How Stable is Stable Price Dispersion? *

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Abstract

Studying price competition within homogeneous good markets where consumers consider differential sets of suppliers, the theoretical framework of Myatt and Ronayne (2023a) asserts the existence of stable price dispersion. Stable list prices are chosen so no firm is interested in undercutting another firm. Allowing for minor differences in marginal costs across firms, despite being more efficient, stable list price equilibria are Pareto-dominated by mixed pricing strategies. The insight that stable price dispersion can emerge in the absence of marginal cost differences is thus not "stable" with respect to the introduction of cost heterogeneities.

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1 Introduction

A homogeneous product is often sold at varying prices. As a result, economists acknowledge that the law of one price does not hold. To explain price discrimination, Varian (1980) assumed a single-stage symmetric setting where firms sell a homogeneous product to informed and uninformed consumers. Uninformed consumers solely consider one firm, whereas informed consumers consider all firms and buy from the cheapest firm. This setup results in a Nash equilibrium solely in mixed strategies. Baye et al. (1992) further expanded on this, revealing multiple mixed strategy equilibria alongside a single symmetric one. More recently, Armstrong and Vickers (2022) examined mixed strategy equilibria with different "consideration sets" for consumers. There are not only consumers who are captive to one firm or consider buying from any firm but also consumers who are partially informed and consider buying from some but not all firms. The actual observed price dispersion is interpreted as a realization of the mixed strategy equilibrium.

Morris et al. (2019) demonstrate sector-dependent frequencies of price changes, ranging from daily adjustments to monthly revisions or even less frequent alterations. Based on the less frequent price changes, Myatt and Ronayne (2023a) show that not only mixed equilibria prevail, but also a pure strategy subgame perfect equilibrium (SPE) with stable price dispersion. They introduce a two-stage game where firms simultaneously post a list price in the first stage. In the second stage, each firm can only decrease the price but cannot increase the price above the chosen list price. Their findings indicate an equilibrium exists where list prices are undercut-proof, and no firm lowers the price in the second stage. The price of firm i is undercut-proof if all firms with a higher list price have no incentive to reduce their price in the second stage below the list price of firm i. They show that firms have no incentive to deviate from such list prices and that the equilibrium profits are equal to the expected profits of a single-stage setting.

Myatt and Ronayne (2023a) show that those list prices are stable with partially informed consumers and no marginal cost differences. Myatt and Ronayne (2023b) also show that these results persist with differentiated marginal costs in the simple framework of Varian (1980) with informed and uninformed consumers. They also indicate that asymmetric costs emerge naturally due to the costs of investing in innovation. Due to the different profits resulting from whether other firms have already invested, they will decide for or against an investment. Additionally, the assumption of identical marginal costs across firms is widely adopted in theoretical Industrial Organization due to convenience. It is often harmless; the equilibrium with asymmetric costs for minor differences closely aligns with the symmetric costs equilibrium. However, this is not true in general.

Using differentiated marginal costs and partially informed consumers, I show that in the second stage, the mixed strategy equilibrium yields greater profits than stable list prices. Stable list prices are payoff dominated by mixed strategy equilibria for (minor) differences in marginal costs. The only surviving SPE is the mixed strategy equilibrium. Even though, in this case, stable list prices are more efficient because the inefficient firm solely sells to its captive customers. Whereas in the mixed strategy equilibrium, the identity of the low-priced firm is not necessarily the most efficient firm. Minor differences in marginal costs cannot explain the identity of the low-priced firm.

First, I will outline the basic framework. Then, I will derive the mixed strategy equilibrium, followed by the stable list prices. Subsequently, I compare the profits from the mixed strategy equilibrium and the stable list prices. Finally, I will conclude.

2 Model: Single-stage mixed equilibrium

Consider a model where three firms are competing in a homogeneous product market. There exists a consumer population with a total measure normalized to 1, where each individual has a unit demand and is willing to pay up to 1 for a unit of the product. Consumers are independently informed about the firms, enabling each firm to access half of the consumer base. This then indicates that $\frac{1}{8}$ consumers considers only firm $i \in \{1, 2, 3\}$, $\frac{1}{8}$ consumers consider firm i and j, where $j \neq i$ and $\frac{1}{8}$ consumer consider all three firms. This symmetric consideration set can be represented by the following Venn diagram.

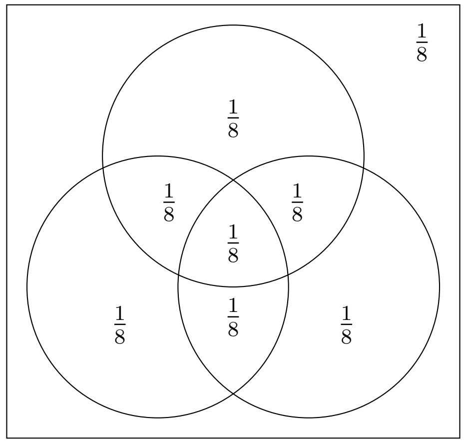

Figure 1: Simplified consideration set.

For simplicity, we assume that solely firm 1 has a marginal cost of $c \in (0, 0.2]$[1]. Firms 2 and 3 have marginal costs normalized to zero. Firms compete in prices. If two firms charge the same price we assume consumers choose at random.

Using Armstrong and Vickers (2022) *Lemma 1* for this special case know that the following has to hold:

Lemma 1 *In any single-stage equilibrium:*

1. *the minimum price $\underline{p}$ ever chosen in the market is strictly positive;*

2. *at least two firms compete at each price in the interval $[\underline{p}, 1]$;*

3. *each distribution is continuous in the half-open set $[\underline{p}, 1)$;*

The lowest price where firm 1 can achieve the monopoly profit is defined as p_1^m:

$$\pi_1 = \frac{1}{8}(1-c) = \frac{4}{8}(p_1^m - c)$$
$$\Leftrightarrow p_1^m = \frac{1+3c}{4}$$

[1]The insights remain valid for values $c > 0.2$. In this case the firms 2 and 3 compete in a larger set of prices than firm 1.

Any price $p < p_1^m$ is dominated for firm 1 by serving the uninformed consumers at the monopoly price. Given $c < 0.2$, all firms compete over the same range of prices $p \in [p_1^m, 1]$.

Lemma 2 *In a single-stage equilibrium, profits are given* $\pi_1 = \frac{1-c}{8}$ *and* $\pi_2 = \pi_3 = \frac{1+3c}{8}$. *Firms choose their price according to following cdf:*

$$F_1(p) = \begin{cases} 0, & \text{if } p < p_1^m \\ 2 - \frac{1+3c}{p\sqrt{\frac{1-c}{p-c}}}, & \text{if } p \in [p_1^m, 1) \\ 1, & \text{if } p \geq 1 \end{cases}$$

$$F_2(p) = F_3(p) = \begin{cases} 0, & \text{if } p < p_1^m \\ 2 - \sqrt{\frac{1-c}{p-c}}, & \text{if } p \in [p_1^m, 1] \\ 1, & \text{if } p > 1 \end{cases}$$

3 Model: Two-stage stable price dispersion

In a two-stage setting as proposed by Myatt and Ronayne (2023a), instead of firms offering a price and then the consumers decide where to buy, the price setting is supposed to be:

(t=1) list price $\tilde{p_i}$ are chosen, and then

(t=2) firms observe all list prices and choose their final retail price $p_i \in [0, \tilde{p_i}]$.

The mixed strategy equilibrium from section 2 is also a SPE in the two-stage setting, simply at t=1, all firms choose $\tilde{p_i} = 1$. This is not a stable list price because a mixed strategy is played in the second stage.

I aim for stable list prices so that no company has a profitable deviation in t=2 from that price. Knowing that a profile of strictly undercut-proof prices is strictly ordered: $\tilde{p_1} \geq \tilde{p_2} \geq \tilde{p_3}$[2].

[2] Note that firms 2 and 3 are identical. Therefore, a second equilibrium exists where firms 1 and 2 are interchanged.

Lemma 3 *Stable list prices are given through* $\tilde{p_1} = 1$, $\tilde{p_2} = \frac{1+c}{2}$ *and* $\tilde{p_3} = \frac{1+c}{4}$. *Associated profits are* $\pi_1 = \frac{1-c}{8}$ *and* $\pi_2 = \pi_3 = \frac{1+c}{8}$.

Proof: Assuming this ordering, it must be that firm 1 is neither interested in undercutting firm 2 nor firm 3. It must hold that $\tilde{p_2}$ is so low that firm 1 has no profitable deviations below the price of firm 2, where firm 1 serves all consumers considering firms 1 and 2 but not firm 3.

$$\begin{aligned} \pi_1 =& \frac{1}{8}(1-c) \geq (\frac{1}{8}+\frac{1}{8})(\tilde{p_2}-c) \\ \Leftrightarrow \tilde{p_2} \leq & \frac{1+c}{2} \end{aligned}$$

Any price lower than $\tilde{p_2} = \frac{1+c}{2}$ but above $\tilde{p_3}$ is dominated by $\tilde{p_2}$ as it decreases the profit of firm 2: $\pi_2 = (\frac{1}{8}+\frac{1}{8})\tilde{p_2} = \frac{1+c}{8}$.

Firm 3 must set its list price so that neither firm 1 nor 2 has an incentive to undercut that price:

$$\begin{aligned} \pi_1 =& \frac{1}{8}(1-c) \geq (\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8})(\tilde{p_3}-c) \\ \Leftrightarrow \tilde{p_3} \leq & \frac{1+3c}{4} \\ \pi_2 =& \frac{1}{8}(1+c) \geq (\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8})\tilde{p_3} \\ \Leftrightarrow \tilde{p_3} \leq & \frac{1+c}{4} \end{aligned}$$

Again, firm 3 will not lower the price below $\tilde{p_3} = \frac{1+c}{4}$ to make firms 1 and 2 indifferent. Therefore, it holds that $\tilde{p_3} = \frac{1+c}{4}$ with $\pi_3 = \frac{1+c}{8}$. Firm 1 cannot have a price in stable strategies that is lower than a competitor's price. Assume $\tilde{p_2} \geq \tilde{p_1} \geq \tilde{p_3}$, then firm 2 sets $\tilde{p_2} = 1$. Firm 2 has no reason to undercut firm 1 if $\tilde{p_1} \leq \frac{1}{2}$, yielding at most a profit of firm 1 which is smaller than serving solely to the uninformed $\pi_1 = \frac{1-2c}{8} < \frac{1-c}{8}$. Assume $\tilde{p_2} \geq \tilde{p_3} \geq \tilde{p_1}$, then firm 2 sets $\tilde{p_2} = 1$ and firm 3 $\tilde{p_3} = \frac{1}{2}$. To make both firms not to undercut firm 1's price $p_1 \leq \frac{1}{4}$, yielding at most a profit of $\pi_1 = \frac{1-4c}{8} < \frac{1-c}{8}$. Firm 1 has no incentive to have a lower stable price than another firm. The proof for no profitable deviation is in the appendix.

4 Results

Table 1 and Figure 2 show the profits of the two-stage stable prices and mixed strategies.

	$\tilde{p_1}$	$\tilde{p_2}$	$\tilde{p_3}$	p_1	p_2	p_3	π_1	π_2	π_3
Mixed strategy	1	1	1	$\in [p_1^m, 1]$	$\in [p_1^m, 1]$	$\in [p_1^m, 1]$	$\frac{1-c}{8}$	$\frac{1+3c}{8}$	$\frac{1+3c}{8}$
Stable list price	1	$\frac{1+c}{2}$	$\frac{1+c}{4}$	1	$\frac{1+c}{2}$	$\frac{1+c}{4}$	$\frac{1-c}{8}$	$\frac{1+c}{8}$	$\frac{1+c}{8}$

Table 1: Comparing mixed strategy and stable list prices.

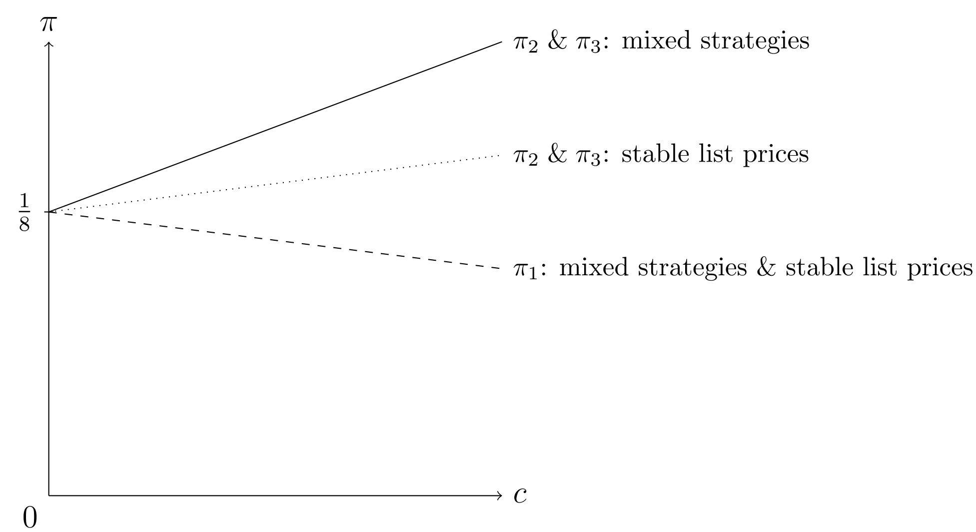

Figure 2: Profit dispersion.

Proposition 1 *Comparing the stable list price equilibrium with the mixed strategy equilibrium, one can see that*

1. *the profit of the inefficient firm is identical;*

2. *the profit of efficient firms is smaller with stable list prices.*

Proposition follows immediately from Lemma 2 and Lemma 3. Firms 2 and 3 have an advantage over firm 1 because of their lower marginal costs. The more consumers they compete for, the greater this advantage. With stable list prices, firm 1 competes for some partially informed consumers (between firms 1 and 2) but not for the other (partially) informed consumers. Therefore, with stable list prices, firm 2's advantage and profit are lower. Since firms 2 and 3 are competing against each other and the profit of firm 2 is lowered, firm 3's profit also decreases. Conditional on firms being able to choose their list price, no firm is incentivized to select a list price below $\tilde{p}_i = 1$.

Proposition 2 *Second-best efficiency can be achieved with stable price dispersion.*

Following the proof of Lemma 3, one can see that firm 1 always has the highest stable list price. With uninformed consumers who shop only at a specific firm, one cannot obtain first-best efficiency. With stable list prices, the identity of the lower-priced firms equals the efficient firms. The identity of the high-priced firm is the inefficient firm 1. No (partially) informed consumers will shop at the inefficient firm. This is second-best efficient as the inefficient firm only serves their uninformed consumers. This is in contrast to a mixed strategy equilibrium, where it occurs that an inefficient firm sells to more than the uninformed consumers with some positive probability. Due to Proposition 1, no firm has incentives to choose a list price below 1. Therefore, I expect to see mixed strategies and inefficiencies in the SPE.

5 Conclusion

This paper analyzes price competition in a market with individual reach where one firm suffers from the disadvantages of higher marginal costs. With these minor differences in marginal costs, I have shown that stable list price dispersion is Pareto dominated by the mixed strategy equilibrium. While stable list prices are more efficient in this setting, firms' profits are lower compared to the mixed strategy equilibrium. This is noteworthy because it implies that the identity of the low-priced firms in the SPE does not necessarily indicate the identity of the efficient firm. Further research has to be done to determine which circumstances can cause prices to be stable.

Appendix

SKETCH PROOF OF LEMMA 1:

1. The minimum price $\underline{p}$ chosen by any firm in the market must be positive. Suppose a firm chooses a negative or zero price, then there exist always a deviation to the monopoly price of 1 to have a profit.

2. Suppose there is a firm competing in the market at price p. Suppose there is no other firm competing at that price. Then, the firm can increase its price locally without losing any demand. In that case, the firm would not have the price p in its support.

3. First, suppose two firms have an atom at the same price; in that case, one firm could slightly undercut this price and receive a discrete jump in demand. Therefore, it cannot be the case that two firms have an atom at the same price. Suppose only firm i has an atom at any price $p \in [0, 1)$. No other firm competing at price p can have an atom at this price. Hence, there is no discrete jump in demand for firm i. Nevertheless, the other firms competing at price p have a discrete jump in demand if they undercut price p. No other firms would price immediately at or above price p. Therefore, if there is an atom at p, it cannot be that two firms compete at this price, which contradicts part 2 of the Lemma 1.

SKETCH PROOF OF LEMMA 2:

Conjecturing that p_1^m is the lowest price ever chosen, in equilibrium, all firms have to be indifferent between choosing all prices $p \in [p_1^m, 1]$. Given Lemma 1.3, no firm can have an atom at p_1^m. Consequently, all firms can secure the profit of serving their reach at a price p_1^m. Profits are $\pi_1 = \frac{1-c}{8}$ and $\pi_2 = \pi_3 = \frac{1+3c}{8}$. To be indifferent, the following profits equation has to hold for all $p \in [p_1^m, 1]$:

$$\pi_1 = \frac{1-c}{8} = (p-c)\Big[\frac{1}{8} + \frac{1}{8}(1-F_2(p)) + \frac{1}{8}(1-F_3(p)) + \frac{1}{8}(1-F_2(p))(1-F_3(p))\Big] \quad (1)$$

$$\pi_2 = \frac{1+3c}{8} = p\Big[\frac{1}{8} + \frac{1}{8}(1-F_1(p)) + \frac{1}{8}(1-F_3(p)) + \frac{1}{8}(1-F_1(p))(1-F_3(p))\Big] \quad (2)$$

$$\pi_3 = \frac{1+3c}{8} = p\Big[\frac{1}{8} + \frac{1}{8}(1-F_1(p)) + \frac{1}{8}(1-F_2(p)) + \frac{1}{8}(1-F_1(p))(1-F_2(p))\Big] \quad (3)$$

From 2 and 3 it follows that $F_2(p) = F_3(p)$. Solving the three equations leads to Lemma 2's distributions. Firms 2 and 3 have no incentive to lower their lowest price below p_1^m as it only decreases the price without increasing demand. Firm 1's cdf has to be a continuously increasing function, therefore it needs to be that the first derivative of $F_1(p)$ at $p = p_1^m$ is increasing in p. This holds for $c \leq 0.2$. No firm wants to deviate from these distributions.

PROOF OF STABLE LIST PRICES EQUILIBRIUM:

In the second stage, no firm can price above $\tilde{p_i}$, and no firm wants to lower their price because they cannot increase profits since lowering the price above the next lower price does not increase demand but lowers the price. Lowering the price underneath other prices cannot be profitable as those prices are chosen to be undercut-proof. In the first stage, a firm could deviate from its list price. For similar reasons, no firm would lower its list price. However, a firm could increase the list price up to the monopoly price of 1. Firm 1 cannot have a possible deviation as it already has the monopoly price as its list price. If firm 2 increases its list price to $\tilde{p_2}^N \in (\tilde{p_2}, 1]$. This price is no longer undercut-proof. In the second stage, firm 1 has incentives to undercut this price. Second-stage price distributions for firms 1 and

2 are given by:

$$F_1(p) = \begin{cases} 0, & \text{if } p < \tilde{p_2} \\ 2 - \frac{1+c}{p}, & \text{if } p \in [\tilde{p_2}, \tilde{p_2}^N] \\ 2 - \frac{1+c}{\tilde{p_2}^N}, & \text{if } p \in [\tilde{p_2}^N, 1) \\ 1, & \text{if } p \geq 1 \end{cases}$$

$$F_2(p) = \begin{cases} 0, & \text{if } p < \tilde{p_2} \\ 2 - \frac{1-c}{p-c}, & \text{if } p \in [\tilde{p_2}, \tilde{p_2}^N) \\ 1, & \text{if } p \geq \tilde{p_2}^N \end{cases}$$

Given those strategies firm 2 is indifferent between all prices $p \in [\tilde{p_2}, \tilde{p_2}^N]$ and firm 1 is indifferent between $p \in [\tilde{p_2}, \tilde{p_2}^N]$ and $p = 1$. Expected profits remain at $\pi_1 = \frac{1-c}{8}$ and $\pi_2 = \frac{1+c}{8}$. Firm 2 has no incentives to deviate in the first stage.

If firm 3 is increasing its list price to $\tilde{p_3}^N$. This price is no longer undercut-proof. Hence, in the second stage firm 2 has incentives to undercut this price. Assume that $\tilde{p_3}^N \leq \tilde{p_2}$. Second-stage price distributions for firm 2 and 3 are given by:

$$F_2(p) = \begin{cases} 0, & \text{if } p < \tilde{p_3} \\ 2 - \frac{1+c}{2p}, & \text{if } p \in [\tilde{p_3}, \tilde{p_3}^N] \\ 2 - \frac{1+c}{2\tilde{p_3}^N}, & \text{if } p \in [\tilde{p_3}^N, \tilde{p_2}) \\ 1, & \text{if } p \geq \tilde{p_2} \end{cases}$$

$$F_3(p) = \begin{cases} 0, & \text{if } p < \tilde{p_3} \\ 2 - \frac{1+c}{2p}, & \text{if } p \in [\tilde{p_3}, \tilde{p_3}^N) \\ 1, & \text{if } p \geq \tilde{p_3}^N \end{cases}$$

If one assumes that firm 3 would deviate its list price above the list price of firm 2: $\tilde{p_3}^N > \tilde{p_2}$. Then, both firms have the following price distribution to make each other indifferent.

$$F_2(p) = F_3(p) = \begin{cases} 0, & \text{if } p < \tilde{p_3} \\ 2 - \frac{1+c}{2p}, & \text{if } p \in [\tilde{p_3}, \tilde{p_2}] \\ 1, & \text{if } p > \tilde{p_3}^N \end{cases}$$

Note that given those strategies, firm 1 strictly prefers to price at her monopoly price of 1. When $\tilde{p_3}^N > \tilde{p_2}$ and firm 1 joins to price in that range $p \in [\tilde{p_3}, \tilde{p_2}]$ her profit is equal to $\tilde{\pi_1} = \frac{(1+c)^2(p-c)}{32p^2} < \frac{1-c}{8} = \pi_1$. In the case that $\tilde{p_3}^N \leq \tilde{p_2}$, there is more weight on lower prices by firm 3. Hence, the range of profitable prices decreases further.

Firm 3's expected profit is equal to $\frac{1+c}{2}$.

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