Imperfect Competition in Markets with Adverse or Advantageous Selection

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Abstract

This paper proposes a spatial model of imperfect competition in markets with adverse or advantageous selection. The model shows that a reduction in competition exacerbates the inefficiency created by adverse selection, but can ameliorate the inefficiency created by advantageous selection. However, reduced competition never corrects the inefficiency perfectly. In contrast, the inefficiency can be corrected perfectly through a corrective tax when there is perfect competition. Our results have implications for competition policy in credit and insurance markets as they caution against viewing imperfect competition as a solution to the inefficiencies created by selection.

Keywords: *Imperfect Competition, Selection Markets, Price Discrimination, Credit Markets, Insurance Markets*

JEL Classification: D82, D43, C7, L1

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1 Introduction

Imperfect competition and asymmetric information are two reasons for market failures. While they have been extensively studied in isolation, there has recently been growing interest in their interaction as many markets exhibit both simultaneously, such as the market for health insurance (Dafny, 2010; Cabral et al., 2018; Einav et al., 2021) or credit markets (Crawford et al., 2018). Competition authorities study these markets closely (European Commision, 2007; Competition and Markets Authority, 2016). While we know that inefficiency can arise in markets with asymmetric information or with imperfect competition, less is known about efficiency in markets with both asymmetric information and imperfect competition. Specifically, we do not know whether, and if so when, imperfect competition exacerbates or ameliorates the inefficiency created by asymmetric information.

This question has recently attracted attention in the context of selection markets, such as credit or insurance markets. While it may appear natural to think that imperfect competition, which in isolation can lead to inefficiency, should exacerbate the inefficiency created by asymmetric information, a string of recent papers argues in the spirit of the theory of the second-best that imperfect competition can ameliorate the inefficiency created by asymmetric information and thus increase welfare (Lester et al., 2019; Crawford et al., 2018; Mahoney and Weyl, 2017).[1]More strongly, in Mahoney and Weyl's model, imperfect competition can even correct the inefficiency perfectly, i.e. result in the efficient allocation. This has led to suggestions that competition policy should tolerate intermediate degrees of competition on the grounds that this can correct inefficiencies arising from asymmetric information.[2]

This paper presents a spatial model of imperfect competition in markets with adverse or advantageous selection to investigate whether imperfect competition exacerbates or ameliorates the inefficiency created by selection. The model shows that while in markets with adverse selection a reduction in competition can exacerbate the inefficiency, in markets with advantageous selection it can ameliorate it, but surprisingly it never results in the efficient allocation. Thus, our results caution against viewing imperfect competition as a solution to inefficiencies introduced by selection.

To build the intuition, start from the microfoundations. Selection can be adverse or advantageous. A market exhibits adverse (advantageous) selection if a firm incurs a higher (lower) cost when selling to agents with a high willingness-to-pay (WTP) than when selling to agents with a low WTP.[3] For example, adverse selection can arise in the market for health insurance as consumers with private information on their poor health have a high WTP for insurance and high expected medical costs, or in credit markets when borrowers have private information on the riskiness of the project they are trying to finance (Stiglitz and Weiss, 1981). Advantageous

[1]In their general theory of the second-best, Lipsey and Lancaster (1956) argue that, in the presence of one irremovable distortion (e.g. monopoly or a tax), adding a second distortion can increase welfare.

[2]For example, Mahoney and Weyl (2017) state: "Policies to correct market power and selection can be misguided when these forces coexist." (p. 637)

[3]The general term 'selection markets' refers to markets where consumers have private information on a fixed characteristic which affects their WTP and the cost a firm incurs when selling to that consumer. Adverse and advantageous selection are the two possible types of selection.

selection can arise in insurance markets when agents who are more risk averse take actions to mitigate their risk (Hemenway, 1990; De Meza and Webb, 2001)[4] or in credit markets when borrowers have privat information on the quality of the project they are trying to finance (De Meza and Webb, 1987).[5]

Empirical evidence demonstrates that both forms of selection are a real world phenomenon and not merely a theoretical artefact. Advantageous selection exists in some markets for health insurance (Fang et al., 2008), life insurance (Cawley and Philipson, 1999), and credit markets (Mahoney and Weyl, 2017).[6] Adverse selection has been detected in various insurance markets (Einav and Finkelstein, 2011) and consumer credit markets (Adams et al., 2009).

Under perfect competition, the equilibrium in markets with selection can be inefficient. In a market with adverse selection, underprovision can arise and the market can even break down completely as shown by Akerlof (1970). In contrast, advantageous selection can lead to overprovision as demonstrated by De Meza and Webb (2001). Firms lower their price to steal the rival firm's existing consumers, which are profitable, even when this lower price also attracts consumers which previously did not purchase from either firm and are not profitable. In equilibrium, there is inefficient overprovision, i.e. some consumers purchase the good even though the cost of selling to these consumers exceeds their WTP.[7]

Model preview: Our model studies imperfect competition by extending models of spatial competition to capture selection. As in canonical models of spatial competition (Hotelling, 1929; Thisse and Vives, 1988), two firms, which are located at the edge of the unit interval, sell differentiated products to consumers who differ in their WTP and their location, which captures brand preferences or geographical location.[8] As in the literature on selection markets, consumers have private information on a characteristic which determines their WTP and the cost a firm incurs when selling to that consumer. Thus, the model merges components which are common in the literature on the respective imperfection.

In this setting, an important distinction arises between two notions of efficiency: the efficient allocation and the efficient quantity. The efficient allocation is reached when a consumer purchases

[4]For example, highly risk averse agents may drive more carefully or choose a healthier lifestyle (Einav and Finkelstein, 2011). We provide micro-foundations for adverse and advantageous selection in insurance markets in Appendix B.2 and in credit markets in Appendix B.1.

[5]Formally, whether adverse or advantageous selection arises in credit markets depends on how the distribution of project returns differs across projects. Stiglitz and Weiss (1981) assume that all projects have the same expected return and that the distribution of returns of a riskier project is a mean preserving spread of the distribution of project returns of a safer project. De Meza and Webb (1987) assume that the return distributions of better projects first order stochastic dominates the return distribution of worse projects. We discuss this in more detail in Appendix B.1.

[6]Fang et al. (2008) study the Medigap market. Medigap is a type of private health insurance in the US which Medicare recipients can purchase to cover health related financial risks which are not covered by Medicare such as deductibles or co-insurance payments. Cawley and Philipson (1999) show that men with life insurance have a lower mortality rate than those without, which is consistent with advantageous selection. Mahoney and Weyl (2017) study the US market for subprime auto loans using data from Einav et al. (2012).

[7]An excellent survey on selection markets under perfect competition and monopoly is provided by Einav and Finkelstein (2011). A recent contribution on perfect competition is Azevedo and Gottlieb (2017).

[8]For insurance markets, Starc (2014) provides empirical evidence that consumers have brand preferences between identical insurance plans and that insurers derive market power from this. For credit markets, Degryse and Ongena (2005) show that banks derive market power from the geographical proximity of their bank branches to a borrower. They also highlight that banks exploit this source of market power in their loan pricing decisions.

from her preferred firm if and only if her WTP exceeds the cost a firm incurs when selling to that consumer and otherwise she does not purchase. The efficient quantity is the quantity traded in the efficient allocation. Thus, in the efficient allocation the quantity must be efficient, but the efficient quantity can be reached by an inefficient allocation. For example, an allocation with underprovision for consumers with some brand preferences and overprovision for consumers with other brand preferences, is an inefficient allocation but can correspond to the efficient quantity. In markets without selection, this distinction is not important as an equilibrium satisfies either both notions of efficiency or neither.[9] In markets with selection, this distinction is important.

Results preview: In our model, imperfect competition never results in the efficient allocation. In markets with adverse selection, perfect competition leads to underprovision and reductions in competition exacerbate this. In markets with advantageous selection, perfect competition leads to overprovision, reductions in competition decrease the equilibrium quantity and thus also decrease overprovision. At some intermediate level of competition, the equilibrium quantity corresponds to the efficient quantity. However, surprisingly, the efficient allocation is never reached. Instead, overprovision for consumers with some brand preferences coexists with underprovision for consumers with other brand preferences.

Which consumers experience overprovision and which underprovision? The answer depends on whether firms price discriminate or use uniform prices. When firms can price discriminate, they compete fiercely for consumers with weak brand preferences while charging high prices to those with strong brand preferences. The result is that overprovision for consumers with weak brand preferences coexists with underprovision for consumers with strong brand preferences. When firms cannot price discriminate, they can only compete for consumers with weak brand preferences by offering a low price to all consumers. The result is overprovision for consumers with strong brand preferences and underprovision for those with weak brand preferences.

While in our model no degree of imperfect competition can reach the efficient allocation, it can be reached through a corrective tax when there is perfect competition. Moreover, in markets with advantageous selection, the corrective tax has the additional benefit of raising government revenue without causing a deadweight loss. This creates room to reduce distortionary taxes in other markets - an additional efficiency gain. Thus, combining the corrective tax with tough competition policy to achieve perfect competition results in higher welfare than tolerating intermediate degrees of competition. Moreover, it has lower informational requirements. Hence, our paper cautions against viewing imperfect competition as a solution to inefficiencies introduced by selection.

More broadly, our model highlights a new mechanism how selection affects a firm's pricing decision and thus refines our understanding of the trade-off a firm faces when considering whether to cut its price in an imperfectly competitive selection market. While in markets without selection firms contemplate how many consumers a price cut would attract, in markets with selection

[9] While in markets without selection there exists inefficient allocations with the efficient quantity, this never arises in equilibrium when firms price discriminate and also not when they use uniform prices. Intuitively, this distinction becomes important only when firms may find it optimal to set prices such that there is overprovision. This is not the case in markets without selection or with adverse selection but is possible in markets with advantageous selection.

firms additionally consider which type of consumer they attract. Our model predicts that, when firms use uniform prices, those consumers who switch to a firm in response to its price cut have on average a higher WTP than the consumers who purchase from the rival firm and thus are more strongly selected. For advantageous (adverse) selection, this magnifies (diminishes) a firm's benefit from cutting its price.

Related literature: Imperfect competition in selection markets has previously been studied by Mahoney and Weyl (2017), Crawford et al. (2018), and Lester et al. (2019). Our paper is distinct since it (i) models spatial competition, (ii) considers price discrimination, (iii) considers alternative policy solutions such as a corrective tax.

The paper closest to us is Mahoney and Weyl (2017) (henceforth MW). Like our paper, they develop a theoretical model to study imperfect competition in markets with adverse or advantageous selection. Unlike our paper, they take a reduced form approach and model equilibrium prices as a weighted average of the price under perfect competition and the price a monopolist would set. Less intense competition corresponds to a larger weight on the price a monopolist would set.[10] In this setting, the efficiency properties of the equilibrium are fully summarised by the equilibrium quantity.

In this reduced form model, imperfect competition can ameliorate the inefficiency introduced by advantageous selection and can even result in the efficient allocation. However, imperfect competition cannot ameliorate the inefficiency introduced by adverse selection. Intuitively, in markets with advantageous selection, perfect competition leads to overprovision, reductions in competition decrease the equilibrium quantity and thus also decrease overprovision. At some intermediate level of competition, the equilibrium quantity corresponds to the efficient quantity. Further reductions in competition lead to underprovision. In markets with adverse selection, perfect competition leads to underprovision and reductions in competition exacerbate this.

Our spatial model nests MW's results as a special case and shows that important results do not generalise.[11] Most notably, the result that, in markets with advantageous selection, there exists a level of imperfect competition which results in the efficient allocation is only valid in the special cases where there is no taste heterogeneity. The reason is that, in the presence of taste heterogeneity, the equilibrium quantity is not a sufficient statistic for the efficiency properties of the equilibrium. Instead, efficiency depends also on the allocation, i.e. on which types of consumers purchase, as a given quantity may not be allocated efficiently. We find that reductions in the intensity of competition lead to a lower equilibrium quantity, but also introduce an inefficiency in the allocation of that quantity. Thus, in our model, like in MW, there exists a level of competition which achieves the efficient quantity but, unlike in MW, it does so via an inefficient allocation where overprovision for some consumers coexists with underprovision for other consumers. Hence, no degree of competition results in the efficient allocation. Our spatial model shows that reductions in competition also introduce an inefficiency

[10] This corresponds to equation (1) on p.640 in Mahoney and Weyl (2017). Their approach is a variant of the conduct parameter approach developed by Bresnahan (1989) and Weyl and Fabinger (2013). They argue that this reduced form approach nests Cournot competition and differentiated Bertrand competition and thus yields "results that are robust to the details of the industrial organization." Quoted from p.638 in Mahoney and Weyl (2017).

[11] A formal proof that our model nests MW's results is provided in Appendix C.

in the allocation in markets with adverse selection. This further exacerbates the inefficiency created by underprovision.

Empirical results support the theoretical prediction that a reduction in competition results in higher prices even in markets with adverse selection. Crawford et al. (2018) develop a structural model of credit markets and estimate it with data on bank loans to small and medium firms in Italy. Their estimates highlight the presence of adverse selection and imperfect competition in this market. In line with theoretical predictions, they find that a reduction in competition, modelled as a merger of two banks, leads to higher prices in their estimated model.[12]

While MW and our model both find that reductions in competition exacerbate the inefficiency created by adverse selection, the literature shows that this prediction can be reversed when firms can make offers which differ not just in price (as in Akerlof (1970), MW, and our model) but also in quantity. Rothschild and Stiglitz (1976) show that this second dimension allows firms to screen consumers, i.e. offer each consumer a menu of price quantity combinations such that consumers self select. Lester et al. (2019) show that, in this setting, increases in competition can reduce welfare as competition makes it harder to sustain pooling contracts. Intuitively, there exist cases where under perfect competition no pooling equilibrium exists as firms can profitably deviate and attract only a subset of consumers (cream skimming), while a monopolist would offer a pooling contract. Since in a pooling contract all gains from trade are realized, monopoly results in higher total surplus than perfect competition. Building a theoretical search model of imperfect competition, Lester et al. (2019) find that in markets with severe adverse selection, intermediate degrees of competition achieve the highest total surplus, while in markets with mild adverse selection, monopoly achieves the highest total surplus.[13] Overall, Lester et al.'s results are complementary to MW's and ours as taken together they highlight that the welfare effects of increases in competition depend on the type of contracts firms can offer. While the results from our model and MW speak to markets with with indivisible goods (e.g. used cars) or where the quantity is regulated (as in some health insurance markets), Lester et al.'s sorting results speak to markets where firms make offers which differ in a second dimension other than price (e.g. insurance with different levels of coverage).[14]

We extend the literature on imperfect competition in selection markets by considering price discrimination. This is increasingly relevant because recent advances in information technology are viewed as making price discrimination feasible in more settings (Vives and Ye, 2021). Our key impossibility result that no degree of competition can achieve the efficient allocation in a selection

[12]While Crawford et al.'s (2018) model permits them to study how a merger, higher funding costs for banks, or stronger selection affect equilibrium prices, their model does not permit them to study how efficiency or social welfare are affected by these changes. Our theoretical model generates insights on efficiency and social welfare.

[13]Formally, Lester et al. (2019) model imperfect competition via search frictions as in Burdett and Judd (1983), i.e. they assume that only a fraction of consumers receives offers from both firms while the remaining consumers receive offers from only one firm. More intense competition corresponds to more consumers receiving offers from both firms. We model imperfect competition via product differentiation. In our model, (i) all consumers receive offers from all firms, (ii) reducing competition corresponds to stronger brand preferences or to a merger of the firms, (iii) each consumer receives only one offer from each firm.

[14]Veiga and Weyl (2016) show that the mechanism that increases in competition make it harder to sustain pooling equilibria also applies when each firm can offer only one contract with two dimensions, rather than offering a menu of contracts as in Lester et al. (2019). Veiga and Weyl (2016) conclude that in markets with adverse selection welfare is maximised when competition is less than perfect.

market holds also when firms can price discriminate. However, the prediction which consumers experience over- or underprovision is reversed depending on whether firms price discriminate or not.[15]

Layout: Section 2 outlines our model. Section 3 characterises the efficient allocation. Section 4 shows that our model nests the existing results for perfect competition and monopoly. Section 5 characterises the equilibrium under imperfect competition. Section 6 outlines policy remedies other than tolerating intermediate degrees of competition. While, for clarity of exposition, section 2 - 6 focus on advantageous selection, section 7 presents the corresponding results for adverse selection. Section 8 discusses extensions of our model and section 9 concludes.

2 The Model

As in the standard Hotelling model, there are two firms which each produce one good. Firms sell to consumers who differ in their taste for the products. This gives firms a degree of market power over consumers who strongly prefer their product to the alternative. We depart from the standard Hotelling model to capture selection. Instead of deriving a firm's cost from its production function, we let the cost a firm incurs when selling to a consumer depend on the consumer's willingness-to-pay (WTP).

2.1 Specification of the Model

Players

There are two firms, which are located at the ends of the unit interval. Thus, we name the firms L (left) and R (right) respectively. Consumers are heterogeneous along two dimensions and are located on the unit square $(x, y) \in [0, 1]^2$. They differ in their taste $x \in [0, 1]$ and in their willingness-to-pay $y \in [0, 1]$. They have unit demand and thus purchase one good or no good. We assume that the distributions of consumers regarding taste x and WTP y are independent and uniform, i.e. $x \sim U[0, 1]$ and $y \sim U[0, 1]$. We denote the respective CDFs as $F(y)$ and $G(x)$.

Consumers differ in how strongly they prefer purchasing from one firm over the other. This can capture geographical distance (e.g. to a bank branch to apply for a loan), taste differences, or switching costs, e.g. created by prior transactions.[16] The common feature in all these cases is

[15] In studying price discrimination, our paper is also related to the industrial organization literature on firm pricing strategies and is distinct through its focus on selection markets. Armstrong (2006) provides an excellent survey. Particularly close to our paper is the literature on price discrimination under imperfect competition. Oligopoly with price discrimination, in the sense that firms may charge different prices to consumers depending on their taste for a firms product or distance from a shop, has been studied by Thisse and Vives (1988) and Corts (1998). Oligopoly with other forms of price discrimination (quantity discounts, prices differing across markets with different elasticities of demand) has been studied e.g. by Armstrong and Vickers (2001).

[16] E.g. banks may be able to charge existing consumers a mark-up as they have access to past payment histories which help the bank judge the customer's credit worthiness but which rival banks cannot access. This consideration was central to the Competition and Markets Authority launching the Open Banking initiative under which consumer data needs to be made accessible to rival banks. This aspect is the focus of the recent literature on open banking (He et al., 2021; Yannelis and Zhang, 2021; Goldstein et al., 2022) and is also prominent in the literature on relationship banking (Sharpe, 1990; Boot and Thakor, 2000). In an insurance context, consumers may have acquired a chronic medical condition which the existing insurer must cover but based on which new insurers could

that while some consumers switch already for small price differentials, others switch only for large price differentials. Formally, consumers face travel costs $T(d)$ when purchasing from a firm which is distance d away, i.e. $d = x$ or $d = 1 - x$ respectively, and where $\frac{dT}{dd} > 0 \; \forall \; d$. Thus, consumers with $x > \frac{1}{2}$ prefer to purchase from R rather than L. $T(d)$ is continuous and differentiable. As a normalisation, let $T(0) = 0$. We present all results assuming that $T(d) = td$ where $t > 0$ and discuss generalizations in the Appendix.

The defining feature of markets with selection is that consumers have private information on one aspect which affects both their willingness-to-pay (WTP) and the cost a firm incurs when selling to that consumer. For example, consumers with private information that their health is poor have a high WTP for health insurance and high expected medical costs.

In our model, consumers' WTP (y) is private information and the expected cost a firm incurs when selling to a consumer $c(y)$ can depend on y. We assume that $c(y) \geq 0 \; \forall \; y$ and that $c(y)$ is continuous and differentiable. In the benchmark case of no selection, $c(y) = \alpha \; \forall \; y$. If $\frac{dc}{dy} < 0 \; \forall \; y$, the market exhibits advantageous selection and if $\frac{dc}{dy} > 0 \; \forall \; y$ there is adverse selection.[17] We assume that the entire market is characterised by the same form of selection. Thus, we study markets with either adverse selection ($\frac{dc}{dy} > 0 \; \forall \; y$), or advantageous selection ($\frac{dc}{dy} < 0 \; \forall \; y$), but not markets where $\frac{dc}{dy}$ changes sign.

We assume that $c(y)$ is linear in y, i.e. $c(y) = \alpha + \beta(1 - y)$. In the case of advantageous selection, $\alpha \geq 0$ is the cost of the lowest cost agent and $\beta > 0$ captures the strength of advantageous selection. Adverse selection is captured by $\beta < 0$. While our model is flexible enough to capture advantageous or adverse selection, for clarity of exposition, we first present the model and our results focusing on markets with advantageous selection. We discuss adverse selection thereafter (in section 7).

Actions

Firms compete in prices. They simultaneously choose prices $p_i(x)$ where $i = \{L, R\}$. We distinguish between a case where firms can price-discriminate based on location x, i.e. $p_i(x)$ can vary across x, and a case of no price-discrimination, i.e. $p_i(x) = p_i \; \forall \; x$. We refer to the latter case as uniform pricing. This case arises when firms do not observe x or when firms are not allowed to condition prices on it. Throughout, we assume that willingness-to-pay (y) is the consumer's private information.

Each consumer (x, y) faces a unique price pair $(p_L(x), p_R(x))$ and chooses whether to buy from L, from R, or not at all.

Pay-offs

Firms maximise profit. Firm L's profit per consumer (x, y) is $\pi_L(x, y) = p_L(x) - c(y)$. Denoting

refuse to offer insurance.

[17] Different micro-foundations for advantageous selection are possible. For the case of insurance markets, see Einav and Finkelstein (2011) or our exposition in Appendix B. In our setting, $c(y)$ would include administrative costs which is one way to rationalise why some consumers may have WTP below $c(y)$.

the set of consumers buying from firm L as $\mathcal{D}_L$, firm L's total profit is

$$\Pi_L = \int\int_{(x,y)\in\mathcal{D}_L} \pi_L(x,y)\, f(y)\, dy\; g(x)\, dx = \int\int_{(x,y)\in\mathcal{D}_L} [p_L(x) - c(y)]\, f(y)\, dy\; g(x)\, dx$$

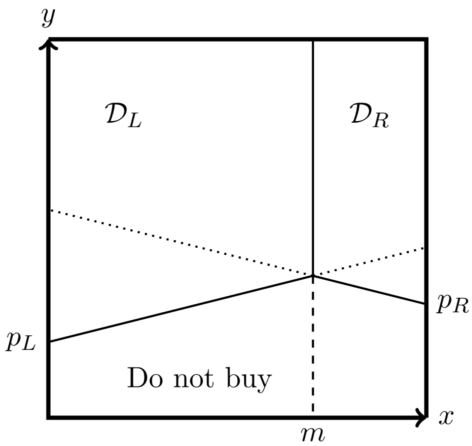
Each consumer maximises his monetary benefit. Normalising the outside option of not purchasing to zero $u_o = 0$, purchasing from L yields $u_L = y - p_L(x) - T(x)$ while purchasing from R results in $u_R = y - p_R(x) - T(1-x)$.

For any pair of uniform prices (p_L, p_R), there exists a location m at which consumers are indifferent between purchasing from L and R. At all locations $x < m$ consumers prefer purchasing from L to purchasing from R. Whether consumers prefer purchasing from L to the outside option depends on their WTP. Thus, consumers purchase from L if $u_L > u_R$ and $u_L > 0$ both hold. Thus,

$$\Pi_L(p_L, p_R) = \int_0^{m(p_L,p_R)} \int_{p_L+T(x)}^{1} [p_L - c(y)]\; f(y)\; dy\; g(x)\; dx$$

The allocation resulting for an arbitrary pair of uniform prices (p_L, p_R) is depicted below.

Figure 1: Purchasing behaviour for uniform prices (p_L, p_R)



At location m consumers are indifferent between purchasing from L or R.

Since our goal is to study competition, we assume that for every location x there exist benefits from trade for some y. Formally, this means that we restrict our attention to pairs of $c(y)$ and $T(d)$ such that $c(1) + T(\frac{1}{2}) \leq 1$ holds. This ensures that the two firms are not monopolists pricing over disjoint sets of demand, but that there exist some consumers who have gains from trade with both firms. Thus, firms will compete for these consumers.

In contrast to many models of spatial competition, our assumptions do not imply that it is efficient to allocate the good to every consumer when $T(d) = 0 \;\forall\; d$.[18] Instead, our assumptions imply that it is efficient to allocate the good only to a fraction of consumers. This departure is

[18] When models of spatial competition assume that a firm's marginal costs are constant and normalized to zero, and that all consumers have a strictly positive WTP (as in Hotelling (1929) and Thisse and Vives (1988)), then for $T(d) = 0 \;\forall\; d$ the efficient allocation is to allocated the good to every consumer. We model costs differently to capture selection.

necessary to allow us to study whether inefficient overprovision arises in equilibrium which is a first-order issue in markets with advantageous selection. Formally, this departure is generated by our assumptions on costs. When $T(d) = 0 \; \forall \; d$, then for markets with advantageous selection, there exist consumers with low WTP who are not allocated the good in the efficient allocation since the cost incurred when selling to these consumers $c(0) = \alpha + \beta > 0$ exceeds their WTP $(y = 0)$. However, there exist consumers with high WTP who are allocated the good in the efficient allocation since $c(1) \leq 1$. Hence, in the efficient allocation a fraction of consumers are allocated the good.

Equilibrium concept

We focus on symmetric Nash equilibria. That means we focus on equilibria where firms set $p_L(x) = p_R(1 - x)$ and consumers respond optimally to these prices.

2.2 Switching Consumers

Our model highlights a new mechanism how selection affects a firm's pricing decision and thus refines our understanding of the trade-off a firm faces when considering whether to cut its price in an imperfectly competitive selection market. More specifically, our model highlights that, relative to markets without selection, firms additionally consider the composition of switching consumers, i.e. consider which types of consumers a price cut would induce to switch from the rival firm. Our model implies that these switching consumers have on average a higher WTP than the consumers who purchase from the rival firm and thus are more strongly selected. For advantageous (adverse) selection, this magnifies (diminishes) a firm's benefit from cutting its price.[19] Thus, our model highlights how imperfect competition and imperfect information interact.

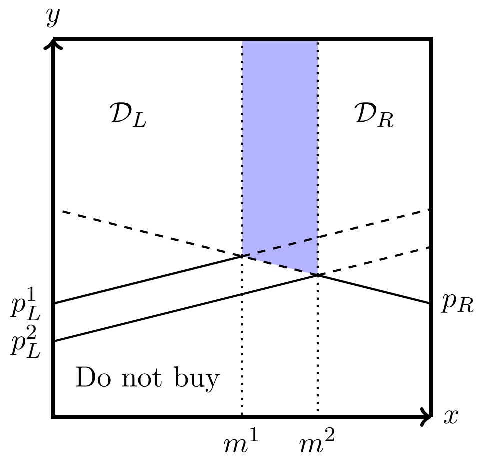
We show that selection introduces a new consideration to the firm's pricing problem: the composition of switching consumers. Like in markets without selection, in markets with selection firms weigh the benefit of a price cut in the form of additional demand (from consumers who previously did not purchase and from some, who previously purchased from the other firm) against the cost in the form of lower profits from selling to those consumers who already purchase from the firm. Unlike in markets without selection, firms additionally consider the composition of the additional demand, i.e. consider which type of consumers a price cut attracts. In markets with selection, the type of consumers a firm sells to is of first-order importance as it determines the cost a firm incurs when selling to that consumer. While it is clear that those consumers who previously did not purchase from either firm must have a lower WTP than existing consumers, it is not immediately clear what type of consumers switch. Our model provides a framework to study the composition of switching consumers.

Our model predicts that switching consumers are more strongly selected than the consumers who purchase from the rival firm in the sense that they have on average a higher WTP and thus lower (higher) costs when there is advantageous (adverse) selection. Intuitively, this arises

[19] This section on switching consumers focuses on the case of uniform pricing. When firms price discriminate, the composition of switching consumers is less surprising as a price cut induces either all consumers at that location to switch or no consumers to switch.

because a price cut only attracts those consumers of the rival firm to switch who had the largest distance to travel and thus incurred the largest transport cost. Therefore, the consumers who switch must have a high WTP as otherwise they would have preferred the outside option. In contrast, consumers who continue to purchase from the rival are those who incur low transport costs and thus prefer the rival over the outside option also at lower WTP. Hence, switching consumers have on average a higher WTP than the consumers who purchase from the rival.[20] Given advantageous selection, this makes switching consumers lower cost than average. This is depicted below:

Figure 2: Switching Consumers are an Advantageous Selection of Demand



Firm L's price cut from p_L^1 to p_L^2 results in consumers in the blue area switching.
These consumers have on average a higher WTP than the consumers in $\mathcal{D}_R$.

Our result on the composition of switching consumers implies that, in a market with advantageous (adverse) selection, a firm's benefit from cutting its price is larger (smaller) than it would be when calculated under the assumption that switching consumers are a random sample of the rival firm's demand. This arises because, for advantageous selection, a firm's cost is lower when selling to more strongly selected consumers and thus profits are higher. For a given quantity of switching consumers, this composition effect increases a firm's benefit from cutting its price.

An additional mechanism how selection affects a firm's pricing decision is via the composition of entering consumers, i.e. consumers who previously did not purchase from either firm but who start to purchase from the firm which cut its price. Our model makes the natural predictions that entering consumers have on average a lower WTP than the consumers who purchased before the price cut and that all entering consumers purchase from the firm which cut its price. This means that, in our model, the pricing decision by one firm affects the other firm only by stealing some consumers, but does not affect the composition of the firm's remaining demand.

[20] This prediction is driven by our assumption of taste heterogeneity. In the special case of our model without taste heterogeneity, i.e. where $G(x)$ is a degenerate distribution, a price cut would attract either all consumers or no consumers (or, in the case of a tie of the sum of price and transport cost, attract half of all consumers). Thus, switching consumers would be on average identical to the rival firm's demand. A formal discussion of this special case is in Appendix C.

3 Efficient Allocation

This section characterises the efficient allocation, which is used as a benchmark throughout the paper. We find that while in markets without selection the efficient allocation can be implemented with price discrimination or uniform pricing, in markets with selection it can only be implemented with price discrimination but not with uniform pricing. Therefore, we also characterise the socially optimal uniform price.[21]

3.1 Welfare

We adopt total surplus as our welfare measure and discuss alternative measures in section 8. Thus, welfare is the sum of consumers' monetary benefit and the firms' profits. Allocating the good to a consumer (x, y) generates surplus $s = y - T(d) - c(y)$. The outside option results in zero surplus.

Total surplus ($\mathcal{S}$) aggregates s across all transactions which take place. Denoting the set of consumers who are allocated the good from firm L as $\mathcal{D}_L$ and from R as $\mathcal{D}_R$, total surplus is:

$$\mathcal{S} = \int\int_{(x,y)\in\mathcal{D}_L} [y - T(x) - c(y)] f(y)\, dy\; g(x)\, dx + \int\int_{(x,y)\in\mathcal{D}_R} [y - T(1-x) - c(y)] f(y)\, dy\; g(x)\, dx \tag{1}$$

3.2 Socially Optimal Allocation

The socially optimal allocation is the allocation which maximises $\mathcal{S}$. Thus, it is to allocate the good to every agent whose WTP exceeds the sum of his costs and his transport costs, and allocate it from the firm where transport costs are lowest. Mathematically, a consumer should be allocated the good from firm L if and only if the following two conditions both hold:

$$y \geq c(y) + T(x) \tag{2}$$

$$x \leq \frac{1}{2} \tag{3}$$

Thus, there exists a unique socially optimal allocation. This allocation has a familiar threshold structure, where the threshold is $y^S(x) = c(y) + T(x)$. Then, all (x, y) with $y > y^S(x)$ and $x \leq \frac{1}{2}$ are allocated the good from L, all (x, y) with $y > y^S(x)$ and $x > \frac{1}{2}$ are allocated the good from R, and all others are not allocated the good.

The gradient of the threshold $y^S(x)$ depends on the presence and strength of selection. While in markets with no selection the threshold increases in line with transport costs ($\frac{dy^S(x)}{dx} = T'(x)$), in markets with advantageous selection, the threshold is flatter than transport costs, but still increasing ($0 < \frac{dy^S(x)}{dx} < T'(x)$).

[21] The socially optimal uniform price results in the constrained efficient allocation in the sense that the planner is constrained by not being able to condition prices on location.

Throughout the paper, we use the socially optimal allocation as the benchmark with which we compare equilibrium allocations as this allows us to isolate the strategic effect of changes in the intensity of competition as capture by transport costs. More generally, in spatial models, changes in transport costs have a strategic effect as well as a direct cost effect. The strategic effect, which we aim to capture, is that changes in transport costs alter a firm's best response to any price of the rival firm and thus change the equilibrium price and allocation. The direct cost effect, which is a confounding effect, is that transport costs reduce the total surplus achieved by any allocation. To avoid biasing our results in favour of competition, we isolate the strategic effect by comparing equilibrium allocations to the allocation a welfare maximising social planner can achieve when she faces the same transport costs.

3.3 Socially Optimal Price

We show that while in markets without selection the efficient allocation can be implemented with price discrimination and uniform pricing, in markets with selection it can only be implemented with price discrimination but not with uniform pricing. Thus, the socially optimal uniform price can only approximate the efficient allocation. In markets with advantageous selection, this approximation results in overprovision for consumers with strong brand preferences and underprovision for consumers with weak brand preferences. Formally:

Proposition 1 *In markets with selection, no uniform price can implement the efficient allocation.*

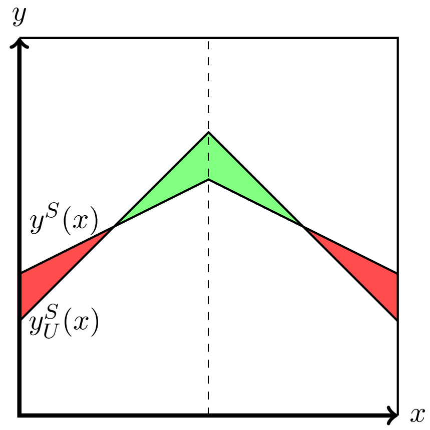
Proof: See Appendix A.1.

Intuition: This impossibility result arises because the threshold characterising the socially optimal allocation ($y^S(x)$) and the threshold describing an allocation created by a uniform price $y_U(x)$ always differ in their slope ($\frac{dy^S(x)}{dx} < T'(x) = \frac{dy_U(x)}{dx}$) and thus never coincide. To see the intuition, suppose the planner sets the uniform price to achieve the efficient allocation at $x = 0$. This means that the consumer $(0, p_L)$ is indifferent between purchasing from L and not purchasing. Moreover, allocating the good to this consumer generates zero surplus by construction. At any $\tilde{x} \in [0, \frac{1}{2}]$, the indifferent consumer $(\tilde{x}, p_L + T(\tilde{x}))$ must have a higher WTP than $(0, p_L)$ which exactly covers his larger transport costs. Crucially, in markets with advantageous selection, a higher WTP also means that the cost a firm incurs when selling to this consumer $c(y)$ is lower. Thus, allocating the good to the marginal consumer at $\tilde{x}$ generates strictly positive surplus. Surplus could thus be increased by allocating the good also to some consumers at $\tilde{x}$ with a WTP below but close to that of the indifferent consumer. Hence, the allocation in not efficient.

Thus, when the planner is limited to setting only uniform prices, she is trying to find the constrained efficient allocation. Since uniform prices result in steeper thresholds than the efficient allocation, the constrained efficient allocation includes overprovision at $x = 0$ and underprovision at $x = \frac{1}{2}$. This is depicted below.[22]

[22]The inefficiency created by the planner not being able to price discriminate is increasing in transport costs and in the degree of selection. Moreover, selection and transport costs magnify each others effect on inefficiency.

Figure 3: Social Planner: Advantageous Selection



$y^S(x)$ is the efficient allocation, $y_U^S(x)$ is the allocation resulting from the socially optimal uniform price. Green areas indicate underprovision, red areas indicate overprovision.

Example: When $c(y) = \alpha + \beta(1 - y)$, $T(d) = t\,d$, $F(y) = y$, $G(x) = x$, then the socially optimal price is:

$$p^S = \frac{\alpha + \beta}{1 + \beta} - \frac{\beta}{4(1 + \beta)}\, t \tag{4}$$

4 Perfect Competition and Monopoly

While the purpose of our model is to study imperfect competition, this section shows that our model nests perfect competition as well as monopoly as special cases and reproduces the results familiar from the literature. In markets with advantageous selection, perfect competition results in overprovision while monopoly results in underprovision.

Perfect Competition

Our model nests perfect competition as the special case of $T(d) = 0 \,\forall\, d$. This means that a firm which undercuts the rival firm's price attracts all consumers who previously purchased from the rival firm. In equilibrium, overprovision arises because firms continue to undercut each other even when the newly entering consumers are loss making as firms try to steal the rival's profitable existing consumers.[23]

Monopoly

Our model offers two ways of studying monopoly and both produce the familiar result of inefficient underprovision. One way is to consider transport costs which are so high that the presence of a

[23] To see this mechanism mathematically, define average costs (AC) and marginal costs (MC) as $AC = \frac{\int_p^1 c(y)f(y)dy}{1-F(p)}$ and $MC = c(p)$. A firm wants to undercut the rival's price as long as industry profits are positive, i.e. $p > AC$. In equilibrium, both firms price such that $p = AC$. That means that there is inefficient overprovision, since in markets with advantageous selection $AC < MC$.

rival firm does not affect pricing decisions. The other way is to consider a firm which sells both products L and R and therefore internalises the effect pricing decisions for one product have on demand and thus profits of the other product. In both cases, the following proposition applies.

Proposition 2 *In monopoly, there is inefficient underprovision at all locations x, regardless of the type and strength of selection and for all available pricing strategies.*

Proof: See Appendix A.2.

When transport costs are sufficiently high, firms effectively compete over disjoint sets of demand and thus act like monopolists. Formally, this arises when there exist some x at which it is socially optimal to not allocate the good to any consumer, i.e. when $t > 2(1-\alpha)$. Then, the presence of firm L does not affect firm R's pricing decision and vice versa. Intuitively, and analogous to markets without selection, the resulting market power leads to underprovision.

When one firm sells both product L and R, it acts like a monopolist as it internalises the effect prices for one product have on demand and thus profits of the other product. When the monopolist can price discriminate, its unique optimal price is naturally the price which equates marginal revenue (MR) and marginal costs (MC) at a given location x. When the monopolist sets a uniform price, it cannot implement the allocation reached under price discrimination and instead chooses a uniform price which approximates it. The result can be extreme underprovision in the sense that the monopolist does not sell to any consumer at an x even though allocating the good to some consumers at this x would generate strictly positive surplus.

To build intuition for the monopolist's problem and highlight that the monopolist partially absorbs transport costs, consider the case without selection where the monopolist can price discriminate. Then, the monopolist's unique optimal price at an x is $p_L^M(x) = \frac{1+\alpha}{2} - \frac{T(x)}{2}$ and the resulting allocation is characterised by threshold $y^M(x) = \frac{1+\alpha}{2} + \frac{T(x)}{2}$. There is underprovision at all x. Notice that the monopolist partially absorbs transport costs ($\frac{dp_L^M(x)}{dx} < 0$) as these costs depress demand. However, the monopolist does not absorb them fully ($0 < \frac{dy^M(x)}{dx} < T'(x)$) which results in the threshold $y^M(x)$ being flatter than the threshold resulting from a uniform price ($\frac{dy_U(x)}{dx} = T'(x)$). This means that there does not exist a uniform price which implements the monopolist's optimal allocation - an issue to which we return below.

A monopolist who operates in a market with advantageous selection and who can price discriminate uses his market power, which is present at all x, to maximise profit which, analogous to the case without selection, results in underprovision at all x. Formally, the monopolist's unique optimal price at a location x is

$$p_L^M(x) \;=\; \frac{1+\alpha+\beta}{2+\beta} - \frac{1+\beta}{2+\beta}\, T(x) \tag{5}$$

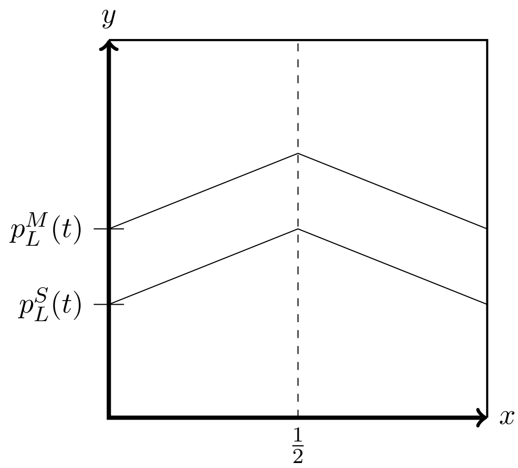
and the resulting allocation is described by $y^M(x) \;=\; \frac{1+\alpha+\beta}{2+\beta} + \frac{1}{2+\beta}\, T(x)$. This means that the monopolist partially absorbs transport costs. A new result specific to the case with selection is that a stronger degree of selection (larger β) leads to higher prices, i.e. $\frac{dp_L^M(x)}{d\beta} > 0$.

When the monopolist sets a uniform price, underprovision can be even more pronounced in the sense that there exist locations x where the monopolist does not sell the good to any consumer even though allocating the good to some consumers would generate strictly positive surplus. This arises because a monopolist, who sets a uniform price, cannot implement the allocation reached under price discrimination and instead chooses a uniform price which approximates it.[24] Formally:

$$p_L^M = \begin{cases} \frac{1+\alpha+\beta}{2+\beta} - \frac{1+\beta}{4\,(2+\beta)}\, t & \text{for} \quad t \leq \frac{4(1-\alpha)}{3+\beta} \\ \frac{1+2\alpha+\beta}{3+\beta} & \text{for} \quad t > \frac{4(1-\alpha)}{3+\beta} \end{cases} \tag{6}$$

Thus, for any t the monopolist's optimal uniform price results in underprovision at all x and exceeds the socially optimal uniform price. However, while for $t \leq \frac{4(1-\alpha)}{3+\beta}$ at all locations some consumers purchase (as depicted in Figure 4), for $t > \frac{4(1-\alpha)}{3+\beta}$ underprovision is more pronounced as no consumer with weak brand preferences purchases even though allocating the good to consumers with a high WTP would generate strictly positive surplus.

Figure 4: Monopoly allocation for uniform prices



Having established that our model reproduces the familiar results for perfect competition and monopoly, we now turn to the novel results on imperfect competition.

5 Imperfect Competition

This section characterises the equilibrium under imperfect competition and shows that reductions in competition can ameliorate the inefficiency created by advantageous selection, but never correct it perfectly. This arises because while reductions in competition can reduce the equilibrium quantity to the efficient quantity, this quantity is always allocated inefficiently. More

[24]The result that a monopolist cannot implement the same allocation as under price discrimination echoes Proposition 1 but differs as it does not depend on the presence of selection. The result from Proposition 1 that no uniform price can implement the socially optimal allocation applies only in markets with selection as the mechanism (that uniform prices cannot account for the effect of higher transport costs on $c(y)$) depends on the presence of selection. In contrast, the result that a monopolist cannot implement the same allocation as under price discrimination applies both in markets with and without selection as the mechanism (that uniform prices cannot absorb transport costs) does not depend on the presence of selection.

precisely, overprovision for consumers with some brand preferences coexists with underprovision for consumers with other brand preferences. The mechanism and the precise result which consumers experience over- or underprovision depend on whether firms use uniform pricing or price discriminate. We consider each in turn.

5.1 Uniform Pricing

When firms use uniform prices, there exists an intermediate degree of competition at which the equilibrium uniform price coincides with the planner's optimal uniform price. However, the equilibrium allocation does not coincide with the efficient allocation for any degree of competition. This arises because no uniform price can implement the efficient allocation.[25] A uniform price can only approximate the efficient allocation. Thus, even when the equilibrium price coincides with the planner's optimal uniform price, there is overprovision for consumers with strong brand preferences and underprovision for consumers with weak brand preferences. First, we characterise the equilibrium formally and then study its efficiency properties in detail.

Competition using uniform prices means that firms must offer the same price to all consumers, i.e. cannot condition p_L on x. Formally, firm L chooses p_L to maximise its profit

$$\Pi_L(p_L, p_R) = \int_0^{m(p_L,p_R)} \int_{p_L+T(x)}^1 [p_L - c(y)]\; f(y)\; dy\; g(x)\; dx \tag{7}$$

where $m(p_L, p_R)$ denotes the location x at which consumers are indifferent between purchasing from either firm. Since $T(d) = td$ where $t > 0$, we get the familiar Hotelling result $m(p_L, p_R) = \frac{1}{2} + \frac{p_R - p_L}{2t}$. Denoting the equilibrium price as p^*, we have the following proposition:

Proposition 3 *There exists a unique equilibrium.*
If $t \leq \frac{4\,(1-\alpha)}{3+\beta}$, it is characterized by the equilibrium price

$$p^* = \frac{1+\alpha+\beta}{2+\beta} + \frac{3+\beta}{(2+\beta)}\,\frac{t}{2} - \frac{1}{2+\beta}\sqrt{(1-\alpha-\frac{t}{2})^2 + \frac{t^2}{2}(\beta+2)(\beta+3)} \tag{8}$$

If $t > \frac{4\,(1-\alpha)}{3+\beta}$, it is characterized by the equilibrium price

$$p^* = \frac{1+2\alpha+\beta}{3+\beta} \tag{9}$$

Proof: See Appendix A.3.

To study efficiency, it is important to distinguish between the efficient, or socially optimal, allocation and the constrained efficient allocation in the sense of the allocation resulting from the socially optimal uniform price.[26] While for markets without selection these notions are

[25] Recall from Section 3 that in markets with advantageous selection the efficient allocation can only be implemented by prices which are decreasing in distance as larger distance means that the consumer who is indifferent between purchasing and not purchasing has a higher WTP which, in markets with advantageous selection, means that the cost a firm incurs when selling to him $c(y)$ is lower. Since a uniform price prevents firms from differentiating prices by distance, the efficient allocation cannot be implemented with a uniform price.

[26] In our model, in the constrained efficient allocation the quantity traded is efficient but it is allocated inefficiently.

identical, they differ in markets with selection.[27] Formally, in the special case of no selection ($\beta = 0$), the equilibrium is neither efficient (there is underprvoision at all x for any $t > 0$) nor constrained efficient ($p^*(t) > p_L^S(t) \, \forall \, t > 0$).[28] The efficiency results for advantageous selection are summarized in Proposition 4.

Proposition 4 *(Advantageous selection)*

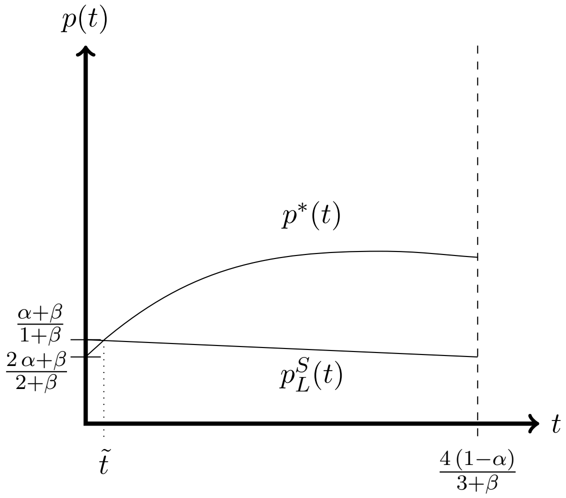
(i) There exists a unique level of transport costs, denoted $\tilde{t}$, at which the equilibrium price $p^*(t)$ and the socially optimal price $p_L^S(t)$ coincide, i.e. $p_L^S(\tilde{t}) = p^*(\tilde{t})$ where $\tilde{t} > 0$.

(ii) However, the equilibrium allocation and the socially optimal allocation do not coincide for any level of transport costs t.

Proof: See Appendix A.5.

The first part of Proposition 4 on the comparison of uniform prices follows from a fixed point argument. Since we know that in the limit case of perfect competition $p^*(0) < p_L^S(0)$, and that for $t \geq \frac{4\,(1-\alpha)}{3+\beta}$ firms price like a monopolist and thus $p^*(t) > p_L^S(t) \, \forall \, t \geq \frac{4\,(1-\alpha)}{3+\beta}$, and that $p^*(t)$ is continuous, there must exist a $\tilde{t}$ at which $p^*(\tilde{t}) = p_L^S(\tilde{t})$. This means that while in markets without selection $p^*(t) > p_L^S(t) \, \forall \, t > 0$, in markets with selection this ordering depends on the degree of competition. This is depicted in Figure 5.[29]

Figure 5: Uniform Price Comparison



The second part of Proposition 4 establishes the impossibility result that there does not exist a degree of competition which perfectly corrects the inefficiency created by advantageous

[27]This arises because a uniform price can implement the efficient allocation in markets without selection, and thus the constraint of using uniform prices is not binding, while no uniform price can implement the efficient allocation in markets with selection and thus the constraint is binding.

[28]Both in markets with selection and without $p_L^M(t) > p^*(t) \, \forall \, t$ and $p_L^M(t) > p_L^S(t) \, \forall \, t$.

[29]In Figure 5, $p^*(t)$ is continuous but not monotone in t. The threshold describing the resulting allocation at $x = \frac{1}{2}$, $p^*(t) + \frac{1}{2}t$, is monotonically increasing in t. The non-monotonicity of $p^*(t)$ arises because, in our spatial model, larger transport costs have two effects. A strategic effect, which tends to raise prices, and a direct cost effect, which alters demand and can lead firms to lower their prices.

selection. This follows directly from the result that no uniform price can implement the efficient allocation (Proposition 1). This means that even when $p^*(\tilde{t}) = p_L^S(\tilde{t})$ - in this case the quantity traded equals the efficient quantity - there is an inefficiency in the allocation. Overprovision for consumers with strong brand preferences and underprovision for consumers with weak brand preferences coexist (as depicted in Figure 3).

Thus, Proposition 4 cautions against viewing imperfect competition as a solution to the inefficiencies created by advantageous selection. Since no degree of competition results in the efficient allocation, other policy interventions which implement the efficient allocation would therefore result in higher total surplus. Section 6 explores alternative policy interventions and finds that combining a corrective tax with perfect competition results in the efficient allocation and thus in higher total surplus. For the formal discussion of a corrective tax in section 6, it is useful to note that Proposition 3 generates a result on cost pass-through.

Corollary 1 *In equilibrium, there is cost pass-through, i.e.* $0 < \frac{dp^*}{d\alpha}$, *regardless of the presence and strength of selection.*

Proof: See Appendix A.4.

Since the arguments above which establish the impossibility result rely on firms using uniform prices, it is natural to wonder whether the impossibility result also applies when firms can price discriminate. Price discrimination gives firms flexibility such that there exist pricing schedules which implement the efficient allocation, but it also changes the firms' strategic interaction. Thus, it is not clear whether the efficient allocation is reached in equilibrium. In the next section, we turn to the case of price discrimination and show that the impossibility result continues to apply even though the mechanism is different and the prediction which consumers experience over- or underprovision is reversed.

5.2 Price Discrimination

When firms price discriminate, the equilibrium does not result in the efficient allocation for any degree of competition. This arises because price discrimination allows firms to target low prices at consumers with weak brand preferences to induce them to switch from the rival firm, while simultaneously charging high prices to consumers with strong brand preferences who would only switch to the rival firm if it offered a considerably lower price. The result is that overprovision for consumers with weak brand preferences coexists with underprovision for consumers with strong brand preferences - the reverse of the case with uniform prices.

Characterisation of the Equilibrium

Formally, price discrimination means that firms choose prices $p_L(x)$ and $p_R(x)$ respectively, which can differ across x. This flexibility allows firms to offer low prices at some x and simultaneously charge high prices at other x, while with uniform pricing firms had to offer the same price at all x. Therefore, when firms price discriminate, they treat each location x as a separate market. The resulting equilibrium is:

Proposition 5 *There exists an equilibrium with the following prices:*

$$
\begin{aligned}
&\textit{For } x \in [0, x_L) && p_L(x) = p_L^M(x) = \tfrac{1+\alpha+\beta}{2+\beta} - \tfrac{1+\beta}{2+\beta}tx \\
& && p_R(x) \geq p_L^M(x) + T(x) - T(1-x) \\
&\textit{For } x \in [x_L, \tfrac{1}{2}] && p_L(x) = \tfrac{2\alpha+\beta}{2+\beta} + \tfrac{2}{2+\beta}t - \tfrac{4+\beta}{2+\beta}tx \\
& && p_R(x) = p_R^B(x) = \tfrac{2\alpha+\beta}{2+\beta} - \tfrac{\beta}{2+\beta}t + \tfrac{\beta t}{2+\beta}tx \\
&\textit{where } x_L = \begin{cases} \frac{2}{3} - \frac{1-\alpha}{3t} & \textit{if} \quad t > \frac{1}{2}(1-\alpha) \\ 0 & \textit{if} \quad t \leq \frac{1}{2}(1-\alpha) \end{cases}
\end{aligned}
$$

This means that at $x < \frac{1}{2}$ consumers purchase either from firm L or do not purchase at all.[30]

Proof: See Appendix A.6.

Thus, in equilibrium the consumers who purchase the good, purchase it from their preferred firm and the firm makes a profit as it charges either the highest price which the rival firm cannot undercut profitably, or the price a monopolist would set. This arises because brand preferences give a firm market power over consumers who prefer its product to the rival firm's product. To see this, consider consumers at an $x < \frac{1}{2}$, i.e. who prefer to purchase from firm L over firm R all else equal. To induce these consumers to purchase from firm R, firm R must offer a price which is below $p_L(x)$ by at least the transport cost differential $T(1-x) - T(x)$, i.e. offer $p_R(x) \leq p_L(x) - [T(1-x) - T(x)]$. However, firm R will only offer $p_R(x)$ which result in non-negative profits, i.e. $p_R(x) \geq p_R^B(x)$ where $p_R^B(x)$ is defined as the lowest possible price at which firm R breaks even if consumers at x purchase from it. Thus, the result is that firm L can set any $p_L(x)$ without losing consumers to the rival firm provided that $p_L(x) \leq p_R^B(x) + T(1-x) - T(x)$, as firm R cannot profitably undercut these $p_L(x)$. For example, in markets without selection, $p_R^B(x) = \alpha$ and firm L can set any $p_L(x)$ up to $p_L(x) \leq \alpha + T(1-x) - T(x)$. Thus, the transport cost differential $T(1-x) - T(x)$ determines how much market power a firm has at location x.

In equilibrium, prices can vary considerably across locations reflecting that a firm's market power differs across locations. It is even possible that consumers with strong brand preferences face the same price they would face under monopoly, while consumers with weak brand preferences face a lower price as a result of competition. This possibility arises when transport costs are sufficiently high ($t > \frac{1}{2}(1-\alpha)$). Then, firm L prices like a monopolist for $x \in [0, x_L)$ as firm R cannot offer a price which both breaks even and attracts consumers.[31] Thus, only consumers at $x \in [x_L, \frac{1}{2}]$ face lower prices under competition than under monopoly. Moreover, for different $x \in [x_L, \frac{1}{2}]$ equilibrium prices are lower if brand preferences are weaker. This indicates that efficiency properties can vary substantially across x - an issue which we investigate formally further below.

A reduction in competition, as captured by an increase in transport costs, changes the equilibrium in two ways. First, it results in x_L shifting to the right, which means that firms

[30] The equilibrium prices on $x > \frac{1}{2}$ and the resulting allocation follow from the symmetric set-up of the model. This means that at $x > \frac{1}{2}$ all consumers either purchase from firm R or do not purchase at all. Firm R sets the monopolist's price for consumers with strong brand preferences and charges a lower price if this is needed to deter firm L from undercutting firm R.

[31] Formally, this means that any $p_R(x)$ which does not attract consumers given $p_L^M(x)$ is an equilibrium price. Hence the inequality in Proposition 5 in $p_R(x) \geq p_L^M(x) + T(x) - T(1-x)$.

engage in monopoly pricing at more locations x. Second, it results in higher $p_L(x)$, which means that prices increase even for those consumers who face lower prices than in monopoly.

The fact that in equilibrium some consumers face the same price they would face in monopoly leads to Corollary 2.

Corollary 2

(i) The equilibrium price is monotonically decreasing in distance from the firm.

(ii) The equilibrium quantity, i.e. the number of consumers purchasing the good, can be non-monotone in distance from the firm.

Proof: See Appendix A.7.

Intuitively, Corollary 2 arises because distance has two effects. First, holding prices constant, distance reduces demand as it increases the transport costs consumers pay. Second, distance intensifies competition as it reduces the transport cost differential. Corollary 2 arises because the effect of distance on demand increases monotonically while the effect of distance on competition appears only for consumers whose brand preferences are sufficiently weak. To see this, note that for $x < x_L$, distance depresses demand but has not effect on competition as firm L prices like a monopolist for all $x \in [0, x_L)$. As monopolist, firm L chooses to partially absorb transport costs because of their effect on demand and thus $\frac{dp_L^M(x)}{dx} < 0$ but $\frac{dy^M(x)}{dx} > 0$.[32] However, when brand preferences are sufficiently weak ($x \in [x_L, \frac{1}{2}]$), distance not just depresses demand but also intensifies competition. Moreover, the effect on competition is so strong that $\frac{dp_L^M(x)}{dx} < 0$ and $\frac{dy(x)}{dx} < 0$, which means that the equilibrium quantity increases with distance.[33]

Efficiency Properties of the Equilibrium

To build intuition about the efficiency properties of the equilibrium, first consider the case without selection which acts as a useful benchmark.[34]

Proposition 6 *In markets without selection, the equilibrium exhibits inefficient underprovision at all x other than $x = \frac{1}{2}$ and efficient provision at $x = \frac{1}{2}$.*

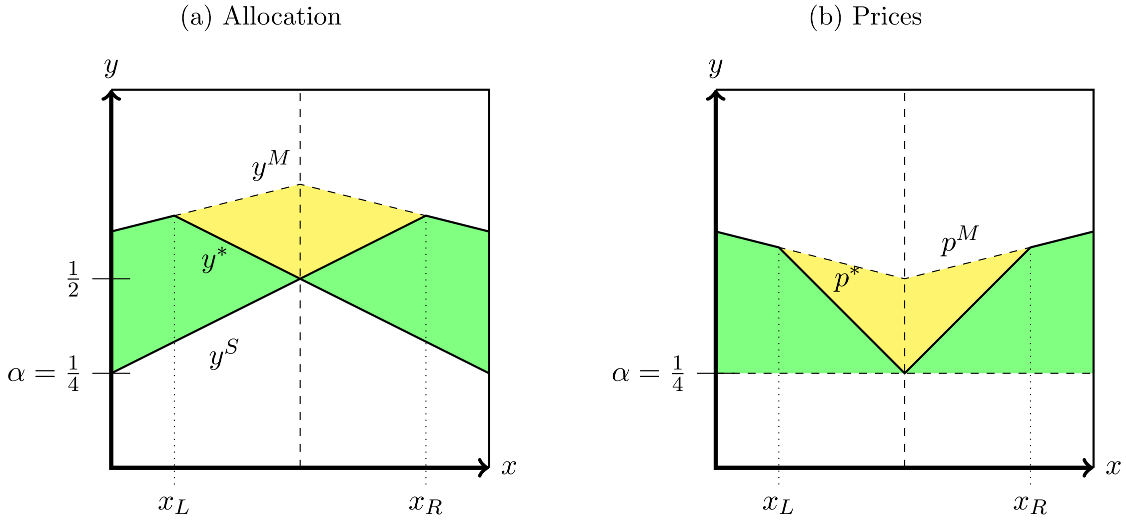
Proof: See Appendix A.8.

[32] The different signs of $\frac{dp_L^M(x)}{dx}$ and $\frac{dy^M(x)}{dx}$ are possible because transport costs drive a wedge between the price consumers pay to the firm ($p(x)$) and the total cost consumers incur ($p(x) + T(x)$) which determines demand $(1 - p(x) - T(x))$.

[33] Mathematically, since $\frac{dp(x)}{dx} < 0$ and $|\frac{dp(x)}{dx}| > |\frac{dT(d)}{dd}| > 0$, we have that $\frac{dy(x)}{dx} < 0$.

[34] We prove the efficiency results of this section not just for $T(d) = td$ where $t > 0$, but for the more general case where we only require $T(d)$ to be strictly increasing in distance travelled ($\frac{dT}{dd} > 0 \,\forall\, d$), i.e. not necessarily linear.

Figure 6: Equilibrium: No Selection



$y^*(x)$ and $p^*(x)$ refer to the equilibrium allocation and price, $y^S(x)$ and $p^S(x)$ to the socially optimal allocation and price, and $y^M(x)$ and $p^M(x)$ to the monopolist's optimal allocation and price. Inefficient underprovision is shaded in green, efficiency gains in equilibrium relative to monopoly are shaded in yellow. For $x \in [0, x_L)$, equilibrium prices are as in monopoly while for $x \in [x_L, \frac{1}{2}]$ equilibrium prices are lower than in monopoly.

To see the intuition, recall that in markets without selection, the efficient allocation is reached when industry profits are zero. However, brand preferences give the preferred firm market power and allow it to make positive profits in equilibrium. Thus, for those consumers who are indifferent between brands ($x = \frac{1}{2}$), firms have no market power, industry profits are zero and the equilibrium results in the efficient allocation. However, for consumers with brand preferences ($x \neq \frac{1}{2}$), the preferred firm has market power, makes a profit in equilibrium and the result is inefficient underprovision.

In markets with advantageous selection, equilibrium firm profits are determined by consumers' brand preferences, which is like in markets without selection. However, the mapping from equilibrium profits to efficiency is unlike in markets without selection because, in markets with advantageous selection, the efficient allocation is reached when industry profits are positive, not when they are zero. This means that efficiency properties differ as over- and underprovision are both possible and can even coexist. Formally,

Proposition 7 *In markets with advantageous selection, the equilibrium allocation has the following efficiency properties:*

(i) If $T(1) \geq \frac{\beta}{2+2\beta}(1-\alpha)$, there exists a unique x_e which solves $\beta(1-\alpha) = (2+2\beta)\, T(1-x_e) - (2+\beta)\, T(x_e)$.[35] *Then*

$$\textit{For all} \quad x \in [0, x_e) \quad \textit{there is inefficient underprovision.}$$
$$\textit{For all} \quad x \in (x_e, \frac{1}{2}] \quad \textit{there is inefficient overprovision.}$$

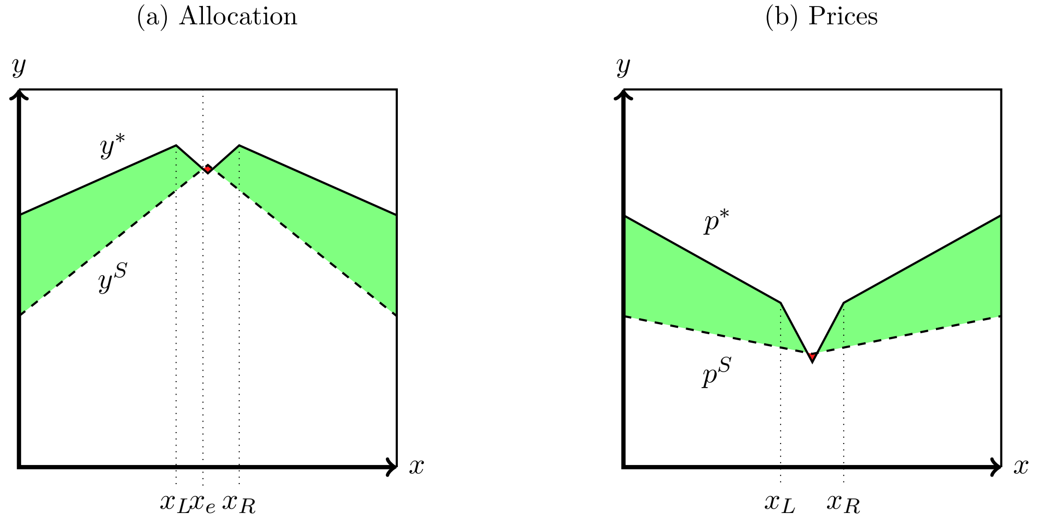
[35] When $T(d) = t\, d$, we have $x_e = \frac{2}{4+3\beta} + \frac{\beta}{t}\, \frac{(2t+\alpha-1)}{(4+3\beta)}$.

(ii) If $T(1) < \frac{\beta}{2+2\beta}(1-\alpha)$, then $x_e = 0$ and there is inefficient overprovision at all x.

Proof: See Appendix A.9.

Proposition 7 highlights that no degree of competition achieves the efficient allocation at all x simultaneously. Over- and underprovision are possible and can even coexist at different x.[36] To see the intuition, recall that brand preferences give the preferred firm market power and that, in markets with advantageous selection, there is inefficient overprovision when firms make zero profits. Thus, for consumers with no or weak brand preferences firms make zero or only small profits and the result is overprovision. For consumers with strong brand preferences, firms have more market power ($x \in [x_L, x_e]$) or even price like a monopolist ($x \in [0, x_L)$) and therefore make larger profits. The result is underprovision. Thus, while at a given x overprovision and underprovision can offset each other, there is no mechanism by which overprovision at some x affects underprovision at other x. Hence, over- and underprovision can coexist.

Figure 7: Equilibrium: Advantageous Selection



$y^*(x)$ and $p^*(x)$ refer to the equilibrium allocation and price, $y^S(x)$ and $p^S(x)$ to the socially optimal allocation and price. Inefficient underprovision is shaded in green, inefficient overprovision is shaded in red and occurs only in a small area around $x = \frac{1}{2}$.

A reduction of competition, as captured by an increase in transport costs, increases underprovision in two ways. First, it shifts x_e to the right which means that underprovision arises also for consumers with weaker brand preferences. Second, it makes underprovision more pronounced for those consumers who already experience underprovision, e.g. by exposing more consumers to the price they would face in monopoly. However, reductions of competition never remove overprovision at all x since at $x = \frac{1}{2}$ there is overprovision for all degrees of competition.[37] Thus, in summary, we have the following impossibility result.

[36] Mathematically, there exists a cut-off location x_e, as defined in Proposition 7, such that for all consumers with stronger brand preferences (at all $x \in [0, x_e)$) there is underprovision and for all consumers with weaker brand preferences (at all $x \in (x_e, \frac{1}{2}]$) there is overprovision.

[37] Only monopoly would remove overprovision at $x = \frac{1}{2}$.

Proposition 8 *Impossibility Results: Advantageous Selection*

In markets with advantageous selection, the equilibrium allocation does not coincide with the socially optimal allocation for any degree of competition.

Proof: See Appendix A.10.

This impossibility result cautions against viewing imperfect competition as a solution to the inefficiencies created by advantageous selection. Moreover, we showed that this impossibility result applies both when firms price discriminate (section 5.2) and when they use uniform prices (section 5.1). Thus, a natural question is whether other policy interventions can achieve the socially optimal allocation. The next section explores this formally.

6 Policy Remedies

Since no degree of imperfect competition can perfectly correct the inefficiency created by advantageous selection, this section discusses alternative policy solutions. First, we study a corrective tax, i.e. a tax or subsidy which applies equally to all trades. Then, we turn to "risk adjustment", which is a tax or subsidy conditioned on an individual's characteristics.

6.1 Corrective Tax

We find that the combination of a corrective tax and fierce competition achieves the socially optimal allocation and thus results in higher total surplus than intermediate degrees of competition, as these can only achieve the constrained efficient allocation. This suggests that competition policy should not be lenient towards firms on the ground that they operate in a market with advantageous selection. Instead, competition policy should continue to focus on achieving perfect competition. The distortions arising from selection can be better addressed through a tax than through altered competition policy. Moreover, the corrective tax has additional benefits. It generates government revenue, holds regardless of whether firms use uniform prices or price discriminate, and has lower informational requirements.

Proposition 9 *There exists a corrective tax which achieves the socially optimal allocation.*

Proof: See Appendix A.11.

A tax per unit sold (τ) is treated by firms like an upward shift in costs. If $c(y) = \alpha + \beta(1-y)$, it corresponds to an increase in α. Since Corollary 1 established that there is cost pass-through, i.e. that $0 < \frac{dp^*(t)}{d\alpha}$, the corrective tax leads firms to raise prices. When appropriately calibrated and combined with perfect competition, the tax can restore the socially optimal allocation.

Since $c(y) = \alpha + \beta(1-y)$ where $\beta > 0$, there is inefficient overprovision under perfect competition, i.e.

$$p_L^S(t=0) = \frac{\alpha+\beta}{1+\beta} > p^*(t=0) = \frac{2\,\alpha+\beta}{2+\beta} \tag{10}$$

Since a per unit tax (τ) results in equilibrium prices $p^*(t=0) = \frac{2\,(\alpha+\tau)+\beta}{2+\beta}$, we can solve for the tax rate which achieves the socially optimal, or first-best efficient, allocation:[38]

$$\tau^* = \frac{\beta\,(1-\alpha)}{2\,(1+\beta)} \tag{11}$$

Regardless of whether firms use uniform pricing or price discriminate, a per unit tax can always achieve the first-best efficient allocation. The optimal tax rate is the same in both cases. Thus, the policy of using a corrective tax is robust in the sense that it does not depend on which pricing strategies firms use.

The corrective tax not only results in larger total surplus than imperfect competition, it also has lower informational requirements. In order to calculate the level of competition which achieves the constrained efficient allocation ($\tilde{t}$), the policy maker needs to know costs $c(y)$ and both type distributions $F(y)$ and $G(x)$. In order to calculate the optimal corrective tax, which achieves the first-best efficient allocation (τ^*), the policy maker only needs to know costs $c(y)$ and the WTP distribution $F(y)$, but not the preference distribution $G(x)$. This arises because perfect competition has the additional advantage that it makes all consumers face the same total cost of purchasing $p + T(x) = p \; \forall \; x$, and thus results in consumers at all x behaving equally. This makes it unnecessary to know $G(x)$.

A corrective tax has additional benefits which are not captured in our model. For example, the tax generates government revenue which can be used to reduce distortionary taxes in other markets and thus enhances efficiency. Attempts to correct overprovision through reductions in competition have no comparable benefit as instead of creating government revenue they increase firm profits and create a deadweight loss (transport costs).

6.2 Risk Adjustment

An alternative policy to the corrective tax or to tolerating intermediate degrees of competition is risk adjustment. Under risk adjustment, firms receive a subsidy conditional on the type of consumers they insure such that the cost a firm incurs from insuring a consumer is independent from the consumer's type. Thus, risk adjustment effectively removes selection.

While risk adjustment is used in practice, e.g. in health insurance markets, a corrective tax has lower informational requirements and is likely more robust to gaming of risk scores by insurance providers. The corrective tax shifts every agent's cost equally and thus preserves heterogeneity in costs. Hence, to implement the tax, neither the policy maker nor the firm needs to know the agent's type y. Under risk adjustment, firms receive a subsidy conditional on y. Thus, both the firm and the policy maker need to know y. In many settings, a consumer's

[38] More generally, for any transport cost t, there exists a tax rate or subsidy rate which achieves the constrained efficient allocation, i.e. for which $p_L^S(t) = p^*(t)$. This tax rate is implicitly defined by $p_L^S(\alpha,\beta,t) = p^*(\alpha+\tau,\beta,t)$ or equally by $\frac{\alpha+\beta}{1+\beta} - \frac{\beta}{4\,(1+\beta)}t = \frac{1+\alpha+\tau+\beta}{2+\beta} + \frac{3+\beta}{2(2+\beta)}t - \frac{1}{2(2+\beta)}\sqrt{(2\beta^2+10\beta+13)t^2 - 4(1-\alpha-\tau)t + 4(1-\alpha-\tau)^2}$. However, the first-best efficient allocation is only achieved under perfect competition ($t=0$) combined with tax rate (11). Moreover, all cases with $t > 0$ are further complicated by considerations whether firms use uniform pricing or price discrimination and by whether the tax rate has to apply equally to all x or can discriminate based on x.

riskiness is not observed and subsidy payments are conditioned on risk scores which insurers assign. This generates an incentive problem as the insurer prefers to make the consumer appear to have large expected claims in order to get a large subsidy. The tax does not require risk scores and therefore does not generate this incentive problem.

Uniform pricing: Our model predicts that the composition of switching consumers matters when calculating the effect of risk adjustment. One way of implementing risk adjustment is to adjust every agent's cost to equal the average cost of all agents who purchase the good. When costs are linear, this is equal to the cost of the average infra-marginal consumer. However, when using uniform prices, firms' pricing decisions are driven by the average marginal consumer, which is calculated across switching and entering consumers. When the average marginal consumer is a lower type than the average infra-marginal consumer, as is the case when distributions are uniform, then in a market with advantageous selection, firms respond to this form of risk adjustment by lowering their prices.

Price discrimination: Calculations of the effect of risk adjustment, or equally comparative statics on the strength of selection, face the obstacle that a change in β corresponds to both altering the slope of marginal costs and also to altering the level of costs for a given demand. That means $\frac{dx_e}{d\beta}$ captures both a pure cost effect, i.e. an increase in total cost for a given level of demand, and an increase in the strength of selection. We need to isolate the effect of increased selection. We achieve this through an indirect approach. $\frac{dx_e}{d\alpha}$ captures a pure cost effect with no change in selection. We find that $\frac{dx_e}{d\alpha} > 0$. We also find that $\frac{dx_e}{d\beta} < 0$. Taken together, this means that the selection effect has the opposite sign of a pure cost effect and more than outweighs it. Thus, an increase in the strength of selection, which does not alter the average cost in the market, likely leads to inefficient overprovision occurring at more locations x. Risk adjustment, i.e. a reduction in the strength of selection, has the opposing effect. Formally:

Corollary 3 *If, in markets with advantageous selection, selection is more pronounced, overprovision occurs at more locations x for given transport costs $T(d)$.*

Proof: See Appendix A.12.

7 Adverse Selection

Turning to markets with adverse selection, our model highlights that reductions of competition exacerbate the inefficiency created by selection via two channels. First, reductions in competition reduce the equilibrium quantity which is already inefficiently low under perfect competition. Second, under imperfect competition, a given quantity is allocated inefficiently. This leads to the result that while in markets with advantageous selection, reductions of competition cannot achieve the efficient allocation, but can achieve the constrained efficient allocation, in markets with adverse selection reductions of competition can achieve neither the efficient, nor the constraint efficient allocation. Therefore, our results caution against viewing imperfect competition as a solution to inefficiencies introduced by selection also for markets with adverse selection, and they caution more strongly than for advantageous selection. As the mechanisms are similar to the case of advantageous selection, we highlight results briefly and focus on the key differences.

The model captures adverse selection when $c(y)$ is increasing in y. We focus on linear costs $c(y) = \alpha + \beta(1 - y)$ where $\beta < 0$ ensures adverse selection.[39]

Switching Consumers: In markets with adverse selection, the consumers who switch to a firm when it cuts its price are more strongly adversely selected than the consumers who purchase from the rival firm. The mechanism is as in markets with advantageous selection. Switching consumers have, on average, higher WTP than the consumers who purchase from the rival firm as switching consumers incurred larger transport costs. Thus, selling to switching consumers is more costly for firms than selling to the average consumer who purchased from the rival firm and than selling to the average consumer in the population. This diminishes a firm's benefit from cutting its price.

Uniform Pricing and Efficiency: As in the case of advantageous selection, in markets with adverse selection no uniform price can implement the efficient allocation (Proposition 1). In both cases, this arises because the socially optimal allocation $y^S(x)$ and the allocation created by a uniform price $y_U(x)$ always differ in their slope and thus never coincide. This arises because the socially optimal allocations takes into account that consumers with a larger distance to travel have higher WTP and therefore are higher (lower) cost when there is adverse (advantageous) selection. A uniform price cannot take this into account. However, while with advantageous selection $\frac{dy^S(x)}{dx} < T'(x) = \frac{dy_U(x)}{dx}$, with adverse selection $\frac{dy^S(x)}{dx} > T'(x) = \frac{dy_U(x)}{dx}$.

The socially optimal uniform price approximates the socially optimal allocation and results in overprovision for some consumers coexisting with underprovision for others. Which consumers experience over- or underprovision is reversed depending on the type of selection. In markets with adverse selection, the socially optimal uniform price results in underprovision for consumers with strong brand preferences and overprovision for consumers with weak brand preferences. This is the reverse of the result for markets with advantageous selection.

Monopoly: In monopoly, there is inefficient underprovision at all locations x. This result, formalised in Proposition 2, holds for both adverse and advantageous selection, for any strength of selection and for both uniform pricing and price discrimination.

Imperfect Competition: Uniform Pricing: While, under adverse selection, the equilibrium uniform price continues to be characterised by Proposition 3 and continues to satisfy Corollary 1 on cost pass-through, the efficiency properties differ from the case of advantageous selection. As in the case with advantageous selection, with adverse selection the equilibrium price does not result in the socially optimal allocation for any degree of competition (Proposition 4 (ii) and Proposition 10 (ii)). However, in contrast to the case with advantageous selection, with adverse selection the equilibrium uniform price does not even result in the constrained efficient allocation (Proposition 4 (i) and Proposition 10 (i)). Thus, for adverse selection we have:

[39]Our assumption that costs are positive implies that $\beta > (-1)$.

Proposition 10 *(Adverse selection)*

(i) The equilibrium price $p^*(t)$ *and the socially optimal price* $p_L^S(t)$ *do not coincide for any level of transport costs* t*.*

(ii) The equilibrium allocation and the socially optimal allocation do not coincide for any level of transport costs t*.*

Proof: See Appendix A.13.

Imperfect Competition: Price Discrimination: While in markets with adverse selection, the equilibrium prices continue to be characterized by Proposition 5 and Corollary 2 continues to apply, the efficiency properties differ from the case of advantageous selection (Proposition 7). Whereas under advantageous selection over- and underprovision coexisted for different consumers, under adverse selection there is underprovision for all consumers.

Proposition 11 *(Adverse selection) In equilibrium, there is inefficient underprovision at all* x*.*

Proof: See Appendix A.14.

Thus, our results caution against viewing imperfect competition as a solution to the inefficiencies introduced by adverse selection, and they caution more strongly than for advantageous selection. When firms use uniform prices, results are stronger because for adverse selection reductions in competition do not even result in the constrained efficient allocation, which can be reached under advantageous competition. When firms price discriminate, results are stronger because while for advantageous selection the inefficiency cancelled out at one location (though not at all locations), for adverse selection the inefficiency is not removed at any location x.

Proposition 12 *Impossibility Result: Adverse Selection*

(i) In markets with adverse selection, the equilibrium allocation does not coincide with the socially optimal allocation for any degree of competition.

(ii) More strongly, in markets with adverse selection, no degree of imperfect competition removes inefficient underprovision at any x*.*

The combination of fierce competition and a corrective tax can achieve the socially optimal allocation also in markets with adverse selection (i.e. Proposition 9 applies). However, whereas in markets with advantageous selection the tax is positive and raises revenue, under adverse selection the tax is negative, i.e. is a subsidy.

8 Discussion

Our results, which argue in favour of competition, hold despite the fact that we make several assumptions which tend to understate the social benefit of competition.

We assume that the social planner maximises total surplus. Competition is even more beneficial under any alternative welfare measure which places more weight on consumer surplus relative to producer surplus. This arises because while overprovision and underprovision both constitute a reduction in total surplus, their distributional implications differ. When there is overprovision, consumer surplus is actually larger than in the efficient allocation but producer surplus is lower.[40] Hence, fierce competition is desirable from a consumer surplus standpoint even when it leads to overprovision.[41]

We focus purely on the short term strategic effect of competition. However, competition can have the additional benefit of spurring innovation and thus lowering costs. This is particularly relevant in markets with advantageous selection, where high costs, especially administrative costs, are often used to rationalize why overprovision occurs in the first place.[42] E.g. consider an insurance market where a firm's cost of providing insurance consists of the agent's expected claims and an administrative cost. When agents are risk-averse, their WTP exceeds their expected claims. However, overprovision can only occur if for some agents it is not efficient to be insured, i.e. if their WTP is below the firm's cost of providing insurance. In an insurance market with rational risk-averse agents, overprovision can therefore be rationalized by administrative costs (Einav and Finkelstein, 2011).[43] If competition erodes administrative costs, e.g. in the extreme to zero, then the costs firms incur when providing insurance would be below the agent's WTP and therefore perfect competition would result in the efficient allocation, not in overprovision. Thus, competition can remove overprovision also by eroding costs.

In spatial models, changes in competition, as captured by changes in transport costs, have a strategic effect as well as a direct cost effect. Our approach was to isolate the strategic effect by comparing equilibrium allocations to the allocation a social planner can achieve when she faces the same transport cost. An alternative approach would be to consider both effects. In this alternative, competition would be even more beneficial as it also reduces direct costs.

Note that our exposition treated costs as identical for all agents with the same WTP. However, our model is more general. We can allow for cost heterogeneity at a given WTP. Then, $c(y)$ can be interpreted as the average cost at WTP y. Since all agents at a given (x, y) will make the same choice, both approaches yield identical results. While the approach with cost heterogeneity at a given y seems more realistic, we phrased this paper in terms of no cost heterogeneity at a given y. This makes the exposition more concise and avoids confusion between heterogeneity in costs *at* y, which is possible but not central to our model, and heterogeneity in costs *across* y, which is the defining feature of markets with selection and thus central to our model.

[40] As total surplus is lower, it must be the case that the reduction in producer surplus is larger than the increase in consumer surplus.

[41] Industry profits are weakly positive, ensuring that it is optimal for firms to provide goods in this market.

[42] In insurance markets, administrative costs are one possible explanation for why firms charge a loading factor.

[43] In other markets, such as credit markets, overprovision can arise also in the absence of administrative costs. See Appendix B for a discussion of microfoundations for advantageous selection.

9 Conclusion

This paper developed a spatial model of imperfect competition in markets with adverse or advantageous selection and used it to investigate whether imperfect competition exacerbates or ameliorates the inefficiency created by selection. In our model, imperfect competition can never perfectly correct the inefficiency introduced by selection. In markets with adverse selection, imperfect competition exacerbates underprovision. In markets with advantageous selection, imperfect competition can ameliorate overprovision, but surprisingly cannot correct this inefficiency perfectly. While reductions in competition reduce the equilibrium quantity and therefore can reach the efficient quantity, the efficient allocation is never reached. Instead, overprovision and underprovision coexist. When firms price discriminate, overprovision for consumers with weak brand preferences coexists with underprovision for those with strong brand preferences. The reverse holds when firms use uniform prices.

While in our model no degree of imperfect competition can reach the efficient allocation, the efficient allocation can be reached through a corrective tax when there is perfect competition. Moreover, the corrective tax has additional benefits. It generates government revenue, holds regardless of whether firms use uniform prices or price discriminate, and has lower informational requirements.

Our results, which argue in favour of competition, hold despite the fact that we make several assumptions which tend to understate the social benefit of competition. We assume that the social planner maximises total surplus. Competition is even more beneficial under any alternative welfare measure which places more weight on consumer surplus relative to producer surplus. We focus purely on the short term strategic effect of competition. Competition can have the additional benefit of spurring innovation and thus lowering costs. We assume that firms incur the same cost when selling to a given consumer. When firms incur different costs when selling to the same consumer, competition can have the additional benefit of allowing the more efficient firm to win market share from the less efficient firm.

Overall, our results caution against viewing imperfect competition as a solution to inefficiencies introduced by selection. While, in markets with selection, no degree of competition reaches the efficient allocation, the efficient allocation is reached when perfect competition is combined with a corrective tax. This suggests that competition policy should not be lenient towards firms on the ground that they operate in a market with advantageous selection. Instead, competition policy should continue to focus on achieving perfect competition. The distortions arising from selection can be better addressed through a tax than through altered competition policy. Thus, if we view the the build up of credit prior to the Global Financial Crisis as inefficient overprovision in a market with advantageous selection, then we should not conclude that competition in credit markets was excessive prior to the Global Financial Crisis, but rather should conclude that we lacked the appropriate corrective taxation.

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A Proofs

A.1 Proof of Proposition 1

Sketch proof: Any uniform price results in an allocation where consumers buy iff $y > p_L + T(x)$. Thus, the WTP threshold above which consumers buy has slope $T'(x)$. Without selection, the efficient allocation is characterised by a threshold with slope $\frac{dy^S(x)}{dx} = T'(x)$ and since the slopes are equal there does exist a uniform price which implements the efficient allocation ($p_L = \alpha$). With advantageous (adverse) selection, the threshold describing the efficient allocation is flatter (steeper) than transport costs. Thus, the threshold resulting from a uniform price and the threshold describing the efficient allocation never coincide. *Q.E.D.*

Formal proof: First, we establish three Lemmata. Then, we combine them to prove Proposition 1. Throughout the proof, we focus on $x \leq \frac{1}{2}$. $x > \frac{1}{2}$ follows by symmetry.

Lemma A.1 *For any uniform price p_L, consumers purchase the good if and only if their WTP exceeds a threshold $y_{L,U}(x)$ where $\frac{dy_{L,U}(x)}{dx} = T'(x)$.*

Proof: A consumer purchases the good if and only if

$$y > p_L + T(x) \tag{12}$$

By definition

$$y_{L,U}(x) = p_L + T(x) \tag{13}$$

Thus

$$\frac{dy_{L,U}(x)}{dx} = T'(x) \tag{14}$$

Q.E.D.

Lemma A.2 *In markets without selection, there exists a unique efficient allocation. In it, consumers are allocated the good if and only if their WTP exceeds a threshold $y^S(x)$ where $\frac{dy^S(x)}{dx} = T'(x)$.*

Proof: It is efficient to allocate the good to a consumer if and only if

$$y > c(y) + T(x) \tag{15}$$

By definition

$$y^S(x) = c\big(y^S(x)\big) + T(x) \tag{16}$$

In the absence of selection, $c(y) = \alpha \; \forall \; y$. Thus,

$$y^S(x) = \alpha + T(x) \tag{17}$$

Therefore,

$$\frac{dy^S(x)}{dx} = T'(x) \tag{18}$$

Q.E.D.

Lemma A.3 *In markets with advantageous selection, there exists a unique efficient allocation. In it, consumers are allocated the good if and only if their WTP exceeds a threshold $y^S(x)$ where $\frac{dy^S(x)}{dx} < T'(x)$.*

Proof: It is efficient to allocate the good to a consumer if and only if

$$y > c(y) + T(x) \tag{19}$$

By definition

$$y^S(x) = c\big(y^S(x)\big) + T(x) \tag{20}$$

In the presence of advantageous selection $\frac{dc}{dy} < 0$. Thus, totally differentiating and rearranging yields

$$\frac{dy^S(x)}{dx} = \frac{1}{1 - \frac{dc}{dy}}\, T'(x) \tag{21}$$

where $\frac{1}{1-\frac{dc}{dy}} < 1$ due to the presence of advantageous selection. *Q.E.D.*

Lemma A.4 *In markets with adverse selection, there exists a unique efficient allocation. In it, consumers are allocated the good if and only if their WTP exceeds a threshold $y^S(x)$ where $\frac{dy^S(x)}{dx} > T'(x)$.*

Proof: It is efficient to allocate the good to a consumer if and only if

$$y > c(y) + T(x) \tag{22}$$

By definition

$$y^S(x) = c\big(y^S(x)\big) + T(x) \tag{23}$$

In the presence of adverse selection $\frac{dc}{dy} > 0$. Thus, totally differentiating and rearranging yields

$$\frac{dy^S(x)}{dx} = \frac{1}{1 - \frac{dc}{dy}}\, T'(x) \tag{24}$$

where $\frac{1}{1-\frac{dc}{dy}} > 1$ due to the presence of adverse selection. *Q.E.D.*

By Lemma A.1 and A.2, in markets without selection, the equilibrium allocation and the socially optimal allocation are characterised by thresholds with the same gradient. Thus, there exists a uniform price at which also the levels coincide at all x.

By Lemma A.1, A.3, and A.4, in markets with selection, the equilibrium allocation and the socially optimal allocation are characterised by thresholds with different gradients. Thus, if the levels of the allocation coincide at one location x, they diverge at all other locations. Hence, there does not exist a uniform price which can implement the socially optimal allocation.[44] *Q.E.D.*

A.2 Proof of Proposition 2

Part i: To capture monopoly, consider one firm which sells both product L and R.

First, we prove this proposition for the case where firms price discriminate. Then, we prove it for the case of uniform pricing.

a): Price discrimination

Lemma A.5 *When firms can price discriminate, the monopolist's optimal price is*

$$p_L^M(x) = \tfrac{1+\alpha+\beta}{2+\beta} - \tfrac{1+\beta}{2+\beta}tx$$

and it sets $p_R(x)$ such that nobody buys from R.

The resulting allocation is that all consumers with $y > \hat{y}(x)$ buy from L where

$$\hat{y}(x) = \frac{1+\alpha+\beta}{2+\beta} + \frac{1}{2+\beta}tx \tag{25}$$

Proof: The monopolist's problem at a location x is to choose a p to maximise

$$\pi(x) = \int_{p+tx}^{1} [p - \alpha - \beta(1-y)]\, dy \tag{26}$$

which simplifies to:

$$\pi = (-\alpha) - \frac{1}{2}\beta - \big(1 + \frac{1}{2}\beta\big)p^2 + (1+\alpha+\beta)p - (1+\beta)txp + (\alpha+\beta)tx - \frac{1}{2}\beta t^2 x^2 \tag{27}$$

The FOC, $\frac{d\pi}{dp} = 0$ yields

$$p_L^M(x) = \frac{1+\alpha+\beta}{2+\beta} - \frac{1+\beta}{2+\beta}tx \tag{28}$$

which is the unique profit maximum since the SOC holds, i.e. $\frac{d^2\pi}{dp^2} = -(2+\beta) < 0$. *Q.E.D.*

Lemma A.6 *When the monopolist can price discriminate, there is inefficient underprovision at all x regardless of the type and strength of selection.*

Proof: To show:

$$\hat{y}(x) > y^S(x) \;\forall\; x \in \left[0, \frac{1}{2}\right] \tag{29}$$

Which equals

$$\frac{1+\alpha+\beta}{2+\beta} + \frac{1}{2+\beta}tx > \frac{\alpha+\beta}{1+\beta} + \frac{tx}{1+\beta} \tag{30}$$

[44] The socially optimal uniform price as depicted for the case of advantageous selection in Figure 3 achieves the efficient allocation at $x = \frac{1}{4}$, results in inefficient overprovision at all $x \in [0, \frac{1}{4})$ and in inefficient underprovision at all $x \in (\frac{1}{4}, \frac{1}{2}]$.

which simplifies to

$$x < \frac{1-\alpha}{t} \tag{31}$$

This is identical to the assumption that, at every x, allocating the good to some consumer generates strictly positive surplus, i.e. that $t < 2(1-\alpha)$. Hence (31) holds. *Q.E.D.*

b): Uniform pricing

Lemma A.7 *When the monopolist sets a uniform price, its optimal price is*

$$p_L^M = \begin{cases} \frac{1+\alpha+\beta}{2+\beta} - \frac{1+\beta}{4(2+\beta)}t & \text{if} \quad t \leq \frac{4(1-\alpha)}{3+\beta} \\ \frac{1+2\alpha+\beta}{3+\beta} & \text{if} \quad t > \frac{4(1-\alpha)}{3+\beta} \end{cases}$$

Proof: The monopolist solves $\max_{p_L} \pi$ where

$$\pi = \int_0^{n(p_L)} \int_{p_L+T(x)}^1 [p_L - c(y)]\, f(y)\, dy\; g(x)\, dx \tag{32}$$

where $n(p_L)$ is defined as solving $p_L + T(n) = 1$ and must satisfy $0 \leq n \leq \frac{1}{2}$. We consider the case of $n = \frac{1}{2}$ and $n < \frac{1}{2}$ separately.

For $n = \frac{1}{2}$, (32) becomes:

$$\pi = -\Big(\frac{2+\beta}{4}\Big)p_L^2 + \frac{1}{2}(1+\alpha+\beta)p_L - \frac{1+\beta}{8}tp_L - \frac{1}{2}\alpha - \frac{1}{4}\beta + \frac{\alpha+\beta}{8}t - \frac{\beta}{48}t^2 \tag{33}$$

Solving the FOC $\frac{d\pi}{dp_L} = 0$ yields the monopolist's optimal uniform price

$$p_L^M = \frac{1+\alpha+\beta}{2+\beta} - \frac{1+\beta}{4(2+\beta)}\, t \tag{34}$$

This is a profit maximum since it satisfies the SOC. I.e. $\frac{d^2\pi}{dp_L^2} = -(1+\frac{1}{2}\beta)$ and thus $\frac{d^2\pi}{dp_L^2} < 0$ is equivalent to $(-2) < \beta$ which holds for any type and strength of selection. $n = \frac{1}{2}$ is satisfied iff $t \leq \frac{4(1-\alpha)}{3+\beta}$.

For $n < \frac{1}{2}$, i.e. $n = \frac{1-p_L}{t}$, (32) becomes

$$\pi = \frac{1}{t}\Big[-\frac{1}{2}a - \frac{1}{6}\beta + \big(\frac{1}{2} + \frac{1}{6}\beta\big)p_L^3 - \big(1 + \frac{1}{2}\alpha + \frac{1}{2}\beta\big)p_L^2 + \big(\frac{1}{2} + \alpha + \frac{1}{2}\beta\big)p_L\Big] \tag{35}$$

Solving the FOC $\frac{d\pi_L}{dp_L} = 0$ yields

$$p_L^M = \frac{2+\alpha+\beta}{3+\beta} \pm \frac{1-\alpha}{3+\beta} \tag{36}$$

where only $p_L^M = \frac{2+\alpha+\beta}{3+\beta} - \frac{1-\alpha}{3+\beta} = \frac{1+2\alpha+\beta}{3+\beta}$ satisfies the SOC that $\frac{d^2\pi}{dp_L^2} < 0$. $n < \frac{1}{2}$ is satisfied if $t > \frac{4(1-\alpha)}{3+\beta}$. *Q.E.D.*

Lemma A.8 *When the monopolist sets a uniform price, its optimal price results in inefficient underprovision at all locations x. This holds for any type and strength of selection.*

Formally, this means that we want to prove that

$$p_L^M + tx > y^S(x)\ \forall\ x \tag{37}$$

Proof: First, we restrict our attention to $t \leq \frac{4(1-\alpha)}{3+\beta}$. Then, using that $y^S(x) = \frac{\alpha+\beta}{1+\beta} + \frac{1}{1+\beta}tx$, inequality (37) becomes

$$\frac{1+\alpha+\beta}{2+\beta} - \frac{1+\beta}{4\,(2+\beta)}t + tx > \frac{\alpha+\beta}{1+\beta} + \frac{1}{1+\beta}tx \tag{38}$$

which simplifies to

$$1-\alpha+\beta(2+\beta)tx - \frac{1}{4}(1+\beta)^2 t > 0 \tag{39}$$

Consider the case of advantageous selection. Then we know from section 3 that the socially optimal allocation is flatter than the threshold describing the allocation resulting from any uniform price. Hence, if there is overprovision at any x, it would be at $x = 0$. Thus, for the case of advantageous selection, showing that (39) holds at $x = 0$ is sufficient to establish that $p_L^M + tx > y^S(x)\ \forall\ x$ holds when $t \leq \frac{4(1-\alpha)}{3+\beta}$. Focusing on $x = 0$, (39) becomes

$$\frac{4(1-\alpha)}{(1+\beta)^2} > t \tag{40}$$

This condition is true for all $t \leq \frac{4(1-\alpha)}{3+\beta}$ if and only if

$$\frac{4(1-\alpha)}{(1+\beta)^2} > \frac{4(1-\alpha)}{3+\beta} \tag{41}$$

which simplifies to $2 > \beta(1+\beta)$. This always holds since for advantageous selection $0 < \beta < 1$. Thus, when there is advantageous selection $p_L^M + tx > y^S(x)\ \forall\ x$ holds when $t \leq \frac{4(1-\alpha)}{3+\beta}$.

Consider the case of adverse selection. Then we know from section 3 that the socially optimal allocation is steeper than the threshold describing the allocation resulting from any uniform price. Moreover, since we are focusing on the case where $t \leq \frac{4(1-\alpha)}{3+\beta}$, we know that the monopolist provides the good at all locations. Thus, if there is overprovision at any x, it would be at $x = \frac{1}{2}$. This means that for the case of adverse selection, showing that (39) holds at $x = \frac{1}{2}$ is sufficient to establish that $p_L^M + tx > y^S(x)\ \forall\ x$ holds when $t \leq \frac{4(1-\alpha)}{3+\beta}$. Focusing on $x = \frac{1}{2}$, (39) simplifies to

$$4(1-\alpha) > (\beta^2 - 3)t \tag{42}$$

Since adverse selection means that $0 > \beta > (-1)$, we must have $0 < \beta^2 < 1$ and therefore $(\beta^2 - 3) < 0$. Thus, the right hand side of (42) is negative and the left hand side is positive. Hence, inequality (42) holds. Thus, when there is adverse selection $p_L^M + tx > y^S(x)\ \forall\ x$ holds when $t \leq \frac{4(1-\alpha)}{3+\beta}$.

Second, we restrict our attention to $t > \frac{4(1-\alpha)}{3+\beta}$. Then, using that $p_L^M = \frac{1+2\alpha+\beta}{3+\beta}$, inequality

(37) becomes

$$\frac{(1-\alpha)(1-\beta)}{(3+\beta)} + \beta t x > 0 \tag{43}$$

Consider the case of advantageous selection. Then we know from section 3 that the socially optimal allocation is flatter than the threshold describing the allocation resulting from any uniform price. Hence, if there is overprovision at any x, it would be at $x = 0$. Thus, for the case of advantageous selection, showing that (43) holds at $x = 0$ is sufficient to show that $p_L^M + tx > y^S(x)\ \forall\, x$ holds when $t > \frac{4(1-\alpha)}{3+\beta}$. Focusing on $x = 0$, (43) becomes

$$\frac{(1-\alpha)(1-\beta)}{(3+\beta)} > 0 \tag{44}$$

This always holds since all three terms are positive when there is advantageous selection. Thus, when there is advantageous selection, $p_L^M + tx > y^S(x)\ \forall\, x$ holds when $t > \frac{4(1-\alpha)}{3+\beta}$.

Consider the case of adverse selection. Then we know from section 3 that the socially optimal allocation is steeper than the threshold describing the allocation resulting from any uniform price. This means that if there is underprovision at the highest possible x at which gains from trade exist, then there also has to be underprovision at all lower x.

Moreover, since we focus on the case where $t > \frac{4(1-\alpha)}{3+\beta}$, and since our parameter space is limited to $t < 2(1-\alpha)$, we know that the highest possible x at which gains from trade exist is $x = \frac{1}{2}$. Therefore, for the case of adverse selection, showing that (43) holds at $x = \frac{1}{2}$ is sufficient to show that $p_L^M + tx > y^S(x)\ \forall\, x$ holds when $t > \frac{4(1-\alpha)}{3+\beta}$. Focusing on $x = \frac{1}{2}$, (43) simplifies to

$$2\,(1-\alpha)\,\frac{(1-\beta)}{(-\beta)(3+\beta)} > t \tag{45}$$

This holds for all $t \in [\frac{4(1-\alpha)}{3+\beta}, 2(1-\alpha)]$ if

$$2\,(1-\alpha)\,\frac{(1-\beta)}{(-\beta)(3+\beta)} > 2(1-\alpha) \tag{46}$$

which simplifies to $(\beta+1)^2 > 0$ which always holds. Thus, when there is adverse selection, $p_L^M + tx > y^S(x)\ \forall\, x$ holds when $t > \frac{4(1-\alpha)}{3+\beta}$.

While the proof above considers four different cases ($t \leq \frac{4(1-\alpha)}{3+\beta}$ and $t > \frac{4(1-\alpha)}{3+\beta}$, each under advantageous selection and adverse selection), in all cases the result from Lemma A.8 holds. This concludes the proof of Lemma A.8. *Q.E.D.*

Part ii: To capture monopoly, consider high transport costs: $t > 2(1-\alpha)$.

a): Price discrimination

Lemma A.9 *The monopolist chooses inefficient underprovision at all locations where some surplus generating trades exist and chooses to not provide the good at locations where no surplus generating trades exist. This holds for advantageous and for adverse selection.*

There exist gains from trade at an x if and only if $y^S(x) < 1$. Using that $y^S(x) = \frac{\alpha+\beta}{1+\beta} + \frac{1}{1+\beta}tx$, we have that gains from trade exist at a location x if and only if $x < \frac{1-\alpha}{t}$. Thus, to prove Lemma A.9, we need to establish that $\hat{y}(x) > y^S(x) \,\forall\, x \leq \frac{1-\alpha}{t}$ and that $\hat{y}(x) \geq 1 \,\forall\, x > \frac{1-\alpha}{t}$.

Proof: From Lemma A.5, we have that the unique profit maximising price is $p_L^M(x) = \frac{1+\alpha+\beta}{2+\beta} - \frac{1+\beta}{2+\beta} tx$ and that the resulting allocation is characterised by threshold $\hat{y}(x) = \frac{1+\alpha+\beta}{2+\beta} + \frac{1}{2+\beta} tx$.

Thus, there is inefficient underprovision if and only if $\hat{y}(x) > y^S(x)$, which equals

$$\frac{1+\alpha+\beta}{2+\beta} + \frac{1}{2+\beta} tx > \frac{\alpha+\beta}{1+\beta} + \frac{1}{1+\beta} tx \tag{47}$$

which simplifies to $x < \frac{1-\alpha}{t}$. This establishes that $\hat{y}(x) > y^S(x) \,\forall\, x \leq \frac{1-\alpha}{t}$.

Moreover, $\hat{y}(\frac{1-\alpha}{t}) = 1$ and since $\hat{y}(x)$ is increasing, we have that the monopolist does not sell to any consumers at $x > \frac{1-\alpha}{t}$. This concludes the proof. *Q.E.D.*

b): Uniform pricing

Lemma A.10 *The monopolist prices such that there is inefficient underprovision at all locations at which gains from trade exist and such that there is zero provision at all locations at which no gains from trade exist.*

Formally, this means that $p_L^M + tx > y^S(x) \,\forall\, x < \frac{1-\alpha}{t}$ and that $p_L^M + tx \geq 1 \,\forall\, x \geq \frac{1-\alpha}{t}$.

For any $t > \frac{4(1-\alpha)}{3+\beta}$, the monopolist has a unique optimal price which is $p_L^M = \frac{1+2\alpha+\beta}{3+\beta}$. Since $2(1-\alpha) > \frac{4(1-\alpha)}{3+\beta}$, $p_L^M = \frac{1+2\alpha+\beta}{3+\beta}$ is also the monopolists optimal price for the high transport costs that capture monopoly, i.e. for $t > 2(1-\alpha)$.

Proof: Start with the second part of Lemma A.10, i.e. the goal is to show that $p_L^M + tx \geq 1 \,\forall\, x \geq \frac{1-\alpha}{t}$.

Substitute $p_L^M = \frac{1+2\alpha+\beta}{3+\beta}$ into $p_L^M + tx \geq 1$ simplifies to

$$x \geq \frac{2(1-\alpha)}{t\,(3+\beta)} \tag{48}$$

Thus, showing that $\frac{2(1-\alpha)}{t\,(3+\beta)} < \frac{1-\alpha}{t}$ is sufficient to prove that $p_L^M + tx \geq 1 \,\forall\, x \geq \frac{1-\alpha}{t}$. Since $\frac{2(1-\alpha)}{t\,(3+\beta)} < \frac{1-\alpha}{t}$ simplifies to $(-1) < \beta$, which is always true, we have established that $p_L^M + tx \geq 1 \,\forall\, x \geq \frac{1-\alpha}{t}$ holds for advantageous and adverse selection.

Turning to the first part of Lemma A.10, i.e. the goal is to show that $p_L^M + tx > y^S(x) \,\forall\, x < \frac{1-\alpha}{t}$. Substituting $p_L^M = \frac{1+2\alpha+\beta}{3+\beta}$ into $p_L^M + tx > y^S(x)$ simplifies to

$$\frac{(1-\alpha)\,(1-\beta)}{(3+\beta)} > (-\beta)\,t\,x \tag{49}$$

First consider the case of advantageous selection. Then, we know from section 3 that the socially optimal allocation is flatter than the threshold describing the allocation resulting from any uniform price. This means that in the case of advantageous selection, showing that (49) holds at $x = 0$ is sufficient to show that $p_L^M + tx > y^S(x) \,\forall\, x < \frac{1-\alpha}{t}$. Focusing on $x = 0$, equation (49) becomes

$$\frac{(1-\alpha)(1-\beta)}{(3+\beta)} > 0 \tag{50}$$

which is true since $1-\alpha > 0$ and $1-\beta > 0$ and $3+\beta > 0$ are all ensured by our assumptions. Hence, for any degree of advantageous selection, (49) holds. This proves that for advantageous selection $p_L^M + tx > y^S(x) \,\forall\, x < \frac{1-\alpha}{t}$.

Consider the case of adverse selection. Then, we know from section 3 that the socially optimal allocation is steeper than the threshold describing the allocation resulting from any uniform price. Moreover, we have already established that at locations where no gains from trade exist, the monopolist does not provide the good. Thus, if there is overprovision anywhere, it would be at the location x at which gains from trade exist for $x-\epsilon$ but at which no gains from trade exist for $x+\epsilon$ for $\epsilon > 0$ but small. This means that if there is overprovision anywhere, it would be at $x = \frac{1-\alpha}{t}$. This means that for the case of adverse selection, showing that (49) holds at $x = \frac{1-\alpha}{t}$ is sufficient to show that $p_L^M + tx > y^S(x) \,\forall\, x < \frac{1-\alpha}{t}$. Focusing on $x = \frac{1-\alpha}{t}$, equation (49) becomes

$$\frac{(1-\alpha)(1-\beta)}{(3+\beta)} > (-\beta)t\frac{(1-\alpha)}{t} \tag{51}$$

which simplifies to:

$$(\beta+1)^2 > 0 \tag{52}$$

which always holds. Hence, for any degree of adverse selection (49) holds. This proves that for adverse selection $p_L^M + tx > y^S(x) \,\forall\, x < \frac{1-\alpha}{t}$.

Hence, we have established that $p_L^M + tx > y^S(x) \,\forall\, x < \frac{1-\alpha}{t}$ holds for both advantageous and adverse selection and that similarly $p_L^M + tx \geq 1 \,\forall\, x \geq \frac{1-\alpha}{t}$ holds. This concludes the proof.

Q.E.D.

A.3 Proof of Proposition 3

Given our assumptions, we can simplify L's profit function:

$$\pi_L(p_L, p_R) = \int_0^{m(p_L,p_R)} \int_{p_L+T(x)}^1 [p_L - c(y)]\; f(y)\, dy\; g(x)\, dx \tag{53}$$

to:

$$\pi_L(p_L, p_R) = \int_0^{m(p_L,p_R)} \int_{p_L+tx}^1 p_L - \alpha - \beta(1-y)\; dy\; dx \tag{54}$$

and to:

$$\begin{aligned} \pi_L(p_L, p_R) =& \frac{1}{2}\, t\, m(p_L,p_R)^2 \left(\alpha+\beta-(1+\beta)p_L\right) \;-\; \frac{1}{6}\,\beta\, t^2\, m(p_L,p_R)^3 \\ &+ m(p_L,p_R)\left((1+\alpha+\beta)p_L - (1+\frac{\beta}{2})p_L^2 - \alpha - \frac{1}{2}\beta\right) \end{aligned} \tag{55}$$

where $m(p_L, p_R)$ is the consumer who is indifferent between consuming from firm L and firm R.

Thus:

$$m(p_L, p_R) = \frac{1}{2} + \frac{p_R - p_L}{2t} \tag{56}$$

and $\frac{dm}{dp_L} = -\frac{1}{2t}$.

Then, firm L's best response function is given by $\frac{d\pi_L}{dp_L} = 0$, i.e.

$$\begin{aligned}\frac{d\pi_L}{dp_L} =& t\, m(p_L,p_R)\, \frac{dm}{dp_L}\, \big(\alpha+\beta-(1+\beta)p_L\big) \; - \; \frac{1}{2}\, t\, m(p_L,p_R)^2\, (1+\beta) \\ &- \; \frac{1}{2}\, \beta\, t^2 m(p_L,p_R)^2\, \frac{dm}{dp_L} \\ &+ \; \frac{dm}{dp_L}\, \big((1+\alpha+\beta)p_L - (1+\frac{\beta}{2})p_L^2 - \alpha - \frac{1}{2}\beta\big) + m(p_L,p_R)\, \big(1+\alpha+\beta-(2+\beta)p_L\big) = 0\end{aligned} \tag{57}$$

This equation simplifies when using that $\frac{dm}{dp_L} = -\frac{1}{2t}$. Moreover, to solve for symmetric Nash equilibria we can use that $p_L = p_R = p^*$ and thus that $m = \frac{1}{2}$. This simplifies (57) to:

$$p_L = \frac{1+\alpha+\beta}{2+\beta} + \frac{3+\beta}{2\,(2+\beta)}t \pm \frac{1}{2\,(2+\beta)}\sqrt{(2\beta^2+10\beta+13)t^2 - 4(1-\alpha)t + 4(1-\alpha)^2} \tag{58}$$

Thus, there are two candidate solutions for a symmetric Nash equilibrium. These candidate solutions were derived assuming that at every location x some consumers buy. To be valid, these candidate solutions need to be such that at all x someone buys the good. To check this, it is sufficient to check that at $x = \frac{1}{2}$ someone buys:

$$p + T\Big(\frac{1}{2}\Big) < 1 \tag{59}$$

Using the negative square-root solution, this holds for all:

$$t < \frac{4\,(1-\alpha)}{3+\beta} \tag{60}$$

The positive square-root solution never satisfies (59).

Thus, the negative square root solution, i.e. equation (8), is the unique candidate symmetric Nash equilibrium. To establish that it actually is a Nash equilibrium, we check the firm's second order condition. To derive $\frac{d^2\pi_L}{dp_L^2}$, start from a version of $\frac{d\pi_L}{dp_L}$ where we used that $\frac{dm}{dp_L} = -\frac{1}{2t}$, but did not use conditions which only hold in equilibrium i.e. that $p_L = p_R = p^*$ or respectively $m = \frac{1}{2}$:

$$\begin{aligned}\frac{d\pi_L}{dp_L} =& -\frac{1}{2}m(p_L,p_R)\big(\alpha+\beta-(1+\beta)p_L\big) - \frac{1+\beta}{2}tm(p_L,p_R)^2 + \frac{1}{4}\beta t m(p_L,p_R)^2 \\ &- \frac{1}{2t}\big((1+\alpha+\beta)p_L - (1+\frac{\beta}{2})p_L^2 - \alpha - \frac{1}{2}\beta\big) + m(p_L,p_R)\,\big(1+\alpha+\beta-(2+\beta)p_L\big)\end{aligned} \tag{61}$$

From (61) we can calculate the second derivative

$$
\begin{aligned}
\frac{d^2\pi_L}{dp_L^2} =& -\frac{1}{2}\frac{dm}{dp_L}\big(\alpha+\beta-(1+\beta)p_L\big)+\frac{1}{2}m(p_L,p_R)(1+\beta)-(1+\beta)\,tm(p_L,p_R)\,\frac{dm}{dp_L}\\
&+\frac{1}{2}\,\beta\,t\,m(p_L,p_R)\,\frac{dm}{dp_L}\\
&-\frac{1}{2t}\big((1+\alpha+\beta)-(2+\beta)p_L\big)+\frac{dm}{dp_L}\big(1+\alpha+\beta-(2+\beta)p_L\big)-m(p_L,p_R)(2+\beta)
\end{aligned}
\tag{62}
$$

which using that $\frac{dm}{dp_L}=-\frac{1}{2t}$ and that $p_L=p_R=p$ and thus $m=\frac{1}{2}$ simplifies to:

$$
\frac{d^2\pi_L}{dp_L^2}=\frac{1}{4t}(-4-3\alpha-3\beta-2t-\frac{1}{2}\beta t+7p+3\beta p) \tag{63}
$$

The second order condition is that $\frac{d^2\pi_L}{dp_L^2}<0$ which equals

$$
p<\frac{4+3\alpha+3\beta}{7+3\beta}+\frac{2+\frac{1}{2}\beta}{7+3\beta}\,t \tag{64}
$$

which holds at $t=0$ and for all other $t<\frac{4\,(1-\alpha)}{3+\beta}$.

Thus, there exists a unique symmetric Nash equilibrium. In this equilibrium, the price is as given in equation (8):

$$
p^*=\frac{1+\alpha+\beta}{2+\beta}+\frac{3+\beta}{(2+\beta)}\,\frac{t}{2}-\frac{1}{2+\beta}\sqrt{(1-\alpha-\frac{t}{2})^2+\frac{t^2}{2}(\beta+2)(\beta+3)}
$$

Q.E.D.

A.4 Proof of Corollary 1

To show: $0<\frac{dp^*}{d\alpha}$. We first consider the cases of $t<\frac{4(1-\alpha)}{3+\beta}$ and then turn to $t>\frac{4(1-\alpha)}{3+\beta}$

If $t<\frac{4(1-\alpha)}{3+\beta}$, then p^* is given by equation (8) and thus

$$
\frac{dp^*}{d\alpha}=\frac{1}{2+\beta}+\frac{1-\alpha-\frac{t}{2}}{\sqrt{(1-\alpha-\frac{t}{2})^2+\frac{t^2}{2}(\beta+2)(\beta+3)}} \tag{65}
$$

Therefore, $\frac{dp^*}{d\alpha}>0$ simplifies to

$$
(1-\alpha-\frac{t}{2})(2+\beta)>-\sqrt{(1-\alpha-\frac{t}{2})^2+\frac{t^2}{2}(\beta+2)(\beta+3)} \tag{66}
$$

Since $(1-\alpha-\frac{t}{2})>0$ by the assumption that $c(1)+T(\frac{1}{2})\leq 1$, and $(2+\beta)>0$ by $\beta>(-1)$, the left side must be positive. The right side must be negative. Hence, the inequality always holds.

If $t > \frac{4(1-\alpha)}{3+\beta}$, then $p^* = \frac{1+2\alpha+\beta}{3+\beta}$ and thus

$$\frac{dp^*}{d\alpha} = \frac{2}{3+\beta} \tag{67}$$

Therefore, $\frac{dp^*}{d\alpha} > 0$ simplifies to $2 > 0$ which is true. *Q.E.D.*

A.5 Proof of Proposition 4

Part i: *There exists a unique level of transport costs, denoted $\tilde{t}$, at which the equilibrium price $p^*(t)$ and the socially optimal* price $p_L^S(t)$ *coincide, i.e.* $p_L^S(\tilde{t}) = p^*(\tilde{t})$ *where* $\tilde{t} > 0$.

Lemma A.11 $p_L^S(0) > p^*(0)$.

Proof: $p_L^S(0) = \frac{\alpha+\beta}{1+\beta}$; $p^*(0) = \frac{2\alpha+\beta}{2+\beta}$. Thus, $p_L^S(0) > p^*(0)$ is equivalent to

$$(1-\alpha)\,\beta > 0 \tag{68}$$

which is true under advantageous selection ($\beta > 0$) since by the assumption that $c(1) + T(\frac{1}{2}) \leq 1$ we must have $\alpha < 1$. *Q.E.D.*

Lemma A.12 $p_L^S(0) < p^*(\frac{4(1-\alpha)}{3+\beta})$.

Proof:

$$p^*(\frac{4(1-\alpha)}{3+\beta}) = \frac{1+\alpha+\beta}{2+\beta} + \frac{2\,(1-\alpha)}{2+\beta} - \frac{(1-\alpha)}{(2+\beta)\,(3+\beta)}(3\beta+7) \tag{69}$$

Thus, $p_L^S(0) < p^*(\frac{4(1-\alpha)}{3+\beta})$ simplifies to

$$(\beta+2)\,(1-\beta) > 0 \tag{70}$$

which holds. *Q.E.D.*

Lemma A.13 $p_L^S(t)$ *is decreasing in* t.

Proof: This statement follows directly from

$$p_L^S(t) = \frac{\alpha+\beta}{1+\beta} - \frac{\beta}{4\,(1+\beta)}\,t \tag{71}$$

Q.E.D.

Lemma A.13 implies that $p_L^S(\frac{4(1-\alpha)}{3+\beta}) < p_L^S(0)$. Thus, combined with Lemma A.12, we have that $p_L^S(\frac{4(1-\alpha)}{3+\beta}) < p^*(\frac{4(1-\alpha)}{3+\beta})$.

Lemma A.14 $p^*(t)$ *is strictly concave.*

Proof: Since

$$p^* = \frac{1+\alpha+\beta}{2+\beta} + \frac{3+\beta}{2\,(2+\beta)}t - \frac{1}{2\,(2+\beta)}\sqrt{(2\beta^2+10\beta+13)t^2 - 4(1-\alpha)t + 4(1-\alpha)^2}$$

we can calculate that

$$\frac{dp^*}{dt} = \frac{3+\beta}{2\,(2+\beta)} + \frac{2(1-\alpha)\ -\ (2\beta^2+10\beta+13)\,t}{2\,(2+\beta)\,\sqrt{(2\beta^2+10\beta+13)t^2 - 4(1-\alpha)t + 4(1-\alpha)^2}} \tag{72}$$

and defining $\phi = (2\beta^2+10\beta+13)t^2 - 4(1-\alpha)t + 4(1-\alpha)^2$ we have that

$$\frac{d^2p^*}{dt^2} = \frac{-(2\beta^2+10\beta+13)\,\phi + \Big[(2\beta^2+10\beta+13)\,t - 2\,(1-\alpha)\Big]^2}{2\,(2+\beta)\,\phi^{\frac{3}{2}}} \tag{73}$$

Then $\frac{d^2p^*}{dt^2} < 0$ is equivalent to

$$\Big[(2\beta^2+10\beta+13)t - 2(1-\alpha)\Big]^2 < (2\beta^2+10\beta+13)\,\phi \tag{74}$$

or equally

$$0 < 2\beta^2+10\beta+13 \tag{75}$$

which holds for any $\beta \geq 0$. *Q.E.D.*

Proof of Proposition 4, Part i: Lemma A.11 - A.14 prove that there exists a unique crossing of $p_L^S(t)$ and $p^*(t)$. Figure 5 provides the visual intuition. *Q.E.D.*

Part ii: *The equilibrium allocation and the socially optimal allocation do not coincide for any level of transport costs t.*

Proof of Proposition 4, Part ii: This follows from proposition 1. In markets with advantageous selection, no uniform price can implement the efficient allocation. Even $p_L^S(t)$ only results in the constrained efficient allocation. Thus, at $\tilde{t}$, we have $p^*(\tilde{t}) = p_L^S(\tilde{t})$, i.e. the equilibrium results in the constrained efficient allocation but by proposition 1 it does not result in the first-best efficient allocation. *Q.E.D.*

A.6 Proof of Proposition 5

The proof proceeds in steps. First, we characterise the price firm L would charge if firm R did not exist and term this monopolist pricing. Second, we characterise the locations x at which firm R can profitably undercut L's monopolist prices. Then, we show how firm L responds and that the resulting equilibrium behaviour is as described in proposition 5.

Step 1: Firm L's pricing if firm R does not exist.

At a location x, firm L's profit is

$$\pi_L(x) = \int_{p_L(x)+T(x)}^{1} [p_L(x) - c(y)]\; f(y)\; dy \tag{76}$$

$$\pi_L(x) = p_L - p_L^2 - p_L T(x) - \left(\alpha + \frac{\beta}{2} - (\alpha+\beta)p_L - (\alpha+\beta)T(x) + \frac{\beta}{2}p_L^2 + \beta p_L T(x) + \frac{\beta}{2}T(x)^2\right) \tag{77}$$

which by solving $\frac{d\pi_L}{dp_L} = 0$ yields

$$p_L^M(x) = \frac{1+\alpha+\beta}{2+\beta} - \frac{1+\beta}{2+\beta}\,T(x) \tag{78}$$

which is the unique profit maximum since $\frac{d^2\pi_L}{dp_L^2} = -(2+\beta) < 0$.

Step 2: Locations at which firm R can profitably undercut the monopolist price $p_L^M(x)$

We introduce two auxiliary concepts. $\hat{p}_R(x)$ is the maximum undercutting price of firm R, i.e. the highest price firm R can set and still attract consumers given that firm L prices like a monopolist. Formally,

$$\hat{p}_R(x) = p_L^M(x) + T(x) - T(1-x) \tag{79}$$

Using (78) this becomes

$$\hat{p}_R(x) = \frac{1+\alpha+\beta}{2+\beta} + \frac{1}{2+\beta}T(x) - T(1-x) \tag{80}$$

As second auxiliary concept, $p_R^B(x)$ is the price firm R needs to charge to make zero profit, i.e. break even, given that firm L does not sell at that location x. This is the lowest possible price firm R will ever offer. Solving the corresponding zero-profit condition $\pi_R(p_R^B(x), x) = 0$ results in

$$p_R^B(x) = \frac{2\alpha+\beta}{2+\beta} - \frac{\beta}{2+\beta}\,T(1-x) \tag{81}$$

Firm R undercuts $p_L^M(x)$ if and only if[45]

$$\hat{p}_R(x) \geq p_R^B(x) \tag{82}$$

which simplifies to

$$1 - \alpha + T(x) \;\geq\; 2\,T(1-x) \tag{83}$$

Since, under the assumptions made, profit functions are well behaved, there exists a unique threshold x_L defined by

$$1 - \alpha + T(x_L) \;=\; 2\,T(1-x_L) \tag{84}$$

For all $x \leq x_L$ firm R does not undercut while for $x > x_L$ firm R does undercut.

Thus, $x_L = 0$ iff $1 - \alpha \geq 2T(1)$, i.e. transport cost are low, and otherwise x_L is the solution to $1 - \alpha + T(x_L) \;=\; 2\,T(1-x_L)$.

[45] If R undercuts, R uses the highest possible price which attracts consumers.

Step 3: Firm L's response to being undercut.

At all $x \in [x_L, \frac{1}{2}]$, firm L will respond to R undercutting its price by in turn undercutting R. Firms continue to undercut each other until a price is reached at which firm R no longer finds it profitable to undercut. Since for $x < \frac{1}{2}$, $T(x) < T(1-x)$, it follows that at all $x \in [x_L, \frac{1}{2}]$ the equilibrium price is:

$$p_L(x) = p_R^B(x) + T(1-x) - T(x) \tag{85}$$

equally

$$p_L(x) = \frac{2\alpha + \beta}{2+\beta} + \frac{2}{2+\beta}T(1-x) - T(x) \tag{86}$$

This proves the existence and uniqueness of the equilibrium described in Proposition 5. *Q.E.D.*

A.7 Proof of Corollary 2

Part i: *The equilibrium price is monotonically decreasing in distance from the firm.*

Proof of Part i:

When $x \in [0, x_L]$: $\frac{dp_L^M}{dx} = -\frac{1+\beta}{2+\beta}\frac{dT(x)}{dd} < 0 \; \forall x$ since $\frac{dT(x)}{dd} > 0$, and for advantageous selection $\beta > 0$, or for adverse selection $0 < \beta < (-1)$.

When $x \in (x_L, \frac{1}{2}]$: $\frac{dp_L}{dx} = -\frac{4+\beta}{2+\beta}\, t < 0 \; \forall \; x$. *Q.E.D.*

Part ii: *The number of consumers purchasing the good can be non-monotone in distance from the firm.*

Proof of Part ii: We prove this by focusing on $p(x) + T(x)$, i.e. the total cost a consumer faces. The total cost and the number of consumers buying are inversely related.

When $x \in [0, x_L]$, the total cost consumers face is:

$$p_L^M(x) + T(x) = \frac{1+\alpha+\beta}{2+\beta} - \frac{1+\beta}{2+\beta}T(x) + T(x) \tag{87}$$

$$= \frac{1+\alpha+\beta}{2+\beta} + \frac{1}{2+\beta}T(x) \tag{88}$$

Thus, defining total cost consumers face as $\bar{p} = p_L^M(x) + T(x)$:

$$\frac{d\bar{p}}{dx} = \frac{1}{2+\beta}\,\frac{dT}{dd} > 0 \; \forall \; x \tag{89}$$

When $x \in (x_L, \frac{1}{2}]$, the total cost consumers face is:

$$\bar{p} = p_L(x) + T(x) = \frac{2\alpha+\beta}{2+\beta} + \frac{2}{2+\beta}\,T(1-x) \tag{90}$$

Thus,

$$\frac{d\bar{p}}{dx} = \frac{2}{2+\beta}\Big(-\frac{dT}{dd}\Big) < 0 \; \forall \; x \tag{91}$$

Since for $x \in [0, x_L]$, $\frac{d\bar{p}}{dx} > 0$ and for $x \in (x_L, \frac{1}{2}]$, $\frac{d\bar{p}}{dx} < 0$, it follows that $\bar{p}$ is non-monotone in x when $x_L > 0$ and thus the number of consumers purchasing is non-monotone. We know from Proposition 5 that for $T(1) > \frac{1-\alpha}{2}$, $x_L > 0$ and then the non-monotonicity arises. If $T(1) \leq \frac{1-\alpha}{2}$, then $x_L = 0$ and the non-monotonicity does not arise. This proves that the non-monotonicity is a possibility, not a certainty, and therefore proves Corollary 2. *Q.E.D.*

A.8 Proof of Proposition 6

Part 1: *In the absence of selection, the equilibrium exhibits inefficient underprovision at all* $x < \frac{1}{2}$.

Part 2: *In the absence of selection, the equilibrium achieves the efficient allocation at* $x = \frac{1}{2}$.

In the no selection case, denote costs as $c(y) = \alpha \; \forall \; y$. For $x \in [0, x_L]$, the equilibrium price is $p_L^M(x) = \frac{1+\alpha-T(x)}{2}$ where x_L is the solution to $T(x_L) = 2T(1-x_L) - 1 + \alpha$. This applies provided that $T(1) > \frac{1-\alpha}{2}$. If $T(1) < \frac{1-\alpha}{2}$, then $x_L = 0$. For $x \in [x_L, \frac{1}{2}]$, $p_L(x) = \alpha + T(1-x) - T(x)$.

Proof of Part 2: To show: $p_L(x) = \alpha$ at $x = \frac{1}{2} \; \forall \; T(d)$.

Substituting $x = \frac{1}{2}$ into $p_L(x) = \alpha + T(1-x) + T(x)$ yields:

$$p_L(x) = \alpha + T(\frac{1}{2}) - T(\frac{1}{2}) = \alpha \tag{92}$$

Q.E.D.

Proof of Part 1: To show: $p_L^M(x) > \alpha \; \forall \; x \leq x_L$ and $p_L(x) > \alpha \; \forall \; x \in (x_L, \frac{1}{2})$.

$$p_L^M(x) = \frac{1+\alpha-T(x)}{2} > \alpha \tag{93}$$

solves to $1 > \alpha + T(x)$, which holds for all x by the the assumption that $c(1) + T(\frac{1}{2}) \leq 1$.

$$p_L(x) = \alpha + T(1-x) - T(x) > \alpha \tag{94}$$

solves to $T(1-x) > T(x)$ which is true for all $x < \frac{1}{2}$. *Q.E.D.*

A.9 Proof of Proposition 7

In the efficient allocation, there is a WTP threshold $y^S(x)$ above which all consumers are allocated the good. The threshold is characterised by

$$y^S(x) = T(x) + c\big(y^S(x)\big) \tag{95}$$

which solves to

$$y^S(x) = \frac{\alpha+\beta}{1+\beta} + \frac{1}{1+\beta}\,T(x) \tag{96}$$

There is inefficient overprovision at a location x if and only if

$$p_L(x) + T(x) < y^S(x) \tag{97}$$

where we can focus on prices for $x > x_L$ since $x < x_L$ results in monopoly pricing which always means inefficient underprovision. Thus, (97) becomes

$$\frac{2\alpha + \beta}{2 + \beta} + \frac{2}{2 + \beta}\, T(1 - x) < \frac{\alpha + \beta}{1 + \beta} + \frac{1}{1 + \beta}\, T(x) \tag{98}$$

which simplifies to

$$\beta\, (1 - \alpha) > (2 + 2\beta)\, T(1 - x) - (2 + \beta)\, T(x) \tag{99}$$

For larger x, the left hand side of this inequality is constant while the right hand side decreases. Thus, there exists at most one crossing. Hence, there are two cases. If $T(1) < \frac{\beta}{(2+2\beta)}(1 - \alpha)$, then there is inefficient overprovision at $x = 0$ and thus at all x. If $T(1) > \frac{\beta}{(2+2\beta)}(1 - \alpha)$, then there exists a unique x_e defined by solving

$$\beta(1 - \alpha) = (2 + 2\beta)\, T(1 - x_e) - (2 + \beta)T(x_e) \tag{100}$$

such that for all $x < x_e$ there is inefficient underprovision and for all $x > x_e$ there is inefficient overprovision. *Q.E.D.*

A.10 Proof of Proposition 8

We prove Proposition 8 by showing that, in markets with advantageous selection, there is always inefficient overprovision at $x = \frac{1}{2}$, i.e. for all levels of transport costs.

The efficient allocation is characterised by the WTP threshold $y^S(x)$ which solves

$$y^S(x) = T(x) + c\big(y^S(x)\big) \tag{101}$$

which simplifies to

$$y^S(x) = \frac{\alpha + \beta}{1 + \beta} + \frac{1}{1 + \beta} T(x) \tag{102}$$

In the equilibrium, the WTP threshold above which consumers buy is $\bar{p}(x) = p(x) + T(x)$. Consumers at $x = \frac{1}{2}$ face $p_L(x)$, not $p_L^M(x)$. Thus, at $x = \frac{1}{2}$ we have

$$\bar{p}\Big(\frac{1}{2}\Big) = p_L\Big(\frac{1}{2}\Big) + T\Big(\frac{1}{2}\Big) \tag{103}$$

$$\bar{p}\Big(\frac{1}{2}\Big) = \frac{2\alpha + \beta}{2 + \beta} + \frac{2}{2 + \beta}\, T\Big(\frac{1}{2}\Big) \tag{104}$$

There is inefficient overprovision if and only if

$$\bar{p}\Big(\frac{1}{2}\Big) \; < y^S\Big(\frac{1}{2}\Big) \tag{105}$$

$$\frac{2\alpha+\beta}{2+\beta} + \frac{2}{2+\beta}\, T\Big(\frac{1}{2}\Big) < \frac{\alpha+\beta}{1+\beta} + \frac{1}{1+\beta}\, T\Big(\frac{1}{2}\Big) \tag{106}$$

which simplifies to

$$\beta\,\Big[1-\alpha-T\Big(\frac{1}{2}\Big)\Big] > 0 \tag{107}$$

which holds in all markets with advantageous selection ($\beta > 0$) since the assumption that $c(1)+T(\frac{1}{2}) \leq 1$ can be restated as $1-\alpha-T\big(\frac{1}{2}\big) > 0$. Thus, provided there is any, possibly very small, degree of advantageous selection ($\beta > 0$), there is inefficient overprovision at $x = \frac{1}{2}$. This overprovision is not removed by any level of transport costs. *Q.E.D.*

A.11 Proof of Proposition 9

We know that, in markets with advantageous (adverse) selection, there is inefficient overprovision (underprovision) if $t = 0$, i.e. $p^S(0) > p^*(0)$. This holds when firms use uniform prices (Proposition 4) and when they price discriminate (Proposition 7). For both pricing strategies, the socially optimal allocation and the equilibrium allocation are characterised by a threshold WTP above which consumers are allocated the good and the threshold is independent of x, i.e. is perfectly horizontal. The level of the threshold differs between the socially optimal allocation and the equilibrium.

Since taxes shift the equilibrium threshold without changing its slope (analogous to cost pass through described in Corollary 1), there exists a tax rate for which the equilibrium allocation and the socially optimal allocation coincide. *Q.E.D.*

Example: Consider a market with advantageous selection, i.e. let $c(y) = \alpha + \beta(1-y)$ with $\beta > 0$. Then, there is inefficient overprovision under perfect competition, i.e.

$$p^S(0) = \frac{\alpha+\beta}{1+\beta} > p^*(0) = \frac{2\,\alpha+\beta}{2+\beta}$$

Since a per unit tax (τ) results in equilibrium prices $p^*(0) = \frac{2\,(\alpha+\tau)+\beta}{2+\beta}$, we can solve for the tax rate which achieves the first-best efficient allocation:[46]

$$\frac{2(\alpha+\tau)+\beta}{2+\beta} = \frac{\alpha+\beta}{1+\beta} \tag{108}$$

[46] More generally, for any transport cost t, there exists a tax rate or subsidy rate which achieves the constrained efficient allocation, i.e. for which $p^S(t) = p^*(t)$. This tax rate is implicitly defined by $p^S(\alpha,\beta,t) = p^*(\alpha+\tau,\beta,t)$ or equally by $\frac{\alpha+\beta}{1+\beta} - \frac{\beta}{4\,(1+\beta)}t = \frac{1+\alpha+\tau+\beta}{2+\beta} + \frac{3+\beta}{2(2+\beta)}t - \frac{1}{2(2+\beta)}\sqrt{(2\beta^2+10\beta+13)t^2 - 4(1-\alpha-\tau)t + 4(1-\alpha-\tau)^2}$. However, the first-best efficient allocation is only achieved under perfect competition ($t = 0$) combined with tax rate (11). Moreover, all cases with $t > 0$ are further complicated by considerations whether firms use uniform prices or price discrimination and by whether the tax rate has to apply equally to all x or can discriminate based on x.

which solves to

$$\tau^* = \frac{\beta\,(1-\alpha)}{2\,(1+\beta)}$$

A.12 Proof of Corollary 3

We established in Proposition 7 that inefficient overprovision occurs at all $x \in [x_e, \frac{1}{2}]$ where x_e is the solution to $\beta(1-\alpha) = (2+2\beta)\,T(1-x_e) - (2+\beta)\,T(x_e)$ provided that $T(1) \geq \frac{\beta}{2+2\beta}(1-\alpha)$, and otherwise $x_e = 0$.

Lemma A.15 $\frac{dx_e}{d\alpha} > 0$.

Proof:

$$\beta(1-\alpha) = (2+2\beta)\,T(1-x_e) - (2+\beta)\,T(x_e) \tag{109}$$

totally differentiate

$$-\beta\,d\alpha = (2+2\beta)\,\frac{dT(1-x_e)}{dd}\,(-dx_e) - (2+\beta)\,\frac{dT(x_e)}{dd} dx_e \tag{110}$$

which solves to

$$\frac{dx_e}{d\alpha} = \frac{\beta}{(2+2\beta)\frac{dT(1-x_e)}{dd} + (2+\beta)\frac{dT(x_e)}{dd}} > 0\;\forall\,x \tag{111}$$

Q.E.D.

Lemma A.16 $\frac{dx_e}{d\beta} < 0$.

Proof:

$$\beta(1-\alpha) = (2+2\beta)\,T(1-x_e) - (2+\beta)\,T(x_e) \tag{112}$$

totally differentiate

$$(1-\alpha)\,d\beta = (2+2\beta)\,\frac{dT(1-x_e)}{dd}\,(-dx_e) + 2\,T(1-x_e)\,d\beta - (2+\beta)\,\frac{dT(x_e)}{dd}\,dx_e - T(x_e)\,d\beta \tag{113}$$

which solves to

$$\frac{dx_e}{d\beta} = -\frac{1-\alpha+T(x_e)-2T(1-x_e)}{(2+2\beta)\frac{dT(1-x_e)}{dd} + (2+\beta)\frac{dT(x_e)}{dd}} \tag{114}$$

where $(2+2\beta)\frac{dT(1-x_e)}{dd} + (2+\beta)\frac{dT(x_e)}{dd} > 0\;\forall\,x$ since $\frac{dT}{dd} > 0$.

We now show that $1-\alpha+T(x_e)-2T(1-x_e) > 0$, which then also proves that $\frac{dx_e}{d\beta} < 0\;\forall\,x$. To show that $1-\alpha+T(x_e)-2T(1-x_e) > 0$, rewrite it as:

$$1-\alpha+T(x_e) > 2T(1-x_e) \tag{115}$$

Recall that x_L is the border between monopoly pricing and effective competition and that overprovision can only arise under effective competition, never under monopoly. Thus, we must have $x_e > x_L$. Recall that x_L was defined by

$$1 - \alpha + T(x_L) = 2T(1 - x_L)$$

Since $1 - \alpha + T(x)$ is increasing in x and $2T(1-x)$ is decreasing in x, for all $x > x_L$ we must have that

$$1 - \alpha + T(x) > 2\,T(1 - x) \tag{116}$$

and since $x_e > x_L$, we must have

$$1 - \alpha + T(x_e) > 2\,T(1 - x_e) \tag{117}$$

Thus, we have shown that $\frac{dx_e}{d\beta} < 0 \;\forall\, x$. *Q.E.D.*

Since $\frac{dx_e}{d\beta}$ captures both a pure cost effect and a selection effect and $\frac{dx_e}{d\alpha}$ captures only a pure cost effect, we interpret the results that $\frac{dx_e}{d\alpha} > 0$ but $\frac{dx_e}{d\beta} < 0$ as indicating that the selection effect has the opposite sign of the pure cost effect and more than outweighs it. *Q.E.D.*

A.13 Proof of Proposition 10

Proof of Part (i): We prove a stronger statement:

Lemma A.17 *The equilibrium uniform price is strictly higher than the socially optimal uniform price.*

Proof: We consider the two regimes $t \leq \frac{4(1-\alpha)}{3+\beta}$ and $t > \frac{4(1-\alpha)}{3+\beta}$ in turn. For each, we show that $p^S < p^*$.

For $t \leq \frac{4(1-\alpha)}{3+\beta}$, $p^S < p^*$ becomes:

$$\frac{\alpha+\beta}{1+\beta} - \frac{\beta}{4(1+\beta)}t < \frac{1+\alpha+\beta}{2+\beta} + \frac{3+\beta}{2+\beta}\frac{t}{2} - \frac{1}{2+\beta}\sqrt{(1-\alpha-\frac{t}{2})^2 + \frac{t^2}{2}(\beta+2)(\beta+3)} \tag{118}$$

which simplifies to

$$0 < (-\beta)\,(2+\beta)\,(1-\alpha-\frac{t}{2})^2 + \frac{t}{2}(3\beta^2+10\beta+8)(1-\alpha-\frac{t}{2}) + \frac{t^2}{16}(\beta+2)\left[\beta^2(\beta+2)+8(\beta+1)\right] \tag{119}$$

which holds sind all terms on the right are positive.

For $t > \frac{4(1-\alpha)}{3+\beta}$, $p^S < p^*$ becomes:

$$\frac{\alpha+\beta}{1+\beta} - \frac{\beta}{4(1+\beta)}t < \frac{1+2\alpha+\beta}{3+\beta} \tag{120}$$

which simplifies to

$$(1-\alpha) > \frac{\beta(3+\beta)}{\beta-1}\,\frac{t}{4} \tag{121}$$

which holds since $(1-\alpha) > 0$ and the right side is negative since $(\beta - 1) < 0$ while the other terms on the right are positive. *Q.E.D.*

Proof of Part (ii): We prove a stronger statement:

Lemma A.18 *The equilibrium uniform price results in inefficient underprovision at all x.*

Proof: Recall that the socially optimal allocation is defined by a threshold $y^S(x) = \frac{\alpha+\beta}{1+\beta} + \frac{tx}{1+\beta}$.

For $t < \frac{4(1-\alpha)}{3+\beta}$, the equilibrium uniform price results in an allocation described by threshold $y_U^*(x)$

$$y_U^*(x) = \frac{1+\alpha+\beta}{2+\beta} + \frac{3+\beta}{2+\beta}\frac{t}{2} + tx - \frac{1}{2+\beta}\sqrt{(1-\alpha-\frac{t}{2})^2 + \frac{t^2}{2}(\beta+2)(\beta+3)} \tag{122}$$

The goal is to show that $y_U^*(x) - y^S(x) > 0 \,\forall\, x \leq \frac{1}{2}$. This simplifies to

$$(-\beta)(2+\beta)(1-\alpha-\frac{t}{2})^2 + t(\beta+2)(2x\beta+\beta+2)(1-\alpha-\frac{t}{2}) + \frac{t^2}{4}(\beta+2)\left[(\beta+2)(2x\beta+\beta+2)^2 - 2(1+\beta)^2(\beta+3)\right] > 0 \tag{123}$$

All terms are positive. To show that $\left[(\beta+2)(2x\beta+\beta+2)^2 - 2(1+\beta)^2(\beta+3)\right]$ is positive it is sufficient to show that the term is positive for $x = \frac{1}{2}$. Substituting in $x = \frac{1}{2}$ simplifies the term to $2(1+\beta)^3$ which is positive since $0 > \beta > (-1)$.

For $t > \frac{4(1-\alpha)}{3+\beta}$ the equilibrium price coincides with the monopolist's optimal price. The monopolist's optimal price results in inefficient overprovision (see Proposition 2). *Q.E.D.*

A.14 Proof of Proposition 11

Denoting the threshold characterising the socially optimal allocation as $y^S(x)$ and the threshold characterising the equilibrium allocation in this case with price discrimination as $y_{PD}^*(x)$, the goal is to show that $y_{PD}^*(x) > y^S(x) \,\forall\, x$.

For $x \in [0, x_L]$, we have $y_{PD}^*(x) = \frac{1+\alpha+\beta}{2+\beta} + \frac{1}{2+\beta}tx$. Thus, $y_{PD}^*(x) > y^S(x)$ becomes

$$\frac{1+\alpha+\beta}{2+\beta} + \frac{1}{2+\beta}tx > \frac{\alpha+\beta}{1+\beta} + \frac{1}{1+\beta}tx \tag{124}$$

which simplifies to $tx < (1-\alpha)$. It is sufficient to show that this inequality holds at $x = \frac{1}{2}$, and $t < 2(1-\alpha)$ indeed holds by the assumption that $c(1) + T(\frac{1}{2}) \leq 1$. Thus, there always exists inefficient underprovision for all $x \in [0, x_L]$.

For $x \in [x_L, \frac{1}{2}]$, we have $y_{PD}^*(x) = \frac{2\alpha+\beta}{2+\beta} + \frac{2}{2+\beta}t - \frac{2}{2+\beta}tx$. Thus, $y_{PD}^*(x) > y^S(x)$ becomes

$$\frac{2\alpha+\beta}{2+\beta} + \frac{2}{2+\beta}t - \frac{2}{2+\beta}tx > \frac{\alpha+\beta}{1+\beta} + \frac{1}{1+\beta}tx \tag{125}$$

which simplifies to

$$(1-2x)2t(1+\beta) > \beta(1-\alpha-tx) \tag{126}$$

Since $(1-2x) > 0$ and $(1+\beta) > 0$, the left side is positive. As $\beta < 0$ and $(1-\alpha-tx) > 0$, the right side is negative. Thus, the inequality always holds.

Thus, there is inefficient underprovision at both $x \in [0, x_L]$ and $x \in [x_L, \frac{1}{2}]$. *Q.E.D.*

A.15 Proof of Proposition 12

Proof: Proposition 11 established that, in equilibrium, there is inefficient underprovision at all x. Since this result holds for any degree of imperfect competition, i.e. any $t > 0$, Proposition 12 (i) holds. Since the result applies at all x for any t, Proposition 12 (ii) holds. *Q.E.D.*

B Microfoundations for Types of Selection and Perfect Competition

B.1 Microfoundations for Selection in Credit Markets

Entrepreneurs seek finance for a project. They have limited liability and raise finance via debt contracts.

Entrepreneurs have private information on the distribution of project returns. The precise assumptions on how the distribution of returns differs across projects determines whether the credit market exhibits adverse or advantageous selection. We consider them in turn.

B.1.1 Assumptions as in Stiglitz and Weiss (1981)

Stiglitz and Weiss (1981) assume that projects differ in their riskiness and that lenders (or banks) cannot distinguish between them. Formally, the distribution of project returns for a riskier project is a mean preserving spread of the distribution of project returns of a safer project. This means that all projects have the same expected return but differ in the dispersion of returns.

For expositional clarity consider the case of binary outcomes, i.e. projects fail or succeed. Normalise the return given failure to zero for all projects. Then the assumption that the returns of a riskier project are a mean preserving spread of the returns of a safer project means that a riskier project has a higher pay-off given success than a safer project, but has a lower probability of success.

Stiglitz and Weiss (1981) establish that under these assumptions there is adverse selection. Entrepreneurs with high risk projects have a higher willingness-to-pay (WTP) for credit (because of limited liability) than entrepreneurs with safer projects. Moreover, lending to riskier projects has lower expected return for the bank. Thus, the borrowers who are willing to borrow at high interest rates are the borrowers who, all else equal, the bank does not like lending to. This means that there is adverse selection.

Our model can be interpreted as capturing such a credit market. Entrepreneurs with risky projects have a high WTP (in our model: high y) and are the agents the bank does not want to lend to, all else equal, as lending to them has a low expected return for the bank. This is analogous to firms having a higher expected cost from selling to agents with a higher WTP, as is the case in our model.

Stiglitz and Weiss (1981) show that in credit markets with adverse selection, prefect competition can result in inefficient underprovision. Stiglitz and Weiss (1981) also discuss conditions under which credit rationing arises, i.e. in equilibrium the market does not clear and there is excess demand for credit. However, De Meza and Webb (1987) show that the result of inefficient underprovision does not depend on whether the equilibrium is market clearing or not, but rather depends on the assumption of project returns.

B.1.2 Assumptions as in De Meza and Webb (1987)

De Meza and Webb (1987) assume that projects differ in their quality and that lenders (or banks) cannot distinguish them. Formally, the distribution of project returns of a better project first-order stochastic dominates the distribution of project returns of a worse project. This means that projects differ in their expected return.

As in Appendix B.1.1, consider the case of binary outcomes, i.e. projects fail or succeed. Let all projects have the same outcome given success and the same outcome given failure. Then, a better project (in the sense that its return distribution first-order stochastic dominates the return distribution of a worse project) is a project with a higher probability of success.

De Meza and Webb (1987) establish that under these assumptions there is advantageous selection. Entrepreneurs with high quality projects have a higher WTP for credit. Moreover, lending to high quality projects has a higher expected return for the bank than lending to low quality projects. Thus, the borrowers who are willing to borrow at high interest rates are the borrowers which, all else equal, the bank likes lending to. This means that there is advantageous selection.

Our model can be interpreted as capturing such a credit market. Entrepreneurs with high quality projects have a high WTP (in our model: high y) and are the agents the banks wants to lend to, all else equal, as lending to them has a high expected return for the bank. This is analogous to firms having a lower cost from selling to agents with a high WTP, as is the case in our model.

De Meza and Webb (1987) show that, in credit markets with advantageous selection, perfect competition can result in inefficient overprovision. As in our model, this arises because firms compete for entrepreneurs with high quality projects by lower the interest rate which in turn draws entrepreneurs with lower quality projects into the market.

B.2 Microfoundations for Selection in Insurance Markets

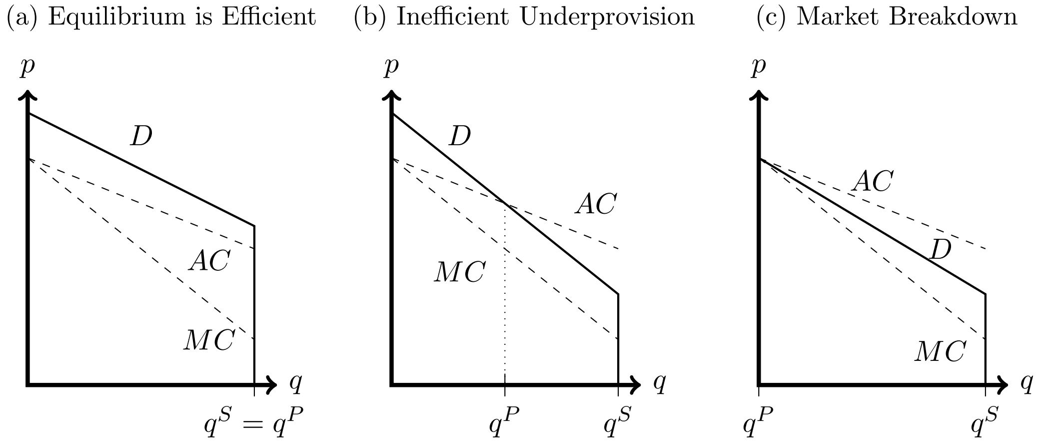
This Appendix provides microfoundations for adverse and advantageous selection in insurance markets, characterises the equilibrium under perfect competition, and discusses the efficiency properties of the perfectly competitive equilibrium. An excellent survey is Einav and Finkelstein (2011). The pictorial exposition in this appendix is borrowed from them, while the pictorial exposition in the remainder of the paper is our own.

B.2.1 Adverse Selection

A market exhibits adverse selection if marginal costs are decreasing in output. This can be microfounded by consumers having private information on a fixed characteristic that affects the firm's cost. For example, in the market for health insurance, a consumer's health is private information and heterogeneous across agents. Consumers with low health have large expected claims and are therefore expensive for the firm to insure. In insurance markets, the WTP of rational consumers corresponds to the sum of their expected claims and a risk premium. Hence, when consumers have identical risk attitudes, low health consumers have the largest willingness-to-pay for insurance. Thus, at high prices only the low health consumers buy while lower prices draw healthier consumers into the market. Marginal costs are decreasing in output. The market exhibits adverse selection.

In insurance markets with adverse selection, the efficiency of the perfectly competitive equilibrium differs depending on the scale of consumers' risk premia relative to the extent of cost heterogeneity. When consumers' WTP far exceed costs, e.g. because they are very risk averse, the equilibrium achieves the efficient allocation (Figure 8a). When consumers' WTP are lower, the equilibrium exhibits inefficient underprovision (Figure 8b) and the market can even break down completely (Figure 8c).

Figure 8: Adverse Selection: Perfect Competition



D denotes demand, MC marginal cost, AC average cost

q^P is the quantity in the perfectly competitive equilibrium

q^S is the efficient or socially optimal quantity

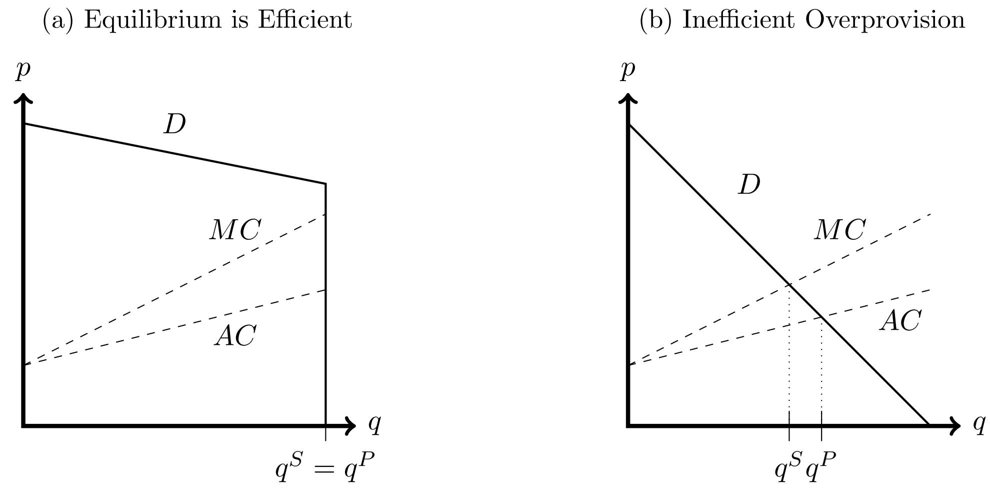
B.2.2 Advantageous Selection

A market exhibits advantageous selection if marginal costs are increasing in output. This arises, for example, when financial risk-taking by consumers is positively correlated with physical risk-taking (Hemenway, 1990). For example, in the market for car accident insurance more risk averse drivers may purchase more generous accident insurance and drive more carefully, thereby reducing the probability of an accident. Thus, it is possible that at high prices only the very risk averse consumers buy - they have low expected costs for the insurer - while lower prices also attract consumers with low risk aversion. Marginal costs are increasing in output.

De Meza and Webb (2001) microfound advantageous selection in insurance markets formally. They consider consumers with private information on either their wealth or their risk-aversion. Moreover, consumers have the possibility to take precautions which reduce the probability of a claim, but which are not observed by the insurance company. They show that more wealthy consumers - modelled as having DARA risk-preferences - or less risk-averse consumer, have a lower benefit from purchasing insurance and from taking precautions. Thus, it is possible that consumers with low wealth (or highly risk-averse agents) buy insurance and take precautions in equilibrium while wealthy consumers (or consumers with a low degree of risk-aversion) neither purchase insurance nor take precautions. The result can be that those who are insured in equilibrium are lower cost to insure than the uninsured. This means that there is advantageous selection.

In insurance markets with advantageous selection, the perfectly competitive equilibrium exhibits inefficient overprovision if for some consumers expected costs exceed their WTP and otherwise results in the efficient allocation. When firms face no administrative costs (then expected costs to the firm are equal to consumers expected claims) and consumers are risk averse (then their WTP exceeds their expected claims by a strictly positive risk premium), all consumers are willing to pay more for insurance than they cost the firm. In this case, the perfectly competitive equilibrium results in the efficient allocation (Figure 9a). When firms have administrative costs or when for behavioural reasons consumers underestimate their future claims, it is possible that some consumers' WTP is below expected cost. In this case, the perfectly competitive equilibrium exhibits inefficient overprovision (Figure 9b).

Figure 9: Advantageous Selection: Perfect Competition



Inefficient overprovision arises because firms undercut each other's price even when the entering consumers are loss making as firms try to steal the rival's existing profitable consumers.

C Taste Homogeneity

The model developed in this paper nests the results of the existing literature on markets with selection and imperfect competition. More specifically, the special case of our model with no taste heterogeneity produces the results known from the existing literature. This Appendix demonstrates that formally.

C.1 Results established by the existing literature:

From the existing literature (e.g. De Meza and Webb, 1987; Mahoney and Weyl, 2017) we have the following key results about markets with advantageous selection:

(a) In markets with advantageous selection, perfect competition can result in inefficient overprovision.

(b) Reductions in competition (starting from perfect competition) reduce this overprovision.

(c) There exists an intermediate level of competition at which the equilibrium results in the efficient allocation.

(d) Further reductions in competition (starting from the level which results in the efficient allocation) lead to underprovision and eventually to firms pricing like a monopolist.

(e) These results are consistent with the composition of switching consumers being identical to the composition of demand, i.e. the types of consumers which a price cut would induce to switch from the rival firm have on average the same WTP as the consumers who purchase from the rival firm.

C.2 Special case of our model: No taste heterogeneity

Let there be no heterogeneity of brand preferences, i.e. all consumers have identical brand preferences. Formally, let

$$g(x) = \begin{cases} 1 & \text{for} \quad \hat{x} \\ 0 & \text{for all} \quad x \neq \hat{x} \end{cases}$$

where $\hat{x}$ is some $x \in [0, 1]$ other than $x = \frac{1}{2}$.[47] To study this special case, we can use the results established in the main paper for the case when firms can price discriminate, as price discrimination makes firms treat each location as an independent market.[48]

[47] Since the goal is to capture changes in competition, we restrict our attention to $x \in [0, 1]$ other than $x = \frac{1}{2}$ and to t which are not too large (in the sense that in the oligopoly equilibrium no firm prices like a monopolist). At $x = \frac{1}{2}$ and if t is too large, further increases in t do not generate market power, thus do not affect competition. Hence, it is natural to exclude these cases here.

[48] Moreover, when there is no taste heterogeneity, there is no need to distinguish the case where firms can price discriminate from the case where they use uniform prices as the constraint implied by uniform pricing cannot be binding.

Socially optimal allocation: The socially optimal allocation is to allocate the good to every consumer whose WTP exceeds the threshold $y^S(x)$ where $y^S(x)$ is defined by

$$y^S(x) = c\big(y^S(x)\big) + T(x)$$

which is

$$y^S(x) = \alpha + \beta\big(1 - y^S(x)\big) + tx$$

which yields

$$y^S(x) = \frac{\alpha + \beta}{1 + \beta} + \frac{1}{1 + \beta}tx \tag{127}$$

Equilibrium allocation: The equilibrium allocation is that every consumer with WTP above threshold $y^*(x)$ buys where

$$y^*(x) = p_L(x) + tx \tag{128}$$

From Proposition 5 we have that for $x \in [x_L, \frac{1}{2}]$

$$p_L(x) = \frac{2\alpha + \beta}{2 + \beta} + \frac{2}{2 + \beta}t - \frac{4 + \beta}{2 + \beta}tx \tag{129}$$

Substituing (129) into (128) and simplifying yields:

$$y^*(x) = \frac{2\alpha + \beta}{2 + \beta} + \frac{2}{2 + \beta}t - \frac{2}{2 + \beta}tx \tag{130}$$

Inefficient overprovision: Inefficient overprovision exists at a location x for a level of transport costs t if and only if $y^*(x) < y^S(x)$. Thus, we define $\Omega = y^S(x) - y^*(x)$ where $\Omega > 0$ captures that inefficient overprovision exists.

Using (127) and (130), $\Omega = y^S(x) - y^*(x)$ becomes:

$$\Omega = \frac{(1 - \alpha)\,\beta}{(1 + \beta)\,(2 + \beta)} - \frac{2}{2 + \beta}\,t + \frac{4 + 3\beta}{(1 + \beta)\,(2 + \beta)}\,tx \tag{131}$$

We use this to prove that our model's special case of no taste heterogeneity yields the results a) - e) which were established by the literature.

Result a): *Perfect competition can result in inefficient overprovision.*

While the literature establishes that overprovision is a possibility, the natural benchmark to compare our model to is that under perfect competition overprovision arises with certainty. To see this, note that De Meza and Webb (1987) and Einav and Finkelstein (2011) show that in markets with advantageous selection, perfect competition leads to inefficient overprovision unless providing the good to every consumer is efficient. Since our model aims to study overprovision and potential remedies, we assume that it is not efficient to provide the good to every consumer. Hence, in order to match the results from the existing literature, the special case of our model should generate the result that there is inefficient overprovision (a stronger result than that there "can be" overprovision). In what follows, we establish that this stronger result holds.

Formally, this means that we need to show that at every x, $t = 0$ results in inefficient overprovision.

Proof: To show: $\Omega > 0$ if $t = 0$.

Using (131) and setting $t = 0$ yields:

$$\Omega = \frac{(1-\alpha)\,\beta}{(1+\beta)\,(2+\beta)}$$

Thus, we need to show that

$$\frac{(1-\alpha)\,\beta}{(1+\beta)\,(2+\beta)} > 0$$

Since $\beta > 0$ (by our focus on advantageous selection), this simplifies to

$$(1-\alpha) > 0$$

Since we assumed that there exists gains from trade at every location x, i.e. that $c(1) + T(\frac{1}{2}) \leq 1$ which for $t = 0$ becomes $\alpha \leq 1$, it follows that $(1-\alpha) > 0$ holds. *Q.E.D.*

Result b): *Reductions in competition (from perfect competition) reduce overprovision.*

Formally, in our model this is equivalent to showing that $\frac{d\Omega}{dt} < 0$.

Proof: Calculating $\frac{d\Omega}{dt}$ from (131) yields that for any $x < \frac{1}{2}$:[49]

$$\frac{d\Omega}{dt} = -\frac{2}{2+\beta} + \frac{4+3\beta}{(1+\beta)\,(2+\beta)}\,x \tag{132}$$

The objective is to show that $\frac{d\Omega}{dt} < 0$ which using (132) becomes

$$x < \frac{2\,(1+\beta)}{4+3\beta} \tag{133}$$

Define the auxiliary concept $\tilde{x}$ which is the x at which the inequality (133) holds with equality, i.e.

$$\tilde{x} = \frac{2\,(1+\beta)}{4+3\beta} \tag{134}$$

We proceed by first establishing that the inequality (133) holds for $\beta = 0$ at all $x < \frac{1}{2}$ and then show that it also holds for all $\beta > 0$, i.e. for any market with advantageous selection.

The case of $\beta = 0$: Substituting $\beta = 0$ into (134) yields $\tilde{x} = \frac{1}{2}$. Thus (133) holds for all $x < \frac{1}{2}$.

The case of $\beta > 0$: Note that

$$\frac{d\tilde{x}}{d\beta} = \frac{2}{(4+3\beta)^2}$$

Therefore, $\frac{d\tilde{x}}{d\beta} > 0$. Since the inequality (133) holds for all $x < \frac{1}{2}$ for $\beta = 0$, and as the right hand side term in (133) is increasing in β and must thus be larger than $\frac{1}{2}$, it follows that the inequality (133) holds for all $x < \frac{1}{2}$ for any $\beta \geq 0$. Thus, $\frac{d\Omega}{dt} < 0$ holds. *Q.E.D.*

[49] $x > \frac{1}{2}$ follow by symmetry.

Result c): *There exists an intermediate level of competition at which the equilibrium results in the efficient allocation.*

Formally, in our model this is equivalent to the claim that at every location x, $\Omega = 0$ is reached for some t which is part of the parameter space we consider.

Proof: Setting $\Omega = 0$ in equation (131) and solving for t yields:

$$t = \frac{(1-\alpha)\,\beta}{2 + 2\beta - (4+3\beta)\,x} \tag{135}$$

As in the main paper, we refer to the t that ensures $\Omega = 0$ as $\tilde{t}$, i.e. (135) defines $\tilde{t}$. To establish Result c), we now show that $\tilde{t}$ is in the parameter space.

Recall that the parameter space we consider is $t \in [0, 2(1-\alpha)]$. $t \geq 0$ is a standard assumption in spatial models. Our assumption that there exist some gains from trade at every location delivers the upper bound on t. Formally, the assumption as stated in the paper is that $c(1) + \frac{t}{2} \leq 1$, which can be rearranged to $t \leq 2(1-\alpha)$. We decompose the proof into two parts.

Part I) To show: $\tilde{t} > 0$ for all $x < \frac{1}{2}$. Using (135) this is

$$\frac{(1-\alpha)\,\beta}{2 + 2\beta - (4+3\beta)\,x} > 0 \tag{136}$$

Since $2 + 2\beta - (4+3\beta)x > 0$ for all $x < \frac{1}{2}$, and since $\beta > 0$ by the assumption that there is advantageous selection, inequality (136) simplifies to $(1-\alpha) > 0$, which is true (see the proof of result a) for details). Hence, we have established that $\tilde{t} > 0$ for all $x < \frac{1}{2}$.

Part II) To show: $\tilde{t} < 2(1-\alpha)$ for all $x < \frac{1}{2}$. $\tilde{t} < 2(1-\alpha)$ is equal to

$$\frac{(1-\alpha)\,\beta}{2\,(1+\beta) - (4+3\beta)\,x} < 2(1-\alpha) \tag{137}$$

which simplifies to

$$2\,(4+3\beta)\,x < 4\,(1+\beta) - \beta$$

and further to

$$x < \frac{1}{2}$$

Hence, we have established that $\tilde{t} < 2(1-\alpha)$ for all $x < \frac{1}{2}$.

Combining the results of Part I) and Part II), we have that, for all $x < \frac{1}{2}$, $\Omega = 0$ is reached for some t, denoted $\tilde{t}$, which is in the parameter space, i.e. $\tilde{t} \in [0, 2(1-\alpha)]$. *Q.E.D.*

Result d): *Further reductions in competition (starting from the level which results in the efficient allocation) lead to underprovision and eventually to firms pricing like a monopolist.*

Proof: This follows directly by combining our proofs of result b) and result c). From result b) we have that $\frac{d\Omega}{dt} < 0$ and from result c) we have that there exists a t, denoted $\tilde{t}$, at which $\Omega = 0$. Thus, increases in t beyond $\tilde{t}$ lead to inefficient underprovision and eventually to firms pricing like a monopolist. *Q.E.D.*

Result e): *These results are consistent with the composition of switching consumers being identical to the composition of demand, i.e. the types of consumers which a price cut would induce to switch from the rival firm have on average the same WTP as the consumers who purchase from the rival firm.*

In the special case of our model with no taste heterogeneity, a price cut by one firm leads to it either serving all consumers who purchase (if its resulting price plus transport costs are lower than the rival's), serving no consumers (if its price plus transport costs are higher), or serving a representative sample of all consumers where the sample size is half the number of consumers who purchase (if its price plus transport costs are equal to those of the rival). This means that, in the special case of no taste heterogeneity, our model implies "perfect switching" as either all consumers switch or no consumer switches (or the case of a tie, a representative sample switch). This means that the consumers who switch have on average the same WTP as the consumers in overall industry demand. Hence, in the special case of no taste heterogeneity, our model's prediction on the composition of switching consumers is consistent with the assumption by Mahoney and Weyl (2017) that the per-consumer cost a firm incurs when selling to switching consumers is on average equal to the per-consumer cost incurred when selling to the average consumer in demand. We term this "representative switching."

In the general case with taste heterogeneity, our model predicts that switching consumers have on average a higher WTP than the average consumer in demand. In this case, the per-consumer cost a firm incurs when selling to switching consumers is on average lower than the per-consumer cost incurred when selling to the average consumer in demand. Thus, in the general case with taste heterogeneity, our model predicts that switching consumers are more strongly selected than overall demand which means that switching is not representative.[50]

Hence, the model developed in the main paper nests the results from the existing literature as a special case with no taste heterogeneity. *Q.E.D.*

[50] The composition of switching consumers in our general model with taste heterogeneity differ not just from the main case in MW ("representative switching"), but also from generalisations discussed by Mahoney and Weyl (2017). On p.640, footnote 7, they state that "Even if this assumption [representative switching] fails, so long as average switching consumers have a cost that is strictly between that of average exiting consumers and average purchasing consumers, most of our results are left unchanged." This generalises representative switching to allow switching consumers to have a *lower* average WTP than demand. Our model predicts a *higher* average WTP than demand, thus differs also from this generalisation.