The Illusion of Competition*

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Abstract

In markets where consumers must search sequentially for prices, whether a firm sells its product under one brand name or two distinct brands is consequential for market outcomes. If each firm sells a single brand then any consumer who receives multiple price quotes places firms in competition with each other. However, a two-brand firm can exhaust the search capacity of a consumer who searches for exactly two price quotes, making them "captive" to that firm. Moreover, if consumers are under an "illusion of competition" in which they believe all brands to be independent competitors, then a two-brand firm can lower consumers' estimates of price dispersion, discouraging them from searching further, by setting identical prices. We extend canonical search models to show that multi-brand firms can raise equilibrium prices by both mechanisms. As a result, breaking the illusion of competition by advertising brand ownership may lower prices. Alternatively, requiring two merging firms to consolidate their brands rather than operate them separately or curtailing brand proliferation by limiting the visibility of such duplicate brands on online platforms can intensify price competition and benefit consumers. In some cases, however, such policies may be counterproductive.

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1 Introduction

Consumer search over prices is crucial for understanding equilibrium outcomes in many markets, and hence, its implications are vital for regulators and antitrust authorities when evaluating mergers and competition policies. In many real-world contexts, consumers do not know the prices offered by every firm in the market. Instead, because they must undertake costly search to discover prices, consumers typically choose from a subset of options, and these consideration sets vary across consumers. This behavior helps explain price dispersion and markups in markets ranging from trash collection (Salz, 2022) to mortgages (Allen, Clark, and Houde, 2019). Importantly, this should be accounted for when predicting the effects of mergers and competition policies in such *search markets*.

In search markets, it is important to distinguish between two types of mergers that are both common in practice. First, there are brand-consolidating mergers, in which merging firms combine and sell under a single name. For instance, when USAir and American Airlines merged, they kept only the American Airlines name. Second, there are brand-preserving mergers, in which the merged firm continues to sell using both pre-merger brand names. For instance, this happened when Uber, the parent company of Uber Eats, acquired the competing food-delivery service Postmates. Uber continues to operate Uber Eats and Postmates as separate brands with their own websites. All else equal, the two types of mergers lead to different outcomes because they generate different numbers of prices for consumers to search over and a different pricing problem for the merged firm.

Moreover, in the case of a brand-preserving merger, it matters whether or not consumers are *aware* that distinct brands such as Uber Eats and Postmates are jointly owned. Sophisticated consumers will be aware of the market structure, but consumers who suffer from the "illusion of competition" will assume that prices for distinct brands are set by independent firms. Consumers' optimal search strategy depends on the distribution of prices, which in turn depends on whether brands are jointly owned. Hence the illusion of competition can lead to suboptimal search, which can reduce competition and raise prices.

We build upon Burdett and Judd's (1983) and Stahl's (1989) canonical models of endogenous consumer search by allowing for a firm to own two brands. Our extensions allow us to model the effects of brand-preserving mergers, compare consumer welfare between brand-preserving and brand-consolidating mergers, and evaluate novel competitions policies including conditioning merger approval either on a requirement for brand consolidation or for parent-company co-branding (to dispel the illusion of competition).

We identify two key reasons why the presence of multi-brand firms can lead to higher prices and, hence, a rationale for novel competition policy. First, there is a *search capture* mechanism because a multi-brand firm can "use up" more of a consumer's search time than a single-brand firm can. Second, there is a *search discouragement* mechanism because a multi-brand firm can depress consumers' beliefs about price dispersion and returns to search by setting similar prices across its brands.

The intuition for the search capture mechanism is that when a consumer who has sufficient time to learn two prices searches in a market with single-brand firms, they will always compare quotes from competing firms. However, with multi-brand firms, both price quotes could come from the same parent company, effectively making them captive to that parent company. This can lower the cross-price elasticity between competing firms and raise prices.

In some cases, informing consumers about the brand ownership structure in a market can reduce this problem, because consumers have an incentive to direct their second searches away from the parent company already searched. Similarly, brand consolidation can lower prices because a second search must be directed to a different parent company when all firms sell only one brand.

The intuition for the search discouragement mechanism is that consumers may be learning about the distribution of prices as they search. If consumers are naively unaware that brands are jointly owned, a multi-brand firm can cause consumers to underestimate price dispersion and returns to search by setting identical prices across brands. This can cause consumers to suboptimally stop and buy from a multi-brand firm without searching further, leading to higher equilibrium prices than with sophisticated consumers or with single-brand firms. Either proposed policy—brand consolidation or information that dispels the illusion of competition—can eliminate this problem and substantially lower prices.

To investigate search capture and search discouragement, we make two different extensions of Stahl's (1989) canonical search model (for the case of unit demand up to product value v). In Stahl's (1989), consumers shop sequentially for a homogeneous good and are either shoppers who observe all prices or non-shoppers who must pay s for each additional quote after their first. To investigate search capture, we introduce consumers who fall between these two extreme types, observing k prices for free before paying cost s for additional quotes. This captures the idea that consumers may have sufficient free time to get k quotes before the opportunity cost of their time rises to s per quote.[1] We focus on the case of three symmetric single-brand firms in which two firms complete a brand-preserving or a brand-consolidating merger and include some additional results for markets with more firms.

To investigate search discouragement, we maintain Stahl's (1989) assumptions about consumer search, but introduce an initial move by nature that affects firms. With probability α_0, nature chooses a "collusive state" in which firms all collude on the monopoly price with certainty. With complementary probability, nature chooses a "competitive state" in which firms choose prices to maximize individual profits. (While we do not model a repeated game, one interpretation of this assumption is that the game is repeated and nature either chooses firms' discount factors to be high, leading to collusion at the monopoly price, or low, leading to maximization of static profits.) Nature's choice is common knowledge to firms but unknown to consumers, who learn about the state as they observe prices. In particular, when consumers observe a price below the monopoly price, they know the state is competitive. However, a multi-brand firm that sets all its prices at the monopoly level could potentially convince a consumer that the state is sufficiently likely to be collusive with no price dispersion that additional search is not worthwhile. We focus on the case of four single-brand firms that complete brand-preserving mergers to become two firms each with two brands, or brand-consolidating mergers to become two single-brand firms.

To develop our results about the effects of search capture, we characterize that model's equilibrium prices under three market structures: (1) three symmetric single-brand firms (pre-merger case); (2) two asymmetric single-brand firms (brand-consolidating merger case);

[1] This is very similar to Burdett and Judd's (1983) "newspaper search" model with Hämäläinen's (2022) interpretation. It differs because our consumers can keep searching one quote at a time at cost s after seeing k prices for free.

and (3) a duopoly in which one firm owns two distinct brands and the other firm owns a single brand (brand-preserving merger case). In the two post-merger market structures, we allow consumers to be sophisticated and aware that a merger has taken place, or to suffer from the illusion of competition and be unaware of the merger. In all cases, we hold fixed the number of initial price quotes consumers are endowed with.[2]

Equilibrium under the first two market structures closely follows existing work. Consumers stop searching once they find a price less than or equal to an endogenous reservation price. Firms randomize over an interval of prices that extends up to consumers' reservation price. Hence, in equilibrium, there is no additional search beyond the price quotes consumers are randomly endowed with. Equilibrium characterization follows Burdett and Judd (1983) for the symmetric case and is similar to Armstrong and Vickers's (2022) characterization for the asymmetric case, except that consumers' reservation prices are endogenous rather than exogenous.

Equilibrium under the third market structure, with joint brand ownership, is novel. We show that there are two different types of equilibrium, depending on the relative fraction of consumers who are endowed with one, two, or three price quotes. (We refer to those endowed with only one price quote as "captive" to a single firm.)

If sufficiently many consumers are either captive to a single firm or consider all of the firms in the market, then there is a "distinct pricing equilibrium" in which the two co-owned brands have distinct prices. The first co-owned brand is priced at the consumers' reservation price (the top of the price distribution), earning high profits on captive consumers and avoiding competing with the second co-owned brand. The second co-owned brand is priced randomly over an interval of lower prices and is in direct competition with the outside brand for the non-captive consumers. Alternatively, if sufficiently many consumers are endowed with exactly two price quotes, then there is a "joint pricing equilibrium" in which the two co-owned brands have the same price. The shared price of the two co-owned brands is randomly drawn over the same interval of prices as the outside brand. This pricing strategy capitalizes on the fact that a large fraction of consumers will only see the prices of the jointly owned brands and are, in a sense, captive to the merged firm. These consumers have their two searches "used up" by the same firm.

Next, we compare consumer welfare across the three studied market structures. (Average transaction price is a sufficient statistic for consumer welfare in this setting.) We consider three levels of consumer sophistication: (1) the illusion of competition (consumers are unaware of the merger); (2) partial sophistication (consumers are aware of the merger but do not know which brands are co-owned; and (3) full sophistication (consumers know which brands are co-owned).

Advertising that dispels the illusion of competition by making consumers aware of joint ownership of brands can reduce consumer harm from mergers.[3] It does so when there are

[2]In our framework, a consumer type corresponds to a number of initial price quotes or searches, whereas in Armstrong and Vickers (2022) a consumer type corresponds to a consideration set. To compare our approaches, consider a consumer who would have searched brands A and B in the absence of a merger. If there is a merger to a consolidated brand AB, Armstrong and Vickers (2022) preserves the consideration set, and the consumer only observes price AB. In contrast, we preserve two initial price quotes so the consumer observes price AB and a competing price.

[3]It is possible that making consumers aware of a merger would lower prices such that a merger would

sufficiently few shoppers who observe all prices relative to lower search intensity consumers. However, when there are sufficiently many shoppers, we find that the opposite can also be true. The illusion of competition can make consumers underestimate the distribution of market prices, inducing them to choose a low reservation price, and thereby leading firms to set lower prices. The effect of information about the market structure on prices therefore depends on the search technology and the nature of the merger.

Conditional on allowing a merger, requiring brand consolidation helps consumers whenever there are sufficiently many consumers who are randomly endowed with exactly two price quotes. Given an illusion of competition, these are the consumers who might observe two prices from the same firm but mistakenly think they have observed competitive price quotes. Nevertheless, the result does not depend on the illusion of competition and is qualitatively the same when consumers are partially or fully sophisticated.

To study the effects of search discouragement we examine equilibrium pricing in our model with the possibility of collusion under three market structures: (1) four symmetric single-brand firms (pre-merger case); (2) two symmetric single-brand firms (brand-consolidating merger case); and (3) a duopoly in which both firms owns two distinct brands (brand-preserving merger case). Here we study the brand-preserving merger both with and without the illusion of competition.

In the four-firm pre-merger case, we provide a novel characterization of equilibrium pricing strategies and consumer search patterns. Once consumers see any price below the monopoly level, they know the market is competitive and continue searching until finding a price below a reservation price that is less than the monopoly price. However, if a consumer sees a sequence of prices all equal to the monopoly price, they begin to think the collusive state is increasingly likely. Eventually, search is no longer worthwhile and they buy at the monopoly price. In the competitive state, independent firms take advantage of this by pricing at the monopoly level with positive probability.

We show that as initial beliefs about the probability of the collusive state (α_0) go to zero, such an equilibrium exists and firm pricing strategies and consumer welfare converge to the Stahl (1989) equilibrium. Importantly, as α_0 goes to zero, observing two monopoly prices becomes a sufficiently strong signal of the collusive state to discourage consumers from searching further and buy at the monopoly price. As a result, following a brand-preserving merger, consumers under the illusion of competition can be captured by a multi-brand firm that sets both of its prices at the monopoly level. This places a lower bound on post-merger profits that implies large increases in prices following a brand preserving merger with the illusion of competition (when the probability of the collusive state is small). In this case, brand consolidation strictly benefits consumers. Moreover, informing consumers about the merger to dispel the illusion of competition is even more effective at lowering prices than brand consolidation.

This paper relates to a large theoretical literature studying how endogenous and costly consumer search affects equilibrium market prices. Early papers establish the frameworks that we build upon in this paper (Burdett and Judd, 1983; Stahl, 1989). A second branch of the literature, begun by Varian (1980), studies models in which consumer search patterns

become unprofitable. In these cases, the threat of such an information remedy may deter certain price increasing mergers from being proposed in the first place.

and consideration sets are exogenous rather than endogenous. Armstrong and Vickers (2022) generalizes these models to a wide class of possible competitive interactions and uses this framework to study the impact of entry/exit and mergers in which the merging brands are consolidated.

Ireland (2007) is the first paper in this literature to extend a model of equilibrium pricing with consumer search to allow for joint ownership of independent brands by the same firm, a common occurrence in real-world markets. Under the assumption that firms are symmetric and consumers check at most two prices, Ireland (2007) shows that brand-preserving mergers always lead to equal pricing for co-owned brands and that brand consolidation benefits consumers. Our work and that by Armstrong and Vickers (2024) both relax the assumption that consumers check at most two prices. As a result, we find alternative equilibria with distinct pricing by co-owned brands and cases in which brand consolidation harms consumers. This paper does so in the context of symmetric but endogenous consumer search, while Armstrong and Vickers (2024) does so while allowing for general asymmetric search patterns but restricting them to be exogenous. Hämäläinen (2022) studies multi-brand pricing when consumers can steer their initial searches but do not search sequentially after hitting a deadline and thus do not have endogenous reservation prices.

Armstrong and Vickers's (2024) allowance for asymmetry leads to a third type of pricing equilibrium that does not arise in our model. Our search capture results applied to mergers were developed independently from and concurrently to Armstrong and Vickers's (2024), while our brand proliferation extension (Appendix A.2) that allows for asymmetric firms builds on their work. The fact that we allow consumer search and reservation prices to be endogenous sometimes reverses results. Moreover, it allows us to compare market outcomes with sophisticated consumers who are aware of mergers to those suffering from the illusion of competition. Our results on search discouragement are completely novel to this paper.

Our search discouragement model is closely related to concurrent work by Heggedal, Moen, and Knutsen (2024). They independently introduce the same model of search with learning about a collusive state for the special case of two single-brand firms. Their laboratory experiments show that equilibrium model predictions are borne out in real behavior, demonstrating the model's usefulness. They also provide a useful summary of other related work on learning in search markets.

2 Search Capture Model

Three symmetric firms with marginal cost c sell a homogeneous product to a unit mass of homogeneous consumers. Consumers have unit demand for the good at a valuation of v. Their utility is their value of the good less the price paid if they buy and zero otherwise. Consumers search for prices sequentially. Fraction μ_k of consumers observe k prices for free before paying cost $s > 0$ for additional quotes. This captures the idea that consumers may have sufficient free time to get k quotes before the opportunity cost of their time rises to s per quote.[4] The search technology is characterized by the vector $\mu = (\mu_1, \mu_2, \mu_3)$ such that $\mu_1 + \mu_2 + \mu_3 = 1$. Each firm (or brand) has an equal probability of being selected by each search. For the special case of $\mu_2 =$ this corresponds to Stahl (1989).

[4]See footnote 1.

It is without loss of generality to consider the case in which $c = 0$ and $s = 1$ (see the proof of Proposition 1 in Westphal (2023)). Other values of production cost and search cost merely shift and scale equilibrium price distributions, respectively. We therefore only reference the more tractable case for the remainder of the paper.

Throughout Section 2, we will use an equilibrium concept of the "reservation price equilibrium" or RPE. This concept is commonly used in the consumer search literature and is a subset of Perfect Bayesian Nash Equilibria. In an RPE, consumers have a reservation price, r. As they search, if they are offered one or more prices at or below r, they purchase the good at the lowest price they have been offered. If they are only offered prices above r, they continue to search. The reservation price r is a best response to the distribution of prices offered by firms. In this type of equilibrium, firms play a mixed strategy and select prices according to an equilibrium distribution. Every price in the support of this distribution is a best response to the consumers' search strategy above and to the distribution of prices played by other firms.

2.1 Pre-merger Baseline: Symmetric Three Firm Case

We begin by characterizing the equilibrium with three competing single-brand firms. In this case, conditional on a RPE existing with reservation price r, each firm's profit function is:

$$\pi(p) = \begin{cases} \sum_{j=1}^{3} \frac{j\mu_j}{3}(1 - F(p))^{j-1}(p - c) & p \leq r \\ 0 & p > r \end{cases}, \tag{1}$$

where $F(p)$ is the equilibrium distribution of prices played by each firm. Throughout Section 2, firms do not set prices above r, so we omit references to this case moving forward.

citetvar80 notes that firms must be indifferent between all prices in the support of the equilibrium price distribution, thus the firms' side of the equilibrium can be solved by setting profits equal to the profits from choosing a price of r. Burdett and Judd (1983) equation (7) gives a polynomial that can be solved for the equilibrium price distribution, F.

The consumers' reservation price r is then the minimum of their valuation of the good, v, and the value implicitly defined by the search indifference condition below.

$$r = 1 + \int_{\underline{p}}^{r} pf(p)dp, \tag{2}$$

where the lowercase of a distribution refers to its corresponding density. Burdett and Judd (1983) show that this equilibrium exists and Johnen and Ronayne (2021) prove that this equilibrium is unique if $\mu_2 > 0$. This "pre-merger" equilibrium will serve as a benchmark to compare to cases with jointly owned brands.

2.2 Brand-Preserving Merger

Next, we consider a merger between two of the three firms, following which the merged firm continues to operate the two brands separately. Here, we assume that the brands continue to be anonymous to the consumer, and the consumer cannot direct their search.

The merged firm's profits are a function of the prices chosen for each of its brands. Without loss of generality, denote p_L as the minimum of the two prices and p_H as the maximum. Then expected profits are

$$\begin{aligned}\pi_{JM}(p_L, p_H) = [\frac{1}{3}\mu_1 + \frac{1}{3}\mu_2 + \frac{1}{3}\mu_2(1 - G(p_L)) + \mu_3(1 - G(p_L))]p_L \\ +[\frac{1}{3}\mu_1 + \frac{1}{3}\mu_2(1 - G(p_H))]p_H,\end{aligned} \tag{3}$$

where $G(p)$ is the equilibrium price distribution for the outside firm. The merged firm's strategy depends on whether this function is increasing or decreasing in p_H. If profits are increasing in p_H, holding p_L fixed, then the firm optimally always sets p_H to r, giving us the distinct pricing equilibrium. If profits are decreasing in p_H, the two-brand firm sets $p_H = p_L$, giving us the joint pricing equilibrium. Thus, the condition for distinct pricing (rather than joint pricing) to be a best response to the outside firm's pricing distribution G is

$$\frac{\partial \pi_{JM}(p_L, p_H)}{\partial p_H} = \frac{1}{3}\mu_1 + \frac{1}{3}\mu_2(1 - G(p_H)) - \frac{1}{3}\mu_2 g(p_H)p_H \geq 0. \tag{4}$$

We will show that at the equilibrium values for G, this condition reduces to $\mu_2^2 \leq 3\mu_1\mu_3$, a simple expression (illustrated in Figure 1) that distinguishes between distinct pricing and joint pricing equilibria based only on the search technology μ.

2.2.1 Distinct Pricing Equilibrium

If the share of consumers who see exactly two prices is sufficiently low, then the two-brand firm has little incentive to compete using *both* of its brands. In this setting, when setting its higher price, p_H, the firm will typically be either facing a consumer with one or three price quotes. If p_H is the only price the consumer will see, the firm should set p_H to the reservation price and maximize its markup. If the consumer sees all three prices, then the firm is already showing the consumer a lower price, p_L, and cannot win the consumer with p_H. By this logic, if μ_2 is sufficiently low, the firm should set two distinct prices, one high monopoly price, and one lower competitive price.

Proposition 1 *If $\mu_2^2 \leq 3\mu_1\mu_3$, then a distinct pricing equilibrium exists*[5]*. The two-brand firm sets one of its brands' prices to the consumers' reservation price, r_D. The two-brand firms' other brand and the outside firm both choose prices on the interval $[\underline{p}_D, r_D]$ according to the distribution F_D. The consumers reservation price is set based on indifference towards an additional draw from F_D. Where $\underline{p}_D$, r_D, and F_D are described by Equations A1 - A3 in Appendix B.1.*

Note that if the consumers' reservation price is set by their indifference towards search, then r_D is pinned down by the distribution F_D, not the overall distribution of prices. The reservation is set accordingly because if the consumer sees a price at or above r_D, they believe that the price must have come from the higher-priced brand of the two-brand firm. Then the remaining prices they could receive must be from the lower pricing brand or the outside firm, both of which set prices according to F_D.

[5]Proofs of all propositions can be found in the appendix.

2.2.2 Joint Pricing Equilibrium

Next, consider the alternative case in which the share of consumers considering exactly two prices is high. One third of these consumers only observe the outside firm's price and the two-brand firm's high price. Hence the two-brand firm cannot rely solely on its low price to win business from the outside firm, but rather wants to set both prices competitively. Another third of these consumers only observe the two-brand firm's prices. These additional "captive" consumers considerably soften price competition. They also make it optimal to price the jointly owned brands identically ($p_L = p_H$) so the two-brand firm avoids undercutting its own prices with these consumers.

Proposition 2 *If $\mu_2^2 \geq 3\mu_1\mu_3$, then a joint pricing equilibrium exists. The two-brand firm sets both of its prices to the same value, $p_L = p_H = p$, chosen from the interval $[\underline{p}_J, r_J]$. With probability λ_J it sets $p = r_J$ and with complement probability, it sets p according to distribution F_J. The outside firm sets its price on the same interval according to distribution G_J. The consumers reservation price is set based on indifference towards an additional draw from G_J. Where $\underline{p}_J$, r_J, λ_J, F_J and G_J are described by Equations A4 - A8 in Appendix B.2.*

Here, consumers set their reservation price according to the outside firms' price distribution, G_J. If a consumer has two price quotes of r_J, they will believe that both of these prices came from the merged firm and that they are facing G_J if they search again, making them indifferent between searching and not. If a two-price consumer receives either one or two prices below r_J, they will believe that they are facing some convex combination of G_J and a degenerate distribution of the price they have already been quoted with certainty. These distributions all first-order stochastically dominate G_J, so the consumer will not search. We will define off-path beliefs such that if a consumer sees a price above r_J, they believe they will face G_J next. This makes beliefs continuous at r_J for consumers with two prices. With these off-path beliefs, consumers with two price quotes above r_J will search. A consumer with a single price quote of r_J or lower will also not search. They believe that they are facing a distribution that is a convex combination of G_J and a degenerate distribution of the price they quoted. Therefore they have negative returns to search and will not search. A consumer with one price above r_J will search because their off-path beliefs are that they are facing G_J moving forward[6].

Figure 1 shows the (weakly) complementary regions of the parameter space for which the distinct pricing equilibrium and joint pricing equilibrium exist. (Along the boundary $\mu_2^2 = 3\mu_1\mu_3$, both equilibria exist.) Note that Ireland (2007) Lemma 3 characterizes the joint pricing equilibrium for an exogenous reservation price and the special case in which consumers check at most two prices ($\mu_3 = 0$) which corresponds to the diagonal upper bound of the parameter space in Figure 1. The distinct pricing equilibrium only arises for $\mu_3 > 0$.

[6]These off-path beliefs constitute an equilibrium, but they are not continuous in price for consumers who only receive a single price.

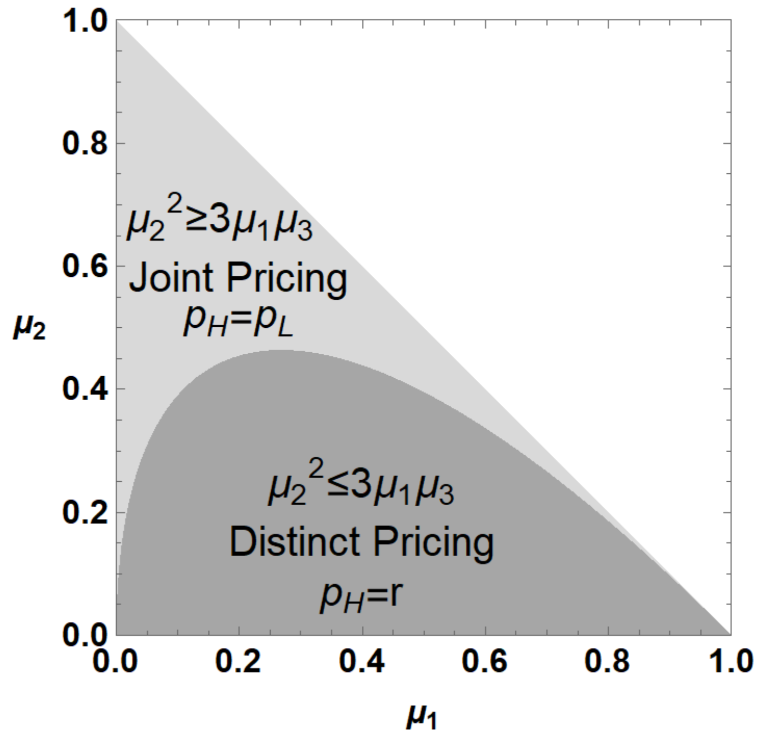


Figure 1: **Brand-preserving merger equilibrium regions:** Parameter μ_k is the fraction of consumers who receive k initial price quotes, where $\mu_3 = 1 - \mu_1 - \mu_2$. Possible values for the vector μ lie in the triangular region. The two prices set by the merged firm following a brand-preserving merger are denoted p_H and p_L, where $p_H \geq p_L$. The consumer's reservation price is denoted r. Distinct pricing equilibria, in which the merged firm sets $p_H = r$ and mixes over p_L, are possible for values of μ within the dark gray inequality region. Joint pricing equilibria, in which the merged firm mixes over both prices and sets $p_H = p_L$, are possible in the remainder of the triangle.

2.3 Brand-Consolidating Equilibrium

We can compare the above equilibria in which the merged firm continues to operate two separate brands to one in which they consolidate into a single brand. Here we assume that consumers (1) continue to receive the same number of initial quotes as pre-merger and (2) each price quote has the same probability of coming from each brand as pre-merger (in other words, the consolidated brand has double the probability of being selected for any given price quote). The merged firm's profit function is therefore:

$$\pi_{JC}(p) = \frac{2}{3}\mu_1 p + (\mu_2 + \mu_3)(1 - G_C(p))p. \tag{5}$$

As in Armstrong and Vickers (2022), an equilibrium exists. The merged firm plays a price of r with probability λ_C or chooses a price on the interval $[\underline{p}_C, r]$ according to distribution F_C. The outside firm sets prices on the same interval with distribution G_C. The consumer sets their reservation price r_C based on the lower of their value of the good or their indifference towards searching when they are facing the outside firm. Equations A9-A13 in Appendix B.3 characterize λ_C, $\underline{p}_C$, F_C, G_C, and r_C.

The consumer's reservation price is set according to G_C because if they see a price at (or above) r_C, they can be certain that it came from the merged firm. Thus, they will face the outside firm moving forward. If they receive a lower price, they will believe they are facing a convex combination of F_C, G_C, and a degenerate distribution of r_C, which first order stochastically dominates G_C. Therefore, consumers who see a price above r_C search, and those who see a price at or below r_C do not.

2.4 Welfare and Antitrust Remedies

To study welfare, we characterize post-merger average transacted prices (a sufficient statistic for consumer welfare in this model), under two settings. In the first, consumers' reservation price is exogenous. One can also think of this setting as one in which the consumer is unaware that the merger has occurred, a setting we refer to as the illusion of competition. If the consumer does not know about the merger, they will set their reservation price according to the symmetric search indifference condition (equation (2)) no matter what game the firms are actually playing. In the second setting, consumers' reservation price is set endogenously by their decision of whether or not to continue searching, given their search cost and the actual equilibrium distribution of prices.

2.4.1 Fixed r/Illusion of Competition

First, we consider the case in which consumers are unaware of the merger and set their reservation price according to the symmetric equilibrium or their valuation of the good v. For brevity, we will denote this common reservation price r.

Joint Pricing Equilibrium

When μ is such that a brand-preserving merger would result in the joint pricing equilibrium (the light gray region in Figure 2), we consider three possible outcomes: a baseline equilibrium (no merger), the joint pricing equilibrium (brand-preserving merger), and the consolidated brands equilibrium (brand-consolidating merger). Consumer welfare is fully captured by the average transacted price in each of these outcomes, which we denote $\bar{p}$, $\bar{p}_J$, and $\bar{p}_C$, respectively.

Proposition 3 *If r is fixed and a joint pricing equilibrium would occur ($\mu_2^2 \geq 3\mu_1\mu_3$), then $\bar{p} \leq \bar{p}_C \leq \bar{p}_J$. The consumer prefers the no-merger outcome to the brand-consolidating merger outcome. They prefer both of these outcomes to the brand-preserving merger.*

Proposition 3 tells us that if enough consumers consider exactly two prices, and consumers do not adjust their reservation prices following the merger, then the outcomes have a clear ranking. Average prices will be highest when firms merge and continue to operate separate brands. If reservation prices are fixed, the merger allows firms to capture additional rent generated by consumers only considering the two merged brands. This is the region of the parameter space that is perhaps most interesting to regulators. If consumers' search technology is as such, firms will want to merge and continue operating multiple brands. However, regulators wishing to protect consumers' interests should either block the merger or at least force the firms to consolidate the brands. (Ireland (2007) finds this result for the special case of $\mu_3 = 0$, which lies along the diagonal upper bound of the parameter space in Figure 2.) These results are consistent with McDonald and Wren's (2018) hypothesis that firms use multiple brands to crowd competitors out of their potential customers' consideration sets. They are also related to Ellison and Wolitzky's (2012) finding that firms have an incentive to raise search costs when consumers have convex search costs (as they do in our model).

Distinct Pricing Equilibrium

When μ is such that a brand-preserving merger would result in the distinct pricing equilibrium (the medium and dark gray regions in Figure 2), we consider three similar outcomes: a baseline equilibrium (no merger), the distinct pricing equilibrium (brand-preserving merger), and the consolidated brands equilibrium (brand-consolidating merger). We denote the average transacted price in each of these outcomes as $\bar{p}$, $\bar{p}_D$, and $\bar{p}_C$, respectively.

Proposition 4 *If r is fixed and a distinct pricing equilibrium would occur ($\mu_2^2 \leq 3\mu_1\mu_3$), then $\bar{p} \leq \bar{p}_C, \bar{p}_D$. The consumer prefers the no merger outcome to either merger outcome. Furthermore, $\bar{p}_C < \bar{p}_D$ iff $\mu_2 > 3\mu_1 * (1 - \mu_1)/(2 * (3 - \mu_1))$. The consumer prefers the brand-consolidating merger to the brand-preserving merger if and only if μ_2 is sufficiently high.*

When the consumers' search technology is such that the distinct pricing equilibrium is realized, the no-merger case is still the best outcome for consumers. If consumers do not change their reservation price, firms are able to raise their average prices following either type of merger.

In the distinct pricing case, it is possible for the brand-preserving merger outcome to be better for consumers than the brand-consolidating merger outcome. In the distinct pricing equilibrium, one-third of the consumers who only see one price will draw a price of r from the higher-priced jointly owned brand. These consumers get "ripped off" while all of the remaining consumers draw at least one price from the fairly competitive F_D distribution. If this fraction of ripped-off consumers is small enough (when μ_3 is relatively high) or if F_D is sufficiently more competitive than the price distributions offered in the brand-consolidating merger equilibrium (when μ_1 is relatively high), then the distinct pricing equilibrium results in lower prices than the consolidated one. The dark gray region in Figure 2 illustrates the parameter region for which consolidating brands actually hurts consumers.

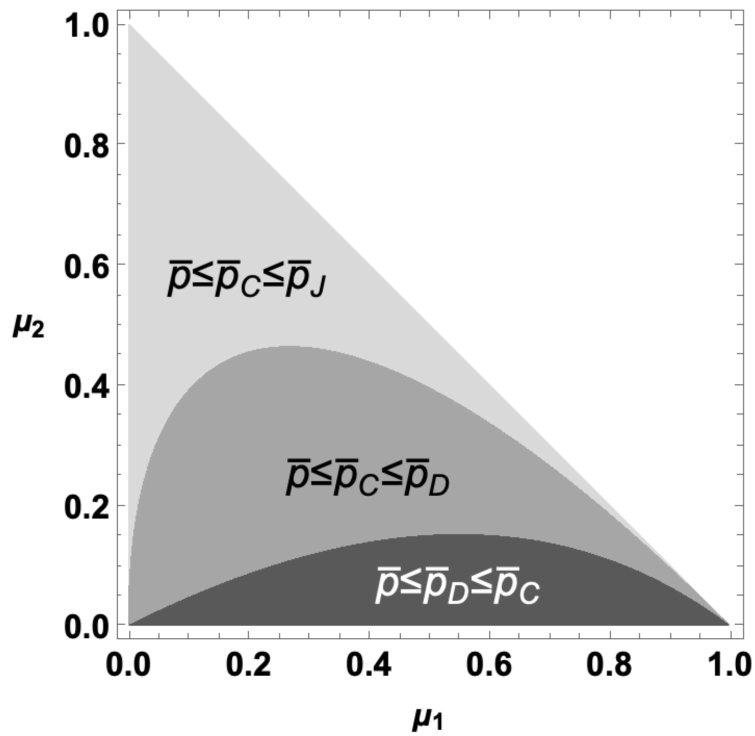


Figure 2: **Ranking equilibrium transaction prices for fixed r / illustion of competition:** Parameter μ_k is the fraction of consumers who receive k initial price quotes, where $\mu_3 = 1 - \mu_1 - \mu_2$. Possible values for the vector μ lie in the triangular region. We compare average transacted prices between the baseline pre-merger symmetric 3 firm equilibrium ($\bar{p}$) to prices following a brand-consolidating merger ($\bar{p}_C$) and to prices of the two possible outcomes of a brand-preserving merger, the distinct pricing equilibrium ($\bar{p}_D$), and the joint pricing equilibrium ($\bar{p}_J$). Here we assume that consumers are under the illusion of competition and do not update their reservation prices following a merger. Average transacted prices are lower for the pre-merger case for all values of μ. The brand-consolidating merger produces lower average transacted prices than the brand-preserving merger for all regions except the darkest gray region.

2.4.2 Endogenous r

Next, we consider consumers who update their reservation price following the merger to reflect the equilibrium of the new game. The following results reflect the equilibria of Section 2 in which r is pinned down by the equilibrium search condition.

The asymmetry of these equilibria allows for certain counter-intuitive results. In the previous subsection, we showed that, for a given reservation price, the overall transaction price distribution is worse for a post-merger equilibrium. However, consumers' equilibrium reservation prices are not generally selected with respect to the overall transaction price distribution. Instead, many of the equilibria involve reservation prices that are chosen based on the lower-priced brands. This can result in consumers setting a lower reservation price post-merger and ultimately lower average transacted prices.

Joint Pricing Equilibrium

We first consider welfare in the parameter space in which the joint pricing equilibrium occurs (that with a relatively high share of consumers considering exactly two prices—the light gray region in Figure 3). We denote average transacted prices (with endogenous r) under the no-merger, equilibrium, brand-consolidating merger equilibrium, and joint pricing equilibrium as $\bar{p}^*$, $\bar{p}^*_C$, and $\bar{p}^*_J$, respectively. Moving forward we distinguish between average transacted prices when consumers are under the illusion of competition (exogenous r) and those where they are not (endogenous r) by adding an asterisk to the endogenous r value.

Proposition 5 *If r is the solution to the sophisticated consumers' search indifference condition and a joint pricing equilibrium would occur ($\mu_2^2 \geq 3\mu_1\mu_3$), then $\bar{p}^* < \bar{p}^*_C < \bar{p}^*_J$. The consumer prefers the brand consolidating-merger to the the brand-preserving merger (joint pricing equilibrium), but prefers no merger to both of these outcomes.*

This proposition tells us that the joint pricing equilibrium is still worse for consumers than the no-merger or brand-consolidating merger equilibria. Whether consumers update their reservation price or not, a joint-pricing equilibrium cannot be optimal for consumers. This suggests that antitrust authorities should be particularly concerned about markets in which one firm operates multiple brands, yet sets the same price across these brands.

This also shows that mergers will not benefit consumers if there are sufficiently many consumers who only consider two prices. Brand-consolidation can mitigate the harm to consumers, but it will not fully offset the increase in prices caused by the merger.

Distinct Pricing Equilibrium

Finally, we consider the case in which consumers update their reservation price, and the search technology is such that the distinct pricing equilibrium would be realized (all except the lightest gray region of the triangle in in Figure 3).

Proposition 6 *If r is the solution to the sophisticated consumers' search indifference condition, then: (1) $\bar{p}^* < \bar{p}^*_C$; (2) $\bar{p}^*_D < \bar{p}^*$ iff Appendix equation (A14) holds; and (3) $\bar{p}^*_D < \bar{p}^*_C$ iff Appendix equation (A15) holds. These are restrictions on the values of μ, which are shown graphically in Figure 3.*

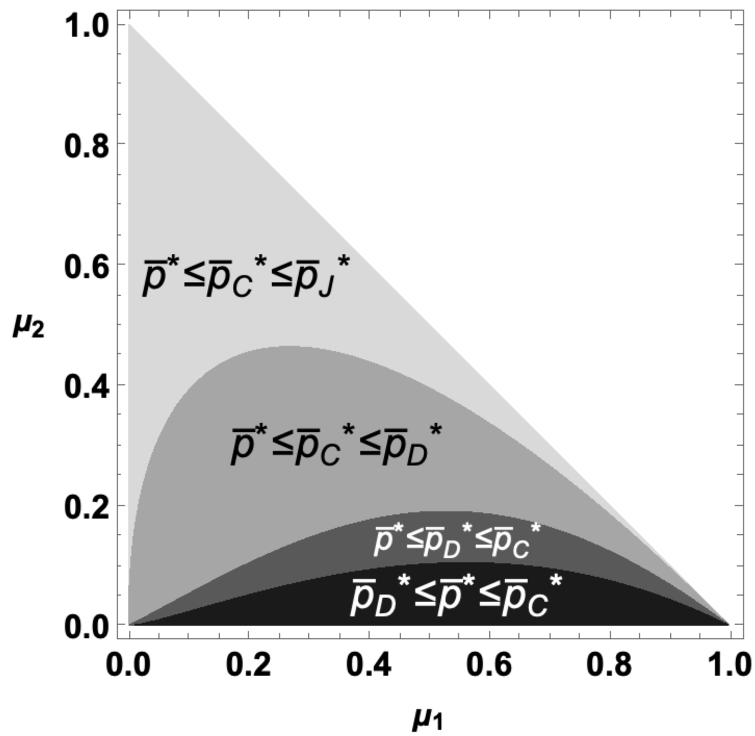


Figure 3: **Ranking equilibrium transaction prices for endogenous r / sophisticated consumers:** Parameter μ_k is the fraction of consumers who receive k initial price quotes, where $\mu_3 = 1 - \mu_1 - \mu_2$. Possible values for the vector μ lie in the triangular region. We compare average transacted prices between a pre-merger symmetric 3 firm equilibrium ($\bar{p}^*$) to prices following a brand-consolidating merger ($\bar{p}_C^*$) and to prices of the two possible outcomes of a brand-preserving merger, the distinct pricing equilibrium ($\bar{p}_D^*$), and the joint pricing equilibrium ($\bar{p}_J^*$). Here we assume that consumers are sophisticated and update their reservation prices following a merger.

Proposition 6 establishes a series of inequalities over the search technology μ that allow us to compare prices under different types of mergers. Again, we find that the no merger case always results in lower prices than the brand-consolidating merger. As we found in the constant reservation price case, there is a region in which the number of consumers considering exactly two prices is low, in which the distinct pricing equilibrium is preferred to the brand-consolidating merger. While the boundary of the inequality changes when reservation prices update, this qualitative statement still holds. When consumers are aware of the merger, and update their reservation price rationally, it is also possible for the distinct pricing equilibrium to result in lower prices than the no-merger case.

Propositions 3-6 allow us to map out the parameters for which different antitrust remedies are consumer optimal. Specifically, in Figure A.1, we plot when brand consolidation and/or informing consumers of a merger (breaking the illusion of competition) will lower prices, conditional on allowing a merger. Consolidation is beneficial when the average price is lower for the consolidated-brands equilibrium than the brand-preserving equilibrium. This is the case when there are sufficiently many consumers considering exactly two prices who become captive consumers following a brand-preserving merger. Consolidation can ensure that these consumers see two competitive prices.

Information about the merger is beneficial to consumers when the reservation price for the merger equilibrium is lower than the reservation price for the no-merger baseline. This happens when there are sufficiently many shoppers. When there is a large population of shoppers, the non-merged firm competes aggressively on price to win over these consumers. Informed consumers' reservation prices are determined by the distribution of prices played by the outside firm, so for high values of μ_3, information can cause consumers to decrease their reservation price, lowering overall transacted prices.

These remedies are only optimal conditional on allowing a merger in the first place. As previously noted, the baseline, no merger case results in the lowest prices for all parameter values if consumers do not update their reservation prices. Furthermore, as seen in Figure 3, prices are lowest in the baseline case unless the share of consumers considering exactly two prices is sufficiently low. A merger in that region of the parameter space would lower prices, but would also lower profits for the merging firms, and would therefore be less likely to be proposed in the first place. For this reason, we consider optimal antitrust remedies conditional on a merger being allowed (perhaps because the merger produces sufficient reductions in fixed costs).

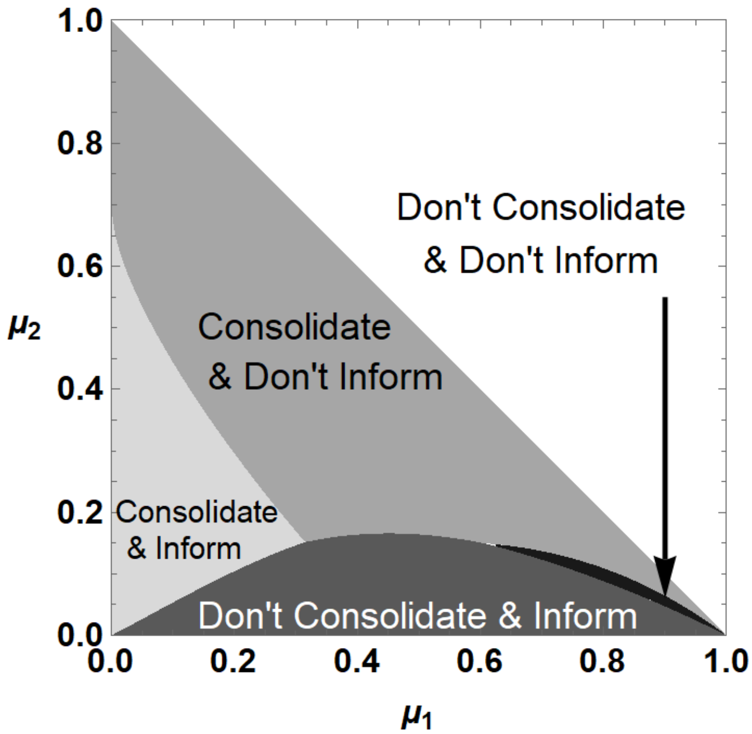


Figure 4: **Optimal policy conditional on a merger:** Parameter μ_k is the fraction of consumers who receive k initial price quotes, where $\mu_3 = 1 - \mu_1 - \mu_2$. Possible values for the vector μ lie in the triangular region. We show the optimal antitrust remedy for a given μ, conditional on allowing a merger. Consolidate refers to forcing the merged firm to consolidate its brands while Inform refers to informing consumers of the market structure, allowing them to update their reservation price.

Collectively, Propositions 3-6 also tell us about the possible harm caused by the illusion of competition. Propositions 3 and 4 state that, if consumers stick to the pre-merger reservation price, the no-merger outcome is always preferred to any merger outcome. Therefore, when consumers suffer from the illusion of competition, mergers are always harmful. However, Proposition 6 shows that this result flips for certain values of μ when consumers rationally update their reservation price to the post-merger equilibrium distribution of prices. In these scenarios, informed consumers can be better off post-merger. This reversal implies that, for these values of μ, the reservation price, and thus transacted prices, fall when consumers are aware of the post-merger equilibrium.

Corollary 6.1 *There exist search technologies μ such that average transacted prices are lower if consumers are aware of a merger and update their reservation price than if they are unaware and use their pre-merger reservation price.*

Corollary 6.1 suggests that regulators may be able to reduce consumer harm from a merger by informing consumers of the new market structure. However, Corollary 6.2 shows that it is also possible for the illusion of competition to benefit consumers.

Corollary 6.2 *There exist search technologies μ such that average transacted prices are lower if consumers are unaware of a merger and do not update their reservation price than if they are aware and use the full-information equilibrium reservation price.*

Consider, for example, the case of $\mu_2 = 1$. Here, pre-merger, all firms are always in competition with another firm, and therefore price at marginal cost, or 0. Therefore, the consumers' pre-merger reservation price is 1 (the average price offered in the degenerate distribution plus their search cost). Following a brand-preserving merger, the merged firm has captive consumers, allowing all brands to earn positive profits. The average offered price following the brand-preserving merger must be positive, meaning the equilibrium reservation price for fully informed consumers must be strictly greater than 1. In such a case, informing consumers that a merger has occurred would be harmful to their welfare.

2.5 Extensions to the model

In Appendix A we consider extensions to our model of search capture.

First, in Appendix A.1, we show that similar results hold when we consider a stronger informational intervention in which consumers know the market structure *and* they can steer their initial searches. Such a search technology could come from a form of convex search costs as in Hämäläinen (2022). In this case, sophisticated consumers endowed with two searches can see the price offered by one brand and decide whether to direct their next search to a brand owned by the same firm, or a different one. While we are unable to characterize equilibria for all possible values of μ in this setting, where we have characterized equilibrium, we find that giving consumers this additional information and ability improves consumer welfare relative to just alleviating the illusion of competition. However, conditional on a allowing a merger, it is still optimal to force brand consolidation whenever there are sufficiently many consumers who initially search twice.

In Appendix A.2, we study multi-brand ownership that arises from brand proliferation. The problem of too-many brands for a homogeneous product can arise without mergers or acquisitions when firms introduce multiple brands to sell the same product. This is increasingly common in online marketplaces. For instance, on Grubhub, a single physical kitchen can easily list its delivery meals using multiple online restaurant names and corresponding menu prices (Hassan, 2023). A simple extension of our primary results (which builds on Armstrong and Vickers's (2024) analysis of the case with exogenous consideration sets) shows that, by banning such brand proliferation, online platform operators can increase competition among their sellers and benefit their consumers. This is true for similar market conditions as when requiring merging firms consolidate their brands is beneficial—when there are sufficiently many consumers who see exactly two price quotes before considering additional search. However, we also show that permitting brand proliferation but limiting visibility for duplicate brands to the bottom of search results can sometimes be even better than a strict ban.

Finally, in Appendix A.3, we show that our results hold qualitatively when there are $N > 3$

firms. It is difficult to characterize all possible equilibria for an arbitrary number of firms. We instead show that, as with 3 firms, requiring the merging firms to consolidate their brands is beneficial to consumers under many circumstances but can be harmful in others.

3 Search Discouragement Model

Consider a setup similar to Stahl (1989), with $N \geq 2$ single-brand firms and consumers with unit demand and value v for a homogeneous good. Marginal costs are normalized to zero. Fraction μ of consumers are shoppers who observe all N prices, and fraction $1-\mu$ have search cost $s > 0$ after receiving their first quote for free. Our tie-breaking rule is that consumers buy from the most recent price among the set of minimum prices observed.

We adjust Stahl's (1989) model by adding an initial stage of the game in which Nature chooses the state to be collusive with probability α_0 and competitive with probability $1-\alpha_0$. Firms observe the state, but consumers do not. In the collusive state, firms are non-strategic and all price at the monopoly level, $p = v$. In the competitive state, firms simultaneously choose prices. We focus on symmetric equilibria in which firms draw prices independently from the distribution F.

Consumers learn about the state (collusive or competitive) from their price quotes. If each firm's equilibrium pricing strategy is to set $p = v$ with probability σ_1, then after observing k prices equal to v, consumers believe they are in the collusive state with probability:

$$\Pr(\text{Collusive State} \mid p_1 = \ldots = p_k = v) \equiv \alpha_k = \frac{\alpha_0}{\alpha_0 + (1-\alpha_0)\sigma^k}. \tag{6}$$

Consumers will then search again with probability ϕ_k consistent with these beliefs. Each additional quote of v increases the likelihood that the state is collusive and reduces the returns to search. Hence if a consumer is indifferent to searching after k quotes of v then they strictly prefer not to search after $k+1$ quotes of v.

The first time a consumer sees a price strictly less than v, they know with certainty that they are in the competitive state and learning stops. From this point onwards, they will follow a constant reservation price strategy with reservation price r satisfying $s = \int_0^r F(p)dp$. They will stop searching and buy if they observe a price less than or equal to r but continue searching otherwise. Thus a consumer's strategy is summarized by reservation price r and the vector $\phi = (\phi_1, \ldots, \phi_{N-1})$.

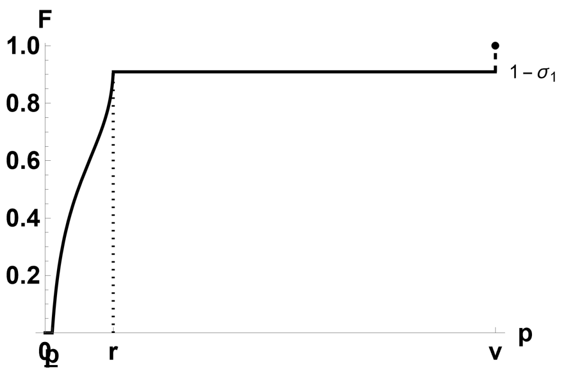
Define a threshold search cost s_N^* as:

$$s_N^*(\mu) \equiv v\left(1 - \int_0^1 \left(\frac{1}{1 + \frac{\mu}{1-\mu} N x^{N-1}}\right) dx\right) \tag{7}$$

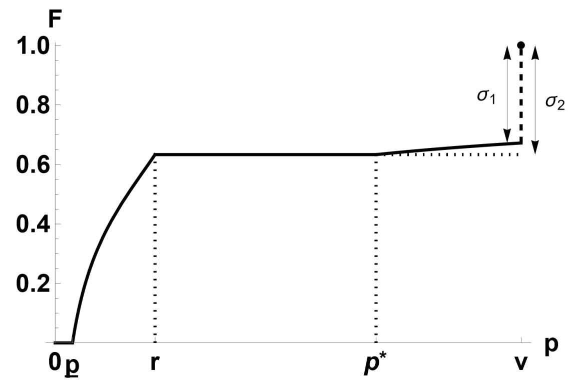
If search cost s is greater than or equal to s_N^*, then a unique symmetric equilibrium corresponds to $Varian$ (1980). Non-shoppers never seek a second quote because $r \geq v$. Firms mix continuously over an interval $[\underline{p}, v]$ with $0 < \underline{p} < v$ and industry profits are $\Pi = v(1-\mu)$.

If search cost $s < s_N^*$ then consumers' equilibrium reservation price r will be strictly less than v. Firms will price at v with probability $\sigma_1 \in (0,1)$, and mix continuously over an interval of prices $[\underline{p}, r]$ below r with probability $(1-\sigma_2) \in (0,1)$. Moreover, in some equilibria, firms may mix continuously over an interval of prices $[p^*, v)$ just below v while refraining from pricing in the gap (r, p^*). These intervals satisfy $0 < \underline{p} < r < p^* \leq v$.

We illustrate these two possible equilibrium structures in Figure 7 for the parameter values $N = 4$, $\alpha_0 = 0.5$, $\mu = 0.7$, and $s/v = 0.07$. At these values there are three equilibria, two of which are depicted in the figure. In Equilibrium A (left column), Firms price at v with probability 0.09 and mix over an interval $[\underline{p}, r]$ otherwise. Consumers respond by searching with probability $\phi_1 = 0.92$ after seeing the first price equal to v but stop searching if they see a second price of v. In Equilibrium B (right column), firms set higher prices, pricing at v with probability 0.33, and otherwise mixing over both $[\underline{p}, r]$ and $[p^*, v)$. Because firms price at v with higher probability, seeing v is a weaker signal of collusion, and consumers continue searching longer—searching with probability 0.042 after seeing two prices of v.



(a) Firm pricing strategy (Eqbm. A)



(b) Firm pricing strategy (Eqbm. B)

(c) Consumer search strategy (Eqbm. A)

(d) Consumer search strategy (Eqbm. B)

Figure 5: This figure depicts equilibrium strategies for firms (top row) and consumers (bottom row) for equilibrium A (left column) and B (right column). These are computed given the example: $N = 4$, $\alpha_0 = 0.5$, $\mu = 0.7$, and $s/v = 0.07$.

The fact that firms randomize between the monopoly price and lower prices in these examples is a general feature of equilibrium. Were firms not to price at v in the competitive state, a price of v would be a perfect signal of the collusive state. Hence any consumer who saw a price of v would stop and buy. As a result, firms would have an incentive to deviate and price at v. Were firms to price at v with probability 1, however, they would want to undercut each other by pricing at $v - \epsilon$. Hence $\sigma_1 \in (0, 1)$, meaning that firms in the competitive state mix between pooling with collusive firms and pricing more competitively. (This model prediction is verified experimentally by Heggedal et al. (2024).)

Notice that in Equilibrium B, a consumer who observes prices $(p_1, p_2) = (v, v)$ will stop searching and buy at v with probability over 95% because they believe the state is likely collusive with no price dispersion and no returns to search. However a consumer who observes prices $(p_1, p_2) = (v, p^*)$ will learn that the state is competitive and there is a high chance that the next price quote will be below r, so will keep searching until they find such a low price or they have learned all the prices. Thus setting prices at v discourages search by lowering consumers' estimate of price dispersion and the returns to search.

Appendix B.6 Proposition 16 provides necessary and sufficient conditions that characterize equilibria of this baseline search discouragement game in which N firms each own a single brand. (These are used to generate the examples above.)

3.1 Limiting Case: A small chance of collusion

An interesting special case of this model is when consumers believe that there is only a very small chance that they are in the collusive state. The set of symmetric equilibria is non-empty and converges to a unique symmetric equilibrium as $\alpha_0 \to 0$.

Proposition 7 *(1) If $s < s_N^*$, then for sufficiently small $\alpha_0 > 0$, there exists a symmetric Nash equilibrium with $\sigma_2 > 0$ and $r < v$. (2) In the limit when $\alpha_0 \to 0$ there is a unique symmetric equilibrium in which $\sigma_1 = \sigma_2 = 0$, $\phi_1 = 1 - s/s_N^*$, $\phi_2 = \ldots = \phi_{N-1} = 0$, and $r = r_{Stahl} = v(s/s_N^*)$. Industry profits (and average transacted prices) are*

$$\Pi_{N,Stahl} = (1-\mu) r_{Stahl} = v(1-\mu)\frac{s}{s_N^*} \tag{8}$$

For low values of α_0, equilibria are similar to Stahl (1989) with two differences. First, as explained above, firms charge the monopoly price with positive probability, $\sigma_1 \in (0, 1)$. Second, consumers whose first quote is the monopoly price v continue searching with probability $\phi_1 \in (0, 1)$ but stop searching and buy at the monopoly price otherwise. A key result from Proposition 7 that our later results rely on is that in the limit $\alpha_0 \to 0$, $\phi_1 = 1 - s/s_N^*$ and $\phi_2 = \ldots = \phi_{N-1} = 0$. Below we sketch the intuition for this result.

For firms to be willing to charge the monopoly price, it must yield positive demand, which means a sequence of monopoly prices must eventually convince consumers that the collusive state is likely enough that they end their search and buy. As the prior probability of the collusive state becomes vanishingly small, monopoly prices must therefore become increasingly strong signals of the collusive state. This means that σ_1 must go to zero with α_0.

As σ_1 goes to zero, demand at the monopoly price, $q(v)$, goes to $(1-\mu)(1-\phi_1)/N$, the share of non-shoppers who see firm i's monopoly price first and then stop searching. (Anyone who sees another price, either before or after, will see a lower price so will not buy from firm i.) For firms to be willing to charge the monopoly price, this demand must be positive, which requires that $\phi_1 < 1$. If consumers are willing to stop searching after seeing one signal of the collusive state, they will strictly prefer to stop searching after seeing more, so $\phi_2 = \ldots = \phi_{N-1} = 0$.

Moreover, in this equilibrium, firms are indifferent between pricing at v and r, so $\pi(v) = \pi(r)$.

$$r * q(r) = v * q(v) \implies r = v * \frac{q(v)}{q(r)}$$

As $\sigma_1 \to 0$, demand at the reservation price, $q(r)$, goes to $(1-\mu)/N$. Hence, as $\alpha_0 \to 0$,

$$r \to v\frac{(1-\mu)(1-\phi_1)/N}{(1-\mu)/N} = v(1-\phi_1)$$

Also as $\sigma_1 \to 0$, pricing below r approaches the distribution of prices played in Stahl (1989), so r approaches the reservation price in that equilibrium, r_{Stahl}. This implies that $\phi_1 \to 1 - r_{\text{Stahl}}/v = 1 - s/s_N^*$ as $\alpha_0 \to 0$. (Reservation price r_{Stahl} equals $= v\frac{s}{s_N^*}$ because r_{Stahl} is proportional to s and—by definition of s_N^*—is equal to v for $s = s_N^*$.)

At the limit $\alpha_0 = 0$, the equilibrium is simply the Stahl (1989) equilibrium—except that consumers only search with probability $\phi_1 = 1 - s/s_N^*$ upon observing the monopoly price once, and stop searching after seeing it twice. At $\alpha_0 = 0$ with single-brand firms, this part of the search strategy is off the equilibrium path because firms never charge the monopoly price. However, as investigated below, this part of the search strategy is nevertheless important if firms merge but consumers are unaware of the merger and do not update their search strategy.

3.2 Merger Under the Illusion of Competition

Next, as we did with the search capture model, we study the effects of brand-preserving mergers. We introduce two symmetric brand preserving mergers (four firms with independent brands become two firms with two brands each). We denote outcomes for this case with **m**ulti-**b**rand firms by a subscript "MB" and compare them to the pre-merger **b**aseline (subscript "B") outcomes and brand-consolidating merger outcomes (subscript "CB" for **c**onsolidated **b**rands).

First, we consider the case in which consumers are under the illusion of competition. Specifically, they believe that all four brands are independent and therefore follow the pre-merger equilibrium search strategy characterized by r and ϕ.[7] Firms know consumers' search strategy (r, ϕ) and set prices for both of their co-owned brands.

Proposition 8 *An equilibrium exists for all r and ϕ. Industry expected profits in the competitive state (and thus average prices) in these equilibria are at least*

$$\Pi_{MB} \geq \underline{\Pi}_{MB} \equiv v(1-\mu)\left((1-\phi_1) + \frac{1}{3}\phi_1(1-\phi_2)\right). \tag{9}$$

Proof. Existence: See Appendix C. Profit lower bound: A lower bound for a single firm's profits in the 4-brand 2-firm brand-preserving merger scenario is the profit achieved by setting both prices equal to v under the worst case for this strategy—that in which the other firm prices at v with zero probability. In this case, the firm wins no shoppers, and only those non-shoppers who view one of the firm's brands first, and either stop searching or continue searching at the firm's other co-owned brand and then stop searching. This yields individual firm profits of $v(1-\mu)\left(\frac{1}{2}(1-\phi_1) + \frac{1}{6}\phi_1(1-\phi_2)\right)$. Doubling them gives the lower bound for profits in equation (9). ■

[7] This search strategy is not optimal given the actual market structure but is optimal conditional on their mis-specified beliefs.

A sketch of the proof of equilibrium existence is as follows. Consider a restricted version of the game in which firms must choose prices from the set $[0, r] \bigcup [r+\epsilon, v-\epsilon] \bigcup \{v\}$ for some small $\epsilon > 0$ (rather than $[0, v]$ as in the actual game). Firms' payoff function is continuous over this region except for along the diagonal in which their lower prices are equal. We can use Dasgupta and Maskin (1986) Theorem 5 to show that this restricted game has an equilibrium. We can then show that this restricted game approximates the actual game as in Fudenberg and Levine (1986), and that the actual game also has an equilibrium.

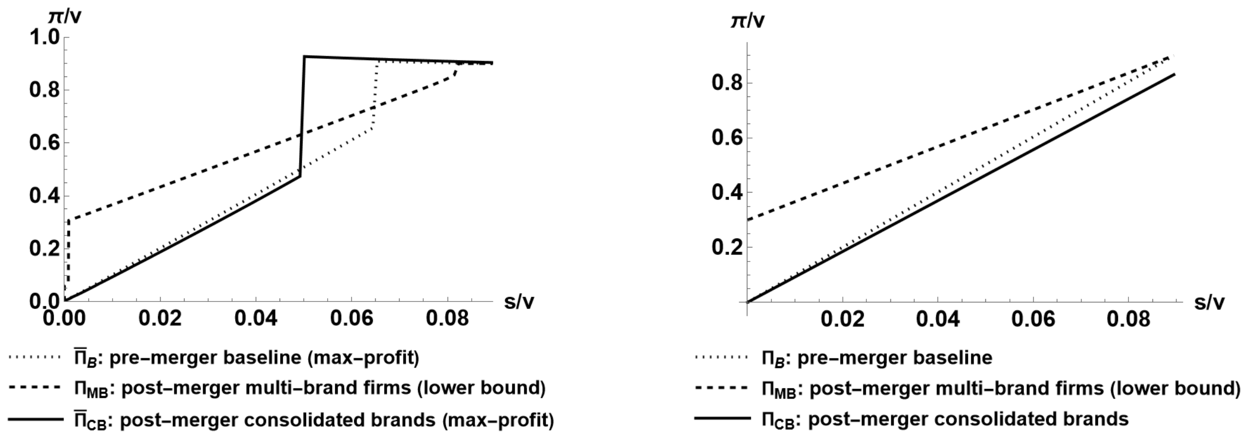
Substituting $\phi_1 = 1 - s/s_N^*$ and $\phi_2 = 0$ from Proposition 7 into the profit lower bound from Proposition 8 yields the following corollary.

Corollary 8.1 *For $s < s_4^*$ and $\alpha \to 0$, the lower bound on industry profits (and thus average prices) goes to*

$$\underline{\Pi}_{MB} = v(1-\mu)\left(\frac{1}{3} + \frac{2}{3}\frac{s}{s_4^*}\right). \tag{10}$$

This is strictly greater than both pre-merger profits ($\Pi_{4,Stahl}$) and brand-consolidating merger profits ($\Pi_{2,Stahl}$) characterized in Proposition 7. Therefore Brand-preserving mergers increase average transacted prices when consumers are under the illusion of competition. Consolidating the brands and informing consumers of the consolidation improves consumer welfare.

When consumers' belief about the probability of collusion, α_0, is sufficiently small, a brand preserving merger under the illusion of competition is harmful to consumers. Forcing the brands to consolidate and informing consumers about the resulting market structure is an effective remedy in lowering prices. Figure 6 plots profits pre-merger, following a brand preserving merger with the illusion of competition, and following a brand consolidating merger with information both for $\alpha = 0.1$ (Panel a) and $\alpha_0 \to 0$ (Panel b).



(a) Competitive-state industry profits vs s by market structure $\mu = \alpha_0 = 0.1$

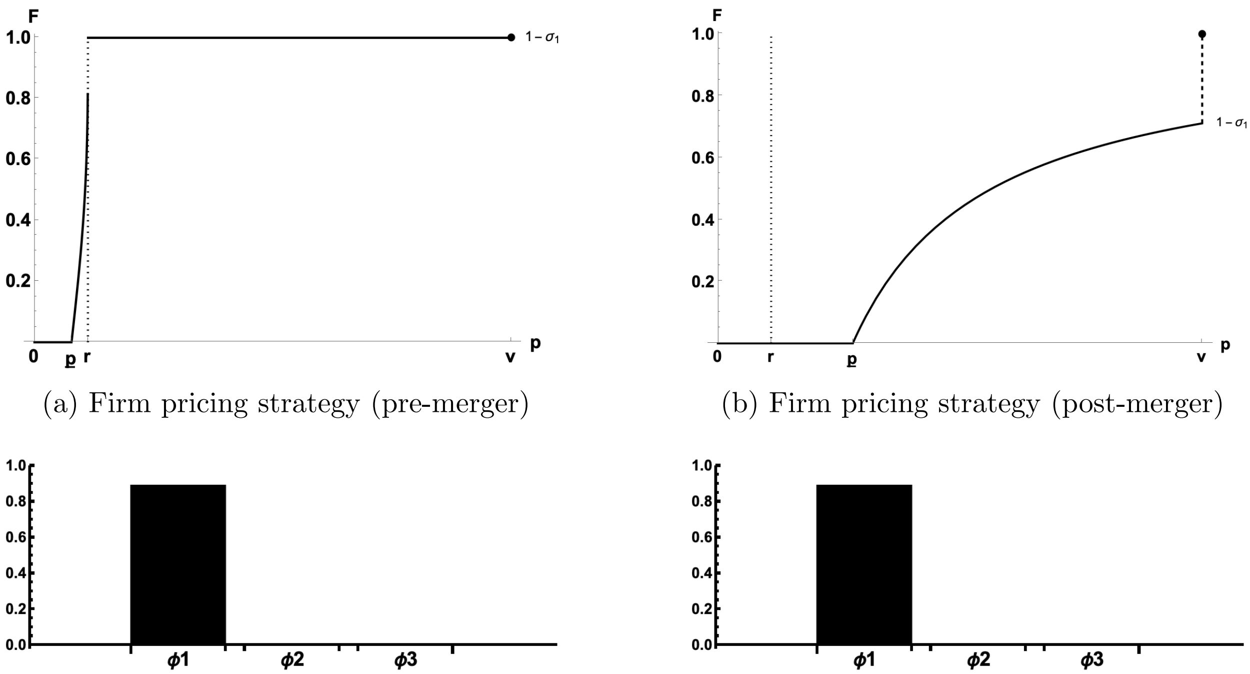
(b) Industry profits vs s by market structure $\mu = 0.1, \alpha_0 \to 0$

Figure 6: This figure plots equilibrium industry profits (average transacted prices) in the competitive state by consumer search cost s for different market structures (axes are normalized by v). Baseline profit $\bar{\Pi}_B$ is the maximum industry profit pre-merger with four single-brand firms. Multi-brand profit Π_{MB} is the lower bound on industry profits after brand preserving mergers to two 2-brand firms under the illusion of competition. Consolidated-brand profit Π_{CB} is the maximum industry profit after a brand-consolidating mergers to two single-brand firms under endogenous consumer search strategies. For $\alpha = 0.1$, $\bar{\Pi}_B$ and $\bar{\Pi}_{CB}$ are maximum industry profits across multiple equilibria, while for $\alpha \to 0$, equilibria are unique.

A clear example of the harm induced by search discouragement occurs when $\alpha_0 = 0.1$, $\mu = 0.1$, and $s/v = 0.01$. In the baseline equilibrium, consumers think that the collusive state is unlikely. Therefore, to make them indifferent towards searching after seeing the monopoly price, firms only play the monopoly price with low probability $\sigma_1 = 0.0013$. Otherwise, they play prices at or below the consumers' reservation price $r = 0.11$, which is low given the low search costs and large share of shoppers. If a consumer sees a single price of v, they are indifferent between searching and not so they are willing to search with a high probability $\phi_1 = 0.89$ that keeps firms indifferent between pricing at v or r. This equilibrium results in low average transacted prices of 0.10. These equilibrium strategies are shown in the left two panels of Figure 7.

Now consider a brand preserving merger in which consumers are under the illusion of competition. Because they think that they are in the pre-merger case described above, they maintain a search strategy of $r = 0.11$ and $\phi_1 = 0.89$. When consumers see the monopoly price, they often search once, but when they see the monopoly price again from the other brand, they do not continue to search, thinking they must be in the collusive state.

For this example, we can characterize an equilibrium and compute profits exactly rather than relying on the lower bound. In equilibrium, firms use a joint pricing strategy (shown in Figure 7 Panel (b)), setting both of their prices to the monopoly price with high probability $\sigma_1 = 0.29$, and otherwise mixing on an interval $[p^*, v]$. This leads to high average transacted prices of 0.55 in the competitive state (50% higher than the lower bound from Proposition 8).



(a) Firm pricing strategy (pre-merger)

(b) Firm pricing strategy (post-merger)

(c) Consumer search strategy (pre-merger)

(d) Consumer search strategy (post-merger)

Figure 7: This figure depicts equilibrium strategies for firms (top row) and consumers (bottom row) before (left column) and after (right column) a brand-preserving merger where consumers are under the illusion of competition. Post merger, firms use a joint-pricing strategy by setting equal prices across brands. The distribution of this joint price is what is shown in panel (b). These are computed given the example: $N = 4$, $\alpha_0 = 0.1$, $\mu = 0.1$, and $s/v = 0.01$.

The pricing strategy is profitable because consumers become discouraged and stop searching after seeing the monopoly price twice. Without understanding the correlation between co-owned brands' prices, two observations of the monopoly price appears to be a very strong signal that they are in the collusive state and that they should stop searching. The illusion of competition and this resulting search discouragement make consumers considerably worse off. Forcing the brands to consolidate and resolving the illusion of competition largely fixes the problem. The equilibrium with consolidated brands and informed consumers looks very similar to the pre-merger case and results in low average transacted prices of 0.09 in the competitive state.

3.3 Dispelling the Illusion of Competition

Section 3.2 shows that brand-preserving mergers can dramatically increase prices when consumers are under the illusion of competition and brand-consolidation can be a beneficial remedy. It is interesting to know whether simply informing consumers about the merger will be similarly beneficial. For small $\alpha_0 > 0$, we expect equilibria to be close to equilibria of the game in which $\alpha_0 = 0$.

If consumers are partially or fully sophisticated and $\alpha_0 = 0$, there always exists a distinct pricing equilibrium in which consumers use a constant reservation price strategy. Each firm sets a high price equal to r and randomizes a low price over the interval $[\underline{p}, r]$ according to distribution from a two-firm Stahl game with an adjusted fraction of shoppers $\hat{\mu} = \frac{2\mu}{1+\mu} > \mu$, $F_{2,\text{Stahl}}(\hat{\mu})$. In equilibrium, non-shoppers never want to get a second quote. Hence, it does not matter whether consumers are only partially sophisticated, or are fully sophisticated and can direct their second search either towards or away from the parent company of their first quote.

The distribution $F_{2,\text{Stahl}}(\hat{\mu})$ is more competitive than that in the consolidated brands equilibrium ($F_{2,\text{Stahl}}(\mu)$). When firms consider lowering their low price, they trade-off a higher probability of winning shoppers against a lower markup on their captive consumers. However, in the sophisticated equilibrium, half of their captive consumers are excluded from this trade-off because they buy the high-priced brand. This means that when setting their low price, firms are more aggressive than under brand consolidation, as if there were $\hat{\mu}$ shoppers. Hence consumers' reservation price is lower than in the consolidated brands equilibrium ($r_{\text{soph}} < r_{CB}$).

In both this sophisticated equilibrium and the consolidated brands equilibrium, industry profits equal the share of non-shoppers times consumers' reservation price, $(1 - \mu)r$. Since the reservation price is lower in the sophisticated equilibrium, consumer welfare is higher. Informing consumers about the joint ownership of brands to make them sophisticated increases consumer welfare more than brand consolidation.

Proposition 9 *Let $\hat{\mu} = \frac{2\mu}{1+\mu}$. If $s < s_4^*(\mu)$, consumers are partially or fully sophisticated, and $\alpha_0 = 0$, there exists a distinct pricing equilibrium in which consumers follow a constant reservation price strategy with $r = r_{2,Stahl}(\hat{\mu})$, each firm sets a high price $p_{H,i} = r$, and draws a low price $p_{L,i}$ from the distribution $F_{2,Stahl}(\hat{\mu})$. Industry profits are*

$$\Pi_{soph} = (1 - \mu) r_{2,Stahl}(\hat{\mu}) = v(1 - \mu)\frac{s}{s_2^*(\hat{\mu})} < \Pi_{CB}. \tag{11}$$

Consumer welfare is strictly higher than under brand consolidation.

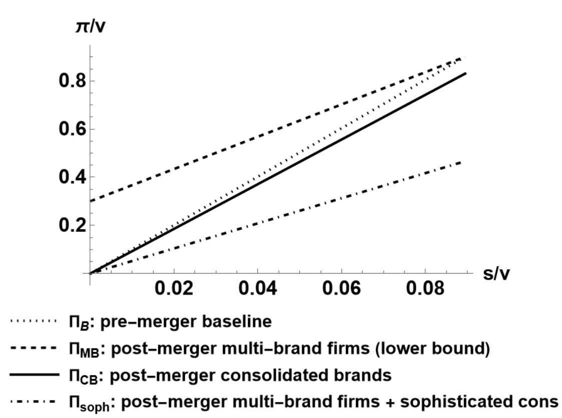


Figure 8: Industry profits vs s by market structure for $\mu = 0.1, \alpha_0 \to 0$. This figure repeats Figure 6 Panel (b) with the addition of profits with sophisticated consumers for the distinct pricing equilibrium characterized in Proposition 9.

We conjecture that a nearby equilibrium will exist for sufficiently small $\alpha_0 > 0$ and $s > 0$. If consumers are partially sophisticated, then in the nearby equilibrium, firms set both their prices to v with small probability σ_1 and otherwise play a similar strategy to that described above, mixing over an interval $[\underline{p}, r]$. In the limit as $\alpha_0 \to 0$, consumers' search strategy is $r = r_{2,\text{Stahl}}(\hat{\mu})$, $\phi_1 = \min\{0, \frac{3}{2}(1 - \frac{s}{s_2^*(\hat{\mu})})\}$, $\phi_2 = \max\{0, 1 - 3(\frac{s}{s_2^*(\hat{\mu})})\}$, and $\phi_3 = 0$. If this conjecture holds, then for small $\alpha_0 > 0$ and $s > 0$, informing consumers about joint ownership of brands can increase consumer welfare and do so more strongly than brand consolidation. (A caveat is that even under this conjecture, we do not rule out the presence of other equilibria which could lead to a less favorable result.)

4 Conclusion

This paper studies novel pricing patterns that emerge when firms operate multiple "brands" in markets characterized by consumer search. We demonstrate two key mechanisms through which multiple brand ownership can harm consumers, "search capture" and "search discouragement". We highlight the role of the "illusion of competition" in which consumers are unaware of joint ownership of distinct brands.

In our model of search capture, when the share of consumers considering exactly two brands is sufficiently high, a joint pricing equilibrium, in which firms set both of their brands' prices to the same value, prevails. If this condition is not met, a distinct pricing equilibrium exists, in which the firm sets two separate prices for its brands. We evaluate novel antitrust interventions—requiring brand consolidation or action to make consumers aware of brand co-ownership as a condition for merging. We show that requiring brand consolidation always benefits consumers (conditional on allowing a merger) if a brand-preserving merger would yield a joint pricing equilibrium. However, requiring brand consolidation can be counter-productive under some conditions when a brand-preserving merger would yield a distinct pricing equilibrium. We study welfare both when consumers are aware and unaware that the merger has taken place and find that ending the illusion of competition by making consumers aware of brand co-ownership may help or hurt consumers.

In our model of search discouragement, brand-preserving mergers are generally harmful to consumers when consumers are under the illusion of competition. Multi-brand firms are able to fool consumers into believing that the returns to search are low by setting their prices to similar values, causing consumers to underestimate price dispersion and search less than a sophisticated consumer would. We show that forcing brand consolidation and dispelling the illusion of competition are both effective remedies for the negative consumer welfare effects of search discouragement, but that dispelling the illusion of competition is most effective.

We present our results as applying to merger policy. However, they can be reinterpreted to address brand proliferation. Each time our model predicts an anti-trust authority should require brand-consolidation, one could equally well say that a market designer should ban brand-proliferation. (For instance a platform operator like UberEats might want to ban Ghost Kitchens from operating multiple branded storefronts for the same kitchen without informing consumers.) A caveat to this reinterpretation is that we model all brands pre-merger as having equal market share. Following introduction of a new brand as part of brand proliferation, the new brand is likely to have smaller market share. With a simple extension (Appendix A.2) our model can address this situation as well.

A limitation of our model is that we assume products are perfectly homogeneous. Exploring the effects of multi-brand ownership and the illusion of competition with differentiated products is an interesting avenue for future work.

References

Allen, Jason, Robert Clark, and Jean-François Houde. 2019. "Search Frictions and Market Power in Negotiated-Price Markets." *Journal of Political Economy* 127 (4): 1550–1598. 10.1086/701684.

Armstrong, Mark, and John Vickers. 2022. "Patterns of Competitive Interaction." *Econometrica* 90 (1): 153–191. 10.3982/ECTA18937.

Armstrong, Mark, and John Vickers. 2024. "Multibrand Price Dispersion." https://drive.google.com/file/d/1j4yBCc-up4gWYYu_RmqCpZAunhUQhqdb/view, accessed on 2024-02-29.

Burdett, Kenneth, and Kenneth L. Judd. 1983. "Equilibrium Price Dispersion." *Econometrica* 51 (4): 955–969. 10.2307/1912045.

Dasgupta, Partha, and Eric Maskin. 1986. "The Existence of Equilibrium in Discontinuous Economic Games, I: Theory." *The Review of Economic Studies* 53 (1): 1–26. 10.2307/2297588.

Ellison, Glenn, and Alexander Wolitzky. 2012. "A Search Cost Model of Obfuscation." *The RAND Journal of Economics* 43 (3): 417–441. 10.1111/j.1756-2171.2012.00180.x.

Fudenberg, Drew, and David Levine. 1986. "Limit Games and Limit Equilibria." *Journal of Economic Theory* 38 (2): 261–279. 10.1016/0022-0531(86)90118-3.

Hämäläinen, Saara. 2022. "Multiproduct Search Obfuscation." *International Journal of Industrial Organization* 85 102863. 10.1016/j.ijindorg.2022.102863.

Hassan, Nahid. 2023. "Ghost Kitchen Update: Grubhub Approves, Students Don't." *The Cornell Review*, https://www.thecornellreview.org/ghost-kitchen-update-grubhub-approves-students-dont/, accessed on 2023-12-13.

Heggedal, Tom-Reiel, Espen R Moen, and Magnus V Knutsen. 2024. "The Effect of Potential Collusion on Equilibrium Prices." https://congresscreator.org/system/files/papers/Heggedal%2C%20Moen%20and%20Knutsen%20EEA%202024.pdf, accessed on 2024-08-27.

Ireland, Norman J. 2007. "Posting Multiple Prices to Reduce the Effectiveness of Consumer Price Search." *The Journal of Industrial Economics* 55 (2): 235–263. 10.1111/j.1467-6451.2007.00310.x.

Janssen, Maarten C. W., José Luis Moraga-Gonz\'{a}lez, and Matthijs R. Wildenbeest. 2005. "Truly Costly Sequential Search and Oligopolistic Pricing." *International Journal of Industrial Organization* 23 (5-6): 451–466. 10.1016/j.ijindorg.2005.01.013.

Johnen, Johannes, and David Ronayne. 2021. "The Only Dance in Town: Unique Equilibrium in a Generalized Model of Price Competition." *The Journal of Industrial Economics* 69 (3): 595–614. 10.1111/joie.12264.

McDonald, Stephen, and Colin Wren. 2018. "Multibrand Pricing as a Strategy for Consumer Search Obfuscation in Online Markets." *Journal of Economics & Management Strategy* 27 (2): 171–187. 10.1111/jems.12239.

Salz, Tobias. 2022. "Intermediation and Competition in Search Markets: An Empirical Case Study." *Journal of Political Economy* 130 (2): 310–345. 10.1086/717349.

Stahl, Dale O. 1989. "Oligopolistic Pricing with Sequential Consumer Search." *The American Economic Review* 79 (4): 700–712.

Varian, Hal R. 1980. "A Model of Sales." *The American Economic Review* 70 (4): 651–659.

Westphal, Ryan. 2023. "What You Don't Know Can't Pass Through: Consumer Beliefs and Pass-through Rates." `https://drive.google.com/file/d/1xf0SqM8OQEgl4DeNRHgaeDYvkZryOHGG/view`, accessed on 2023-10-07.

Appendix A Search Capture Model Extensions

A.1 Additional Consumer Information

While our baseline model allows us to study the impact of consumers who have been informed of the market structure and adjust their reservation price strategy accordingly, we could also consider an even stronger consumer information intervention. In our baseline model, consumers receive their first j quotes at random, and then they are able to optimally direct their future sequential searches. Here, we model consumers who receive their first quote at random, but then can optimally direct any future searches (even those initial j searches). This could occur in a model in which the initial consideration set is not exogenous, but instead is formed as the result of a sequential search process with convex costs (as in Hämäläinen (2022), for example). An informational intervention that might give rise to such a search technology would be one in which merging brands must advertise that they belong to the same parent company, allowing consumers who see one of the co-owned brands with their first search to direct their second search elsewhere.

Proposition 10 *If consumers are able to redirect their second initial search and μ_2 meets Inequality A16 in Appendix B, an equilibrium in which the merged firm plays a distinct pricing strategy exists. For all such values, average transacted prices are lower than when they can only adjust their reservation price. For sufficiently low μ_2 that meet inequality A17 in Appendix B, average transacted prices are lower than in the consolidated brands equilibrium.*

In this equilibrium, the merged firm plays a distinct price strategy in which it sets one of its prices to the consumers' reservation price and the other according to the same distribution as the outside firm. If a consumer who receives two initial price quotes gets a first price of r, they know that this came from the high priced brand and thus, they are indifferent between the two remaining brands for their additional search. If their first price is lower than r, they know that they should search the competing firm next. Reservation prices are pinned down by indifference towards search by consumers who receive a single quote of r. These consumers know that they are facing the same distribution of prices at both of the remaining brands, hence the reservation price is the average price offered in this distribution plus the consumers search cost.

Perhaps unsurprisingly, giving consumers additional information and ability to direct their search results in lower average prices than the case in which consumers can update their reservation price strategy, but cannot direct their initial searches. As before, breaking the illusion of competition with this stronger consumer information intervention can benefit consumers more than brand consolidation if the share of consumers who search exactly twice is sufficiently low. The set of parameters under which this information intervention is more beneficial than the consolidated equilibrium is a superset of the parameters under which the previously discussed information information leads to lower prices. Qualitatively, the conclusions of the previous model remain unchanged when we strengthen the information intervention in this way. Information about the merger can benefit consumers when the share of consumers who search twice is sufficiently low. Otherwise, forcing the merged firm to consolidate its brands is more helpful.

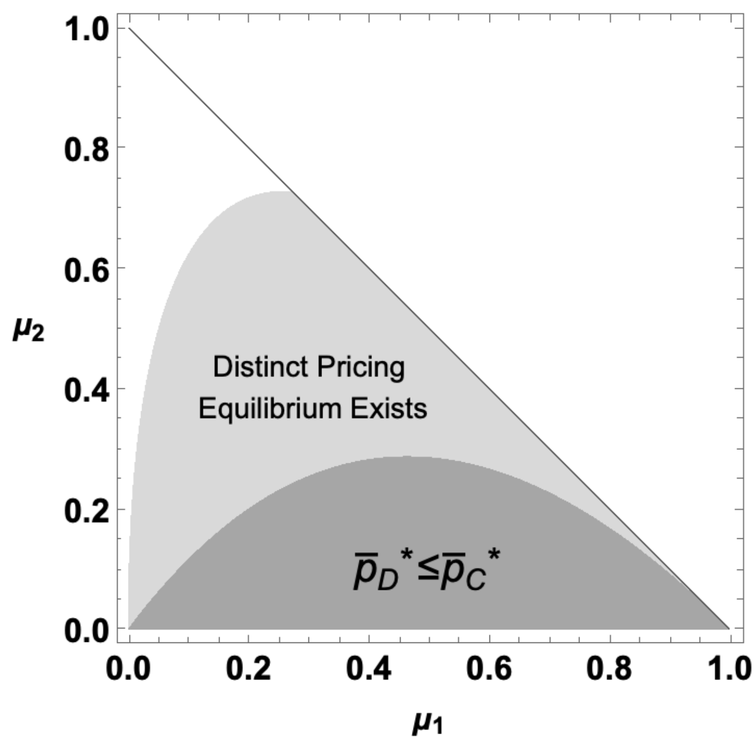


Figure A.1: **Equilibrium existence and pricing with stronger consumer information:** Parameter μ_k is the fraction of consumers who receive k initial price quotes, where $\mu_3 = 1 - \mu_1 - \mu_2$. Possible values for the vector μ lie in the triangular region. We plot the region for which an equilibrium in which firms play a distinct pricing strategy exists. We also compare average prices under this equilibrium ($\bar{p}_D$) to those under the consolidated brands equilibrium ($\bar{p}_C$).

A.2 Brand Proliferation

Several of our preceding results evaluate the benefits of requiring firms to consolidate brands following a merger. These results (Propositions 3–6) can also be reinterpreted as measuring the benefits of restricting brand proliferation by preventing a single-brand firm from introducing an additional brand for its existing product. This reinterpretation suggests that restricting brand proliferation benefits consumers either when they suffer from the illusion of competition or when sufficiently many see exactly two prices before considering additional search—and hence are at risk of capture by a multi-brand firm.

In this section, we develop additional insights by adapting our assumptions to better fit the brand-proliferation interpretation of our model. First, we assume that the status-quo is a symmetric duopoly with two single-brand firms. Equilibrium (characterized by Burdett and Judd (1983) and documented in Appendix Section B.5) is similar to the symmetric three-firm case presented in Section 2.1. We compare this to the case in which one of the two firms has introduced an additional brand (resulting in a market with 3 brands).

We first consider a *symmetric brand proliferation* in which the newly introduced brand is symmetric to both the existing co-owned brand and the existing independent brand. It receives the same share of searches as either of the existing brands. This results in the same equilibria discussed in Section 2.2.

Below, we characterize prices under the symmetric two-firm equilibrium and the symmetric brand proliferation equilibrium. Denote average transacted prices in these cases as $\bar{p}^*$ and $\bar{p}^*_D$ (which corresponds to the distinct pricing equilibrium in Proposition 4), respectively.

Proposition 11 *If r is the solution to the sophisticated consumers' search indifference condition, then following symmetric brand proliferation, $\bar{p}^* > \bar{p}^*_D$ iff Appendix equation (A18) holds. This condition holds only when μ_2 is sufficiently low and is shown graphically in Appendix Figure C.1.*

If consumers update their reservation prices following symmetric brand proliferation, the results are qualitatively similar to those about brand-preserving mergers. Allowing a firm to operate two brands will be worse than the baseline symmetric equilibrium unless the share of consumers considering exactly two prices is sufficiently low. Regulators or platforms interested in consumer welfare should therefore be hesitant to allow brand proliferation in settings where new brands would receive equal consideration to existing brands.

We can also compare this outcome to one in which the newly introduced brand is only seen by consumers who have already searched the existing two brands, an outcome we call *obscure brand proliferation.* Armstrong and Vickers (2024) are the first to introduce this possibility in Section 5.1 of their paper (see their discussion of "inferior" brand proliferation). Here the newly introduced brand does not receive any captive consumers or consumers who are considering exactly two prices. It is only able to compete for shoppers that consider all three prices. This assumption can capture a brand introduction that isn't heavily advertised, or a hypothetical policy by an online platform such as Grubhub to rank 'duplicate' brands at the bottom of search results.

Given exogenous consideration sets, Armstrong and Vickers (2024) are the first to prove the existence of and characterize the pricing equilibrium in this case. The firms' equilibrium pricing strategies follow directly from their work. Lemma 3 in Appendix 2 restates the pricing strategy for firms in our context (it follows from Armstrong and Vickers's (2024) Proposition 2 and Appendix). In this equilibrium, the new brand has no captive consumers so it can only win shoppers. The two-brand firm therefore optimally sets the price of its new brand below the price of its old brand. It selects its two prices from two adjoining but non-overlapping intervals. The outside firm then competes with both of these brands and mixes over prices in both intervals.

This allows us to compare symmetric brand proliferation to obscure brand proliferation for a fixed reservation price (when consumers are under the illusion of competition. Figure A.2 shows that if sufficiently many consumers consider only two brands (and thus are at risk of being captive to the proliferating firm), then the obscure brand proliferation results in lower prices than the symmetric brand proliferation. For regulators, or a platform that is interested in its consumers' welfare, this implies that, when consumers are under the illusion of competition, limiting the visibility of "duplicate" brands can sometimes reduce the harm to consumers introduced by brand proliferation.

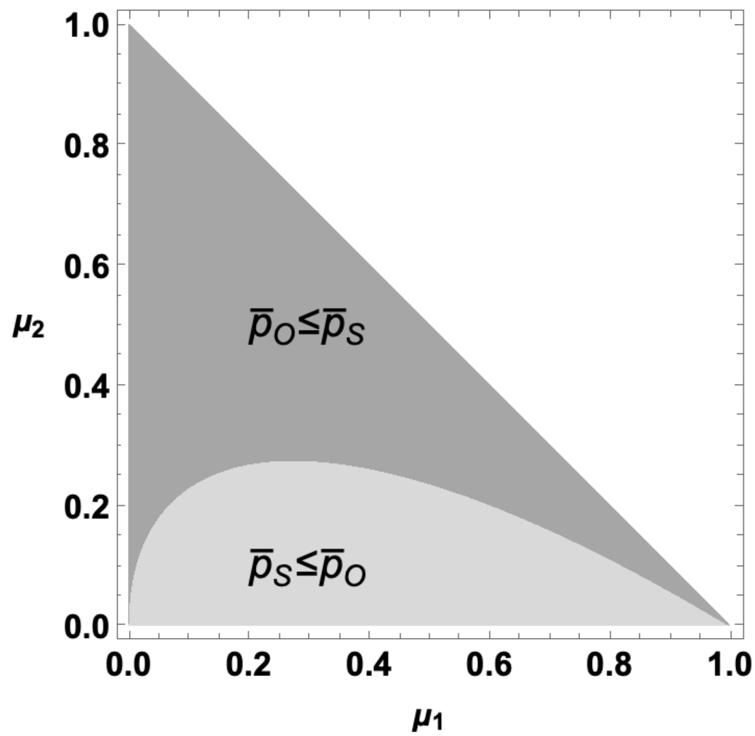


Figure A.2: **Ranking equilibrium transacted prices for fixed r / illusion of competition following brand proliferation:** Parameter μ_k is the fraction of consumers who receive k initial price quotes, where $\mu_3 = 1 - \mu_1 - \mu_2$. We compare average transacted prices between the symmetric brand equilibrium (p_S) and the obscure brand equilibrium (p_O). The obscure brand equilibrium results in lower average transacted prices than the symmetric brand equilibrium for sufficiently high values of μ_2 (the darker gray region).

We then extend these results to allow for endogenous search: We prove that there exists a consumer reservation price strategy that is a best response to the equilibrium price distributions set by the firms and thus, a reservation price equilibrium exists. We then characterize this reservation price, allowing us to compare outcomes when consumers are sophisticated and optimally set a reservation price in response to the actual market structure to the case when consumers are under the illusion of competition.

Proposition 12 *For all μ, following obscure brand proliferation, a reservation price equilibrium exists with consumer reservation price r_O characterized by equation (19) and price distributions characterized by equations (12)–(18) in Appendix B.4.*

If consumers endogenously respond to the introduction of the new brand, they set their reservation prices based on the expected price offered by the new brand (the most favorable of the three distributions). A consumer who sees two prices in the upper interval of prices is certain that these two prices came from the two existing brands, therefore they would draw a price from the new brand if they searched. A consumer who has only seen one price must have more pessimistic beliefs about the distribution of prices they are facing upon search, therefore they will not search if they receive a single price of r_O. Off-path beliefs are such that if a consumer sees a price above r_O, they think their next price will come from distribution F_N.

Next, we present results comparing the two-firm baseline to obscure brand proliferation when consumers update their reservation price following the proliferation. With exogenous consideration sets, studied by Armstrong and Vickers (2024), brand proliferation always leads to higher consumer prices. With endogenous search, this is only true under the illusion of competition. When sophisticated consumers update their reservation prices to reflect the true market structure, brand proliferation can lead to lower average transacted prices. Denote average transacted prices with sophisticated consumers as $\bar{p}^*$ under the two-firm symmetric equilibrium, $\bar{p}^*_O$ under the obscure brand proliferation equilibrium, $\bar{p}^*_D$ under symmetric brand proliferation (which corresponds to the distinct pricing equilibrium in Proposition 6).

Proposition 13 *If r is the solution to the sophisticated consumers' search indifference condition, then following obscure brand proliferation, $\bar{p}^* > \bar{p}^*_O$ iff Appendix equation (A19) holds. This condition holds only for sufficiently low μ_3. Additionally, $\bar{p}^*_O > \bar{p}^*_D$ iff Appendix equation (A20) holds. This condition holds only for sufficiently low μ_2. These are restrictions on the values of μ, which are shown graphically in Appendix Figure C.2.*

If consumers update their reservation values following an obscure brand proliferation, prices will increase unless the share of shoppers is sufficiently low. The intuition behind this is that the share of shoppers (μ_3) determines how much of an influence the new brand can have on the existing brands' pricing strategy. At one extreme, if $\mu_3 = 0$, the new brand receives zero initial searches. This means that for a given value of r, the new brand does not affect the pricing strategy of the existing two brands at all. As the share of shoppers increases, the new brand's potential market share does as well, and the existing brands move towards competing over higher prices. On the other hand, the introduction of the new brand causes consumers to set their reservation price with the belief that the next price will come from the new brand. This asymmetry allows for the possibility of lower reservation prices. If μ_3 is sufficiently low, this reservation price effect can outweigh the change in pricing patterns and average transacted prices can fall.

The second statement in the proposition demonstrates that prices are lower following obscure brand proliferation than they are following symmetric brand proliferation, unless the share of consumers considering exactly two prices is sufficiently low. If there are many consumers considering exactly two prices, then the policy of making duplicate brands less visible to searchers could be effective at lowering prices when proliferation cannot be blocked entirely.

Together, these propositions give us the optimal regulatory policy for all possible search technology. In Figure A.3 we plot the regions of μ for which different policies are optimal. Brand proliferation with a fixed reservation price can never lead to lower prices than the baseline. Therefore if regulators allow proliferation, it is always optimal to inform consumers of the new market structure, breaking the illusion of competition and allowing them to update their reservation prices. When there are sufficiently few shoppers who consider all three brands (the top right black region), it is optimal to allow brand proliferation but to limit the visibility of new, duplicate brands. When many consumers consider exactly two prices and more shoppers (the dark gray region), banning proliferation is first best, but reducing visibility can still reduce harm to consumers. When there are a low to intermediate amount of consumers considering two prices (the lightest region), banning proliferation is optimal; but conditional on allowing proliferation, symmetric brand proliferation leads to lower prices. Lastly, when very few consumers consider exactly two prices (the bottom gray region), allowing symmetric brand proliferation is first best for consumers.

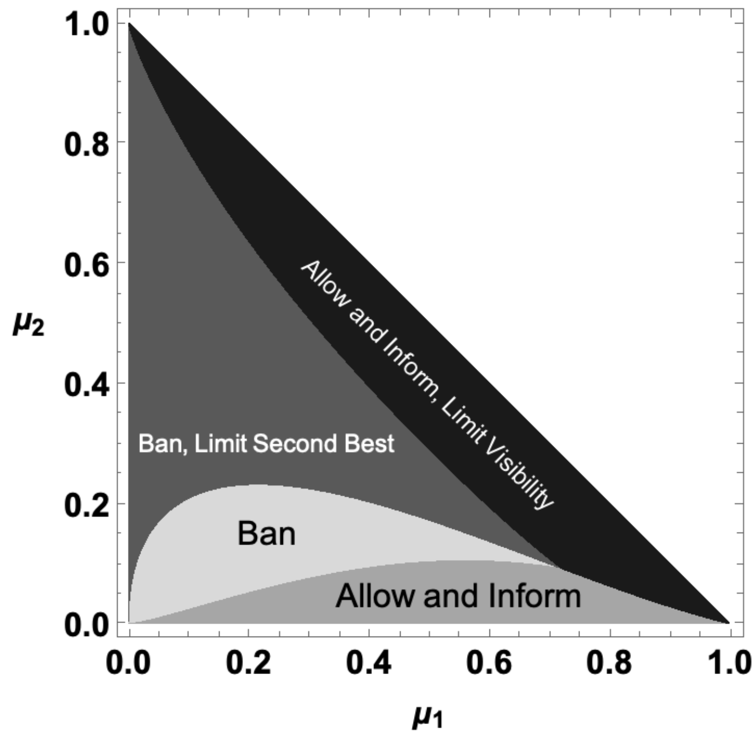


Figure A.3: **Optimal policy on brand proliferation:** Parameter μ_k is the fraction of consumers who receive k initial price quotes, where $\mu_3 = 1 - \mu_1 - \mu_2$. Possible values for the vector μ lie in the triangular region. The terms Ban and Allow refer to whether banning brand proliferation or allowing it is first best for consumer welfare. In all cases where proliferation is allowed, it is optimal to inform consumers that proliferation has occurred, breaking the illusion of competition. In the areas labeled Limit Visibility and Limit Second Best, obscure brand proliferation results in lower average transacted prices than symmetric brand proliferation.

A.3 $N > 3$ Firms

Studying the case of a merger between two firms in a three firm market allows us to study outcomes for all possible parameterizations of the search technology, but certain conclusions continue to hold when we consider a market with N single-brand firms of which K are considering a merger. Specifically, we can demonstrate that requiring the merging firms to consolidate their brands is beneficial to consumers under many circumstances but can be harmful in others.

Proposition 14 *In a market with N single-brand firms of which K are merging, where $2 \leq K < N$, there exist vectors of the search technology μ for which the average transacted* *price is higher in the brand-preserving merger equilibrium than in the brand-consolidating merger equilibrium.*

Consider the following example of a case in which consolidation would benefit consumers. Some consumers in the market search and receive K price quotes ($\mu_K > 0$) and all consumers in the market search and receive K or more price quotes ($\sum_{i=K}^{N} \mu_i = 1$). Prior to the merger, this would result in all firms pricing at marginal cost, a la Bertrand competition, and earning zero profits. If the firms merge and the merged firm continues to operate K separate brands, they will face some consumers who searched K times and only saw co-owned brands. This means that the jointly owned brands could set positive markups and earn profits. If, on the other hand, the firms merge but are forced to consolidate their brands, all consumers will see prices from at least $\min\{K, N-K+1\}$ independently owned brands. This will restore Bertrand competition, in which firms price at marginal cost, firms receive zero profits, and all surplus goes to consumers. More generally, if operating multiple brands causes sufficiently many consumers to flip from seeing multiple competing firms to only the merged firm, then consolidation would benefit consumers.

It is also possible to come to the opposite conclusion with more than 3 firms. It is possible for prices to be lower in a brand-preserving merger than in a brand-consolidating merger.

Proposition 15 *In a market with N firms, there exist vectors of the search technology μ, and a number of firms merging K, for which the average transacted price is higher in the brand-consolidating equilibrium than in the brand-preserving equilibrium.*

One such market in which consolidation could harm consumers is when 2 of N firms are merging and consumers' search technology is that considered by Stahl (1989) ($\mu_N = \mu, \mu_1 = 1-\mu$). In such a scenario, the average price, conditional on a given reservation price r, is the same across the brand-preserving and brand-consolidating equilibria. However, sophisticated consumers have a lower reservation price in the brand-preserving equilibrium than in the brand-consolidating equilibrium. Therefore, in this case, consumer welfare is higher if the merged firm continues to operate separate brands than if it consolidates them.

Appendix B Additional Results

B.1 Distinct Pricing Equilibrium Characterization

The price distribution described by Proposition 1 are characterized by Equations A1 - A3.

$$\underline{p}_D = \frac{(\mu_1 + \mu_2)r_D}{\mu_1 + 2\mu_2 + 3\mu_3}, \tag{A1}$$

$$r_D = \min\{1 + \int_{\underline{p}_D}^{r_D} p f_D(p)dp, v\}, \tag{A2}$$

and

$$F_D(p) = G_D(p) = 1 - \frac{(\mu_1 + \mu_2)(r_D - p)}{(\mu_2 + 3\mu_3)p}. \tag{A3}$$

B.2 Joint Pricing Equilibrium Characterization

The price distribution described by Proposition 2 are characterized by Equations A4 - A8.

$$\underline{p}_J = \frac{(2\mu_1 + \mu_2)r_J}{2\mu_1 + 3\mu_2 + 3\mu_3}, \tag{A4}$$

$$r_J = \min\{1 + \int_{\underline{p}_J}^{r_J} p g_J(p) dp, v\}, \tag{A5}$$

$$\lambda_J = \frac{\mu_1 + \mu_2}{3 - \mu_1}, \tag{A6}$$

$$F_J(p) = 1 - \frac{[\mu_1 + \lambda_J(2\mu_2 + 3\mu_3)](r_J - p)}{(2\mu_2 + 3\mu_3)(1 - \lambda_J)p}, \tag{A7}$$

and

$$G_J(p) = 1 - \frac{(2\mu_1 + \mu_2)(r_J - p)}{(2\mu_2 + 3\mu_3)p}. \tag{A8}$$

B.3 Joint Pricing Equilibrium Characterization

The price distribution described in Subsection 2.3 are characterized by Equations A9 - A13.

$$\lambda_C = \frac{\mu_1}{3 - \mu_1}, \tag{A9}$$

$$\underline{p}_C = \frac{2\mu_1 r_C}{2\mu_1 + 3\mu_2 + 3\mu_3}, \tag{A10}$$

$$F_C(p) = 1 - \frac{(\mu_1 + 3\lambda_C(\mu_2 + \mu_3))(r_C - p)}{3(1 - \lambda_C)(\mu_2 + \mu_3)p}. \tag{A11}$$

$$G_C(p) = 1 - \frac{2}{3}\frac{\mu_1(r_C - p)}{(\mu_2 + \mu_3)p}. \tag{A12}$$

$$r = \min\{1 + \int_{\underline{p}_C}^{r} q g_C(q) dq, v\}. \tag{A13}$$

B.4 Obscure brand proliferation

The price distributions described by Proposition 12 are characterized by equations (12)–(19), where we denote the probability that the single-brand firm plays a price in the higher interval by $\gamma = G_O(p^\dagger)$.

$$\lambda_O = \frac{2(1 - \mu_1 - \mu_2)}{2 - \mu_1}, \tag{12}$$

$$\gamma_O = \frac{\mu_1 \lambda_O}{\mu_1 + 2\mu_2(1 - \lambda_O)}, \tag{13}$$

$$p_O^\dagger = \frac{\mu_1}{\mu_1 + 2\mu_2(1-\lambda_O)} r_O, \tag{14}$$

$$\underline{p}_O = \frac{\mu_1 + 2\mu_2\gamma_O}{\mu_1 + 2\mu_2 + 2\mu_3} r_O, \tag{15}$$

$$F_O(p) = 1 - \gamma_O - \frac{(\mu_1 + 2\mu_2\gamma_O)(r_O - p)}{2\mu_2 p}, \tag{16}$$

$$F_N(p) = 1 - \frac{(\mu_1 + 2\mu_2)(p_O^\dagger - p)}{2\mu_3 p}, \tag{17}$$

$$G_O(p) = \begin{cases} 1 - (1-\lambda_O)\frac{p_O^\dagger}{p} & \underline{p}_O \leq p \leq p_O^\dagger \\ 1 - \frac{\mu_1(r_O - p)}{2\mu_2 p} & p_O^\dagger < p \leq r_O \end{cases}, \tag{18}$$

and

$$r_O = \min\{1 + \int_{\underline{p}_O}^{p_O^\dagger} (f_N(p) * p) dp, v\}. \tag{19}$$

B.5 Two firm symmetric equilibrium

When two symmetric firms compete in a market with search technology μ, the unique equilibrium is for both firms to mix over the interval $[\underline{p}, r]$ according to the common distribution F, where

$$\underline{p} = \frac{\mu_1}{2 - \mu_1} r, \tag{20}$$

$$F(p) = 1 - \frac{\mu_1(r-p)}{2\mu_2 p}, \tag{21}$$

and

$$r = \min\{1 + \int_{\underline{p}}^{r} f(p) * p dp, v\} \tag{22}$$

B.6 Search discouragement model—equilibrium characterization

The following Proposition 16 gives necessary and sufficient conditions that characterize equilibria of the pre-merger search discouragement game in which N firms each control a single brand. Prior to stating the proposition, it is helpful to introduce the notation $\sigma_2 = \Pr(p > r)$, the probability that a firm sets a price higher than consumers reservation price, and the following condition:

Condition 1 ***Monotonicity of*** ϕ***:*** *For some* $k \in \{1, \ldots, N-1\}$*,* ϕ *satisfies* $\phi_1 = \ldots = \phi_{k-1} = 1$*,* $\phi_k \in [0, 1)$*, and* $\phi_{k+1} = \ldots = \phi_N = 0$*.*

Condition 1 implies that ϕ_k is weakly decreasing in k, consumers can be mixing for at most one value of k, and $\phi_{N-1} < 1$.

Proposition 16 ***(1)*** *If $s \geq s_N^*$ then the unique symmetric equilibrium coincides with that in Varian's (1980) "Model of Sales". In particular, all non-shoppers stop searching after observing one price ($r \geq v$, $\phi_1 = \ldots \phi_{N-1} = 0$) and firms choose prices with support $[\underline{p}, v]$ for $\underline{p} = v / \left(1 + \frac{\mu}{1-\mu} N\right)$ and distribution*

$$F(p) = 1 - \left(\frac{1-\mu}{\mu} \frac{1}{N} \frac{v-p}{p} \right)^{\frac{1}{N-1}}, \tag{23}$$

earning industry profits $\pi = v(1-\mu)$.

(2) *If $s < s_N^*$ then:* ***(i)*** *As a function of $r < v$, ϕ, σ_1, and σ_2, demand at price p is*

$$q(p) = \begin{cases} \mu(1-F(p))^{N-1} + q_{NS} & \text{if } p \leq r \\ (1-F(p))^{N-1} - \frac{1}{N}(1-\mu)\sum_{n=1}^{N-1}(N-n)\sigma_1^n(1-F(p))^{N-1-n}\left(\Pi_{j=1}^{n-1}\phi_j\right)(1-\phi_n) & \text{if } r < p < v \\ \frac{1}{N}\left(\mu\sigma_1^{N-1} + (1-\mu)\sum_{n=1}^{N}(1-\phi_n)\Pi_{j=1}^{n-1}\sigma_1\phi_j\right) & \text{if } p = v \\ 0 & \text{if } p > v \end{cases} \tag{24}$$

where $\phi_N = 0$ and q_{NS} is non-shopper demand at any price $p \leq r$:

$$q_{NS} = \frac{1}{N}(1-\mu)\left(1 + \sum_{n=1}^{N-1}\left(\Pi_{j=1}^{n}\sigma_1\phi_j + (\sigma_2 - \sigma_1)\sum_{m=0}^{n-1}\left(\sigma_2^{n-m-1}\Pi_{j=1}^{m}\sigma_1\phi_j\right)\right)\right) \tag{25}$$

(ii) *Given ϕ satisfying Condition 1 and $\sigma_2 \in (0,1)$, the unique reservation price r and pricing distribution $F(p)$ such that firms' strategies are symmetric mutual best responses to each other and to consumers are characterized by equations (24)–(32):*

$$\sigma_{1A} = \{\sigma_1 : \sigma_1 > 0 \cap q(v, \sigma_1) = \lim_{p \to v^-} q(p, \sigma_1)\} \tag{26}$$

$$\sigma_1 = \min\{\sigma_2, \sigma_{1A}\} \tag{27}$$

$$\pi = vq(v) \tag{28}$$

$$p^* = \frac{\pi}{\sigma_2^{N-1} - \frac{1}{N}(1-\mu)\sum_{n=1}^{N-1}(N-n)\sigma_1^n\sigma_2^{N-1-n}\left(\Pi_{j=1}^{n-1}\phi_j\right)(1-\phi_n)} \tag{29}$$

$$r = \frac{\pi}{\mu\sigma_2^{N-1} + q_{NS}} \tag{30}$$

$$\underline{p} = \frac{\pi}{\mu + q_{NS}} \tag{31}$$

$$F(p) = \begin{cases} 0 & \text{if } p \leq \underline{p} \\ 1 - \left(\frac{1}{\mu}\frac{\pi}{p} - \frac{1}{\mu}q_{NS}\right)^{\frac{1}{N-1}} & \text{if } \underline{p} \leq p \leq r \\ \sigma_2 & \text{if } r \leq p \leq p^* \\ \{F(p) : F(p) \in [1-\sigma_2, 1-\sigma_1] \cap \pi = pq(p)\} & \text{if } p^* \leq p < v \\ 1 & \text{if } v \leq p \end{cases} \tag{32}$$

Note that $p \in [p^*, v)$ is only a relevant interval if $\sigma_2 > \sigma_{1A}$.

(iii) *The reservation price r specified above in equation (30) is a best response by consumers to the firm pricing distribution $F(p)$ specified above in equation (32) if and only if $s = s_r(\phi, \sigma_2)$, where*

$$s_r(\phi, \sigma_2) \equiv \pi \int_{\sigma_2}^{1} \left(\frac{1}{\hat{q}(\sigma_2)} - \frac{1}{\hat{q}(x)} \right) dx, \tag{33}$$

π is determined by equation (28), and $\hat{q}(x)$ is demand for $p \leq r$ written as a function of $x = 1 - F(p) \in [\sigma_2, 1]$ as

$$\hat{q}(x) = \mu x^{N-1} + q_{NS}. \tag{34}$$

Let k be the index characterized by Condition 1. The search strategy ϕ is a best response to firms' pricing distribution $F(p)$ if and only if

$$\begin{cases} s = s_k & \text{if } \phi_k \in (0,1) \\ s_k \leq s \leq s_{k-1} & \text{if } \phi_k = 0 \end{cases} \tag{35}$$

where $s_0 \equiv s_N^*$ and for $k \in \{1, \ldots, N-1\}$.

$$s_k \equiv \frac{1-\alpha_0}{\alpha_0} \sigma_1^k \int_r^v \left((1-\sigma_1) - (1 - F(p) - \sigma_1)\,(1 - F(p))^{N-k-1} \right) dp \tag{36}$$

which, in the special case $\sigma_1 = \sigma_2 \leq \sigma_{1A}$, reduces to

$$s_k \equiv \frac{1-\alpha_0}{\alpha_0} \sigma_1^k (1-\sigma_1)(v-r). \tag{37}$$

Therefore the strategies described above in (ii) form a symmetric Nash equilibrium if and only if $s = s_r(\phi, \sigma_2)$ and equation (35) both hold.

Appendix C Proofs of Propositions

C.1 Proposition 1

One can show that this is an equilibrium by first showing that all of the prices between $\underline{p}_D$ and r_D are best responses for the outside firm. The outside firm's profit function (conditional on the inside firm playing r_D with one of its prices) is

$$\pi_{OM}(p) = \frac{1}{3}[\mu_1 + \mu_2 + (\mu_2 + 3\mu_3)(1 - F(p))]p \tag{38}$$

Profits from playing r_D are then:

$$\pi_{OM}(r_D) = \frac{r_D}{3}[\mu_1 + \mu_2] \tag{39}$$

Likewise profits from playing $\underline{p}_D$ from Equation A1 are:

$$\pi_{OM}(\underline{p}_D) = \frac{1}{3}[\mu_1 + 2\mu_2 + 3\mu_3] \frac{(\mu_1 + \mu_2) r_D}{\mu_1 + 2\mu_2 + 3\mu_3} = \pi_{OM}(r) \tag{40}$$

The firm is therefore indifferent between these two prices. Playing a price above r_D results in zero profits as the consumer will search or purchase from the other firm. The merged firm plays prices weakly below r_D so the outside firm will lose a customer with certainty. The firm also will earn strictly lower profits from playing a price below $\underline{p}$ as they cannot gain any customers, but can only lower their markups. Therefore, the outside firm will not price above or below the interval. To show that they are indifferent over all prices in the interval, plug in $F(p)$ from Equation A3 to get

$$\pi_{OM}(p) = \frac{1}{3}[\mu_1 + \mu_2 + (\mu_2 + 3\mu_3)(\frac{(\mu_1 + \mu_2)(r_D - p)}{(\mu_2 + 3\mu_3)p})]p = \pi_{OM}(r) \tag{41}$$

Therefore, the outside firm is indifferent over all prices in $[\underline{p}_D, r_D]$.

The merged firm's profit function is Equation 3. If

$$G_D(p) = 1 - \frac{(\mu_1 + \mu_2)(r_D - p)}{(\mu_2 + 3\mu_3)p}. \tag{42}$$

We can plug this into Equation 3 and get

$$\begin{aligned} \pi_{JM}(p_L, p_H) = [\frac{1}{3}\mu_1 + \frac{1}{3}\mu_2 + (\frac{1}{3}\mu_2 + \mu_3)(\frac{(\mu_1 + \mu_2)(r_D - p_L)}{(\mu_2 + 3\mu_3)p_L})]p_L \\ +[\frac{1}{3}\mu_1 + \frac{1}{3}\mu_2(1 - G(p_H))]p_H. \end{aligned} \tag{43}$$

Simplifying:

$$\begin{aligned} \pi_{JM}(p_L, p_H) = \frac{1}{3}(\mu_1 + \mu_2)r \\ +[\frac{1}{3}\mu_1 + \frac{1}{3}\mu_2(1 - G(p_H))]p_H. \end{aligned} \tag{44}$$

This is no longer a function of p_L (in the support of G), therefore the merged firm is indifferent over values of p_L. Therefore, all p_L in $[\underline{p}_D, r]$ are in the best response correspondence for the merged firm.

To show that the merged firm maximizes profits by choosing $p_H = r_D$, we can show that the profit function is increasing in p_H, leading to the corner solution of r_D. The derivative of π_{JM} with respect to p_H is

$$\frac{\partial \pi_{JM}(p_L, p_H)}{\partial p_H} = \frac{1}{3}[[\mu_1 - \frac{\mu_2(\mu_1 + \mu_2)}{(\mu_2 + 3\mu_3)}]. \tag{45}$$

Which is weakly positive when:

$$\mu_1 - \frac{\mu_2(\mu_1 + \mu_2)}{(\mu_2 + 3\mu_3)} \geq 0 \iff 3\mu_1\mu_3 \geq \mu_2^2 \tag{46}$$

Which is the region of interest for μ. Therefore, if the consumer has a reservation price of r_D, this pricing strategy constitutes an equilibrium for both the merged and outside firm.

To show that the consumer will optimally choose a reservation price strategy with reservation price r_D from Equation A2, first note that $G(p)$ (and $F(p)$ which equals $G(p)$) has a lower average price than any other distribution of prices the consumer could face. If the consumer has not seen $p_H = r_D$ from the merged firm yet, then they face some mixture distribution of $G(p)$ and a degenerate distribution of r_D. Given this fact, it is obvious that a consumer will never search if they see a price at or below r_D as defined by Equation A2.

That they will search when they hold prices above r_D comes from the consumers' off path beliefs. If a consumer sees a price greater than r_D, they believe this price came from p_H. Therefore, they are facing $G(p)$ moving forward and therefore have positive value of search. This constitutes a reservation price strategy with reservation price r_D on the consumers' side. Therefore, the equilibrium exists.

C.2 Proposition 2

Start by considering the merged firms' profit maximization problem. Their profit function is again Equation 3. Consider first the constrained problem in which the firm must set both of its prices to p. Then its profits from playing $p = r_J$ are

$$\pi_{JM}(r_J, r_J) = [\frac{2}{3}\mu_1 + \frac{1}{3}\mu_2]r_J. \tag{47}$$

Profits from playing $p = \underline{p}_J$ from Equation A4 are then

$$\pi_{JM}(\underline{p}_J, \underline{p}_J) = [\frac{2}{3}\mu_1 + \mu_2 + \mu_3]\frac{(2\mu_1 + \mu_2)r_J}{2\mu_1 + 3\mu_2 + 3\mu_3} = \pi_{JM}(r_J, r_J) \tag{48}$$

So the firm is indifferent over r_J and $\underline{p}_J$ (in the constrained problem. To see that they are indifferent over the interval between these values, we can plug in the outside firm's distribution of prices from Equation A8 into the constrained profit function to get

$$\pi_{JM}(p, p) = [\frac{2}{3}\mu_1 + \frac{1}{3}\mu_2 + (\frac{2}{3}\mu_2 + \mu_3)(\frac{(2\mu_1 + \mu_2)(r_J - p)}{(2\mu_2 + 3\mu_3)p})]p = \pi_{JM}(r_J, r_J) \tag{49}$$

The firm is then indifferent between any price vector (p, p) where $p \in [\underline{p}_J, r_J]$. To see that this constrained problem is equivalent to the unconstrained profit maximization, we must show that holding $p_H = p_L$, profits are increasing in p_L, and holding $p_L = p_H$, profits are decreasing in p_H. The first of these expressions is

$$\frac{\partial \pi_{JM}(p_L, p_H)}{\partial p_L} = \frac{1}{3}[(\mu_1 + \mu_2) - (\mu_2 + 3\mu_3)(\frac{(2\mu_1 + \mu_2)}{(2\mu_2 + 3\mu_3)})]. \tag{50}$$

Which is positive when:

$$\frac{1}{3}[(\mu_1 + \mu_2) - (\mu_2 + 3\mu_3)(\frac{(2\mu_1 + \mu_2)}{(2\mu_2 + 3\mu_3)})] \geq 0 \iff \mu_2^2 \geq 3\mu_1\mu_3. \tag{51}$$

Which is what we have assumed. Likewise the derivative of profits with respect to p_H are

$$\frac{\partial \pi_{JM}(p_L, p_H)}{\partial p_H} = \frac{1}{3}[\mu_1 - \mu_2(\frac{(2\mu_1 + \mu_2)}{(2\mu_2 + 3\mu_3)})]. \tag{52}$$

Which is negative iff:

$$\frac{1}{3}[\mu_1 - \mu_2(\frac{(2\mu_1 + \mu_2)}{(2\mu_2 + 3\mu_3)})] \leq 0 \iff 3\mu_1\mu_3 \leq \mu_2^2 \tag{53}$$

Which again is what we assumed about μ. Therefore the merged firm optimally sets $p_L = p_H = p$ where $p \in [\underline{p}_J, r_J]$ (conditional on the consumers having a reservation price strategy with reservation price r_J and the outside firm mixing according to $G_J(p)$).

Next we must show that the outside firm is indifferent over all prices in the interval. Profits for the outside firm, conditional on the merged firm choosing $p_L = p_H = p$ where $p = r_J$ with probability λ_J and $p = p$ according to $F_J(p)$ otherwise, are

$$\pi_{OM}(p) = \frac{1}{3}[\mu_1 + (2\mu_2 + 3\mu_3)\lambda_J + (1 - \lambda_J)(2\mu_2 + 3\mu_3)(1 - F(p))]p. \tag{54}$$

Plugging in λ_J from Equation A6 to get profits from playing $p = r$:

$$\pi_{OM}(r) = \frac{1}{3}[\mu_1 + (2\mu_2 + 3\mu_3)(\frac{\mu_1 + \mu_2}{2\mu_1 + 3\mu_2 + 3\mu_3})]r_j. \tag{55}$$

Then profits from playing $p = \underline{p}_J$ are

$$\pi_{OM}(\underline{p}_J) = \frac{1}{3}[\mu_1 + 2\mu_2 + 3\mu_3]\frac{(2\mu_1 + \mu_2)r_J}{2\mu_1 + 3\mu_2 + 3\mu_3} = \pi_{OM}(r). \tag{56}$$

Lastly, profits from playing any price in between can be found by plugging in $F_J(p)$ from Equation A7.

$$\pi_{OM}(p) = \frac{1}{3}[\mu_1 + (2\mu_2 + 3\mu_3)\lambda_J + (2\mu_2 + 3\mu_3)(\frac{[\mu_1 + \lambda_J(2\mu_2 + 3\mu_3)](r_J - p)}{(2\mu_2 + 3\mu_3)p})]p = \pi_{OM}(r) \tag{57}$$

Therefore, the equilibrium holds from the firms' side. To see that this holds from the consumers' side as well, first note that consumers prefer to pull a price from G_J rather than any other possible mixture distribution of prices.

First consider the problem facing a consumer holding a single price of r_J as defined by Equation A5. This consumer knows that they drew this from the merged firm with certainty and therefore they are facing a mixture distribution of G_J and a degenerate distribution of r_J. This results in a higher average price than the average of G_J so they will not search. Prices above r_J do not occur in equilibrium so this is off-path. The consumer must believe that they will face the distribution G_J moving forward if they see a single price above r_J. Given the construction of r_J, this means they would search for any price above r_J.

Table 1: Mean transaction price

Equilibrium	Mean Price, Fixed r	Mean Price, Equilibrium r	Existence
Baseline	$\mu_1 r$	$\frac{\mu_1}{1-A\mu_1}$	Everywhere
Consolidated	$\frac{2(2-\mu_1)}{3-\mu_1}\mu_1 r$	$(\frac{2(2-\mu_1)}{3-\mu_1}\mu_1)(\frac{3(1-\mu_1)}{3(1-\mu_1)-2\mu_1\log(\frac{3-\mu_1}{2\mu_1})})$	Everywhere
Distinct	$(\mu_1+\frac{2}{3}\mu_2)r$	$(\mu_1+\frac{2}{3}\mu_2)(\frac{3(1-\mu_1)-2\mu_2}{3(1-\mu_1)-\mu_1\log(\frac{3-2\mu_1-\mu_2}{\mu_1+\mu_2})-2\mu_2-\mu_2\log(\frac{3-2\mu_1-\mu_2}{\mu_1+\mu_2})})$	$\mu_2^2 \leq 3\mu_1\mu_3$
Joint	$\frac{-6\mu_1^2+(6-\mu_2)\mu_2+\mu_1(12-5\mu_2)}{3(3-\mu_1)}r$	$(\frac{-6\mu_1^2+(6-\mu_2)\mu_2+\mu_1(12-5\mu_2)}{3(3-\mu_1)})(\frac{3*(1-\mu_1)-\mu_2}{3(1-\mu_1)-2\mu_1\log(\frac{3-\mu_1}{2\mu_1+\mu_2})-\mu_2-\mu_2\log(\frac{3-\mu_1}{2\mu_1+\mu_2})})$	$\mu_2^2 \geq 3\mu_1\mu_3$

Notes: A is given by equation 60.

Next consider a consumer holding two prices of r_J, this consumer believes these came from the merged firm with certainty, therefore they know they are facing G_J moving forward and won't search. Consumers who hold two prices above r_J believe they both came from the inside firm and therefore would search, thinking the price will come from G_J. The consumer therefore optimally plays a reservation price search strategy with r_J as the reservation price. This means that the joint pricing equilibrium as defined exists.

C.3 Propositions 3 - 6

The remaining propositions compare average transacted prices across equilibria. When the reservation price is fixed, this is straightforward. The distribution of transacted prices is a mixture distribution of the order distributions that each type of consumer faces. For example, in the symmetric equilibrium, the distribution of transacted prices is

$$T(p) = \mu_1 F(p) + \mu_2(1-(1-F(p))^2) + \mu_3(1-(1-F(p))^3). \tag{58}$$

Then, to calculate the average transacted price, one can take the derivative of this to find the density and integrate over the density times price. In this example, the average transacted price is

$$\bar{p} = \int_{\underline{p}}^{r} t(p) * p * dp = \mu_1 * r \tag{59}$$

This can be repeated for each of the four possible equilibria (symmetric, distinct pricing, joint pricing, consolidated brands). This gives us expressions for the average transacted price under each equilibrium in terms of the common, exogenous reservation price.

When the reservation price is endogenously determined by the consumers' search problem, we must first solve for the fixed point of the search indifference condition (Equations 2, A2, A5, and A13, respectively). This gives us a reservation price for each equilibrium. This can then be plugged into the average transacted price equation. The results of each of these calculations is listed in Table 1. Where:

$$A = \frac{\arctan[\frac{3-3\mu_1-2\mu_2}{\sqrt{3\mu_1(1-\mu_1-\mu_2)-\mu_2^2}}] - \arctan[\frac{\mu_2}{\sqrt{3\mu_1(1-\mu_1-\mu_2)-\mu_2^2}}]}{\sqrt{3\mu_1(1-\mu_1-\mu_2)-\mu_2^2}}. \tag{60}$$

Proposition 3 compares column 2, rows 1, 2, and 4. It is straightforward to show that

$$\mu_1 r \leq \frac{2(2-\mu_1)}{3-\mu_1}\mu_1 r \leq \frac{-6\mu_1^2+(6-\mu_2)\mu_2+\mu_1(12-5\mu_2)}{3(3-\mu_1)}r. \tag{61}$$

Therefore the proposition holds.

Proposition 4 is similar. The first part comes from

$$\mu_1 r \leq \frac{2(2-\mu_1)}{3-\mu_1}\mu_1 r, \tag{62}$$

and

$$\mu_1 r \leq (\mu_1 + \frac{2}{3}\mu_2) r. \tag{63}$$

The second part compares the distinct pricing equilibrium and shows that the distinct pricing equilibrium results in lower prices iff:

$$(\mu_1 + \frac{2}{3}\mu_2) r \leq \frac{2(2-\mu_1)}{3-\mu_1}\mu_1 r \iff \mu_2 \leq \frac{3\mu_1(1-\mu_1)}{2(3-\mu_1)} \tag{64}$$

Which is the statement.

Propositions 5 and 6 use column 3 of the table. In particular, $\bar{p}^* < \bar{p}_C^*$ iff equation (65) holds. Further, $\bar{p}_J^* > \bar{p}^*$ iff equation (66) holds and $\bar{p}_J^* > \bar{p}_C^*$ iff equation (67) holds. Both conditions are implied by the condition of Proposition 5 for a joint pricing equilibrium to exist ($\mu_2^2 \geq 3\mu_1\mu_3$). Finally, $\bar{p}_D^* < \bar{p}^*$ iff equation (A14) holds and $\bar{p}_D^* < \bar{p}_C^*$ iff equation (A15) holds. Both conditions are stricter than the condition of Proposition 6 for a distinct pricing equilibrium to exist ($\mu_2^2 \leq 3\mu_1\mu_3$). (In these equations, A is given by equation 60.)

$$\frac{\mu_1}{1-A\mu_1} < (\frac{2(2-\mu_1)}{3-\mu_1}\mu_1)(\frac{3(1-\mu_1)}{3(1-\mu_1)-2\mu_1\log(\frac{3-\mu_1}{2\mu_1})}) \tag{65}$$

$$(\frac{-6\mu_1^2+(6-\mu_2)\mu_2+\mu_1(12-5\mu_2)}{3(3-\mu_1)})(\frac{3*(1-\mu_1)-\mu_2}{3(1-\mu_1)-2\mu_1\log(\frac{3-\mu_1}{2\mu_1+\mu_2})-\mu_2-\mu_2\log(\frac{3-\mu_1}{2\mu_1+\mu_2})} > \frac{\mu_1}{1-A\mu_1} \tag{66}$$

$$(\frac{-6\mu_1^2+(6-\mu_2)\mu_2+\mu_1(12-5\mu_2)}{3(3-\mu_1)})(\frac{3*(1-\mu_1)-\mu_2}{3(1-\mu_1)-2\mu_1\log(\frac{3-\mu_1}{2\mu_1+\mu_2})-\mu_2-\mu_2\log(\frac{3-\mu_1}{2\mu_1+\mu_2})} > (\frac{2(2-\mu_1)}{3-\mu_1}\mu_1)(\frac{3(1-\mu_1)}{3(1-\mu_1)-2\mu_1\log(\frac{3-\mu_1}{2\mu_1})}) \tag{67}$$

$$(\mu_1+\frac{2}{3}\mu_2)(\frac{3(1-\mu_1)-2\mu_2}{3(1-\mu_1)-\mu_1\log(\frac{3-2\mu_1-\mu_2}{\mu_1+\mu_2})-2\mu_2-\mu_2\log(\frac{3-2\mu_1-\mu_2}{\mu_1+\mu_2})} < \frac{\mu_1}{1-A\mu_1} \tag{A14}$$

$$(\mu_1+\frac{2}{3}\mu_2)(\frac{3(1-\mu_1)-2\mu_2}{3(1-\mu_1)-\mu_1\log(\frac{3-2\mu_1-\mu_2}{\mu_1+\mu_2})-2\mu_2-\mu_2\log(\frac{3-2\mu_1-\mu_2}{\mu_1+\mu_2})} < (\frac{2(2-\mu_1)}{3-\mu_1}\mu_1)(\frac{3(1-\mu_1)}{3(1-\mu_1)-2\mu_1\log(\frac{3-\mu_1}{2\mu_1}} \tag{A15}$$

These expressions are complicated algebraically and are plotted using Mathematica in Figures 3 and A.1. In particular, Figure 3 plots the separate regions for which equations 65, A14, A15, and $\mu_2^2 \leq 3\mu_1\mu_3$ hold. Figure A.1 plots the regions for which no merger is best (equation 65 holds, and equation A14 fails), and for which a brand preserving merger is best (equations A14 and A15 hold). Code is available by request.

C.4 Proposition 7

We start by proving two initial lemmas before proceeding to Prove Proposition 7.

C.4.1 Lemma 1: Limit of r as $\sigma_2 \to 0$.

Lemma 1 *In equilibrium, if $s \leq s_N^*$ then $\lim_{\sigma_2 \to 0} r = v(1 - \phi_1)$.*

Proof. The fact that firm profits are equal at v and r in equilibrium implies that $r = \frac{vq(v)}{q(r)}$. Lemma 6 equation (24) implies $q(r) = \mu\sigma_2^{N-1} + q_{NS}$ and hence

$$\lim_{\sigma_2 \to 0} r = v \frac{\lim_{\sigma_2 \to 0} q(v)}{\lim_{\sigma_2 \to 0}(\mu\sigma_2^{N-1} + q_{NS})} \tag{68}$$

Lemma 7 equations (109)–(110) therefore imply the result (they apply since σ_1 goes to zero when σ_2 goes to zero by equation (27):

$$\lim_{\sigma_2 \to 0} r = v \frac{\frac{1}{N}(1-\mu)(1-\phi_1)}{\frac{1}{N}(1-\mu)} = v(1-\phi_1) \tag{69}$$

■

C.4.2 Lemma lem:single-sk-properties: Properites of s_k

Lemma 2 *Properties of s_k defined by equation (36) in Proposition 16 include:*

1. $s_k(\phi, \sigma_2 = 0) = 0 < s_k(\phi, \sigma_2 > 0)$
2. *For $\sigma_2 \in (0, \sigma_{1A}(\phi))$:*
   (a) $ds_k/d\phi_k > 0$
   (b)
   $$\lim_{\sigma_2 \to 0} \frac{ds_k}{d\sigma_2} = \begin{cases} \frac{1-\alpha_0}{\alpha_0} v\phi_1 & \text{if } k = 1 \\ 0 & \text{if } k > 1 \end{cases} \tag{70}$$

   $ds_k/d\sigma_2 =$ *and* $\lim_{\sigma_2 \to 0} \frac{ds_k}{d\sigma_2} =$

Proof.

Proof of Part 1: For $\sigma_2 = 0$, $\sigma_1 = \min\{\sigma_2, \sigma_{1A}\} = 0$. Moreover, by definition of σ_2, $F(p)$ must equal 1 for all $p \geq r$. Hence, equation (36) simplifies to

$$s_k(\sigma_2 = 0) = \frac{1-\alpha_k}{\alpha_k} \int_r^v \left(1 - (1 - F(p))^{N-k}\right) dp = 0. \tag{71}$$

For any $\sigma_2 > 0$, however, $\sigma_1 \in (0, 1)$ and $0 \leq 1 - \sigma_2 \leq F(p) \leq 1 - \sigma_1 < 1$ for all $p \in [r, v)$. Hence the integrand of s_k, which we denote by $\psi_k(p)$ is strictly positive:

$$\psi_k = (1 - \sigma_1) - (1 - F(p) - \sigma_1)(1 - F(p))^{N-k-1}$$
$$> (1 - \sigma_1) - (1 - F(p) - \sigma_1) = F(p) \geq 0. \tag{72}$$

Moreover, the limits of the integral satisfy $r < v$ (). Hence $s_k(\sigma_2 > 0) > 0$.

Proof of Part 2a: A restatement of Lemma 9 Part 6.

Proof of Part 2b: For $\sigma_1 = \sigma_2 = \sigma$, equation (37) shows that s_k changes with σ via r and σ. Thus

$$\frac{ds_k}{d\sigma_2} = \frac{\partial s_k}{\partial \sigma_1} + \frac{\partial s_k}{\partial r}\frac{dr}{d\sigma_2} \tag{73}$$

However, in the limit as σ goes to zero, the term $\frac{\partial s_k}{\partial r} = -\frac{1-\alpha_0}{\alpha_0}\sigma_1^k(1 - \sigma_1)$ also goes to zero and hence:

$$\lim_{\sigma_2 \to 0} \frac{ds_k}{d\sigma_2} = \lim_{\sigma_2 \to 0} \frac{\partial s_k}{\partial \sigma_1} = \lim_{\sigma_2 \to 0} \frac{1-\alpha_0}{\alpha_0}(v - r)(k\sigma^{k-1} - (k+1)\sigma^k)$$
$$= \begin{cases} \lim_{\sigma_2 \to 0} \frac{1-\alpha_0}{\alpha_0}(v - r) & \text{if } k = 1 \\ 0 & \text{if } k > 1 \end{cases} \tag{74}$$

The result follows by substituting $\lim_{\sigma_2 \to 0} r = v(1 - \phi_1)$ from Lemma 1.

■

C.4.3 Proof of Proposition 7 Part (1): Existence for sufficiently small α_0.

Proof.

Let $\hat{\sigma}_2$ be the smallest value of σ_2 for which s_r and s_1 intersect given $\phi_1 = 0$:

$$\hat{\sigma}_2 = \min\{\sigma_2 : s_r(\phi_1 = 0, \sigma_2) = s_1(\phi_1 = 0, \sigma_2)\} \tag{75}$$

We begin by stating and proving two initial claims.

Claim 1 *(1) The value $\hat{\sigma}_2$ exists and satisfies $0 < \hat{\sigma}_2 < 1$ and $s_r(\phi_1 = 0, \hat{\sigma}_2) \in (0, s^*)$. (2) For sufficiently small α_0, $\hat{\sigma}_2 < \sigma_{1A}$.*

Proof. (1) By Lemmas 10–2, at $\sigma_2 = 0$, $s_r(\phi_1 = 0, \sigma_2 = 0) = s^* > 0 = s_1(\phi_1 = 0, \sigma_2 = 0)$, while at $\sigma_2 = 1$, $s_r(\phi_1 = 0, \sigma_2 = 1) = 0 < s_1(\phi_1 = 0, \sigma_2 = 1)$. Thus s_r and s_1 cross at least once for $\sigma_2 \in (0, 1)$ and $s \in (0, s^*)$. $\hat{\sigma}_2$ is defined by the first such crossing.

(2) s_k is proportional to $\frac{1-\alpha_0}{\alpha_0}$, so is increasing without bound as α_0 decreases towards zero for all $\sigma_2 > 0$ (for which s_k is strictly positive by Lemma 2). In contrast, s_r is independent of α_0 and is bounded above by s^* for all σ_2 (Lemma 10). Thus for sufficiently small α_0, $s_1(\phi_1 = 0, \sigma_2 = \sigma_{1A}) > s_r(\phi_1 = 0, \sigma_2 = \sigma_{1A})$. In this case, $\hat{\sigma}_2 < \sigma_{1A}$. ■

Claim 2 *For sufficiently small α_0, $s_1(\phi_1 = 1, \sigma_2) > s_r(\phi_1 = 1, \sigma_2)$ for all $\sigma_2 > 0$.*

Proof. By Lemmas 10 and 2, for sufficiently small α_0, $\frac{d}{d\sigma_2} s_1(\phi_1 = 1, \sigma_2 = 0) > \frac{d}{d\sigma_2} s_r(\phi_1 = 1, \sigma_2 = 0)$, and hence $s_1(\phi_1 = 1, \sigma_2) > s_r(\phi_1 = 1, \sigma_2)$ for $\sigma_2 \in (0, \delta)$ in a neighborhood of zero for some $\delta > 0$. Next, s_k is proportional to $\frac{1-\alpha_0}{\alpha_0}$, so is increasing without bound as α_0 decreases towards zero for all $\sigma_2 > 0$ (for which s_k is strictly positive by Lemma 2). In contrast, s_r is independent of α_0 and is bounded above by s^* (Lemma 10). Thus, for sufficiently small α_0, it holds that $s_1(\phi_1 = 1, \sigma_2) > s_r(\phi_1 = 1, \sigma_2)$ for all σ_2 in the compact set $[\delta, 1]$. Therefore the result holds. ■

We next show existence separately for each of two exhaustive cases.

Case 1: For all $s \in [s_r(\hat{\sigma}_2), s^*]$, there exists an equilibrium with $\phi^* = 0$ and σ_2^* defined implicitly by $s_r(\phi_1 = 0, \sigma_2) = s$. We know that this σ_2^* exists and is unique for all $s \in [s_r(\hat{\sigma}_2), s^*]$ because s_r is continuous in σ_2 and Lemma 10 shows that $s_r(\phi_1 = 0, \sigma_2)$ strictly decreases in σ_2 from s^* to $s_r(\hat{\sigma}_2)$ over $\sigma_2 \in [0, \hat{\sigma}_2]$.

Case 2: For sufficiently small α_0, it holds that for all $s \in (0, s_r(\hat{\sigma}_2))$, there exists an equilibrium with $\phi_1^* \in (0,1)$ and ϕ_1^* and σ_2^* defined implicitly by $s_r(\phi_1^*, \sigma_2^*) = s_1(\phi_1^*, \sigma_2^*) = s$. We show existence of (ϕ_1^*, σ_2^*) in two steps.

First we show that, for sufficiently small α_0, it holds that for any $\sigma_2 \in [0, \hat{\sigma}_2]$, there exists $\phi_1^*(\sigma_2)$ that satisfies $s_r(\phi_1^*(\sigma_2), \sigma_2) = s_1(\phi_1^*(\sigma_2), \sigma_2)$ and that $\phi_1^*(\sigma_2)$ is a continuous function of σ_2. To do so, note that (1) $s_r(\phi_1, \sigma_2)$ is strictly decreasing in ϕ_1 (Lemma 10) while $s_1(\phi_1, \sigma_2)$ is strictly increasing in ϕ_1 for all $\sigma_2 \in (0, \sigma_{1A}$; (2) $s_r(\phi_1 = 0, \sigma_2) > s_1(\phi_1 = 0, \sigma_2)$ for all $sigma_2 < \hat{\sigma}_2$ (since by definition $\hat{\sigma}_2$ is the first crossing of $s_r(\phi_1 = 0, \sigma_2)$ and $s_1(\phi_1 = 0, \sigma_2)$); and (3) for sufficiently small α_0, $s_r(\phi_1 = 1, \sigma_2) < s_1(\phi_1 = 1, \sigma_2)$ for all $sigma_2 \in (0, \hat{\sigma}_2)$ (Claim 2). Points (2) and (3) imply that (for sufficiently small α_0), $s_1(\phi_1, \sigma_2$ and $s_r(\phi_1, \sigma_2)$ cross as ϕ_1 varies between 0 and 1 for all $\sigma_2 \in (0, \hat{\sigma}_2)$. Point (1) implies that (for sufficiently small α_0 for which $\hat{\sigma}_2 < \sigma_{1A}$ by Claim 1) there is most a single crossing, which satisfies the conditions for the implicit function theorem. Hence, the implicit function theorem implies that $\phi_1^*(\sigma_2)$ is a continuous function on $(0, \hat{\sigma}_2)$. This extends to the closed interval $[0, \hat{\sigma}_2]$ where $\phi_1^*(\sigma_2 = \hat{\sigma}_2) = 0$ (by definition of $\hat{\sigma}_2$ and Claim 1 and $\phi_1^*(\sigma_2 = 0) = 1$ by Lemmas 10–2).

Second, we note that $s_1(\phi_1^*(\sigma_2), \sigma_2)$ varies continuously in σ_2 from $s_1(\phi_1 = 0, \hat{\sigma}_2)$ to $s_1(\phi_1 = 1, \sigma_2 = 0) = 0$. Thus, by the intermediate value theorem, a solution to $s_r(\phi_1^*, \sigma_2^*) = s_1(\phi_1^*, \sigma_2^*) = s$ exists for all $s \in (0, s_r(\phi_1 = 0, \hat{\sigma}_2))$.

Together, Cases 1 and 2 demonstrate the result. ■

C.4.4 Proof of Proposition 7 Part (2): Limiting equilibrium as $\alpha_0 \to 0$.

Proof. By Proposition 16, in any equilibrium it holds that $s \geq \min_\phi s_k(\phi, \sigma_2^*)$. Because s_k is proportional to $\frac{1-\alpha_0}{\alpha_0}$, it increases without bound as α_0 goes to zero if $\sigma_2 > 0$ (for which s_k is strictly positive). Thus, for any $\sigma_2 > 0$, the necessary equilibrium condition $s \geq \min_\phi s_k(\phi, \sigma_2^*)$ fails for sufficiently small α_0. Thus, σ_2 must go to zero in all equilibria as α_0 goes to zero. By equation (27), this implies that σ_1 goes to zero as well.

For $\sigma_2 = 0$, $q(r) = \hat{q}(\sigma_2) = \hat{q}(0) = q_{NS} = \frac{1}{N}(1-\mu)$ and $q(v) = \frac{1}{N}(1-\mu)(1-\phi_1)$. Thus profits are $\pi = v\frac{1}{N}(1-\mu)(1-\phi_1)$. Plugging these expressions into equation (33) yields

$$s_r(\sigma_2 = 0) = v\frac{1}{N}(1-\mu)(1-\phi_1)\int_0^1 \left(\frac{1}{\frac{1}{N}(1-\mu)} - \frac{1}{\mu x^{N-1} + \frac{1}{N}(1-\mu)} \right) dx, \qquad (76)$$

which, using the definition of s^* in equation (7), can be re-written as

$$s_r = (1 - \phi_1)s^*. \tag{77}$$

Solving this equation for ϕ_1 yields

$$\phi_1 = 1 - \frac{s}{s^*} < 1. \tag{78}$$

By monotonicity Condition 1, $\phi_1 < 1$ implies $\phi_2 = \ldots = \phi_{N-1} = 0$. Given unique limiting values of ϕ and σ_2, Proposition 16 implies that the limiting equilibrium is unique.

As Lemma 1 states that $\lim_{\sigma_2 \to 0} r = v(1 - \phi_1)$, we can substitute in $\phi_1 = 1 - \frac{s}{s^*}$ yielding

$$r = v\frac{s}{s^*}. \tag{79}$$

Similarly, substituting for ϕ_1 in $\pi = v\frac{1}{N}(1 - \mu)(1 - \phi_1)$ yields

$$\pi = \frac{s}{s^*}v\frac{1}{N}(1 - \mu). \tag{80}$$

■

C.5 Proposition 8

For $n = 1, \ldots, \infty$ Take a sequence $\{\epsilon_n\} \to 0$. Define $P_n \equiv [0, r] \bigcup [r + \epsilon_n, v - \epsilon_n] \bigcup \{v\}$. Let $R_{i,n}$ be the set of probability measures over P_n^2 with a finite number of atoms. Let S_i be the set of probability measures over $[0, v]^2$ with a finite number of atoms. Let R be the Cartesian product of both firms' R_i, $R \equiv X_{i \in \{1,2\}} R_i$. Similarly, let S be the Cartesian product of both firms' S_i, $S \equiv X_{i \in \{1,2\}} S_i$. Let G be the original game extended to the mixed strategy space S. Let G_n be a modification of game G with the restricted strategy space R_n.

C.5.1 G_n has an equilibrium

We show that the game with the restricted strategy space has a Nash Equilibrium by Dasgupta and Maskin's (1986), Theorem 5. Let the subset A^* as defined there of strategy profiles at which payoff functions are discontinuous be the diagonal on which the lower price of the firms are equal. Payoffs are otherwise continuous everywhere on the restricted strategy space (they are not continuous at r from above and v from below, but we have removed these from the restricted strategy space). Payoffs are bounded at v and are weakly lower semi-continuous in actions. Therefore, by Dasgupta and Maskin's (1986), Theorem 5, G_n has a Nash equilibrium in mixed strategies for all n.

C.5.2 $\{R_n\}$ approximates S

Next, we show that $\{R_n\}$ approximates S under the m-topology according to Fudenberg and Levine's (1986) Definitions 3.6 and 7.2. We can show that R_n approaches S uniformly by showing that $\sup_{r_n \in R_n} [\epsilon(r_n, S) - \epsilon(r_n, R_n)] \to 0$. This can be rewritten as follows for all $r_n \in R_n$

$$[\epsilon(r_n, S) - \epsilon(r_n, R_n)] = [\pi(r_n^{**}) - \pi(r_n)] - [\pi(r_n^*) - \pi(r_n)] = [\pi(r_n^{**}) - \pi(r_n^*)] \leq \epsilon_n,$$

where $r_N^{**} = \arg\max_{r \in S} E[\pi(r, r_{n,-i})]$ and $r_N^* = \arg\max_{r \in R_n} E[\pi(r, r_{n,-i})]$.

This is bounded by ϵ_n. If all of the prices played in r_N^{**} are in the support of R_N, this term is equal to 0. However, if r_N^{**} contains prices that are not in R_n, these prices cannot result in larger demand than at $v - \epsilon_n$. Therefore the largest change in profits would be ϵ_n multiplied by the demand at $v - \epsilon_n$ which is bounded by 1. As $\epsilon_n \to 0$, this component goes to 0. Therefore, $\{R_n\}$ approximates S.

C.5.3 $\exists$ a subsequence $\{r_{n_k}\}$ that converges to $s \in S$

Next we show that there exists a subsequence $\{r_{n_k}$ that converges to $s \in S$. To do so, first note that because S is a compact subset of Euclidean space, then the probability measures on it are tight. Then by a corollary of Prokhorov's theorem, there exist a subsequence $\{r_{n_k}\}$ that converges weakly to $s \in S$.

C.5.4 Equilibrium Existence

Then by Fudenberg and Levine's (1986) Proposition 7.1, $\epsilon(r_n, R_n) \to \epsilon(s, S)$. We know that r_n is a Nash equilibrium of R_n, therefore $\epsilon(r_n, R_n) = 0$. This implies that $\epsilon(s, S) = 0$, and thus s is also a Nash equilibrium of S. The original game has an equilibrium in mixed strategies.

C.6 Proof of Corollary 8.1

Proof. In the limit as $\alpha_0 \to 0$, we can substitute $\phi_1 = 1 - \frac{s}{s_N^*}$ and $\phi_2 = 0$ into Proposition 8 equation (9), which yields equation (**??**). Industry profits from the 2-firm consolidated-brands scenario follow from Proposition **??**.

The inequality $\lim_{\alpha_0 \to 0} \Pi_{CB} < \lim_{\alpha_0 \to 0} \Pi_{LB}$ holds for

$$s < \frac{s_2^* s_4^*}{3s_4^* - 2s_2^*} \tag{81}$$

Notice that $s \leq s_4^*$ is a sufficient condition for this to hold because $s_4^* \leq s_2^*$ implies that $s_4^* \leq \frac{s_2^* s_4^*}{3s_4^* - 2s_2^*}$. Moreover, letting $f(N) = 1 + \frac{\mu}{1-\mu} N x^{N-1}$,

$$\frac{ds_N^*}{dN} = v \int_0^1 \left(\frac{f'(N)}{f^2(N)} \right) dx \tag{82}$$

and the integrated is weakly negative because

$$\begin{aligned} f'(N) = \frac{\mu}{1-\mu} \left(x^{N-1} + N \left(x^N \ln(x) x^{-1} + x^N (-1) x^{-2} \right) \right) \\ = \frac{\mu}{1-\mu} x^{N-2} \left(x + N \left(x \ln(x) - 1 \right) \right) \\ = \frac{\mu}{1-\mu} x^{N-2} \left(x + N \left(x \ln(x) - 1 \right) \right) \leq 0. \end{aligned} \tag{83}$$

■

C.7 Proposition 9

Proof. Existence: The condition $s < s_4^*(\mu)$ ensures that $r = r_{2,\text{Stahl}}(\hat{\mu}) < v$ because s^* is increasing in μ and decreasing in N, and hence $s < s_2^*(\hat{\mu}) < s_2^*(\mu) < s_4^*(\mu)$. At this equilibrium, consumers who have seen $p_1 = p_2 = r$ expect to draw future prices from $F_{2,\text{Stahl}}(\hat{\mu})$, and so are indifferent to searching, as that condition determines $r = r_{2,\text{Stahl}}(\hat{\mu})$ in the two-firm game with $\hat{\mu}$ shoppers. Consumers who have seen $p_1 = r$ face a worse distribution, because one of the three remaining prices will be r rather than a draw from $F_{2,\text{Stahl}}(\hat{\mu})$, and hence strictly prefer not to search. At prices above r, search is deterred by off equilibrium path beliefs. Hence, consumers' strategy is a best response.

Firms pricing their lower price in the region $(\underline{p}, r)$ earn profits from that offer of $\pi = \mu F(p)p + \frac{1}{4}(1-\mu)p$, which by substituting $\mu = \hat{\mu}/(2-\hat{\mu})$, can be re-written as $\pi = \frac{1}{2-\hat{\mu}}\left(\hat{\mu}F(p)p + \frac{1}{2}(1-\hat{\mu})p\right)$. This is the same as profits in the two-firm Stahl game with $\hat{\mu}$ shoppers rescaled by a constant. Hence, firms must be indifferent to mixing their lower price over $[\underline{p}, r]$. Finally, setting their higher price at r is optimal because only one price is needed to compete for shoppers. The other is optimally set at r to exploit captive consumers. Hence, firms are also playing a best response.

Profits In the sophisticated equilibrium, indifference implies profits are equal to those from setting $p_1 = p_2 = r$, which yields $(1-\mu)r$. Similarly, in the consolidated brands equilibrium, profits are equal to those from setting $p = r$, or $(1-\mu)r$. Profits are different in the two cases because the reservation prices differ. In the sophisticated equilibrium the reservation price is $r = r_{2,\text{Stahl}}(\hat{\mu}) = vs/s_2^*(\hat{\mu})$, which is strictly less than the consolidated brands equilibrium reservation price $r = r_{2,\text{Stahl}}(\mu) = vs/s_2^*(\mu)$. (The fact that r is decreasing in μ follows from the fact that s_N^* is increasing in μ, which is apparent from inspection of equation (7). ■

C.8 Proposition 10

Denote the probability that a type 2 consumer who sees a price of r searches the outside brand with their second search as γ. The profit function for the outside brand is:

$$\pi(p) = [\frac{1}{3}\mu_1 + \frac{1}{3}\gamma\mu_2 + \frac{1}{3}\mu_2(1-F(p) + \frac{1}{6}\mu_2 + \frac{1}{6}\mu_2(1-F(p)) + \mu_3(1-F(p))]p$$

Profits for the co-owned firm are:

$$\pi(p_L, p_H) = \begin{aligned} &[\frac{1}{3}\mu_1 + \frac{1}{6}\mu_2(1-G(p_H))]p_H \\ &+[\frac{1}{3}\mu_1 + \frac{1}{3}(1-\gamma)\mu_2 + \frac{1}{3}\mu_2(1-G(p_L)) + \frac{1}{6}\mu_2(1-G(p_L)) + \mu_3(1-G(p_L))]p_L \end{aligned}$$

For this equilibrium to work, $F(p)$ must equal $G(p)$. For that to happen,

$$[\frac{1}{3}\mu_1+\frac{1}{3}\gamma\mu_2+\frac{1}{3}\mu_2(1-F(p)+\frac{1}{6}\mu_2+\frac{1}{6}(1-F(p))+\mu_3(1-F(p))]p = [\frac{1}{3}\mu_1+\frac{1}{3}(1-\gamma)\mu_2+\frac{1}{3}\mu_2(1-F(p))+\frac{1}{6}\mu_2(1-F$$

Which implies

$$\gamma = \frac{1}{4}.$$

Then profits for the outside brand are

$$\pi(p) = [\frac{1}{3}\mu_1 + \frac{1}{4}\mu_2 + \frac{1}{2}\mu_2(1 - F(p) + \mu_3(1 - F(p))]p.$$

Profits from playing r are therefore

$$\pi(r) = [\frac{1}{3}\mu_1 + \frac{1}{4}\mu_2]r.$$

Setting this equal to profits from any p,

$$F(p) = G(p) = 1 - (\frac{4\mu_1 + 3\mu_2}{6\mu_2 + 12\mu_3})(\frac{r - p}{p}).$$

So,

$$f(p) = (\frac{4\mu_1 + 3\mu_2}{6\mu_2 + 12\mu_3})\frac{r}{p^2}.$$

This equilibrium then holds if the merged firms' profits are increasing in p_H.

$$\frac{\partial \pi}{\partial p_H} = [\frac{1}{3}\mu_1 + \frac{1}{6}\mu_2(1 - F(p_H))] - \frac{1}{6}\mu_2 f(p_H)p_H$$

Which is positive if

$$[\frac{1}{3}\mu_1 + \frac{1}{6}\mu_2(\frac{4\mu_1 + 3\mu_2}{6\mu_2 + 12\mu_3})(\frac{r - p}{p})] - \frac{1}{6}\mu_2(\frac{4\mu_1 + 3\mu_2}{6\mu_2 + 12\mu_3})\frac{r}{p} > 0$$

$$2\mu_1 > \mu_2(\frac{4\mu_1 + 3\mu_2}{6\mu_2 + 12\mu_3}) \tag{A16}$$

If this statement is true, the merged firm optimally sets its higher price to r. The merged firm is also indifferent over setting their lower price to any price in the equilibrium interval as is the outside firm. The equilibrium holds from the firms' side and all that remains is that a reservation price strategy for the consumer is optimal.

Consumers set their reservation price based on the distribution F. If they see a price of r, they know with certainty that it came from the higher priced brand of the merged firm and that they are facing the distribution F moving forward. Therefore,

$$r = s + \int_{\underline{p}}^{r} f(p) * p * dp. \tag{84}$$

If a consumer sees a price lower than r, they know they are facing either r or F and therefore they will not have incentive to search. Their off-path beliefs are such that if they see a price greater than r, they assume it came from the high priced brand of the merged firm and therefore they are facing F if they search. If they saw such a price they would search.

If an equilibrium exists, prices are lower under this information setting than the endogenous r case previously considered in the distinct pricing region of the parameter space if

$$\frac{4\mu_1 + 3\mu_2}{6\mu_2 + 12\mu_3} \leq \frac{\mu_1 + \mu_2}{\mu_2 + 3\mu_3} = \frac{4\mu_1 + 4\mu_2}{4\mu_2 + 12\mu_3}.$$

This is always true, therefore this intervention always lowers prices relative to just informing consumers of the market structure (removing the illusion of competition).

If an equilibrium exists, prices are lower under this information setting than the endogenous r case previously considered in the distinct pricing region of the parameter space if

$$\frac{4\mu_1 + 3\mu_2}{6\mu_2 + 12\mu_3} \leq \frac{(2\mu_1 + \mu_2)}{2\mu_2 + 3\mu_3} = \frac{6\mu_1 + 3\mu_2}{6\mu_2 + 9\mu_3}.$$

Which again, is always true.

Prices are lower under this information setting than the consolidated brands equilibrium with endogenous r if

$$\frac{4\mu_1 + 3\mu_2}{6\mu_2 + 12\mu_3} \leq \frac{2\mu_1}{3\mu_2 + 3\mu_3}$$

$$\mu_2^2 + \mu_2(1 - \mu_1 - \mu_2) \leq \frac{4}{3}\mu_1(1 - \mu_1 - \mu_2) \tag{A17}$$

Which is true for sufficiently low μ_2.

C.9 Proposition 11

First, it is worth noting that the reservation price for the baseline two firm symmetric equilibrium is

$$r = \frac{2(1 - \mu_1)}{2(1 - \mu_1) - \mu_1 * \log(\frac{2-\mu_1}{\mu_1})} \tag{85}$$

Then average transacted prices in the baseline are $\mu_1 r$. When $\mu^2 \geq 3\mu_1\mu_3$ and the joint pricing equilibrium is played following symmetric brand proliferation, then average transacted prices are found in Table 1 row 4. Prices are lower in the baseline than following proliferation to the joint pricing equilibrium if

$$\mu_1 \frac{2(1 - \mu_1)}{2(1 - \mu_1) - \mu_1 * \log(\frac{2-\mu_1}{\mu_1})} \leq (\frac{-6\mu_1^2 + (6 - \mu_2)\mu_2 + \mu_1(12 - 5\mu_2)}{3(3 - \mu_1)})(\frac{3 * (1 - \mu_1) - \mu_2}{3(1 - \mu_1) - 2\mu_1 \log(\frac{3-\mu_1}{2\mu_1+\mu_2}) - \mu_2 - \mu_2 \ldots} \tag{86}$$

Which is always true. When $\mu^2 \leq 3\mu_1\mu_3$ and the distinct pricing equilibrium holds following symmetric brand proliferation, then average transacted prices are found in Table 1 row 3. Then prices are higher in the baseline than following symmetric brand proliferation iff

$$\mu_1 \frac{2(1 - \mu_1)}{2(1 - \mu_1) - \mu_1 * \log(\frac{2-\mu_1}{\mu_1})} > (\mu_1 + \frac{2}{3}\mu_2)(\frac{3(1 - \mu_1) - 2\mu_2}{3(1 - \mu_1) - \mu_1 \log(\frac{3-2\mu_1-\mu_2}{\mu_1+\mu_2}) - 2\mu_2 - \mu_2 \log(\frac{3-2\mu_1-\mu_2}{\mu_1+\mu_2})}) \tag{A18}$$

This is plotted in Figure C.1. This covers all cases of μ for symmetric brand proliferation with consumers who update their reservation prices to the equilibrium value.

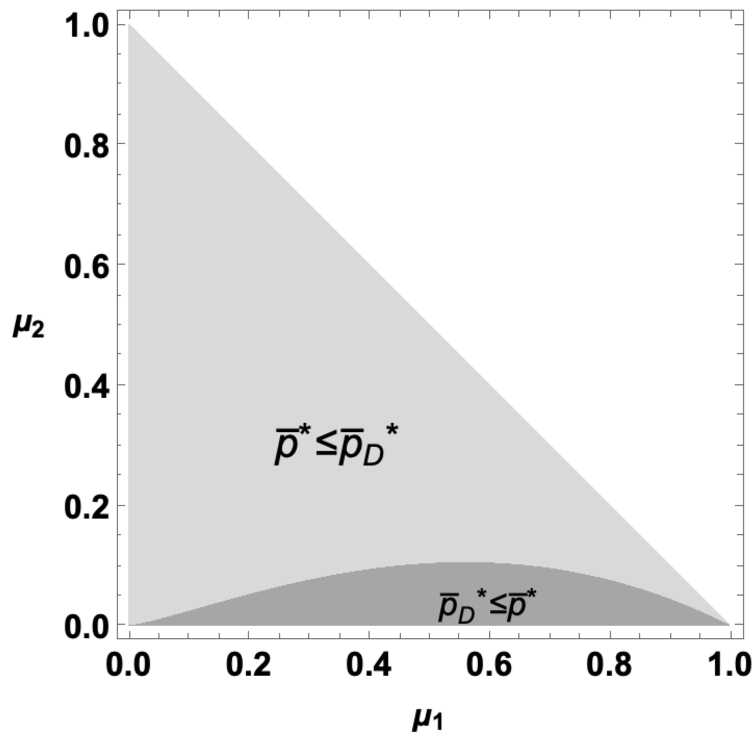


Figure C.1: **Ranking equilibrium transaction prices for symmetric brand proliferation:** Parameter μ_k is the fraction of consumers who receive k initial price quotes, where $\mu_3 = 1 - \mu_1 - \mu_2$. Possible values for the vector μ lie in the triangular region. We compare average transacted prices between a symmetric 2 firm equilibrium ($\bar{p}^*$) and prices following a symmetric brand proliferation ($\bar{p}^*_D$). Here we assume that consumers are sophisticated and update their reservation prices following brand proliferation.

C.10 Proposition 12

Given an exogenous reservation price, Armstrong and Vickers (2024) characterize equilibrium under obscure brand proliferation (a special case of their Proposition 2 discussed in their Section 5.1). The following lemma follows directly from that work.

Lemma 3 *For all μ, and a fixed reservation price r_O, following obscure brand proliferation, a pricing equilibrium exists. The single-brand firm sets its price in the interval $[\underline{p}_O, r_O]$, according to distribution G_O. Price $p^\dagger \in (\underline{p}, v)$ divides this interval into lower and upper regions. The two-brand firm sets the price of its new brand in the lower region $[\underline{p}, p^\dagger]$, according to distribution F_N. It sets the price of its existing brand in the upper region $[p^\dagger_O, r_O]$, pricing at the top, $p = v$, with probability γ_O and mixing over the interval according to distribution F_O with complement probability. Equations (12)–(18) in Appendix B.4 characterize these price distributions.*

When consumers' reservation price r_O is endogenous, it is characterized by equation (19). This equation is an indifference condition that ensures consumers are exactly indifferent between paying a price of r_O and receiving an additional draw from the distribution F_N.

A consumer who has seen two price quotes of r_O knows for certain that they came from the outside brand and the multi-brand firm's old brand. This consumer would know with certainty that they are drawing from F_N next. Any consumer who is holding at least one lower price would strictly prefer not to search because a) they are paying a lower price and b) their beliefs about the distribution of prices they are drawing from can only be worse than F_N. Any consumer with only a single price at or below r would similarly not prefer to search because they face a convex combination of F_N and a distribution of higher prices. This means that no consumer who is holding a price at or below r_O will ever choose to search. Prices above r_O do not occur in equilibrium, but off-path beliefs are such that consumers believe they will draw from F_N if they search. Therefore, if they did see a price above r_O, they would prefer to search. This constitutes a reservation price strategy with r_O as the reservation price. Hence, this constitutes a Perfect Bayesian Nash Equilibrium.

C.11 Proposition 13

Baseline profits are again $\mu_1 r$ where r is defined in Equation 85. Profits for a given reservation price following obscure brand proliferation are given by

$$\bar{p}_O = (\mu_1 + \mu_2\gamma + \mu_3(\frac{\mu_1 + 2\mu_2\gamma}{\mu_1 + 2\mu_2 + 2\mu_3}))r \geq \mu_1 r.$$

The reservation price following obscure brand proliferation is given by

$$r_O = \frac{1}{1 - \frac{(\mu_1+2\mu_2)}{2\mu_3}(\frac{\mu_1}{\mu_1+2\mu_2(1-\lambda_O)})\log(\frac{1}{1-\lambda_O})}. \tag{87}$$

Therefore prices are higher in the baseline than following obscure brand proliferation iff

$$\mu_1\frac{2(1-\mu_1)}{2(1-\mu_1) - \mu_1 * \log(\frac{2-\mu_1}{\mu_1})} > (\mu_1+\mu_2\gamma_O+\mu_3(\frac{\mu_1 + 2\mu_2\gamma_O}{\mu_1 + 2\mu_2 + 2\mu_3}))(\frac{1}{1 - \frac{(\mu_1+2\mu_2)}{2\mu_3}(\frac{\mu_1}{\mu_1+2\mu_2(1-\lambda_O)})\log(\frac{1}{1-\lambda_O})}) \tag{A19}$$

Average transacted prices are higher in the obscure brand proliferation case than in the symmetric brand proliferation case iff

$$\begin{aligned}(\mu_1 + \frac{2}{3}\mu_2)(&\frac{3(1-\mu_1) - 2\mu_2}{3(1-\mu_1) - \mu_1\log(\frac{3-2\mu_1-\mu_2}{\mu_1+\mu_2}) - 2\mu_2 - \mu_2\log(\frac{3-2\mu_1-\mu_2}{\mu_1+\mu_2})}) \\ > (\mu_1 + \mu_2\gamma_O + \mu_3(\frac{\mu_1+2\mu_2\gamma_O}{\mu_1+2\mu_2+2\mu_3}))(&\frac{1}{1 - \frac{(\mu_1+2\mu_2)}{2\mu_3}(\frac{\mu_1}{\mu_1+2\mu_2(1-\lambda_O)})\log(\frac{1}{1-\lambda_O})})\end{aligned} \tag{A20}$$

Both of these are plotted in Figure C.2. This covers all cases of μ for obscure brand proliferation with consumers who update their reservation prices to the equilibrium value.

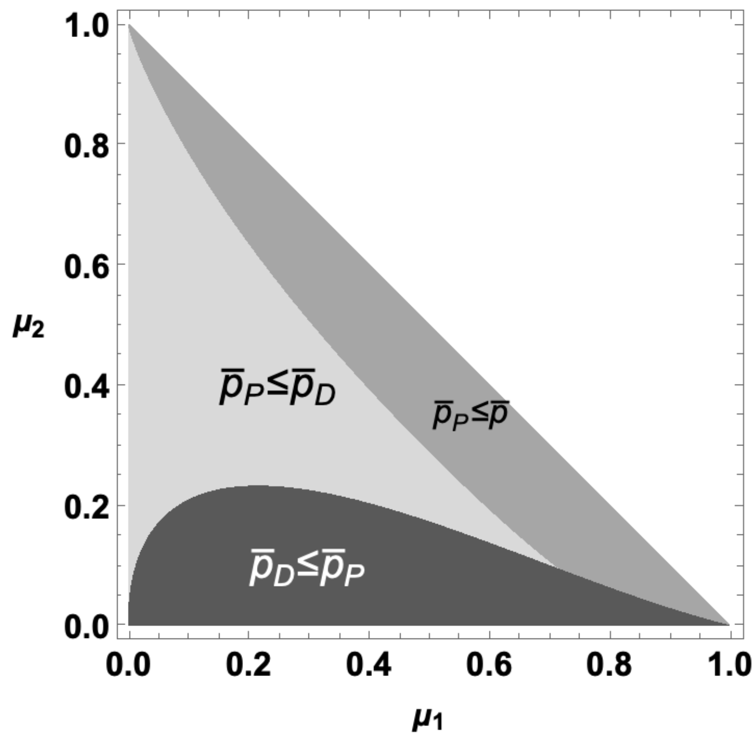


Figure C.2: **Ranking equilibrium transaction prices for obscure brand proliferation:** Parameter μ_k is the fraction of consumers who receive k initial price quotes, where $\mu_3 = 1 - \mu_1 - \mu_2$. Possible values for the vector μ lie in the triangular region. We compare average transacted prices between a symmetric 2 firm equilibrium ($\bar{p}^*$), prices following an obscure brand proliferation ($\bar{p}_O^*$), and prices following a symmetric brand proliferation ($\bar{p}_D^*$). Here we assume that consumers are sophisticated and update their reservation prices following brand proliferation.

C.12 Proposition 14

We will prove existence by construction. Consider a market in which $\mu_K = 1$. In the consolidated equilibrium, all consumers receive $\min\{K, N - K + 1\} \geq 2$ prices, each from an independent firm. In this equilibrium, firms (including the consolidated firm) cannot profitably deviate from marginal cost pricing, $p = 0$. If they play a price greater than 0, they will not sell to any consumers as all consumers have an offer from a firm offering a price of 0.

In the multi-brand case, the merged firm chooses prices for each of its K brands. A fraction of the consumers will receive their K price quotes all from the merged firms' brands. These consumers are captive to the merged firm, and given a non-zero search cost, allow the firm to earn positive profits by charging a price of $r > 0$ at each of these brands. Given this possibility, the firm must earn positive profits in equilibrium. Therefore average transacted prices must be greater than 0, and thus greater than in the consolidated equilibrium.

C.13 Proposition 15

We will, again, prove existence by construction. Consider a market in which $\mu_N = \mu$ and $\mu_1 = 1 - \mu$ and $K = 2$.

First, consider the equilibrium in the multi-brand case. Without loss of generality, order the prices offered by the multi-brand firm's brands such that $p_L \leq p_H$. This firm can only win shoppers with price p_L. Every shopper is considering all N brands (including the 2 offered by the multi-brand firm) and will purchase at the lowest of these prices. By this naming convention, it is not possible for brand 2 to transact with shoppers. This means that the only consumers that brand 2 could possibly interact with are non-shoppers who are only considering one brand. The multi-brand firm therefore maximizes profits by setting $p_H = r$.

Conditional on setting price 2 to r, brand 1 of the multi-brand firm and each of the outside firms are symmetric. They each face the following profit function:

$$\pi_M(p) = (\frac{1}{N}(1-\mu) + \mu(1 - F_M(p))^{N-2}) * p \tag{88}$$

Solving for the equilibrium distribution in the usual manner (setting profits equal to profits from playing r (or $\pi(r) = \frac{1}{N}(1-\mu)r$):

$$F_M(p) = 1 - [\frac{(1-\mu)}{N\mu}\frac{r-p}{p}]^{\frac{1}{N-2}} \tag{89}$$

As this is a market with perfectly inelastic demand and no extensive margin, consumer surplus and total firm profits add to a constant. Therefore, if we demonstrate that firm profits are higher (lower) in one equilibrium than another, we have also demonstrated that consumer surplus is lower (higher). Total firm profits in this equilibrium are:

$$\Pi_M = \frac{(1-\mu)r}{N} + (N-1) * \frac{(1-\mu)r}{N} = (1-\mu)r \tag{90}$$

Average price is then just $r - \Pi$ or μr. The same as in the distinct price equilibrium with 3 firms (and this search technology).

Next consider the equilibrium for the consolidated-brand case. Here the profit function for the merged firm is:

$$\pi_{JC}(p) = (\frac{2}{N}(1-\mu) + \mu(1 - F_{OC}(p))^{N-2})p \tag{91}$$

From our typical equilibrium condition this firm will earn profits of $\pi(r) = \frac{2}{N}(1-\mu)p$. In order to induce the outside brands to mix over the same interval of prices, the merged firm needs to choose a price of r with positive probability λ_C. The outside firm(s)' profit function is then:

$$\pi_{OC}(p) = \frac{1}{N}(1-\mu) + \mu(\lambda_C + (1-\lambda_C)(1 - F_{JC}(p)))(1 - F_{OC}(p))^{N-3} * p \tag{92}$$

As per usual the outside firms will be indifferent over all prices so their profits are equal to their profits from playing r which are:

$$\pi_{OC}(r) = \frac{(1-\mu)r}{N} \tag{93}$$

Therefore total firm profits in this market are:

$$\Pi_C = (1-\mu)r \tag{94}$$

Therefore, when r is fixed and $K = 2$, profits and thus average transacted prices are the same in the multi-brand and consolidated equilibria.

When r is determined endogenously by non-shopper consumers' indifference to search upon seeing a price of r, we need to consider the non-shopper consumers' beliefs upon seeing a price of r. In both equilibria, seeing a price of r, causes the consumer to know with certainty that this price came from the merged firm.

In the consolidated equilibrium, this means that they are facing the distribution $F_{OC}(p)$ should they search. In the brand-preserving equilibrium this means that they are facing the distribution $F_M(p)$. In both cases, all remaining firms are identical and play the same distribution. To compare the reservation prices consider the distribution F_{OC}. Setting profits equal to the profits from playing r_C in the case of $K = 2$:

$$\frac{1}{N}(1-\mu) + \mu(\lambda_C + (1-\lambda_C)(1-F_{JC}(p)))(1-F_{OC}(p))^{N-3} * p = \frac{1}{N}(1-\mu)r_M \tag{95}$$

If $\lambda_C = 0$ and $F_{JC}(p) = F_{OC}(p)$ for all p, then this would be the same condition as in the multi-brand case, and the distribution F_OC would be equal to the distribution F_M (conditional on a reservation price). However, the distribution played by the consolidated firm is higher than that of the outside firms in the multi-brand case, therefore F_{OC} has a higher mean than F_M. Therefore, for $K = 2$, $r_M < r_C$ and overall average transacted prices are lower for the multi-brand equilibrium than the consolidated equilibrium.

C.14 Proposition 16

Before proceeding to prove Proposition 16, we state and prove a sequence of supporting lemmas.

C.14.1 Lemma 4: Optimal Consumer Search

Lemma 4 characterizes optimal consumer search.

Lemma 4 *Consumers' optimal search strategy is characterized by the following conditions:*

1. *Once a consumer sees any price strictly below v, they know that they are in the competitive state, and they follow a stationary reservation price strategy for the rest of the game. They continue searching until either (1) buying at the first price they observe*

less than or equal to r, or (2) if all prices are strictly above r, they search all prices and buy at the minimum price. The reservation price r is the unique solution to

$$s = \int_{\underline{p}}^{r} F(p)dp \tag{96}$$

if $\int_{\underline{p}}^{v} F(p)dp \leq s$ and is $r = +\infty$ otherwise. This implies $r > \underline{p}$.

2. *If $\sigma_1 \in \{0, 1\}$, then it is optimal to stop searching if the first price is v ($\phi_1 = \ldots = \phi_{N-1} = 0$).*

3. *If $\sigma_1 \in (0, 1)$, then:*

   (a) *ϕ satisfies Condition 1 or $\phi_1 = \ldots = \phi_{N-1} = 1$.*

   (b) *Let $\hat{B}_k$ be the net benefit of one additional search conditional on having observed $p_1 \ldots p_k = v$ and a continuation search strategy to use the optimal reservation price strategy if $p_{k+1} < v$ but to stop searching if $p_{k+1} = v$ (i.e. $\phi_{k+1}, \ldots, \phi_{N-1} = 0$) regardless if that is optimal. $\hat{B}_k$ is strictly decreasing in k and s (for $k \in \{1, \ldots, N-1\}$).*

   - *Necessary and sufficient conditions for ϕ to be optimal are that for all $k \in \{1, \ldots, N-1\}$: If $\hat{B}_k < 0$ then $\phi_k = 0$. If $\hat{B}_k > 0$ then $\phi_k = 1$. If $\hat{B}_k = 0$ then $\phi_k \in [0, 1]$.*
   - *Necessary and sufficient conditions for ϕ to be optimal are that for all $k \in \{1, \ldots, N-1\}$: If $\phi_k = 0$ then $\hat{B}_k \leq 0$. If $\phi_k = 1$ then $\hat{B}_k \geq 0$. If $\phi_k \in (0, 1)$ then $\hat{B}_k = 0$.*
   - *Equivalently:*
     - *Strategy $\phi_1 = \ldots = \phi_{N-1} = 0$ is optimal iff $\hat{B}_1 \leq 0$.*
     - *For $k \in \{2, \ldots, N-1\}$, strategy $\phi_1 = \ldots = \phi_{k-1} = 1$ and $\phi_k = \ldots = \phi_{N-1} = 0$ is optimal iff $\hat{B}_k \leq 0 \leq \hat{B}_{k-1}$.*
     - *Strategy $\phi_1 = \ldots = \phi_{N-1} = 1$ is optimal iff $\hat{B}_{N-1} \geq 0$.*
     - *For $k \in \{1, \ldots, N-1\}$, strategy $\phi_1 = \ldots = \phi_{k-1} = 1 > \phi_k > 0 = \phi_{k+1}, \ldots, \phi_{N-1}$ is optimal iff $\hat{B}_k = 0$.*

     No other strategies can be optimal.
   - *Equivalently: Let $\hat{s}_k$ be the unique solution to $\hat{B}_k(\hat{s}_k) = 0$. $\hat{s}_k$ is strictly positive and strictly decreasing in k.*
     - *Strategy $\phi_1 = \ldots = \phi_{N-1} = 0$ is optimal iff $s \geq \hat{s}_1$.*
     - *For $k \in \{2, \ldots, N-1\}$, strategy $\phi_1 = \ldots = \phi_{k-1} = 1$ and $\phi_k = \ldots = \phi_{N-1} = 0$ is optimal iff $s_k \leq s \leq s_{k-1}$.*
     - *Strategy $\phi_1 = \ldots = \phi_{N-1} = 1$ is optimal iff $s \leq \hat{s}_{N-1}$.*
     - *For $k \in \{1, \ldots, N-1\}$, strategy $\phi_1 = \ldots = \phi_{k-1} = 1 > \phi_k > 0 = \phi_{k+1}, \ldots, \phi_{N-1}$ is optimal iff $s = \hat{s}_k$.*

     No other strategies can be optimal.

   (c) *If $r \geq v$ then $\hat{B}_k < 0$ for all k and hence $\phi_1 = \ldots = \phi_{N-1} = 0$.*

(d) *For $r < v$:*

$$\hat{B}_k = -\alpha_k s + (1-\alpha_k)\int_r^v \Big((1-\sigma_1) - (1-F(p)-\sigma_1)(1-F(p))^{N-k-1}\Big)\, dp \quad (97)$$

Which, in the case that $\sigma_2 = \sigma_1$ simplifies to the value of the search for p_{k+1} since it cannot lie in the only interval (r, v) that triggers additional search (given $\phi_{k+1} = \ldots = \phi_{N-1} = 0$).

$$\hat{B}_k = -\alpha_k s + (1-\alpha_k)(1-\sigma_2)(v-r) \quad (98)$$

Proof. Proof of part (1): Once a consumer sees any price strictly below v, uncertainty is resolved and the subgame corresponds to the classic consumer search problem in Stahl (1989). Hence, Stahl's (1989) characterization of optimal consumer search applies. For the special case of unit demand, in which $D(p) = 1 \iff p \leq v$ and $D(p) = 0$ otherwise, this corresponds to part (1). If $s = \int_{\underline{p}}^r F(p)dp$ then $s > 0$ implies $r > \underline{p}$. Otherwise $r = \infty > \underline{p}$.

Proof of part (2): Suppose the consumer observes $p_1 = v$ (or $p_1 = \ldots = p_k = v$ for any $k \geq 1$). If $\sigma_1 = 0$, this reveals the state to be collusive and all remaining prices to be v. If $\sigma_1 = 1$, all remaining prices must be v regardless of the state. Hence, there are no benefits to search, only the cost $s > 0$.

Proof of part (3): Let B_k be the net benefit of an additional search given $p_1 \ldots p_k = v$ and an optimal continuation search strategy (meaning an optimal r and $\phi_{k+1} \ldots \phi_{N-1}$). Before continuing we state and prove two claims about the properties of

Claim 3 *B_k and $\hat{B}_k$ are strictly decreasing in k and s.*

Proof of Claim 3. The net benefit of learning p_{k+1} is strictly decreasing in the cost of the search s and the probability that search is futile due to a collusive state α_k. The option value of conducting additional searches could be zero, so is weakly rather than strictly decreasing in s and α_k. Moreover, α_k is strictly increasing with k (given $\alpha_0 < 1$ and $\sigma_1 \in (0, 1)$). Finally, B_k and $\hat{B}_k$ are weakly increasing in the number of remaining products $N - k$ at which there is an option for further search (which is decreasing in k). Thus B_k and $\hat{B}_k$ are both strictly decreasing in k and s. ■

Claim 4 *If $B_k \leq 0$, then $B_k = \hat{B}_k$.*

Proof Claim 4. If $B_k \leq 0$ then $B_{k+1}, \ldots, B_{N-1} \leq 0$ because B_k is decreasing in k (by Claim 3). Therefore the optimal continuation search strategy is $\phi_{k+1} = \ldots = \phi_{N-1} = 0$ and the definitions of $\hat{B}_k$ and B_k coincide. ■

Proof of part (3a): Part (3a) holds because B_k is strictly decreasing in k (Claim 3).

Proof of part (3b): If B_k replaced $\hat{B}_k$ in the statements within the first bullet, they would hold given the definition of B_k. $\hat{B}_k < s$ and $\hat{B} = s$ are sufficient conditions for for $B_k < s$ and $B_k = s$, respectively, following Claim 4. Moreover, $\hat{B}_k > s$ implies $B_k > s$ because $B_k \geq \hat{B}_k$ by their definitions. Hence, the inequalities involving $\hat{B}$ in the first bullet are sufficient to show the corresponding inequalities with B_k. Therefore, the statements in bullet 1 are true as stated.

The second bullet point is logically equivalent to the first. The third bullet point is implied by the first two given that $\hat{B}_k$ is strictly decreasing in k (Claim 3). The fourth bullet point is equivalent to the third given that $\hat{B}_k$ decreases in s (Claim 3).

Proof of part (3c): If $r \geq v$, then there is no option value for future search conditional on learning that the state is competitive. Hence, following part (1), $\hat{B}_k$ captures the gross benefit of one search $\hat{B}_k = (1-\alpha_k)\int_{\underline{p}}^{v} F(p)dp$ which equals $(1-\alpha_k)s$ if $r = v$ and is less than $(1-\alpha_k)s$ if $r > v$. In either case $\hat{B}_k < 0$ and hence $\phi_1 = \ldots = \phi_{N-1} = 0$.

Proof of part (3d) (Derivation of equations (97)–(98)): Suppose the consumer has observed $p_1 = \ldots = p_k = v$. $\hat{B}_k$ is the net benefit of searching one more time given $\phi_{k+1} = \ldots = \phi_{N-1} = 0$ but optimal future search if $p_{k+1} < v$. To calculate the benefit of search, let us break the possible realizations into three sets. (1) First, with probability $(1-\alpha_k)\sigma_1 + \alpha_k$, $p_{k+1} = v$ and search ends with zero benefit.

(2) Second, with probability $(1-\alpha_k)(\sigma_2-\sigma_1)\sigma_2^{N-k-1}$, $p_{k+1} \in (r, v)$ and $p_{k+2}, \ldots, p_N > r$, triggering search of all prices and expected benefit $v - E[\min\{p_{k+1}, \ldots, p_N > r\} \mid p_{k+1} \in (r,v) \;\;\&\;\; p_{k+2}, \ldots, p_N > r]$. To compute the conditional expected value, note that the distribution of p_{k+1} conditional on $p_{k+1} \in (r,v)$ is $\frac{F(p)-(1-\sigma_2)}{\sigma_2-\sigma_1}$ and the distribution of p_{k+j} for $j > 1$ conditional on $p_{k+j} \in (r, v]$ is $\frac{F(p)-(1-\sigma_2)}{\sigma_2}$. Since $\left(1 - \frac{F(p)-(1-\sigma_2)}{\sigma_2-\sigma_1}\right) = \left(\frac{1-F(p)-\sigma_1}{\sigma_2-\sigma_1}\right)$ and $\left(1 - \frac{F(p)-(1-\sigma_2)}{\sigma_2}\right) = \left(\frac{1-F(p)}{\sigma_2}\right)$, the conditional distribution of the first-order statistic is

$$1 - \left(\frac{1-F(p)-\sigma_1}{\sigma_2-\sigma_1}\right)\left(\frac{1-F(p)}{\sigma_2}\right)^{N-k-1}. \tag{99}$$

As a result, this case contributes

$$\begin{aligned}&(1-\alpha_k)(\sigma_2-\sigma_1)\sigma_2^{N-k-1}\int_r^v \left(1 - \left(\frac{1-F(p)-\sigma_1}{\sigma_2-\sigma_1}\right)\left(\frac{1-F(p)}{\sigma_2}\right)^{N-k-1}\right)dp \\ &\quad = (1-\alpha_k)\left((\sigma_2-\sigma_1)\sigma_2^{N-k-1}(v-r) - \int_r^v (1-F(p)-\sigma_1)(1-F(p))^{N-k-1}dp\right)\end{aligned} \tag{100}$$

to the expected benefits.

(3) Third, with probability $(1-\alpha_k)\Big(1-\sigma_1-(\sigma_2-\sigma_1)\sigma_2^{N-k-1}\Big)$, search yields $p_{k+j} \leq r$ after $j \in \{1, \ldots, N-k\}$ additional searches. In this case, the expected benefit is $v - E[p \mid p \leq r] = v - r + (r - E[p \mid p \leq r])$. Notably, given $r \leq v$ and the definition of r, $s = (1-\sigma_2)(r - E[p \mid p \leq r])$ which implies that$(r - E[p \mid p \leq r]) = s/(1-\sigma_2)$. This means that we can rewrite the benefit in case 3 as $v - r + s/(1-\sigma_2)$. Hence case 3 contributes $(1-\alpha_k)\Big(1-\sigma_1-(\sigma_2-\sigma_1)\sigma_2^{N-k-1}\Big)\Big(v-r+s/(1-\sigma_2)\Big)$ to the expected benefits.

The expected number of additional searches is the first additional search for p_{k+1}, a second additional search for p_{k+2} with probability $(1-\alpha_k)(\sigma_2-\sigma_1)$, and additional searches for p_j with probability $(1-\alpha_k)(\sigma_2-\sigma_1)\sigma_2^{j-2}$ for $j \in \{3, \ldots, N-k\}$. This yields expected search costs of

$$\left(1 + (1-\alpha_k)(\sigma_2-\sigma_1)\sum_{j=2}^{N-k}\sigma_2^{j-2}\right)s \tag{101}$$

Putting expected costs and benefits together,

$$\hat{B}_k = +(1-\alpha_k)\left((\sigma_2-\sigma_1)\sigma_2^{N-k-1}(v-r) \ -\int_r^v \left(1-F(p)-\sigma_1\right)\left(1-F(p)\right)^{N-k-1} dp\right)$$
$$+(1-\alpha_k)\Big(1-\sigma_1-(\sigma_2-\sigma_1)\sigma_2^{N-k-1}\Big)\Big(v-r+s/(1-\sigma_2)\Big)-\left(1+(1-\alpha_k)(\sigma_2-\sigma_1)\sum_{j=2}^{N-k}\sigma_2^{j-2}\right)s \tag{102}$$

Rearranging and canceling terms yields

$$\hat{B}_k = (1-\alpha_k)\left(\int_r^v \left((1-\sigma_1)-\left(1-F(p)-\sigma_1\right)\left(1-F(p)\right)^{N-k-1}\right) dp\right)$$
$$-\left(\alpha_k+(1-\alpha_k)(\sigma_2-\sigma_1)\left(\left(\sum_{j=2}^{N-k}\sigma_2^{j-2}\right)-\frac{1-\sigma_2^{N-k-1}}{1-\sigma_2}\right)\right)s, \tag{103}$$

which reduces to equation (97) by noticing that $\sum_{j=2}^{N-k}\sigma_2^{j-2} = \frac{1-\sigma_2^{N-k-1}}{1-\sigma_2}$.

In the case that $\sigma_2 = \sigma_1$, this further simplifies to the value of the search for p_{k+1} since it cannot lie in the only interval (r, v) that triggers additional search (given $\phi_1 = \ldots = \phi_{N-1} = 0$).

$$\hat{B}_k = -s + (1-\alpha_k)(1-\sigma_2)\Big(v-r+s/(1-\sigma_2)\Big) \tag{104}$$

Which after rearranging and canceling terms yields equation (98. ∎

C.14.2 Lemma 5

Lemma 5 provides an initial set of necessary conditions for a symmetric equilibrium. Recall that $\sigma_1 = \Pr(p = v)$, $\sigma_2 = \Pr(p > r)$. Let p^* be the supremum price at which $1 - F(p) = \sigma_2$ (the point above r at which the gap in firm pricing stops and firms start pricing with positive probability again).

Lemma 5 *Necessary conditions for a symmetric equilibrium include:*

1. $\sigma_1 < 1$.
2. *Firm profits are strictly positive and hence so is the lower bound of firms' price distribution:* $\Pi > 0$ *and* $\underline{p} > 0$.
3. $F(p)$ *has no atoms below* v.
4. *If* $r \geq v$ *then* $F(p)$ *has no gaps in its support. If* $r < v$*, there is a single gap in the support of* $F(p)$*, with no pricing on the interval* (r, p^*) *for* $r < p^* \leq v$.
5. $\sigma_1 = 0 \iff r \geq v$.

6. $\sigma_2 < 1$.

7. $\phi_{N-1} < 1$

Proof. Proof of Lemma 5:

1. Proof of part (1): If $\sigma_1 = 1$, then a firm can profitably deviate to $v - \epsilon$ and steal $\mu\frac{N-1}{N}$ additional shoppers while only losing ϵ markup on its other customers.

2. Proof of part (2): Consider two cases: (1) $\sigma_1 > 0$. In this case, profits from setting $p = v$ are at least $\Pi \geq \frac{N-1}{N}\sigma_1^{N-1}v > 0$. (In this expression, σ_1^{N-1} is the probability of tying the other $N-1$ firms at $p = v$ and $\frac{N-1}{N}$ is the share of consumers that the firm would win in such a tie.) (2) $\sigma_1 = 0$. In this case, $\phi_1 = 0$ because consumers infer from observing v that the state is collusive and additional search has no benefit. Profits are bounded below by the profit from setting $p = v$ and selling to the firm's share of non-shoppers: $\Pi \geq (1-\mu)\frac{1}{N}v > 0$.

3. Proof of part (3): Suppose firms price at $p' < v$ with probability $\sigma > 0$. Then it is strictly more profitable to price at $p' - \epsilon$ than p'. Undercutting the atom gains $\mu\frac{N-1}{N}\sigma^{N-1} > 0$ additional shoppers while only losing ϵ markup on its other customers. This contradicts symmetry. Importantly, lowering the price does not increase search because consumers follow a stationary reservation price strategy for all $p < v$ (Lemma 4).

4. Proof of part (4): Claim (i): There are no gaps below r. Proof: Suppose there is a gap with no pricing on (p_L, p_H) for some $\bar{p} < p_L < p_H < r$. This cannot be an equilibrium because it is strictly more profitable to price at p_H than at p_L. Demand is the same at either price—always including non-shoppers who visit the firm and including all shoppers if the other firms all price above p_L (which implies they price above p_H). Hence the higher price is more profitable.

   Claim (ii): $F(p)$ has no gaps on $[p^*, v]$. Specifically, if $F(p)$ is strictly increasing on $[p^*, p_L]$ for some $r < p^* < p_L < v$ then it is strictly increasing on $[p^*, v]$. Proof: Suppose not and there is a gap in pricing on $[p_L, p_H]$ for $p^* < p_L < p_H < v]$. Then pricing at p_L is strictly dominated by pricing at p_H because the higher price achieves identical demand. Demand is the same at either price—including both non-shoppers who visit the firm and all shoppers in the event that the other firms all price above p_L (which implies they price above p_H).

   Claim (iii): $r < v$ implies there is a gap in pricing with $F(p)$ constant over (r, p^*) for some $p^* \in (r, v]$. Proof: There is a discrete drop in demand from pricing at r to just above at r^+. In particular, with probability $(1-\sigma_2^{N-1})$ another firm prices at or below r. In this event, the $\frac{1}{N}(1-\mu)$ non-shoppers who visit firm i first would buy at price r but keep searching and buy elsewhere at price r^+, leading to a drop in demand of at least $(1-\sigma_2^{N-1})\frac{1}{N}(1-\mu)$. Hence, pricing in a neighborhood above r cannot be optimal.

   Claims (i), (ii), and (iii) jointly imply part (4).

5. Proof of part (5): (1) $\sigma_1 = 0 \rightarrow r \geq v$: Suppose $\sigma_1 = 0$. Suppose the supremum price chosen by firms is $\bar{p} < v$. Demand must be positive at $\bar{p}$ to satisfy positive profits

(part (2)). By part (3), there is no atom at $\bar{p}$ and hence demand at $\bar{p}$ includes no shoppers. For demand to be positive at $\bar{p}$, therefore, requires $\bar{p} \leq r$, which leads to demand of $\frac{1}{N}(1-\mu)$ non-shoppers who visit firm i first. However, deviating to $p = v$ still wins the same number of non-shoppers at a higher price because $p = v$ convinces all non-shoppers to stop searching because the state must be collusive. A contradiction. Therefore $\bar{p} = v$. Moreover, firms price in $(v - \epsilon, v)$ because $\sigma_1 = 0$ means there is no atom at v. This requires that profits, and hence demand, are weakly higher in a neighborhood below v than at v. Since $\phi_1 = \ldots = \phi_{N-1} = 0$ by Lemma 4, this requires $r \geq v$.

(2) $r \geq v \rightarrow \sigma_1 = 0$: Suppose $r \geq v$. Suppose that $\sigma_1 > 0$. Then profits, and hence demand, must be weakly higher at $p = v$ than in a neighborhood below v. Because $r \geq v$, demand from non-shopper is $\frac{1}{N}\mu$ at either price. However, pricing just below v wins additional expected share of shoppers $\sigma_1^{N-1}\frac{N-1}{N} > 0$. This is a contradiction, hence $\sigma_1 = 0$.

6. Proof of part (6): Lemma 4 part (1) implies $r > \underline{p}$. If $\sigma_2 = 1$, then $p^* = \underline{p}$ and hence $r > p^*$, which contradicts part (4).

7. Proof of part(7): Suppose not and $\phi_{N-1} = 1$. There are two cases. (1) $\sigma_1 > 0$. In this case, it is strictly more profitable to undercut and price at $p = v - \epsilon$ than at $p = v$ since doing so does not trigger more search and wins $\mu\frac{N-1}{N}\sigma_1^{N-1} > 0$ shoppers. (2) $\sigma_1 = 0$. In this case, only $\phi_k = 0$ for all k is consistent with equilibrium by part (5).

■

C.14.3 Lemma 6: Demand

Lemma 6 *Given $r < v$, ϕ, σ_1, and σ_2, demand is given by equations (24)–(25) in Proposition 16.*

Proof. (1) $p \leq r$: Consider non-shopper demand at $p \leq r$. Consider all the search paths that non-shoppers who see firm i's price n^{th} might take to get to firm i. Quantity $\frac{1}{N}(1-\mu)$ simply search firm i first. For $n > 1$ and $m \in \{0, \ldots, n-1\}$ they could see m prices equal to v ($p_1 = \ldots = p_m = v$), then, if $m \leq n-2$ see $p_{m+1} \in (\bar{p}, v)$, and then, if $m \leq n-3$, see $(n-m-2)$ prices $p \in (\bar{p}, v]$. The probability that a consumer follows a search path for a particular n and m is

$$q(n,m) = \frac{1}{N}(1-\mu)(\sigma_2 - \sigma_1)^{1_{m<n-1}}\sigma_2^{\max\{0,n-m-2\}}\Pi_{j=1}^{m}\sigma_1\phi_j \tag{105}$$

Adding $1/N$ to the sum of $q(n,m)$ over all $n \in \{2, \ldots, N\}$ and $m \in \{0, \ldots, n-2\}$ yields equation (25). Shopper demand is $\mu(1-F(p))^{N-1}$ because there are no atoms for $p \leq r$ (Lemma 5). Adding shopper and non-shopper demand yields equation (24) for $p \leq r$.

(2) $p \in (r, v)$: In this region, demand is zero unless p is the lowest price because any consumer who observes $p \in (r, v)$ will keep searching until they find a lower price $p' \leq r$ or they have searched all prices. Conditional on p being the lowest price (which occurs

with probability $(1-F(p))^{N-1}$), all consumers will buy except for those discouraged non-shoppers who give up searching after viewing a sequence of prices equal to v. Fraction $1/N$ non-shoppers would see firm i's price n^{th} if they were not discouraged from searching earlier. The conditional probability that such a consumer gets discouraged after seeing the first $m<n$ prices equal to v is $\left(\frac{\sigma_1}{1-F(p)}\right)^m \left(\Pi_{j=1}^{m-1}\phi_j\right)(1-\phi_m)$. Conditional on p being the lowest price, demand is 1 less the sum over n and m of these discouraged consumers. Expected demand is then:

$$q(p)=(1-F(p))^{N-1}\left(1-\frac{1}{N}(1-\mu)\sum_{n=2}^{N}\sum_{m=1}^{n-1}\left(\frac{\sigma_1}{1-F(p)}\right)^m\left(\Pi_{j=1}^{m-1}\phi_j\right)(1-\phi_m)\right) \quad (106)$$

Simplifying this expression yields equation (24) for $p\in(r,v)$.

(3) $p=v$: If all $N-1$ other firms also price at v, firm i wins share $1/N$ of them in the tie, yielding $q=\frac{1}{N}\mu\sigma_1^{N-1}$ expected shoppers. Following the tie-breaking rule that consumers buy from the last firm searched in the case of ties, non-shopper demand comes only from consumers who only see prices of v and decide to stop searching at firm i. Quantity $\frac{1}{N}(1-\mu)$ non-shoppers would see firm i's price n^{th} if they were not discouraged from searching earlier. Of these, fraction $(1-\phi_n)\Pi_{j=1}^{n-1}\sigma_1\phi_j$ see a price of v and decide to keep searching $n-1$ times before giving up and buying at firm i's price of v. Summing shopper demand and non-shopper demand for each n yields equation (24) for $p=v$. ■

C.14.4 Lemma 7: Additional Properties of Demand

Lemma 7 *Given $r<v$, ϕ, σ_1, and σ_2, demand satisfies the following properties:*

1. *Given ϕ satisfies Condition 1 for index k, demand can be expressed as:*

$$q(p;k)=\begin{cases}\mu(1-F(p))^{N-1}+q_{NS} & \text{if } p\le r\\ (1-F(p))^{N-1}-\frac{1}{N}(1-\mu)\begin{pmatrix}(N-k)\sigma_1^k(1-F(p))^{N-1-k}(1-\phi_k)\\ +(N-1-k)\sigma_1^{k+1}(1-F(p))^{N-2-k}\phi_k\end{pmatrix} & \text{if } r<p<v\\ \frac{1}{N}\left(\mu\sigma_1^{N-1}+(1-\mu)\left((1-\phi_k)\sigma_1^{k-1}+\phi_k\sigma_1^k\right)\right) & \text{if } p=v\\ 0 & \text{if } p>v\end{cases} \quad (107)$$

where q_{NS} is:

$$q_{NS}=\frac{1}{N}(1-\mu)\left(1+\sum_{n=1}^{k-1}\sigma_1^n+\sigma_1^k\phi_k+(\sigma_2-\sigma_1)\left(\sum_{n=0}^{N-2}\sum_{j=0}^{\min\{k-1,n\}}\sigma_2^{n-j}\sigma_1^j+\sum_{n=k}^{N-2}\sigma_2^{n-k}\sigma_1^k\phi_k\right)\right) \quad (108)$$

2. *In the limit as $\sigma_1,\sigma_2\to 0$, it holds that:*

$$\lim_{\sigma_1,\sigma_2\to 0}q(p;k)=\begin{cases}\mu(1-F(p))^{N-1}+\lim_{\sigma_1,\sigma_2\to 0}q_{NS} & \text{if } p\le r\\ 0 & \text{if } r<p<v\\ \frac{1}{N}(1-\mu)(1-\phi_1) & \text{if } p=v\\ 0 & \text{if } p>v\end{cases} \quad (109)$$

where $\lim_{\sigma_1,\sigma_2\to 0} q_{NS}$ *is:*

$$\lim_{\sigma_1,\sigma_2\to 0} q_{NS} = \frac{1}{N}(1-\mu). \tag{110}$$

3. *It holds that*

$$q(v^-;k) \equiv \lim_{p\to v^-} q(p;k) = \left(1 - \frac{1}{N}(1-\mu)(N-k-\phi_k)\right)\sigma_1^{N-1}. \tag{111}$$

For $\sigma_1 = 1$,

$$q(v^-;k,\sigma_1=1) = 1 - \frac{1}{N}(1-\mu)(N-k-\phi_k) > q(v;k,\sigma_1=1) = \frac{1}{N}. \tag{112}$$

4. *The partial derivatives of demand with respect to* ϕ_k, σ_1, *and* $(1-F(p))$ *are*

$$\frac{\partial}{\partial\phi_k}q(p;k) = \begin{cases} 0 & \text{if } p \le r \\ \frac{1}{N}(1-\mu)\sigma_1^k(1-F(p))^{N-1-k}\left(1+(N-1-k)\left(1-\frac{\sigma_1}{1-F(p)}\right)\right) \ge 0 & \text{if } r < p < v \\ -\frac{1}{N}(1-\mu)\sigma_1^{k-1}(1-\sigma_1) \le 0 & \text{if } p = v \\ 0 & \text{if } p > v \end{cases} \tag{113}$$

$$\frac{\partial q_{NS}}{\partial\phi_k} = \frac{1}{N}(1-\mu)\left(\sigma_1^k + (\sigma_2-\sigma_1)\sigma_1^k\sum_{n=k}^{N-2}\sigma_2^{n-k}\right) \ge 0 \tag{114}$$

$$\frac{\partial q(v^-;k)}{\partial\phi_k} = \frac{1}{N}(1-\mu)\sigma_1^{N-1} \ge 0 \tag{115}$$

$$\frac{\partial q(p;k)}{\partial\sigma_1} = \begin{cases} 0 & \text{if } p \le r \\ -\frac{1}{N}(1-\mu)\begin{pmatrix} (N-k)k\sigma_1^{k-1}(1-F(p))^{N-1-k}(1-\phi_k) \\ +(N-1-k)(k+1)\sigma_1^k(1-F(p))^{N-2-k}\phi_k \end{pmatrix} \le 0 & \text{if } r < p < v \\ \frac{1}{N}\left(\mu(N-1)\sigma_1^{N-2} + (1-\mu)\left((1-\phi_k)(k-1)\sigma_1^{k-2} + \phi_k k\sigma_1^{k-1}\right)\right) \ge 0 & \text{if } p = v \\ 0 & \text{if } p > v \end{cases} \tag{116}$$

$$\frac{\partial q_{NS}}{\partial\sigma_1} = -\left(k\sigma_1^{k-1}(1-\phi_k)\sum_{n=k+1}^{N}\sigma_2^{n-k-1}\right) - \left((k+1)\sigma_1^k\phi_k\sum_{n=k+2}^{N}\sigma_2^{n-k-2}\right) \le 0 \tag{117}$$

$$\frac{\partial q(v^-;k)}{\partial\sigma_1} = \frac{N-1}{\sigma_1}q(v^-;k) \ge 0 \tag{118}$$

$$\frac{\partial q(p;k)}{\partial(1-F(p))} = \begin{cases} \mu(N-1)(1-F(p))^{N-2} \ge 0 & \text{if } p \le r \\ \begin{pmatrix} (N-1)(1-F(p))^{N-2} - \frac{1}{N}(1-\mu)\cdot \\ \begin{pmatrix} (N-k)(N-1-k)\sigma_1^k(1-F(p))^{N-2-k}(1-\phi_k) \\ +(N-1-k)(N-2-k)\sigma_1^{k+1}(1-F(p))^{N-3-k}\phi_k \end{pmatrix} \end{pmatrix} \ge 0 & \text{if } r < p < v \\ 0 & \text{if } p \ge v \end{cases} \tag{119}$$

It holds that $\frac{\partial q(v)}{\partial \sigma_1} \leq \frac{N-1}{\sigma_1} q(v)$. This and the inequalities in equations (113)–(118) are strict for $\sigma_1 \in (0,1)$.

Proof.

Proof of part (1): Follows directly from equation (24) in Lemma 6.

Proof of part (2): Follows directly from substituting $\sigma_1 = \sigma_2 = 0$ and, for $p \geq r$, $(1 - F(p)) = 0$ into equations (107)–(108).

Proof of part (3): Equation (111) follows from equation (24) in Lemma 6. Equation (112) follows by substituting $\sigma_1 = 1$ into equation (111) for $q(v^-; k)$ and equation (107) for $q(v; k)$. The inequality holds strictly because $(N - k - \phi_k) \leq N - 1$ and $(1 - \mu) < 1$.

Proof of part (4): Equations (113)–(116) and (118)–(119) follow from differentiating equations (107)–(111) and simplifying the resulting expressions. Equation (117) is derived in three steps. First, equation (25) is equivalently expressed as:

$$q_{NS} = \sum_{n=1}^{N} \sigma_2^{n-1} - \sum_{m=2}^{N} \left(\sigma_1^{m-1} \left(\Pi_{j=1}^{m-2} \phi_j \right) (1 - \phi_{m-1}) \sum_{n=m}^{N} \sigma_2^{n-m} \right) \tag{120}$$

Second, we differentiate this expression

$$\frac{\partial q_{NS}}{\partial \sigma_1} = - \sum_{m=2}^{N} \left((m-1) \sigma_1^{m-2} \left(\Pi_{j=1}^{m-2} \phi_j \right) (1 - \phi_{m-1}) \sum_{n=m}^{N} \sigma_2^{n-m} \right) \leq 0 \tag{121}$$

Third, we simplify the expression using Condition 1 for index k, which yields equation (117).

It only remains to prove the stated inequalities. Most are apparent by inspection. Three are not:

(1) $\frac{\partial}{\partial \phi_k} q(p; k)$ for $p \in (r, v)$: For $p \in (r, v)$, it holds that $(1 - F(p)) \in [\sigma_1, \sigma_2]$, which implies $(1 - \frac{\sigma_1}{1 - F(p)}) \geq 0$ and hence $\frac{\partial}{\partial \phi_k} q(p; k)$ is greater than zero (strictly for $\sigma_1 > 0$).

(2) $\frac{dq(v)}{d\sigma_1} \leq \frac{N-1}{\sigma_1} q(v)$: It holds that $\frac{dq(v)}{d\sigma_1} \leq \frac{N-1}{\sigma_1} q(v)$ because $\frac{dq(v)}{d\sigma_1}$ is equal to the three additive terms of $q(v)$ multiplied by $\frac{N-1}{\sigma_1}$, $\frac{k-1}{\sigma_1}$, and $\frac{k}{\sigma_1}$. Since $1 \leq k \leq N - 1$, and the three terms are weakly positive, the result follows.

(3) $\frac{\partial}{\partial (1_F(p))} q(p; k)$ for $p \in (r, v)$: Note that while it is intuitive that demand should be increasing in the probability of one's price being lower than a competitiors', it is not obviously true because non-shoppers can be captured at the monopoly price v. However it is true because we hold σ_1, the probability the monopoly price is offered, constant while varying x.

More formally, following equation (119) in Lemma 7, we can reexpress $\frac{\partial q(p;k)}{\partial(1_F(p))}$ for $p \in (r, v)$ as a function of $x = 1 - F(p)$ as

$$\frac{d}{dx}\hat{\hat{q}}(x;k) = (N-1)x^{N-2} - \frac{1}{N}(1-\mu)\Bigg((N-1-k)(N-k)\sigma_1^k x^{N-2-k}(1-\phi_k) \\ + (N-2-k)(N-1-k)\sigma_1^{k+1}x^{N-3-k}\phi_k\Bigg) \quad (122)$$

For $x \in [\sigma_1, \sigma_2]$, we know that $x \geq \sigma_1$, which implies the first inequality in equation (123). Since $(1-\mu) < 1$, $k \geq 1$, and $\phi_k \geq 0$ the second inequality holds.

$$\frac{d}{dx}\hat{\hat{q}}(x;k) \geq (N-1)x^{N-2} - \frac{1}{N}(1-\mu)\Bigg((N-1-k)(N-k)x^{N-2}(1-\phi_k) + (N-2-k)(N-1-k)x^{N-2}\phi_k\Bigg) \\ = x^{N-2}\Bigg((N-1) - \frac{1}{N}(1-\mu)(N-1-k)\Bigg((N-k) - 2\phi_k\Bigg)\Bigg) \\ > x^{N-2}\Bigg((N-1) - \frac{1}{N}(N-2)(N-1)\Bigg) = x^{N-2}(N-1)\frac{2}{N} > 0 \quad (123)$$

■

C.14.5 Lemma 8: σ_{1A}

In a slight abuse of notation, let $q(v^-)$ be shorthand for the left-hand limit of $q(v)$, or $\lim_{p\to v^-} q(p)$, and $\pi(v^-1)$ be shorthand for the left-hand limit of $\pi(v)$, or $\lim_{p\to v^-} \pi(p)$. Define $\sigma_{1A}(\phi)$ to be the value be the value of $\sigma_1 > 0$ which makes a firm indifferent between pricing at v and just below v (for which $q(v) = q(v^-)$ and hence $\pi(v) = \pi(v^-)$ given consumer strategy ϕ. Lemma 8 derives properties of $\sigma_{1A}(\phi)$ and shows that it is an upper bound for σ_1 that can be used to characterize σ_1 as a function of σ_2.

Lemma 8 *Let $\sigma_{1A}(\phi)$ be the value of $\sigma_1 > 0$ which makes a firm indifferent between pricing at v and just below v at v^- given consumer strategy ϕ (satisfying Condition 1) and $r < v$. $\sigma_{1A}(\phi)$ exists and is a differentiable and strictly decreasing function of ϕ that satisfies $\sigma_{1A}(\phi) \in (0,1)$. A necessary condition for firms to be playing a best response is that $\sigma_1 = \min\{\sigma_2, \sigma_{1A}\}$, which implies that $q(v^-) \leq q(v)$.*

Proof. By Lemma 7, $q(v;k)$ and $q(v^-;k)$ are given by equations (107) and (108), and at $\sigma_1 = 1$, $q(v^-;k) > q(v;k)$. Moreover, $q(v) > q(v^-)$ in a neighborhood $\sigma_1 \in (0,\delta)$. To see why this must hold, consider two cases. (i) For $k < N-1$, both the σ_1^k and the σ_1^{k-1} terms in $q(v)$ have lower order than the lone σ_1^{N-1} term in $q(v^-)$ and at least one must have a positive coefficient. (ii) For $k = N-1$, we know that $\phi_k < 1$ (Lemma 5 Part 7), so the σ_1^{k-1} term in $q(v)$ is lower order than σ_1^{N-1} and has a positive coefficient. In either case, $q(v)$ must grow faster than $q(v^-)$ as σ_1 increases above zero. Since $q(v;\sigma_1 = 0) \geq 0 = q(v^-;\sigma_1 = 0)$, this implies $q(v) > q(v^-)$ in a neighborhood $\sigma_1 \in (0,\delta)$.

The fact that $q(v) > q(v^-)$ in a neighborhood $\sigma_1 \in (0, \delta)$ and $q(v^-; \sigma_1 = 1) > q(v; \sigma_1 = 1)$ implies that $q(v)$ and $q(v^-)$ must cross for some $\sigma_{1A} \in (0, 1)$. To show that the crossing is unique, consider the derivatives of $q(v; k)$ and $q(v^-; k)$ with respect to σ_1 for $\sigma_1 > 0$:

$$\frac{dq(v;k)}{d\sigma_1} = (N-1)\frac{1}{N}\mu\sigma_1^{N-2} + k\frac{1}{N}(1-\mu)\phi_k\sigma_1^{k-1} + (k-1)\frac{1}{N}(1-\mu)(1-\phi_k)\sigma_1^{k-2} < \frac{N-1}{\sigma_1}q(v;k) \tag{124}$$

$$\frac{dq(v^-;k)}{d\sigma_1} = (N-1)\left(1 - \frac{1}{N}(1-\mu)(N-k-\phi_k)\right)\sigma_1^{N-2} = \frac{N-1}{\sigma_1}q(v^-;k) \tag{125}$$

(Note that the inequality $\frac{dq(v;k)}{d\sigma_1} < \frac{N-1}{\sigma_1}q(v;k)$ holds because $k \leq N-1$ and, by Lemma 5 Part 7, $\phi_{N-1} < 1$.) As a result, for all $\sigma_1 > 0$ at which $q(v^-) \geq q(v)$, it must hold that $\frac{dq(v^-;k)}{d\sigma_1} > \frac{dq(v;k)}{d\sigma_1}$. This implies $q(v^-)$ crosses $q(v)$ at most once for $\sigma_1 > 0$, and hence σ_{1A} is unique. Moreover, it implies that $q(v^-) \leq q(v)$ for all $\sigma_1 \leq \sigma_{1A}$.

Given that $\sigma_1 > 0$ and $q(v^-) = q(v)$ imply $\frac{dq(v^-;k)}{d\sigma_1} > \frac{dq(v;k)}{d\sigma_1}$, the implicit function theorem holds at the unique solution for each ϕ. Hence σ_{1A} is a continuous function of ϕ.

Moreover, by the implicit function theorem,

$$\frac{d\sigma_{1A}}{d\phi_k} = -\frac{\frac{\partial q(v^-)}{\partial \phi_k} - \frac{\partial q(v)}{\partial \phi_k}}{\frac{\partial q(v^-)}{\partial \sigma_1} - \frac{\partial q(v)}{\partial \sigma_1}} < 0. \tag{126}$$

Note that the terms on the right-hand side of equation (126) are evaluated at $\sigma_1 = \sigma_{1A} \in (0, 1)$. Hence, by the logic above, $\frac{\partial q(v^-)}{\partial \sigma_1} > \frac{\partial q(v)}{\partial \sigma_1}$ and the denominator is positive. Moreover, by Lemma 7, $\frac{\partial q(v^-)}{\partial \phi_k} < 0$, $\frac{\partial q(v)}{\partial \phi_k} > 0$ and $\frac{\partial q(v^-)}{\partial \sigma_1} = \frac{N-1}{\sigma_1}q(v^-)$, meaning the numerator is positive. Hence, $\frac{d\sigma_{1A}}{d\phi_k}$ is negative due to the negative sign in front of the expression.

A necessary condition for firms to be playing a best response is that $\sigma_1 = \min\{\sigma_2, \sigma_{1A}\}$: If $\sigma_1 > \sigma_{1A}$, then the fact that $q(v^-)$ crosses $q(v)$ at most once for $\sigma_1 > 0$ implies that $\pi(v^-) > \pi(v)$, which contradicts $\sigma_1 > 0$ being part of a firm's best response. Hence $\sigma_1 \leq \sigma_{1A}$. If $0 < \sigma_1 < \sigma_{1A}$, then by the same logic $\pi(v^-) < \pi(v)$ and hence firms do not price in a neighborhood below v, which implies they do not price on (r, v) given the single gap condition (Lemma 5 Part 4) and hence that $\sigma_1 = \sigma_2$. Since $\sigma_1 \leq \sigma_2$ by definition, this means that $\sigma_1 = \min\{\sigma_2, \sigma_{1A}\}$. ■

C.14.6 Lemma 9: Derivatives with respect to ϕ_k

Lemma 9 *For π, q_{NS}, r, and $(1 - F(p)$ characterized by Proposition 16 part (2) as a function of ϕ and σ_1, it holds that: (1) $\frac{d\pi}{d\phi_k} < 0$; (2) $\frac{d(1-F(p))}{d\phi_k}$ for $p \in (p^*, v)$; (3) $\frac{dq_{NS}}{d\phi_k} \geq 0$; (4) $\frac{dr}{d\phi_k} < 0$; (5) $\frac{ds_r}{d\phi_k} < 0$ for $\sigma_1 > 0$ and $\sigma_2 < 1$; and (6) $\frac{ds_k}{d\phi_k} > 0$ for $\sigma_1 = \sigma_2 < \sigma_{1A}(\phi)$.*

Proof. Part (1) $\frac{d\pi}{d\phi_k} < 0$: As $\pi = vq(v)$, $\frac{d\pi}{d\phi_k}$ is:

$$\frac{d\pi}{d\phi_k} = v\frac{dq(v)}{d\phi_k} = v\Bigg(\underbrace{\frac{\partial q(v)}{\partial \phi_k}}_{-} + \underbrace{\frac{\partial q(v)}{\partial \sigma_1}}_{+}\underbrace{\frac{d\sigma_1}{d\phi_k}}_{-}\Bigg) < 0 \tag{127}$$

The inequality follows from $\partial q(v)/\partial\phi_k < 0$ and $\partial q(v)/\partial\sigma_1 \geq 0$ (Lemma 7) as well as $d\sigma_1/d\phi_k \leq 0$ (Lemma 8, the inequality is strict iff $\sigma_1 = \sigma_{1A} < \sigma_2$).

Part (2) $\frac{d(1-F(p))}{d\phi_k}$ **for** $p \in (p^*, v)$**:** By taking the total derivative $dq(p)/d\phi_k$ for $p \in (p^*, v)$, and we can rearrange the expression to solve for $\frac{d(1-F(p))}{d\phi_k}$ as follows:

$$\frac{d(1-F(p))}{d\phi_k} = \underbrace{\Big(\underbrace{\frac{dq(p)}{d\phi_k}}_{-} - \underbrace{\frac{\partial q(p)}{\partial\phi_k}}_{+} - \underbrace{\frac{\partial q(p)}{\partial\sigma_1}}_{-}\underbrace{\frac{d\sigma_1}{d\phi_k}}_{-}\Big)}_{-} \Big/ \underbrace{\frac{\partial q(p)}{\partial(1-F(p))}}_{+} < 0 \tag{128}$$

The partial derivatives on the right hand side are signed by Lemma 7 (for $p \in (r, v)$, $\frac{\partial q(p)}{\partial\phi_k} \geq 0$, $\frac{\partial q(p)}{\partial\sigma_1} \leq 0$, $\frac{\partial q(p)}{\partial(1-F(p))} \geq 0$). Since $pq(p) = \pi$, $\frac{dq(p)}{d\phi_k} = \frac{1}{p}\frac{d\pi}{d\phi_k} < 0$. Finally, $\frac{d\sigma_1}{d\phi_k} \leq 0$ because either $\sigma_1 = \sigma_2$, and is held fixed as we vary ϕ_k, or $\sigma_1 = \sigma_{1A}$ which is strictly decreasing in ϕ_k (Lemma 8. Thus the result holds.

Part (3) $\frac{dq_{NS}}{d\phi_k} \geq 0$**:** It holds that $\frac{dq_{NS}}{d\phi_k} \geq 0$ because $\frac{\partial q_{NS}}{\partial\phi_k} \geq 0$ and $\frac{\partial q_{NS}}{\partial\sigma_1} \leq 0$ (Lemma 7) and $\frac{d\sigma_1}{d\phi_k} < 0$ (Lemma 8).

$$\frac{dq_{NS}}{d\phi_k} = \underbrace{\frac{\partial q_{NS}}{\partial\phi_k}}_{+} + \underbrace{\frac{\partial q_{NS}}{\partial\sigma_1}}_{-}\underbrace{\frac{d\sigma_1}{d\phi_k}}_{-} \geq 0 \tag{129}$$

Part (4) $\frac{dr}{d\phi_k} < 0$**:** Notice that $r = \pi/\hat{q}(\sigma_2) = \frac{\pi}{\mu\sigma_2^{N-1} + q_{NS}}$. Therefore $\frac{d\pi}{d\phi_k} < 0$ (part 1) and $\frac{dq_{NS}}{d\phi_k} \geq 0$ (part 3) imply $\frac{dr}{d\phi_k} < 0$.

Part (5) $\frac{ds_r}{d\phi_k} < 0$**:** Differentiating equation (33) with respect to ϕ_k yields

$$\frac{ds_r(\phi,\sigma_2)}{d\phi_k} = \frac{d\pi}{d\phi_k}\int_{\sigma_2}^{1}\left(\frac{1}{\hat{q}(\sigma_2)} - \frac{1}{\hat{q}(x)}\right)dx - \pi\frac{dq_{NS}}{d\phi_k}\int_{\sigma_2}^{1}\left(\frac{1}{\hat{q}^2(\sigma_2)} - \frac{1}{\hat{q}^2(x)}\right)dx < 0, \tag{130}$$

which is negative because $d\pi/d\phi_k < 0$ (part 1) and $dq_{NS}/d\phi_k > 0$ (part 3) and $\hat{q}(\sigma_2) > \hat{q}(x)$ for $x > \sigma_2$ (as $d\hat{q}(x)/dx > 0$ by inspection of equation (34)).

Part (6) $\frac{ds_k}{d\phi_k} > 0$ **for** $\sigma_1 = \sigma_2 < \sigma_{1A}(\phi)$**:** Given the assumption that $\sigma_1 = \sigma_2 \leq \sigma_{1A}$, it holds that σ_1 is constant for changes in ϕ_k. Therefore, equation (37) shows that s_k changes with ϕ_k only via $r(\phi_k)$, where $\frac{\partial s_k}{\partial r} = -\frac{1-\alpha_k}{\alpha_k}(1-\sigma_1) \leq 0$. Because $\frac{dr}{d\phi_k} < 0$ (part 4), this implies $\frac{ds_k}{d\phi_k} > 0$:

$$\frac{ds_k(\sigma_2 < \sigma_{1A})}{d\phi_k} = \underbrace{\frac{\partial s_k}{\partial r}}_{-}\underbrace{\frac{dr}{d\phi_k}}_{-} + \frac{\partial s_k}{\partial\sigma_1}\underbrace{\frac{d\sigma_1}{d\phi_k}}_{=0} > 0 \tag{131}$$

■

C.14.7 Lemma 10: Properties of s_r

Lemma 10 *Properties of s_r defined by equation (33) in Proposition 16 include:*

1. $s_r(\phi_1 = 0, \sigma_2 = 0) = s^* > 0$, $s_r(\phi_1 = 1, \sigma_2 = 0) = 0$, *and* $s_r(\phi, \sigma_2 = 1) = 0$.

2. $ds_r/d\phi_k < 0$ for $\sigma_1 > 0$ and $\sigma_2 < 1$.

3. For $\phi = 0$, s_r is strictly decreasing in $\sigma_2 < 1$.

4. $s_r \leq s^*$

5. For $\phi_1 = 0$ and $\phi_2 = \ldots = \phi_{N-1} = 0$, $\lim_{\sigma_2 \to 0} \frac{d}{d\sigma_2} s_r$ is finite and strictly positive.

Proof.

Proof of part (1): $s_r(\phi_1 = 0, \sigma_2 = 0) = s^* > 0$ holds by comparing definitions of s^* and s_r. $s_r(\phi_1 = 1, \sigma_2 = 0) = 0$ holds because for $(\phi_1 = 1, \sigma_2 = 0)$ $q(v) = 0$ (see equation (107)) and hence $\pi = 0$. $s_r(\phi, \sigma_2 = 1) = 0$ holds because the limits of the integral are equal.

Proof of part (2): This is a restatement of Lemma 9 Part 5.

Proof of part (3): Equation 33 defines $s_r(\phi, \sigma_2)$. By inspection, $s_r = ds_r/d\sigma_2 = 0$ if $\sigma_2 = 1$ for all ϕ. By definition of s_r and s^*, $s_r(\phi = 0, \sigma_2 = 0) = s^*$.

Equation 33 shows that $s_r(\phi, \sigma_2)$ depends on $\pi = vq(v)$ and $\hat{q}(x) = \mu x^{N-1} + q_{NS}$, which in turn depend on $q(v)$ and q_{NS}. Substituting $\phi = 0$ (or equivalently $k = 1$ and $\phi_k = 0$) into expressions for these terms in equations (107)–(108) yields:

$$q(v; \phi = 0) = \frac{1}{N}\left(\mu\sigma_1^{N-1} + (1-\mu)\right) \tag{132}$$

$$q_{NS}(\phi_1 = 0) = \frac{1}{N}(1-\mu)\left(1 + (\sigma_2 - \sigma_1)\sum_{n=1}^{N-1}\sigma_2^{n-1}\right) \tag{133}$$

There are two cases to consider. First, by inspection, if $\sigma_2 > \sigma_1 = \sigma_{1A}(\phi = 0)$, then $\pi = vq(v)$ is constant in σ_2 but q_{NS} is increasing in σ_2. In this case, for $\sigma_2 < 1$,

$$\frac{d}{d\sigma_2}s_r(\phi = 0, \sigma_2) = -\pi\int_{\sigma_2}^{1}\left(\frac{\mu(N-1)\sigma_2^{N-2} + \frac{dq_{NS}}{d\sigma_2}}{\hat{q}^2(\sigma_2)} - \frac{\frac{dq_{NS}}{d\sigma_2}}{\hat{q}^2(x)}\right)dx < 0 \tag{134}$$

The inequality follows because the integrand is strictly positive and hence the integral is strictly positive for all $\sigma_2 < 1$. The integrated is strictly positive because (1) numerators and denominators of both terms are positive (as $dq_{NS}/d\sigma_2 > 0$); (2) $\hat{q}(\sigma_2) < \hat{q}(x)$ for all $x > \sigma_2$, making the denominator of the first term of the integrated smaller than that of the second term; and (2) the numerator of the first term is larger due to the additional $\mu(N-1)\sigma_2^{N-2}$.

Second, if $\sigma_2 = \sigma_1 < 1$, then $q_{NS}(\phi_1 = 0) = \frac{1}{N}(1-\mu)$ is independent of σ_2 but $\pi = vq(v)$ is strictly increasing in σ_2. In this case it is useful to express $q(v)$ as

$$q(v) = \frac{1}{N}\mu\sigma_2^{N-1} + \frac{1}{N}(1-\mu) = \frac{1}{N}\Big(\hat{q}(\sigma_2) - q_{NS}\Big) + q_{NS} = \frac{1}{N}\hat{q}(\sigma_2) + \frac{N-1}{N}q_{NS} \tag{135}$$

Letting

$$\psi(\sigma_2, x) = \left(\frac{\frac{1}{N}\hat{q}(\sigma_2) + \frac{N-1}{N}q_{NS}}{\hat{q}(\sigma_2)} - \frac{\frac{1}{N}\hat{q}(\sigma_2) + \frac{N-1}{N}q_{NS}}{\hat{q}(x)} \right) \tag{136}$$

We can rewrite equation (33) for s_r as $s_r = v \int_{\sigma_2}^{1} \psi(\sigma_2, x)dx$. Because q_{NS} is independent of σ_2, $\frac{d\psi(\sigma_2,x)}{d\sigma_2} = \frac{\partial \psi}{\partial \hat{q}(\sigma_2)} \frac{d\hat{q}(\sigma_2)}{d\sigma_2}$, and $\frac{d\hat{q}(v)}{d\sigma_2} = \mu(N-1)\sigma_2^{N-2} > 0$. The derivative $\frac{\partial \psi}{\partial \hat{q}(\sigma_2)}$ is negative:

$$\frac{\partial \psi}{\partial \hat{q}(\sigma_2)} = -\int_{\sigma_2}^{1} \left(\frac{\frac{N-1}{N}q_{NS}}{\hat{q}^2(\sigma_2)} + \frac{\frac{1}{N}\hat{q}(x)}{\hat{q}^2(x)} \right) dx < 0 \tag{137}$$

Thus $\frac{ds_r(\phi=0,\sigma_2)}{d\sigma_2} < 0$ when $\sigma_2 = \sigma_1 < 1$.

Proof of part (4): A direct implication of Parts 1–3.

Proof of part (5): Equation 33 defines $s_r(\phi, \sigma_2)$. Equation 33 shows that $s_r(\phi, \sigma_2)$ depends on $\pi = vq(v)$ and $\hat{q}(x) = \mu x^{N-1} + q_{NS}$, which in turn depend on $q(v)$ and q_{NS}. Substituting $\sigma_1 = \sigma_2 = \sigma$ into expressions for these terms in equations (107)–(108) yields:

$$q(v;k) = \frac{1}{N}\left(\mu\sigma^{N-1} + (1-\mu)\left((1-\phi_k)\sigma^{k-1} + \phi_k\sigma^k\right)\right) \tag{138}$$

$$q_{NS} = \frac{1}{N}(1-\mu)\left(1 + \sum_{n=1}^{k-1}\sigma^n + \sigma^k\phi_k\right) \tag{139}$$

Differentiating with respect to σ yields:

$$\frac{d}{d\sigma}q(v;k) = \frac{1}{N}\left(\mu(N-1)\sigma^{N-2} + (1-\mu)\left((1-\phi_k)(k-1)\sigma^{k-2} + \phi_k k\sigma^{k-1}\right)\right) \tag{140}$$

$$\frac{d}{d\sigma}q_{NS} = \frac{1}{N}(1-\mu)\left(\sum_{n=1}^{k-1} n\sigma^{n-1} + k\sigma^{k-1}\phi_k\right) \tag{141}$$

Taking the limit as $\sigma \to 0$ yields:

$$\lim_{\sigma\to 0}\frac{d}{d\sigma}q(v;k) = \begin{cases} \frac{1}{N}(1-\mu)\phi_1 & \text{if } k=1 \\ \frac{1}{N}(1-\mu)(1-\phi_2) & \text{if } k=2 \\ 0 & \text{if } k>2 \end{cases} \tag{142}$$

$$\lim_{\sigma\to 0}\frac{d}{d\sigma}q_{NS} = \frac{1}{N}(1-\mu)\phi_1 \tag{143}$$

Differentiation equation (33) with respect to σ_2 yields:

$$\frac{d}{d\sigma_2}s_r = -vq(v)\int_{\sigma_2}^{1}\left(\frac{\mu(N-1)\sigma_2^{N-2} + \frac{dq_{NS}}{d\sigma_2}}{\hat{q}^2(\sigma_2)} - \frac{\frac{dq_{NS}}{d\sigma_2}}{\hat{q}^2(x)} \right) dx + v\frac{dq(v)}{d\sigma_2}\int_{\sigma_2}^{1}\left(\frac{1}{\hat{q}(\sigma_2)} - \frac{1}{\hat{q}(x)}\right)dx \tag{144}$$

Since $\lim_{\sigma_2 \to 0} q(v) = \frac{1}{N}(1-\mu)(1-\phi_1)$ (Lemma 7), for $\phi_1 = 1$ we have $\lim_{\sigma_2 \to 0} q(v; \phi_1 = 1) = 0$. This means that the first term is zero. Then, substituting equation (142) for $dq(v)/d\sigma_2$, $\lim_{\sigma_2 \to 0} \hat{q}(\sigma_2) = q_{NS}$, and $\lim_{\sigma \to 0} q_{NS} = \frac{1}{N}(1-\mu)$ (Lemma 7) into the second term yields:

$$\lim_{\sigma \to 0} \frac{d}{d\sigma_2} s_r(\phi_1 = 1) = v \left(\begin{cases} \frac{1}{N}(1-\mu) & \text{if } k = 1 \\ \frac{1}{N}(1-\mu)(1-\phi_2) & \text{if } k = 2 \\ 0 & \text{if } k > 2 \end{cases} \right) \int_0^1 \left(\frac{1}{\frac{1}{N}(1-\mu)} - \frac{1}{\mu x^{N-1} + \frac{1}{N}(1-\mu)} \right) dx$$
$$= \quad (145)$$

For $\phi_2 < 1$ this is positive but finite, otherwise it is zero.

■

C.14.8 Proof of Proposition 16

Proof.

Proof of Part 2(i): This is the result of Lemma 6.

Proof of Proposition 16 Part 2 (ii):

Step 1. Show that these conditions are necessary: Necessity follows from Lemmas 5–8 and the requirement that a firm be indifferent over all prices in the support of its strategy. Given σ_2 and ϕ, Lemma 8 implies that equations (26)–(27) characterize the unique σ_1 consistent with firms' best responding and that $\sigma_{1A} \in (0,1)$. Given the assumption that $\sigma_2 > 0$, equation (27) and $\sigma_{1A} > 0$ imply that $\sigma_1 > 0$ and hence v is in the support of $F(p)$. Since expected firm profits must be equal at any price in the support of the $F(p)$, firm profits are given by equation (28). Moreover, given the structure of firms' strategy specified by Lemma 5, $pq(p)$ must equal π for all $p \in [\underline{p}, r] \cap [p^*, v]$. p^*, r, and $\underline{p}$ can thus be solved by substituting $q(p)$ from equation 24 into $p = \pi/q(p)$ for each of the three values of p: $p^* = \pi/\hat{\hat{q}}(\sigma_2)$, $r = \pi/\hat{q}(\sigma_2)$, $\underline{p} = \pi/\hat{q}(1)$. This yields equations (29)–(31). Similarly, $pq(p) = \pi$ can be solved explicitly for $F(p)$ for $p \in [\underline{p}, r]$, and implicitly defines $F(p)$ for $p \in [p^*, v)$, yielding the second and fourth lines of the expression for $F(p)$ in equation (32).

Step 2. Show that these conditions characterize a unique r and function $F(p)$ that is strictly increasing on $[\underline{p}, r] \cap [p^*, v]$, continuous for all $p \neq v$, and is a valid CDF.

Claim: $p^* < v$ if and only if $\sigma_2 > \sigma_1$. The proof of the claim has two parts: (1) $\sigma_2 = \sigma_1 \to p^* \geq v$: By Lemma 8, equation (27) implies that $q(v^-) \leq q(v)$. Note that for $\sigma_2 = \sigma_1$, $1 - F(p^*) = \sigma_2 = \sigma_1 = 1 - F(v^-1)$ and hence $q(p^*) = q(v^-)$. Hence, for $\sigma_1 = \sigma_2$, $p^* = \pi/q(p^*) = \pi/q(v^-) \geq \pi/q(v) = v$. (2) $\sigma_2 > \sigma_1 \to p^* < v$: For the alternative case of $\sigma_2 > \sigma_1$, we have $\sigma_1 = \sigma_{1A}$, which implies $q(v^-) = q(v)$ by definition of σ_{1A}. Moreover, $q(p^*) = \hat{\hat{q}}(\sigma_2) > \hat{\hat{q}}(\sigma_1) = q(v^-1)$ because $\hat{\hat{q}}(x)$ is strictly increasing. Hence $p^* = \pi/q(p^*) < \pi/q(v^-) = \pi/q(v) = v$.

Further, the intervals over which $F(p)$ is defined partition prices because the definitions of $\underline{p}$, r, and p^* imply that $\underline{p} < r < p^*$ and $r < v$. The inequality $\underline{p} < r < p^*$ follows because $\hat{q}(x)$ and $\hat{\hat{q}}(x)$ are both strictly increasing and $\hat{q}(\sigma_2) = q(r) > q(p^*) = \hat{\hat{q}}(\sigma_2)$, meaning that $\hat{q}(1) > \hat{q}(\sigma_2) > \hat{\hat{q}}(\sigma_2)$. Moreover, $q(r) > q(v)$ implies $r < v$. The inequality $q(r) > q(v)$ holds because reducing p from v to r wins at least as many non-shoppers and strictly more shoppers by breaking the tie when all $N-1$ firms price at v (a positive probability event for given $\sigma_1 > 0$). More formally, from equation (24),

$$q(r) = \mu\sigma_2^{N-1} + q_{NS} \geq \mu\sigma_1^{N-1} + \frac{1}{N}(1-\mu)\left(1 + \sum_{n=1}^{N-1} \Pi_{j=1}^{n}\sigma_1\phi_j\right) = \mu\sigma_1^{N-1} + \frac{1}{N}(1-\mu)\sum_{n=1}^{N} \Pi_{j=1}^{n-1}\sigma_1\phi_j$$

$$> \frac{1}{N}\mu\sigma_1^{N-1} + \frac{1}{N}(1-\mu)\sum_{n=1}^{N}(1-\phi_n)\Pi_{j=1}^{n-1}\sigma_1\phi_j = q(v) \quad (146)$$

$F(p)$ is continuous at $\underline{p}$, r, and p^* by construction. Similarly, for $\sigma_2 > \sigma_1$, $\lim_{p\to v^-} F(p) = 1 - \sigma_1$ by construction.

Existence and uniqueness of σ_{1A} follows from Lemma 8. Existence and uniqueness of the terms in equations (27)–(31) follows by inspection. For $\sigma_2 > \sigma_1$, the region $[p^*, v)$ is non-empty and $F(p) = 1 - x$ is implicitly defined by $\hat{\hat{q}}(x) = \pi/p$. Because $\hat{\hat{q}}(x)$ is continuous and strictly increasing, this has a unique solution for which x is continuous and strictly decreasing in p (meaning $F(p)$ is continuous and strictly increasing on $p \in [p^*, v)$). $F(p)$ is strictly increasing for $p \in [\underline{p}, r]$ by inspection. Thus these conditions characterize a unique $F(p)$ that is strictly increasing on $[\underline{p}, r] \cap [p^*, v]$.

Step 3. Show that these conditions are sufficient for $F(p)$ to be a mutual best response by firms to each other and to consumers.

By construction of r and $F(p)$, firm expected profits are equal over the support of $F(p)$. It only remains to show that profits are weakly lower for any price outside the support of $F(p)$. Prices $p > v$ yield zero demand and zero profit, so are strictly lower. Prices $p < \underline{p}$ yield the same demand as at $\underline{p}$, so strictly lower profit. For $\sigma_2 > \sigma_1$, demand is constant on the interval $(r, p^*]$, so profits from $p \in (r, p^*)$ must be strictly lower than at p^*. Similarly, for $\sigma_2 = \sigma_1$, demand is constant on the interval (r, v), so profits from $p \in (r, v)$ must be strictly increasing over that interval. Moreover, since $\sigma_1 = \sigma_{1A}$ we know that $q(v^-) = q(v)$ by definition of σ_{1A}. Hence, profits for any $p \in (r, v)$ must be strictly lower than at $p = v$.

Proof of Proposition 16 Part 2 (iii):

Condition for optimal r: Equation (96) from Lemma 4 part (1) can be equivalently rewritten as:

$$s = \int_{\underline{p}}^{r} (r-p)\frac{f(p)}{1-\sigma_2}dp \quad (147)$$

For $p \in [\underline{p}, r]$, $F(p)$ is strictly increasing (equation (32). Hence, we can rewrite this again using a change of variables as an integral over $x = 1 - F(p)$.[8]To do so, we need to express p as a function of x. For firms to be indifferent over all prices in the support of F, profits must be constant at $\pi = vq(v)$, which implies that $p = \pi/q(p)$, or equivalently that $p = \pi/\hat{q}(x)$. This yields $s = s_r$ for s_r given by equation (33). For $s = s_r$, the consumers' reservation price will be $r = \pi/\hat{q}(\sigma_2)$, consistent with equation (96) which specifies the r for which firms are indifferent between $p = r$ and $p = v$.

Condition for optimal ϕ: The conditions on s for ϕ to be optimal are implied by Lemma 4. The equation for s_k is derived from equation 97 by setting $\hat{B}_k = 0$ and noticing that equation (6) implies

$$\frac{1-\alpha_k}{\alpha_k} = \frac{1-\alpha_0}{\alpha_0}\sigma_1^k. \tag{148}$$

Proof of Proposition 16 Part 1: Consider s sufficiently high that $r \geq v$. Following Lemmas 4–5, $\sigma_1 = \phi_1 = \ldots = \phi_{N-1} = 0$. In this case, demand in equation (24) reduces to that assumed by Varian (1980):

$$q(p) = \begin{cases} \mu(1-F(p))^{N-1} + \frac{1}{N}(1-\mu) & \text{if } p \leq v \\ 0 & \text{if } p > v \end{cases} \tag{149}$$

Therefore this yields Varian's (1980) equilibrium firm pricing as described in the Proposition.

This firm pricing strategy is consistent with the consumer strategy $r \geq v$ for all $s \geq s^*$ (where s^* is defined in equation (7)). This is because $s_r(\phi = 0, \sigma_2 = 0) = s^*$, as is apparent by substituting $\phi = \sigma_2 = 0$ into equation (33). To do so, note that equations (25) and (34) reduce to $q_{NS}(\phi = 0, \sigma_2 = 0) = \frac{1}{N}(1-\mu)$ and $\hat{q}(\phi = 0, \sigma_2 = 0) = q_{NS}$. Substituting these expressions, as well as $\hat{q}(x)$ from equation (34), and $\pi = vq(v)$ for profits using $q(v)$ from equation (24), into equation (96) yields s^*.

No other equilibrium can exist for $s \geq s^*$ because all equilibria with $r < v$ must satisfy $s_r(\phi, \sigma_2) = s$ (by part 2 of the proposition), and s^* is the upper bound for s_r, which it only attains at $\phi = \sigma_2 = 0$ (as s_r is strictly decreasing in σ_2 for $\phi = 0$ and decreasing in ϕ by Lemma 10). ■

[8]The change of variables formula for $x = g(t)$ is $\int_{g(a)}^{g(b)} h(x)\,dx = \int_a^b h(g(t))\,g'(t)\,dt$. Here $t = p$, $x = g(p) = 1 - F(p)$, and $h(x) = \pi/\hat{q}(x)$. Thus $\int_{1-F(\underline{p})}^{1-F(\bar{p})} h(x)\,dx = -\int_{\underline{p}}^{\bar{p}} pf(p)\,dp$, or equivalently, $\int_{\underline{p}}^{\bar{p}} pf(p)\,dp = \int_{\sigma_2}^{1} h(x)\,dx$. This approach is based on Janssen, Moraga-Gonz\'{a}lez, and Wildenbeest (2005).