Multiproduct Firms and Refunds

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Abstract

We determine how a multi-product firm optimally sells one of its products to consumers who have to pay an inspect cost to find out a product's match value. The firm chooses product prices and return policies (refunds) in case the consumer finds out she does not like the product after she has bought it. One strategy e-commerce firms have adopted is to stimulate consumers to order many products at once, inspect their fit at home, and then decide what to return. These policies introduce a trade-off as they may result in consumers acquiring products that better fit their taste, at the expense of the private and social costs associated with returns. We determine the conditions under which firms find it optimal to offer these "Buy Many" policies and show that the optimal alternative sequential purchase policy may introduce asymmetric prices. We also analyze the efficiency properties of market outcomes and, surprisingly, find that these policies may actually lead to fewer returns.

JEL Classification: D40, D83, L10

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1 Introduction

Product returns play an increasingly important role in retail markets. A recent report of the National Retail Federation estimates that in the USA across different retail channels $743 billion of merchandise value is returned in 2023, which is around 14,5% of total retail sales. In the online segment of the retail market even 17,6% of product value is returned.[1] Given the importance of product returns, firms have started to treat returns strategically by developing optimal return policies. One of these developments is that firms, like Amazon and Zalando, offer consumers the possibility to order multiple items at the same time, inspect them at home to see whether they like them, and to return all items that are considered not to be a good fit.[2]

In this paper we ask how a firm's product return policy could help generating profits and what the welfare consequences of such policies are. For the welfare analysis it is also important to ask how frequently products are returned as product returns are associated with environmental costs that are paid by agents not involved in the transaction, while returned products often also cannot be easily resold in the market.[3]

To study product returns, a consumer search framework is appropriate. Products have a consumer-specific match value and consumers have to inspect a product at a cost to determine its value. In standard consumer search models consumers have to pay this search cost up front to learn their match value before purchase (see, for example, the seminal contributions by Wolinsky (1986), Anderson and Renault (1999) and Armstrong (2017)). We augment these models by allowing consumers to order (or buy) products without inspecting them before purchase and only inspect after purchase. As inspecting after purchase can usually be done in a more comfortable environment at a time that suits the consumer best, the inspection cost after purchase is lower. Firms may stimulate that consumers inspect after purchase by offering generous return policies, i.e., refunds. Thus, consumers may find it optimal to inspect products after purchase if the inspection cost difference is sufficiently large and/or the firm has a sufficiently generous return policy. However, offering refunds is costly as the salvage value of products that are returned is typically lower than the production cost. The difference

[1] See, https://www.digitalcommerce360.com/2023/12/27/online-returns-2023-nrf-appriss-retail-report/.

[2] Amazon now labels this 'Prime Try Before You Buy', which previously was called 'Amazon Prime Wardrobe'. See, https://www.amazon.com/gp/help/customer/display.html?nodeId=GCQDLMG7C2YEXSM4 for more details.

[3] These environmental costs include greenhouse gas emissions, non-recycled packaging and products filling up landfills (see, e.g. Tian and Sarkis (2022)), where some websites estimate that only 54 percent of all packaging gets recycled and 5 billion pounds of returned goods end up in landfills each year.

between the production cost and the salvage value is the second important dimension of our analysis.

To study the optimal selling policy of the firm, and when a firm may find it optimal to offer consumers to simultaneously order multiple products and return as many as they like, we consider a multi-product monopolist. We focus on two products, but the qualitative results continue to hold for a broader range of products. Not to bias our results in favour of ordering products simultaneously, consumers will only buy one product as their valuation for both products is the same as the maximum valuation of the two products separately. Thus, if the monopolist finds it optimal to engage in offering consumers to order multiple items at once, it is not because it can sell more products. The firm can offer different prices and refunds for different products, but also condition these on whether or not a consumer orders multiple products simultaneously. The firm cannot, however, condition prices or refunds on whether or not a consumer inspected a product as (certainly in online markets) firms do not know this. Prices and refunds do, of course, determine whether consumers find it optimal to inspect products before or after purchase.

Since Morgan and Manning (1985) it is well-known that if consumers can choose to search sequentially or simultaneously at the same prices, they find it optimal to search items sequentially.[4] This result also applies to our setting if prices and refunds are identical across inspection modes. By offering different prices and refunds if a consumer orders multiple items at once, the firm may, however, incentivize the consumer to search simultaneously. If the consumer takes this option, she will necessarily return at least one product.

We have two main substantial results and a significant methodological contribution. First, we consider that the inspection cost before purchase is sufficiently large such that consumers will never find it optimal to search before purchase. In this case, the optimal sequential selling policy after purchase involves asymmetric contracts. To show that we redefine the strategy of the firm as follows. As the difference between the product's price and the refund is a "price" the consumer always pays if she inspects the product, no matter whether she eventually buys the product or not, we call this difference the inspection fee the firm chooses, i.e., the price the consumer pays for inspecting the product. The cost for the firm related to inspection only is the product degradation when the product is returned. Once the consumer inspected the product, the relevant decision is whether she returns the product or not. The price the firm charges for *not* returning the product is the refund, while the cost for the firm of *not*

[4]That is if the searcher is patient enough or there is no delay due to sequential search. As we do not want our result that a firm induces consumers to search simultaneously to depend on an exogenously imposed delay because of sequential search, we assume that there is no delay.

returning the product is the salvage value of the product when it is returned.[5]

Using this redefinition we show that the optimal sequential selling policy after purchase when the inspection cost before purchase is sufficiently large is such that for the second product the refund is set equal to the salvage value and the inspection fee is chosen such that all consumer surplus from inspecting the second product is attracted. For the first product, the refund is set equal to the opportunity cost of selling the second product, while the inspection fee is chosen to extract all surplus from the whole search process.[6] These prices are such that the consumer finds it optimal to inspect the first product first and this product has a lower inspection fee and a higher refund relative to the second product. The policy is such that if the consumer returns the first product, she never comes back to buy this product again. The optimal selling policy has the flavour of a two-part tariff (in the sense that it is also used to extract surplus), but the inspection fee for the second product is not really a fixed fee as it influences the decision whether or not to inspect the second product. Actually, from a social efficiency perspective, the optimal selling policy sets this inspection fee too high and the second product is not inspected often enough.

Instead, in the optimal simultaneous contract the firm sets the refund equal to the salvage value and sets a fixed fee that extract the expected maximum consumer value (given that it is larger than the salvage value). The consumers' return decision for both products is socially optimal, but from a social efficiency perspective, there is too much search, especially when the product degradation is relatively large.

When the inspection cost before purchase is sufficiently large the firm finds it profitable to have the consumer order multiple items at once and return all products the consumer does not want to keep if product degradation and/or the inspection cost after purchase is sufficiently small. In this case, "buying many" always leads to more returns and a policy that forbids such policies would reduce the environmental costs related to returns (while consumers are equally well off as they obtain zero surplus in both solutions).

We next consider the case where the inspection cost before purchase is sufficiently small so that it severely constrains the sequential selling strategy of the firm.[7] The smaller the search

[5] Thus, the selling price and the product cost are then implicitly defined as the inspection fee plus the refund and the product degradation plus the salvage value, respectively.

[6] If the inspection cost after purchase is positive, then this cost should be deducted from the inspection fees.

[7] In the limit when this inspection cost equals 0, the consumer will always want to search the products sequentially before purchase. More generally, the question is whether the firm wants to induce the consumer to inspect products before or after purchase. This question boils down to under which inspection form social surplus is higher and how much of that surplus the firm is able to extract. It is clear that social surplus is potentially higher under inspection after purchase if the difference in inspection costs is relatively large and

cost before purchase, the more credible the threat of the consumer to search before purchase and to induce the consumer to inspect after purchase the firm has to give a larger refund for every given price. When this constraint becomes binding for both products, the firm sets identical prices and refunds for both products.

In this case, and perhaps surprisingly, even though it is guaranteed that consumers return at least one item if they order multiple items at the same time, the expected number of returns may be lower than if consumers order items sequentially. The reason is as follows. First, under the "Buy Many" strategy, consumers will always return at least one product, and return also the second product if both products have a value smaller than the refund (which is set equal to the salvage value). Second, under the alternative sequential inspection strategy, consumers return products if their value is below their reservation value, which -if the search cost is small- may actually be high. This would imply that consumers may almost surely return products under sequential search (whether it is before or after purchase). Thus, no matter how the frequency of returns is measured (as the expected number of products that are returned or as the expected number of returns) there will be less returns under "buying many and return" if the search cost and/or the salvage value is small.

Related literature. The paper combines two strands of literature. The papers most closely related to ours are Janssen and Williams (2024), Jerath and Ren (2023) and Matthews and Persico (2007) in that they also study product returns in a consumer search setting. However, all these papers study a single product firm sell and consumers searching sequentially (where the former paper studies a competitive setting, while the latter two analyze monopoly behavior. They find that the number of refunds is either inefficiently high or low. None of these papers consider a firm that incentivizes consumers to search simultaneously among its multiple products.[8] Petrikaitė (2018a) studies search with returns in a duopoly setting, but also does not consider multiple products per seller or simultaneous search. The second strand of literature is on multi-product search (Rhodes (2015), Shelegia (2012) and Zhou (2014)),

the production degradation is relatively small. When consumers search before purchase the firm is generically not able to extract all surplus as it sets price in such a way that consumers find it beneficial to search. When consumers inspect products after purchase the firm is better able to extract all surplus by setting prices and refunds appropriately. Only when the search cost of inspecting products before purchase is relatively small and the threat of inspecting before purchase is more severe, the firm has to offer consumers prices and refunds so that they make positive surplus. Thus, the firm may induce consumers to inspect after purchase even if this is not socially optimal.

[8] Another difference with Janssen and Williams (2024) is that we study a setting where consumers can learn the prices and refunds the firm sets without any cost. This is a feature the paper has in common with the recent literature on price directed search; see, e.g., Armstrong (2017), Choi, Dai, and Kim (2018).

but the focus of these papers is on consumers searching for multiple products, creating a joint search effect in that once a consumer is at a store it has a lower search cost to buy other products at that store. These papers do not study product returns or simultaneous search.

The optimal behaviour of the firm if it wants to induce sequential search after purchase has features that also arise in Petrikaitė (2018b) and Gamp (2022) in that a multi-product firm has an incentive to obfuscate search among its products. These papers study a setting where consumers have to inspect products before purchasing one of them and where (together with prices) the firm chooses consumers' search cost directly. They show that the firm has an incentive to set a positive search cost and asymmetric prices so as to induce consumers to search the products in a particular order. In contrast, we allow consumers to order (or buy) products before inspecting them[9] and have a setting where the firm cannot affect the inspection cost of consumers directly. However, by choosing a refund that is smaller than the price, the firm effectively sets an inspection fee that the consumer pays upfront when deciding to inspect. This inspection fee is part of the firm's profits, which makes for another important difference to the above mentioned papers.

The rest of the paper is organized as follows. The next section introduces the model. Section 3 discusses the case when the inspection cost of inspection before purchase is large, while Section 4 considers the opposite case when this cost is small. Section 5 concludes with a discussion.

2 The Model

A monopoly firm sells two products. Each product has a production cost $c \geq 0$ and a salvage value $\eta \in [0, c]$ to the firm in case the product is bought and then returned. We will define $k = c - \eta$ as the value lost if the product is returned after it is inspected and we will refer to k as the *product degradation.* The firm can set different prices and refunds for the different products $i = 1, 2$ and we denote price by $p_i \geq 0$ and refund by $\tau_i \in [0, p_i]$.[10] As (certainly in online markets) a firm cannot verify whether the consumer has inspected the value of the product before purchase or not, it cannot charge different prices for when consumers inspect

[9] Doval (2018) allows consumers to buy blindly, that is without inspecting the product at all. Buying and inspecting after purchase has features that can be considered a generalization of blind buying in the sense that if the refund that the firm gives is zero, the consumer will never inspect the product afterwards and will then also not return the product. However, in our framework the consumer has to pay the inspection fee, a feature that is absent in Doval (2018).

[10] Note that to prevent arbitrage the firm would never set a refund larger than price.

products before or after purchase. It can only set prices and refunds such that it incentivizes consumers to inspect products in one or the other way. As the firm *does know* whether or not a consumer buys multiple products at once, it can offer different prices and refunds for this situation and we denote them by (p_{sim}, τ_{sim}) with $p_{sim} \geq \tau_{sim}$.[11] We will sometimes refer to a set of prices and refunds as a *contract* or somewhat imprecisely simply as *prices*.

There is a representative consumer with unit demand. The two products are ex-ante identical to the consumer with each product having a valuation that is independently and identically distributed by $v_i \sim F[\underline{v}, \bar{v}]$, with a density $f(v)$ that is positive, continuously differentiable and where f is logconcave.[12] To have an interesting model, we require $\bar{v} > c$. The consumer knows the prices and refunds the firm offers, but has to pay an inspection cost of $s_B > 0$ to learn a product's value before purchase and a cost of s_A if she wants to learn the product's value after purchase, with $s_A \leq s_B$. Thus, if consumers simultaneously buy two products they will always inspect them after purchase as this comes at a lower inspection cost. The outside option of the consumer is normalized to 0. For future reference, it will be useful to write $\hat{v}_b$ as the reservation value of inspecting before purchase and $\hat{v}_{ai}$ as the reservation value of inspecting product i after purchase. They are implicitly defined through the following equations:[13]

$$\int_{\hat{v}_b}^{\infty} (v - \hat{v}_b) f(v) dv = s_B \quad \text{and} \quad \int_{\hat{v}_{ai}}^{\infty} (v - \hat{v}_{ai}) f(v) dv = s_A + p_i - \tau_i. \tag{1}$$

Note that $\hat{v}_{ai}$ is not only a function of exogenous parameters but also of p_i and τ_i, the two strategic variables of the firm for product i. When we write $\hat{v}_{ai}$ we implicitly mean the function $\hat{v}_{ai}(p_i - \tau_i)$.

Given the firm's choices, the consumer can take one of the following actions:[14] (i) Inspect the products sequentially, (ii) Inspect the products simultaneously after purchase or (iii) Leave and take the outside option with a pay-off of 0. Under (i), the consumer decides in which

[11] As the firm will not benefit from setting different prices under simultaneous search, we do not use subscripts for the price and refund of the different products.

[12] It is well-known that this implies that the associated distribution function F and $1 - F$ are then also logconcave; see, e.g., Bagnoli and Bergstrom (2005).

[13] In general, we define $\hat{v}(\tilde{s})$ implicitly through $\int_{\hat{v}}^{\infty} (v - \hat{v}) f(v) dv = \tilde{s}$. Then $\hat{v}_b = \hat{v}(s_B)$ and $\hat{v}_{ai} = \hat{v}(s_A + p_i - \tau_i)$.

[14] Note that we have left two possible consumer strategies out of the above list. First, it turns out that it is never optimal for the firm to set prices such that the consumer would choose to buy a product without inspecting it at all (as in Doval (2018)). Second, simultaneous inspection before purchase is also never chosen. In the case of inspection before purchase, at a given price the firm receives the same payoff irrespective of whether the consumer inspects sequentially or simultaneously, while simultaneous search is never optimal for the consumer. Note that, in contrast, the firm's payoffs for simultaneous and sequential search after purchase do differ as firms can make a profit or a loss over their returns.

order to inspect the products and can inspect each product either before purchase or after purchase. Inspecting a product before purchase entails paying the inspection cost of s_B to learn that product's value and then deciding whether to buy it at price p_1 or, in case of the first product, continuing to inspect the second product. Inspecting a product after purchase entails paying the inspection cost of s_A to learn that product's value, deciding whether to keep it and pay the price p_1, or, in case of the first product, continuing to inspect the second product, and finally returning and paying $p_i - \tau_i$ for all products inspected after purchase that are not kept[15]. If consumers search sequentially, they have perfect recall. Under (ii), the consumer inspects both products simultaneously after purchase for inspection cost of s_A each and decides whether to buy at most one of the products at the contract (p_{sim}, τ_{sim}) of the products and returns at least one. In the following we will refer to (ii) in short as A^{sim}, while we refer to (i) if both products are inspected after purchase as A^{seq}.

It is important to note that it is possible to redefine inspection after purchase as a structurally simpler problem, which will facilitate the analysis. From the consumer's view inspection after purchase can be re-written as inspection before purchase with certain inspection costs and prices. In particular, at the moment the consumer pays the inspection cost s_A to learn the value of product i after purchase, she commits to paying at least $p_i - \tau_i$ – which is the part of the price she does not get back if she returns the product. If she instead wants to keep the product she pays the additional τ_i. Thus, we can redefine inspection after purchase as inspection before purchase with a *redefined inspection cost* of $s_A + p_i - \tau_i$ and a *redefined price* of τ_i. Note that while s_A is lost, $p_i - \tau_i$ is the part of the redefined inspection cost that is paid to the firm. It is thus as if the firm was offering product i for inspection before purchase at an *inspection fee* of $\sigma_i := p_i - \tau_i$ and a price for keeping the product $\rho_i := \tau_i$. In line with this redefinition, we can also split the production cost c into a part (the product's degradation k) the firm incurs when the consumer inspects the product and a part the firm incurs when the consumer decides to keep the product (the salvage value η, with $c = k + \eta$. Overall, it is as if the firm chooses for each product an inspection fee σ_i with the associated opportunity cost k and a price (refund) τ with the associated opportunity cost η.

3 Large Inspection Cost before Purchase

When the inspection cost before purchase s_B is relatively large, the consumer will not choose this option and when designing the optimal contract conditional on the consumer searching

[15]Note that it does not matter if the price p_i is paid before inspecting the product or after deciding which products to keep and which to return.

sequentially, the firm's strategy focuses on a consumer that inspects the product sequentially after purchase. In this section, we first construct the optimal contracts for both simultaneous search and sequential search. We then compare profits under both contracts to determine the conditions under which a contract is optimal for the firm, before we compare the number of returns under sequential and simultaneous search.

Consider first the optimal contract under sequential search. The next proposition summarizes the result.

Proposition 1 *If s_B is large and the firm induces consumers to inspect sequentially after purchase the optimal strategy is as follows:*

$$(\sigma_1^*, \rho_1^*) = (\mathbb{E}[\max(v - ES_I - \eta, 0)] - s_A, ES_I + \eta) \text{ and } (\sigma_2^*, \rho_2^*) = (ES_I + k, \eta)$$

with profits $\pi^*_{Aseq} = \mathbb{E}[\max(v - \eta, ES_I)] - s_A - k$ and where:

$$ES_I = \mathbb{E}[\max(v - \eta, 0)] - s_A - k. \tag{2}$$

The intuition behind the optimality of the strategy seems clear. If the firm incentivizes A^{seq} then Weitzman (1979) implies that the consumer first inspects the product with the higher net reservation value $\hat{v}_{a1} - \rho_1 \geq \hat{v}_{a2} - \rho_2$ and only inspects product i if it has a non-negative net reservation value $\hat{v}_{ai} - \rho_i \geq 0$ (as this is a necessary condition for non-negative utility). Without loss of generality consider that product $i = 1$ is inspected first. Then, as the inspection fee σ_1 for the first inspected product is committed to be paid before inspection starts, the firm can increase it (without distorting consumer decisions) as long as the above inequalities are not violated. This implies that in the optimal contract we should have that $\hat{v}_{a1} - \rho_1 = \hat{v}_{a2} - \rho_2$, i.e. the net reservation values of the two products will be equal.[16] *If* the firm will choose the contracts for both products such that the net reservation values will be equal to zero $\hat{v}_{ai} - \rho_i = 0$, implying that the consumer will buy the first product that has a positive observed net value, $v_i - \rho_i > 0$, then it is clear what is the optimal contract. For the last product in this order, the firm sets the refund (or the price for keeping the product) equal to the opportunity cost, i.e., $\rho_2 = \eta$ and the inspection fee σ_2 such that it extracts ES_I, the efficient surplus from inspection of the second product. Turning to the first product that is inspected, the firm's strategy follows the same principle, but here ρ_1 is priced at the "opportunity cost of selling the first product", which is the sum of the salvage value and the profit that the firm foregoes if the consumer does not inspect the second product. Thus, the firm (realizing it can make a profit of ES_I and is getting the salvage value if the consumer

[16] From (1) it follows that $\partial \hat{v}_{ai} / \partial \sigma_i = -1/[1 - F(\hat{v}_{ai})] \leq -1$.

continues to inspect the second product) will set the refund price such that $\rho_1^* = ES_I + \eta$ and an inspection fee σ_1^* that extracts all remaining surplus, with $\sigma_2^* \geq \sigma_1^* \geq k$.[17,18]

What is less clear is why it is optimal to set $\hat{v}_{ai} - \rho_i = 0$. At one level, this seems obvious as the firm extracts all consumer surplus. However, this is not the efficient surplus as (i) the inspection fee for the second product causes an inefficiency as the first product may be kept, ending search, even though the second product has a higher (net) value, while (ii) the difference in refunds for the first and second product also creates an inefficiency as it may well happen that the first product is returned, while the second product turns out to have a lower net value.[19]

The issue can also be illustrated by means of Figure 1. In the optimal solution, we have that the whole value area can be divided into three parts as in the left part of the figure: (i) if the consumer has a value $v_1 > \rho_1$ she will buy product 1, (ii) if the consumer has a value $v_1 < \rho_1$ she will continue to search the second product and purchase that product if $v_2 > \rho_2$, and (iii) if the consumer has a value $v_1 < \rho_1$ and $v_2 < \rho_2$, she will buy none of the products. In the right part of the figure, we indicate the different consumer behaviours in case $\hat{v}_{ai} - \rho_i > 0$. Here, after inspecting the first product, the consumer may decide not to buy the product immediately even if she discovers that $v_1 > \rho_1$. Inspecting the second product delivers another inspection fee of σ_2 to the firm and the consumer may still decide to buy product 1. The largest part of the proof in the appendix is dedicated to showing that this is not optimal and the firm indeed wants to set $\hat{v}_{ai} - \rho_i = 0$ if $f(v)$ is logconcave.

We finalize the discussion of the optimal sequential contract after purchase with a numerical example and a few general remarks.

Example. The following example illustrates the nature of the optimal solution under A^{seq} *and shows why the optimal solution involves an asymmetric contract even if the products are ex ante symmetric. Suppose that* $s_A = c = \eta = 0$ *and that values are uniformly distributed over* $[0, 1]$. *If the firm would have one product to sell, it is clear that the optimal contract would have* $\tau = \rho = 0$ *and* $p = \sigma = 1/2$. *The firm sets the refund efficiently, namely equal to the salvage value, and then extracts all surplus by setting the price equal to the expected surplus of searching. This is also the optimal contract for the second product if the firm sells two products. Consider then the first product. The firm knows it can make a profit of* $1/2$ *and*

[17] $\sigma_1^* = \mathbb{E}[\max(v - ES_I - \eta), 0] - s_A = \mathbb{E}[\max(v - \eta, ES_I)] - \mathbb{E}[\max(v - \eta, 0)] + k \geq k$.

[18] It is relatively easy to see how this optimal solution can be generalized to selling one out of n products.

[19] Note that even if the first product is returned only after the second is inspected, the consumer would still return the first product as it has a higher refund.

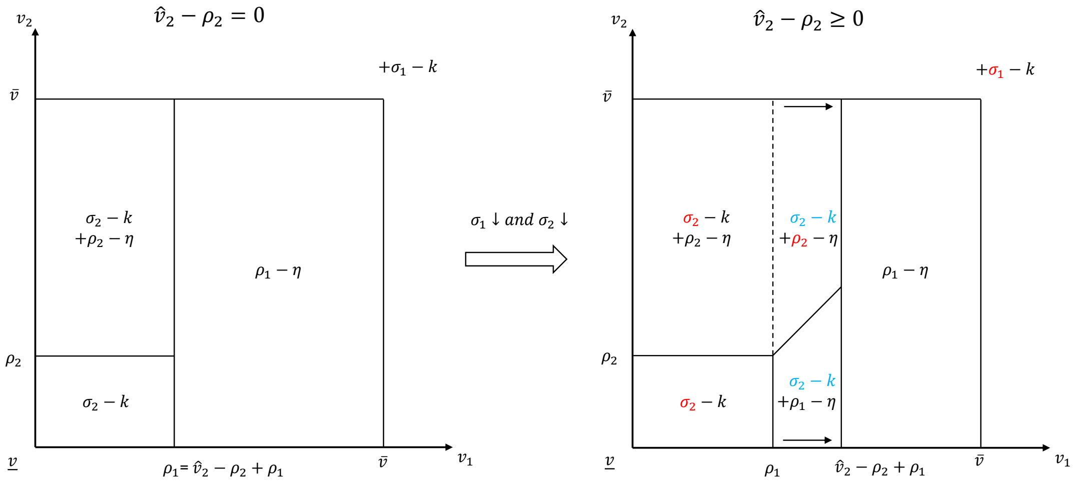

Figure 1: Possible deviation from the optimal A^{seq} strategy.

that the consumer gets an expected surplus of zero if the consumer continues to inspect the second product. It is then optimal to set the refund in the first period $\tau_1 = \rho_1 = 1/2$ *as this is the opportunity cost of the refund: a higher refund yields some extra consumers returning the product with a refund that is larger than the profit it generates. Given the choice of the refund and a price* p_1 *in the first period consumers start searching if their expected surplus is nonnegative, which yields the following constraint:* $-\sigma_1 + 1/2 * (3/4 - \rho_1) + 1/2 * 0 \geq 0$. *It is optimal for the firm to set the largest price given this constraint, yielding* $p_1 = \sigma_1 - 1/2 = 5/8$. *The total profit is thus equal to* $5/8$ *as the consumer pays the first inspection fee* σ_1 *of 1/8 and then pays the additional price* τ_1 *of 1/2 if the valuation is larger than 1/2 (which happens with probability 1/2) and if the valuation is smaller than 1/2 the consumer continues to search the second product, pays the inspection fee* σ_2 *of 1/2 and always keeps the product.*

Thus, the firm finds it optimal to make inspection costly by creating an inspection fee σ_i, which is the difference between the selling price and the refund, that consumers know they lose when they inspect a product. The example shows that even though the actual inspection cost equals 0, this optimal inspection fee can actually be quite large, especially for the second product. Second, it is interesting to see that the resulting profit under A^{seq} equals $\mathbb{E}[\max(v - \eta, ES_I)] - s_A - k$, which is exactly identical to the efficient surplus if there was no recall. In addition, the firm makes this profit independent of whether the consumer eventually purchases product 1, 2 or no product at all, i.e., even if the consumer returns both products the firm makes the same profit as when it sells. Third, as in Petrikaitė (2018b), the profit maximizing strategy of the firm distorts the consumer's optimal search behavior in such a way as to remove their ability to recall any earlier inspected product. However, in our case it is further able to extract all that surplus by setting the inspection fees appropriately. The fact that the inspection fees are another source of revenue create the technical complications alluded to above to show that indeed the firm wants to set $\hat{v}_{ai} - \rho_i = 0$.

We now consider the optimal contract and profits when consumers search simultaneously after inspection so that the consumer pays the inspection fee σ_{sim} and the inspection cost s_A for both products upfront as long as their expected utility is non-negative. Recall that the consumer can buy at the terms of contract $(\sigma_{sim}, \rho_{sim})$ only if she chooses the action A^{sim}. The firm does not have to consider therefore a potential deviation of the consumer when incentivizing A^{sim} as it can in principle set very unattractive terms for the consumer to search sequentially. When consumers search simultaneously, they will buy the product with the higher net value $v_i - \rho_{sim}$, as long as either of them is non-negative. So, the profit-maximizing contract is essentially a two-part tariff where the optimal price ρ^*_{sim} is set at marginal cost η and the optimal inspection fee σ^*_{sim} extracts all surplus. In particular, as the expected social surplus is given by

$$\mathbb{E}[\max(v_1 - \eta, v_2 - \eta, 0)] - 2(s_A + k) \tag{3}$$

the profit $\pi^*_{Asim} = 2(\sigma^*_{sim} - k)$ is equal to this expression.[20] From an efficiency standpoint, the number of inspections is too large, but products are returned at an efficient level: the product with the lowest valuation will always be returned and this is efficient as the consumer has no (additional) value for it, while the firm has a salvage value and the product with the highest valuation will be returned if its value is smaller than the firm's salvage value.

Example continued. Keeping the same parameter values, it is clear that under A^{sim}, the firm wants to set $\rho_{sim} = \eta = 0$. The firm then wants to set the price for the two products such that it attracts $\mathbb{E}[\max(v_1, v_2)] = 2/3$. Thus, it will set the price for each product equal to $1/3$.

Finally, we are able to compare the profits for A^{sim} to those for A^{seq} and evaluate the impact of A^{sim} on the number of products returned. We find the following:

[20] Note that any other contract with asymmetric prices σ^i_{sim} satisfying $\sigma^1_{sim} + \sigma^2_{sim} = 2\sigma^*_{sim}$ would have resulted qualitatively in the same outcome.

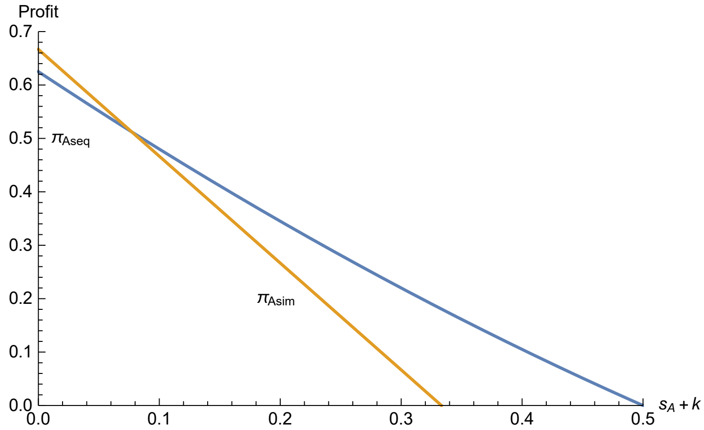

Figure 2: Profits π_{Asim} and π_{Aseq} as functions of the sum of inspection and degradation costs $s_A + k$ for uniformly on $[0, 1]$ distributed values and $\eta = 0$.

Proposition 2 *If s_B is large enough,*[21] *then there exists a function $\underline{S}_A(\eta) > 0$ such that for all (s_A, k, η):*

$$s_A + k \leq \underline{S}_A \Leftrightarrow \pi^*_{Asim} \geq \pi^*_{Aseq}.$$

Moreover, the expected number of returns under A^{sim} is larger than under A^{seq}.

The intuition behind the proposition is clear. Under both search protocols the firm extracts all surplus. However, the surplus is quite different. Under simultaneous search, the consumer inspects both products and chooses the one with the higher net value. The potential loss in surplus is due to inspection costs and product degradation related to the purchase and return of at least one product. Under sequential search, the consumer inspects the first product and keeps it if it has a higher net value than the expected value of the second inspection, including the inspection fee the firm imposes. Compared to simultaneous search, surplus is lower if the consumer decides not to inspect the second product even though it would have had a higher net value if she would have done so, or if the consumer continues to inspect the second product, but then does not keep the product with the highest value due to the difference in refunds. If the loss in surplus under simultaneous search due to unnecessary inspection costs and product degradation is relatively small, simultaneous search leads to higher profits. If, on the other hand, $s_A + k$ is relatively large, then A^{seq} yields more profits as one can find a good fit already with the first product and save on inspection cost and product degradation. Figure 2 presents a numerical example. What is interesting is that for a single product firm

[21] It is clear that how large s_B should be for it not to impose a constraint on the contract the firm can offer under A^{seq} depends on the other parameters, most notably s_A. If s_A is fairly large itself, then s_B itself should be relatively large for this to be true. If s_B is not large enough, then obviously the profits of the firm under A^{seq} will be lower and it may also be that these profits are smaller than under inspection before purchase.

both solutions yield the same outcome and that the outcome is efficient. The inefficiencies that are created under both A^{sim} and A^{seq} are due to the multi-product nature of the firm and the associated search.[22]

Thus, the firm induces consumers to "Buy Many and Return" if the sum of inspection and degradation costs s_A+k is small and this leads too more product returns than in an alternative contract if $1+F^2(\eta) > F(\rho_1^*)(1+F(\eta))$. The proof of the proposition shows that this condition follows from the logconcavity of $1 - F(v)$. The proposition focuses on the case where the alternative contract is A^{seq}, but the same result would obviously apply if the alternative contract is one where the consumer only inspects before purchase, as then no products will be returned at all. A policy where such simultaneous contracts would be forbidden would therefore reduce the number of returns if s_B is large enough. The next section shows that this is not necessarily the case if s_B is small.

Alternatively, a regulator could choose to impose that consumers get full refunds. In our framework this would imply that $\sigma_i = 0, i = 1, 2$. It is not difficult to see that in that case the firm's profits when setting a price p are equal to

$$(1 - F^2(p))(p - c - k) - 2F^2(p)k = (1 - F^2(p))(p - \eta) - 2k,$$

for the simultaneous search contract, and

$$(1 - F^2(p))(p - c - k) - 2F^2(p)k + F(\hat{v}_a)k = (1 - F^2(p))(p - \eta) - 2k + F(\hat{v}_a)k,$$

for the sequential contract, where $\hat{v}_a$ is defined as the usual reservation price relative to the search cost s_A. Thus in both cases the firm optimally sets the price such that it maximizes

[22]It can be argued that the consumer may use the contract of A^{sim} in a different way: While she does have to pay σ_{sim} for both products upfront, once she has them "at home" she does not necessarily have to inspect both simultaneously, but can do so sequentially instead. This is indeed optimal if some part of the inspection cost s_A comes from the effort of "testing the product at home". If instead s_A only comes from the effort of selecting a product before ordering it, then nothing changes in our analysis as presented in this section. We now show that the extreme opposite, where that effort is zero and all of s_A instead comes from testing the product at home, does not change our result in a substantial way. In that case, before any inspection the consumer pays $2\sigma_{sim}$ to the firm, who anticipates a cost of $2k$. Then the consumer inspects the two products sequentially at their inspection cost s_A and if she decides to keep one product, she pays ρ_{sim} to the firm, who realizes an additional cost of η in that event. From an efficiency view, the maximum surplus is realized if the firm sets $\rho_{sim} = \eta$, and the firm is able to extract all that surplus using σ_{sim}. This surplus - and therefore firm profit - is bigger than what we derived in the above section as sequential search is more efficient than simultaneous. This implies that the threshold of Proposition 2, below which $\pi^*_{Asim} > \pi^*_{Aseq}$ would be "higher" - note, however, since now s_A and k are not both invested at the same time, we would need to adjust Proposition 2 such that $k \leq \tilde{S}_A(s_A, \eta) \Leftrightarrow \pi^*_{Asim} > \pi^*_{Aseq}$.

joint monopoly profits given a cost η, and the profit in case of sequential search after purchase is higher as the firm may economize on the cost related to product degradation. Unless, the reservation value $\hat{v}_a < \rho_1^*$, it is clear that mandating full refunds leads to an increase in product returns as it leads to much higher refunds. In the absence of inspection fees, consumers are able, however, to enjoy more surplus.

4 Small Inspection Cost before Purchase

When the inspection cost before purchase s_B is relatively small, the consumer may choose this option instead of inspecting a product after purchase while she is searching sequentially. As we argued before, the firm cannot observe whether the consumer has inspected a product before or after purchase and therefore cannot set different prices in each of those cases. In this section, we continue to focus on the case where the firm wants the consumer to inspect its products after purchase instead of before purchase. We will identify when that is beneficial for the firm. Note that if s_B is relatively small, then the firm cannot set the same contract as in the previous section. As the consumer gets zero expected utility under that contract, she would deviate to inspecting before purchase. Therefore the firm has to adjust its contract accordingly to ensure the consumer does not deviate. Naturally, this implies reduced profits under A^{seq} compared to the previous section. However, that loss in profits is not the only implication of a relatively small s_B. In this section, we will show, perhaps surprisingly, that "Buy Many and Return" contracts can actually lead to a lower number of returns than sequential inspection after purchase when s_B is relatively small. The presence of the threat of the consumer deviating to inspection before purchase turns out to be important in facilitating this result.[23]

Before continuing, we provide the following example, which shows that if $s_B = 0$ the firm acts as a "standard" multi-product firm and incorporates the positive externality selling the products impose on each other.

Example continued. Suppose now in addition that $s_B = 0$. If the inspection costs are the same whether the consumer inspects before or after purchase, the firm has to provide full refunds, and wants to induce the consumer to inspect the products before purchase. It is also not profitable to set asymmetric prices. Setting a price p for each of the products, it makes a profit of

$$[p(1-p) + (1-p)^2/2]p$$

[23]Results in this section are still work in progress.

over each product. This expression can be understood as follows. There is a probability p that the value of the other product is smaller than p and in that case the product under consideration is sold if it has a value larger than p, which happens with probability $1-p$. *With the remaining probability* $1-p$ *the value of the other product is larger than p and in that case the product under consideration is sold if it has the largest of the two values. Maximizing this expression with respect to p yields the FOC* $3p^2 = 1$, *or* $p = \sqrt{1/3}$. *Thus, the total profit of the firm is* $\frac{2}{3}\sqrt{1/3}$. *Note that both the price and the profit is larger than the profit of a single product monopolist, but that the profit is considerably smaller than the profit under* A^{seq} *we derived in the previous section. The reason is that the firm has to leave quite a bit of surplus to the consumer as the consumer knows his value for buying. Under* A^{seq} *the firm transforms the demand of the consumer and makes it less price sensitive as she has to decide to commit to pay the inspection fee before knowing the value.*[24] *As we argued above, if* s_B *is small, then the firm cannot achieve the same profits under* A^{seq} *as in the previous section. Note that in fact if* $s_B = 0$, *the maximal profit it can achieve under* A^{seq} *is equal to the profit we have derived here for inspection before purchase.*

If s_B is relatively small, we find that the firm still prefers to induce the consumer to inspect after purchase if the sum of inspection cost and degradation cost, $s_A + k$, associated with inspection after purchase is smaller than s_B, the counterpart for inspection before purchase.[25] This is only a sufficient condition. Thus, we consider in this section that $s_A + k \leq s_B$.

The threat to deviate to inspect before purchase introduces upper bounds $\bar{\sigma}_i$ on any contract the firm can set to induce the consumer to search after purchase. If the firm sets a contract with $\sigma_i \geq \bar{\sigma}_i$, then the consumer prefers inspecting that product before purchase. This prevents the firm from setting the optimal contract of the previous section if s_B is small. For $s_B = 0$ we find that $\bar{\sigma}_i = 0$, implying that the firm has to set $\sigma_i = 0$. This means the firm can only set the same contract it can set if it induces the consumer to inspect before purchase. Therefore, the firm only makes positive profits from product sales in that case, but none from inspection. For positive s_B, the firm can set strictly positive σ_i, improving their profit over the alternative of inspection before purchase, as we have shown above.

We focus the analysis on this case where the firm prefers to incentivize A^{seq} instead of inspection before purchase and ask when the number of returns is smaller under the optimal

[24]This, in a sense, rotates the demand curve and makes it more flat. See, Johnson and Myatt (2006).

[25]Suppose the optimal price for inspection before purchase at s_B is p^*, then the firm can instead offer inspection after purchase with $\sigma_i = s_B - s_A > k$ and $\rho_i = p^*$, providing an offer that is identical to the consumer to what it would offer under inspection before purchase, but making additional profits from inspection.

simultaneous contract A^{sim}. As we argued above, small values of s_B force the firm to set low σ_i. The reason is that if consumers had to pay a relatively large inspection fee upfront, they rather inspect before purchase without paying the inspection fee. To compensate the firm sets a high refund price ρ_i.

The expected number of returns under the two search modes for small s_B are given by

$$n_{Asim} = 1 + F(\eta)^2 \quad \text{and} \quad n_{Aseq} = F(\hat{v}_{a2} - \rho_2 + \rho_1) + F(\rho_1)F(\rho_2).$$

The number of returns under A^{sim} does not depend on s_A. Both products are always inspected, implying that one product is returned with certainty. Both are returned only if their values are both below η, the efficient return price and the lowest price the firm will ever set. In comparison, the number of returns under A^{seq} depends on s_A: a consumer certainly returns one product if she inspects the second product, which happens if $v_1 - \rho_1 \leq \hat{v}_{a2} - \rho_2$, while she will also return the other product if both turn out to have a negative net value $v_i - \rho_i < 0$.

The following proposition states when A^{sim} or A^{seq} create more returns.

Proposition 3 *If $s_A + k < s_B$, then there exists an $\bar{s}$ such that the "Buy Many and Return" contracts lead to less expected returns than sequential contracts for all $s_B < \bar{s}$.*

Thus, banning "Buy Many and Return" may actually lead to more rather than to less returns. The intuition behind this result is the following. Under A^{seq}, the low inspection costs and inspection fee σ_2 makes inspection of the second product attractive to the consumer, while the high refund price ρ_1 makes it unlikely that the consumer will consider the first product a good enough fit. Thus, there is a high chance that the second product will be inspected, in which case again at least one product will be returned with certainty. Due to the similarly high refund price ρ_2, it is however also likely that the consumer finds neither of the two products a good enough fit, implying that both would be returned. For small s_B this effect is most severe, leading to a higher expected number of returns under A^{seq} than under A^{sim}.

While we derived Proposition 2 for large values of s_B, the implication for when A^{sim} leads to higher profits than A^{seq} also holds true for small s_B. The reason is that the profit from A^{seq} will be strictly smaller for small s_B than what we derived in the previous section for large s_B. Then Propositions 2 and 3 together imply the following:

Proposition 4 *There exists a function $\underline{S}_A(\eta) > 0$ and an $\bar{s}$ such that for all (s_A, s_B, k, η) with $s_A + k \leq \min[s_B, \underline{S}_A(\eta)]$ and $s_B < \bar{s}$ the firm induces the consumer to "Buy Many and Return", leading to higher profits and a lower number of returns than sequential inspection after purchase.*

Note that the two conditions of Proposition 4 are independent of each other. If $s_A + k$ is small, then the firm induces A^{sim}, while it might be the case that $s_B \geq \bar{s}$ is large and A^{seq} would lead to less returns.[26] On the other hand, if $s_A + k$ is large, then the firm may induce A^{seq}, while it might be the case that s_B is small, and therefore A^{sim} would lead to less returns than A^{seq}. Both propositions together simply imply that if inspection costs $s_A + k$ and s_B both are small enough, then A^{sim} will be induced and it will lead to less returns than A^{seq} would.

5 Discussion and Conclusion

This paper showed that multi-product firms may induce consumers to buy many products simultaneously and get a refund for the products they want to return. Especially in online markets this may be an interesting proposition for consumers as they may then inspect products at their own ease at home. Presented with this option, consumers buy the product with the highest valuation and are willing to pay a higher price.

To show that this may be a profitable strategy for firms, we also had to consider the alternative, which is for consumers to inspect products sequentially. Sequential inspection may be done either before or after purchase. An interesting subsidiary result of our paper is that the characterization of the optimal contracts under sequential search may be to induce consumers to inspect after purchase and that the way to do so is to set asymmetric contracts where the contract for the first product to be inspected has a lower inspection fee and a higher refund price. These contracts have features in common with optimal obfuscation contracts as in Petrikaitė (2018b), with the main difference that the optimal contracts here have features of a two-part tariff where the firm benefits from having an inspection fee.

Our final result is that despite the appearance of creating unnecessary refunds, "buying and returning many" contracts may actually lead to fewer (rather than more) products being returned. This has interesting implications for environmental policy as the question is not so much to abandon all these "buying and returning many" contracts, but rather to investigate in more detail in what type of markets they are more likely to lead to more or less returns.

[26] Note that Proposition 3 is a sufficient condition - it does not imply that for all $s_B \geq \bar{s}$ A^{seq} leads to less returns. However, it can be shown that for large s_B where the firm sets asymmetric contracts and for either very small or very large η, A^{seq} does lead to less returns.

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A Appendix

A.1 Proof of Proposition 1

Proof. For clarity, we denote $\hat{v}_{ai}$ as $\hat{v}_i$ in this proof. We further define $\overline{\rho} = \eta + \frac{1-F(\overline{\rho})}{f(\overline{\rho})}$. The proof is in several steps. First, note that as long as the consumer continues to inspect the first product first we can always increase σ_1 to increase profits. Thus, we should have $\widehat{v}_1 - \rho_1 = \widehat{v}_2 - \rho_2 \geq 0$. It is easy to show that *if* $\widehat{v}_2 - \rho_2 = 0$, the optimal contract is as specified in the Proposition. If $\widehat{v}_2 - \rho_2 = 0$, the firm's profit equals

$$\begin{aligned} & \sigma_1 - k + (1 - F(\rho_1))(\rho_1 - \eta) + F(\rho_1)(1 - F(\rho_2))(\sigma_2 + \rho_2 - c) + F(\rho_2)(F(\rho_1))(\sigma_2 - k) \\ = \; & \sigma_1 - k + (1 - F(\rho_1))(\rho_1 - \eta) + F(\rho_1)\left(\int_{\rho_2}(1 - F(v))dv - s_A + \rho_2 - c - F(\rho_2)(\rho_2 - \eta)\right). \end{aligned}$$

The derivative wrt ρ_2 equals $-f(\rho_2)(\rho_2 - \eta)$. Thus, we should have $\rho_2 = \eta$ and it then follows from $\widehat{v}_2 - \rho_2 = 0$ that $\sigma_2 = \int_\eta (1 - F(v))dv - s_A$. Thus, the profit on the second product equals $\int_\eta (1 - F(v))dv - s_A - k$ and overall profit is then equal to

$$\int_{\rho_1}(1 - F(v))dv - s_A - k + (1 - F(\rho_1))(\rho_1 - \eta) + F(\rho_1)\left(\int_\eta (1 - F(v))dv - s_A - k\right).$$

The derivative wrt ρ_1 yields

$$-f(\rho_1)(\rho_1 - \eta) + f(\rho_1)\left(\int_\eta (1 - F(v))dv - s_A - k\right),$$

which implies that the optimal ρ_1 is

$$\rho_1 = \eta + \int_\eta (1 - F(v))dv - s_A - k.$$

The rest of the proof shows that it cannot be the case that $\widehat{v}_1 - \rho_1 = \widehat{v}_2 - \rho_2 > 0$. This part of the proof is by contradiction. If $\widehat{v}_2 - \rho_2 > 0$ the firm can increase either σ_2 and ρ_1 or ρ_2 and ρ_1 or σ_2 and σ_1 such that $\widehat{v}_1 - \rho_1 = \widehat{v}_2 - \rho_2 > 0$. By analyzing these joint increases in turn, we successively rule out different subcases that together imply that it cannot be that $\widehat{v}_2 - \rho_2 > 0$.

First consider that we jointly increase σ_2 and ρ_1 such that $\widehat{v}_1 - \rho_1 = \widehat{v}_2 - \rho_2$. We can do that by changing them such that $(1 - F(\widehat{v}_2))d\rho_1 = d\sigma_2$. The profit function is equal to

$$\begin{aligned}&\sigma_1 - k + F(\widehat{v}_2 + \rho_1 - \rho_2)(\sigma_2 - k) + \\ &\left[\int_{\rho_1}^{\widehat{v}_2+\rho_1-\rho_2} F(v_1 - \rho_1 + \rho_2)f(v_1)dv_1 + 1 - F(\widehat{v}_2 + \rho_1 - \rho_2)\right](\rho_1 - \eta) + \\ &\left[\int_{\rho_2}^{\widehat{v}_2} F(v_2 + \rho_1 - \rho_2)f(v_2)dv_2 + F(\widehat{v}_2 + \rho_1 - \rho_2)\left[1 - F(\widehat{v}_2)\right]\right](\rho_2 - \eta).\end{aligned}$$

The increase in profits equals

$$\begin{aligned}F(\widehat{v}_1)\left(1 - F(\widehat{v}_2)\right) + \quad &\int_{\rho_1}^{\widehat{v}_1} F(v_1 - \rho_1 + \rho_2)f(v_1)dv_1 + 1 - F(\widehat{v}_1) - \\ &\left[\int_{\rho_1}^{\widehat{v}_1} f(v_1 - \rho_1 + \rho_2)f(v_1)dv_1 + F(\rho_2)f(\rho_1)\right](\rho_1 - \eta) + \\ &\left[\int_{\rho_2}^{\widehat{v}_2} f(v_2 + \rho_1 - \rho_2)f(v_2)dv_2\right](\rho_2 - \eta),\end{aligned}$$

which can be rewritten as

$$\begin{aligned}&\int_{\rho_1}^{\widehat{v}_1} F(v_1 - \rho_1 + \rho_2)f(v_1)dv_1 + 1 - F(\widehat{v}_1)F(\widehat{v}_2) - F(\rho_2)f(\rho_1)(\rho_1 - \eta) + \\ - \quad &\left[\int_{\rho_1}^{\widehat{v}_1} f(v_1 - \rho_1 + \rho_2)f(v_1)dv_1\right](\rho_1 - \rho_2).\end{aligned}$$

This is equal to

$$\begin{aligned}&\int_{\rho_1}^{\widehat{v}_1} \left[F(v_1 - \rho_1 + \rho_2) - F(\rho_2)\right] f(v_1)dv_1 + 1 - F(\widehat{v}_2)F(\widehat{v}_1) - F(\rho_2)(1 - F(\widehat{v}_1)) \\ &+F(\rho_2)\left[(1 - F(\rho_1) - f(\rho_1)(\rho_1 - \eta)\right] - \int_{\rho_1}^{\widehat{v}_1} f(v_2 + \rho_1 - \rho_2)(\rho_1 - \rho_2)f(v_2)dv_2,\end{aligned}$$

which, as $1 - F$ is logconcave and $1 - F(\widehat{v}_2)F(\widehat{v}_1) - F(\rho_2)(1 - F(\widehat{v}_1)) = (1 - F(\rho_2))(1 - F(\widehat{v}_1)) + F(\widehat{v}_1)(1 - F(\widehat{v}_2)) > 0$, is strictly larger than 0 if $\rho_1 \leq \min\{\rho_2, \overline{\rho}\}$. Thus, if $\widehat{v}_2 - \rho_2 > 0$ we should have $\rho_1 > \min\{\rho_2, \overline{\rho}\}$.

Next, we argue that raising both ρ_1 and ρ_2 to the same extent (keeping $\widehat{v}_1$ and $\widehat{v}_2$ constant) increases in profits if $\rho_i \leq \overline{\rho}, i = 1, 2$. The increase in profits in this case is equal to

$$\begin{aligned}
& (1 - F(\rho_1 + \widehat{v}_2 - \rho_2)) + F(\rho_1 + \widehat{v}_2 - \rho_2)(1 - F(\widehat{v}_2)) + \\
& \int_{\rho_1}^{\rho_1 + \widehat{v}_2 - \rho_2} f(v)F(v + \rho_2 - \rho_1)dv - (\rho_1 - \eta)f(\rho_1)F(\rho_2) + \\
& \int_{\rho_2}^{\widehat{v}_2} f(v)F(v - \rho_2 + \rho_1)dv - (\rho_2 - \eta)f(\rho_2)F(\rho_1) \\
= \; & 1 - F(\rho_1)F(\rho_2) - (\rho_1 - \eta)f(\rho_1)F(\rho_2) - (\rho_2 - \eta)f(\rho_2)F(\rho_1) \\
= \; & F(\rho_2)(1 - F(\rho_1) - f(\rho_1)(\rho_1 - \eta)) + F(\rho_1)(1 - F(\rho_2) - f(\rho_2)(\rho_2 - \eta)) \\
& + (1 - F(\rho_1))(1 - F(\rho_2)),
\end{aligned} \tag{4}$$

which by logconcavity of $1 - F$ is clearly positive if $\rho_i \leq \overline{\rho}$, $i = 1, 2$.[27] Moreover if $\rho_2 = \eta$ this is positive if $F(\rho_2)(1 - F(\rho_1) - f(\rho_1)(\rho_1 - \eta)) + 1 - F(\rho_2) > 0$, which is the case if $1 - F(\rho_2)F(\rho_1) - F(\rho_2)f(\rho_1)(\rho_1 - \eta) > 0$.

We next argue that $\rho_2 \geq \eta$. If not, then a decrease in σ_2 and an increase in ρ_2 such that $\widehat{v}_2 - \rho_2$ is constant (so that $d\sigma_2 = -(1 - F(\widehat{v}_2))d\rho_2$ increases profits. Profits can be written as

$$\begin{aligned}
& \sigma_1 - k + F(\rho_1)[\sigma_2 - k + (1 - F(\rho_2)(\rho_2 - \eta)] + (F(\rho_1 + \widehat{v}_2 - \rho_2) - F(\rho_1))(\sigma_2 - k) + \\
& (\rho_1 - \eta)\int_{\rho_1}^{\rho_1 + \widehat{v}_2 - \rho_2} f(v)F(v + \rho_2 - \rho_1)dv + (\rho_2 - \eta)\int_{\rho_1}^{\rho_1 + \widehat{v}_2 - \rho_2} f(v)(1 - F(v + \rho_2 - \rho_1))dv \\
& + (1 - F(\rho_1 + \widehat{v}_2 - \rho_2))(\rho_1 - \eta)
\end{aligned}$$

so that the increase in profits is equal to

$$\begin{aligned}
& F(\rho_1)[-(1 - F(\widehat{v}_2)) + (1 - F(\rho_2)) - f(\rho_2)(\rho_2 - \eta)] \\
& -(F(\rho_1 + \widehat{v}_2 - \rho_2) - F(\rho_1))(1 - F(\widehat{v}_2)) \\
& + \int_{\rho_1}^{\rho_1 + \widehat{v}_2 - \rho_2} f(v)(1 - F(v + \rho_2 - \rho_1))dv \\
& + (\rho_1 - \rho_2)\int_{\rho_1}^{\rho_1 + \widehat{v}_2 - \rho_2} f(v)f(v + \rho_2 - \rho_1)dv \\
= \; & F(\rho_1)[F(\widehat{v}_2) - F(\rho_2) - f(\rho_2)(\rho_2 - \eta)] \\
& + \int_{\rho_1}^{\rho_1 + \widehat{v}_2 - \rho_2} f(v)(F(\widehat{v}_2) - F(v + \rho_2 - \rho_1))dv \\
& + (\rho_1 - \rho_2)\int_{\rho_1}^{\rho_1 + \widehat{v}_2 - \rho_2} f(v)f(v + \rho_2 - \rho_1)dv
\end{aligned}$$

which is clearly positive if $\rho_2 - \eta \leq 0$ and $\rho_1 \geq \rho_2$. Thus, the optimal solution can only involve $\rho_2 \leq \eta$ and $\widehat{v}_2 - \rho_2 > 0$ if $\rho_1 < \rho_2 \leq \eta$, which cannot be the case as $\rho_1 > \min\{\rho_2, \overline{\rho}\}$.

[27] Note by the way that at $\rho_1 = \rho_2$ this equals 0 if $\rho_1 = \rho_2$ is equal to the joint monopoly price that solves $\rho = \eta + \frac{1 - F^2(\rho)}{2f(\rho)F(\rho)}$.

Consider then an increase in σ_1 and σ_2 so that $\widehat{v}_1-\rho_1=\widehat{v}_2-\rho_2\geq 0$. As $-(1-F(\widehat{v}_i))\frac{\partial \widehat{v}_i}{\partial \sigma_i}=1$, this implies that $\frac{(1-F(\widehat{v}_1))}{(1-F(\widehat{v}_2))}=\frac{d\sigma_1}{d\sigma_2}$. We write the firm's profit as

$$
\begin{aligned}
&\sigma_1-k+(1-F(\rho_1+\widehat{v}_2-\rho_2))(\rho_1-\eta)+F(\rho_1+\widehat{v}_2-\rho_2)(1-F(\widehat{v}_2))(\sigma_2+\rho_2-c)+\\
&F(\rho_2)F(\rho_1)(\sigma_2-k)+(\sigma_2+\rho_1-c)\int_{\rho_1}^{\rho_1+\widehat{v}_2-\rho_2} f(v)F(v+\rho_2-\rho_1)dv+\\
&(\sigma_2+\rho_2-c)\int_{\rho_2}^{\widehat{v}_2} f(v)F(v-\rho_2+\rho_1)dv.
\end{aligned}
$$

So that the increase in profit equals

$$
\begin{aligned}
&\frac{(1-F(\widehat{v}_1))}{(1-F(\widehat{v}_2))}+\frac{f(\rho_1+\widehat{v}_2-\rho_2)}{(1-F(\widehat{v}_2))}(\rho_1-\eta)+F(\rho_1+\widehat{v}_2-\rho_2)(1-F(\widehat{v}_2))+\\
&\frac{F(\rho_1+\widehat{v}_2-\rho_2)f(\widehat{v}_2)-f(\rho_1+\widehat{v}_2-\rho_2)(1-F(\widehat{v}_2))}{(1-F(\widehat{v}_2))}(\sigma_2+\rho_2-c)+\\
&F(\rho_2)F(\rho_1)+\int_{\rho_1}^{\rho_1+\widehat{v}_2-\rho_2} f(v)F(v+\rho_2-\rho_1)dv+\int_{\rho_2}^{\widehat{v}_2} f(v)F(v-\rho_2+\rho_1)dv\\
&-(\sigma_2+\rho_1-c)\frac{f(\rho_1+\widehat{v}_2-\rho_2)F(\widehat{v}_2)}{(1-F(\widehat{v}_2))}-(\sigma_2+\rho_2-c)\frac{F(\rho_1+\widehat{v}_2-\rho_2)f(\widehat{v}_2)}{(1-F(\widehat{v}_2))}.
\end{aligned}
$$

This can be rewritten as

$$
\begin{aligned}
&\frac{(1-F(\widehat{v}_1))}{(1-F(\widehat{v}_2))}+\frac{f(\widehat{v}_1)}{(1-F(\widehat{v}_2))}(\rho_1-\eta)+F(\widehat{v}_1)\\
&-(\rho_1-\rho_2)\frac{f(\widehat{v}_1)F(\widehat{v}_2)}{(1-F(\widehat{v}_2))}-\frac{f(\widehat{v}_1)}{(1-F(\widehat{v}_2))}(\sigma_2+\rho_2-c)\\
=\;&\frac{(1-F(\widehat{v}_1))}{(1-F(\widehat{v}_2))}+\frac{f(\widehat{v}_1)}{(1-F(\widehat{v}_2))}(\rho_1-\eta)+F(\widehat{v}_1)\\
&-(\sigma_2+\rho_1-c)\frac{f(\widehat{v}_1)F(\widehat{v}_2)}{(1-F(\widehat{v}_2))}-\frac{f(\widehat{v}_1)(1-F(\widehat{v}_2)}{(1-F(\widehat{v}_2))}(\sigma_2+\rho_2-c)\\
=\;&\frac{1-F(\widehat{v}_1)}{(1-F(\widehat{v}_2))}-(\sigma_2-k)\frac{f(\widehat{v}_1)F(\widehat{v}_2)}{(1-F(\widehat{v}_2))}+F(\widehat{v}_1)-f(\widehat{v}_1)(\sigma_2-k-\rho_1+\rho_2)\\
=\;&\frac{1-F(\widehat{v}_1)}{(1-F(\widehat{v}_2))}-\frac{(\sigma_2-k)f(\widehat{v}_1)}{(1-F(\widehat{v}_2))}+F(\widehat{v}_1)+f(\widehat{v}_1)(\rho_1-\rho_2),
\end{aligned}
$$

which because $\sigma_2=\int_{\widehat{v}_2}(1-F(v))dv-s_A$ is equal to

$$
\frac{1-F(\widehat{v}_1)}{(1-F(\widehat{v}_2))}-\frac{(\int_{\widehat{v}_2}(1-F(v))dv-s_A-k)f(\widehat{v}_1)}{(1-F(\widehat{v}_2))}+F(\widehat{v}_1)+f(\widehat{v}_1)(\widehat{v}_1-\widehat{v}_2).
$$

This is positive if

$$
\frac{1-F(\widehat{v}_1)F(\widehat{v}_2)}{f(\widehat{v}_1)}-\left(\int_{\widehat{v}_2}(1-F(v))dv-s_A-k\right)+(1-F(\widehat{v}_2))(\widehat{v}_1-\widehat{v}_2)>0. \tag{5}
$$

That is certainly the case if $\widehat{v}_2=\overline{v}$. The derivative of this expression wrt $\widehat{v}_2$ equals

$$
-\frac{F(\widehat{v}_1)f(\widehat{v}_2)}{f(\widehat{v}_1)}+(1-F(\widehat{v}_2))-(1-F(\widehat{v}_2))-f(\widehat{v}_2))(\widehat{v}_1-\widehat{v}_2).
$$

This is clearly nonpositive if $-\frac{F(\widehat{v}_1)f(\widehat{v}_2)}{f(\widehat{v}_1)} - f(\widehat{v}_2))(\widehat{v}_1 - \widehat{v}_2) \leq 0$, which is the case if $\widehat{v}_1 \geq \widehat{v}_2 - \frac{F(\widehat{v}_1)}{f(\widehat{v}_1)}$. So, if we decrease $\widehat{v}_2$ starting from $\widehat{v}_2 = \overline{v}$, then 5 remains positive if $\widehat{v}_1 \geq \widehat{v}_2 - \frac{F(\widehat{v}_1)}{f(\widehat{v}_1)}$. So, the only possibility for an equilibrium with $\widehat{v}_2 > \rho_2$ is that $\widehat{v}_1 < \widehat{v}_2 - \frac{F(\widehat{v}_1)}{f(\widehat{v}_1)}$.

To rule out that $\widehat{v}_1 < \widehat{v}_2 - \frac{F(\widehat{v}_1)}{f(\widehat{v}_1)}$ we finally consider that we increase σ_2 and decrease ρ_2 such that $\widehat{v}_2 - \rho_2$ is constant. We can do that by changing them such that $-(1 - F(\widehat{v}_2))d\rho_1 = d\sigma_2$. The profit function is equal to

$$\begin{aligned}
&\sigma_1 - k + F(\widehat{v}_2 + \rho_1 - \rho_2)(\sigma_2 - k) + \\
&\left[\int_{\rho_1}^{\widehat{v}_2+\rho_1-\rho_2} F(v_1 - \rho_1 + \rho_2)f(v_1)dv_1 + 1 - F(\widehat{v}_2 + \rho_1 - \rho_2)\right](\rho_1 - \eta) + \\
&\left[\int_{\rho_2}^{\widehat{v}_2} F(v_2 + \rho_1 - \rho_2)f(v_2)dv_2 + F(\widehat{v}_2 + \rho_1 - \rho_2)\left[1 - F(\widehat{v}_2)\right]\right](\rho_2 - \eta).
\end{aligned}$$

The increase in profits equals

$$\begin{aligned}
&F(\widehat{v}_1)\left(1 - F(\widehat{v}_2)\right) - \left[\int_{\rho_2}^{\widehat{v}_2} F(v_2 + \rho_1 - \rho_2)f(v_2)dv_2 + F(\widehat{v}_1)\left[1 - F(\widehat{v}_2)\right]\right] \\
&- \left[\int_{\rho_1}^{\widehat{v}_1} f(v_1 - \rho_1 + \rho_2)f(v_1)dv_1\right](\rho_1 - \eta) + \left[-F(\widehat{v}_1)f(\widehat{v}_2)\right](\rho_2 - \eta) + \\
&\left[\int_{\rho_2}^{\widehat{v}_2} f(v_2 + \rho_1 - \rho_2)f(v_2)dv_2 + F(\rho_1)f(\rho_2) + F(\widehat{v}_1)f(\widehat{v}_2)\right](\rho_2 - \eta),
\end{aligned}$$

which can be rewritten as

$$\begin{aligned}
- &\int_{\rho_2}^{\widehat{v}_2} F(v_2 + \rho_1 - \rho_2)f(v_2)dv_2 \\
&+ \left[\int_{\rho_2}^{\widehat{v}_2} f(v_1 + \rho_1 - \rho_2)f(v_2)dv_2\right](\rho_2 - \rho_1) + F(\rho_1)f(\rho_2)(\rho_2 - \eta) \\
\geq &\int_{\rho_2}^{\widehat{v}_2} \left[\frac{f(v_1 + \rho_1 - \rho_2)}{F(v_2 + \rho_1 - \rho_2)}\frac{F(\widehat{v}_1)}{f(\widehat{v}_1)} - 1\right] F(v_2 + \rho_1 - \rho_2)f(v_2)dv_2 + F(\rho_1)f(\rho_2)(\rho_2 - \eta).
\end{aligned}$$

As F is logconcave, f/F is decreasing and therefore $\frac{f(v_1+\rho_1-\rho_2)}{F(v_2+\rho_1-\rho_2)} > \frac{f(\widehat{v}_1)}{F(\widehat{v}_1)}$. Thus, the term in square brackets is positive and the whole expression is strictly positive as $\rho_2 > \eta$. ■

A.2 Proof of Proposition 2

Proof. The profits under the two search modes are

$$\pi^*_{Asim} = \mathbb{E}[\max(v_1 - \eta, v_2 - \eta, 0)] - 2(s_A + k),$$

and

$$\pi^*_{Aseq} = \mathbb{E}[\max(v_1 - \eta, \mathbb{E}[\max(v_2 - \eta, 0)] - s_A - k)] - s_A - k$$

respectively, where in the second equation it is important to note that the second product is only inspected if inspection of the first product results in a low value. Thus, we have that $\pi^*_{Asim} \geq \pi^*_{Aseq}$, if and only if,

$$\mathbb{E}[\max(v_1 - \eta, v_2 - \eta, 0)] \geq \mathbb{E}[\max(v_1 - \eta + s_A + k, \mathbb{E}[\max(v_2 - \eta, 0)])]$$

It is immediately evident that for $s_A + k = 0$ and for any value of η, A^{sim} leads to strictly higher profits. Thus, by continuity of the RHS in $s_A + k$, it follows that there exists a threshold $\underline{S}_A(\eta)$ such that A^{sim} yields larger profit if $s_A + k \leq \underline{S}_A(\eta)$. On the other hand, as the RHS of the above inequality is weakly increasing in $s_A + k$, and strictly increasing in $s_A + k$ if $s_A + k$ is large enough, it also follows that A^{seq} yields larger profit if $s_A + k > \underline{S}_A(\eta)$. This proves the first part of the proposition.

For the second part we need to prove that

$$1 + F^2(\eta) > F(\rho_1^*)(1 + F(\eta)).$$

This can be written as

$$\frac{(1 + F(\eta))^2 - 2F(\eta)}{1 + F(\eta)} = 1 + F(\eta) - \frac{2F(\eta)}{1 + F(\eta)} > F(\rho_1^*). \tag{6}$$

The LHS equals 1 at $\eta \leq \underline{v}$ and at $\eta = \overline{v}$. The RHS equals 1 at $\eta = \overline{v}$ and is smaller than 1 at $\eta \leq \underline{v}$. The derivatives of the LHS and the RHS wrt η are respectively $f(\eta) - \frac{2f(\eta)}{(1+F(\eta))^2} = f(\eta)\left(1 - \frac{2}{(1+F(\eta))^2}\right)$, which is first decreasing and then from $F(\eta) = \sqrt{2} - 1$ it is increasing, and $f(\rho_1^*)F(\eta) > 0$. At $\eta = \overline{v}$ the derivative of the LHS is smaller than that of the RHS.

The derivatives are equal to each other if $\left(\frac{1}{F(\eta)} - \frac{2}{F(\eta)(1+F(\eta))^2}\right) = f(\rho_1^*)/f(\eta)$. As $1 - F$ is logconcave, $\frac{1-F(v)}{f(v)}$ is decreassing in v and thus $\frac{f(\rho_1^*)}{f(\eta)} > \frac{1-F(\rho_1^*)}{1-F(\eta)}$ as $\rho_1^* > \eta$. Thus, the derivatives can only be equal to each other if

$$\left(\frac{1}{F(\eta)} - \frac{2}{F(\eta)\,(1 + F(\eta))^2}\right) > \frac{1 - F(\rho_1^*)}{1 - F(\eta)},$$

which can be rewritten as

$$\left(\frac{1}{F(\eta)} - \frac{2(1 - F(\eta))}{F(\eta)\,(1 + F(\eta))^2}\right) = \frac{-1 + 4F(\eta) + F^2(\eta)}{F(\eta)\,(1 + F(\eta))^2} > 2 - F(\rho_1^*),$$

or

$$-1 + 3F(\eta) - 2F^2(\eta) - F^3(\eta) > (1 - F(\rho_1^*))\,(1 + F(\eta))^2\,F(\eta).$$

As the LHS is negative for any $0 \leq F(\eta) \leq 1$, while the RHS is positive, this inequality can never hold. Thus, the the derivative of the LHS of (6) is always smaller than the derivative of its RHS and therefore (6) holds. ■

A.3 Proof of Proposition 3

Proof. An optimal A^{seq} contract for small s_B must exist: As the firm first sets its contract and the consumer searches only afterwards, the firm can fully predict the behavior of the consumer. The firm's expected profit is continuous in all variables and bounded (an upper bound is the efficient surplus, a lower bound is $-2c$). The domain of the expected profit function is closed and bounded. Further the value distribution is continuous and its domain is also closed and bounded. Then by the extreme value theorem a maximum must exist.

For $s_B = s_A = k = 0$ we can identify the optimal contract: As search is free, the consumer will never pay a positive inspection fee and therefore $\sigma_i = 0$. Then the optimal price is $\rho_i = p^{JM}$ with $p^{JM} := c + \frac{1-F(p^{JM})^2}{2F(p^{JM})f(p^{JM})}$, the joint monopoly price. Note that $p^{JM} > c \geq \eta$. Then $n_{Aseq} = 1 + F(p^{JM})$, which is strictly larger than n_{Asim}. Note that at this lower bound for the parameters the firm and consumer are indifferent between inducing the consumer to inspect before and after purchase. However, once $0 < s_A + k < s_B$, the firm will prefer to induce the consumer to inspect after purchase, as we argue in the following.

We now argue that for small positive s_B it will similarly be the case that $n_{Aseq} > n_{Asim}$. First, note that, depending on the contract, it may be optimal for the consumer to only decide after observing the value of the first product whether to inspect the second product before or after purchase. This optimal decision rule is known and predictable by the firm. By contrast, the decision whether to inspect the first product before or after purchase cannot be conditioned on anything learned during search or otherwise, and therefore will always be the same. Then the firm can choose the contract such that the consumer always starts by inspecting the first product after purchase and this will be optimal for the firm as we assume $s_A + k \leq s_B$. As σ_1 is paid before anything else happens, the firm optimally sets it to the highest possible value for which the consumer is indifferent to starting search by inspecting the first product before purchase. Therefore there exists an upper bound on σ_1 which depends on the firm's other strategic variables ρ_1, ρ_2 and σ_2, but importantly also on the value of s_B. A small value of s_B close to zero will result in a tight bound on σ_1 that goes to zero as s_B goes to zero.

As the consumer starts search with the product with the biggest net reservation value $\hat{v}_i - \rho_i$, it follows that σ_2 cannot be too different from σ_1 if also ρ_1 and ρ_2 are close to each other. Then there will also exist an upper bound for σ_2, which will be similarly affected by s_B. In particular, if s_B is very small, then σ_2 will also have to be very small. Compared to the case where $s_B = 0$, the firm will also want to increase the σ_i somewhat at the expense of slightly decreasing the ρ_i. The reason is that as we argued above, the upper bound on the σ_i

depends negatively on the ρ_i. Further, the profit function is strictly increasing in σ_i, while $\rho_i = p^{JM}$ is the profit maximizing value, for which by the envelope theorem a small change does not affect profit in a substantial way. Then it is profitable to slightly increase σ_i and slightly decrease ρ_i. Overall, for small s_B the effect will be small, implying that ρ_i will still be close to p^{JM} and certainly larger than η, while σ_i will be close to zero.

Putting everything together, we know that for a small positive s_B, σ_2 will be small, implying $\hat{v}_{a2}$ will be close to $\bar{v}$ and $F(\hat{v}_{a2}) \sim 1$. At the same time, ρ_i will be very close to p^{JM} and certainly larger than η. Together, this implies that there exists a $\bar{s}$ such that for $s_B < \bar{s}$ it holds that $n_{Aseq} > n_{Asim}$. ■