Flexclusivity: Exclusive Agreements and Competitive Flexibility*

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Abstract

Sellers often face a critical choice: open the bidding to multiple buyers to maximize potential offers, or favor a single, initially preferred buyer by limiting competition. Our key finding is that a powerful seller may accept a lock-in agreement from that buyer and hence give up the possibility of running an optimal auction. As a result, the seller chooses with positive probability to completely disregard alternative buyers. Simple option contracts are very effective at increasing the expected joint profit of the contracting parties. The joint gain from such contracts may represent 75% of what can be achieved by vertical integration.

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1 Introduction

A recurring phenomenon in the business world is the departure from competitive bidding practices. Frequently, buyers and sellers opt for direct negotiations without soliciting offers from multiple parties. For instance, in the realm of private sector nonresidential building construction projects in Northern California, Bajari, McMillan, and Tadelis (2009) observed that more than 43% of projects were awarded through one-on-one negotiations with suppliers, while the remainder followed some form of competitive bidding process. In a different context, Aktas, de Bodt, and Roll (2010) examined SEC filings spanning from 1994 to 2007 and concluded that approximately half of the deals were secured without the involvement of a competitive process. Similarly, Boone and Mulherin (2007) conducted an analysis of 400 takeovers involving major U.S. corporations in 1998 and 1999. Their findings revealed that 198 of these deals were settled through private takeover negotiations, often featuring only one potential buyer.

These observations pose a challenge to economists who typically praise the virtues of competition. Bulow and Klemperer (1996) seminal work, "Auctions Versus Negotiations" show, in a symmetric IPV environment that a revenue maximizing auction with n bidders provides a seller less profit than a standard auction with $n+1$ bidders. They underscore the value of auctions in maximizing shareholder value, citing the case of the 1993 attempt to sell Paramount to Viacom, where QVC's interest in bidding for Paramount prompted a legal dispute. Paramount and Viacom reached an agreement that effectively excluded other bidders, leading to a contested deal. This case serves as a stark example of the tension between negotiations and auctions in the pursuit of optimal outcomes, with the Delaware courts ultimately siding with the proponents of auctions.

To solve the puzzle, recent studies generally rely on the existence of moral hazard problems (see the literature review below). In the present paper, we put forward a different perspective, keeping the pure adverse selection environment of Bulow and Klemperer (1996). We simply add to their setting the possibility of contracting with a preferred buyer before valuations are known (henceforth, “ex ante”). We show that the contracting parties can combine the stability of an exclusive agreement (i.e., prohibiting solicitation of offers from other buyers) with the adaptability of an auction (i.e., allowing for the solicitation of offers from other buyers).

In practice, the seller and the preferred buyer agree on a buying option made of two elements: a fixed fee that determines the sharing their joint expected surplus and a strike price that governs the level of exclusivity. We call this strategic approach "flexclusivity" because it affords the parties the flexibility to determine their preferred level of exclusivity. Importantly, flexclusivity is established ex ante, and the degree (or probability) of exclusivity remains independent of other buyers' types. Once a flexclusivity contract is agreed upon, the ex ante partner privately discovers their type and decides whether to exercise the option.

In equilibrium, when the ex ante partner's valuation is sufficiently high, they opt to proceed with the purchase, resulting in exclusivity (a deal is reached without competitive bidding). Conversely, when the ex ante partner's valuation is relatively low, they opt not to proceed, and the seller initiates an auction.

The profitability of flexclusivity becomes evident when the ex ante partner, who has declined the exclusive deal, participates in the auction. In such auctions, it is commonly understood that the ex ante partner is in a weaker position, signifying a lower valuation. This scenario allows the seller to extract higher rents from other buyers, as a Myersonian optimal auction enables the seller to discriminate against strong bidders.

It is crucial to emphasize that this mechanism is neither collusive nor favoritism-based. When the seller reverts to competitive bidding, all buyers are treated equitably. In any auction, the seller maximizes her individual profit and treats her ex ante partner no differently from any other buyer.

Related literature Bulow and Klemperer (2009) examine the efficiency and revenue generation of auctions versus sequential sales mechanisms in asset selling, finding that auctions, despite being less efficient, often generate higher revenue for sellers. This preference is underscored by the tendency of auctions to (inefficiently) attract more bidders, thereby increasing competition and potential selling price. Their model has been extended by Roberts and Sweeting (2013) (see also Gentry and Stroup (2019)) who allow potential bidders to receive a signal of their valuation before their costly entry decision. They emphasize that the sequential mechanism can give both buyers and sellers significantly higher payoffs than the commonly used simultaneous bid auction.

Economists have met the challenge by emphasizing at least three forces. First, the good a firm want to procure might be complex. Combining cost dimensions which can be revealed by an auction with quality or design dimensions which are better dealt with through a negotiation. Goldberg (1977) is an early contribution along these lines. Manelli and Vincent (1995) show that when quality concerns are strong enough, the optimal procurement mechanism is a series of sequential take-it-or-leave-it offers (i.e., some extreme form of negotiation) made by the buyer to potential sellers rather than an auction (even when reserve prices are feasible). Bajari and Tadelis (2001) theoretically study the tension between providing ex ante incentives and avoiding ex post transaction costs due to costly renegotiation. The procurement literature investigates the complexity of the contract due to quality concerns. Second, the relational contract literature has highlighted the role of a long-term relationship in building trust and reputation in environments where contract enforcement is limited. Board (2011) studies a dynamic version of the holdup game under complete information. He considers a buyer who designs a contract to maximize her profit and must invest in at most one of the potential suppliers, with the chosen supplier having ex post all the bargaining power. The distribution of bargaining power changes over the course of the game and contracts are incomplete at the first stage. Calzolari and Spagnolo (2009, 2020) show, under incomplete information, that a buyer optimally restricts the number of selected suppliers to maintain suppliers' incentives to provide quality. They also have collusion. [1]

Our study may also provide a new explanation for the stickiness in business relationships found in the trade literature. As noted by Martin, Mejean, and Parenti (2023), stickiness has mostly been analysed in models featuring relationship-specific investments, search costs in the market for suppliers, market incompleteness, trust and reputation.[2] Our results also apply to procurement environments where the buyer has strong bargaining power and suppliers are affected by i.i.d. costs at each period. Allowing the buyer and the incumbent supplier to agree at the current period on a partial exclusivity agreement increases the probability that they still trade together at the following period.

The paper is organized as follows. Section 2 explains the role of flexclusivity with a simple example. Section 3 shows how it can be implemented with an option contract. Section 4 examines the extent to which more general contracts improve the contracting parties' joint profit and allow to get closer of what vertical integration can achieve.

[1] Empirical papers have provided support to the relational contract approach. In particular, Corts and Singh (2004) for the drilling industry and Macchiavello and Morjaria (2015) for Kenyan rose exporters.

[2] See Baker, Gibbons, and Murphy (2002), MacLeod (2007), and Malcomson (2012).

Macchiavello and Morjaria (2015). The cost of designing and running the auction, the risk of collusion between bidders (see for instance McAfee and McMillan (1992) or Marshall and Marx (2009)). Monarch (2021) study relationship-specific investments or search costs in the market for suppliers or switching costs see for an empirical study of these costs for U.S. firms importing from Chinese suppliers, may also justify to prefer dealing directly with the incumbent buyer (or supplier) rather than relying on a fully competitive mechanism.

2 Simple example

A seller, S, of a single good faces two potential symmetric buyers, C_0 and C_1. Their valuations, w_0 and w_1, are independently uniformly distributed between $[12, 24]$. Assume that for all $12 \leq x \leq 24$, an ex ante option contract exists such that if $w_0 < x$, then C_0 does not buy directly and an auction is organized, whereas if $w_0 > x$, then C_0 buys directly. When $x = 12$, S and C_0 have an exclusivity contract, whereas when $x = 24$ there is full competition –it is as if S and C_0 have no contract, as S always runs an auction.

In the revenue maximizing auction –when $w_0 < x$– S allocates the good to the buyer with the largest virtual valuation. C_1 wins when

$$w_1 - (24 - w_1) > w_0 - (x - w_0) \;\text{ or }\; w_1 > w_0 + (24 - x)/2 = m_1(w_0; x)$$

which for $x = 24$, simplifies into $w_1 > w_0$. Under flexclusivity, i.e. $12 < x < 24$, when C_1 wins the auction, he pays $m_1(w_0; x) > w_0$.[3] The trade-off is clear, conditional of C_1 winning, the lower x the greater the price paid. On the other hand, a lower x means that C_1 wins less often (even when $w_1 > w_0$).

To see if flexclusivity is worth it, let $\Pi_{S0}(x)$ denote the expected joint payoff of S and C_0. With only two symmetric buyers, it is easy to check that $\Pi_{S0}(12) = \Pi_{S0}(24) = 18$. The joint profit is the same under exclusivity or with full competition.[4] Subtracting $\Pi_{S0}(12) = \int_{12}^{24} \int_{12}^{24} w_0 dw_1 dw_0/144$, we have

$$\begin{aligned}
\Pi_{S0}(x) - \Pi_{S0}(12) &= \int_{12}^{x} \int_{m_1(w_0;x)}^{24} (m_1(w_0; x) - w_0) \frac{dw_1}{12} \frac{dw_0}{12} \\
&= \int_{12}^{x} \int_{m_1(w_0;x)}^{24} \frac{24 - x}{2} \frac{dw_1}{12} \frac{dw_0}{12} \\
&= \frac{1}{48}(24 - x)(x - 12)
\end{aligned}$$

which emphasizes the contribution of the event "C_1 buys" to the joint profit of S and C_0. Therefore $\Pi_{S0}(x)$ is a concave function of x, which is maximized for $x = 18$, and

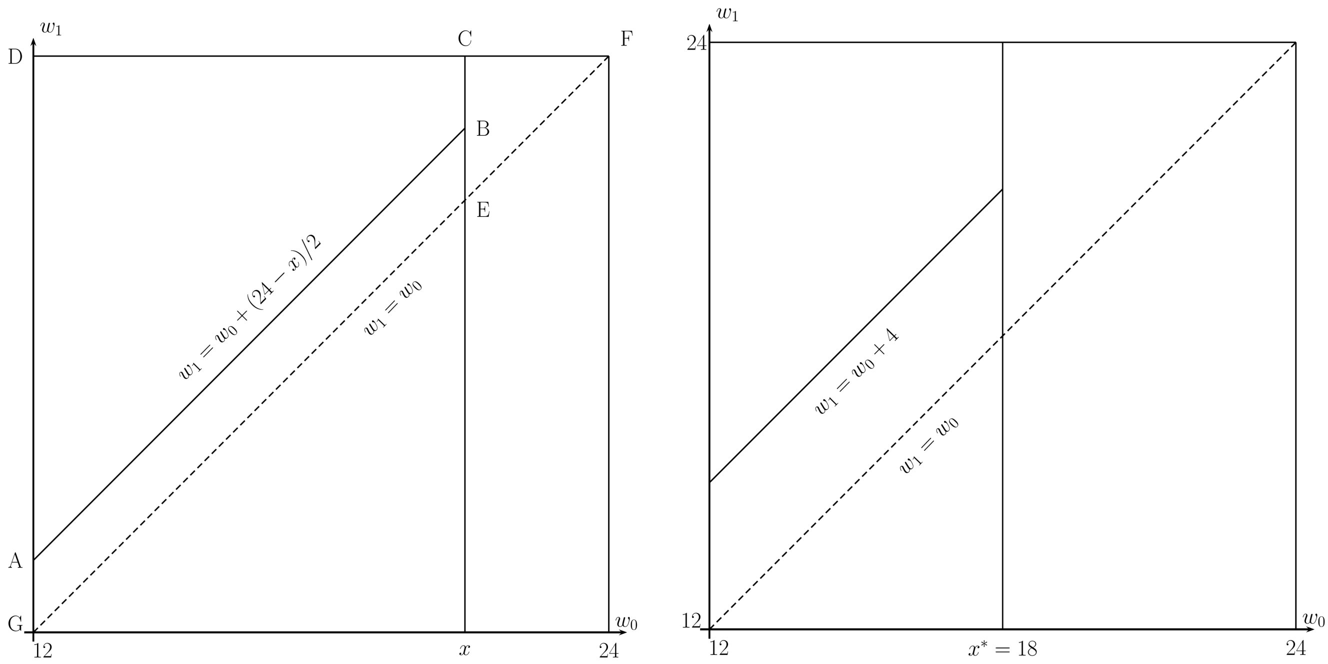
$$\Pi_{S0}(18) = 18.75 > 18 = \Pi_{S0}(24)$$

Thus flexclusivity is profitable. The optimal ex ante contract allocates, on expectation, the good to C_0 half the time (i.e. for $18 \leq w_0 \leq 24$). Otherwise S runs an optimal auction using the information that $12 \leq w_0 \leq 18$. This contract outperforms both exclusivity and full competion, and increases the expected joint profit by 4.16%.

Figure 1 illustrates this example. On Figure 1a, C_0 wins whenever $w_0 > x$ and when $w_0 < x$, whenever $w_1 < m_1(w_0; x) = w_0 + (24 - x)/2$. In comparison with the absence of an ex ante contract (i.e. $x = 24$) there are only three areas where the joint profit is potentially changed. First, the ABCD trapeze where C_1 wins whether there is a contract or not. But with a contract C_1 pays $m_1(w_0; x)$ whereas without a contract C_1 pays $w_0 < m_1(w_0; x)$. This is the area where the joint profit increases unambiguously. Second, in the ABEG parallelogram, C_0 wins the auction when the ex ante contract is in place whereas C_1 wins in the absence of contract. As, in this area, $w_1 > w_0$, it might seems that the S-C_0 pair is losing an opportunity. However, in the absence of contract, C_1 pays exactly w_0 and therefore the exclusion of C_1 is costless in this area. The third area, the CEF triangle is similar. Consequently, the goal of the S-C_0 pair is to choose, ex

[3] The lowest valuation, 12, is high enough compared to 24 that there is no binding reserve price here.

[4] Indeed, with two symmetric buyers, when C_1 wins –under full competition– the price is w_0, which is exactly the joint profit of S and C_0 under exclusivity.



(a) Revenue Maximizing Auction ex post for a given x

(b) Optimal x

Figure 1: Illustration of an ex ante contract

ante, x such as to maximize the expected value of $m_1(w_0; x)$ on the ABCD trapeze.

Table 1 details the payoffs of each player (assuming correctly –as shown in Section 3– that to implement $x = 18$, S and C_0 have to choose a strike price of $p = 15$ in their option contract). Whereas flexclusivity increases the join profit (239+61=300>288=256+32) S suffers a loss and C_0 enjoys a gain. For the implementation of flexclusivity it is then crucial that C_0 pays S ex ante. This ex ante payment should be above 17 and below 29 for both parties to agree. C_1's expected profit is deeply reduced due to both exclusion (half the time) and a higher price. Overall, the flexclusivity scheme reduces welfare (due to the symmetry and the absence of reserve price, the Myersonian auction implements the first best here).

Table 1: Payoffs comparison ($\times 16$)

	Myerson				Option ($p = 15$)			
	Π_S	U_0	U_1	W	Π_S	U_0	U_1	W
$w_0 < 18$	116	4	28	148	119	13	13	145
$w_0 > 18$	140	28	4	172	120	48	0	168
Total	256	32	32	320	239	61	13	313

Notice that the example ignores three forces. First, competition is minimal as C_1 is the sole competitor of C_0. A weak competition makes shutting down the auction less costly. In the presence of more competitors, the S-C_0 pair would lose profits in the areas ABEG and CEF. Because in an auction the winner would pay the second highest valuation which would not always be w_0. Second, the support of the valuations is such that there is no reserve price in the revenue maximizing auction. With reserve prices, the introduction of x would reduce the reserve price imposed by S to C_0 in case of an auction and therefore would reduce the inefficient area where both competitors have valuation below the reserve prices. Consequently, binding reserve prices would make the ex ante contract more attractive. Finally, the valuation of C_0 and C_1 have been assumed symmetrical here. Asymmetry could make the ex ante contract less attractive as we see below.

3 Option contract

A monopolistic firm S and an ex ante partner C_0 contemplate a future business opportunity. This framework, written in terms of surpluses, encompasses both the case where S is a seller as well as the procurement case where S wants to buy a fixed quantity –see Appendix A. These two cases can be viewed as extreme cases of more general business relationships where two firms work together to create a surplus.

There are two time periods. At time 1 ("ex ante"), S and C_0 have the ability to agree on a buyer/selling option characterized by a strike price p. That is, they can implement flexclusivity. At time 2 (i.e. "ex post"), C_0 and $n \geq 1$ competitors $C_1, \ldots, C_n$ privately receive signals about the surpluses they can create with S. These surpluses are denoted w_0, $w_1, \ldots, w_n$, and are drawn from distributions F_j with supports $[\underline{w_j}, \overline{w_j}]$, $j = 0, \ldots, n$. We assume that for all j the virtual valuation function

$$\Psi_j(w) = w - \frac{1 - F_j(w)}{f_j(w)}$$

increases with w. At time 2 ("ex post"), C_0, after observing his type w_0, decides whether to exercise the option. If he does, his profit is $w_0 - p$. If he does not, S runs an optimal auction à la Myerson (1981) involving all potential partners, including C_0. Importantly, once C_0 has declined to use the option, S and C_0 are no longer bound by a contract: S updates her beliefs about C_0's type and maximizes her own profit at the auction stage. Similarly, C_0 participates freely to the mechanism proposed by S.

Ex ante, there is no asymmetric information: S and C_0 choose the strike price p to maximize the (expected) sum of their profits.[5]

Ex post, a Bayesian equilibrium is characterised by the ex ante partner's decision to exercise the option and by the principal's belief regarding the probability distribution of C_0's types w_0 that are present in the auction. In equilibrium, decisions and beliefs are consistent: (i) C_0's decision is optimal given the belief of S; (ii) the belief of S respects the Bayes rule given C_0's decision.

Proposition 1 shows that the strike price p controls the degree of flexclusivity.

Proposition 1 (Ex post equilibrium)**.** *For any strike price p, there exists a unique Bayesian equilibrium ex post. The equilibrium is characterized by a threshold x^* such that the ex ante partner exercises the option if and only if w_0 is greater than or equal to x^*. The threshold x^* increases from $\underline{w}_0$ to $\overline{w}_0$ as p rises from $\underline{w}_0$ to $\overline{w}_0$.*

Proof. See Appendix B.1 □

Thus, there exists a one-to-one relationship between the strike price p and the exercise threshold $x^*(p)$, and the ex ante partners can maximize their expected joint surplus by choosing a threshold x. To enjoy full exclusivity, S and C_0 can set the threshold at $\underline{w}_0$. On the other hand there is full competition when the threshold is set at $\overline{w}_0$. Finally, when x is set at an intermediate value, flexclusivity rules. The option allows

[5] We do not have to specify how they share the expected joint profit. If S has the upper hand in the ex ante negotiation, she would make a take-it-or-leave-it offer to C_0. However, one can also envision configurations where C_0 is the one with all the bargaining power, as well as any intermediary situations.

the ex ante partners to control the probability that S considers alternative partners. We now show that under broad conditions on supports of the surpluses, this probability is indeed below one in equilibrium.

3.1 One alternative partner

We start the analysis with the case $n = 1$, generalizing our simple example to asymmetric distributions. For now, we assume that the optimal Myerson auction features no reserve prices.[6] If S and C_0 have agreed on the exclusivity threshold $x \in [\underline{w}_0, \overline{w}_0]$, and C_0 has not excised the option, the belief of S regarding w_0 obtains by truncation of F_0

$$\tilde{F}_0(w_0; x) = \frac{F_0(w_0)}{F_0(x)}$$

for $w_0 \leq x$. When the auction takes place ($w_0 \leq x$), we denote by

$$\tilde{\Psi}_0(w_0; x) = w_0 - \frac{1 - \tilde{F}_0(w_0; x)}{\tilde{f}_0(w_0; x)} = w_0 - \frac{F_0(x) - F_0(w_0)}{f_0(w_0)}$$

the virtual valuation of C_0. The competitor C_1 wins the Myerson auction when $\Psi_1(w_1) \geq \tilde{\Psi}_0(w_0; x)$ and then pays

$$m_1(w_0; x) = \Psi_1^{-1}\left(\tilde{\Psi}_0(w_0; x)\right). \tag{1}$$

Conditionally on w_0, the profit of the ex ante partners is therefore

$$F_1(m_1)w_0 + [1 - F_1(m_1)]\, m_1 = w_0 + \Pi_1(m_1; w_0),$$

where

$$\Pi_1(m; w_0) = (m - w_0)\ [1 - F_1(m)] \tag{2}$$

is the standard monopoly profit for a cost w_0 and demand $1 - F_1(m)$. For any given w_0, the profit $\Pi_1(m_1(w_0; x); w_0)$ represents the ex ante partners' net gain of running the Myerson auction relative to remaining exclusive.

When the auction takes place, the ex ante partners are independent and the seller does not know the precise value of w_0 (she only knows that w_0 is lower than x). It follows that $m_1(w_0; x)$ does *not* maximize $\Pi(m; w_0)$. Specifically, for any $w_0 < x$, we have

$$\begin{aligned}
\frac{\partial \Pi_1(m_1(w_0; x); w_0)}{\partial m} &= f_1(m_1(w_0; x)) \left[w_0 - m_1(w_0; x) - \frac{1 - F_1(m_1(w_0; x))}{f_1(m_1(w_0; x))}\right] \\
&= f_1(m_1(w_0; x))\ [w_0 - \Psi_1(m_1(w_0; x))] \\
&= f_1(m_1(w_0; x))\, \frac{F_0(x) - F_0(w_0)}{f_0(w_0)} > 0.
\end{aligned} \tag{3}$$

As a result, when $w_0 < x$, the payment $m_1(w_0; x)$ is lower than the value of m that maximizes $\Pi_1(m; x)$, which we denote hereafter by $m_1^*(x) = m_1(x; x) = \Psi_1^{-1}(x)$. Only when $w_0 = x$, does the payment $m_1(x; x)$ maximize $\Pi_1(m; x)$, and we denote the maximum value of the profit by $\Pi_1^*(x) = \max_m \Pi_1(m; x)$.

The expected joint gain of the ex ante partners is given by

$$\mathbb{E}\,\Pi_{S0} = \int_{\underline{w}_0}^{\overline{w}_0} w_0 \,\mathrm{d}F_0(w_0) + \int_{\underline{w}_0}^{x} \Pi_1(m_1(w_0; x); w_0) \,\mathrm{d}F_0(w_0). \tag{4}$$

[6] The role of reserve prices is discussed in Section 4.1.

A slight increase in the threshold x has two effects on the expected joint profit:

$$\frac{\mathrm{d}\,\mathbb{E}\,\Pi_{S0}}{\mathrm{d}x} = \Pi_1(m_1(x;x);x)f_0(x) + \int_{\underline{w}_0}^{x} \frac{\partial \Pi(m_1;w_0)}{\partial m}\frac{\partial m_1(w_0;x)}{\partial x}\,\mathrm{d}F_0(w_0). \tag{5}$$

The first term reflects the gain of the profit caused by running the auction more frequently. The second term reflects the loss in profit due to reduced competitive pressure placed on the alternative buyer. That term is negative because

$$\frac{\partial m_1(w_0;x)}{\partial x} = -\frac{1}{\Psi_1'(m_1(w_0;x))}\frac{f_0(x)}{f_0(w_0)} \leq 0. \tag{6}$$

Lemma 1 summarizing our findings so far on the derivative of the expected joint profit with respect to the flexclusivity threshold x.

Lemma 1. *Suppose there is one alternative buyer ($n = 1$). Suppose furthermore that reserve prices are not binding. The derivative of the ex ante partners' expected joint profit is*

$$\frac{\mathrm{d}\,\mathbb{E}\,\Pi_{S0}}{\mathrm{d}x} = f_0(x)\left\{\Pi_1^*(x) - \int_{\underline{w}_0}^{x} \frac{f_1(m_1(w_0;x))}{\Psi_1'(m_1(w_0;x))}\frac{F_0(x)-F_0(w_0)}{f_0(w_0)}\,\mathrm{d}w_0\right\}. \tag{7}$$

Proof. See Appendix B.2 □

We can now state our main result.

Proposition 2. *If $\overline{w}_1$ is lower than or equal to $\overline{w}_0$, the auction takes place with probability strictly lower than one. If $\overline{w}_1$ is lower than or equal to $\underline{w}_0$, it never takes place.*

Proof. When $\overline{w}_1 \leq \overline{w}_0$, we have $\Pi_1^*(\overline{w}_0) = 0$, and it follows from Lemma 1 that the derivative of the expected joint profit is negative at $x = \overline{w}_0$. The optimal exclusivity threshold x^* must therefore satisfy $x^* < \overline{w}_0$.

When $\overline{w}_1 \leq \underline{w}_0$, we have $\Pi_1^*(x) = 0$ for all $x \in [\underline{w}_0, \overline{w}_0]$, implying that the expected joint profit decreases with x on the whole interval, hence $x^* = \underline{w}_0$. The ex ante partners remain exclusive with probability one. □

Proposition 2 implies that with ex ante symmetric buyers the seller is willing to commit to ignore the alternative buyer with a positive probability. The intuition is that by committing not to consider the alternative buyer when the ex ante partner is strong, the seller manipulates the strength of that partner should an auction take place. Specifically the seller knows that the ex ante partner (with whom contractual ties no longer exist at the auction stage) is a weak bidder. She has therefore an incentive to bias the auction in favor of that buyer, i.e., to be relatively more aggressive with the other supplier. Weakening the ex ante partner allows to extract more surplus from the alternative buyer.

Example Assume w_0 and w_1 are independently and uniformly distributed on $[\underline{w}_0, \overline{w}_0]$ and $[\underline{w}_1, \overline{w}_1]$, with $\underline{w}_0 < \overline{w}_1$. We check in Appendix B.4 that the optimal exclusivity threshold is given by

$$x^* = \min\left\{\frac{\underline{w}_0 + \overline{w}_1}{2}, \overline{w}_0\right\} \tag{8}$$

as $\underline{w}_0 < \overline{w}_1$, x^* is larger than $\underline{w}_0$. If $\overline{w}_1 < \overline{w}_0 + (\overline{w}_0 - \underline{w}_0)$, then x^* is also lower than $\overline{w}_0$. If $\overline{w}_1 < \overline{w}_0 + (\overline{w}_0 - \underline{w}_0)$, then an auction always takes place ($x^* = \overline{w}_0$). The probability that the seller considers the alternative buyer, $\min\{(x^* - \underline{w}_0)/(\overline{w}_0 - \underline{w}_0), 1\}$, is one when $\overline{w}_1$ is large enough and tends to zero as $\overline{w}_0$ grows.

Except for the required condition that $x^* \leq \overline{w}_0$, the value of x^* does not depend on $\overline{w}_0$.

For instance if $\underline{w}_0 = \underline{w}_1 = .5$ and $\overline{w}_1 = 1$, x remains equal to .75 as long as $\overline{w}_0 \geq .75$.

For instance if $\overline{w}_0 = .9$, the seller works under exclusivity with C_0 when $.75 \leq w_0 \leq .9$; it follows that the seller commit not to consider the alternative buyer C_1 with probability $.15/.4 = 37.5\%$ even though C_0 is known to be ex ante less efficient.

For instance if $\underline{w}_0 = .5$, $\overline{w}_0 = \underline{w}_1 = 1$, and $\overline{w}_1 = 1.3$, then $x^* = 0.9$, and the seller commit not to consider the alternative buyer C_1 with probability $.1/.5 = 20\%$ even though C_0 is known to be ex ante less efficient.

3.2 Multiple alternative buyers

We now assume that there are $n \geq 1$ alternative buyers. The expected joint profit of the seller and the ex ante buyer is given by

$$\mathbb{E}\,\Pi_{S0} = \int_{\underline{w}_0}^{\overline{w}_0} w_0 \, \mathrm{d}F_0(w_0) + \int_{\underline{w}_0}^{x} \left[A(w_0;x) + B(w_0;x)\right] \mathrm{d}F_0(w_0), \tag{9}$$

where $A(w_0;x)$ and $B(w_0;x)$ respectively account for the cases

1. $\tilde{\Psi}_0(w_0) \leq \Psi_1(w_{(n-1)})$: two alternative buyers are stronger than the ex ante partner, so the winner pays $w_{(n-1)}$;
2. $\Psi_1(w_{(n-1)}) \leq \tilde{\Psi}_0(w_0;x) \leq \Psi_1(w_n)$: only one alternative buyer is stronger than the ex ante partner, so the winner pays $m_1(w_0;x)$.

Using the density of $w_{(n-1)}$, namely $n(n-1)F_1(w)^{n-2}f_1(w)(1-F_1(w))$, we have

$$A(w_0;x) = \int_{m_1(w_0;x)}^{\overline{w}_1} (w - w_0) n(n-1) F_1(w)^{n-2} f_1(w)(1-F_1(w))\,\mathrm{d}w.$$

Using $\Pr(w_{(n)} \geq m_1 | w_{(n-1)} = w) = [1 - F_1(m_1)]/[1 - F_1(w)]$, we get

$$\begin{aligned} B(w_0;x) &= \int_{\underline{w}_1}^{m_1} (m_1 - w_0) \frac{1 - F_1(m_1)}{1 - F_1(w)} n(n-1) F_1(w)^{n-2} f_1(w)(1-F_1(w))\,\mathrm{d}w \\ &= n\Pi_1(m_1;w_0)F_1(m_1)^{n-1}, \end{aligned}$$

where $m_1 = m_1(w_0;x)$ is given by (2).

It follows that Proposition 2 generalizes with multiple alternative partners. The seller is willing to commit to ignore those partners with a positive probability.

Proposition 3. *Suppose there are $n \geq 1$ alternative buyers with iid valuations. Suppose furthermore that reserve prices are not binding. If $\overline{w}_1 \leq \overline{w}_0$ the auction takes place with probability strictly lower than one.*

Proof. See Appendix B.3. □

Example (cont'd) Suppose the valuations of all buyers are uniformly distributed and the reserve pricing are not binding. Then the probability that an auction takes place is reported in Table 2.

Figure 2 plots the expected joint profits of S and C_0 as functions of the exclusivity threshold x for $n = 1$ to 5. The top curve is for $n = 5$ and the bottom curve, for $n = 1$. The ex ante partners benefit from the presence of more potential competitors, and the larger n, the larger the probability the auction stage is

Table 2: Equilibrium when buyer valuations are iid and uniformly distributed

Number of alternative buyers	1	2	3	4	5
Proba. auction takes place (in %)	50.00	60.47	69.46	75.71	79.92
Gain in expected joint profit (in %)	4.17	2.34	1.30	0.80	0.54

Note: Gains are relative to the Myerson revenue-maximizing auction (independent seller). The support of the distribution is such that the sell always occurs (no reserve price).

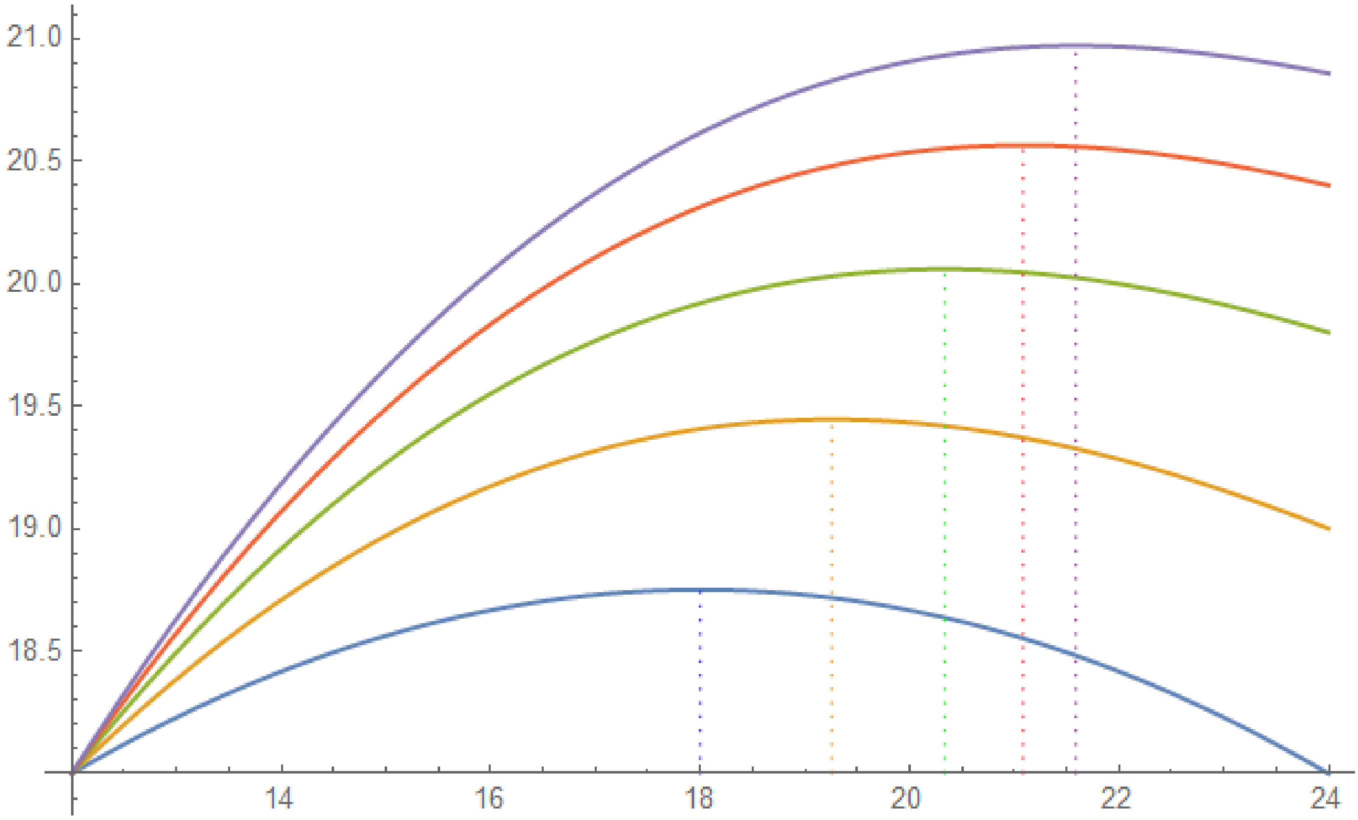


Figure 2: Expected joint profits for $n = 1$ (bottom curve) to 5 (top curve)

reached. Yet, in the case of iid uniform distributions, even for $n = 5$ there is 20% chances that the deal is closed between S and C_0 without looking at the competition.

Finally, we examine the robustness of our results when reserve prices are binding.

4 Extensions

We first examine the welfare impact of the option contract, allowing for reserves prices and asymmetric distributions and then discuss stochastic contracts and vertical integration. For all extensions, we assume that $n = 1$.

4.1 Welfare analysis

In the absence of any contractual arrangement, when w_0 and w_1 are iid, and the reserve price is not biding, S's market power does not affect the allocation nor welfare. In such a symmetric framework without reserve prices, the option contract reduces welfare by allocating too often the market to C_0. Indeed, when $x < w_0 < w_1$, the market is inefficiently allocated to C_0. Inefficiency is also present when the auction stage is reached, i.e. when $w_0 < w_1 < m_1(w_0; x)$. So even if the ex ante contract does not directly interfere with the competitive stage, it creates efficiency concerns.

Asymmetric distributions In an asymmetric environment S's market power (i.e., the ability to organize a revenue maximizing auction), distorts welfare, and our ex ante option contract can reduce or amplify this distortion.

Lemma 2. *Assume w_0 and w_1 are uniformly distributed on $[\underline{w_0}, \overline{w_0}]$ and $[\underline{w_1}, \overline{w_1}]$, respectively. If $\overline{w_0} < \overline{w_1}$, the revenue maximizing auction is distorted in favor of C_0 and this distortion is exacerbated by the option contract. On the other hand, if $\overline{w_0} > \overline{w_1}$, the revenue maximizing auction is distorted in favor of C_1 and this distortion is reduced by the option contract.*

Reserves prices Weaking the ex ante partner type leads to lower the reserve price for that buyer, hence a positive effect on the welfare.

Lemma 3. *The ex ante option contract can increase welfare when reserve prices are binding.*

Proof. For symmetric uniform distributions over $[0, b]$, the welfare is $W_{RMA} = 7b/12$ for a revenue maximizing auction, whereas for the option contract the welfare is $W_{Opt} = (343 + 13\sqrt{13})b/648$ which is always larger. Both are lower than $2b/3$ which is the first-best welfare. □

The idea behind Lemma 3 is simply that the reserve price is lowered for C_0, e.g. from $b/2$ to $x/2$ for uniform distribution. This generates a welfare gain as welfare increases from zero to w_0. The welfare losses are still present but their magnitude is $w_1 - w_0$ and in the case of uniform distributions they are more than compensated by the gain.

4.2 Choice of partner

4.3 Bargaining weights

We have assumed above that if the ex ante partner does not exercise the option, the seller has all the bargaining power vis-à-vis the buyers. We now consider more balanced bargaining environments. Following Loertscher and Marx (2019) and Loertscher and Marx (2022), we model the bargaining process (should it take place) as the incentive-compatible mechanism that maximizes the weighted surplus $\mathbb{E}\,\Pi_S + \mu\mathbb{E}\,U_0 + \mu\mathbb{E}\,U_1$, where the bargaining weights are one for the seller and $\mu < 1$ for the buyers. In this context, the virtual valuations $\Psi_1(w_1)$ and $\tilde{\Psi}_0(w_0; x)$ must be replaced with respectively

$$\Psi_1(w_1; \mu) = w_1 - (1-\mu)\frac{1 - F_1(w_1)}{w_1} \quad \text{and} \quad \tilde{\Psi}_0(w_0; x, \mu) = w_0 - (1-\mu)\frac{F_0(x) - F_0(w_0)}{f_0(w_0)},$$

and the payment $m_1(w_0; x, \mu)$ given by (1) must be changed accordingly. All the above analysis, including Proposition 1, carries over to that case. The important point is that $\tilde{\Psi}_0(w_0; x, \mu)$ decreases with x for any $\mu < 1$. Equation (6) becomes

$$\frac{\partial m_1(w_0; x)}{\partial x} = -(1-\mu)\frac{1}{\Psi_1'(m_1(w_0; x))}\frac{f_0(x)}{f_0(w_0)} \leq 0. \tag{10}$$

The derivative of the ex ante partners' expected joint profit is changed as follows:

$$\frac{\mathrm{d}\,\mathbb{E}\,\Pi_{S0}}{\mathrm{d}x} = f_0(x)\left\{\Pi_1^*(x^*) - (1-\mu)\int_{\underline{w}_0}^{x^*} \frac{f_1(m_1(w_0;x^*,\mu))}{\Psi_1'(m_1(w_0;x^*,\mu))}\,\frac{F_0(x^*)-F_0(w_0)}{f_0(w_0)}\,\mathrm{d}w_0\right\}, \tag{11}$$

which yields Proposition 2. When the valuations w_0 and w_1 are independently and uniformly distributed on $[\underline{w}_0, \overline{w}_0]$ and $[\underline{w}_1, \overline{w}_1]$, with $\underline{w}_0 < \overline{w}_1$, we check in Appendix B.4 that for $\mu \in [0,1]$ the optimal exclusivity threshold is given by

$$x^* = \frac{\underline{w}_0\,\sqrt{1-\mu} + \overline{w}_1}{\sqrt{1-\mu}+1}, \tag{12}$$

provided that the solution is interior. Because the exclusivity threshold increases with μ, exclusivity is more likely when the seller is more powerful. If the distributions are symmetric, the threshold increases from $(\underline{w}+\overline{w})/2$ to $\overline{w}$ as μ rises from 0 to 1. The case $\mu = 1$ corresponds to the second-price auction, a situation where weak and strong buyers are treated the exact same way.[7]

4.4 Stochastic ex ante contracting

Our simple option contract leads to a partition of the interval $[\underline{w}_0, \overline{w}_0]$ into two sub-intervals $[\underline{w}_0, x^*]$ and $[x^*, \overline{w}_0]$. If w_0 lies in the former sub-interval, S runs a revenue-maximizing auction for sure. If w_0 lies in the second sub-interval, S ignores alternative suppliers and allocates the good to C_0 for sure. It is natural to ask whether the ex ante partners can increase their expected joint profit by refining the partition and using stochastic contracts.

As before, C_0 and S contract ex ante, and once the market is open to competition they are no longer linked by any contractual relationship. Events unfold as follows:

1. Ex ante contracting stage: C_0 and S may agree on
   - an increasing sequence $(x_k)_{k=0}^{K}$ with $x_0 = \underline{w}_0$ and $x_K = \overline{w}_0$;
   - a corresponding sequence of price p_k;
   - a sequence of probabilities $\pi_k \in [0,1]$;
2. Nature determines costs;
3. Interim phase:
   - Communication: C_0 reports that w_0 belongs to $I_k = (x_k; x_{k+1})$;
   - Application of the contract: with probability π_k, the good is sold to C_0 at price p_k ;
4. End of contractual relationship between C_0 and S;
5. If no deal has been struck bilaterally, the market is open to competition. At this point, S and C_0 are independent players, free of any contractual obligation: S maximizes her profit independently and C_0 freely decides whether to participate in any auction organized by S.

The option contract studied in section 3 corresponds to a simple sequence $(x_0 = \underline{w}_0,\ x_1 = x^*,\ x_2 = \overline{w}_0)$, with $\pi_0 = 0$ and $\pi_1 = 1$, and p_1 is the exercise price.

[7] By contrast, if a first-price auction takes place in the event the ex ante partner does not exercise the option, then there is exclusivity with positive probability (close to .5 in the uniform case). Details are available upon request.

Assuming the good has not been allocated to C_0, S maximizes her profit knowing that $w_0 \in I_k$. We model the revenue-maximizing auction S as a direct mechanism $(\mathbf{Q}(\mathbf{w};k), \mathbf{M}(\mathbf{w};k))$, asking each participant j to report his type and allocating the good with $Q_j(\mathbf{w};k)$ and payment $M_j(\mathbf{w};k)$. If C_0 chooses to participate, his indirect utility is:

$$U_0(w_0;k) = \max_{\hat{w}_0 \in I_k} q_0(\hat{w}_0;k)w_0 - m_0(\hat{w}_0;k)$$

where $q_0(\hat{w}_0;k)$ and $m_0(\hat{w}_0;k)$ are the expected probability of winning and the expected payment conditional on w_0.

Interim incentive compatibility If the buyer of type w_0 announces $w_0 \in I_k$ at the communication stage, and does not get the good interim, then he has the possibility but not the obligation to participate in the subsequent auction with the alternative buyers. He will therefore earn $\max(0, U(w_0;k)$. The indirect utility of the ex ante partner at the interim stage is

$$V_0(w_0) = \max_k \pi_k(w_0 - p_k) + (1 - \pi_k)\max(0, U_0(w_0;k))$$

Proposition 4. *Incentive compatibility at the interim stage requires that the probability of striking a private deal satisfies*

$$\pi_{k+1} - \pi_k \geq q_0(x_{k+1};k)(1 - \pi_k), \tag{13}$$

where $q_0(x_{k+1};k)$ is the probability that the buyer wins the good at the auction organized for $w_0 \in I_k = (x_k, x_{k+1})$ when his type is $w_0 = x_{k+1}$.

Proof. Consider two values, $w_0 \in I_k$ and $w_0' \in I_{k+1}$, for the type of the ex ante partner. The following two incentive compatibility constraints should hold (they are not the only ones)

$$\begin{aligned} \pi_k(w_0 - p_k) + U_0(w_0;k) &\geq \pi_{k+1}(w_0 - p_{k+1}) + \max(0, U_0(w_0;k+1)) \\ \pi_{k+1}(w_0' - p_{k+1}) + U_0(w_0';k+1) &\geq \pi_k(w_0' - p_k) + \max(0, U_0(w_0';k)). \end{aligned}$$

Observe first that $U_0(w_0;k+1) < 0$. The reason is that C_0 cannot be better than reporting x_{k+1} in the auction, thus earning a negative profit, so if C_0 falsely reports $w_0 \in I_k$ he does not participate in the subsequent auction (should it take place). Second, because the lowest type has no rent in the auction, $U_0(w_0';k+1)$ is arbitrarily small when w_0' arbitrarily close to x_{k+1}. In this circumstance, adding the two inequalities above yields

$$(\pi_{k+1} - \pi_k)(w_0' - w_0) \geq U_0(w_0';k) - U_0(w_0;k).$$

If w_0' falsely reports $w_0' \in I_k$, we have seen in the proof of Proposition 1 that his optimal report is $\hat{w}_0 = x_{k+1}$ and that for this reason $U_0(w_0;k)$ is linear with slope $q_0(x_{k+1};k)$ on I_{k+1}. For w_0 close to x_{k+1}, we have, by continuity of q_0: $U_0(w_0';k) - U_0(w_0;k) \approx (w_0' - w_0)q_0(x_{k+1};k)$. Dividing by $w_0' - w_0 > 0$ yields (13). □

The expected joint profit of the ex ante partners is

$$\mathbb{E}\,\Pi_{S0} = \int_{\underline{w}_0}^{\overline{w}_0} w_0 \,\mathrm{d}F_0(w_0) + \sum_k (1 - \pi_k) \int_{x_k}^{x_{k+1}} \Pi_1(m_1(w_0;k);w_0)\,\mathrm{d}F(w_0), \tag{14}$$

where the profit function $\Pi_1(m;w_0)$ is given by (2) and the amount $m_1(w_0;k)$ paid by the alternative buyer is given by

$$m_1(w_0;k) = \Psi_1^{-1}(\tilde{\Psi}_0(w_0;k)),$$

with $\tilde{\Psi}_0(w_0; k)$ being the virtual valuation of C_0 in interval I_k. We can rewrite the incentive constraint (13) as

$$1 - \pi_{k+1} \leq [1 - \pi_k] \, [1 - q_0(x_{k+1}; k)] \tag{15}$$

As we want to maximize the probability of the auction, we see that the above incentive constraints bind and hence

$$1 - \pi_k = [1 - q_0(x_1; 0)] \, [1 - q_0(x_2; 1)] \ \ldots \ [1 - q_0(x_k; k_1)]. \tag{16}$$

Table 3 reports the expected gain when the buyers' valuations are uniformly distributed. The threshold $x^* = x_1^*$ that partitions the interval $[12, 24]$ are $x_1 = 18$ for the option contract, which coincides with the optimal two-part stochastic contract. The optimal three-part contract has thresholds $(x_1^*, x_2^*) = (17.53, 20.76)$, with the probability if interim allocation being $\pi_1^* = .87$ when w_0 belongs to the middle interval (x_1^*, x_2^*). The optimal four-part contract has thresholds $(x_1^*, x_2^*, x_3^*) = (17.51, 20.64, 22.32)$ with the interim probabilities being $\pi_1^* = .73$ and $\pi_2^* = .96$. Whatever the number of sub-intervals, the interim probability π^* is zero in the bottom interval and is constrained by incentive compatibility to be higher than some lower bound in the top interval.[8]

Table 3 shows that when the buyers' valuations are uniformly distributed the incremental gain brought by refining the partition from three to four sub-intervals is negligible.

Table 3: Expected joint profit under different contracting arrangements

Contractual environment	Gain in expected joint profit (in %)
Option contract	4.17
Stochastic contract (three-part)	4.32
Stochastic contract (four-part)	4.32
Vertical integration	5.56

Note: Gains are relative to the Myerson auction (independent seller). One alternative potential buyer. Buyers' valuations are i.i.d. with uniform distribution. (The support of the distribution is such that the sell always occurs (no reserve price).)

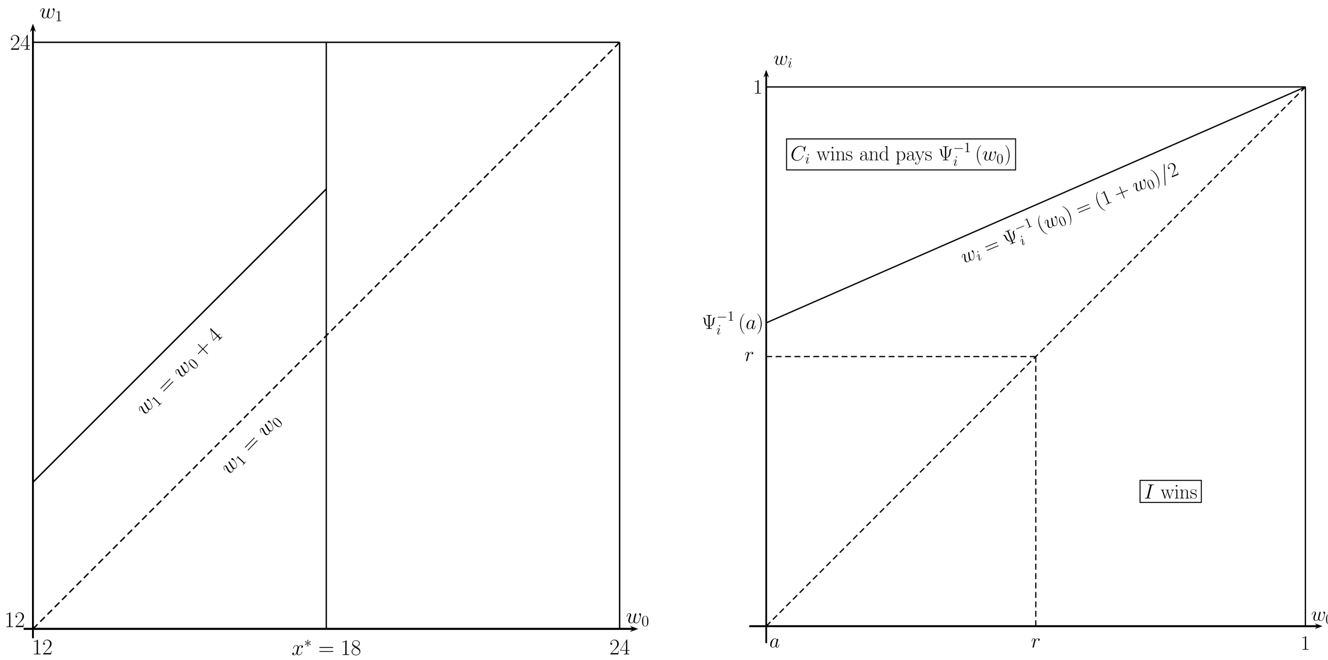
4.5 Vertical integration

The maximal expected joint profit that S and C_0 can achieve together obtains under vertical integration. When S and C_0 maximize their joint profit, the informational asymmetry between S and C_0 about w_0 plays no role because that information extraction by S is costless, hence S imposes a reserve price $\Psi_i^{-1}(w_0)$ in the auction, see Appendix B.5.

Proposition 5. *If S and C_0 are vertically integrated, S runs a revenue-maximizing auction with reserve prices $\Psi_i^{-1}(w_0)$. The joint expected profit obtained under vertical integration cannot be achieved by a contract ending before the seller considers alternative buyers.*

Proof. Imposing the reserve price w_0 in the auction requires that S perfectly knows w_0 at the end of the contractual period, i.e., that the contracting equilibrium is fully separating. But this is not incentive

[8]With only one alternative buyer and with iid valuations, the alternative buyer pays $m_1 = w_0$ when he wins the auction in the top interval. It follows that the exact value of the interim probability π does not affect the expected joint profit.



(a) Option contract

(b) Joint Revenue Maximizing Auction

Figure 3: Comparison for $n = 1$ and symmetric uniform distributions

compatible. Take $w_0 < w_0'$. If w_0 mimics w_0', he would gain a negative payoff in the auction and therefore will not take part in it. If w_0' mimics w_0, he will report w_0 in the auction and earn $(w_0' - w_0)q_0(w_0; w_0)$ where $q_0(w_0; w_0) = \Pr(w_0 \geq \max_{j=1,\ldots,n} \Psi_j(w_j) \,|w_0)$, hence the incentive constraints:

$$\begin{aligned} \pi(w_0)(w_0 - p(w_0)) &\geq \pi(w_0')(w_0 - p(w_0')) \\ \pi(w_0')(w_0' - p(w_0')) &\geq \pi(w_0)(w_0' - p(w_0)) + (w_0' - w_0)q_0(w_0; w_0). \end{aligned}$$

Adding up, we get

$$(\pi(w_0') - \pi(w_0))(w_0' - w_0) \geq (w_0' - w_0)q_0(w_0; w_0),$$

or $\pi(w_0') - \pi(w_0) \geq q_0(w_0; w_0)$. The interim probability π should jump at any point $w_0 \in [\underline{w}_0, \overline{w}_0]$, with the jump being positive and increasing in w_0, which is impossible. □

Achieving the maximal joint profit implies for S and C_0 to fully share the information about w_0. Figure 3 illustrates the difference between the vertical merger and ex ante contracting.

This maximal joint profit can be obtained without integration by a contracting arrangement suggested in Burguet and Perry (2009), whereby S pays C_0 in return for the information about w_0 and the now-informed S runs a revenue maximizing auction *to which C_0 is contractually forced to participate*. Hence this contract, which replicates vertical integration, requires S and C_0 to be contractually tied during the competitive phase when S interacts with the alternative buyers.[9]

Table 3 shows the gain in expected joint profit from the option represents in this example 75% of what can be achieved by vertical integration. Ex ante contracting can thus go a long way towards the full

[9]Another implementation proposed by Hua (2007) consists in delegating the decision to the informed party: S would sell her the project to C_0who could invite the competitors to participate in a tender. This requires buyer power to be transferable from S to C_0 – a very strong condition in practice. The most intuitive implementation, however, is a discriminatory ascending auction followed by a take-it-or-leave-it offer by S to the auction winner (???).

maximization of the parties' joint profit.

APPENDIX

A The role of S

Selling.– A monopolistic seller S has one unit of an indivisible good for sale. She can sell it to her current partner C_0 or to n potential competing buyers. The incumbent's valuation for the good is v_I while that of buyer i's is v_i. Assuming S has no cost, when the good is sold at price p_i to buyer i, the seller gets p_i and the buyer $v_i - p_i$. In terms of the common framework, buyer i provides utility $u_i = p_i$ to S, the total surplus is $w_i = v_i$ and the buyer profit $w_i - u_i = v_i - p_i$.

Procurement.– A monopolistic buyer S needs to procure a good in fixed quantity. She can source it from her incumbent supplier C_0 or from n other potential suppliers. Supplier C_0's production cost is denoted by c_i. When the buyer purchases from supplier i at price p_i, she earns $\theta - p_i$, while the supplier earns $p_i - c_i$. In terms of the common framework, the deal generates a total surplus $w_i = \theta - c_i$, the utility of S is $u_i = \theta - p_i$ and the profit of supplier i is, indeed, $w_i - u_i$.

B Proofs

B.1 Proof of Proposition 1

Suppose that S and C_0 have agreed ex ante on an option with strike price p.

First, we characterize an ex post equilibrium (assuming its existence). Suppose that the preferred partner has not exercised the option and denote by $\tilde{F}_0$ the belief of S regarding the distribution of w_0.

The principal runs a Myerson auction which we model as a direct mechanism $(\mathbf{M}(\hat{\mathbf{w}}; \tilde{F}_0), \mathbf{Q}(\hat{\mathbf{w}}; \tilde{F}_0))$, where $\hat{\mathbf{w}} = (\hat{w}_0, \ldots, \hat{w}_n)$ is the vector of reports, $\mathbf{M} = (M_0, \ldots, M_n)$ and $\mathbf{Q} = (Q_0, \ldots, Q_n)$ denote the payments and the probabilities to award the contract. She expects that the announcement $\hat{w}_0$ made by C_0 belongs to the support of $\tilde{F}_0$. We assume that if C_0 announces $\hat{w}_0$ outside that support then she offers him the payment $\overline{w}_0$. Because this yields a zero utility to the preferred partner, his announcement $\hat{w}_0$ always belongs to support of $\tilde{F}_0$. The interim expected utility of C_0 is therefore

$$U_0(w_0; \tilde{F}_0) = \sup_{\hat{w}_0 \in \text{ supp } \tilde{F}_0} \mathbb{E}\left\{\left[w_0 - M_0(\hat{w}_0, w_1 \ldots, w_n; \tilde{F}_0)\right] \; Q_0(\hat{w}_0, w_1 \ldots, w_n; \tilde{F}_0) \,|\, w_0\right\}, \tag{17}$$

which is convex on $[\underline{w}_0; \overline{w}_0]$, with derivative

$$\frac{\mathrm{d}U_0}{\mathrm{d}w_0} = q_0(w_0; \tilde{F}_0) = \mathbb{E}\left(Q_0(w_0, \ldots, w_n; \tilde{F}_0) | w_0\right)$$

being the probability $q_0(w_0; \tilde{F}_0)$ that C_0 is awarded the project. (This probability increases with w_0.)

The preferred partner C_0 exercises the option if and only if $w_0 - p \geq U_0(w_0; \tilde{F}_0)$. To avoid uninteresting complications, we assume that when indifferent he does exercise the option. Since $q_0(w_0; \tilde{F}_0)$ lies between 0 and 1, the inequality is equivalent to w_0 being higher than some threshold x. We define x as the lowest value $w_0 \in [\underline{w}_0, \overline{w}_0]$ such that $w_0 - p \geq U_0(w_0; \tilde{F}_0)$.[10]

Accordingly, the principal's belief regarding the distribution of w_0 should the auction take place obtains by right-truncation of the initial distribution F_0: $\tilde{F}_0(w_0) = F_0(w_0)/F_0(x)$ for $w_0 \in [\underline{w}_0, x]$. We may therefore denote the interim utility as $U_0(w_0; x)$ instead of $U_0(w_0; \tilde{F}_0)$. If out of equilibrium the ex ante partner with

[10]By convention, we set $x = \overline{w}_0$ if $U_0(w_0; \tilde{F}_0)$ is greater than $w_0 - p$ on $[\underline{w}_0, \overline{w}_0]$.

$w_0 > x$ were to participate in the auction, he would choose his report $\hat{w}_0$ according to (17), where the support of $\tilde{F}_0$ is the interval $[\underline{w}_0, x]$. Because the preferred report is $\hat{w}_0 = x$ if $w_0 = x$, this is a fortiori true if $w_0 > x$. It follows that U_0 is linear on $[x, \overline{w}_0]$ with slope $q_0(x; x)$. At a Bayesian equilibrium, we must have

$$x^* - p = U_0(x^*; x^*). \tag{18}$$

We now show that (18) defines a unique equilibrium threshold x^*, which is increasing in the strike price p.

To this aim,, consider a particular x and a belief $\tilde{F}_0(w_0) = F_0(w_0)/F_0(x)$ for $w_0 \in [\underline{w}_0, x]$. We then define $\hat{x}(x; p)$ as the lowest value $w_0 \in [\underline{w}_0, \overline{w}_0]$ such that $w_0 - p \geq U_0(w_0; x)$, with the convention that $\hat{x} = \overline{w}_0$ if $U_0(w_0, x)$ is greater than $w_0 - p$ on $[\underline{w}_0, \overline{w}_0]$. The equilibrium condition (18) is equivalent to $\hat{x}(x^*; p) = x^*$.

In the Myerson auction, the virtual valuation of the preferred partner

$$\Psi_0(w_0; x) = w_0 - \frac{1 - \tilde{F}_0(w_0)}{\tilde{f}_0(w_0)} = w_0 - \frac{F_0(x) - F_0(w_0)}{f_0(w_0)}$$

decreases in x. It follows that the expected probability that C_0 wins the auction

$$q_0(w_0; x) = \Pr\left(\Psi_0(w_0; x) \geq \max_{j=1,\ldots,n} \Psi_j(w_j)\, | w_0\right)$$

and his interim utility

$$U_0(w_0; x) = \int_{\underline{w}_0}^{w_0} q_0(t; x) dt$$

are continuous and decreasing in x. It follows that the function $\hat{x}(x; p)$ is continuous and non-increasing in x on $[\underline{w}_0, \overline{w}_0]$. Because the function takes its values in that same interval $[\underline{w}_0, \overline{w}_0]$, there exists a unique fixed point $x^* = \hat{x}(x^*; p)$.

Finally, differentiating the condition $\hat{x} - p = U_0(\hat{x}; x)$ with respect to p and using $\partial U_0/\partial \hat{x} = q_0 \leq 1$, we see that $\hat{x}(x; p)$ increases with p, which shows that the equilibrium threshold x^* increases with p. We have: $x^*(\underline{w}_0) = \underline{w}_0$ and $x^*(\overline{w}_0 - U_0(\overline{w}_0; \overline{w}_0)) = \overline{w}_0$, where $U_0(\overline{w}_0; \overline{w}_0)$ denotes the ex ante partner's expected utility in the standard optimal Myerson auction, i.e., in the absence of ex ante contract.

B.2 Proof of Lemma 1

Because $m_1(x; x) = \Psi_1^{-1}(x)$, we have $\Pi_1(m_1(x; x); x) = \Pi^*(x)$. Using (3) and (6), we can rewrite the derivative of the expected joint profit (5) with respect to the exclusivity threshold as

$$\frac{\mathrm{d}\,\mathbb{E}\,\Pi_{S0}}{\mathrm{d}x} = f_0(x)\left[\Pi_1^*(x) - \int_{\underline{w}_0}^{x} \frac{f_1(m_1(w_0; x))}{\Psi_1'(m_1(w_0; x))}\, \frac{F_0(x) - F_0(w_0)}{f_0(w_0)}\, \mathrm{d}w_0\right].$$

B.3 Proof of Proposition 3

The proof of the proposition follows from Lemma B.1 below, and from observing that $K_1(\overline{w}_0) < 0$, $K_2(\overline{w}_0) = 0$, and $K_3(\overline{w}_0) < 0$.

Lemma B.1. *The derivative of the ex ante partners' expected joint profit is given by*

$$\frac{\mathrm{d}\,\mathbb{E}\,\Pi_{S0}}{\mathrm{d}x} = f_0(x)\left[K_1(x) + K_2(x) + K_3(x)\right], \tag{19}$$

with

$$K_1(x) = n(n-1)\int_{m_1^*(x)}^{\overline{w}_1} \left[w - w_0\right] F_1(w)^{n-2} f_1(w)(1 - F_1(w))\,\mathrm{d}w$$

$$K_2(x) = n\Pi_1^*(x^*)F_1(m_1^*(x))^{n-1}$$

$$K_3(x) = -n\int_{\underline{w}_0}^{x^*} \frac{f_1(m_1)}{\Psi_1'(m_1)}\,\frac{F_0(x^*) - F_0(w_0)}{f_0(w_0)} F_1(m_1)^{n-1}\,\mathrm{d}w_0,$$

where $m_1 = m_1(w_0; x)$ *and* $m_1^*(x) = m_1(x; x) = \Psi_1^{-1}(x)$.

The first two terms represent the marginal variation in expected joint profit achieved by running the auction more frequently. More precisely, the first term $K_1(x)$ comes from the cases where the second highest valuation of the alternative suppliers is above $m_1^*(x)$. The second term $K_2(x)$ comes for the cases where the second highest valuation of the alternative suppliers is below $m_1^*(x)$ but the highest valuation is above $m_1^*(x)$. The last term $K_3(x)$ reflects the loss in profit due to reduced competitive pressure placed on the alternative buyer. The rent shifting effect of a change in x occurs only when $(n-1)$ alternative suppliers (to be chosen among n) have their valuation below m_1.

Proof. Differentiating (9) with respect to x yields

$$\frac{\mathrm{d}\,\mathbb{E}\,\Pi_{S0}}{\mathrm{d}x} = A(x;x)f_0(x) + B(x;x)f_0(x) + \int_{\underline{w}_0}^{x}\left[\frac{\partial A}{\partial x} + \frac{\partial B}{\partial x}\right]\mathrm{d}F_0(w_0).$$

Because the derivatives with respect to the lower bound of the integral in A and to the upper bound of the integral in B cancel out, we get from (3) and (6) that

$$\begin{aligned}\frac{\partial A}{\partial x} + \frac{\partial B}{\partial x} &= nF_1(m_1)^{n-1}\frac{\partial \Pi_1}{\partial m_1}\frac{\partial m_1}{\partial x}\\ &= -nF_1(m_1)^{n-1}\frac{f_1(m_1)}{\Psi_1'(m_1)}\,\frac{F_0(x^*) - F_0(w_0)}{f_0(w_0)}\,\frac{f_0(x)}{f_0(w_0)},\end{aligned}$$

which yields

$$\int_{\underline{w}_0}^{x}\left[\frac{\partial A(w_0;x)}{\partial x} + \frac{\partial B(w_0;x)}{\partial x}\right]\mathrm{d}F_0(w_0) = f_0(x)K_3(x).$$

Noticing that $K_1(x) = A(x,x)$ and $K_2(x) = B(x,x)$ achieves the proof of (19).

□

B.4 Uniform distributions with different supports

Suppose F_0 and F_1 are uniform. Then $1-F_1 = (\overline{w}_1 - w_1)/(\overline{w}_1 - \underline{w}_1)$, $f_1 = 1/(\overline{w}_1 - \underline{w}_1)$, $(1-F_1)/f_1 = \overline{w}_1 - w_1$, $\Psi_1 = 2w_1 - \overline{w}_1$, $\Psi_1' = 2$. We have

$$\Pi^*(x) = \max_p (1 - F_1)(p - x) = \frac{1}{\overline{w}_1 - \underline{w}_1}\max_p(\overline{w}_1 - p)(p - x) = \frac{1}{4}\,\frac{(\overline{w}_1 - x)^2}{\overline{w}_1 - \underline{w}_1}$$

and

$$\int_{\underline{w}_0}^{x} \frac{f_1(m_1(w_0;x))}{\Psi_1'(m_1(w_0;x))} \frac{F_0(x)-F_0(w_0)}{f_0(w_0)} \,\mathrm{d}w_0 = \frac{1}{4} \frac{(x-\underline{w}_0)^2}{\overline{w}_1-\underline{w}_1}$$

After eliminating $f_1(m_1) = 1/(\overline{w}_1 - \underline{w}_1)$, the first-order condition reads

$$\frac{(\overline{w}_1 - x)^2}{4} = \frac{(x-\underline{w}_0)^2}{4}, \tag{20}$$

which yields (8).

When the buyers have bargaining weighs $\mu \leq 1$, the first-order condition (20) is changed into

$$\frac{(\overline{w}_1 - x)^2}{4} = (1-\mu)\frac{(x-\underline{w}_0)^2}{4}, \tag{21}$$

which yields (12).

B.5 Vertical merger (Proposition 5)

Consider any direct mechanism $(q_i(w_i, w_{-i}), m_i(w_i, w_{-i}))$, where $q_i(w_i, w_{-i})$ is the probability that i gets the good and $m_i(w_i, w_{-i})$ is the associated payment. Below, $\mathbb{E}$ is the expectation against the distribution of $w_0, w_1, \ldots, w_n$ and $\mathbb{E}_{w_{-i}}$ is the expectation against the distribution of w_{-i} given w_i. The expected joint profit of the pair $S - C_0$ is

$$\mathbb{E}\,\Pi_{S0} = \mathbb{E}\left[w_0 q_0 + \sum_{i=1}^{n} m_i q_i\right].$$

We show here that when the above joint surplus is maximized the contract is awarded to the buyer whose type achieves the maximum of w_0 and $\Psi_j(w_j)$.

The expected utility of supplier i is

$$U_i(w_i) = \max_{\hat{w}_i} \mathbb{E}_{w_{-i}} \left\{ q_i(\hat{w}_i, w_{-i}) \left[w_i - m_i(\hat{w}_i, w_{-i})\right] \right\}$$

By the envelope theorem, we have

$$U'(w_i) = \mathbb{E}_{w_{-i}} q_i(w_i, w_{-i}).$$

Substituting for m_i

$$\mathbb{E}_{w_{-i}} \left\{ m_i(w_i, w_{-i})\, q_i(w_i, w_{-i}) \right\} = \mathbb{E}_{w_{-i}} \left\{ q_i(w_i, w_{-i})\, w_i \right\} - U_i(w_i)$$

By integration by parts

$$\begin{aligned}
\mathbb{E}\, m_i q_i &= \int_{w_i} \mathbb{E}_{w_{-i}}(m_i q_i)\,\mathrm{d}F_i(w_i) = \mathbb{E}\,(q_i w_i) - \int U_i(w_i)\,\mathrm{d}F_i(w_i) \\
&= \mathbb{E}\,(q_i w_i) - \int \mathbb{E}_{w_{-i}} q_i(w_i, w_{-i}) \; \frac{1-F_i(w_i)}{f_i(w_i)} \,\mathrm{d}F_i(w_i) \\
&= \mathbb{E}\, q_i \left[w_i - \frac{1-F_i(w_i)}{f_i(w_i)}\right] = \mathbb{E}\,\Psi_i(w_i) q_i.
\end{aligned}$$

Rewriting the expected joint profit yields

$$\mathbb{E}\,\Pi_{S0} = \mathbb{E}\left\{w_0 q_0 + \sum_{i=1}^{n} \Psi_i(w_i) q_i\right\}$$

It is therefore optimal for the pair that the contract goes to the buyer that achieves the maximum of w_0 and $\max \Psi_j(w_j)$, $j = 1, \ldots, n$.

The outcome is implemented under dominant strategies with the payments

$$m_0(w_{-0}) = \max_{j\geq 1} \Psi_j(w_j) \quad \text{and} \quad m_j(w_{-j}) = \Psi_j^{-1}(\max(w_0, \max_{k\neq j, k\geq 1} \Psi_k(w_k))).$$

These payments do not depend on the agents' types. It follows easily that the mechanism induces truthful revelation.[11]

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[11] The utility of agent j if she reports $\hat{w}_j$ can be written as $(w_j - m_j)\mathbb{1}_{\hat{w}_j \geq m_j}$, which is maximal for $\hat{w}_j = w_j$.

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