}$$

The equilibrium profit of platform i is

$$\pi_i = \frac{(t+\alpha_i+\vartheta-c)^2}{4t} - \alpha_i \frac{\vartheta-c}{t} - F. \tag{13}$$

Let us now comment on the results of this extended version. First, as observed in (9), the subscription fee increases after including the extra features. The value of the increase is $\frac{\vartheta+c}{2}$ which is lower than the increase in the value for the consumer if $\vartheta > c$, so, when the latter holds, the added features increase the consumer surplus for the users of the premium version. The subscription fee is higher the higher the marginal cost of adding features, c, as the platform needs to increase the price of the premium version for the latter to be profitable, and the fee is also higher the higher the increased value of the premium version, ϑ, as the latter increases the willingness of users to pay for the premium version. Second, from (10) and (11), we observe that the number of subscribers increases by $\frac{\vartheta-c}{2t}$ so there are more subscribers than there were without the extra features if $\vartheta > c$ and less subscribers otherwise. Third, as a consequence of the extra features there may be more subscribers than users of the free version, an event which occurs when $\vartheta > c + \alpha_i$. Regarding the advertising price, and comparing (12) with (7), the price charged to advertisers increases if $c > \vartheta$ and decreases otherwise, due to the fact that this price depends linearly on the number of users of the free version. Finally, by comparing (13) and (8), the increase in profit due to the extra content is $\frac{\vartheta-c}{2}\left(1+\frac{\vartheta-c-2\alpha_i}{2t}\right)-F$, which is positive if $F<\tilde{F}=\frac{\vartheta-c}{2}\left(1+\frac{\vartheta-c-2\alpha_i}{2t}\right)$, as the firm is better off by adding extra features to the premium version if the fixed cost of producing those features is not too high.

4.2. Platform advertising of the premium version

Let us now analyze an alternative extension of the base model of Section 2. In most streaming services, the platform introduces advertisements of the premium version in the free version. These ads may have a double objective: first, they inform about the existence and features of the premium version; also, in some cases, they intentionally reduce the utility of the free version by creating an added nuisance, which makes it more appealing for the user to switch to the premium version.[6] We now extend the base model by considering that the platform introduces advertising of the premium version which reduces the utility of the free version by a value a, which is identical for all users.

Once again, we compare the utility enjoyed by a user of the free version, which is now $u_i^f(x)=v-tx-a$, with the utility enjoyed by a user of the premium version, $u_i^p(x)=v-p_i$. The demand for the free and premium versions are $n_i^f=\frac{(p_i-a)}{t}$ and $n_i^p=1-\left[\frac{(p_i-a)}{t}\right]$ respectively.

The advertising price is $p_i^a=\frac{\alpha_i(p_i-a)}{t}$, so we can rewrite the problem of the firm as a function of the subscription fee p_i in the following way:

$$max_{p_i}\pi_i=\alpha_i\frac{p_i-a}{t}+p_i\left(1-\frac{p_i-a}{t}\right).$$

The equilibrium price for the premium version obtained from the first order condition is

$$p_i=\frac{t+\alpha_i+a}{2}, \tag{14}$$

the number of subscribers of the premium version is now

[6] Nan et al. (2018) affirm in their discussion that the purpose of providng a freemium service model is to transform free users into paid users.

$$n_i^p = \frac{1}{2} - \frac{\alpha_i - a}{2t}, \tag{15}$$

the number of users of the free version is

$$n_i^f = \frac{1}{2} + \frac{\alpha_i - a}{2t}, \tag{16}$$

the advertising price is

$$p_i^a = \alpha_i \frac{t + \alpha_i - a}{2t} \tag{17}$$

and the equilibrium profit of platform i is

$$\pi_i = \frac{(t + \alpha_i + a)^2}{4t} - \frac{\alpha_i a}{t}. \tag{18}$$

Let us discuss the results. First, the price of the premium version increases by $\frac{a}{2}$ (comparing (14) with (4)), and the price is higher the higher the disutility caused by the platform's premium version advertising as the latter reduces the utility of the free version increasing the willingness to pay for the premium version. Second, by (15) and (16), the platform's self-advertising activity increases the number of subscribers by $\frac{a}{2}t$. The number of users of the premium version will end up being higher than those in the free version if $a > \alpha_i$. Third, by comparing (17) and (7), the advertising price decreases by $\frac{\alpha_i a}{2}t$.

Therefore, this self-advertising activity by the platform unambiguously increases the income from the premium version, as it increases both the number of subscribers and the subscription fee, but reduces unambiguously the income from the free version, as it reduces the advertising income. Depending on which effect dominates, the net effect on profits will be positive or negative. By comparing (18) and (8), the platform increases its profit by posting ads of the premium version in the free version streaming by $B' = \frac{a}{2}\left(\frac{a}{2t} - \frac{\alpha_i}{t} + 1\right)$, which is positive if $a > \tilde{a} = 2(\alpha_i - t)$.

The extra profit caused by self-advertising, B', has two additional features: $\frac{\partial B'}{\partial \alpha_i} = -\frac{1}{2t} < 0$ and $\frac{\partial B'}{\partial t} = -\frac{a}{2t^2}\left(\frac{a}{2} + \alpha_i\right) < 0$, which means that the extra profit is lower the higher the externality from users to advertisers and the higher the nuisance cost coefficient. Regarding the former, as α_i increases, users of the free version are more profitable for the platform, so there is less need to switch them to the premium version. With respect to the nuisance cost, the higher t the more disturbing is the hired advertising, so the lower the need of adding extra nuisance. A high t already ensures a high subscription fee without the platform's advertising of the premium version.

The results of this subsection deserve a discussion on the grounds of consumer protection laws. We have observed that the subscription fee, the number of subscribers and the increase in profits for the platform all increase in the value of the disutility added by this activity. This evidence may help explain why the advertisements of the premium version in some platforms may be so frequent and disturbing and may open the debate about whether regulators should impose a limit on the maximum amount of advertising of this nature that could be introduced by platforms. This self-advertising activity is welfare enhancing if $a > \frac{2(t+\alpha_i)}{3}$ and it increases consumer surplus when $a > 6t + 2\alpha_i$, implying that this practice damages consumers except in the unlikely case that both the disutility caused by paid advertising and the externality of users to advertising are both negligible.

4.3. Own-group network externalities

Let us now perform another extension of the base model of the streaming platform by considering that the extra features of the premium version depend on the number of subscribers as, for example, those features may allow users to share songs or playlists with other users of that version. If this feature increases the utility of the premium version for its users it thus leads to the existence of (positive) own-group network externalities among the users of the premium version as the utility of that version increases with the number of other users with which they can share their songs or playlists. This corresponds to the network externalities analyzed in the early literature (see Katz and Shapiro (1985) and Farrell and Saloner (1985)).

Let us denote θ_i the value in which the use of the premium version increases the utility of the user from an additional subscriber of that version. The utility of a premium version's user becomes $u_i^p(x) = v - p_i + \theta_i\, n_i^p$ for all $x \in [0,1]$. The marginal consumer who is indifferent between the free and the premium versions of the streaming service is the one with a nuisance value $x = 1 - n_i^p$. That nuisance value satisfies the following indifference condition: $u_i^f(x) = v - tx = u_i^p(x) = v - p_i + \theta_i\,(1 - x)$, which holds for $x = \frac{(p_i - \theta_i)}{(t - \theta_i)}$, so the number of users of the free version and subscribers of the premium version will be, respectively, $n_i^f = \frac{(p_i - \theta_i)}{(t - \theta_i)}$ and $n_i^p = \frac{(t - p_i)}{(t - \theta_i)}$.

The advertising price is $p_i^a = \alpha_i \frac{(p_i - \theta_i)}{(t - \theta_i)}$, so we can rewrite the problem of the firm as a function of the subscription fee p_i in the following way:

$$max_{p_i}\pi_i = \alpha_i \frac{p_i - \theta_i}{t - \theta_i} + p_i \frac{t - p_i}{t - \theta_i}.$$

The equilibrium price for the premium version, obtained from the first-order condition, is

$$p_i = \frac{t + \alpha_i}{2}. \tag{19}$$

The number of subscribers of the premium version is

$$n_i^p = \frac{t - \alpha_i}{2(t - \theta_i)}, \tag{20}$$

the number of users of the free version is

$$n_i^f = \frac{1}{2} + \frac{\alpha_i - \theta_i}{2(t - \theta_i)} \tag{21}$$

and the advertising price becomes

$$p_i^a = \alpha_i \frac{t + \alpha_i - 2\theta_i}{2(t - \theta_i)}. \tag{22}$$

The equilibrium profit of platform i is

$$\pi_i = \frac{(t + \alpha_i)^2}{4(t - \theta_i)} - \frac{\alpha_i \theta_i}{t - \theta_i}. \tag{23}$$

Let us now comment on the results of this extended version of the model. First, as we observe in (19), these own-group network externalities do not change the subscription fee, which remains as it was without those effects. Second, regarding the number of users of each version ((20) and (21)), we observe that the number of subscribers of the premium version increases by $\frac{\theta_i(t - \alpha_i)}{2t(t - \theta_i)}$ and becomes higher than the number of users of the free version if $\theta_i > \alpha_i$, i.e. if the own-group network effects are more relevant than the cross-group network effects. Third, by comparing (22) and (7), the price of advertising, p_i^a, decreases by $\frac{\alpha_i \theta_i (t-\alpha_i)}{2t(t-\theta_i)}$. Finally, comparing (23) and (8), the increase in profit due to these own-group network effects is $B'' = \frac{\theta_i (t-\alpha_i)^2}{4t(t-\theta_i)}$, which has the features: $B > 0$ and $\frac{\partial B}{\partial \theta_i} = \frac{(t-\alpha_i)^2}{4(t-\theta_i)^2} > 0$ if $\theta_i < t$ so, provided the latter, a higher level of own-group network effects increases the profitability of those effects, so the platform will try to introduce features having this property.

4.4. Consumers' income heterogeneity

So far, we have considered that, from the consumer's viewpoint, the decision of subscribing to the premium version only depends on the value of the nuisance suffered. Our intuition suggests that this decision also depends on the consumer's income as using the free version may be the consequence of a consumer having a low income despite a high nuisance cost (Weyl (2010) incorporated this type of user heterogeneity in the model of two-sided markets).

I therefore extend the benchmark model of subsection 2.2 by considering that users are heterogeneous in the level of income. Let us define the variable w ($0 \leq w \leq 1$), which is the net disposable income (wealth) available for consumers to spend on streaming services, once they have spent the rest of their income on first-need (non-leisure) goods and services. We assume that consumers (of mass 1) are uniformly distributed on the (x, w)–space. A consumer with nuisance value $\tilde{x}$ and disposable income $\widetilde{w}$ subscribes to the premium version if the two following conditions are simultaneously satisfied: $u_i^p(\tilde{x}) > u_i^f(\tilde{x})$ and $\widetilde{w} > p_i$.

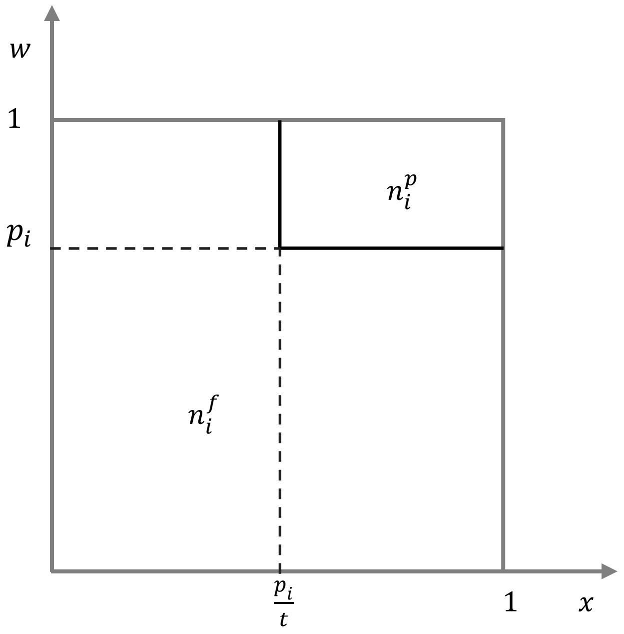

Figure 4: Demands for free and premium versions with uniform distribution of users regarding nuisance and disposable income

Figure 4 shows the space of distribution of consumers, which is two-dimensional. There is a mass n_i^p of consumers who obtain a higher utility from the premium version than from the free version (those with a nuisance cost value $x > \frac{p_i}{t}$) and a disposable income higher than the subscription price ($w > p_i$). The reminder, n_i^f, of consumers use the free version.

The demands for the premium and free versions are, respectively, $n_i^p = (1 - p_i)\left(1 - \frac{p_i}{t}\right)$ and $n_i^f = 1 - (1 - p_i)\left(1 - \frac{p_i}{t}\right)$. Therefore the problem of the firm can be written as

$$max_{p_i}\pi_i = \alpha_i\left[1 - (1 - p_i)\left(1 - \frac{p_i}{t}\right)\right] + p_i(1 - p_i)\left(1 - \frac{p_i}{t}\right)$$

Solving the first-order condition we obtain the subscription price:

$$p_i = \frac{1+t+\alpha_i-\sqrt{(t-\alpha_i)^2+(1-\alpha_i)(1-t)}}{3} \tag{24}$$

the number of subscribers of the premium version is now

$$n_i^p = \frac{4t-t^2-1-2\alpha_i(1+t-\alpha_i)+(1+t-2\alpha_i)\sqrt{(t-\alpha_i)^2+(1-\alpha_i)(1-t)}}{9t} \tag{25}$$

the number of users of the free version is

$$n_i^f = \frac{5t+t^2+1+2\alpha_i(1+t-\alpha_i)-(1+t-2\alpha_i)\sqrt{(t-\alpha_i)^2+(1-\alpha_i)(1-t)}}{9t} \quad (26)$$

the advertising price is

$$p_i^a = \alpha_i \frac{5t+t^2+1+2\alpha_i(1+t-\alpha_i)-(1+t-2\alpha_i)\sqrt{(t-\alpha_i)^2+(1-\alpha_i)(1-t)}}{9t} \quad (27)$$

and the equilibrium profit of platform i is

$$\pi_i = \frac{(t+\alpha_i)^3+3\left[t\left(1+t-t^2\right)+\alpha_i\left(1+\alpha_i-\alpha_i{}^2\right)\right]+15t\alpha_i-2+2\left[(t-\alpha_i)^2+(1-\alpha_i)(1-t)\right]^{\frac{3}{2}}}{27t} \quad (28)$$

Let us now comment on the results: first, if we compare the subscription fee in (24) with the one in (4), when there was no income restriction, the one in (24) is lower. As we could expect, facing a minimum income restriction leads the platform to reduce the fee in an attempt to do the service affordable to more users. Second, the number of subscribers of the premium version (in (25)) is higher than under consumer income homogeneity (in (5)) when $[(t-\alpha_i)^2+(1-\alpha_i)^2]^2-4(1-t)^2(1-\alpha_i)(t-\alpha_i)-\frac{81}{4}(1+t-2\alpha_i)^2(t-\alpha_i)^2[(t-\alpha_i)^2+(1-\alpha_i)(1-t)]>0$, condition which is satisfied when α_i and t take very similar values. Third, the derivatives of p_i with respect to α_i and t ($\frac{\partial p_i}{\partial \alpha_i}=\frac{1}{3}+\frac{1+t-2\alpha_i}{6\sqrt{(t-\alpha_i)^2+(1-\alpha_i)(1-t)}}$ and $\frac{\partial p_i}{\partial t}=\frac{1}{3}+\frac{1+\alpha_i-2t}{6\sqrt{(t-\alpha_i)^2+(1-\alpha_i)(1-t)}}$) are both positive, which imply, as before, that the subscription fee increases with both the nuisance suffered by users and the level of user attractiveness to advertisers.

4.5. Consumers' heterogeneity regarding externality on advertisers

The last extension to the benchmark monopoly model is done in this subsection by considering that all consumers are not alike in terms of the externality caused on advertisers but those with a higher disposable income are more appealing for the firms which advertise their products through the platform.[7] The analysis in this subsection is built upon the one in 3.4 by considering that, instead of having that all consumers exert an externality α_i on advertisers, a consumer with a disposable income w exerts an

[7] Prasad et al. (2003) and Lin (2020) analyzed heterogeneous externality from users to advertisers. The modeling of that externality in the present paper is different than their as both works consider two possible values for the externality whereas in this paper I assume that the externality value is uniformly distributed within a range.

externality equal to $\alpha_i + w - \frac{1}{2}$. Therefore, the level of externality in uniformly distributed on $\left[\alpha_i - \frac{1}{2}, \alpha_i + \frac{1}{2}\right]$. The average externality is still α_i, so the results in this subsection can be straightforwardly compared with the ones in 4.4.

The demand areas in Figure 4 totally apply for the current case but the average externality of free users is now different from the one before. We can divide the mass of free version users in two groups: those with a nuisance value lower than $\frac{p_i}{t}$ and those with a higher value than that. The externality of users in the former group is uniformly distributed on $\left[\alpha_i - \frac{1}{2}, \alpha_i + \frac{1}{2}\right]$ with an average externality α_i, while user's externality in the latter group is uniformly distributed on $\left[\alpha_i - \frac{1}{2}, \alpha_i + p_i - \frac{1}{2}\right]$ with an average externality equal to $\alpha_i - \frac{1}{2}(1 - p_i)$.

As in the previous subsection, the demands for the premium and free versions are, respectively, $n_i^p = (1 - p_i)\left(1 - \frac{p_i}{t}\right)$ and $n_i^f = 1 - (1 - p_i)\left(1 - \frac{p_i}{t}\right)$ but, in the current case, platform's income from advertising is not $\alpha_i n_i^f$ but $\alpha_i n_i^f\left(x < \frac{p_i}{t}\right) + \left[\alpha_i - \frac{1}{2}(1 - p_i)\right] n_i^f\left(x \geq \frac{p_i}{t}\right)$. From Figure 1 we can observe that $n_i^f\left(x < \frac{p_i}{t}\right) = \frac{p_i}{t}$ and $n_i^f\left(x \geq \frac{p_i}{t}\right) = p_i\left(1 - \frac{p_i}{t}\right)$. The problem of the firm becomes

$$max_{p_i}\pi_i = \alpha_i \frac{p_i}{t} + \left[\alpha_i - \frac{1}{2}(1 - p_i)\right] p_i\left(1 - \frac{p_i}{t}\right) + p_i(1 - p_i)\left(1 - \frac{p_i}{t}\right).$$

Solving the first-order condition we obtain the subscription price:

$$p_i = \frac{1 + t + 2\alpha_i - \sqrt{(t - 2\alpha_i)^2 + (1 - 2\alpha_i)(1 - t)}}{3}, \quad (29)$$

the number of subscribers of the premium version is now

$$n_i^p = \frac{4t - t^2 - 1 - 4\alpha_i(1 + t - 2\alpha_i) + (1 + t - 4\alpha_i)\sqrt{(t - 2\alpha_i)^2 + (1 - 2\alpha_i)(1 - t)}}{9t}, \quad (30)$$

the number of users of the free version is

$$n_i^f = \frac{5t + t^2 + 1 + 4\alpha_i(1 + t - 2\alpha_i) - (1 + t - 4\alpha_i)\sqrt{(t - 2\alpha_i)^2 + (1 - 2\alpha_i)(1 - t)}}{9t}, \quad (31)$$

the advertising price is

$$p_i^a = \frac{2[(40\alpha_i^2-18\alpha_i-3)\alpha_i-(1+t^3)]-3t[12\alpha_i^2+10\alpha_i-1+(2\alpha_i-1)t]}{54t} +$$

$$\frac{[t^2+(4\alpha_i-1)t-20\alpha_i^2+4\alpha_i+1]\sqrt{(t-2\alpha_i)^2+(1-2\alpha_i)(1-t)}}{27t} \quad (32)$$

and the equilibrium profit of platform i is

$$\pi_i = \frac{3t[4\alpha_i^2+10\alpha_i+1+(2\alpha_i+1)t]-2[(8\alpha_i^2-6\alpha_i-3)\alpha_i+(1+t^3)]}{54t} +$$

$$\frac{[(1-t)^2+(t-2\alpha_i)^2+(1-2\alpha_i)^2]\sqrt{(t-2\alpha_i)^2+(1-2\alpha_i)(1-t)}}{54t} \quad (33)$$

Let us now comment on the results: first, if we compare the subscription fee in (29), when user's externality on advertisers increases with user's income, with the one in (24), when the externality was identical for all users, the one in (29) is higher. Higher-income consumers are more appealing for advertisers so the platform increases the subscription price to increase the number of high-income consumers using the free version thus obtaining a higher advertising revenue. As premium version subscribers are those with highest income, the platform is able to increase the subscription fee and there are still users ready to pay that higher fee. Second, as a consequence of the previous, the number of subscribers decreases. Third, the derivatives of p_i with respect to α_i and t ($\frac{\partial p_i}{\partial t} = \frac{1}{3} + \frac{1+2(\alpha_i-t)}{6\sqrt{(t-2\alpha_i)^2+(1-2\alpha_i)(1-t)}}$ and $\frac{\partial p_i}{\partial \alpha_i} = \frac{2}{3} + \frac{1+t-4\alpha_i}{3\sqrt{(t-2\alpha_i)^2+(1-2\alpha_i)(1-t)}}$) are both positive, which imply, as before, that the subscription fee increases with both the nuisance suffered by users and the level of user attractiveness to advertisers. Fourth, the profit in (33) is higher than in (28) when $(1+t)(2-t)(1-2t) + [(1-t)^2 + (t-2\alpha_i)^2 + (1-2\alpha_i)^2]\sqrt{(t-2\alpha_i)^2 + (1-2\alpha_i)(1-t)} - 4[(t-\alpha_i)^2 + (1-\alpha_i)(1-t)]\sqrt{(t-\alpha_i)^2 + (1-\alpha_i)(1-t)} + 6(1+t-2\alpha_i)\alpha_i^2 > 0$. That value may be either positive or negative. That condition is more likely to be held (and thus the platform earns a higher profit than in the case that all users are equally appealing to advertisers) the higher the value of $\frac{\alpha_i}{t}$. Finally, the platform increases its advertising income due to the non-uniform network externality (with the value in (32) being higher than the one in (27)) when $(1+t)(2-t)(1-2t) + 6\alpha_i(1+t-2\alpha_i)[4\alpha_i + \sqrt{(t-\alpha_i)^2 + (1-\alpha_i)(1-t)}] - 2[(t+4\alpha_i-1)t - 20\alpha_i^2 + 4\alpha_i + 1]\sqrt{(t-2\alpha_i)^2 + (1-2\alpha_i)(1-t)} - 20\alpha_i^2 > 0$. The latter condition holds (except in the case where $t = \alpha_i = 1$, where the change is 0), implying that the non-uniform network externality allows the firm to earn a higher advertising income as the number of users of the free version increases.

5. Duopoly model

In this section I extend the base model analyzed in the previous sections to consider competition among streaming platforms. I now consider two duopolists who enter the market simultaneously. In contrast to the base model analyzed above, we now allow for horizontal product differentiation in terms of the streamed content.

We consider a mass 1 of users who differ in two dimensions. First, analogously to the monopoly model, each consumer has a value x ($0 \leq x \leq 1$), which represents the nuisance she suffers from the advertising in the free version. Additionally, there is another variable y ($0 \leq y \leq 1$) which represents the space in terms of the content streamed in the platform. This dimension may refer to the music style or the movie genders. Intuitively a platform's choice of location close to $\frac{1}{2}$ means that the platform screens the content preferred by the 'average user' (probably blockbusters or most popular songs) while a value close to either 0 or 1 may mean a more specialized content. Sato (2019) and Zennyo (2020), which are the previous works analyzing platform competition in the freemium model, both assumed that platforms are located at 0 and 1, so this work is the first to predict the expected degree of platform differentiation.

Let us assume that consumers are uniformly distributed in the (x, y)-space. In addition to t (the nuisance cost coefficient) there is a transport cost coefficient in the content space, τ. All else equal, the higher τ the more a user dislikes using a platform with a content different from her favorite one. Analogously to t (and for the sake of restricting the set of likely equilibria to a tractable set) I assume that $v < \tau$.

There are two platforms, 1 and 2, entering the market simultaneously, each running both a free and a premium version. Decisions are taken in the following sequence: in stage 1, platforms simultaneously decide their values for the streamed content, y_1 and y_2; and, in stage 2, they simultaneously decide the prices for their premium versions, p_1 and p_2. With respect to advertising revenues, I assume that each platform i has users who exert a positive externality α_i to the advertiser who hires advertising content in that platform. We keep the assumption above that the price charged by platform i to its advertiser is $p_i^a = \alpha_i n_i^f (i = 1,2)$. We therefore consider, in this simple version of the model, that the streaming platforms compete for users but not for advertisers, over whom platforms possess all the bargaining power as they extract from them the whole externality obtained from users.

I analyze two alternative cases. First, users singlehome (in 5.1) so each potential customer may only use either platform 1 or 2 but not both. Alternatively, users are allow to multihome (in 5.2) so a user of a free platform (either 1 or 2) may also use the free version of the other platform, and the same applies to a subscriber of a platform who may also use the premium version of the other platform. For the sake of computational tractability we neglect the possibility of using the free version of a platform and the premium version of the other one.

5.1. Singlehoming users

We start by considering that consumers may only use a single platform and plan. The utility of a consumer with a nuisance cost $\tilde{x}$, $0 \leq \tilde{x} \leq 1$, and a preference for streaming content $\tilde{y}$, $0 \leq \tilde{y} \leq 1$, is:

- $v - t\tilde{x} - \tau|\tilde{y} - y_1|$, if she uses the free version of platform 1.
- $v - \tau|\tilde{y} - y_1| - p_1$, if she uses the premium version of platform 1.
- $v - t\tilde{x} - \tau|\tilde{y} - y_2|$, if she uses the free version of platform 2.
- $v - \tau|\tilde{y} - y_2| - p_2$, if she uses the premium version of platform 2.

We assume, without loss of generality, that $y_1 \leq y_2$. To simplify the analysis, we assume that the reservation value v is the same for all users and identical for both platforms.

The location decisions of firms, y_1 and y_2, joint with the premium version price decisions, p_1 and p_2, determine the four values of demand, which correspond to the number of users of the free and the premium versions of each platform: n_1^f, n_1^p, n_2^f and n_2^p. The shape of those demand functions depend primarily on whether p_1 is higher than, lower than or equal to p_2^p. The latter depends basically on the relative values of α_1 and

α_2. Each demand function corresponds to the set of consumers for whom the utility of that platform and plan is highest.

In this section I solve for the symmetric case, where $\alpha_1 = \alpha_2$, which eventually leads to identical prices for the premium versions of platforms 1 and 2. In Appendix 1 we present the demand functions for the case where $\alpha_1 > \alpha_2$ and Appendix 2 reports those corresponding to $\alpha_2 > \alpha_1$.

Let us now assume that the users of both platforms exert the same externality on the advertisers, therefore $\alpha_1 = \alpha_2 = \alpha$. The areas corresponding to the demands of the free versions of platforms 1 and 2 (n_1^f and n_2^f, respectively) and the premium versions (n_1^p and n_2^p) are shown in Figure 5.

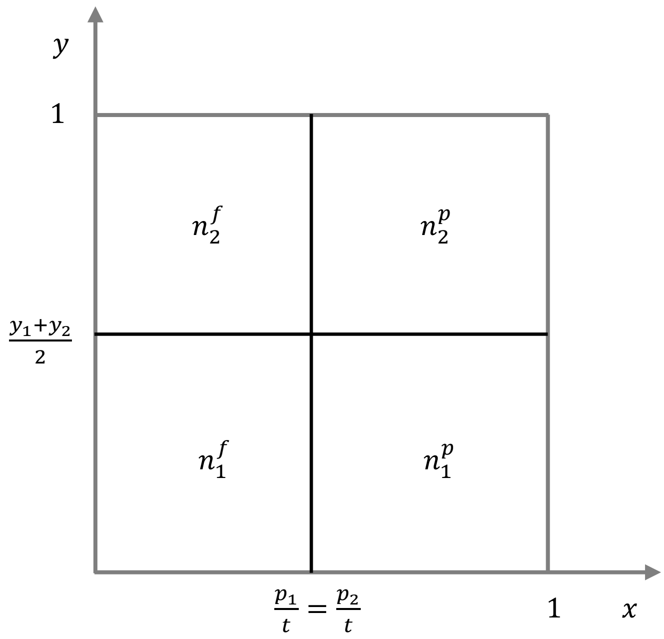

Figure 5: Duopoly model with $\alpha_1 = \alpha_2$

The demand function of each platform and version, as a function of prices and location in the content space, are:

$$n_1^f = \frac{p_1}{t}\frac{y_1 + y_2}{2}$$

$$n_1^p = \left(1 - \frac{p_1}{t}\right)\frac{y_1 + y_2}{2}$$

$$n_2^f = \frac{p_2}{t}\left(1 - \frac{y_1 + y_2}{2}\right)$$

$$n_2^p = \left(1 - \frac{p_2}{t}\right)\left(1 - \frac{y_1 + y_2}{2}\right)$$

yielding the following second-stage profit functions:

$$\pi_1 = \alpha \frac{p_1}{t}\frac{y_1+y_2}{2} + p_1\left(1 - \frac{p_1}{t}\right)\frac{y_1+y_2}{2} \text{ and}$$

$$\pi_2 = \alpha \frac{p_2}{t}\left(1 - \frac{y_1+y_2}{2}\right) + p_2\left(1 - \frac{p_2}{t}\right)\left(1 - \frac{y_1+y_2}{2}\right).$$

The first-order conditions of those profit functions with respect to the price of the premium versions yield the following values for those prices:

$$p_1 = p_2 = \frac{t+\alpha}{2}. \tag{34}$$

Substituting those prices in the profit functions just described yield the following first-stage profit functions:

$$\pi_1 = \frac{(y_1+y_2)(t+\alpha)^2}{8t} \text{ and } \pi_2 = \frac{(2-y_1-y_2)(t+\alpha)^2}{8t}.$$

Given the assumption that $y_1 \leq y_2$ and the values of the first order conditions, $\frac{\partial\pi_1}{\partial y_1} = \frac{(t+\alpha)^2}{8t} > 0$ and $\frac{\partial\pi_2}{\partial y_2} = -\frac{(t+\alpha)^2}{8t} < 0$, we obtain the unique solution for the platforms' decision of location in the content space:

$$y_1 = y_2 = \frac{1}{2}, \tag{35}$$

the number of premium versions' subscribers are

$$n_1^p = n_2^p = \frac{1}{4} - \frac{\alpha}{4t}, \tag{36}$$

the equilibrium number of free versions' users are

$$n_1^f = n_2^f = \frac{1}{4} + \frac{\alpha}{4t}, \tag{37}$$

yielding the following advertising prices:

$$p_1^a = p_2^a = \alpha\frac{t+\alpha}{4t} \tag{38}$$

and profits

$$\pi_1 = \pi_2 = \frac{(t+\alpha)^2}{8t}. \tag{39}$$

This is an interesting result. Solution (35) implies minimum product differentiation in terms of the content streamed as both platforms are eventually located at the center of the content space. In contrast to Hotelling (1929) and other works where that equilibrium solution implied zero prices and profits, platforms in this equilibrium charge the monopoly price (as the price in (34) is the same as the equilibrium price of the monopoly model, in (4)). This is due to the heterogeneity of users in terms of the nuisance suffered from advertising. This heterogeneity avoids the Bertrand paradox and does not lead to a reduction of prices. The number of users of the free and premium versions of the streaming service and the platforms' profits are the same as in the monopoly model of Section 2 but are equally split by platforms. The advertising price is the same for both platforms and is half the one in the monopoly model, so the overall advertising payment ($p_1^a + p_2^a$) is the same as the one in (7).

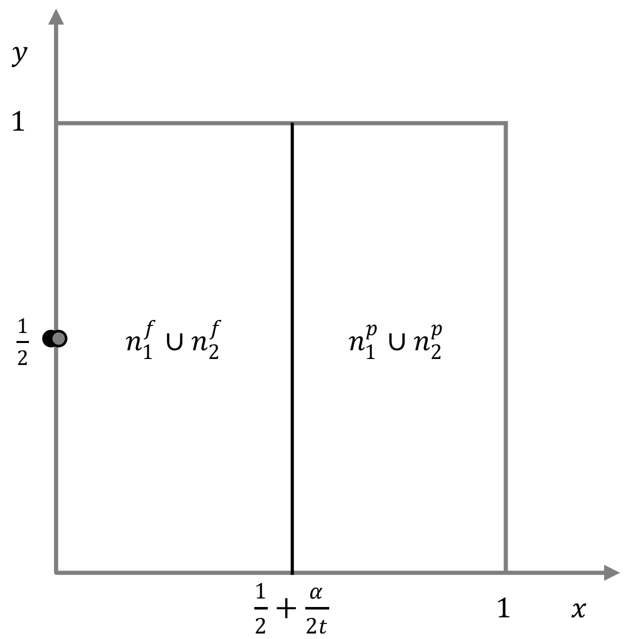

Figure 6: Symmetric duopoly model equilibrium

Figure 6 shows the solution of the symmetric duopoly model. Both platforms are located at the center of the content space (at $y = \frac{1}{2}$). Consumers with a nuisance value $x < \frac{1}{2} + \frac{\alpha}{2t}$ choose a free platform and are equally split by platforms 1 and 2. Consumers with a nuisance value $x > \frac{1}{2} + \frac{\alpha}{2t}$ choose a premium platform and are equally split by platforms 1 and 2 as well.

Let us now discuss about the plausibility of the main results: minimum platform differentiation and monopoly price. Regarding the price, anecdotal evidence of the music streaming market tends to support this result as we usually observe that platforms who started offering the service first tend to charge the same price when they were the only provider as well as when they compete with a new entrant. Moreover, the new entrants tend to charge the same price as the incumbent. This is supported by the data on fees reported in Colbjornsen et al. (2021) where authors show that Spotify and Tidal have identical prices in the different markets. The minimum differentiation result is also quite well supported by evidence in the same market as we observe that different services have an almost identical portfolio in terms of the most popular tracks and may differ in content of interest for some particular groups like podcasts or audiobooks.

The minimum differentiation result is obtained in this model as a consequence of the assumption of homogeneity of users of both platforms in terms of the externality they exert on advertisers (that I have used in order to obtain a solution). It is the symmetry of users in terms of this externality which leads to symmetry of platforms.[8]

5.2. Multihoming users

We now allow customers to use more than one platform in either the free or the premium plan. The utility of a consumer with a nuisance cost $\tilde{x}$, $0 \leq \tilde{x} \leq 1$, and a preference for streaming content $\tilde{y}$, $0 \leq \tilde{y} \leq 1$, is:

- $v - t\tilde{x} - \tau|\tilde{y} - y_1|$, if she uses the free version of platform 1.
- $v - \tau|\tilde{y} - y_1| - p_1$, if she uses the premium version of platform 1.
- $v - t\tilde{x} - \tau|\tilde{y} - y_2|$, if she uses the free version of platform 2.
- $v - \tau|\tilde{y} - y_2| - p_2$, if she uses the premium version of platform 2.

[8] Ambrus and Argenziano (2009) analyze the case where users are heterogeneous in the externality they cause on advertisers and this externality leads to asymmetric platforms, such as Visa and MasterCard in the credit card market, with different price structures.

- $v + v|y_2 - y_1| - t\tilde{x} - \tau|\tilde{y} - y_1| - \tau|\tilde{y} - y_2|$, if she uses the free versions of platforms 1 and 2.
- $v + v|y_2 - y_1| - \tau|\tilde{y} - y_1| - \tau|\tilde{y} - y_2| - p_1 - p_2$, if she uses the premium versions of platforms 1 and 2.

Looking at the last two bullets we observe that using two platforms in terms of one (i.e. multihoming) has an increased value for the user of $v|y_2 - y_1|$, which depends on the degree of differentiation across platforms. When both platforms stream the same content ($y_1 = y_2$) there is no increase in utility for the user ($v|y_2 - y_1| = 0$). At the other extreme, when differentiation across platforms is highest (when $y_1 = 0$ and $y_2 = 1$), using both platform doubles the value of the user ($v|y_2 - y_1| = v$), neglecting the *transport* costs and the prices paid in the case of the premium version.

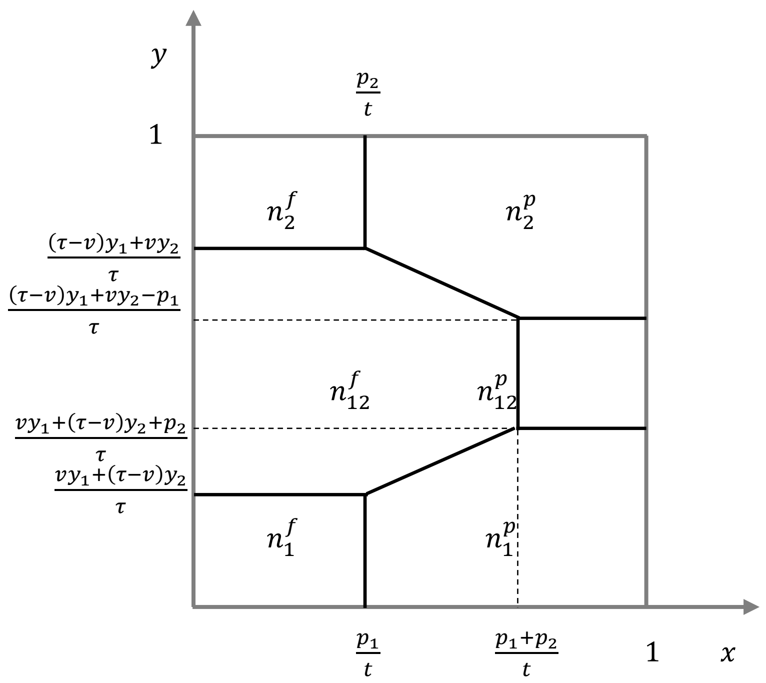

Figure 7: Duopoly model with multihoming users with $p_1, p_2 < \left(v - \frac{\tau}{2}\right)(y_2 - y_1)$

For the sake of computational tractability I restrict to solving the symmetric case alone, when users of both platforms exert the same level of network externality to advertisers: $\alpha_1 = \alpha_2 = \alpha$. Under that case, we have two alternative equilibria, depending on the values of parameters, each one yielding a map of platform versions' demands. The first equilibrium (depicted in Figure 7) takes place when premium version prices satisfy the condition: $p_1 < \left(v - \frac{\tau}{2}\right)(y_2 - y_1)$, $p_2 < \left(v - \frac{\tau}{2}\right)(y_2 - y_1)$. In that case, there are users multihoming in both the free and the premium versions.

The values of the demand areas are:

$$n_1^f = \frac{p_1}{t}\frac{vy_1 + (\tau - v)y_2}{\tau}$$

$$n_1^p = \frac{p_2}{\tau}\left(1 - \frac{p_1}{t} - \frac{p_2}{2t}\right) + \left(1 - \frac{p_1}{t}\right)\frac{vy_1 + (\tau - v)y_2}{\tau}$$

$$n_2^f = \frac{p_2}{t}\left[1 - \frac{(\tau - v)y_1 + vy_2}{\tau}\right]$$

$$n_2^p = \frac{p_1}{\tau}\left(1 - \frac{p_1}{2t} - \frac{p_2}{t}\right) + \left(1 - \frac{p_2}{t}\right)\left[1 - \frac{(\tau - v)y_1 + vy_2}{\tau}\right]$$

$$n_{12}^f = \frac{p_1 + p_2}{t}\frac{(2v - \tau)(y_2 - y_1)}{\tau} - \frac{{p_1}^2 + {p_2}^2}{2t\tau}$$

$$n_{12}^p = \left(1 - \frac{p_1 + p_2}{t}\right)\left[\frac{(2v - \tau)(y_2 - y_1)}{\tau} - \frac{p_1 + p_2}{\tau}\right]$$

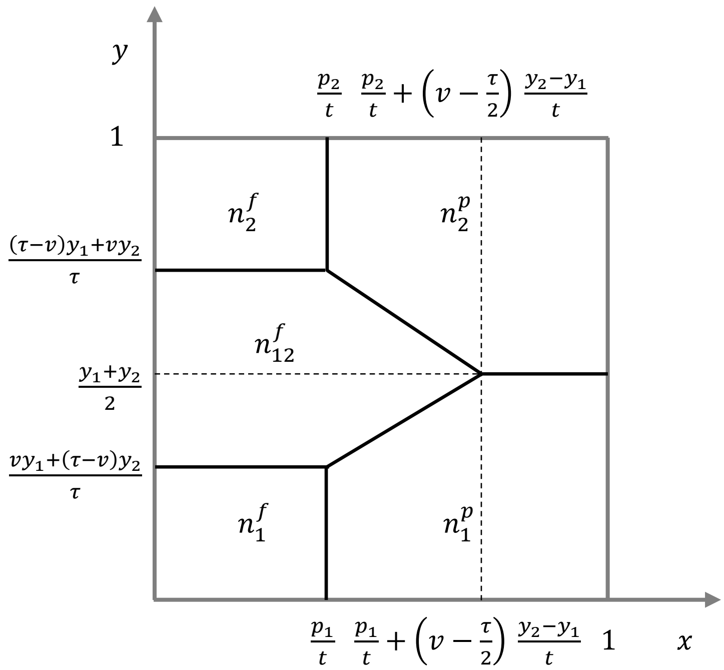

Figure 8: Duopoly model with multihoming users with $p_1, p_2 \geq \left(v - \frac{\tau}{2}\right)(y_2 - y_1)$

Alternatively, when prices of the premium versions satisfy the conditions $p_1 \geq \left(v - \frac{\tau}{2}\right)(y_2 - y_1)$ and $p_2 \geq \left(v - \frac{\tau}{2}\right)(y_2 - y_1)$ the equilibrium implies multihoming in the free version alone, as depicted in Figure 8.

The values of the demand areas are:

$$n_1^f = \frac{p_1}{t}\frac{vy_1 + (\tau - v)y_2}{\tau}$$

$$n_1^p = \left(1 - \frac{p_1}{t}\right)\frac{y_1 + y_2}{2} - \left(v - \frac{\tau}{2}\right)^2 \frac{(y_2 - y_1)^2}{2t\tau}$$

$$n_2^f = \frac{p_2}{t}\left[1 - \frac{(\tau - v)y_1 + vy_2}{\tau}\right]$$

$$n_2^p = \left(1 - \frac{p_2}{t}\right)\frac{2 - y_1 - y_2}{2} - \left(v - \frac{\tau}{2}\right)^2 \frac{(y_2 - y_1)^2}{2t\tau}$$

$$n_{12}^f = \frac{p_2}{t}\frac{(\tau - v)y_1 + vy_2}{\tau} - \frac{p_1}{t}\frac{vy_1 + (\tau - v)y_2}{\tau} - \frac{p_2 - p_1}{t}\frac{y_1 + y_2}{2} + \left(v - \frac{\tau}{2}\right)^2 \frac{(y_2 - y_1)^2}{t\tau}$$

We first analyze the case depicted in Figure 7, where users in equilibrium multihome in both the free and the premium versions. Firms' profits are:

$$\pi_1 = \alpha\left[\frac{p_1}{t}\frac{vy_1 + (\tau - v)y_2}{\tau} + \frac{p_1 + p_2}{t}\frac{(2v - \tau)(y_2 - y_1)}{\tau} - \frac{{p_1}^2 + {p_2}^2}{2t\tau}\right] + p_1\left[\frac{p_2}{\tau}\left(1 - \frac{p_1}{t} - \frac{p_2}{2t}\right) + \left(1 - \frac{p_1}{t}\right)\frac{vy_1 + (\tau - v)y_2}{\tau} + \left(1 - \frac{p_1 + p_2}{t}\right)\left(\frac{(2v - \tau)(y_2 - y_1)}{\tau} - \frac{p_1 + p_2}{\tau}\right)\right]$$

$$\pi_2 = \alpha\left[\frac{p_2}{t}\left(1 - \frac{(\tau - v)y_1 + vy_2}{\tau}\right) + \frac{p_1 + p_2}{t}\frac{(2v - \tau)(y_2 - y_1)}{\tau} - \frac{{p_1}^2 + {p_2}^2}{2t\tau}\right] + p_2\left[\frac{p_1}{\tau}\left(1 - \frac{p_1}{2t} - \frac{p_2}{t}\right) + \left(1 - \frac{p_2}{t}\right)\left(1 - \frac{(\tau - v)y_1 + vy_2}{\tau}\right) + \left(1 - \frac{p_1 + p_2}{t}\right)\left(\frac{(2v - \tau)(y_2 - y_1)}{\tau} - \frac{p_1 + p_2}{\tau}\right)\right]$$

Solving the first-order conditions with respect to premium version prices yield the following second-stage prices (as a function of locations in content space y_1 and y_2):

$$p_1 = \frac{(3\tau - 4v)y_1 + (4v - \tau)y_2 + \alpha + 2t}{11} - \frac{\sqrt{[(3\tau - 4v)y_1 + (4v - \tau)y_2 - (\alpha + 2t)]^2 + 2v(5t - 3\alpha)(y_2 - y_1) + 2\tau[(t - 5\alpha)y_1 - 2(3\alpha + 2t)y_2]}}{11}$$

$$p_2 = \frac{\alpha + 2(t + \tau) - (4v - \tau)y_1 - (3\tau - 4v)y_2}{11} - \frac{\sqrt{[(4v - \tau)y_1 + (3\tau - 4v)y_2 + \alpha + 2(t - \tau)]^2 + 2\{2[(2t + \alpha)\tau - (5t - 3\alpha)v]y_1 + (5\alpha - t)(\tau - v)y_2 - (3t + 7\alpha)\tau\}}}{11}$$

When we introduce those price functions in the profit functions, obtaining the first-stage profit functions $\pi_1(y_1, y_2)$ and $\pi_2(y_1, y_2)$, and solve for the first-order conditions with respect to locations y_1 and y_2, we do not reach any symmetric equilibria for y_1 and y_2 in the region $[0,1]$ satisfying the condition $p_1, p_2 < \left(v - \frac{\tau}{2}\right)(y_2 - y_1)$. We thus rule out the equilibrium where users multihome in both the free and the premium version.

We next analyze the case depicted in Figure 8, where users in equilibrium multihome in the free version alone. Firms' profits are:

$$\pi_1 = \alpha\left[\frac{p_2}{t}\frac{(\tau - v)y_1 + vy_2}{\tau} - \frac{p_2 - p_1}{t}\frac{y_1 + y_2}{2} + \left(v - \frac{\tau}{2}\right)^2\frac{(y_2 - y_1)^2}{t\tau}\right] + p_1\left[\left(1 - \frac{p_1}{t}\right)\frac{y_1 + y_2}{2} - \left(v - \frac{\tau}{2}\right)^2\frac{(y_2 - y_1)^2}{2t\tau}\right]$$

$$\pi_2 = \alpha\left[\frac{p_2}{t} - \frac{p_1}{t}\frac{vy_1 + (\tau - v)y_2}{\tau} - \frac{p_2 - p_1}{t}\frac{y_1 + y_2}{2} + \left(v - \frac{\tau}{2}\right)^2\frac{(y_2 - y_1)^2}{t\tau}\right] + p_2\left[\left(1 - \frac{p_2}{t}\right)\frac{2 - y_1 - y_2}{2} - \left(v - \frac{\tau}{2}\right)^2\frac{(y_2 - y_1)^2}{2t\tau}\right]$$

Solving the first-order conditions with respect to premium version prices yield the following second-stage prices (as a function of locations in content space y_1 and y_2):

$$p_1 = \frac{t + \alpha}{2} - \left(v - \frac{\tau}{2}\right)^2\frac{(y_2 - y_1)^2}{2\tau(y_1 + y_2)}$$

$$p_2 = \frac{t + \alpha}{2} - \left(v - \frac{\tau}{2}\right)^2\frac{(y_2 - y_1)^2}{2\tau(2 - y_1 - y_2)}$$

We can observe from second-stage prices that, when there is no differentiation across platforms ($y_1 = y_2$), premium prices are same as in the monopoly case ($p_1 = p_2 = \frac{t+\alpha}{2}$). When platforms stream an identical content, users singlehome so prices are those of the singlehoming case in 5.1. By contrast, when platforms stream content with some degree of differentiation ($y_1 \neq y_2$) prices decrease below $\frac{t+\alpha}{2}$. Platforms therefore face a tradeoff: when they differentiate ($y_1 \to 0, y_2 \to 1$) premium versions' prices are reduced as the transport cost in terms of content increases for the average user but the number of users multihoming in the free version increases which allows platforms to increase their advertising income. The optimal location depends on the values of the relevant parameters of the model.

When $v \leq \frac{\tau}{2}$, the equilibrium is the singlehoming equilibrium with no platform differentiation: $y_1 = y_2 = \frac{1}{2}$, so all results are identical to the ones in 5.1.

When $v > \frac{\tau}{2}$, the equilibrium implies platform differentiation in the content space.

Let us analyse the derivatives of the first-stage profit functions with respect to y_1 and y_2 (as I restrict to symmetric equilibria, I impose the condition $y_1 + y_2 = 1$):

- $\frac{\partial \pi_1}{\partial y_1} < 0$ if $(t+\alpha)[(t+3\alpha)\tau - 4\alpha v] - \frac{(2v-\tau)^2(1-2y_1)}{16\tau}\{(2v-\tau)^2(1-2y_1)^2(7-6y_1) + 8[4\alpha v(1+y_1) + 2t\tau y_1 + 3\tau(\alpha - t)]\} < 0$.
- $\frac{\partial \pi_2}{\partial y_2} > 0$ if $(t+\alpha)[(3\alpha - t)\tau + 2\alpha v] + \frac{(2v-\tau)^2(2y_2-1)}{16\tau}\{(2v-\tau)^2(2y_2-1)^2(2y_2+3) + 8[(5\tau - 2v)\alpha + (2y_2-1)t\tau + 4\alpha(v-\tau){y_2}^2]\} > 0$.

The two former conditions are satisfied for all y_1 and y_2 in the region $[0,1]$ except for some cases where simultaneously α is small (close to 0) and $\frac{v}{\tau}$ is high (close to 1). In such cases, platforms differentiate in terms of content without choosing maximum differentiation (with $0 < y_1 < \frac{1}{2}$ and $\frac{1}{2} < y_2 < 1$). In all the other cases, platforms choose maximum differentiation (with $y_1 = 0$ and $y_2 = 1$). This high (if not full) degree of differentiation explains why there is no multihoming in the premium version: users who are most likely to multihome are those who are close to the center of the location space (with $\tilde{y} \approx \frac{1}{2}$). As platforms locate at the extremes of the content space (or very close to the extremes) the transport cost suffered by those central users in terms of content is relevant so they cannot afford incurring in such a cost twice and also pay the subscription prices of both streaming services.

I now summarize the values of prices, demands and profits which take place in the most frequent equilibrium, which corresponds to the case of maximum differentiation between platforms, with $y_1 = 0$ and $y_2 = 1$.

Premium versions' prices are

$$p_1 = p_2 = \frac{t+\alpha}{2} - \frac{1}{2\tau}\left(v - \frac{\tau}{2}\right)^2 \tag{40}$$

the number of premium versions' subscribers are

$$n_1^p = n_2^p = \frac{t-\alpha}{4t} - \frac{1}{4t\tau}\left(v - \frac{\tau}{2}\right)^2 \tag{41}$$

the number of free versions' singlehoming users for each platform are

$$n_1^f = n_2^f = \frac{\tau - v}{2t\tau}\left[t + \alpha - \frac{1}{\tau}\left(v - \frac{\tau}{2}\right)^2\right] \tag{42}$$

the number of users who multihome in the free versions are

$$n_{12}^f = \frac{(t+\alpha)(2v-\tau)}{2t\tau} + \frac{3\tau - 2v}{2t\tau^2}\left(v - \frac{\tau}{2}\right)^2 \tag{43}$$

yielding the following advertising prices:

$$p_1^a = p_2^a = \frac{(t+\alpha)\alpha v}{2t\tau} + \frac{(2\tau - v)\alpha}{2t\tau^2}\left(v - \frac{\tau}{2}\right)^2 \tag{44}$$

and profits

$$\pi_1 = \pi_2 = \frac{(t+\alpha)(4\alpha v + t\tau - \alpha\tau)}{8t\tau} + \frac{4\alpha\tau - 2\alpha v - t\tau}{4t\tau^2}\left(v - \frac{\tau}{2}\right)^2 + \frac{1}{8t\tau}\left(v - \frac{\tau}{2}\right)^4. \tag{45}$$

By comparing these results with those in the previous subsection, we can highlight the main implications of allowing multihoming streaming users: First, the subscription fee decreases and is now lower than the monopoly. Second, fees are lower the more platforms differentiate. Third, the degree of differentiation is maximal except in some cases where simultaneously the user externality on advertisers is sufficiently small and the value of the service relative to the transport cost in terms of content is sufficiently high (a combination which occurs in a very limited number of cases). Fourth, the total number of subscribers is reduced. Fifth, the advertising income increases if $\alpha > \frac{(v-2\tau)\left(v-\frac{\tau}{2}\right)}{\tau^3} - t$. Finally, platforms are better off with multihoming users if the following condition holds: $4\alpha\tau(t+\alpha) + 2(4\alpha\tau - 2\alpha v - t\tau)\left(v - \frac{\tau}{2}\right) + \tau\left(v - \frac{\tau}{2}\right)^3 > 0$.

6. Conclusions

This paper analyses, through a theoretical two-sided market model, the pricing decision of a platform which provides a streaming service and considers whether to run a premium (advertising-free) version in addition to a free version, financed from advertising revenues. When pricing the premium version, the platform faces a trade-off as a lower subscription fee increases the number of subscribers but reduces the number of users of the free version and thus the ability to obtain profits from advertisers. The key assumption of the model is that platform users are heterogeneous in the degree of nuisance suffered from advertising.

The results show that the platform runs a premium version only if the value of the multimedia content for users and/or the network effects from users to advertisers are not too high. Otherwise the platform earns a higher profit by running the free version only. By adding the premium version, the platform attracts those users with a higher nuisance from advertising and the subscription fee is higher the higher the average level of nuisance suffered by users and the higher the externality from users to advertisers, as the platform has to compensate the income lost from lower advertising income with a higher income from users of the premium version.

Adding extra features to the premium version is profitable for the firm if this does not require a high fixed cost. The extra content always increases the subscription fee and allows the platform to have more subscribers if the marginal utility of those features for users is higher than the marginal cost for the firm. The platform may also increase the number of subscribers and the subscription fee by introducing advertising of the premium version in the free version, but this reduces the advertising income from the free version. The firm increases its profits by doing this when the level of nuisance caused by this self-advertising activity is considerably high, which helps explain why the ads of premium versions that we usually observe in free versions are so frequent and disturbing.

With the aim of doing the model more realistic, we analyze an alternative setup where the decision to subscribe to the premium version depends on the user's disposable income in addition to externality nuisance. The subscription fee for the premium version is lower than in the benchmark case in order to foster more subscriptions. In the last monopoly extension, we allow higher income consumers to be more appealing for advertisers. The subscription fee increases as the platform tries to induce some high-income users to use the free version to increase the advertising revenue. This price increase is feasible as the remaining subscribers are the ones with the highest income. Platform's profit may be either higher or lower in the latter case than in the one with independence of user income on network externality depending on the values of nuisance and the average network externality.

The monopoly model is finally extended by considering competition by means of a duopoly model where firms compete for users but not for advertisers. The symmetric version (where users of both platforms are equally appealing for advertisers) is solved showing minimum differentiation of platforms in equilibrium as they choose the same location in the streaming content space. Despite this lack of differentiation, user heterogeneity allows the platforms to charge the monopoly price, so each duopolist earns a profit equal to half the platform's profit in the monopoly model.

Even though it may seem surprising, anecdotal evidence for the music streaming market tends to support this result as we usually observe that platforms which started offering the service first do not change their prices when a new platform enters the market and competes with the incumbent. Additionally, new entrants tend to match the incumbent's price. The minimum differentiation result is also quite well supported by evidence as we observe that different streaming platforms do not tend to specialize in different content like music or movie gender, but they all try to stream the most popular contents. In any case, platforms in real life are differentiated in more than one dimension (the model, as typically happens, is a simplification of reality) and there may be some differences across platforms in terms of the extra features but there are no big differences in the main features of platforms.

Finally, we allow users to multihome, i.e. to use both platforms either in the free or in the premium version. In this case, platforms differentiate from each other and subscription fees are reduced when the value of externality from users to advertisers is not very small (in such a case, with $\alpha \leq \frac{1}{2}$) there is no multihoming users in equilibrium.

When there is multihoming, the degree of differentiation tends to be maximum to incentivize more users to use both platform but this happens only in the free version. Platform differentiation increases the average transport cost in terms of content space reducing the utility of premium multihoming users. This result of maximum differentiation across platforms is not well supported by anecdotal evidence for the most popular music or movies-and-series streaming services. This may actually imply that the externality to advertisers from users of streaming services is not significantly high so most services obtain most of their income from premium users so they don't choose a high degree of differentiation from other platforms.

Appendix 1. Duopoly model with $\alpha_1 > \alpha_2$

Let us now describe the demand functions of platforms 1 and 2 when the users of platform 1 are more appealing for advertisers than those of platform 2 ($\alpha_1 > \alpha_2$). As we have seen in the model in both the monopoly and duopoly setups, with results in (4) and (24), the subscription fee is always positively related to the level of the externality from users to advertisers. This implies that the previous inequality eventually leads to a higher subscription fee in platform 1 than in platform 2 ($p_1 > p_2$).

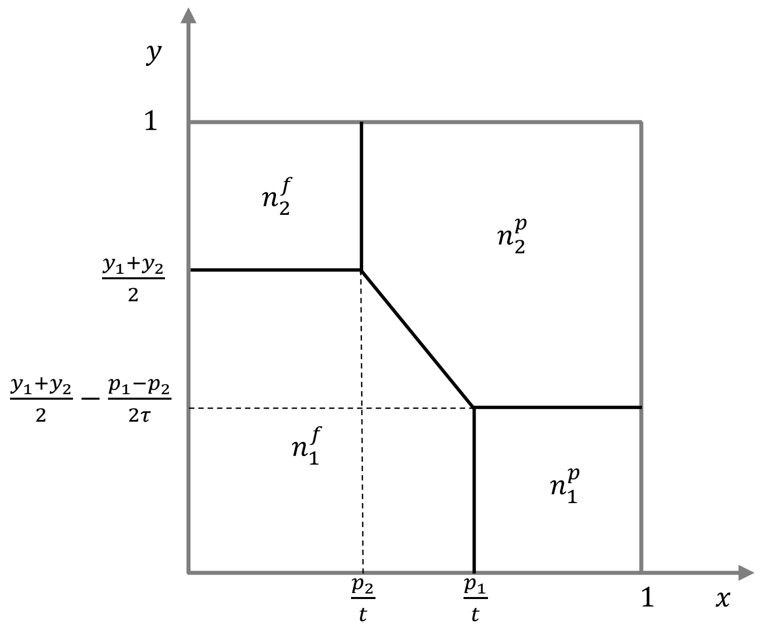

Figure A1: Duopoly model with $\alpha_1 > \alpha_2$

The demand functions, as a function of subscription fees 2 (p_1 and p_2) and location variables (y_1 and y_2), are:

$$n_1^f = \frac{p_1}{t}\frac{y_1 + y_2}{2} - \frac{(p_1 - p_2)^2}{4t\tau}$$

$$n_1^p = \left(1 - \frac{p_1}{t}\right)\left(\frac{y_1 + y_2}{2} - \frac{p_1 - p_2}{2\tau}\right)$$

$$n_2^f = \frac{p_2}{t}\left(1 - \frac{y_1 + y_2}{2}\right)$$

$$n_2^p = \left(1 - \frac{p_2}{t}\right)\left(1 - \frac{y_1 + y_2}{2}\right) + \frac{p_1 - p_2}{2\tau}\left(1 - \frac{p_1 + p_2}{2t}\right)$$

Appendix 2. Duopoly model with $\alpha_2 > \alpha_1$

Let us now describe the demand functions of platforms 1 and 2 when the users of platform 2 are more appealing for advertisers than those of platform 1 ($\alpha_2 > \alpha_1$). In this case, the subscription fee in platform 2 than in platform 1 ($p_2 > p_1$).

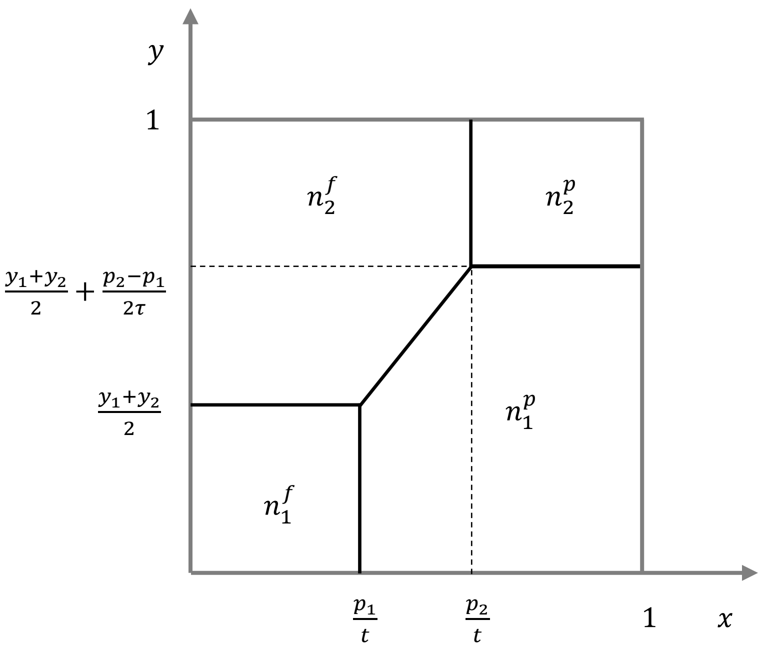

Figure A2: Duopoly model with $\alpha_2 > \alpha_1$

The demand functions, as a function of subscription fees 2 (p_1 and p_2) and location variables (y_1 and y_2), are:

$$n_1^f = \frac{p_1}{t}\frac{y_1 + y_2}{2}$$

$$n_1^p = \left(1 - \frac{p_1}{t}\right)\frac{y_1 + y_2}{2} + \left(1 - \frac{p_1 + p_2}{2t}\right)\frac{p_2 - p_1}{2\tau}$$

$$n_2^f = \frac{p_2}{t}\left(1 - \frac{y_1 + y_2}{2}\right) - \frac{(p_2 - p_1)^2}{4t\tau}$$

$$n_2^p = \left(1 - \frac{p_2}{t}\right)\left(1 - \frac{y_1 + y_2}{2} - \frac{p_2 - p_1}{2\tau}\right)$$

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