Do firms gain from managerial overconfidence? Managerial bargaining power and the role of severance pay*

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Abstract

We analyze the effects of optimism and overconfidence when the manager has bargaining power and the compensation package includes severance pay. If the manager can renegotiate the initial contract, the advantage of using severance pay to induce the manager to invest, commonly found in the literature, is reduced by the presence of the biases. Optimism increases severance pay and managerial entrenchement with a negative effect on profit. Overconfidence reduces incentive pay, as shown by the previous literature, but its effect on severance pay depends on the intensity of the bias. When overconfidence is moderate, severance pay is also reduced and profit increases. When overconfidence is high, however, the increase in severance pay may offset the beneficial effect on incentive pay. Thus, the attempt to exploit managerial overconfidence to reduce incentive pay may backfire if the manager is replaced and severance agreements come into effect. Our model suggests that the large severance payments documented by the empirical literature represent a form of efficient contracting when the optimistic and overconfident manager has bargaining power.

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1 Introduction

Severance agreements specify the payments the executives receive in case of departure. They are used by a vast majority of firms and both the number of firms signing such contracts and the average amount of the severance payments have increased in the last decades (Cadman, Campbell and Klasa 2016, Callahan 2023, Huand 2011).[1] Despite the widespread use of these contracts, according to Callahan (2016; p.2), "severance pay is perhaps the most controversial yet least understood form of executive compensation". Indeed, theoretical research investigating severance agreements, has provided mixed results so far. Severance payments have been criticized because they occur when the board of directors dismisses the incumbent manager after a period of poor performance. Thus, these payments seem a "reward for failure" that violates the pay-for-performance principle of agency theory (Bebchuk and Fried 2004). Criticisms are particularly severe when the dismissed manager receives a payment in excess of the severance amount specified by the contract, a documented and widespread practice (Goldman and Huang 2015). Other scholars however, suggest that severance payments are part of an efficient contract because they provide CEOs with insurance for their human capital and, by offsetting the manager's risk

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[1] See also Cowen, King, Marcel (2016).

aversion, they offer incentives for investments in valuable risky projects. Furthermore, the separation pay in addition to the contractual amount may be part of an implicit contract.

The present paper investigates the efficiency of severance payments by studying how managerial optimism and overconfidence influence the optimal design of the agreement when the manager has bargaining power and can renegotiate the initial severance contract. Following de la Rosa (2011) we distinguish between *optimism* and *overconfidence*. Optimism occurs when the manager has a subjective belief on the probability of success higher than the "true" probability, while overconfidence distorts the manager's assessment of the increase in the probability of success due to her effort. In our context, effort takes the form of a firm-specific investment that is observable but unverifiable. We investigate how optimism and overconfidence affect the amount of severance pay necessary to induce the manager to leave when this is profit enhancing but the manager can oppose replacement. We build upon the analysis of Almazan and Suarez (2003) who suggest that renegotiating severance pay when separation occurs may be optimal because it allows to establish the exact amount of the payment ex-post, once the board knows whether the investment has been made. This is cheaper than just motivating the manager through an incentive pay that has to satisfy an ex-ante incentive compatibility constraint.

Similarly to what happens in the previous literature on the optimality of severance agreements, in our model severance pay mitigates the incentive to dismiss the manager when the replacement is only marginally better. This, in turn, helps inducing the manager to undertake the level of investment desired by the board. We assume that the incumbent manager has some bargaining power and can credibly threaten to resist being replaced. The idea underlying this assumption is that the manager can oppose replacement by making it a costly and contentious process so that valuable opportunities are missed and firm value decreases. To allow a smooth replacement, the board is willing to renegotiate the separation agreement and consent to a payment high enough to avoid costly opposition. As in Almazan and Suarez (2003) the severance pay is renegotiated when the board knows whether the investment has been undertaken and can be tailored to its level. However, overconfidence and optimism create a wedge between board's and manager's beliefs on expected profit and they increase the amount of severance pay asked by the manager. This, in turn, leads to entrenchment as the board retains the incumbent manager even when the expected probability of success is significantly lower than that of the replacement. Then, a new trade-off emerges between the saving on (ex-ante) incentive compensation resulting from the relaxed incentive compatibility constraint and the higher cost for the (ex-post) severance payment.

The main findings of the paper can be summarized as follows. First, the advantage of using severance pay to incentivize the manager to invest is reduced by managerial optimism and overconfidence if the manager has bargaining power. These biases, that usually are beneficial for the firm when only the incentive pay is considered (de la Rosa 2011; Santos-Pinto 2008), may turn out to be detrimental when turnover and severance pay are taken into account. Second, both the degree and the kind of managerial bias matter because overconfidence and optimism have different impact on managerial compensation package and firm expected profit. In our model, optimism does not affect incentive pay but raises contractual severance pay and thus reduces firm profit. Overconfidence, on the other hand, may be either advantageous or detrimental for the firm depending on the degree of the bias. Moderate overconfidence reduces both incentive pay and severance pay without affecting the investment choice, thus increasing expected profit. Extreme overconfidence, on the contrary, may have a negative impact on profit. In fact, it reduces the incentive pay but such positive effect may be offset by the distortion in the investment choice resulting in a higher renegotiated severance pay. Thus, firm profit is non-monotonic in the degree of overconfidence and our model shows that the attempt to exploit executive overconfidence through a heavy use of incentive pay, documented for example by Humphery-Jenner et al. (2016), can backfire when the investment choice and the opportunity of replacing a manager who holds some bargaining power are considered. Given that the managerial bargaining power is a key assumption in our model, we study the robustness of our findings under alternative specifications of the payment that the manager can obtain in case of dismissal. We show that our results are robust to alternative assumptions on the bargaining power as long as it allows the manager to renegotiate the severance pay and obtain a positive payment. When instead the manager cannot oppose replacement, zero severance pay is optimal.

Finally, our model helps explaining the common practice of granting a separation pay largely exceeding its contractual level, as reported by empirical studies and anecdotal evidence. We rationalize such discretionary payment in forced turnover as result of the bargaining between the board and the manager and we show that paying the manager who has been fired a sum in excess of the contractual severance pay may be optimal for shareholders. When instead the manager has no bargaining power as it is usually the case in voluntary turnover, zero severance payment is optimal. Our results are in line with the empirical evidence of Goldman and Huang (2015) who show that discretionary separation pay in forced turnover is motivated by the desire to facilitate a smooth transfer of power from a poorly performing CEO to a potentially better replacement, while in voluntary turnover it signals weak internal corporate governance.

This paper contributes to two streams of literature. First, it is related to the literature on the role of severance pay from an optimal contracting point of view. In particular, in addition to Almazan and Suarez (2003), several other papers suggest that severance agreements, by inducing a more lenient replacement policy, mitigate the moral hazard problem and induce the manager to make a risky but profitable investment. Wu and Weng (2018) consider a setting where the contract designed by the board depends on the informativeness of an observed signal on the manager ability. In their setting both entrenchment and anti-entrenchment may emerge in the optimal contract. Laux (2015) shows that the optimal pay mix consists of restricted stock, to combat excessive risk-taking, and severance pay, to combat excessive conservatism.[2] In a similar vein, a few papers demonstrate that severance pay, by protecting the manager from the cost of dismissal, can alleviate information revelation problems. For instance, Inderst and Mueller (2010) find that offering a combination of severance pay and steep incentive pay may be the cheapest way to induce the manager to disclose information that may lead to her dismissal. Green and Taylor (2016) show that severance pay may be necessary to induce truthtelling when the manager has an informational advantage over the principal and the latter has to decide whether to terminate a multistage project on the basis of such information. Similarly, Vladimirov (2021) focuses on the interplay between severance pay and contract length and indicates that severance pay may discourage managers from using window dressing or information concealment to avoid replacement.[3] Our paper contributes to this literature by showing that the quasi-rent necessary to induce the manager to leave are likely to be larger when the manager is optimistic and highly overconfident than in the absence of such biases, thus limiting the optimality of severance pay.

Second, we contribute to the research on managerial overconfidence and optimism in a principal-agent relationship (see, for example, Santos-Pinto 2008, de la Rosa 2011, Otto 2014) by showing that models that do not account for the possibility of managerial turnover and severance payment, may overstate the positive contribution of overconfidence and optimism. In a standard agency model with moral hazard and risk averse agent the optimal contract trades off risk insurance and incentive provision. Managerial overconfidence, and the resulting divergence of beliefs between principal and agent, makes it easier to satisfy the participation and the incentive compatibility constraints, thus reducing the cost of eliciting effort (Santos-Pinto 2008, de La Rosa 2011, Gervais et al. 2011, Otto 2014, Köszegi, 2014). Firms can

[2] The positive role of severance pay in encouraging risk-taking investment is confirmed by several empirical studies (see among others Cadman et al. 2023).

[3] Empirical evidence suggesting that severance pay may curb managers' incentive to misreport financial information reducing agency costs is provided by Brown (2015).

take advantage of this effect either by inducing the same level of effort required to an unbiased (rational) manager at a lower cost, or by offering a compensation structure with a particularly heavy incentive pay (the so-called exploitation hypothesis).[4]

We depart from this literature by assuming risk neutrality and bargaining power on the manager's side. Managerial bargaining power implies that the manager can oppose being fired so that it may be necessary to renegotiate the severance payment to induce her to leave. In a standard principal/agent setting, managerial bargaining power would generally result in a different division of the gains from trade. In our framework, the outcome of the renegotiation on the severance pay depends also on the managerial perceived probability of success, so that a trade-off between the ex-ante saving on incentive pay and the ex-post higher payment for the severance payment arises. We show that the impact of the managerial biases in this setting is different from that in a standard principal-agent model. Optimism does not affect incentive pay but increases renegotiated severance pay. This has a negative impact on expected profits, contrary to what happens in the principal-agent literature where optimism makes high-powered incentives more profitable for the firm. As to overconfidence, the size of the bias matters. We find a positive effect of moderate overconfidence on expected profit in line with the results obtained in the agency literature, but a high overconfidence may have a negative impact on firm profit.

Our paper complements the previous literature where the bias on beliefs leads to systematic differences in "payoffs, effort levels and incentives between over- and underconfident agents" (Sautmann, 2013), by showing that the belief bias impacts also the amount of severance pay and entrenchment. Moreover, our results suggest that different degree of managerial biases may be beneficial for the firm according to the bargaining position of the manager: moderate overconfidence benefits the firm when the manager has a strong bargaining position while extreme overconfidence is beneficial when the manager has no bargaining power.

The rest of the paper is organized as follows. Section 2 presents the model. In Section 3 we discuss the difference between contractual and renegotiated severance pay in a simplified setting. Section 4 studies the replacement decision and the renegotiation stage. Section 5 analyzes the optimal compensation package. Given the optimal severance and incentive pay, Section 6 investigates the separate effects of optimism and overconfidence on the optimal contract and on firm expected profit. Section 7 presents an extension of the model where we discuss different assumptions on managerial bargaining power and we show that our findings are robust to different bargaining settings as long as the manager has some bargaining power. Finally, Section 8 concludes.

2 The model

Consider a board that perfectly represents the shareholders and maximizes firm's value. The board hires a risk neutral manager to implement a project. The cash flow generated by the project can take two values, either 0 or $R > 0$. The probability of success of the project, denoted by p_k, depends on a firm-specific investment I_k, $k = L, M, H$, made by the manager after joining the firm. In the absence of investment ($I = I_L = 0$), the probability of success is $p_L > 0$. If the manager makes investment I_M, the probability of success increases to $p_M > p_L$, while it becomes $p_H > p_M$ when the larger investment $I_H > I_M$ is chosen. The cost of the investment $c(I_k) = I_k$ is borne by the manager. The investment

[4] Note, however, that several papers highlight also the downsides of optimism and overconfidence. De La Rosa (2011) finds that principal's expected profit decreases in agent overconfidence when the agent is significantly optimistic and no effort is implemented. Overconfidence may also induce managers to take too much risk in project choice (Malmendier and Tate, 2008, Gervais et al. 2011) or it may reduce the incentive to gather information about a project leading to inefficient implementation, (Downs 2023). For a comprehensive survey of the large literature on managerial optimism and overconfidence see Malmendier and Tate (2015) and Santos-Pinto and de la Rosa (2020).

is unverifiable, though it is observable by the board that, consequently, comes to know the manager's probability of success before the results of the project become publicly known.

Only after the manager has undertaken the investment, a new manager materializes. We denote the probability of success of the new manager by $q \in [0,1]$ and, since we have no reason to consider any particular value of q more/less likely to occur than other values, we assume that q is uniformly distributed.[5] Both the board and the incumbent manager observe the realization of q which may be higher than the probability of success of the incumbent even when the latter makes the investment desired by the board. The new manager does not need to make any investment when joining the firm. A high value of q may result, for example, from a better match between the new manager's ability and the skills required by the firm (possibly she has made elsewhere an investment in human capital that is valuable also in this firm). In other words, the firm-specific investment of the incumbent may not be sufficient to avoid being less productive than the replacement. In such a case, the board may prefer to fire the incumbent and hire the new manager. Note that this implies that we are focusing on forced, not voluntary, turnover. Consistently with the evidence that CEOs have some power over the board, we model a situation where the manager can oppose dismissal. In other words, the incumbent has the ability to prolong the firing process and make it difficult for the company to move on in a different direction with a new manager. Thus, replacement can occur only with mutual agreement between board and incumbent. Specifically, we assume that the latter has a high enough bargaining power to oppose replacement if contractual severance pay is smaller than what the manager believes she would receive by staying with the firm, an amount that can be considered her "outside option" in the bargaining process. This is meant to capture the fact that actual severance payments are indeed related to the annual managerial compensation.[6] In Section 7, several alternative assumptions are discussed.

The incumbent manager and the board hold heterogeneous beliefs regarding the probability of success and are aware of such divergence that affects both the original contract and subsequent renegotiation, if any. Following the previous literature (de la Rosa, 2011), we decompose the managerial bias into two components: optimism, θ, that is independent of the manager's action and is always at work, and overconfidence Δ_i, $i = M, H$, that captures the manager's distorted belief on the productivity of her investment. The effect of such biases on beliefs is made explicit by the following assumption

Assumption 1: The manager's beliefs about the probability of success are:
i) $p_L + \theta$, if no investment is made, $I = I_L = 0$;
ii) $p_M + \theta + \Delta_M$, if the manager makes investment I_M;
iii) $p_H + \theta + \Delta_H$, if the manager makes investment I_H, where $\Delta_H = \Delta_M(1+z)$ denotes the high overconfidence resulting from the high investment, with $z > \frac{I_H - I_M}{I_M} > 0$ and $1 - p_H - \theta \geq \Delta_H > 0$.

Assumption 1 specifies that manager's beliefs differ from the "true" probabilities used by the board in two dimensions: level (optimism) and differences (overconfidence). Optimism has a uniform effect on the manager's beliefs, while overconfidence increases in the investment level: it is higher in case of I_H than in case of I_M and it is null in case of I_L. In other words, overconfidence and investment are complements.[7] We consider the rate z of increase in overconfidence when moving from I_M to I_H as a parameter and we refer to an increase in Δ_M as an increase in overconfidence. Given $z > 0$, the slope of the manager's beliefs of success (considered as a function of Δ_M) is everywhere steeper in case of I_H

[5] This assumption is not necessary for most of our results, it is only used in the proof of Corollary 6 and in Section 7.
[6] "A typical severance contract would detail payments, usually equaling multiple times the CEO's base salary and bonus, as well as continuing/immediate vesting of existing executive stocks and options." (Goldman and Huang 2015; 1110). See also Brown (2015).
[7] The complementarity between overconfidence and effort (investment) is a common assumption in the theoretical literature. Chen and Schilberg-Horisch (2019) provide experimental evidence that supports it.

than in case of I_M.[8] Then, a rise in overconfidence results in a higher increase in the manager's beliefs of success if the latter chooses I_H than if she chooses I_M. The assumption $z > \frac{I_H - I_M}{I_M}$ implies that the increase in the manager's subjective belief of success is higher than the relative increase in the cost of the investment making the high investment particularly attractive. In the following, we will show that this assumption together with a high value of Δ_M can lead to a situation where the managerial bias is large enough to result in a distorted choice of the investment. A situation that we call extreme overconfidence. We also consider the cases where the manager is not optimistic ($\theta = 0$) or not overconfident ($\Delta_M = 0$) or both ($\theta = \Delta_M = 0$). In the latter case we call her a rational manager.

The following assumption completes the framework.

Assumption 2: *Investment I_M is efficient: $(p_M - p_L)R > I_M$ while investment I_H is not: $(p_H - p_M)R < I_H - I_M$, though $(p_H - p_L)R > I_H$.*

Note that Assumptions 2 implies $\frac{I_H - I_M}{p_H - p_M} \geq R \geq \frac{I_M}{p_M - p_L}$ which can be satisfied if and only if

$$\frac{I_M}{I_H} \leq \frac{p_M - p_L}{p_H - p_L}.$$

Assumption 2 ensures that I_M is more efficient than both I_L and I_H. Indeed, I_M is more efficient than $I_L = 0$ because its cost is lower than the increase in the expected return. The high investment I_H instead, is better than no investment but it is inefficient when compared to I_M because the additional cost of choosing I_H rather than I_M is larger than the increment in expected return from the high investment.

If the incumbent remains in office, she enjoys benefit of control $B \geq 0$.[9]

The contract offered by the board maximizes the expected final cash flow of the project net of managerial compensation. Recalling that the new manager is not necessarily more productive than an incumbent manager who has made a positive investment, we focus on cases where the profit-maximizing board wants to provide the incumbent with the incentive to invest even if replacement may occur with positive probability. To this end, we consider a simple incentive contract with base salary, incentive pay w contingent on the high return R, and severance pay s. We normalize the reservation level of utility to 0, so that the base salary of the incumbent takes value 0, as well as the compensation of the new manager when replacement occurs. Our results would not change if we assumed a positive level of reservation utility as long as such level is smaller than the expected compensation in case of no investment (see discussion in footnote 12) and would remain qualitatively the same even with a larger reservation value.

The timing of the model can be summarized as follows:

$t = 0$: The board observes whether the manager is overconfident, optimistic or both and, accordingly, offers a compensation contract aimed at inducing investment I_i. The contract is enforceable. We denote by $\{w(\theta, \Delta_i; I_i), s(\theta, \Delta_i; I_i)\}$, $i = M, H$ the contract of an optimistic and overconfident manager. In general, the first or (and) the second argument of the two functions can take value zero if the manager is either not optimistic or not overconfident (or both). The manager decides whether to accept the offer.

$t = 1$: If the contract is accepted, the manager decides how much to invest.

$t = 2$: The board observes the investment level and deduces the probability of success.

$t = 3$: A rival manager appears. Board and incumbent manager observe the rival's ability. The board evaluates whether it is profitable to replace the incumbent. If this is the case, and contractual

[8] Similarly to de La Rosa (2011) and Santos-Pinto (2022), we are assuming that the marginal contribution of the investment to the probability of success is increasing. Note that also the assumption that optimism has a uniform effect on managerial belief is common to de La Rosa and Santos-Pinto.

[9] We allow for the presence of such a non-monetary benefit for the sake of the comparison with the literature on severance pay. Contrary to what happens in the corporate governance literature where the benefit usually incentivizes the manager to exert effort at the cost of a distorted replacement decision, in our setting no such trade-off occurs and the benefit plays no role in the results.

$s(\theta, \Delta_i; I_i)$ is too low for the incumbent to accept replacement, renegotiation occurs and a new level of severance pay s' is agreed upon. Note that, at this stage, the board knows whether the level of investment is $I_j = I_i$ meaning that the manager complies with the contract or whether it is $I_j \neq I_i$ meaning that she has chosen a different level of investment. Thus, $s'(\theta, \Delta_j | I_j \neq I_i)$ indicates that the initial contract was meant to induce investment I_i while the manager has chosen a level of investment, $I_j \neq I_i$, resulting in a level of overconfidence Δ_j. In the case in which the manager has indeed undertaken the investment required by the board we simplify the notation writing $s'(\theta, \Delta_i | I_i)$ rather than $s'(\theta, \Delta_i | I_i = I_i)$[10]

$t = 4$: Cash flow realizes. The manager is paid the compensation/severance pay agreed upon.

The model is solved by working backwardly. We first discuss renegotiation and we find the outcome of the renegotiation under an arbitrary initial contract. Then, we discuss the board's replacement decision. Given the replacement decision, we derive the incentive compatible contract $\{w(\theta, \Delta_i; I_i), s(\theta, \Delta_i; I_i)\}$ that maximizes the firm final cash flow and we determine the investment level chosen by the manager. Before developing our model we discuss the role of contractual and renegotiated severance pay in the incentive contract.

3 Contractual versus renegotiated severance pay: an example

Before analyzing the optimal contract in the model described above, let us briefly illustrate the role of severance pay with a simple example. Specifically, we discuss the different characteristics of the contractual and the renegotiated severance pay and their impact on incentive pay. Recall that the manager is risk neutral, so that there is no gain from reducing the uncertainty to which she is exposed. On the contrary, paying the manager a pre-contracted sum at the moment when she is fired, reduces the incentive to choose a positive investment because the difference in the expected payment under the different investment choices is reduced. A key assumption in our the model is that the board can fire the incumbent manager if a new and more productive manager materializes. In such a case, the incumbent manager will be paid either the contractual severance pay or a renegotiated amount according to her bargaining power and to contract provisions. A full discussion of this issue is presented in the following section. Here, we limit ourselves to underline the implications of the different information available to the board when the contractual and the renegotiated severance pay are agreed upon.

Given that the investment is unobservable, the board cannot condition the contract on the investment level and it has to offer a combination of incentive and severance pay in order to induce the manager to make the desired investment. Consider a simplified framework with no private benefits ($B = 0$) and only two possible investment levels: $I = I_L = 0$ or $I = I_M$, with the cash flow generated by the project specified above, i.e. either 0 or $R > 0$. The associated probabilities of success are: $p_L > 0$ if $I = 0$ and $p_M > p_L$ if $I = I_M$. Assume further that the manager shares the same beliefs regarding the probability of success of the board so that no divergence arises. This assumption does not affect the qualitative difference between contractual and renegotiated severance pay that is our focus here.

The optimal value of the probability of success used by the board as a cutoff in the dismissal/retention decision will be derived in the following section. Here, we simplify the analysis by assuming that the board uses an exogenously given value as a threshold for dismissal. Specifically, we assume that it dismisses the incumbent if the new manager has a probability of success q higher than p_M when investment $I = I_M$ is undertaken, and higher than p_L if $I = I_L = 0$.

[10] The notation used for the contractual components differs from the one used for the renegotiated pay because the contractual payments (both incentive and severance pay) are agreed before the investment choice and therefore the investment level specified in these payments (and the resulting overconfidence) is the one desidered by the board: $\{w(\theta, \Delta_i; I_i), s(\theta, \Delta_i; I_i)\}$. Conversely the renegotiated severance pay takes into account the possible divergence between the investment undertaken and the one desidered by the board.

Consider first the case with a positive contractual severance pay $s > 0$ and no renegotiation, $(s' = 0)$. Since s is decided before the investment stage, it cannot be contingent on I. This implies that, in case of dismissal, the manager will receive the same payment irrespective of the level of I previously chosen. As in standard agency problems, the optimal compensation has to induce the manager to accept the contract (the participation constraint) and to choose investment I_M rather than $I = I_L = 0$ (the incentive compatibility constraint). Under the zero reservation utility assumption and the assumption that q is uniformly distributed, the participation constraint (PC) can be written as

$$\begin{aligned} \int_0^{p_M} p_M w f(q) dq + \int_{p_M}^1 s f(q) dq - I_M &= \\ (p_M w) p_M + s(1 - p_M) - I_M &\geq 0, \end{aligned}$$

where the first integral represents the expected compensation in case of retention $(q \leq p_M)$, and the second integral represents the expected payment in case of dismissal $(q > p_M)$ when the manager chooses I_M.

The incentive compatibility constraint (ICC) is

$$\int_0^{p_M} p_M w f(q) dq + \int_{p_M}^1 s f(q) dq - I_M = (p_M w) p_M + s(1 - p_M) - I_M$$

$$\geq (p_L w) p_L + s(1 - p_L) = \int_0^{p_L} p_L w f(q) dq + \int_{p_L}^1 s f(q) dq - I_M,$$

where the first line is the expected compensation when the manager chooses I_M and the second line is the expected compensation when she chooses $I_L = 0$. It is immediate to see that the assumption of zero reservation utility implies that PC is not binding. Consequently, we can focus our attention on the ICC.

The lowest incentive pay that satisfies (ICC) is

$$w(s > 0, s' = 0) = \frac{I_M}{p_M^2 - p_L^2} + \frac{s}{p_M + p_{L.}}.$$

Given that the amount s is paid when the manager is dismissed, the severance pay reduces the penalty provided by the higher probability of dismissal when $I = 0$ is chosen, and therefore increases the incentive pay necessary to induce the manager to invest I_M, i.e. $\frac{\partial w}{\partial s} > 0$. Thus, in case of a positive contractual severance payment, the contract offered to incentivize I_M is more expensive.

Consider now the situation in which the original contract does not include any severance pay $(s = 0)$ but the manager has bargaining power so that she can oppose the board decision and ask a positive payment $s' > 0$ in order to leave the firm. Note that there is no reason from an incentive point of view to pay the manager once the investment has already been made. In other words, the renegotiated severance pay can arise only in a situation where the manager has bargaining power and can oppose replacement if she does not receive any payment when fired. This, per se, does not imply that the payment should be renegotiated, but a key implication of the fact that at the renegotiation stage the board knows the level of the investment is that the renegotiated severance payment can be contingent on I. Thus, if the bargaining power of the manager positively depends on the investment level, the amount paid when the investment is I_M will be higher than the amount paid when no investment is undertaken: $s'_M > s'_L$. Suppose for simplicity that in case of no investment the bargaining power goes to zero, so that $s'_L = 0$.

Then, the ICC becomes $(p_M w)p_M + s'_M(1-p_M) - I_M \geq (p_L w)p_L$, and the lowest incentive pay that satisfies it, is

$$w(s = s'_L = 0, s'_M > 0) = \frac{I_M}{p_M^2 - p_L^2} - \frac{(1-p_M)s'_M}{p_M^2 - p_L^2}.$$

The term that is subtracted on the RHS is the expected value of the severance pay. It is immediate to see that it is easier to satisfy the ICC and that the incentive pay to induce $I = I_M$ is reduced. Thus, there is a negative relation between incentive pay and renegotiated severance pay, $\frac{\partial w}{\partial s'} < 0$, because the higher renegotiated severance pay when $I = I_M$ is chosen (which is anticipated at the contracting stage) contributes to incentivize the manager to choose this investment level. Observe that there is an important difference between incentive pay and renegotiated severance pay because the former is contingent on the return R which is a noisy signal of the investment, while the latter is directly contingent on the investment.

The qualitative results derived here under the assumption that the manager is rational hold also if the manager is optimistic and overconfident. The optimal incentive pay will be different, but the sign of $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial s'}$ are still positive and negative respectively, confirming that contractual severance pay makes the ICC more stringent while the renegotiated severance pay contingent on I relaxes the constraint.

4 Renegotiation and replacement decision

Let us now analyze the problem in the general framework introduced in Section 2, starting from the renegotiation of a contract designed to induce a positive level of investment I_i, with i equal either to M or to H. We first determine under which condition the manager accepts dismissal. Given that she can oppose such decision, the severance pay must compensate her for the loss she will suffer when fired, corresponding to her belief of what she can obtain if she remains with the firm, $(p_i + \theta + \Delta_i)w_i(\theta, \Delta_i; I_i) + B$. In the present section we consider the case where contractual severance pay does not meet this requirement. Consequently, a board willing to fire the incumbent, renegotiates the contract by making a take-it-or-leave-it offer,

$$s'(\theta, \Delta_i | I_i) \geq (p_i + \theta + \Delta_i)w(\theta, \Delta_i; I_i) + B, \tag{1}$$

that is acceptable by the manager. The optimal level of renegotiated severance pay $s'(\theta, \Delta_i | I_i)$ is clearly equal to the RHS of the above inequality. Given that the investment decision has already been made, there is in fact no reason for the board to increase the renegotiated payment above the minimum level necessary to overcome the incumbent's opposition to replacement. Such payment is correctly anticipated by both the manager and the board when the contract is signed and thus contributes to the expected compensation or expected cost calculated by the manager and the board respectively.

The board wants to replace the incumbent whenever the expected profit is higher under the new manager, i.e., when the gain from replacement, computed by using the "right" probability of success, is higher than the cost. Anticipating that severance pay will be renegotiated, the board will then fire the manager when

$$qR - s'(\theta, \Delta_i | I_i) \geq p_i(R - w(\theta, \Delta_i; I_i)), \qquad i = M, H, \tag{2}$$

where the LHS represents the expected profit under the new manager and the RHS the expected profit under the incumbent. Note that, since the new manager does not invest, she does not receive any incentive pay. The above inequality implies that the manager is fired when

$$q \geq p_i - \left(\frac{p_i w(\theta, \Delta_i; I_i) - s'(\theta, \Delta_i | I_i)}{R} \right), \tag{3}$$

meaning that the firing decision is based on the difference between the probability of success of the replacement with respect to that of the incumbent, "adjusted" for the difference in the payment to the incumbent manager in case of retention and dismissal. When the firing cost represented by the severance pay exceeds the sum saved by replacing the manager, i.e., when $p_i w(\theta,\Delta_i;I_i) < s_i'(\theta,\Delta_i|I_i)$, the board will replace the manager for higher values of q than in the opposite case. We are here discussing renegotiation but the above expression allows us to consider also what happens if no severance pay (either renegotiated or contractual) is paid. It is immediate to see that, in case $s_i'(\theta,\Delta_i|I_i) = 0$, the manager is fired even when the replacement has a lower probability of success than her own. In other words, if the contract does not contemplate any severance payment the manager is dismissed too frequently. This occurs because the new manager is not given any incentive pay in relation to the current investment project.

For replacement to occur, both condition (2) and condition (1), the latter in the form of an equality, must be simultaneously satisfied

$$(q-p_i)R + p_i w(\theta,\Delta_i;I_i) \geq s'(\theta,\Delta_i|I_i) = (p_i+\theta+\Delta_i)w(\theta,\Delta_i;I_i) + B.$$

Note that in the LHS of this inequality, the payment to the manager in case of retention is what the board expects to pay and therefore is computed using the "right" probability of success, while in the RHS the payment required by the manager in case of dismissal is computed using her subjective beliefs of success.

The above condition, taken in the form of an equality, determines the cutoff value of q, above which the board will replace the incumbent.

$$\widehat{q}(\theta,\Delta_i|I_i) = p_i + \frac{B + (\theta+\Delta_i)w(\theta,\Delta_i;I_i)}{R}. \tag{4}$$

Since the severance pay required by the manager in order to leave is the monetary equivalent of her expected utility in case she stays with the firm, the above cutoff value is increasing in $w(\theta,\Delta_i;I_i)$ and in B. Note that the term $\frac{B+(\theta+\Delta_i)w(\theta,\Delta_i;I_i)}{R}$ represents the difference between the two expected payment in case of dismissal and in case of retention (see, 3).

To fully characterize the contract offered to the manager, we have to determine the incentive pay $w(\theta,\Delta_i;I_i)$ and the contractual severance pay $s(\theta,\Delta_i;I_i)$ (which in turn determine the outcome of the renegotiation process, $s'(\theta,\Delta_i|I_i)$, and the cutoff value $\widehat{q}(\theta,\Delta_i|I_i)$). This is done in the following section. Note, however, that in the above discussion we have taken it for granted that the incumbent makes investment I_i, $i = M, H$, when the contract has an incentive component equal to $w(\theta,\Delta_i;I_i)$. In order to determine the optimal contract, we also need to consider the hypothetical case where the incumbent makes investment I_j when the contract requires her to choose I_i, so that the renegotiated severance pay becomes $s'(\theta,\Delta_j|I_j \neq I_i)$ and the cutoff value above which the manager is dismissed becomes $\widehat{q}(\theta,\Delta_j|I_j \neq I_i)$. This never occurs in equilibrium but, precisely to provide the appropriate incentives to discourage such behavior, we need to take into account what payment the incumbent obtains in such a case.

The next lemma summarizes the above discussion, and indicates the optimal payments that induce the incumbent to leave both when she has made investment I_i and when she has made investment I_j, despite the incentive pay offered at $t = 0$, $w(\theta,\Delta_i;I_i)$, was meant to induce I_i. Note that the latter case may occur if either the manager has not invested ($I_j = I_L = 0$) or if she has chosen a positive level of investment different from the one required by the board (I_H when I_M is required or viceversa).

Lemma 1. *If renegotiation between the board and the incumbent manager takes place under a contract with incentive pay equal to* $w(\theta,\Delta_i;I_i)$*, the optimal renegotiated severance payment is*

i) $s'(\theta,\Delta_i|I_i) = (p_i+\theta+\Delta_i)w(\theta,\Delta_i;I_i) + B$ *if the incumbent has made investment* I_i*,* $i = M, H$*;*

ii) $s'(\theta,\Delta_j|I_j \neq I_i) = (p_j+\theta+\Delta_j)w(\theta,\Delta_i;I_i) + B$ *if the incumbent has made investment* $I_j \neq I_i$ $i = M, H$*,* $j = L, M, H$*.*

4.1 Renegotiation in the case of a rational manager

In order to establish a benchmark for our analysis, we consider a rational manager whose subjective beliefs are equal to the "true" probabilities, that is we consider the case where $\theta = \Delta_M = 0$. In such a case, the cutoff value of q above which the manager is dismissed is

$$\widehat{q}(0,0|I_i) = p_i + \frac{B}{R}.$$

Again, the replacement decision is determined by the difference between the probability of success of the replacement with respect to that of the incumbent, "adjusted" for the difference in what should be paid to the incumbent manager in case of dismissal and retention. The additional cost paid in case of dismissal with respect to retention is only given by the private benefit enjoyed by the manager if staying with the firm. In other words the difference $p_i w(0,0;I_i) - s'(0,0|I_i)$ reduces to $\frac{B}{R}$.

Note that the cutoff $\widehat{q}(0,0|I_i)$ is lower than the cutoff determined in the case of a biased manager $\widehat{q}(\theta,\Delta_i|I_i)$ and does not depend on the incentive pay. In fact, optimism and overconfidence give rise to an entrenchment effect by distorting the replacement decision: $\widehat{q}(\theta,\Delta_i|I_i)$ is higher than $\widehat{q}(0,0|I_i)$ because of the higher severance pay (depending in turn on the incentive pay) required by the overconfident and optimistic manager to accept replacement. Since it is more costly to replace a biased manager, the board will replace her only when the probability of success of the replacement is higher.

As far as the renegotiated severance pay is concerned, by setting $\theta = \Delta_i = 0$, we have from Lemma 1 that if the incumbent has made investment I_i

$$s_i'(0,0|I_i) = p_i w_i(0,0;I_i) + B, \qquad i = M,H.$$

As in the case of an overconfident manager, the renegotiated severance pay corresponds to the amount that the manager expects to obtain by staying with the firm, because any payment lower than that would be rejected.

5 Manager's investment and optimal compensation

Having established the optimal renegotiated severance pay, we can now determine the contractual severance pay and the incentive pay necessary to induce a positive level of investment. Then, we will be able to determine which is the optimal level.

Recall that we have normalized to zero the reservation level of utility and consider that the firm wants to induce the manager to choose investment I_i, where i can alternatively be M or H. Anticipating that in equilibrium the severance pay will be renegotiated,[11] the participation constraint is

$$\int_0^{\widehat{q}(\theta,\Delta_i|I_i)} [(p_i+\theta+\Delta_i)w(\theta,\Delta_i;I_i)+B]f(q)dq + \int_{\widehat{q}(\theta,\Delta_i|I_i)}^1 s'(\theta,\Delta_i|I_i)f(q)dq - I_i \geq 0, \tag{PC}$$

where the first integral represents the expected compensation in case of retention ($q \leq \widehat{q}(\theta,\Delta_i|I_i)$), and the second integral represents the expected payment in case of dismissal ($q > \widehat{q}(\theta,\Delta_i|I_i)$), provided that the manager has undertaken the investment desired by the board.

[11] Such result is proved in Proposition 1. Anticipating it here greatly simplifies the notation and the statement of the constraints that, in fact, have the form that they will take in the problem of the board (where the result of Proposition 1 is considered).

Moreover, two incentive constraints must be satisfied. The first one, (ICC 1), guarantees that the manager prefers I_i to not investing

$$\int_0^{\widehat{q}(\theta,\Delta_i|I_i)} [(p_i+\theta+\Delta_i)w(\theta,\Delta_i;I_i)+B]f(q)dq + \int_{\widehat{q}(\theta,\Delta_i|I_i)}^{1} s'(\theta,\Delta_i|I_i)f(q)dq - I_i \geq \tag{ICC 1}$$

$$\int_0^{\widehat{q}(\theta,0|0\neq I_i)} [(p_L+\theta)w(\theta,\Delta_i,I_i)+B]f(q)dq + \int_{\widehat{q}(\theta,0|0\neq I_i)}^{1} s'(\theta,0|0\neq I_i)f(q)dq,$$

where, according to the notation introduced in Section 2, $s'(\theta,\Delta_i|I_i)$ and $\widehat{q}(\theta,\Delta_i|I_i)$ are, respectively, the renegotiated severance pay and the cutoff when the manager complies with the contract aimed to induce investment I_i, while $s'(\theta,0|0\neq I_i)$ and $\widehat{q}(\theta,0|0\neq I_i)$ would be the values of these variables if the manager were to undertake no investment. In the latter case, the manager would not be overconfident, i.e., $\Delta_i = 0$. However, this would not affect the value of incentive pay $w(\theta,\Delta_i;I_i)$ as such value was established *ex ante* at the contractual stage, on the ground of the required level of investment I_i.

The second incentive constraint, (ICC 2) requires that the manager prefers I_i to I_j when the contract prescribes investment I_i

$$\int_0^{\widehat{q}(\theta,\Delta_i|I_i)} [(p_i+\theta+\Delta_i)w(\theta,\Delta_i;I_i)+B]f(q)dq + \int_{\widehat{q}(\theta,\Delta_i|I_i)}^{1} s'(\theta,\Delta_i|I_i)f(q)dq - I_i \geq \tag{ICC 2}$$

$$\int_0^{\widehat{q}(\theta,\Delta_j|I_j\neq I_i)} [(p_j+\theta+\Delta_j)w(\theta,\Delta_i;I_i)+B]f(q)dq + \int_{\widehat{q}(\theta,\Delta_j|I_j\neq I_i)}^{1} s'(\theta,\Delta_j|I_j\neq I_i)f(q)dq - I_j.$$

Here $\widehat{q}(\theta,\Delta_j|I_j\neq I_i)$ denotes the cutoff when the contract aims to induce I_i but the manager undertakes investment I_j. In the latter case, the manager's overconfidence is $\Delta_j \neq \Delta_i$ but again this, while affecting the renegotiated severance pay and the *ex post* condition for dismissal, does not affect the incentive pay that is determined *ex ante* when the contract is offered.

By comparing (PC) and (ICC 1), we can immediately prove the following lemma.

Lemma 2. If incentive compatibility constraint (ICC 1) is satisfied, the participation constraint (PC) is satisfied as well.

Proof: Note that, by substituting $s'(\theta,0|0\neq I_i)$ from Lemma 1, the RHS of (ICC 1) becomes $(p_L+\theta)w(\theta,\Delta_i;I_i)+B>0$ in the case of a biased manager and $p_L w(0,0;I_i)+B>0$ in the case of a rational one. Consequently, if incentive compatibility constraint (ICC 1) is satisfied, the participation constraint (PC) holds in the form of an inequality.□

Given that the manager has enough bargaining power to obtain the same utility both in case of retention and in case of dismissal, the participation constraint is never binding.[12] We then focus our attention on the ICCs.

[12] A positive level of the reservation utility would not affect this result as long as such level is smaller than the expected compensation in case of no investment, $(p_L+\theta)w(\theta,\Delta_i,I_i)+B$. Note that if it were $(p_L+\theta)w(\theta,\Delta_i,I_i)+B<\overline{u}$, the reverse would hold because (ICC 1) would always be satisfied and could be disregarded in the following analysis.

However, in the latter case, the participation constraint would become binding which would make i) the case of overinvestment more likely and ii) incentive pay also depend on optimism.

5.1 Contractual severance pay

When writing the ICCs, we have anticipated that severance pay will always be renegotiated, so that the relevant value is $s'(\theta, \Delta_i|I_i)$. We now want to prove that this is the case. The following proposition establishes that there is an entire range of optimal contractual severance payments $s(\theta, \Delta_i; I_i)$, but that all such values are lower than (or equal to) the minimum payment the manager can get at the renegotiation stage $s'(\theta, 0|0 \neq I_i)$ corresponding to the case where she makes no investment under a contract prescribing $I_i > 0$. Consequently, contractual severance pay will always be renegotiated.

Proposition 1. *Any level of contractual severance pay $s(\theta, \Delta_i; I_i)$ such that $0 \leq s(\theta, \Delta_i; I_i) \leq s'(\theta, 0|0 \neq I_i)$ is optimal. Given that $s'(\theta, 0|0 \neq I_i) < s'(\theta, \Delta_i|I_i)$, renegotiation occurs at $t = 3$ whenever the manager is replaced after a positive investment.*

Proof: See Appendix 1.

The intuition comes from the fact that the amount established at the contracting stage cannot be contingent on the investment. Thus, it reduces the expected penalty from not investing and implies a higher incentive pay to induce the manager to invest. Moreover, since the contract is enforceable, the board must pay any amount specified in the contract even if the (unverifiable) investment, is not the desired one. Conversely, at the renegotiation stage, the board knows the investment chosen by the manager, so that the severance payment can be contingent on the investment. It follows that, at the contracting stage, the board doesn't want to commit to any level of severance pay higher than the minimum the manager can receive at the renegotiation stage. This minimum amount is what the manager can obtain in the case where the contract requires a positive investment but the manager does not comply and chooses $I_L = 0$. A contractual severance pay $s(\theta, \Delta_i; I_i)$ higher than the minimum level required at the renegotiation stage by a manager who had not invested $s'(\theta, 0; 0 \neq I_i)$, is not profitable for the firm because it makes the ICCs more binding with no positive effect on the participation constraint (that is slack). Hence, the total expected compensation would increase with no positive counterbalancing effect.

To better understand why even a contractual severance pay equal to zero can be optimal, consider that the manager anticipates that any payment lower than what she can obtain by staying with the firm will be renegotiated. In other words, the manager anticipates that, when $s(\theta, \Delta_i; I_i)$ is low, an additional payment will be agreed upon at the renegotiation stage so that the overall amount of the renegotiated severance pay will offer her the amount required to accept replacement. This clearly provides the incentive to invest even when contractual severance pay is zero. Hence, what really matters for the investment is the severance pay that the manager can bargain in case of dismissal, not the amount specified in the contract. In Section 7 where we discuss alternative bargaining assumptions we show that a zero contractual severance pay can still be optimal even with different levels of bargaining power.

Finally, note that this result is in contrast with the generous severance payments specified by managerial contracts. While any level between zero and $s'(\theta, 0; 0 \neq I_i)$ is equally optimal in our model, in the real world a zero contractual severance payment is the exception rather than the rule. A possible reason to set the contractual severance pay at a higher level, included the highest optimal level, $s'(\theta, 0; 0 \neq I_i)$, is that shareholders generally oppose a large renegotiated payment in excess of the contractual agreements. Shareholders usually consider such a payment an indicator of managerial power that allows the departing manager to obtain more than its contractual entitlement with no economic justification other than its influence on the board and consequently try to limit its amount. However, as explained above, this can give rise to costly ligation. Then, the board may want to keep the additional component at the minimum by setting the contractual component at the highest among the optimal levels.

5.1.1 Contractual severance pay for a rational and for a biased manager

All what we said on contractual severance pay holds both for an optimistic ($\theta > 0$) and/or overconfident ($\Delta_i > 0$) manager as well as for a rational one ($\theta = \Delta_i = 0$). The only difference is obviously in the values taken by the maximum level of contractual severance pay. In the case of a biased manager this is $s'(\theta, 0; 0 \neq I_i) = p_L w(\theta, \Delta_i; I_i) + B$ while in the case of a rational manager it is $s'(0, 0|0 \neq I_i) = p_L w(0, 0; I_i) + B$

5.2 Incentive pay and investment level I_M

In this section we determine the incentive pay necessary to induce the level of investment requested by the board. Consider first the case where the board wants to induce the efficient investment level I_M. The compensation must satisfy incentive compatibility constraints (ICC 1) and (ICC 2). By substituting $s'(\theta, \Delta_M|I_M)$ and $s'(\theta, 0|0 \neq I_M)$ from Lemma 1, (ICC 1) can be written as

$$\int_0^{\widehat{q}(\theta,\Delta_M|I_M)} [(p_M + \theta + \Delta_M)w(\theta, \Delta_M; I_M) + B]f(q)dq + \int_{\widehat{q}(\theta,\Delta_M|I_M)}^{1} [(p_M + \theta + \Delta_M)w(\theta, \Delta_M; I_M) + B]f(q)dq - I_M \geq$$

$$\int_0^{\widehat{q}(\theta,0|0\neq I_M)} [(p_L + \theta)w(\theta, \Delta_M; I_M) + B]f(q)dq + \int_{\widehat{q}(\theta,0|0\neq I_M)}^{1} [(p_L + \theta)\, w(\theta, \Delta_M; I_M) + B]f(q)dq$$

or:

$$(p_M + \theta + \Delta_M)w(\theta, \Delta_M; I_M) + B - (p_L + \theta)w(\theta, \Delta_M; I_M) - B \geq I_M,$$

implying that the incentive pay must satisfy

$$w(\theta, \Delta_M; I_M) \geq \frac{I_M}{(p_M + \Delta_M - p_L)}. \tag{5}$$

Consider then (ICC 2) with $i = M$ and $j = H$. Substituting for $s'(\theta, \Delta_M|I_M)$ and $s'(\theta, \Delta_H|I_H \neq I_M)$ from Lemma 1, (ICC 2) becomes

$$(p_H + \Delta_H - p_M - \Delta_M)w_M(\theta, \Delta_M; I_M) \leq I_H - I_M.$$

Therefore, to guarantee that both ICC1 and ICC2 are satisfied, it must be the case that

$$\frac{I_M}{p_M + \Delta_M - p_L} \leq w_M(\theta, \Delta_M; I_M) \leq \frac{I_H - I_M}{p_H + \Delta_H - p_M - \Delta_M}. \tag{6}$$

Recalling that $\Delta_H = \Delta_M(1 + z)$ such inequality can be satisfied if and only if

$$\frac{I_M}{I_H} \leq \frac{p_M + \Delta_M - p_L}{p_H + \Delta_M(1 + z) - p_L}. \tag{7}$$

When the value of Δ_M is high, the value of the RHS of the above inequality is small so that it can be difficult to satisfy such condition and guarantee that the manager chooses investment I_M.[13] In other words, when the degree of overconfidence is large, the manager even if offered $w(\theta, \Delta_M; I_M)$ to incentivize I_M, will choose I_H. This happens because a high level of overconfidence increases the subjective probability of success to such an extent that, by choosing I_H, the manager expects an increase

[13] Note that $\frac{\partial}{\partial \Delta_M}\left(\frac{p_M+\Delta_M-p_L}{p_H+\Delta_M(1+z)-p_L}\right) = \frac{(p_H-p_L)-(1+z)(p_M-p_L)}{(p_H+\Delta_M(1+z)-p_L)^2} < 0$. It is in fact $1 + z > \frac{I_H}{I_M}$ from Assumption 1, and $\frac{I_H}{I_M} \geq \frac{p_H-p_L}{p_M-p_L}$ from Assumption 2.

in her compensation that more than compensates the additional cost of the investment. This implies that, for a high enough level of overconfidence, ICC2 does not hold.

Consider the following definition.

Definition 1. *The manager is moderately overconfident when* $\Delta_M \leq \Delta_M^* = \frac{I_M(p_H - p_L) - I_H(p_M - p_L)}{I_H - I_M(1+z)}$ *so that ICC 2 is satisfied. Conversely, when* $\Delta_M > \Delta_M^*$ *the manager is extremely overconfident and ICC 2 does not hold.*

Note that $\Delta_M^* > 0$ follows from $z > \frac{I_H - I_M}{I_M}$.[14] When the manager is moderately overconfident, I_M is incentive compatible and a board willing to induce such a level of investment will offer the lowest possible level of $w(\theta, \Delta_M; I_M)$ satisfying the incentive compatibility constraint, that is

$$w(\theta, \Delta_M; I_M) = \frac{I_M}{(p_M + \Delta_M - p_L)}. \tag{8}$$

When instead the manager is extremely overconfident, I_M cannot be implemented. This highlights the potential downside of overconfidence that may induce the manager to choose the inefficient investment.

5.3 *Incentive pay and investment level* I_H

Consider then the incentive pay necessary to implement the investment level I_H. The incentive compatibility constraints for the high level of investment, I_H, ICC1 and ICC2, respectively imply

$$w(\theta, \Delta_H; I_H) \geq \frac{I_H}{(p_H + \Delta_H - p_L)} \tag{9}$$

and

$$w(\theta, \Delta_H; I_H) \geq \frac{I_H - I_M}{(p_H + \Delta_H - p_M - \Delta_M)} = \frac{I_H - I_M}{p_H - p_M + z\Delta_M}. \tag{10}$$

The first inequality guarantees that the manager prefers I_H to zero investment, while the second one guarantees that I_H is preferred to I_M. The minimum incentive pay required to induce investment I_H depends on which constraint is binding.

Let us first consider the case in which the manager is extremely overconfident so that I_M is not implementable. It is immediate to verify that the binding constraint in order to implement I_H is the first one. In fact, Definition 1 says that extreme overconfidence, $\Delta_M > \Delta_M^*$, occurs when

$$\frac{I_M}{I_H} > \frac{p_M + \Delta_M - p_L}{p_H + \Delta_M(1+z) - p_L}, \tag{11}$$

which corresponds to the case where (9) is the binding constraint because condition (11) is equivalent to $\frac{I_H}{(p_H + \Delta_H - p_L)} > \frac{I_H - I_M}{(p_H + \Delta_H - p_M - \Delta_M)}$. In this case, the increase in the perceived probability of success due to the choice of I_H is so high that the manager always prefers I_H to I_M so that the optimal incentive pay is given by the lowest value of w satisfying (9)[15]

$$w(\theta, \Delta_H; I_H) = \frac{I_H}{p_H + \Delta_H - p_L}.$$

Let us then investigate whether investment I_H can be incentive compatible even under moderate overconfidence. The next proposition proves that even if this is possible it is generally unprofitable. Thus,

[14] In fact if the rate of increase in overconfidence when moving from I_M to I_H were low ($z < \frac{I_H - I_M}{I_M}$), (7) would always be satisfied. Here we want to analyze the interesting case where overconfidence results in the distortion in the investment level.

[15] Note that if $\frac{I_M}{I_H} > \frac{p_M + \Delta_M - p_L}{p_H + \Delta_M(1+z) - p_L}$, it follows that $w(\theta, \Delta_H; I_H) = \frac{I_H}{p_H + \Delta_H - p_L} < \frac{I_M}{p_M + \Delta_M - p_L} = w(\theta, \Delta_M; I_M)$.

Proposition 2 allows us to restrict our attention to two mutually exclusive cases: extreme overconfidence with investment level I_H, and zero or moderate overconfidence with investment level I_M.[16]

Proposition 2. *The incentive pay offered by the board depends on the degree of managerial overconfidence while it is independent of optimism:*

- $w(\theta, \Delta_M; I_M) = w(0, \Delta_M; I_M) = \frac{I_M}{(p_M + \Delta_M - p_L)}$ *is generally offered when the manager is moderately overconfident* ($\Delta_M \leq \Delta_M^*$) *so that the manager chooses* I_M
- $w(\theta, \Delta_H; I_H) = w(0, \Delta_H; I_H) = \frac{I_H}{(p_H + \Delta_H - p_L)}$ *is offered when the manager is extremely overconfident* ($\Delta_M > \Delta_M^*$) *so that the manager chooses* I_H.

Proof: See Appendix 1.

The effect of overconfidence follows immediately from the discussion above. Optimism has no impact on the bonus because it uniformly shifts upwards the probabilities of success with no distortion in the marginal probabilities. A similar "no distortion" result is found in de la Rosa (2011) where, however, optimism tends to make the incentive pay steeper by relaxing the participation constraint. Note that, since in our model optimism increases the return of both investment levels without changing their relative profitability, it does not affect the choice of the investment.

5.3.1 Incentive pay when the manager is rational

When the manager is rational ($\theta = \Delta_M = 0$), the efficient level of investment I_M can always be implemented while I_H is not implementable. Consider condition (6). This implies that the two incentive compatibility constraints for investment level I_M are now satisfied when

$$\frac{I_M}{p_M - p_L} \leq w_M(0, 0; I_M) \leq \frac{I_H - I_M}{p_H - p_M}. \tag{12}$$

For such condition to hold it must be

$$\frac{I_M}{I_H} \leq \frac{p_M - p_L}{p_H - p_L},$$

which, however, is always the case by assumption 2. On the other hand, it can be easily proved that I_H cannot be incentive compatible, so that I_M will always be implemented in the case of a rational manager.

Corollary 1. When *the manager is rational, only* I_M *is incentive compatible and the board offers incentive pay* $w(0, 0|I_M) = \frac{I_M}{(p_M - p_L)}$.

Proof: To prove that, when the manager is rational, I_H is not incentive compatible, consider that ICC2 in this case implies

$$w(\theta, \Delta_H; I_H) \geq \frac{I_H - I_M}{(p_H - p_M)}$$

but we know that $\frac{I_H - I_M}{(p_H - p_M)} > R$ by Assumption 2, so that the board will never offer such incentive pay. Then I_M will be incentivized with the lowest level of w that satisfies (12).□

Proposition 2 and Corollary 1 highlight the role of overconfidence in the investment choice. When the manager is rational and holds correct beliefs about the probability of success, it is not possible to implement I_H because the manager is aware that the additional cost cannot be compensated by the

[16] The terms moderate and extreme overconfidence in our model only refer to the bias in the belief of the manager's investment productivity (overconfidence in a strict sense). This is different from the distinction in de la Rosa who considers "slight" or "significant overconfidence overall" referring to the sum of the two biases.

increase in the expected compensation. In fact, Assumption 2 guarantees that the rise in the cost is higher than the gain in expected return. However, this may not be enough to prevent an overconfident manager from choosing the inefficient investment because of the biased assessment of the probability of success. Thus, our model accounts for the possibility, documented by a large literature (see, among others, Malmendier and Tate 2005 and 2015), that an optimistic and overconfident manager may choose an investment level higher than the optimal one. The differential effects of optimism and overconfidence are analyzed in more detail in the following subsection.

6 The effects of Optimism and Overconfidence on the optimal contract and firm expected profits

From the above discussion there emerges that overconfidence and optimism have different impacts on the contract offered by the board. In this section we analyze in detail how each single bias affects the contractual components (incentive pay and severance pay), the cutoff used by the board in the replacement decision and the firm's expected profit.

6.1 Optimistic, but not overconfident, manager

Consider first a manager who is only optimistic ($\theta > 0, \Delta_M = 0$). The following corollary clarifies the impact of a change in optimism on the contractual components.

Corollary 2. *Optimism has no effect on incentive pay,* $\frac{\partial w(\theta,0;I_i)}{\partial \theta} = 0$, *while it increases the renegotiated severance pay and the cutoff value for the replacement decision:* $\frac{\partial s'(\theta,0|I_i)}{\partial \theta} > 0$ *and* $\frac{\partial \widehat{q}(\theta,0|I_i)}{\partial \theta} > 0$, $i = M, H$.

Proof: see Appendix 1.

Optimism shifts the perceived probability of success upward by the same amount for all investment levels and therefore does not affect the incentive pay. Hence, an optimistic manager has no reason to prefer the high investment I_H when I_M is required. In fact, given that incentive pay is not affected by optimism, the optimal level of the bonus for an optimistic (but not overconfident) manager is equal to that of a rational one

$$w(\theta,0;I_M) = \frac{I_M}{(p_M - p_L)} = w(0,0;I_M).$$

An increase in optimism (θ), however, raises the renegotiated severance pay because the higher perceived probability of success induces the manager to overvalue the expected incentive pay if confirmed[17]

$$s'(\theta,0|I_M) = (p_M + \theta)w(\theta,0;I_M) + B > p_M w(0,0;I_M) + B = s'(0,0|I_M).$$

The higher renegotiated severance pay increases the cutoff for the retention/dismissal decision (4)

$$\widehat{q}(\theta,0|I_M) = p_M + \frac{B}{R} + \frac{\theta \cdot w(\theta,0;I_M)}{R} > p_M + \frac{B}{R} = \widehat{q}(0,0|I_M).$$

Thus, a positive entrenchment effect is at work even if the manager is only optimistic. Observe that $\widehat{q}(\theta,0;I_M)$ has an intermediate value between the corresponding cutoff value for a rational manager $\widehat{q}(0,0|I_M) = p_M + \frac{B}{R}$ and that for an optimistic and overconfident manager, $\widehat{q}(\theta,\Delta_M|I_M) = p_i + \frac{B+(\theta+\Delta_i)w(\theta,\Delta_i;I_i)}{R}$.

[17] This has no effect on the incentive compatibility constraints because $s'(\theta,0|0 \neq I_M)$ and $s'(\theta,0|I_j \neq I_M)$ are raised by the same proportion.

Consider now the expected profits of the firm

$$V(\theta,0|I_M)=\int_0^{\widehat{q}(\theta,0|I_M)}\left[p_M(R-w(\theta,0;I_M))\right]f(q)dq+\int_{\widehat{q}(\theta,0|I_M)}^{1}(qR-(p_M+\theta)w(\theta,0;I_M))f(q)dq. \quad (13)$$

A natural question concerns the overall impact of optimism on expected profits resulting from the combined effects on expected incentive pay, retention policy and expected severance payment. This is established by following corollary.

Corollary 3. *The expected profit of the firm is decreasing in optimism* θ : $\frac{dV(\theta,0|I_M)}{d\theta}<0$.

Proof: See Appendix 1.

The negative relation between optimism and expected profits in our model is in contrast with the positive effect found in de la Rosa and in the previous principal/agent literature where optimism, by relaxing the incentive/insurance trade-off due to the agent being risk-averse, makes high-powered incentives more attractive and has a positive influence on expected profits. The difference is that in our model optimism increases the expected payment to be made to the manager through the severance pay.[18] Being the manager risk neutral, there is no positive insurance effect.

The corollary above suggests that, if the board were to know the type of the manager, it would prefer a rational manager to an optimistic (but not overconfident) one. Indeed, considering that in both cases the investment level I_M will always be chosen, the above discussion shows that the expected profit is lower under an optimistic (but not overconfident) manager than under a rational one: $V(\theta,0|I_i)<V(0,0|I_i)$.

6.2 Overconfident but not optimistic manager

Consider now the opposite case where the manager is overconfident but not optimistic. The following corollary shows how a change in overconfidence affects the contract offered by the board when $\theta=0$ and $\Delta_i>0$.

Corollary 4. *Overconfidence continuously reduces incentive pay, and increases the cutoff value for dismissal. For a given investment level I_i, $i=M,H$, it also reduces the renegotiated severance payment:* $\frac{\partial s'(0,\Delta_i|I_i)}{\partial\Delta_i}|_{I_i}<0$. *Only at Δ_M^* where the shift from I_M to I_H occurs, overconfidence has an increasing impact on severance pay.*

Proof: See Appendix 1.

That the incentive bonus is continuously decreasing in Δ_M for a given level of investment I_i, $i=M,H$ immediately follows from the expression for $w(0,\Delta_i;I_i)$. The bonus is also decreasing at Δ_M^* where there is the shift from I_M to I_H. This implies that the incentive pay is always lower than that of a rational manager

$$w(0,\Delta_i;I_i)=\frac{I_i}{(p_i+\Delta_i-p_L)}<\frac{I_M}{(p_M-p_L)}=w(0,0;I_M).$$

The result that incentive pay is smaller when the manager is overconfident is in line with previous theoretical literature and with empirical evidence (Otto 2014, Humphery-Jenner et al. 2016). This simply derives from the fact that overconfidence induces the manager to overestimate the effect of her

[18] Even if $\widehat{q}(\theta,0|I_M)$ increases with θ, contributing to the entrenchment phenomenon, expected profit is not affected by such increase because $\widehat{q}(\theta,0|I_M)$ is adjusted in order to maximize profits (see the proof of the Corollary).

investment on the probability of success so that a lower bonus is needed to incentivize the same level of investment.

The inverse relation between overconfidence and the renegotiated severance pay is the result of two contrasting effects. On the one hand, there is an increase in the manager's perceived probability of success that in turn increases the required severance pay. On the other hand, there is a reduction in the incentive pay that is the main component of the severance payment. This latter effect dominates and the reduction in incentive pay more than compensates the increase in the subjective belief of success. Only at Δ_M^*, where the shift from I_M to I_H induces a spike in the belief of success, there is an increase in s'. For $\Delta_M > \Delta_M^*$, however, $s'(0, \Delta_H|I_H)$ is again decreasing in Δ_M even if we cannot say whether the rise occurring at Δ_M^* will make $s'(0, \Delta_H|I_H)$ everywhere higher than $s'(0, \Delta_M^*|I_M)$. It may happen that for very high values of Δ_M, $s'(0, \Delta_H|I_H)$ falls below $s'(0, \Delta_M^*|I_M)$.

By comparing the renegotiated severance payment of a moderately overconfident manager with that of a rational one we can say that the former is lower than the latter

$$s'(0, \Delta_M|I_M) = (p_M + \Delta_M)w(0, \Delta_M; I_M) + B < p_M w(0, 0; I_M) + B = s'(0, 0|I_M).$$

Overconfidence results in higher entrenchment because

$$\widehat{q}(0, \Delta_i|I_i) = p_i + \frac{B}{R} + \frac{\Delta_i \cdot w(0, \Delta_i; I_i)}{R} > p_M + \frac{B}{R} = \widehat{q}(0, 0|I_M)$$

both for a given value of investment I_i and at Δ_M^* where the shift from I_M to I_H occurs. Again there are two contrasting effects at work. On the one hand, overconfidence has a positive direct impact through the higher belief of success that multiplies the incentive pay. On the other hand, it has a negative indirect effect due to the fact that the incentive pay $w_i(0, \Delta_i; I_i)$ is decreasing in Δ_i. The latter effect dominates creating an upward distortion in the replacement decision.

Assumption 1 indicates that Δ_M is the parameter that measures the "basic" overconfidence while z measures the increase in overconfidence due to a higher investment level, i.e., when moving from I_M to I_H. In fact, z is another route through which overconfidence affects incentive pay. Thus, we may wonder what is the effect of z on the contractual components, keeping the "basic" overconfidence Δ_M constant. Obviously, this is relevant only in the case in which the manager chooses I_H. It can be easily verified that in such a case the impact of a rise in z is similar to the impact of Δ_M just analyzed: when z rises, the incentive pay and the renegotiated severance pay decrease, while the entrenchment increases. Again the higher entrenchment follows from the dominance of the incentive pay effect that makes the retention of the manager more convenient.

Let us now evaluate the total effect on expected profit of an increase in overconfidence as proved in the following:

Corollary 5. *For a given investment level I_i, $i = M, H$, expected profit is increasing in overconfidence. When $\Delta_M = \Delta_M^*$ and a further increase in Δ_M implies a shift from I_M to I_H, there is a discontinuity: the expected profit generally has an initial drop and then resumes an increasing trend.*

Proof: See Appendix 1.

Profit is increasing in overconfidence as long as overconfidence is moderate and does not lead to a change in the investment level. This also implies that expected profit is higher with a moderately overconfident but not optimistic manager ($\theta = 0$, $0 < \Delta_M \leq \Delta_M^*$) than with a rational manager. When a rise in overconfidence induces the shift from I_M to I_H, there is a discontinuity and expected profit suddenly drops because, due to the rise in the subjective belief of success, the manager requires a higher renegotiated severance pay. Observe that the difference between renegotiated and contractual severance pay is at least equal to the value of the investment.[19] Thus, when the manager switches from investment I_M to I_H, the severance pay suddenly increases by at least $I_H - I_M$. Once the new level of investment, I_H, is chosen, expected profit is again increasing in overconfidence. Note, however, that there is no guarantee that it will reach again the level corresponding to Δ_M^*, because the increase in Δ_M is bounded by the constraint that the belief of success cannot exceed one.

Summarizing, a moderate level of overconfidence that does not distort the investment choice, is beneficial for the firm but this may not hold true for extreme levels of overconfidence. In particular, while a moderately overconfident but not optimistic manager yields higher expected profits than a rational one, the reverse may hold in the case of an extremely overconfident manager. This issue is analyzed by the following corollary.

Corollary 6. *For $\Delta_M \leq \Delta_M^*$ it is $V(0, \Delta_M | I_M) > V(0, 0 | I_M)$, but the drop in expected profit occurring at Δ_M^* can be high enough to result in $V(0, \Delta_H | I_H) < V(0, 0 | I_M)$ for $\Delta_M^* = \Delta_M^* + \varepsilon$, when $z > \frac{\frac{p_L}{(p_M - p_L)} + p_H + \frac{\Delta_H^* w(0, \Delta_H^*; I_H)}{2R}}{1 - p_H - \frac{\Delta_H^* w(0, \Delta_H^*; I_H)}{2R}}$ and $p_H - p_M$ is sufficiently small.*

Proof. See Appendix 1.

Corollary 6 highlights the negative impact on expected profit resulting from the shift in the investment from I_M to I_H and provides an indication as to the size of the drop that takes place at Δ_M^*. In particular, if the manager believes that investment I_H will greatly raise the probability of success with respect to I_M (z is high) even if the objective probability of success is not much higher ($p_H - p_M$ is small), it may occur that a rational manager choosing I_M yields higher expected profit than a biased one choosing I_H. In other words, a rational manager may lead to higher expected profits than an extremely overconfident one when there is a relevant discrepancy in the evaluations of the board and the manager as to the effects of the increase in the investment level. Notice that Corollary 6 is proved for $\theta = 0$, that is, without considering the negative impact of optimism. This leads to the conclusion that, even in the absence of optimism, the firm benefits from hiring a moderately overconfident manager while it may prefer a rational manager to an extremely overconfident one.

6.3 The combined effects of optimism and overconfidence

Much of the above analysis also holds in the case of a manager who is both optimistic and overconfident. In particular, corollaries 2 and 3 are also valid in the case of a given positive value of Δ_M (see the proofs of the corollaries in Appendix 1), indicating that optimism has no effect on the incentive pay while it increases the renegotiated severance pay and the cutoff value for the replacement decision for any given level of investment I_i, $i = M, H$. This, in turn, implies that no matter the (given) level of overconfidence, expected profits are decreasing in θ. Similarly, corollaries 4 and 5 also hold in case of a given positive value of θ (see again the proofs of the corollaries in Appendix 1) so that overconfidence continuously reduces the incentive pay while it raises the cutoff value for dismissal and it also reduces the renegotiated severance payment for any given investment level I_i, $i = M, H$. Only at Δ_M^* where the shift from I_M to I_H occurs, overconfidence has an increasing impact on severance pay. This implies that for a given investment level I_i, $i = M, H$, expected profit is increasing in overconfidence but at $\Delta_M = \Delta_M^*$ (where a further increase in Δ_M implies a shift from I_M to I_H) the expected profit has an initial drop.

[19] By substituting for $w(\theta, \Delta_i; I_i)$ it is immediate to verify that the difference between the renegotiated and the contractual severance payment, when the latter is at its maximum level, $s(\theta, \Delta_i | I_i) = s'(\theta, 0; 0 \neq I_i)$ is given by the cost of the investment I_i. Hence the minimum increase in severance payment at Δ_M^* is given by the difference $I_H - I_M$.

Summarizing, the two managerial biases affect the contract in a very different way: while overconfidence reduces its cost by lowering the incentive pay necessary to satisfy the incentive compatibility constraints and reducing also the severance pay except at $\Delta_M = \Delta_M^*$, optimism always increases the cost of the contract through its effect on severance pay. Then, if we consider a manager who is both optimistic and overconfident, the negative impact on profit found in Corollary 6 for an extremely overconfident manager can be generalized to a moderately overconfident one. Indeed, the negative impact of a sufficiently large degree of optimism can overcome even the positive effect of moderate overconfidence. As a result, the firm may be worse off when hiring a biased manager compared to a rational one.[20]

7 The role of managerial bargaining power

Anecdotal evidence suggests that CEOs are powerful and hold a strong bargaining position vis-a-vis the board. Hence, we believe our assumption that the manager can oppose replacement captures many situations of CEO turnover. Observe that the manager's bargaining power derives from the possibility to oppose replacement, i.e. to resist a decision taken against her will. This implies that we are focusing on forced turnovers rather than voluntary turnovers where, since it is the manager the one who wants to leave, her bargaining position is much weaker.[21] Given that the possibility to oppose replacement and the resulting payment in case of dismissal is a key feature of our model, we discuss here the robustness of our findings to alternative bargaining assumptions. Specifically, we extend our analysis to consider two different settings. First, we illustrate a more flexible setting where the bargaining power allows the manager to obtain a severance payment that may be either greater or smaller than the one considered in our model. In this first extension we modify the payment the manager obtains in case of dismissal by assuming that the manager gets a payment equal to the severance payment determined in the above model plus or minus either a fixed amount or a fraction of the firm's gain from replacement. Then, in a second extension, we analyze the extreme case in which the manager has no bargaining power at all and the board can fire her at will with no possibility of her opposing such decision.

7.1 Positive bargaining power

Let us discuss how a more flexible assumption on the manager's bargaining power would change our results. Suppose the manager has greater bargaining power so that at the renegotiation stage she can obtain a payment in excess of what she would get by staying with the firm. Consider first the case where, in case of dismissal, she can obtain a fixed additional payment no matter the level of investment she has undertaken. Let $X \leq (R - p_i w(\theta, \Delta_i; I_i))$ represent such additional amount resulting from the increase in the bargaining power. Specifically, when investment I_i is undertaken, at the renegotiation stage the manager can obtain

$$s'_{PB}(\theta, \Delta_i | I_i) = s'(\theta, \Delta_i | I_i) + X = (p_i + \theta + \Delta_i) w(\theta, \Delta_i; I_i) + B + X,$$

where X is the additional amount in excess to what she believes her expected compensation would be and the subscript PB denotes a positive level of bargaining power even if different from that of the previous sections. Note that in such a case, the incentive pay is unaffected by the higher renegotiated severance pay because the amount X is paid irrespective of the undertaken investment. In particular,

[20] This result is quite intuitive, a formal proof that there always exist a value $\widetilde{\theta}$: $1 - p_H > \widetilde{\theta} \geq 0$ such that $V(\theta > \widetilde{\theta}, \Delta_M | I_M) < V(0, 0 | I_M)$ can be provided upon request.

[21] Specifically, we are looking at forced turnover "without cause" where "for cause" usually refers to conditions such as willful misconduct or breach of fiduciary duties.

it is also paid if no investment is made and consequently the incentive compatibility constraints are not affected.

In such a case, the cutoff will be higher as the dismissal decision will be more costly because of the additional new payment. Rewriting condition (4) then yields

$$\widehat{q}_{PB}(\theta, \Delta_i|I_i) = p_i + \frac{w(\theta, \Delta_i; I_i)(\theta + \Delta_i) + B + X}{R} = \widehat{q}(\theta, \Delta_i|I_i) + \frac{X}{R}.$$

As expected, the higher bargaining power results in a higher cost for replacing the manager, which in turn reduces the probability of replacement, thus increasing entrenchment. Observe that if X is sufficiently large $\widehat{q}(\theta, \Delta_i|I_i) + \frac{X}{R}$ may be equal to 1.

The qualitative results of our model are still valid in this modified setting though the renegotiated payment is modified. Hence, the optimal renegotiated severance payment is not the one specified in Lemma 1 but it is $s'_{PB}(\theta, \Delta_i|I_i) = s'(\theta, \Delta_i|I_i) + X$ if investment I_i desired by the board is undertaken and $s'_{PB}(\theta, \Delta_i|I_j \neq I_i) = s'(\theta, \Delta_i|I_j \neq I_i) + X$ if the manager does not comply with the contract and chooses a different investment. Similarly, Proposition 1 needs to be adjusted for the new component but it is still true that it is optimal to establish the contractual severance pay at any level between zero and the amount the manager can obtain at the renegotiation stage when she does not invest: $0 \leq s_{PB}(\theta, \Delta_i; I_i) \leq s'_{PB}(\theta, \Delta_i|I_j \neq I_i)$. Also the analysis of the selection of the investment with the possible distortion due to extreme overconfidence is unchanged. Finally, note that since X is not related to optimism and overconfidence the analysis carried over in Section 6 is still valid. In summary, a higher bargaining power that results in a larger fixed payment in case of dismissal leads to a higher cost for the firm and a higher entrenchment with no major qualitative changes in the model. Finally, note that the amount X can also be negative implying a reduction of the bargaining power with respect to the analysis of the previous sections. This alternative case leads to lower renegotiated severance pay and lower entrenchment, but same incentive pay. Again our qualitative findings hold also in this case.

Such conclusions, however, partly depend on the specification used to represent the additional gains that can be obtained from a higher managerial bargaining power, which were assumed to be independent of the investment level. If we consider a case where the manager obtains a fraction k, with $0 < k \leq 1$,[22] of the gains from replacement in addition to what she could obtain by staying with the firm, the renegotiated severance payment can be written as

$$s'_{PB}(\theta, \Delta_i|I_i) = (p_i + \theta + \Delta_i)w_{PB}(\theta, \Delta_i; I_i) + B + k(q - p_i)(R - w_{PB}(\theta, \Delta_i; I_i)),$$

where $(q - p_i)(R - w_{PB}(\theta, \Delta_i; I_i)$ represents the gains from replacement. As expected, an increases in the severance payment leads to an increase in entrenchment as it is

$$\widehat{q}_{PB}(\theta, \Delta_i|I_i) = p_i + \frac{w_{PB}(\theta, \Delta_i; I_i)(\theta + \Delta_i) + B}{R(1 - k)}.$$

More interestingly, we also have an increase in the incentive bonus as $w_{PB}(\theta, \Delta_i; I_i) > w(\theta, \Delta_i; I_i)$. This is due to the fact that, as the fraction of the gains from replacement that can be appropriated by the manager increases, the latter needs more powerful incentives in order to choose a positive level of investment. In other words, contrary to what happens in our main analysis, now the manager gains more from being replaced when $I = 0$ than in case of a positive level of I_i. As a consequence a higher bonus is needed to have the manager invest.

7.2 No Bargaining power

[22] As in the previous case we could also consider a negative value of k. This would reverse the results.

Let us now consider the main findings in a framework where the manager has no bargaining power at all. We discuss the differences with the case analyzed in Sections 4-5 and we relegate all formal results to Appendix 2 where a complete analysis of the optimization problem determines the optimal values for the incentive pay $w_{NB}(\theta, \Delta_i; I_i)$, the severance pay $s_{NB}(\theta, \Delta_i; I_i)$, and the cutoff $\widehat{q}_{NB}(\theta, \Delta_i|I_i)$. The subscript NB indicates no bargaining power. In section 3 we have already seen that there is no point in paying a severance pay in excess of the contractual one if the manager has no bargaining power. We now want to show that this is a general result that follows from the fact that the contractual severance pay is established simultaneously with the incentive pay before the investment decision so that it cannot be contingent on the investment level. Clearly, if the manager has no bargaining power and cannot oppose replacement, no renegotiation will occur and only contractual severance pay, if any, will be paid in case of replacement. Thus, we can restrict our attention to the optimal contractual amount. To simplify the analysis we here assume that the manager's private benefit is zero: $B = 0$.

As discussed in section 4, the board wants to hire the new manager whenever the latter yields higher expected profit than retaining the incumbent

$$qR - s \geq p_i(R - w_{NB}(\theta, \Delta_i; I_i))$$

which results in a cutoff value equal to

$$\widehat{q}_{NB}(\theta, \Delta_i|I_i) = p_i - \frac{p_i w_{NB}(\theta, \Delta_i; I_i) - s_{NB}(\theta, \Delta_i; I_i)}{R} \tag{14}$$

This expression is the equivalent of (3) where the cutoff is negatively related to the incentive pay and positively related to the severance pay, with the difference that here we have contractual severance pay s rather than the renegotiated one s'. A further difference is that the level of s' in expression (3) was exogenously determined by the manager bargaining power so that we could substitute such value to obtain equation (4). In that case, the assumption that the manager could credibly reject any payment smaller than what she would get if confirmed, resulted in a cutoff based on the minimum payment acceptable by the incumbent expressed in terms of the incentive pay. Conversely, here, the optimal value of s has to be determined together with the other components of the contract.

Let us consider the firm's profit when I_i is the desired level of investment

$$V_{NB} = \int_0^{\widehat{q}_{NB}(\theta,\Delta_i|I_i)} \left[p_i(R - w_{NB}(\theta, \Delta_i; I_i))\right] f(q)dq + \int_{\widehat{q}_{NB}(\theta,\Delta_i|I_i)}^1 (qR - s_{NB}(\theta, \Delta_i; I_i))f(q)dq.$$

This is the objective function to be maximized subject to participation and incentive compatibility constraints analogous to (PC), (ICC 1) and (ICC 2), where we have contractual severance pay in the place of the renegotiated one. As in the baseline model, the participation constraint is not binding and the optimal value of the incentive pay $w_{NB}(\theta, \Delta_i; I_i)$ will be determined by the binding incentive compatibility constraint ICC1. This implies that the optimal value of the bonus will depend on the level of the severance pay since, as argued above, now we cannot substitute for s.

In order to show that the optimal severance pay is equal to zero, consider that the severance pay has both a direct (negative) effect on profit and an indirect effect through the cutoff value and through the incentive pay (derived from ICC1). Thus, the overall effect of the severance pay on firm's profit is given by

$$\frac{dV_{NB}}{ds_{NB}} = \underbrace{\frac{\partial V_{NB}}{\partial s_{NB}(\theta,\Delta_i;I_i)}}_{<0} + \underbrace{\frac{\partial V_{NB}}{\partial w_{NB}(\theta,\Delta_i;I_i)}}_{<0}\underbrace{\frac{\partial w_{NB}(\theta,\Delta_i;I_i)}{\partial s_{NB}(\theta,\Delta_i;I_i)}}_{?} + \underbrace{\frac{\partial V_{NB}}{\partial \widehat{q}_{NB}(\theta,\Delta_i|I_i)}}_{=0}\left(\frac{\partial \widehat{q}_{NB}(\theta,\Delta_i|I_i)}{\partial w_{NB}}\frac{\partial w_{NB}(\theta,\Delta_i;I_i)}{\partial s_{NB}(\theta,\Delta_i;I_i)} + \frac{\partial \widehat{q}_{NB}(\theta,\Delta_i|I_i)}{\partial s_{NB}}\right). \tag{15}$$

The first term on the right hand side of (15) and the first component of the second term measure the direct impact on profit of severance and incentive pay, respectively. These are negative because both payments represent a cost for the firm. However, $\frac{\partial V}{\partial w_{NB}(\theta,\Delta_i;I_i)}$, is multiplied by $\frac{\partial w_{NB}(\theta,\Delta_i;I_i)}{\partial s_{NB}(\theta,\Delta_i;I_i)}$ which measures the (indirect) effect of the severance pay on the incentive pay. The third term, represents the indirect effect of contractual severance pay on the cutoff value and is equal to zero because the optimal cutoff automatically adjust to offset any change in the level of s_{NB}.[23]

When the total effect $\frac{dV}{ds_{NB}}$ is negative, the optimal contract will offer zero severance pay, while in the opposite case a positive severance will be included in the contract. Looking at expression (15) it is immediate to see that a non negative sign of $\frac{\partial w_{NB}(\theta,\Delta_i;I_i)}{\partial s_{NB}(\theta,\Delta_i;I_i)}$ is a sufficient condition for an overall negative sign of the expected profit, $\frac{dV_{NB}}{ds_{NB}(\theta,\Delta_i;I_i)} < 0$. In the example in section 3, we saw that, in the absence of renegotiation, the contractual severance pay had a positive effect on $w_{NB}(\theta,\Delta_i;I_i)$ because it was more difficult to induce the manager to choose the desired level of investment when the contractual severance pay was higher. This result, derived in a simplified framework with an exogenous cutoff value for dismissal, is confirmed in the more general setting. In Appendix 2 we prove that $\frac{\partial w_{NB}(\theta,\Delta_M;I_M)}{\partial s_{NB}(\theta,\Delta_M;I_M)} > 0$ for any level of $s_{NB} \geq 0$, provided a mild condition on the parameters is satisfied.[24] When we consider a rational manager we always have $\frac{\partial w_{NB}(0,0;I_M)}{\partial s_{NB}(0,0;I_M)} > 0$. We can then conclude that the optimal contractual severance pay s_{NB} is equal to zero.

As in the baseline model, the question whether investment I_M is incentive compatible arises. Also in the absence of managerial bargaining power, high overconfidence may lead to overinvestment. In Appendix 2 a condition analogous to (11) is derived so that for values of Δ_M higher than the threshold derived from such condition, the manager chooses investment I_H. Despite the apparent similarity, there is however a crucial difference with respect to the case analyzed in Section 5. Here, with no renegotiation and no severance pay, it is the manager who eventually bears the cost of the investment. Then, the manager's benefit from choosing investment I_H is reduced by its high cost, even if it is not cancelled. This is in contrast with the case with bargaining power where the manager is able to transfer the investment cost to the firm. Observe that now the firm benefits from the choice of I_H because it enjoys a higher probability of success for free. In fact, with no managerial bargaining power, the expected profit is continuously decreasing in overconfidence contrary to what happens in the model with bargaining power. Here, the switch from investment I_M to investment I_H does not lead to any increase in the severance payment, contrary to what happens in the model with bargaining power. This has an important implication because it implies that different degrees of overconfidence may be optimal for the firm according to whether the manager has bargaining power: moderate overconfidence is optimal when the manager has bargaining power while extreme overconfidence may be optimal when the board can fire the manager at will.

Interestingly, in the absence of managerial bargaining power, the optimal incentive pay is decreasing

[23] This term was absent in our discussion in section 3 because there we assumed an exogenously given cutoff.

[24] The condition is slightly more restrictive that the one that allows to determine the value for the bonus. Note that, in any case, $\frac{\partial w_{NB}(\theta,\Delta_i;I_i)}{\partial s_{NB}(\theta,\Delta_i;I_i)} > 0$ is just a sufficient condition for the optimal s to be equal to zero. The overall effect is negative even when it is $\frac{\partial w_{NB}(\theta,\Delta_i;I_i)}{\partial s_{NB}(\theta,\Delta_i;I_i)} < 0$ but small in absolute value, $\left|\frac{\partial V_{NB}}{\partial s_{NB}(\theta,\Delta_i;I_i)}\right| > \left|\frac{\partial V_{NB}}{\partial w_{NB}(\theta,\Delta_i;I_i)}\frac{\partial w_{NB}(\theta,\Delta_i;I_i)}{\partial s_{NB}(\theta,\Delta_i;I_i)}.\right|$.

in both optimism and overconfidence: $\frac{\partial w_{NB}(\theta,\Delta_i;I_i)}{\partial\theta} < 0$ and $\frac{\partial w_{NB}(\theta,\Delta_i;I_i)}{\partial\Delta_M} < 0$. Thus, the effect of overconfidence is in line with the result in section 5: increasing the subjective beliefs of success makes it easier to satisfy the incentive compatibility constraint. The negative impact of optimism, instead, is in contrast with the findings in the model with managerial bargaining power. The difference is due to the fact that, in the baseline model, optimism increases managerial beliefs by the same amount irrespective of the level of the investment, so that it does not play any role in the incentive constraints. Conversely, in the present case, optimism increases what the manager expects to receive when investing more than what she expects to receive with no investment (due to the effect on the cutoff values for dismissal) relaxing the incentive compatibility constraint.

The negative relationships between incentive pay and managerial biases imply that the optimal incentive pay for the optimist and overconfident manager is lower than the one for the rational manager: $w_{NB}(\theta,\Delta_i;I_i) < w_{NB}(0,0;I_i)$. This finding is consistent with the results of the principal/agent literature when the agent is optimistic and /or overconfident confirming that, in the absence of managerial bargaining power, there are no negative effects from hiring an optimistic and/or overconfident manager.

8 Conclusion

The paper examines the interplay between managerial biases and remuneration when the board can fire the manager if a better replacement appears after the manager has undertaken a firm-specific and unverifiable investment. The board offers a contract that comprises incentive pay and severance pay. Severance pay helps motivating the manager to invest despite the anticipated possibility of being replaced. We assume that the manager has bargaining power so that she can oppose replacement. It turns out that the best way to motivate the manager is to offer a low contractual severance pay and to renegotiate the payment ex post in case replacement becomes profitable. The manager anticipates that, by renegotiating the contractual severance agreements, she will be able to recover the cost of the investment. This provides the incentive to invest, despite the risk of being replaced. In other words, renegotiation allows the board to provide the necessary ex-ante incentive by reimbursing the manager for the investment only if this has been actually undertaken.

We show that the degree and the kind of managerial bias matter. Indeed, optimism and overconfidence have different effects on the components of the compensation package. Optimism does not affect incentive pay but raises severance pay (contractual and renegotiated), consequently the expected profit is lower than the one obtained when the manager is unbiased. Overconfidence, instead, decreases incentive pay with a positive effect on profits, while its impact on severance pay depends on the degree of the bias. A moderate level of overconfidence reduces severance pay with no distortion in the investment, but a sufficiently high level of overconfidence induces the manager to choose an inefficiently high investment, which in turn results in a very high renegotiated severance pay, lowering expected profits. Hence, there is a discontinuity with a drop in expected profits at the level of overconfidence that induces the switch from the efficient to the inefficient investment. In summary, the firm benefits from moderate overconfidence while extreme overconfidence and optimism are detrimental.

Overall, our model indicates that when the manager has bargaining power there may be a trade-off between the lower cost of the incentive compensation and the higher cost of the severance payment. Hence, it is important to consider severance agreements when studying the effect of managerial optimism and overconfidence because their beneficial impact may be overstated otherwise. We consider alternative assumptions on the manager's bargaining power and we show that our qualitative results hold as long as the manager can obtain some positive payment in order to accept replacement. When instead the board can fire the manager at will the optimal severance pay is zero.

Our findings explain the high payments observed in several turnover events as the result of optimal contract provisions in the presence of managerial biases coupled with some bargaining power originated by the possibility to oppose replacement. We show that some bargaining power is necessary to explain the severance agreements. However, such payments should not be considered a "reward for failure" explained only by the control of a powerful CEOs over weak board. Indeed, they facilitate unverifiable and risky investment by the manager and therefore they may be efficient. Our model suggests that it is optimal to have a minimum of contractual severance pay and then renegotiate this amount when the firm has more information consistently with the empirical evidence provided by Goldman and Huang (2015).

9 References

Almazan, A. and J. Suarez (2003) "Entrenchment and Severance Pay in Optimal Governance Structures", *The Journal of Finance*, 58(2), 519–547.

Bebchuk, L.A. and J. Fried (2004) *Pay without Performance: the unfulfilled Promise of Executive Compensation*, Cambridge, Harvard University Press.

Brown, K. E. (2015) "Ex Ante Severance Agreements and Earnings Management", *Contemporary Accounting Research*, 32(3), 897–940,

Cadman, B.D., Campbell J.L., and S. Klasa (2016) "Are Ex Ante CEO Severance Pay Contracts Consistent with Efficient Contracting?" *Journal of Financial Quantitative Analysis*, 51(3), 737–769.

Cadman, B. D. and Campbell, J.L. and R.G. Johnson (2023) "Time-Series Variation in the Efficacy of Executive Risk-Taking Incentives: The Role of Uncertainty". Available at SSRN: https://ssrn.com/abstract=3719591, forthcoming at *The Accounting Review.*

Callahan, C. (2023) "Arranging Golden Goodbyes for Executive Exits: A Review and Agenda for Severance Pay" Journal of Management, 1-30, 01492063231172181.

Chen, S., and H. Schildberg-Hörisch (2019) "Looking at the bright side: The motivational value of confidence", *European Economic Review,* 120, 103302.

Cowen, A. P., King, A. W., & Marcel, J. J. (2016). "CEO severance agreements: A theoretical examination and research agenda". Academy of Management Review, 41(1), 151-169.

de La Rosa, L. E. (2011) "Overconfidence and moral hazard" *Games and Economic Behavior*, 73, 429-451.

Downs, J. (2023) "Information gathering by overconfident agents" *Journal of Economics & Management Strategy, 30 554-568.*

Gervais, S., J.B. Heaton and T. Odean (2011) "Overconfidence, Compensation Contracts and Capital Budgeting", *The Journal of Finance*, 66, (5), 1735- 1777.

Goldman, E. M. and P. P. Huang (2015) "Contractual vs. Actual Separation Pay Following CEO Turnover". *Management Science* 61(5), 1108-1120.

Green, B., and C.R. Taylor (2016) "Breakthroughs, deadlines, and self-reported progress: Contracting for multistage projects". *American Economic Review*, 106(12), 3660-99.

Humphery-Jenner, M., Lisic, L., V. Nanda and S. D. Silveri (2016) "Executive overconfidence and compensation structure" *Journal of Financial Economics*, 119, 533-558.

Köszegi, B. (2014), "Behavioral Contract Theory", *Journal of Economic Literature*, 52(4), 1075-1118.

Inderst, R. and H. Mueller (2010) "CEO replacement under private information", *Review of Financial Studies,* 23, 2935-2969.

Laux, V. (2015) "Executive pay, Innovation, and Risk-taking" *Journal of Economics & Management Strategy*, 24(2), 275-303.

Malmendier, U. and G. Tate (2005) "CEO overconfidence and corporate investment", *The Journal of Finance,* 60, 2661-2700.

Malmendier, U. and G. Tate (2008) "Who makes acquisitions? CEO overconfidence and the market's reaction", *Journal of Financial Economics* 89(1), 20-43.

Malmendier, U. and G. Tate (2015) "Behavioral CEOs: The Role of Managerial Overconfidence", *Journal of Economic Perspectives*, 29 (4), 37-60.

Muscarella, C.J. and J. Zhao (2015) "Promoting the Quiet Life or Risk-Taking? CEO Severance Contracts and Managerial Decision-Making", (July 1, 2015) AFA 2012 Chicago Meetings Paper. Available at SSRN: https://ssrn.com/abstract=1787221.

Otto, C. (2014) "CEO Optimism and Incentive Compensation" *Journal of Financial Economics*, 114, 366-404.

Santos-Pinto, L. (2008) "Positive self-image and incentives in organizations", *The Economic Journal*, 118, 1315-1332.

Santos-Pinto, L. (2022) "Are overconfident workers more likely to win bonuses or to be promoted? "

Santos-Pinto, L. and L. E. de la Rosa (2020) "Overconfidence in labor markets" in K. Zimmerman (ed.), *Handbook of Labor, Human Resources and Population Economics,* 1-42, Springer.

Sautmann, A. (2013). "Contracts for agents with biased beliefs: Some theory and an experiment". *American Economic Journal: Microeconomics*, 5(3), 124-156.

Vladimirov, V. (2021), "Contract Length and Severance Pay", CEPR DP 16288.

Wu, Z., & Weng, X. (2018) "Managerial turnover and entrenchment", *Journal of Economics & Management Strategy*, 27(4), 742-771.

10 Appendix 1.

Proof of Proposition 1

For any given level of investment, the board maximizes its profit by keeping both incentive and severance pay as low as possible, considering the incentive compatibility constraints (recall that the participation constraint is never binding). We then want to prove that raising contractual $s(\theta, \Delta_i|I_i)$ above $(p_L + \theta)\, w(\theta, \Delta_i; I_i) + B \equiv s'(\theta, 0|0 \neq I_i)$ makes the ICCs more binding, thus raising both $w(\theta, \Delta_i; I_i)$ and the severance pay (either contractual or renegotiated) that is paid in case of replacement.

Consider that, if contractual severance pay is not higher than $s'(\theta, 0|0) = (p_L{+}\theta{+})w(\theta, \Delta_i; I_i){+}B$, the contract will be renegotiated in case the manager is dismissed after undertaking I_i. Then the severance pay becomes $s'(\theta, \Delta_i|I_i) = (p_i + \theta + \Delta_i)w(\theta, \Delta_i; I_i) + B$ by Lemma 1, and this is anticipated at the contracting stage so that ICC1 takes the form

$$\int_0^{\widehat{q}(\theta,\Delta_i|I_i)} [(p_i + \theta + \Delta_i)w(\theta, \Delta_i; I_i) + B]f(q)dq + \int_{\widehat{q}(\theta,\Delta_i|I_i)}^1 s'(\theta, \Delta_i|I_i)]f(q)dq - I_i =$$

$$[(p_i + \theta + \Delta_i)w(\theta, \Delta_i; I_i) + B] - I_i \geq$$

$$[(p_L + \theta)w(\theta, \Delta_i; I_i) + B] =$$

$$\int_0^{\widehat{q}(\theta,|0\neq I_i)} [(p_L + \theta +)w(\theta, \Delta_i; I_i) + B]f(q)dq + \int_{\widehat{q}(\theta,0|0\neq I_i)}^1 s'(\theta, 0|0)]f(q)dq.$$

First of all note that, setting contractual $s(\theta, \Delta_i; I_i)$ below $s'(\theta, 0|0)$ does not help relaxing the ICCs because it results in the same renegotiated severance pay equal either to $s'(\theta, \Delta_i|I_i)$ if the manager complies with the contract or to $s'(\theta, 0|0)$ if she does not comply.

Consider then a level of contractual severance pay $s = s'(\theta, 0|0) + t$ such that $s'(\theta, 0|0) < s \leq s'(\theta, \Delta_i|I_i)$ and define $\widehat{\widehat{q}}(\theta, \Delta_i|I_i) \equiv p_i + \frac{s - w(\theta,\Delta_i;I_i)}{R} \geq \widehat{q}(\theta, \Delta_i|I_i) \equiv p_i + \frac{(\theta+\Delta_i)w(\theta,\Delta_i;I_i)}{R}$ and $\widehat{\widehat{q}}(\theta, \Delta_K|I_k \neq I_i) \equiv p_k + \frac{s - w(\theta,\Delta_i;I_i)}{R}$, $i = M, H,\ k = L, M, H$.

Note that in case the manager is dismissed after choosing I_i, $i = M, H$, the contract will be renegotiated and the severance pay will still be set at $s\prime(\theta, \Delta_i|I_i) = (p_i+\theta+\Delta_i)w(\theta, \Delta_i; I_i)+B$ resulting in a cutoff value of $\widehat{q}(\theta, \Delta_i|I_i) \equiv p_i + \frac{(\theta+\Delta_i)w(\theta,\Delta_i;I_i)}{R}$. On the other hand if she were to be dismissed after choosing $I_L = 0$, she would be paid $s = s'(\theta, 0|0)+t$ resulting in a cutoff equal to $\widehat{\widehat{q}}(\theta, 0|0 \neq I_i) \equiv p_L + \frac{s - w(\theta,\Delta_i|I_i)}{R}$. Substituting these values in ICC1, the latter becomes

$$\int_0^{\widehat{q}(\theta,\Delta_i|I_i)} [(p_i + \theta + \Delta_i)w(\theta, \Delta_i; I_i) + B]f(q)dq + \int_{\widehat{q}(\theta,\Delta_i|I_i)}^{1} s'(\theta, \Delta_i|I_i)]f(q)dq - I_i = \tag{16}$$

$$[(p_i + \theta + \Delta_i)w(\theta, \Delta_i; I_i) + B] - I_i \geq$$

$$[(p_L + \theta)w(\theta, \Delta_i; I_i) + B] + \int_{\widehat{\widehat{q}}(\theta,0|0\neq I_i)}^{1} tf(q)dq =$$

$$\int_0^{\widehat{\widehat{q}}(\theta,0|0\neq I_i)} [(p_i + \theta)w(\theta, \Delta_i; I_i) + B]f(q)dq + \int_{\widehat{\widehat{q}}(\theta,0|0\neq I_i)}^{1} s(\theta, \Delta_i; I_i)]f(q)dq$$

which is clearly more stringent than (16).

Consider finally a level of $s = s'(\theta, \Delta_i|I_i) + t$. Note that, in case the manager is dismissed, she will now be paid s independently of the level of investment she has undertaken. This results in a cutoff value equal to $\widehat{\widehat{q}}(\theta, \Delta_i|I_i) \equiv p_i + \frac{s - p_i w(\theta,\Delta_i;I_i)}{R}$ if she chooses I_i and in a cutoff equal to $\widehat{\widehat{q}}(\theta, 0|0 \neq I_i) \equiv p_L + \frac{s - p_L w(\theta,\Delta_i|I_i)}{R}$ if she were to choose $I_L = 0$. Substituting these values in ICC1, the latter becomes

$$\int_0^{\widehat{\widehat{q}}(\theta,\Delta_i|I_i)} [(p_i + \theta + \Delta_i)w(\theta, \Delta_i; I_i) + B]f(q)dq + \int_{\widehat{\widehat{q}}(\theta,\Delta_i|I_i)}^{1} sf(q)dq - I_i =$$

$$[(p_i + \theta + \Delta_i)w(\theta, \Delta_i; I_i) + B] + \int_{\widehat{\widehat{q}}(\theta,\Delta_i|I_i)}^{1} tf(q)dq - I_i \geq$$

$$[(p_L + \theta)w(\theta, \Delta_i; I_i) + B] + \int_{\widehat{\widehat{q}}(\theta,0|0\neq I_i)}^{1} [(p_i + \Delta_i - p_L)w(\theta, \Delta_i; I_i) + t]f(q)dq =$$

$$\int_0^{\widehat{\widehat{q}}(\theta,0|0\neq I_i)} [(p_L + \theta)w(\theta, \Delta_i; I_i) + B]f(q)dq + \int_{\widehat{\widehat{q}}(\theta,0|0\neq I_i)}^{1} sf(q)dq$$

which, considering that it is $\widehat{\widehat{q}}(\theta, \Delta_i|I_i) > \widehat{\widehat{q}}(\theta, 0|0 \neq I_i)$, can also be written as

$$[(p_i + \theta + \Delta_i)w(\theta, \Delta_i; I_i) + B] - I_i \geq$$

$$[(p_L + \theta)w(\theta, \Delta_i; I_i) + B] + \int_{\widehat{\widehat{q}}(\theta,0|0\neq I_i)}^{1} [(p_i + \Delta_i - p_L)w(\theta, \Delta_i; I_i)]f(q)dq + \int_{\widehat{\widehat{q}}(\theta,0|0\neq I_i)}^{\widehat{\widehat{q}}(\theta,\Delta_i|I_i)} tf(q)dq$$

thus making it clear that such expression is more stringent than (16).

Let us then discuss ICC2. Consider first the case where the board offers a contract aimed at incentivizing I_M. ICC2 is

$$\int_0^{\widehat{q}(\theta,\Delta_M|I_M)} [(p_M+\theta+\Delta_i)w(\theta,\Delta_M;I_M)+B]f(q)dq + \int_{\widehat{q}(\theta,\Delta_M|I_M)}^{1} sf(q)dq - I_M \geq$$

$$\int_0^{\widehat{q}(\theta,\Delta_H|I_H\neq I_M)} [(p_H+\theta+\Delta_H)w(\theta,\Delta_M;I_M)+B]f(q)dq + \int_{\widehat{q}(\theta,\Delta_H|I_H\neq I_M)}^{1} sf(q)dq$$

Now the three relevant cases are the following.

i) if $s \leq s'(\theta,\Delta_M|I_M) = (p_M+\theta+\Delta_M)w(\theta,\Delta_M;I_M)+B]$: renegotiation occurs both if the manager chooses I_M and if she chooses I_H so that contractual severance pay does not affect the constraint

ii) if $(p_H+\theta+\Delta_H)w(\theta,\Delta_M;I_M)+B > s = s'(\theta,\Delta_M|I_M)+t = (p_M+\theta+\Delta_M)w(\theta,\Delta_M;I_M)+B+t$ so that there occurs renegotiation if I_H is chosen but not if I_M is chosen then ICC2 takes the form

$$\int_0^{\widehat{q}(\theta,\Delta_M|I_M)} [(p_M+\theta+\Delta_M)w(\theta,\Delta_M;I_M)+B]f(q)dq + \int_{\widehat{q}(\theta,\Delta_M|I_M)}^{1} sf(q)dq - I_M =$$

$$[(p_M+\theta+\Delta_M)w(\theta,\Delta_M;I_M)+B] - I_M + \int_{\widehat{q}(\theta,\Delta_M|I_M)}^{1} tf(q)dq$$

$$\geq [(p_H+\theta+\Delta_H)w(\theta,\Delta_M;I_M)+B] - I_H =$$

$$\int_0^{\widehat{q}(\theta,\Delta_H|I_H\neq I_M)} [(p_H+\theta+\Delta_H)w(\theta,\Delta_M;I_M)+B]f(q)dq + \int_{\widehat{q}(\theta,\Delta_H|I_H\neq I_M)}^{1} s'(\theta,\Delta_H|I_H\neq I_M)f(q)dq - I_H$$

and is then relaxed by a positive value of t.

iii) if $s = s'(\theta,\Delta_H|I_H\neq I_M)+t' = (p_H+\theta+\Delta_H)w(\theta,\Delta_M;I_M)+B+t'$ there is no renegotiation. ICC2 is equal to

$$[(p_M+\theta+\Delta_M)w(\theta,\Delta_M;I_M)+B] - I_M + \int_{\widehat{q}(\theta,\Delta_M|I_M)}^{1} [(p_H+\Delta_H-p_M-\Delta_M)w(\theta,\Delta_M;I_M)+B+t']f(q)dq$$

$$\geq \quad [(p_H+\theta+\Delta_H)w(\theta,\Delta_M;I_M)+B] + \int_{\widehat{q}(\theta,\Delta_H|I_H\neq I_M)}^{1} t'f(q)dq - I_H, \qquad i,j=M,H, \quad j\neq i.$$

and is again relaxed by a positive value of t' as $\widehat{q}(\theta,\Delta_M|I_M) < \widehat{q}(\theta,\Delta_H|I_H\neq I_M)$.

The fact that the ICC2 constraints are less stringent in cases 2 and 3 raises the question whether the increase in contractual severance pay could be beneficial for the firm. We can however verify than the negative effect on ICC1 dominates the positive effect on ICC2. Consider case 2 with $(p_H+\theta+\Delta_H)w(\theta,\Delta_M;I_M)+B > s = s'(\theta,\Delta_M|I_M)+t = (p_M+\theta+\Delta_M)w(\theta,\Delta_M;I_M)+B+t$. From ICC1 we obtain that $w(\theta, \Delta_M; I_M)$ must satisfy

$$w(\theta, \Delta_M; I_M) \geq \frac{I_M + \int_{\widehat{q}(\theta,0|0\neq I_M)}^{\widehat{q}(\theta,\Delta_M|I_M)} tf(q)dq}{(p_M + \Delta_M - p_L) \int_0^{\widehat{q}(\theta,0|0\neq I_M)} f(q)dq}$$

while from ICC2 we have

$$\frac{I_H - I_M + \int_{\widehat{q}(\theta,\Delta_M|I_M)}^{1} tf(q)dq}{p_H + \Delta_H - p_M - \Delta_M} \geq w(\theta, \Delta_M; I_M).$$

So that for both constraints to be simultaneously satisfied it must the case that

$$\frac{I_H - I_M + \int_{\widehat{q}(\theta,\Delta_M|I_M)}^{1} tf(q)dq}{p_H + \Delta_H - p_M - \Delta_M} \geq \frac{I_M + \int_{\widehat{q}(\theta,0|0\neq I_M)}^{\widehat{q}(\theta,\Delta_M|I_M)} tf(q)dq}{(p_M + \Delta_M - p_L) \int_0^{\widehat{q}(\theta,0|0\neq I_M)} f(q)dq}$$

Note that in the case where $s \leq s'(\theta, 0|0)$ and the board renegotiates whenever it wants to dismiss the manager, the relevant inequality that can be derived from the corresponding forms of ICC1 and ICC2 (the details are also found in Section 5.3 in the text) is

$$\frac{I_H - I_M}{p_H + \Delta_H - p_M - \Delta_M} \geq \frac{I_M}{(p_M + \Delta_M - p_L)}$$

which is less stringent than the inequality above implying that, by choosing $s \leq s'(\theta, 0|0)$ the board can maximize over a larger set of values for w.

For case iii) where $s = s'(\theta, \Delta_H|I_H \neq I_M) + t' = (p_H + \theta + \Delta_H)w(\theta, \Delta_M; I_M) + B + t'$ we have that it is not possible to satisfy ICC1 unless the denominator of the following inequality is positive (which depends on the values of the parameters). Clearly if IIC1 is not satisfied such level of s is not feasible. If the parameters are such that ICC1 can be satisfied, $w(\theta, \Delta_M; I_M)$ must now simultaneously satisfy

$$w(\theta, \Delta_M; I_M) \geq \frac{I_M + \int_{\widehat{q}(\theta,0|0\neq I_M)}^{\widehat{q}(\theta,\Delta_M|I_M)} t'f(q)dq}{(p_M + \Delta_M - p_L) \int_0^{\widehat{q}(\theta,0|0\neq I_M)} f(q)dq - (p_H + \Delta_H - p_M - \Delta_M) \int_{\widehat{q}(\theta,0|0\neq I_M)}^{\widehat{q}(\theta,\Delta_M|I_M)} f(q)dq}$$

and

$$\frac{I_H - I_M + \int_{\widehat{q}(\theta,\Delta_M|I_M)}^{\widehat{q}(\theta,\Delta_H|I_H\neq I_M)} t'f(q)dq}{(p_H + \Delta_H - p_M - \Delta_M) \int_0^{\widehat{q}(\theta,\Delta_M|I_M)} f(q)dq} \geq w(\theta, \Delta_M; I_M).$$

and we can apply the same line of reasoning as above in order to show that, by choosing $s \leq s'(\theta, 0|0)$ the board can maximize over a larger set of values for w.

Consider then ICC2 for the case where the board offers a contract aimed at incentivizing I_H.

i) if $s < s'(\theta, \Delta_M|I_M \neq I_H) = (p_M + \theta + \Delta_M)w(\theta, \Delta_H; I_H) + B]$: renegotiation occurs both is the manager chooses I_H and if she were to choose I_M so that the value of contractual s is irrelevant.

ii) if $(p_H+\theta+\Delta_H)w(\theta, \Delta_H; I_H)+B > s = s'(\theta, \Delta_M|I_M \neq I_H)+x = (p_M+\theta+\Delta_M)w(\theta, \Delta_H; I_H)+B+x$ we observe renegotiation if I_H is chosen but not if I_M is chosen. ICC2 can then be written as

$$[(p_H + \theta + \Delta_H)w(\theta, \Delta_H; I_H) + B] - I_H \geq$$

$$[(p_M + \theta + \Delta_M)w(\theta, \Delta_H; I_H) + B] + \int_{\widehat{q}(\theta, \Delta_M|I_M \neq I_H)}^{1} xf(q)dq - I_M, \quad i, j = M, H, \ \ j \neq i.$$

which becomes more stringent with a positive x.

iii) if $s = (p_H + \theta + \Delta_H)w(\theta, \Delta_H; I_H) + B + x'$, ICC2 is given by

$$[(p_H + \theta + \Delta_H)w(\theta, \Delta_H; I_H) + B] + \int_{\widehat{q}(\theta, \Delta_H|I_H)}^{1} x'f(q)dq - I_H \geq$$

$$[(p_M + \theta + \Delta_M)w(\theta, \Delta_H; I_H) + B] - I_M + \int_{\widehat{q}(\theta, \Delta_M|I_M \neq I_H)}^{1} [(p_H + \Delta_H - p_M - \Delta_M)w(\theta, \Delta_H; I_H) + B + x']f(q)dq$$

which is again made more stringent the larger is x' as $\widehat{q}(\theta, \Delta_M|I_M \neq I_H) < \widehat{q}(\theta, \Delta_H|I_H)$.

Proof of Proposition 2

The part of the proposition concerning extreme overconfidence is proved in the text. In order to prove the part on moderate overconfidence, recall that in this case both I_M and I_H can be made incentive compatible. Incentive compatibility of I_M has been discussed in the text where we have shown that it implies offering $w(\theta, \Delta_M; I_M) = \frac{I_M}{(p_M+\Delta_M-p_L)}$. Consider now the incentive compatibility constraints for I_H and note that the condition for moderate overconfidence (7) can be written as $\frac{I_H}{(p_H+\Delta_H-p_L)} \leq \frac{I_H-I_M}{(p_H+\Delta_H-p_M-\Delta_M)}$ implying that, in case the board wants to implement I_H, (10) is binding, and the lowest value of incentive pay is

$$w(\theta, \Delta_H; I_H) = \frac{I_H - I_M}{(p_H + \Delta_H - p_M - \Delta_M)}.$$

Observe that, given moderate overconfidence, that is given $\frac{I_M}{I_H} < \frac{p_M+\Delta_M-p_L}{p_H+\Delta_H-p_L}$, it is $w(\theta, \Delta_H; I_H) = \frac{I_H-I_M}{p_H+\Delta_H-(p_M+\Delta_M)} > \frac{I_M}{(p_M+\Delta_M-p_L)} = w(\theta, \Delta_M; I_M)$. This also implies that $\widehat{q}(\theta, \Delta_M|I_M) = p_M + \frac{B}{R} + \frac{\theta+\Delta_M}{R}w(\theta, \Delta_M; I_M) < p_H + \frac{B}{R} + \frac{\theta+\Delta_H}{R}w(\theta, \Delta_H; I_H) = \widehat{q}(\theta, \Delta_H|I_H)$. We must prove that these two effects make I_H generally unprofitable for the firm that will consequently offer the manager $w(\theta, \Delta_M; I_M)$ to induce I_M. In other words we must prove that the expected profit of the firm is higher under a contract based on $w(\theta, \Delta_M; I_M) = \frac{I_M}{(p_M+\Delta_M-p_L)}$ (to induce the choice of I_M) than under a contract based on $w(\theta, \Delta_H; I_H) = \frac{I_H-I_M}{p_H+\Delta_H-(p_M+\Delta_M)}$ (to induce the choice of I_H).

Taking into account that renegotiation occurs when the manager is replaced (Proposition 1) and that $s'(\theta, \Delta_i|I_i) = (p_i + \theta + \Delta_i)w(\theta, \Delta_i; I_i) + B$, the expected profit of the firm $V(\theta, \Delta_i|I_i)$ $i = M, H$, can be written as

$$V(\theta,\Delta_i|I_i)=\int\limits_0^{\widehat{q}(\theta,\Delta_i|I_i)}[p_i(R-w(\theta,\Delta_i;I_i))]\,f(q)dq+\int\limits_{\widehat{q}(\theta,\Delta_i|I_i)}^{1}qR-(p_i+\theta+\Delta_i)\,w(\theta,\Delta_i;I_i)-B)f(q)dq.$$

The difference between the expected profit that can be obtained by offering $w(\theta,\Delta_H;I_H)=\frac{I_H-I_M}{p_H+\Delta_H-(p_M+\Delta_M)}$ and $w(\theta,\Delta_M;I_M)=\frac{I_M}{(p_M+\Delta_M-p_L)}$ can be written as

$$V(\theta,\Delta_H|I_H)-V(\theta,\Delta_M|I_M)= \tag{17}$$

$$\int\limits_0^{\widehat{q}(\theta,\Delta_M|I_M)}(p_{H-}p_M)\,(R-w(\theta,\Delta_M;I_M))f(q)dq-\int\limits_0^{\widehat{q}(\theta,\Delta_M|I_M)}p_H(w(\theta,\Delta_H;I_H)-w(\theta,\Delta_M;I_M))f(q)dq+$$

$$+\int\limits_{\widehat{q}(\theta,\Delta_M|I_M)}^{\widehat{q}(\theta,\Delta_H|I_H)}\{p_H(R-(w(\theta,\Delta_M;I_M))-[qR-(p_M+\Delta_M+\theta)\,w(\theta,\Delta_M;I_M)-B]\}\,f(q)dq+$$

$$-\int\limits_{\widehat{q}(\theta,\Delta_M|I_M)}^{\widehat{q}(\theta,\Delta_H|I_H)}p_H(w(\theta,\Delta_H;I_H)-w(\theta,\Delta_M;I_M))f(q)dq+$$

$$-\int\limits_{\widehat{q}(\theta,\Delta_H|I_H)}^{1}(p_H+\Delta_H-p_M-\Delta_M)w(\theta,\Delta_M;I_M)f(q)dq+$$

$$-\int\limits_{\widehat{q}(\theta,\Delta_H|I_H)}^{1}(p_H+\Delta_H+\theta)[w(\theta,\Delta_H,I_H)-w(\theta,\Delta_M;I_M)]f(q)dq$$

which, by summing and subtracting p_MR in the first integral of the second line becomes

$$V(\theta,\Delta_H|I_H)-V(\theta,\Delta_M|I_M)=$$

$$\int\limits_0^{\widehat{q}(\theta,\Delta_M|I_M)}(p_{H-}p_M)\,(R-w(\theta,\Delta_M;I_M))f(q)dq-\int\limits_0^{\widehat{q}(\theta,\Delta_M|I_M)}p_H(w(\theta,\Delta_H;I_H)-w(\theta,\Delta_M;I_M))f(q)dq+$$

$$+\int\limits_{\widehat{q}(\theta,\Delta_M|I_M)}^{\widehat{q}(\theta,\Delta_H|I_H)}(p_H-p_M)Rf(q)dq-\int\limits_{\widehat{q}(\theta,\Delta_M|I_M)}^{\widehat{q}(\theta,\Delta_H|I_H)}p_H(w(\theta,\Delta_H;I_H)-w(\theta,\Delta_M;I_M))f(q)dq+$$

$$-\int\limits_{\widehat{q}(\theta,\Delta_M|I_M)}^{\widehat{q}(\theta,\Delta_H|I_H)}\{p_Hw(\theta,\Delta_M;I_M)-p_MR-[qR-(p_M+\Delta_M+\theta)\,w(\theta,\Delta_M;I_M)-B]\}\,f(q)dq+$$

$$-\int\limits_{\widehat{q}(\theta,\Delta_H|I_H)}^{1}(p_H+\Delta_H-p_M-\Delta_M)w(\theta,\Delta_M;I_M)f(q)dq-\int\limits_{\widehat{q}(\theta,\Delta_H|I_H)}^{1}(p_H+\Delta_H+\theta)[w(\theta,\Delta_H,I_H)-w(\theta,\Delta_M;I_M)]f(q)dq$$

Summing $p_M(w(\theta,\Delta_H;I_H)-w(\theta,\Delta_M;I_M))$ to the second and fourth integral and summing the integrals in the first two lines we can write

$$V(\theta,\Delta_H|I_H) - V(\theta,\Delta_M|I_M) \leq \int\limits_0^{\widehat{q}(\theta,\Delta_H|I_H)} (p_{H-}p_M)\left(R - w(\theta,\Delta_M;I_M)\right) f(q)dq +$$

$$-\int\limits_0^{\widehat{q}(\theta,\Delta_H|I_H)} (p_H - p_M)(w(\theta,\Delta_H;I_H) - w(\theta,\Delta_M;I_M)) f(q)dq +$$

$$-\int\limits_{\widehat{q}(\theta,\Delta_M|I_M)}^{\widehat{q}(\theta,\Delta_H|I_H)} \left\{p_H w(\theta,\Delta_M;I_M) - p_M R + \left[qR - (p_M + \Delta_M + \theta)\, w(\theta,\Delta_M;I_M) - B\right]\right\} f(q)dq +$$

$$-\int\limits_{\widehat{q}(\theta,\Delta_H|I_H)}^{1} (p_H + \Delta_H - p_M - \Delta_M) w(\theta,\Delta_M;I_M) f(q)dq +$$

$$-\int\limits_{\widehat{q}(\theta,\Delta_H|I_H)}^{1} (p_H + \Delta_H + \theta)[w(\theta,\Delta_H,I_H) - w(\theta,\Delta_M;I_M)] f(q)dq$$

Considering also that $I_H - I_M \geq (p_{H-}p_M)\, R$ we have

$$V(\theta,\Delta_H|I_H) - V(\theta,\Delta_M|I_M) \leq \int\limits_0^{\widehat{q}(\theta,\Delta_H|I_H)} \left[I_H - I_M - (p_{H-}p_M)\, w(\theta,\Delta_H;I_H)\right] f(q)dq +$$

$$-\int\limits_{\widehat{q}(\theta,\Delta_M|I_M)}^{\widehat{q}(\theta,\Delta_H|I_H)} \left\{p_H w(\theta,\Delta_M;I_M) - p_M R + \left[qR - (p_M + \Delta_M + \theta)\, w(\theta,\Delta_M;I_M) - B\right]\right\} f(q)dq +$$

$$-\int\limits_{\widehat{q}(\theta,\Delta_H|I_H)}^{1} (p_H + \Delta_H - p_M - \Delta_M) w(\theta,\Delta_M;I_M) f(q)dq +$$

$$-\int\limits_{\widehat{q}(\theta,\Delta_H|I_H)}^{1} (p_H + \Delta_H + \theta)[w(\theta,\Delta_H;I_H) - w(\theta,\Delta_M;I_M)] f(q)dq$$

Substituting $I_H - I_M = [p_H + \Delta_H - (p_M + \Delta_M)]\, w(\theta,\Delta_H;I_H)$, and recalling that $\Delta_H - \Delta_M = z\Delta_M$, the above inequality becomes

$$V(\theta,\Delta_H|I_H) - V(\theta,\Delta_M|I_M) \leq \int\limits_0^{\widehat{q}(\theta,\Delta_H|I_H)} z\Delta_M w(\theta,\Delta_H|I_H) f(q)dq$$

$$-\int\limits_{\widehat{q}(\theta,\Delta_M|I_M)}^{\widehat{q}(\theta,\Delta_H|I_H)} \left\{p_H w(\theta,\Delta_M;I_M) - p_M R + \left[qR - (p_M + \Delta_M + \theta)\, w(\theta,\Delta_M;I_M) - B\right]\right\} f(q)dq +$$

$$-\int\limits_{\widehat{q}(\theta,\Delta_H|I_H)}^{1} (p_H + \Delta_H - p_M - \Delta_M) w(\theta,\Delta_M;I_M) f(q)dq +$$

$$-\int\limits_{\widehat{q}(\theta,\Delta_H|I_H)}^{1} (p_H + \Delta_H + \theta)[w(\theta,\Delta_H,I_H) - w(\theta,\Delta_M;I_M)] f(q)dq$$

Considering that $z\Delta_M$ is very small and all the other terms are negative, $V(\theta, \Delta_H|I_H) - V(\theta, \Delta_M|I_M)$ will generally be negative. Recall that we are here considering the case of moderate overconfidence, that is values of $\Delta_M \leq \Delta_M^*$.

Proof of Corollary 2

The following proof shows that the corollary holds for any value of $\Delta_i \geq 0$,including $\Delta_i = 0$.

Part i) immediately follows from the expression for $w(\theta, \Delta_i; I_i)$ which does not depend on θ so that $\frac{\partial w(\theta,\Delta_i;I_i)}{\partial \theta} = 0$; for part ii) it is immediate that $\frac{\partial s'(\theta,\Delta_i|I_i)}{\partial \theta} = (p_i + \Delta_i)\, w(\theta, \Delta_i; I_i) > 0$; finally, part iii) can be verified by substituting $w(\theta, \Delta_i; I_i) = \frac{I_i}{(p_i+\Delta_i-p_L)}$ in (4) thus obtaining $\widehat{q}(\theta, \Delta_i|I_i) = p_i + \frac{B}{R} + \frac{(\theta+\Delta_i)I_i}{R(p_i+\Delta_i-p_L)}$ which is clearly increasing in θ, i.e. $\frac{\partial \widehat{q}(\theta,\Delta_i|I_i)}{\partial \theta} > 0$.

Proof of Corollary 3

As for the previous corollary the proof considers any possible value of $\Delta_i \geq 0$ including $\Delta_i = 0$. The overall impact of optimism on expected profit, resulting from the combined effects on expected incentive pay, retention policy and expected severance pay is given by

$$\frac{dV(\theta, \Delta_i|I_M)}{d\theta} = \frac{\partial V}{\partial w(\theta, \Delta_i; I_i)} \frac{\partial w(\theta, \Delta_i; I_i)}{\partial \theta} + \frac{\partial V}{\partial s'(\theta, \Delta_i|I_i)} \frac{\partial s'(\theta, \Delta_i|I_i)}{\partial \theta} + \frac{\partial V}{\partial \widehat{q}(\theta, \Delta_i|I_i)} \frac{\partial \widehat{q}(\theta, \Delta_i|I_i)}{\partial \theta}$$

where the first term on RHS is zero (incentive pay is not affected by θ) as well as the last term because $\widehat{q}$ is optimally determined by balancing what is gained from replacement and the payment necessary to have the incumbent leave. It is in fact $\frac{\partial V}{\partial \widehat{q}(\theta,\Delta_i|I_i)} = \underbrace{[(\widehat{q}(\theta, \Delta_i|I_i) - p_i)R - ((\theta + \Delta_i)w(\theta, \Delta_i|I_i) + B)]}_{=0} f(\widehat{q}(\theta, \Delta_i|I_i))$.

Then, the total effect on firm profit is negative because we are left with only the effect of severance pay which s indeed negative: $\frac{\partial V}{\partial s'(\theta,\Delta_i;I_i)} \frac{\partial s'(\theta,\Delta_i|I_i)}{\partial \theta} = -\int_{\widehat{q}(\theta,\Delta_i|I_i)}^{1} w(\theta, \Delta_i|I_i) f(q) dq < 0.$

Proof of Corollary 4

We provide the proof for values of $\theta \leq p_i - p_L$ including $\theta = 0$. That the incentive pay is continuously decreasing in Δ_M immediately follows from $w(\theta, \Delta_M; I_M) = \frac{I_M}{(p_M+\Delta_M-p_L)}$ and $w(\theta, \Delta_H; I_H) = \frac{I_H}{(p_H+\Delta_M(1+z)-p_L)}$, considering that at Δ_M^* it is $\frac{I_M}{(p_M+\Delta_M^*-p_L)} = \frac{I_H}{(p_H+\Delta_M^*(1+z)-p_L)}$.

That $s'(\theta, \Delta_i|I_i)$ is decreasing in overconfidence for a given level of investment I_i with $i = M, H$ follows from

$$\begin{aligned} \frac{\partial s'(\theta, \Delta_M|I_M)}{\partial \Delta_M} &= -\left(\frac{I_M}{(p_M + \Delta_M - p_L)}\right)\left(\frac{(p_L + \theta)}{(p_M + \Delta_M - p_L)}\right) < 0. \\ \frac{\partial s'(\theta, \Delta_H|I_H)}{\partial \Delta_M} &= -\left(\frac{(1+z)\, I_H}{(p_H + \Delta_M (1+z) - p_L)}\right)\left(\frac{(p_L + \theta)}{(p_H + \Delta_M (1+z) - p_L)}\right) < 0. \end{aligned}$$

To verify that $s'(\theta, \Delta_i|I_i)$ is increasing in overconfidence at Δ_M^*, where the shift from I_M to I_H occurs, note that at Δ_M^* it is $w(\theta, \Delta_M|I_M) = w(\theta, \Delta_H|I_H) \equiv w$ implying $s'(\theta, \Delta_M|I_M) = (p_M + \theta + \Delta_M^*)w < (p_H + \theta + \Delta_M^*(1+z))w = s'(\theta, \Delta_H|I_H)$.

To evaluate the effect of overconfidence on the cutoff value for dismissal, substitute $w(\theta, \Delta_i|I_i) = \frac{I_i}{(p_i+\Delta_i-p_L)}$ in (4) thus obtaining $\widehat{q}(\theta, \Delta_i|I_i) = p_i + \frac{B}{R} + \frac{(\theta+\Delta_i)I_i}{R(p_i+\Delta_i-p_L)}$. For a given level of investment I_i, $\widehat{q}(\theta, \Delta_i|I_i)$ is increasing in the overconfidence parameter Δ_M as

$$\frac{\partial \widehat{q}(\theta, \Delta_i|I_i)}{\partial \Delta_M} = \frac{I_i R\,(p_i + \Delta_i - p_L) - I_i R(\theta + \Delta_i)}{[R\,(p_i + \Delta_i - p_L)]^2} = \frac{I_i(p_i - p_L - \theta)}{R\,(p_i + \Delta_i - p_L)^2} > 0, \qquad i = M, H$$

which is satisfied for $p_i - p_L > \theta$.

To evaluate what happens at Δ_M^* where the shift from I_M to I_H occurs, note that at Δ_M^* it is $w(\theta, \Delta_i|I_M) = w(\theta, \Delta_i|I_H) \equiv w$ implying that $\widehat{q}(\theta, \Delta_i|I_M) = p_M + \frac{B}{R} + \frac{\theta+\Delta_M^*}{R} w < p_H + \frac{B}{R} + \frac{\theta+\Delta_M^*(1+z)}{R} w = \widehat{q}(\theta, \Delta_i|I_H)$. Then the cutoff value is increasing also in this point (even if there is a discontinuity).

Proof of Corollary 5

Note that the proof considers any possible value of $\theta \geq 0$ including $\theta = 0$. In order to prove the corollary we must show that a) when I_i is chosen, profits are increasing in Δ_i and b) when (7) holds as an equality, profits are generally higher if I_M is chosen. Taking into account that renegotiation occurs when the manager is replaced (Proposition 1) and that $s'(\theta, \Delta_i|I_i) = (p_i + \theta + \Delta_i) w(\theta, \Delta_i; I_i) + B$, the expected profit of the firm $V(\theta, \Delta_i|I_i)$ $i = M, H$, can be written as

$$V(\theta, \Delta_i|I_i) = p_i(R - w(\theta, \Delta_i; I_i)) + \int_{\widehat{q}(\theta,\Delta_i|I_i)}^{1} [(q - p_i)R - (\theta + \Delta_i)\, w(\theta, \Delta_i; I_i) - B]\, f(q)dq$$

a) For a given $i = M, H$ it is

$$\begin{aligned}\frac{\partial V(\theta, \Delta_i|I_i)}{\partial \Delta_i} &= -p_i \frac{\partial w(\theta, \Delta_i; I_i)}{\partial \Delta_i} \\ &- \int_{\widehat{q}(\theta,\Delta_i|I_i)}^{1} [w(\theta, \Delta_i; I_i) + (\theta + \Delta_i)\frac{\partial w(\theta, \Delta_i; I_i)}{\partial \Delta_i}] f(q)dq + \\ &- \frac{\partial \widehat{q}(\theta, \Delta_i|I_i)}{\partial \Delta_M} \underbrace{[(\widehat{q}(\theta, \Delta_i|I_i) - p_i)R - ((\theta + \Delta_i) w(\theta, \Delta_i; I_i) + B)]}_{=0} f(\widehat{q}(\theta, \Delta_i|I_i)).\end{aligned}$$

In fact, by substituting $\widehat{q}(\theta, \Delta_i|I_i) = p_i + \frac{B}{R} + \frac{(\theta+\Delta_i)w(\theta,\Delta_i;I_i)}{R}$, we can immediately verify that the square bracket in the last term of the RHS is equal to zero. Substituting $\frac{\partial w(\theta,\Delta_i;I_i)}{\partial \Delta_i} = -\frac{I_i}{(p_i+\Delta_\theta-p_L)^2} = -\frac{w(\theta,\Delta_i;I_i)}{(p_i+\Delta_i-p_L)} < 0$, we then obtain:

$$\begin{aligned}\frac{\partial V(\theta, \Delta_i|I_i)}{\partial \Delta_i} &= p_i \frac{w(\theta, \Delta_i; I_i)}{(p_i + \Delta_i - p_L)} - \int_{\widehat{q}(\theta,\Delta_i|I_i)}^{1} [w(\theta, \Delta_i; I_i) - \frac{(\theta + \Delta_i) w(\theta, \Delta_i; I_i)}{(p_i + \Delta_i - p_L)}] f(q)dq \\ &= \frac{w(\theta, \Delta_i; I_i)}{p_i + \Delta_i - p_L} \left[p_i - \int_{\widehat{q}(\theta,\Delta_i|I_i)}^{1} (p_i - \theta - p_L) f(q)dq \right] > 0, \; i = M, H.\end{aligned}$$

b) When (7) holds as an equality, the manager is indifferent between I_M and I_H but profits are generally higher in the former case. Define $\Delta_H^* \equiv \Delta_M^*(1+z)$. Note that $w(\theta, \Delta_M^*; I_M) = w(\theta, \Delta_H^*; I_H) = w$ while $\widehat{q}(\theta, \Delta_M^*|I_M) = p_M + \frac{B}{R} + \frac{\theta+\Delta_M^*}{R} w < p_H + \frac{B}{R} + \frac{\theta+\Delta_H^*}{R} w = \widehat{q}(\theta, \Delta_H^*|I_H)$. Consider that the difference

$V(\theta,\Delta_H^*|I_H)-V(\theta,\Delta_M^*|I_M)$ can be written as

$$V(\theta,\Delta_H^*|I_H)-V(\theta,\Delta_M^*|I_M)=\int_0^{\widehat{q}(\theta,\Delta_M^*|I_M)}(p_{H-}p_M)(R-w)f(q)dq$$

$$+\int_{\widehat{q}(\theta,\Delta_M^*|I_M)}^{\widehat{q}(\theta,\Delta_H^*|I_H)}p_H(R-w)f(q)dq-\int_{\widehat{q}(\theta,\Delta_H^*|I_H)}^{1}(p_H+\Delta_H^*-p_M-\Delta_M^*)wf(q)dq$$

$$-\int_{\widehat{q}(\theta,\Delta_M^*|I_M)}^{\widehat{q}(\theta,\Delta_H^*|I_H)}\left[qR-(p_M+\Delta_M+\theta)\,w-B\right]f(q)dq$$

Which is equivalent to (17) in the proof of Proposition 2, where we have substituted $w(\theta,\Delta_M^*;I_M)=w(\theta,\Delta_H^*;I_H)=w$. We can then apply the same line of reasoning to show that

$$V(\theta,\Delta_H^*|I_H)-V(\theta,\Delta_M^*|I_M)=$$

$$=\int_0^{\widehat{q}(\theta,\Delta_H^*|I_H)}z\Delta_M wf(q)dq-\int_{\widehat{q}(\theta,\Delta_M^*|I_M)}^{\widehat{q}(\theta,\Delta_H^*|I_H)}\left\{\left[q-(p_M+\Delta_M+\theta)\,w-B\right]-p_MR+p_Hw\right\}f(q)dq+$$

$$-\int_{\widehat{q}(\theta,\Delta_H^*|I_H)}^{1}(p_H+\Delta_H-p_M-\Delta_M)wf(q)dq,$$

and, considering that $z\Delta_M$ is very small and both the second and the third term are negative, $V(\theta,\Delta_H^*|I_H)-V(\theta,\Delta_M^*|I_M)$ will generally be negative..

Proof of Corollary 6

Consider that expected profits for $\theta=0$ can be written as

$$V(0,\Delta_i|I_i)=p_i(R-w(0,\Delta_i;I_i))+\int_{\widehat{q}(0,\Delta_i|I_i)}^{1}\left[(q-p_i)R-\Delta_i w(0,\Delta_i;I_i)-B)\right]f(q)dq.$$

Then, using the assumption of uniform distribution of q, the difference between $V(0,\Delta_H|I_H)$ and $V(0,0|I_M)$ is equal to

$$V(0,\Delta_H|I_H)-V(0,0|I_M)=(p_H-p_M)\,R-p_Hw(0,\Delta_H;I_H))+p_Mw(0,0;I_M)+$$
$$-\frac{\widehat{q}(0,\Delta_H|I_H)^2-\widehat{q}(0,0|I_M)^2}{2}-(p_HR+\Delta_Hw(0,\Delta_H;I_H)+B)\,(1-\widehat{q}(0,\Delta_H|I_H))+$$
$$+(p_MR+B)\,(1-\widehat{q}(0,0|I_M)).$$

After some manipulation we can then write

$$V(0,\Delta_H|I_H)-V(0,0|I_M)=$$
$$p_H\widehat{q}(0,\Delta_H|I_H)-p_M\widehat{q}(0,0|I_M)]R-\frac{\widehat{q}(0,\Delta_H|I_H)^2-\widehat{q}(0,0|I_M)^2}{2}$$
$$-\Delta_Hw(0,\Delta_H;I_H)(1-\widehat{q}(0,\Delta_H|I_H))+[\widehat{q}(0,\Delta_H|I_H)-\widehat{q}(0,0|I_M)]B$$
$$-p_Hw(0,\Delta_H;I_H))+p_Mw(0,0;I_M)$$

Recalling that $\widehat{q}(0,\Delta_H|I_H) = p_H + \Delta_H \frac{w(0,\Delta_H;I_H)}{R} + \frac{B}{R}$ and $\widehat{q}(0,0|I_M) = p_M + \frac{B}{R}$, such expression is equal to

$$V(0,\Delta_H|I_H) - V(0,0|I_M) = -\frac{[\Delta_H w(0,\Delta_H;I_H)]^2}{2R} + \frac{(p_H)^2 - (p_M)^2}{2}R - \Delta_H \frac{w(0,\Delta_H;I_H)}{R}B$$
$$-\Delta_H w(0,\Delta_H;I_H)(1 - p_H - \Delta_H \frac{w(0,\Delta_H;I_H)+B}{R}) + (p_H - p_M + \Delta_H \frac{w(0,\Delta_H;I_H)}{R})B+$$
$$+p_M w(0,0;I_M) - p_H w(0,\Delta_H;I_H).$$

To prove the corollary, consider the difference in profits at $\Delta_M^* + \varepsilon$ for $\varepsilon \to 0$. Recall that we have defined $\Delta_H^* \equiv \Delta_M^*(1+z)$ and that $w(0,\Delta_H^*;I_H) = w(0,\Delta_M^*;I_M) = \frac{I_M}{p_M+\Delta_M^*-p_L}$ while $w(0,0;I_M) = \frac{I_M}{p_M-p_L}$. Then, if we consider the case where $B \to 0$, it is

$$V(0,\Delta_H^*|I_H) - V(0,0|I_M) = -\frac{[\Delta_H^* w(0,\Delta_H^*;I_H)]^2}{2R} +$$
$$+\frac{(p_H)^2-(p_M)^2}{2}R - \Delta_H^* w(0,\Delta_H^*;I_H)(1 - p_H - \Delta_H^* \frac{w(0,\Delta_H^*;I_H)}{R})$$
$$+\frac{p_M \Delta_M^* I_M}{(p_M + \Delta_M^* - p_L)(p_M - p_L)} - \frac{(p_H - p_M)I_M}{(p_M + \Delta_M^* - p_L)} =$$
$$-\Delta_H w(0,\Delta_H^*;I_H)(1 - p_H - \frac{\Delta_H^* w(0,\Delta_H^*;I_H)}{2}) + \frac{(p_H)^2-(p_M)^2}{2}R$$
$$+\frac{p_M \Delta_M^* w(0,\Delta_H^*;I_H)}{(p_M - p_L)} - (p_H - p_M) w(0,\Delta_H^*;I_H).$$

which can also be written as

$$V(0,\Delta_H^*|I_H) - V(0,0|I_M) =$$
$$\Delta_M^* w(0,\Delta_H^*;I_H)\left[\frac{p_M}{(p_M-p_L)} - 1 + p_H + \frac{\Delta_H w(0,\Delta_H^*;I_H)}{2R}\right]$$
$$-z\Delta_M^* w(0,\Delta_H^*;I_H)\left[1 - p_H - \frac{\Delta_H^* w(0,\Delta_H;I_H)}{2R}\right] + \frac{(p_H)^2-(p_M)^2}{2}R.$$

This expression can be negative for $p_H - p_M$ small, possibly $p_H - p_M \to 0$, if

$$z > \frac{\frac{p_L}{(p_M-p_L)} + p_H + \frac{\Delta_H^* w(0,\Delta_H^*;I_H)}{2R}}{1 - p_H - \frac{\Delta_H^* w(0,\Delta_H^*;I_H)}{2R}}.$$

11 Appendix 2: The case with no managerial bargaining power

We here derive the results discussed in subsection 7.2 for the case where the manager has no bargaining power. Assume for simplicity that $B = 0$ and consider investment I_M. Then incentive compatibility constraint ICC 1 becomes:

$$\int_0^{\widehat{q}_{NB}(\theta,\Delta_M|I_M)} (p_M + \Delta_M + \theta)\, w_{NB}(\theta,\Delta_M;I_M) f(q)dq + \int_{\widehat{q}_{NB}(\theta,\Delta_M|I_M)}^1 s_{NB}(\theta,\Delta_M;I_M) f(q)dq - I_M \geq$$
$$\int_0^{\widehat{q}_{NB}(\theta,0|0\neq I_M)} p_L w_{NB}(\theta,\Delta_M;I_M) f(q)dq + \int_{\widehat{q}_{NB}(\theta,0|0\neq I_M)}^1 s_{NB}(\theta,\Delta_M;I_M) f(q)dq$$

where

$\widehat{q}_{NB}(\theta,\Delta_M|I_M)=p_M-\frac{(p_M w_{NB}(\theta,\Delta_M;I_M)-s_{NB}(\theta,\Delta_M;I_M))}{R}$ and $\widehat{q}_{NB}(\theta,0|0\neq I_M)=p_L-\frac{(p_L w_{NB}(\theta,\Delta_M;I_M)-s_{NB}(\theta,\Delta_M;I_M))}{R}$.
Using the assumption of uniform distribution we can write this constraint as:

$$w_{NB}(\theta,\Delta_M;I_M)[\widehat{q}_{NB}(\theta,\Delta_M|I_M)x-\widehat{q}_{NB}(\theta,0|I_L\neq I_M)y]-s(\widehat{q}_{NB}(\theta,\Delta_M|I_M)-\widehat{q}_{NB}(\theta,0|I_L\neq I_M))-I_M\geq 0 \tag{18}$$

where $x=(p_M+\theta+\Delta_M)$ and $y=(p_L+\theta)$. Substituting the expression for $\widehat{q}_{NB}(\theta,\Delta_M|I_M)$ and $\widehat{q}_{NB}(\theta,0|0\neq I_M)$ we obtain

$$-[w_{NB}(\theta,\Delta_M;I_M)]^2\left(\frac{xp_M-yp_L}{R}\right)+$$
$$w_{NB}(\theta,\Delta_M;I_M)\left[(xp_M-yp_L)+\frac{s_{NB}(\theta,\Delta_M;I_M)}{R}[2(p_M-p_L)+\Delta_M]\right]-s_{NB}(\theta,\Delta_M;I_M)(p_M-p_L)-I_M\geq 0.$$

Suppose that ICC2 is satisfied. Then the incentive bonus is represented by the minimum value that satisfies the condition

$$w_{NB}(\theta,\Delta_M;I_M)=\frac{R}{2}+\frac{s_{NB}(\theta,\Delta_M;I_M)[2(p_M-p_L)+\Delta_M]}{2(xp_M-yp_L)}-\frac{R}{2(xp_M-yp_L)}* \tag{19}$$
$$\left\{\left((xp_M-yp_L)+\frac{s_{NB}(\theta,\Delta_M;I_M)[2(p_M-p_L)+\Delta_M]}{R}\right)^2-\frac{4(xp_M-yp_L)}{R}\left[s_{NB}(\theta,\Delta_M;I_M)(p_M-p_L)+I_M\right]\right\}^{\frac{1}{2}}.$$

It is immediate to verify that the value of the discriminant is increasing in s. Consequently the condition for the discriminant to be positive, for any value of $s_{NB}(\theta,\Delta_M;I_M)\geq 0$, is $R\frac{(p_M^2-p_L^2+\theta(p_M-p_L)+p_M\Delta_M)}{4}>I_M$.

The derivative of the incentive pay with respect to s_{NB} is

$$\frac{\partial w_{NB}(\theta,\Delta_M;I_M)}{\partial s_{NB}(\theta,\Delta_M;I_M)}=\frac{2(p_M-p_L)+\Delta_M}{2(xp_M-yp_L)}+$$
$$-\frac{1}{2}\left\{\left((xp_M-yp_L)+\frac{s_{NB}(\theta,\Delta_M;I_M)[2(p_M-p_L)+\Delta_M]}{R}\right)^2-\frac{4(xp_M-yp_L)}{R}\left[s(p_M-p_L)+I_M\right]\right\}^{-\frac{1}{2}}$$
$$\left(\frac{s_{NB}(\theta,\Delta_M;I_M)[2(p_M-p_L)+\Delta_M]^2}{R(xp_M-yp_L)}+\Delta_M]\right).$$

Evaluated at $s=0$, the above expression becomes

$$\left.\frac{\partial w_{NB}(\theta,\Delta_M;I_M)}{\partial s_{NB}(\theta,\Delta_M;I_M)}\right|_{s=0}=\frac{2(p_M-p_L)+\Delta_M}{2(xp_M-yp_L)}-\left((xp_M-yp_L)^2-\frac{4(xp_M-yp_L)I_M}{R}\right)^{-\frac{1}{2}}\left(\frac{\Delta_M}{2(xp_M-yp_L)}\right)$$

Then, $\frac{\partial w_{NB}(\theta,\Delta_M;I_M)}{\partial s_{NB}(\theta,\Delta_M;I_M)}>0$ if and only if

$$(xp_M-yp_L)(1-\frac{\Delta_M^2}{[2(p_M-p_L)+\Delta_M]^2})>\frac{4I_M}{R} \tag{20}$$

which is satisfied for I_M sufficiently small with respect to R. Such condition is slightly more restrictive than the condition for the discriminant to be positive. Notice however that the LHS of (20) is increasing in Δ_M, implying that the higher is overconfidence, the easier it is to satisfy it. It is in fact

$$\frac{\partial\left\{(xp_M-yp_L)[1-\frac{\Delta_M^2}{[2(p_M-p_L)+\Delta_M]^2}]\right\}}{\partial\Delta_M}>0\quad\text{iff}\quad\theta<p_M+\frac{(p_M-p_L)(2p_M+\Delta_M)}{\Delta_M}.$$

As to the effects of optimism and overconfidence on incentive pay, they can be immediately derived from (19). Having established that it is optimal not to pay any severance pay, we evaluate these effects at $s^{NB}(\theta, \Delta_M; I_M) = 0$

$$\left.\frac{\partial w_{NB}(\theta, \Delta_M; I_M)}{\partial \theta}\right|_{s^{NB}(\theta,\Delta_M;I_M)=0} =$$

$$\frac{R(p_M - p_L)}{4(xp_M - yp_L)^2}\left\{(xp_M - yp_L)^2 - \frac{4(xp_M - yp_L)I_M}{R}\right\}^{\frac{1}{2}} +$$

$$-\frac{R[2(p_M - p_L)(xp_M - yp_L) + 2(p_M - p_L)(\Delta_M - 4I_M)/R]}{4(xp_M - yp_L)}\left\{(xp_M - yp_L)^2 - \frac{4(xp_M - yp_L)I_M}{R}\right\}^{-\frac{1}{2}}$$

which is negative as

$$sign\left.\frac{\partial w_{NB}(\theta, \Delta_M; I_M)}{\partial \theta}\right|_{s^{NB}(\theta,\Delta_M;I_M)=0} = sign - [(xp_M - yp_L) - \frac{4I_M}{R} + \frac{2\Delta_M}{R}] < 0.$$

$$\left.\frac{\partial w_{NB}(\theta, \Delta_M; I_M)}{\partial \Delta_M}\right|_{s^{NB}(\theta,\Delta_M;I_M)=0} = \frac{2Rp_M}{4(xp_M - yp_L)^2}\left\{(xp_M - yp_L)^2 - \frac{4(xp_M - yp_L)I_M}{R}\right\}^{\frac{1}{2}} + -\frac{Rp_M[2(xp_M - yp_L) - 4I_M/R]}{4(xp_M - yp_L)}\left\{(xp_M - yp_L)^2 - \frac{4(xp_M - yp_L)I_M}{R}\right\}^{-\frac{1}{2}}$$

which is negative as

$$sign\left.\frac{\partial w_{NB}(\theta, \Delta_M; I_M)}{\partial \Delta_M}\right|_{s^{NB}(\theta,\Delta_M;I_M)=0} = sign - \frac{4(xp_M - yp_L)I_M}{R} < 0.$$

We can then conclude that $w_{NB}(\theta, \Delta_M; I_M)$ is decreasing in both θ and Δ_M implying that $w_{NB}(\theta, \Delta_M; I_M) < w_{NB}(0, 0; I_M)$. In other words, also when the manager has no bargaining power, incentive pay for a biased manager is lower than for a rational one. The difference is that here also optimism reduces incentive pay.

Finally, consider the issue of whether the above solution with investment I_M is incentive compatible with respect to the choice of I_H. Setting $s_{NB}(\theta, \Delta_M; I_M) = 0$, ICC2 can now be written as

$$\int_0^{\widehat{q}_{NB}(\theta,\Delta_M|I_M)} (p_M + \Delta_M + \theta)\, w_{NB}(\theta, \Delta_M; I_M) f(q)dq - I_M \geq$$

$$\int_0^{\widehat{q}_{NB}(\theta,\Delta_H|I_H \neq I_M)} (p_H + \Delta_H + \theta)\, w_{NB}(\theta, \Delta_M; I_M) f(q)dq - I_H$$

or

$$w_{NB}(\theta, \Delta_M; I_M)[\widehat{q}_{NB}(\theta, \Delta_M|I_M)x - \widehat{q}_{NB}(\theta, \Delta_H|I_H \neq I_M)j] \geq I_M - I_H$$

where $j = (p_H + \theta + \Delta_H)$.

For the above solution to be incentive compatible, the following must then hold

$$\frac{I_M}{\widehat{q}_{NB}(\theta, \Delta_M|I_M)x - \widehat{q}_{NB}(\theta, 0|I_L \neq I_M)y} = w_{NB}(\theta, \Delta_M; I_M) \leq \frac{I_H - I_M}{\widehat{q}_{NB}(\theta, \Delta_H|I_H)x_H - \widehat{q}_{NB}(\theta, \Delta_M|I_M)x}$$

which requires

$$\frac{I_M}{I_H} \leq \frac{xp_M - yp_L}{jp_H - yp_L}. \tag{21}$$

Thus, the manager chooses I_M when the above condition is satisfied while she chooses I_H otherwise. Comparing the RHS of the condition to the RHS of (7), the corresponding condition in the baseline model, note that i) now the condition also depends on θ; and ii) $\frac{xp_M - yp_L}{jp_H - yp_L} < \frac{p_M + \Delta_M - p_L}{p_H + \Delta_H - p_L}$ implying that I_M is not incentive compatible for lower values of the biases if compared to the baseline model.

11.1 The case of a rational manager

Let us now consider the case of a rational manager ($\theta = \Delta_M = 0$). Expression (18) for ICC1 takes the form

$$w_{NB}(0,0;I_M)[\widehat{q}_{NB}(0,0|I_M)p_M - \widehat{q}_{NB}(0,0|0 \neq I_M)p_L] - s_{NB}(0,0;I_M)(\widehat{q}_{NB}(0,0|I_M) - \widehat{q}_{NB}(0,0|0 \neq I_M)) - I_M \geq 0$$

Substituting the expression for $\widehat{q}_{NB}(0,0|I_M)$ and $\widehat{q}_{NB}(0,0|0 \neq I_M)$ we obtain:

$$-[w^{NB}(0,0;I_M)]^2 \left(\frac{p_M^2 - p_L^2}{R}\right) +$$
$$w^{NB}(0,0;I_M)\left[(p_M^2 - p_L^2) + \frac{s_{NB}(0,0;I_M)}{R} 2(p_M - p_L)\right] - s_{NB}(0,0;I_M)(p_M - p_L) - I_M \geq 0$$

and the minimum value of the incentive bonus that satisfies this condition is:

$$\begin{aligned} w^{NB}(0,0;I_M) &= \frac{R}{2} + \frac{s_{NB}(0,0;I_M)}{(p_M + p_L)} - \frac{R}{2}\sqrt{1 + \frac{4\left[s_{NB}(0,0;I_M)\right]^2}{R^2(p_M + p_L)^2} - \frac{4I_M}{R(p_M^2 - p_L^2)}} = \\ & \frac{R}{2} + \frac{s_{NB}(0,0;I_M)}{(p_M + p_L)} - \sqrt{\frac{R^2}{4} + \frac{\left[s_{NB}(0,0;I_M)\right]^2}{(p_M + p_L)^2} - \frac{RI_M}{p_M^2 - p_L^2}}. \end{aligned}$$

In this case the condition for the discriminant to be positive, for any value of $s_{NB}(0,0;I_M) \geq 0$, is $R\frac{(p_M^2 - p_L^2)}{4} > I_M$ which is more restrictive than in the case of a biased manager.[25] The derivative of $w^{NB}(0,0;I_M)$ with respect to $s_{NB}(0,0;I_M)$ is

$$\frac{\partial w^{NB}(0,0;I_M)}{\partial s_{NB}(0,0;I_M)} = \frac{1}{(p_M + p_L)} +$$
$$-\left\{1 + \frac{4\left[s_{NB}(0,0;I_M)\right]^2}{R^2(p_M + p_L)^2} - \frac{4I_M}{R(p_M^2 - p_L^2)}\right\}^{-\frac{1}{2}} \left(\frac{s_{NB}(0,0;I_M)}{R(p_M + p_L)^2}\right).$$

Evaluated at $s_{NB}(0,0;I_M) = 0$, such derivative becomes

$$\left.\frac{\partial w^{NB}(0,0;I_M)}{\partial s}\right|_{s_{NB}(0,0;I_M)=0} = \frac{1}{(p_M + p_L)} > 0.$$

Recalling that the positive sign of $\frac{\partial w}{\partial s}$ is a sufficient condition to have a negative impact of severance pay on firm profit, we can say that the optimal contractual severance pay when the manager is rational is zero.

Let us finally check the condition that makes I_M incentive compatible with respect to I_H for a rational manager. Condition (21) now takes the form

$$\frac{I_M}{I_H} \leq \frac{p_M^2 - p_L^2}{p_H^2 - p_L^2}$$

which represents a slightly more restrictive condition with respect the one following from Assumption 2: $\frac{I_M}{I_H} \leq \frac{p_M - p_L}{p_H - p_L}$.

[25] This is also more stringent than Assumption 2 since $\frac{(p_M + p_L)}{4} < 1$.