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Economic Growth when Knowledge is Concentrated

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June 2024

PRELIMINARY AND INCOMPLETE

Abstract

Firms' innovation outcomes crucially depend on their ability to screen, attract and retain talented inventors. The efficient functioning of the market for inventors is essential to ensure that firms with high innovation potential are matched with high-productivity inventors. What frictions prevent such efficient sorting? And what is their impact on aggregate innovation and growth? We address these questions both empirically and theoretically. Empirically, we show that firms facing strong competition in the product market employ more productive inventors, while firms that are far away from their competitors employ less productive researchers. Theoretically, we rationalize these findings by embedding a frictional labor market for inventors in an endogenous-growth model of strategic interaction and innovation. In the model, firms must hire researchers in order to advance their productivity, and hiring incentives are shaped by the intensity of competition that firms face. Market leaders retain talented inventors to discourage competitors' innovation efforts. We show that the model delivers predictions that align with empirical evidence and use it to quantify the growth and welfare implications of inventor-firm sorting. A preliminary qualitative exercise suggests that increasing matching efficiency in the market for inventors favors the reallocation of high-productivity inventors to firms with high R&D intensity. Aggregate R&D spending and the rate of economic growth therefore increase as misallocation in the market for inventors falls.

JEL CODES: L16, J6, O3, O4.

KEYWORDS: Inventors, Innovation, R&D Productivity, Growth, Misallocation, Search.

[*]Andrea Guccione acknowledges financial support from Grant PRE2022-102132 funded by MCIN/AEI/10.13039/501100011033 and by "ESF +", the Maria de Maeztu Unit of Excellence CEMFI MDM-2016-0684 and CEMFI CEX2020-001104-M, funded by MCIN/AEI/10.13039/501100011033, and CEMFI.

1 Introduction

In idea-based growth models, economic growth arises from people generating new ideas. Long-run growth in these models is given by the product of two terms: the total amount of R&D inputs in the economy and the productivity of these inputs. Thus, understanding productivity growth requires understanding not only firms' incentives to advance their productivity, but also the efficiency with which they utilize their R&D inputs.

Inventors are the key inputs in firms' R&D processes. Inventors' human capital is crucial for innovation and firms' innovation outcomes critically depend on their ability to screen, attract and retain talented inventors (Jones (2009), Bhaskarabhatla, Cabral, Hegde and Peeters (2021)). Aggregate innovation and growth in an economy are highest when firms with highest innovation potential and R&D productivity are matched with the best inventors. The functioning of the market for inventors is essential for ensuring such an efficient allocation of inventors across firms.

In this paper, we study the labor market for inventors to understand the sorting patterns of inventors into firms and quantify their aggregate implications. Empirically, we present new findings on inventors' mobility and their allocation across firms. Theoretically, we develop an endogenous-growth model featuring a frictional market for inventors, where firms competing in the product market strive to match with the best inventors. Efficiency in the market for inventors determines the extent to which firms with high R&D capabilities are able to hire and retain talented inventors. Our model therefore allows to quantify the growth and welfare effects of matching frictions that prevent an efficient allocation of inventors across firms.

We begin by documenting novel empirical facts about the market for inventors and their allocation across firms. By combining patent data from the USPTO with firms' financial information from Compustat North America, we identify which inventors work for which firms and relate inventors' characteristics to balance sheet information about their employers. We document that there is significant turnover in the market for inventors: on average 6.6% of US inventors change employer every year and mobility rates are higher for inventors with better past innovation outcomes. Inventors' mobility is important for firms' innovativeness: we document that employing more productive inventors improves firms' innovation outcomes, especially for firms that face intense competition in the product market. As a result, we find that there is an hump-shaped relationship between hiring rates of inventors and firms' relative sales within industries, which is particularly pronounced in the top quintiles of the inventors' productivity distribution.

Motivated by these findings, we build an innovation-based growth model with a frictional market for researchers. In particular, we merge a model of competitive innovation and a dynamic directed search model. A standard feature of innovation-based growth models is that firms can hire R&D inputs on frictionless spot markets. In our model, by contrast, in order to innovate, firms must access the frictional market for inventors. Inventors are heterogeneous in terms of human capital and firms compete to attract the best inventors and implement their ideas.

The model economy consists of many sectors. In each sector, two firms produce imperfect substitute varieties of the same good and compete in prices à la Bertrand for market leadership. Firms are heterogeneous in productivity, which evolves endogenously through successive innovations. Firms that employ inventors of higher ability can achieve greater productivity advancements by implementing their ideas. Firms' innovation incentives are driven by competition in the product market: market leaders try to innovate to widen their lead, while followers try to catch up with the leader. Intense competition in the product market induces more aggressive innovation investments by firms, while innovation efforts are lower in sectors where the productivity gap between leader and follower is larger. Our framework thus allows for heterogeneity along both R&D intensity and R&D productivity, and links them together. In this framework, it is not only the aggregate arrival rate of innovations that matters for economic growth, but also the matching of inventors and firms: growth will be highest when inventors with the best ideas work for the firms that have the highest incentives to implement them.

In our model, the matching between inventors and firms occurs in a frictional labor market. Search in this market is directed on both the inventor and the firm side: firms announce wage contracts in order to attract unattached inventors, who optimally direct their search based on these offers. Meeting rates in in the market for inventors are therefore determined by the optimal choices of inventors and firms and the model generates endogenously heterogeneous job-filling rates across firms. In particular, firms that operate in highly competitive sectors offer higher wages to high-productivity inventors and are more likely to hire them as a result. However, dominant firms in concentrated sectors also promise high values to these inventors, knowing that employing them will deter competitors' innovation efforts.

We validate our model by showing that this theoretical framework delivers predictions that are in line with the empirical evidence. Specifically, the model replicates the allocation of inventors across firms and the patterns' of inventors' mobility that we observe in the data. As in the data, firms that employ more productive inventors invest more in R&D, and the effect is hump-shaped in the firm's market share. Moreover, the model reproduces the empirical hump-shaped relationship between hiring rates of inventors and firms' relative sales. In equilibrium, firms that are far from their competitors tend to employ low-productivity inventors, while firms that are close to their competitors tend to employ high-productivity inventors. There are however many high-productivity inventors that are inefficiently employed by dominant firms, with little R&D intensity. This happens because of the strategic nature of firms' competition for inventors: dominant firms employ talented inventors to discourage innovation by their competitors and defend their market leadership.

We use the model to study the growth and welfare implications of inventor-firm sorting. In a preliminary qualitative exercise, we show that increasing matching efficiency in the market for inventors favors the reallocation of high-productivity inventors to firms with high R&D intensity. Aggregate R&D spending and the rate of economic growth therefore increase as misallocation in the market for inventors falls. As matching efficiency improve, we also observe an increase in the degree of concentration in the product market, as firm with high R&D intensity separate themselves from competition. This force pushes down aggregate output and consumption. Despite this, due to the effect on growth, welfare is higher in an economy with larger matching efficiency.

Related literature. This paper builds on and contributes to several literatures. Theoretically, our model builds on the branch of the endogenous-growth literature that incorporates product market competition and strategic interaction between firms in general equilibrium models of economic growth (Aghion, Harris, Howitt and Vickers (2001), Aghion, Bloom, Blundell, Griffith and Howitt (2005), Acemoglu and Akcigit (2012), Akcigit and Ates (2023)). We contribute to this literature by embedding a frictional labor market for inventors in a growth model of strategic interaction and innovation. Our modeling of the market for inventors borrows from the dynamic directed search literature with heterogeneous firms (Kaas and Kircher (2015), Schaal (2017)). Thus we also contribute to the literature on frictional search by applying it to the market for inventors.

By studying the matching between inventors and firms, our paper is closely related to the recent literature, initiated by Bloom, Jones, Van Reenen and Webb (2020), that investigates the determinants of aggregate research productivity. Within this literature, some authors have pointed to the role of the allocation of inventors across firms as a potentially important determinant of research productivity (Lehr (2022), Manera (2022), Akcigit and Goldschlag (2023)). While the focus of these papers is on the changing patterns of firm-inventor sorting over time and their potential to explain the recent productivity slowdown in the US economy, we still know very little about the determinants of the sorting of inventors into firms in the cross-section and its quantitative implications for innovative capacity and growth. Our paper tries to fill this gap.

Closest to our paper is Babalievsky (2023). As we do, Babalievsky (2023) proposes a growth model that features a frictional labor market for inventors to study the consequences for growth of the allocation of inventors across firms. However, in his framework, search is random, so that meeting rates between inventors and firms are exogenous. By contrast, in our model, search is directed and meeting rates are determined endogenously by the values that firms promise to workers. Our model can therefore rationalize the novel micro evidence on inventors' mobility and firms' hiring policies that we document. Moreover, it allows for a quantification of the macroeconomic impact of this micro behaviour and is particularly suited for policy analysis, as it endogenizes firms' and inventors' responses to policy changes.

The rest of the paper is structured as follows. Section 2 describes the data, while Section 3 presents the results of our empirical analysis. Section 4 introduces the theoretical model and Section 5 describes its equilibrium. In Section 6 we present some of the model results, showing that the model's predictions align with the empirical evidence. In Section 7, we present the results of a preliminary assessment of the growth and welfare implications of the market for inventors. Section 8 concludes.

2 Data

2.1 Data sources

We combine data from two main sources: (i) the US Patent and Trademark Office (USPTO) PatentsView database; and (ii) Compustat North America.

The USPTO database provides comprehensive information on all patents granted by the USPTO since 1976. We leverage data on utility patents granted up to 2018, which allows us at least five years to observe forward citations to the latest patents. For each patent, the dataset includes detailed information about both the inventors who contributed to the innovation and on its assignees, and allow us to track them over time. Our dataset contains 6,203,450 patents granted between 1976 and 2018, and 3,180,835 unique inventors. We assign patents to specific years according to their filing date to the USPTO, which ensures that the patent is assigned to the year closest to the actual innovation.

We rely on Compustat North American Fundamentals for financial statement information about

US-listed firms. We map patent assignees to Compustat firms using the crosswalk by Dyèvre and Seager (2023). Doing so, allows us to identify which inventors work for which firms and relate inventors' characteristics to balance sheet information about their employers[1]. The crosswalk includes 2,564,512 patents, that is 41% of all patents granted by the USPTO since 1976, assigned to 8,182 unique Compustat firms. 1,127,381 inventors are attached to these firms.

2.2 Variable construction

Inventors' productivity. The main inventor characteristic that we are interested in is some measure of their productivity. In what follows, we explain how we construct this measure.

As is standard in the literature, we measure the quality of a patent by the number of forward citations it receives in a 5-year window from its grant date:

$$q_p = \sum_{\tau=g}^{g+4} \text{citations}_\tau$$

We define the flow productivity of inventor j in year t as the total quality of the patents that she produced in that year:

$$p_{j,t} = \sum_p q_{p(j,t)}$$

Given this flow productivity measure, we construct the stock, or cumulative, productivity of inventor j at time t as her citations-weighted patent stock at the beginning of period t:

$$P_{j,t} = \frac{\sum_{s=t_0(j)}^{t-1} p_{j,s}}{t - t_0(j)}$$

This is our benchmark measure of an individual inventor's productivity and takes into account the quantity and quality of an inventor's past innovations. We normalize it by the inventor's age to adjust for the fact that older inventors are expected to have higher cumulative productivity due to having more years to accumulate citations.

For robustness, we also consider alternative measures of inventors' productivity based on different definitions of patent quality: forward citations in a 3-year window from the grant date; forward citations excluding self-citations; and citations per author. At the firm level, for each firm and year, we then compute the average productivity of the inventors employed by the firm in that year.

[1] We say that inventor j is employed by firm i in year t if the majority of the patents that j applied for in that year are assigned to firm i.

Inventors' employment histories. As we do not have access to actual employment information for inventors, we analyze their mobility by identifying their affiliation over time from the information contained in the patents. Based on the application dates and assignees of their patents, we create each inventor's employment history. As in Akcigit, Caicedo, Miguelez, Stantcheva and Sterzi (2018), we assume that inventor j is employed in firm i from the year of her first patent in firm i to either (1) the year before the first patent invented in firm k or (2) the year of the last patent in firm i in case she does not have any further patent afterwards. We assign zero patents to the "unproductive" years of inventors, instead of treating them as missing.

Based on inventors' employment histories, we identify inventors who move across firms. We then compute hiring and separation rates of inventors at the firm level, defined as the number of inventors who move to/leave firm i in year t as a fraction of the total number of inventors in firm i in year $t-1$.

3 Empirical analysis

To motivate our model's assumptions and test its predictions, we document new empirical facts about the market for inventors and the allocation of inventors across firms.

Inventors' mobility. We begin by presenting some descriptive evidence about inventors' mobility across firms.

1. In the full USPTO sample, on average 6.6% of inventors change employer every year. This share has been increasing over time, from 4-5% in the 1980s to 9-10% in the 2010s.

2. In the full USPTO sample, the average number of years between an inventor's first and last patent is 6.7. Inventors have on average 5.3 patents across 1.5 firms over their career. This suggests a non-negligible amount of turnover: a move occurs once every 4.6 years on average.

3. Mobility rates are higher among inventors with better past innovation outcomes (Figure 1).

Firms' hiring policies. We analyze how hiring and separation rates of inventors change across firms of different market share. We estimate the following regression at the firm level:

$$Y_{it} = \beta_0 + \beta_1 MktShare_{it} + \beta_2 MktShare_{it}^2 + \beta_3' X_{it} + \gamma_{s(i)t} + u_{it} \tag{1}$$

The outcome variables that we consider are: the firm hiring rate, separation rate and net hiring rate of inventors, defined as the difference between the first two. X_{it} includes controls for the firm's: age, employment, R&D stock, profitability, leverage and market-to-book ratio. $\gamma_{s(i)t}$ are industry-by-year fixed effects. Results are in Table 1. We find that, within industries, there is an hump-shaped relationship between hiring rates and firms' relative sales and a U-shaped relationship between separation rates and firms' relative sales. As a result, we observe an hump-shaped relationship between a firm's *net* hiring rate of inventors and its market share.

We also compute hiring and separation rates separately for groups of inventors of different productivity, identified by the quintiles of the inventors' productivity distribution in a given year. This allows us to characterize how hiring and separation patterns change for inventors of different productivity across firms with different market share. As shown in Table 2, the hump-shaped relationship between net hiring rates and firms' relative sales is preserved within each quintile of the inventor's productivity distribution, but tends to get more pronounced for the top quintiles. For a given market share, net hiring rates tend to be increasing in the researchers' productivity.

Distribution of inventors across firms. We analyze the empirical patterns of inventor-firm sorting by looking at how firms that differ in their market share differ in terms of the quality of the inventors that they employ. To do so, we estimate the following regression specification:

$$\log InventorsProductivity_{it} = \beta_0 + \beta_1 MktShare_{it} + \beta_2 MktShare_{it}^2 + \beta_3' X_{it} + \gamma_{s(i)t} + u_{it} \tag{2}$$

where X_{it} includes the same firm-level controls as above. The estimation results are reported in Table 3 for alternative measures of inventors' productivity. Across these different measures, we see that there is a hump-shaped relationship between inventors' quality and firms' relative sales within industries. This tells us that, on average, firms that are close to their competitors employ better inventors than firms that are far away from their competitors. This is the allocation of inventors to firms that we observe in the data and that we would want our model to replicate.

Inventors' productivity and firms' innovation outcomes. To rationalize the allocation of inventors to firms documented above, we look at the relationship between inventors' productivity and firms' innovation outcomes. We show that this relationship is heterogeneous across firms that face different degrees of competition in the product market.

We begin by estimating the following regression specification:

$$Y_{it} = \beta_0 + \beta_1 \log InventorsProductivity_{it} + \beta_2' X_{it} + \gamma_{s(i)t} + u_{it} \tag{3}$$

We consider the following innovation outcomes at the firm level: log R&D expenditure, number of patent applications, number of citations to these patents, average number of citations per patent application. X_{it} includes controls for the number of inventors and the firm's: market share and its square, age, employment, R&D stock, profitability, leverage, market-to-book ratio. $\gamma_{s(i)t}$ are industry-by-year fixed effects. The estimation results in Table 4 show that firms that employ more productive inventors spend more in R&D, are granted more patents, have more total citations and average citations on their patents.

To study heterogeneity in the effects of inventors' productivity, we estimate equation (3) augmented with the interactions between $InventorsProductivity_{it}$, $MarketShare_{it}$ and $MarketShare^2_{it}$. The estimates, reported in Table 5, show that employing more productive inventors improves innovation outcomes especially for firms that are close to neck-to-neck and less so for firms that are far from their competitors. This relationship holds and is statistically significant for the firm's R&D expenditure, patent applications and patent citations, while we do not detect any significant heterogeneity for the average number of citations per patent application.

4 Model

Motivated by the empirical findings, we build a framework that merges an endogenous-growth model of competitive innovation with a directed search model for the market for researchers.

Demographics. Time is continuous and infinite. The model economy is populated by three groups of agents:

1. *Firms:* a representative final good firm, and a continuum of intermediate good firms (two firms per industry, and a continuum of industries). In each industry, two firms produce imperfect substitute varieties of the good and compete in prices à la Bertrand.

2. *A representative household:* with log preferences, composed of household members providing production labor, with inelastic labor supply $\boldsymbol{L} = 1$.

3. *Inventors:* with linear preferences, who differ in their knowledge capital, $\kappa \in \mathbb{K} \equiv \{0, 1, 2, \ldots, \overline{\kappa}\}$. Researchers of higher κ have better ideas, i.e. ideas that if implemented by the firm would advance its productivity by more steps.

Preferences and technology. The representative household maximizes:

$$\int_0^{+\infty} e^{-\rho t} \ln(\boldsymbol{C}_t) \mathrm{d}t$$

subject to

$$\dot{\boldsymbol{A}}_t = r_t \boldsymbol{A}_t + w_t^C - \boldsymbol{C}_t$$

with the final good as the numeraire, where w_t^C is the wage for production labor, earned in a competitive labor market.

The final good, $\boldsymbol{Y}_t$, is produced competitively combining a continuum of intermediate goods Y_{jt}. Industry j's output, Y_{jt}, is a CES aggregate of the varieties produced by the two active firms in the industry, with CES parameter σ. Final output and industry j's output are therefore:

$$\boldsymbol{Y}_t = \exp\left(\int_0^1 \ln(Y_{jt}) \mathrm{d}j\right), \quad \text{where } Y_{jt} = \left(y_{ijt}^{\frac{\varepsilon-1}{\varepsilon}} + y_{-ijt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} := \left(y_{ijt}^{\sigma} + y_{-ijt}^{\sigma}\right)^{\frac{1}{\sigma}}$$

Intermediate good firms produce with a technology that is linear in production labor:

$$y_{ijt} = q_{ijt} l_{ijt}$$

where q_{ijt} is labor productivity, which evolves endogenously through successive innovations on a ladder with step size $\lambda > 1$.

We denote by $n_{ijt} \in \mathbb{N} \equiv \{-\bar{n}, \ldots, -1, 0, 1, \ldots, \bar{n}\}$ the technology gap between firms i and $-i$ in industry j. If $n_{ijt} > 0$, firm i is the market leader in industry j; if $n_{ijt} < 0$, firm i is the market follower in industry j; if $n_{ijt} = 0$, firm i and $-i$ are neck-to-neck. Given the quality ladder assumption, the relative productivity of firm i with respect to its competitor is then:

$$\frac{q_{ijt}}{q_{-ijt}} = \lambda^{n_{ijt}}$$

As we will show later, n_{ijt} is a sufficient statistic for firm i's static payoffs. Indeed, demand for the variety produced by firm i depends on its price relative to that of the competitor. Since firms compete in prices, the firm with higher labor productivity obtains a cost advantage and captures a larger share of the market for good j in the equilibrium, thereby charging a higher markup and making higher profits.

In order to advance productivity, a firm must access the frictional market for researchers. If a researcher of type κ finds an innovation, the firm must choose the implementation rate, x. To implement an innovation at rate x, the firm must pay $\xi x^{\phi} \boldsymbol{Y}$ units of the final good, where $\xi > 0$ and $\phi > 1$. If an idea is implemented, the firm's productivity q advances by κ steps: $q_{ij(t+\Delta t)} = \lambda^{\kappa} q_{ijt}$. Thus, by implementing their researchers' ideas, firms advance their productivity and extend/reduce their gap with respect to their competitor. We also assume that market followers benefit from exogenous knowledge diffusion at rate $\tilde{\delta}$, in which case they catch up with the market leader.

Inventors. An inventor has a stock of knowledge capital $\kappa \in \mathbb{K} \equiv \{0, 1, 2, \ldots, \overline{\kappa}\}$. Researchers are therefore heterogeneous in the quality of their ideas[2].

When unattached (i.e. not working for a firm), a researcher gets an exogenous income stream equal to $b\boldsymbol{Y}$, where $b > 0$. Regardless of employment status, the knowledge capital of the researcher depreciates (e.g. the researcher loses the patent on the κ-th innovation) at exogenous rate $\delta > 0$, and her state goes from κ to $\kappa - 1$.

Regardless of employment status, the researcher finds an innovation (advancing κ to $\kappa + 1$) at Poisson rate z and cost $\chi z^{\psi} \boldsymbol{Y}$ in units of final good, where $\chi > 0$ and $\psi > 1$, which the researcher must finance on her own. As explained above, if the researcher is employed, the firm can choose to implement her idea. To implement an idea at rate x, the firm must pay $x^{\phi} \boldsymbol{Y}$ units of final good.

4.1 Search markets

Firms and inventors match in a frictional labor market, which is segmented by κ. At each point in time one firm is matched with one inventor only. Only unattached inventors search for employment, while firms can keep searching for inventors while they are matched.

Search is directed on both the inventor and the firm side: at every point in time firms announce wage contracts in order to attract unattached inventors; unattached inventors can observe each contract and direct their search optimally to the wages offered by firms.

Contracts are complete, fully state-contingent and long-term, with full commitment on the

[2] Absent this heterogeneity, the allocation of researchers across firms would be irrelevant for research productivity and growth.

firm's side and no commitment on the researcher's side. Given the assumption that inventors are risk-neutral, a sufficient statistic for each contract is the net present value that the firm promises to the researcher, which we denote by R. Therefore, each segment κ of the labor market can be described as a continuum of submarkets indexed by the utility that firms promise to researchers.

The frequency of matches in each submarket is determined by a CRS matching function $\mathcal{M}(f, u)$, where f is the measure of firms and u is the measure of unattached researchers. Given a market tightness $\theta \equiv f/u$, researchers match with a firm at Poisson rate $\mu(\theta) \equiv \mathcal{M}(\theta, 1)$, while firms find a researcher with Poisson intensity $\eta(\theta) \equiv \mathcal{M}(1, \theta^{-1})$, so that $\mu(\theta) = \theta\eta(\theta)$. We assume: (i) $\mu(\theta)$ is increasing and concave in θ; (ii) $\eta(\theta)$ is decreasing and convex in θ; (iii) Inada conditions apply, i.e. $\mu(0) = \lim_{\theta \to +\infty} \eta(\theta) = 0$, and $\lim_{\theta \to +\infty} \mu(\theta) = \lim_{\theta \to 0} \eta(\theta) = +\infty$.

4.2 Entry

Entry is directed at labor market segment κ. Every period, potential entrants pay a flow cost $c_\kappa \boldsymbol{Y}$ to enter this submarket and offer a set of promised values to unattached researchers. We assume these promised values are contingent on the state of the industry where the entrant will land, which makes entry effectively directed at intermediate good industries. A successful entrant hires a researcher of type κ and replaces the follower in its product line. The displaced follower exits the industry and its researcher goes to unemployment. We assume free-entry in any labor market segment $\kappa \in \mathbb{K}$ and normalize the measure of the pool of potential entrants to 1.

5 Equilibrium

5.1 Static Equilibrium Conditions

Households. The household's problem results in the Euler equation, which equates the growth rate of consumption to the interest rate net of the discount rate:

$$\frac{\dot{\boldsymbol{C}}_t}{\boldsymbol{C}_t} = r_t - \rho$$

Firms. The optimization of the representative final good producer generates the following demand for the variety of intermediate good j produced by firm i:

$$y_{ijt} = \left(\frac{p_{ijt}^{\frac{1}{\sigma-1}}}{p_{ijt}^{\frac{\sigma}{\sigma-1}} + p_{-ijt}^{\frac{\sigma}{\sigma-1}}} \right) \boldsymbol{Y}_t$$

implying that $P_{jt}Y_{jt} = p_{ijt}y_{ijt} + p_{-ijt}y_{-ijt} = \boldsymbol{Y}_t, \forall j$. Given this demand schedule, the market share of firm i in industry j, s_{ijt} is:

$$s_{ijt} = \frac{p_{ijt}y_{ijt}}{P_{jt}Y_{jt}} = \frac{p_{ijt}^{\frac{\sigma}{\sigma-1}}}{p_{ijt}^{\frac{\sigma}{\sigma-1}} + p_{-ijt}^{\frac{\sigma}{\sigma-1}}}$$

In the Bertrand equilibrium, firm i sets a markup:

$$M_{ijt} \equiv \frac{p_{ijt}}{w_t^C / q_{ijt}} = \frac{1 - \sigma s_{ijt}}{\sigma(1 - s_{ijt})}$$

Firm i's demand for production labor is:

$$l_{ijt} = s_{ijt} \left(\frac{M_{ijt}}{\boldsymbol{M}_t} \right)^{-1} = \boldsymbol{M}_t \frac{\sigma s_{ijt}(1 - s_{ijt})}{1 - \sigma s_{ijt}}$$

where $\boldsymbol{M}_t \equiv \frac{\boldsymbol{Y}_t}{w_t^C}$ is the aggregate markup, or the inverse of the aggregate labor share. To obtain the formula for the aggregate markup, note that by labor market clearing $\int_0^1 (l_{ijt} + l_{-ijt})\mathrm{d}j = 1$, and thus:

$$\boldsymbol{M}_t = \left[\int_0^1 (s_{ijt} M_{ijt}^{-1} + s_{-ijt} M_{-ijt}^{-1})\mathrm{d}j \right]^{-1}$$

so the aggregate markup is also a sales-weighted harmonic mean of firm-level markups.

The sales of the firm are $s_{ijt}\boldsymbol{Y}_t$ and its static profits are:

$$\Pi_{ijt} = s_{ijt}(1 - M_{ijt}^{-1})\boldsymbol{Y}_t = \left(\frac{1 - \sigma}{1 - \sigma s_{ijt}} \right) s_{ijt} \boldsymbol{Y}_t$$

We can now show that the technology gap n_{ijt} is a sufficient statistic for firms' static payoffs. To see this, note that we can write firm i's market share as:

$$s_{ijt} = \left[1 + \left(\frac{p_{-ijt}}{p_{ijt}} \right)^{\frac{\sigma}{\sigma-1}} \right]^{-1}$$

where, using the optimal pricing conditions and $s_{-ijt} = 1 - s_{ijt}$:

$$\frac{p_{-ijt}}{p_{ijt}} = \underbrace{\frac{1-\sigma(1-s_{ijt})}{1-\sigma s_{ijt}}\frac{1-s_{ijt}}{s_{ijt}}}_{:=f(s_{ijt})}\lambda^{n_{ijt}}$$

As a result, firm i's market share, markup and profits only depend on n_{ijt}:

$$\begin{aligned}
s_{ijt} &= s(n_{ijt}) \\
M_{ijt} &= M(n_{ijt}) = \frac{1-\sigma s(n_{ijt})}{\sigma(1-s(n_{ijt}))} \\
\Pi_{ijt} &= \pi(n_{ijt})\mathbf{Y}_t = s(n_{ijt})\Big(1-\big(M(n_{ijt})\big)^{-1}\Big)\mathbf{Y}_t
\end{aligned}$$

5.2 Dynamic Equilibrium Conditions

Contracts. The firm's dynamic problem has three state variables. The first is n, the technology gap with respect to its competitor, which determines the firm's static profits. The second is κ, the type of researcher that the firm employs, which determines the gains from innovation. Then, given the strategic interaction between firms, the firm needs to take the policies of its competitor into account, and these policies depend on the researcher that the competitor employs, κ^-.

A dynamic contract offered at time t by a firm with current technology gap n_t who's employing a researcher with knowledge capital κ_t, and facing a competitor employing a researcher of type κ_t^-, specifies a sequence of state-contingent wages

$$w\big(n_h, \kappa_h, \kappa_h^- : h \in [t, t+j]\big)$$

which determines a wage at each possible tenure $j > 0$.

Contracts can be written in recursive form, as specifying the current wage and some future promised utility, contingent on next period state. In a Markov Perfect Equilibrium, a recursive contract offered by a firm in state (n, κ, κ^-) in the market for researchers of human capital κ is the object:

$$\mathcal{C}_\kappa = \Big(w_\kappa, \{R'_\kappa(n', \kappa', \kappa^{-\prime})\}\Big) := \Big(\omega_\kappa \cdot \mathbf{Y}, \{P'_\kappa(n', \kappa', \kappa^{-\prime}) \cdot \mathbf{Y}\}\Big)$$

where $(n', \kappa', \kappa^{-\prime})$ is the next state, while ω_κ and $P'_\kappa(n', \kappa', \kappa^{-\prime})$ are the normalized wage and continuation value, respectively.

Unattached inventors. Since unattached inventors direct their job search towards the most attractive offers, the value while unattached, denoted $\boldsymbol{U}_\kappa$, is:

$$\boldsymbol{U}_\kappa = \max_P \; U_\kappa(P)$$

where $U_\kappa(P)$ is the value of searching in submarket P and solves the HJB equation:

$$\begin{aligned} rU_\kappa(P) - \dot{U}_\kappa(P) = b\boldsymbol{Y} \; &+ \mu(\theta_\kappa(P)) \max\Big(P\boldsymbol{Y} - U_\kappa(P), 0\Big) \\ &+ \max_{z_\kappa^U} \left\{ z_\kappa^U \Big(U_{\min(\kappa+1,\bar{\kappa})}(P) - U_\kappa(P) \Big) - \chi (z_\kappa^U)^\psi \boldsymbol{Y} \right\} \\ &+ \delta \Big(U_{\max(\kappa-1,0)}(P) - U_\kappa(P) \Big) \end{aligned}$$

The first-order condition for innovating while unattached is:

$$z_\kappa^U(P) = \left(\frac{U_{\min(\kappa+1,\bar{\kappa})}(P) - U_\kappa(P)}{\chi\psi\boldsymbol{Y}} \right)^{\frac{1}{\psi-1}}$$

Free-entry of researchers of type κ implies the following complementarity-slackness condition:

$$\forall (P,\kappa) \in \mathbb{P}_+ \times \mathbb{K} : \quad U_\kappa(P) \leq \boldsymbol{U}_\kappa, \text{ with equality if, and only if, } \mu\big(\theta_\kappa(P)\big) > 0$$

In words, since inventors can choose where to search, the value of being unattached must be equal in all markets where job finding is positive. So we have an indifference condition on the researcher's side: since unattached researchers choose the best market to search in, all active markets must be equally attractive ex ante.

Therefore, we can replace $U_\kappa(P)$ by $\boldsymbol{U}_\kappa$ (or else market segment P would remain idle), which allows us to write the value of being unattached and type κ as:

$$r\boldsymbol{U}_\kappa - \dot{\boldsymbol{U}}_\kappa = b\boldsymbol{Y} + \mu(\theta_\kappa(P))\Big(P\boldsymbol{Y} - \boldsymbol{U}_\kappa\Big) + \boldsymbol{Z}_\kappa$$

where

$$\boldsymbol{Z}_\kappa \equiv \delta\Big(\boldsymbol{U}_{\max(\kappa-1,0)} - \boldsymbol{U}_\kappa\Big) + z_\kappa^U\Big(\boldsymbol{U}_{\min(\kappa+1,\bar{\kappa})} - \boldsymbol{U}_\kappa\Big) - \chi(z_\kappa^U)^\psi \boldsymbol{Y}$$

and

$$z_{\kappa}^{U} = \left(\frac{\boldsymbol{U}_{\min(\kappa+1,\bar{\kappa})} - \boldsymbol{U}_{\kappa}}{\chi \psi \boldsymbol{Y}} \right)^{\frac{1}{\psi-1}}$$

From here, we can get the equilibrium market tightness for researchers of type κ, denoted by $\theta_{\kappa} : (P, \boldsymbol{U}) \to \mathbb{R}_{+}$ and given by:

$$\theta_{\kappa}(P) = \mu^{-1} \left(\boldsymbol{\Phi}_{\kappa}(P) \right)$$

where $\boldsymbol{\Phi}_{\kappa}(P) \equiv \frac{r\boldsymbol{U}_{\kappa} - \dot{\boldsymbol{U}}_{\kappa} - b\boldsymbol{Y} - \boldsymbol{Z}_{\kappa}}{P\boldsymbol{Y} - \boldsymbol{U}_{\kappa}}$. Market tightness is a decreasing function of P: more ex-post profitable offers attract a larger measure of inventors. In equilibrium firms design contracts for which a low meeting rate for inventors is compensated with higher promised value.

Employed inventors and firms. We denote the value of an inventor of type κ employed in firm with technology gap n, facing a competitor employing an inventor of type κ^{-}, under contract $\mathcal{C}_{\kappa} \equiv$ by $\boldsymbol{R}(n, \mathcal{C}_{\kappa})$. This value solves HJB equation (A1) in the Appendix. We denote the value of a firm of type n employing a researcher of type $\kappa \in \mathbb{K}$, facing a competitor with researcher of type κ^{-}, under promised value P, by $\boldsymbol{V}_{\kappa,\kappa^{-}}(n, P)$. This value solves HJB equation (A2) in the Appendix.

The dynamic problem of the firm has three choice variables: the current wage, ω_{κ}, the promised values $\{P_{\kappa}'(n', \kappa', \kappa^{-\prime})\}$ and the rate of implementation of the researcher's idea. In choosing the optimal contract, the firm is constrained by a promise-keeping and a worker's participation constraint. Promise-keeping arises from the assumption of commitment on the firm side, which implies that contracts must deliver at least the promised utility to the worker. The assumption of no commitment on the inventor side means that contracts are also subject to a participation constraint: he value that the inventor obtains in any future state cannot be lower than its outside option.

Joint surplus problem. Define the joint surplus as the sum of the firm's and the inventor's values:

$$\boldsymbol{\Omega}_{\kappa,\kappa^{-}}(n, P) = \boldsymbol{V}_{\kappa,\kappa^{-}}(n, P) + P \cdot \boldsymbol{Y}$$

We write the researcher's value as $P \cdot \boldsymbol{Y}$ because the promise-keeping constraint holds with equality in equilibrium: by monotonicity of preferences, the firm will always choose to offer the lowest possible value to the researcher so that initial promises are still honored.

Define the normalized joint surplus as:

$$\boldsymbol{O}_{\kappa,\kappa^-}(n,P) = \boldsymbol{V}_{\kappa,\kappa^-}(n,P)/\boldsymbol{Y} + P$$

and the normalized value functions

$$\boldsymbol{v}_{\kappa,\kappa^-}(n,P):\ \boldsymbol{V}_{\kappa,\kappa^-}(n,P) = \boldsymbol{v}_{\kappa,\kappa^-}(n,P)\cdot\boldsymbol{Y}$$

$$\boldsymbol{r}(n,\mathcal{C}_\kappa):\ \boldsymbol{R}(n,\mathcal{C}_\kappa) = \boldsymbol{r}(n,C_\kappa)\cdot\boldsymbol{Y}$$

$$\boldsymbol{u}_\kappa:\ \boldsymbol{U}_\kappa = \boldsymbol{u}_\kappa\cdot\boldsymbol{Y}$$

It can be shown that $\boldsymbol{O}_{\kappa,\kappa^-}(n,P)$ is independent of P and solves the following HJB equation[3]:

$$\begin{aligned}
\rho\,\boldsymbol{O}_{\kappa,\kappa^-}(n) = \max_{\substack{\{P'_\kappa(n',\kappa',\kappa^{-\prime})\}\\ x_{n,\kappa,\kappa^-}>0,\\ z_{n,\kappa,\kappa^-}>0}} \Bigg\{ & \pi(n) - \xi x^{\phi}_{n,\kappa,\kappa^-} - \chi z^{\psi}_{n,\kappa,\kappa^-} \\
& + \delta\Big(\boldsymbol{O}_{\kappa-1,\kappa^-}(n) - \boldsymbol{O}_{\kappa,\kappa^-}(n) + \boldsymbol{O}_{\kappa,\kappa^- -1}(n) - \boldsymbol{O}_{\kappa,\kappa^-}(n)\Big) \\
& + z_{n,\kappa,\kappa^-}\Big(\boldsymbol{O}_{\min(\kappa+1,\bar{\kappa}),\kappa^-}(n) - \boldsymbol{O}_{\kappa,\kappa^-}(n)\Big) \\
& + \sum_{\kappa'\in\mathbb{K}} \nu\Big(P'_\kappa(n,\kappa',\kappa^-)\Big)\Big(\boldsymbol{O}_{\kappa',\kappa^-}(n) - \boldsymbol{O}_{\kappa,\kappa^-}(n) + \boldsymbol{u}_\kappa - P'_\kappa(n,\kappa',\kappa^-)\Big) \\
& + z_{-n,\kappa^-,\kappa}\Big(\boldsymbol{O}_{\kappa,\min(\kappa^-+1,\bar{\kappa})}(n) - \boldsymbol{O}_{\kappa,\kappa^-}(n)\Big) \\
& + \sum_{\kappa^{-\prime}\in\mathbb{K}} \nu\Big(P'_{\kappa^-}(-n,\kappa^{-\prime},\kappa)\Big)\Big(\boldsymbol{O}_{\kappa,\kappa^{-\prime}}(n) - \boldsymbol{O}_{\kappa,\kappa^-}(n)\Big) \\
& + x_{n,\kappa,\kappa^-}\Big(\boldsymbol{O}_{\kappa,\kappa^-}(n+\kappa) - \boldsymbol{O}_{\kappa,\kappa^-}(n)\Big) \\
& + x_{-n,\kappa^-,\kappa}\Big(\boldsymbol{O}_{\kappa,\kappa^-}(n-\kappa^-) - \boldsymbol{O}_{\kappa,\kappa^-}(n)\Big) \\
& + \tilde{\delta}\Big(\boldsymbol{O}_{\kappa,\kappa^-}(0) - \boldsymbol{O}_{\kappa,\kappa^-}(n)\Big) \\
& + \mathbb{1}\{n>0\}\cdot\sum_{\kappa'\in\mathbb{K}} \nu_{\kappa'}(P^E_{\kappa'}(n,\kappa,\kappa^-))\Big(\boldsymbol{O}_{\kappa,\kappa'}(n) - \boldsymbol{O}_{\kappa,\kappa^-}(n)\Big) \\
& + \mathbb{1}\{n<0\}\cdot\sum_{\kappa'\in\mathbb{K}} \nu_{\kappa'}(P^E_{\kappa'}(-n,\kappa,\kappa^-))\big(\boldsymbol{u}_\kappa - \boldsymbol{O}_{\kappa,\kappa^-}(n)\big)
\end{aligned}$$

[3]The intuition for this independence result is that P is simply a transfer from the firm to the inventor. As such, it only affects how the joint surplus is shared between the two parties, but not the total size of the pie.

$$+\mathbb{1}\{n=0\}\cdot\sum_{\kappa'\in\mathbb{K}}\nu_{\kappa'}(P^E_{\kappa'}(n,\kappa,\kappa^-))\left(\frac{1}{2}\boldsymbol{O}_{\kappa,\kappa'}(n)+\frac{1}{2}u_\kappa-\boldsymbol{O}_{\kappa,\kappa^-}(n)\right)\Bigg\}$$

(4)

$$\text{s.t. } P'_\kappa(n',\kappa',\kappa^{-\prime})\geq\boldsymbol{u}_\kappa\ \ \forall(n',\kappa',\kappa^{-\prime})$$

Equation (4) tells us that the joint surplus is composed of the flow surplus, plus the changes in joint surplus value due to worker separation, researcher innovation, own idea implementation and human capital depreciation, plus the changes due to competitor's hiring, innovation, idea implementation and entry. The flow surplus is in turn given by the firm's flow profit, plus the researcher's outside option, net of R&D costs and commitment costs.

Optimal policies. It can be shown that the firm's and joint surplus problems are equivalent. This implies that the contract that maximizes the firm's value can be found by maximizing the joint surplus. Specifically, we can solve the firm's problem in two stages: (i) we solve problem (4) to find the optimal promises and innovation policies; (ii) we find wages residually by ensuring that the promise-keeping constraint is binding and the worker-participation constraint is satisfied at all points of the state space.

Taking first-order conditions of (4), the optimal implementation rate of a firm in state n employing a researcher of type κ, and facing a competitor with researcher of type κ^- is:

$$x_{n,\kappa,\kappa^-}=\left(\frac{\boldsymbol{O}_{\kappa,\kappa^-}(n+\kappa)-\boldsymbol{O}_{\kappa,\kappa^-}(n)}{\xi\phi}\right)^{\frac{1}{\phi-1}}$$

and the optimal innovation rate of the researcher employed by this firm is:

$$z_{n,\kappa,\kappa^-}=\left(\frac{\boldsymbol{O}_{\min(\kappa+1,\bar{\kappa}),\kappa^-}(n)-\boldsymbol{O}_{\kappa,\kappa^-}(n)}{\chi\psi}\right)^{\frac{1}{\psi-1}}$$

Taking the first order condition with respect to $P'_\kappa(n,\kappa',\kappa^-)$:

$$\frac{\partial\nu(P)}{\partial P}_{|P=P'_\kappa(n,\kappa',\kappa^-)}\Big(\boldsymbol{O}_{\kappa',\kappa^-}(n)-\boldsymbol{O}_{\kappa,\kappa^-}(n)\Big)=\frac{\partial\nu(P)}{\partial P}_{|P=P'_\kappa(n,\kappa',\kappa^-)}\Big(P'_\kappa(n,\kappa',\kappa^-)-\boldsymbol{u}_\kappa\Big)+\nu(P'_\kappa(n,\kappa',\kappa^-))$$

(5)

Under a Cobb-Douglas matching function, $M(f,u)=Af^\gamma u^{1-\gamma}$, from (5), the promised value has the following expression:

$$P'_\kappa(n,\kappa',\kappa^-) = (1-\gamma)\Big(\boldsymbol{O}_{\kappa',\kappa^-}(n) - \boldsymbol{O}_{\kappa,\kappa^-}(n)\Big) + \boldsymbol{u}_\kappa \tag{6}$$

The promise made by the firm to a researcher of type κ' is the outside option of the incumbent researcher κ plus a share $(1-\gamma)$ of the gain in joint surplus from the match[4]. The equilibrium job-filling rate is then:

$$\nu(P'_\kappa(n,\kappa',\kappa^-)) = A^{\frac{1}{\gamma}}\left(\frac{(1-\gamma)\Big(\boldsymbol{O}_{\kappa',\kappa^-}(n) - \boldsymbol{O}_{\kappa,\kappa^-}(n)\Big) + \boldsymbol{u}_\kappa - \boldsymbol{u}_{\kappa'}}{\rho\boldsymbol{u}_{\kappa'} - b - \boldsymbol{z}_{\kappa'}}\right)^{\frac{1-\gamma}{\gamma}} \tag{7}$$

From (4), given $\{\boldsymbol{u}_\kappa\}_{\kappa\in\mathbb{K}}$, we can solve for $\boldsymbol{O}_{\kappa,\kappa^-}(n)$, $\forall(n,\kappa,\kappa^-)\in\mathbb{N}\times\mathbb{K}\times\mathbb{K}$ using value function iteration. Indeed, given $\{\boldsymbol{u}_\kappa\}_{\kappa\in\mathbb{K}}$, the joint surplus is a contraction and has a unique fixed point.

Entrants. At any given point in time, the state of an industry can be summarized by three variables: m, the gap between leader and follower; κ^L, the inventor employed by the leader; ans κ^F, the inventor employed by the follower.

Every period, potential entrants pay a flow cost $c_\kappa \boldsymbol{Y}$ to enter labor market segment κ and offer a set of promised values $\{P^E_\kappa(m,\kappa^L,\kappa^F)\}$ to unattached researchers. At rate $\nu_\kappa(P^E(m,\kappa^L,\kappa^F))$, a potential entrant hires a researcher of type κ and replaces the follower in product line (m,κ^L,κ^F)[5]. Therefore, the value of a potential entrant aiming at labor market segment κ is:

$$\begin{aligned} r\boldsymbol{V}^E_\kappa - \dot{\boldsymbol{V}}^E_\kappa = -c_\kappa\boldsymbol{Y} + \max_{\{P^E_\kappa(m,\kappa^L,\kappa^F)\}}\Bigg\{ & \sum_{\kappa_L=0}^{\bar{\kappa}}\sum_{\kappa_F=0}^{\bar{\kappa}} \nu_\kappa(P^E_\kappa(0,\kappa^L,\kappa^F))\cdot\frac{1}{2}\Big(\boldsymbol{V}_{\kappa,\kappa_L}\Big(0,P^E_\kappa(0,\kappa^L,\kappa^F)\Big) \\ & + \boldsymbol{V}_{\kappa,\kappa_F}\Big(0,P^E_\kappa(0,\kappa^L,\kappa^F)\Big)\Big) \\ & + \sum_{m=1}^{\bar{n}}\sum_{\kappa_L=0}^{\bar{\kappa}}\sum_{\kappa_F=0}^{\bar{\kappa}} \nu_\kappa(P^E_\kappa(m,\kappa^L,\kappa^F))\cdot\boldsymbol{V}_{\kappa,\kappa_L}\Big(-m,P^E_\kappa(m,\kappa^L,\kappa^F)\Big) - \boldsymbol{V}^E_\kappa\Bigg\} \\ \text{s.t. } & P^E_\kappa(m,\kappa^L,\kappa^F)\cdot\boldsymbol{Y} \geq \boldsymbol{U}_\kappa \;\; \forall(m,\kappa^L,\kappa^F) \end{aligned} \tag{8}$$

By free entry, in an equilibrium with positive entry, $\boldsymbol{V}^E_\kappa = 0$ for all $\kappa\in\mathbb{K}$. Then, we can write the

[4]Note also that the researcher's participation constraint is slack, since $P'_\kappa(n',\kappa',\kappa^{-\prime}) > \boldsymbol{u}_\kappa$.
[5]If the entrant enter an industry with $m=0$, we assume that it randomly replaces one of the two incumbents.

free entry condition in terms of normalized joint surplus as:

$$c_\kappa = \max_{\{P^E_\kappa(m,\kappa^L,\kappa^F)\}} \Bigg\{ \sum_{\kappa_L=0}^{\bar{\kappa}} \sum_{\kappa_F=0}^{\bar{\kappa}} \nu_\kappa(P^E_\kappa(0,\kappa^L,\kappa^F)) \cdot \Big(\frac{1}{2}\Big(\boldsymbol{O}_{\kappa,\kappa_L}(0) + \boldsymbol{O}_{\kappa,\kappa_F}(0)\Big) - P^E_\kappa(0,\kappa^L,\kappa^F)\Big)$$
$$+ \sum_{m=1}^{\bar{n}} \sum_{\kappa_L=0}^{\bar{\kappa}} \sum_{\kappa_F=0}^{\bar{\kappa}} \nu_\kappa(P^E_\kappa(m,\kappa^L,\kappa^F)) \cdot \Big(\boldsymbol{O}_{\kappa,\kappa_L}(-m) - P^E_\kappa(m,\kappa^L,\kappa^F)\Big)\Bigg\}$$

Taking first order conditions, under a Cobb-Douglas matching function:

$$P^E_\kappa(m,\kappa^L,\kappa^F) = (1-\gamma)\boldsymbol{O}_{\kappa,\kappa_L}(-m) + \gamma\boldsymbol{u}_\kappa, \quad \forall(m,\kappa^L,\kappa^F), m>0$$
$$P^E_\kappa(0,\kappa^L,\kappa^F) = (1-\gamma)\frac{1}{2}\Big(\boldsymbol{O}_{\kappa,\kappa_L}(0) + \boldsymbol{O}_{\kappa,\kappa_F}(0)\Big) + \gamma\boldsymbol{u}_\kappa, \quad \forall(0,\kappa^L,\kappa^F)$$

and the entry rate in labor market segment κ and industry (m,κ^L,κ^F) is

$$\nu_\kappa(P^E_\kappa(m,\kappa^L,\kappa^F)) = A^{\frac{1}{\gamma}} \left(\frac{(1-\gamma)\left(\boldsymbol{O}_{\kappa,\kappa_L}(-m) - \boldsymbol{u}_\kappa\right)}{\rho\boldsymbol{u}_\kappa - b - \boldsymbol{z}_\kappa}\right)^{\frac{1-\gamma}{\gamma}} \quad \forall(m,\kappa^L,\kappa^F), m>0$$
$$\nu_\kappa(P^E_\kappa(0,\kappa^L,\kappa^F)) = A^{\frac{1}{\gamma}} \left(\frac{(1-\gamma)\left(\frac{1}{2}\Big(\boldsymbol{O}_{\kappa,\kappa_L}(0) + \boldsymbol{O}_{\kappa,\kappa_F}(0)\Big) - \boldsymbol{u}_\kappa\right)}{\rho\boldsymbol{u}_\kappa - b - \boldsymbol{z}_\kappa}\right)^{\frac{1-\gamma}{\gamma}} \quad \forall(0,\kappa^L,\kappa^F)$$

Then, given optimal entry choices, the free entry condition reads:

$$c_\kappa = \gamma \cdot A^{\frac{1}{\gamma}} \left(\frac{1-\gamma}{\rho\boldsymbol{u}_\kappa - b - \boldsymbol{z}_\kappa}\right)^{\frac{1-\gamma}{\gamma}} \Bigg[\sum_{m=1}^{\bar{n}} \sum_{\kappa_L=0}^{\bar{\kappa}} \sum_{\kappa_F=0}^{\bar{\kappa}} \left(\boldsymbol{O}_{\kappa,\kappa_L}(-m) - \boldsymbol{u}_\kappa\right)^{\frac{1}{\gamma}}$$
$$+ \sum_{\kappa_L=0}^{\bar{\kappa}} \sum_{\kappa_F=0}^{\bar{\kappa}} \left(\frac{1}{2}\Big(\boldsymbol{O}_{\kappa,\kappa_L}(0) + \boldsymbol{O}_{\kappa,\kappa_F}(0)\Big) - \boldsymbol{u}_\kappa\right)^{\frac{1}{\gamma}} \Bigg]$$

Notice that the right-hand side of this equation is monotonically decreasing in $\boldsymbol{u}_\kappa$ for all $\kappa \in \mathbb{K}$. Therefore, the free entry condition allows us to pin down the value of unattached researchers $\{\boldsymbol{u}_\kappa\}_{\kappa\in\mathbb{K}}$ in equilibrium.

5.3 Further equilibrium conditions on a BGP

Output and growth. Let $q_{jt}^{max} := \max\{q_{ijt}, q_{-ijt}\}$ and $Q_t := \exp\left(\int_0^1 \ln q_{jt}^{max} dj\right)$ be the productivity frontier of the economy. From the definition of the aggregate price index

$$P_t = \exp\left(\int_0^1 \ln\left(p_{ijt}^{\frac{\sigma}{\sigma-1}} + p_{-ijt}^{\frac{\sigma}{\sigma-1}}\right)^{\frac{\sigma-1}{\sigma}} dj\right)$$

and given the normalization $P_t = 1$, the wage for production labor is given by:

$$w_t^C = Q_t \cdot \exp\left(\frac{1-\sigma}{\sigma}\int_0^1 \ln\left(\left(\frac{\sigma(1-s_{ijt})}{1-\sigma s_{ijt}}\right)^{\frac{\sigma}{1-\sigma}} + \left(\lambda^{-n_{jt}}\frac{\sigma s_{ijt}}{1-\sigma(1-s_{ijt})}\right)^{\frac{\sigma}{1-\sigma}}\right) dj\right) \tag{9}$$

Aggregate output, $\boldsymbol{Y}_t$, is:

$$\begin{aligned}\boldsymbol{Y}_t &= Q_t \cdot M_t \cdot \exp\left(\int_0^1 \ln\left(\left(\frac{\sigma s_{ijt}(1-s_{ijt})}{1-\sigma s_{ijt}}\right)^{\sigma} + \left(\lambda^{-n_{jt}}\frac{\sigma s_{ijt}(1-s_{ijt})}{1-\sigma(1-s_{ijt})}\right)^{\sigma}\right)^{\frac{1}{\sigma}}\right) dj \equiv \\ &\equiv Q_t \cdot \lambda^{-\int_0^1 n_{jt} dj} \cdot M_t \cdot \exp\left(\int_0^1 \ln\left(\left(\lambda^{n_{jt}}\frac{\sigma s_{ijt}(1-s_{ijt})}{1-\sigma s_{ijt}}\right)^{\sigma} + \left(\frac{\sigma s_{ijt}(1-s_{ijt})}{1-\sigma(1-s_{ijt})}\right)^{\sigma}\right)^{\frac{1}{\sigma}}\right) dj\end{aligned} \tag{10}$$

The evolution of Q_t can be characterized as follows:

- In a product line where the leader is $m \geq 0$ steps ahead, employs researcher κ^L and the followe employs researcher $\kappa^- F > m$:

$$\ln(q_{jt+\Delta t}^{max}) = \begin{cases} \ln(\lambda^{\kappa^L} \cdot q_{jt}^{max}) & \text{w.p.} \quad x_{(m+\bar{n}+1),\kappa^L,\kappa^F} \cdot \Delta t + o(\Delta t) \\ \ln(\lambda^{\kappa^F - m} \cdot q_{jt}^{max}) & \text{w.p.} \quad x_{-(m+\bar{n}+1),\kappa^F,\kappa^L} \cdot \Delta t + o(\Delta t) \\ \ln(q_{jt}^{max}) & \text{w.p.} \quad 1 - (x_{(m+\bar{n}+1),\kappa^L,\kappa^F} + x_{-(m+\bar{n}+1),\kappa^F,\kappa^L}) \cdot \Delta t + o(\Delta t) \end{cases}$$

- In a product line where the leader is $m \geq 0$ steps ahead, employs a researcher of type κ^L and the follower employs a researcher of type $\kappa^F \leq m$:

$$\ln(q_{jt+\Delta t}^{max}) = \begin{cases} \ln(\lambda^{\kappa^L} \cdot q_{jt}^{max}) & \text{w.p.} \quad x_{(m+\bar{n}+1),\kappa^L,\kappa^F} \cdot \Delta t + o(\Delta t) \\ \ln(q_{jt}^{max}) & \text{w.p.} \quad 1 - x_{(m+\bar{n}+1),\kappa^L,\kappa^F} \cdot \Delta t + o(\Delta t) \end{cases}$$

Let $\mu_{m,\kappa^L,\kappa^F,t}$ be the measure of product lines where, at time t, the leader is $m \geq 0$ steps ahead, employs a researcher of type κ^L and the follower employs a researcher κ^F. Then the growth rate of the productivity frontier is:

$$\frac{\dot{Q}_t}{Q_t} := \lim_{\Delta t \to 0} \frac{\ln Q_{t+\Delta t} - \ln Q_t}{\Delta t} =$$
$$= \sum_{m=0}^{\bar{n}} \sum_{\kappa^L=0}^{\bar{\kappa}} \sum_{\kappa^F=0}^{\bar{\kappa}} \mu_{m,\kappa^L,\kappa^F} \cdot \ln(\lambda^{\kappa^L}) \cdot x_{(m+\bar{n}+1),\kappa^L,\kappa^F} + \sum_{m=0}^{\bar{n}} \sum_{\kappa^L=0}^{\bar{\kappa}} \sum_{\kappa^F=m+1}^{\bar{\kappa}} \mu_{m,\kappa^L,\kappa^F} \cdot \ln(\lambda^{\kappa^F-m}) \cdot x_{(-m+\bar{n}+1),\kappa^L,\kappa^F} \tag{11}$$

On a balanced-growth path, output grows as the same rate as the productivity frontier. Output growth therefore depends on the aggregate arrival rate of innovations; these are due to both leaders' and followers' innovation efforts, weighted by the quality of their innovations.

Market clearing. The market-clearing condition for production labor is:

$$1 = \int_0^1 (l_{ijt} + l_{-ijt}) dj \tag{12}$$

which equates labor supply to the sum of intermediate-good producers' production worker demand. Since the representative household owns all firms in the economy, asset market-clearing is:

$$A_t = \int_{\mathcal{F}} V_{ft} df \tag{13}$$

Finally, the aggregate resource constraint of the economy is:

$$\boldsymbol{Y}_t = C_t + \int_0^1 \xi(x_{ijt}^{\phi} + x_{-ijt}^{\phi}) \boldsymbol{Y}_t \, dj + \int_0^1 \chi(z_{ijt}^{\psi} + z_{-ijt}^{\psi}) \boldsymbol{Y}_t \, dj + \sum_{\kappa \in \mathbb{K}} \left[\mu_{\kappa t}^U \cdot \left(\chi(z_{\kappa t}^U)^{\psi} \boldsymbol{Y}_t + b\boldsymbol{Y}_t \right) + c_\kappa \boldsymbol{Y}_t \right] \tag{14}$$

which states that final output is used for consumption, firms' R&D, researchers' investment in human capital, both when attached and when unattached, unemployment benefits and entry costs. We denote by $\mu_{\kappa t}^U$ the measure of unattached researchers of type κ at time t.

Welfare. Along a balanced-growth path, consumption C_t grows at the same rate as $\boldsymbol{Y}_t$. Therefore, the welfare of the representative household can be summarized by C_0 and g:

$$W = \int_0^\infty e^{-\rho t} \ln(C_t) dt = \frac{1}{\rho} \left(\ln(C_0) + \frac{g}{\rho} \right)$$

where, from (14) at $t = 0$:

$$C_0 = \mathbf{Y}_0 \cdot \frac{C_0}{\mathbf{Y}_0} = \mathbf{Y}_0 \left(1 - \int_0^1 \xi(x^{\phi}_{ij0} + x^{\phi}_{-ij0})\, dj - \int_0^1 \chi(z^{\psi}_{ij0} + z^{\psi}_{-ij0})\, dj - \sum_{\kappa \in \mathbb{K}} \left[\mu^U_{\kappa 0} \cdot \left(\chi(z^U_{\kappa 0})^{\psi} + b \right) + c_{\kappa} \right] \right)$$

Equilibrium measures. To close the model, we have two sets of flow equations that determine the measures of industries, $\{\mu_{m,\kappa^L,\kappa^F}\}$, and unattached inventors, $\{\mu^U_{\kappa}\}$, in a stationary balanced-growth path equilibrium. To save space, we report these equations in the Appendix.

6 Model Results

In this section, we present some of the model results, highlighting that our framework delivers predictions that are in line with the empirical evidence. To facilitate the intuition, we consider a simplified version of the model with two types of inventors only: $\kappa = \{\kappa^L, \kappa^H\}$, with $\kappa^L < \kappa^H$. The results reported in this section are obtained under a parametrization of the model as in Table A1[6].

In the top-left panel of Figure 2, we plot the difference in innovation rates between firms that employ inventors of type κ^H and firms that employ inventors of type κ^L, as a function of the firm's market share. We see that this difference is everywhere non-negative, meaning that firms that employ better inventors always innovate more than firms that employ worse inventors. Moreover, we see that the difference is small for firms which have either very small or very large market share and is largest for firms that are close to neck-to-neck with their competitors. Indeed, these are the firms that have the highest incentives to innovate to escape competition from their rivals.

In the top-right panel of Figure 2, we plot the difference in innovation rates between firms that face competitors who employ κ^H inventors and firms that face competitors who employ κ^L inventors. We see that the effect of the competitor employing a better researcher is negative for market followers and positive for market leaders. Market leaders innovate more than they would if their competitor employed a worse researcher, because they know their small lead is at higher risk and they need to defend it. By contrast, followers innovate less than they would if their competitor employed a worse researcher: when the leader employs a better researcher, it is harder for the follower to obtain market leadership. As a result, the follower gets discouraged and innovates less.

In the bottom panel of Figure 2, we plot job filling rates in labor market κ^H as a function of the

[6] For the moment, we set the model parameters externally. Later we will calibrate the model to match the salient features of the market for researchers and the sorting between inventors and firms documented in Section 4.

firm's market share. We see that job-filling rates are lowest for firms that are either far behind or far ahead of their competitor and highest for firms that are close to neck-to-neck with their competitors. The intuition is that these firms are those that gain the most from hiring better researchers, as they can escape competition from their rivals. By contrast, firms that are far ahead of their competitor have little incentives to hire good researchers, as their lead is already large; and firms that are far behind of their competitor also have small incentives as they have a very large gap to overcome.

Finally, we look at what the model delivers in terms of firm-inventor matching. In particular, we look at how firms with different market share differ in terms of the researchers that they employ. In Figure 3, we plot, for different values of market share, the share of firms with that market share that employ researchers of type κ^L (blue bars) and the share of firms that employ κ^H researchers (orange bars). In the right panel, we plot the difference between the two. We see that this difference is negative for firms with small market share, meaning that most of these firms employ low productivity inventors. Then, as we approach intermediate values of market share, this difference becomes smaller and smaller in absolute value, as followers that are close to their rivals tend to employ more high productivity researchers. The difference then becomes positive, as most of the leaders with a small lead over their rivals employ good researchers. And finally, as we approach a market share of 1, the difference turns negative again, as firms that are very far ahead of their competitors hire low productivity researchers. Indeed, the lead of these firms is large enough that they do not need to employ high-quality researchers to defend it.

7 Growth and welfare effects of the market for inventors

In our model, the efficiency of the matching technology governs the extent of misallocation of inventors across firms. Therefore, to gauge the importance of the market for inventors for growth and welfare, we vary the scale factor of the matching function, A, on a grid from 0, which is equivalent to shutting down the market, to 1.

Notice that, in our model, market shares are not directly affected by the functioning of the market for inventors. Therefore, changes in A have no direct effect on static allocative efficiency. The market for researchers has however dynamic effects, due to the endogenous responses of firms and inventors in terms of R&D investment, which lead to changes in the technology gap distribution. Through changes in this distribution, changes in A affect the initial level of output and consumption in the economy. Moreover, through changes in innovation policies and the allocation of inventors across firms, changes in A affect the growth rate of the economy.

The results of our qualitative exercise are displayed in Figure 4. As efficiency in the market for researchers improves, the measure of employed inventors of type κ^H increases, while the measure of employed inventors of type κ^L decreases. Moreover, the distribution of employed inventors across firms within sectors also changes. In particular, the larger A, the easier it is for firms with high R&D intensity to hire good researchers: misallocation in the market for inventors is lower. As a result of these two forces, aggregate innovation in the economy increases with A. The fact that there are more employed researchers of type κ^H also implies that innovations in the economy are on average more impactful. This, combined with the increase in aggregate R&D, implies that the growth rate is higher in an economy with larger A.

At the same time, the fact that innovations now advance productivity by more steps, pushes the technology gap distribution towards larger gaps. So an economy with larger A is an economy with a higher degree of concentration in the product market. This force pushes down initial output, Y_0. The fall in initial output, combined with the increase in R&D spending, reduces the level of initial consumption, C_0, which has a negative impact on welfare. Despite the fall in initial consumption, welfare is higher in an economy with larger A, due to the increase in the growth rate.

8 Conclusions

This paper studies the labor market for inventors to understand the importance of efficient inventor-firm matching in fostering aggregate innovation and economic growth. Empirical evidence from patent data reveals significant turnover in the inventor market. This mobility enhances firms' innovation outcomes, particularly for those operating in highly-competitive sectors. Our findings show a hump-shaped relationship between firms' hiring rates of inventors and their relative sales within industries, especially pronounced within the top quintiles of the inventors' productivity distribution.

Theoretically, we develop an endogenous growth model incorporating a frictional labor market for inventors, where firms compete strategically to attract high-productivity inventors. We use the model to study the growth and welfare implications of the market for inventors. Our preliminary results indicate that improved matching efficiency in the market for inventors reallocates high-productivity inventors to firms with high R&D intensity, boosting aggregate innovation, growth and welfare. Addressing frictions in the inventor market can thus lead to significant gains in R&D productivity, driving long-term economic growth.

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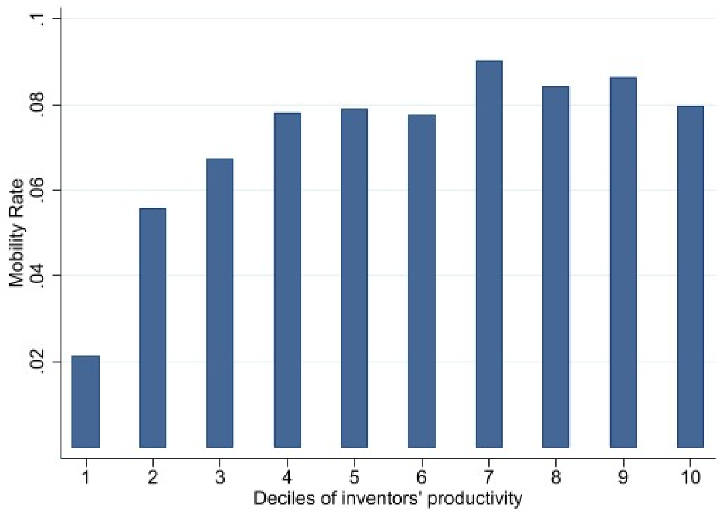
Tables and Figures

Figure 1: Mobility rates by inventors' productivity deciles



Note: We group inventors into deciles based on the distribution of inventor productivity, $P_{j,t}$, in year t. We compute mobility rates year-by-year for each decile and then average across years.

Table 1: Hiring rates of inventors and firms' relative sales

	(1) Hiring rate	(2) Separation rate	(3) Net hiring rate
Market Share	0.204***	-0.044***	0.248***
	(0.052)	(0.015)	(0.055)
Market Share Squared	-0.197***	0.037**	-0.234***
	(0.059)	(0.017)	(0.063)
N	24,551	24,551	24,551
R^2	0.120	0.131	0.118
Controls	Yes	Yes	Yes
Year-industry FE	Yes	Yes	Yes

Notes: Standard errors clustered at firm and year level in parentheses.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2: Hiring rates of inventors and firms' relative sales, by inventors' productivity

	(1) Full sample	(2) Q1 Inv. Prod.	(3) Q2 Inv. Prod.	(4) Q3 Inv. Prod.	(5) Q4 Inv. Prod.	(6) Q5 Inv. Prod.
Market Share	0.204***	0.026***	0.053***	0.053***	0.067***	0.049**
	(0.052)	(0.009)	(0.013)	(0.015)	(0.017)	(0.017)
Market Share Squared	-0.197***	-0.023**	-0.051***	-0.055***	-0.063***	-0.043**
	(0.059)	(0.010)	(0.015)	(0.018)	(0.020)	(0.019)
N	24,551	24,551	24,551	24,551	24,551	24,551
R^2	0.120	0.109	0.104	0.120	0.099	0.085
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Year-industry FE	Yes	Yes	Yes	Yes	Yes	Yes

Notes: Standard errors clustered at firm and year level in parentheses.
* p<0.10, ** p<0.05, *** p<0.01

Table 3: Inventors' productivity and firms' relative sales

	(1) Baseline	(2) 3-year window	(3) No self-citations	(4) Citations per author
Market Share	1.765***	1.914***	1.655***	1.577***
	(0.447)	(0.393)	(0.423)	(0.360)
Market Share Squared	-1.766***	-1.727***	-1.681***	-1.522***
	(0.503)	(0.469)	(0.471)	(0.425)
N	15,858	13,691	15,654	19,174
R^2	0.142	0.183	0.136	0.120
Controls	Yes	Yes	Yes	Yes
Year-industry FE	Yes	Yes	Yes	Yes

Notes: Standard errors clustered at firm and year level in parentheses. Controls: employment, age, R&D stock, leverage, profitability, market-to-book ratio. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 4: Inventors' productivity and firms' innovation outcomes

	(1) R&D expenditure	(2) Patents	(3) Citations	(4) Citations per patent
Inventors' productivity	0.200***	0.624***	0.945***	1.209***
	(0.016)	(0.036)	(0.039)	(0.121)
Market Share	2.120***	2.359***	2.969***	1.198
	(0.653)	(0.843)	(0.895)	(1.178)
Market Share Squared	-2.100***	-2.339**	-3.211***	-1.380
	(0.729)	(1.013)	(1.074)	(1.315)
N	14,104	15,858	15,828	15,858
R^2	0.794	0.882	0.876	0.049
Controls	Yes	Yes	Yes	Yes
Year-industry FE	Yes	Yes	Yes	Yes

Notes: Standard errors clustered at firm and year level in parentheses. Poisson regressions in (2), (3)
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 5: Inventors' productivity and firms' innovation outcomes, interactions with market share

	(1) R&D expenditure	(2) Patents	(3) Citations	(4) Citations per patent
Inventors' productivity	0.195***	0.579***	0.829***	1.229***
	(0.026)	(0.085)	(0.084)	(0.143)
Inventors' Prod. X Mkt Share	1.188**	2.424***	3.844***	-0.876
	(0.420)	(0.756)	(1.087)	(2.310)
Inventors' Prod. X Mkt Share Sq.	-2.356***	-4.474***	-6.115***	1.448
	(0.646)	(1.113)	(1.481)	(3.248)
N	14,106	15,862	15,832	15,862
R^2	0.566	0.820	0.816	0.049
Controls	Yes	Yes	Yes	Yes
Year-industry FE	Yes	Yes	Yes	Yes

Notes: Standard errors clustered at firm and year level in parentheses. Poisson regressions in (2), (3)
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Figure 2: Model results: policy functions

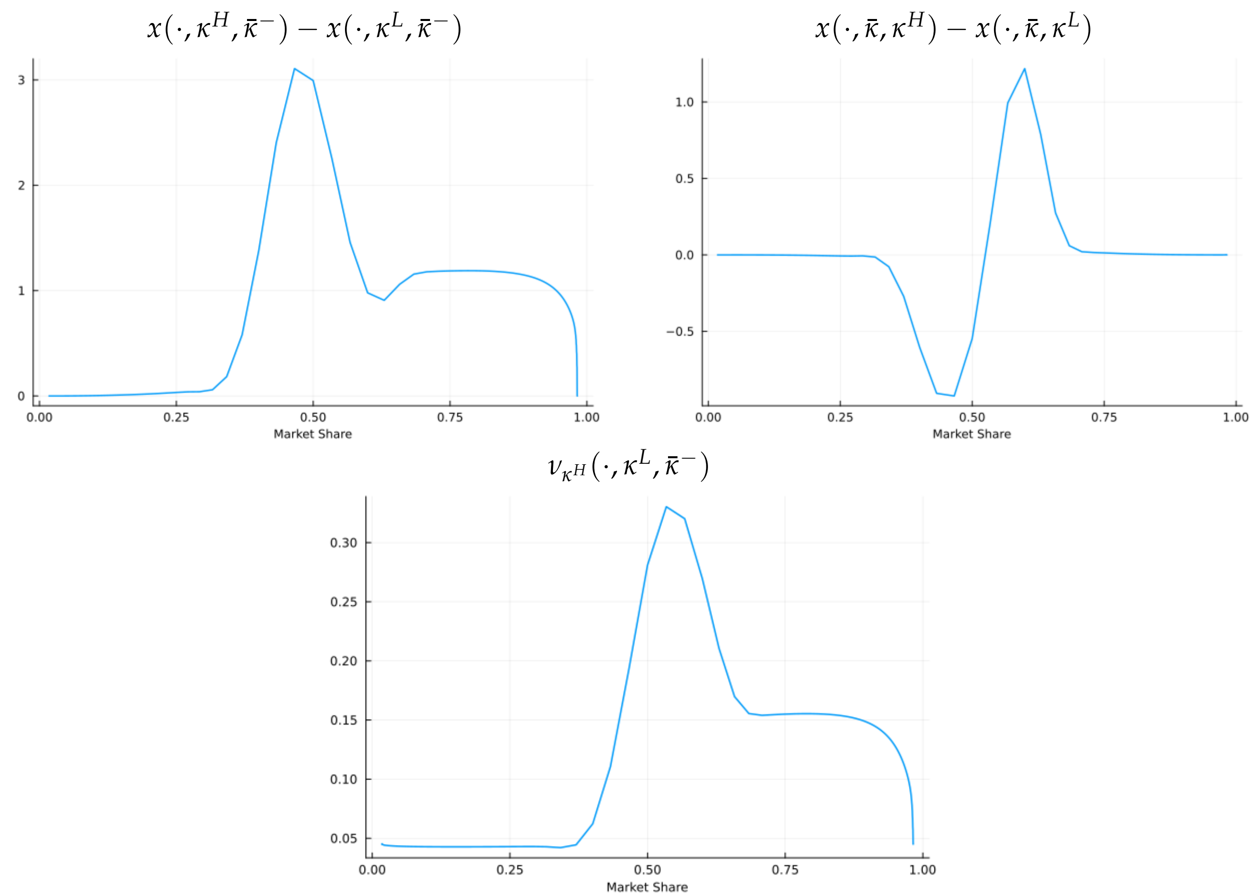


Figure 3: Model results: Firm-inventor sorting

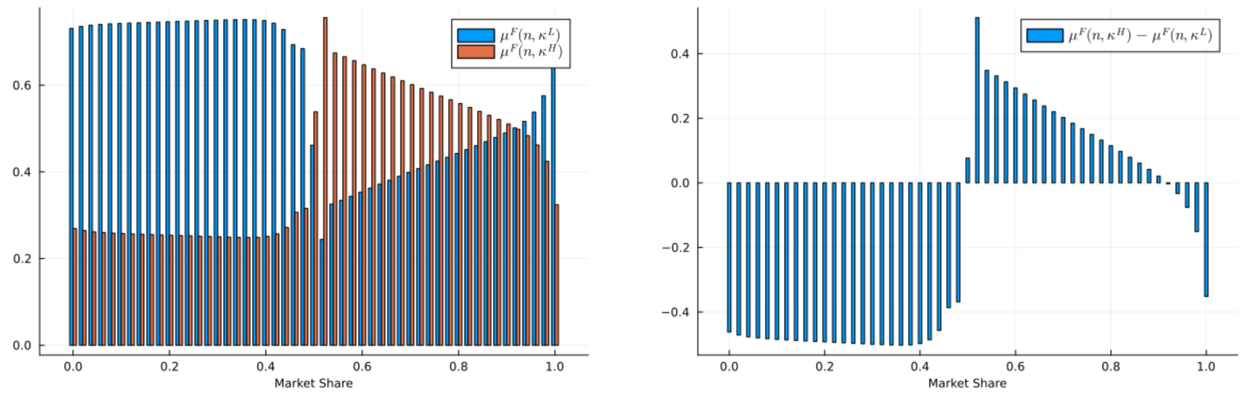
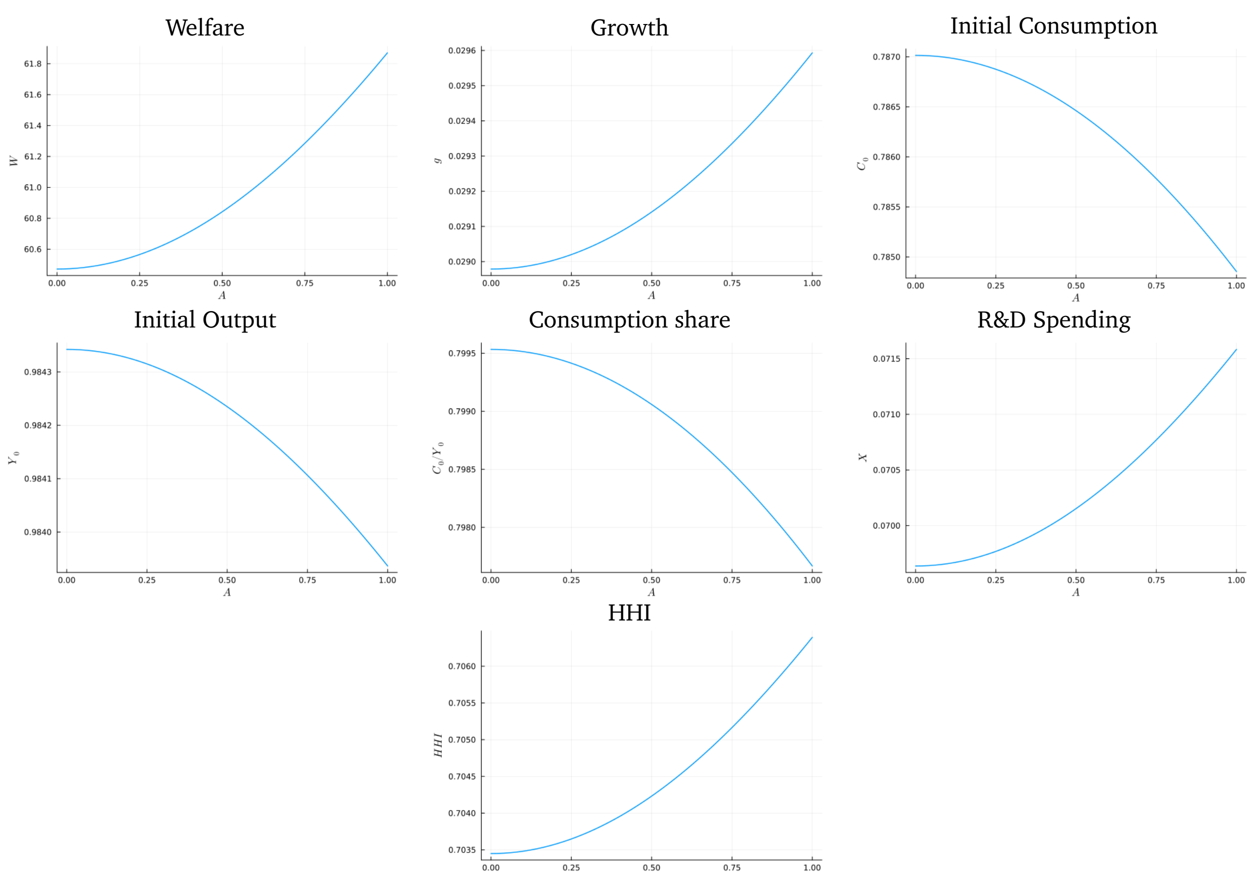


Figure 4: Effects of changes in A on growth and welfare



A Appendix

A.1 Model derivations

Employed researchers and firms. Define the short-hand notation for the job-filling rate of the firm:

$$\nu_\kappa(P) \equiv \eta \circ \mu^{-1}(\boldsymbol{\Phi}_\kappa(P))$$

The value of a researcher of type κ employed in firm with technology gap n, facing a competitor employing a researcher of type κ^-, under contract $\mathcal{C}_\kappa \equiv \left(\omega_\kappa \cdot \boldsymbol{Y}, \{P'_\kappa(n', \kappa', \kappa^{-\prime}) \cdot \boldsymbol{Y}\}\right)$ by $\boldsymbol{R}(n, \mathcal{C}_\kappa)$ solves:

$$\begin{aligned}
r\boldsymbol{R}(n,\mathcal{C}_\kappa) - \dot{\boldsymbol{R}}(n,\mathcal{C}_\kappa) = \omega_\kappa \boldsymbol{Y} + \max_{z_{n,\kappa,\kappa^-}} \Bigg\{ & z_{n,\kappa,\kappa^-}\left(P'_\kappa\left(n, \min(\kappa+1,\overline{\kappa}), \kappa^-\right) \cdot \boldsymbol{Y} - \boldsymbol{R}(n,\mathcal{C}_\kappa)\right) - \chi z^{\psi}_{n,\kappa,\kappa^-}\boldsymbol{Y} \Bigg\} \\
& + \delta\left(P'_\kappa\left(n,\kappa-1,\kappa^-\right) \cdot \boldsymbol{Y} - \boldsymbol{R}(n,\mathcal{C}_\kappa) + P'_\kappa\left(n,\kappa,\kappa^- - 1\right) \cdot \boldsymbol{Y} - \boldsymbol{R}(n,\mathcal{C}_\kappa)\right) \\
& + \sum_{\kappa' \in \mathbb{K}} \nu_{\kappa'}\left(P'_\kappa(n,\kappa',\kappa^-)\right)\left(\boldsymbol{U}_\kappa - \boldsymbol{R}(n,\mathcal{C}_\kappa)\right) \\
& + z_{-n,\kappa^-,\kappa}\left(P'_\kappa\left(n,\kappa,\min(\kappa^- + 1,\overline{\kappa})\right) \cdot \boldsymbol{Y} - \boldsymbol{R}(n,\mathcal{C}_\kappa)\right) \\
& + \sum_{\kappa^{-\prime} \in \mathbb{K}} \nu_{\kappa^{-\prime}}\left(P'_{\kappa^-}(-n,\kappa^{-\prime},\kappa)\right)\left(P'_\kappa\left(n,\kappa,\kappa^{-\prime}\right) \cdot \boldsymbol{Y} - \boldsymbol{R}(n,\mathcal{C}_\kappa)\right) \\
& + (x_{n,\kappa,\kappa^-}) \cdot \left(P'_\kappa\left(n+\kappa,\kappa,\kappa^-\right) \cdot \boldsymbol{Y} - \boldsymbol{R}(n,\mathcal{C}_\kappa)\right) \\
& + (x_{-n,\kappa^-,\kappa}) \cdot \left(P'_\kappa\left(n-\kappa^-,\kappa,\kappa^-\right) \cdot \boldsymbol{Y} - \boldsymbol{R}(n,\mathcal{C}_\kappa)\right) \\
& + \tilde{\delta} \cdot \left(P'_\kappa\left(0,\kappa,\kappa^-\right) \cdot \boldsymbol{Y} - \boldsymbol{R}(n,\mathcal{C}_\kappa)\right) \\
& + \mathbb{1}\{n > 0\} \cdot \sum_{\kappa' \in \mathbb{K}} \nu_{\kappa'}(P^E_{\kappa'}(n,\kappa,\kappa^-))\left(P'_\kappa(n,\kappa,\kappa') \cdot \boldsymbol{Y} - \boldsymbol{R}(n,\mathcal{C}_\kappa)\right) \\
& + \mathbb{1}\{n < 0\} \cdot \sum_{\kappa' \in \mathbb{K}} \nu_{\kappa'}(P^E_{\kappa'}(-n,\kappa,\kappa^-))(\boldsymbol{U}_\kappa - \boldsymbol{R}(n,\mathcal{C}_\kappa)) \\
& + \mathbb{1}\{n = 0\} \cdot \sum_{\kappa' \in \mathbb{K}} \nu_{\kappa'}(P^E_{\kappa'}(n,\kappa,\kappa^-))\left(\frac{1}{2}P'_\kappa(n,\kappa,\kappa') \cdot \boldsymbol{Y} + \frac{1}{2}\boldsymbol{U}_\kappa - \boldsymbol{R}(n,\mathcal{C}_\kappa)\right) \Bigg\}
\end{aligned} \tag{A1}$$

The first-order condition for innovating while employed is:

$$z_{n,\kappa,\kappa^-} = \left(\frac{P'_\kappa\Big(n, \min(\kappa+1,\overline{\kappa}), \kappa^-\Big) \cdot \boldsymbol{Y} - \boldsymbol{R}(n, \mathcal{C}_\kappa)}{\chi \psi \boldsymbol{Y}} \right)^{\frac{1}{\psi-1}}$$

The value of a firm of type n employing a researcher of type $\kappa \in \mathbb{K}$, facing a competitor with researcher of type κ^-, under promised value P, solves:

$$\begin{aligned}
r\boldsymbol{V}_{\kappa,\kappa^-}(n,P) - \dot{\boldsymbol{V}}_{\kappa,\kappa^-}(n,P) = \max_{\substack{\omega_\kappa, \{P'_\kappa(n',\kappa',\kappa^{-\prime})\} \\ x_{n,\kappa,\kappa^-}>0}} & \Bigg\{ \pi(n)\boldsymbol{Y} - \omega_\kappa \boldsymbol{Y} - \xi x^{\phi}_{n,\kappa,\kappa^-}\boldsymbol{Y} \\
& + z_{n,\kappa,\kappa^-}\left(\boldsymbol{V}_{\min(\kappa+1,\bar{\kappa}),\kappa^-}\Big(n, P'_\kappa(n, \min(\kappa+1,\bar{\kappa}), \kappa^-\Big) - \boldsymbol{V}_{\kappa,\kappa^-}(n,P) \right) \\
& + \delta\left(\boldsymbol{V}_{\kappa-1,\kappa^-}\Big(n, P'_\kappa(n,\kappa-1,\kappa^-)\Big) + \boldsymbol{V}_{\kappa,\kappa^- -1}\Big(n, P'_\kappa(n,\kappa,\kappa^- -1)\Big) - 2\boldsymbol{V}_{\kappa,\kappa^-}(n,P) \right) \\
& + \sum_{\kappa'\in\mathbb{K}} \nu_{\kappa'}\Big(P'_\kappa(n,\kappa',\kappa^-)\Big)\left(\boldsymbol{V}_{\kappa',\kappa^-}\Big(n, P'_\kappa(n,\kappa',\kappa^-)\Big) - \boldsymbol{V}_{\kappa,\kappa^-}(n,P) \right) \\
& + z_{-n,\kappa^-,\kappa}\left(\boldsymbol{V}_{\kappa,\min(\kappa^-+1,\bar{\kappa})}\Big(n, P'_\kappa(n,\kappa,\min(\kappa^-+1,\bar{\kappa})\Big) - \boldsymbol{V}_{\kappa,\kappa^-}(n,P) \right) \\
& + \sum_{\kappa^{-\prime}\in\mathbb{K}} \nu_{\kappa^{-\prime}}\Big(P'_{\kappa^-}(-n,\kappa^{-\prime},\kappa)\Big)\left(\boldsymbol{V}_{\kappa,\kappa^{-\prime}}\Big(n, P'_\kappa(n,\kappa,\kappa^{-\prime})\Big) - \boldsymbol{V}_{\kappa,\kappa^-}(n,P) \right) \\
& + x_{n,\kappa,\kappa^-}\left(\boldsymbol{V}_{\kappa,\kappa^-}\Big(n+\kappa, P'_\kappa(n+\kappa,\kappa,\kappa^-)\Big) - \boldsymbol{V}_{\kappa,\kappa^-}(n,P) \right) \\
& + (x_{-n,\kappa^-,\kappa}) \cdot \left(\boldsymbol{V}_{\kappa,\kappa^-}\Big(n-\kappa^-, P'_\kappa(n-\kappa^-,\kappa,\kappa^-)\Big) - \boldsymbol{V}_{\kappa,\kappa^-}(n,P) \right) \\
& + \tilde{\delta} \cdot \left(\boldsymbol{V}_{\kappa,\kappa^-}\Big(0, P'_\kappa(0,\kappa,\kappa^-)\Big) - \boldsymbol{V}_{\kappa,\kappa^-}(n,P) \right) \\
& + \mathbb{1}\{n>0\} \cdot \sum_{\kappa'\in\mathbb{K}} \nu_{\kappa'}(P^E_{\kappa'}(n,\kappa,\kappa^-))\Big(\boldsymbol{V}_{\kappa,\kappa'}\big(n, P'_\kappa(n,\kappa,\kappa')\big) - \boldsymbol{V}_{\kappa,\kappa^-}(n,P) \Big) \\
& + \mathbb{1}\{n<0\} \cdot \sum_{\kappa'\in\mathbb{K}} \nu_{\kappa'}(P^E_{\kappa'}(-n,\kappa,\kappa^-))\big(0 - \boldsymbol{V}_{\kappa,\kappa^-}(n,P)\big) \\
& + \mathbb{1}\{n=0\} \cdot \sum_{\kappa'\in\mathbb{K}} \nu_{\kappa'}(P^E_{\kappa'}(n,\kappa,\kappa^-))\Big(\frac{1}{2}\boldsymbol{V}_{\kappa,\kappa'}\big(n, P'_\kappa(n,\kappa,\kappa')\big) - \boldsymbol{V}_{\kappa,\kappa^-}(n,P) \Big) \Bigg\}
\end{aligned} \tag{A2}$$

The problem is subject to promise-keeping and participation constraints:

$$\boldsymbol{R}(n, \mathcal{C}_\kappa) \geq P \cdot \boldsymbol{Y}$$

$$P'_\kappa(n', \kappa', \kappa^{-\prime}) \cdot \boldsymbol{Y} \geq \boldsymbol{U}_\kappa, \quad \forall (n', \kappa', \kappa^{-\prime})$$

Equilibirum measures. The evolution of $\mu_{m,\kappa^L,\kappa^F,t}$ is described by the following equation:

$$\begin{aligned}
\frac{\mu_{m,\kappa^L,\kappa^F,t+\Delta t} - \mu_{m,\kappa^L,\kappa^F,t}}{\Delta t} = \; & \mu_{m-\kappa^L,\kappa^L,\kappa^F,t} \cdot x_{(m-\kappa^L+\bar{n}+1),\kappa^L,\kappa^F,t} \\
& + \mu_{m,\kappa^L-1,\kappa^F,t} \cdot z_{m+\bar{n}+1,\kappa^L-1,\kappa^F,t} \\
& + \sum_{\kappa' \in \mathbb{K}} \mu_{m,\kappa',\kappa^F,t} \cdot \nu_{\kappa^L} \left(P'_{\kappa'}(m+\bar{n}+1, \kappa^L, \kappa^F) \right) \\
& + \mu_{m+\kappa^F,\kappa^L,\kappa^F,t} \cdot x_{-(m+\kappa^F)+\bar{n}+1,\kappa^F,\kappa^L,t} \\
& + \mu_{\kappa^F-m,\kappa^L,\kappa^F,t} \cdot x_{-(\kappa^- - m)+\bar{n}+1,\kappa^-,\kappa,t} \\
& + \mu_{m,\kappa^L,\kappa^F-1,t} \cdot z_{-m+\bar{n}+1,\kappa^F-1,\kappa^L,t} \\
& + \sum_{\kappa' \in \mathbb{K}} \mu_{m,\kappa^L,\kappa',t} \cdot \nu_{\kappa^F} \left(P'_{\kappa'}(-m+\bar{n}+1, \kappa^F, \kappa^L) \right) \\
& + \sum_{\kappa' \in \mathbb{K}} \mu_{m,\kappa^L,\kappa',t} \cdot \nu_{\kappa^F} (P^E_{\kappa^F}(m, \kappa^L, \kappa')) \\
& - \mu_{m,\kappa^L,\kappa^F,t} \cdot \Big(x_{m+\bar{n}+1,\kappa^L,\kappa^F,t} + x_{-m+\bar{n}+1,\kappa^F,\kappa^L,t} + z_{m+\bar{n}+1,\kappa^L,\kappa^F,t} + z_{-m+\bar{n}+1,\kappa^F,\kappa^L,t} \\
& + \sum_{\kappa' \in \mathbb{K}} \left(\nu_{\kappa'} \left(P'_{\kappa^L}(m+\bar{n}+1, \kappa', \kappa^F) \right) + \nu_{\kappa'} \left(P'_{\kappa^F}(-m+\bar{n}+1, \kappa', \kappa^L) \right) \right) \\
& + \nu_{\kappa'} (P^E_{\kappa'}(m, \kappa^L, \kappa^F) \Big) \\
& + \frac{o(\Delta t)}{\Delta t}
\end{aligned} \tag{A3}$$

The terms on the right-hand side represent, respectively: additions due to innovations of leaders at $(n-\kappa, \kappa, \kappa^-)$; additions due to innovations of leaders' researchers at $(m, \kappa-1, \kappa^-)$; additions due to leaders hiring researchers of type κ; additions due to innovations of followers at $(n + \kappa^-, \kappa, \kappa^-)$; additions due to innovations of followers at $(\kappa^- - m, \kappa, \kappa^-)$; additions due to innovations of followers' researchers at $(m, \kappa, \kappa^- - 1)$; additions due to followers hiring researchers of type κ^-; additions due to entry; subtractions due to innovation and hiring decisions of leaders and followers at (m, κ, κ^-); subtractions due to entry[7].

[7]Note that in equation (A3): the first line is absent whenever $n - \kappa < 0$; the second line is absent if $\kappa = \underline{\kappa}$; the fourth is absent whenever $n + \kappa^- > \bar{n}$; the sixth is present only if $\kappa^- - n \geq 0 \wedge \kappa = \kappa^-$; and the sixth is absent if $\kappa^- = \underline{\kappa}$.

Finally, the following flow equation pins down the stationary measure of unattached inventors of type κ, μ_κ^U, for each $\kappa \in \mathbb{K}$:

$$
\begin{aligned}
\frac{\mu^U_{\kappa,t+\Delta t} - \mu^U_{\kappa,t}}{\Delta t} &= \mu^U_{\kappa-1,t} \cdot z^U_{\kappa-1,t} + \mu^U_{\kappa+1,t} \cdot \delta \\
&+ \sum_n \sum_{\kappa^-} \mu^E_\kappa(n,\kappa^-) \cdot \left(\sum_{\kappa'} \left[\nu_{\kappa'}(P(n,\kappa,\kappa^-)) + \mathbb{1}\{n<0\} \cdot \nu_{\kappa'}(P^E(-n,\kappa^L=\kappa,\kappa^F=\kappa^-)) \right] \right) \\
&- \mu^U_{\kappa,t} \cdot \Big(z^U_{\kappa,t} + \delta \\
&+ \sum_m \sum_{\kappa^L} \sum_{\kappa^F} \mu_{m,\kappa^F,\kappa^L} \cdot \Big(\mu(\theta_\kappa(P_\kappa(m+\bar{n}+1,\kappa^L,\kappa^F)) + \mu(\theta_\kappa(P_\kappa(-m+\bar{n}+1,\kappa^F,\kappa^L)) \Big) \Big) \\
&+ \frac{o(\Delta t)}{\Delta t}
\end{aligned} \tag{A4}
$$

A.2 Model parametrization

Table A1: Model parametrization

Parameter	*Value*	*Description*	*Source*
ρ	0.02	Discount rate	External
σ	0.078	CES parameter	External
λ	1.009	Innovation step size	External
γ	0.5	Matching elasticity	External
A	1.0	Matching efficiency	External
ξ	0.02	R&D scale parameter, Firms	External
χ	0.02	R&D scale parameter, Inventors	External
ϕ	1/0.55	R&D cost curvature, Firms	External
ψ	1/0.55	R&D cost curvature, Inventors	External
δ	0.8	Depreciation rate	External
b	0.0001	Unemployment benefits	External
$\tilde{\delta}$	0.17	Exogenous catch-up	External
c_κ	[0.35,0.42]	Entry costs	External