Start-up Acquisitions and the Entrant's and Incumbent's Innovation Portfolios*

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First draft: March 2023

Abstract

An entrant and an incumbent engage in an investment portfolio problem where each chooses how to allocate its research funds across a rival market, where they compete with one another, and a non-rival market, where they do not interact. Allowing for acquisitions distorts both players' incentives to allocate funding across their rival and non-rival projects. We show conditions under which the incumbent, anticipating the rents that accrue from the monopolization of the rival market, moves R&D resources from other markets to the rival market. This "incumbency for buyout effect" lowers the expected rents the entrant may obtain from the contestable market, which gives it incentives to move its investment portfolio away from the rival market. We show that this strategic effect dominates the usual "innovation for buyout effect" when the entrant's bargaining power is below a threshold. Allowing for acquisitions may improve the direction of innovation of each of the players and consumer surplus.

JEL Classification: O31, L13, L41

Keywords: start-up acquisitions, innovation portfolios, incumbency for buyout, innovation for buyout, strategic effect

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*This paper has benefited from presentations at the Frankfurt School of Finance and Management and Goethe University Frankfurt, and the VII Workshop on Knowledge, Innovation and Internationalization Strategies (University of Valencia, November 2022).

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1 Introduction

> "I remember your internal post about how Instagram was our threat and not Google+. You were basically right. One thing about startups though is you can often acquire them."
>
> *Mr. Zuckerberg on April 9, 2012, the day Facebook announced it was acquiring Instagram, cited in FTC vs. Facebook, Case No.: 1:20-cv-03590.*

> "Examples of things we could scale back or cancel: . . . Mobile photos app (since we're acquiring Instagram)."
>
> *Mr. Zuckerberg on April 22, 2012, cited in FTC vs. Facebook, Case No.: 1:20-cv-03590.*

The study of the (anti-)competitive effects of start-up acquisitions has recently received a great deal of attention. While some authors have emphasized the "killer acquisitions" phenomenon (see e.g. Cunningham et al. (2021) and Motta and Peitz (2021)) by which incumbent firms that buy start-ups discontinue some of their innovation projects, others have put forward the "innovation for buyout" effect (e.g. Cabral (2021); Hollenbeck (2020) and Katz (2021)), which refers to the idea that allowing start-ups to exit via acquisitions boost their incentives to innovate in order to appropriate a significant share of the acquisition rents.

With some exceptions, notably Katz (2021), the literature on start-up acquisitions has focused on the effects of acquisitions on the entrants' investment incentives and has, to a large extent, ignored their impact on the acquiring firms. This omission is important because of at least two reasons. First, incumbent firms do pursue their own innovative projects and, when acquisitions are allowed, they may distort their investment plans to strengthen their competitive position vis-à-vis the entrant in order to merge on more favorable terms. This is what Katz (2021) calls the "incumbency for buyout" effect. On the other hand, by acquiring a start-up, an incumbent may forgo its own research effort in the area of business interaction with the entrant and focus on other business areas. This is the "buy vs. build" trade-off mentioned by Caffarra et al. (2020) which has the potential to cause undesirable "reverse killer acquisitions", as the above quote on *Facebook* scaling back its work on mobile photos app suggests. In this paper, by examining how start-up acquisitions affect not only the investment plan of the target firm but also that of the acquirer, we give the aspect of strategic interaction in the investment market a central role.

We present a model of an industry where an incumbent and a start-up entrant interact in the innovation and product markets. Both firms have a fixed R&D budget, or alternatively a fixed number of scientist-hours. Initially, the incumbent is active as a monopolist in two markets. In each of these markets, the incumbent originally sells low-quality products but can make investments to improve their quality. One of these markets is alluded to as the *rival* market and the other as the *non-rival* market. This nomenclature is meant to refer to the idea that the entrant can challenge the position of the incumbent in the former market but not in the latter. In fact, the entrant can make investments to enter the rival market and another, third, non-rival market. The outcomes of the research projects are stochastic. A project may turn successful or unsuccessful, with the probability of project success being increasing in the amount of investment allocated to it. A successful outcome in a project allows the entrant to enter the market corresponding to that project. When the entrant successfully enters the rival market, it enters with a high-quality product. Likewise, a successful project allows the incumbent to improve the quality of its offering in the corresponding market. When the incumbent succeeds in the rival market, it also offers a high-quality product. We are interested in how permitting acquisitions distorts the players' incentives to allocate funding across rival and non-rival projects and how this bears on the direction of innovation and consumer surplus.

The interaction between incumbent and entrant is modeled as a three-stage game, with an additional bargaining stage if an acquisition is allowed. In the first stage, the start-up allocates its research budget over the rival and non-rival projects. In the second stage, upon observing the outcome of the entrant's projects, the incumbent apportions its R&D resources over the rival and non-rival projects. In the last stage, if acquisitions are not allowed, the start-up and the incumbent engage in strategic competition to serve the rival market, while each player serves its non-rival market. If acquisitions are allowed, they always take place, the merged entity serves all markets and the incumbent and entrant bargain to appropriate a share of the monopolization rents generated by the acquisition.

We first examine the impact of acquisitions on the investment portfolios chosen by the target and the acquirer. The key to understand how acquisitions distort the incentives to allocate funding across projects is to realize that firms distribute their funding across their investment opportunities so as to equalize the marginal returns from their investments. Hence, if permitting acquisitions makes a project relatively more attractive for a player, then acquisitions will imply a shift of resources towards the project whose relative profitability rises, to the detriment of the alternative project.

This insight helps us easily explain how the incumbent adapts its investment portfolio in anticipation of the acquisition of the entrant. First, notice that an acquisition alters the returns from the rival project (because of the acquisition rents created by monopolization of the rival market) but does not affect those from the non-rival project. Hence, when the acquisition rents accruing to the incumbent in case of successfully innovating in the rival market are greater than when failing to innovate, permitting acquisitions results in the incumbent shifting resources from the independent market project to the rival market project. Otherwise, allowing acquisitions results in the incumbent giving up the rival market and focusing its research effort on its independent market. Therefore, the so-called "incumbency for buyout" effect of Katz (2021) manifests itself here as a change in the investment portfolio –and consequently in the incumbent's direction of innovation– that depends on how the expected acquisition rents in case of project success and failure rank. We provide a micro-founded example of interaction in the product market showing that the incumbent acquisition rents in case of success are higher than in case of failure provided that the high-quality product's quality is sufficiently high compared to the low-quality product.

We now discuss how the entrant adjusts its investment portfolio in anticipation of its acquisition. We observe that the entrant's change in its investment incentives is driven by two economic forces. The first force is the "innovation for buyout effect", that is, the mere anticipation of being bought by the incumbent gives a start-up incentives to increase investment (Rasmusen (1988); Cabral (2021); Motta and Peitz (2021)). Because the rents from the entrant's independent project are not affected by the prospect of an acquisition, by this effect the entrant tends to move resources from the non-rival project to the rival project. The second effect is a strategic effect that arises because the entrant's and the incumbent's investments in the rival project are strategic substitutes. By this effect, the entrant, anticipating the incumbent will decrease investment in the rival project, will tend to strategically increase it, but anticipating the opposite the entrant will tend to strategically decrease its investment in the rival project. Hence, the innovation for buyout effect and the strategic effect operate in the same direction when the incumbent's expected acquisition rents in case of project failure are higher than in case of project success, in which case the entrant moves its investment portfolio towards the rival project. When the opposite holds and the incumbent's expected acquisition rents in case of project success are higher than in case of project failure, then the innovation for buyout effect and the strategic effects operate in opposite directions. We show that the relative strength of these two effects depends on the bargaining power of the target and the acquirer. Specifically, the innovation for buyout effect dominates the strategic effect when the entrant's bargaining power is relatively large. In such a case, the incumbent changes its investment portfolio little in anticipation of the acquisition of the target and, hence, the strategic effect is of limited size. Consequently, acquisitions result in both the entrant and incumbent tilting their research portfolios towards the contestable market. By contrast, when the entrant's bargaining power is low, the strategic effect dominates the innovation for buyout effect and acquisitions result in the incumbent investing more in the rival market and the entrant shying away from it and focusing on other markets.

Having described how acquisitions alter the investment portfolios of the players, we now relate how acquisitions impact the efficiency of their investment portfolios. We show that circumstances exist under which allowing for acquisitions improves the direction of innovation for both the entrant and incumbent, though there also exist other contexts where the opposite occurs. Specifically, when an acquisition causes the entrant to incline its investment portfolio towards the contestable market and the incumbent to give it up and move research resources away from it, then the direction of innovation improves provided that the surplus of consumers in the entrant's alternative market is low and that in the incumbent's alternative market is high. The conditions simply require that resources are moved towards markets with high returns for consumers. Hence, the direction of innovation also improves in the opposite situation where an acquisition causes the entrant to give up on the contestable market and the incumbent to invest in it more aggressively provided that the surplus of consumers in the entrant's alternative market is high and that in the incumbent's alternative market is low. Last but not least, we show that the efficiency enhancing effect of acquisitions on the direction of innovation may be sufficiently large so as to dominate the negative price effects of acquisitions. In those cases, thus, permitting acquisitions improves the direction of innovation and increases consumer surplus.

To conclude, we draw two important insights for antitrust from our paper. First, we find that tighter regulations on acquisitions may either positively or negatively impact the direction of innovation and consumer welfare. As such, we suggest avoiding blanket prohibitions on start-up acquisitions and instead conducting a case-by-case assessment to determine the potential benefits and drawbacks. Second, we argue that examining the acquisition of start-ups by innovative incumbents in multi-project settings using the traditional *definition-of-the-market* approach overlooks a crucial factor: the shift of R&D resources towards or away from non-overlapping areas of business of the target and the acquirer, which causes significant welfare effects.

The remainder of the paper is structured as follows. In the next subsection, we provide an overview of the relevant literature. Section 2 presents the model. Section 3 offers a general characterization of the solution to the investment portfolio problem. This general solution applies irrespective of whether the decision maker is the start-up, incumbent, joint entity or social planner. Section 4 derives the socially optimal investment portfolios of target and acquirer and Sections 5 and 6 the equilibria of the no-acquisition and acquisition games. In Section 7, we investigate how the prospect of an acquisition distorts the start-up's and incumbent's investment portfolios. Section 8 evaluates how acquisitions impact the players' direction of innovation activity from a social welfare point of view, while Section 9 assesses the impact of permitting acquisitions on consumer welfare. Section 10 concludes the paper by offering some policy recommendations. All the proofs are relegated to the Appendix.

1.1 Related literature

Our paper is a contribution to the understanding of the effects of start-up acquisitions on innovation. Specifically, by modelling investment portfolios, our paper focuses on the effect of permitting acquisitions on the direction of innovation; moreover, by allowing both the target and the acquirer to invest in R&D projects, our paper captures strategic interaction in the innovation market in the context of start-up acquisitions.

The study of start-up acquisitions has attracted much effort in recent years, and our study builds upon existing work in this area. In a seminal contribution, Cunningham et al. (2021) demonstrated the possible occurrence of the "killer acquisitions" phenomenon whereby, owing to the force of the well-known "Arrow replacement effect", incumbents discontinue the research projects of acquired firms. Although Greenstein and Ramey (1998), Chen and Schwartz (2013) and Motta and Peitz (2021) point out that it is possible that the replacement effect is weaker for incumbents than for entrants and hence killer acquisitions need not materialize, Cunningham et al. (2021) and Gautier and Lamesch (2021) provide empirical evidence for this phenomenon in the pharmaceutical and digital markets, respectively.[1]

Several authors have highlighted a potentially positive aspect of start-up acquisitions: the "innovation for buyout" effect. This refers to the increased incentives for start-ups to innovate when they anticipate being acquired (see Rasmusen (1988); Cabral (2018); Hollenbeck (2020); Katz (2021)). However, other authors have pointed out that this effect need not be definitely beneficial. For instance, Bryan and Hovenkamp (2020) present a dynamic multiple-incumbents model in which start-ups choose to innovate in a way that benefits the market leader rather than the laggards, thereby perpetuating the dominance of the leader firm. Further, Kamepalli et al. (2020) suggest that start-up acquisitions may discourage entry into markets with network externalities because consumers may be less inclined to adopt a new technology from a new entrant if they anticipate the entrant will be acquired by an incumbent. Finally, Denicolò and Polo (2021) argue that while the innovation for buyout effect may boost innovation in the short-term, repeated acquisitions can reinforce the incumbent's dominance over time, leading to an "entrenchment of monopoly" effect, which can stifle innovation in the long run. Our contribution to this line of work is showing that the "innovation for buyout" effect may be offset by a strategic effect by which the entrant, anticipating the incumbent to "defend" its dominance in the contestable market, gives it up and focuses on other non-rival activities.

The innovation for buyout effect affects not only the intensity of innovation but also its nature. For example, Callander and Matouschek (2021) and Gilbert and Katz (2022) show that start-ups, anticipating being acquired by an incumbent, tend to choose products that are more similar to those of the incumbent, rather than products that are more horizontally differentiated. Moreover, Warg (2022) finds that start-ups are more likely to develop substitute products rather than complementary ones when they anticipate being acquired by an incumbent. Our paper is complementary to this line of inquiry because it, rather than asking how acquisitions affect the nature of the product developed by the entrant, it asks which products get developed.

Our contribution is more closely related to a group of papers in the literature that explore the effects of start-up acquisitions not only on entrants but also incumbents. For instance, as

[1] A related study is Fumagalli et al. (2020), which shows that an acquisition may be beneficial for consumers if the entrant is severely financially constrained despite incumbents having less incentive to innovate than entrants due to the Arrow replacement effect.

far as we know, Katz (2021) is the first paper mentioning the "incumbency for buyout" effect, whereby an incumbent may make investments to extract rents from an entrant through a merger. Further, Motta and Shelegia (2021) allow incumbents to use strategies to protect their market power and show that the possibility of imitation by the incumbent pushes the entrant to develop a complement product, rather than a substitute. Permitting acquisitions enhances start-ups' incentives to enter the incumbent's market by introducing a rival product. Furthermore, in a model with multiple start-ups, Teh et al. (2022) show that allowing acquisitions may create "kill zones" for non-targeted start-ups. This occurs because non-acquired start-ups, anticipating tougher competition from the incumbent upon acquiring the latest technology, choose to develop a weak substitute or even a non-rival product.

Finally, our paper relates to a cluster of papers studying firm decision-making in multi-project settings. The study of multi-project settings is a central focus in the works of Gilbert (2019); Letina et al. (2020); Letina (2016), as well as other studies on the direction of innovation (Bryan and Lemus, 2017; Bryan et al., 2022; Chen et al., 2018; Hopenhayn and Squintani, 2021). The main difference between these papers and ours is that these studies typically assume firms work on developing one project only but must explore multiple research avenues to potentially discover a successful one. In contrast, our study specifically examines the impact of start-up acquisitions on multi-project R&D firms. In this sense, the closest paper to the current one is our own previous work Dijk et al. (2022). While our earlier contribution focuses solely on the impact of start-up acquisitions on the entrant's investment incentives and direction of innovation while assuming the incumbent remains passive, our current paper takes the strategic interaction between the entrant and the incumbent in the innovation market to its heart. Combining the well-known "innovation for buyout" and "incumbency for buyout" effects into a single framework delivers the importance insight that strategic effects may completely offset the incentives to invest in rivalry projects.

2 The model

We study an industry with an incumbent (I) and a start-up entrant (E). Initially, the incumbent is active as a monopolist in markets A and C. In both markets, the incumbent originally sells products of low quality but can make investments to improve the quality of its products. Likewise, the entrant can invest to enter in one of these markets, say market A, and in another market B. The focus of our paper is on how the incumbent and the entrant allocate funding to their research projects and how this allocation is affected by acquisitions. We assume that both I and E have fixed R&D budgets, which we normalize to one.[2] The R&D budget of a firm can also be interpreted as the fixed number of scientist-hours it has at its disposal.

[2]JL: Think about I and E having different budgets.

We model the interaction between the incumbent and the start-up as a three-stage game. In the first stage of the game, *the entrant's innovation stage*, the start-up chooses its investment portfolio. Specifically, the entrant chooses how much funding to allocate to projects A and B. The entrant can only enter these markets upon successful completion of the projects. Let x^E be the start-up's investment in project A and, correspondingly, let $1-x^E$ be its investment in project B. Following Moraga-González et al. (2022) and Dijk et al. (2022), the probabilities the entrant successfully completes the projects are given by the Tullock specifications:

$$p(x^E, \epsilon_A) = \frac{x^E}{x^E + \epsilon_A} \text{ and } q(1-x^E, \epsilon_B) = \frac{1-x^E}{1-x^E+\epsilon_B}. \tag{1}$$

The parameters ϵ_A and ϵ_B proxy for the innovation difficulty of the projects. The success probabilities increase in investment and decrease in innovation difficulty. Adopting the Tullock functional form is useful because the investment portfolio problem is strictly concave in own investment. Moreover, when $\epsilon_A \to 0$ and $\epsilon_B \to 0$ the maximization problem of the entrant has an interior solution and can be computed in closed form by solving the first order condition (FOC) for expected profit maximization (see Section 3). If project A is successful, the entrant enters market A with a product of higher quality than the basic quality of the incumbent. If project A is unsuccessful, the entrant stays out of market A and gets zero profits. Likewise, if project B is successfully concluded, the entrant enters market B; otherwise, it stays out.

In the second stage of the game, *the incumbent's innovation stage*, the incumbent chooses its investment portfolio. Specifically, the incumbent chooses how much money to allocate to projects A and C. The investment in the common market A will naturally depend on whether the entrant has failed (f) of succeeded (s) to enter market A in stage 1. Hence, let x_j^I be the incumbent's (conditional) investment in project A and, correspondingly, let $1-x_j^I$ be its investment in project C, where $j=f,s$. Similarly, as for the entrant, the probabilities the incumbent successfully completes the projects are given by:

$$p(x_j^I, \epsilon_A) = \frac{x_j^I}{x_j^I + \epsilon_A} \text{ and } q(1-x_j^I, \epsilon_C) = \frac{1-x_j^I}{1-x_j^I+\epsilon_C}, \ j=f,s. \tag{2}$$

Successful completion of project A allows the incumbent to increase its product quality. Specifically, we assume that this quality improvement allows the incumbent to match the quality of the entrant in case the latter has entered the market. Likewise, if project C is successfully terminated, the incumbent increases the quality of its offering in market C.

If acquisitions are allowed there is a third stage of the game, *the bargaining stage*, in which the incumbent and the start-up (Nash-)bargain over the rents created by the acquisition. We denote the bargaining power of the entrant by $1-\delta$ and that of the incumbent by δ. If acquisitions are not allowed, this stage is skipped.

Finally, the last stage of the game, *the market stage*, depends on whether the entrant enters markets A and B and whether acquisitions are allowed or not. If the start-up enters the rival market A and acquisitions are not allowed, the start-up and the incumbent engage in strategic interaction to serve consumers. Likewise, if the start-up enters the non-rival market B, then it serves its consumers. The incumbent serves consumers in the non-rival market C. Otherwise, if acquisitions are allowed, the merged entity serves all markets A, B, and C.

We solve the game by backward induction. In principle, we do not explicitly model the last stage of the game and specify the payoffs from the strategic interaction in market A as follows. There are four sub-games. First, suppose that both the start-up and the incumbent successfully complete project A. In that case, both firms compete in market A to sell a high-quality product so we have a continuation symmetric duopoly game. We denote their payoff as π_{hh}. Second, suppose now that the start-up successfully completes project A but the incumbent fails. In that case, the continuation game is asymmetric because the entrant sells a high-quality product while the incumbent the basic low-quality one. Let $\pi_{h\ell}$ and $\pi_{\ell h}$ denote the payoffs of the start-up and the incumbent, respectively. Naturally, $\pi_{h\ell} \geq \pi_{hh} \geq \pi_{\ell h}$. Third, suppose the start-up fails to complete project A but the incumbent succeeds. In that case, the entrant does not enter market A and the incumbent operates in market A as a monopolist selling a high-quality product. We denote the payoff it obtains by π_h^m. Finally, suppose both the start-up and the incumbent fail to complete project A. In that case, the entrant does not enter and the incumbent operates as a monopolist in market A selling a low-quality product. We denote the payoff it obtains by π_ℓ^m. Naturally, $\pi_h^m \geq \pi_\ell^m$.

We provide a schematic representation of our model in Figure 1. We refer to market A as the "rival market" to emphasize that successful completion of the project by the entrant breaks the initial monopoly position of the incumbent. Projects B and C are "non-rival" or independent projects in the sense that project B has nothing to do with the incumbent's business and similarly project C does not affect the entrant's profits. We specify the payoffs from successfully completing those projects as π_B and π_C. Projects B and C shape the optimal portfolios of the entrant and the incumbent, respectively, but they do not affect the acquisition rents directly. However, as we have explained in the introduction, they play a very important role when assessing the social welfare implications of acquisitions.

When necessary to obtain further insights from our model, we impose additional structure. Specifically, we sometimes invoke an explicit model of strategic interaction in market A. When we do this, we assume that demand in market A stems from a unit mass of consumers with the well-known quality-augmented quadratic utility function introduced in Sutton (1997) (see also Sutton (2001)):

$$U^A = \sum_{i=1}^{2}\left[\alpha q_i - \left(\frac{\beta q_i}{s_i}\right)^2\right] - \sigma \sum_{i=1}^{2}\sum_{j<i}\frac{\beta q_i}{s_i}\frac{\beta q_j}{s_j} - \sum_{i=1}^{2} p_i q_i.$$

For tractability reasons, we assume away horizontal product differentiation by setting $\sigma = 2$. The incumbent's basic product has quality $s_\ell > 0$. If the start-up's investment effort in project

A turns out to be successful, we assume that the start-up enters the incumbent's market offering a product of higher quality s_h than that of the incumbent, with $s_\ell < s_h < 2s_\ell$.[3] Otherwise, the start-up does not enter. The start-up and the incumbent engage in quantity competition in market A. We normalize the marginal cost of production to zero.

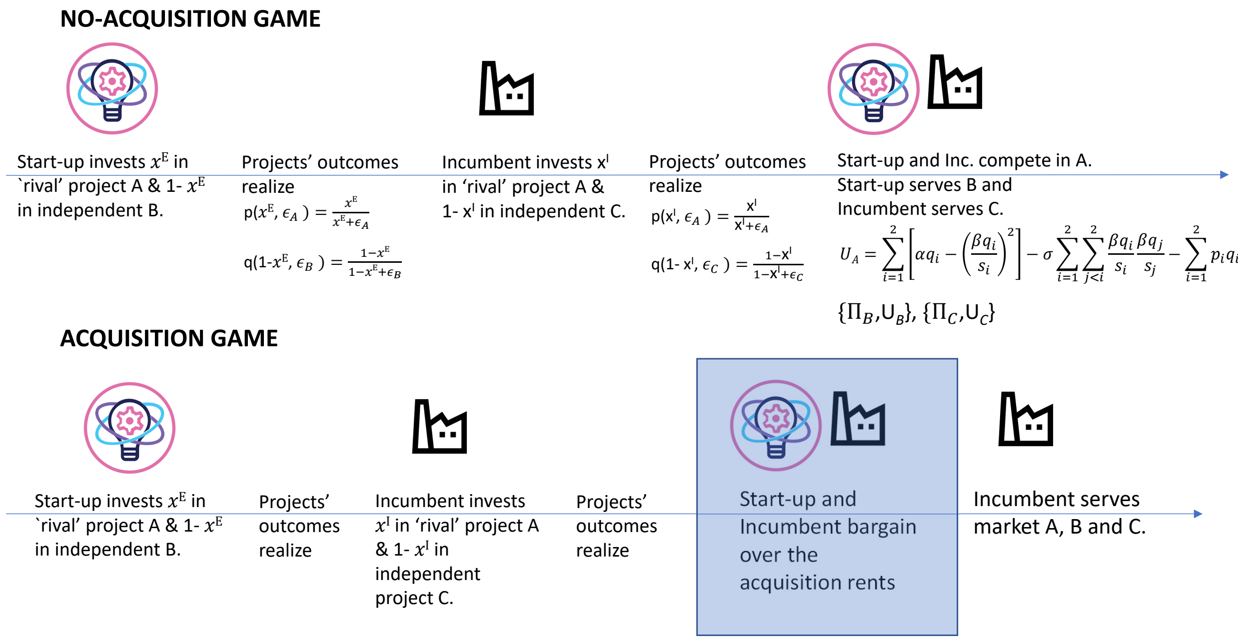

Figure 1: Schematic representation of the games.

3 The investment decision

This section provides an auxiliary result that will be used repeatedly in the rest of the paper. The auxiliary result describes how the agents choose their investment portfolios to maximize their expected payoffs. The result is valid in all stages of the game and for all the decision makers (incumbent, entrant and social planner).

Consider a decision maker (incumbent, entrant or social planner) who picks its investment portfolio $(x, 1-x)$ to maximize its objective function (profits for the incumbent/entrant or

[3]The restriction $s_h < 2s_\ell$ rules out drastic innovations.

social welfare for the social planner) anticipating the expected (private or social) returns on the projects in which it invests. The returns on a project equal the difference between its reward in case of success and its reward in case of failure. Correspondingly, let R_A^S denote the rewards, be private or social, from investing in the rival project A when it turns out successful, and R_A^F the rewards when it fails. Define R_z^S and R_z^F (which we have normalized to zero) similarly. Then, the problem of a decision maker is to maximize an expected returns expression of the form:

$$\mathbb{E}R(x) = \frac{x}{x+\epsilon_A} R_A^S + \frac{\epsilon_A}{x+\epsilon_A} R_A^F + \frac{1-x}{1-x+\epsilon_z} R_z^S, \tag{3}$$

where z can be either B or C, depending on which decision maker is considered. The first term of this expression is the probability that the rival project A turns successful, times its corresponding payoff. The second term is the probability that the rival project A fails, times its payoff in such a case. Finally, the third term is the probability that the non-rival project z succeeds times its payoff in that event.

The expression in (3) is strictly concave in x. Therefore, the first-order condition (FOC) for maximization of (3) suffices for a maximum. Taking the FOC and solving for x gives

$$x\left(R_A/R_z; \epsilon_A, \epsilon_z\right) = \frac{1 + \epsilon_z - \epsilon_A \sqrt{\frac{\epsilon_z}{\epsilon_A}\frac{R_z}{R_A}}}{1 + \sqrt{\frac{\epsilon_z}{\epsilon_A}\frac{R_z}{R_A}}}, \tag{4}$$

where we have used the notation $R_A \equiv R_A^S - R_A^F$ and $R_z \equiv R_z^S - R_z^F$.

The function given in expression (4) will repeatedly be used in the rest of the paper. In choosing the optimal portfolio of investments, what matters for the decision maker is the ratio of relative returns R_A/R_z. Depending on the decision maker or the market structure, the returns R_A and R_z will take on different values, which we will specify later. However, the solution of the optimization problem of the decision maker will always have the form given in expression (4). Hence, we will no longer write this expression but instead refer to equation (4) and indicate the returns that have to be plugged in for projects A, B, and C.

Inspection of (4) immediately reveals that the optimal investment level in the rival project A is increasing in the relative returns on the projects R_A/R_z for all non-rival projects z and all decision makers. Hence, a decision maker will move its investment portfolio toward a particular project when that project's innovation returns increase relative to those of the alternative one. This observation makes it relatively easy to examine how investment portfolios compare across decision makers and market structures. For example, to examine whether the private equilibrium is efficient the only thing we have to do is to compare the relative returns of the entrant and the incumbent with the relative returns of the social planner. Finally, we note also that the optimal investment level is decreasing in ϵ_A and increasing in ϵ_z for all z so when the difficulty of a project decreases relative to the difficulty of the alternative one the decision maker tilts its investment portfolio toward the simpler project.

4 Socially optimal investments

Before analyzing the (no-acquisition and acquisition) games outlined above, we characterize the socially optimal investment portfolios for the entrant and incumbent. While doing this, we take the case in which acquisitions are not allowed as a benchmark and further we assume that the planner cannot control the firms' production levels.

We start by characterizing the socially optimal choice of incumbent investments. This choice occurs after the results of the start-up's projects are realized and hence depends on whether the entrant has successfully entered the rival market A or not.

Suppose the entrant has not entered market A. Then, anticipating that in case of success the incumbent will stay as a monopolist in market A selling high quality and in case of failure too but selling low quality, the socially optimal incumbent's portfolio follows from (4) and is given by:

$$x_f^{I,o} = x\left(\frac{U_C}{U_h^m - U_\ell^m}\right), \tag{5}$$

where U_h^m and U_ℓ^m are the surpluses consumers obtain in market A when it is served by a monopolist selling high or low quality, respectively.

Suppose now that the entrant has successfully entered market A. In that case, the socially optimal incumbent portfolio is given by:

$$x_s^{I,o} = x\left(\frac{U_C}{U_{hh} - U_{\ell h}}\right) \tag{6}$$

Here U_{hh} is the consumer surplus corresponding to the case in which both the start-up and the incumbent produce high quality, thereby the sub-index hh. The expression $U_{\ell h}$ refers to the case in which the incumbent sells low quality and the start-up sells high quality.

We now move to stage 1 where the planner chooses the entrant's portfolio that maximizes consumer surplus. In doing so, the planner anticipates the possible outcomes of the continuation game. Again, making use of (4), the socially optimal entrant's portfolio is:

$$x^{E,o} = x\left(\frac{U_B}{\frac{x_s^{I,o}U_{hh}+\epsilon_A U_{h\ell}}{x_s^{I,o}+\epsilon_A} + \frac{1-x_s^{I,o}}{1-x_s^{I,o}+\epsilon_C}U_C - \left(\frac{x_f^{I,o}U_h^m+\epsilon_A U_\ell^m}{x_f^{I,o}+\epsilon_A} + \frac{1-x_f^{I,o}}{1-x_f^{I,o}+\epsilon_C}U_C\right)}\right). \tag{7}$$

In this expression, notice that the social returns from the entrant successfully entering market A are given by the first two summands in the denominator of the RHS of (7). The first of these two summands equals the expected surplus from market A. This expected surplus "integrates" over the outcomes that may realize after incumbent investment. While deriving this expected value, we take into account the socially optimal investment (conditional) investment $x_s^{I,o}$. Meanwhile, the second summand is the expected surplus from market C. Likewise, the social returns from the entrant failing to enter market A are given by the third and fourth summands in the denominator of the RHS of (7). The third summand is the expected surplus from market A where we now "integrate" over the outcomes that may realize after incumbent (conditional) investment $x_f^{I,o}$ is put in. The fourth summand is again the expected surplus from market C.

The expressions for $x_f^{I,o}$, $x_s^{I,o}$ and $x^{E,o}$ are the socially optimal investment for the entrant and the incumbent. In what follows, we compare these investments with the private investments corresponding to the no-acquisition and acquisition games. Such comparisons give rise to our results on the (in-)efficiency of the private equilibria and on the impact of acquisitions on the direction of the innovative efforts of the entrant and the incumbent.

5 The no-acquisition game

In this section, we solve the no-acquisition game. As mentioned above, this game has three stages. In the first stage, the start-up invests x^E in the rival project A and $1 - x^E$ in the independent project B. In the second stage, once the results of the entrant's projects have been realized, the incumbent invests x^I in the rival project A and $1 - x^I$ in the independent project C. Finally, in the third stage, once the results of the incumbent's projects have been realized, if the start-up has successfully entered market A, then the entrant and the incumbent compete in market A; otherwise, if the start-up has failed to entered the market, the incumbent serves it on its own. Moreover, if the the start-up has entered market B, it serves market B. Finally, the incumbent serves market C.

To characterize the equilibrium (conditional) investments of the incumbent, we again take advantage of equation (4). Suppose the entrant has failed to enter market A in which case the incumbent will stay as the only supplier of market A. Anticipating that a successful project A will return a profit level π_h^m and an unsuccessful project A will yield profits equal to π_ℓ^m, the equilibrium investment in project A equals:

$$x_f^I = x\left(\frac{\pi_C}{\pi_h^m - \pi_\ell^m}\right) \tag{8}$$

The rest of the budget $1 - x_f^I$ is invested in project C. Suppose now that the entrant has successfully entered market A. Then, the incumbent, anticipating that a successful project A will return a profit level π_{hh} and an unsuccessful project A will yield profits equal to $\pi_{\ell h}$, chooses to invest in project A an amount equal to:

$$x_s^{I,na} = x\left(\frac{\pi_C}{\pi_{hh} - \pi_{\ell h}}\right) \tag{9}$$

The rest of the budget $1 - x_s^{I,na}$ is invested in project C.[4]

[4]We need to compare x_f^I, $x_s^{I,na}$ and $x_s^{I,a}$

We now move to the first stage of the game where the entrant chooses its investment portfolio. Plugging the difference between the entrant's expected returns in case of a successful and an unsuccessful project A in (4) we obtain the equilibrium investment of the entrant in project A:

$$x^{E,na} = x\left(\frac{\pi_B}{\frac{x_s^{I,na}\pi_{hh}+\epsilon_A\pi_{h\ell}}{x_s^{I,na}+\epsilon_A}}\right). \tag{10}$$

In this expression, $x_s^{I,na}$ is the anticipated investment in project A of the incumbent given by (9).

Notice that, because $\pi_{h\ell} > \pi_{hh}$, the entrant's effort put into project A is a decreasing function of the anticipated incumbent's effort in the same project. We return to this observation at the end of Section 6.

6 The acquisition game

In this section, we solve the acquisition game. As described above, this game has four stages. In the first stage, the start-up chooses its portfolio of investments. In the second stage, once the results of the entrant's projects have been realized, the incumbent picks its investment portfolio. In the third stage, once the results of the incumbent's projects have been realized, the start-up and the incumbent bargain over the expected acquisition rents. This stage is only meaningful if the entrant's investment in project A turns successful, in which case an acquisition always occurs; otherwise, this stage is void of meaning. Finally, in the fourth stage, the joint entity serves markets A, B and C.

To characterize the equilibrium (conditional) investments of the incumbent in the acquisition game we again use equation (4). In this equation we need to factor the difference between the incumbent's returns from a successful and an unsuccessful project A. Compared to the no-acquisition game, this difference varies due to the monopoly rents an acquisition generates in market A. (An acquisition does not generate additional rents in independent markets B and C.) Hence, suppose first the incumbent's investment in project A is successful. In such a case, the acquisition rents equal the extra profits from monopolizing a market initially served by two sellers of high quality, i.e. $\pi_h^m - 2\pi_{hh} > 0$. By contrast, when the incumbent's investment in project A is unsuccessful, the acquisition rents equal the excess profits of monopolizing a market with one seller of high quality and one seller of low quality, i.e. $\pi_h^m - \pi_{\ell h} - \pi_{h\ell} > 0$.

The Nash bargaining solution implies that the start-up and the incumbent divide the available surplus in proportions corresponding to their bargaining powers. Hence, using (4), in the acquisition game the incumbent's investment in project A conditional on the entrant's successfully entering market A is given by:

$$x_s^{I,a} = x\left(\frac{\pi_C}{\pi_{hh} + \delta(\pi_h^m - 2\pi_{hh}) - (\pi_{\ell h} + \delta(\pi_h^m - \pi_{\ell h} - \pi_{h\ell}))}\right). \tag{11}$$

Notice that the reason why the investment of the incumbent in the acquisition case (11) differs from that in the no-acquisition case (9) is purely driven by rent-seeking. In fact, the profits of the joint entity equal π_h^m no matter whether the incumbent invests in project A or not. The only reason why the incumbent puts effort into project A is to enhance its bargaining position vis-à-vis the entrant. Naturally, such an incentive is modulated by the bargaining power δ. If the incumbent could not capture any of the monopoly rents, it would not change its investment portfolio despite anticipating the acquisition of the start-up. Obviously, if the entrant fails to enter market A, the incumbent's investment in project A is the same as in the no-acquisition scenario and is given by (8). For completeness, we write:

$$x_f^{I,a} = x_f^{I,na}.$$

We now move back to the first stage of the game where the start-up chooses its portfolio of investments. In doing so, the entrant must anticipate the incumbent's equilibrium investment portfolio and the induced bargaining rents brought about by the acquisition. Plugging in (4) the difference between the entrant's expected returns in case of a successful and an unsuccessful project A we get the start-up's equilibrium investment portfolio:

$$x^{E,a} = x\left(\frac{\pi_B}{\frac{x_s^{I,a}[\pi_{hh}+(1-\delta)(\pi_h^m-2\pi_{hh})]+\epsilon_A[\pi_{h\ell}+(1-\delta)(\pi_h^m-\pi_{h\ell}-\pi_{\ell h})]}{x_s^{I,a}+\epsilon_A}}\right) \tag{12}$$

In this expression $x_s^{I,a}$ is the anticipated incumbent's investment in project A and is given by (11). The entrant's returns from a successful project A depend, on the one hand, on the outcome of the investment effort of the incumbent and, on the other hand, the outcome of the Nash bargaining with the incumbent over the monopolization rents. When the incumbent's project is successful, the entrant's bargaining rents amount to $(1-\delta)(\pi_h^m - 2\pi_{hh})$. However, when the incumbent's project is unsuccessful, the start-up's bargaining rents equal $(1-\delta)(\pi_h^m - \pi_{h\ell} - \pi_{\ell h})$. The denominator of the argument of $x(\cdot)$ on the RHS of (12) thus gives the entrant's expected returns from investing in project A.

We finish this section by pointing out the strategic substitutability between the incumbent's investment in the rival market A and the entrant's investment in the same market.

Lemma 1 *Irrespective of whether acquisitions are permitted or not, the entrant will cut its investment in the rival project A if it anticipates an increase in the incumbent's effort into the same project.*

Lemma 1 implies that the incumbent's investment in the rival project A is a strategic substitute to the entrant's investment in the same project. Hence, an anticipation that the incumbent will invest more in the rival project A to defend its monopoly power in such a market will result in the entrant "giving up" and cutting investment in that project.

7 The impact of acquisitions on the entrant's and incumbent's investment portfolios

We are now ready to examine the impact acquisitions have on the investment portfolios of the entrant and the incumbent.

Proposition 1 *(a) Suppose that acquisitions are allowed and $\pi_h^m - 2\pi_{hh} < \pi_h^m - \pi_{h\ell} - \pi_{\ell h}$. Then, the incumbent will invest less in the rival project A (and hence more in the alternative project C) while the entrant will invest more in the rival project A (and thus less in the alternative project B).*

(b) Suppose that acquisitions are allowed and, alternatively, $\pi_h^m - 2\pi_{hh} > \pi_h^m - \pi_{h\ell} - \pi_{\ell h}$. Then:

(i) If the following condition holds:

$$\delta > \bar{\delta}(\epsilon_A, \epsilon_C, \pi_C, \pi_h^m, \pi_{hh}, \pi_{h\ell}, \pi_{\ell h}), \tag{13}$$

the incumbent will invest more in the rival project A (and hence less in C) while the entrant will invest less in the rival project A (and thus more in B).

(ii) Otherwise, if (13) does not hold, both the incumbent and the entrant will invest more in the rival project A (and so less in the alternative projects B and C).

Proposition 1 shows that acquisitions have a bearing on both the investment portfolios of target and acquirer. As explained above, both players' incentives to invest in the rival and non-rival markets are shaped by the expected relative returns from these projects. Compared to the case in which acquisitions are forbidden, acquisitions modify the returns from these projects because the players anticipate getting a share of the rents from monopolization of the rival market. If for a player these (expected) rents are larger when project A is successful than when it is not then the player in question will invest more in project A in anticipation of an acquisition.

We now provide intuition for the conditions in Proposition 1. Consider first the incumbent. Anticipating the acquisition of the entrant, the incumbent will invest more in project A (and so less in C) whenever $\pi_h^m - 2\pi_{hh} > \pi_h^m - \pi_{h\ell} - \pi_{\ell h}$. This condition simply says that the monopolization rents from market A are greater when the incumbent produces high quality than when it produces low quality. In such a case, because the incumbent shares in the monopolization rents, its incentives to invest in project A will go up if acquisitions are permitted. In contrast, when this condition does not hold, the monopolization rents from market A are lower when the incumbent produces high quality than when it produces low quality, in which case the incumbent will invest less in the rival market A when acquisitions are allowed. The impact of permitting acquisitions on the incumbent's incentives to allocate funding across the rival and non-rival projects is the analog of Katz's (2021) "*incumbency for buyout*" effect, whereby an incumbent invests to strengthen its competitive position vis-á-vis the entrant in order to merge on more favorable terms. In our setting, the "incumbency for buyout" effect manifests itself as a change in the incumbent's direction of innovation and we see that the nature of this effect depends on market fundamentals.

Consider now the entrant. The impact of allowing acquisitions on the entrant's incentives to invest is a bit more complex because of strategic interaction. To see this, it is useful to assume momentarily the case of a "naive" incumbent that would not change its investment anticipating the acquisition of the entrant (or else an incumbent with no bargaining power whatsoever). If the incumbent did not alter its investment portfolio at all, the entrant, anticipating its acquisition by the incumbent, would always increase its investment in the rival market A. This is what Cabral (2021) and Motta and Peitz (2021) call the "*innovation for buyout effect*": anticipating to gain monopolization rents in market A as a result of integration (and not in the alternative market B), the entrant's incentives to invest in the rival market go up compared to when acquisitions are not allowed. The "innovation for buyout effect" thus incentivizes the entrant to move research resources from the non-rival market to the rival market.

However, because the incumbent can also invest to protect its rents from the rival market A, we have an additional "*strategic effect*". This strategic effect arises due to the strategic substitutability of the players' investments in market A. By this strategic effect, anticipating that the incumbent will invest more (less) in the rival market A, the entrant will reduce (raise) its investment in such a market.

Hence, expecting the incumbent to cut investment in market A, the "innovation for buyout effect" and the strategic effect operate together to boost the entrant's incentives to move funding towards the rival market A. This is the result in Proposition 1(a). By contrast, when the entrant anticipates the incumbent to be increasing its investment in market A, the "innovation for buyout effect" and the strategic effect operate in opposite directions. Proposition 1(b) provides the condition (13) under which the strategic effect has a dominating influence and therefore the entrant reduces its investment in the rival market. When condition (13) does not hold, the "innovation for buyout effect" is stronger than the strategic effect and the opposite occurs.

We illustrate Proposition 1 in Figure 2 using the micro-founded model with quadratic utility function and Cournot competition. To construct this figure, we need expressions for the payoffs that appear in the proposition. These payoffs are straightforward to compute and we omit the details:

$$\pi_{hh} = \frac{\alpha^2 s_h^2}{18\beta^2},\ \pi_h^m = \frac{\alpha^2 s_h^2}{8\beta^2},\ \pi_{h\ell} = \frac{\alpha^2(2s_h - s_\ell)^2}{18\beta^2},\ \pi_{\ell h} = \frac{\alpha^2(2s_\ell - s_h)^2}{18\beta^2}.$$

Using these formulas, the condition $\pi_h^m - 2\pi_{hh} < \pi_h^m - \pi_{h\ell} - \pi_{\ell h}$ in the proposition is equivalent to $s_h < \frac{5}{3}s_\ell$. This condition is depicted by the dashed vertical threshold in Figure 2. Therefore, when $s_h < \frac{5}{3}s_\ell$ anticipating the acquisition of the entrant the incumbent will cut investment in the rival project (and increase it in the non-rival one) while the entrant will do exactly the opposite. This occurs in Region I of Figure 2. Otherwise, when $s_h > \frac{5}{3}s_\ell$ the incumbent will increase investment in the rival project (and cut it in the non-rival one) while the entrant, depending on whether the parameters satisfy condition (13), which is represented by the dashed blue curve in the graph, will do the same or the opposite. Hence, in Region II of Figure 2 the entrant will reduce investment in project A and the incumbent will increase it, while in Region III both the entrant and the incumbent will raise investment in the rival project A.

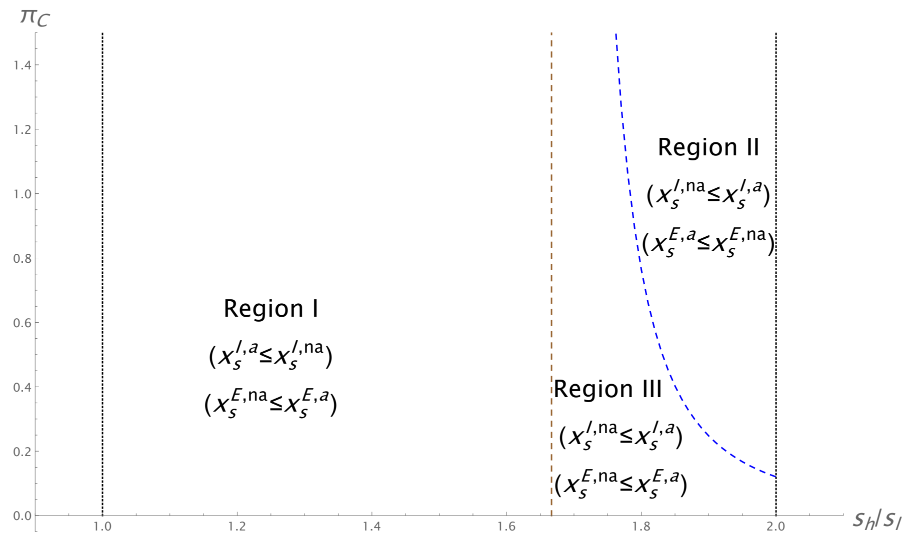

Figure 2: The impact of acquisitions on the entrant's and incumbent's investment portfolios.

We finish this section by commenting on the importance of the bargaining power parameter δ in shaping the players' incentives to adjust their investment portfolios in anticipation of an acquisition. Specifically, we note that the threshold dashed blue curve that separates Regions II and III shifts south-westwards as the incumbent bargaining power goes up. To see what this means, recall that if δ were very small the incumbent would almost not change its investment portfolio and, hence, absence the strategic effect the entrant would definitely tilt its investment portfolio towards the rival market. Region II would simply be empty in the limit when $\delta \to 0$. As δ increases, the strategic effect starts playing a significant role and Region II begins to exist. The higher δ the more important is the strategic effect and the less important is the innovation for buyout effect. Hence, region II expands and region III shrinks, thereby making it more likely that the entrant reduces investment in anticipation of its acquisition. In the limit when δ approaches 1, region III vanishes because the bargaining power of the entrant is negligible so that the innovation for buyout effect does no longer play a role.

8 On the (in-)efficiency of entrant's and incumbent's investment portfolios

In this section, we compare the entrant's and incumbent's investment portfolios in the no-acquisition and acquisition games to the socially optimal portfolios in order to derive conditions under which acquisitions improve or worsen the direction of innovation.

Consider first the incumbent's investment portfolio and focus on the case in which the entrant enters market A for otherwise allowing or disallowing acquisitions is inconsequential. Comparing (6) with (9), it is straightforward to conclude that when acquisitions are forbidden the incumbent excessively tilts its investment portfolio towards market A if and only if:

$$\frac{\pi_C}{U_C} < \frac{\pi_{hh} - \pi_{\ell h}}{U_{hh} - U_{\ell h}}. \tag{14}$$

The condition is rather intuitive. On the LHS we have a measure of appropriability of social surplus in market C, while on the RHS we have a similar measure but for market A. When social surplus appropriability in market C is small compared to appropriability in market A, the incumbent inefficiently invests too little in market C (and hence too much in A). If the condition holds with the opposite sign, then the incumbent inefficiently invests too little in project A.

Comparing now (6) with (11), we obtain a condition under which the incumbent, anticipating the acquisition of the entrant, inefficiently invests too much in project A if and only if:

$$\frac{\pi_C}{U_C} < \frac{\pi_{hh} + \delta(\pi_h^m - 2\pi_{hh}) - \pi_{\ell h} - \delta(\pi_h^m - \pi_{\ell h} - \pi_{h\ell})}{U_{hh} - U_{\ell h}}. \tag{15}$$

Like before, if the condition holds with the opposite sign, then the incumbent inefficiently invests too little in project A. The interpretation of this condition is similar to the one in the no-acquisition case.

We now examine the (in)-efficiency of the entrant's investment portfolios. Considering first the no-acquisition game, a comparison of (7) and (10) immediately yields the conclusion that the entrant over-invests in the rival project A if and only if:

$$\frac{\pi_B}{U_B} < \frac{\frac{x_s^{I,na}\pi_{hh} + \epsilon_A \pi_{h\ell}}{x_s^{I,na} + \epsilon_A}}{\frac{x_s^{I,o} U_{hh} + \epsilon_A U_{h\ell}}{x_s^{I,o} + \epsilon_A} + \frac{1 - x_s^{I,o}}{1 - x_s^{I,o} + \epsilon_C} U_C - \left(\frac{x_f^{I,o} U_h^m + \epsilon_A U_\ell^m}{x_f^{I,o} + \epsilon_A} + \frac{1 - x_f^{I,o}}{1 - x_f^{I,o} + \epsilon_C} U_C \right)} \tag{16}$$

If the inequality holds the other way around, then the entrant over-invests in the independent project B. Although this condition is more intricate than that for the incumbent, the intuition is similar. The condition basically states that the entrant will over-invest in the rival project when the surplus appropriability in such a project is higher than in the alternative project.

Finally, in the acquisition game, comparing (7) and (12) we come to the conclusion that entrant over-invests in the rival project A if and only if:

$$\frac{\pi_B}{U_B} < \frac{\frac{x_s^{I,a}[\pi_{hh}+(1-\delta)(\pi_h^m-2\pi_{hh})]+\epsilon_A[\pi_{h\ell}+(1-\delta)(\pi_h^m-\pi_{h\ell}-\pi_{\ell h})]}{x_s^{I,a}+\epsilon_A}}{\frac{x_s^{I,o}U_{hh}+\epsilon_A U_{h\ell}}{x_s^{I,o}+\epsilon_A}+\frac{1-x_s^{I,o}}{1-x_s^{I,o}+\epsilon_C}U_C-\left(\frac{x_f^{I,o}U_h^m+\epsilon_A U_\ell^m}{x_f^{I,o}+\epsilon_A}+\frac{1-x_f^{I,o}}{1-x_f^{I,o}+\epsilon_C}U_C\right)} \tag{17}$$

As before, if this condition holds with the opposite sign then the entrant under-invests in A and over-invests in B.

8.1 The overall impact of acquisitions on the direction of innovation

Equipped with the conditions presented above on the occurrence of excessive or insufficient investment in the rival project A, we can now show that circumstances exist under which allowing for acquisitions improves the direction of innovation for both the entrant and incumbent. We start with a case in which in the absence of acquisitions the incumbent over-invests in A while the entrant under-invests in A.

Proposition 2 *Assume that $\pi_h^m - 2\pi_{hh} < \pi_h^m - \pi_{h\ell} - \pi_{\ell h}$ so that by Proposition 1(a), anticipating an acquisition, the incumbent reduces investment in the rival market and the entrant increases it. Assume further that*

$$\frac{\pi_C}{U_C} < \frac{\pi_{hh}+\delta(\pi_h^m-2\pi_{hh})-\pi_{\ell h}-\delta(\pi_h^m-\pi_{\ell h}-\pi_{h\ell})}{U_{hh}-U_{\ell h}} \tag{18}$$

and that

$$\frac{\pi_B}{U_B} > \frac{\frac{x_s^{I,a}[\pi_{hh}+(1-\delta)(\pi_h^m-2\pi_{hh})]+\epsilon_A[\pi_{h\ell}+(1-\delta)(\pi_h^m-\pi_{h\ell}-\pi_{\ell h})]}{x_s^{I,a}+\epsilon_A}}{\frac{x_s^{I,o}U_{hh}+\epsilon_A U_{h\ell}}{x_s^{I,o}+\epsilon_A}+\frac{1-x_s^{I,o}}{1-x_s^{I,o}+\epsilon_C}U_C-\left(\frac{x_f^{I,o}U_h^m+\epsilon_A U_\ell^m}{x_f^{I,o}+\epsilon_A}+\frac{1-x_f^{I,o}}{1-x_f^{I,o}+\epsilon_C}U_C\right)}. \tag{19}$$

Then, if acquisitions are allowed, both the incumbent and the entrant improve the direction of their innovation portfolios.

Proposition 2 provides a first set of conditions under which permitting acquisitions increases the efficiency of the investment portfolios of both the acquirer and the target. Specifically, when conditions (18) and (19) hold, this boost in efficiency occurs in Region I of Figure 2 where, anticipating an acquisition, the incumbent reduces investment in the rival market and the entrant increases it. Conditions (18) and (19) basically state that, compared to project A, the appropriability of social surplus for project C is small while for project B is large, which ensure that in both the no-acquisition and acquisition games, the incumbent over-invests in A while the entrant under-invests in A. Because the incumbent's acquisition rents satisfy the inequality $\pi_h^m - 2\pi_{hh} < \pi_h^m - \pi_{h\ell} - \pi_{\ell h}$ the incumbent, anticipating the acquisition of the start-up, moves funds away from the rival project to the independent project. This increases the efficiency of its innovation portfolio given that it over-invests in A. Likewise, the entrant, anticipating its acquisition and the change in the incumbent's investment portfolio, raises its effort in the rival project and reduces it in the alternative project. Given that the entrant under-invests in A, allowing for acquisitions increases the efficiency of its innovation portfolio.

We illustrate Proposition 2 in Figure 3 using the micro-founded model with quadratic utility function and Cournot competition. This figure shows the effect of acquisitions on the efficiency of the incumbent's and entrant's investment portfolios. We build this figure for parameters satisfying the inequality $\pi_h^m - 2\pi_{hh} < \pi_h^m - \pi_{h\ell} - \pi_{\ell h}$, thereby corresponding to region I in Figure 2 where the incumbent cuts investment in the rival market and the entrant increases it. On the vertical axis, we place the payoff π_C of the incumbent in case its investment in the non-rival project turns out successful. Because in this figure we fix the value of U_C, a higher value of π_C can be interpreted as higher social surplus appropriability in market C. On the horizontal axis we place π_B/U_B, i.e. a measure of appropriability in the entrant's non-rival project. The blue-dashed horizontal line is condition (14) and hence delimits the lower parameter space for which the incumbent's investment in the contestable market in the no-acquisition game is excessive from a social welfare viewpoint. Likewise, the red-dashed horizontal line is condition (15) thereby demarcating the lower region of parameters for which the incumbent's investment in the contestable market in the acquisition game is excessive from an efficiency point of view. Together, these two observations mean that if parameters fall in the area below the red-dashed horizontal line, then an acquisition enhances the efficiency of the incumbent's investment portfolio (because its investment in the rival market is excessive and, anticipating the acquisition of the entrant, the incumbent decreases it).

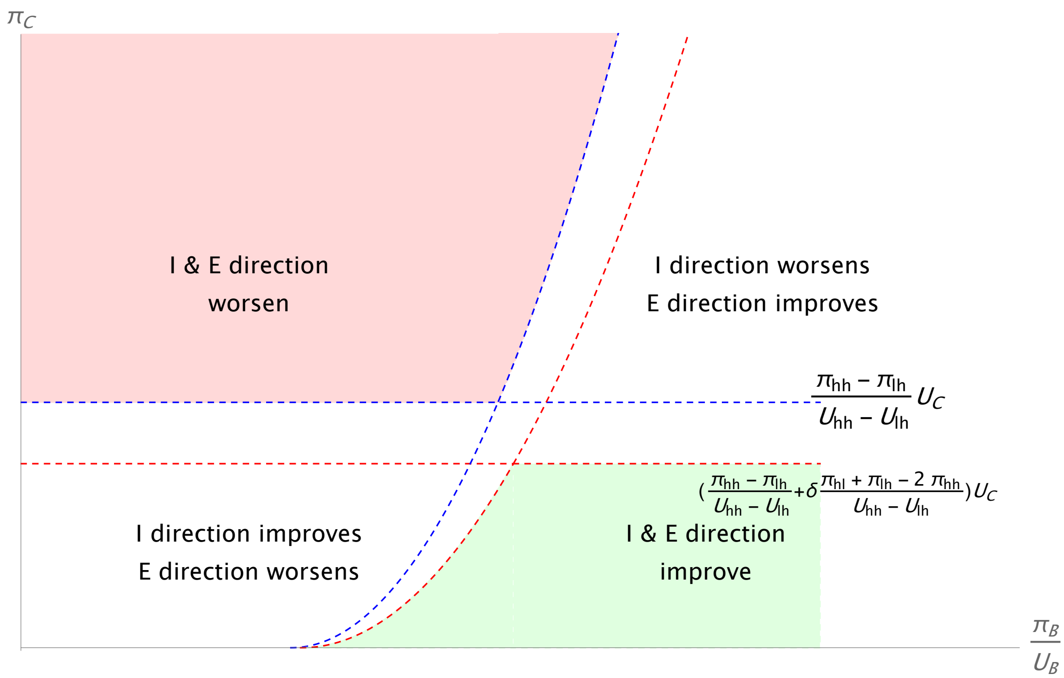

Figure 3: Parameters areas where acquisitions improve (green) or worsen (red) both target's and acquirer's innovation direction ($\pi_h^m - 2\pi_{hh} < \pi_h^m - \pi_{h\ell} - \pi_{\ell h}$)

In the figure there are also two dashed increasing curves; these two curves refer to the entrant. The blue-dashed one is condition (16) which demarcates the right parameter space for which the entrant's investment in the rival market in the no-acquisition game is insufficient from an efficiency point of view. Likewise, the red-dashed curve is condition (17) which delimits the right region of parameters for which the entrant's investment in the rival market in the acquisition game is insufficient. Together, these two observations imply that if parameters fall in the area to the right of the red-dashed curve, then an acquisition enhances the efficiency of the entrant's investment portfolio (because its investment in the rival market is insufficient and, anticipating its acquisition, the entrant increases it).

Overall, we conclude that permitting acquisitions increases the efficiency of both the entrant's and the incumbent's investment portfolios in the green area. Having explained this, it is straightforward to see that in the red area, both players investment portfolios get worse from an efficient point of view. Moreover, in the most south-west parameter area the incumbent's direction of innovation improves while that of the entrant worsens. The opposite occurs in the most north-east parameter area.

Our next two results show that it is also possible that acquisitions increase the efficiency of the acquirer's and target's investment portfolios when the parameters fall in Regions II and III of Figure 2. Recall that these regions arise when the incumbent's acquisition rents in case of a successful project A are higher than in case the project turns out unsuccessful. Anticipating such conditional flows of rents, the incumbent raises its investment in the rival project and decreases it in the non-rival one.

Proposition 3 *(a) Assume that* $\pi_h^m - 2\pi_{hh} > \pi_h^m - \pi_{h\ell} - \pi_{\ell h}$ *and that condition (13) holds so that by Proposition 1(b)(i), anticipating an acquisition, the incumbent raises investment in the rival market and the entrant cuts it. Assume further that*

$$\frac{\pi_C}{U_C} > \frac{\pi_{hh} + \delta(\pi_h^m - 2\pi_{hh}) - \pi_{\ell h} - \delta(\pi_h^m - \pi_{\ell h} - \pi_{h\ell})}{U_{hh} - U_{\ell h}} \tag{20}$$

and that

$$\frac{\pi_B}{U_B} < \frac{\frac{x_s^{I,a}[\pi_{hh}+(1-\delta)(\pi_h^m-2\pi_{hh})]+\epsilon_A[\pi_{h\ell}+(1-\delta)(\pi_h^m-\pi_{h\ell}-\pi_{\ell h})]}{x_s^{I,a}+\epsilon_A}}{\frac{x_s^{I,o}U_{hh}+\epsilon_A U_{h\ell}}{x_s^{I,o}+\epsilon_A} + \frac{1-x_s^{I,o}}{1-x_s^{I,o}+\epsilon_C}U_C - \left(\frac{x_f^{I,o}U_h^m+\epsilon_A U_\ell^m}{x_f^{I,o}+\epsilon_A} + \frac{1-x_f^{I,o}}{1-x_f^{I,o}+\epsilon_C}U_C\right)}. \tag{21}$$

Then, if acquisitions are allowed, both the incumbent and the entrant improve the direction of their innovation portfolios.

(b) Assume again that $\pi_h^m - 2\pi_{hh} > \pi_h^m - \pi_{h\ell} - \pi_{\ell h}$ *and that condition (13) does not hold so that by Proposition 1(b)(ii), anticipating an acquisition, the incumbent and the entrant both raise their investments in the rival market (and lower than in the independent markets). Further, assume that (20) holds while (21) holds with the opposite sign. Then, if acquisitions are allowed, both the incumbent and the entrant improve the direction of their innovation portfolios.*

The first part of Proposition 3 states conditions under which the efficiency of the innovation portfolios of both the start-up and the incumbent improve because the incumbent raises investment in the rival market and the entrant cuts it. This occurs in Region II of Figure 2. Condition (20) means that the incumbent's investment in project A in the no-acquisition and acquisition games is insufficient from the point of view of social welfare maximization. Because the acquisition rents the incumbent gets when the rival project is successful are higher than when the project is unsuccessful, the incumbent, anticipating the acquisition of the entrant, invests more in A and less in C. This adjustment makes its innovation portfolio more efficient than when acquisitions are not permitted.

Condition (21) signifies that the start-up's investment in project A in the no-acquisition and acquisition games is excessive from the point of view of social welfare maximization. As discussed after Proposition 1, when the strategic effect of acquisitions is sufficiently strong (condition (13) holds), the entrant, anticipating its acquisition and the incumbent's move of research funds from its non-rival project to the rival one, cuts its effort in the rival project. This cut improves the efficiency of its investment portfolio.

The second part of Proposition 3 states conditions under which the efficiency of the innovation portfolios of both the start-up and the incumbent improve because both move funds from

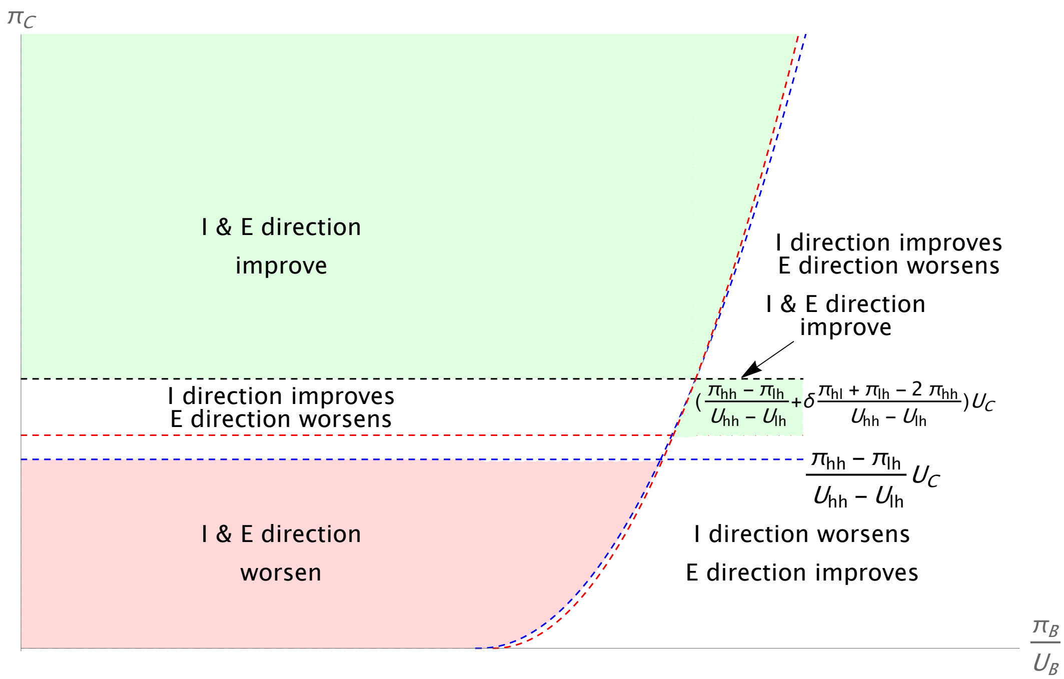

Figure 4: Parameters areas where acquisitions improve (green) or worsen (red) both target's and acquirer's innovation direction ($\pi_h^m - 2\pi_{hh} > \pi_h^m - \pi_{h\ell} - \pi_{\ell h}$)

their independent projects to the rival project. This occurs in Region III of Figure 2. The interpretation of the conditions provided is similar to Part (a) and we omit it to save space.

Similarly as we did before, we now illustrate Proposition 3 in Figure 4 using the micro-founded model. The figure, which is constructed in the same way as Figure 3, shows the effect of acquisitions on the efficiency of the portfolio of the incumbent and entrant when $\pi_h^m - 2\pi_{hh} > \pi_h^m - \pi_{h\ell} - \pi_{\ell h}$. This parameter constellation corresponds to regions II and III in Figure 2. The black-dashed horizontal line corresponds to the threshold that separates regions II and III in Figure 2. Hence, in the parameter space above this line the entrant decreases investment in the rival market in anticipation of its acquisition. The blue-dashed horizontal line again represents condition (14) so in the area above it the incumbent under-invests in the rival project in the no-acquisition game. Likewise, the red-dashed horizontal line represents condition (15) so in the area above it the incumbent under-invests in the rival project in the acquisition game. Together these conditions imply that when parameters fall above the red-dashed horizontal line, an acquisition enhances the efficiency of the incumbent's investment portfolio (because there is under-investment in the rival project and the incumbent, anticipating the acquisition of the entrant, increases it). Next, we describe the increasing dashed curves. The blue-dashed curve represents condition (16), to the right of which the entrant under-invests in the rival project in the no-acquisition game. Likewise, the red-dashed curve is condition (17), to the right of which the entrant under-invests in the rival project in the acquisition game. Together, these observations imply that if the parameters fall in the area to the right of the red-dashed curve and below the black-dashed horizontal line, then the entrant's direction of innovation will improve (because the entrant under-invests in the rival project and, anticipatingc its acquisition, the entrant invests more in it). This parameter area corresponds to Region III in Figure 2. Likewise, if the parameters fall to the left of the red-dashed curve and above the blacked-dashed horizontal line, an acquisition also enhances the efficiency of the entrant's investment portfolio (in this case because the entrant over-invests in the rival market and, anticipating its acquisition, reduces its investment in it). This parameter area corresponds to Region II in Figure 2.

Overall, we conclude that permitting acquisitions increases the efficiency of both the incumbent and entrant's portfolios in the green parameter areas. Further, both players' investment portfolios get worse from an efficiency point of view in the red area. Finally, we observe that the direction of innovation of the entrant improves, while that of the incumbent worsens in the south-east corner of the figure. The opposite happens in the north-east and mid-west part of the figure.

9 Consumer Surplus Analysis

To explore the impact of start-up acquisitions on the welfare of consumers, we compare the expressions for the *overall* consumer surplus in the no-acquisition and acquisition cases.

Consumer surplus in the no-acquisition game is given by:

$$\begin{aligned}\mathbb{E}U^{na}(x_s^{I,na}, x_f^{I}, x^{E,na}) = {} & \frac{x^{E,na}}{x^{E,na}+\epsilon_A}\left[\frac{x_s^{I,na}}{x_s^{I,na}+\epsilon_A}U_{hh} + \frac{\epsilon_A}{x_s^{I,na}+\epsilon_A}U_{hl} + \frac{1-x_s^{I,na}}{1-x_s^{I,na}+\epsilon_C}U_C\right] \\ & + \frac{\epsilon_A}{x^{E,na}+\epsilon_A}\left[\frac{x_f^{I}}{x_f^{I}+\epsilon_A}U_h^m + \frac{\epsilon_A}{x_f^{I}+\epsilon_A}U_l^m + \frac{1-x_f^{I}}{1-x_f^{I}+\epsilon_C}U_C\right] \\ & + \frac{1-x^{E,na}}{1-x^{E,na}+\epsilon_B}U_B, \end{aligned} \tag{22}$$

This expression for consumer surplus can be interpreted as follows. In the first line, we write out the expected consumer surplus resulting from the entrant entering the contestable market, which is equal to the probability of the entrant entering the market multiplied by the expected consumer surplus generated in markets A and C (given in square brackets). The latter expected consumer surplus (in the brackets) is given by the probability that the incumbent successfully innovates or fails to do so in those markets times the corresponding consumer surpluses. The second line is written following a similar logic but it corresponds to the case in which the entrant stays out of the contestable market. Finally, the last line gives the expected consumer surplus from market B, which is independent of the entrant entering or staying out of the rival market.

$$\begin{aligned}\mathbb{E}U^a(x_s^{I,a}, x_f^I, x^{E,a}) = & \frac{x^{E,a}}{x^{E,a}+\epsilon_A}\left[U_h^m + \frac{1-x_s^{I,a}}{1-x_s^{I,a}+\epsilon_C}U_C\right] \\ & + \frac{\epsilon_A}{x^{E,a}+\epsilon_A}\left[\frac{x_f^I}{x_f^I+\epsilon_A}U_h^m + \frac{\epsilon_A}{x_f^I+\epsilon_A}U_l^m + \frac{1-x_f^I}{1-x_f^I+\epsilon_C}U_C\right] \\ & + \frac{1-x^{E,a}}{1-x^{E,a}+\epsilon_B}U_B. \end{aligned} \tag{23}$$

The interpretation of this consumer surplus expression is similar.

The following result provides the consumer surplus implications of prohibiting start-up acquisitions.

Proposition 4 *(a) Assume that $\pi_h^m - 2\pi_{hh} < \pi_h^m - \pi_{h\ell} - \pi_{\ell h}$ so that, by Propositions 1(a), $x_s^{I,a} < x_s^{I,na} < x_f^I$ and $x^{E,a} > x^{E,na}$. Then, there exists $\widetilde{U}_C > 0$ such that for all $U_C > \widetilde{U}_C$, a prohibition of acquisitions results in a decrease in consumer surplus. Otherwise, a prohibition of acquisitions increases consumer surplus.*

(b) Suppose that alternatively $\pi_h^m - 2\pi_{hh} > \pi_h^m - \pi_{h\ell} - \pi_{\ell h}$, so that, by Proposition 1(b), $x_s^{I,na} < x_s^{I,a} < x_f^I$. Then, if (13) holds so that $x^{E,a} < x^{E,na}$, there exists $\widehat{U}_B > 0$ such that for all $U_B > \widehat{U}_B$, a prohibition of acquisitions results in a decrease in consumer surplus. Otherwise, a prohibition of acquisitions increases consumer surplus.

Proposition 4 examines the overall impact of permitting start-up acquisitions on the welfare of consumers by putting together its effects on both the direction of innovation and consumer prices. When an acquisition worsens the direction of innovation for both players (red-colored regions in Figures 3 and 4), consumer surplus cannot increase because acquisitions have both detrimental innovation and price effects. However, when an acquisition improves the direction of innovation for both players (green-colored regions in Figures 3 and 4) it is possible that permitting acquisitions results in an increase in consumer surplus because the positive effects on the direction of innovation may outweigh the negative price effects.

Proposition 4 puts forward two types of circumstances (parts (*a*) and (*b*)) under which the positive direction of innovation effects of an acquisition dominate the detrimental price effects. The first situation, described in Proposition 4(a), arises in a subset of the parameters of Region I of Figure 2 and is depicted by the green area at the south-west of Figure 5.[5] In this parameter space, the incumbent, in anticipation of an acquisition, moves investment funds away from the rival market while the entrant does the opposite and focuses more on the contestable market. When this occurs, under the conditions outlined in Proposition 2, the "industry" direction

[5]This figure is built from Figure 2. It uses the expressions for profits and utility from the micro-founded model with quadratic utility and Cournot competition. Parameter values are $\alpha = 4$, $\beta = 5$, $\delta = 0.85$, $\epsilon_A = 0.3$, $\epsilon_B = 5$, $\epsilon_C = 1$, $\pi_B = 1$, $U_B = 40$ and $U_C = 10$.

of innovation improves. Proposition 4(a) simply posits that when consumer surplus in the independent market C that receives additional research funds is sufficiently large, the decrease in the innovation distortion has a dominating influence over the increase in the price distortion and overall consumer surplus increases when acquisitions are allowed.

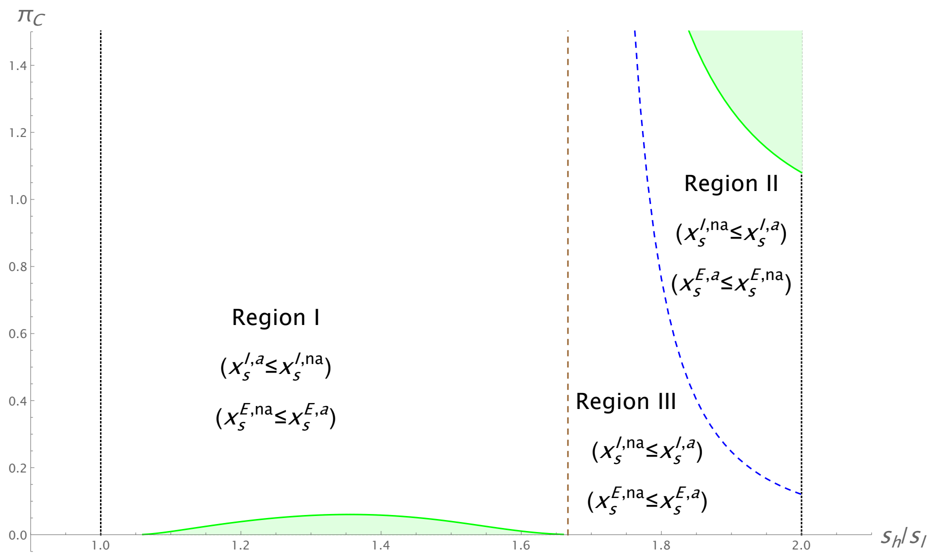

Figure 5: Parameter regions (green) for which permitting acquisitions raises consumer surplus

The second situation arises in a subset of Region II of Figure 2 and is shown by the green area at the north-east of Figure 5. In this parameter area, the incumbent, in anticipation of the acquisition of the entrant, moves investment funds towards the rival market. Anticipating this defensive strategy, the entrant, despite the incentive provided by the innovation for buyout effect, strategically reduces its investment in the rival project. In such a case, under the condition in Proposition 3(a), the "industry" direction of innovation improves. Proposition 4(b) simply states that when project B, which is receiving additional research funds in the acquisition game, is sufficiently valuable for society then the efficiency improvement in the direction of innovation dominates the negative price effects of acquisitions.

We finish this section by pointing out to the reader that the circumstances under which acquisitions are consumer welfare improving are not exhausted by those described in Proposition 4. As pointed out in Section 8.1, there exist parameter constellations for which acquisitions make the investment portfolio of one of the players more efficient while that of the other player less efficient (see regions labeled GR or RG in Figures 3 and 4). In such situations, and under conditions similar to those in Proposition 4, the improvement in the direction of innovation of just one player could be sufficient to outweigh the negative price effects of acquisitions.

10 Conclusions and policy recommendations

Our paper has provided two important insights for antitrust. The first insight is that tighter regulations on acquisitions may have a positive or negative impact on the direction of innovation and consumer welfare. Therefore, we recommend against implementing blanket prohibitions of start-up acquisitions and instead suggest a case-by-case assessment to determine the potential benefits and drawbacks. The second insight is that examining integration processes of innovative multi-project firms using the traditional *definition-of-the-market* approach overlooks the crucial fact that it is precisely the shift of resources towards and away from non-rival projects what causes significant gains and losses. We have referred to these effects as the impact on the direction of innovation.

More specifically, we provide the following guidelines for the assessment of start-up acquisitions in multi-project settings. First, because start-up acquisitions result in a shift of R&D resources away from (towards) non-overlapping areas of business and towards (and away from) overlapping ones, key variables for the assessment of start-up acquisitions are the levels of social surplus appropriability in the different business lines. This is because acquisitions that result in a move of research funds towards areas that deliver large social gains and small private gains are more likely to improve the direction of innovation and consumer surplus.

When the innovation is incremental and the incumbent moves its portfolio away from the rival market while the entrant moves it towards it, the direction of innovation improves provided that social surplus appropriability in the entrant's alternative market is high and in the incumbent's alternative market is low. If this improvement in the efficiency in the direction of innovation is sufficiently large compared to the detrimental price effects, then permitting an acquisition will result in consumer surplus increases. When the innovation is sizeable and the incumbent moves its investment portfolio towards the contestable market, two things can happen. First, the entrant may choose to also allocate more resources to the contestable market. In that case, which occurs when the incumbent's bargaining power is low, the overall direction of innovation improves when appropriability in the incumbent's alternative market is intermediate and in the entrant's alternative market is high. Second, the entrant may shy away from the contestable market. In that case, which happens when the incumbent's bargaining power is high, the industry direction of innovation improves when appropriability in the incumbent's alternative market is high and in the entrant's is low. These situations are likely to result in a consumer surplus increase when the price effects associated with the acquisition are bounded.[6]

[6]Assessing the bargaining power of a startup that does not yet have any output can be challenging, as many of the traditional indicators of bargaining power, such as market share and price-cost margins, may not be applicable. However, antitrust authorities may still be able to assess the startup's bargaining power by considering other factors. For example, they may consider the strength of the startup's intellectual property, such as patents or trademarks. If the startup's intellectual property is particularly valuable or unique, it may give the startup more bargaining power. Further, the level of investor interest in the startup can be an indicator of bargaining power. If the startup has a significant amount of funding from investors, this may indicate that investors see potential in

Appendix

Proof of Lemma 1. We prove this result for the acquisition game and notice that setting $\delta = 1$ proves it for the no-acquisition game. We first observe that

$$\pi_{h\ell} + (1-\delta)(\pi_h^m - \pi_{h\ell} - \pi_{\ell h}) > \pi_{hh} + (1-\delta)(\pi_h^m - 2\pi_{hh}).$$

To see this, rewrite this inequality as

$$\delta\pi_{h\ell} + (1-\delta)(\pi_h^m - \pi_{\ell h}) > \delta\pi_{hh} + (1-\delta)(\pi_h^m - \pi_{hh})$$

and notice that this is always true because $\pi_{h\ell} > \pi_{hh}$ and $\pi_h^m - \pi_{\ell h} > \pi_h^m - \pi_{hh}$.

The implication of this observation is that the entrant's expected returns from investing in project A, which are given by the denominator of the argument of the function $x(\cdot)$ in (12), are decreasing in $x_s^{I,a}$. Hence, $x^{E,a}$ decreases as $x_s^{I,a}$ goes up.

In the no-acquisition case, we can set $\delta = 1$ and notice that the entrant's expected returns from investing in project A, given by the denominator of the argument of the function $x(\cdot)$ in (10), are decreasing in $x_s^{I,na}$ when $\pi_{h\ell} > \pi_{hh}$. Hence, $x^{E,na}$ decreases as $x_s^{I,na}$ goes up. ■

Proof of Proposition 1. (a) When comparing (9) and (11) we see that the latter is lower when

$$\pi_h^m - 2\pi_{hh} < \pi_h^m - \pi_{h\ell} - \pi_{\ell h}.$$

Further $x^{E,na} < x^{E,a}$ holds, when (10) is lower than (12), which is equivalent to

$$\frac{x_s^{I,na}\pi_{hh} + \epsilon_A\pi_{h\ell}}{x_s^{I,na} + \epsilon_A} < \frac{x_s^{I,a}[\pi_{hh} + (1-\delta)(\pi_h^m - 2\pi_{hh})] + \epsilon_A[\pi_{h\ell} + (1-\delta)(\pi_h^m - \pi_{h\ell} - \pi_{\ell h})]}{x_s^{I,a} + \epsilon_A}. \quad (24)$$

Note that because $0 < x_s^{I,a}[(1-\delta)(\pi_h^m - 2\pi_{hh})] + \epsilon_A[(1-\delta)(\pi_h^m - \pi_{h\ell} - \pi_{\ell h})]$, we have

$$\frac{x_s^{I,a}\pi_{hh} + \epsilon_A\pi_{h\ell}}{x_s^{I,a} + \epsilon_A} < \frac{x_s^{I,a}[\pi_{hh} + (1-\delta)(\pi_h^m - 2\pi_{hh})] + \epsilon_A[\pi_{h\ell} + (1-\delta)(\pi_h^m - \pi_{h\ell} - \pi_{\ell h})]}{x_s^{I,a} + \epsilon_A}$$

Finally, since $\frac{x_s^{I,na}\pi_{hh}+\epsilon_A\pi_{h\ell}}{x_s^{I,na}+\epsilon_A} < \frac{x_s^{I,a}\pi_{hh}+\epsilon_A\pi_{h\ell}}{x_s^{I,a}+\epsilon_A}$ for all $x_s^{I,na} > x_s^{I,a}$ and $\pi_{hh} < \pi_{h\ell}$ we conclude that (24) holds and, hence, we have shown that $x^{E,na} < x^{E,a}$.

(b) When comparing (9) and (11) we see that the latter is higher when

$$\pi_h^m - 2\pi_{hh} > \pi_h^m - \pi_{h\ell} - \pi_{\ell h}.$$

the startup and believe it will have bargaining power in the future. Lastly, antitrust authorities may also consider the competitive landscape in the startup's industry. If the startup is entering a market with few competitors, it may be able to negotiate more favorable terms.

The condition (13) follows from comparison of $x^{E,na}$ in (10) and $x^{E,a}$ in (12). [7]

b(i). $x^{E,a} < x^{E,na}$ is equivalent to

$$\frac{x_s^{I,a}[\pi_{hh} + (1-\delta)(\pi_h^m - 2\pi_{hh})] + \epsilon_A[\pi_{h\ell} + (1-\delta)(\pi_h^m - \pi_{h\ell} - \pi_{\ell h})]}{x_s^{I,a} + \epsilon_A} < \frac{x_s^{I,na}\pi_{hh} + \epsilon_A\pi_{h\ell}}{x_s^{I,na} + \epsilon_A}, \quad (25)$$

which can be rewritten as

$$\delta > \frac{\left(x_s^{I,a}\left(\pi_{hh} + \pi_h^m - 2\pi_{hh}\right) + \epsilon_A\left(\pi_{h\ell} + \pi_h^m - \pi_{h\ell} - \pi_{\ell h}\right)\right)\left(x_s^{I,na} + \epsilon_A\right) - \left(x_s^{I,na}\pi_{hh} + \epsilon_A\pi_{h\ell}\right)\left(x_s^{I,a} + \epsilon_A\right)}{\left(x_s^{I,na} + \epsilon_A\right)\left(x_s^{I,a}(\pi_h^m - 2\pi_{hh}) + \epsilon_A(\pi_h^m - \pi_{h\ell} - \pi_{\ell h})\right)}.,$$

where

$$\delta > \bar{\delta}(\epsilon_A, \epsilon_C, \pi_C, \pi_h^m, \pi_{hh}, \pi_{h\ell}, \pi_{\ell h}).$$

b(ii). While we have $x^{E,a} > x^{E,na}$, when condition in (25) does not hold.

It is also possible to show that both conditions in b(i) and b(ii) are not empty. Observe that the LHS of (25) is a decreasing function of δ. And note that when $\delta = 1$, (25) becomes

$$\frac{x_s^{I,a}\pi_{hh} + \epsilon_A\pi_{h\ell}}{x_s^{I,a} + \epsilon_A} < \frac{x_s^{I,na}\pi_{hh} + \epsilon_A\pi_{h\ell}}{x_s^{I,na} + \epsilon_A}.$$

This surely holds for all $x_s^{I,na} < x_s^{I,a}$ and $\pi_{hh} < \pi_{h\ell}$. This implies that the set of conditions identified in Proposition 1(b)(i) is not empty for sufficiently large δ.

Next, note that when δ approaches 0, the LHS of condition (25) approaches

$$\frac{x_s^{I,na}[\pi_{hh} + (\pi_h^m - 2\pi_{hh})] + \epsilon_A[\pi_{h\ell} + (\pi_h^m - \pi_{h\ell} - \pi_{\ell h})]}{x_s^{I,na} + \epsilon_A}.$$

This expression is greater than the RHS of (25). Hence, (25) holds with the opposite sign and we can show that the set of conditions identified in Proposition 1(b)(ii) is not empty. This holds for sufficiently small δ. ■

Proof of Proposition 2. We start with the assumption that $\pi_h^m - 2\pi_{hh} < \pi_h^m - \pi_{h\ell} - \pi_{\ell h}$ and so by Proposition 1, $x_s^{I,a} < x_s^{I,na}$. When we compare expressions (6), (11) and (9), we find that

$$\frac{\pi_C}{U_C} < \frac{\pi_{hh} + \delta(\pi_h^m - 2\pi_{hh}) - \pi_{\ell h} - \delta(\pi_h^m - \pi_{\ell h} - \pi_{h\ell})}{U_{hh} - U_{\ell h}} < \frac{\pi_{hh} - \pi_{\ell h}}{U_{hh} - U_{\ell h}}. \quad (26)$$

This holds because $\pi_h^m - 2\pi_{hh} < \pi_h^m - \pi_{h\ell} - \pi_{\ell h}$. As a result, the incumbent initially over-invests in project A in the no-acquisition and acquisition cases. The prospect of acquisition leads to a portfolio move away from project A. Hence, under conditions specified in Proposition 2 the

[7] to change

incumbent's portfolio moves closer to that of the social planner and thereby the direction of innovation improves ($x_s^{I,o} < x_s^{I,a} < x_s^{I,na}$).

Because of Proposition 1 we know that when the incumbent reduces its investment level in anticipation of the acquisition, the entrant will increase it, or $x^{E,na} < x^{E,a}$. Comparing expressions (12), (10) and (7) we obtain

$$\frac{\pi_B}{U_B} > \frac{\frac{x_s^{I,a}[\pi_{hh}+(1-\delta)(\pi_h^m-2\pi_{hh})]+\epsilon_A[\pi_{h\ell}+(1-\delta)(\pi_h^m-\pi_{h\ell}-\pi_{\ell h})]}{x_s^{I,a}+\epsilon_A}}{\frac{x_s^{I,o}U_{hh}+\epsilon_A U_{h\ell}}{x_s^{I,o}+\epsilon_A}+\frac{1-x_s^{I,o}}{1-x_s^{I,o}+\epsilon_C}U_C-\left(\frac{x_f^{I,o}U_h^m+\epsilon_A U_\ell^m}{x_f^{I,o}+\epsilon_A}+\frac{1-x_f^{I,o}}{1-x_f^{I,o}+\epsilon_C}U_C\right)}$$
$$> \frac{\frac{x_s^{I,na}\pi_{hh}+\epsilon_A\pi_{h\ell}}{x_s^{I,na}+\epsilon_A}}{\frac{x_s^{I,o}U_{hh}+\epsilon_A U_{h\ell}}{x_s^{I,o}+\epsilon_A}+\frac{1-x_s^{I,o}}{1-x_s^{I,o}+\epsilon_C}U_C-\left(\frac{x_f^{I,o}U_h^m+\epsilon_A U_\ell^m}{x_f^{I,o}+\epsilon_A}+\frac{1-x_f^{I,o}}{1-x_f^{I,o}+\epsilon_C}U_C\right)}.$$

This holds because $\frac{x_s^{I,a}[\pi_{hh}+(1-\delta)(\pi_h^m-2\pi_{hh})]+\epsilon_A[\pi_{h\ell}+(1-\delta)(\pi_h^m-\pi_{h\ell}-\pi_{\ell h})]}{x_s^{I,a}+\epsilon_A} > \frac{x_s^{I,na}\pi_{hh}+\epsilon_A\pi_{h\ell}}{x_s^{I,na}+\epsilon_A}$. This is true because $x_s^{I,a} < x_s^{I,na}$ and $[\pi_{h\ell} + (1-\delta)(\pi_h^m - \pi_{h\ell} - \pi_{\ell h})] > [\pi_{hh} + (1-\delta)(\pi_h^m - 2\pi_{hh})]$, see also Proposition 1(a). As a result, the entrant initially under-invests in project A in the no-acquisition and acquisition cases. Anticipating the acquisition entrant increases its investment in project A. Hence, conditions of Proposition 2 imply that the entrant's portfolio moves closer to that of the social planner and thereby the direction of innovation improves ($x^{E,na} < x^{E,a} < x^{E,o}$). ■

Proof of Proposition 3.

(a) We start with the assumption that $\pi_h^m - 2\pi_{hh} > \pi_h^m - \pi_{h\ell} - \pi_{\ell h}$ so that by Proposition 1(b), anticipating an acquisition, the incumbent raises investment. When we compare expressions (6), (11) and (9), we find that

$$\frac{\pi_C}{U_C} > \frac{\pi_{hh} + \delta(\pi_h^m - 2\pi_{hh}) - \pi_{\ell h} - \delta(\pi_h^m - \pi_{\ell h} - \pi_{h\ell})}{U_{hh} - U_{\ell h}} > \frac{\pi_{hh} - \pi_{\ell h}}{U_{hh} - U_{\ell h}} \tag{27}$$

This holds because $\pi_h^m - 2\pi_{hh} > \pi_h^m - \pi_{h\ell} - \pi_{\ell h}$. As a result, the incumbent initially under-invests in project A in the no-acquisition and acquisition cases. The prospect of acquisition leads to a portfolio move toward project A. Hence, under the conditions specified in Proposition 2, the incumbent's portfolio moves closer to that of the social planner, and thereby the direction of innovation improves ($x_s^{I,na} < x_s^{I,a} < x_s^{I,o}$).

Assume that (13) holds so that by Proposition 1(b)(i), anticipating an acquisition, the entrant cuts investment in the rival market, $x^{E,a} < x^{E,na}$. Comparing expressions (12),

(10) and (7) we obtain.

$$\frac{\pi_B}{U_B} < \frac{\frac{x_s^{I,a}[\pi_{hh}+(1-\delta)(\pi_h^m-2\pi_{hh})]+\epsilon_A[\pi_{h\ell}+(1-\delta)(\pi_h^m-\pi_{h\ell}-\pi_{\ell h})]}{x_s^{I,a}+\epsilon_A}}{\frac{x_s^{I,o}U_{hh}+\epsilon_A U_{h\ell}}{x_s^{I,o}+\epsilon_A}+\frac{1-x_s^{I,o}}{1-x_s^{I,o}+\epsilon_C}U_C-\left(\frac{x_f^{I,o}U_h^m+\epsilon_A U_\ell^m}{x_f^{I,o}+\epsilon_A}+\frac{1-x_f^{I,o}}{1-x_f^{I,o}+\epsilon_C}U_C\right)}$$
$$< \frac{\frac{x_s^{I,na}\pi_{hh}+\epsilon_A\pi_{h\ell}}{x_s^{I,na}+\epsilon_A}}{\frac{x_s^{I,o}U_{hh}+\epsilon_A U_{h\ell}}{x_s^{I,o}+\epsilon_A}+\frac{1-x_s^{I,o}}{1-x_s^{I,o}+\epsilon_C}U_C-\left(\frac{x_f^{I,o}U_h^m+\epsilon_A U_\ell^m}{x_f^{I,o}+\epsilon_A}+\frac{1-x_f^{I,o}}{1-x_f^{I,o}+\epsilon_C}U_C\right)}.$$

This holds because $\frac{x_s^{I,a}[\pi_{hh}+(1-\delta)(\pi_h^m-2\pi_{hh})]+\epsilon_A[\pi_{h\ell}+(1-\delta)(\pi_h^m-\pi_{h\ell}-\pi_{\ell h})]}{x_s^{I,a}+\epsilon_A} < \frac{x_s^{I,na}\pi_{hh}+\epsilon_A\pi_{h\ell}}{x_s^{I,na}+\epsilon_A}$. This is true because $x_s^{I,a} > x_s^{I,na}$ and $[\pi_{h\ell}+(1-\delta)(\pi_h^m-\pi_{h\ell}-\pi_{\ell h})] > [\pi_{hh}+(1-\delta)(\pi_h^m-2\pi_{hh})]$. As a result, the entrant initially over-invests in project A in the no-acquisition and acquisition cases. Anticipating the acquisition, the entrant decreases its investment in project A. Hence, the conditions of Proposition 2 imply that the entrant's portfolio moves closer to that of the social planner, and thereby, the direction of innovation improves ($x^{E,o} < x^{E,a} < x^{E,na}$).

(b) Assume again that $\pi_h^m - 2\pi_{hh} > \pi_h^m - \pi_{h\ell} - \pi_{\ell h}$ and that (13) does not hold so that by Proposition 1(b)(ii), anticipating an acquisition, the incumbent and the entrant both raise their investments in the rival market (and lower than in the independent markets), $x_s^{I,na} < x_s^{I,a}$ and $x^{E,na} < x^{E,a}$. The direction of innovation of the incumbent improves under the same condition as in part (a), it initially under-invests, and anticipating the acquisition moves the portfolio closer to the social optimum. The direction of innovation of the entrant improves when condition (21) holds with the opposite sign. The entrant initially under-invests and anticipating its acquisition moves the portfolio closer to the social optimum.

■

Proof of Proposition 4. (a) Consider

$$\lim_{U_C\to\infty}\frac{\mathbb{E}U^a(x_s^{I,a}, x_f^I, x^{E,a})}{\mathbb{E}U^{na}(x_s^{I,na}, x_f^I, x^{E,na})} = \frac{\frac{x^{E,a}}{x^{E,a}+\epsilon_A}\frac{1-x_s^{I,a}}{1-x_s^{I,a}+\epsilon_C}+\frac{\epsilon_A}{x^{E,a}+\epsilon_A}\frac{1-x_f^I}{1-x_f^I+\epsilon_C}}{\frac{x^{E,na}}{x^{E,na}+\epsilon_A}\frac{1-x_s^{I,na}}{1-x_s^{I,na}+\epsilon_C}+\frac{\epsilon_A}{x^{E,na}+\epsilon_A}\frac{1-x_f^I}{1-x_f^I+\epsilon_C}}. \tag{28}$$

Note that $\frac{x^{E,na}}{x^{E,na}+\epsilon_A}\frac{1-x_s^{I,na}}{1-x_s^{I,na}+\epsilon_C}+\frac{\epsilon_A}{x^{E,na}+\epsilon_A}\frac{1-x_f^I}{1-x_f^I+\epsilon_C}$ can be represented as $pa+(1-p)b$, where $p=\frac{x^{E,na}}{x^{E,na}+\epsilon_A}$, $1-p=\frac{\epsilon_A}{x^{E,na}+\epsilon_A}$, $a=\frac{1-x_s^{I,na}}{1-x_s^{I,na}+\epsilon_C}$ and $b=\frac{1-x_f^I}{1-x_f^I+\epsilon_C}$. Note also that $a>b$.[8] Hence,

[8]To show that $\frac{1-x_s^{I,na}}{1-x_s^{I,na}+\epsilon_C} > \frac{1-x_f^I}{1-x_f^I+\epsilon_C}$, we need to show that $x_s^{I,na} < x_f^I$. Recall $x_f^I = x\left(\frac{\pi_C}{\pi_h^m-\pi_\ell^m}\right) = \frac{1+\epsilon_C-\epsilon_A\sqrt{\frac{\epsilon_C\pi_C}{\epsilon_A\left(\pi_h^m-\pi_\ell^m\right)}}}{1+\sqrt{\frac{\epsilon_C\pi_C}{\epsilon_A\left(\pi_h^m-\pi_\ell^m\right)}}}$. While $x_s^{I,na} = x\left(\frac{\pi_C}{\pi_{hh}-\pi_{\ell h}}\right) = \frac{1+\epsilon_C-\epsilon_A\sqrt{\frac{\epsilon_C\pi_C}{\epsilon_A(\pi_{hh}-\pi_{\ell h})}}}{1+\sqrt{\frac{\epsilon_C\pi_C}{\epsilon_A(\pi_{hh}-\pi_{\ell h})}}}$. Under assumption $\pi_{hh}-\pi_{\ell h} < \pi_h^m - \pi_\ell^m$, which is satisfied in our micro-founded example, we have $x_s^{I,na} < x_f^I$.

we can conclude that $pa + (1-p)b$ is an increasing function of p. This implies that for all $p' > p$, we should have $pa + (1-p)b < p'a + (1-p')b$. Furthermore, for any $a' > a$,[9] we have $pa + (1-p)b < p'a + (1-p')b < p'a' + (1-p')b$. This shows that the limit in expression (28) is bigger than 1.

So, for sufficiently large U_C, allowing acquisitions can increase consumer surplus, provided other parameters are such that the improvement in the direction of innovation identified in Proposition 2 is possible.

(b) Consider

$$\lim_{U_B \to \infty} \frac{\mathbb{E}U^{a}(x_s^{I\ ,a}, x_f^{I}, x^{E\ ,a})}{\mathbb{E}U^{na}(x_s^{I\ ,na}, x_f^{I}, x^{E\ ,na})} = \frac{\frac{1-x^{E\ ,a}}{1-x^{E\ ,a}+\epsilon_B}}{\frac{1-x^{E\ ,na}}{1-x^{E,na}+\epsilon_B}}. \tag{29}$$

And note that $\frac{1-x}{1-x+\epsilon_B}$ is decreasing function of x. Then it is straightforward to show that this limit is bigger than 1, when $x^{E\ ,a} < x^{E\ ,na}$.

So, for sufficiently large U_B, allowing acquisitions can increase consumer surplus, provided other parameters are such that the improvement in the direction of innovation identified in Proposition 3(a) is possible.

[9] Which holds when $x^{E,a} > x^{E,na}$.

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