# A class of misspecified profit functions for the firm

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**Abstract**

We study the behavior of a firm that consistently maximizes a misspecified profit function. We provide an equilibrium concept where the misspecification error remains undetected. We examine the uniqueness and stability of the equilibria. The model of the price-taking firm belongs to this class. In one of these models, the cost-taking firm, the equilibrium price increases with fixed costs. This price can be lower or higher than the rational price, meaning consumers can benefit from the lack of rationality. Finally in a long-run perspective where the cost function is endogenous, we show that both the cost-taking and rational firms end with the same level of output.

**Keywords:** Monopoly ; Misspecified profit function ; Fixed costs

**JEL codes:** L12, L21, L23, L25, M41.

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Economic theory formulates thoughts via what we call "models." The word model sounds more scientific than the word fable or tale, but I think we are talking about the same thing. Ariel Rubinstein, *Economic Fables*, 2012.

# 1 Introduction

According to the Merriam-Webster dictionary, a model is "a system of postulates, data, and inferences presented as a mathematical description of an entity or state of affairs". At the heart of the theory of the firm, lies the profit function, which the firm aims to maximize. Although profit is easily understood as revenue minus costs, for which numerous indicators are provided by accountants, even the brightest MBA student would struggle to write down her firm's profit function. Despite these practical difficulties, many economists share the optimistic belief that if a firm gets her profit function wrong, the day of reckoning would soon come, and the firm would either exit the market or adjusts its profit function in the right direction.

At least since the seminal work of Esponda and Pouzo (2016) on the Berk-Nash equilibrium concept, we know, however, that such a reckoning might never come. In this paper, we propose a simple (i.e. with no noise) class of models of the firm in which the profit function is misspecified. In equilibrium the firm's wrong belief about its profit function is reinforced (i.e. goes undetected). In our framework, the reality check is the accounting profit. The equilibrium concept is based on the notion of rational expectations. The firm makes her quantity decision based on expectations regarding the value of a key variable (e.g. unit costs), and these expectations are correct in equilibrium. This simple concept can be seen as a full-information version of the Berk-Nash equilibrium.[1]

To construct our class of models, we note that the standard profit function embodies two fundamental economic constraints. The first is a market constraint, which links price and quantity through the demand function. The second constraint is technological in nature, linking unit cost and quantity. When a firm produces more (or less), it can expect that both the selling price and the unit cost will be affected. A rational firm does but the

[1] Martimort and Stole (2020) followed a similar methodology to study average-price bias by consumers in a nonlinear pricing context. Most applications of misspecified models study a behavioral bias of the consumers.

firm in our models does not fully understand these constraints. Consequently, we develop a variety of models to explore the implications of this misspecification assumption, and we analyze in detail two of them.

A first model in our class is the well known price-taking firm. The firm, in that case, wrongly believes that its selling price is fixed (the market constraint is therefore not understood), and maximizes its profit accordingly. This mistake remains unnoticed, in equilibrium, as the firm rationally anticipates the right price equalizing supply and demand.[2] In terms of profits, the misperception that price is fixed is not inconsequential as the firm cannot achieve the optimal profit.

The counterpart of the price-taking firm is the cost-taking firm. In that case, the firm understands the market constraint but wrongly believes that the unit cost of production is constant. Again, in equilibrium, the mistake remains unnoticed as the firm rationally anticipates the right average-cost. The implied misspecification equilibrium is new to the literature and presents the intuitive (outside the economists' realm) property that price varies with fixed costs. When these costs increase, a price raise follows.

From the perspective of consumers and welfare, the models have different predictions. First, assume that the fixed costs are low enough so the firm produces in equilibrium. Consumers can benefit from a mistaken profit function. In particular, the price-taking-firm's price is always lower than the rational price. For relatively low values of the fixed costs, cost-taking behavior also leads to a lower price.

Next, it is worth to emphasize that as profits are lower than the rational profits, the firm finds it unprofitable to produce when fixed costs become large. The cost-taking firm is the first to drop out, followed by the price-taking firm.

We also show that in the long run, when the firm can duplicate its technology, the investments of both the rational and the cost-taking firm are similar. Moreover they are such that the quantities and profits are the same in both models. Meaning that in the long run, the cost-taking firm would behave optimally. An intriguing result that underlines that even if a firm holds the entrenched belief that fixed-costs should impact the selling price, there could be no distortion in terms of profit maximization.

After a literature review which concludes the introduction, the article is organized

[2] In the model of competitive firms, one of the pillar of microeconomics, the rationality of the price taking behavior is sidestepped by assuming firms are atomistic.

as follows. Section 2 presents the general logic of our models for the misspecified-profit-maximizing firm. Then in section 3 we introduce our simple class of misspecified models and characterize the misspecification equilibrium of two models. In section **??**, we solve completely a parametric example. Section 6 presents a couple of additional models that fit in our general framework and provide a complementary view to our simple class. Section 5 study the long run investment strategy of a cost-taking firm and compare it to the choice of the rational firm. Finally, section 7 concludes.

**Literature review.** In several of our misspecification equilibria, fixed costs have an influence on the price. Thus, our work is related to the "full-cost" pricing literature. Despite Economics 101, there has been a controversy for at least 80 years between economists on whether prices depend or not on fixed cost.[3] Staged as an anti-marginalists vs marginalists debate, it started with Hall and Hitch (1939) who interviewed 38 U.K. entrepreneurs without finding evidence they equalized marginal revenue to marginal costs. Hall and Hitch concluded that economic theory should be re-thought in light of their findings. Lester (1946) shares their conclusion: "The conventional explanation of the output and employment policies of individual firms runs in terms of maximizing profits by equating marginal revenue and marginal cost. Student protests that their entrepreneurial parents claim not to operate on the marginal principle have apparently failed to shake the confidence of the textbook writers in the validity of the marginal analysis."

The conversation at cross purposes continued with Lester (1947) and two "rejoinders" Machlup (1947) and Stigler (1946). Later, Machlup (1967) recounted the battle.[4] The influential Friedman (1953) put an end to the debate at least from the point of view of the marginalists. Already present in Friedman and Savage (1948) the famous analogy of the billiard player[5] seems to have been the decisive blow. Mongin (1992) is an excellent

[3]See the survey Ellison (2006). As well as Nubbemeyer (2010), a Ph.D. on full-cost pricing. The author is fair but sympathetic to the anti-marginalist point of view.

[4]See also, in the *Journal of Post Keynesian Economics*, a journal (according to one of his Editor) "devoted primarily to criticizing destructively the analytical foundations of neoclassical theory" Davidson (1990). First, Langlois (1989), then a symposium at the end of 1990 on "The marginalist controversy and Post Keynesian price theory". *Journal of Post Keynesian Economics*, volume 13, number 2.

[5]The mathematical formulae which explain the best hits are extremely complicated. Yet, the assumption that an expert billiard player makes his shots as if he knew the formulas, should give good predictions of what is observed. In 1947 Machlup used a similar analogy with a driver on a highway who ponders to overtake a truck or not

discussion of this controversy.

Recently, Altomonte, Barattieri, and Basu (2015) use a survey of 14,000 European firms. They asked these firms whether "their prices are fixed by the market, set as a margin over a measure of total (including fixed) cost, or fixed as a margin over a measure of variable cost." Among the 60% of firms which are not price takers, 75% of them set their prices according to full cost pricing. Next, focusing on U.S. data, they show that the correlation between changes in output prices and changes in variable inputs prices is significantly lower when fixed costs are likely to be more important.

Our results are connected to the literature building on or extending Esponda and Pouzo (2016). The focus of this literature is on beliefs and their convergence through repeated Bayesian learning. In Appendix F, we discuss briefly how to introduce noise in our model and apply the insight of Esponda and Pouzo. There is a growing theoretical literature building on and refining the Berk-Nash equilibrium concept. The focus is on the formation/convergence of beliefs or how to reach an equilibrium whereas our focus is on the properties of the equilibrium. Heidhues, Kőszegi, and Strack (2018) focus, as we do, on a single player framework and combine the idea of overconfidence with learning in a misspecified model. Heidhues, Kőszegi, and Strack (2021) derive further convergence results in a more general framework for the objective and subjective production functions (which play the role of our profit functions) while restricting on Gaussian shocks.

Whereas the idea of full-cost pricing lost traction among economists[6] it always remained present in the accounting literature[7] which still refers to surveys where firms declare using the full cost of a product when setting their list prices. E.g. Govindarijian and Anthony (1983), Drury, Braund, Osborne, and Tayles (1993) and Shim and Sudit (1995). Balakrishnan and Sivaramakrishnan (2002) and Göx and Schiller (2006) present this literature. Finally, Bouwens and Steens (2016) is an empirical study of a single firm.

[6]The idea that in some circumstances sunk costs might influence a rational decision maker has, however, been illustrated in several papers: Friedman, Pommerenke, Lukose, Milam, and Huberman (2007) survey the literature on the sunk cost fallacy. More recently, McAfee, Mialon, and Mialon (2010) list several explanations compatible with the use of sunk cost and rational behavior and Baliga and Ely (2011) model a simple two-period investment game with a single decision maker where 'sunk cost verity' holds.

[7]See the literature review in Al-Najjar, Baliga, and Besanko (2008).

# 2 Models with a misspecified profit function

A cornerstone principle in economics is that choices are made in order to maximize an objective. Even a mistaken firm has a profit function which it maximizes. Yet, instead of maximizing the real profit function, which we call the objective profit, the firm might maximize a misspecified profit function, which we call the subjective profit function. Nevertheless, even a mistaken firm cashes in profits and only objective profits can be cashed in –the reality check.

The challenge arises when a firm continues to maximize its subjective profit function instead of the objective profit function. To do so, the objective profits earned must be equal to the subjective profits the firm had expected to earn. This consistency requirement is crucial since it ensures that the firm has no reason to suspect that its profit function has been misspecified. Such a configuration is called a misspecification equilibrium.

## 2.1 The objective profit function

To keep the matter simple, we consider a firm which produces and sells a single product, and we take the quantity as its strategic variable. The objective profit writes (for $q > 0$)

$$\Pi^{\mathcal{O}}(q) = P(q)q - C(q) - \phi$$

where $P(q)$ the inverse demand, $\phi \geq 0$ is the fixed cost of production,[8] $C(q)$ is the variable cost with $C(0) = 0$. Irrespectively of its beliefs, when a firm produces $q$ the accounting profits are $\Pi^{\mathcal{O}}(q)$. Let $AC(q\,;\phi) = (C(q) + \phi)/q$ denote the average cost. We make the usual technical assumptions: i) increasing and convex cost $C' \geq 0$, and $C'' \geq 0$, ii) convex and U-shaped average cost, iii) decreasing marginal revenue $MR(q) = P'(q)q + P(q)$.

**Parametric example:** Throughout, we use the following parametric example lying on a linear demand $P(q) = a - bq$, where both $a$ and $b$ are positive, and a quadratic cost function $C(q) = c_1 q + c_2 q^2/2$, where both $c_1$ and $c_2$ are positive. Moreover for the firm

[8]The fixed cost $\phi$ is often called manufacturing overhead by managers. It includes capital depreciation, repairs, insurance, wages of workers not directly involved in production, etc…

to be profitable it is necessary to assume that $a > c_1$.

We denote the profit maximizing quantity $q^m$. It is solution to $MR(q) = C'(q)$. Let $\phi^m = P(q^m)q^m - C(q^m)$. The rational profit is $\phi^m - \phi$, and the rational quantity, $q^m$, is independent of $\phi$ as long as $\phi \leq \phi^m$. If $\phi$ is larger than $\phi^m$, the market is unprofitable even for a rational firm. Let $\underline{q}(\phi)$ be the lowest root and $\overline{q}(\phi)$ the highest root of $P(q) = AC(q\,;\phi)$. These roots exist as long as $\phi \leq \phi^m$. The profit $\Pi^{\mathcal{O}}(q)$ is positive for $q$ between $\underline{q}$ and $\overline{q}$.

**Parametric example (continued):** The rational quantity and profits are

$$q^m = \frac{a - c_1}{2b + c_2} \text{ and } \Pi^m = \frac{(a - c_1)^2}{2(2b + c_2)} - \phi = \phi^m - \phi\,.$$

For a quantity $q$, the profit $\Pi^{\mathcal{O}}(q)$ is positive only if $q \in \left[\underline{q}, \overline{q}\right]$ where the quantities $\underline{q}$ and $\overline{q}$ are such that $\Pi^{\mathcal{O}}(q) = 0$:

$$\underline{q} = \left(1 - \sqrt{1 - \phi/\phi^m}\right) q^m \text{ and } \overline{q} = \left(1 + \sqrt{1 - \phi/\phi^m}\right) q^m\,.$$

## 2.2 Misspecification equilibrium

It is convenient to assume that the subjective profit function of the firm is part of a family and writes $\Pi^{\mathcal{S}}(q\,;\theta)$ where $\theta$ is a parameter (possibly a list of parameters but as having more parameters makes it easier to generate a misspecification equilibrium we prefer to be conservative). Notice that the objective profit function $\Pi^{\mathcal{O}}(q)$ is not necessarily part of this family.

**Definition 1.** *For a family of misspecified profit functions $\Pi^{\mathcal{S}}(q;\theta)$, the quantity choice $q^{\mathcal{S}}$ and the parameter $\theta^{\mathcal{S}}$ form a misspecification equilibrium if and only if:*

$$(i)\ q^{\mathcal{S}} \in \arg\max_q \Pi^{\mathcal{S}}\left(q\,;\theta^{\mathcal{S}}\right)$$

*and*

$$(ii)\ \Pi^{\mathcal{O}}\left(q^{\mathcal{S}}\right) = \Pi^{\mathcal{S}}\left(q^{\mathcal{S}}\,;\theta^{\mathcal{S}}\right).$$

Condition (i) insures optimality and condition (ii) consistency. This simple (two-part) idea (maximization on the one hand, reality check on the other) defines our equilibrium concept.[9]

# 3 A simple class of misspecified models

The objective profit function which writes

$$\Pi^{\mathcal{O}}(q) = (P(q) - AC(q\,; \phi))\, q \tag{1}$$

reflects two economic constraints. First, a market constraint: price and quantity are linked by the demand function as consumers cannot be forced to buy and the firm cannot profit from rationing them in this set-up. Second, a technological constraint: the unit cost varies with $q$. The firm should anticipate that producing more (or less) would modify the unitary cost. Writing (1) as

$$(P(q_1) - AC(q_2\,; \phi))\, q_3\,,$$

the market constraint is $q_1 = q_3$ and the technological constraint is $q_2 = q_3$. Our approach is to assume that the firm does not fully understand these forces.

Assuming these constraints are active or not is a simple way to introduce the parameter $\theta$ and this allows us to define the following two models.

## 3.1 Price-taking firm

The simplest illustration of a misspecified profit function is given by the classical price-taking firm. Indeed, assume:

$$\Pi^{\mathcal{S}}\,(q\,; \theta) = (P(\theta) - AC(q\,; \phi))\, q\ , \tag{2}$$

and the firm produces only if it expects a non negative profit.

In this well known model, the firm wrongly believes that the inverse demand function

[9]Notice that whenever it exists $\theta$ such that it is optimal not to produce, i.e. $q^{\mathcal{S}} = 0$, then a misspecification equilibrium exists as $\Pi^{\mathcal{O}}\,(0) = 0 = \Pi^{\mathcal{S}}\,(0\,; \theta)$. That is, if the firm anticipates the worst, it is optimal not to produce and to expect no profit. Our goal is to study misspecification equilibria where the firm does produce.

$P(.)$ is constant.[10] The market constraint is not understood, i.e. $q_1 \neq q_3$, while the technological constraint is active, i.e. $q_2 = q_3$. To derive the misspecification equilibrium, let $q^c$ be the unique quantity such that $P(q^c) = C'(q^c)$. Moreover let $\phi^c = P(q^c)q^c - C(q^c)$.

**Lemma 1.** *In the model of the price-taking firm, when $\phi \leq \phi^c$, there exists a unique misspecification equilibrium: the quantity is $q^{\mathcal{S}} = q^c > q^m$ and the parameter $\theta^{\mathcal{S}} = q^c$.*

*Proof.* We can check that the quantity $q^{\mathcal{S}} = q^c$ and the parameter $\theta^{\mathcal{S}} = q^c$ form a misspecification equilibrium for this model. By construction condition (i) of definition 1 is satisfied. Condition (ii) also holds as $\Pi^{\mathcal{S}}\left(q^{\mathcal{S}}\,;\theta^{\mathcal{S}}\right) = \left(P(q^c) - AC(q^c\,;\phi)\right) q^c$ which is exactly $\Pi^{\mathcal{O}}(q^c)$. That $q^c > q^m$ follows from $MR(q) < P(q)$ for the relevant values of $q$. □

The profit made by the price-taking firm is $\max\{0\,;\,\phi^c - \phi\}$. It is always strictly lower than the rational profit $\max\{0\,;\,\phi^m - \phi\}$ as long as the market is profitable $\phi < \phi^m$. Notice that the quantity $q^c$ and the profit difference $\phi^m - \phi^c$ (which is the profit loss due to the misspecification) are both independent of $\phi$.

**Parametric example (continued):** The price-taking quantity and profit are

$$q^c = \frac{a - c_1}{b + c_2} \quad \text{and} \quad \Pi^c = \frac{c_2(a - c_1)^2}{2(b + c_2)^2} - \phi = \phi^c - \phi\,.$$

## 3.2 Cost-taking firm

When the technology constraint is not understood, i.e. $q_2 \neq q_3$, while the market constraint is understood, i.e. $q_1 = q_3$, the firm wrongly believes that its cost is constant by which we mean its unit-cost (or average cost) is constant. Formally,

$$\Pi^{\mathcal{S}}\left(q\,;\theta\right) = \left(P(q) - AC(\theta;\phi)\right) q\ , \tag{3}$$

and the firm produces only if it expects a non negative profit.

[10] Other misspecification of the demand function have been studied. McLennan (1984) studies. Nyarko (1991)

To characterize a misspecification equilibrium of this model, let $q^{\mathcal{S}}$ denote a quantity solution of

$$MR(q^{\mathcal{S}}) = AC(q^{\mathcal{S}}\,;\phi)\,. \tag{4}$$

Under the assumption that the marginal revenue is less convex than the average cost on the interval where the average cost function is decreasing, (4) has at most two roots. If $\phi$ is too large, the average cost curve is everywhere above the marginal revenue curve and (4) has no solution. In Appendix A we characterize $\phi^{\mathcal{AC}}$ such that (4) has two solutions whenever $\phi < \phi^{\mathcal{AC}}$. We denote the largest root of (4) $q_H^{\mathcal{AC}}$ and the lowest root $q_L^{\mathcal{AC}}$.

**Lemma 2.** *In the model of the cost-taking firm, there is at most two misspecification equilibrium: $q^{\mathcal{S}} = \theta^{\mathcal{S}} = q_L^{\mathcal{AC}}$ and $q^{\mathcal{S}} = \theta^{\mathcal{S}} = q_H^{\mathcal{AC}}$. The equilibrium at $q_H^{\mathcal{AC}}$ is better for the consumers and the firm.*

*Proof.* Notice the optimality condition is $MR(q^{\mathcal{S}}) = AC(\theta^{\mathcal{S}};\phi)$ and the consistency condition is $C(q^{\mathcal{S}}) + \phi = AC(\theta^{\mathcal{S}};\phi)q^{\mathcal{S}}$. Combining these two conditions leads to (4). In Appendix B we show that the profits are larger at $q_H^{\mathcal{AC}}$ than $q_L^{\mathcal{AC}}$. □

As long as $\phi \leq \phi^{\mathcal{AC}}$, the profit of the cost-taking firm is positive. Both the quantity and the profits are positive for $\phi = \phi^{\mathcal{AC}}$. But the firm collapses (no production and no profit) for $\phi = \phi^{\mathcal{AC}} + \varepsilon$ with $\varepsilon > 0$. In Appendix D, it is further shown that $q_H^{\mathcal{AC}}$ (resp. $q_L^{\mathcal{AC}}$) is a decreasing (resp. increasing) and concave (resp. convex) function of $\phi$.

**Parametric example (continued):** The equation (4) has two solutions when $\phi$ is low enough. More precisely, if $\phi < \phi^{\mathcal{AC}} = \frac{(a-c_1)^2}{2(4b+c_2)}$ then

$$q_H^{\mathcal{AC}} = \frac{a-c_1}{4b+c_2} + \sqrt{2\,\frac{\phi^{\mathcal{AC}}-\phi}{4b+c_2}} \quad \text{and} \quad q_L^{\mathcal{AC}} = \frac{a-c_1}{4b+c_2} - \sqrt{2\,\frac{\phi^{\mathcal{AC}}-\phi}{4b+c_2}}$$

with $q_L^{\mathcal{AC}} < q_H^{\mathcal{AC}}$. Notice that when $\phi = 0$, then $q_L^{\mathcal{AC}} = 0$ and $q_H^{\mathcal{AC}} = \frac{a-c_1}{2b+c_2/2}$ which is larger than $q^m$ whenever $c_2 > 0$. If $c_2 > 2b$, then $q_H^{\mathcal{AC}}$ is also larger than $q^c$ for $\phi = 0$. Substituting in (1) $q$ for these values allows to compute the profits $\Pi_L^{\mathcal{AC}}$ and $\Pi_H^{\mathcal{AC}}$.

$$\Pi_H^{\mathcal{AC}} = \frac{2b\phi^{\mathcal{AC}}}{4b+c_2}\left(1+\sqrt{\frac{\phi^{\mathcal{AC}}-\phi}{\phi^{\mathcal{AC}}}}\right)^2 \quad \text{and} \quad \Pi_L^{\mathcal{AC}} = \frac{2b\phi^{\mathcal{AC}}}{4b+c_2}\left(1-\sqrt{\frac{\phi^{\mathcal{AC}}-\phi}{\phi^{\mathcal{AC}}}}\right)^2$$

### 3.3 *Tâtonnement*

In the spirit of a Walrasian *tâtonnement*, we look for a dynamic sequence of quantities converging to the equilibrium. Quite generally, a given fixed point $x^* = f(x^*)$ is locally stable, if starting from a neighborhood of $x^*$ any sequence $x_{t+1} = f(x_t)$ converge to $x^*$. This property holds if and only if $|f'(x^*)| < 1$. For the price-taking, the cost-taking, as well as the rational firm such a dynamic sequence can be defined and studied.

Starting with a $\theta_0 \in \left[\underline{q}, \overline{q}\right]$, we have the following three *tâtonnement*:

- The price-taking firm starts with a price $p_0 = P(\theta_0)$ which leads to a quantity choice $q_0$ equalizing marginal cost with $p_0$. This quantity can only be sold at price $p_1 = P(q_0)$. Then the firm takes $p_1$ as the new price and maximizes again by choosing a quantity $q_1$. The process repeats itself until the fixed point is reached.

- The cost-taking firm starts with a cost $c_0 = AC(\theta_0\,; \phi)$, which leads to a quantity choice $q_0$ equalizing marginal revenue with $c_0$. This quantity can only be produced at an average cost $c_1 = AC(q_0\,; \phi)$. Then the firm takes $c_1$ as the new cost and maximizes again by choosing a quantity $q_1$. The process repeats itself until the fixed point is reached.

- The rational firm starts a cost $c_0 = C'(\theta_0)$, which leads to a quantity choice $q_0$ equalizing marginal revenue with $c_0$. This quantity can only be produced at a marginal cost $c_1 = C'(q_0)$. Then the firm takes $c_1$ as the new cost and maximizes again by choosing a quantity $q_1$. The process repeats itself until the fixed point is reached.

**Proposition 1.** *The price-taking equilibrium quantity $q^c$ is locally stable if $|P'(q^c)| < C''(q^c)$. The rational quantity $q^m$ is locally stable for the equation $q^m = (MR)^{-1}\left(C'(q^m)\right)$ if $C''(q^m) < |MR'(q^m)|$. The quantity $q_L^{AC}$ is a non-stable fixed point. For the largest cost-taking quantity:*

- *If $q_H^{AC}$ is lower than $q^m$, then it is locally stable.*

- *If $q_H^{AC}$ is close enough to $q^m$, then it is locally stable.*

- *If $q_H^{AC}$ is larger than $q^m$, then it is locally stable if and only if $AC'(q_H^{AC}\,; \phi) < \left|MR'(q_H^{AC})\right|$. In particular, if $q_H^{AC} \leq q^c$, and $qP''(q)/P'(q) > -1$, then it is locally stable.*

*Proof.* See Appendix C. □

**Parametric example (continued):** The price-taking equilibrium quantity $q^c$ is locally stable if $b < c_2$. The rational quantity $q^m$ is locally stable for the equation $q^m = (MR)^{-1}(C'(q^m))$ if $c_2 < 2b$, a condition which is also sufficient for $q_H^{\mathcal{AC}}$ to be locally stable even when it is larger than $q^m$.

# 4 Comparative statics

We have so far defined four equilibrium quantities. The rational profit-maximizing quantity, $q^m$, and three quantities compatible with a misspecified model of a maximizing-profit firm. The price-taking quantity, $q^c$, and the cost-taking quantities, $q_H^{\mathcal{AC}}$ and $q_L^{\mathcal{AC}}$. How do these quantities vary with changes in the environment?

## 4.1 Variation of the fixed costs

The question of whether an increase in the fixed costs of production $\phi$ leads to an increase or no change in the selling price is central to our study. For each one of our quantities the answer is unambiguous and Proposition 2 presents the results.

**Proposition 2.** *The cost-taking stable quantity, $q_H^{\mathcal{AC}}$, is the only one decreasing with $\phi$, and thus the corresponding price is the only one increasing. In particular, $\partial q_H^{\mathcal{AC}}/\partial\phi \to -\infty$ when $\phi \to \phi^{\mathcal{AC}}$. The quantities $q^m$ and $q^c$ are invariant with $\phi$ while $q_L^{\mathcal{AC}}$ is increasing with $\phi$.*

The proof is in Appendix B. As a consequence, among our models, the only one compatible with the "full-cost" pricing intuition –that the price should increase with fixed costs– is the cost-taking model, assuming that in this model the stable quantity, $q_H^{\mathcal{AC}}$, is the one which is selected.

It is also worth comparing $q_H^{\mathcal{AC}}$ with $q^m$ and $q^c$ when $\phi$ varies from 0 to $\phi^{\mathcal{AC}}$. Let define $\phi^* = q^m MR(q^m) - C(q^m)$, and let $q^*(\phi)$ be the arg min of the average cost, i.e. $C'(q^*) = AC(q^*;\phi)$. Notice that $\phi^* < \phi^{\mathcal{AC}} = \max_q qMR(q) - C(q)$, see Appendix A.

**Proposition 3.** *When $0 \leq \phi < \phi^*$, then $q_H^{\mathcal{AC}} > q^m > q^*$, when $\phi = \phi^*$, $q_H^{\mathcal{AC}} = q^m = q^* < q^c$, and the profits are the same, and when $\phi^* < \phi < \phi^{\mathcal{AC}}$, then $q_H^{\mathcal{AC}} < q^m < q^* \leq q^c$.*

*Proof.* When $\phi = \phi^*$,

$$AC(q^m; \phi^*) = \frac{C(q^m) + q^m MR(q^m) - C(q^m)}{q^m} = MR(q^m)$$

and therefore $q_H^{\mathcal{AC}} = q^m$.[11] Moreover, when $\phi$ is close enough to zero, the average cost is mainly $C(q)/q$ (the $\phi/q$ part becomes negligible). Now as $C(q)$ is a convex function, the average-variable-cost, $C(q)/q$ is lower than the marginal cost $C'(q)$ and therefore $q_H^{\mathcal{AC}} > q^m$. $\square$

The cost-taking stable quantity, $q_H^{\mathcal{AC}}$, is not systematically lower nor larger than $q^m$. It is lower when $\phi$ is relatively small and greater when $\phi$ is relatively large. Consumers prefer the cost-taking firm to the rational firm when $\phi < \phi^*$ but they prefer the rational firm when $\phi^* < \phi < \phi^{\mathcal{AC}}$. Finally, they (have no choice but to) prefer the rational firm when $\phi^{\mathcal{AC}} < \phi < \phi^m$.

It is intriguing that $q_H^{\mathcal{AC}}$ and $q^m$ can coincide –for $\phi = \phi^*$, but it seems this would happen only by chance. In section 5 we come back to this issue in a long run perspective.

As long as $\phi \geq \phi^*$, the price-taking quantity, $q^c$, is larger than the cost-taking stable quantity, $q_H^{\mathcal{AC}}$. For $\phi < \phi^*$ and close to 0, however, $q_H^{\mathcal{AC}}$ can be larger than $q^c$.

**Parametric example (continued):** Here

$$\phi^* = \frac{c_2(a - c_1)^2}{2(2b + c_2)^2}$$

and $q_H^{\mathcal{AC}}$ can be larger than $q^c$ –for $\phi$ small enough– only if $c_2 > 2b$, a condition which is not compatible with the stability of $q^m$ whereas for $\phi$ close to 0, $q_H^{\mathcal{AC}}$ is stable as long as $c_2 < 4b$.

Figure 1 depicts the different quantities as functions of $\phi$. The cost-taking quantity, $q_H^{\mathcal{AC}}$, is the dark blue decreasing curve. For $0 \leq \phi \leq \phi^*$, it is larger than the rational quantity $q^m$. For this example, when $\phi$ is close enough to zero, then $q_H^{\mathcal{AC}}$ is even larger than $q^c$. Meaning that from the welfare point of view the cost-taking inefficiently produces too much.

The market is profitable as long as $0 \leq \phi \leq \phi^m$ but only $q^m$ remains positive up to

[11] Recall that the lowest root $q_L^{\mathcal{AC}}$ is strictly lower than $q^m$.

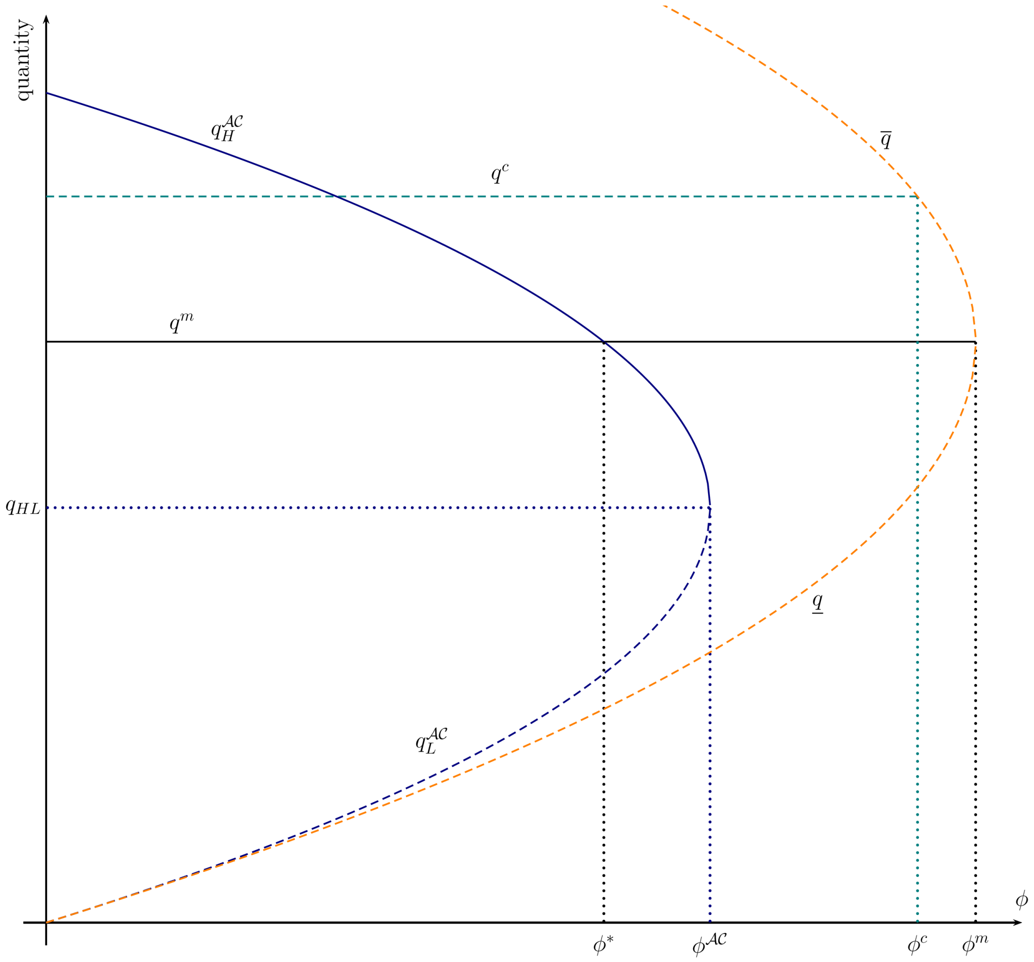


**Figure 1:** Quantities: Comparative static on $\phi$, for $a = 64$, $b = 1$, $c_1 = 0$, and $c_2 = 3$.

$\phi = \phi^m$. The other quantities collapse for some lower values of $\phi$. The cost-minimizing quantity collapses when $\phi = \phi^c$, and the cost-taking quantities collapse at $\phi = \phi^{\mathcal{AC}}$.

Notice that for the parameter values of Figure 1 and 2, we have $\phi^{\mathcal{AC}} < \phi^c$. This is not always the case, however. It is readily confirmed that if $c_2 < b/2$ then $\phi^c < \phi^{\mathcal{AC}}$ (in fact $\phi^c \to 0$ when $c_2 \to 0$). Whereas $\phi^{\mathcal{AC}} < \phi^c$ when $b/2 < c_2$.

Figure 2 complements Figure 1 by plotting the equilibrium profits as functions of $\phi$. The rational profit is linear in $\phi$ as well as the price-taking-firm profit. The figure underlines that when $\phi$ is in a neighborhood of $\phi^*$ the two profits $\pi^m$ and $\pi_H^{\mathcal{AC}}$ are very close.

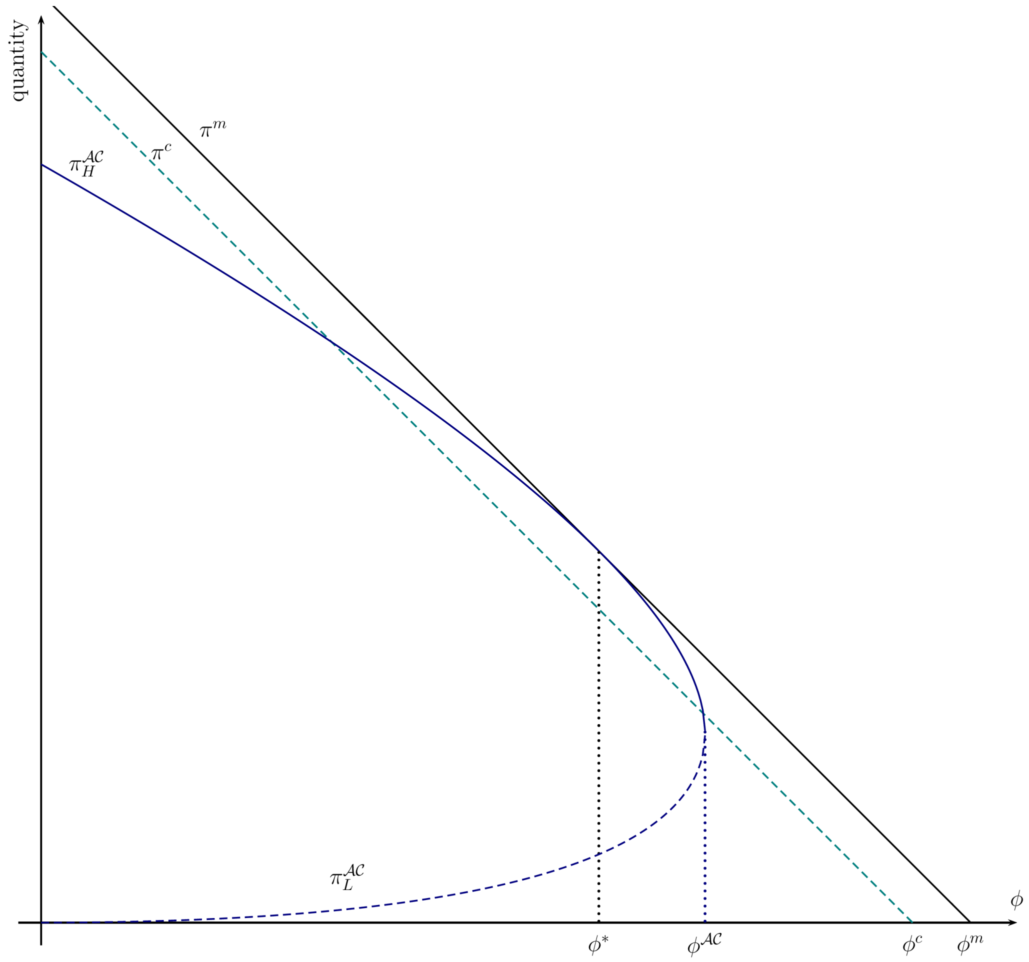


**Figure 2:** Profits: Comparative static on $\phi$, for $a = 64$, $b = 1$, $c_1 = 0$, and $c_2 = 3$.

## 4.2 Variation of the marginal cost

To study the effects of a change in the marginal cost, assume the variable cost takes the form $C(q;\gamma) = C(q) + \gamma q$. Notice that a variation of $\gamma$ is exactly equivalent to a variation of $\alpha$ when the inverse demand takes the form $P(q;\alpha) = P(q) - \alpha$.

For each of the three quantities –$q^c$, $q^m$, and $q_H^{AC}$– the pass-through is given by

$$\frac{\partial p^*}{\partial \gamma} = \frac{\partial P(q^*)}{\partial \gamma} = P'(q^*)\frac{\partial q^*}{\partial \gamma}$$

where $q^* = q^c$, $q^m$, or $q_H^{AC}$.

| Now, from, | it follows that |
|---|---|
| • $P(q^c) = C'(q^c) + \gamma$ | • $\frac{\partial q^c}{\partial \gamma}\left[P'(q^c) - C''(q^c)\right] = 1$ |
| • $MR(q^m) = C'(q^m) + \gamma$ | • $\frac{\partial q^m}{\partial \gamma}\left[MR'(q^m) - C''(q^m)\right] = 1$ |
| • $MR(q_H^{AC}) = AC(q_H^{AC}) + \gamma$ | • $\frac{\partial q_H^{AC}}{\partial \gamma}\left[MR'(q_H^{AC}) - AC'(q_H^{AC})\right] = 1$ |

and therefore

- $\dfrac{\partial p^c}{\partial \gamma} = \dfrac{P'(q^c)}{P'(q^c) - C''(q^c)}$
- $\dfrac{\partial p^m}{\partial \gamma} = \dfrac{P'(q^m)}{MR'(q^m) - C''(q^m)} \geq 0$
- $\dfrac{\partial p_H^{AC}}{\partial \gamma} = \dfrac{P'(q_H^{AC})}{MR'(q_H^{AC}) - AC'(q_H^{AC})} \geq 0$

Obviously all three are positive, and $\partial p^c/\partial\gamma$ is at most 1. Otherwise, it turns out that comparing the values of the pass-through rates, $\partial p^c/\partial\gamma$, $\partial p^m/\partial\gamma$, and $\partial p_H^{AC}/\partial\gamma$, is not, in general, an obvious exercise. Not only are the ratios different functions but they are also evaluated at different quantities, making the comparisons trickier. Yet a very general result emerges:

**Lemma 3.** *If $\phi \to \phi^{AC}$, then $\partial p_H^{AC}/\partial\gamma \to +\infty$, the cost-taking firm has a larger pass-through rate than both the price-taking and the rational firm.*

*Proof.* The quantity $q_H^{AC}$ is such that $MR'(q_H^{AC}) - AC'(q_H^{AC}) \leq 0$ and $\phi^{AC}$ is such that $MR'(q_H^{AC}) - AC'(q_H^{AC}) = 0$. □

To go further, let

$$\Theta(q) = qP''(q)/P'(q) \in \,]-2, +\infty[$$

be the elasticity of the derivative of the inverse demand. The condition $\Theta > -2$ is to ensure that the objective monopoly profit is concave when $C'' = 0$.

To go further, in most papers, this type of analysis is undertook for a linear cost function[12] –i.e. $C(q) = cq$, in which case $C'' = 0$ and $AC' = -\phi/q^2$, consequently the formulae above become:

- $\dfrac{\partial p^c}{\partial \gamma} = 1 \; ; \; \dfrac{\partial p^m}{\partial \gamma} = \dfrac{1}{2 + \Theta(q^m)}$
- $\dfrac{\partial p_H^{\mathcal{AC}}}{\partial \gamma} = \dfrac{1}{2 + \Theta(q_H^{\mathcal{AC}}) + \phi(q_H^{\mathcal{AC}})^2/P'(q_H^{\mathcal{AC}})}$

where

**Proposition 4.** *If $\phi \to \phi^{\mathcal{AC}}$, then $\partial q_H^{\mathcal{AC}}/\partial \gamma \to -\infty$, and the cost-taking firm has the largest pass-through rate.*

*Otherwise, assume $MR''(q) \leq 0$, and $\phi^* < \phi < \phi^{\mathcal{AC}}$, then the cost-taking firm has a larger pass-through rate than the rational firm.*

*Proof.* When $\phi^* < \phi < \phi^{\mathcal{AC}}$, then $q_H^{\mathcal{AC}} \leq q^m$ and $MR''(q) \leq 0$ implies $0 > MR'(q_H^{\mathcal{AC}}) \geq MR'(q^m)$. Moreover, $q_H^{\mathcal{AC}} \leq q^*$ and therefore $AC'(q_H^{\mathcal{AC}}) \leq 0 \leq C''(q^m)$, thus

$$MR'(q_H^{\mathcal{AC}}) - AC'(q_H^{\mathcal{AC}}) \geq MR'(q^m) - AC'(q_H^{\mathcal{AC}}) \geq MR'(q^m) - C''(q^m)$$

furthermore $0 \geq MR'(q_H^{\mathcal{AC}}) - AC'(q_H^{\mathcal{AC}})$. Therefore

$$0 \geq \frac{\partial q^m}{\partial \gamma} \geq \frac{\partial q_H^{\mathcal{AC}}}{\partial \gamma}$$

□

**Parametric example (continued):** Here the three equations write: $\frac{\partial q^c}{\partial \gamma}[-b - c_2] = 1$, $\frac{\partial q^m}{\partial \gamma}[-2b - c_2] = 1$, and $\frac{\partial q_H^{\mathcal{AC}}}{\partial \gamma}\left[-2b - c_2/2 + \phi/(q_H^{\mathcal{AC}})^2\right] = 1$. Therefore, the pass-through rate of the price-taking firm is larger than the one of the rational firm –a result in line with the intuition. Similarly, the pass-through rate of the cost-taking firm is

[12] E.g. Weyl and Fabinger (2013) and Miklos-Thal and Shaffer (2021).

larger than the one of the rational firm. Finally, let $\widetilde{c} = 2b\left(1 - 9b\phi/(a - c_1)^2\right)$. If $c_2 > \widetilde{c}$, then the pass-through rate of the cost-taking firm is larger than the one price-taking firm and the reverse holds when $c_2 < \widetilde{c}$.

# 5 Long term view of the cost function

Whereas technology is given in the short run, with time a firm should be able to duplicate it in order to produce more efficiently. In this section, we show that in a longer run, our firm makes the same choices as a rational one. At first, this result might seems surprising. However the intuition is fairly simple. In the long run, a rational firm produces at the minimum of the average cost function. Thus behaving as our firm. It remains to show that both type of firms follow the same investment strategy.

Let assume that the firm can build $n$ plants. Each plant produces according to the total cost function $TC(q;\phi) = \phi + C(q)$. Assume the firm wants to produce $Q$. Given the convexity of $C(.)$ it is optimal for the firm to produce $Q/n$ in every plant. Integer issues are neglected and $n$ is treated as a continuous variable. Thus its total cost is:

$$\mathcal{TC}(Q;\phi,n) = n\phi + nC\left(Q/n\right)$$

and its average and marginal costs are respectively

$$\mathcal{AC}(Q\,;\phi,n) = \mathcal{TC}(Q\,;\phi,n)/Q = \frac{\phi}{Q/n} + \frac{C\left(Q/n\right)}{Q/n} = AC\left(Q/n\,;\phi\right)$$

and

$$\mathcal{MC}(Q\,;\phi,n) = \frac{\partial\mathcal{TC}(Q\,;\phi,n)}{\partial Q} = C'\left(Q/n\right)\ .$$

**Proposition 5.** *In the long run, both the rational and the cost-taking firm invest in the same number of plants and produce the same quantity, thus achieve the same profit.*

*Proof.* The rational firm would thus choose $n$ and $Q$ to maximize $\Pi^{\mathcal{O}}(Q\,;\phi,n) = P(Q)Q - \mathcal{TC}(Q\,;\phi,n)$ leading to the f.o.c.:

$$\begin{cases} \frac{\partial\Pi^{\mathcal{O}}}{\partial Q} = 0 & \Rightarrow MR(Q) = C'(Q/n) \\ \\ \frac{\partial\Pi^{\mathcal{O}}}{\partial n} = 0 & \Rightarrow C'(Q/n) = AC\left(Q/n\,;\phi\right) \end{cases}$$

these conditions hold whether the choice of $n$ and $Q$ are simultaneous or sequential. Let $n^{LT}$ and $Q^{LT}$ denote the rational firm's long term choices. Given (23) the second f.o.c. writes $Q^{LT}/n^{LT} =$ and the first one is $MR(Q^{LT}) = C'()$. The long term quantity is chosen as if the firm had a constant marginal cost equal to $C'()$. We assume (for simplicity) that $q^m$ (and therefore $q_H^{\mathcal{AC}}$) is larger than $Q^{LT}$, ensuring that it is optimal to build more than one plant.

As $MR(Q^{LT}) = AC\left(Q^{LT}/n^{LT};\phi\right) = \mathcal{AC}(Q^{LT};\phi,n^{LT})$, the quantity $Q^{LT}$ is a misspecification equilibrium quantity for a cost-taking firm with $n^{LT}$ factories. The question is then: Would such a firm build this number of plants? As such a firm has a wrong belief about the profit function, it is, at first sight, more difficult to model how it should anticipate future profits. To circumvent the difficulty, we assume the firm follow an iterative process where the firm is divided into a marketing unit and a production unit. Both behave myopically.

Starting with $n_0 = 1$ (one plant), the cost-taking marketing unit produces $Q_1 = q_H^{\mathcal{AC}}$ defined by (4). Next, the production unit takes for granted this quantity and choose a number $n_1$ of plants to minimize the total cost of production: $\mathcal{TC}(Q_1;\phi,n)$. That is $n_1$ such that $C'(Q_1/n) = AC\left(Q_1/n\,;\phi\right)$. Therefore $n_1 = Q_1/$, which is larger than 1 under the assumption $q_H^{\mathcal{AC}} >$. The process then starts again with the quantity choice by the marketing unit but this time for an average cost function $\mathcal{AC}(Q\,;\phi,n_1)$.

This defines a sequence $Q_t$ such that $Q_1 = q_H^{\mathcal{AC}}$ and $Q_t$ is the largest root of

$$MR(Q_t) = \mathcal{AC}(Q_t;\phi,n_{t-1}) = AC(\frac{Q_t}{n_{t-1}};\phi) = AC(\frac{Q_t}{Q_{t-1}};\phi)$$

In consequence, the sequence $Q_t$ (resp. $n_t$) is increasing and converges to $Q^{LT}$ (resp. $n^{LT}$). To show that $Q_t$ is increasing, remark that the function $\mathcal{AC}(Q;\phi,n_{t-1})$ is by construction minimal for $Q = Q_{t-1}$, increasing for $Q > Q_{t-1}$, and such that $MR(Q_{t-1}) > \mathcal{AC}(Q_{t-1};\phi,n_{t-1})$. $\square$

# 6 Variants and extensions

The previous models were based on a particular misspecification: either the market constraint or the technological constraint was not understood. Many variants are possible and we present a few here.

### 6.1 Average-variable-cost and average-fixed-cost

It is instructive to distinguish in the unit-cost, the role played by the average-variable-cost, $C(q)/q$ from the one played by the average-fixed-cost $\phi/q$. For that purpose, assume first, that the firm understand the technological constraint for the former but not the latter. Then the misspecified profit writes

$$\Pi^{\mathcal{S}}(q\,;\theta) = \left(P(q) - \frac{C(q)}{q} - \frac{\phi}{\theta}\right) q \;\; \text{with } \theta \in \left[\underline{q}, \overline{q}\right] \, . \tag{5}$$

Consequently, the misspecification equilibrium quantity is given by $\theta^{\mathcal{S}} = q^{\mathcal{S}}$ and

$$MR(q^{\mathcal{S}}) = C'(q^{\mathcal{S}}) + \frac{\phi}{q^{\mathcal{S}}} \tag{6}$$

which always leads to quantities smaller than $q^m$ and also smaller than $q_H^{\mathcal{AC}}$ (because for $C$ convex, $C(q)/q$ is smaller than $C'(q)$). This emphasizes that in the model of the cost-taking firm, the average-fixed-cost-taking part always pushes to a lower quantity (compared to the choice of the rational firm) and a higher price. Neglecting that the average-fixed-cost decreases with the quantity unambiguously leads to a higher price. It is also readily confirmed that the largest root of (6) is decreasing with $\phi$ a desirable property of full-cost-pricing, at least from an intuitive point of view.

At the other extreme, the firm could perfectly understand how the average-fixed-cost varies with $q$ but wrongly believe that the average-variable-cost is constant. Leading to

$$\Pi^{\mathcal{S}}(q\,;\theta) = \left(P(q) - \frac{C(\theta)}{\theta} - \frac{\phi}{q}\right) q \;\; \text{with } \theta \in \left[\underline{q}, \overline{q}\right] \, , \tag{7}$$

and therefore a misspecification equilibrium quantity given by $\theta^{\mathcal{S}} = q^{\mathcal{S}}$ and

$$MR(q^{\mathcal{S}}) = \frac{C(q^{\mathcal{S}})}{q^{\mathcal{S}}} \, . \tag{8}$$

This quantity is always larger, because $C(q)/q < C'(q)$, than the rational quantity and it is invariant with $\phi$. In the model of the cost-taking firm, these two forces are combined which explains why $q_H^{AC}$ can be larger or smaller than $q^m$.

## 6.2 More parameters

Similar results as the ones for the cost-taking firm are obtained for the following family of misspecified profit functions. Let $\theta = (\theta_0, \theta_1)$ and

$$\Pi^{\mathcal{S}}(q\,;\theta) = (P(q) - AC(\theta_1\,;\phi))\,q - \theta_0$$

The firm behaves as if it had a fixed cost $\theta_0$ and a constant marginal cost $C'(\theta_1)$. It is readily confirmed that a misspecification equilibrium obtains when

$$MR(q) = AC(q\,;\phi - \theta_0).$$

A condition similar to the one obtained for the cost-taking firm. The condition is exactly the same for $\theta_0 = 0$. For other values of $\theta_0$ the equilibrium as the same flavor but the interpretation slightly differs. In particular, if $\theta_0 = \phi$ (no misspecification of the fixed cost), then the equilibrium quantity is defined by $MR(q) = C(q)/q$.

The main advantage of the introduction of a second parameter $\theta_0$ is that now, the equilibrium exists for all $0 \leq \phi \leq \phi^m$.

## 6.3 De/Inflating the true cost function

Let assume, that the firm de/inflates its true marginal cost by a constant because it is difficult to assess if some costs are fixed or variable.[13] In this spirit, the misspecified profit writes

$$\Pi^{\mathcal{S}}(q\,;\theta) = \left(P(q) - \frac{C(q)}{q} - \theta_1 - \frac{\theta_0}{q}\right) q \;\text{ with } \theta \in \left[\underline{q}, \overline{q}\right]\,. \tag{9}$$

[13] This is what Al-Najjar, Baliga, and Besanko (2008) do, yet in a different context. Moreover, they restrict their analysis to the case where $C(q) = cq$.

Consequently, the misspecification equilibrium quantity is given by $\theta_1^{\mathcal{S}} q^{\mathcal{S}} + \theta_0 = \phi$ (where $\theta_0$ is simply kept as an exogenous parameter) and

$$MR(q^{\mathcal{S}}) = C'(q^{\mathcal{S}}) + \frac{\phi - \theta_0}{q^{\mathcal{S}}} \tag{10}$$

which is very close to (6). When $\phi - \theta_0 > 0$ (the intuitive case), then $q^{\mathcal{S}} < q^m$. The marginal cost is inflated which leads to a lower quantity and a higher price. On the contrary, if $\phi - \theta_0 < 0$ (a less intuitive case), then $q^{\mathcal{S}} > q^m$. The marginal cost is deflated which leads to a higher quantity and a lower price.

A firm would not distort its production choice and would produce the rational quantity only if $\theta_0 = \phi$ (which in this context amounts to assume that the firm is rational).

## 6.4 Pass-through

Let assume the variable cost function increases with a parameter $\gamma$: $C(q;\gamma)$. A first question is how does the cost pass-through varies with the fixed cost $\phi$? That is what is the sign of $\partial^2 q_H^{\mathcal{AC}}/\partial\gamma\partial\phi$?

**Proposition 6.** *For the cost-taking firm,*

$$\frac{\partial q_H^{\mathcal{AC}}}{\partial \gamma} < 0\ ,\ \frac{\partial q_H^{\mathcal{AC}}}{\partial \phi} < 0\ ,\ \frac{\partial^2 q_H^{\mathcal{AC}}}{\partial \phi^2} < 0 \quad \textit{and}$$

$$\frac{\partial^2 q_H^{\mathcal{AC}}}{\partial\gamma\partial\phi} = \frac{\partial C'(q_H^{\mathcal{AC}};\gamma)}{\partial\gamma}\left(\frac{\partial q_H^{\mathcal{AC}}}{\partial\phi}\right)^2 + \frac{\partial C(q_H^{\mathcal{AC}};\gamma)}{\partial\gamma}\frac{\partial^2 q_H^{\mathcal{AC}}}{\partial\phi^2} \tag{11}$$

*Proof.* See Appendix D □

The first term on the right-hand-side of (11) is positive whereas the second term is negative. In our parametric example, assuming further that $C(q;\gamma) = C(q) + \gamma q$ or $C(q;\gamma) = (1+\gamma)C(q)$, the negative term dominates meaning that an increase of $\gamma$ leads to a larger increase of the price (i.e. a larger pass-through) when $\phi$ is larger. In Altomonte, Barattieri, and Basu (2015) (see their section 3.2), they find empirically that the pass-through is lower in industry where $\phi/q$ is larger.

Another question is the comparison of the pass-through of the rational firm and the one of the price- or cost-taking firm. A priori $q^m$ and $q_H^{\mathcal{AC}}$ are different which complicates the interpretation of these equations. However, in the long run (as well as for $\phi = \phi^*$) we have seen that $q^m = q_H^{AC}$ which allows us to derive the following proposition.

**Proposition 7.** *Assume that both the cost-taking and rational firms produce at the production efficient level and that $C(q;\gamma) = C(q) + \gamma q$ or $C(q;\gamma) = (1+\gamma)C(q)$ , then the cost-taking firm reacts more to a shock on the cost function than a rational one.*

*Proof.* See Appendix E

□

## 7 Conclusion

A natural extension of our models is the inclusion of competition. This is all the more natural that the Berk-Nash approach of Esponda and Pouzo (2016) is build for several players. Although we are confident that it is possible to extend our work in that direction, that would include additional strategic reasons not to maximize the objective profit function. Indeed, building on the logic of Brander and Spencer (1985) and Eaton and Grossman (1986),[14] a number of papers showed that, in an oligopoly context, firms have an incentive to inflate internal transfer prices. That is, firms organize within themselves a vertical structure where a production center sell for a transfer price the good to a marketing division which sells to final consumers. In such a context, a transfer price above marginal cost softens competition downstream. See Alles and Datar (1998), Göx (2000), Arya and Mittendorf (2008), and Thépot and Netzer (2008). See also Buchheit and Feltovich (2011) for an experimental study. Al-Najjar, Baliga, and Besanko (2008) develop a model where firms' total costs of production have two parts: a constant marginal cost and a fixed cost. They assume firms have a distorted view of these costs leading to inflate marginal cost by an exogenous amount. Firms maximize (the framework is one of reinforcement learning rather than Berk-Nash) their profits using this inflated marginal cost they are assumed to be boundedly rational players following

[14]See also Katz (1991). In these seminal articles, two countries (one firm per country) compete for a market in a third country. Governments subsidize or tax exports in order to help their national firm. Under Cournot competition, Brander and Spencer show that subsidies are optimal but that they deteriorate the welfare of the exporting countries. The subsidy game is like a prisoner dilemma. By contrast, under Bertrand competition (with differentiated goods) Eaton and Grossman show that taxes are optimal and that they improve the welfare of the exporting countries.

an adaptive pricing process. The main result is that in a world of price competition and differentiated products, firms benefit from basing their pricing decision on an inflated marginal cost.

# APPENDIX

## A Solutions of (4) and proof of Lemma 2

**Number of solutions** Equation (4) also writes:

$$qP(q) = (-P'(q))q^2 + C(q) + \phi$$

The left-hand-side is the revenue function $qP(q)$ which is assumed to be a concave function of $q$ (i.e. $qP''(q)/P'(q) \geq -2$), a common assumption is the IO literature. Let

$$H(q) + \phi = (-P'(q))q^2 + C(q) + \phi \tag{12}$$

denote the right-hand-side which is like a cost function. It is readily confirmed that $H$ is increasing with $q$ as

$$H'(q) = C'(q) + (-qP'(q))\,(2 + qP''(q)/P'(q)) > 0$$

The functions $qP(q)$ and $H(q)+\phi$ intersect at most twice as long as $H$ is everywhere less concave than the revenue $qP(q)$ which is our assumption throughout. As the convexity of $C$ tends to make $H$ convex, our assumption is certainly intuitive. It trivially holds for a linear demand or for a constant elasticity function $P(q) = q^{-\sigma}$ with $0 < \sigma < 1$ (the usual assumption on $\sigma$ for an inverse demand function).

**Existence** Even under the above assumption on $H$, (4) has no solution if $\phi$ is too large. The limit case is characterized by both $MR(q) = AC(q\,;\phi)$ and $MR'(q) = AC'(q\,;\phi)$ when the two curves intersect at a tangency point. Using $AC'(q\,;\phi) = (C'(q) - AC(q\,;\phi))\,/q$ the two conditions imply that

$$MR(q) + qMR'(q) = C'(q) \;\text{ and }\; qMR(q) - C(q) = \phi$$

that is, $\phi^{\mathcal{AC}} = \max_q qMR(q) - C(q)$, and $q$ is $q^{dm}$ the double marginalization quantity (notice that $q^{dm}$ is independent of $\phi$).[15]

[15] In a chain of monopolies, it is as if the demand of the upstream firm is $MR(q)$.

By construction when $\phi = \phi^{AC}$ then $q_H^{AC} = q_L^{AC} = q^{dm}$. Notice that the profit of the cost-taking firm is $(P(q) - AC(q\,;\phi)q$ which means that it is positive for $\phi = \phi^{AC}$.

# B Proof of Proposition 2

Only for the cost-taking firm is the comparative statics with respect to $\phi$ not straightforward. Using Differentiating $qP(q) = H(q) + \phi$ with respect to $\phi$ gives:

$$[MR(q) - H'(q)]\,\frac{\partial q}{\partial \phi} = 1$$

as $MR(q_L^{AC}) - H'(q_L^{AC}) > 0$ a $MR(q_H^{AC}) - H'(q_H^{AC}) < 0$ it follows that $q_L^{AC}$ increases and $q_H^{AC}$ decreases with $\phi$. It is straightforward to show that $q_L^{AC}$ is always lower that $q^m$ whereas $q_H^{AC}$ is larger than $q^m$ if $\phi$ is small enough.

Rewriting (4) as:

$$qP(q) - C(q) - \phi = (-P'(q))q^2$$

and given that $qP''(q)/P'(q) \geq -2$ implies $(-P'(q))q^2$ increases with $q$ it is immediate that the profit for $q_H^{AC}$ is larger than the profit for $q_L^{AC}$.

# C Proof of Proposition 1

The derivative of $(MR)^{-1}(AC(q\,;\phi))$ is $AC'(q\,;\phi)/MR'(q)$ for $q = q_L^{AC}$ or $q = q_H^{AC}$. When $q = q_L^{AC}$, $AC'(q_L^{AC}\,;\phi) < MR'(q_L^{AC}) < 0$ and thus $\left|\frac{AC'(q_L^{AC}\,;\phi)}{MR'(q_L^{AC})}\right| > 1$ and $q_L^{AC}$ is non-stable.

On the contrary, when $q_H^{AC} \leq q^m$ then $MR'(q_H^{AC}) < AC'(q_H^{AC}\,;\phi) < 0$ and $q_H^{AC}$ is locally stable.

When $q^m < q_H^{AC}$ then $MR'(q_H^{AC}) < 0 < AC'(q_H^{AC}\,;\phi)$ and $q_H^{AC}$ is locally stable if

$$AC'(q_H^{AC}\,;\phi) < \left|MR'(q_H^{AC})\right|$$

As the slope of $MR'$ could be arbitrarily close to zero, this condition cannot hold for all demand functions. Yet, let $\varepsilon > 0$ such that for all $q$, $|MR'(q)| > \varepsilon > 0$, if $q_H^{AC}$ is close enough to (which happens when $\phi$ is close enough to $\phi^*$) then $AC'(q_H^{AC}\,;\phi)$ is arbitrarily close to zero and therefore $AC'(q_H^{AC}\,;\phi) < \varepsilon < \left|MR'(q_H^{AC})\right|$ ensuring the local stability property.

Otherwise, using $AC(q_H^{AC}\,;\phi) = MR(q_H^{AC})$ it is readily confirmed that for $q = q_H^{AC}$:

$$\left|\frac{AC'(q\,;\phi)}{MR'(q)}\right| = \frac{\frac{C'(q)-P(q)}{q}}{-MR'(q)} + \frac{-P'(q)}{-MR'(q)}$$

Now, the first term is negative if $q_H^{AC} \leq q^c$ (it is null for $q_H^{AC} = q^c$). Moreover $MR' = 2P' + qP''$ which means that

$$\left|\frac{AC'(q\,;\phi)}{MR'(q)}\right| < \frac{-P'(q)}{2P'(q) + qP''(q)} = \frac{1}{2 + \frac{qP''(q)}{P'(q)}}$$

therefore the condition $qP''(q)/P'(q) > -1$ ensures that the stability ratio is lower than one even in the neighborhood of $q^c$. This condition is stronger than the usual $qP''(q)/P'(q) > -2$ (which ensures the concavity of the revenue function).

# D Proof of Proposition 6

The quantity $q_H^{AC}$ is characterized by

$$MR(q) = AC(q\,;\gamma,\phi) \text{ as in Appendix A, it writes } R(q) = H(q\,;\gamma) + \phi \tag{13}$$

with

$$H(q\,;\gamma) + \phi = (-P'(q))q^2 + C(q\,;\gamma) + \phi$$

which behaves like a cost function. In the following we use the notation $H'(q\,;\gamma) = \partial H(q\,;\gamma)/\partial q$, and similarly for $C'$. Differentiating once w.r.t. $\phi$ gives

$$\frac{\partial q}{\partial \phi}\left(R'(q) - H'(q\,;\gamma)\right) = 1 \tag{14}$$

Differentiating (14) w.r.t. $\phi$ establishes

$$\frac{\partial^2 q}{\partial \phi^2}\left(R'(q) - H'(q\,;\gamma)\right) + \left(\frac{\partial q}{\partial \phi}\right)^2\left(R''(q) - H''(q\,;\gamma)\right) = 0$$

thus $\frac{\partial^2 q_H^{AC}}{\partial \phi^2} < 0$, using that for $q = q_H^{AC}$, $R' - H' < 0$ and under the general assumption that $R'' - H'' < 0$.

Similarly, differentiating (13) once w.r.t. $\gamma$

$$\frac{\partial q}{\partial \gamma}\left(R'(q) - H'(q\,;\gamma)\right) = \frac{\partial H(q\,;\gamma)}{\partial \gamma} = \frac{\partial C(q\,;\gamma)}{\partial \gamma} \tag{15}$$

Combining (14) and (15) leads to (still for $q = q_H^{AC}$)

$$\frac{\partial q}{\partial \gamma} = \frac{\partial C(q\,;\gamma)}{\partial \gamma}\frac{\partial q}{\partial \phi} \tag{16}$$

Differentiating (16) w.r.t. $\phi$ gives

$$\frac{\partial^2 q}{\partial \gamma \partial \phi} = \frac{\partial C'(q\,;\gamma)}{\partial \gamma}\left(\frac{\partial q}{\partial \phi}\right)^2 + \frac{\partial C(q\,;\gamma)}{\partial \gamma}\frac{\partial^2 q}{\partial \phi^2} \tag{17}$$

Differentiating (14) w.r.t. $\phi$ establishes that $\frac{\partial^2 q}{\partial \phi^2} < 0$.

# E Proof of Proposition 7

For a rational firm, the f.o.c. is (where the prime in $C'$ denotes the derivative with respect to $q$)

$$MR(q^m) = C'(q^m;\gamma)$$

therefore differentiating w.r.t. $\gamma$ and rearranging terms leads to

$$\frac{\partial q^m}{\partial \gamma} = \frac{\frac{\partial C'(q^m;\gamma)}{\partial \gamma}}{MR'(q^m) - C''(q^m;\gamma)}$$

For a cost-taking firm, the equilibrium condition is (ignoring in the notation the dependence of $AC$ on $\phi$)

$$MR(q_H^{AC}) = AC(q_H^{AC};\gamma)$$

therefore differentiating w.r.t. $\gamma$, noting that $\frac{\partial AC(q_H^{AC};\gamma)}{\partial \gamma} = \frac{\partial C(q_H^{AC};\gamma)/q_H^{AC}}{\partial \gamma}$ and rearranging terms leads to

$$\frac{\partial q_H^{AC}}{\partial \gamma} = \frac{\frac{\partial C(q_H^{AC};\gamma)/q_H^{AC}}{\partial \gamma}}{MR'(q^m) - AC'(q_H^{AC};\gamma)}$$

Now, we have assumed that $q^m = q_H^{\mathcal{AC}}$ and $AC'(q_H^{\mathcal{AC}};\gamma) = 0$, i.e. both type of firms produce at the minimum of the average cost, see Section 5. Moreover, if $C(q;\gamma) = C(q) + \gamma q$, then $\frac{\partial C(q_H^{\mathcal{AC}};\gamma)/q_H^{\mathcal{AC}}}{\partial \gamma} = 1$ and $\frac{\partial C'(q^m;\gamma)}{\partial \gamma} = 1$ also. Therefore

$$\left|\frac{\partial q_H^{\mathcal{AC}}}{\partial \gamma}\right| = \left|\frac{1}{MR'(q^m)}\right| > \left|\frac{1}{MR'(q^m) - C''(q^m;\gamma)}\right| = \left|\frac{\partial q^m}{\partial \gamma}\right|$$

If $C(q;\gamma) = (1+\gamma)C(q)$, then $\frac{\partial C(q^{\mathcal{S}};\gamma)/q^{\mathcal{S}}}{\partial \gamma} = C(q^{\mathcal{S}};\gamma)/q^{\mathcal{S}}$ and $\frac{\partial C'(q^m;\gamma)}{\partial \gamma} = C'(q^m;\gamma)$, and the same result follows.

In the general case:

$$\left|\frac{\partial q_H^{\mathcal{AC}}}{\partial \gamma}\right| = \left|\frac{\frac{\partial C(q_H^{\mathcal{AC}};\gamma)/q_H^{\mathcal{AC}}}{\partial \gamma}}{MR'(q^m)}\right| \gtrless \left|\frac{\frac{\partial C'(q^m;\gamma)}{\partial \gamma}}{MR'(q^m) - C''(q^m;\gamma)}\right| = \left|\frac{\partial q^m}{\partial \gamma}\right|$$

The intuition is that if the cost shift influences more the average cost than the marginal cost, i.e. $\frac{\partial C(q_H^{\mathcal{AC}};\gamma)/q_H^{\mathcal{AC}}}{\partial \gamma} > \frac{\partial C'(q^m;\gamma)}{\partial \gamma}$ then the cost-taking firm unambiguously reacts more to a shock on the cost function than a rational one. However, if the cost shift impacts more the marginal cost than the average cost the reverse could happens. To illustrate, one can imagine no impact on the average cost if the marginal cost is impacted only from $q_H^{\mathcal{AC}} - \varepsilon$. In that case the cost-taking firm would not react at all while the rational one would.

# F Bayesian learning

We now account for shocks affecting the firm cost function, incorporating such costs within the framework of Esponda and Pouzo (2016). We assume a constant marginal cost $c$. While the fixed cost $\phi$ is deterministic, the marginal cost $c$ is drawn from the normal distribution with mean $\bar{c}$ and variance 1, or

$$c = \bar{c} + \omega,$$

where the cost shock $\omega$ is drawn from the standard normal distribution. The true data generating process, or "objective model", for the average cost function is thus

$$AC^{\mathcal{O}}(q;\phi) = \bar{c} + \omega + \frac{\phi}{q}.$$

The firm believes that the average cost follows the Gaussian distribution of mean $\theta$ and variance 1. The subjective, misspecified model is thus

$$AC^{\mathcal{S}}(q;\phi\,|\,\theta) = \theta + \varepsilon,$$

where $\varepsilon$ is drawn from the standard normal distribution. The firm chooses quantity $q$, observes the realized average cost $AC(q;\phi)$ after it has been affected by the shock $\omega$, and infers $\theta$ from that observation.

The Berk-Nash equilibrium $(\theta^*, q^*)$ is defined by two conditions. First, the firm belief $\theta^*$ minimizes the Kullback-Leibler divergence between the objective and subjective models, i.e.

$$\theta^* = \operatorname{argmin}_\theta \mathbb{E}_\omega \ln \frac{\varphi\left[AC^{\mathcal{O}}(q^*;\phi) - \bar{c} - \phi/q^*\right]}{\varphi\left[AC^{\mathcal{O}}(q^*;\phi) - \theta\right]}, \tag{18}$$

where $\varphi$ denotes the density function of the standard normal distribution.[16] Second, the firm optimally chooses output given its belief:

$$q^* = \operatorname{argmax}_q \mathbb{E}_\varepsilon \left[P(q)q - AC^{\mathcal{S}}(q;\phi\,|\,\theta^*)q\right]. \tag{19}$$

In this particular context, the minimization problem (18) is simple because $\ln\,\varphi\left[AC^{\mathcal{O}}(q^*;\phi) - \bar{c} - \phi/q^*\right]$ does not depend on $\theta$. Hence the problem boils down to

$$\theta^* = \operatorname{argmax}_\theta \mathbb{E}_\omega \ln\varphi\left[AC^{\mathcal{O}}(q^*;\phi) - \theta\right] = \operatorname{argmin}_\theta \mathbb{E}_\omega \left[\bar{c} + \omega + \frac{\phi}{q^*} - \theta\right]^2 = 1 + \left[\bar{c} + \frac{\phi}{q^*} - \theta\right]^2.$$

It follows that $\theta^*$ is given by

$$\theta^* = \bar{c} + \frac{\phi}{q^*}. \tag{20}$$

The quantity choice in (19) is equally simple as the firm objective is linear in average

[16]The numerator is the true likelihood of the average cost, while the denominator is the subjective likelihood that reflects the firm belief.

cost:

$$q^* = \text{argmax}_q \; P(q)q - \theta^* q,$$

hence

$$MR(q^*) = \theta^*. \tag{21}$$

Equations (20) and (21), which characterizes the Berk-Nash equilibrium, are the same as (4) in section 3.2.

Esponda and Pouzo (2016) show that the above equilibrium can be achieved as the result of a learning process where a Bayesian firm at each period myopically maximizes its profit and then updates its belief about its average production cost. Specifically, let $\mu_0$ be the firm's prior belief about $\theta$ at date 0. Consider an iid sequence of cost shocks $(\omega_t)_{t\geq 0}$ drawn from the standard normal distribution. Let $q_t$ be the outcome produced at date $t$, $t \geq 0$. For $t \geq 1$, the firm beliefs at the beginning of period $t$ are described by the posterior distribution $\mu_t$ which, by Bayes rule, is proportional to

$$\mu_t(\theta) \propto \mu_0(\theta) \prod_{n=0}^{t-1} \varphi \left[ AC^{\mathcal{O}}(q_n; \phi) - \theta \right].$$

At date $t$, the firm chooses output $q_t$ to maximize its current profit

$$q_t = \text{argmax}_q \mathbb{E}_{\mu_t} \; [qP(q) - \theta q]$$

hence

$$MR(q_t) = \mathbb{E}_{\mu_t} \theta$$

Esponda and Pouzo (2016) demonstrate that the quantity $q_t$ tends to $q^*$ and the posterior distribution $\mu_t$ tends to the mass point at $\theta^*$ as $t$ tends to infinity, where $(q^*, \theta^*)$ form a Berk-Nash defined by (20) and (21).

# G Two other simple misspecified models

Within the same class of models as those of section 3, one possibility is the margin-maximizing firm. The firm understands that both the price and the unit-cost vary with the chosen quantity but wrongly believes that the sold quantity is fixed. Admittedly a strange belief, but nevertheless, in equilibrium, the firm rationally anticipates the correct sold quantity and does not realize its profits could be larger. In this model also the price varies with fixed costs but in an opposite direction. When these costs increase, a price reduction follows.

Next, we present the cost-minimizing firm. Here the firm takes both the price and the sold quantity as fixed, and thus maximizes profits by minimizing the average costs. This is the mirror case of the cost-taking firm. As in the margin-maximizing firm model, the equilibrium price decreases with the fixed costs.

For completeness, we also have the quantity-maximizing and the quantity-minimizing firm: $(P(\theta) - AC(\theta; \phi))\, q$ and $(P(q) - AC(\theta; \phi))\, \theta$. But both of them would lead to zero profits. It is easy to check that when the firm maximizes the quantity the only misspecification equilibrium is $q = \overline{q}$ and no profit as $P(\overline{q}) = AC(\overline{q}; \phi)$. Whereas when the firm maximizes the price (it minimizes the quantity) the only misspecification equilibrium is $q = \underline{q}$ and no profit as $P(\underline{q}) = AC(\underline{q}; \phi)$.

The cost-minimizing firm's price is lower than the rational price when the fixed costs are relatively large. Finally, the margin-maximizing quantity is always lower than the rational quantity. So, in this model, the lack of rationality of the firm hurts both the firm and the consumers. Moreover, the margin-maximizing firm remains active as long as the rational firm is active.

## G.1 Margin-maximizing firm

A third model is obtained by assuming that the firm understand that its unit margin is given by $P(q) - AC(q; \phi)$ but wrongly believes that the total quantity is constant. That is, neither the market nor the technological constraints are understood, i.e. $q_1 \neq q_3$ and $q_2 \neq q_3$, yet the maximizing variable is $q_1 = q_2$. Formally,

$$\Pi^{\mathcal{S}}\left(q\,; \theta\right) = \left(P(q) - AC(q; \phi)\right) \theta \quad \text{with } \theta \in \left[\underline{q}, \overline{q}\right] . \tag{22}$$

The mistake in this model, although less intuitive, is easy to explain. The behavior of the firm is simply to maximize its unitary margin.

Let  be the unique solution to $P'(q) = AC'(q;\phi)$ which also writes:

$$MR(q) = C'(q) + (P(q) - AC(q\,;\phi)) \; . \tag{23}$$

To show uniqueness, notice that the derivative of the right-hand side is $C''(q) + P'(q) - AC'(q\,;\phi)$ which simplifies into $C''() \geq 0$ for $q =$.

**Lemma G.1.** *In the model of the margin-maximizing firm, when $\phi \leq \phi^m$, there exists a unique misspecification equilibrium: $q^{\mathcal{S}} = \theta^{\mathcal{S}} =$.*

*Proof.* The optimality condition is $P'(q^{\mathcal{S}}) = AC'(q^{\mathcal{S}};\phi)$ and the consistency is again $\theta^{\mathcal{S}} = q^{\mathcal{S}}$. Rearranging the optimality condition, using $AC'(q;\phi) = (C'(q) - AC(q;\phi))/q$ we have (23). □

Whenever the rational firm is active (i.e. $\phi \leq \phi^m$) so is the margin-maximizing firm. In particular when $\phi \to \phi^m$, $\to q^m$ and the choice of the margin-maximizing firm becomes rational allowing survival.

## G.2 Cost-minimizing firm

A fourth model can be derived by assuming, as in the cost-taking case, that the market constraint is satisfied, i.e. $q_1 = q_3$ and the technological one is not, i.e. $q_2 \neq q_3$ but assuming that the maximizing variable is $q_2$ (whereas it is $q_1 = q_3$ in the cost-taking case). Formally,

$$\Pi^{\mathcal{S}}(q\,;\theta) = (P(\theta) - AC(q;\phi))\,\theta \;\; \text{with } \theta \in \left[\underline{q}, \overline{q}\right] \; . \tag{24}$$

Here the firm minimizes its average cost of production. Let  be the unique solution to

$$C'(q) = AC(q\,;\phi)\,. \tag{25}$$

**Lemma G.2.** *In the model of the cost-minimizing firm, when $\phi \leq \phi^c$, there exists a unique misspecification equilibrium: $q^{\mathcal{S}} = \theta^{\mathcal{S}} =$.*

*Proof.* As $AC'(q;\phi) = (C'(q) - AC(q;\phi))/q$ the optimality condition is $C'(q) = AC(q^{\mathcal{S}};\phi)$, implying $q =$, and the consistency is again $\theta^{\mathcal{S}} =$. □

**For the margin-maximizing firm** quantity and profits are

$$= \sqrt{\frac{2\phi}{2b + c_2}} \quad \text{and} \quad \Pi^{\mathcal{MA}} = 2\sqrt{\phi}\left(\sqrt{\phi^m} - \sqrt{\phi}\right)$$

these values are well defined for $0 \leq \phi \leq \phi^m$. The quantity is increasing with $\phi$ from zero for $\phi = 0$ up to $q^m$ when $\phi = \phi^m$. The profit is increasing for $0 \leq \sqrt{\phi} \leq \sqrt{\phi^m}/2$ and then decreasing.

**For the cost-minimizing firm,**

$$= \sqrt{\frac{2\phi}{c_2}} \quad \text{and} \quad \Pi^{\mathcal{MI}} = \frac{2(b + c_2)}{c_2}\sqrt{\phi}\left(\sqrt{\phi^c} - \sqrt{\phi}\right)$$

these values are well defined for $0 \leq \phi \leq \phi^c$.

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