# Buying Many, Returning (How) Many? 

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#### Abstract

E-commerce firms have adopted policies stimulating consumers to order many products at once, inspect their fit at home, and then decide what to return. These policies introduce a trade-off as they may result in consumers acquiring products that better fit their taste, at the expense of the private and social costs associated with returns. We ask under which conditions firms find it optimal to offer these "Buy Many" policies, whether they lead to more efficient market outcomes and how the social surplus is divided. Surprisingly, we also find that these policies may actually lead to fewer returns. The paper also makes a methodological contribution to the consumer search literature by characterizing equilibrium behaviour of a multi-product firm when refunds are strategically chosen.


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## 1 Introduction

Product returns play an increasingly important role in retail markets. A recent report of the National Retail Federation estimates that in the USA across different retail channels $\$ 743$ billion of merchandise value is returned in 2023, which is around $14,5 \%$ of total retail sales. In the online segment of the retail market even $17,6 \%$ of product value is returned. ${ }^{1}$ Given the importance of product returns, firms have started to treat returns strategically by developing optimal return policies. One of these developments is that firms, like Amazon and Zalando, offer consumers the possibility to order multiple items at the same time, inspect them at home to see whether they like them, and to return all items that are considered not to be a good fit. ${ }^{2}$

In this paper we ask how a firm's product return policy could help generating profits and what the welfare consequences of such policies are. For the welfare analysis it is also important to ask how frequently products are returned as product returns are associated with environmental costs that are paid by agents not involved in the transaction, while returned products often also cannot be easily resold in the market. ${ }^{3}$

To study product returns, a consumer search framework is appropriate. Products have a consumer-specific match value and consumers have to inspect a product at a cost to determine its value. In standard consumer search models consumers have to pay this search cost up front to learn their match value before purchase (see, for example, the seminal contributions by Wolinsky (1986), Anderson and Renault (1999) and Armstrong (2017)). We augment these models by allowing consumers to order (or buy) products without inspecting them before purchase and only inspect after purchase. As inspecting after purchase can usually be done in a more comfortable environment at a time that suits the consumer best, the inspection cost after purchase is lower. The difference in inspection cost before and after purchase is one important dimension of our analysis. Firms may stimulate that consumers inspect after purchase by offering generous return policies, i.e., refunds. Thus, consumers may find it optimal to inspect products after purchase if the inspection cost difference is sufficiently large and/or the firm has a sufficiently generous return policy. However, offering refunds is costly

[^1]as the salvage value of products that are returned is typically lower than the production cost. The difference between the production cost and the salvage value is the second important dimension of our analysis.

To study when a firm may find it optimal to offer consumers to simultaneously order multiple products and return as many as they like, we consider a multi-product monopolist. We focus on two products, but the qualitative results continue to hold for a broader range of products. Not to bias our results in favour of ordering products simultaneously, consumers will only buy one product as their valuation for both products is the same as the maximum valuation of the two products separately. Thus, if the monopolist finds it optimal to engage in offering consumers to order multiple items at once, it is not because it can sell more products. The firm can offer different prices and refunds for different products, but also condition these on whether or not a consumer orders multiple products simultaneously. The firm cannot, however, condition prices or refunds on whether or not a consumer inspected a product as (certainly in online markets) firms do not know this. Prices and refunds do, of course, determine whether consumers find it optimal to inspect products before or after purchase.

Since Morgan and Manning (1985) it is well-known that if consumers can choose to search sequentially or simultaneously at the same prices, they find it optimal to search items sequentially. ${ }^{4}$ This result also applies to our setting if prices and refunds are identical across inspection modes. By offering different prices and refunds if a consumer orders multiple items at once, the firm may, however, incentivize the consumer to search simultaneously. If the consumer takes this option, she will necessarily return at least one product.

We have two main substantial results and a significant methodological contribution. First, we show that there are indeed circumstances under which it is profitable for the firm to have the consumer order multiple items at once and return all products the consumer does not want to keep. This happens when the difference between production cost and salvage value and/or the inspection cost after purchase is sufficiently small. The firm will set the refund equal to the salvage value so that consumers' return decision for both products is socially optimal and sets a price that extracts all surplus. The consumer is just indifferent between starting to search and not searching at all.

Second, and perhaps surprisingly, even though it is guaranteed that consumers return at least one item if they order multiple items, the expected number of returns may be lower

[^2]than if consumers order items sequentially. To explain this result we first have to address the methodological contribution this paper makes in characterizing equilibrium behaviour under sequential search with product returns. In order to show when offering simultaneous search is optimal, one should also characterize the maximal profits under the alternative.

A first question in this regard is whether the firm wants to induce the consumer to inspect products before or after purchase. This question boils down to under which inspection form social surplus is higher and how much of that surplus the firm is able to extract. It is clear that social surplus is potentially higher under inspection after purchase if the difference in inspection costs is relatively large and the difference between production cost and salvage value is relatively small. When consumers search before purchase the firm is generically not able to extract all surplus as it sets price in such a way that consumers find it beneficial to search. When consumers inspect products after purchase the firm is better able to extract all surplus by setting prices and refunds appropriately. Only when the search cost of inspecting products before purchase is relatively small and the threat of inspecting before purchase is more severe, the firm has to offer consumers prices and refunds so that they make positive surplus. Thus, the firm may induce consumers to inspect after purchase even if this is not socially optimal.

A second question is whether the firm wants to set identical prices and refunds or not. By offering different prices and refunds, the firm may steer consumers in inspecting a particular product first. We find that the firm indeed wants to set different prices and refunds if, and only if, the search cost of inspecting before purchase is large enough. By offering different prices and refunds, the firm is able to extract all surplus from the consumer. However, when the search cost becomes smaller, the threat of the consumer to search before purchase becomes more credible and to induce the consumer to inspect after purchase the firm has to give a larger refund for every given price. When this constraint becomes binding for both products, the firm sets identical prices and refunds for both products.

The reason why "buying many and return" may actually yield fewer returns is the following. First, under this strategy, consumers will always return at least one product, and return also the second product if both products have a value smaller than the refund (which is set equal to the salvage value). Second, under the alternative sequential inspection strategy, consumers return products if their value is below their reservation value, which -if the search cost is small- may actually be high. This would imply that consumers may almost surely return products under sequential search (whether it is before or after purchase). Thus, no matter how the frequency of returns is measured (as the expected number of products that
are returned or as the expected number of returns) there will be less returns under "buying many and return" if the search cost and/or the salvage value is small.

Related literature. The paper combines two strands of literature. The papers most closely related to ours are Janssen and Williams (2023), Jerath and Ren (2023) and Matthews and Persico (2007) in that they also study product returns in a consumer search setting. However, all these papers study a single product firm sell and consumers searching sequentially (where the former paper studies a competitive setting, while the latter two analyze monopoly behavior. They find that the number of refunds is either inefficiently high or low. None of these papers consider a firm that incentivizes consumers to search simultaneously among its multiple products. ${ }^{5}$ Petrikaitė (2018a) studies search with returns in a duopoly setting, but also does not consider multiple products per seller or simultaneous search. The second strand of literature is on multi-product search (Rhodes (2015), Shelegia (2012) and Zhou (2014)), but the focus of these papers is on consumers searching for multiple products, creating a joint search effect in that once a consumer is at a store it has a lower search cost to buy other products at that store. These papers do not study product returns or simultaneous search.

The optimal behaviour of the firm if it wants to induce sequential search after purchase has features that also arise in Petrikaite (2018b) and Gamp (2022) in that a multi-product firm has an incentive to obfuscate search among its products. These papers study a setting where consumers have to inspect products before purchasing one of them and where (together with prices) the firm chooses consumers' search cost directly. They show that the firm has an incentive to set a positive search cost and asymmetric prices so as to induce consumers to search the products in a particular order. In contrast, we allow consumers to order (or buy) products before inspecting them ${ }^{6}$ and have a setting where the firm cannot affect the inspection cost of consumers directly. However, by choosing a refund that is smaller than the price, the firm effectively sets an additional inspection fee that the consumer has to pay if she wants to return the product. This inspection fee adds to the firm's profits, which adds another difference to the above mentioned papers.

The rest of the paper is organized as follows. The next section introduces the model features. Section 3 discusses when the firm wants to incentivize the consumer to "buy and

[^3]return many". It does so by also studying the alternatives of searching sequentially either before or after purchase. Section 4 analyzes under what conditions "buying and returning many" yields fewer returns than the alternative options and we show that the incentives of the firm to incentivize "buying and returning many" partly coincide with the social incentives to reduce the number of returns. Section 5 concludes.

## 2 The Model

A monopoly firm sells two products. Each product has a production cost $c \geq 0$ and a salvage value $\eta \in[0, c]$ to the firm in case the product is bought and then returned. We will define $k=c-\eta$ as the value lost if the product is returned after it is inspected and we will refer to $k$ as the product degradation. The firm can set different prices and refunds for the different products $i=1,2$ and we denote price by $p_{i} \geq 0$ and refund by $\tau_{i} \in\left[0, p_{i}\right]^{7}$ As (certainly in online markets) a firm cannot verify whether the consumer has inspected the value of the product before purchase or not, it cannot charge different prices for when consumers inspect products before or after purchase. It can only set prices and refunds such that it incentivizes consumers to inspect products in one or the other way. As the firm does know whether or not a consumer buys multiple products at once, it can offer different prices and refunds for this situation and we denote them by ( $p_{\text {sim }}, \tau_{\text {sim }}$ ) with $p_{\text {sim }} \geq \tau_{\text {sim }} .{ }^{8}$ If consumers simultaneously buy two products they will always inspect them after purchase as this is at lower inspection costs. We will sometimes refer to a set of prices and refunds as a contract or somewhat imprecisely simply as prices.

There is a representative consumer with unit demand. The two products are ex-ante identical to the consumer with each product having a valuation that is independently and identically distributed by $v_{i} \sim F[\underline{v}, \bar{v}] .{ }^{9}$ To have an interesting model, we require $\bar{v}>c$. The consumer knows the prices and refunds the firm offers, but has to pay an inspection cost of $s>0$ to learn a product's value before purchase and a cost of $\beta s$ if she wants to learn the product's value after purchase, with $\beta \in[0,1]$. Thus, $(1-\beta) s$ measures the difference in inspection cost. The outside option of the consumer is normalized to 0 . For future reference, it will be useful to write $\hat{v}_{b}$ as the reservation value of inspection before purchase and $\hat{v}_{a i}$ as

[^4]the reservation value of inspection of product $i$ after purchase. They are implicitly defined through the following equations: ${ }^{10}$
\[

$$
\begin{equation*}
\int_{\hat{v}_{b}}^{\infty}\left(v-\hat{v}_{b}\right) f(v) d v=s \text { and } \int_{\hat{v}_{a i}}^{\infty}\left(v-\hat{v}_{a i}\right) f(v) d v=\beta s+p_{i}-\tau_{i} . \tag{1}
\end{equation*}
$$

\]

Note that $\hat{v}_{a i}$ is not only a function of exogenous parameters but also of $p_{i}$ and $\tau_{i}$, the two strategic variables of the firm for product $i$. When we write $\hat{v}_{a i}$ we implicitly mean the function $\hat{v}_{a i}\left(p_{i}, \tau_{i}\right)$.

Given the firm's choices, the consumer can take one of the following actions: ${ }^{11}$ (i) Inspect the products sequentially before purchase, which entails choosing which product to inspect first, paying the inspection cost of $s$ to learn that product's value and then deciding whether to buy it at price $p_{1}$, or to inspect the second product, (ii) Inspect the products sequentially after purchase, which entails choosing which product to inspect first, paying the inspection cost of $\beta s$ to learn that product's value, deciding whether to keep it and pay the price $p_{1}$, or to inspect the second product, and finally returning and paying $p_{i}-\tau_{i}$ for all products that are not kept ${ }^{12}$ (iii) Inspect the products simultaneously after purchase for inspection cost of $\beta s$ each and decide whether to buy at most one of the products at the contract ( $p_{\text {sim }}, \tau_{\text {sim }}$ ) of the products and return at least one, or finally, (iv) Leave and take the outside option with a pay-off of 0 . If consumers search sequentially, they have perfect recall. In the following we will refer to the first three actions in short as $B^{s e q}, A^{\text {seq }}$ and $A^{\text {sim }}$ respectively.

It is important to note that it is possible to redefine inspection after purchase as a structurally simpler problem, which will facilitate the analysis. From the consumer's view inspection after purchase can be re-written as inspection before purchase with certain inspection costs and prices. In particular, once the consumer pays the inspection cost $\beta s$ to learn the value of product $i$ after purchase, she commits to paying at least $p_{i}-\tau_{i}-$ this is the return cost in the case where she wants to return the product. If instead she wants to keep the product she needs to pay $p_{i}$. Thus, the part $p_{i}-\tau_{i}$ of the cost of the product is sunk at the time of inspection, while the remainder $\tau_{i}$ is being paid only in case the consumer keeps the

[^5]product. Thus, we can redefine inspection after purchase as inspection before purchase with a redefined inspection cost of $\beta s+p_{i}-\tau_{i}$ and a redefined price of $\tau_{i}$. Note that while $\beta s$ is lost, $p_{i}-\tau_{i}$ is the part of the redefined inspection cost that is paid to the firm. It is thus as if the firm was offering product $i$ for inspection before purchase at price $\tau_{i}$ and inspection cost $\beta s$ and with an additional inspection fee of $p_{i}-\tau_{i}$. To avoid confusion, we define the inspection fee $\sigma_{i}:=p_{i}-\tau_{i}$ and the redefined price $\rho_{i}:=\tau_{i}$. Whenever we use the redefined interpretation we will use ( $\sigma_{i}, \rho_{i}$ ) instead of ( $p_{i}, \tau_{i}$ ) as the firm's strategic variables.

From the firm's point of view, it loses $k=c-\eta$, the product degradation, with every inspection after purchase, which is a sunk cost at the moment the consumer is induced to inspect after purchase. The salvage value $\eta$ is then the "real" opportunity cost of the firms as it is the value of the product to the firm that is kept by the consumer and not returned.

To summarize: if the firm offers a contract that induces the consumer to inspect product $i$ after purchase, it offers an inspection fee of $\beta s+\sigma_{i}$, associated with a cost of $k$. The consumer discovers the value $v_{i}$ and can buy $i$ at price $\rho_{i}$, whereas the value of the unsold product to the firm is $\eta$. The consumer can decide to continue searching by inspecting another product in the same way, or to buy the product at price $\rho_{i}$, ending search.

## 3 When Buying Many Simultaneously is Maximizing Profit

We now identify when the firm wants to incentivize the consumer to engage in simultaneous inspection after purchase instead of engaging in sequential inspection before or after purchase. To this end, we first derive the maximum profits for each of the three options the firm can incentivize: $A^{\text {sim }}, A^{\text {seq }}$ and $B^{\text {seq }}$. As will become clear in our discussion, even though for given contracts the consumer can choose whether to engage in $A^{\text {seq }}$ or $B^{\text {seq }}$ so that these two options are not really independent, the firm can choose certainly elements of these contracts (in particular the refund) so as to (not) stimulate consumers to inspect before or after.

As the optimal contract under sequential search turn out to be the most interesting in its own right, let us consider this first. As discussed, the consumer can choose between $A^{\text {seq }}$ and $B^{\text {seq }}$ for any pair of contracts $\left\{\left(p_{i}, \tau_{i}\right)\right\}_{i=1,2}$. It is clear that if the firm wants to incentivize the consumer to choose $B^{\text {seq }}$, it can set unattractive refunds $\tau_{i} \rightarrow 0$ so that the consumer will never consider choosing $A^{\text {seq }}$. However, if the firm wants to incentivize $A^{\text {seq }}$, this is not the case. Generally, deterring the consumer from switching to $B^{s e q}$ forces the firm to compromise its ability to extract the highest profits and the threat of $B^{\text {seq }}$ only does not play a role when it is incredible, which is when the difference in inspection cost $(1-\beta) s$ is large. Denoting the firm's maximum profit under $A^{\text {seq }}$ by $\pi_{\text {Aseq }}^{*}$, we first derive the upper bound $\bar{\pi}_{A s e q}^{*}$ of this
profit, which is attained when the firm does not have to compromise its pricing decision in order to prevent the consumer from deviating to $B^{\text {seq }}$. Generally speaking, $\pi_{\text {Aseq }}^{*} \leq \bar{\pi}_{\text {Aseq }}^{*}$, with equality when the threat of $B^{s e q}$ is incredible, which is when $(1-\beta) s$ is large.

Proposition 1 If the firm induces consumers to inspect sequentially after purchase the maximal profit $\bar{\pi}_{\text {Aseq }}^{*}$ is attained by choosing the following strategy: ${ }^{13}$

$$
\left(\sigma_{1}^{*}, \rho_{1}^{*}\right)=\left(\mathbb{E}\left[\max \left(v-E S_{I}-\eta\right), 0\right]-\beta s, E S_{I}+\eta\right) \text { and }\left(\sigma_{2}^{*}, \rho_{2}^{*}\right)=\left(E S_{I}+k, \eta\right)
$$

with profits $\bar{\pi}_{\text {Aseq }}^{*}=\mathbb{E}\left[\max (v-\eta), E S_{I}\right]-\beta s-k$ and where:

$$
\begin{equation*}
E S_{I}=\mathbb{E}[\max (v-\eta, 0)]-\beta s-k . \tag{2}
\end{equation*}
$$

The proposition can intuitively be understood in the following way. It is optimal for the firm to raise the inspection fees so high that the consumer will not make use of their ability to recall, i.e. she will never go back to buy an earlier inspected product. Then the firm can use the contract $\left(\sigma_{2}, \rho_{2}\right)$ as a two-part tariff, with $\rho_{2}=\eta$ priced at marginal cost and $\sigma_{2}$ chosen such that it extracts $E S_{I}$, the efficient surplus from inspection of the second product. For the first product, the firm does the same, only here $\rho_{1}$ is priced at "marginal opportunity cost", which is the profit that the firm foregoes if the consumer does not inspect the second product.

If the firm incentivizes $A^{s e q}$ then Weitzman (1979) implies that the consumer first inspects the product with the higher net reservation value $\hat{v}_{a 1}-\rho_{1} \geq \hat{v}_{a 2}-\rho_{2}$ and only inspects product $i$ if it has a non-negative net reservation value $\hat{v}_{a i}-\rho_{i} \geq 0$ (as this is a necessary condition for non-negative utility). Without loss of generality consider that product $i=1$ is inspected first. The following observations determine the optimal contract under sequential search after purchase. First, as the inspection fee $\sigma_{1}$ for the first inspected product is committed to be paid before inspection starts, the firm can increase it as long as the above inequalities are not violated. This implies that in equilibrium $\hat{v}_{a 1}-\rho_{1}=\hat{v}_{a 2}-\rho_{2}$, i.e. the net reservation values of the two products will be equal. ${ }^{14}$ Second, the firm will choose the contracts for both products such that the net reservation values will be equal to zero $\hat{v}_{a i}-\rho_{i}=0$ implying that the consumer will buy the first product that has a positive observed net value, $v_{i}-\rho_{i}>0$. It follows that the firm sets the refund of the last product equal to the marginal cost of a return, $\rho_{2}^{*}=\eta$, and the corresponding inspection fee $\sigma_{2}^{*}$ as high as possible. In this way the firm is able to extract the efficient surplus $E S_{I}$, from search of the last product, with $\sigma_{2}^{*}=E S_{I}+k$, where $E S_{I}$ is defined by (2). Note that the refund price is efficient, but the inspection fee is too high as efficiency requires $\sigma_{i}=k$. Turning to the first product that is inspected, the firm (realizing

[^6]it can make a profit of $E S_{I}$ if the consumer continues to inspect the second product) will set the refund price such that $\rho_{1}^{*}=E S_{I}+\eta$ and an inspection fee $\sigma_{1}^{*}$ that extracts all remaining surplus, with $\sigma_{2}^{*} \geq \sigma_{1}^{*} \geq k .{ }^{15}$ Thus, the firm sets $\rho_{1}$ such that it is indifferent between selling immediately to the consumer and the consumer continuing to inspect the second product. The resulting upper bound of the firm's profit under $A^{\text {seq }}$ (that is, in the absence of the threat of the consumer switching to $\left.B^{s e q}\right)$ is $\bar{\pi}_{A s e q}^{*}=\mathbb{E}\left[\max \left(v-\eta, E S_{I}\right)\right]-\beta s-k$. Interestingly, this expression is identical to the efficient surplus if there was no recall: As in Petrikaite (2018b), the profit maximizing strategy of the firm distorts the consumer's optimal search behavior in such a way as to remove their ability to recall any earlier inspected product. However, here it is further able to extract all that surplus at the same time. Note that both $\sigma_{1}^{*}$ and $\rho_{1}^{*}$ are inefficiently high implying that the total surplus from search of both products will not be maximized. In particular, search is inefficient because the decision whether to return the first product and the decision whether to inspect the second product are not independent.

Example. The following example illustrates the nature of the optimal solution under $A^{\text {seq }}$ and shows why the optimal solution involves an asymmetric contract even if the products are ex ante symmetric. Suppose that $\beta=c=\eta=0$ and that values are uniformly distributed over $[0,1]$. If the firm would have one product to sell, it is clear that the optimal contract would have $\tau=\rho=0$ and $p=\sigma=1 / 2$. The firm sets the refund efficiently, namely equal to the salvage value, and then extracts all surplus by setting the price equal to the expected surplus of searching. This is also the optimal contract for the second product if the firm sells two products. Consider then the first product. The firm knows it can make a profit of $1 / 2$ and that the consumer gets an expected surplus of zero if the consumer continues to inspect the second product. It is then optimal to set the refund in the first period $\tau_{1}=\rho_{1}=1 / 2$ as this is the opportunity cost of the refund: a higher refund yields some extra consumers returning the product with a refund that is larger than the profit it generates. Given the choice of the refund and a price $p_{1}$ in the first period consumers start searching if their expected surplus is nonnegative, which yields the following constraint: $-\sigma_{1}+1 / 2 *\left(3 / 4-\rho_{1}\right)+1 / 2 * 0 \geq 0$. It is optimal for the firm to set the largest price giving this constraint, yielding $p_{1}=\sigma_{1}-1 / 2=5 / 8$. The total profit is thus equal to $5 / 8$ as the consumer pays the first inspection fee $\sigma_{1}$ of $1 / 8$ and then pays the additional price $\tau_{1}$ of $1 / 2$ if the valuation is larger than $1 / 2$ (which happens with probability 1/2) and if the valuation is smaller than 1/2 the consumer continues to search the second product, pays the inspection fee $\sigma_{2}$ of $1 / 2$ and always keeps the product.

[^7]Note that the profit that is derived here is the upper bound $\bar{\pi}_{\text {Aseq }}^{*}$ of the profit under sequential search after purchase as the constraint imposed by $B^{s e q}$ is not taken into account. However, this upper bound is realized when the threat of $B^{\text {seq }}$ is not credible, which is when $s$ is large.

The example shows that even though the actual inspection cost equals 0 (as $\beta=0$ ), the firm makes inspection costly by creating an inspection fee $\sigma_{i}$ that consumers know they lose when they inspect a product. The inspection fee for the second product causes an inefficiency as the first product may be kept, ending search, even though the second product has a higher (net) value. The difference in refunds for the first and second product also creates an inefficiency as it may well happen that the first product is returned (if, in the example, its value is smaller than $1 / 2$ ), while the second product turns out to have a lower net value. In the example, this could even arise when the second product's value is close to 0 . Note that even if the first product is returned only after the second is inspected, the consumer would still return the first product as it has a higher refund.

Second, consider the optimal contract and profits when consumers search simultaneously after inspection so that the consumer pays the inspection fee $\sigma_{\text {sim }}$ and the inspection cost $\beta s$ for both products upfront as long as their expected utility is non-negative. Recall that the consumer can buy at the terms of contract ( $\sigma_{\text {sim }}, \rho_{\text {sim }}$ ) only if she chooses the action $A^{\text {sim }}$. The firm does not have to consider therefore a potential deviation of the consumer when incentivizing $A^{s i m}$ as it can in principle set very unattractive terms for the consumer to search sequentially. When consumers search simultaneously, they will buy the product with the higher net value $v_{i}-\rho_{\text {sim }}$, as long as either of them is non-negative. So, the profitmaximizing contract is essentially a two part tariff where the optimal price $\rho_{\text {sim }}^{*}$ is set at marginal cost $\eta$ and the optimal inspection fee $\sigma_{\text {sim }}^{*}$ extracts all surplus. In particular, as the expected social surplus is given by

$$
\begin{equation*}
\mathbb{E}\left[\max \left(v_{1}-\eta, v_{2}-\eta, 0\right)\right]-2(\beta s+k) \tag{3}
\end{equation*}
$$

the profit $\pi_{\text {Asim }}^{*}=2\left(\sigma_{\text {sim }}^{*}-k\right)$ is equal to this expression. ${ }^{16}$ From an efficiency standpoint, the number of inspections is too large, but products are returned at an efficient level: the product with the lowest valuation will always be returned and this is efficient as the consumer has no (additional) value for it, while the firm has a salvage value and the product with the highest valuation will be returned if its value is smaller than the firm's salvage value.

[^8]Example continued. Keeping the same parameter values, it is clear that under $A^{\text {sim }}$, the firm wants to set $\rho_{\text {sim }}=\eta=0$. The firm then wants to set the price for the two products such that it attracts $\mathbb{E}\left[\max \left(v_{1}, v_{2}\right)\right]=2 / 3$. Thus, it will set the price for each product equal to $1 / 3$.

Comparing the profits for $A^{\text {sim }}$ to those for $A^{\text {seq }}$, we find the following:
Lemma 1 There exists a function $\underline{S}_{A}(\eta)>0$ such that for all $(\beta, s, k, \eta)$ :

$$
\beta s+k \leq \underline{S}_{A} \Leftrightarrow \pi_{A s i m}^{*} \geq \bar{\pi}_{\text {Aseq }}^{*} .
$$

The intuition behind the lemma is clear. Under both search protocols the firm extracts all surplus. However, the surplus is quite different. Under simultaneous search, the consumer inspects both products and chooses the one with the higher net value. The potential loss in surplus is due to inspection costs and product degradation related to the purchase and return of at least one product. Under sequential search, the consumer inspects the first product and keeps it if it has a higher net value than the expected value of the second inspection, including the inspection fee the firm imposes. Compared to simultaneous search, the consumer loses if the consumer decides not to inspect the second product even though it would have had a higher net value if he would have done so, or if the consumer continues to inspect the second product, but then does not keep the product with the highest value due to the difference in refunds. If the loss in surplus under simultaneous search due to unnecessary inspection costs and product degradation is relatively small, simultaneous search leads to higher profits. If, on the other hand, $\beta s+k$ is relatively large, then $A^{\text {seq }}$ yields more profits as one can find a good fit already with the first product and save on inspection cost and product degradation. Note that as $\bar{\pi}_{\text {Aseq }}^{*}$ is the upper bound on profits, $\underline{S}_{A}(\eta)$ is a lower bound of the threshold below which profits of $A^{s i m}$ are higher than those of $A^{s e q}$.

Figure 1 presents an example.
Next, we consider when the firm's profit under sequential search after purchase is higher than the firm's profit under sequential search before purchase. The following example shows that the firm acts as a "standard" multi-product firm and incorporates the positive externality selling the products impose on each other.

Example continued. Suppose now in addition that $s=0$. If the firm wants to induce $B^{\text {seq }}$, it cannot gain by setting asymmetric prices. If it sets a price $p$ for each of the products, then it makes a profit of

$$
\begin{equation*}
\left[p(1-p)+(1-p)^{2} / 2\right] p \tag{4}
\end{equation*}
$$



Figure 1: Profits $\pi_{\text {Asim }}$ and $\bar{\pi}_{\text {Aseq }}$ as functions of the sum of inspection and degradation costs $\beta s+k$ for uniformly distributed values and $\eta=0$.
over each product. This expression can be understood as follows. There is a probability $p$ that the value of the other product is smaller than $p$ and in that case the product under consideration is sold if it has a value larger than $p$, which happens with probability $1-p$. With the remaining probability $1-p$ the value of the other product is larger than $p$ and in that case the product under consideration is sold if it has the largest of the two values. Maximizing this expression with respect to $p$ yields the $F O C 3 p^{2}=1$, or $p=\sqrt{1 / 3}$. Thus, the total profit of the firm is $\frac{2}{3} \sqrt{1 / 3}$. Note that both the price and the profit is larger than the profit of a single product monopolist, but that the profit is considerably smaller than the upper bound of the profit under $A^{\text {seq }}$ we derived before. The reason is that the firm has to leave quite a bit of surplus to the consumer as the consumer knows his value for buying. Under $A^{\text {seq }}$ the firm transforms the demand of the consumer and makes it less price sensitive as he has to decide to commit to pay the inspection fee before knowing the value. ${ }^{17}$ Note also, that under $A^{\text {seq }}$ the firm cannot achieve the upper bound of profits if $s$ is small and that in fact if $s=0$ the maximal profit it can achieve under $A^{\text {seq }} s$ is equal to the profit we have derived here for $B^{\text {seq }}$.

As the profits under $B^{s e q}$ are fairly standard, we immediately provide the following result. ${ }^{18}$

Lemma 2 There exists a function $S_{B}(\beta, s, k, \eta)>0$ such that for all $(\beta, s, k, \eta)$

$$
\beta s+k \leq S_{B}(\beta, s, k, \eta) \Rightarrow \pi_{A s e q}^{*} \geq \pi_{B s e q}^{*}
$$

The firm prefers $A^{\text {seq }}$ to $B^{s e q}$ when the sum of inspection and degradation costs $\beta s+k$ under $B^{\text {seq }}$ is small compared to the inspection cost $s$ under $B^{\text {seq }}$. This is natural as these are

[^9]the respective costs associated with the two forms of sequential search. Moreover, the firm's ability to extract profits under $B^{s e q}$ is in general fairly limited due to only having one strategic variable available. We will see in the next section, that for the extreme case of zero inspection costs under both schemes, $\beta s+k=s=0$, both $A^{s e q}$ and $B^{s e q}$ lead to the same profits.

The following proposition then follows immediately from Lemmas 1 and 2 by setting $\underline{S}(\beta, s, k, \eta):=\min \left\{\underline{S}_{A}(\eta), S_{B}(\beta, s, k, \eta)\right\}$.

Proposition 2 There exists a function $\underline{S}(\beta, s, k, \eta)>0$ such that for all $(\beta, s, k, \eta)$ with $\beta s+k \leq \underline{S}(\beta, s, k, \eta)$

$$
\pi_{\text {Asim }}^{*} \geq \bar{\pi}_{\text {Aseq }}^{*} \geq \pi_{\text {Aseq }}^{*} \geq \pi_{\text {Bseq }}^{*}
$$

Thus, the firm induces consumers to "Buy Many and Return" if the sum of inspection and degradation costs $\beta s+k$ is small. The optimal contract is a two part tariff where $\tau_{\text {sim }}^{*}=\eta$ ("marginal cost") and $p_{\text {sim }}^{*}$ is used to extract all surplus. Note that Proposition 2 presents a sufficient condition for when the firm wants to incentivize $A^{\operatorname{sim}}$, as $\underline{S}(\beta, s, k, \eta)$ is a lower bound for the threshold for the actual profit $\pi_{\text {Aseq }}^{*}$.

## 4 The Number of Returns

In the previous section, we derived an upper bound on the profits under $A^{\text {seq }}$, which arises if consumers cannot deviate to $B^{s e q}$. We have argued that the the threat of $B^{s e q}$ requires the firm to adapt its contracts, especially when $s$ is small, so that consumers do not want to choose $A^{\text {seq }}$, reducing profits. However, the loss in profits is not the only implication of the threat of $B^{s e q}$. In this section, we will show, perhaps surprisingly, that "Buy Many and Return" contracts can actually lead to a lower number of returns than sequential inspection after purchase. The presence of the threat of the consumer deviating to $B^{s e q}$ turns out to be important in facilitating this result. ${ }^{19}$

The threat of $B^{s e q}$ introduces an upper bound $\bar{\sigma}=\hat{v}(s)-\hat{v}(\beta s+\bar{\sigma})$ on any contract the firm can set to induce the consumer to search after purchase. If the firm sets a contract with $\sigma_{i} \geq \bar{\sigma}$, then the consumer prefers to inspecting that product before purchase. This prevents the firm from setting its preferred contract if $s$ is small. For $s=0$ we find that $\bar{\sigma}=0$, implying that the firm has to set $\sigma_{i}=0$. This in turn implies that the firm makes a loss from inspecting after purchase for any $k>0$. In that case, the firm only makes positive profits from product sales. For positive $s$, the firm can set contracts for $A^{s e q}$ that are different from those under $B^{\text {seq }}$, which may lead to higher or lower profits, depending on the difference in inspection

[^10]costs $(\beta)$ and product degradation $(k)$. For now, we do not identify the full profit maximizing strategy of the firm, but focus on the case where the sum of inspection and degradation costs $\beta s+k$ is small, as in the previous section. Lemma 2 states that if the sum of these costs are small enough the firm prefers to incentivize $A^{\text {seq }}$ and not $B^{\text {seq }}$.

We focus the analysis on this case where the firm prefers to incentivize $A^{\text {seq }}$ instead of $B^{\text {seq }}$ and ask when the number of returns is smaller under the optimal simultaneous contract. We do so for small values of $s$. One can show that small values of $s$ force the firm to set symmetric contracts under $A^{\text {seq }}$ and in particular a low $\sigma$. The reason is that if consumers had to pay a relatively large inspection fee upfront, they rather inspect before purchase without paying the inspection fee. To compensate the firm sets a high refund price $\rho$.

The expected number of returns under the two search modes for small $s$ are given by

$$
n_{\text {Asim }}=1+F(\eta)^{2} \text { and } n_{\text {Aseq }}=F\left(\hat{v}_{a 2}\right)+F(\rho)^{2} .
$$

The number of returns under $A^{\text {sim }}$ does not depend on $s$. Both products are always inspected, implying that one product is returned with certainty. Both are returned only if their values are both below $\eta$, the efficient return price and the lowest price the firm will ever set. In comparison, the number of returns under $A^{\text {seq }}$ depends on $s$ : a consumer returns the first product if it turns out to have a relatively low value, while she may also return the second product if its value is too low.

The following proposition states when $A^{\text {sim }}$ or $A^{\text {seq }}$ create more returns.
Proposition 3 If $\beta s+\sigma$ is small enough, then there exists an $\bar{s}$ such that the "Buy Many and Return" contracts lead to less expected returns than sequential contracts for all $s<\bar{s}$.

Thus, banning "Buy Many and Return" may actually lead to more rather than to less returns. The intuition behind this result is the following. The low inspection costs and inspection fee $\sigma$ make inspection of the second product attractive to the consumer, while the high refund price $\rho$ makes it unlikely that the consumer will consider the first product a good enough fit. Thus, there is a high chance that the second product will be inspected, in which case again at least one product will be returned with certainty. Due to the high refund price, it is however also likely that the consumer finds neither of the two products a good enough fit, implying that both would be returned. For small $s$ this effect is most severe, leading to a higher expected number of returns under $A^{\text {seq }}$ than under $A^{\text {sim }}$.

Figure 2 illustrates the proposition for values being uniformly distributed and $\beta=k=$ $\eta=0$.


Figure 2: The number of returns $n_{\text {Asim }}$ and $n_{\text {Aseq }}$ as a function of the inspection cost $s$ for uniformly distributed values and $\beta=k=\eta=0$.

Propositions 2 and 3 together imply the following:
Proposition 4 There exists a function $\underline{S}(\beta, s, k, \eta)>0$ and an $\bar{s}$ such that for all $(\beta, s, k, \eta)$ with $\beta s+k \leq \underline{S}(\beta, s, k, \eta)$ and $s<\bar{s}$ the firm induces the consumer to "Buy Many and Return", leading to higher profits and a lower number of returns than sequential inspection after purchase.

Note that the two conditions of Proposition 4 are independent of each other. If $\beta s+k$ is small, then the firm induces $A^{\text {sim }}$, while it might be the case that $s \geq \bar{s}$ is large and $A^{\text {seq }}$ would lead to less returns. ${ }^{20}$ On the other hand, if $\beta s+k$ is large, then the firm induces $A^{s e q}$ or $B^{\text {seq }}$, while it might be the case that $s$ is small, and therefore $A^{s i m}$ would lead to less returns than $A^{\text {seq }}$. Both propositions together simply imply that if inspection costs $\beta s+k$ and $s$ both are small enough, then $A^{s i m}$ will be induced and it will lead to less returns than $A^{s e q}$ would.

## 5 Discussion and Conclusion

This paper showed that multi-product firms may induce consumers to buy many products simultaneously and get a refund for the products they want to return. Especially in online markets this may be an interesting proposition for consumers as they may then inspect products at their own ease at home. Presented with this option, consumers buy the product with the highest valuation and are willing to pay a higher price.

[^11]To show that this may be a profitable strategy for firms, we also had to consider the alternative, which is for consumers to inspect products sequentially. Sequential inspection may be done either before or after purchase. An interesting subsidiary result of our paper is that the characterization of the optimal contracts under sequential search may be to induce consumers to inspect after purchase and that the way to do so is to set asymmetric contracts where the contract for the first product to be inspected has a lower inspection fee and a higher refund price. These contracts have features in common with optimal obfuscation contracts as in Petrikaite (2018b), with the main difference that the optimal contracts here have features of a two-part tariff where the firm benefits from having an inspection fee.

Our final result is that despite the appearance of creating unnecessary refunds, "buying and returning many" contracts may actually lead to fewer (rather than more) products being returned. This has interesting implications for environmental policy as the question is not so much to abandon all these "buying and returning many" contracts, but rather to investigate in more detail in what type of markets they are more likely to lead to more or less returns.

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## A Appendix

## A. 1 Proof of Proposition 1

For clarity, we denote $\hat{v}_{a i}$ as $\hat{v}_{i}$ in this proof.
The firm's profit function under $A^{s e q}$ is similar to that in Petrikaite (2018b), but with
additional terms capturing the profits from inspections:

$$
\begin{aligned}
& \pi_{\text {Aseq }}\left(\rho_{1}, \rho_{2}, \sigma_{1}, \sigma_{2}\right)=\sigma_{1}-k+F\left(\hat{v}_{2}-\rho_{2}+\rho_{1}\right)\left(\sigma_{2}-k\right)+ \\
& \quad+\left[\int_{\rho_{1}}^{\hat{v}_{2}-\rho_{2}+\rho_{1}} F\left(v_{1}-\rho_{1}+\rho_{2}\right) f\left(v_{1}\right) d v_{1}+\left[1-F\left(\hat{v}_{2}-\rho_{2}+\rho_{1}\right)\right]\right]\left(\rho_{1}-\eta\right)+ \\
& \quad+\left[F\left(\hat{v}_{2}-\rho_{2}+\rho_{1}\right)\left[1-F\left(\hat{v}_{2}\right)\right]+\int_{\rho_{2}}^{\hat{v}_{2}} F\left(v_{2}-\rho_{2}+\rho_{1}\right) f\left(v_{2}\right) d v_{2}\right]\left(\rho_{2}-\eta\right) .
\end{aligned}
$$

The first line represents the profits the firm makes from consumers purely inspecting the products. The second line represents the profit from sales of the first product and the third line represents the profit from sales of the second product. The constraints the firm faces are (i) $\hat{v}_{i}-\rho_{i} \geq 0$, to ensure that the consumer inspects product $i$, and (ii) $\hat{v}_{1}-\rho_{1} \geq \hat{v}_{2}-\rho_{2}$, to ensure that the consumer inspects the products in the intended order.

We immediately see that $\sigma_{1}$ only occurs once in the expression, implying that its value will be chosen maximally, implying that the constraint on the relation between the net reservation values will be binding, i.e. $\hat{v}_{1}-\rho_{1}=\hat{v}_{2}-\rho_{2}$. We can use this fact to eliminate $\sigma_{1}$ from the function. We know that $\hat{v}_{1}-\rho_{1}=\hat{v}_{2}-\rho_{2} \Leftrightarrow U_{1}=U_{2}$ with $U_{i}:=\int_{\hat{v}_{i}}^{\infty}(1-F(v)) d v-$ $\sigma_{i}-\beta s$, the utility from inspecting product $i$ alone. From this last identity we derive $\sigma_{1}=$ $\sigma_{2}-\int_{\hat{v}_{2}}^{\hat{v}_{2}-\rho_{2}+\rho_{1}}(1-F(v)) d v$. Replacing $\sigma_{1}$ by this expression, we obtain the profit function $\pi_{\text {Aseq }}\left(\rho_{1}, \rho_{2}, \sigma_{2}\right)$. Using the fact that we can express $\sigma_{2}$ as a function of $\hat{v}_{2}$, we obtain the following profit function the firm wants to maximize:

$$
\begin{aligned}
& \pi_{\text {Aseq }}\left(\rho_{1}, \rho_{2}, \hat{v}_{2}\right)=\left[1+F\left(\hat{v}_{2}-\rho_{2}+\rho_{1}\right)\right]\left(\sigma_{2}\left(\hat{v}_{2}\right)-k\right)-\int_{\hat{v}_{2}}^{\hat{v}_{2}-\rho_{2}+\rho_{1}}(1-F(v)) d v+ \\
& \quad+\left[\int_{\rho_{1}}^{\hat{v}_{2}-\rho_{2}+\rho_{1}} F\left(v_{1}-\rho_{1}+\rho_{2}\right) f\left(v_{1}\right) d v_{1}+\left[1-F\left(\hat{v}_{2}-\rho_{2}+\rho_{1}\right)\right]\right]\left(\rho_{1}-\eta\right)+ \\
& \quad+\left[F\left(\hat{v}_{2}-\rho_{2}+\rho_{1}\right)\left[1-F\left(\hat{v}_{2}\right)\right]+\int_{\rho_{2}}^{\hat{v}_{2}} F\left(v_{2}-\rho_{2}+\rho_{1}\right) f\left(v_{2}\right) d v_{2}\right]\left(\rho_{2}-\eta\right) .
\end{aligned}
$$

Taking the derivative with regard to $\hat{v}_{2}$ and using the fact that $\frac{\partial \sigma_{2}}{\partial \hat{v}_{2}}=-\left[1-F\left(\hat{v}_{2}\right)\right]$ yields:

$$
\frac{\partial \pi_{\text {Aseq }}}{\partial \hat{v}_{2}}=f\left(\hat{v}_{2}-\rho_{2}+\rho_{1}\right)\left[\sigma_{2}\left(\hat{v}_{2}\right)-k\right]-\left[1-F\left(\hat{v}_{2}\right) F\left(\hat{v}_{2}-\rho_{2}+\rho_{1}\right)\right]-\left(\rho_{1}-\rho_{2}\right) f\left(\hat{v}_{2}-\rho_{2}+\rho_{1}\right)\left[1-F\left(\hat{v}_{2}\right)\right] .
$$

The last term comes from the sales part of profit, as in Petrikaite (2018b), while the first two terms arise from the profits the firm makes from consumers inspecting the products. As is the case there, by demonstrating that this derivative is always negative, we can conclude that the firm will want to set $\hat{v}_{2}$ as low as possible, or equivalently $\sigma_{2}$ as high as possible. ${ }^{21}$

[^12]That implies that $\hat{v}_{2}-\rho_{2}=0$. The proof that this derivative is negative for the case of logconcave distributions is in progress. We present here the proof for the uniform distribution $F \sim U[0,1]:$

$$
\frac{\partial \pi_{\text {Aseq }}}{\partial \hat{v}_{2}}=\sigma_{2}-k-\left[1-\hat{v}_{2}\left(\hat{v}_{2}-\rho_{2}-\rho_{1}\right)\right]-\left(\rho_{1}-\rho_{2}\right)\left[1-\hat{v}_{2}\right]
$$

Using $\hat{v}_{2}=1-\sqrt{2\left(\beta s+\sigma_{2}\right)}$ we can express $\sigma_{2}$ through $\hat{v}_{2}$, implying:

$$
\frac{\partial \pi_{\text {Aseq }}}{\partial \hat{v}_{2}}=\frac{1}{2}\left[3 \hat{v}_{2}^{2}-2\left[1+2\left(\rho_{1}-\rho_{2}\right)\right] \hat{v}_{2}-\left[1+2\left(\rho_{1}-\rho_{2}\right)\right]-2(\beta s+k)\right]
$$

As we know that $\rho_{1} \geq \rho_{2}$, it is easy to verify that the derivative is negative for all $\hat{v}_{2} \in[0,1] \Leftrightarrow$ $\sigma_{2} \in[0,1 / 2-\beta s] .{ }^{22}$ As we now know that $\hat{v}_{2}-\rho_{2}=0$, the consumer will only continue to inspect product 2 if she observed a utility $v_{1} \leq \rho_{1}$. But in that case, she will never go back to buy the first product. Thus we can split up the profit maximization by products:

$$
\pi_{\text {Aseq }}\left(\rho_{1}, \rho_{2}, \sigma_{1}, \sigma_{2}\right)=\sigma_{1}-k+\left[1-F\left(\rho_{1}\right)\right]\left(\rho_{1}-\eta\right)+F\left(\rho_{1}\right) \pi_{2}\left(\rho_{2}, \sigma_{2}\right)
$$

and

$$
\pi_{2}\left(\rho_{2}, \sigma_{2}\right)=\sigma_{2}-k+\left[1-F\left(\rho_{2}\right)\right]\left(\rho_{2}-\eta\right) .
$$

Maximizing $\pi_{2}$ with regard to ( $\sigma_{2}, \rho_{2}$ ), plugging the result into the above expression of $\pi_{\text {Aseq }}$ and maximizing that with regard to ( $\sigma_{1}, \rho_{1}$ ) yields the solution as presented in Proposition 1.

## A. 2 Proof of Lemma 1

The profits under the two search modes are

$$
\pi_{\text {Asim }}^{*}=\mathbb{E}\left[\max \left(v_{1}-\eta, v_{2}-\eta, 0\right)\right]-2(\beta s+k),
$$

and

$$
\bar{\pi}_{\text {Aseq }}^{*}=\mathbb{E}\left[\max \left(v_{1}-\eta, \mathbb{E}\left[\max \left(v_{2}-\eta, 0\right)\right]-\beta s-k\right)\right]-\beta s-k
$$

respectively, where in the second equation it is important to note that the second product is only inspected if inspection of the first product results in a low value. Thus, we have that $\pi_{\text {Asim }}^{*} \geq \bar{\pi}_{\text {Aseq }}^{*}$, if and only if,

$$
\mathbb{E}\left[\max \left(v_{1}-\eta, v_{2}-\eta, 0\right)\right] \geq \mathbb{E}\left[\max \left(v_{1}-\eta+\beta s+k, \mathbb{E}\left[\max \left(v_{2}-\eta, 0\right)\right]\right)\right]
$$

It is immediately evident that for $\beta s+k=0$ and for any value of $\eta, A^{s i m}$ leads to strictly higher profits. Thus, by continuity of the RHS in $\beta s+k$, it follows that there exists a threshold $\underline{S}_{A}(\eta)$ such that $A^{s i m}$ yields larger profit if $\beta s+k \leq \underline{S}_{A}(\eta)$. On the other hand, as the RHS of the above inequality is weakly increasing in $\beta s+k$, and strictly increasing in $\beta s+k$ if $\beta s+k$ is large enough, it also follows that $A^{\text {seq }}$ yields larger profit if $\beta s+k>\underline{S}_{A}(\eta)$.

[^13]
## A. 3 Proof of Lemma 2

Take for example $S_{B}(\beta, s, k, \eta)=s$. Then the sum of inspection and degradation costs $\beta s+k$ and the firm's cost of selling a product $\eta$ are both smaller for $A^{\text {seq }}$ than the equivalent values for $B^{\text {seq }}, s$ and $c$. That means whatever value $p^{*}$ would maximize profits under $B^{\text {seq }}$, the firm can recreate the same offer with $A^{\text {seq }}$ by setting $\sigma_{i}=(1-\beta) s$ and $\rho_{i}=p^{*}$. From the consumer's perspective, both are identical. The firm, however, makes more profits under $A^{\text {seq }}$, as it has a lower selling cost $\eta \leq c$ and as $k \leq(1-\beta) s$ given the above function, the firm also makes a profit or at least no loss from inspections. The consumer will also not deviate to $B^{\text {seq }}$ as $p_{i}=\sigma_{i}+\rho_{i} \geq \rho_{i}=p^{*}$.

## A. 4 Proof of Proposition 3

Note that we use that for small $s$ the contracts under $A^{\text {seq }}$ will be symmetric, i.e. $\sigma_{i}=\sigma$ and $\rho_{i}=\rho$ for both $i$. For the comparison we first consider the case $s=0$. Then $\bar{\sigma}=0$ and thus $\hat{v}_{a 2}=\hat{v}(\beta s+\sigma)=\hat{v}(0)=\bar{v}$, the maximum of the value distribution. This implies $F\left(\hat{v}_{a 2}\right)=1$. We also know that for the profit maximizing $\rho$ it must hold that $\rho \geq \eta$. In particular, for $s=0, \rho$ will be at the value that maximizes profits for no search costs, a value that the firm will never surpass, as it would always lower profits. Then we immediately see that for $s=0$ there are strictly less returns under $A^{\text {sim }}$. If we increase $s$, then $\sigma$ will be increased as $\bar{\sigma}$ becomes more and more positive. This implies that $\hat{v}_{a 2}$ can only fall. At the same time, $\rho$ will also only be lowered. Together, this implies that the number of returns under $A^{\text {seq }}$ will steadily decrease in $s$. Then there must exist a threshold $\bar{s}$, below which $A^{s i m}$ and above which $A^{\text {seq }}$ will lead to a lower number of returns. Note once again that this conclusion is for symmetric contracts under $A^{\text {seq }}$. We know that for large $s$, the profit maximizing contracts will be asymmetric. As we have not fully characterized the optimal strategy under $A^{\text {seq }}$, we restrict this argument to say that $A^{\text {sim }}$ leads to less returns below $\bar{s}$. Therefore the proposition presents only a sufficient condition for when $A^{s i m}$ leads to less returns.

Note that should the $\bar{s}$ derived in the above way lie in the region where the firm sets asymmetric contracts, then we can instead choose the highest $s$ for which optimal contracts are symmetric as the threshold.


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[^1]:    ${ }^{1}$ See, https://www.digitalcommerce360.com/2023/12/27/online-returns-2023-nrf-appriss-retail-report/.
    ${ }^{2}$ Amazon now labels this 'Prime Try Before You Buy', which previously was called 'Amazon Prime Wardrobe'. See, https://www.amazon.com/gp/help/customer/display.html?nodeId=GCQDLMG7C2YEXSM4 for more details.
    ${ }^{3}$ These environmental costs include greenhouse gas emissions, non-recycled packaging and products filling up landfills (see, e.g. Tian and Sarkis (2022)), where some websites estimate that only 54 percent of all packaging gets recycled and 5 billion pounds of returned goods end up in landfills each year.

[^2]:    ${ }^{4}$ That is if the searcher is patient enough or there is no delay due to sequential search. As we do not want our result that a firm induces consumers to search simultaneously to depend on an exogenously imposed delay because of sequential search, we assume that there is no delay.

[^3]:    ${ }^{5}$ Another difference with Janssen and Williams (2023) is that we study a setting where consumers can learn the prices and refunds the firm sets without any cost. This is a feature the paper has in common with the recent literature on price directed search; see, e.g., Armstrong (2017), Choi, Dai, and Kim (2018).
    ${ }^{6}$ Doval (2018) allows consumers to buy blindly, that is without inspecting the product at all. Buying and inspecting after purchase may be considered a generalization of blind buying in the sense that if the refund that the firm gives is zero, the consumer will never inspect the product afterwards and will then also not return the product.

[^4]:    ${ }^{7}$ Note that to prevent arbitrage the firm would never set a refund larger than price.
    ${ }^{8}$ As the firm will not benefit from setting different prices under simultaneous search, we do not use subscripts for the price and refund of the different products.
    ${ }^{9}$ We do not provide specific assumption on the distribution at this point. Some results -where noted- have so far been derived only for the uniform distribution.

[^5]:    ${ }^{10}$ In general, we define $\hat{v}(\tilde{s})$ implicitly through $\int_{\hat{v}}^{\infty}(v-\hat{v}) f(v) d v=\tilde{s}$. Then $\hat{v}_{b}=\hat{v}(s)$ and $\hat{v}_{a i}=\hat{v}\left(\beta s+p_{i}-\tau_{i}\right)$.
    ${ }^{11}$ Note that we have left two possible consumer strategies out of the above list. First, it turns out that it is never optimal for the firm to set prices such that the consumer would choose to buy a product without inspecting it at all (as in Doval (2018)). Second, simultaneous inspection before purchase is also never chosen. In the case of inspection before purchase, at a given price the firm receives the same payoff irrespective of whether the consumer inspects sequentially or simultaneously, while simultaneous search is never optimal for the consumer. Note that, in contrast, the firm's payoffs for simultaneous and sequential search after purchase do differ as firms can make a profit or a loss over their returns.
    ${ }^{12}$ Note that it does not matter if the price $p_{i}$ is paid before inspecting the product or after deciding which products to keep and which to return.

[^6]:    ${ }^{13} \mathrm{~A}$ part of the proof is so far only shown for the uniform distribution, as noted in the appendix.
    ${ }^{14}$ From (1) it follows that $\partial \hat{v}_{a i} / \partial \sigma_{i}=-1 /\left[1-F\left(\hat{v}_{a i}\right)\right] \leq-1$.

[^7]:    ${ }^{15} \sigma_{1}^{*}=\mathbb{E}\left[\max \left(v-E S_{I}-\eta\right), 0\right]-\beta s$, which is certainly larger than $k$.

[^8]:    ${ }^{16}$ Note that any other contract with asymmetric prices $\sigma_{\text {sim }}^{i}$ satisfying $\sigma_{\text {sim }}^{1}+\sigma_{\text {sim }}^{2}=2 \sigma_{s i m}^{*}$ would have resulted qualitatively in the same outcome.

[^9]:    ${ }^{17}$ This, in a sense, rotates the demand curve and makes it more flat. See, Johnson and Myatt (2006).
    ${ }^{18}$ Note that this result holds for the actual profit of $A^{\text {seq }}$ and not only for the upper bound.

[^10]:    ${ }^{19}$ Results in this section are still work in progress.

[^11]:    ${ }^{20}$ Note that Proposition 3 is a sufficient condition - it does not imply that for all $s \geq \bar{s} A^{\text {seq }}$ leads to less returns. However, it can be shown that for large $s$ where the firm sets asymmetric contracts and for either very small or very large $\eta, A^{\text {seq }}$ does lead to less returns.

[^12]:    ${ }^{21}$ In Petrikaite $(2018 \mathrm{~b})$, the derivative could be zero if $\hat{v}_{2}=\bar{v}$, which is equivalent to $\sigma_{2}=0$, but even there that is not profitable. In our setting, the firm additionally benefits from the inspection fee, making that choice even worse. Note that in our case it would also be necessary that $\beta s=k=0$ for the derivative to be zero.

[^13]:    ${ }^{22}$ Note that for the uniform distribution, $1 / 2-\beta s$ is the upper bound for $\sigma$ above which inspection after purchase would not be socially optimal.

