# Endogenous common ownership\*

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#### Abstract

We develop a model where the level of common ownership (when an investor holds stakes in competing firms) and cross ownership (when a company holds a stake in a competitor) are endogenously determined. Both common and cross ownership lessen the competition among the firms, raising industry profits. What limits the level of overlapping ownership is its negative impact on corporate governance. We analyze various possible mechanisms that generate this impact, all of which share the property that a decrease in the ownership share left in the hands of the controlling stakeholder reduces his incentives to exert actions that improve firm value. In this framework, we analyze the determinants of the equilibrium level of overlapping ownership, such as the intensity of product market competition and the quality of corporate governance. A feature of the model is that common ownership may not emerge as a smooth process as corporate governance improves, but rather with a sudden, discrete jump. This may help explain why the last decades have witnessed a large increase in the level of overlapping ownership even if the underlying factors do not seem to have changed significantly, and why common ownership accounts for the lion's share of such increase.

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# 1 Introduction

Common ownership is when an investor holds stakes in firms that ought to compete with each other. Firms involved in such overlapping-ownership structure may behave differently from standard profit-maximizing agents; in particular, they may compete less intensely.

A growing literature, reviewed below, has documented a significant increase in common ownership in the last decades and raised concerns about the possible anticompetitive effects. This literature has generally regarded the degree of common ownership as something exogenous to the industry, being determined by investment funds' strategies of portfolio diversification. Accordingly, it has focused on the consequences rather than the causes of the phenomenon.

But if common ownership does relax competition and increase industry profits, there must be incentives to acquire stakes in competing firms that go beyond the diversification motive. The objective of this paper is to analyze these incentives and the way they endogenously shape the ownership structure of industries. In other words, we ask what determines common ownership when investors' portfolios are already well diversified, and, in particular, what limits its extent.

Given that policy remains permissive,<sup>1</sup> and common ownership is good for firms' profits, one may indeed wonder why we do not observe even more of it. We propose that the reason for this is that common ownership aggravates the agency problems inside the firm. The idea, in a nutshell, is that when outsiders acquire a stake in a company, the share left in the hands of the controlling stakeholder falls. This reduces his incentives to choose actions that improve firm value, such as monitoring the managers, exerting non-contractible efforts that reduce costs or increase demand, or refraining from extracting private benefits from his control position. The equilibrium level of common ownership balances the increase in profits due to softer product market competition and the reduction in firm value due to a less effective governance of the companies.

More in detail, our baseline model considers two symmetric firms that compete in the same product market. Each firm is run by a manager and is controlled by a large blockholder. An institutional investor is interested in acquiring a stake in both firms. Such acquisition softens product market competition: the larger the common owner's stakes, the less intensely firms compete, and hence the higher industry profits. However, shareholders cannot appropriate all the profits, as managers divert part of the firm's cash flow to themselves. The blockholders can limit such rent diversion by monitoring the managers. However, monitoring is privately costly and not contractible. As a result, minority shareholders free ride on the blockholders' efforts without engaging in monitoring themselves. Therefore, the smaller the blockholders' stakes, the lower their monitoring efforts.

This implies that when the common owner holds a stake in both firms, product market competition becomes less intense, which is good for profits, but monitoring

<sup>&</sup>lt;sup>1</sup>Proposals to limit the degree of overlapping ownership have been put forward in the scholarly debate (see e.g. Elhauge 2016, Posner 2017 and Rock and Rubinfield, 2018) but have had little impact on antitrust policy so far.

is reduced, with an adverse effect on share value. This trade-off determines the equilibrium ownership structure.

The trade-off would not arise if the investor acquired its stakes from dispersed shareholders. But another well known free-riding problem (Grossman and Hart, 1980) implies that this acquisition would not be profitable. Things are different when the investor buys its stakes from the blockholders, who are large enough to internalize the effect of the acquisition on share values.

Whether the model's equilibrium exhibits common ownership, and to what extent, depends on several factors. Among these, the most important ones are perhaps the intensity of product market competition and the quality of corporate governance rules and institutions. We show that the more intense competition, the higher the degree of common ownership. Furthermore, the smaller the fraction of profits that an unmonitored manager can appropriate, the higher the degree of common ownership. The latter result implies that improvements in corporate governance, that reduce the need for monitoring the managers, benefit shareholders but are detrimental to consumers, as they lead to more common ownership and hence higher prices.

A simple variant of the model demonstrates that the equilibrium ownership structure may include dispersed owners. In this variant, the blockholder sells part of his stake to dispersed shareholders before dealing with the investor. As in Zingales (1995), the blockholder does so in order to extract the full value of the shares sold earlier. When negotiating with the investor, on the other hand, he must leave some of the surplus to it (unless he has all the bargaining power). However, the stake optimally sold to dispersed shareholders is limited by the fact that they do not contribute to softening competition.

In two other variants, we consider different types of agency problem. In the first one, it is the blockholders who can appropriate private benefits of control at the expense of the other shareholders. If such appropriation is inefficient, as in Burkart, Gromb and Panunzi (1998), overlapping ownership will again be limited by the fact that the smaller the blockholders' stakes, the lower the internalization of the deadweight losses created by rent extraction. In the second variant, overlapping ownership reduces the blockholders' incentive to exert efforts that increase firms' profits but are privately costly, as in Anton et al. (2022). The trade-off between softer product market competition and more acute agency problems emerges also in these new frameworks.

A similar logic applies to the case of cross ownerships, when a company acquires a stake in a rival. However, there are two important differences between cross and common ownership. First, cross ownership can be profitable even if it is unilateral; i.e., only one firm acquires a stake in the rival. Second, while a small change in the underlying economic factors will always cause a small change in the equilibrium level of cross ownership, it can make the equilibrium level of common ownership jump from zero to a relatively large positive level (or vice versa). This property of the equilibrium may help explain why a large increase in the degree of overlapping ownership has been observed in the last decades, even though the underlying factors do not seem to have changed significantly, and why common ownership seems to be responsible for the lion's share of the increase. The rest of the paper is structured as follows. Section 2 briefly reviews the related literature. Section 3 presents the baseline model of common ownership. Section 4 derives the equilibrium of the model. Section 5 provides a characterization of the equilibrium ownership structure, and section 6 analyzes the comparative statics of the equilibrium. Section 7 analyzes the alternative agency problems mentioned above. Section 8 turns to the analysis of cross ownership. Finally, Section 9 concludes the paper by discussing several model extensions and the possible implications for policy. All proofs are collected in Appendix A, whereas Appendix B presents some specific examples.

# 2 Relation to the literature

The notion that overlapping ownership mitigates the intensity of product market competition was first put forward by Rotemberg (1984) and O'Brien and Salop (2000). This notion is quite natural in the case of cross ownership, as a firm that holds a stake in a rival has an obvious incentive to compete less aggressively not to impair its rival's profitability. For the case of common ownership, on the other hand, things are less obvious. Rotemberg (1984) assumes that companies act in the interest of their shareholders, and that the possible heterogeneity in shareholders' interests is accounted for by forming a weighted average of their payoffs, with weights given by their respective ownership shares. As a result, under common ownership each firm maximizes a linear combination of own and rivals' profits. The higher the relative weight given to rivals' profits, which is commonly referred to as the "lambda," the less aggressively firms compete in product markets, and hence the higher prices and profits.<sup>2</sup>

This weighted-payoff approach has been adopted broadly in applied work, and we follow it in this paper. The literature has proposed various mechanisms that may lead firms' managers to internalize the interests of minority shareholders. For example, Azar (2017) develops a theory where a company's management proposes a strategic plan to its shareholders and dislikes their disapproval or opposition.<sup>3</sup> As another example, Anton et al. (2022) study a mechanism based on managerial incentives. They argue that firms with common owners tolerate managerial slack to a higher degree in order to keep prices and profits high. Piccolo and Schneemeier (2020) analyze a model where some investors acquire a diversified portfolio while others hold undiversified portfolios. The crowding out of undiversified investors leads to an anticompetitive effect. Schmalz (2021) reviews these and other possible governance mechanisms whereby common ownership may affect competitive outcomes. Shekita (2021) analyzes the channels through which common ownership influences firm behavior empirically. Studying 30 cases of common ownership, he documents three main corporate governance mechanisms – voice and engagement, executive

<sup>&</sup>lt;sup>2</sup>Overlapping ownerships may affect not only prices but also other strategic choices of the firms, such as for instance investment in R&D: see e.g. Lopez and Vives (2019).

 $<sup>^{3}</sup>$ Azar (2020) argues the anticompetitive effects of common ownership are mitigated when managers are entrenched. Yet, they only disappear in the extreme case where managers are fully insulated from shareholders dissent.

compensation, and voting – as a conduit to influence firm decision-making.

Some scholars, on the other hand, continue to adhere to the traditional view that index funds are passive investors that do not intervene in their portfolio companies and thus cannot facilitate anticompetitive behavior. See e.g. Bebchuk et al. (2017) for a recent articulation of this view. Banal-Estañol et al. (2020) distinguish between passive and active investment funds, the former being more diversified than the latter. They document that in two sectors, department stores and publishers, when the holdings of the passive investors decrease, common-ownership incentives decrease. On the contrary, when the holdings of the passive investors increase, common-ownership incentives increase. Heath et al. (2022) show that, relative to active funds, index funds are more likely to vote against firm management on contentious governance issues. Passive investors also tend to promote less board independence, and worse pay-performance sensitivity, at their portfolio companies.

Although the last few years have witnessed a blooming of empirical studies on the effects of common ownership, the empirical literature is also unsettled. In a pioneering contribution, Azar, Schmalz and Tecu (2018) show that common ownership increases market concentration in the U.S. airline industry, and that within-route increases in common ownership correlate with route-level increases in ticket prices. These findings are confirmed in the analysis carried out by Park and Seo (2019). However, Kennedy et al. (2017) and Dennis et al. (2022), using a different structural model of the US airline industry or different measures of investor control of airlines operating in bankruptcy, do not find evidence that common ownership raises airline prices.<sup>4</sup> Azar, Schmalz and Tecu (2021) address these critiques and argue that in fact they do not invalidate their main finding.

Looking at other industries, Torshizi and Clapp (2021) document that the rise of common ownership explains a sizeable fraction of the increase in soy, corn, and cotton seed prices over the 1997-2017 period. He and Huang (2017), using a sample of U.S. public firms from 1980 to 2014, find evidence suggesting that institutional cross-ownership facilitates explicit forms of product market collaboration (e.g., within-industry joint ventures, strategic alliances, or within-industry acquisitions) and improves innovation productivity and operating profitability. On the other hand, Backus et al. (2021a) find little support for markup effects of common ownership in the ready-to-eat cereal industry.

In spite of the ongoing controversies, a consensus seems to be emerging that common ownership does affect product market competition.<sup>5</sup> Some authors have

 $<sup>{}^{4}</sup>$ In a similar vein, Lewellen and Lowry (2021) contend that the effects that are commonly attributed to common ownership are caused by other factors, such as differential responses of firms (or industries) to the 2008 financial crisis. Controlling for thes factors, they find little robust evidence that common ownership affects firm behavior.

<sup>&</sup>lt;sup>5</sup>While most of the literature focuses on a single industry, Azar and Vives (2021) analyze common ownership in a general equilibrium oligopoly model. They argue that when common ownerships extends from a specific sector to the whole economy, it will reduce markups and prices. This follows from the fact that when an industry expands, it creates positive externalities for other industries. Inter-industry common ownership allows firms to better internalize these externalities, creating incentives for firms to expand output and reduce prices. In an attempt to empirically test this prediction, Azar and Vives (2022) reconsider the US airline industry and find that common ownership

ventured to quantify the welfare effects of common ownership. For example, Ederer and Pellegrino (2022) estimate that the welfare cost of common ownership, measured as the ratio of deadweight loss to total surplus, has increased more than tenfold in about 20 years, from 0.3% in 1994 to over 4% in 2018.

What is not controversial, in any case, is that overlapping ownership, and in particular common ownership, has been on the rise in the last decades. Backus at al. (2021b) calculate the weight that S&P 500 companies would place on rivals' profits in their objective function under the standard proportional-control assumption and show that the average weight tripled in the last decades, from 0.2 in the 1980s to almost 0.7 in 2017.

# **3** Baseline model

In this section, we present the assumptions of our basic model of common ownership. We focus on investors that are already well diversified and therefore acquire a stake in competing firms purely in the anticipation of capital gains – the increase in firm value due to less intense competition.

After developing this baseline model, we shall consider several variants in the subsequent sections.

## 3.1 Agents

Consider an industry with two *ex-ante* symmetric firms, indexed by i = 1, 2, that compete in the same product market. Initially, firm i is owned by blockholder  $\mathcal{B}_i$ , who holds a fraction b of the firm's shares, and a mass of dispersed shareholders, who taken together own the remaining fraction 1 - b. For now, we take b as given, but later we shall endogenize this variable.<sup>6</sup>

An institutional investor, denoted  $\mathcal{I}$ , may acquire a stake  $s_i$  of firm *i* from its initial owners. We assume that  $\mathcal{I}$ 's portfolio is already well diversified,<sup>7</sup> so  $\mathcal{I}$  proceeds with the acquisition only if it can obtain a capital gain. This is possible because of the lessening-of-competition effect of common ownership, which is a key element of the analysis.

Dispersed shareholders are forward looking and correctly anticipate the *ex post* value of their shares. This creates the well-known free-rider problem first analyzed in Grossman and Hart (1980). The implication is that  $\mathcal{I}$  cannot gain by purchasing shares from dispersed shareholders.

The blockholder, on the other hand, internalizes the effect of  $\mathcal{I}$ 's acquisition on share value, so there is room for a profitable deal with him. Therefore, if the investor

by the "Big Three" (BlackRock, Vanguard and State Street), taken as a proxy for economy-wide common ownership, is associated with lower airline prices, whereas common ownership by other investors, taken as a proxy for industry-specific common ownership, is associated with higher prices.

<sup>&</sup>lt;sup>6</sup>As a rule, we use latin letters to denote endogenous variables, calligraphic letters for agents, and greek letters for exogenous parameters. The one exception is profits, which are denoted by  $\pi$ .

<sup>&</sup>lt;sup>7</sup>See Shy and Stenbacka (2020) for a model that instead emphasizes the diversification motive for common ownership.

acquires any stake  $s_i$  in firm *i*, it will acquire it from  $\mathcal{B}_i$ . After the acquisition,  $\mathcal{B}_i$  is left with a residual ownership share of  $b - s_i$ .

Firm *i* is run by a manager,  $\mathcal{M}_i$ . If left unchecked, manager  $\mathcal{M}_i$  diverts part of the firm's revenues to herself, appropriating a fraction  $\xi$  of the profits  $\pi_i$  in the form of private benefits. (To avoid confusion, we use feminine pronouns for managers, masculine pronouns for blockholders, and neutral pronouns for institutional investors).

Private-benefit extraction is limited by shareholders' monitoring efforts. We assume that only blockholders engage in monitoring minority shareholders free ride on the blockholder's monitoring efforts and do not exert any effort of their own.<sup>8</sup>

 $\mathcal{B}_i$ 's monitoring,  $m_i$ , reduces the manager's private benefits from  $\xi \pi_i$  to  $\xi(1 - m_i)\pi_i$ . The private cost of monitoring,  $C(m_i)\pi_i$ , is taken to be proportional to the firm's profit. This simplifies the analysis by making  $\mathcal{B}_i$ 's optimal choice of  $m_i$  independent of product market competition. The function  $C(m_i)$  is increasing and convex, with C(0) = 0. To guarantee an interior solution, we assume C'(0) = 0 and  $C'(1) > \xi$ .

### 3.2 Payoffs

Under the above assumptions, blockholder  $\mathcal{B}_i$ 's payoff is

$$B_i = (b - s_i) \left[ 1 - \xi (1 - m_i) \right] \pi_i - C(m_i) \pi_i + P_i(s_i), \tag{1}$$

where  $P_i(s_i)$  denotes the acquisition price of the entire block  $s_i$ . The first term on the right-hand side is the value of  $\mathcal{B}_i$ 's residual stake  $b - s_i$ , the second is the cost of monitoring, and the last is the revenue from the sale of stake  $s_i$  to the investor. Likewise, the investor's payoff is

$$I = \sum_{i=1}^{2} \left\{ s_i \left[ 1 - \xi (1 - m_i) \right] \pi_i - P_i(s_i) \right\}.$$
 (2)

As for the managers, we assume that they appropriate what private benefits they can, subject to blockholders' monitoring. Besides, they are responsible for product market competition choices. Following the recent literature on common ownership reviewed in section 2, we assume that when managers make these strategic decisions, they maximize a linear combination of the shareholders' payoffs, with weights proportional to their respective ownership shares:

$$O_i = (b - s_i) B_i + \theta s_i I.$$
(3)

<sup>&</sup>lt;sup>8</sup>There may be various reason for this. For example, one may think of the monitoring activity of different agents as entirely duplicative. In this case, when effort levels are chosen non-cooperatively, in equilibrium only the largest shareholder engages in monitoring. Another possibility is that dispersed shareholders and institutional investors are simply less capable of monitoring the managers than the blockholders. In any case, the assumption that only blockholders engage in monitoring simplifies the analysis but is not really necessary for our results: all that matters is that blockholders exert more effort per share owned.

In equation (3), the degree of influence of minority shareholders is measured by the parameter  $\theta \leq 1$ . As discussed in section 2, different mechanisms have been proposed that may lead managers to internalize, at least partially, the interests of minority shareholders. Our results do not depend on the specific mechanism and hold as long as  $\theta > 0$ . The case  $\theta = 0$ , on the other hand, would correspond to the traditional view that institutional investors do not influence managerial strategies. (The assumption that  $\theta > 0$  is not necessary in the model with cross ownership of section 8, nor in the model with privately costly investment of section 7.2. In these models, the results continue to hold even if minority shareholders have no influence on firms' strategic choices.)

The payoff of dispersed shareholders does not appear in (3) because the term that would correspond to a generic dispersed shareholder  $\mathcal{D}_h$  who holds a share  $\varepsilon_{hi}$ of firm i is  $\theta \varepsilon_{hi} \times \varepsilon_{hi} [1 - \xi(1 - m_i)] \pi_i$ , so for  $\varepsilon_{hi} \approx 0$  it would be negligible. This implies that the interests of dispersed shareholders do not affect managerial choices even if  $\theta > 0$ .

In equation (3), managers assess shareholders' interests accounting for profit diversion and monitoring costs.<sup>9</sup> This may seem far-fetched; it is probably more natural to assume that managers disregard these components of the shareholders' payoffs.<sup>10</sup> That is,  $B_i$  is replaced by  $(b - s_i) \pi_i$  and I by  $\sum_{j=1}^2 s_j \pi_j$ . In this simpler and

more standard formulation, the firm's objective function becomes:

$$\tilde{O}_{i} = (b - s_{i})^{2} \pi_{i} + \theta s_{i} \sum_{j=1}^{2} s_{j} \pi_{j}.$$
(4)

Our main results hold with both specifications, (3) and (4), but for simplicity the presentation focuses on the latter.

The objective function (4) may be rewritten as:

$$O_i = \pi_i + \lambda_i \pi_j,\tag{5}$$

where:

$$\lambda_i = \frac{\theta s_i s_j}{\left(b - s_i\right)^2 + \theta s_i^2}.^{11} \tag{6}$$

Thus, when  $\theta > 0$  firms maximize a weighted average of own and rival's profits. The weight attached to the rival's profit,  $\lambda_i$ , is nil if either  $s_i$  or  $s_j$ , or both, vanish.

<sup>11</sup>With the alternative specification (3), one would have:

$$\lambda_{i} = \theta \frac{s_{i}s_{j} \left[1 - \xi_{j}(1 - m_{j}^{*})\right]}{\left[\left(b_{i} - s_{i}\right)^{2} + \theta s_{i}^{2}\right] \left[1 - \xi_{i}(1 - m_{i}^{*})\right] - \left(b_{i} - s_{i}\right)C_{i}(m_{i}^{*})}$$

<sup>&</sup>lt;sup>9</sup>In the formulation (3), acquisition prices also affects the company's objective function, even if they represent a pure transfer between different shareholders.

<sup>&</sup>lt;sup>10</sup>In particular, it seems questionable that managers discount the profits they appropriate; if anything, they should give to these profits a greater weight than to those left to the shareholders.

#### 3.3 Product market competition

For sake of generality, we consider a reduced-form model of product market competition. Each firm *i* chooses a strategic variable  $x_i$  (e.g., price or quantity) and these choices determine firms' profits  $\pi_i(x_i, x_j)$ . (For ease of notation we treat  $x_i$ as a scalar but nothing would change if it were a vector). Since firms are *ex-ante* symmetric, the functions  $\pi_i(x_i, x_j)$  are taken to be symmetric. We also assume that they are quasi-concave and twice continuously differentiable over the relevant range. In Appendix B, we present specific models of product market competition where these assumptions hold.

### 3.4 Bargaining

The acquisition prices  $P_i$  are determined through a bargaining process between the investor and the blockholders. We shall consider both the case where the terms of  $\mathcal{I}$ 's agreement with  $\mathcal{B}_i$  may be conditioned on the agreement reached with  $\mathcal{B}_j$ , and the case where they may not.

#### 3.5 Timing

The game proceeds in three stages. In the first stage, investor  $\mathcal{I}$  bargains with blockholders  $\mathcal{B}_1$  and  $\mathcal{B}_2$  over the stakes to be acquired,  $s_1$  and  $s_2$ , and the acquisition prices,  $P_1$  and  $P_2$ . In the second stage, firms engage in product market competition, determining the equilibrium profits  $\pi_i$ . Finally, in the last stage of the game blockholders choose their monitoring efforts  $m_i$ , and payoffs are realized.

## 4 Equilibrium

We are interested in the subgame perfect equilibrium of the game, so we solve the model backwards.

#### 4.1 Monitoring

In the last stage of the game, blockholder  $\mathcal{B}_i$  chooses  $m_i$  so as to maximize his payoff  $B_i$ . At this stage,  $s_i$ ,  $P_i$  and profits  $\pi_i$  are pre-determined, so the blockholder's objective function reduces to:

$$\{(b-s_i) \left[1-\xi_i(1-m_i)\right] - C_i(m_i)\} \pi_i.$$

Since our assumptions guarantee an interior solution, the equilibrium level of monitoring is determined by the first-order condition:

$$C'(m_i) = (b - s_i)\xi.$$

$$\tag{7}$$

Convexity of  $C(m_i)$  implies that the equilibrium level of monitoring  $m_i^*$  increases with the blockholder's residual ownership share,  $b - s_i$ , and the manager's ability to steal,  $\xi$ .

As noted, our specification of the monitoring costs implies that  $m^*$  does not depend on  $\pi_i$ . Note also that monitoring is inefficiently low from the aggregate viewpoint of the shareholders. From this viewpoint, the optimal level of monitoring is given by condition  $C'(m) = \xi$ .

#### 4.2 Product market competition

Manager  $\mathcal{M}_i$  chooses  $x_i$  so as to maximize  $\tilde{O}_i = \pi_i + \lambda_i \pi_j$ . For simplicity, we assume that there exists a unique, interior Nash equilibrium, which is characterized by following first- and second-order conditions:<sup>12</sup>

$$\frac{\partial \pi_i}{\partial x_i} + \lambda_i \frac{\partial \pi_j}{\partial x_i} = 0 \tag{8}$$

$$\left(\frac{\partial^2 \pi_i}{\partial x_i^2} + \lambda_i \frac{\partial^2 \pi_j}{\partial x_i^2}\right) < 0.$$
(9)

Equilibrium profits are  $\pi_i^*(\lambda_i, \lambda_j)$ . Since the weights  $\lambda_i$  depend on the stakes  $s_i$ , we shall also denote equilibrium profits by  $\pi_i^*(s_i, s_j)$ .

## 4.3 Bargaining

In the first stage of the game, the investor bargains with the blockholders over the stakes to be acquired and their respective prices,  $(s_1, P_1)$  and  $(s_2, P_2)$ .

To fix ideas, we assume that either the buyer or the sellers make a take-it-orleave-it offer. Denote by  $\alpha$  the probability that the investor makes the offers and the blockholders receive them; with probability  $1-\alpha$ , these roles are reversed. Thus,  $\alpha$  is the share of the bargaining surplus obtained on average by the investor – a measure of its bargaining power.<sup>13</sup>

At this point, one can distinguish between two bargaining protocols, depending on whether the offers made by  $\mathcal{I}$  to  $\mathcal{B}_i$ , or by  $\mathcal{B}_i$  to  $\mathcal{I}$ , may be conditioned on the terms of the agreement between  $\mathcal{I}$  and  $\mathcal{B}_j$  or not. In principle, this might affect the players' reservation payoffs. In fact, however, these payoffs do not depend on whether offers may be conditional or not.

To see this, consider the blockholders' reservation payoffs first. With conditional offers,  $\mathcal{I}$  would not purchase its target stake in firm *i* unless an agreement with  $\mathcal{B}_j$  is also reached,<sup>14</sup> so the outside option for blockholder  $\mathcal{B}_i$  is his equilibrium payoff in the equilibrium arising when  $s_i = s_j = 0$ :

$$\bar{B}_i = b \left[ 1 - \xi (1 - m^*(b)) \right] \pi_i^*(0, 0) - C(m^*(b)) \pi_i^*(0, 0).$$
(10)

<sup>&</sup>lt;sup>12</sup>The analysis could be extended to the case of multiple equilibria by using monotone comparative statics techniques.

<sup>&</sup>lt;sup>13</sup>We adopt a strategic approach to the bargaining process in order to avoid mixing notions from cooperative and non-cooperative game theory. In any case, many different bargaining solutions would lead to the same expected outcome as our non-cooperative assumptions.

<sup>&</sup>lt;sup>14</sup>This is proved formally below.

If instead offers cannot be conditioned on common acceptance,  $\mathcal{I}$  must purchase from  $\mathcal{B}_i$  even if there is no agreement with  $\mathcal{B}_j$ . In this case, the outside option for blockholder  $\mathcal{B}_i$  is his equilibrium payoff when  $s_i = 0$ , so his reservation payoff is:

$$\bar{B}'_i(s_j) = b \left[ 1 - \xi (1 - m^*(b)) \right] \pi^*_i(0, s_j) - C(m^*(b))\pi^*_i(0, s_j).$$
(11)

But it appears from (6) that both weights  $\lambda_i$  and  $\lambda_j$  vanish as soon as either stake,  $s_i$  or  $s_j$ , vanishes. This means that  $\pi_i^*(0, s_j) = \pi_i^*(0, 0)$ . As a result, the two reservation payoffs actually coincide:  $\bar{B}'_i(s_j) = \bar{B}_i$  for all  $s_j \ge 0$ . By the same reasoning, the investor's reservation payoff is nil both with conditional or unconditional offers. In the baseline model, therefore, it does not matter whether bargaining is bilateral or multilateral: the two protocols lead to the same outcome.

This outcome is *constrained-efficient*; in other words, it is efficient from the viewpoint of the large shareholders only. (In this paper, the expression "large shareholders" refers to the institutional investor  $\mathcal{I}$  and the blockholders  $\mathcal{B}_1$  and  $\mathcal{B}_2$ .)

**Proposition 1** The equilibrium ownership structure  $(s_1^*, s_2^*)$  maximizes the joint payoff of the investor and the blockholders:

$$S = I + B_1 + B_2 = \sum_{i=1}^{2} \left\{ b \left[ 1 - \xi (1 - m^*(b - s_i)) \right] - C \left( m^*(b - s_i) \right) \right\} \pi_i^*$$
(12)

This result rests on the fact that  $\mathcal{B}_i$ 's disagreement payoff does not depend on  $s_j$ . If it did, the investor could distort the offer to  $\mathcal{B}_j$  in order to improve its bargaining position vis-à-vis  $\mathcal{B}_i$ .<sup>15</sup> In the absence of such strategic motives, the bargaining solution maximizes the aggregate surplus of the large shareholders, S. The acquisition prices then divide this surplus among the players.

These prices are set as follows. When the investor makes the offers, which happens with probability  $\alpha$ , the prices  $P_i$  are such that the blockholders' participation constraints  $B_i \geq \bar{B}_i$  bind; that is,

$$P_i^L = \{ b [1 - \xi(1 - m^*(b))] - C(m^*(b)) \} \pi_i^*(0, 0) + \\ - \{ (b - s_i) [1 - \xi(1 - m^*(b - s_i))] - C(m^*(b - s_i)) \} \pi_i^*(s_i, s_j).$$
(13)

When instead the blockholders make the offers, which happens with probability  $1-\alpha$ , it is the investor's payoff that is set to its reservation value, i.e., zero. Therefore:

$$P_i^H = s_i \left[1 - \xi (1 - m^*(b - s_i))\right] \pi_i^*(s_i, s_j).$$
(14)

<sup>&</sup>lt;sup>15</sup>With three or more firms, Proposition 1 would hold only if the investor could make conditional offers. With unconditional offers, the offer made to a certain blockholder could be distorted in order to affect the outside option of the other blockholders.

On average, the acquisition prices therefore are:

$$P_{i}^{*}(s_{i}, s_{j}) = \alpha P_{i}^{L} + (1 - \alpha) P_{i}^{H}$$
  
=  $\alpha \{ b [1 - \xi(1 - m^{*}(b))] - C(m^{*}(b)) \} \pi_{i}^{*}(0, 0) +$   
 $-\alpha \{ b [1 - \xi(1 - m^{*}(b - s_{i}))] - C(m^{*}(b - s_{i})) \} \pi_{i}^{*}(s_{i}, s_{j}) + (15)$   
 $+ s_{i} [1 - \xi(1 - m^{*}(b - s_{i}))] \pi_{i}^{*}(s_{i}, s_{j}).$ 

## 5 Ownership structure

We are now ready to analyze the equilibrium ownership structure of the industry.

Notice first of all that in our model the investor acquires a stake in one firm, only if it can also acquire a stake in the other.<sup>16</sup> More generally, since firms are symmetric, and the joint payoff (12) is concave, Proposition 1 implies that the equilibrium ownership structure is symmetric:  $s_1^* = s_2^* = s^*$ . Accordingly, we shall henceforth focus on the case where the investor acquires the same stake s in both firms.

The equilibrium level of common ownership  $s^*$  therefore maximizes:

$$S = \nu^* \Pi^*. \tag{16}$$

where

$$\nu^* = b \left\{ 1 - \xi \left[ 1 - m^*(b - s) \right] \right\} - C \left[ m^*(b - s) \right]$$
(17)

and

$$\Pi^* = \pi_1^* + \pi_2^*. \tag{18}$$

The first factor in expression (16),  $\nu^*$ , is the large shareholders' aggregate payoff per unit of profit, net of what is appropriated by the managers and of the monitoring costs; the second factor,  $\Pi^*$ , is industry profits.

We now show that a change in the degree of common ownership s affects the two factors in opposite directions.

#### 5.1 The softening-of-competition effect

A key element of the model is the fact that an increase in the degree of common ownership increases industry profits  $\Pi^*$  by softening product market competition. This

$$s_i \left[1 - \xi(1 - m^*(b - s_i))\right] \pi_i^*(0, 0) - P_i^*(s_i, 0)$$
  
=  $\left\{b \left[1 - \xi(1 - m^*(b - s_i))\right] - C(m^*(b - s_i))\right\} \pi_i^*(0, 0) +$   
 $- \left\{b \left[1 - \xi(1 - m^*(b))\right] - C(m^*(b))\right\} \pi_i^*(0, 0) < 0,$ 

where the equality follows from (15) and the inequality from the fact that  $m^*(b)$  maximizes  $b[1 - \xi(1 - m_i)] - C(m_i)$ . Intuitively, when  $s_j = 0$ ,  $\mathcal{I}$ 's acquisition of a stake in firm *i* does not affect the product market equilibrium but reduces the blockholder's incentive to monitor. Since there is too little monitoring when  $s_i > 0$ , the acquisition destroys value and hence never occurs in equilibrium.

<sup>&</sup>lt;sup>16</sup>To see this, suppose that  $s_i > 0$  and  $s_j = 0$  and consider the case most favorable to the investor,  $\alpha = 1$ . Using the fact that  $\pi_i^*(s_i, 0) = \pi_i^*(0, 0)$ , the investor's net payoff then is:

effect has been at the center stage of the recent literature on overlapping ownership.

**Lemma 1** If  $\theta > 0$ , industry profits are monotonically increasing in the degree of common ownership s:

$$\frac{\partial \Pi^*}{\partial s} \ge 0.$$

The derivative is strictly positive for 0 < s < b and vanishes at s = 0 and s = b.

A symmetric increase in the stakes owned by the common owner increases profits because each firm attaches a greater weight to the rival's profits and thus becomes less aggressive.

The effect is well known. For our purposes, however, it is important to note that the effect vanishes when common ownership is very small (s = 0) or very high (s = b). At s = 0, the effect vanishes because the weight  $\lambda$  depends on the product of the two stakes:

$$\lambda = \frac{\theta s^2}{\left(b-s\right)^2 + \theta s^2},\tag{19}$$

and thus the effect of an increase in s on  $\lambda$  is second order at s = 0. At s = b, on the other hand, the effect vanishes for two reasons. First, the impact of s on the weight attached to the rival's profit,  $\lambda$ , vanishes when s approaches b; this follows from the fact that:

$$\frac{\partial \lambda}{\partial s} = \frac{2\theta s b \left(b-s\right)}{\left[\left(b-s\right)^2 + \theta s^2\right]^2}.$$
(20)

Second, when s = b the weight  $\lambda$  is equal to one, so firms collude perfectly. Therefore, in the proximity of that point industry profits are close to a maximum, implying that a small change in s has a second-order effect.

While we cannot rule out more complex shapes, the derivative  $\frac{\partial \Pi^*}{\partial s}$  therefore tends to be inverted-U shaped: the effect of an increase in the level of common ownership on industry profits tends to be largest for intermediate levels of common ownership. An inverted-U shape is indeed obtained under various commonly used specifications of the product market: see Appendix B for details.

#### 5.2 The effect on monitoring

The second key element of the model is that an increase in the degree of common ownership reduces monitoring, as the blockholder is left with a lower residual stake. This affects the large shareholders' aggregate payoff per unit of profit,  $\nu^*$ .

When s = 0, monitoring is at the efficient level from the viewpoint of the blockholders (and hence of the large shareholders). When s > 0, on the other hand, monitoring becomes inefficiently low, reducing the net payoff per unit of profit  $\nu^*$ . Formally:

$$\frac{\partial v^*}{\partial s} = [\xi b - C'(m^*)] \frac{\partial m^*}{\partial s}$$
$$= -\frac{\xi [\xi b - C'(m^*)]}{C''(m^*)} < 0.$$

By the envelope theorem, the monitoring effect is second order at s = 0, where the term inside square brackets vanishes. When s > 0, on the other hand, the effect is first order. Its strength generally tends to increase with s; a sufficient condition for this is  $C'''(m) \ge 0$ .

#### 5.3 The limits to common ownership

From the above analysis, it appears that the choice of the ownership structure involves a trade-off between softer product market competition and lower monitoring.

The existence of this trade-off explains why common ownership is limited. Formally, we have:

**Proposition 2** Common ownership is always partial:  $0 \le s^* < b$ .

What poses a limit to the equilibrium level of common ownership is the fact that as common ownership increases, the negative effect on monitoring gets stronger and stronger whereas the positive effect on industry profits eventually fades away. Therefore, the former effect must eventually dominate the latter. At that point, any further increase in s necessarily decreases the aggregate payoff of the large shareholders.

#### 5.4 The emergence of common ownership

Our model may help shed light on the process whereby common ownership, starting from a hardly noticeable level, has become as prevalent as it is today. In analytical terms, the question may be posed as follows. Suppose that we start from a situation where the equilibrium level of common ownership is nil, and we then gradually change the model's parameters in such a way that it becomes positive. What is the pattern of transition from a zero to a positive level of common ownership?

To answer this question, note that the function  $S(s) = \nu^*(s)\Pi^*(s)$  is always flat at s = 0. However, the point s = 0 may be a global maximum, a local (but not global) maximum, or a local minimum. The three possibilities are illustrated in Figure 1. They all may occur for some combinations of parameter values. As the model parameters vary, the shape of the function S(s) moves from one extreme to the other, passing through the intermediate case.<sup>17</sup>

Figure 2 illustrates these possibilities from a different angle, by depicting the marginal effects of a change in the level of common ownership. From (16), the derivative  $\frac{\partial S}{\partial s}$  may be rewritten as:

$$\frac{\partial S}{\partial s} = \nu^* \Pi^* \left( \frac{\partial \nu^*}{\partial s} \frac{1}{\nu^*} + \frac{\partial \Pi^*}{\partial s} \frac{1}{\Pi^*} \right) \tag{21}$$

The first term in brackets is negative and represents the marginal cost of common ownership in term of reduced monitoring, whereas the second term (i.e., the semielasticity of industry profits) is the marginal benefit. As argued above, under mild

<sup>&</sup>lt;sup>17</sup>The direction of change will be analyzed, separately for each of the model's parameters, in the following section.



Figure 1: The function S(s) for different parameter configurations. The picture is drawn for Example 1 in Appendix B, with the following parameter values:  $\gamma = 2$ ,  $b = \frac{4}{5}$ ,  $\theta = \frac{4}{5}$ and  $\delta = \frac{3}{5}$ , whereas the parameter  $\xi$  varies across curves. Specifically, the black curve, which represents the case where s = 0 is a local minimum, is obtained for  $\xi = \frac{1}{2}$ ; the blue curve, which represents the case where s = 0 is a local but not global maximum, is obtained for  $\xi = \frac{19}{30}$ ; and the red curve, which represents the case where s = 0 is a global maximum, is obtained for  $\xi = \frac{3}{4}$ .

regularity conditions the marginal cost is increasing in s, whereas the marginal benefit is inverted-U shaped.

The marginal cost and marginal benefit curves always intersect at s = 0. Leaving apart degenerate cases, and keeping in mind that at s = b the marginal cost is positive and the marginal benefit is nil, it appears that three possibilities may arise, illustrated in the three panels of the figure. First, the marginal cost curve may lie everywhere above the marginal benefit curve (the red curve in Figure 2; in this case s = 0 is a global maximum). Second, it may intersect the marginal benefit curve an even number of times, starting from the above (the blue curve in Figure 2; in this case s = 0 is a local maximum, but not necessarily a global one). Third, it may intersect the marginal benefit curve an odd number of times, starting from below (the black curve in Figure 2; in this case s = 0 is a minimum).

The transition to common ownership occurs when, in the second case, the two areas between the marginal cost and marginal benefit curves are of the same size and thus the two local maxima,  $s^* = 0$  and  $s^* = s^+ > 0$ , are both global maxima. At this point, any perturbation in the underlying parameters will cause a jump in  $s^*$  from 0 to  $s^+$ , or the other way around.

This implies that in our model common ownership emerges in discrete amounts, not infinitesimal ones. This is a notable property of the equilibrium. It implies that one can observe a significant change in the level of common ownership even if the change in the underlying conditions is small.



Figure 2: The marginal cost and marginal benefit of cross ownership. The picture is drawn for the same examples as Figure 1. Since the only parameter that varies is  $\xi$ , and  $\xi$  does not affect the marginal benefit of cross ownership, only the marginal cost curve changes.

# 6 The determinants of common ownership

In this section, we analyze the factors that determine whether in equilibrium common ownership is nil or positive and, when it is positive, its level.

It is convenient to analyze separately the factors that affect the impact of common ownership on the residual income per unit of profit  $\nu^*$ , and those that affect its impact on industry profits,  $\Pi^*$ . Broadly speaking, the first set of factors pertains to corporate governance, the second to product market competition. We consider each category in turn, and we then turn to those factors that affect both  $\nu^*$  and  $\Pi^*$  simultaneously.

## 6.1 Corporate governance

Generally speaking, corporate governance rules and institutions determine the controlling party' ability to appropriate a fraction of the firm's profits, and how costly it is to monitor it. In legal systems that provide more protection to shareholders, insiders generally have less opportunities to extract private rents ( $\xi$  is lower), and monitoring is easier (the cost C(m) is smaller).

The large increase in the degree of common ownership observed in the last decades may have been triggered by changes in the effectiveness of corporate governance institutions. To analyze this issue more formally, it is convenient to use a quadratic specification of the monitoring-cost function:

$$C(m) = \frac{1}{2}\gamma m^2. \tag{22}$$

With this specification, the costliness of monitoring is measured by parameter  $\gamma$ .<sup>18</sup>

<sup>&</sup>lt;sup>18</sup>With this specification, condition  $C'(1) > \xi$ , which guarantees an interior solution for  $m^*$ ,

Both parameters  $\xi$  and  $\gamma$  are inversely related to the "quality" of corporate institutions, in the sense that for any given level of monitoring m, the residual payoff per unit of profit,  $\nu = b \left[1 - \xi \left(1 - m\right)\right] - \frac{1}{2}\gamma m^2$ , decreases with both  $\xi$  and  $\gamma$ .<sup>19</sup> However, the impact of  $\xi$  and  $\gamma$  on the equilibrium level of common ownership is different. When managers have fewer opportunities to divert revenues into private benefits (lower  $\xi$ ), common ownership increases; on the other hand, when it is easier to monitor the managers (lower  $\gamma$ ), common ownership decreases.

**Proposition 3** The equilibrium level of common ownership  $s^*$  is monotonically decreasing in  $\xi$  and monotonically increasing in  $\gamma$ .

To understand why  $\xi$  and  $\gamma$  have opposite effects on the equilibrium level of common ownership, it may be useful to compare the level of monitoring chosen in equilibrium by the blockholder and the level that would be optimal from the aggregate viewpoint of the large shareholders. With the quadratic specification (22), the former is

$$m^* = \frac{\xi(b-s)}{\gamma},\tag{23}$$

the latter  $\frac{\xi b}{\gamma}$ . The gap between the two,  $\frac{\xi s}{\gamma}$ , increases with  $\xi$  but decreases with  $\gamma$ . This implies that if  $\xi$  decreases, the cost of common ownership in terms of reduced monitoring falls, and thus the equilibrium entails more common ownership. If instead  $\gamma$  decreases, the cost of common ownership in terms of reduced monitoring increases. In this case, better corporate governance translates into less common ownership.

The above observations raise an interesting policy issue: assuming that it is feasible to improve corporate governance, is it always desirable to do so? Suppose that the policymaker's objective is to maximize the welfare of a representative agent, who is both a dispersed shareholder and a consumer of the products supplied by the two firms. In his capacity as investor, the representative agent obtains a payoff proportional to  $[1 - \xi(1 - m^*)] \Pi^*$ . As  $\xi$  decreases, he gains both directly, as the term inside square brackets increases, and indirectly, as the equilibrium level of common ownership increases with a positive effect on industry profits. However, the representative agent loses in his capacity as consumer, as his consumer surplus  $CS^*$ decreases with the degree of common ownership. Therefore, if the relative weight of  $CS^*$  in the representative agent's payoff is large enough, the policymaker may not want to lower  $\xi$  even if this move were feasible.<sup>20</sup>

### 6.2 Product market competition

We now turn to the factors that impact the way common ownership affects industry profits  $\Pi^*$ . One such factor is  $\theta$ , which affects  $\frac{d\lambda}{ds}$  and hence  $\frac{\partial \Pi^*}{\partial s}$ . Another is the

becomes  $\gamma > \xi$ .

 $<sup>^{19}</sup>$  On the other hand, neither  $\xi$  nor  $\gamma$  affect on industry profits  $\Pi^*.$ 

<sup>&</sup>lt;sup>20</sup>On the other hand, the policymaker will always set  $\gamma$  as small as possible, as reducing  $\gamma$  benefits the representative agent both in his capacity as consumer and as investor. This follows from the fact that as product market competition becomes softer, consumer surplus falls by more than indutry profits increase.

intensity of product market competition. This variable has not been parametrized so far but it clearly affects the way  $\Pi^*$  depend on  $s^{21}$ 

To proceed, we parametrize the intensity of product market competition. There are different ways to do so, but to keep the analysis as general as possible, we consider a generic parameter  $\sigma$  that captures the strength of competition in the sense that an increase in  $\sigma$  increases the semi-elasticity  $\frac{\partial \Pi^*}{\partial \lambda} \frac{1}{\Pi^*}$ :

$$\frac{\partial}{\partial\sigma} \left( \frac{\partial\Pi^*}{\partial\lambda} \frac{1}{\Pi^*} \right) > 0.$$
(24)

Intuitively, the stronger the competition, the lower the profits  $\Pi^*$  firms obtain when  $\lambda = 0$  (i.e., when each firm maximizes exclusively its own profits). On the other hand, when  $\lambda = 1$  firms do not compete at all and obtain monopoly profits. Therefore, the stronger the competition, the bigger the increase in industry profits as we move from  $\lambda = 0$  to  $\lambda = 1$ . Condition (24) posits that this holds true not only for the jump from  $\lambda = 0$  to  $\lambda = 1$  but also for any small increase in  $\lambda$ : it is, essentially, a monotonicity condition. Appendix B shows that in standard models, commonly used measures of the intensity of competition, such as an increase in the degree of product substitutability, or a switch from Cournot to Bertrand, accord with our definition (24).

**Proposition 4** The equilibrium level of common ownership  $s^*$  is monotonically increasing in  $\theta$  and in the intensity of product market competition  $\sigma$ .

The intuition is simple. When  $\theta$  is higher, i.e., the interests of minority shareholders have a bigger weight in the company's strategic choices, the lessening-ofcompetition effect of common ownership is stronger, and therefore the investor has a bigger incentive to acquire stakes in the competing firms. Likewise, the lessening-ofcompetition effect of common ownership is more relevant when competition is strong. For example, in the limiting case in which the two companies operate in separate markets, there is no competition even if  $\lambda = 0$ , so common ownership entails no gains.

### 6.3 Dispersed shareholders

In this subsection, we show that our model can produce a rich and diverse ownership structure, with controlling blockholders, dispersed shareholders and common owners, for purely non-financial reasons. The analysis also uncovers another mechanism that limits the equilibrium level of common ownership.

The new mechanism relies on the fact that even if initially blockholder  $\mathcal{B}_i$  owns 100% of the shares of his company, he will find it profitable to sell a positive fraction of his shares to dispersed shareholders before dealing with the investor. This reduces b, and since  $s^*$  is an increasing function of b, it reduces the equilibrium level of common ownership.

<sup>&</sup>lt;sup>21</sup>Note that neither of these factors affects  $\nu^*$ .

We begin by showing that the equilibrium level of common ownership is, indeed, an increasing function of b:

**Proposition 5** The equilibrium level of common ownership  $s^*$  is monotonically increasing in b.

The proof of the proposition identifies several subtle effects of a change in b, but the main force driving the result is purely mechanical: the lower b, the fewer the shares the blockholder can sell to the institutional investor.

Consider now an extended game, with a preliminary stage where blockholder  $\mathcal{B}_i$ , who initially holds 100% of his company, may sell some of his shares to dispersed shareholders. The rest of the game proceeds as in the baseline model. That is,  $\mathcal{I}$  bargains with  $\mathcal{B}_1$  and  $\mathcal{B}_2$ , firms compete in the product market, and stakeholders choose their monitoring efforts.

In the extended game, blockholder  $\mathcal{B}_i$  will choose  $b_i$  so as to maximize the revenue from the sale of the fraction  $(1 - b_i)$  of the company plus his continuation payoff. Dispersed shareholders are forward looking and anticipate that the blockholders, after selling a fraction  $(1 - b_i)$  of their shares to them, will sell a further  $s_i^*(b_i, b_j)$ to the institutional investor. (For this analysis, we must allow for the possibility that  $b_i \neq b_j$ ; we then denote by  $(s_i^*(b_i, b_j), s_j^*(b_j, b_i))$  the corresponding equilibrium ownership structure.) Therefore, dispersed shareholders are willing to pay a price of

$$[1 - \xi(1 - m^*(b_i - s_i^*(b_i, b_j)))] \pi_i^*(s_i^*(b_i, b_j), s_j^*(b_j, b_i))$$

per share.

The calculation of the continuation payoff is simplest when the investor  $\mathcal{I}$  has all the bargaining power ( $\alpha = 1$ ). In this case, the blockholders will obtain his reservation payoffs:

$$\bar{B}_i = \{b_i \left[1 - \xi(1 - m^*(b_i))\right] - C \left[m^*(b_i)\right]\} \pi_i^*(0, 0).$$

Therefore, blockholder  $\mathcal{B}_i$ 's payoff in the preliminary stage of the game is (omitting the dependence of  $s_i^*$  on  $b_i$  and  $b_j$  to simplify the notation):

$$B_{i} = (1 - b_{i}) \left[ 1 - \xi (1 - m^{*}(b_{i} - s_{i}^{*})) \right] \pi_{i}^{*}(s_{i}^{*}, s_{j}^{*}) + \left\{ b_{i} \left[ 1 - \xi (1 - m^{*}(b_{i})) \right] - C(m^{*}(b_{i})) \right\} \pi_{i}^{*}(0, 0).$$

$$(25)$$

The equilibrium value of  $b_i$  is obtained by maximizing this expression. When  $\alpha > 0$ , the algebra is more cumbersome but the logic does not change.

**Proposition 6** If  $s^*(1) > 0$  and  $\alpha > 0$ , in equilibrium blockholders sell a positive amount of their initial stakes to dispersed shareholders:  $0 < b^* < 1$ .

The intuition for this result is similar to Zingales (1995). When the investor has all the bargaining power,<sup>22</sup> if the blockholder sells his shares to it, he obtains

<sup>&</sup>lt;sup>22</sup>This is the case where the mechanism is most powerful; however, the incentive to sell to dispersed shareholders would disappear only in the investor had no bargaining power at all.

only the value that the shares would have if firms competed fiercely: the benefits of the lessening-of-competition effect of common ownership are reaped entirely by the investor. On the other hand, since dispersed shareholders are forward looking but have no bargaining power, selling to them allows the blockholder to capture the full value of common ownership. However, the blockholder cannot sell his shares only to the dispersed shareholders, who do not contribute to increasing share value neither by monitoring nor by softening competition. If it did, the benefits from common ownership would simply not materialize.

# 7 Other mechanisms

The contention of this paper is that while the upside of overlapping ownership, from the viewpoint of the shareholders, is that it lessens competition and increases profits, the downside is that it worsens the companies' governance. So far, the analysis has focused on a specific mechanism: that is, the reduction of blockholders' incentives to monitor the managers. However, the corporate governance literature has highlighted other mechanisms whereby share value may decrease when blockholders' ownership is diluted. In this section, we discuss two such mechanisms, which can generate a trade-off similar to that analyzed in the previous sections.

### 7.1 Extraction of private benefits by the blockholder

This theory posits that blockholders themselves can extract private benefits from the company they control, at the expenses of minority shareholders (Burkart, Gromb and Panunzi, 1998). In this variant of the model, therefore, managers no longer divert cash flow to themselves: their only role is to make the strategic choices at the product market competition stage.

#### 7.1.1 Assumptions

Let  $a_i$  denote the fraction of profits privately appropriated by blockholder  $\mathcal{B}_i$ ; the remaining fraction  $(1 - a_i)$  being distributed among all shareholders. Following Burkart, Gromb and Panunzi (1998), assume that the diversion of profit entails deadweight losses  $D(a_i)\pi_i$ , which are an increasing, convex function of  $a_i$ .

#### 7.1.2 Analysis

Under these assumptions, the payoff of a blockholder  $\mathcal{B}_i$  who has sold a stake  $s_i$  to investor  $\mathcal{I}$  is :

$$B_i = [a_i + (b - s_i)(1 - a_i) - D(a_i)]\pi_i + P_i(s_i).$$
(26)

The blockholder then chooses  $a_i$  so as to maximize  $B_i$ . The first-order condition for a maximum is:

$$D'(a_i) = 1 - (b - s_i).$$
(27)

From this we get:

$$\frac{\partial a_i^*}{\partial s_i} = \frac{1}{D''(a_i)} > 0.$$
(28)

That is, the lower the blockholder's residual stake  $(b - s_i)$ , the higher the fraction of firm's profits he will privately appropriate.

The analysis then proceeds as in the baseline model. The only difference is that the large shareholders' joint payoff now is:

$$S = I + B_1 + B_2$$
  
=  $\sum_{i=1}^{2} [a_i + b(1 - a_i) - D(a_i)] \pi_i.$  (29)

Exploiting symmetry, one sees that the joint surplus may again be written as:

$$S = \nu^* \Pi^*, \tag{30}$$

where now:

$$\nu^* = a^*(s) + b \left[ 1 - a^*(s) \right] - D \left[ a^*(s) \right].$$
(31)

As in the baseline model, the upside of common ownership is that it increases industry profits  $\Pi^*$ , the downside is that it decreases the net payoff per unit of profit,  $\nu^*$ .

Assuming that at the product market competition stage managers maximize the objective function (4), the marginal benefit of common ownership still vanishes both at s = 0 and at s = b, and thus tends to be an inverted-U shaped function of s. As for the marginal cost, it is easy to see that:

$$\frac{\partial \nu^*}{\partial s} = \left[ (1-b) - D'(a^*) \right] \frac{\partial a^*}{\partial s}$$
$$= -s \frac{\partial a^*}{\partial s} \le 0$$
(32)

The derivative is negative because the lower the blockholders' residual stakes, the higher their incentive to privately appropriate part of the companies' profits. This is bad for the total surplus of the large shareholders, because from their viewpoint the level of profit extraction  $a^*$  is inefficiently large. Note that here, as in the baseline model, the marginal cost of common ownership vanishes at s = 0.

#### 7.1.3 Example

A convenient specification of the deadweight loss function is:

$$D(a_i) = \frac{1}{2}\phi a_i^2,\tag{33}$$

where  $\phi$  is a parameter that measures how difficult it is for the blockholder to dilute minority shareholders. Therefore, in this version of the model  $\phi$  represents the quality of corporate governance. Under this specification, one obtains:

$$a_i^* = \frac{1 - (b - s_i)}{\phi}.$$
(34)

and

$$\nu^* = b + \frac{1}{2} \frac{(1-b)}{\phi} \left[ (1-b)^2 - s^2 \right].$$
(35)

All our results hold also in this version of the model. The main difference is that we now have only one corporate governance parameter,  $\phi$ . The comparative statics of the equilibrium level of common ownership with respect to this new parameter is as follows.

**Proposition 7** The equilibrium level of common ownership  $s^*$  is monotonically increasing in  $\phi$ .

That is, an improvement in the quality of corporate governance institutions has a negative side effects in terms of common ownership. This produces the same trade-off as the one we have analyzed in Section 6.

#### 7.2 Non-contractible investment

Next, we consider a model based on Anton et al. (2022), where common ownership not only lessens product market competition but also reduces the incentives to exert efforts that increase the firm's profits but are privately costly. Examples are efforts devoted to reducing the firm's costs, or to improving the quality of its products.

Assuming that these efforts are exerted by managers, Anton et al. (2022) show that common ownership makes it optimal to offer to such managers less high-powered incentive schemes. For simplicity, here we assume instead that the efforts are exerted directly by blockholders. In this case, common ownership mechanically decrease the incentives to exert such efforts by diluting the blockholder's ownership stakes. On the other hand, we endogenize the level of common ownership, while Anton et al. (2022) take it as exogenous.

While in the baseline model, as well as in the variant considered in the previous subsection, the fact that common ownership worsens corporate governance is always bad for the large shareholders' payoff, here things are different. A higher level of common ownership reduces the blockholders' efforts, but this effect may be either good or bad for industry profits, depending on the initial level of common ownership.

The reason for this is that reducing a firm's cost (or improving the quality of its products) increases the firm's profits but decreases those of the rival. Therefore, the individual choice of effort levels entails externalities at the industry level.

Without common ownership, when b = 1 the privately optimal efforts are always excessively large from the viewpoint of industry-profit maximization. Therefore, an increase in common ownership initially raises aggregate profits, even abstracting from its collusive effect. In this region, an increase in common ownership is a win-win move. However, when common ownership exceeds a critical level, the incentives to invest become so low that efforts are too small from the viewpoint of industry-profit maximization. At this point, there emerges a trade-off: a further increase in the level of common ownership leads to inefficiently low efforts, but on the other hand it softens competition and thereby increases industry profits.

This implies that in this variant of the model the equilibrium level of common ownership is always positive if b = 1. Similarly to the baseline model, however, common ownership is limited; the limiting factor here is the need to preserve the incentives to invest.

#### 7.2.1 Assumptions

To be specific, let us assume that blockholder  $\mathcal{B}_i$  may reduce the firm's marginal costs from c to  $c-r_i$  at a private cost of  $C(r_i)$ , with  $C(0) = 0, C' > 0, C'' > 0^{23}$  The timing of the game now is the following: first, the common owner and the blockholders bargain over the stakes to be acquired by the common owner,  $s_i$ . Blockholder i then decides his effort and the consequent cost reduction  $r_i$ , which is observed by the rival firm.<sup>24</sup> Finally, firms compete in the product market, as in the baseline. To highlight the new effects arising in this variant of the model, we focus on the case where blockholders initially own 100% of their companies (b = 1).

Equilibrium profits now are  $\pi_i^*(\lambda_i, \lambda_j, r_i, r_j)$ , where the function  $\pi_i^*$  is increasing in  $r_i$  and decreasing in  $r_j$ . The equilibrium profits depend on the stakes  $s_i$  both because the weight  $\lambda_i$  depend on  $s_i$  and  $s_j$ , as in the baseline, and because the investment in cost reduction, and hence  $r_i$ , depend on  $s_i$ .

To analyze the way it does, consider the optimal choice of  $r_i$ . When selecting  $r_i$ ,  $\mathcal{B}_i$  maximizes

$$B_i = (1 - s_i)\pi_i^*(\lambda_i, \lambda_j r_i, r_j) - C(r_i).$$
(36)

Let us assume that the payoff  $B_i$  is a concave function of  $r_i$ . (This property holds in the examples considered in Appendix B.) Assuming an interior solution, the firstorder condition with respect to  $r_i$  gives:

$$(1-s_i)\frac{\partial \pi_i^*(\lambda_i, \lambda_j, r_i, r_j)}{\partial r_i} = C'(r_i).$$
(37)

The left-hand side is the increase in profits due to the cost reduction; the right-hand side is the marginal cost of effort. By the implicit function theorem, we have:

$$\frac{dr_i}{ds_i} = -\frac{-\frac{\partial \pi_i^*(\lambda_i, \lambda_j, r_i, r_j)}{\partial r_i} + (1 - s_i) \left[\frac{\partial^2 \pi_i^*(\lambda_i, \lambda_j, r_i, r_j)}{\partial \lambda_i \partial r_i} \frac{d\lambda_i}{ds_i} + \frac{\partial^2 \pi_i^*(\lambda_i, \lambda_j, r_i, r_j)}{\partial \lambda_j \partial r_i} \frac{d\lambda_j}{ds_i}\right]}{\frac{\partial^2 \pi_i^*(\lambda_i, \lambda_j, r_i, r_j)}{\partial r_i^2} - C''(r_i)}.$$
(38)

<sup>&</sup>lt;sup>23</sup>Note that even if the common owner were capable of investing in cost reduction, it would have a lower incentive to do so than the blockholder. The reason for this is that a reduction in the marginal cost makes the firm more competitive and therefore more aggressive in the product market, causing a loss in profits for competing firms, of which the common owner owns a share.

 $<sup>^{24}</sup>$ The analysis can be easily extended to the case where a firm's cost reduction is not observed by the rival, as in Lopez and Vives (2019).

At  $s_i = 0$ , the term inside square brackets vanishes and thus  $r_i^*$  is a decreasing function of  $s_i$ . This property holds for any  $s_i \ge 0$  provided that the marginal benefit from a cost reduction increases with the intensity of competition, as is the case in many standard models. In this condition holds, the term inside square brackets is always negative.

Proceeding backwards, consider now the bargaining stage. For simplicity, let us assume that the bargaining outcome is efficient and that the common owner has all the bargaining power.<sup>25</sup> As in the baseline model, the equilibrium ownership structure then maximizes the large shareholders' aggregate payoffs:

$$S = \sum_{i=1}^{2} \pi_i^*(\lambda_i, \lambda_j, r_i^*, r_j^*) - \sum_{i=1}^{2} C(r_i^*).$$
(39)

Given the symmetry of the model, we can restrict attention to the case  $s_i = s_j = s$ with no loss of generality. In this case, S rewrites as

$$S = b\Pi^*(\lambda, r) - 2C^*(r), \tag{40}$$

where  $\Pi^*$  is again industry profits and  $2C^*$  is the aggregate cost of the investment in cost reduction.

The first-order condition for surplus maximization is:

$$\frac{dS}{ds} = \frac{\partial \Pi^*}{\partial \lambda} \frac{d\lambda}{ds} + \left(\frac{\partial \Pi^*}{\partial r} - 2\frac{\partial C^*}{\partial r}\right) \frac{dr}{ds} = 0.$$
(41)

Let us consider first the case where  $\theta = 0$  and hence  $\lambda \equiv 0$ , as in Anton et al. (2022). In this case, the first term on the right-hand side of (41) vanishes, and hence the equilibrium level of common ownership is implicitly given by the condition

$$\frac{\partial \Pi^*}{\partial r} - 2\frac{\partial C^*}{\partial r} = 0. \tag{42}$$

That is, common ownership is such that the equilibrium level of cost reduction maximizes industry profits net of the cost of effort.<sup>26</sup> To put it differently, the equilibrium level of common ownership re-produces the investment in cost reduction that would be chosen if the firms perfectly coordinated their efforts, while continuing to compete in the product market.

<sup>&</sup>lt;sup>25</sup>If one assumes that the common owner makes takes-it-or-leave-it offers to the blockholders, efficiency now requires that such offers be conditional. In other words, the offer made to blockholder  $\mathcal{B}_i$  is valid only if the offer made to  $\mathcal{B}_j$  is accepted. Unlike the baseline model, with non-conditional offers efficiency is not guaranteed. The reason for this is that acceptance of an offer by a blockholder creates a positive externality on the other blockholder, a sthe accepting blockholder's incentives to reduce the marginal cost are hindered. If the offers are not conditional, these externalities are not internalized and prevent efficiency of the outcome.

<sup>&</sup>lt;sup>26</sup>Thus, in our model common ownership increases S but decreases  $\Pi^*$ . This marks a difference with respect to Anton et al. (2022), where the cost of effort is borne by managers, who are rewarded by shareholders. In their model, therefore, common ownership raises industry profits  $\Pi^*$ , net of the manager wages.

Evidently, condition (41) requires that the equilibrium level of common ownership be strictly positive, as when s = 0 the equilibrium efforts are excessively high from the viewpoint of industry-profit maximization. On the other hand, common ownership is partial, for otherwise the investment in cost reduction would vanish,<sup>27</sup> and this would violate condition (41).

When instead  $\theta > 0$ , the term  $\frac{\partial \Pi^*}{\partial \lambda} \frac{d\lambda}{ds}$  is positive, and hence at equilibrium we have;

$$\frac{\partial \Pi^*}{\partial r} - 2\frac{\partial C^*}{\partial r} > 0, \tag{43}$$

meaning that the investment in cost reduction would be below the industry-profit maximizing level. In this case, the equilibrium level of common ownership is determined as the optimal resolution to the trade-off between collusion and cost reduction: more common ownership lessens competition, but this effect is achieved at the cost of distorting the investment in cost reduction below the efficient level (from the firm's viewpoint).<sup>28</sup>

In any case, the "collusive" effect of common ownership vanishes when s = 1, as we have shown in section 5.1 above. Since the effect on efforts is always strictly negative as soon as (41) holds, the equilibrium level of common ownership is always limited by the need to preserve the incentives to exert effort.

#### 7.2.2 Example

To analyze the comparative statics of this model, it is convenient to consider the case where firms supply differentiated products, the demand for which is

$$q_i = 1 - p_i + \gamma p_j, \tag{44}$$

where  $\gamma \in [0, 1]$  is a parameter that measures the degree of product differentiation. Further assume that marginal costs are nil and that firms compete in prices.<sup>29</sup>

In this example, it is easy to verify that when  $\theta = 0$  one has

$$s^* = \frac{\gamma}{2 - \gamma^2},$$

and therefore  $s^*$  increases with the degree of product differentiation  $\gamma$  – a measure of the intensity of competition. It can be shown that this is true also when  $\theta > 0$ , consistently with our previous results that point to a positive impact of the intensity of competition on the level of common ownership.

## 8 Cross ownership

In this section, we apply our theoretical framework to the case of cross ownership.

 $<sup>^{27}</sup>$ (as is evident from the first-order condition ())

<sup>&</sup>lt;sup>28</sup>From the consumer perspective, on the other hand, both collusion and higher costs have an adverse impact on welfare.

<sup>&</sup>lt;sup>29</sup>This is example 2 in Appendix B.

#### 8.1 Assumptions

The model is as in Section 3; the only difference is that now there is no external investor. Instead, we consider the possibility that each blockholder may acquire a stake in the competing company.<sup>30</sup>

Differently from common ownership, cross ownership relaxes competition even if  $\theta = 0$ ; that is, even if, at the product market competition stage, managers pursue exclusively the interests of their controlling stakeholder. The reason for this is that cross ownership changes the behavior of the acquiring firm rather than that of the acquired one. To mark this difference from the case of common ownership, in this section we shall assume that  $\theta = 0$ .

As in the model of common ownership, dispersed shareholders are forward looking and correctly anticipate the *ex post* value of their shares. Therefore, blockholder  $\mathcal{B}_j$ cannot profitably acquire a stake  $s_i$  in company *i* from the dispersed shareholders but will acquire it from blockholder  $\mathcal{B}_i$ . As a result, blockholder  $\mathcal{B}_i$  will be left with a residual stake of  $b - s_i$ .

Denoting again by  $P_i(s_i)$  the total price paid for the stake  $s_i$ , blockholder  $\mathcal{B}_i$ 's payoff now becomes:

$$B_{i} = \{(b - s_{i}) [1 - \xi(1 - m(b - s_{i}))] - C [m(b - s_{i})]\} \pi_{i}^{*} + P_{i}(s_{i}) + s_{j} [1 - \xi(1 - m(b - s_{i}))] \pi_{j}^{*} - P_{j}(s_{j}).$$

$$(45)$$

The first term on the right-hand side is the value of  $\mathcal{B}_i$ 's residual stake  $b - s_i$ , net of profit diversion by the managers and monitoring costs, the second term is the revenue from the sale of the share  $s_i$  to  $\mathcal{B}_j$ , and the last two terms are the net revenue from the acquisition of share  $s_j$  in company j.

Since  $\theta = 0$ , at the product market competition stage managers now pursue exclusively the interests of their controlling shareholders. Thus, firm *i* chooses  $x_i$  so as to maximize  $B_i$ . As in the case of common ownership, however, we shall focus on a simpler formulation of the objective function, which disregards the "spurious" components of  $B_i$ . In this formulation, manager  $\mathcal{M}_i$  maximizes:

$$O_i = (b - s_i) \pi_i + s_j \pi_j.$$
(46)

This may be rewritten as:

$$\tilde{O}_i = \pi_i + \lambda_i \pi_j, \tag{47}$$

where the lambdas now are simply:

$$\lambda_i = \frac{s_j}{b - s_i}.\tag{48}$$

As noted, the weight  $\lambda_i$  is now positive even if  $\theta = 0$ . Another notable difference with the case of common ownership is that when  $s_i > 0$ ,  $\lambda_i$  is positive even if  $s_i = 0$ .

<sup>&</sup>lt;sup>30</sup>The stake in firm j could be acquired by company i or directly by blockholder  $\mathcal{B}_i$ . The two formulations lead to the same outcome. To fix ideas, in what follows we shall focus on the case where the acquisition is made by blockholder  $\mathcal{B}_i$ .

As we shall see, this implies that even unilateral acquisitions may be profitable with cross ownership.

As for the bargaining over the stakes  $s_i$  and  $s_j$ , we now assume, for simplicity, that each blockholder has a probability of 50% of making a take-it-or-leave-it offer to the other. The offer specifies the shares  $s_i$  and  $s_j$  and the net compensation to be paid by the receiver (which may be interpreted as the difference between the acquisition prices for the two stakes).

#### 8.2 Analysis

Like in the case of common ownership, the model's equilibrium is constrained efficient.

**Proposition 8** The equilibrium ownership structure  $(s_1^*, s_2^*)$  maximizes the joint payoff of the blockholders:

$$S = B_1 + B_2 = \sum_{i=1}^{2} \left\{ b \left[ 1 - \xi (1 - m^*(b - s_i)) \right] - C \left[ m^*(b - s_i) \right] \right\} \pi_i^*(s_i, s_j)$$
(49)

Proceeding as in the model of common ownership, the symmetry and concavity of the blockholders' joint payoff implies that the equilibrium ownership structure is symmetric:  $s_1^* = s_2^* = s^*$ . Therefore, the equilibrium level of cross ownership  $s^*$ maximizes:

$$S = \nu^* \Pi^*. \tag{50}$$

However, an important difference with the case of common ownership is that the weight  $\lambda$  now is:

$$\lambda = \frac{s}{b-s},\tag{51}$$

which implies

$$\frac{d\lambda}{ds} = \frac{b}{\left(b-s\right)^2}.$$
(52)

Therefore,  $\frac{d\lambda}{ds}$  is now an increasing, convex function of s rather than being inverted-U shaped.

This implies that the equilibrium level of cross ownership is always strictly positive.

## **Proposition 9** In equilibrium, cross ownership is positive but partial: $0 < s^* < \frac{b}{2}$ .

Intuitively, the equilibrium level of cross ownership balances the positive effect of cross ownership on industry profits,  $\Pi^*$ , and the negative impact on the blockholders' income per unit of profits,  $\nu^*$ , due to the effect of cross ownership on monitoring. However, the positive effect becomes second order as s approaches  $\frac{b}{2}$ , where  $\lambda = 1$  and therefore industry profits are maximized. On the other hand, the negative effect becomes second order as s approaches 0, where the equilibrium level of monitoring is at the efficient level from the blockholders' viewpoint. Therefore,  $s^*$  is always positive but lower than  $\frac{b}{2}$ .

Under mild regularity conditions, the function S(s) now is everywhere concave. As a consequence,  $s^*$  is a continuous function of the exogenous parameters of the model. This marks an important difference with respect to the case of common ownership: a small change in the underlying conditions can only cause a small change in the level of cross ownership.

Apart from this difference, the comparative statics of cross ownership is exactly the same as that of common ownership.

**Proposition 10** The equilibrium level of common ownership  $s^*$  is monotonically decreasing in the managers' ability to steal  $\xi$  and monotonically increasing in the costliness of monitoring  $\gamma$ , the intensity of product market competition  $\sigma$ , and the blockholders' initial stakes b.

The intuition is exactly the same as in the case of common ownership. One difference with that case, however, is that cross ownership may emerge also in a unilateral way: if, for some reason, blockholder  $\mathcal{B}_i$  is prevented from acquiring a stake in company j, blockholder  $\mathcal{B}_j$  may still find it profitable to acquire a stake in company i.<sup>31</sup>

## 9 Conclusion

This paper has argued that common and cross ownership come with costs and benefits for firms. The benefit, as argued by a blossoming theoretical and empirical literature, is softer product market competition that leads to higher profits. The cost, and this is the novel contribution of the paper, is a less effective corporate governance.

In our model, "active" and "passive" investors play a complementary role. Active investors, such as individuals or families holding a large block, have stronger incentives to monitoring the manager, limiting private benefits extraction or stimulating efforts aimed at reducing costs or increasing market share. Investment funds such as the Big Three, with their stakes in several firms in the same industry, have

$$\left. \frac{\partial S}{\partial s_i} \right|_{s_i, s_j = 0} > 0.$$

Using the envelope theorem, the derivative writes as

$$\frac{\partial S}{\partial s_i}\Big|_{s_i,s_j=0} = \left\{ b \left[ 1 - \xi (1 - m^*((b))) \right] - C(m^*((b))) \right\} \left. \frac{\partial \Pi^*(s_i,0)}{\partial s_i} \right|_{s_i=0}$$

Proceeding as in the proof of Lemma 1, it is easy to show that  $\frac{\partial \Pi^*(s_i,0)}{\partial s_i}\Big|_{s_i=0}$ . This implies that it is always optimal to acquire a stake in the rival's firm, if the rival does not hold a stake in yours.

<sup>&</sup>lt;sup>31</sup>To show this, let us suppose, to fix ideas, that only  $\mathcal{B}_j$  can acquire a stake in firm  $\mathcal{B}_j$   $(s_j = 0)$ . In this case,  $s_i$  is positive in equilibrium if

instead an incentive to soften product market competition. The equilibrium ownership structure emerges as the optimal response to these different forces. Factors that make monitoring more valuable, such as the manager's ability to divert resources as private benefits, limit the extent of common ownership. More intense product market competition, on the other hand, favors the emergence of common ownership.

A noteworthy feature of our model is that common ownership does not emerge as a continuous and smooth process, but rather with an initial jump. A sequence of improvements in shareholder legal protection that mitigate the need for active monitoring may have no impact on common ownership for some time, and then suddenly lead to a discrete change of the ownership structure, with a larger presence of institutional investors.

The same forces are at play in the case of cross ownership, where the product market decisions of the rival are not affected directly but only indirectly, through a softer competitive stance. Again, a higher stake in the rival firms mitigates the intensity of product market competition but creates a less efficient governance in the rival firm.

The main trade-off of the paper does not depend on the exact way the agency problem is modelled: we have considered both the case where private benefits of control can be extracted by the manager or directly by a large blockholder. In both cases, softer product market competition comes at the expense of restrained ability to prevent the extraction of private benefits of control.

The model has interesting implications for the political economy of corporate governance. As long as improvements in shareholder protection do not lead to the emergence of common ownership, consumers are not affected and thus have no reason to oppose such improvements. But once corporate governance is strong enough to reduce the need for an active monitoring, so that common ownership emerges in equilibrium, shareholder and consumer interests become opposed. Voters with little or no wealth invested in stocks may prefer corporate governance institutions that offer less protection to minority shareholders.

To highlight the trade-off between the costs and benefits of common ownership, we have considered a simplified set-up with two symmetric firms, a single common owner, and efficient bargaining. In future work, it would be interesting to relax these assumptions. We believe that a similar trade-off will emerge also in more complex frameworks, but the exact way the different effects play out may change.

Another assumption that could be relaxed is that firms and their shareholders are driven only by profits. An important trend witnessed in recent years is the diffusion of socially responsible investors, i.e., investors that are concerned with goals, such as environmental preservation or the protection of human rights, different from profit maximization. The literature has pointed out an important issue related to these "responsible" strategies, the so-called leakage problem. That is, if a green technology, say, is associated with higher marginal costs, the reduction in emissions of one firm may be partially offset by more emissions by rivals that continue to use brown technologies. Common ownership may be a way to mitigate the leakage problem and increase the effectiveness of socially responsible investment strategies. But if this is so, and softer product market competition is the ultimate outcome of the involvement of socially motivated investors, consumers may foot the bill of social responsibility. Exploring these new trade-offs is an exciting avenue for future research.

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# Appendix A: Proofs

This appendix collects the proofs omitted in the main text.

*Proof of Proposition 1.* Investor  $\mathcal{I}$  chooses its offers so as to maximize its net payoff I. Since the participation constraints  $B_i \geq \overline{B}_i$  must bind in equilibrium, we have

$$I + B_1 + B_2 = I + \bar{B}_1 + \bar{B}_2.$$

Inspection of (10) reveals that  $\bar{B}_i$  does not depend on the investor's stakes,  $s_1$  and  $s_2$ . It follows immediately that maximization of I is equivalent to maximization of  $I + B_1 + B_2$ , and hence of  $I + B_1 + B_2$ .

Proof of Lemma 1. Clearly:

$$\frac{d\Pi^*}{ds} = \frac{\partial\Pi^*}{\partial\lambda}\frac{\partial\lambda}{\partial s}.$$

From (19), we know that  $\frac{\partial \lambda}{\partial s}$  is always non-negative but vanishes at s = 0 and s = b. Furthermore,  $\frac{\partial \lambda}{\partial s}$  is inverted-U shaped in s. Next, consider the factor  $\frac{\partial \Pi^*}{\partial \lambda}$ . Using the first-order conditions (8), we obtain:

$$\frac{\partial \Pi^*}{\partial \lambda} = \left(-\lambda \frac{\partial \pi_2^*}{\partial x_1} + \frac{\partial \pi_2^*}{\partial x_1}\right) \frac{\partial x_1}{\partial \lambda} + \left(\frac{\partial \pi_1^*}{\partial x_2} - \lambda \frac{\partial \pi_1^*}{\partial x_2}\right) \frac{\partial x_2}{\partial \lambda}$$

Symmetry implies  $\frac{\partial x_1}{\partial \lambda} = \frac{\partial x_2}{\partial \lambda} = \frac{\partial x}{\partial \lambda}$ , so the above expression may be rewritten as:

$$\frac{\partial \Pi^*}{\partial \lambda} = 2\left(1 - \lambda\right) \frac{\partial \pi_i^*}{\partial x_i} \frac{\partial x}{\partial \lambda}.$$
 (A1)

To calculate  $\frac{\partial x}{\partial \lambda}$ , we use once again the first-order conditions (8), obtaining:

$$\frac{\partial \lambda}{\partial x} = \frac{\frac{\partial^2 \pi_i^*}{\partial x_i^2} + \lambda \frac{\partial^2 \pi_j^*}{\partial x_i^2}}{\frac{\partial \pi_j^*}{\partial x_i}},$$

which implies:

$$\frac{\partial x}{\partial \lambda} = -\frac{\frac{\partial \pi_j^*}{\partial x_i}}{\left(\frac{\partial^2 \pi_i^*}{\partial x_i^2} + \lambda \frac{\partial^2 \pi_j^*}{\partial x_i^2}\right)}.$$

Plugging this expression into (A1) we eventually get:

$$\frac{\partial \Pi^*}{\partial \lambda} = -2\left(1-\lambda\right) \frac{\left(\frac{\partial \pi_j^*}{\partial x_i}\right)^2}{\left(\frac{\partial^2 \pi_i^*}{\partial x_i^2} + \lambda \frac{\partial^2 \pi_j^*}{\partial x_i^2}\right)} \ge 0,$$

which is positive by the second-order conditions (9). The derivative is strictly positive for  $\lambda < 1$ , i.e., for s < b. Therefore, the sign of  $\frac{\partial \pi^*}{\partial s}$  coincides with the sign of  $\frac{\partial \lambda}{\partial s}$ . The result then follows from the observation that  $\frac{\partial \lambda}{\partial s}$  is always non negative and vanishes only at s = 0 and s = b.

Proof of Proposition 2. From (16) we have (omitting for notational convenience the dependence of  $m^*$  on b - s):

$$\begin{split} \frac{\partial S}{\partial s} &= \left. \Pi^* \frac{\partial m^*}{\partial s} \left. \frac{\partial \left\{ b \left[ 1 - \xi(1-m) \right] - C\left(m\right) \right\}}{\partial m} \right|_{m=m^*} + \\ &+ \left\{ b \left[ 1 - \xi(1-m^*) \right] - C\left(m^*\right) \right\} \frac{\partial \Pi^*}{\partial s} \\ &= \left. -\Pi^* \frac{s\xi^2}{C''\left(m^*\right)} + \left\{ b \left[ 1 - \xi(1-m^*) \right] - C\left(m^*\right) \right\} \frac{\partial \Pi^*}{\partial s} \end{split}$$

where the equality follows from condition (7), which implies  $\frac{\partial m^*}{\partial s} = -\frac{\xi}{C''(m^*)}$  and  $\frac{\partial \{b [1 - \xi(1 - m)] - C(m^*)\}}{\partial m} \Big|_{m=m^*} = \xi s.$  Since  $\frac{\partial \Pi^*}{\partial s} \Big|_{s=b} = 0$  by Lemma 1, we have  $\frac{\partial S}{\partial s} \Big|_{s=b} < 0,$ 

which implies that  $s^* < b$ .

Proof of Proposition 3. Monotonicity requires that  $\frac{\partial s^*}{\partial \xi} < 0$  (resp.,  $\frac{\partial s^*}{\partial \gamma} > 0$ ) when  $s^* > 0$ , and that  $s^*$  jumps downwards (resp., upwards) as  $\xi$  (resp.,  $\gamma$ ) increases.

To show this, note first of all that with the quadratic specification (22) of the monitoring cost function, the equilibrium level of monitoring is (23). Therefore, the derivative  $\frac{\partial S}{\partial s}$  becomes:

$$\frac{\partial S}{\partial s} = \frac{\xi^2}{\gamma} H \Pi^*, \tag{A2}$$

where

$$H \equiv -s + \left[\gamma b \frac{1-\xi}{\xi^2} + \frac{1}{2}(b^2 - s^2)\right] \frac{\partial \Pi^*}{\partial s} \frac{1}{\Pi^*}$$
(A3)

To proceed, consider first the case in which  $s^* > 0$ . By Proposition 2, in this case  $s^*$  is an interior maximum of the function S(s), and thus it must satisfy the first-order condition H = 0. By implicit differentiation, we then obtain:

$$\frac{\partial s^*}{\partial \xi} = -\frac{\frac{\partial H}{\partial \xi}}{\frac{\partial H}{\partial s}} < 0$$

and

$$\frac{\partial s^*}{\partial \gamma} = -\frac{\frac{\partial H}{\partial \gamma}}{\frac{\partial H}{\partial s}} < 0$$

where the sign follows from the fact that  $\frac{\partial H}{\partial s} < 0$  by the second order condition, whereas  $\frac{\partial H}{\partial \xi} < 0$  and  $\frac{\partial H}{\partial \gamma} > 0$ . (These latter inequalities follows immediately from (A3)).

Next, consider the possibility that as  $\xi$  or  $\gamma$  changes,  $s^*$  may jump from an interior solution where  $s^* \equiv s^+ > 0$  to a corner solution where  $s^* = 0$ . At the switching point, we must have:

$$\Delta S \equiv S(s^+) - S(0) = \frac{\xi^2}{\gamma} K \Pi_+^*, \tag{A4}$$

where  $\Pi_0^*$  is industry profits at  $s = 0, \Pi_+^*$  is industry profits at  $s = s^+$ , and

$$K \equiv \left\{ \left[ 2\gamma b \frac{1-\xi}{\xi^2} + b^2 \right] \frac{\Pi_+^* - \Pi_0^*}{\Pi_+^*} - s^{+2} \right\}.$$
 (A5)

It follows that:

$$\left. \frac{\partial \Delta S}{\partial \xi} \right|_{\Delta S=0} \propto \frac{\partial K}{\partial \xi} < 0,$$

where the symbol  $\propto$  means "has the same sign has." This implies that when  $\Delta S = 0$ , an increase in  $\xi$  makes  $\Delta S$  become negative, causing a downward jump of the equilibrium level of common ownership from  $s^+$  to 0.

Likewise, we have

$$\left.\frac{\partial\Delta S}{\partial\gamma}\right|_{\Delta S=0}\propto\frac{\partial K}{\partial\gamma}>0,$$

implying that when  $\Delta S = 0$ , an increase in  $\gamma$  makes  $\Delta S$  become positive, causing an upward jump of the equilibrium level of common ownership from 0 to  $s^+$ . *Proof of Proposition 4.* We proceed as in the proof of Proposition 3. The function H and K defined in (A3) and (A5) clearly depend on the derivative  $\frac{\partial \Pi^*}{\partial s} \frac{1}{\Pi^*}$ , or its discrete analog  $\frac{\Pi^*_+ - \Pi^*_0}{\Pi^*_+}$ . We have:

$$\frac{\partial \Pi^*}{\partial s} \frac{1}{\Pi^*} = \frac{\partial \Pi^*}{\partial \lambda} \frac{1}{\Pi^*} \frac{\partial \lambda}{\partial s} \\ = \frac{2\theta s b (b-s)}{\left[ (b-s)^2 + \theta s^2 \right]^2} \frac{\partial \Pi^*}{\partial \lambda} \frac{1}{\Pi^*}.$$

It follows immediately that:

$$\frac{\partial s^*}{\partial \theta} = -\frac{\frac{\partial H}{\partial \theta}}{\frac{\partial H}{\partial s}} > 0$$

and

$$\frac{\partial s^*}{\partial \sigma} = -\frac{\frac{\partial H}{\partial \sigma}}{\frac{\partial H}{\partial s}} < 0$$

A similar logic applies to the direction of the jump from  $s^* = 0$  to  $s^* = s^+$ : in both cases, the jump is upwards, as  $\frac{\Pi^*_+ - \Pi^*_0}{\Pi^*_+}$  is increasing in both  $\theta$  and  $\sigma$ .

*Proof of Proposition 5.* We proceed as in the proof of Proposition 3. Consider first the function H:

$$H \equiv -s + \left[\gamma b \frac{1-\xi}{\xi^2} + \frac{1}{2}(b^2 - s^2)\right] \frac{\partial \Pi^*}{\partial s} \frac{1}{\Pi^*}$$
$$= -s + \left[\gamma b \frac{1-\xi}{\xi^2} + \frac{1}{2}(b^2 - s^2)\right] \left\{\frac{2\theta s b (b-s)}{\left[(b-s)^2 + \theta s^2\right]^2}\right\} \frac{\partial \Pi^*}{\partial \lambda} \frac{1}{\Pi^*}$$

The blockholder's stake b affects H in a complex way, but it is easy to confirm that  $\frac{\partial H}{\partial b} > 0$ . By a standard argument, this implies that  $\frac{\partial s^*}{\partial b} > 0$ , so  $s^*$  increases with b when  $s^*$  is strictly positive.

That  $s^*$  jumps upward at the discontinuity point when b increases follows from the fact that  $\frac{\partial K}{\partial b} > 0$ , which again is easy to confirm using (A5). *Proof of Proposition 6.* Using the envelope theorem, differentiating (24) we obtain:

$$\begin{aligned} \frac{dB_i}{db_i} &= \left[1 - \xi(1 - m^*(b_i))\right] \pi_i^*(0, 0)\right] - \left[1 - \xi(1 - m^*(b_i - s_i^*))\right] \pi_i^*(s_i^*, s_j^*) + \\ &- (1 - b_i) \frac{\partial m^*}{\partial (b_i - s_i^*)} \left(1 - \frac{ds_i^*}{db_i}\right) \pi_i^*(s_i^*, s_j^*) \\ &+ (1 - b_i) \left[1 - \xi(1 - m^*(b_i - s_i^*))\right] \left[\frac{\partial \pi_i^*(s_i^*, s_j^*)}{\partial s_i} \frac{ds_i^*}{db_i} + \frac{\partial \pi_i^*(s_i^*, s_j^*)}{\partial s_j} \frac{ds_i^*}{db_i}\right] \end{aligned}$$

The term inside square brackets is the difference in share value between the case where the investor  $\mathcal{I}$  has a positive stake and when its stake is 0. Generally speaking, this term can be positive or negative, but in a symmetric equilibrium where the investor does acquire positive stakes in both firms it must be negative, for otherwise the investor would not make the acquisition.

The last two terms vanish at  $b_i = 1$ , implying that stakeholders always sell at least a fraction of their initial stake. For  $b_i < 1$ , however, the second term is negative (as  $\frac{ds_i^*}{db_i} < 1$ ): selling to dispersed shareholders reduces total monitoring and thus reduces share value. The third term instead is positive, but in a symmetric equilibrium it vanishes when  $s^*$  is close to zero by Lemma 1. And since the first term obviously vanishes when  $b_i = 0$ , it follows that at  $b_i = 0$  the derivative is negative, implying that  $b^* > 0$ .

Proof of Proposition 7. Suppose, to fix ideas, that  $\mathcal{B}_i$  makes a take-it-or-leave-it offer

to  $\mathcal{B}_j$ . Clearly,  $\mathcal{B}_j$ 's reservation payoff is:

$$\hat{B}_{j} = b \left[ 1 - \xi (1 - m^{*}(b)) \right] \pi_{j}^{*}(0, 0) - C(m^{*}(b)) \pi_{j}^{*}(0, 0) \,.$$

If  $\mathcal{B}_j$  sells a stake  $s_j$  to  $\mathcal{B}_i$  at price  $P_j$  and  $\mathcal{B}_i$  sells a stake  $s_i$  to  $\mathcal{B}_j$  at price  $P_i$ ,  $\mathcal{B}_j$ 's payoff becomes

$$B_{j} = (b - s_{j}) \left[ 1 - \xi (1 - m^{*}(b - s_{j})) \right] \pi_{j}^{*}(s_{j}, s_{i}) - C(m^{*}(b - s_{j})) \pi_{j}^{*}(s_{j}, s_{i}) + s_{i} \left[ 1 - \xi (1 - m^{*}(b - s_{i})) \right] \pi_{i}^{*}(s_{i}, s_{j}) + (P_{j} - P_{i}).$$

A similar expression holds for  $\mathcal{B}_i$ . The net compensation obtained by  $\mathcal{B}_i$ ,  $P_i - P_j$ , is set such that  $\mathcal{B}_j$  is indifferent between accepting or not:  $B_j = \hat{B}_j$ . Therefore,  $\mathcal{B}_i$ 's payoff becomes:

$$B_i = \sum_{i=1}^{2} \left\{ b \left[ 1 - \xi (1 - m^*(b - s_i)) \right] - C \left[ m^*(b - s_i) \right] \right\} \pi_i^*(s_i, s_j) - \hat{B}_j.$$

The stakes to be transferred,  $s_i$  and  $s_j$ , are chosen so as to maximize  $B_i$ . Since  $\hat{B}_j$  does not depend on  $s_i$  and  $s_j$ ,  $\mathcal{B}_i$  will choose the stakes that maximize S. The same is true of  $\mathcal{B}_i$ , when it comes his turn to make the offer.

Proof of Proposition 8. From (16) we have:

$$\frac{\partial S}{\partial s} = -\Pi^* \frac{s\xi^2}{C''(m^*)} + \left\{ b \left[ 1 - \xi(1 - m^*) \right] - C(m^*) \right\} \frac{\partial \Pi^*}{\partial s}$$

The first term of the derivative is negative and represents the marginal cost of cross ownership, the second term is positive and represents the marginal benefit.

At  $s = \frac{b}{2}$ , the weight  $\lambda$  is equal to 1, so industry profits are maximized. This implies that  $\frac{\partial \Pi^*}{\partial s}\Big|_{s=\frac{b}{2}} = 0$ , so  $\frac{\partial S}{\partial s}\Big|_{s=\frac{b}{2}} < 0$ . This proves that cross ownership is always partial:  $s^* < \frac{b}{2}$ .

On the other hand, at s = 0 the first term vanishes whereas the second term does not, as  $\frac{d\lambda}{ds}\Big|_{s=0} = \frac{1}{b} > 0$ . It follows that  $\frac{\partial S}{\partial s}\Big|_{s=0} > 0$ , proving that cross ownership is always positive:  $s^* > 0$ .

*Proof of Proposition 9.* The proof is identical to the proofs of Propositions 3, 4 and 5 and is therefore omitted.  $\blacksquare$ 

# Appendix B: Examples

Here we consider various specific models of product market competition and show that standard measures of the intensity of competition accord with our condition (23). **Example 1**. The firms supply differentiated products, the inverse demand for which is:

$$p_i = 1 - q_i - \delta q_j$$

where  $\delta \in [0, 1]$  is a parameter that captures the degree of product differentiation: products are independent for  $\delta = 0$ , perfect substitutes for  $\delta = 1$ . Marginal costs are nil, and firms compete in quantities  $(x_i = q_i)$ .

It is easy to verify that the profit functions are well-behaved and that the equilibrium is unique. Equilibrium prices and profits are:

$$q_i^* = \frac{1}{2 + \delta + \delta \lambda}$$
$$\Pi^* = \frac{1 + \delta \lambda}{(2 + \delta + \delta \lambda)^2}.$$

Using (19), one obtains

$$\frac{\partial \Pi^*}{\partial s} = \frac{4\theta \delta^2 bs \left(b-s\right)^3}{\left[\left(2+\delta\right) \left(b-s\right)^2 + 2\theta \left(1+\delta\right) s^2\right]^3},$$

whence it is easy to verify that the derivative is positive and inverted-U shaped.

In this example, a natural index of the intensity of competition is the degree of product substitutability  $\delta$ . Indeed, we have

$$\frac{\partial}{\partial \delta} \left[ \frac{\partial \Pi^*}{\partial \lambda} \frac{1}{\Pi^*} \right] = \frac{\delta \left( 1 - \lambda \right) \left( 4 + \delta + 3\delta \lambda \right)}{\left( 1 + \delta \lambda \right)^2 \left( 2 + \delta + \delta \lambda \right)^2} > 0,$$

consistently with our condition (23).

**Example 2**. Under the same assumptions as in Example 1, suppose that firms compete in prices. The Bertrand equilibrium is:

$$p_i^* = \frac{1-\delta}{2-\delta-\delta\lambda}$$
$$\Pi^* = \frac{(1-\delta)(1-\delta\lambda)}{(1+\delta)(2-\delta-\delta\lambda)^2}$$

Using (19), one then obtains

$$\frac{\partial \Pi^*}{\partial s} = \frac{4\theta \left(1-\delta\right) \delta^2 b s \left(b-s\right)^3}{\left(1+\delta\right) \left[\left(2-\delta\right) \left(b-s\right)^2 + 2\theta \left(1-\delta\right) s^2\right]^3}.$$

As in the case of quantity competition, the derivative is positive and inverted-U shaped.

As above, it is natural to take  $\delta$  as a measure of the intensity of competition. This measure accords with our condition (23), as

$$\frac{\partial}{\partial \delta} \left[ \frac{\partial \Pi^*}{\partial \lambda} \frac{1}{\Pi^*} \right] = \frac{\delta \left( 1 - \delta \right) \left( 4 - \delta - 3\delta \lambda \right)}{\left( 1 - \delta \lambda \right)^2 \left( 2 - \delta - \delta \lambda \right)^2} > 0.$$

It is also generally recognized that competition is more intense when firms choose prices than if they choose output levels. This notion of the intensity of competition also accords with (23), as

$$\frac{\partial \Pi^*}{\partial \lambda} \frac{1}{\Pi^*} \bigg|_{\text{Bertrand}} - \frac{\partial \Pi^*}{\partial \lambda} \frac{1}{\Pi^*} \bigg|_{\text{Cournot}} = \frac{2\delta^3 \left(1 + 2\lambda - 3\lambda^2\right)}{\left(1 - \delta\lambda\right) \left(1 + \delta\lambda\right) \left(2 - \delta - \delta\lambda\right) \left(2 + \delta + \delta\lambda\right)} > 0.$$