The Illusion of Competition[*]

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Abstract

Existing models of price competition in the presence of endogenous consumer search restrict attention to single-brand firms, ensuring that any consumer who receives multiple price quotes places firms in competition with each other. We extend these models to allow a single firm to own two brands, meaning that a consumer who receives exactly two price quotes may receive two from the same firm, and hence be "captive" to that firm. If there are sufficiently many such consumers, requiring two merging firms to consolidate their brands rather than operate them separately would increase competition and benefit consumers. Similarly, curtailing brand proliferation by restricting firms from introducing multiple brands for the same product or limiting the visibility of such duplicate brands on online platforms can intensify price competition and benefit consumers. These results are also true if consumers operate under an "illusion of competition" in which they are unaware that separate brands may be co-owned by the same firm, believing them all to be independent competitors. Breaking such an illusion of competition by advertising the brand ownership structure may help or hurt consumers.

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1 Introduction

Consumer search over prices is crucial for understanding equilibrium outcomes in many markets, and hence, its implications are vital for regulators and antitrust authorities when evaluating mergers and competition policies. In many real-world contexts, consumers do not know the prices offered by every firm in the market. Instead, because they must undertake costly search to discover prices, consumers typically choose from a subset of options, and these consideration sets vary across consumers. This behavior helps explain price dispersion and markups in markets ranging from trash collection (Salz, 2022) to mortgages (Allen, Clark, and Houde, 2019). Importantly, this should be accounted for when predicting the effects of mergers and competition policies in such *search markets*.

In search markets, it is important to distinguish between two types of mergers that are both common in practice. First, there are brand-consolidating mergers, in which merging firms combine and sell under a single name. For instance, when USAir and American Airlines merged, they kept only the American Airlines name. Second, there are brand-preserving mergers, in which the merged firm continues to sell using both pre-merger brand names. For instance, this happened when Uber, the parent company of Uber Eats, acquired the competing food-delivery service Postmates. Uber continues to operate Uber Eats and Postmates as separate brands with their own websites. All else equal, the two types of mergers lead to different outcomes because they generate different numbers of prices for consumers to search over and a different pricing problem for the merged firm.

Existing models in the literature either treat consumer search patterns and consideration sets as exogenous (following (Varian, 1980)) or let them be endogenous functions of search costs and equilibrium prices (following Burdett and Judd (1983) and Stahl (1989)). Within these branches of the literature, most models assume each firm owns a single brand and hence can be used to evaluate brand-consolidating mergers (as in Armstrong and Vickers's (2022) model with exogenous consumer consideration sets) but cannot be used to evaluate brand-preserving mergers. Ireland (2007) and Armstrong and Vickers (2024) are important exceptions. This paper is the first that we are aware of to allow for both ownership of multiple brands and endogenous sequential search. (Our results related to mergers were developed independently and concurrently to Armstrong and Vickers (2024) but our extension to brand proliferation builds on their work.)

We build upon Burdett and Judd's (1983) and Stahl's (1989) canonical models of endogenous consumer search by allowing for a single firm to own two brands. Our extension allows us to model the effects of brand-preserving mergers, compare consumer welfare between brand-preserving and brand-consolidating mergers, and evaluate a novel competition policy that requires brand consolidation as a condition for merger approval. We consider a search technology very similar to that of the "newspaper search" of Burdett and Judd (1983). Consumers with inelastic unit demand are shopping for a homogeneous good. They are randomly endowed with one or more price quotes and then must decide whether to continue searching sequentially, one price quote at a time. (If consumers are randomly endowed with *one or all* price quotes—rather than *one or more*—this corresponds to search in Stahl (1989).)

We focus on the case of three symmetric single-brand firms in which two firms complete a brand-preserving or a brand-consolidating merger and include some additional results for markets with more firms.

Our results vary with consumer awareness about brand-preserving mergers. If consumers are sophisticated, they will know that the prices of distinct but jointly-owned brands are set by the same firm rather than competitors. If consumers suffer from the "illusion of competition," however, they will assume that prices for all brands are set by independent firms. This illusion of competition can reduce consumer surplus, meaning that making consumers aware of the post-merger brand ownership structure could be beneficial.

To develop our results, we characterize equilibrium prices under three market structures: (1) three symmetric single-brand firms (pre-merger case); (2) two asymmetric single-brand firms (brand-consolidating merger case); and (3) a duopoly in which one firm owns two distinct brands and the other firm owns a single brand (brand-preserving merger case). In the two post-merger market structures, we allow consumers to be sophisticated and aware that a merger has taken place, or to suffer from the illusion of competition and be unaware of the merger. In all cases, we hold fixed the number of initial price quotes consumers are endowed with.[1]

Equilibrium under the first two market structures closely follows existing work. Consumers stop searching once they find a price less than or equal to an endogenous reservation price. Firms randomize over an interval of prices that extends up to consumers' reservation price. Hence, in equilibrium, there is no additional search beyond the price quotes consumers are randomly endowed with. Equilibrium characterization follows Burdett and Judd (1983) for the symmetric case and is similar to Armstrong and Vickers's (2022) characterization for the asymmetric case, except that consumers' reservation prices are endogenous rather than exogenous.

Equilibrium under the third market structure, with joint brand ownership, is novel. We show that there are two different types of equilibrium, depending on the relative fraction of consumers who are endowed with one, two, or three price quotes. (We refer to those endowed with only one price quote as "captive" to a single firm.)

If sufficiently many consumers are either captive to a single firm or consider all of the firms in the market, then there is a "distinct pricing equilibrium" in which the two co-owned brands have distinct prices. The first co-owned brand is priced at the consumers' reservation price (the top of the price distribution), earning high profits on captive consumers and avoiding competing with the second co-owned brand. The second co-owned brand is priced randomly

[1] In our framework, a consumer type corresponds to a number of initial price quotes or searches, whereas in Armstrong and Vickers (2022) a consumer type corresponds to a consideration set. To compare our approaches, consider a consumer who would have searched brands A and B in the absence of a merger. If there is a merger to a consolidated brand AB, Armstrong and Vickers (2022) preserves the consideration set, and the consumer only observes price AB. In contrast, we preserve two initial price quotes so the consumer observes price AB and a competing price.

over an interval of lower prices and is in direct competition with the outside brand for the non-captive consumers. Alternatively, if sufficiently many consumers are endowed with exactly two price quotes, then there is a "joint pricing equilibrium" in which the two co-owned brands have the same price. The shared price of the two co-owned brands is randomly drawn over the same interval of prices as the outside brand. This pricing strategy capitalizes on the fact that a large fraction of consumers will only see the prices of the jointly owned brands and are, in a sense, captive to the merged firm.

Next, we compare consumer welfare across the three studied market structures, both for the case of sophisticated consumers and for the case of an illusion of competition. To do so, we characterize and compare average transaction prices, which are sufficient statistics for consumer welfare in this setting. With an illusion of competition, a merger (whether it is brand preserving or brand consolidating) is always harmful to consumers. In contrast, with sophisticated consumers, results are sometimes reversed. With sophisticated consumers, a merger of the right type can increase welfare when there are sufficiently few consumers who are endowed with exactly two price quotes. A brand-preserving merger improves welfare when there are many who observe all prices and many are captive to a single firm.

A direct implication of the preceding results is that the illusion of competition can reduce consumer surplus following a merger. Hence, advertising that makes consumers aware of joint ownership of brands can reduce consumer harm from mergers.[2] However, we find that the opposite can also be true. Under certain settings, the illusion of competition makes consumers believe that if they continue to search, prices will be lower. This incorrect belief that the market is more competitive than it really is can cause consumers to have a lower reservation price, leading firms to set lower prices. The effect of information about the market structure on prices therefore depends on the search technology and the nature of the merger.

Conditional on allowing a merger, requiring brand consolidation helps consumers whenever there are sufficiently many consumers who are randomly endowed with exactly two price quotes. Given an illusion of competition, these are the consumers who might observe two prices from the same firm but mistakenly think they have observed competitive price quotes. Nevertheless, the result does not depend on the illusion of competition and is qualitatively the same in the sophisticated case.

The problem of too-many brands for a homogeneous product can arise without mergers or acquisitions when firms introduce multiple brands to sell the same product. This is increasingly common in online marketplaces. For instance, on Grubhub, a single physical kitchen can easily list its delivery meals using multiple online restaurant names and corresponding menu prices (Hassan, 2023). A simple extension of our primary results (which builds on Armstrong and Vickers's (2024) analysis of the case with exogenous consideration

[2]It is possible that making consumers aware of a merger would lower prices such that a merger would become unprofitable. In these cases, the threat of such an information remedy may deter certain price increasing mergers from being proposed in the first place.

sets) shows that, by banning such brand proliferation, online platform operators can increase competition among their sellers and benefit their consumers. This is true for similar market conditions as when requiring merging firms consolidate their brands is beneficial—when there are sufficiently many consumers who see exactly two price quotes before considering additional search. However, we also show that permitting brand proliferation but limiting visibility for duplicate brands to the bottom of search results can sometimes be even better than a strict ban.

This paper relates to a large theoretical literature studying how endogenous and costly consumer search affects equilibrium market prices. Early papers establish the frameworks that we build upon in this paper (Burdett and Judd, 1983; Stahl, 1989). A second branch of the literature, begun by Varian (1980), studies models in which consumer search patterns and consideration sets are exogenous rather than endogenous. Armstrong and Vickers (2022) generalizes these models to a wide class of possible competitive interactions and uses this framework to study the impact of entry/exit and mergers in which the merging brands are consolidated.

Ireland (2007) is the first paper in this literature to extend a model of equilibrium pricing with consumer search to allow for joint ownership of independent brands by the same firm, a common occurrence in real-world markets. Under the assumption that firms are symmetric and consumers check at most two prices, Ireland (2007) shows that brand-preserving mergers always lead to equal pricing for co-owned brands and that brand consolidation benefits consumers. Our work and that by Armstrong and Vickers (2024) both relax the assumption that consumers check at most two prices. As a result, we find alternative equilibria with distinct pricing by co-owned brands and cases in which brand consolidation harms consumers. This paper does so in the context of symmetric but endogenous consumer search, while Armstrong and Vickers (2024) does so while allowing for general asymmetric search patterns but restricting them to be exogenous.

Armstrong and Vickers's (2024) allowance for asymmetry leads to a third type of pricing equilibrium that does not arise in our model. Our results applied to mergers were developed independently from and concurrently to Armstrong and Vickers's (2024), while our brand proliferation extension that allows for asymmetric firms builds on their work. The fact that we allow consumer search and reservation prices to be endogenous sometimes reverses results. Moreover, it allows us to compare market outcomes with sophisticated consumers who are aware of mergers to those suffering from the illusion of competition.

2 Theoretical Model

We consider a market for a homogeneous good with $N = 3$ symmetric risk-neutral firms and a unit mass of consumers. All firms can produce a unit of the good at a marginal cost of c. Consumers are homogeneous and have unit demand for the good at a valuation of v. Their utility is risk-neutral and quasi-linear and can be expressed simply as the valuation for the good less the price paid for the good. The market is characterized by search frictions. We assume that the first set of price quotes is free but that any further price quotes come at a cost of s. As in Burdett and Judd (1983), the initial search is "noisy;" consumers receive between 1 and N initial, free price quotes. This initial search technology is characterized by the vector $\mu = (\mu_1, \mu_2, \ldots, \mu_N)$ such that $\sum_{j=1}^{N} \mu_j = 1$ where μ_j is the probability that a consumer receives j initial, free price quotes. Each firm (or brand) has an equal probability of being selected by each search.

Throughout this paper, we will use an equilibrium concept of the "reservation price equilibrium" or RPE. This concept is commonly used in the consumer search literature and is a subset of Perfect Bayesian Nash Equilibria. In an RPE, consumers have a reservation price, r. As they search, if they are offered one or more prices at or below r, they purchase the good at the lowest price they have been offered. If they are only offered prices above r, they continue to search. The reservation price r is a best response to the distribution of prices offered by firms. In this type of equilibrium, firms play a mixed strategy and select prices according to an equilibrium distribution. Every price in the support of this distribution is a best response to the consumers' search strategy above and to the distribution of prices played by other firms.

2.1 Symmetric Three Firm Case

Before jumping into the case of co-owned brands, it is useful to characterize the equilibrium with three competing single-brand firms. In this case, conditional on a RPE existing with reservation price r, each firm's profit function is:

$$\pi(p) = \begin{cases} \sum_{j=1}^{3} \frac{j\mu_j}{3}(1 - F(p))^{j-1}(p - c) & p \leq r \\ 0 & p > r \end{cases}, \tag{1}$$

where $F(p)$ is the equilibrium distribution of prices played by each firm. In all of the models considered in this paper, firms will not set prices above r. We therefore will omit references to this case moving forward. Also, it is without loss of generality to consider the case in which $c = 0$ and $s = 1$ (see the proof of Proposition 1 in Westphal (2023)). Other values of production cost and search cost merely shift and scale equilibrium price distributions, respectively. We therefore only reference the more tractable case for the remainder of the paper.

Varian (1980) notes that firms must be indifferent between all prices in the support of the equilibrium price distribution, thus the firms' side of the equilibrium can be solved by setting profits equal to the profits from choosing a price of r. Burdett and Judd (1983) gives us a polynomial that can be solved for the equilibrium price distribution:

$$F(p) = 1 - \frac{\sqrt{\frac{3\mu_1\mu_3(r-p)}{p} + \mu_2^2} - \mu_2}{3\mu_3}. \tag{2}$$

The consumers' reservation price r is then the minimum of their valuation of the good, v, and the value implicitly defined by the search indifference condition below.

$$r = 1 + \int_{\underline{p}}^{r} pf(p)dp, \tag{3}$$

where the lowercase of a distribution refers to its corresponding density. Burdett and Judd (1983) show that this equilibrium exists and Johnen and Ronayne (2021) prove that this equilibrium is unique if $\mu_2 > 0$. This "pre-merger" equilibrium will serve as a benchmark to compare to cases with jointly owned brands.

2.2 Brand-Preserving Merger

Next, we consider a merger between two of the three firms, following which the merged firm continues to operate the two brands separately. Here, we assume that the brands continue to be anonymous to the consumer, and the consumer cannot direct their search.

The merged firm's profits are a function of the prices chosen for each of its brands. Without loss of generality, denote p_L as the minimum of the two prices and p_H as the maximum. Then expected profits are

$$\begin{aligned}\pi_{JM}(p_L, p_H) = [\frac{1}{3}\mu_1 + \frac{1}{3}\mu_2 + \frac{1}{3}\mu_2(1 - G(p_L)) + \mu_3(1 - G(p_L))]p_L \\ +[\frac{1}{3}\mu_1 + \frac{1}{3}\mu_2(1 - G(p_H))]p_H,\end{aligned} \tag{4}$$

where $G(p)$ is the equilibrium price distribution for the outside firm. The merged firm's strategy depends on whether this function is increasing or decreasing in p_H. If profits are increasing in p_H, holding p_L fixed, then the firm optimally always sets p_H to r, giving us the distinct pricing equilibrium. If profits are decreasing in p_H, the jointly owned firm sets $p_H = p_L$, giving us the joint pricing equilibrium. Thus, the condition for distinct pricing (rather than joint pricing) to be a best response to the outside firm's pricing distribution G is

$$\frac{\partial \pi_{JM}(p_L, p_H)}{\partial p_H} = \frac{1}{3}\mu_1 + \frac{1}{3}\mu_2(1 - G(p_H)) - \frac{1}{3}\mu_2 g(p_H)p_H \geq 0. \tag{5}$$

We will show that at the equilibrium values for G, this condition reduces to $\mu_2^2 \leq 3\mu_1\mu_3$, a simple expression (illustrated in Figure 1) that distinguishes between distinct pricing and joint pricing equilibria based only on the search technology μ.

2.2.1 Distinct Pricing Equilibrium

If the share of consumers who see exactly two prices is sufficiently low, then the jointly owned firm has little incentive to compete using *both* of its brands. In this setting, when setting its higher price, p_H, the firm will typically be either facing a consumer with one or three price quotes. If p_H is the only price the consumer will see, the firm should set p_H to the reservation price and maximize its markup. If the consumer sees all three prices, then the firm is already showing the consumer a lower price, p_L, and cannot win the consumer with p_H. By this logic, if μ_2 is sufficiently low, the firm should set two distinct prices, one high monopoly price, and one lower competitive price.

Proposition 1 *If $\mu_2^2 \leq 3\mu_1\mu_3$, then a distinct pricing equilibrium exists*[3]*. The jointly owned firm sets one of its brands' prices to the consumers' reservation price, r_D. The jointly owned*

[3] Proofs of all propositions can be found in the appendix.

firms' other brand and the outside firm both choose prices on the interval $[\underline{p}_D, r_D]$ according to the distribution F_D. Where

$$\underline{p}_D = \frac{(\mu_1 + \mu_2) r_D}{\mu_1 + 2\mu_2 + 3\mu_3}, \tag{6}$$

$$r_D = \min\{1 + \int_{\underline{p}_D}^{r_D} p f_D(p) dp, v\}, \tag{7}$$

and

$$F_D(p) = G_D(p) = 1 - \frac{(\mu_1 + \mu_2)(r_D - p)}{(\mu_2 + 3\mu_3) p}. \tag{8}$$

Note that if the consumers' reservation price is set by their indifference towards search, then r_D is pinned down by the distribution F_D, not the overall distribution of prices. The reservation is set accordingly because if the consumer sees a price at or above r_D, they believe that the price must have come from the higher pricing brand of the jointly owned firm. Then the remaining prices they could receive must be from the lower pricing brand or the outside firm, both of which set prices according to F_D.

2.2.2 Joint Pricing Equilibrium

Next, consider the alternative case in which the share of consumers considering exactly two prices is high. Here, the jointly owned firm frequently finds itself in situations where p_H is in direct competition with only the outside firm. The firm will therefore want to set both prices in competition with the outside firm. However, there are also a large number of consumers who are only seeing the prices of the jointly owned brands. These additional "captive" consumers considerably soften price competition.

Proposition 2 *If $\mu_2^2 \geq 3\mu_1\mu_3$, then a joint pricing equilibrium exists. The jointly owned firm sets both of its prices to the same value, $p_L = p_H = p$, chosen from the interval $[\underline{p}_J, r_J]$. With probability λ_J it sets $p = r_J$ and with complement probability, it sets p according to distribution F_J. The outside firm sets its price on the same interval according to distribution G_J. Where*

$$\underline{p}_J = \frac{(2\mu_1 + \mu_2) r_J}{2\mu_1 + 3\mu_2 + 3\mu_3}, \tag{9}$$

$$r_J = \min\{1 + \int_{\underline{p}_J}^{r_J} p g_J(p) dp, v\}, \tag{10}$$

$$\lambda_J = \frac{\mu_1 + \mu_2}{3 - \mu_1}, \tag{11}$$

$$F_J(p) = 1 - \frac{[\mu_1 + \lambda_J(2\mu_2 + 3\mu_3)](r_J - p)}{(2\mu_2 + 3\mu_3)(1 - \lambda_J) p}, \tag{12}$$

and

$$G_J(p) = 1 - \frac{(2\mu_1 + \mu_2)(r_J - p)}{(2\mu_2 + 3\mu_3) p}. \tag{13}$$

Here, consumers set their reservation price according to the outside firms' price distribution, G_J. If a consumer has two price quotes of r_J, they will believe that both of these prices came from the merged firm and that they are facing G_J if they search again, making them indifferent between searching and not. If a two-price consumer receives either one or two prices below r_J, they will believe that they are facing some convex combination of G_J and a degenerate distribution of the price they have already been quoted with certainty. These distributions all first-order stochastically dominate G_J, so the consumer will not search. We will define off-path beliefs such that if a consumer sees a price above r_J, they believe they will face G_J next. This makes beliefs continuous at r_J for consumers with two prices. With these off-path beliefs, consumers with two price quotes above r_J will search. A consumer with a single price quote of r_J or lower will also not search. They believe that they are facing a distribution that is a convex combination of G_J and a degenerate distribution of the price they quoted. Therefore they have negative returns to search and will not search. A consumer with one price above r_J will search because their off-path beliefs are that they are facing G_J moving forward[4].

These two equilibria exist in (weakly) complementary regions of the parameter space. Figure 1 shows the values of μ for which the distinct pricing equilibrium and joint pricing equilibrium exist. It is worth noting that, along the locus $\mu_2^2 = 3\mu_1\mu_3$, both the joint pricing and the distinct pricing equilibria exist. Note that Ireland (2007) Lemma 3 characterizes the joint pricing equilibrium for an exogenous reservation price and the special case in which consumers check at most two prices ($\mu_3 = 0$) which corresponds to the diagonal upper bound of the parameter space in Figure 1. The distinct pricing equilibrium only arises for $\mu_3 > 0$.

[4]These off-path beliefs constitute an equilibrium, but they are not continuous in price for consumers who only receive a single price.

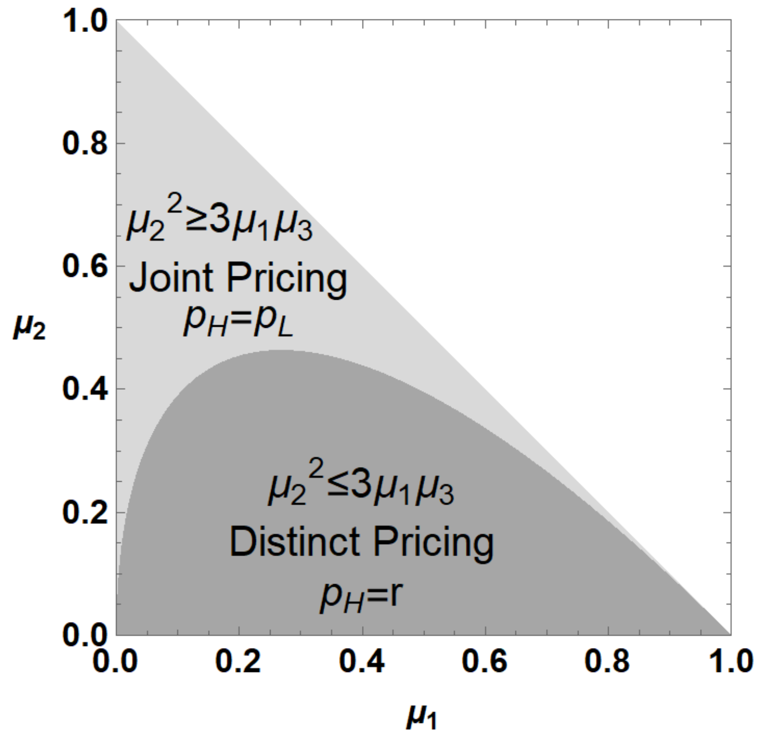

Figure 1: **Brand-preserving merger equilibrium regions:** Parameter μ_k is the fraction of consumers who receive k initial price quotes, where $\mu_3 = 1 - \mu_1 - \mu_2$. Possible values for the vector μ lie in the triangular region. The two prices set by the merged firm following a brand-preserving merger are denoted p_H and p_L, where $p_H \geq p_L$. The consumer's reservation price is denoted r. Distinct pricing equilibria, in which the merged firm sets $p_H = r$ and mixes over p_L, are possible for values of μ within the dark gray inequality region. Joint pricing equilibria, in which the merged firm mixes over both prices and sets $p_H = p_L$, are possible in the remainder of the triangle.

2.3 Brand-Consolidating Equilibrium

We can compare the above equilibria in which the merged firm continues to operate two separate brands to one in which they consolidate into a single brand. Here we assume that consumers (1) continue to receive the same number of initial quotes as pre-merger and (2) each price quote has the same probability of coming from each brand as pre-merger (in other words, the consolidated brand has double the probability of being selected for any given price quote). The merged firm's profit function is therefore:

$$\pi_{JC}(p) = \frac{2}{3}\mu_1 p + (\mu_2 + \mu_3)(1 - G_C(p))p. \tag{14}$$

As in Armstrong and Vickers (2022) an equilibrium exists. The merged firm plays a price of r with probability λ_C or chooses a price on the interval $[\underline{p}_C, r]$ according to distribution F_C, where

$$\lambda_C = \frac{\mu_1}{3 - \mu_1}, \tag{15}$$

$$\underline{p}_C = \frac{2\mu_1 r_C}{2\mu_1 + 3\mu_2 + 3\mu_3}, \tag{16}$$

and

$$F_C(p) = 1 - \frac{(\mu_1 + 3\lambda_C(\mu_2 + \mu_3))(r_C - p)}{3(1-\lambda_C)(\mu_2 + \mu_3)p}. \tag{17}$$

The outside firm sets prices on the same interval with distribution G_C:

$$G_C(p) = 1 - \frac{2}{3}\frac{\mu_1(r_C - p)}{(\mu_2 + \mu_3)p}. \tag{18}$$

The consumer sets their reservation price based on the lower of their value of the good or their indifference towards searching when they are facing the outside firm:

$$r = \min\{1 + \int_{\underline{p}_C}^{r} q g_C(q) dq, v\}. \tag{19}$$

The consumer's reservation price is set according to G_C because if they see a price at (or above) r_C, they can be certain that it came from the merged firm. Thus, they will face the outside firm moving forward. If they receive a lower price, they will believe they are facing a convex combination of F_C, G_C, and a degenerate distribution of r_C, which first order stochastically dominates G_C. Therefore, consumers who see a price above r_C search, and those who see a price at or below r_C do not.

3 Welfare and Antitrust Remedies

To study welfare, we characterize average transacted prices (a sufficient statistic for consumer welfare in this model), under two settings. In the first, consumers' reservation price is exogenous. One can also think of this setting as one in which the consumer is unaware that the merger has occurred, a setting we refer to as the illusion of competition. If the consumer does not know about the merger, they will set their reservation price according to the symmetric search indifference condition (Equation 3) no matter what game the firms are actually playing. In the second setting, consumers' reservation price is set endogenously by their decision of whether or not to continue searching, given their search cost and the actual equilibrium distribution of prices.

3.1 Fixed r/Illusion of Competition

First, we consider the case in which consumers are unaware of the merger and set their reservation price according to the symmetric equilibrium or their valuation of the good v. For brevity, we will denote this common reservation price r.

3.1.1 Joint Pricing Equilibrium

When μ is such that a brand-preserving merger would result in the joint pricing equilibrium (the light gray region in Figure 2), we consider three possible outcomes: a baseline equilibrium (no merger), the joint pricing equilibrium (brand-preserving merger), and the consolidated brands equilibrium (brand-consolidating merger). Consumer welfare is fully captured by the average transacted price in each of these outcomes, which we denote $\bar{p}$, $\bar{p}_J$, and $\bar{p}_C$, respectively.

Proposition 3 *If r is fixed and a joint pricing equilibrium would occur ($\mu_2^2 \geq 3\mu_1\mu_3$), then $\bar{p} \leq \bar{p}_C \leq \bar{p}_J$. The consumer prefers the no-merger outcome to the brand-consolidating merger outcome. They prefer both of these outcomes to the brand-preserving merger.*

Proposition 3 tells us that if enough consumers consider exactly two prices, and consumers do not adjust their reservation prices following the merger, then the outcomes have a clear ranking. Average prices will be highest when firms merge and continue to operate separate brands. If reservation prices are fixed, the merger allows firms to capture additional rent generated by consumers only considering the two merged brands. This is the region of the parameter space that is perhaps most interesting to regulators. If consumers' search technology is as such, firms will want to merge and continue operating multiple brands. However, regulators wishing to protect consumers' interests should either block the merger or at least force the firms to consolidate the brands. (Ireland (2007) finds this result for the special case of $\mu_3 = 0$, which lies along the diagonal upper bound of the parameter space in Figure 2.) These results are consistent with McDonald and Wren's (2018) hypothesis that firms use multiple brands to crowd competitors out of their potential customers' consideration sets.

3.1.2 Distinct Pricing Equilibrium

When μ is such that a brand-preserving merger would result in the distinct pricing equilibrium (the medium and dark gray regions in Figure 2), we consider three similar outcomes: a baseline equilibrium (no merger), the distinct pricing equilibrium (brand-preserving merger), and the consolidated brands equilibrium (brand-consolidating merger). We denote the average transacted price in each of these outcomes as $\bar{p}$, $\bar{p}_D$, and $\bar{p}_C$, respectively.

Proposition 4 *If r is fixed and a distinct pricing equilibrium would occur ($\mu_2^2 \leq 3\mu_1\mu_3$), then $\bar{p} \leq \bar{p}_C, \bar{p}_D$. The consumer prefers the no merger outcome to either merger outcome. Furthermore, $\bar{p}_C < \bar{p}_D$ iff $\mu_2 > 3\mu_1 * (1 - \mu_1)/(2 * (3 - \mu_1))$. The consumer prefers the brand-consolidating merger to the brand-preserving merger if and only if μ_2 is sufficiently high.*

When the consumers' search technology is such that the distinct pricing equilibrium is realized, the no-merger case is still the best outcome for consumers. If consumers do not change their reservation price, firms are able to raise their average prices following either type of merger.

In the distinct pricing case, it is possible for the brand-preserving merger outcome to be better for consumers than the brand-consolidating merger outcome. In the distinct pricing equilibrium, one-third of the consumers who only see one price will draw a price of r from the higher-priced jointly owned brand. These consumers get "ripped off" while all of the remaining consumers draw at least one price from the fairly competitive F_D distribution. If this fraction of ripped-off consumers is small enough (when μ_3 is relatively high) or if F_D is sufficiently more competitive than the price distributions offered in the brand-consolidating merger equilibrium (when μ_1 is relatively high), then the distinct pricing equilibrium results in lower prices than the consolidated one. The dark gray region in Figure 2 illustrates the parameter region for which consolidating brands actually hurts consumers.

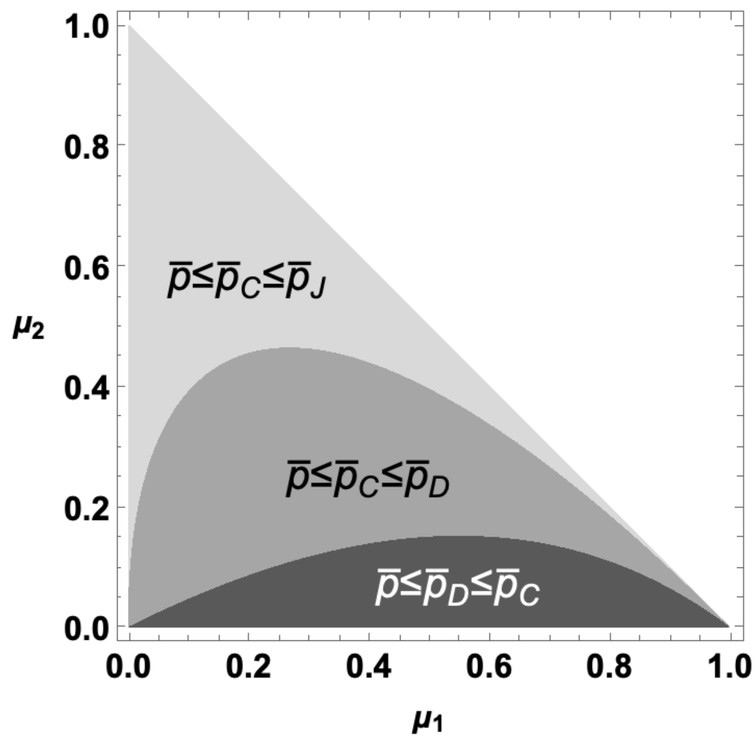

Figure 2: **Ranking equilibrium transaction prices for fixed r / illustion of competition:** Parameter μ_k is the fraction of consumers who receive k initial price quotes, where $\mu_3 = 1 - \mu_1 - \mu_2$. Possible values for the vector μ lie in the triangular region. We compare average transacted prices between the baseline pre-merger symmetric 3 firm equilibrium ($\bar{p}$) to prices following a brand-consolidating merger ($\bar{p}_C$) and to prices of the two possible outcomes of a brand-preserving merger, the distinct pricing equilibrium ($\bar{p}_D$), and the joint pricing equilibrium ($\bar{p}_J$). Here we assume that consumers are under the illusion of competition and do not update their reservation prices following a merger. Average transacted prices are lower for the pre-merger case for all values of μ. The brand-consolidating merger produces lower average transacted prices than the brand-preserving merger for all regions except the darkest gray region.

3.2 Endogenous r

Next, we consider consumers who update their reservation price following the merger to reflect the equilibrium of the new game. The following results reflect the equilibria of Section 2 in which r is pinned down by the equilibrium search condition.

The asymmetry of these equilibria allows for certain counter-intuitive results. In the previous subsection, we showed that, for a given reservation price, the overall transaction price distribution is worse for a post-merger equilibrium. However, consumers' equilibrium reservation prices are not generally selected with respect to the overall transaction price distribution. Instead, many of the equilibria involve reservation prices that are chosen based on the lower-priced brands. This can result in consumers setting a lower reservation price post-merger and ultimately lower average transacted prices.

3.2.1 Joint Pricing Equilibrium

We first consider welfare in the parameter space in which the joint pricing equilibrium occurs (that with a relatively high share of consumers considering exactly two prices—the light gray region in Figure 3). We denote average transacted prices (with endogenous r) under the no-merger, equilibrium, brand-consolidating merger equilibrium, and joint pricing equilibrium as $\bar{p}^*$, $\bar{p}_C^*$, and $\bar{p}_J^*$, respectively. Moving forward we distinguish between average transacted prices when consumers are under the illusion of competition (exogenous r) and those where they are not (endogenous r) by adding an asterisk to the endogenous r value.

Proposition 5 *If r is the solution to the sophisticated consumers' search indifference condition and a joint pricing equilibrium would occur ($\mu_2^2 \geq 3\mu_1\mu_3$), then $\bar{p}^* < \bar{p}_C^* < \bar{p}_J^*$. The consumer prefers the brand consolidating-merger to the the brand-preserving merger (joint pricing equilibrium), but prefers no merger to both of these outcomes.*

This proposition tells us that the joint pricing equilibrium is still worse for consumers than the no-merger or brand-consolidating merger equilibria. Whether consumers update their reservation price or not, a joint-pricing equilibrium cannot be optimal for consumers. This suggests that antitrust authorities should be particularly concerned about markets in which one firm operates multiple brands, yet sets the same price across these brands.

This also shows that mergers will not benefit consumers if there are sufficiently many consumers who only consider two prices. Brand-consolidation can mitigate the harm to consumers, but it will not fully offset the increase in prices caused by the merger.

3.2.2 Distinct Pricing Equilibrium

Finally, we consider the case in which consumers update their reservation price, and the search technology is such that the distinct pricing equilibrium would be realized (all except the lightest gray region of the triangle in in Figure 3).

Proposition 6 *If r is the solution to the sophisticated consumers' search indifference condition, then: (1) $\bar{p}^* < \bar{p}_C^*$; (2) $\bar{p}_D^* < \bar{p}^*$ iff Appendix equation (A1) holds; and (3) $\bar{p}_D^* < \bar{p}_C^*$ iff Appendix equation (A2) holds. These are restrictions on the values of μ, which are shown graphically in Figure 3.*

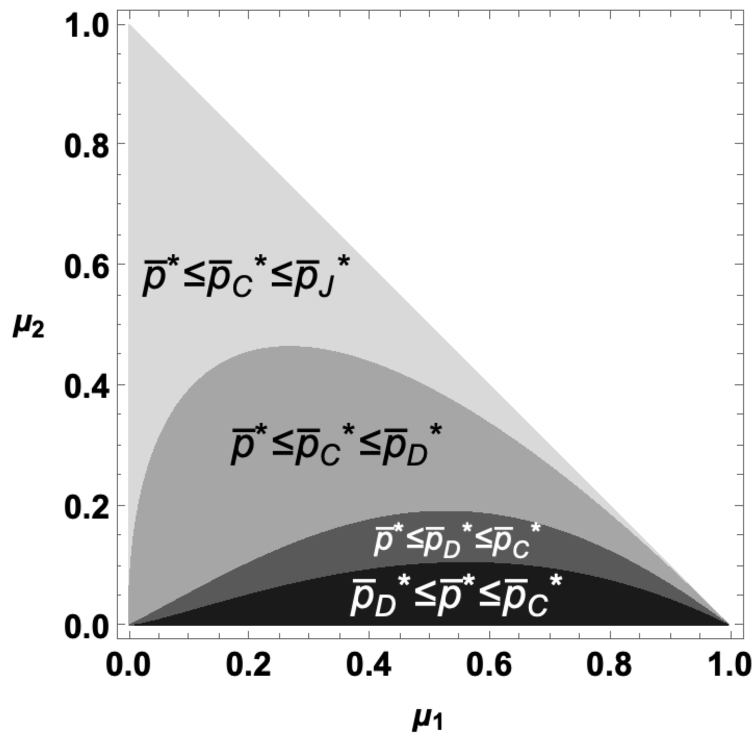

Figure 3: **Ranking equilibrium transaction prices for endogenous r / sophisticated consumers:** Parameter μ_k is the fraction of consumers who receive k initial price quotes, where $\mu_3 = 1 - \mu_1 - \mu_2$. Possible values for the vector μ lie in the triangular region. We compare average transacted prices between a pre-merger symmetric 3 firm equilibrium ($\bar{p}^*$) to prices following a brand-consolidating merger ($\bar{p}^*_C$) and to prices of the two possible outcomes of a brand-preserving merger, the distinct pricing equilibrium ($\bar{p}^*_D$), and the joint pricing equilibrium ($\bar{p}^*_J$). Here we assume that consumers are sophisticated and update their reservation prices following a merger.

Proposition 6 establishes a series of inequalities over the search technology μ that allow us to compare prices under different types of mergers. Again, we find that the no merger case always results in lower prices than the brand-consolidating merger. As we found in the constant reservation price case, there is a region in which the number of consumers considering exactly two prices is low, in which the distinct pricing equilibrium is preferred to the brand-consolidating merger. While the boundary of the inequality changes when reservation prices update, this qualitative statement still holds. When consumers are aware of the merger, and update their reservation price rationally, it is also possible for the distinct pricing equilibrium to result in lower prices than the no-merger case.

Propositions 3-6 allow us to map out the parameters for which different antitrust remedies are consumer optimal. Specifically, in Figure 4, we plot when brand consolidation and/or informing consumers of a merger (breaking the illusion of competition) will lower prices, conditional on allowing a merger. Consolidation is beneficial when the average price is lower for the consolidated-brands equilibrium than the brand-preserving equilibrium. This is the case when there are sufficiently many consumers considering exactly two prices who become captive consumers following a brand-preserving merger. Consolidation can ensure that these consumers see two competitive prices.

Information about the merger is beneficial to consumers when the reservation price for the merger equilibrium is lower than the reservation price for the no-merger baseline. This happens when there are sufficiently many shoppers. When there is a large population of shoppers, the non-merged firm competes aggressively on price to win over these consumers. Informed consumers' reservation prices are determined by the distribution of prices played by the outside firm, so for high values of μ_3, information can cause consumers to decrease their reservation price, lowering overall transacted prices.

These remedies are only optimal conditional on allowing a merger in the first place. As previously noted, the baseline, no merger case results in the lowest prices for all parameter values if consumers do not update their reservation prices. Furthermore, as seen in Figure 3, prices are lowest in the baseline case unless the share of consumers considering exactly two prices is sufficiently low. A merger in that region of the parameter space would lower prices, but would also lower profits for the merging firms, and would therefore be less likely to be proposed in the first place. For this reason, we consider optimal antitrust remedies conditional on a merger being allowed (perhaps because the merger produces sufficient reductions in fixed costs).

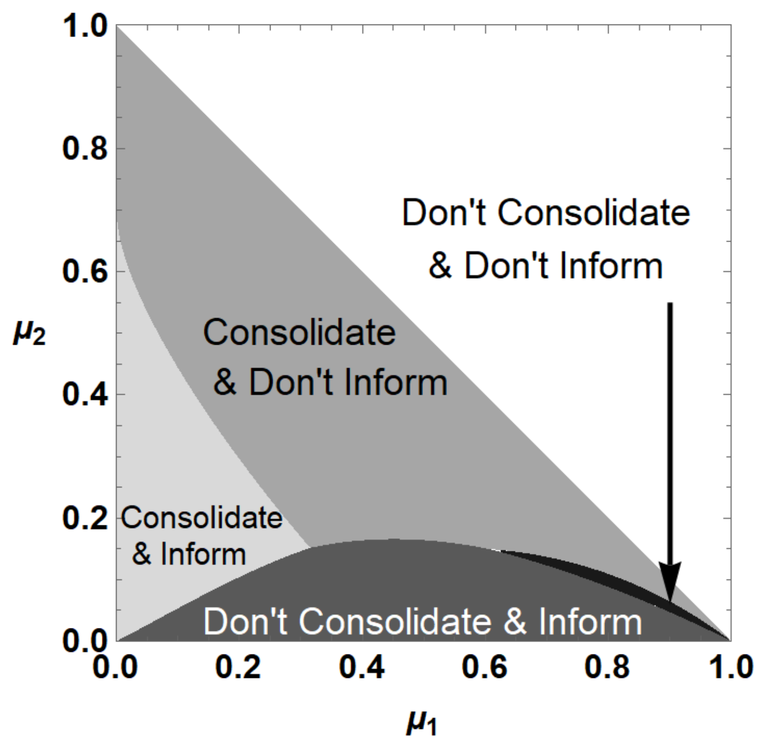

Figure 4: **Optimal policy conditional on a merger:** Parameter μ_k is the fraction of consumers who receive k initial price quotes, where $\mu_3 = 1 - \mu_1 - \mu_2$. Possible values for the vector μ lie in the triangular region. We show the optimal antitrust remedy for a given μ, conditional on allowing a merger. Consolidate refers to forcing the merged firm to consolidate its brands while Inform refers to informing consumers of the market structure, allowing them to update their reservation price.

Collectively, Propositions 3-6 also tell us about the possible harm caused by the illusion of competition. Propositions 3 and 4 state that, if consumers stick to the pre-merger reservation price, the no-merger outcome is always preferred to any merger outcome. Therefore, when consumers suffer from the illusion of competition, mergers are always harmful. However, Proposition 6 shows that this result flips for certain values of μ when consumers rationally update their reservation price to the post-merger equilibrium distribution of prices. In these scenarios, informed consumers can be better off post-merger. This reversal implies that, for these values of μ, the reservation price, and thus transacted prices, fall when consumers are aware of the post-merger equilibrium.

Corollary 6.1 *There exist search technologies μ such that average transacted prices are lower if consumers are aware of a merger and update their reservation price than if they are unaware and use their pre-merger reservation price.*

Corollary 6.1 suggests that regulators may be able to reduce consumer harm from a merger by informing consumers of the new market structure. However, Corollary 6.2 shows that it is also possible for the illusion of competition to benefit consumers.

Corollary 6.2 *There exist search technologies μ such that average transacted prices are lower if consumers are unaware of a merger and do not update their reservation price than if they are aware and use the full-information equilibrium reservation price.*

Consider, for example, the case of $\mu_2 = 1$. Here, pre-merger, all firms are always in competition with another firm, and therefore price at marginal cost, or 0. Therefore, the consumers' pre-merger reservation price is 1 (the average price offered in the degenerate distribution plus their search cost). Following a brand-preserving merger, the merged firm has captive consumers, allowing all brands to earn positive profits. The average offered price following the brand-preserving merger must be positive, meaning the equilibrium reservation price for fully informed consumers must be strictly greater than 1. In such a case, informing consumers that a merger has occurred would be harmful to their welfare.

4 Extension: Brand Proliferation

Several of our preceding results evaluate the benefits of requiring firms to consolidate brands following a merger. These results (Propositions 3–6) can also be reinterpreted as measuring the benefits of restricting brand proliferation by preventing a single-brand firm from introducing an additional brand for its existing product. This reinterpretation suggests that restricting brand proliferation benefits consumers either when they suffer from the illusion of competition or when sufficiently many see exactly two prices before considering additional search—and hence are at risk of capture by a multi-brand firm.

In this section, we develop additional insights by adapting our assumptions to better fit the brand-proliferation interpretation of our model. First, we assume that the status-quo is a symmetric duopoly with two single-brand firms. Equilibrium (characterized by Burdett and Judd (1983) and documented in Appendix Section A.2) is similar to the symmetric three-firm case presented in Section 2.1. We compare this to the case in which one of the two firms has introduced an additional brand (resulting in a market with 3 brands).

We first consider a *symmetric brand proliferation* in which the newly introduced brand is symmetric to both the existing co-owned brand and the existing independent brand. It receives the same share of searches as either of the existing brands. This results in the same equilibria discussed in Section 2.2.

Below, we characterize prices under the symmetric two-firm equilibrium and the symmetric brand proliferation equilibrium. Denote average transacted prices in these cases as $\bar{p}^*$ and $\bar{p}^*_D$ (which corresponds to the distinct pricing equilibrium in Proposition 4), respectively.

Proposition 7 *If r is the solution to the sophisticated consumers' search indifference condition, then following symmetric brand proliferation, $\bar{p}^* > \bar{p}^*_D$ iff Appendix equation (A3) holds. This condition holds only when μ_2 is sufficiently low and is shown graphically in Appendix Figure B.1.*

If consumers update their reservation prices following symmetric brand proliferation, the results are qualitatively similar to those about brand-preserving mergers. Allowing a firm to operate two brands will be worse than the baseline symmetric equilibrium unless the share of consumers considering exactly two prices is sufficiently low. Regulators or platforms interested in consumer welfare should therefore be hesitant to allow brand proliferation in settings where new brands would receive equal consideration to existing brands.

We can also compare this outcome to one in which the newly introduced brand is only seen by consumers who have already searched the existing two brands, an outcome we call *obscure brand proliferation*. Armstrong and Vickers (2024) are the first to introduce this possibility in Section 5.1 of their paper (see their discussion of "inferior" brand proliferation). Here the newly introduced brand does not receive any captive consumers or consumers who are considering exactly two prices. It is only able to compete for shoppers that consider all three prices. This assumption can capture a brand introduction that isn't heavily advertised, or a hypothetical policy by an online platform such as Grubhub to rank 'duplicate' brands at the bottom of search results.

Given exogenous consideration sets, Armstrong and Vickers (2024) are the first to prove the existence of and characterize the pricing equilibrium in this case. The firms' equilibrium pricing strategies follow directly from their work. Lemma 1 in Appendix 2 restates the pricing strategy for firms in our context (it follows from Armstrong and Vickers's (2024) Proposition 2 and Appendix). In this equilibrium, the new brand has no captive consumers so it can only win shoppers. The two-brand firm therefore optimally sets the price of its new brand below the price of its old brand. It selects its two prices from two adjoining but non-overlapping intervals. The outside firm then competes with both of these brands and mixes over prices in both intervals.

This allows us to compare symmetric brand proliferation to obscure brand proliferation for a fixed reservation price (when consumers are under the illusion of competition. Figure 5 shows that if sufficiently many consumers consider only two brands (and thus are at risk of being captive to the proliferating firm), then the obscure brand proliferation results in lower prices than the symmetric brand proliferation. For regulators, or a platform that is interested in its consumers' welfare, this implies that, when consumers are under the illusion of competition, limiting the visibility of "duplicate" brands can sometimes reduce the harm to consumers introduced by brand proliferation.

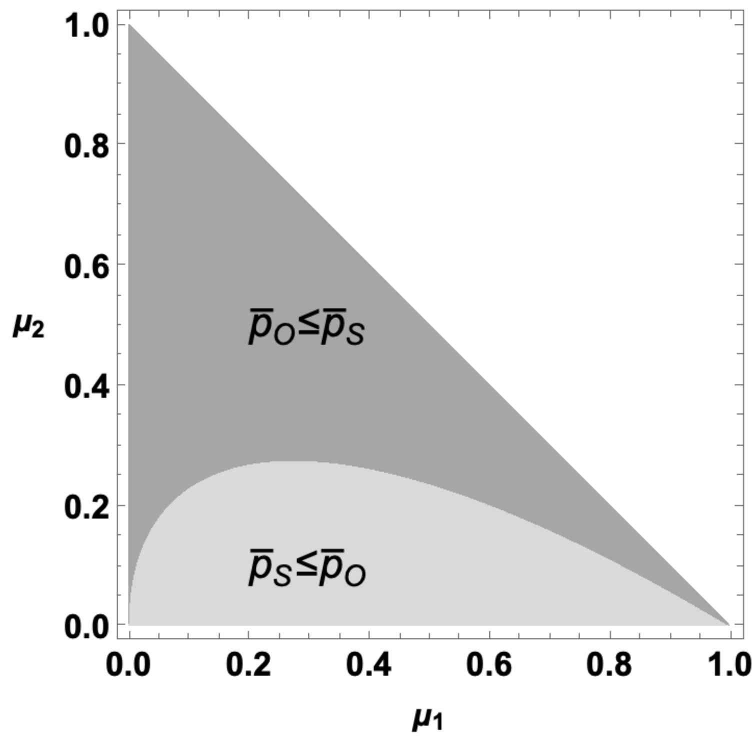

Figure 5: **Ranking equilibrium transacted prices for fixed r / illusion of competition following brand proliferation:** Parameter μ_k is the fraction of consumers who receive k initial price quotes, where $\mu_3 = 1 - \mu_1 - \mu_2$. We compare average transacted prices between the symmetric brand equilibrium (p_S) and the obscure brand equilibrium (p_O). The obscure brand equilibrium results in lower average transacted prices than the symmetric brand equilibrium for sufficiently high values of μ_2 (the darker gray region).

We then extend these results to allow for endogenous search: We prove that there exists a consumer reservation price strategy that is a best response to the equilibrium price distributions set by the firms and thus, a reservation price equilibrium exists. We then characterize this reservation price, allowing us to compare outcomes when consumers are sophisticated and optimally set a reservation price in response to the actual market structure to the case when consumers are under the illusion of competition.

Proposition 8 *For all μ, following obscure brand proliferation, a reservation price equilibrium exists with consumer reservation price r_O characterized by equation (27) and price distributions characterized by equations (20)–(26) in Appendix A.1.*

If consumers endogenously respond to the introduction of the new brand, they set their reservation prices based on the expected price offered by the new brand (the most favorable of the three distributions). A consumer who sees two prices in the upper interval of prices is certain that these two prices came from the two existing brands, therefore they would draw a price from the new brand if they searched. A consumer who has only seen one price must have more pessimistic beliefs about the distribution of prices they are facing upon search, therefore they will not search if they receive a single price of r_O. Off-path beliefs are such that if a consumer sees a price above r_O, they think their next price will come from distribution F_N.

Next, we present results comparing the two-firm baseline to obscure brand proliferation when consumers update their reservation price following the proliferation. With exogenous consideration sets, studied by Armstrong and Vickers (2024), brand proliferation always leads to higher consumer prices. With endogenous search, this is only true under the illusion of competition. When sophisticated consumers update their reservation prices to reflect the true market structure, brand proliferation can lead to lower average transacted prices. Denote average transacted prices with sophisticated consumers as $\bar{p}^*$ under the two-firm symmetric equilibrium, $\bar{p}^*_O$ under the obscure brand proliferation equilibrium, $\bar{p}^*_D$ under symmetric brand proliferation (which corresponds to the distinct pricing equilibrium in Proposition 6).

Proposition 9 *If r is the solution to the sophisticated consumers' search indifference condition, then following obscure brand proliferation, $\bar{p}^* > \bar{p}^*_O$ iff Appendix equation (A4) holds. This condition holds only for sufficiently low μ_3. Additionally, $\bar{p}^*_O > \bar{p}^*_D$ iff Appendix equation (A5) holds. This condition holds only for sufficiently low μ_2. These are restrictions on the values of μ, which are shown graphically in Appendix Figure B.2.*

If consumers update their reservation values following an obscure brand proliferation, prices will increase unless the share of shoppers is sufficiently low. The intuition behind this is that the share of shoppers (μ_3) determines how much of an influence the new brand can have on the existing brands' pricing strategy. At one extreme, if $\mu_3 = 0$, the new brand receives zero initial searches. This means that for a given value of r, the new brand does not affect the pricing strategy of the existing two brands at all. As the share of shoppers increases, the new brand's potential market share does as well, and the existing brands move towards competing over higher prices. On the other hand, the introduction of the new brand causes consumers to set their reservation price with the belief that the next price will come from the new brand. This asymmetry allows for the possibility of lower reservation prices. If μ_3 is sufficiently low, this reservation price effect can outweigh the change in pricing patterns and average transacted prices can fall.

The second statement in the proposition demonstrates that prices are lower following obscure brand proliferation than they are following symmetric brand proliferation, unless the share of consumers considering exactly two prices is sufficiently low. If there are many consumers considering exactly two prices, then the policy of making duplicate brands less visible to searchers could be effective at lowering prices when proliferation cannot be blocked entirely.

Together, these propositions give us the optimal regulatory policy for all possible search technology. In Figure 6 we plot the regions of μ for which different policies are optimal. Brand proliferation with a fixed reservation price can never lead to lower prices than the baseline. Therefore if regulators allow proliferation, it is always optimal to inform consumers of the new market structure, breaking the illusion of competition and allowing them to update their reservation prices. When there are sufficiently few shoppers who consider all three brands (the top right black region), it is optimal to allow brand proliferation but to limit the visibility of new, duplicate brands. When many consumers consider exactly two prices and more shoppers (the dark gray region), banning proliferation is first best, but reducing visibility can still reduce harm to consumers. When there are a low to intermediate amount of consumers considering two prices (the lightest region), banning proliferation is optimal; but conditional on allowing proliferation, symmetric brand proliferation leads to lower prices. Lastly, when very few consumers consider exactly two prices (the bottom gray region), allowing symmetric brand proliferation is first best for consumers.

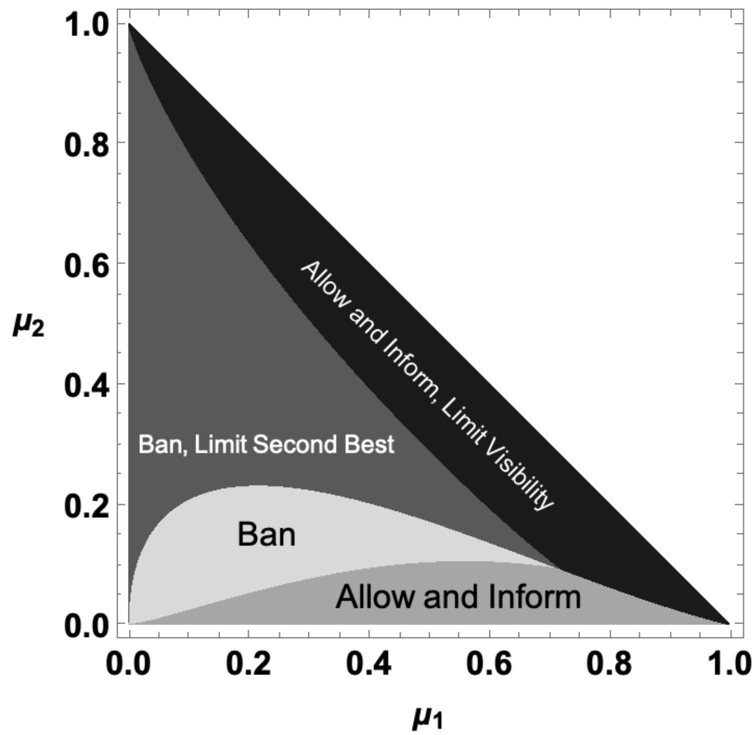

Figure 6: **Optimal policy on brand proliferation:** Parameter μ_k is the fraction of consumers who receive k initial price quotes, where $\mu_3 = 1 - \mu_1 - \mu_2$. Possible values for the vector μ lie in the triangular region. The terms Ban and Allow refer to whether banning brand proliferation or allowing it is first best for consumer welfare. In all cases where proliferation is allowed, it is optimal to inform consumers that proliferation has occurred, breaking the illusion of competition. In the areas labeled Limit Visibility and Limit Second Best, obscure brand proliferation results in lower average transacted prices than symmetric brand proliferation.

5 Extension: $N > 3$ Firms

Studying the case of a merger between two firms in a three firm market allows us to study outcomes for all possible parameterizations of the search technology, but certain conclusions continue to hold when we consider a market with N single-brand firms of which K are considering a merger. Specifically, we can demonstrate that requiring the merging firms to consolidate their brands is beneficial to consumers under many circumstances but can be harmful in others.

Proposition 10 *In a market with N single-brand firms of which K are merging, where $2 \leq K < N$, there exist vectors of the search technology μ for which the average transacted* *price is higher in the brand-preserving merger equilibrium than in the brand-consolidating merger equilibrium.*

Consider the following example of a case in which consolidation would benefit consumers. Some consumers in the market search and receive K price quotes ($\mu_K > 0$) and all consumers in the market search and receive K or more price quotes ($\sum_{i=K}^{N} \mu_i = 1$). Prior to the merger, this would result in all firms pricing at marginal cost, a la Bertrand competition, and earning zero profits. If the firms merge and the merged firm continues to operate K separate brands, they will face some consumers who searched K times and only saw co-owned brands. This means that the jointly owned brands could set positive markups and earn profits. If, on the other hand, the firms merge but are forced to consolidate their brands, all consumers will see prices from at least $\min\{K, N-K+1\}$ independently owned brands. This will restore Bertrand competition, in which firms price at marginal cost, firms receive zero profits, and all surplus goes to consumers. More generally, if operating multiple brands causes sufficiently many consumers to flip from seeing multiple competing firms to only the merged firm, then consolidation would benefit consumers.

It is also possible to come to the opposite conclusion with more than 3 firms. It is possible for prices to be lower in a brand-preserving merger than in a brand-consolidating merger.

Proposition 11 *In a market with N firms, there exist vectors of the search technology μ, and a number of firms merging K, for which the average transacted price is higher in the brand-consolidating equilibrium than in the brand-preserving equilibrium.*

One such market in which consolidation could harm consumers is when 2 of N firms are merging and consumers' search technology is that considered by Stahl (1989) ($\mu_N = \mu, \mu_1 = 1-\mu$). In such a scenario, the average price, conditional on a given reservation price r, is the same across the brand-preserving and brand-consolidating equilibria. However, sophisticated consumers have a lower reservation price in the brand-preserving equilibrium than in the brand-consolidating equilibrium. Therefore, in this case, consumer welfare is higher if the merged firm continues to operate separate brands than if it consolidates them.

6 Conclusion

This paper studies novel pricing patterns that emerge when firms operate multiple "brands" in markets characterized by consumer search. We establish conditions under which each type of equilibrium exists. When the share of consumers considering exactly two brands is sufficiently high, a joint pricing equilibrium, in which firms set both of their brands' prices to the same value, prevails. If this condition is not met, a distinct pricing equilibrium exists, in which the firm sets two separate prices for its brands. We evaluate novel antitrust interventions—requiring brand consolidation or action to make consumers aware of brand co-ownership as a condition for merging. We show that requiring brand consolidation always benefits consumers (conditional on allowing a merger) if a brand-preserving merger would yield a joint pricing equilibrium. However, requiring brand consolidation can be counterproductive under some conditions when a brand-preserving merger would yield a distinct pricing equilibrium. We study welfare both when consumers are aware and unaware that the merger has taken place and find that ending the illusion of competition by making consumers aware of brand co-ownership may help or hurt consumers. We also evaluate related policies for market platform operators to restrict brand proliferation—limiting firms to list their product under at most one brand or limiting the visibility of duplicate brands. These policies benefit consumers when sufficiently many check exactly two prices, and hence are at risk of becoming captive to a single firm via brand proliferation.

This paper is an important first step in understanding how firms operate multiple brands in markets with search. These arrangements are common in real markets and understanding their effects on prices is critical for economists and regulators. This paper is limited by the fact that it primarily considers the case of three firms with two considering a merger. While adding firms reduces tractability, extending the model in this direction would be useful for understanding actual markets. Additionally, our model assumes that search is random and that consumers' awareness of the merger allows them to optimally set their reservation price, not steer their search. Additional theoretical work could help clarify outcomes under alternative consumer search technologies.

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Appendix A Additional Results

A.1 Obscure brand proliferation

The price distributions described by Proposition 8 are characterized by equations (20)–(27), where we denote the probability that the single-brand firm plays a price in the higher interval by $\gamma = G_O(p^\dagger)$.

$$\lambda_O = \frac{2(1-\mu_1-\mu_2)}{2-\mu_1}, \tag{20}$$

$$\gamma_O = \frac{\mu_1 \lambda_O}{\mu_1 + 2\mu_2(1-\lambda_O)}, \tag{21}$$

$$p_O^\dagger = \frac{\mu_1}{\mu_1 + 2\mu_2(1-\lambda_O)} r_O, \tag{22}$$

$$\underline{p}_O = \frac{\mu_1 + 2\mu_2\gamma_O}{\mu_1 + 2\mu_2 + 2\mu_3} r_O, \tag{23}$$

$$F_O(p) = 1 - \gamma_O - \frac{(\mu_1 + 2\mu_2\gamma_O)(r_O - p)}{2\mu_2 p}, \tag{24}$$

$$F_N(p) = 1 - \frac{(\mu_1 + 2\mu_2)(p_O^\dagger - p)}{2\mu_3 p}, \tag{25}$$

$$G_O(p) = \begin{cases} 1 - (1-\lambda_O)\frac{p_O^\dagger}{p} & \underline{p}_O \leq p \leq p_O^\dagger \\ 1 - \frac{\mu_1(r_O - p)}{2\mu_2 p} & p_O^\dagger < p \leq r_O \end{cases}, \tag{26}$$

and

$$r_O = \min\{1 + \int_{\underline{p}_O}^{p_O^\dagger} (f_N(p) * p) dp, v\}. \tag{27}$$

A.2 Two firm symmetric equilibrium

When two symmetric firms compete in a market with search technology μ, the unique equilibrium is for both firms to mix over the interval $[\underline{p}, r]$ according to the common distribution F, where

$$\underline{p} = \frac{\mu_1}{2-\mu_1} r, \tag{28}$$

$$F(p) = 1 - \frac{\mu_1(r-p)}{2\mu_2 p}, \tag{29}$$

and

$$r = \min\{1 + \int_{\underline{p}}^{r} f(p) * p dp, v\} \tag{30}$$

Appendix B Proofs of Propositions

B.1 Proposition 1

One can show that this is an equilibrium by first showing that all of the prices between $\underline{p}_D$ and r_D are best responses for the outside firm. The outside firm's profit function (conditional on the inside firm playing r_D with one of its prices) is

$$\pi_{OM}(p) = \frac{1}{3}[\mu_1 + \mu_2 + (\mu_2 + 3\mu_3)(1 - F(p))]p \tag{31}$$

Profits from playing r_D are then:

$$\pi_{OM}(r_D) = \frac{r_D}{3}[\mu_1 + \mu_2] \tag{32}$$

Likewise profits from playing $\underline{p}_D$ from Equation 6 are:

$$\pi_{OM}(\underline{p}_D) = \frac{1}{3}[\mu_1 + 2\mu_2 + 3\mu_3]\frac{(\mu_1 + \mu_2)r_D}{\mu_1 + 2\mu_2 + 3\mu_3} = \pi_{OM}(r) \tag{33}$$

The firm is therefore indifferent between these two prices. Playing a price above r_D results in zero profits as the consumer will search or purchase from the other firm. The merged firm plays prices weakly below r_D so the outside firm will lose a customer with certainty. The firm also will earn strictly lower profits from playing a price below $\underline{p}$ as they cannot gain any customers, but can only lower their markups. Therefore, the outside firm will not price above or below the interval. To show that they are indifferent over all prices in the interval, plug in $F(p)$ from Equation 8 to get

$$\pi_{OM}(p) = \frac{1}{3}[\mu_1 + \mu_2 + (\mu_2 + 3\mu_3)(\frac{(\mu_1 + \mu_2)(r_D - p)}{(\mu_2 + 3\mu_3)p})]p = \pi_{OM}(r) \tag{34}$$

Therefore, the outside firm is indifferent over all prices in $[\underline{p}_D, r_D]$.

The merged firm's profit function is Equation 4. If

$$G_D(p) = 1 - \frac{(\mu_1 + \mu_2)(r_D - p)}{(\mu_2 + 3\mu_3)p}. \tag{35}$$

We can plug this into Equation 4 and get

$$\begin{aligned}\pi_{JM}(p_L, p_H) = [\frac{1}{3}\mu_1 + \frac{1}{3}\mu_2 + (\frac{1}{3}\mu_2 + \mu_3)(\frac{(\mu_1 + \mu_2)(r_D - p_L)}{(\mu_2 + 3\mu_3)p_L})]p_L \\ +[\frac{1}{3}\mu_1 + \frac{1}{3}\mu_2(1 - G(p_H))]p_H.\end{aligned} \tag{36}$$

Simplifying:

$$\begin{aligned}\pi_{JM}(p_L, p_H) = \frac{1}{3}(\mu_1 + \mu_2)r \\ +[\frac{1}{3}\mu_1 + \frac{1}{3}\mu_2(1 - G(p_H))]p_H.\end{aligned} \tag{37}$$

This is no longer a function of p_L (in the support of G), therefore the merged firm is indifferent over values of p_L. Therefore, all p_L in $[\underline{p}_D, r]$ are in the best response correspondence for the merged firm.

To show that the merged firm maximizes profits by choosing $p_H = r_D$, we can show that the profit function is increasing in p_H, leading to the corner solution of r_D. The derivative of π_{JM} with respect to p_H is

$$\frac{\partial \pi_{JM}(p_L, p_H)}{\partial p_H} = \frac{1}{3}[[\mu_1 - \frac{\mu_2(\mu_1 + \mu_2)}{(\mu_2 + 3\mu_3)}]. \tag{38}$$

Which is weakly positive when:

$$\mu_1 - \frac{\mu_2(\mu_1 + \mu_2)}{(\mu_2 + 3\mu_3)} \geq 0 \iff 3\mu_1\mu_3 \geq \mu_2^2 \tag{39}$$

Which is the region of interest for μ. Therefore, if the consumer has a reservation price of r_D, this pricing strategy constitutes an equilibrium for both the merged and outside firm.

To show that the consumer will optimally choose a reservation price strategy with reservation price r_D from Equation 7, first note that $G(p)$ (and $F(p)$ which equals $G(p)$) has a lower average price than any other distribution of prices the consumer could face. If the consumer has not seen $p_H = r_D$ from the merged firm yet, then they face some mixture distribution of $G(p)$ and a degenerate distribution of r_D. Given this fact, it is obvious that a consumer will never search if they see a price at or below r_D as defined by Equation 7.

That they will search when they hold prices above r_D comes from the consumers' off path beliefs. If a consumer sees a price greater than r_D, they believe this price came from p_H. Therefore, they are facing $G(p)$ moving forward and therefore have positive value of search. This constitutes a reservation price strategy with reservation price r_D on the consumers' side. Therefore, the equilibrium exists.

B.2 Proposition 2

Start by considering the merged firms' profit maximization problem. Their profit function is again Equation 4. Consider first the constrained problem in which the firm must set both of its prices to p. Then its profits from playing $p = r_J$ are

$$\pi_{JM}(r_J, r_J) = [\frac{2}{3}\mu_1 + \frac{1}{3}\mu_2]r_J. \tag{40}$$

Profits from playing $p = \underline{p}_J$ from Equation 9 are then

$$\pi_{JM}(\underline{p}_J, \underline{p}_J) = [\frac{2}{3}\mu_1 + \mu_2 + \mu_3]\frac{(2\mu_1 + \mu_2)r_J}{2\mu_1 + 3\mu_2 + 3\mu_3} = \pi_{JM}(r_J, r_J) \tag{41}$$

So the firm is indifferent over r_J and $\underline{p}_J$ (in the constrained problem. To see that they are indifferent over the interval between these values, we can plug in the outside firm's distribution of prices from Equation 13 into the constrained profit function to get

$$\pi_{JM}(p,p) = [\frac{2}{3}\mu_1 + \frac{1}{3}\mu_2 + (\frac{2}{3}\mu_2 + \mu_3)(\frac{(2\mu_1+\mu_2)(r_J - p)}{(2\mu_2+3\mu_3)p})]p = \pi_{JM}(r_J, r_J) \tag{42}$$

The firm is then indifferent between any price vector (p,p) where $p \in [\underline{p}_J, r_J]$. To see that this constrained problem is equivalent to the unconstrained profit maximization, we must show that holding $p_H = p_L$, profits are increasing in p_L, and holding $p_L = p_H$, profits are decreasing in p_H. The first of these expressions is

$$\frac{\partial \pi_{JM}(p_L, p_H)}{\partial p_L} = \frac{1}{3}[(\mu_1+\mu_2) - (\mu_2+3\mu_3)(\frac{(2\mu_1+\mu_2)}{(2\mu_2+3\mu_3)})]. \tag{43}$$

Which is positive when:

$$\frac{1}{3}[(\mu_1+\mu_2) - (\mu_2+3\mu_3)(\frac{(2\mu_1+\mu_2)}{(2\mu_2+3\mu_3)})] \geq 0 \iff \mu_2^2 \geq 3\mu_1\mu_3. \tag{44}$$

Which is what we have assumed. Likewise the derivative of profits with respect to p_H are

$$\frac{\partial \pi_{JM}(p_L, p_H)}{\partial p_H} = \frac{1}{3}[\mu_1 - \mu_2(\frac{(2\mu_1+\mu_2)}{(2\mu_2+3\mu_3)})]. \tag{45}$$

Which is negative iff:

$$\frac{1}{3}[\mu_1 - \mu_2(\frac{(2\mu_1+\mu_2)}{(2\mu_2+3\mu_3)})] \leq 0 \iff 3\mu_1\mu_3 \leq \mu_2^2 \tag{46}$$

Which again is what we assumed about μ. Therefore the merged firm optimally sets $p_L = p_H = p$ where $p \in [\underline{p}_J, r_J]$ (conditional on the consumers having a reservation price strategy with reservation price r_J and the outside firm mixing according to $G_J(p)$).

Next we must show that the outside firm is indifferent over all prices in the interval. Profits for the outside firm, conditional on the merged firm choosing $p_L = p_H = p$ where $p = r_J$ with probability λ_J and $p = p$ according to $F_J(p)$ otherwise, are

$$\pi_{OM}(p) = \frac{1}{3}[\mu_1 + (2\mu_2+3\mu_3)\lambda_J + (1-\lambda_J)(2\mu_2+3\mu_3)(1-F(p))]p. \tag{47}$$

Plugging in λ_J from Equation 11 to get profits from playing $p = r$:

$$\pi_{OM}(r) = \frac{1}{3}[\mu_1 + (2\mu_2+3\mu_3)(\frac{\mu_1+\mu_2}{2\mu_1+3\mu_2+3\mu_3})]r_j. \tag{48}$$

Then profits from playing $p = \underline{p}_J$ are

$$\pi_{OM}(\underline{p}_J) = \frac{1}{3}[\mu_1 + 2\mu_2 + 3\mu_3]\frac{(2\mu_1+\mu_2)r_J}{2\mu_1+3\mu_2+3\mu_3} = \pi_{OM}(r). \tag{49}$$

Lastly, profits from playing any price in between can be found by plugging in $F_J(p)$ from Equation 12.

$$\pi_{OM}(p) = \frac{1}{3}[\mu_1+(2\mu_2+3\mu_3)\lambda_J+(2\mu_2+3\mu_3)(\frac{[\mu_1 + \lambda_J(2\mu_2 + 3\mu_3)](r_J - p)}{(2\mu_2 + 3\mu_3)p})]p = \pi_{OM}(r) \quad (50)$$

Therefore, the equilibrium holds from the firms' side. To see that this holds from the consumers' side as well, first note that consumers prefer to pull a price from G_J rather than any other possible mixture distribution of prices.

First consider the problem facing a consumer holding a single price of r_J as defined by Equation 10. This consumer knows that they drew this from the merged firm with certainty and therefore they are facing a mixture distribution of G_J and a degenerate distribution of r_J. This results in a higher average price than the average of G_J so they will not search. Prices above r_J do not occur in equilibrium so this is off-path. The consumer must believe that they will face the distribution G_J moving forward if they see a single price above r_J. Given the construction of r_J, this means they would search for any price above r_J.

Next consider a consumer holding two prices of r_J, this consumer believes these came from the merged firm with certainty, therefore they know they are facing G_J moving forward and won't search. Consumers who hold two prices above r_J believe they both came from the inside firm and therefore would search, thinking the price will come from G_J. The consumer therefore optimally plays a reservation price search strategy with r_J as the reservation price. This means that the joint pricing equilibrium as defined exists.

B.3 Propositions 3 - 6

The remaining propositions compare average transacted prices across equilibria. When the reservation price is fixed, this is straightforward. The distribution of transacted prices is a mixture distribution of the order distributions that each type of consumer faces. For example, in the symmetric equilibrium, the distribution of transacted prices is

$$T(p) = \mu_1 F(p) + \mu_2(1 - (1 - F(p))^2) + \mu_3(1 - (1 - F(p))^3). \quad (51)$$

Then, to calculate the average transacted price, one can take the derivative of this to find the density and integrate over the density times price. In this example, the average transacted price is

$$\bar{p} = \int_{\underline{p}}^{r} t(p) * p * dp = \mu_1 * r \quad (52)$$

This can be repeated for each of the four possible equilibria (symmetric, distinct pricing, joint pricing, consolidated brands). This gives us expressions for the average transacted price under each equilibrium in terms of the common, exogenous reservation price.

When the reservation price is endogenously determined by the consumers' search problem, we must first solve for the fixed point of the search indifference condition (Equations 3, 7,

Table 1: Mean transaction price

Equilibrium	Mean Price, Fixed r	Mean Price, Equilibrium r	Existence
Baseline	$\mu_1 r$	$\frac{\mu_1}{1-A\mu_1}$	Everywhere
Consolidated	$\frac{2(2-\mu_1)}{3-\mu_1}\mu_1 r$	$(\frac{2(2-\mu_1)}{3-\mu_1}\mu_1)(\frac{3(1-\mu_1)}{3(1-\mu_1)-2\mu_1\log(\frac{3-\mu_1}{2\mu_1})})$	Everywhere
Distinct	$(\mu_1+\frac{2}{3}\mu_2)r$	$(\mu_1+\frac{2}{3}\mu_2)(\frac{3(1-\mu_1)-2\mu_2}{3(1-\mu_1)-\mu_1\log(\frac{3-2\mu_1-\mu_2}{\mu_1+\mu_2})-2\mu_2-\mu_2\log(\frac{3-2\mu_1-\mu_2}{\mu_1+\mu_2})})$	$\mu_2^2 \leq 3\mu_1\mu_3$
Joint	$\frac{-6\mu_1^2+(6-\mu_2)\mu_2+\mu_1(12-5\mu_2)}{3(3-\mu_1)}r$	$(\frac{-6\mu_1^2+(6-\mu_2)\mu_2+\mu_1(12-5\mu_2)}{3(3-\mu_1)})(\frac{3*(1-\mu_1)-\mu_2}{3(1-\mu_1)-2\mu_1\log(\frac{3-\mu_1}{2\mu_1+\mu_2})-\mu_2-\mu_2\log(\frac{3-\mu_1}{2\mu_1+\mu_2})})$	$\mu_2^2 \geq 3\mu_1\mu_3$

Notes: A is given by equation 53.

10, and 19, respectively). This gives us a reservation price for each equilibrium. This can then be plugged into the average transacted price equation. The results of each of these calculations is listed in Table 1. Where:

$$A = \frac{\arctan[\frac{3-3\mu_1-2\mu_2}{\sqrt{3\mu_1(1-\mu_1-\mu_2)-\mu_2^2}}] - \arctan[\frac{\mu_2}{\sqrt{3\mu_1(1-\mu_1-\mu_2)-\mu_2^2}}]}{\sqrt{3\mu_1(1-\mu_1-\mu_2)-\mu_2^2}}. \tag{53}$$

Proposition 3 compares column 2, rows 1, 2, and 4. It is straightforward to show that

$$\mu_1 r \leq \frac{2(2-\mu_1)}{3-\mu_1}\mu_1 r \leq \frac{-6\mu_1^2+(6-\mu_2)\mu_2+\mu_1(12-5\mu_2)}{3(3-\mu_1)}r. \tag{54}$$

Therefore the proposition holds.

Proposition 4 is similar. The first part comes from

$$\mu_1 r \leq \frac{2(2-\mu_1)}{3-\mu_1}\mu_1 r, \tag{55}$$

and

$$\mu_1 r \leq (\mu_1 + \frac{2}{3}\mu_2)r. \tag{56}$$

The second part compares the distinct pricing equilibrium and shows that the distinct pricing equilibrium results in lower prices iff:

$$(\mu_1 + \frac{2}{3}\mu_2)r \leq \frac{2(2-\mu_1)}{3-\mu_1}\mu_1 r \iff \mu_2 \leq \frac{3\mu_1(1-\mu_1)}{2(3-\mu_1)} \tag{57}$$

Which is the statement.

Propositions 5 and 6 use column 3 of the table. In particular, $\bar{p}^* < \bar{p}_C^*$ iff equation (58) holds. Further, $\bar{p}_J^* > \bar{p}^*$ iff equation (59) holds and $\bar{p}_J^* > \bar{p}_C^*$ iff equation (60) holds. Both conditions are implied by the condition of Proposition 5 for a joint pricing equilibrium to exist ($\mu_2^2 \geq 3\mu_1\mu_3$). Finally, $\bar{p}_D^* < \bar{p}^*$ iff equation (A1) holds and $\bar{p}_D^* < \bar{p}_C^*$ iff equation (A2) holds. Both conditions are stricter than the condition of Proposition 6 for a distinct pricing equilibrium to exist ($\mu_2^2 \leq 3\mu_1\mu_3$). (In these equations, A is given by equation 53.)

$$\frac{\mu_1}{1-A\mu_1} < (\frac{2(2-\mu_1)}{3-\mu_1}\mu_1)(\frac{3(1-\mu_1)}{3(1-\mu_1)-2\mu_1\log(\frac{3-\mu_1}{2\mu_1})}) \tag{58}$$

$$(\frac{-6\mu_1^2 + (6-\mu_2)\mu_2 + \mu_1(12-5\mu_2)}{3(3-\mu_1)})(\frac{3*(1-\mu_1)-\mu_2}{3(1-\mu_1) - 2\mu_1 \log(\frac{3-\mu_1}{2\mu_1+\mu_2}) - \mu_2 - \mu_2 \log(\frac{3-\mu_1}{2\mu_1+\mu_2})} > \frac{\mu_1}{1-A\mu_1} \tag{59}$$

$$(\frac{-6\mu_1^2 + (6-\mu_2)\mu_2 + \mu_1(12-5\mu_2)}{3(3-\mu_1)})(\frac{3*(1-\mu_1)-\mu_2}{3(1-\mu_1) - 2\mu_1 \log(\frac{3-\mu_1}{2\mu_1+\mu_2}) - \mu_2 - \mu_2 \log(\frac{3-\mu_1}{2\mu_1+\mu_2})} > (\frac{2(2-\mu_1)}{3-\mu_1}\mu_1)(\frac{3(1-\mu_1)}{3(1-\mu_1) - 2\mu_1 \log(\frac{3-\mu_1}{2\mu_1})}) \tag{60}$$

$$(\mu_1 + \frac{2}{3}\mu_2)(\frac{3(1-\mu_1)-2\mu_2}{3(1-\mu_1) - \mu_1 \log(\frac{3-2\mu_1-\mu_2}{\mu_1+\mu_2}) - 2\mu_2 - \mu_2 \log(\frac{3-2\mu_1-\mu_2}{\mu_1+\mu_2})} < \frac{\mu_1}{1-A\mu_1} \tag{A1}$$

$$(\mu_1 + \frac{2}{3}\mu_2)(\frac{3(1-\mu_1)-2\mu_2}{3(1-\mu_1) - \mu_1 \log(\frac{3-2\mu_1-\mu_2}{\mu_1+\mu_2}) - 2\mu_2 - \mu_2 \log(\frac{3-2\mu_1-\mu_2}{\mu_1+\mu_2})} < (\frac{2(2-\mu_1)}{3-\mu_1}\mu_1)(\frac{3(1-\mu_1)}{3(1-\mu_1) - 2\mu_1 \log(\frac{3-\mu_1}{2\mu_1}} \tag{A2}$$

These expressions are complicated algebraically and are plotted using Mathematica in Figures 3 and 4. In particular, Figure 3 plots the separate regions for which equations 58, A1, A2, and $\mu_2^2 \leq 3\mu_1\mu_3$ hold. Figure 4 plots the regions for which no merger is best (equation 58 holds, and equation A1 fails), and for which a brand preserving merger is best (equations A1 and A2 hold). Code is available by request.

B.4 Proposition 7

First, it is worth noting that the reservation price for the baseline two firm symmetric equilibrium is

$$r = \frac{2(1-\mu_1)}{2(1-\mu_1) - \mu_1 * \log(\frac{2-\mu_1}{\mu_1})} \tag{61}$$

Then average transacted prices in the baseline are $\mu_1 r$. When $\mu^2 \geq 3\mu_1\mu_3$ and the joint pricing equilibrium is played following symmetric brand proliferation, then average transacted prices are found in Table 1 row 4. Prices are lower in the baseline than following proliferation to the joint pricing equilibrium if

$$\mu_1 \frac{2(1-\mu_1)}{2(1-\mu_1) - \mu_1 * \log(\frac{2-\mu_1}{\mu_1})} \leq (\frac{-6\mu_1^2 + (6-\mu_2)\mu_2 + \mu_1(12-5\mu_2)}{3(3-\mu_1)})(\frac{3*(1-\mu_1)-\mu_2}{3(1-\mu_1) - 2\mu_1 \log(\frac{3-\mu_1}{2\mu_1+\mu_2}) - \mu_2 - \mu} \tag{62}$$

Which is always true. When $\mu^2 \leq 3\mu_1\mu_3$ and the distinct pricing equilibrium holds following symmetric brand proliferation, then average transacted prices are found in Table 1 row 3. Then prices are higher in the baseline than following symmetric brand proliferation iff

$$\mu_1 \frac{2(1-\mu_1)}{2(1-\mu_1) - \mu_1 * \log(\frac{2-\mu_1}{\mu_1})} > (\mu_1 + \frac{2}{3}\mu_2)(\frac{3(1-\mu_1)-2\mu_2}{3(1-\mu_1) - \mu_1 \log(\frac{3-2\mu_1-\mu_2}{\mu_1+\mu_2}) - 2\mu_2 - \mu_2 \log(\frac{3-2\mu_1-\mu_2}{\mu_1+\mu_2})}) \tag{A3}$$

This is plotted in Figure B.1. This covers all cases of μ for symmetric brand proliferation with consumers who update their reservation prices to the equilibrium value.

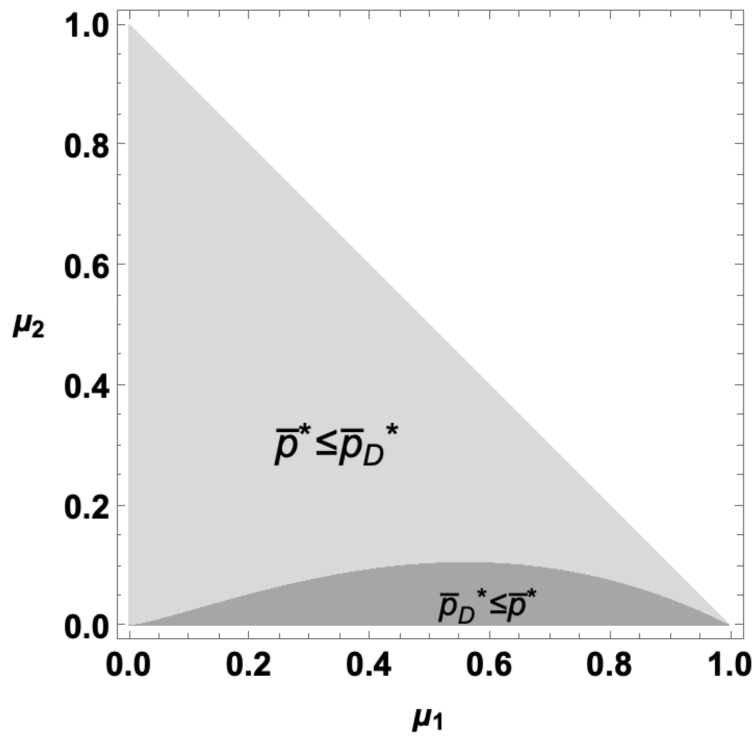

Figure B.1: **Ranking equilibrium transaction prices for symmetric brand proliferation:** Parameter μ_k is the fraction of consumers who receive k initial price quotes, where $\mu_3 = 1 - \mu_1 - \mu_2$. Possible values for the vector μ lie in the triangular region. We compare average transacted prices between a symmetric 2 firm equilibrium ($\bar{p}^*$) and prices following a symmetric brand proliferation ($\bar{p}_D^*$). Here we assume that consumers are sophisticated and update their reservation prices following brand proliferation.

B.5 Proposition 8

Given an exogenous reservation price, Armstrong and Vickers (2024) characterize equilibrium under obscure brand proliferation (a special case of their Proposition 2 discussed in their Section 5.1). The following lemma follows directly from that work.

Lemma 1 *For all μ, and a fixed reservation price r_O, following obscure brand proliferation, a pricing equilibrium exists. The single-brand firm sets its price in the interval $[\underline{p}_O, r_O]$, according to distribution G_O. Price $p^\dagger \in (\underline{p}, v)$ divides this interval into lower and upper regions. The two-brand firm sets the price of its new brand in the lower region $[\underline{p}, p^\dagger]$, according to distribution F_N. It sets the price of its existing brand in the upper region $[p_O^\dagger, r_O]$, pricing at the top, $p = v$, with probability γ_O and mixing over the interval according to distribution F_O with complement probability. Equations (20)–(26) in Appendix A.1 characterize these price distributions.*

When consumers' reservation price r_O is endogenous, it is characterized by equation (27). This equation is an indifference condition that ensures consumers are exactly indifferent between paying a price of r_O and receiving an additional draw from the distribution F_N.

A consumer who has seen two price quotes of r_O knows for certain that they came from the outside brand and the multi-brand firm's old brand. This consumer would know with certainty that they are drawing from F_N next. Any consumer who is holding at least one lower price would strictly prefer not to search because a) they are paying a lower price and b) their beliefs about the distribution of prices they are drawing from can only be worse than F_N. Any consumer with only a single price at or below r would similarly not prefer to search because they face a convex combination of F_N and a distribution of higher prices. This means that no consumer who is holding a price at or below r_O will ever choose to search. Prices above r_O do not occur in equilibrium, but off-path beliefs are such that consumers believe they will draw from F_N if they search. Therefore, if they did see a price above r_O, they would prefer to search. This constitutes a reservation price strategy with r_O as the reservation price. Hence, this constitutes a Perfect Bayesian Nash Equilibrium.

B.6 Proposition 9

Baseline profits are again $\mu_1 r$ where r is defined in Equation 61. Profits for a given reservation price following obscure brand proliferation are given by

$$\bar{p}_O = (\mu_1 + \mu_2\gamma + \mu_3(\frac{\mu_1 + 2\mu_2\gamma}{\mu_1 + 2\mu_2 + 2\mu_3}))r \geq \mu_1 r.$$

The reservation price following obscure brand proliferation is given by

$$r_O = \frac{1}{1 - \frac{(\mu_1+2\mu_2)}{2\mu_3}(\frac{\mu_1}{\mu_1+2\mu_2(1-\lambda_O)})\log(\frac{1}{1-\lambda_O})}. \tag{63}$$

Therefore prices are higher in the baseline than following obscure brand proliferation iff

$$\mu_1 \frac{2(1-\mu_1)}{2(1-\mu_1) - \mu_1 * \log(\frac{2-\mu_1}{\mu_1})} > (\mu_1 + \mu_2\gamma_O + \mu_3(\frac{\mu_1 + 2\mu_2\gamma_O}{\mu_1 + 2\mu_2 + 2\mu_3}))(\frac{1}{1 - \frac{(\mu_1+2\mu_2)}{2\mu_3}(\frac{\mu_1}{\mu_1+2\mu_2(1-\lambda_O)})\log(\frac{1}{1-\lambda_O})}) \tag{A4}$$

Average transacted prices are higher in the obscure brand proliferation case than in the symmetric brand proliferation case iff

$$\begin{aligned} &(\mu_1 + \frac{2}{3}\mu_2)(\frac{3(1-\mu_1) - 2\mu_2}{3(1-\mu_1) - \mu_1\log(\frac{3-2\mu_1-\mu_2}{\mu_1+\mu_2}) - 2\mu_2 - \mu_2\log(\frac{3-2\mu_1-\mu_2}{\mu_1+\mu_2})}) \\ &> (\mu_1 + \mu_2\gamma_O + \mu_3(\frac{\mu_1 + 2\mu_2\gamma_O}{\mu_1 + 2\mu_2 + 2\mu_3}))(\frac{1}{1 - \frac{(\mu_1+2\mu_2)}{2\mu_3}(\frac{\mu_1}{\mu_1+2\mu_2(1-\lambda_O)})\log(\frac{1}{1-\lambda_O})}) \end{aligned} \tag{A5}$$

Both of these are plotted in Figure B.2. This covers all cases of μ for obscure brand proliferation with consumers who update their reservation prices to the equilibrium value.

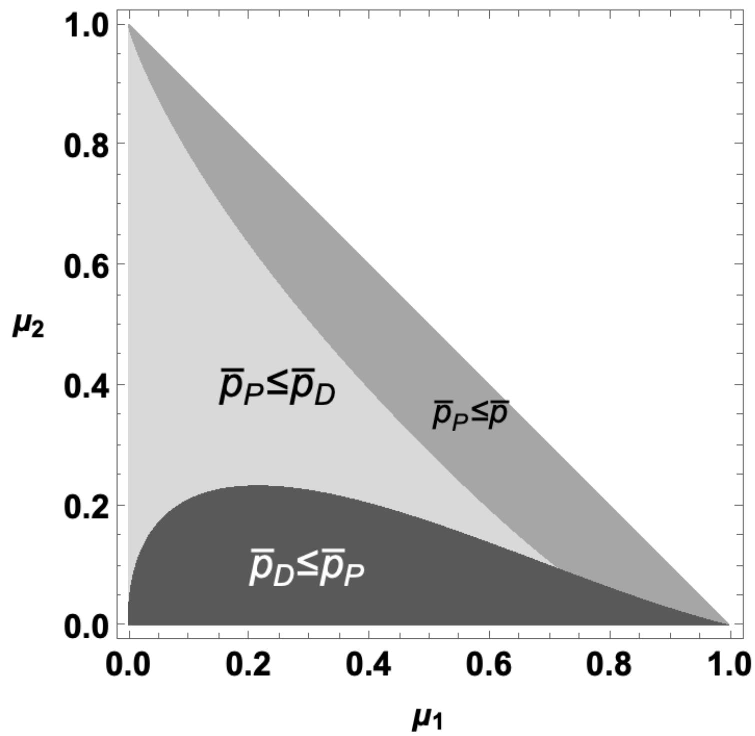

Figure B.2: **Ranking equilibrium transaction prices for obscure brand proliferation:** Parameter μ_k is the fraction of consumers who receive k initial price quotes, where $\mu_3 = 1 - \mu_1 - \mu_2$. Possible values for the vector μ lie in the triangular region. We compare average transacted prices between a symmetric 2 firm equilibrium ($\bar{p}^*$), prices following an obscure brand proliferation ($\bar{p}^*_O$), and prices following a symmetric brand proliferation ($\bar{p}^*_D$). Here we assume that consumers are sophisticated and update their reservation prices following brand proliferation.

B.7 Proposition 10

We will prove existence by construction. Consider a market in which $\mu_K = 1$. In the consolidated equilibrium, all consumers receive $\min\{K, N - K + 1\} \geq 2$ prices, each from an independent firm. In this equilibrium, firms (including the consolidated firm) cannot profitably deviate from marginal cost pricing, $p = 0$. If they play a price greater than 0, they will not sell to any consumers as all consumers have an offer from a firm offering a price of 0.

In the multi-brand case, the merged firm chooses prices for each of its K brands. A fraction of the consumers will receive their K price quotes all from the merged firms' brands. These consumers are captive to the merged firm, and given a non-zero search cost, allow the firm to earn positive profits by charging a price of $r > 0$ at each of these brands. Given this possibility, the firm must earn positive profits in equilibrium. Therefore average transacted prices must be greater than 0, and thus greater than in the consolidated equilibrium.

B.8 Proposition 11

We will, again, prove existence by construction. Consider a market in which $\mu_N = \mu$ and $\mu_1 = 1 - \mu$ and $K = 2$.

First, consider the equilibrium in the multi-brand case. Without loss of generality, order the prices offered by the multi-brand firm's brands such that $p_L \leq p_H$. This firm can only win shoppers with price p_L. Every shopper is considering all N brands (including the 2 offered by the multi-brand firm) and will purchase at the lowest of these prices. By this naming convention, it is not possible for brand 2 to transact with shoppers. This means that the only consumers that brand 2 could possibly interact with are non-shoppers who are only considering one brand. The multi-brand firm therefore maximizes profits by setting $p_H = r$.

Conditional on setting price 2 to r, brand 1 of the multi-brand firm and each of the outside firms are symmetric. They each face the following profit function:

$$\pi_M(p) = (\frac{1}{N}(1-\mu) + \mu(1 - F_M(p))^{N-2}) * p \tag{64}$$

Solving for the equilibrium distribution in the usual manner (setting profits equal to profits from playing r (or $\pi(r) = \frac{1}{N}(1-\mu)r$):

$$F_M(p) = 1 - [\frac{(1-\mu)}{N\mu}\frac{r-p}{p}]^{\frac{1}{N-2}} \tag{65}$$

As this is a market with perfectly inelastic demand and no extensive margin, consumer surplus and total firm profits add to a constant. Therefore, if we demonstrate that firm profits are higher (lower) in one equilibrium than another, we have also demonstrated that consumer surplus is lower (higher). Total firm profits in this equilibrium are:

$$\Pi_M = \frac{(1-\mu)r}{N} + (N-1) * \frac{(1-\mu)r}{N} = (1-\mu)r \tag{66}$$

Average price is then just $r - \Pi$ or μr. The same as in the distinct price equilibrium with 3 firms (and this search technology).

Next consider the equilibrium for the consolidated-brand case. Here the profit function for the merged firm is:

$$\pi_{JC}(p) = (\frac{2}{N}(1-\mu) + \mu(1 - F_{OC}(p))^{N-2})p \tag{67}$$

From our typical equilibrium condition this firm will earn profits of $\pi(r) = \frac{2}{N}(1-\mu)p$. In order to induce the outside brands to mix over the same interval of prices, the merged firm needs to choose a price of r with positive probability λ_C. The outside firm(s)' profit function is then:

$$\pi_{OC}(p) = \frac{1}{N}(1-\mu) + \mu(\lambda_C + (1-\lambda_C)(1 - F_{JC}(p)))(1 - F_{OC}(p))^{N-3} * p \tag{68}$$

As per usual the outside firms will be indifferent over all prices so their profits are equal to their profits from playing r which are:

$$\pi_{OC}(r) = \frac{(1-\mu)r}{N} \tag{69}$$

Therefore total firm profits in this market are:

$$\Pi_C = (1-\mu)r \tag{70}$$

Therefore, when r is fixed and $K = 2$, profits and thus average transacted prices are the same in the multi-brand and consolidated equilibria.

When r is determined endogenously by non-shopper consumers' indifference to search upon seeing a price of r, we need to consider the non-shopper consumers' beliefs upon seeing a price of r. In both equilibria, seeing a price of r, causes the consumer to know with certainty that this price came from the merged firm.

In the consolidated equilibrium, this means that they are facing the distribution $F_{OC}(p)$ should they search. In the brand-preserving equilibrium this means that they are facing the distribution $F_M(p)$. In both cases, all remaining firms are identical and play the same distribution. To compare the reservation prices consider the distribution F_{OC}. Setting profits equal to the profits from playing r_C in the case of $K = 2$:

$$\frac{1}{N}(1-\mu) + \mu(\lambda_C + (1-\lambda_C)(1-F_{JC}(p)))(1-F_{OC}(p))^{N-3} * p = \frac{1}{N}(1-\mu)r_M \tag{71}$$

If $\lambda_C = 0$ and $F_{JC}(p) = F_{OC}(p)$ for all p, then this would be the same condition as in the multi-brand case, and the distribution F_OC would be equal to the distribution F_M (conditional on a reservation price). However, the distribution played by the consolidated firm is higher than that of the outside firms in the multi-brand case, therefore F_{OC} has a higher mean than F_M. Therefore, for $K = 2$, $r_M < r_C$ and overall average transacted prices are lower for the multi-brand equilibrium than the consolidated equilibrium.