# Information design for colluding firms*

Alessandro Bonatti† Raffaele Fiocco‡ Salvatore Piccolo§

February 2024

**Abstract**

Firms often acquire information through common intermediaries, such as data analytics companies. We characterize the optimal design of information for colluding firms in a market where information is about the state of demand under secret price cuts. Information can be provided either before price setting — i.e., ex ante information provision — or after price setting — i.e., ex post information provision. The two modes of information provision generate a trade-off between a price sophistication effect and a profit dissipation effect. Such trade-off relies upon different factors, namely, the heterogeneity of demand, the efficiency of the information technology, and the number of firms.

Keywords: collusion, data analytics, ex ante information provision, ex post information provision, information design.
JEL Classification: D43, D83, L13, L41.

*We are indebted to Volker Nocke and Patrick Rey for valuable comments and suggestions. We also thank the participants at Unibg Industrial Organization Winter Symposium in Passo del Tonale (December 2022) and Unibg Economics Winter Symposium in Spiazzi di Gromo (February 2023).

†MIT Sloan School of Management. Email address: bonatti@mit.edu
‡University of Bergamo. Email address: raffaele.fiocco@unibg.it
§University of Bergamo, Compass Lexecon and CSEF. Email address: salvapiccolo@gmail.com

## 1 Introduction

The continuous advancement of data science technologies has promoted the development of sophisticated techniques to collect and analyze large amounts of data, especially through the use of artificial intelligence, machine learning and data mining. Firms have increasingly resorted to common intermediaries, such as data analytics companies, whose specialized algorithms and software programs can examine data sets in order to provide firms with accurate information that allows more effective business decisions. The data analytics market is growing significantly. Its size was valued at USD 30 billion in 2022 and is projected to grow at 29.4% per year during the current decade.[1] Two major forms of data analytics are predictive analytics and diagnostic analytics. As its name suggests, predictive analytics is used to make forecasts about unknown future events, such as market demand predictions. Diagnostic analytics focuses on the identification of factors and events behind a certain outcome, such as a change in sales.

Although more accurate information can improve firms' decision making, antitrust practitioners and scholars have devoted increasing attention to the anticompetitive effects associated with the possibility of facilitating collusion. Such concern is even more severe in light of the observation that in a number of sectors large competitors are served by the same data analytics company. Examples abound. Alteryx offers data analytics services to General Motors, Ford, Daimler, Honda, Toyota and Hyundai in the automotive industry, to Johnson&Johnson, Pfizer and Hoffmann La Roche in the pharmaceuticals industry, to Comcast and Verizon in the telecommunications industry, to Procter&Gamble and Unilever in the consumer goods industry, as well as to Philips and Siemens in the medical equipment industry. Samsung, Sony and Fujitsu, which provide consumer electronics, are clients of Sisense, the insurance companies Allianz and ERGO resort to SAS, whereas in food processing General Mills, Kellogg's and Conagra are served by First Analytics.

In this paper, we characterize the optimal information design in a setting where firms are prone to collusion but lack information about some relevant market features. Inspired by the relevance of data analytics in modern economies, we allow firms to acquire information about market demand. Two alternative modes of information provision are investigated, referred to as *ex ante information provision* and *ex post information provision*. Under ex ante information provision, firms receive information prior to price setting. Under ex post information provision, they obtain information after price setting. Essentially, ex ante information provision reflects predictive analytics, whereas ex post information provision corresponds to diagnostic analytics. A range of challenging questions arise. What are the distinguishing features of such modes of information provision? What is the optimal information design for collusion purposes? What are the driving forces behind the results? What are the main testable predictions and policy implications?

To address these issues, we consider a market where firms aim to collude but cannot directly observe demand fluctuations. An appealing feature of this framework, stemming from the seminal paper of Green and Porter (1984), is that it predicts periodic price wars, consistently with some relevant empirical evidence. Rather than assuming quantity competition as in Green

[1] Further details are available at https://www.precedenceresearch.com/data-analytics-market (last retreived in February 2024).

and Porter (1984), we follow Tirole's (1988) stylized but equally insightful approach that firms engage in price competition and, in line with Stigler's (1964) classical idea, cannot observe rivals' prices. For instance, a firm may offer secret price discounts to its customers. Under price secrecy, each firm can only rely on the observation of its own market share or demand in order to detect the deviation by a rival that undercuts the collusive price. Thus, punishment is implemented (through reversion to the noncooperative equilibrium) after a firm faced low demand (or low profit). When market demand is stochastic and shocks are unobservable, however, a deviation cannot be perfectly inferred. In particular, the realization of low demand (or low profit) may stem from a slack in demand rather than from price undercutting. To enforce collusion, firms must not only coordinate on a certain price but also determine the length of punishment because, under uncertainty, mistakes are unavoidable and the maximal punishment (through eternal reversion to the noncooperative equilibrium) is not necessarily optimal.

In this framework, more accurate information about demand can definitely facilitate collusion. We consider an information provider that supplies firms with information in the form of a signal about the state of demand.[2] Information can be provided either before price setting — i.e., ex ante information provision — or after price setting — i.e., ex post information provision. As information before price setting allows firms to update both the pricing strategy and the punishment strategy, one might think that ex ante information provision should unambiguously be more effective in sustaining collusion. However, we show that the two modes of information provision generate a trade-off between two effects, referred to as *price sophistication effect* and *profit dissipation effect*. On the one hand, the opportunity to make prices contingent on the information received enables colluding firms to conduct a more refined pricing strategy. This yields a price sophistication effect, which renders ex ante information provision more profitable for collusion purposes. On the other hand, more accurate information prior to price setting magnifies a firm's incentives to deviate when it increases the probability of low demand, because each firm anticipates that a punishment phase is more likely to start anyway and thus the loss from deviation becomes smaller. To alleviate deviation incentives, under ex ante information provision colluding firms prefer to set their prices at the marginal cost after receiving the signal of low demand, irrespective of what previously occurred. Hence, firms forgo some profits when, despite of the information received, demand is indeed not low. This yields a profit dissipation effect, which exacerbates the cost of distorting information accuracy under ex ante information provision. When the information technology is relatively inefficient, ex post information provision promotes the reduction in information accuracy to curb the associated costs, which increases the profitability of collusion. We find that the trade-off between the price sophistication effect and the profit dissipation effect relies upon different factors, namely, the heterogeneity of demand, the efficiency of the information technology, and the number of firms.

Our analysis conveys novel insights into the anticompetitive effects of information provision. Specifically, we provide testable predictions about the relative profitability of different forms

[2]Notably, the data analytics market is rather concentrated. According to the latest available estimates, in 2019 SAS was the market leader with a share of 27.9%, followed by IBM with a share of 13.1%. These figures have been quite stable over recent years. Further details are available at https://www.statista.com/statistics/475644/advanced-and-predictive-analytics-software-market-share-worldwide-by-vendor/ (last retrieved in February 2024).

of data analytics in markets plagued by collusion. We also deliver some potentially significant implications for antitrust policy. As different modes of information provision encourage collusion to a different extent and antitrust authorities are typically endowed with scarce resources, our work can contribute to the identification of compelling criteria and protocols that assign priorities in the antitrust assessment of data analytics services.

The rest of the paper is organized as follows. Section 2 describes the related literature. Section 3 sets out the formal model. Section 4 derives the equilibrium features of ex ante and ex post information provision with two demand states and compare the two modes of information provision. Section 5 turns to the case of three demand states. Section 6 concludes the analysis. The Appendix collects the formal proofs. The Supplementary Appendix extends the analysis to an arbitrary number of demand states and firms.

## 2 Related literature

As discussed in the introduction, our paper relies on the well-established literature about tacit collusion under imperfect monitoring. The pioneering work of Stigler (1964) investigates the problem of collusion enforcement when firms can secretly cut their prices. Building on Stigler's (1964) influential approach, Green and Porter (1984) provide a seminal formalization of collusion under secret price cutting in a model where firms set their quantities and cannot directly observe demand fluctuations. The reversion to Cournot competition occurs for some amount of time because of low demand, without any defection from the collusive agreement. Hence, price wars are involuntary and triggered by a recession. Rotemberg and Saloner (1986) examine the scope for collusion in a different setting where in every period firms learn the current state of demand before choosing their actions, which leads to perfect monitoring of rivals' past actions. Contrary to Green and Porter (1984), they find that the cartel experiences price wars during booms. In a repeated game under imperfect monitoring, Abreu at al. (1991) show that reducing the interest rate to zero leads to asymptotically efficient equilibria, whereas shortening the period of a fixed action removes any possibility of cooperation. A delay in the release of information can generate a higher equilibrium payoff for all players. In a framework à la Green and Porter (1984), Sannikov and Skrzypacz (2007) find that collusion cannot be sustained in the presence of flexible production technologies that allow firms to quickly respond to new information. Harrington and Skrzypacz (2007) consider an environment where firms privately know their prices but quantities are public information. Collusion can be achieved with asymmetric punishments that involve transfers through which firms selling too much compensate those selling too little. In a subsequent work, Harrington and Skrzypacz (2011) derive the conditions for the sustainability of collusion when firms' prices and quantities are private information.

Our work also pertains to the literature on information acquisition and disclosure. Bergemann and Välimäki (2002) examine the agent's incentives to acquire information before participating in a mechanism and investigate whether there exists a mechanism that induces efficient information acquisition. When two principals contract sequentially with the same agent, Calzolari and Pavan (2006) show that under certain circumstances the upstream principal prefers to grant the agent full privacy. In a setting where a monopolist sells an indivisible object to risk-neutral buyers that only have an estimate of their private valuations, Eső and Szentes (2007) find that the monopolist's expected revenue maximizing mechanism fully reveals the seller's information. When information is endogenous, Li and Shi (2017) show that discriminatory disclosure, which consists of releasing different amounts of additional information to different buyer types, dominates full disclosure in terms of the seller's revenue. In a model where a sender randomly draws a 'prospect' and a receiver observes only a signal provided by the sender, Rayo and Segal (2010) demonstrate that the sender's profit maximizing disclosure rule typically exhibits partial information. In a framework that allows for strategic complementarity or substitutability in actions, Colombo et al. (2014) characterize the precision of private information acquired in equilibrium and how it differs from the socially optimal one. Argenziano et al. (2016) show that, in a setting with costly information and endogenous choice of information acquisition, a decision maker can induce a biased expert to acquire more information than what would be directly acquired. Hörner and Skrzypacz (2016) derive the conditions under which an agent finds it optimal to gradually reveal information to a firm. Bergemann et al. (2018) analyze the sale of supplementary information to a data buyer that faces a decision problem under uncertainty. They show that the data seller's revenue maximizing menu of information products generally contains both the fully informative experiment and partially informative 'distorted' experiments. Kastl et al. (2018) study the impact of market competition on information accuracy and find that a monopolistic information provider may prefer to supply imprecise information to perfectly competitive firms, even when information is costless. Mathevet et al. (2020) show how agents' beliefs can be manipulated through information disclosure and describe the structure of the optimal belief distributions. In a dynamic model of information acquisition, Zhong (2022) demonstrates that a decision maker finds it optimal to acquire a signal that arrives according to a Poisson process. Denti (2023) develops a model of information acquisition in games that incorporate players' incentives to learn what others know and endogenizes the information structure with applications to rational inattention and global games. We refer to Bergemann and Morris (2019) for an excellent overview of the literature on information design through a unified perspective.

Our paper is also related to the literature that explores the links between information and collusion. In a setting with a minimal information structure such that a firm only observes whether it gets the sale, Hörner and Jamison (2007) demonstrate that full collusion can be approximated irrespective of the number of competitors as long as firms are sufficiently patient. Rojas (2012) provides experimental evidence that, for a sufficiently high discount rate, information (about demand realization in the following period) may facilitate collusion to a larger extent than monitoring (about rivals' past actions). Miller (2012) shows that, if the cartel's member firms have private information about their costs, an optimal robust equilibrium predicts price wars under certain circumstances. In a repeated oligopoly with secret price cuts, Awaya and Krishna (2016) study the role of communication within a cartel and identify equilibria with 'cheap talk' that result in near-perfect collusion. Sugaya and Wolitzky (2018) characterize the conditions under which colluding firms benefit from maintaining privacy of their actions and market outcomes.

A fast-growing strand of literature relevant to our work deals with the impact of artificial intelligence, machine learning and algorithms on the sustainability of collusion. In a framework with perfect monitoring where in every period firms obtain a signal about the current state of demand before setting their prices, Miklós-Thal and Tucker (2019) find that better demand prediction driven by algorithms generates nuanced effects on firms' ability to collude. More precise demand forecasting can lead to lower prices and higher consumer welfare. Adopting an experimental approach, Calvano et al. (2020) show that pricing algorithms systematically learn to play collusive strategies by trial and error. Collusion is sustained through a finite period of punishment after a defection, with a gradual return to cooperation. In a subsequent work, Calvano et al. (2021) find that pricing algorithms can collude even under imperfect monitoring and collusive strategies significantly resemble those considered by Green and Porter (1984). Along these lines, Klein (2021) shows that competing reinforcement learning algorithms can converge to collusive equilibria when the number of discrete prices is limited. Hansen et al. (2021) find that, if the informational value of price experiments is high, long-run prices are supracompetitive and the full information joint monopoly outcome can be achieved. Abada and Lambin (2023) observe that machine learning algorithms for a storable good learn quickly to achieve seemingly collusive decisions and indicate that collusion could stem from imperfect exploration. Harrington (2022) shows that outsourcing a pricing algorithm to a third party developer makes prices more sensitive to demand variations, which harms consumers and increases industry profit. In a market where horizontally differentiated firms can price discriminate based on private information and receive private, noisy signals about consumer preferences, Peiseler et al. (2022) find a non-monotonic relationship between signal quality and the sustainability of collusion. When price discrimination depends on whether consumers belong to a firm's inherited market share, Colombo and Pignataro (2022) also show that information accuracy has a non-monotonic impact on the stability of collusion.

# 3 The model

**Environment.** Our framework relies on Green and Porter's (1984) seminal model of collusion under secret price cuts, following Tirole's (1988) stylized but equally insightful version of direct price setting. Two identical firms sell a homogeneous good and simultaneously set their prices in every period over an infinite time horizon. Each firm aims to maximize its present discounted profit, with the common discount factor $\delta \in (0, 1)$. In every period, there exists a unit mass of consumers with homogeneous willingness to pay. Consumers all buy from the firm that charges the lowest price. Demand is equally shared if both firms set the same price. The magnitude of consumers' willingness to pay in every period identifies the state of demand.[3] The evolution of demand over time follows a stochastic process and the realizations of demand are independently and identically distributed over time. Furthermore, all demand states are equally likely.[4] We first consider the case where in every period the state of demand $\theta_i \in \Theta$ can assume two values, high or low, i.e., $\theta_i \in \Theta = \Theta_2 \triangleq \{\overline{\theta}, \underline{\theta}\}$, where $\overline{\theta} > \underline{\theta}$. Then, we turn to the

[3] Our qualitative results remain unaltered if demand states correspond to different downward sloping demand functions instead of different levels of consumers' willingness to pay.

[4] Assigning different probabilities to demand states would complicate the analysis without providing any additional useful insights.

case where in every period the state of demand $\theta_i \in \Theta$ can also assume an intermediate value, i.e., $\theta_i \in \Theta = \Theta_3 \triangleq \left\{\overline{\theta}, \widehat{\theta}, \underline{\theta}\right\}$, where $\overline{\theta} > \widehat{\theta} > \underline{\theta}$.[5] These two cases reflect different degrees of heterogeneity or dispersion of demand. A firm does not know the state of demand and cannot observe the rival's price. The low demand state $\underline{\theta}$ corresponds to zero demand.[6] In line with Stigler's (1964) classical idea, each firm faces a nontrivial inference problem about the rival's price. A firm that collects zero profit cannot perfectly know whether such event arises from the realization of the low demand state or rather from price undercutting by the rival. Notably, the fact that at least one firm makes zero profit is always common knowledge. If this occurs in the absence of a price cut, both firms make zero profit. If this stems from a price cut, the undercutting firm knows that the rival makes zero profit. A collusive strategy prescribes that each firm charges the collusive price (which may depart from the static monopoly level) at the outset of the game and continues to do so until at least one firm makes zero profit. The occurrence of zero profit (which is common knowledge) triggers a punishment phase during which both firms revert to the noncooperative equilibrium by setting their prices at the (constant) marginal cost. At the end (if any) of the punishment phase, firms start colluding again as long as they obtain positive profits.

**Information provision.** A monopolistic information provider supplies firms with information about the state of demand. Information can be provided before price setting at the beginning of every period. This is referred to as *ex ante information provision.* Alternatively, information can be provided after price setting at the end of every period. This is referred to as *ex post information provision.* As discussed in the introduction, such information design is reasonably suitable for data analytics services, which can assume either the form of predictive analytics — i.e., ex ante information provision — or the form of diagnostic analytics — i.e., ex post information provision. At the outset of the game, the information provider determines an information disclosure policy $\{E, \rho\}$ that specifies an experiment $E$ and the associated price $\rho$. The information provider offers the same experiment to both firms, which purchase it if this is profitable. An experiment $E \triangleq \{S, f\}$ consists of a set of signals $S \subseteq \mathbb{R}$, with a generic element $s_i \in S$, and of a likelihood function $f : \Theta \to \Delta(S)$ that maps demand states into signals. Conditionally on the demand state, signals are independent. In line with the relevant literature (e.g., Bergemann et al., 2018; Calzolari and Pavan, 2006; Kastl et al., 2018; Miklós-Thal and Tucker, 2019), without any loss of generality we confine attention to experiments that exhibit a number of signals equal to the number of demand states, i.e., $s_i \in S = S_2 \triangleq \{\overline{s}, \underline{s}\}$ if $\Theta = \Theta_2$ and $s_i \in S = S_3 \triangleq \{\overline{s}, \widehat{s}, \underline{s}\}$ if $\Theta = \Theta_3$. For the sake of convenience, signals are symmetric (e.g., Miklós-Thal and Tucker, 2019). The parameter $\alpha = \Pr[s_i | \theta_i] \in [\alpha_0, 1]$ measures the degree of accuracy, or precision, of information. For $\alpha = \alpha_0$ the experiment is uninformative, i.e., $\Pr[\theta_i | s_i] = \Pr[\theta_i]$, whereas for $\alpha = 1$ the experiment fully reveals the state of demand, i.e., $\Pr[\theta_i | s_i] = 1$. As a standard convention, given the signal $s_i$, the probability of the state of demand $\theta_i$ is (weakly) higher than the corresponding unconditional probability. In other terms, the signal $s_i$ is a 'good news' about the state of demand $\theta_i$ and increases the belief in $\theta_i$.

[5] We refer to Section 6 for a more general framework with an arbitrary number of demand states and firms.

[6] As firms do not have any incentive to set a price lower than the (constant) marginal cost, the low demand state $\underline{\theta}$ can assume any value below the marginal cost.

The information provider's cost for a degree $\alpha$ of information accuracy is $C(\alpha) = c(\alpha - \alpha_0)$.[7] The unit cost of information provision $c > 0$ constitutes an inverse measure of efficiency of the information technology. The same cost parameter $c$ under the two modes of information provision allows a more insightful analysis by neutralizing any technology-driven effect.

**Timing and equilibrium concept.** At the outset of the game, the information provider determines an information disclosure policy. In every period of the infinitely repeated game, the timing unfolds as follows.

1. The state of demand is realized and unknown to both firms.
2. If firms accepted the information disclosure policy, under ex ante information provision they receive a signal about the state of demand and then simultaneously set prices. Under ex post information provision they simultaneously set prices and then receive a signal about the state of demand. A firm cannot directly observe either the state of demand or the rival's price.
3. Consumers purchase the good and firms' profits materialize.

In line with the relevant literature (e.g., Miklós-Thal and Tucker, 2019), we look for a symmetric pure-strategy subgame perfect Nash equilibrium where collusion generates the highest profits among all symmetric pure-strategy subgame perfect Nash equilibria. We refer to this equilibrium as the *most cooperative equilibrium*.

# 4 Two states of demand

We first examine the case where the state of demand $\theta_i$ can assume two values, high or low, i.e., $\theta_i \in \Theta_2 \triangleq \{\overline{\theta}, \underline{\theta}\}$, where $\overline{\theta} > \underline{\theta}$. An experiment exhibits two signals, i.e., $s_i \in S_2 \triangleq \{\overline{s}, \underline{s}\}$. The degree of information accuracy is $\alpha = \Pr[s_i | \theta_i] \in [\alpha_0, 1]$, where $\alpha_0 = \Pr[\theta_i] = 1/2$.

## 4.1 No information

We start our analysis with the benchmark case where no information is provided. As described in Section 3, a collusive strategy prescribes that each firm charges the collusive price at the outset of the game and continues to do so until at least one firm makes zero profit. This triggers a punishment phase for $T$ periods, during which firms revert to the noncooperative equilibrium by setting their prices at the marginal cost. Consider the collusive price at the static monopoly level $\overline{\theta}$. A firm's present discounted profit in the collusive phase is given by

$$V = \frac{1}{2}\left(\frac{\overline{\theta}}{2} + \delta V\right) + \frac{1}{2}\delta^{T+1}V.$$

In the current period, firms equally share the entire collusive profit when facing high demand, which occurs with probability 1/2. In this case, they keep on colluding in the following period. With complementary probability 1/2, firms face zero demand and thus make zero profit. This

[7] The (constant) term $-c\alpha_0$ ensures the natural requirement that producing the perfectly uniformative signal is costless. It can be omitted without affecting our analysis at all.

leads to a punishment phase that lasts for $T$ periods. At the end of the punishment phase, firms start colluding again.

A deviating firm slightly undercuts the collusive price $\overline{\theta}$ and appropriates the entire expected collusive profit in the current period. This generates a punishment for $T$ periods. The incentive constraint that ensures the sustainability of collusion writes as

$$V \geq \frac{1}{2}\overline{\theta} + \delta^{T+1}V.$$

Straightforward calculations reveal that the incentive constraint fails to hold and hence there is no scope for collusion.[8] We summarize these results in the following remark.

**Remark 1** *Suppose that the state of demand can assume two values — i.e.,* $\theta_i \in \Theta_2 \triangleq \{\overline{\theta}, \underline{\theta}\}$. *Then, in the absence of information provision, firms cannot sustain collusion.*

With two states of demand, a necessary condition for collusion among $N \geq 2$ firms to emerge is that the likelihood of the low demand state is below $1/N$. The idea is that a high probability of low demand reduces a firm's loss from deviation, which discourages collusion and tightens the incentive constraint. This condition for the sustainability of collusion is never satisfied when the two states of demand are equally likely, regardless of the number of firms. Hence, collusion cannot be supported in the absence of information provision. Clearly, this does not affect the comparison between the two modes of information provision.

## 4.2 Ex ante information provision

We consider the collusive strategy according to which in every period each firm sets the price at $\overline{\theta}$ after receiving the signal $\overline{s}$ and at the marginal cost after receiving the signal $\underline{s}$. If at least one firm obtained zero profit in the previous period (which is common knowledge), the market enters a punishment phase, whose duration $T_i \in \{\overline{T}, \underline{T}\}$ is contingent on the signal received $s_i \in \{\overline{s}, \underline{s}\}$, respectively. During the punishment phase, firms revert to the noncooperative equilibrium by setting their prices at the marginal cost.[9]

We define by $V_i \in \{\overline{V}, \underline{V}\}$ the present discounted profit of a firm in the collusive phase after receiving the signal $s_i \in \{\overline{s}, \underline{s}\}$, respectively. As demand states are equally likely, the degree $\alpha$ of information accuracy is such that $\alpha = \Pr[s_i | \theta_i] = \Pr[\theta_i | s_i]$. Upon receiving the signal $\overline{s}$, each firm charges the price at $\overline{\theta}$ and obtains a present discounted profit equal to

$$\begin{aligned}\overline{V} &= \Pr\left[\overline{\theta} \middle| \overline{s}\right]\left[\frac{\overline{\theta}}{2} + \delta\left(\Pr[\overline{s}]\,\overline{V} + \Pr[\underline{s}]\,\underline{V}\right)\right] + \Pr\left[\underline{\theta} \middle| \overline{s}\right]\delta^{\overline{T}+1}\left(\Pr[\overline{s}]\,\overline{V} + \Pr[\underline{s}]\,\underline{V}\right) \\ &= \alpha\left(\frac{\overline{\theta}}{2} + \delta\frac{\overline{V}+\underline{V}}{2}\right) + (1-\alpha)\,\delta^{\overline{T}+1}\frac{\overline{V}+\underline{V}}{2}. \end{aligned} \tag{1}$$

In the current period, firms equally share the entire collusive profit when facing high demand, which occurs with probability $\Pr\left[\overline{\theta} \middle| \overline{s}\right] = \alpha$. In this case, they continue to collude in the following period and obtain the expected present discounted profit $\left(\overline{V} + \underline{V}\right)/2$. With complementary

[8]Notably, collusion cannot be sustained for any price lying between $\underline{\theta}$ and $\overline{\theta}$, either. This price reduces the per period collusive profit and does not modify the probability of zero demand in the collusive phase, which leaves the incentive constraint unchanged.

[9]We refer to the discussion after Lemma 1 for the optimality of this collusive strategy.

probability $\Pr[\underline{\theta}|\,\overline{s}] = 1-\alpha$, firms face zero demand and thus make zero profit, which triggers a punishment phase for $\overline{T}$ periods. At the end of the punishment phase, firms start colluding again.

Upon receiving the signal $\underline{s}$, each firm charges the price at the marginal cost and obtains a present discounted profit equal to

$$\begin{aligned}\underline{V} &= \delta^{\underline{T}+1}\left(\Pr[\overline{s}]\,\overline{V} + \Pr[\underline{s}]\,\underline{V}\right)\\ &= \delta^{\underline{T}+1}\frac{\overline{V}+\underline{V}}{2},\end{aligned} \tag{2}$$

where $\underline{T}$ is the length of punishment conditional on the signal $\underline{s}$, which occurs after receiving zero profit.[10]

Upon receiving the signal $\overline{s}$, a deviating firm slightly undercuts the collusive price $\overline{\theta}$ and appropriates the entire expected collusive profit in the current period. This generates a punishment for $\overline{T}$ periods. The incentive constraint that ensures the sustainability of collusion writes as

$$\begin{aligned}\overline{V} &\geq \Pr\left[\overline{\theta}\middle|\,\overline{s}\right]\overline{\theta} + \delta^{\overline{T}+1}\left(\Pr[\overline{s}]\,\overline{V} + \Pr[\underline{s}]\,\underline{V}\right)\\ &= \alpha\overline{\theta} + \delta^{\overline{T}+1}\frac{\overline{V}+\underline{V}}{2}.\end{aligned} \tag{3}$$

We summarize the main results in the following lemma.

**Lemma 1** *Suppose that the state of demand can assume two values — i.e., $\theta_i \in \Theta_2 \triangleq \left\{\overline{\theta}, \underline{\theta}\right\}$. Then, under ex ante information provision, in the most cooperative equilibrium, each firm sets the price at $\overline{\theta}$ after receiving the signal $\overline{s}$ and at the marginal cost after receiving the signal $\underline{s}$, provided that both firms obtained positive profits in the previous period. Otherwise, a punishment phase is triggered, during which each firm sets the price at the marginal cost for a number of periods equal to $\overline{T}^a(\alpha)$ after receiving the signal $\overline{s}$ and $\underline{T}^a$ after receiving the signal $\underline{s}$, where $\overline{T}^a(\alpha) > \underline{T}^a = 0$.*

Lemma 1 shows that, under ex ante information provision, collusion can be sustained through a strategy that prescribes the static monopoly price $\overline{\theta}$ conditional on the signal $\overline{s}$.[11] Upon receiving the signal $\underline{s}$, each firm sets the price at the marginal cost, which generates zero profit in the current period as in the noncooperative equilibrium, irrespective of what previously occurred. This removes any incentives to deviate in response to the signal $\underline{s}$. Sustaining a collusive price above the marginal cost would introduce an additional incentive constraint. As

[10] As marginal cost pricing ipso facto leads to zero profit, the event of zero demand (instead of zero profit) might be perceived as a better measure upon which a punishment should rely. Intuitively, given the signal $\overline{s}$, a firm's present discounted profit is still given by (1), because a firm makes zero profit if and only if demand is zero. Given the signal $\underline{s}$, a firm faces high demand with probability $\Pr\left[\overline{\theta}\middle|\,\underline{s}\right]$ and low demand with complementary probability $\Pr[\underline{\theta}|\,\underline{s}]$, although marginal cost pricing yields zero profit anyhow. If the punishment phase starts after the event of zero demand, a firm's present discounted profit becomes $\underline{V}' = \Pr\left[\overline{\theta}\middle|\,\underline{s}\right]\delta\left(\Pr[\overline{s}]\,\overline{V} + \Pr[\underline{s}]\,\underline{V}\right) + \Pr[\underline{\theta}|\,\underline{s}]\,\delta^{\underline{T}+1}\left(\Pr[\overline{s}]\,\overline{V} + \Pr[\underline{s}]\,\underline{V}\right)$. As discussed below, no firm has any incentives to deviate after receiving the signal $\underline{s}$ and thus the corresponding punishment vanishes in equilibrium, i.e., $\underline{T} = 0$. This implies that $\underline{V}' = \underline{V}$, where $\underline{V}$ is given by (2).

[11] Any price between $\underline{\theta}$ and $\overline{\theta}$ reduces the per period collusive profit, without affecting deviation incentives because the probability of zero demand does not change.

the signal $\underline{s}$ increases the belief that demand is low, it turns out that, in line with the rationale for the result in Remark 1, a firm's loss from deviation is relatively small and thus a price above the marginal cost conditional on the signal $\underline{s}$ would be excessively costly to enforce.

To sustain collusion, firms revert to the noncooperative equilibrium after at least one firm has obtained zero profit. The punishment phase lasts for a number of periods $\overline{T}^a(\cdot) > 0$ after receiving the signal $\overline{s}$ of high demand.[12] This is because the signal $\overline{s}$ reinforces the belief that the event of zero profit is attributable to price undercutting rather than to low demand. Clearly, no punishment is implemented after the signal $\underline{s}$ of low demand, i.e., $\underline{T}^a = 0$, because the collusive strategy dictates the price at the marginal cost and thus eliminates any deviation incentives.

### 4.3 Ex post information provision

We consider the collusive strategy such that each firm sets the static monopoly price $\overline{\theta}$ at the outset of the game and continues to do so as long as both firms obtained positive profits in the previous period. If at least one firm obtained zero profit in the previous period (which is common knowledge), firms revert to the noncooperative equilibrium during a punishment phase that lasts for $T_i \in \{\overline{T}, \underline{T}\}$ periods according to the signal $s_i \in \{\overline{s}, \underline{s}\}$, respectively.[13]

In the collusive phase, each firm charges the price at $\overline{\theta}$ and obtains a present discounted profit equal to

$$\begin{aligned} V &= \Pr\left[\overline{\theta}\right]\left(\frac{\overline{\theta}}{2} + \delta V\right) + \Pr\left[\underline{\theta}\right] V \left(\Pr\left[\overline{s}\right|\underline{\theta}\right]\delta^{\overline{T}+1} + \Pr\left[\underline{s}\right|\underline{\theta}\right]\delta^{\underline{T}+1}\right) \\ &= \frac{1}{2}\left(\frac{\overline{\theta}}{2} + \delta V\right) + \frac{1}{2} V\left[(1-\alpha)\,\delta^{\overline{T}+1} + \alpha\delta^{\underline{T}+1}\right]. \end{aligned} \tag{4}$$

When demand is high, which occurs with probability 1/2, firms equally share the entire collusive profit in the current period and continue to collude in the following period. Otherwise, they face zero demand and thus make zero profit, which triggers a punishment for $T_i \in \{\overline{T}, \underline{T}\}$ periods conditionally on the signal received $s_i \in \{\overline{s}, \underline{s}\}$.

A deviating firm slightly undercuts the collusive price $\overline{\theta}$ and appropriates the entire expected collusive profit in the current period, which leads to a punishment for $T_i \in \{\overline{T}, \underline{T}\}$ periods. The incentive constraint that ensures the sustainability of collusion writes as

$$\begin{aligned} V &\geq \Pr\left[\overline{\theta}\right]\overline{\theta} + V\left(\Pr\left[\overline{s}\right]\delta^{\overline{T}+1} + \Pr\left[\underline{s}\right]\delta^{\underline{T}+1}\right) \\ &= \frac{1}{2}\overline{\theta} + \frac{1}{2}V\left(\delta^{\overline{T}+1} + \delta^{\underline{T}+1}\right). \end{aligned} \tag{5}$$

We summarize the main results in the following lemma.

**Lemma 2** *Suppose that the state of demand can assume two values — i.e., $\theta_i \in \Theta_2 \triangleq \{\overline{\theta}, \underline{\theta}\}$. Then, under ex post information provision, in the most cooperative equilibrium, each firm sets the price at $\overline{\theta}$, provided that both firms obtained positive profits in the previous period. Otherwise, a punishment phase is triggered, during which each firm sets the price at the marginal cost for*

[12] When $\overline{T}^a(\cdot)$ is not an integer, a randomizing device can be used in order to determine whether the length of punishment is the smallest integer above $\overline{T}^a(\cdot)$ or the largest integer below $\overline{T}^a(\cdot)$, where the corresponding probabilities make the incentive constraint (3) binding.

[13] We refer to the discussion after Lemma 2 for the optimality of this collusive strategy.

*a number of periods equal to $\overline{T}^p(\alpha)$ after receiving the signal $\overline{s}$ and $\underline{T}^p$ after receiving the signal $\underline{s}$, where $\overline{T}^p(\alpha) > \underline{T}^p = 0$.*

As Lemma 2 indicates, under ex post information provision, collusion can be sustained at the static monopoly price $\overline{\theta}$.[14] The punishment phase is such that, after at least one firm has obtained zero profit, firms revert to the noncooperative equilibrium for a number of periods $\overline{T}^p(\cdot) > 0$ in response to the signal $\overline{s}$ of high demand. As discussed after Lemma 1 for the case of ex ante information provision, the signal $\overline{s}$ reinforces the conjecture that the event of zero profit arises from price undercutting rather than from low demand. Upon receiving the signal $\underline{s}$ of low demand, no punishment is implemented, i.e., $\underline{T}^p = 0$, because the signal $\underline{s}$ makes it more likely that the event of zero profit stems from low demand rather than from a deviation. Notably, for a given degree $\alpha$ of information accuracy, the length of punishment coincides under the two modes of information provision.[15] As punishment is enforced after at least one firm has obtained zero profit and the signal $\overline{s}$ has been received — which occurs with probability $\Pr[\underline{\theta}|\overline{s}]\Pr[\overline{s}]$ on the equilibrium path — firms establish the same punishment irrespective of the mode of information provision as long as they benefit from equally accurate information.

## 4.4 Equilibrium information provision

The information provider designs the accuracy of the information $\alpha \in [\alpha_0, 1]$ and determines the price for information $\rho$ in order to capture the firms' aggregate collusive profits net of their outside option.[16] As shown in Remark 1, collusion cannot be sustained in the absence of information provision, which brings the firms' outside option to zero. The information provider's profit maximization problem writes as

$$\max_{\alpha \in [\alpha_0, 1]} \Pi^c(\alpha) - C(\alpha), \tag{6}$$

where $\Pi^c(\alpha) \in \{\Pi^a(\alpha), \Pi^p(\alpha)\}$ identifies the firms' aggregate collusive profits under ex ante and ex post information provision respectively, and $C(\alpha) = c(\alpha - \alpha_0)$ represents the cost of information provision. In the following proposition, we consider the case of ex ante information provision.

**Proposition 1** *Suppose that the state of demand can assume two values — i.e., $\theta_i \in \Theta_2 \triangleq \{\overline{\theta}, \underline{\theta}\}$. Then, in the most cooperative equilibrium, ex ante information provision exhibits the following features: (i) for $c < \widetilde{c}$ full information is provided — i.e., $\alpha^a = 1$ — and (ii) for $c \geq \widetilde{c}$ no information is provided — i.e., $\alpha^a = \alpha_0$.*

Proposition 1 indicates that, under ex ante information provision, the equilibrium information disclosure policy possesses an intuitive 'bang-bang' property according to which firms

[14] Any price between $\underline{\theta}$ and $\overline{\theta}$ reduces the per period collusive profit, without affecting deviation incentives because the probability of zero demand does not change.

[15] We refer to the proofs of Lemmas 1 and 2 for technical details.

[16] Our analysis fully extends to different allocations of bargaining power in the negotiations between the information provider and the firms. Nothing substantial would change even in the presence of contractual externalities, for instance associated with firms' private information vis-à-vis the information provider.

obtain full information about the state of demand as long as the information technology is efficient enough. Otherwise, no information is supplied.

Now, we turn to the case of ex post information provision.

**Proposition 2** *Suppose that the state of demand can assume two values — i.e.,* $\theta_i \in \Theta_2 \triangleq \{\overline{\theta}, \underline{\theta}\}$. *Then, in the most cooperative equilibrium, ex post information provision exhibits the following features:* $(i)$ *for* $c \leq \underline{c}$ *full information is provided — i.e.,* $\alpha^p = 1$ *—* $(ii)$ *for* $\underline{c} < c < \overline{c}$ *partial information is provided — i.e.,* $\alpha^p\left(\overline{\theta}\right) \in (\alpha_0, 1)$ *— and* $(iii)$ *for* $c \geq \overline{c}$ *no information is provided — i.e.,* $\alpha^p = \alpha_0$.

Proposition 2 shows that the equilibrium information disclosure policy under ex post information provision is more nuanced than under ex ante information provision. Intuitively, firms receive full information about the state of demand, i.e., $\alpha^p = 1$, when the information technology is efficient enough. Differently from the case of ex ante information provision, there exists an intermediate range of efficiency of the information technology where the degree of information accuracy is distorted downward according to the magnitude of the collusive price $\overline{\theta}$, i.e., $\alpha^p\left(\overline{\theta}\right) \in (\alpha_0, 1)$. The information provider prefers to deliver partial information in order to curb the associated costs. Clearly, when information provision is excessively costly, no information is supplied.

We now compare the information provider's profits under the two modes of information provision.[17] Our results are formalized in the following proposition.

**Proposition 3** *Suppose that the state of demand can assume two values — i.e.,* $\theta_i \in \Theta_2 \triangleq \{\overline{\theta}, \underline{\theta}\}$. *Then, ex post information provision is at least as profitable as ex ante information provision. Specifically, it holds that* $(i)$ *for* $c \leq \underline{c}$ *the two modes of information provision are equally profitable and* $(ii)$ *for* $c > \underline{c}$ *ex post information provision is more profitable.*

As ex ante information provision allows firms to make not only the punishment strategy but also the pricing strategy contingent on the signal received, the results in Proposition 3 might appear prima facie counterintuitive. When the information technology is relatively efficient, i.e., $c \leq \min\{\widetilde{c}, \underline{c}\} = \underline{c}$, it follows from Propositions 1 and 2 that the equilibrium disclosure policy is fully informative irrespective of the mode of information provision, i.e., $\alpha^a = \alpha^p = 1$. Hence, firms perfectly learn the state of demand and collect the same collusive profits under the two modes of information provision. However, when the information technology is relatively inefficient, i.e., $c > \underline{c}$, full information still occurs under ex ante information provision, whereas partial information emerges under ex post information provision. To gain some insights, it is helpful to consider the same degree $\alpha$ of information accuracy under the two modes of information provision. We find that, for any $\alpha < 1$, the firms' aggregate collusive profits are higher under ex post information provision.[18] Despite the same punishment phase, the two modes of information provision crucially differ in the collusive phase. This occurs when under ex ante information provision firms set their prices at the marginal cost after receiving the signal

[17] For the sake of concreteness, we omit the uninteresting extreme case where the information technology is so inefficient that no information is provided irrespective of the mode of information provision.

[18] We refer to the proofs of Lemmas 1 and 2 for technical details.

$\underline{s}$ but demand is indeed high, whereas under ex post information provision the static monopoly price $\overline{\theta}$ is charged. Information provision before price setting harms firms because it makes deviation so attractive conditionally on the signal $\underline{s}$ that the entire current collusive profits are dissipated through marginal cost pricing. Thus, ex ante information provision generates a profit dissipation effect that exacerbates the cost of distorting the accuracy of information. When the information technology is relatively inefficient, i.e., $c > \underline{c}$, the information provider mitigates the precision of information only under ex post information provision in order to curb the associated costs. As established in Proposition 3, in this case ex post information provision enhances the profitability of collusion. Below, we show that the results in Proposition 3 need to be substantially qualified in the presence of more than two demand states.

# 5 Three states of demand

We now consider a framework where in every period the demand state can assume three values, high, intermediate, or low, i.e., $\theta_i \in \Theta_3 \triangleq \left\{\overline{\theta}, \widehat{\theta}, \underline{\theta}\right\}$, where $\overline{\theta} > \widehat{\theta} > \underline{\theta}$. As before, demand states are independently and identically distributed over time, and they share equal probabilities. An experiment is characterized by a signal $s_i \in S_3 \triangleq \{\overline{s}, \widehat{s}, \underline{s}\}$. Without loss of insights, we impose the following assumption.

**Assumption 1** $\Pr[\theta_j | s_i] = \Pr[\theta_k | s_i]$, for $i \neq j \neq k$ and $i \neq k$.

According to Assumption 1, after receiving the signal $s_i$, firms revise the probability of the state of demand $\theta_i$ but still assign equal probabilities to the remaining states of demand, as prior to receiving the signal $s_i$. This reflects the natural idea that the signal $s_i$ is a 'good news' about the state of demand $\theta_i$ and reinforces the belief in $\theta_i$ but does not alter the relative probabilities of the other states of demand. The parameter $\alpha = \Pr[s_i | \theta_i] \in [\alpha_0, 1]$ measures the degree of information accuracy, where $\alpha_0 = \Pr[\theta_i] = 1/3$. It follows from Assumption 1 that $\Pr[s_i | \theta_j] = (1 - \alpha)/2$, for $i \neq j$.

## 5.1 No information

As in the setting with two demand states, we start our analysis with the benchmark case where no information is provided. We consider the collusive strategy according to which each firm charges the collusive price at the outset of the game and continues to do so as long as both firms received positive profits in the previous period. Otherwise, a punishment phase is triggered, which lasts for $T$ periods. Suppose first that the collusive price is $\overline{\theta}$. A firm's present discounted profit in the collusive phase is given by

$$V = \frac{1}{3}\left(\frac{\overline{\theta}}{2} + \delta V\right) + \frac{2}{3}\delta^{T+1} V.$$

In the current period, firms equally share the entire collusive profit when facing high demand, which occurs with probability 1/3. In this case, they continue to collude in the following period. With complementary probability 2/3, firms face zero demand and thus make zero profit. This yields a punishment phase that lasts for $T$ periods, at the end of which firms start colluding again.

A deviating firm sets the price below $\overline{\theta}$ in order to capture the expected static monopoly profit. Slightly undercutting the price $\overline{\theta}$ constitutes an optimal deviation when $\overline{\theta}$ corresponds to the static monopoly price, i.e., $\overline{\theta} > \widetilde{\theta}^m$, where $\widetilde{\theta}^m \triangleq 2\widehat{\theta}$. Otherwise, an optimal deviation is to set the price at $\widehat{\theta}$. This generates a punishment for $T$ periods. The incentive constraint that ensures the sustainability of collusion writes as

$$V \geq \begin{cases} \frac{1}{3}\overline{\theta} + \delta^{T+1}V & \text{if} \quad \overline{\theta} > \widetilde{\theta}^m \\ \\ \frac{2}{3}\widehat{\theta} + \delta^{T+1}V & \text{if} \quad \overline{\theta} \leq \widetilde{\theta}^m. \end{cases}$$

Straightforward calculations reveal that the incentive constraint is violated. Hence, collusion cannot be sustained with a price equal to $\overline{\theta}$.[19]

We now turn to the collusive strategy that prescribes the price $\widehat{\theta}$. A firm's present discounted profit in the collusive phase is given by

$$V = \frac{2}{3}\left(\frac{\widehat{\theta}}{2} + \delta V\right) + \frac{1}{3}\delta^{T+1}V. \tag{7}$$

In the current period, firms equally share the entire collusive profit when facing either high or intermediate demand, which occurs with probability 2/3. In this case, they continue to collude in the following period. With complementary probability 1/3, firms face zero demand and thus make zero profit, which triggers a punishment for $T$ periods.

The incentive constraint that ensures the sustainability of collusion writes as

$$V \geq \frac{2}{3}\widehat{\theta} + \delta^{T+1}V. \tag{8}$$

We summarize the main results in the following remark.

**Remark 2** *Suppose that the state of demand can assume three values — i.e., $\theta_i \in \Theta_3 \triangleq \left\{\overline{\theta}, \widehat{\theta}, \underline{\theta}\right\}$. Then, in the absence of information provision, in the most cooperative equilibrium, each firm sets the price at $\widehat{\theta}$, provided that both firms obtained positive profits in the previous period. Otherwise, a punishment phase is triggered, during which each firm sets the price at the marginal cost for a number of periods $T^n > 0$.*

With three demand states, firms can sustain collusion even in the absence of information provision by setting the price at $\widehat{\theta}$.[20] The collusive outcome cannot be achieved at any price above $\widehat{\theta}$ because the event of zero demand is so likely to occur in the collusive phase that firms prefer to deviate. The price $\widehat{\theta}$ allows firms to support collusion through a reduction in the probability of zero demand. For $\overline{\theta} > \widetilde{\theta}^m$, collusion is enforced at the cost of a price distortion below the static monopoly level $\overline{\theta}$. Clearly, the opportunity to collude even in the absence of

[19] Any price lying between $\widehat{\theta}$ and $\overline{\theta}$ cannot be sustained, either. This price reduces the per period collusive profit without affecting deviation incentives because the probability of low demand does not change.

[20] Any price between $\underline{\theta}$ and $\widehat{\theta}$ reduces the per period collusive profit without affecting deviation incentives.

information provision endows firms with a positive outside option when information is supplied. As in the case of two demand states, this does not affect the comparison between the two modes of information provision.

### 5.2 Ex ante information provision

We consider the collusive strategy according to which in every period each firm sets the price at $\overline{\theta}$ after receiving the signal $\overline{s}$, at $\widehat{\theta}$ after the signal $\widehat{s}$, and at the marginal cost after the signal $\underline{s}$. If at least one firm obtained zero profit in the previous period (which is common knowledge), the market enters a punishment phase, whose duration $T_i \in \left\{\overline{T}, \widehat{T}, \underline{T}\right\}$ is contingent on the signal received $s_i \in \{\overline{s}, \widehat{s}, \underline{s}\}$, respectively. During the punishment phase, firms revert to the noncooperative equilibrium by setting their prices at the marginal cost.[21]

We define by $V_i \in \left\{\overline{V}, \widehat{V}, \underline{V}\right\}$ the present discounted profit of a firm in the collusive phase after receiving the signal $s_i \in \{\overline{s}, \widehat{s}, \underline{s}\}$, respectively. As with two demand states, the degree of information accuracy is $\alpha = \Pr[s_i | \theta_i] = \Pr[\theta_i | s_i]$. Upon receiving the signal $\overline{s}$, each firm charges the price at $\overline{\theta}$ and obtains a present discounted profit equal to

$$\begin{aligned}\overline{V} &= \Pr\left[\overline{\theta}\middle|\,\overline{s}\right]\left[\frac{\overline{\theta}}{2} + \delta\left(\Pr[\overline{s}]\,\overline{V} + \Pr[\widehat{s}]\,\widehat{V} + \Pr[\underline{s}]\,\underline{V}\right)\right] \\ &\quad + \left(\Pr\left[\widehat{\theta}\middle|\,\overline{s}\right] + \Pr\left[\underline{\theta}\middle|\,\overline{s}\right]\right)\delta^{\overline{T}+1}\left(\Pr[\overline{s}]\,\overline{V} + \Pr[\widehat{s}]\,\widehat{V} + \Pr[\underline{s}]\,\underline{V}\right) \\ &= \alpha\left(\frac{\overline{\theta}}{2} + \delta\frac{\overline{V} + \widehat{V} + \underline{V}}{3}\right) + (1-\alpha)\,\delta^{\overline{T}+1}\frac{\overline{V} + \widehat{V} + \underline{V}}{3}. \end{aligned} \tag{9}$$

In the current period, firms equally share the entire collusive profit when facing high demand, which occurs with probability $\Pr\left[\overline{\theta}\middle|\,\overline{s}\right] = \alpha$. In this case, they continue to collude in the following period and obtain the expected present discounted profit $\left(\overline{V} + \widehat{V} + \underline{V}\right)/3$. With complementary probability $\Pr\left[\widehat{\theta}\middle|\,\overline{s}\right] + \Pr\left[\underline{\theta}\middle|\,\overline{s}\right] = 1-\alpha$, firms face zero demand and thus make zero profit. This generates a punishment phase for $\overline{T}$ periods. At the end of the punishment phase, firms start colluding again.

Upon receiving the signal $\widehat{s}$, each firm charges the price at $\widehat{\theta}$ and obtains a present discounted profit equal to

$$\begin{aligned}\widehat{V} &= \left(\Pr\left[\overline{\theta}\middle|\,\widehat{s}\right] + \Pr\left[\widehat{\theta}\middle|\,\widehat{s}\right]\right)\left[\frac{\widehat{\theta}}{2} + \delta\left(\Pr[\overline{s}]\,\overline{V} + \Pr[\widehat{s}]\,\widehat{V} + \Pr[\underline{s}]\,\underline{V}\right)\right] \\ &\quad + \Pr\left[\underline{\theta}\middle|\,\widehat{s}\right]\delta^{\widehat{T}+1}\left(\Pr[\overline{s}]\,\overline{V} + \Pr[\widehat{s}]\,\widehat{V} + \Pr[\underline{s}]\,\underline{V}\right) \\ &= \frac{1+\alpha}{2}\left(\frac{\widehat{\theta}}{2} + \delta\frac{\overline{V} + \widehat{V} + \underline{V}}{3}\right) + \frac{1-\alpha}{2}\delta^{\widehat{T}+1}\frac{\overline{V} + \widehat{V} + \underline{V}}{3}. \end{aligned} \tag{10}$$

In the current period, firms equally share the entire collusive profit when facing either high or intermediate demand. This occurs with probability $\Pr\left[\overline{\theta}\middle|\,\widehat{s}\right] + \Pr\left[\widehat{\theta}\middle|\,\widehat{s}\right] = (1-\alpha)/2 + \alpha = (1+\alpha)/2$. In this case, firms continue to collude in the following period. With complementary probability $\Pr\left[\underline{\theta}\middle|\,\widehat{s}\right] = (1-\alpha)/2$, firms face zero demand and thus make zero profit, which

[21] We refer to the discussion after Lemma 3 for the optimality of this collusive strategy.

triggers a punishment for $\widehat{T}$ periods.

Upon receiving the signal $\underline{s}$, each firm charges the price at the marginal cost and obtains a present discounted profit equal to

$$\underline{V} = \delta^{\underline{T}+1}\left(\Pr[\overline{s}]\,\overline{V} + \Pr[\widehat{s}]\,\widehat{V} + \Pr[\underline{s}]\,\underline{V}\right)$$
$$= \delta^{\underline{T}+1}\frac{\overline{V}+\widehat{V}+\underline{V}}{3}, \tag{11}$$

where $\underline{T}$ is the duration of the punishment phase conditional on the signal $\underline{s}$, which occurs after making zero profit.

Given the signal $\overline{s}$, a deviating firm slightly undercuts the collusive price $\overline{\theta}$ and appropriates the entire expected collusive profit in the current period.[22] This generates a punishment phase that lasts for $\overline{T}$ periods. The incentive constraint that ensures the sustainability of collusion writes as

$$\overline{V} \geq \Pr\left[\overline{\theta}\middle|\,\overline{s}\right]\overline{\theta} + \delta^{\overline{T}+1}\left(\Pr[\overline{s}]\,\overline{V} + \Pr[\widehat{s}]\,\widehat{V} + \Pr[\underline{s}]\,\underline{V}\right)$$
$$= \alpha\overline{\theta} + \delta^{\overline{T}+1}\frac{\overline{V}+\widehat{V}+\underline{V}}{3}. \tag{12}$$

Along the same lines, given the signal $\widehat{s}$, a deviating firm slightly undercuts the collusive price $\widehat{\theta}$ and appropriates the entire expected collusive profit in the current period. This leads to a punishment for $\widehat{T}$ periods. The incentive constraint that ensures the sustainability of collusion writes as

$$\widehat{V} \geq \left(\Pr\left[\overline{\theta}\middle|\,\widehat{s}\right] + \Pr\left[\widehat{\theta}\middle|\,\widehat{s}\right]\right)\widehat{\theta} + \delta^{\widehat{T}+1}\left(\Pr[\overline{s}]\,\overline{V} + \Pr[\widehat{s}]\,\widehat{V} + \Pr[\underline{s}]\,\underline{V}\right)$$
$$= \frac{1+\alpha}{2}\widehat{\theta} + \delta^{\widehat{T}+1}\frac{\overline{V}+\widehat{V}+\underline{V}}{3}. \tag{13}$$

We summarize the main results in the following lemma.

**Lemma 3** *Suppose that the state of demand can assume three values — i.e., $\theta_i \in \Theta_3 \triangleq \left\{\overline{\theta}, \widehat{\theta}, \underline{\theta}\right\}$. Then, under ex ante information provision, in the most cooperative equilibrium, each firm sets the price at $\overline{\theta}$ after receiving the signal $\overline{s}$, at $\widehat{\theta}$ after the signal $\widehat{s}$, and at the marginal cost after the signal $\underline{s}$, provided that both firms obtained positive profits in the previous period. Otherwise, a punishment phase is triggered, during which each firm sets the price at the marginal cost for a number of periods equal to $\overline{T}^a(\alpha)$ after receiving the signal $\overline{s}$, $\widehat{T}^a(\alpha)$ after the signal $\widehat{s}$, and $\underline{T}^a$ after the signal $\underline{s}$, where $\overline{T}^a(\alpha) > \widehat{T}^a(\alpha) > \underline{T}^a = 0$.*

Lemma 3 shows that, under ex ante information provision, firms can sustain collusion through a strategy that makes the collusive price contingent on the signal received.[23] Following the same rationale as in the setting with two demand states, the price is set at the marginal

[22]Slightly undercutting the collusive price $\overline{\theta}$ constitutes an optimal deviation as long as information is sufficiently accurate. In particular, it must hold that, conditionally on the signal $\overline{s}$, the collusive price $\overline{\theta}$ maximizes the current expected profit, i.e., $\Pr\left[\overline{\theta}\middle|\,\overline{s}\right]\overline{\theta} > \left(\Pr\left[\overline{\theta}\middle|\,\overline{s}\right] + \Pr\left[\widehat{\theta}\middle|\,\overline{s}\right]\right)\widehat{\theta}$, which yields $\alpha > \widehat{\theta}/\left(2\overline{\theta}-\widehat{\theta}\right)$.

[23]Intuitively, this strategy ensures the highest present discounted collusive profit as long as information is sufficiently precise. As it will be clear below, the information provider has stronger incentives to deliver accurate information with multiple demand states.

cost after receiving the signal $\underline{s}$ in order to prevent any deviation. The punishment phase also depends on the signal received. After at least one firm has obtained zero profit, punishment is longer in response to the signal $\overline{s}$ than in response to the signal $\widehat{s}$, i.e., $\overline{T}^a(\cdot) > \widehat{T}^a(\cdot)$. Note from (12) and (13) that a deviation is more attractive with the collusive price $\overline{\theta}$ conditional on the signal $\overline{s}$ rather than with the collusive price $\widehat{\theta}$ conditional on the signal $\widehat{s}$, which calls for a more severe punishment.[24] Clearly, no punishment is implemented after receiving the signal $\underline{s}$, i.e., $\underline{T}^a = 0$, because marginal cost pricing removes any deviation incentives.

### 5.3 Ex post information provision

For the sake of exposition, we disentangle the analysis according to the magnitude of the high state of demand $\overline{\theta}$. First, suppose that $\overline{\theta} > \widetilde{\theta}^m$, where $\widetilde{\theta}^m \triangleq 2\widehat{\theta}$ (see Section 5.1). A price candidate for the most cooperative equilibrium is the static monopoly price $\overline{\theta}$.[25] In the collusive phase, each firm charges the price at $\overline{\theta}$ and obtains a present discounted profit equal to

$$\begin{aligned} V &= \Pr\left[\overline{\theta}\right]\left(\frac{\overline{\theta}}{2} + \delta V\right) + \Pr\left[\widehat{\theta}\right] V\left(\Pr\left[\overline{s}\middle|\,\widehat{\theta}\right]\delta^{\overline{T}+1} + \Pr\left[\widehat{s}\middle|\,\widehat{\theta}\right]\delta^{\widehat{T}+1} + \Pr\left[\underline{s}\middle|\,\widehat{\theta}\right]\delta^{\underline{T}+1}\right) \\ &\quad + \Pr\left[\underline{\theta}\right] V\left(\Pr\left[\overline{s}\middle|\,\underline{\theta}\right]\delta^{\overline{T}+1} + \Pr\left[\widehat{s}\middle|\,\underline{\theta}\right]\delta^{\widehat{T}+1} + \Pr\left[\underline{s}\middle|\,\underline{\theta}\right]\delta^{\underline{T}+1}\right) \\ &= \frac{1}{3}\left(\frac{\overline{\theta}}{2} + \delta V\right) + \frac{1}{3} V\left[(1-\alpha)\,\delta^{\overline{T}+1} + \frac{1+\alpha}{2}\delta^{\widehat{T}+1} + \frac{1+\alpha}{2}\delta^{\underline{T}+1}\right]. \end{aligned} \tag{14}$$

When demand is high, which occurs with probability 1/3, firms equally share the entire collusive profit in the current period and continue to collude in the following period. Otherwise, they face zero demand and thus make zero profit, which leads to a punishment whose duration $T_i \in \left\{\overline{T}, \widehat{T}, \underline{T}\right\}$ is contingent on the signal received $s_i \in \{\overline{s}, \widehat{s}, \underline{s}\}$, respectively.

A deviating firm slightly undercuts the collusive price $\overline{\theta}$ (which corresponds to the static monopoly price as $\overline{\theta} > \widetilde{\theta}^m$) and appropriates the entire expected collusive profit in the current period. This triggers a punishment for $T_i \in \left\{\overline{T}, \widehat{T}, \underline{T}\right\}$ periods. The incentive constraint that ensures the sustainability of collusion writes as

$$\begin{aligned} V &\geq \Pr\left[\overline{\theta}\right]\overline{\theta} + V\left(\Pr\left[\overline{s}\right]\delta^{\overline{T}+1} + \Pr\left[\widehat{s}\right]\delta^{\widehat{T}+1} + \Pr\left[\underline{s}\right]\delta^{\underline{T}+1}\right) \\ &= \frac{1}{3}\overline{\theta} + \frac{1}{3}V\left(\delta^{\overline{T}+1} + \delta^{\widehat{T}+1} + \delta^{\underline{T}+1}\right). \end{aligned} \tag{15}$$

We summarize the main results in the following lemma.

**Lemma 4** *Suppose that the state of demand can assume three values — i.e., $\theta_i \in \Theta_3 \triangleq \left\{\overline{\theta}, \widehat{\theta}, \underline{\theta}\right\}$. Then, for $\overline{\theta} > \widetilde{\theta}^m$, under ex post information provision, there exists a candidate for the most cooperative equilibrium such that each firm sets the price at $\overline{\theta}$, provided that both firms obtained positive profits in the previous period. Otherwise, a punishment phase is triggered, during which each firm sets the price at the marginal cost for a number of periods equal*

[24] This holds as long as information is sufficiently precise that slightly undercutting the collusive price $\overline{\theta}$ constitutes an optimal deviation, i.e., $\alpha > \widehat{\theta}/\left(2\overline{\theta} - \widehat{\theta}\right)$, as previously discussed.

[25] Provided that the most profitable collusive price is above $\widehat{\theta}$, this price must be $\overline{\theta}$. Any price between $\widehat{\theta}$ and $\overline{\theta}$ reduces the per period collusive profit without affecting deviation incentives because the probability of low demand does not change. We refer to Lemma 4 for further details.

*to $\overline{T}^p(\alpha)$ after receiving the signal $\overline{s}$, $\widehat{T}^p$ after the signal $\widehat{s}$, and $\underline{T}^p$ after the signal $\underline{s}$, where $\overline{T}^p(\alpha) > \widehat{T}^p = \underline{T}^p = 0$.*

As Lemma 4 indicates, for $\overline{\theta} > \widetilde{\theta}^m$, a collusive strategy dictates the price at the static monopoly level $\overline{\theta}$. Collusion is sustained through a punishment phase that commences after at least one firm has received zero profit, conditionally on the signal $\overline{s}$ of high demand, i.e., $\overline{T}^p(\cdot) > 0$. Otherwise, no punishment takes place, i.e., $\widehat{T}^p = \underline{T}^p = 0$. This is because the signals $\widehat{s}$ and $\underline{s}$ reinforce the belief that the event of zero profit stems from a level of consumers' willingness to pay below the collusive price rather than from price undercutting. Notably, the collusive strategy formalized in Lemma 4 constitutes a candidate for the most cooperative equilibrium. As explained below, for $\overline{\theta} > \widetilde{\theta}^m$, firms may prefer to sustain collusion by reducing the price at $\widehat{\theta}$, which lies below the static monopoly level $\overline{\theta}$.

Now, suppose that $\overline{\theta} \leq \widetilde{\theta}^m$. In the most cooperative equilibrium, the collusive price unambiguously reflects the static monopoly level $\widehat{\theta}$.[26] In the collusive phase, each firm charges the price at $\widehat{\theta}$ and obtains a present discounted profit equal to

$$\begin{aligned} V &= \left(\Pr\left[\overline{\theta}\right] + \Pr\left[\widehat{\theta}\right]\right)\left(\frac{\widehat{\theta}}{2} + \delta V\right) + \Pr\left[\underline{\theta}\right] V \left(\Pr\left[\overline{s}\middle|\, \underline{\theta}\right]\delta^{\overline{T}+1} + \Pr\left[\widehat{s}\middle|\, \underline{\theta}\right]\delta^{\widehat{T}+1} + \Pr\left[\underline{s}\middle|\, \underline{\theta}\right]\delta^{\underline{T}+1}\right) \\ &= \frac{2}{3}\left(\frac{\widehat{\theta}}{2} + \delta V\right) + \frac{1}{3} V \left(\frac{1-\alpha}{2}\delta^{\overline{T}+1} + \frac{1-\alpha}{2}\delta^{\widehat{T}+1} + \alpha\delta^{\underline{T}+1}\right). \end{aligned} \tag{16}$$

When demand is at least at the intermediate level, which occurs with probability $2/3$, firms equally share the entire collusive profit in the current period and continue to collude in the following period. Otherwise, they face zero demand and thus make zero profit, which generates a punishment phase for $T_i \in \left\{\overline{T}, \widehat{T}, \underline{T}\right\}$ periods according to the signal received $s_i \in \{\overline{s}, \widehat{s}, \underline{s}\}$, respectively.

A deviating firm slightly undercuts the collusive price $\widehat{\theta}$ and appropriates the entire expected collusive profit in the current period, which generates a punishment for $T_i \in \left\{\overline{T}, \widehat{T}, \underline{T}\right\}$ periods. The incentive constraint that ensures the sustainability of collusion writes as

$$\begin{aligned} V &\geq \left(\Pr\left[\overline{\theta}\right] + \Pr\left[\widehat{\theta}\right]\right)\widehat{\theta} + V\left(\Pr\left[\overline{s}\right]\delta^{\overline{T}+1} + \Pr\left[\widehat{s}\right]\delta^{\widehat{T}+1} + \Pr\left[\underline{s}\right]\delta^{\underline{T}+1}\right) \\ &= \frac{2}{3}\widehat{\theta} + \frac{1}{3}V\left(\delta^{\overline{T}+1} + \delta^{\widehat{T}+1} + \delta^{\underline{T}+1}\right). \end{aligned} \tag{17}$$

We summarize the main results in the following lemma.

**Lemma 5** *Suppose that the state of demand can assume three values — i.e., $\theta_i \in \Theta_3 \triangleq \left\{\overline{\theta}, \widehat{\theta}, \underline{\theta}\right\}$. Then, for $\overline{\theta} \leq \widetilde{\theta}^m$, under ex post information provision, in the most cooperative equilibrium, each firm sets the price at $\widehat{\theta}$, provided that both firms obtained positive profits in the previous period. Otherwise, a punishment phase is triggered, during which each firm sets the price at the marginal cost for a number of periods equal to $\overline{T}^p(\alpha)$ after receiving the signal $\overline{s}$, $\widehat{T}^p(\alpha)$ after the signal $\widehat{s}$, and $\underline{T}^p$ after the signal $\underline{s}$, where $\overline{T}^p(\alpha) > 0$, $\widehat{T}^p(\alpha) > 0$, and $\underline{T}^p = 0$. For $\overline{\theta} > \widetilde{\theta}^m$, such a collusive strategy is a candidate for the most cooperative equilibrium.*

[26]Intuitively, firms do not gain from setting any price above $\widehat{\theta}$, which reduces the per period collusive profit and magnifies deviation incentives. We refer to Lemma 5 for further details.

Lemma 5 indicates that, for $\overline{\theta} \leq \widetilde{\theta}^m$, in the most cooperative equilibrium, the collusive strategy prescribes the static monopoly price $\widehat{\theta}$.[27] Notably, for $\overline{\theta} > \widetilde{\theta}^m$, firms may still prefer this collusive strategy. The benefit of charging a price lower than the static monopoly level $\overline{\theta}$ stems from the reduction in the probability of zero demand in the collusive phase, which makes collusion more attractive and thus mitigates deviation incentives. After at least one firm has obtained zero profit, the punishment periods $\overline{T}^p(\cdot)$ and $\widehat{T}^p(\cdot)$ respectively conditional on the signals $\overline{s}$ and $\widehat{s}$ are designed in order to establish the aggregate length of punishment such that $\delta^{\overline{T}^p+1} + \delta^{\widehat{T}^p+1}$ makes the incentive constraint (17) binding.[28] When the collusive price $\widehat{\theta}$ is charged, the signals $\overline{s}$ and $\widehat{s}$ are equally informative for the punishment strategy, because they increase to the same extent the belief that the event of zero profit arises from price undercutting rather than from low demand.

It is worth noting that ex post information provision prevents firms from making the pricing strategy contingent on the signal about the demand state but allows a more parsimonious punishment strategy with respect to ex ante information provision. Specifically, we know from Lemma 3 that, under ex ante information provision, firms set three possible price levels according to the signal received. This requires three different signals in order to determine the punishment phase. As established in Lemmas 4 and 5, under ex post information provision, it suffices for the sustainability of collusion to render the punishment length conditional only upon two signals, which indicate whether consumers' willingness to pay is at least equal to the collusive price or falls below it.

### 5.4 Equilibrium information provision

The information provider designs the accuracy of the experiment $\alpha \in [\alpha_0, 1]$ and extracts through the price $\rho$ the firms' aggregate collusive profits net of their outside option. As stated in Remark 2, in the absence of information provision, firms can obtain positive collusive profits, i.e., $\Pi^n > 0$. The information provider's profit maximization problem writes as

$$\max_{\alpha \in [\alpha_0, 1]} \Pi^c(\alpha) - C(\alpha) - \Pi^n, \tag{18}$$

where $\Pi^c(\alpha) \in \{\Pi^a(\alpha), \Pi^p(\alpha)\}$ denotes the firms' aggregate collusive profits under ex ante and ex post information provision respectively, and $C(\alpha) = c(\alpha - \alpha_0)$ represents the cost of information provision. First, we consider the case of ex ante information provision.

**Proposition 4** *Suppose that the state of demand can assume three values — i.e., $\theta_i \in \Theta_3 \triangleq \left\{\overline{\theta}, \widehat{\theta}, \underline{\theta}\right\}$. Then, in the most cooperative equilibrium, ex ante information provision exhibits the following features: (i) for $c < \widetilde{c}'$ full information is provided — i.e., $\alpha^a = 1$ — and (ii) for $c \geq \widetilde{c}'$ no information is provided — i.e., $\alpha^a = \alpha_0$.*

Proposition 4 indicates that, as in the setting with two demand states, the equilibrium information disclosure policy under ex ante information provision perfectly reveals the state of

[27] Any price between $\underline{\theta}$ and $\widehat{\theta}$ reduces the per period collusive profit without affecting deviation incentives because the probability of low demand does not change.

[28] Technical details can be found in the proof of Lemma 5.

demand as long as the information technology is relatively efficient. Otherwise, no information is supplied. The threshold $\widetilde{c}'$ for the (unit) cost of information provision $c$ is lower than the threshold $\widetilde{c}$ with two demand states (formalized in Proposition 1) because firms have the opportunity to collude even in the absence of information provision.

Now, we turn to the case of ex post information provision. The analysis is disentangled according to the magnitude of the high state of demand $\overline{\theta}$.

**Proposition 5** *Suppose that the state of demand can assume three values — i.e., $\theta_i \in \Theta_3 \triangleq \left\{\overline{\theta}, \widehat{\theta}, \underline{\theta}\right\}$. Then, in the most cooperative equilibrium, ex post information provision exhibits the following features:*

*(a) if $\overline{\theta} \geq \overline{\theta}'$, it holds that (i) for $c \leq \underline{c}'$ full information is provided — i.e., $\alpha^p = 1$ — (ii) for $\underline{c}' < c < \overline{c}'$ partial information is provided — i.e., $\alpha^p\left(\overline{\theta}\right) \in (\alpha_0, 1)$ — and (iii) for $c \geq \overline{c}'$ no information is provided — i.e., $\alpha^p = \alpha_0$;*

*(b) if $\overline{\theta} \in \left(\widetilde{\theta}^m, \overline{\theta}'\right)$, it holds that (i) for $c \leq \underline{c}''$ full information is provided — i.e., $\alpha^p = 1$ — (ii) for $\underline{c}'' < c < \overline{c}''$ partial information is provided — i.e., $\alpha^p\left(\widehat{\theta}\right) \in (\alpha_0, 1)$ — and (iii) for $c \geq \overline{c}''$ no information is provided — i.e., $\alpha^p = \alpha_0$;*

*(c) if $\overline{\theta} \leq \widetilde{\theta}^m$, it holds that (i) for $c \leq \underline{c}'''$ full information is provided — i.e., $\alpha^p = 1$ — (ii) for $\underline{c}''' < c < \overline{c}'''$ partial information is provided — i.e., $\alpha^p\left(\widehat{\theta}\right) \in (\alpha_0, 1)$ — and (iii) for $c \geq \overline{c}'''$ no information is provided — i.e., $\alpha^p = \alpha_0$.*

Proposition 5 shows that, if the information technology is efficient enough, in equilibrium the information provider delivers a fully informative signal, whereas for intermediate values of efficiency of the information technology it mitigates the degree of information accuracy in order to curb the associated costs. Differently from the setting with two demand states, the information disclosure policy and the firms' collusive strategy depend on the magnitude of the high state of demand $\overline{\theta}$. For sufficiently large values of the high state of demand, i.e., $\overline{\theta} \geq \overline{\theta}'$, where $\overline{\theta}' > \widetilde{\theta}^m$, firms charge the static monopoly price $\overline{\theta}$, as formalized in Lemma 4. If the value of the high state of demand is not so large but continues to reflect the static monopoly level, i.e., $\overline{\theta} \in \left(\widetilde{\theta}^m, \overline{\theta}'\right)$, the collusive price still corresponds to $\overline{\theta}$ as long as the information technology is efficient enough that firms receive full information. For intermediate values of efficiency of the information technology, the information provider delivers partial information and firms adopt the collusive strategy according to which the price is reduced at $\widehat{\theta}$, which lies below the static monopoly level $\overline{\theta}$, as established in Lemma 5. Given that partial information makes collusion more difficult to enforce, firms prefer to coordinate on a lower collusive price, which decreases the probability of zero demand in the collusive phase and thus mitigates deviation incentives. Notably, with partial information, the equilibrium degree of information accuracy is larger at the collusive price $\overline{\theta}$ than at collusive price $\widehat{\theta}$, i.e., $\alpha^p\left(\overline{\theta}\right) > \alpha^p\left(\widehat{\theta}\right)$.[29] The reason is that a higher collusive price calls for more precise information in order to be implemented. When the value of the high state of demand is sufficiently small that the static monopoly price becomes $\widehat{\theta}$, i.e., $\overline{\theta} \leq \widetilde{\theta}^m$, it follows from Lemma 5 that firms set the collusive price at $\widehat{\theta}$ even when the information technology is relatively efficient.

[29] Technical details can be found in the proof of Proposition 5.

We are now in a position to compare the two modes of information provision.[30] Our results are formalized in the following proposition.

**Proposition 6** *Suppose that the state of demand can assume three values — i.e., $\theta_i \in \Theta_3 \triangleq \left\{\overline{\theta}, \widehat{\theta}, \underline{\theta}\right\}$. Then, in the most cooperative equilibrium,*

*(a) if $\overline{\theta} \geq \overline{\theta}'$, ex ante information provision is more profitable than ex post information provision if and only if $c < c'$;*

*(b) if $\overline{\theta} \in \left[\overline{\theta}'', \overline{\theta}'\right)$, ex ante information provision is more profitable than ex post information provision;*

*(c) if $\overline{\theta} < \overline{\theta}''$, ex ante information provision is more profitable than ex post information provision if and only if $c < c''$.*

The results in Proposition 6 substantially differ from those in Proposition 3. With three states of demand, the two modes of information provision generate a trade-off between a price sophistication effect and a profit dissipation effect. When the information technology is efficient enough to generate full information irrespective of the mode of information provision, we find that ex ante information provision makes firms unambiguously better off. Providing information prior to price setting allows firms to select the collusive price at $\overline{\theta}$ or $\widehat{\theta}$ according to the signal received. Under ex post information provision, however, the collusive price is set at $\overline{\theta}$ or $\widehat{\theta}$ without any additional information. Hence, if the collusive price under ex post information provision is $\overline{\theta}$ but intermediate demand materializes, firms make positive profits only under ex ante information provision. Alternatively, if the collusive price under ex post information provision is $\widehat{\theta}$ but high demand occurs, ex ante information provision enhances firms' profits. As ex ante information provision enables firms to conduct a more refined pricing strategy, there exists a price sophistication effect that improves the profitability of collusion compared to ex post information provision.

When the cost of the information technology is not too small, it follows from Propositions 4 and 5 that under ex ante information provision the equilibrium disclosure policy is still fully informative, whereas partial information emerges under ex post information provision. As in the setting with two demand states, there exists a profit dissipation effect that inflates the cost of distorting information under ex ante information provision. Given that firms set their prices at the marginal cost after receiving the signal $\underline{s}$ of low demand, imprecise information implies that firms forgo some profits in the collusive phase when demand is indeed not low. Hence, as a result of the trade-off between the price sophistication effect and the profit dissipation effect, we obtain that, under certain circumstances, ex post information provision increases the profitability of collusion, provided that the technology of information is relatively inefficient and thus firms receive partial information. As Proposition 6 indicates, this occurs for non-monotonic values of the high state of demand $\overline{\theta}$ such that $\overline{\theta}$ is either high enough or low enough, i.e., $\overline{\theta} \geq \overline{\theta}'$ or $\overline{\theta} < \overline{\theta}''$. For intermediate values of $\overline{\theta}$, i.e., $\overline{\theta} \in \left[\overline{\theta}'', \overline{\theta}'\right)$, ex ante information provision is more profitable irrespective of the magnitude of the cost of information provision. This is because

[30] For the sake of concreteness, as in the setting with two demand states, we omit the uninteresting extreme case where the information technology is so inefficient that no information is supplied irrespective of the mode of information provision.

ex ante information provision allows firms to select an additional valuable price (contingent on the signal received), which magnifies the price sophistication effect. To gain some insights, as $\overline{\theta}'' < \widetilde{\theta}^m$, we split the interval $\left[\overline{\theta}'', \overline{\theta}'\right)$ into two subintervals. First, we consider the case $\overline{\theta} \in \left(\widetilde{\theta}^m, \overline{\theta}'\right)$, where the static monopoly price $\overline{\theta}$ does not significantly differ from $\widehat{\theta}$. Combining Lemmas 3 through 5 with Propositions 4 and 5, we find that, when under ex post information provision the collusive price is at the static monopoly level $\overline{\theta}$, under ex ante information provision firms substantially benefit from charging the valuable price $\widehat{\theta}$ (which does not differ too much from $\overline{\theta}$) in addition to $\overline{\theta}$. When under ex post information provision the collusive price is set at $\widehat{\theta}$, under ex ante information provision firms can also charge the static monopoly price $\overline{\theta}$. Now, we turn to the case $\overline{\theta} \in \left[\overline{\theta}'', \widetilde{\theta}^m\right]$, where the price $\overline{\theta}$ is sufficiently higher than the static monopoly price $\widehat{\theta}$. It follows from Lemmas 3 and 5 as well as Propositions 4 and 5 that under ex ante information provision firms can select the valuable price $\overline{\theta}$ (which is relatively high) in addition to the static monopoly price $\widehat{\theta}$ charged under ex post information provision.

# 6 Concluding remarks

Nowadays, firms commonly use data analytics services to improve their knowledge of the business environment. In this paper, we characterize the optimal information design in a market where colluding firms cannot directly observe demand fluctuations but have the opportunity to acquire information in the form of a signal about the state of demand. Specifically, we investigate two modes of information provision, called ex ante information provision and ex post information provision. Under ex ante information provision, firms receive information prior to price setting, whereas under ex post information provision they obtain information after price setting. Such modes of information provision reflect two major forms of data analytics, i.e., predictive analytics and diagnostic analytics, respectively. We find that the two modes of information provision generate a trade-off between a price sophistication effect and a profit dissipation effect. This trade-off implies that, when the information technology is efficient enough, ex ante information provision can facilitate collusion to a further extent than ex post information provision, whereas the opposite tends to occur with a relatively inefficient information technology.

As shown in the Supplementary Appendix, the trade-off between the price sophistication effect and the profit dissipation effect hinges upon a number of factors. A higher degree of heterogeneity of demand (as implied by a higher number of demand states) raises the benefit of conducting a more sophisticated pricing strategy, which increases the profitability of collusion under ex ante information provision. A less efficient information technology inflates the cost of providing information. It follows from our analysis that, under ex ante information provision, a lower level of information accuracy magnifies profit dissipation because marginal cost pricing is implemented in response to a signal that is more likely to erroneously indicate low demand. Thus, ex post information provision becomes more attractive for collusion purposes. The number of firms also plays a relevant role. In a less concentrated market, information is more valuable to alleviate deviation incentives and thus collusion can be better enforced under ex ante information provision.

In light of the relevance of data analytics for firms' decision making in modern economies, our work provides some testable predictions about the main features of different forms of data analytics and their economic effects. This can help antitrust authorities in the assessment of anticompetitive practices in markets where firms resort to data analytics services.

## Appendix

**Proof of Lemma 1.** The firms' aggregate present discounted collusive profits under ex ante information provision are given by

$$\Pi(\alpha) = \overline{V}(\alpha) + \underline{V}(\alpha), \tag{A1}$$

where $\overline{V}(\cdot)$ and $\underline{V}(\cdot)$ in (1) and (2) are respectively equal to

$$\overline{V}(\alpha) = \frac{\alpha\overline{\theta}\left(2 - \delta^{\underline{T}+1}\right)}{2\left[2 - \alpha\delta - (1-\alpha)\,\delta^{\overline{T}+1} - \delta^{\underline{T}+1}\right]} \tag{A2}$$

and

$$\underline{V}(\alpha) = \frac{\alpha\overline{\theta}\delta^{\underline{T}+1}}{2\left[2 - \alpha\delta - (1-\alpha)\,\delta^{\overline{T}+1} - \delta^{\underline{T}+1}\right]}. \tag{A3}$$

It follows from (A2) and (A3) that the incentive constraint (3) can be rewritten as

$$\alpha\overline{\theta}\frac{2 - \delta^{\overline{T}+1} - \delta^{\underline{T}+1}}{2\left[2 - \alpha\delta - (1-\alpha)\,\delta^{\overline{T}+1} - \delta^{\underline{T}+1}\right]} - \alpha\overline{\theta} \geq 0. \tag{A4}$$

Differentiating the left-hand side of (A4) with respect to $\underline{T}$ yields after some manipulation

$$\frac{\alpha^2\overline{\theta}\left(\delta - \delta^{\overline{T}+1}\right)\delta^{\underline{T}+1}\ln\delta}{2\left[2 - \alpha\delta - (1-\alpha)\,\delta^{\overline{T}+1} - \delta^{\underline{T}+1}\right]^2} < 0, \tag{A5}$$

which implies that a lower $\underline{T}$ relaxes the incentive constraint (A4). Furthermore, substituting (A2) and (A3) into (A1) and differentiating (A1) with respect to $\underline{T}$ gives

$$\frac{\alpha\overline{\theta}\delta^{\underline{T}+1}\ln\delta}{\left[2 - \alpha\delta - (1-\alpha)\,\delta^{\overline{T}+1} - \delta^{\underline{T}+1}\right]^2} < 0, \tag{A6}$$

which implies that $\Pi(\cdot)$ in (A1) decreases with $\underline{T}$. It follows from (A5) and (A6) that, for any given $\alpha$, the equilibrium punishment phase conditional on the signal $\underline{s}$ under ex ante information provision lasts for a number of periods equal to

$$\underline{T}^a = 0. \tag{A7}$$

Differentiating the left-hand side of (A4) with respect to $\overline{T}$ yields after some manipulation

$$-\frac{\alpha^2\overline{\theta}\left(2 - \delta - \delta^{\underline{T}+1}\right)\delta^{\overline{T}+1}\ln\delta}{2\left[2 - \alpha\delta - (1-\alpha)\,\delta^{\overline{T}+1} - \delta^{\underline{T}+1}\right]^2} > 0, \tag{A8}$$

which implies that a lower $\overline{T}$ tightens the incentive constraint (A4). Furthermore, substituting (A2) and (A3) into (A1) and differentiating (A1) with respect to $\overline{T}$ gives

$$\frac{\alpha\left(1-\alpha\right)\overline{\theta}\delta^{\overline{T}+1}\ln\delta}{\left[2-\alpha\delta-\left(1-\alpha\right)\delta^{\overline{T}+1}-\delta^{\underline{T}+1}\right]^{2}}\leq 0, \tag{A9}$$

which implies that $\Pi\left(\cdot\right)$ in (A1) decreases with $\overline{T}$ (the equality holds if and only if $\alpha=1$). It follows from (A8) and (A9) that the equilibrium value for $\overline{T}$ is such that the incentive constraint (A4) is binding. Using (A7), we find that, for any given $\alpha$, the equilibrium punishment phase conditional on the signal $\overline{s}$ under ex ante information provision lasts for a number of periods equal to

$$\overline{T}^{a}\left(\alpha\right)=\left(\ln\delta\right)^{-1}\ln\left(\delta-\frac{2\left(1-\delta\right)}{2\alpha-1}\right)-1>0. \tag{A10}$$

Substituting (A2), (A3), (A7) and (A10) into (A1), we obtain that, for any given $\alpha$, the firms' equilibrium aggregate present discounted collusive profits under ex ante information provision are given by

$$\Pi^{a}\left(\alpha\right)=\frac{2\alpha-1}{2\left(1-\delta\right)}\overline{\theta}.\quad\blacksquare \tag{A11}$$

**Proof of Lemma 2.** It follows from (4) that a firm's present discounted collusive profit under ex post information provision is given by

$$V\left(\alpha\right)=\frac{\overline{\theta}}{2\left[2-\delta-\left(1-\alpha\right)\delta^{\overline{T}+1}-\alpha\delta^{\underline{T}+1}\right]}. \tag{A12}$$

Using (A12), the incentive constraint (5) becomes

$$\overline{\theta}\frac{2-\delta^{\overline{T}+1}-\delta^{\underline{T}+1}}{4\left[2-\delta-\left(1-\alpha\right)\delta^{\overline{T}+1}-\alpha\delta^{\underline{T}+1}\right]}-\frac{\overline{\theta}}{2}\geq 0. \tag{A13}$$

Differentiating the left-hand side of (A13) with respect to $\overline{T}$ yields after some manipulation

$$\frac{\left[\delta-\delta^{\underline{T}+1}-2\alpha\left(1-\delta^{\underline{T}+1}\right)\right]\overline{\theta}\delta^{\overline{T}+1}\ln\delta}{4\left[2-\delta-\left(1-\alpha\right)\delta^{\overline{T}+1}-\alpha\delta^{\underline{T}+1}\right]^{2}}>0,$$

which implies that a lower $\overline{T}$ tightens the incentive constraint (A13) (the inequality follows from the assumptions on the parameters of the model). As $V\left(\cdot\right)$ in (A12) decreases with $\overline{T}$, the equilibrium value for $\overline{T}$ is such that the incentive constraint (A13) is binding. This yields

$$\overline{T}\left(\alpha\right)=\left(\ln\delta\right)^{-1}\ln\left(\frac{\left(2\alpha-1\right)\delta^{\underline{T}+1}-2\left(1-\delta\right)}{2\alpha-1}\right)-1. \tag{A14}$$

Substituting (A14) into (A12) and differentiating with respect to $\underline{T}$ yields

$$\frac{\left(2\alpha-1\right)^{2}\overline{\theta}\delta^{\underline{T}+1}\ln\delta}{2\left[\delta-\delta^{\underline{T}+1}-2\alpha\left(1-\delta^{\underline{T}+1}\right)\right]^{2}}\leq 0,$$

which implies that $V\left(\cdot\right)$ in (A12) decreases with $\underline{T}$ (the equality holds if and only if $\alpha=1/2$).

Thus, for any given $\alpha$, the equilibrium punishment phase conditional on the signal $\underline{s}$ under ex post information provision lasts for a number of periods equal to

$$\underline{T}^p = 0. \tag{A15}$$

Replacing (A15) into (A14), we find that, for any given $\alpha$, the equilibrium punishment phase conditional on the signal $\overline{s}$ under ex post information provision lasts for a number of periods equal to

$$\overline{T}^p(\alpha) = (\ln \delta)^{-1} \ln\left(\delta - \frac{2(1-\delta)}{2\alpha - 1}\right) - 1 > 0. \tag{A16}$$

Substituting (A15) and (A16) into (A12), we obtain that, for any given $\alpha$, the firms' equilibrium aggregate present discounted collusive profits under ex post information provision are given by

$$\Pi^p(\alpha) = \frac{2\alpha - 1}{2\alpha(1-\delta)}\overline{\theta}. \quad \blacksquare \tag{A17}$$

**Proof of Proposition 1.** Let $\Psi^a(\alpha) \triangleq \Pi^a(\alpha) - C(\alpha)$ be the information provider's profit under ex ante information provision, where $\Pi^a(\alpha)$ is given by (A11) and $C(\alpha) = c(\alpha - \alpha_0)$. The information provider's profit maximization problem is described in (6) for $\Pi^c(\alpha) = \Pi^a(\alpha)$. Taking the derivative of $\Psi^a(\alpha)$ with respect to $\alpha$ yields $\overline{\theta}/(1-\delta) - c$. As $\Psi^a(\alpha)$ increases with $\alpha$ for $c < \overline{\theta}/(1-\delta)$ and decreases with $\alpha$ otherwise, the equilibrium value for $\alpha \in [\alpha_0, 1]$ is either $\alpha^a = 1$ or $\alpha^a = \alpha_0$, where $\alpha_0 = 1/2$. The information provider's equilibrium profit at $\alpha^a = 1$ under ex ante information provision is given by

$$\Psi^a\big|_{\alpha^a=1} = \frac{\overline{\theta} - (1-\delta)c}{2(1-\delta)}. \tag{A18}$$

It holds $\Psi^a\big|_{\alpha^a=1} > 0$ if and only if $c < \overline{\theta}/(1-\delta)$. Then, $\Psi^a(\alpha)$ increases with $\alpha$ throughout the entire interval where $\Psi^a\big|_{\alpha^a=1} > 0$. This implies that under ex ante information provision there exists a threshold $\widetilde{c} \triangleq \overline{\theta}/(1-\delta)$ such that $(i)$ for $c < \widetilde{c}$ the equilibrium value for $\alpha$ is $\alpha^a = 1$ and $(ii)$ for $c \geq \widetilde{c}$ the equilibrium value for $\alpha$ is $\alpha^a = \alpha_0$. $\blacksquare$

**Proof of Proposition 2.** Let $\Psi^p(\alpha) \triangleq \Pi^p(\alpha) - C(\alpha)$ be the information provider's profit under ex post information provision, where $\Pi^p(\alpha)$ is given by (A17) and $C(\alpha) = c(\alpha - \alpha_0)$. The information provider's profit maximization problem is described in (6) for $\Pi^c(\alpha) = \Pi^p(\alpha)$. The first-order condition for $\alpha$ associated with an interior solution is $\overline{\theta}/\left[2\alpha^2(1-\delta)\right] - c = 0$.[31] The equilibrium value for $\alpha$ in an interior solution under ex post information provision is given by

$$\alpha^p\left(\overline{\theta}\right) = \sqrt{\frac{\overline{\theta}}{2c(1-\delta)}}. \tag{A19}$$

This constitutes an interior solution, i.e., $\alpha^p\left(\overline{\theta}\right) \in (\alpha_0, 1)$, where $\alpha_0 = 1/2$, if and only if $c > \overline{\theta}/[2(1-\delta)]$ and $c < 2\overline{\theta}/(1-\delta)$. This implies that under ex post information provision there exist two thresholds $\underline{c} \triangleq \overline{\theta}/[2(1-\delta)]$ and $\overline{c} \triangleq 2\overline{\theta}/(1-\delta)$ such that $(i)$ for $c \leq \underline{c}$ the equilibrium value for $\alpha$ is $\alpha^p = 1$, $(ii)$ for $\underline{c} < c < \overline{c}$ the equilibrium value for $\alpha$ is $\alpha^p\left(\overline{\theta}\right) \in (\alpha_0, 1)$

[31]Differentiating the left-hand side of this expression with respect to $\alpha$ yields $-\overline{\theta}/\left[\alpha^3(1-\delta)\right] < 0$, which ensures that the second-order condition for a maximum is satisfied.

in (A19), and $(iii)$ for $c \geq \overline{c}$ the equilibrium value for $\alpha$ is $\alpha^p = \alpha_0$. ■

**Proof of Proposition 3.** It follows from the proof of Proposition 1 that under ex ante information provision the information provider's equilibrium profit is $\Psi^a|_{\alpha^a=1}$ in (A18) for $c < \widetilde{c}$ and $\Psi^a|_{\alpha^a=\alpha_0} = 0$ for $c \geq \widetilde{c}$. Furthermore, it follows from the proof of Proposition 2 that under ex post information provision the information provider's equilibrium profit is $\Psi^p|_{\alpha^p=1}$ for $c \leq \underline{c}$, $\Psi^p|_{\alpha^p(\overline{\theta})\in(\alpha_0,1)}$ for $\underline{c} < c < \overline{c}$, and $\Psi^p|_{\alpha^p=\alpha_0} = 0$ for $c \geq \overline{c}$, where $\Psi^p|_{\alpha^p=1}$ and $\Psi^p|_{\alpha^p(\overline{\theta})\in(\alpha_0,1)}$ are respectively given by

$$\Psi^p|_{\alpha^p=1} = \frac{\overline{\theta} - (1-\delta)\, c}{2\,(1-\delta)} \quad \text{and} \quad \Psi^p|_{\alpha^p(\overline{\theta})\in(\alpha_0,1)} = \frac{2\overline{\theta} + (1-\delta)\, c - 2\sqrt{2c\,(1-\delta)\,\overline{\theta}}}{2\,(1-\delta)}. \tag{A20}$$

Using (A18) and (A20), we obtain that there exists a threshold $\underline{c}$ (where $\underline{c} < \widetilde{c}$, as defined in the proofs of Propositions 1 and 2) such that $(i)$ for $c \leq \underline{c}$ it holds $\Psi^a|_{\alpha^a=1} = \Psi^p|_{\alpha^p=1}$ and $(ii)$ for $c > \underline{c}$ it holds $\Psi^p|_{\alpha^p(\overline{\theta})\in(\alpha_0,1)} > \Psi^a|_{\alpha^a=1}$.[32] ■

**Proof of Remark 2.** We find from (7) that a firm's present discounted collusive profit is given by

$$V = \frac{\widehat{\theta}}{3 - 2\delta - \delta^{T+1}}. \tag{A21}$$

Using (A21), the incentive constraint (8) reduces to

$$\widehat{\theta}\frac{4\delta - \delta^{T+1} - 3}{3\,(3 - 2\delta - \delta^{T+1})} \geq 0. \tag{A22}$$

Differentiating the left-hand side of (A22) with respect to $T$ yields after some manipulation

$$-\frac{2\widehat{\theta}\,(1-\delta)\,\delta^{T+1}\ln\delta}{\left(3 - 2\delta - \delta^{T+1}\right)^2} > 0,$$

which implies that a lower $T$ tightens the incentive constraint (A22). As $V$ in (A21) decreases with $T$, we find that the equilibrium value for $T$ is such that the incentive constraint (A22) is binding. Then, the equilibrium punishment phase with no information lasts for a number of periods equal to

$$T^n = (\ln\delta)^{-1}\ln(4\delta - 3) - 1 > 0.$$

Using (A21), we obtain that the firms' equilibrium aggregate present discounted collusive profits with no information are given by

$$\Pi^n = \frac{\widehat{\theta}}{3\,(1-\delta)}. \quad ■ \tag{A23}$$

**Proof of Lemma 3.** The firms' aggregate present discounted collusive profits under ex ante information provision are given by

$$\Pi(\alpha) = 2\frac{\overline{V}(\alpha) + \widehat{V}(\alpha) + \underline{V}(\alpha)}{3}, \tag{A24}$$

[32] For $c \geq \overline{c}$ (where $\overline{c}$ is defined in the proof of Proposition 2), we have $\alpha^a = \alpha^p = \alpha_0$, which yields the trivial case $\Psi^a|_{\alpha^a=\alpha_0} = \Psi^p|_{\alpha^p=\alpha_0} = 0$.

where $\overline{V}(\alpha)$, $\widehat{V}(\alpha)$ and $\underline{V}(\alpha)$ in (9), (10) and (11) are respectively equal to

$$\overline{V}(\alpha)=\frac{\alpha\overline{\theta}\left[6-(1+\alpha)\left(\delta-\delta^{\widehat{T}+1}\right)-2\delta^{\widehat{T}+1}-2\delta^{\underline{T}+1}\right]+(1+\alpha)\widehat{\theta}\left[\delta^{\overline{T}+1}+\alpha\left(\delta-\delta^{\overline{T}+1}\right)\right]}{2\left[6-\delta(1+3\alpha)-2(1-\alpha)\delta^{\overline{T}+1}-(1-\alpha)\delta^{\widehat{T}+1}-2\delta^{\underline{T}+1}\right]}, \tag{A25}$$

$$\widehat{V}(\alpha)=\frac{(1+\alpha)\widehat{\theta}\left[3-(1-\alpha)\delta^{\overline{T}+1}-\delta^{\underline{T}+1}-\alpha\delta\right]+\alpha\overline{\theta}\left[(1-\alpha)\delta^{\widehat{T}+1}+\delta(1+\alpha)\right]}{2\left[6-\delta(1+3\alpha)-2(1-\alpha)\delta^{\overline{T}+1}-(1-\alpha)\delta^{\widehat{T}+1}-2\delta^{\underline{T}+1}\right]} \tag{A26}$$

and

$$\underline{V}(\alpha)=\frac{\left[2\alpha\overline{\theta}+(1+\alpha)\widehat{\theta}\right]\delta^{\underline{T}+1}}{2\left[6-\delta(1+3\alpha)-2(1-\alpha)\delta^{\overline{T}+1}-(1-\alpha)\delta^{\widehat{T}+1}-2\delta^{\underline{T}+1}\right]}. \tag{A27}$$

Using (A25), (A26) and (A27), the incentive constraints (12) and (13) become respectively

$$\begin{aligned}&\frac{\alpha^2\left(\delta-\delta^{\overline{T}+1}\right)\overline{\theta}}{6-\delta(1+3\alpha)-2(1-\alpha)\delta^{\overline{T}+1}-(1-\alpha)\delta^{\widehat{T}+1}-2\delta^{\underline{T}+1}}-\frac{\alpha\overline{\theta}}{2}\\&+\frac{\alpha(1+\alpha)\left(\delta-\delta^{\overline{T}+1}\right)\widehat{\theta}}{2\left[6-\delta(1+3\alpha)-2(1-\alpha)\delta^{\overline{T}+1}-(1-\alpha)\delta^{\widehat{T}+1}-2\delta^{\underline{T}+1}\right]}\geq 0\end{aligned} \tag{A28}$$

and

$$\frac{(1+\alpha)\widehat{\theta}\left[(1-\alpha)\delta^{\overline{T}+1}-\alpha\delta^{\widehat{T}+1}+\delta^{\underline{T}+1}-3\right]-(1+\alpha)\left[\alpha\delta\left(\overline{\theta}+\widehat{\theta}\right)+(1+\alpha)\delta\widehat{\theta}-\alpha\overline{\theta}\delta^{\widehat{T}+1}\right]}{2\left[6-\delta(1+3\alpha)-2(1-\alpha)\delta^{\overline{T}+1}-(1-\alpha)\delta^{\widehat{T}+1}-2\delta^{\underline{T}+1}\right]}\geq 0. \tag{A29}$$

Differentiating the left-hand side of (A28) with respect to $\underline{T}$, we obtain after some manipulation

$$\frac{\alpha\left[2\alpha\overline{\theta}+(1+\alpha)\widehat{\theta}\right]\left(\delta-\delta^{\overline{T}+1}\right)\delta^{\underline{T}+1}\ln\delta}{\left[6-\delta(1+3\alpha)-2(1-\alpha)\delta^{\overline{T}+1}-(1-\alpha)\delta^{\widehat{T}+1}-2\delta^{\underline{T}+1}\right]^2}<0, \tag{A30}$$

which implies that a lower $\underline{T}$ relaxes the incentive constraint (A28). Differentiating the left-hand side of (A29) with respect to $\underline{T}$ yields after some manipulation

$$\frac{(1+\alpha)\left[2\alpha\overline{\theta}+(1+\alpha)\widehat{\theta}\right]\left(\delta-\delta^{\widehat{T}+1}\right)\delta^{\underline{T}+1}\ln\delta}{2\left[6-\delta(1+3\alpha)-2(1-\alpha)\delta^{\overline{T}+1}-(1-\alpha)\delta^{\widehat{T}+1}-2\delta^{\underline{T}+1}\right]^2}<0, \tag{A31}$$

which implies that a lower $\underline{T}$ relaxes the incentive constraint (A29). Substituting (A25), (A26) and (A27) into (A24) and differentiating (A24) with respect to $\underline{T}$ yields

$$\frac{2\left[2\alpha\overline{\theta}+(1+\alpha)\widehat{\theta}\right]\delta^{\underline{T}+1}\ln\delta}{\left[6-\delta(1+3\alpha)-2(1-\alpha)\delta^{\overline{T}+1}-(1-\alpha)\delta^{\widehat{T}+1}-2\delta^{\underline{T}+1}\right]^2}<0, \tag{A32}$$

which implies that $\Pi(\cdot)$ in (A24) decreases with $\underline{T}$. It follows from (A30), (A31) and (A32) that, for any given $\alpha$, the equilibrium punishment phase conditional on the signal $\underline{s}$ under ex ante information provision lasts for a number of periods equal to

$$\underline{T}^a = 0. \tag{A33}$$

Differentiating the left-hand side of (A28) with respect to $\overline{T}$, we obtain after some manipulation

$$-\frac{\alpha\left[2\alpha\overline{\theta}+(1+\alpha)\widehat{\theta}\right]\left[6-\delta(3+\alpha)-(1-\alpha)\delta^{\widehat{T}+1}-2\delta^{\underline{T}+1}\right]\delta^{\overline{T}+1}\ln\delta}{2\left[6-\delta(1+3\alpha)-2(1-\alpha)\delta^{\overline{T}+1}-(1-\alpha)\delta^{\widehat{T}+1}-2\delta^{\underline{T}+1}\right]^2}>0, \tag{A34}$$

which implies that a lower $\overline{T}$ tightens the incentive constraint (A28) (the inequality follows from the assumptions on the parameters of the model). Differentiating the left-hand side of (A29) with respect to $\overline{T}$ yields after some manipulation

$$\frac{(1-\alpha)(1+\alpha)\left[2\alpha\overline{\theta}+(1+\alpha)\widehat{\theta}\right]\left(\delta-\delta^{\widehat{T}+1}\right)\delta^{\overline{T}+1}\ln\delta}{2\left[6-\delta(1+3\alpha)-2(1-\alpha)\delta^{\overline{T}+1}-(1-\alpha)\delta^{\widehat{T}+1}-2\delta^{\underline{T}+1}\right]^2}\leq 0, \tag{A35}$$

which implies that a lower $\overline{T}$ relaxes the incentive constraint (A29) (the equality holds if and only if $\alpha = 1$).

Differentiating the left-hand side of (A28) with respect to $\widehat{T}$, we obtain after some manipulation

$$\frac{\alpha(1-\alpha)\left[2\alpha\overline{\theta}+(1+\alpha)\widehat{\theta}\right]\left(\delta-\delta^{\overline{T}+1}\right)\delta^{\widehat{T}+1}\ln\delta}{2\left[6-\delta(1+3\alpha)-2(1-\alpha)\delta^{\overline{T}+1}-(1-\alpha)\delta^{\widehat{T}+1}-2\delta^{\underline{T}+1}\right]^2}\leq 0, \tag{A36}$$

which implies that a lower $\widehat{T}$ relaxes the incentive constraint (A28) (the equality holds if and only if $\alpha = 1$). Differentiating the left-hand side of (A29) with respect to $\widehat{T}$ yields after some manipulation

$$-\frac{(1+\alpha)\left[2\alpha\overline{\theta}+(1+\alpha)\widehat{\theta}\right]\left[3-\delta(1+\alpha)-(1-\alpha)\delta^{\overline{T}+1}-\delta^{\underline{T}+1}\right]\delta^{\widehat{T}+1}\ln\delta}{2\left[6-\delta(1+3\alpha)-2(1-\alpha)\delta^{\overline{T}+1}-(1-\alpha)\delta^{\widehat{T}+1}-2\delta^{\underline{T}+1}\right]^2}>0, \tag{A37}$$

which implies that a lower $\widehat{T}$ tightens the incentive constraint (A29) (the inequality follows from the assumptions on the parameters of the model).

Substituting (A25), (A26) and (A27) into (A24) and differentiating (A24) with respect to $\overline{T}$ and $\widehat{T}$ yields respectively

$$\frac{2(1-\alpha)\left[2\alpha\overline{\theta}+(1+\alpha)\widehat{\theta}\right]\delta^{\overline{T}+1}\ln\delta}{\left[6-\delta(1+3\alpha)-2(1-\alpha)\delta^{\overline{T}+1}-(1-\alpha)\delta^{\widehat{T}+1}-2\delta^{\underline{T}+1}\right]^2}\leq 0 \tag{A38}$$

and

$$\frac{(1-\alpha)\left[2\alpha\overline{\theta}+(1+\alpha)\widehat{\theta}\right]\delta^{\widehat{T}+1}\ln\delta}{\left[6-\delta(1+3\alpha)-2(1-\alpha)\delta^{\overline{T}+1}-(1-\alpha)\delta^{\widehat{T}+1}-2\delta^{\underline{T}+1}\right]^2}\leq 0, \tag{A39}$$

which implies that $\Pi(\cdot)$ in (A24) decreases with $\overline{T}$ and $\widehat{T}$ (the equalities hold if and only if

$\alpha = 1$). It follows from (A34) through (A39) that the equilibrium values for $\overline{T}$ and $\widehat{T}$ are such that the incentive constraints (A28) and (A29) are binding. Using (A33), we find that, for any given $\alpha$, the equilibrium punishment phases conditional on the signals $\overline{s}$ and $\widehat{s}$ under ex ante information provision respectively last for a number of periods equal to

$$\overline{T}^a(\alpha) = (\ln \delta)^{-1} \ln \left( \delta - \frac{3(1-\delta)\overline{\theta}}{(2\alpha - 1)\overline{\theta} + \alpha\widehat{\theta}} \right) - 1 > 0 \tag{A40}$$

and

$$\widehat{T}^a(\alpha) = (\ln \delta)^{-1} \ln \left( \delta - \frac{3(1-\delta)\widehat{\theta}}{(2\alpha - 1)\overline{\theta} + \alpha\widehat{\theta}} \right) - 1 > 0, \tag{A41}$$

where $\overline{T}^a(\alpha) > \widehat{T}^a(\alpha)$. Substituting (A25), (A26), (A27), (A33), (A40) and (A41) into (A24), we obtain that, for any given $\alpha$, the firms' equilibrium aggregate present discounted collusive profits under ex ante information provision are given by

$$\Pi^a(\alpha) = \frac{(2\alpha - 1)\overline{\theta} + \alpha\widehat{\theta}}{3(1-\delta)}. \quad \blacksquare \tag{A42}$$

**Proof of Lemma 4.** It follows from (14) that a firm's present discounted collusive profit under ex post information provision is given by

$$V(\alpha) = \frac{\overline{\theta}}{6 - 2\delta - 2(1-\alpha)\delta^{\overline{T}+1} - (1+\alpha)\left(\delta^{\widehat{T}+1} + \delta^{\underline{T}+1}\right)}. \tag{A43}$$

Using (A43), the incentive constraint (15) becomes

$$\frac{\left(3 - \delta^{\overline{T}+1} - \delta^{\widehat{T}+1} - \delta^{\underline{T}+1}\right)\overline{\theta}}{3\left[6 - 2\delta - 2(1-\alpha)\delta^{\overline{T}+1} - (1+\alpha)\left(\delta^{\widehat{T}+1} + \delta^{\underline{T}+1}\right)\right]} - \frac{\overline{\theta}}{3} \geq 0. \tag{A44}$$

Differentiating the left-hand side of (A44) with respect to $\overline{T}$ yields after some manipulation

$$-\frac{\left[2\delta + (3\alpha - 1)\left(\delta^{\widehat{T}+1} + \delta^{\underline{T}+1}\right) - 6\alpha\right]\overline{\theta}\delta^{\overline{T}+1}\ln\delta}{3\left[6 - 2\delta - 2(1-\alpha)\delta^{\overline{T}+1} - (1+\alpha)\left(\delta^{\widehat{T}+1} + \delta^{\underline{T}+1}\right)\right]^2} > 0,$$

which implies that a lower $\overline{T}$ tightens the incentive constraint (A44) (the inequality follows from the assumptions on the parameters of the model). Then, as $V(\cdot)$ in (A43) decreases with $\overline{T}$, the equilibrium value for $\overline{T}$ is such that the incentive constraint (A44) is binding. This gives

$$\overline{T}(\alpha) = (\ln \delta)^{-1} \ln \left( \frac{2\delta + \alpha\left(\delta^{\widehat{T}+1} + \delta^{\underline{T}+1}\right) - 3}{2\alpha - 1} \right) - 1. \tag{A45}$$

Substituting (A45) into (A43) and differentiating (A43) with respect to $\widehat{T}$ and $\underline{T}$ yields respectively

$$\frac{\overline{\theta}(3\alpha - 1)(2\alpha - 1)\delta^{\widehat{T}+1}\ln\delta}{\left[2\delta - 6\alpha + (3\alpha - 1)\left(\delta^{\widehat{T}+1} + \delta^{\underline{T}+1}\right)\right]^2} < 0$$

and

$$\frac{\overline{\theta}\left(3\alpha-1\right)\left(2\alpha-1\right)\delta^{\underline{T}+1}\ln\delta}{\left[2\delta+6\alpha+\left(3\alpha-1\right)\left(\delta^{\widehat{T}+1}+\delta^{\underline{T}+1}\right)\right]^{2}}<0,$$

which implies that $V(\cdot)$ in (A43) decreases with $\widehat{T}$ and $\underline{T}$ (the inequalities hold as long as $V(\cdot)$ is positive). Then, for any given $\alpha$, the candidate equilibrium punishment phases conditional on the signals $\widehat{s}$ and $\underline{s}$ under ex post information provision last for a number of periods equal to

$$\widehat{T}^{p}=\underline{T}^{p}=0. \tag{A46}$$

Replacing (A46) into (A45), we find that, for any given $\alpha$, the candidate equilibrium punishment phase conditional on the signal $\overline{s}$ under ex post information provision lasts for a number of periods equal to

$$\overline{T}^{p}\left(\alpha\right)=\left(\ln\delta\right)^{-1}\ln\left(\delta-\frac{3\left(1-\delta\right)}{2\alpha-1}\right)-1>0. \tag{A47}$$

Substituting (A46) and (A47) into (A43), we obtain that, for any given $\alpha$, the firms' candidate equilibrium aggregate present discounted collusive profits under ex post information provision are given by

$$\Pi^{p}\left(\alpha\right)=\frac{2\alpha-1}{3\alpha\left(1-\delta\right)}\overline{\theta}. \quad \blacksquare \tag{A48}$$

**Proof of Lemma 5.** It follows from (16) that a firm's present discounted collusive profit under ex post information provision is given by

$$V\left(\alpha\right)=\frac{2\widehat{\theta}}{6-4\delta-\left(1-\alpha\right)\left(\delta^{\overline{T}+1}+\delta^{\widehat{T}+1}\right)-2\alpha\delta^{\underline{T}+1}}. \tag{A49}$$

Using (A49), the incentive constraint (17) becomes

$$\frac{2\widehat{\theta}\left(3-\delta^{\overline{T}+1}-\delta^{\widehat{T}+1}-\delta^{\underline{T}+1}\right)}{3\left[6-4\delta-\left(1-\alpha\right)\left(\delta^{\overline{T}+1}+\delta^{\widehat{T}+1}\right)-2\alpha\delta^{\underline{T}+1}\right]}-\frac{2}{3}\widehat{\theta}\geq0. \tag{A50}$$

Differentiating the left-hand side of (A50) with respect to $\overline{T}$ yields after some manipulation

$$-\frac{2\widehat{\theta}\left[3\left(1+\alpha\right)-4\delta-\left(3\alpha-1\right)\delta^{\underline{T}+1}\right]\delta^{\overline{T}+1}\ln\delta}{3\left[6-4\delta-\left(1-\alpha\right)\left(\delta^{\overline{T}+1}+\delta^{\widehat{T}+1}\right)-2\alpha\delta^{\underline{T}+1}\right]^{2}}>0,$$

which implies that a lower $\overline{T}$ tightens the incentive constraint (A50) (the inequality follows from the assumptions on the parameters of the model). As $V(\cdot)$ in (A49) decreases with $\overline{T}$, for any given $\alpha$, the equilibrium value for $\overline{T}$ is such that the incentive constraint (A50) is binding. This yields

$$\overline{T}^{p}\left(\alpha\right)=\left(\ln\delta\right)^{-1}\ln\left(\frac{4\delta-\alpha\delta^{\widehat{T}+1}+\left(2\alpha-1\right)\delta^{\underline{T}+1}-3}{\alpha}\right)-1. \tag{A51}$$

Substituting (A51) into (A49) and differentiating (A49) with respect to $\underline{T}$ yields

$$\frac{2\alpha\widehat{\theta}\left(3\alpha-1\right)\delta^{\underline{T}+1}\ln\delta}{\left[3\left(1+\alpha\right)-4\delta-\left(3\alpha-1\right)\delta^{\underline{T}+1}\right]^{2}}<0,$$

which implies that $V(\cdot)$ in (A49) decreases with $\underline{T}$ (the inequality holds as long as $V(\cdot)$ is positive). Then, we obtain that, for any given $\alpha$, the equilibrium punishment phase conditional on the signal $\underline{s}$ under ex post information provision lasts for a number of periods equal to

$$\underline{T}^p = 0. \tag{A52}$$

Replacing (A52) into (A51), we find that, for any given $\alpha$, the equilibrium punishment phases conditional on the signals $\overline{s}$ and $\widehat{s}$ under ex post information provision last respectively for a number of periods such that

$$\overline{T}^p(\alpha) = (\ln\delta)^{-1}\ln\left(\frac{2\alpha\delta - 3(1-\delta)}{\alpha} - \delta^{\widehat{T}^p(\alpha)+1}\right) - 1 > 0. \tag{A53}$$

and

$$\widehat{T}^p(\alpha) = (\ln\delta)^{-1}\ln\left(\frac{2\alpha\delta - 3(1-\delta)}{\alpha} - \delta^{\overline{T}^p(\alpha)+1}\right) - 1 > 0. \tag{A54}$$

Substituting (A52), (A53) and (A54) into (A49), we obtain that, for any given $\alpha$, the firms' equilibrium aggregate present discounted collusive profits under ex post information provision are given by

$$\Pi^p(\alpha) = \frac{4\alpha\widehat{\theta}}{3(1+\alpha)(1-\delta)}. \quad \blacksquare \tag{A55}$$

**Proof of Proposition 4.** Let $\Psi^a(\alpha) \triangleq \Pi^a(\alpha) - C(\alpha)$ be the information provider's profit under ex ante information provision (gross of the firms' outside option $\Pi^n$ in (A23)), where $\Pi^a(\alpha)$ is given by (A42) and $C(\alpha) = c(\alpha - \alpha_0)$. The information provider's profit maximization problem is described in (18) for $\Pi^c(\alpha) = \Pi^a(\alpha)$. Taking the derivative of $\Psi^a(\alpha)$ with respect to $\alpha$ yields $\left(2\overline{\theta} + \widehat{\theta}\right)/[3(1-\delta)] - c$. As $\Psi^a(\alpha)$ increases with $\alpha$ for $c < \left(2\overline{\theta} + \widehat{\theta}\right)/[3(1-\delta)]$ and decreases with $\alpha$ otherwise, the equilibrium value for $\alpha \in [\alpha_0, 1]$ is either $\alpha^a = 1$ or $\alpha^a = \alpha_0$, where $\alpha_0 = 1/3$. The information provider's equilibrium profit at $\alpha^a = 1$ under ex ante information provision (gross of the firms' outside option $\Pi^n$ in (A23)) is given by

$$\Psi^a\big|_{\alpha^a=1} = \frac{\overline{\theta} + \widehat{\theta} - 2c(1-\delta)}{3(1-\delta)}. \tag{A56}$$

Given the firms' outside option $\Pi^n$ in (A23), it holds $\Psi^a\big|_{\alpha^a=1} - \Pi^n > 0$ if and only if $c < \overline{\theta}/[2(1-\delta)]$. As $\overline{\theta}/[2(1-\delta)] < \left(2\overline{\theta} + \widehat{\theta}\right)/[3(1-\delta)]$, we find that $\Psi^a(\alpha)$ increases with $\alpha$ throughout the entire interval where $\Psi^a\big|_{\alpha^a=1} - \Pi^n > 0$. This implies that under ex ante information provision there exists a threshold $\widetilde{c}' \triangleq \overline{\theta}/[2(1-\delta)]$ such that $(i)$ for $c < \widetilde{c}'$ the equilibrium value for $\alpha$ is $\alpha^a = 1$ and $(ii)$ for $c \geq \widetilde{c}'$ the equilibrium value for $\alpha$ is $\alpha^a = \alpha_0$. ■

**Proof of Proposition 5.** Let $\Psi^p(\alpha) \triangleq \Pi^p(\alpha) - C(\alpha)$ be the information provider's profit under ex post information provision (gross of the firms' outside option $\Pi^n$ in (A23)), where $\Pi^p(\alpha)$ is given by (A48) or (A55) and $C(\alpha) = c(\alpha - \alpha_0)$. Two possible collusive outcomes emerge. First, we consider the collusive outcome in Lemma 4. The information provider's profit maximization problem is described in (18) for $\Pi^c(\alpha) = \Pi^p(\alpha)$, where $\Pi^p(\alpha)$ is given by (A48). The first-order condition for $\alpha$ associated with an interior solution is $\overline{\theta}/\left[3\alpha^2(1-\delta)\right] - c = 0$.[33]

[33]Differentiating the left-hand side of this expression with respect to $\alpha$ yields $-2\overline{\theta}/\left[3\alpha^3(1-\delta)\right] < 0$, which

The equilibrium value for $\alpha$ in an interior solution is given by

$$\alpha^p\left(\overline{\theta}\right) = \sqrt{\frac{\overline{\theta}}{3c(1-\delta)}}. \tag{A57}$$

This constitutes an interior solution, i.e., $\alpha^p\left(\overline{\theta}\right) \in (\alpha_0, 1)$, where $\alpha_0 = 1/3$, if and only if $c > \overline{\theta}/[3(1-\delta)]$ and $c < 3\overline{\theta}/(1-\delta)$. We find that the information provider's equilibrium profits (gross of the firms' outside option $\Pi^n$ in (A23)) evaluated at $\alpha^p = 1$ and $\alpha^p\left(\overline{\theta}\right) \in (\alpha_0, 1)$ in (A57) are respectively given by

$$\Psi^p|_{\alpha^p=1} = \frac{\overline{\theta} - 2c(1-\delta)}{3(1-\delta)} \quad \text{and} \quad \Psi^p|_{\alpha^p\left(\overline{\theta}\right)\in(\alpha_0,1)} = \frac{2\overline{\theta} + c(1-\delta) - 2\sqrt{3c(1-\delta)\overline{\theta}}}{3(1-\delta)}. \tag{A58}$$

The analysis proceeds through the following cases.

(Ia) Let $c \leq \overline{\theta}/[3(1-\delta)]$. We have $\alpha^p = 1$ as long as $\Psi^p|_{\alpha^p=1} - \Pi^n > 0$. Using (A23) and (A58), we find that $\Psi^p|_{\alpha^p=1} - \Pi^n > 0$ for $c < \left(\overline{\theta} - \widehat{\theta}\right)/[2(1-\delta)]$.

(Ib) Let $\overline{\theta}/[3(1-\delta)] < c < 3\overline{\theta}/(1-\delta)$. We have $\alpha^p\left(\overline{\theta}\right) \in (\alpha_0, 1)$ in (A57) as long as $\Psi^p|_{\alpha^p\left(\overline{\theta}\right)\in(\alpha_0,1)} - \Pi^n > 0$. Using (A23) and (A58), we find that $\Psi^p|_{\alpha^p\left(\overline{\theta}\right)\in(\alpha_0,1)} - \Pi^n > 0$ for $c < \left[4\overline{\theta} + \widehat{\theta} - 2\sqrt{3\overline{\theta}\left(\overline{\theta} + \widehat{\theta}\right)}\right]/(1-\delta)$.

(Ic) Alternatively, we have $\alpha^p = \alpha_0$.

Now, we turn to the collusive outcome in Lemma 5. The information provider's profit maximization problem is described in (18) for $\Pi^c(\alpha) = \Pi^p(\alpha)$, where $\Pi^p(\alpha)$ is given by (A55). The first-order condition for $\alpha$ associated with an interior solution is $4\widehat{\theta}/\left[3(1+\alpha)^2(1-\delta)\right] - c = 0$.[34] The equilibrium value for $\alpha$ in an interior solution is given by

$$\alpha^p\left(\widehat{\theta}\right) = 2\sqrt{\frac{\widehat{\theta}}{3c(1-\delta)}} - 1. \tag{A59}$$

This constitutes an interior solution, i.e., $\alpha^p\left(\widehat{\theta}\right) \in (\alpha_0, 1)$, where $\alpha_0 = 1/3$, if and only if $c > \widehat{\theta}/[3(1-\delta)]$ and $c < 3\widehat{\theta}/[4(1-\delta)]$. We find that the information provider's equilibrium profits (gross of the firms' outside option $\Pi^n$ in (A23)) evaluated at $\alpha^p = 1$ and $\alpha^p\left(\widehat{\theta}\right) \in (\alpha_0, 1)$ in (A59) are respectively given by

$$\Psi^{p\prime}|_{\alpha^p=1} = 2\frac{\widehat{\theta} - c(1-\delta)}{3(1-\delta)} \quad \text{and} \quad \Psi^{p\prime}|_{\alpha^p\left(\widehat{\theta}\right)\in(\alpha_0,1)} = \frac{4\widehat{\theta} + 4c(1-\delta) - 4\sqrt{3c(1-\delta)\widehat{\theta}}}{3(1-\delta)}. \tag{A60}$$

The analysis proceeds through the following cases.

(IIa) Let $c \leq \widehat{\theta}/[3(1-\delta)]$. We have $\alpha^p = 1$ (it follows from (A23) and (A60) that $\Psi^{p\prime}|_{\alpha^p=1} - \Pi^n > 0$).

(IIb) Let $\widehat{\theta}/[3(1-\delta)] < c < 3\widehat{\theta}/[4(1-\delta)]$. We have $\alpha^p\left(\widehat{\theta}\right) \in (\alpha_0, 1)$ in (A59) (it follows from (A23) and (A60) that $\Psi^{p\prime}|_{\alpha^p\left(\widehat{\theta}\right)\in(\alpha_0,1)} - \Pi^n > 0$).

(IIc) Alternatively, we have $\alpha^p = \alpha_0$.

ensures that the second-order condition for a maximum is satisfied.

[34] Differentiating the left-hand side of this expression with respect to $\alpha$ yields $-8\widehat{\theta}/\left[3(1+\alpha)^3(1-\delta)\right] < 0$, which ensures that the second-order condition for a maximum is satisfied.

Using (A58) and (A60), we obtain that $\Psi^p|_{\alpha^p=1} > \Psi^{p\prime}|_{\alpha^p=1}$ if and only if $\overline{\theta} > \widetilde{\theta}^m$ (where $\widetilde{\theta}^m \triangleq 2\widehat{\theta}$, as defined in Section 5.1) and that $\Psi^p|_{\alpha^p=1} > \Psi^{p\prime}|_{\alpha^p(\overline{\theta})\in(\alpha_0,1)}$ if $\overline{\theta} \geq \overline{\theta}'$, where $\overline{\theta}' \triangleq 3\widehat{\theta}$. It follows from cases (Ia) through (Ic) that, for $\overline{\theta} \geq \overline{\theta}'$, there exist two thresholds $\underline{c}' \triangleq \overline{\theta}/\left[3\left(1-\delta\right)\right]$ and $\overline{c}' \triangleq \left[4\overline{\theta}+\widehat{\theta}-2\sqrt{3\overline{\theta}\left(\overline{\theta}+\widehat{\theta}\right)}\right]/\left(1-\delta\right)$ such that $(i)$ for $c \leq \underline{c}'$ the equilibrium value for $\alpha$ is $\alpha^p = 1$, $(ii)$ for $\underline{c}' < c < \overline{c}'$ the equilibrium value for $\alpha$ is $\alpha^p\left(\overline{\theta}\right) \in (\alpha_0, 1)$ in (A57), and $(iii)$ for $c \geq \overline{c}'$ the equilibrium value for $\alpha$ is $\alpha^p = \alpha_0$. For $\overline{\theta} \in \left(\widetilde{\theta}^m, \overline{\theta}'\right)$, we obtain from (A58) and (A60) that $\Psi^p|_{\alpha^p=1} > \Psi^{p\prime}|_{\alpha^p=1}$ and that $\Psi^p|_{\alpha^p=1} > \Psi^{p\prime}|_{\alpha^p(\widehat{\theta})\in(\alpha_0,1)}$ if and only if $c < \min\left\{3\widehat{\theta}/\left[4\left(1-\delta\right)\right], \left[\overline{\theta}+2\sqrt{2\widehat{\theta}\left(\overline{\theta}-2\widehat{\theta}\right)}\right]/\left[6\left(1-\delta\right)\right]\right\}$. We find from cases (Ia), (IIb) and (IIc) that there exist two thresholds $\underline{c}'' \triangleq \min\left\{3\widehat{\theta}/\left[4\left(1-\delta\right)\right], \left[\overline{\theta}+2\sqrt{2\widehat{\theta}\left(\overline{\theta}-2\widehat{\theta}\right)}\right]/\left[6\left(1-\delta\right)\right]\right\}$ and $\overline{c}'' \triangleq 3\widehat{\theta}/\left[4\left(1-\delta\right)\right]$ such that $(i)$ for $c \leq \underline{c}''$ the equilibrium value for $\alpha$ is $\alpha^p = 1$, $(ii)$ for $\underline{c}'' < c < \overline{c}''$ the equilibrium value for $\alpha$ is $\alpha^p\left(\widehat{\theta}\right) \in (\alpha_0, 1)$ in (A59), and $(iii)$ for $c \geq \overline{c}''$ the equilibrium value for $\alpha$ is $\alpha^p = \alpha_0$.[35] For $\overline{\theta} \leq \widetilde{\theta}^m$, the collusive outcome in Lemma 5 applies. It follows from cases (IIa) through (IIc) that there exist two thresholds $\underline{c}''' \triangleq \widehat{\theta}/\left[3\left(1-\delta\right)\right]$ and $\overline{c}''' \triangleq 3\widehat{\theta}/\left[4\left(1-\delta\right)\right]$ such that $(i)$ for $c \leq \underline{c}'''$ the equilibrium value for $\alpha$ is $\alpha^p = 1$, $(ii)$ for $\underline{c}''' < c < \overline{c}'''$ the equilibrium value for $\alpha$ is $\alpha^p\left(\widehat{\theta}\right) \in (\alpha_0, 1)$ in (A59), and $(iii)$ for $c \geq \overline{c}'''$ the equilibrium value for $\alpha$ is $\alpha^p = \alpha_0$. ■

**Proof of Proposition 6.** It follows from the information provider's profit in (A56) under ex ante information provision and in (A58) or (A60) under ex post information provision that $(i)$ $\Psi^a|_{\alpha^a=1} > \Psi^p|_{\alpha^p=1}$, $(ii)$ $\Psi^a|_{\alpha^a=1} > \Psi^{p\prime}|_{\alpha^p=1}$, $(iii)$ there exists a threshold $c' \triangleq \min\left\{\overline{\theta}/\left[2\left(1-\delta\right)\right], \left[\overline{\theta}+\widehat{\theta}+2\sqrt{\overline{\theta}\widehat{\theta}}\right]/\left[3\left(1-\delta\right)\right]\right\}$ such that it holds $\Psi^a|_{\alpha^a=1} > \Psi^p|_{\alpha^p(\overline{\theta})\in(\alpha_0,1)}$ if and only if $c < c'$, $(iv)$ for $\overline{\theta} \geq \overline{\theta}''$, with $\overline{\theta}'' \triangleq (3/2)\,\widehat{\theta}$, it holds $\Psi^a|_{\alpha^a=1} > \Psi^p|_{\alpha^p(\widehat{\theta})\in(\alpha_0,1)}$, whereas for $\overline{\theta} < \overline{\theta}''$ there exists a threshold $c'' \triangleq \left[\overline{\theta}+\widehat{\theta}+2\sqrt{2\widehat{\theta}\left(\overline{\theta}-\widehat{\theta}\right)}\right]/\left[6\left(1-\delta\right)\right]$ such that it holds $\Psi^a|_{\alpha^a=1} > \Psi^p|_{\alpha^p(\widehat{\theta})\in(\alpha_0,1)}$ if and only if $c < c''$.[36] Combining the findings in points $(i)$ through $(iv)$ with those in Propositions 4 and 5 yields the results in the proposition. ■

# References


Abada, I., and Lambin, X. (2023). Artificial intelligence: Can seemingly collusive outcomes be avoided? *Management Science*, 69(9), 5042-5065.

Abreu, D., Milgrom, P., and Pearce, D. (1991). Information and timing in repeated partnerships. *Econometrica*, 59(6), 1713-1733.

Argenziano, R., Severinov, S., and Squintani, F. (2016). Strategic information acquisition and transmission. *American Economic Journal: Microeconomics*, 8(3), 119-155.


[35] Note that the interval $\underline{c}'' < c < \overline{c}''$ is non-empty if and only if $\left[\overline{\theta}+2\sqrt{2\widehat{\theta}\left(\overline{\theta}-2\widehat{\theta}\right)}\right]/\left[6\left(1-\delta\right)\right] < 3\widehat{\theta}/\left[4\left(1-\delta\right)\right]$, i.e., $\overline{\theta} < (5/2)\,\widehat{\theta}$. For $\overline{\theta} \geq (5/2)\,\widehat{\theta}$, it follows from cases (Ia) through (Ic) that $\alpha^p = 1$ for $c \leq \underline{c}'' = \overline{c}''$ and $\alpha^p = \alpha_0$ otherwise, where $\underline{c}'' = \overline{c}'' = \left(\overline{\theta}-\widehat{\theta}\right)/\left[2\left(1-\delta\right)\right]$.

[36] It follows from Propositions 4 and 5 that, when $c$ is sufficiently large, the equilibrium value for $\alpha$ is $\alpha^a = \alpha^p = \alpha_0$, which yields the trivial case $\Psi^a|_{\alpha^a=\alpha_0} = \Psi^p|_{\alpha^p=\alpha_0} = 0$.

Awaya, Y., and Krishna, V. (2016). On communication and collusion. *American Economic Review*, 106(2), 285-315.

Bergemann, D., Bonatti, A., and Smolin, A. (2018). The design and price of information. *American Economic Review*, 108(1), 1-48.

Bergemann, D., and Morris, S. (2019). Information design: A unified perspective. *Journal of Economic Literature*, 57(1), 44-95.

Bergemann, D., and Välimäki, J. (2002). Information acquisition and efficient mechanism design. *Econometrica*, 70(3), 1007-1033.

Calvano, E., Calzolari, G., Denicolò, V., and Pastorello, S. (2020). Artificial intelligence, algorithmic pricing, and collusion. *American Economic Review*, 110(10), 3767-3297.

Calvano, E., Calzolari, G., Denicolò, V., and Pastorello, S. (2021). Algorithmic collusion with imperfect monitoring. *International Journal of Industrial Organization*, 79, 102712.

Calzolari, G., and Pavan, A. (2006). On the optimality of privacy in sequential contracting. *Journal of Economic Theory*, 130(1), 168-204.

Colombo, L., Femminis, G., and Pavan, A. (2014). Information acquisition and welfare. *Review of Economic Studies*, 81(4), 1438-1483.

Colombo, S., and Pignataro, A. (2022). Information accuracy and collusion. *Journal of Economics and Management Strategy*, 31(3), 638-656.

Denti, T. (2023). Unrestricted information acquisition. *Theoretical Economics*, 18(3), 1101-1140.

Ező, P., and Szentes, B. (2007). Optimal information disclosure in auctions and the handicap auction. *Review of Economic Studies*, 74(3), 705-731.

Green, E. J., and Porter, R. H. (1984). Noncooperative collusion under imperfect price information. *Econometrica*, 52(1), 87-100.

Hansen, K. T., Misra, K., and Pai, M. M. (2021). Frontiers: Algorithmic collusion: Supra-competitive prices via independent algorithms. *Marketing Science*, 40(1), 1-12.

Harrington, J. E., Jr. (2022). The effect of outsourcing pricing algorithms on market competition. *Management Science*, 68(9), 6889-6906.

Harrington, J. E., Jr., and Skrzypacz, A. (2007). Collusion under monitoring of sales. *Rand Journal of Economics*, 38(2), 314-331.

Harrington, J. E., Jr., and Skrzypacz, A. (2011). Private monitoring and communication in cartels: Explaining recent collusive practices. *American Economic Review*, 101(6), 2425-2449.

Hörner, J., and Jamison, J. (2007). Collusion with (almost) no information. *Rand Journal of Economics*, 38(3), 804-822.

Hörner, J., and Skrzypacz, A. (2016). Selling information. *Journal of Political Economy*, 124(6), 1515-1562.

Kastl, J., Pagnozzi, M., and Piccolo, S. (2018). Selling information to competitive firms. *Rand Journal of Economics*, 49(1), 254-282.

Klein, T. (2021). Autonomous algorithmic collusion: Q-learning under sequential pricing. *Rand Journal of Economics*, 52(3), 538-558.

Li, H., and Shi, X. (2017). Discriminatory information disclosure. *American Economic Review*, 107(11), 3363-3385.

Mathevet, L., Perego, J., and Taneva, I. (2020). On information design in games. *Journal of Political Economy*, 128(4), 1370-1404.

Miklós-Thal, J., and Tucker, C. (2019). Collusion by algorithm: Does better demand prediction facilitate coordination between sellers? *Management Science*, 65(4), 1552-1561.

Miller, D. A. (2012). Robust collusion with private information. *Review of Economic Studies*, 79(2), 778–811.

Peiseler, F., Rasch, A., and Shekhar, S. (2022). Imperfect information, algorithmic price discrimination, and collusion. *Scandinavian Journal of Economics*, 124(2), 516-549.

Rayo, L., and Segal, I. (2010). Optimal information disclosure. *Journal of Political Economy*, 118(5), 949-987.

Rojas, C. (2012). The role of demand information and monitoring in tacit collusion. *Rand Journal of Economics*, 43(1), 78-109.

Rotemberg, J. J., and Saloner, G. (1986). A supergame-theoretic model of price wars during booms. *American Economic Review*, 76(3), 390-407.

Sannikov, Y., and Skrzypacz, A. (2007). Impossibility of collusion under imperfect monitoring with flexible production. *American Economic Review*, 97(5), 1794-1823.

Stigler, G. J. (1964). A theory of oligopoly. *Journal of Political Economy*, 72(1), 44-61.

Sugaya, T., and Wolitzky, A. (2018). Maintaining privacy in cartels. *Journal of Political Economy*, 126(6), 2569-2607.

Tirole, J. (1988). *The theory of industrial organization*. MIT Press, Cambridge, MA.

Zhong, W. (2022). Optimal dynamic information acquisition. *Econometrica*, 90(4), 1537-1582.

# Information design for colluding firms

Supplementary Appendix

Alessandro Bonatti* Raffaele Fiocco† Salvatore Piccolo‡

February 2024

## 1 Introduction

This Supplementary Appendix complements the paper and generalizes the analysis to a setting with $M \geq 2$ demand states and $N \geq 2$ firms. In every period the demand state is $\theta_i \in \Theta_M \triangleq \{\theta_0, \theta_1, \ldots, \theta_{M-1}\}$, where $\theta_{M-1} > \ldots > \theta_1 > \theta_0$. Given that $\theta_0$ corresponds to zero demand, for the sake of exposition we impose $\theta_0 = 0$.[1] As in the baseline model, demand states are independently and identically distributed over time, and they share equal probabilities. An experiment is characterized by a signal $s_i \in S_M \triangleq \{s_0, s_1, \ldots, s_{M-1}\}$. With symmetric signals, the parameter $\alpha = \Pr[s_i | \theta_i]$ measures the degree of accuracy, or precision, of information. It follows from Assumption 1 of the paper that $\Pr[s_i | \theta_j] = (1-\alpha)/(M-1)$, for $i \neq j$. Given the signal $s_i$, the probability of the state of demand $\theta_i$ is (weakly) higher than the corresponding unconditional probability. This implies that $\alpha \in [\alpha_0, 1]$, where $\alpha_0 = 1/M$. The information provider's cost for $\alpha$ is $C(\alpha) = c(\alpha - \alpha_0)$, where $c > 0$ constitutes an inverse measure of efficiency of the information technology. As the benchmark case of no information does not affect the comparison between the two modes of information provision, without loss of insights we abstract from the corresponding analysis.

## 2 Ex ante information provision

We consider the collusive strategy according to which in every period each firm sets the price $\theta_i \in \{\theta_0, \theta_1, \ldots, \theta_{M-1}\}$ after receiving the signal $s_i \in \{s_0, s_1, \ldots, s_{M-1}\}$, respectively. If at least one firm obtained zero profit in the previous period (which is common knowledge), the market enters a punishment phase, whose duration $T_i \in \{T_0, T_1, \ldots, T_{M-1}\}$ is contingent on the signal received $s_i \in \{s_0, s_1, \ldots, s_{M-1}\}$, respectively. During the punishment phase, firms revert to the noncooperative equilibrium by setting their prices at the marginal cost.[2]

We define by $V_i \in \{V_0, V_1, \ldots, V_{M-1}\}$ the present discounted profit of a firm in the collusive phase after receiving the signal $s_i \in \{s_0, s_1, \ldots, s_{M-1}\}$, respectively. As demand states are

*MIT Sloan School of Management. Email address: bonatti@mit.edu
†University of Bergamo. Email address: raffaele.fiocco@unibg.it
‡University of Bergamo, Compass Lexecon and CSEF. Email address: salvapiccolo@gmail.com

[1] Each firm's (constant) marginal cost is set at zero accordingly.

[2] We refer to the discussion after Lemma 6 for the optimality of this collusive strategy.

equally likely, the degree $\alpha$ of information accuracy is such that $\alpha = \Pr[s_i | \theta_i] = \Pr[\theta_i | s_i]$. Upon receiving the signal $s_i$, each firm charges the price at $\theta_i$ and obtains a present discounted profit equal to

$$V_i = \sum_{h=i}^{M-1} \Pr[\theta_h | s_i] \left( \frac{\theta_i}{N} + \delta \sum_{j=0}^{M-1} \Pr[s_j] V_j \right) + \sum_{h=0}^{i-1} \Pr[\theta_h | s_i] \delta^{T_i+1} \sum_{j=0}^{M-1} \Pr[s_j] V_j$$
$$= \left[ \alpha + \frac{1-\alpha}{M-1} (M-1-i) \right] \left( \frac{\theta_i}{N} + \frac{\delta}{M} \sum_{j=0}^{M-1} V_j \right) + i \frac{1-\alpha}{M-1} \frac{\delta^{T_i+1}}{M} \sum_{j=0}^{M-1} V_j. \quad \text{(S1)}$$

In the current period, firms equally share collusive profits when the demand state is at least equal to $\theta_i$, which occurs with probability $\sum_{h=i}^{M-1} \Pr[\theta_h | s_i] = \alpha + (1-\alpha)(M-1-i)/(M-1)$, where $i \in \{0, 1, \ldots, M-1\}$. In this case, they continue to collude in the following period and obtain the expected present discounted profit $\sum_{j=0}^{M-1} V_j / M$. With complementary probability $\sum_{h=0}^{i-1} \Pr[\theta_h | s_i] = i(1-\alpha)/(M-1)$, firms face zero demand and thus make zero profit. This generates a punishment for $T_i \in \{T_0, T_1, \ldots, T_{M-1}\}$ periods according to the signal received $s_i \in \{s_0, s_1, \ldots, s_{M-1}\}$. At the end of the punishment phase, firms start colluding again.

Given signal $s_i$, a deviating firm slightly undercuts the collusive price $\theta_i$ and appropriates the entire expected collusive profit in the current period.[3] This leads to a punishment for $T_i$ periods. The incentive constraint that ensures the sustainability of collusion writes as

$$V_i \geq \sum_{h=i}^{M-1} \Pr[\theta_h | s_i] \theta_i + \delta^{T_i+1} \sum_{j=0}^{M-1} \Pr[s_j] V_j$$
$$= \left[ \alpha + \frac{1-\alpha}{M-1} (M-1-i) \right] \theta_i + \frac{\delta^{T_i+1}}{M} \sum_{j=0}^{M-1} V_j. \quad \text{(S2)}$$

We summarize the main results in the following lemma.

**Lemma 6** *Suppose that the state of demand can assume $M \geq 2$ values — i.e., $\theta_i \in \Theta_M \triangleq \{\theta_0, \theta_1, \ldots, \theta_{M-1}\}$ — and $N \geq 2$ firms operate in the market. Then, under ex ante information provision, in the most cooperative equilibrium, each firm sets the price at $\theta_i \in \{\theta_0, \theta_1, \ldots, \theta_{M-1}\}$ after receiving the signal $s_i \in \{s_0, s_1, \ldots, s_{M-1}\}$ respectively, provided that all firms obtained positive profits in the previous period. Otherwise, a punishment phase is triggered, which which each firm sets the price at the marginal cost for a number of periods equal to $T_i^a \in \{T_0^a, T_1^a, \ldots, T_{M-1}^a\}$ after receiving the signal $s_i \in \{s_0, s_1, \ldots, s_{M-1}\}$ respectively, where $T_{M-1}^a(\alpha) > \ldots > T_1^a(\alpha) > T_0^a = 0$.*

**Proof of Lemma 6.** Using (S1), we find that

$$\sum_{j=0}^{M-1} V_j = \sum_{i=0}^{M-1} \left[ \alpha + \frac{1-\alpha}{M-1} (M-1-i) \right] \left( \frac{\theta_i}{N} + \frac{\delta}{M} \sum_{j=0}^{M-1} V_j \right)$$
$$+ \frac{1-\alpha}{M(M-1)} \sum_{j=0}^{M-1} V_j \sum_{i=0}^{M-1} i \delta^{T_i+1}.$$

[3] As in the baseline model, slightly undercutting the collusive price $\theta_i$ constitutes an optimal deviation as long as information is sufficiently accurate. In particular, it must hold that, conditionally on the signal $s_i$, the collusive price $\theta_i$ maximizes the current expected profit, i.e., $\Pr[\theta \geq \theta_i | s_i] \theta_i > \Pr[\theta \geq \theta_j | s_i] \theta_j$, for any $\theta_i > \theta_j$, which yields $[\alpha + (1-\alpha)(M-1-i)/(M-1)] \theta_i > [\alpha + (1-\alpha)(M-1-j)/(M-1)] \theta_j$ and thus $\alpha > 1 - (M-1)(\theta_i - \theta_j)/(i\theta_i - j\theta_j)$.

This yields

$$\sum_{j=0}^{M-1} V_j(\alpha) = \frac{2M \sum_{i=0}^{M-1} [M-1-i(1-\alpha)]\,\theta_i}{N\left[M(M-1)(2-\delta-\alpha\delta) - 2(1-\alpha)\sum_{i=0}^{M-1} i\delta^{T_i+1}\right]}. \tag{S3}$$

Substituting (S3) into the incentive constraint (S2), we obtain after some manipulation

$$\begin{aligned} &-\frac{M-1-i(1-\alpha)}{N(M-1)\left[M(M-1)(2-\delta-\alpha\delta) - 2(1-\alpha)\sum_{j=0}^{M-1} j\delta^{T_j+1}\right]} \\ &\quad \times \left\{2\left(\delta-\delta^{T_i+1}\right)\sum_{j=0}^{M-1}[M-1-j(1-\alpha)]\,\theta_j \right. \\ &\quad \left. +(N-1)\,\theta_i\left[2(1-\alpha)\sum_{j=0}^{M-1} j\delta^{T_j+1} - (2-\delta-\alpha\delta)\,M(M-1)\right]\right\} \geq 0. \end{aligned} \tag{S4}$$

Differentiating the left-hand side of (S4) with respect to $T_i$ yields after some manipulation

$$\begin{aligned} &-\frac{2[M-1-i(1-\alpha)]\,\delta^{T_i+1}\ln\delta \sum_{j=0}^{M-1}[M-1-j(1-\alpha)]\,\theta_j}{N(M-1)\left[M(M-1)(2-\delta-\alpha\delta) - 2(1-\alpha)\sum_{j=0,\, j\neq i}^{M-1} j\delta^{T_j+1} - 2(1-\alpha)\,i\delta^{T_i+1}\right]^2} \\ &\quad \times \left[M(M-1)(2-\delta-\alpha\delta) - 2i(1-\alpha)\,\delta - 2(1-\alpha)\sum_{j=0,\, j\neq i}^{M-1} j\delta^{T_j+1}\right] > 0. \end{aligned} \tag{S5}$$

To demonstrate the sign of (S5), a preliminary step is to note that it corresponds to the sign of the expression in square brackets in the second line, which does not depend on $T_i$. First, suppose that the sign of this expression is negative. Then, a lower $T_i$ relaxes the incentive constraint (S4). Substituting the lowest value for $T_i$, i.e., $T_i = 0$, into the incentive constraint (S4) gives

$$-\frac{M-1-i(1-\alpha)}{N(M-1)}(N-1)\,\theta_i < 0,$$

where the inequality follows for any $\theta_i \in \{\theta_1, \theta_2, \ldots, \theta_{M-1}\}$.[4] Then, collusion can be sustained only if the expression in square brackets in the second line of (S5) is positive.[5] This implies that a lower $T_i$ tightens the incentive constraint (S4). Noting from (S3) that the firms' aggregate present discounted collusive profits $\Pi(\alpha) = (N/M)\sum_{j=0}^{M-1} V_j(\alpha)$ decrease with $T_i$, we find that the incentive constraint (S4) is binding in equilibrium. This yields

$$\sum_{j=0}^{M-1} j\delta^{T_j+1} = \frac{(2-\delta-\alpha\delta)\,M(M-1)(N-1)\,\theta_i - 2\left(\delta-\delta^{T_i+1}\right)\sum_{j=0}^{M-1}[M-1-j(1-\alpha)]\,\theta_j}{2(1-\alpha)(N-1)\,\theta_i}.$$

Substituting this expression into (S3), we obtain that

$$\sum_{j=0}^{M-1} V_j(\alpha) = \frac{M(N-1)\,\theta_i}{N\left(\delta-\delta^{T_i(\alpha)+1}\right)}. \tag{S6}$$

[4] As shown below, for $\theta_0 = 0$ in equilibrium it holds $T_0^a = 0$ conditional on the signal $s_0$.

[5] It follows from (S5) that this occurs for $\alpha$ large enough.

It follows from $M(N-1)\theta_1/\left[N\left(\delta-\delta^{T_1(\alpha)+1}\right)\right]=M(N-1)\theta_2/\left[N\left(\delta-\delta^{T_2(\alpha)+1}\right)\right]=...=M(N-1)\theta_{M-1}/\left[N\left(\delta-\delta^{T_{M-1}(\alpha)+1}\right)\right]$ that, for any given $\alpha$, the equilibrium punishment phase conditional on the signal $s_i\in\{s_1,s_2,\dots,s_{M-1}\}$ under ex ante information provision lasts for a number of periods equal to

$$T_i^a(\alpha)=(\ln\delta)^{-1}\ln\left(\frac{\theta_i\delta^{T_1^a(\alpha)+1}-\delta(\theta_i-\theta_1)}{\theta_1}\right)-1,$$

which implies that $\partial T_i^a(\cdot)/\partial\theta_i>0$ and thus $T_{M-1}^a(\alpha)>...>T_1^a(\alpha)$. Using the binding incentive constraint (S4), we find that

$$T_1^a(\alpha)=(\ln\delta)^{-1}\ln\left(\delta-\frac{M(M-1)(N-1)(1-\delta)\theta_1}{\sum_{j=0}^{M-1}[M-1-j(1-\alpha)]\theta_j-(1-\alpha)(N-1)\sum_{j=0}^{M-1}j\theta_j}\right)>0. \tag{S7}$$

Furthermore, it holds $T_0^a=0$ because firms do not have any incentives to deviate when the price is set at $\theta_0=0$.

Substituting (S7) into (S6) and recalling $\Pi(\alpha)=(N/M)\sum_{j=0}^{M-1}V_j(\alpha)$ we obtain that, for any given $\alpha$, the firms' equilibrium aggregate present discounted collusive profits under ex ante information provision are given by

$$\Pi^a(\alpha)=\frac{\sum_{j=0}^{M-1}[M-1-j(1-\alpha)]\theta_j-(1-\alpha)(N-1)\sum_{j=0}^{M-1}j\theta_j}{M(M-1)(1-\delta)}.\quad\blacksquare \tag{S8}$$

# 3 Ex post information provision

We consider the collusive strategy such that each firm charges the price $\theta_k\in\{\theta_1,\theta_2,\dots,\theta_{M-1}\}$ at the outset of the game and continues to do so as long as all firms obtained positive profits in the previous period.[6] If at least one firm obtained zero profit in the previous period (which is common knowledge), firms revert to the noncooperative equilibrium during a punishment phase that lasts for $T_i\in\{T_0,T_1,\dots,T_{M-1}\}$ periods according to the signal $s_i\in\{s_0,s_1,\dots,s_{M-1}\}$, respectively.

In the collusive phase, each firm sets the price at $\theta_k\in\{\theta_1,\theta_2,\dots,\theta_{M-1}\}$ and obtains a present discounted profit equal to

$$\begin{aligned}V&=\sum_{h=k}^{M-1}\Pr[\theta_h]\left(\frac{\theta_k}{N}+\delta V\right)+V\sum_{h=0}^{k-1}\Pr[\theta_h]\sum_{j=0}^{M-1}\Pr[s_j|\theta_h]\delta^{T_j+1}\\&=\frac{M-k}{M}\left(\frac{\theta_k}{N}+\delta V\right)+\frac{V}{M}\left\{k\frac{1-\alpha}{M-1}\sum_{j=k}^{M-1}\delta^{T_j+1}+\left[\alpha+\frac{1-\alpha}{M-1}(k-1)\right]\sum_{j=0}^{k-1}\delta^{T_j+1}\right\}.\end{aligned} \tag{S9}$$

When the demand state is at least equal to $\theta_k$, which occurs with probability $\sum_{h=k}^{M-1}\Pr[\theta_h]=(M-k)/M$, firms equally share the entire collusive profit in the current period and continue

[6] Clearly, the collusive price must be higher than $\theta_0=0$.

to collude in the following period. Otherwise, they face zero demand and thus make zero profit, which triggers a punishment for $T_i \in \{T_0, T_1, \ldots, T_{M-1}\}$ conditionally on the signal received $s_i \in \{s_0, s_1, \ldots, s_{M-1}\}$.

A deviating firm slightly undercuts the collusive price $\theta_k$ and appropriates the entire expected collusive profit in the current period, which leads to a punishment for $T_i \in \{T_0, T_1, \ldots, T_{M-1}\}$ periods. The incentive constraint that ensures the sustainability of collusion writes as

$$\begin{aligned} V &\geq \sum\nolimits_{h=k}^{M-1} \Pr[\theta_h]\, \theta_k + V \sum\nolimits_{j=0}^{M-1} \Pr[s_j]\, \delta^{T_j+1} \\ &= \frac{M-k}{M}\theta_k + \frac{V}{M}\sum\nolimits_{j=0}^{M-1} \delta^{T_j+1}. \end{aligned} \tag{S10}$$

We summarize the main results in the following lemma.

**Lemma 7** *Suppose that the demand state can assume $M \geq 2$ values — i.e., $\theta_i \in \Theta_M \triangleq \{\theta_0, \theta_1, \ldots, \theta_{M-1}\}$ — and $N \geq 2$ firms operate in the market. Then, under ex post information provision, there exists a candidate for the most cooperative equilibrium such that each firm sets the price at $\theta_k \in \{\theta_1, \theta_2, \ldots, \theta_{M-1}\}$, provided that all firms obtained positive profits in the previous period. Otherwise, a punishment phase is triggered, during which each firm sets the price at the marginal cost for a number of periods equal to $T_i^p \in \{T_0^p, T_1^p, \ldots, T_{M-1}^p\}$ after receiving the signal $s_i \in \{s_0, s_1, \ldots, s_{M-1}\}$ respectively, where $T_i^p(\alpha) > 0$ for $i \in \{k, ..., M-1\}$ and $T_i^p = 0$ for $i \in \{0, ..., k-1\}$.*

**Proof of Lemma 7.** It follows from (S9) that a firm's present discounted collusive profit is given by

$$V(\alpha) = \frac{(M-1)(M-k)\theta_k}{N\left[M(M-1)(1-\delta) + \delta k(M-1) - (1-\alpha)k\sum_{j=k}^{M-1}\delta^{T_j+1} - (k+\alpha M - \alpha k - 1)\sum_{j=0}^{k-1}\delta^{T_j+1}\right]}. \tag{S11}$$

Using (S11), the incentive constraint (S10) becomes

$$\begin{aligned} &\frac{(M-k)\theta_k}{MN\left[M(M-1)(1-\delta) + \delta k(M-1) - (1-\alpha)k\sum_{j=k}^{M-1}\delta^{T_j+1} - (k+\alpha M - \alpha k - 1)\sum_{j=0}^{k-1}\delta^{T_j+1}\right]} \\ &\times \Big\{(M-1)(M - MN + \delta MN - \delta kN) - [M - k(1-\alpha)N - 1]\sum\nolimits_{j=k}^{M-1}\delta^{T_j+1} \\ &- [M(1-\alpha N) - (k-1-\alpha k)N - 1]\sum\nolimits_{j=0}^{k-1}\delta^{T_j+1}\Big\} \geq 0. \end{aligned} \tag{S12}$$

Differentiating the left-hand side of (S12) with respect to $\sum_{j=k}^{M-1}\delta^{T_j+1}$ yields

$$\frac{(M-1)(M-k)\left[\delta + kM - \alpha kM - \delta kM - M(M-1)(1-\delta) - (1-\alpha M)\sum_{j=0}^{k-1}\delta^{T_j+1}\right]\theta_k}{MN\left[M(M-1)(1-\delta) + \delta k(M-1) - (1-\alpha)k\sum_{j=k}^{M-1}\delta^{T_j+1} - (k+\alpha M - \alpha k - 1)\sum_{j=0}^{k-1}\delta^{T_j+1}\right]^2} < 0,$$

which implies that a lower $\sum_{j=k}^{M-1}\delta^{T_j+1}$ tightens the incentive constraint (S12) (the inequality follows from the assumptions on the parameters of the model). As $V(\cdot)$ in (S11) increases with $\sum_{j=k}^{M-1}\delta^{T_j+1}$, the incentive constraint (S12) is binding in equilibrium. This gives

$$\sum_{j=k}^{M-1}\delta^{T_j+1} = \frac{(M-1)\left[M - MN(1-\delta) - \delta kN\right] - \left[M(1-\alpha N) - (k - \alpha k - 1)N - 1\right]\sum_{j=0}^{k-1}\delta^{T_j+1}}{M - 1 - k(1-\alpha)N}. \tag{S13}$$

Substituting (S13) into (S11) and differentiating (S11) with respect to $\sum_{j=0}^{k-1}\delta^{T_j+1}$ yields

$$-\frac{(M-k)(1-\alpha M)(M-1-kN+\alpha kN)\theta_k}{N\left[M(M-1)(1-\delta) - \delta k - kM + \alpha kM + \delta kM + (1-\alpha M)\sum_{j=0}^{M-1}\delta^{T_j+1}\right]^2} \geq 0,$$

which implies that $V(\cdot)$ in (S11) increases with $\sum_{j=0}^{k-1}\delta^{T_j+1}$ (the equality holds if and only if $\alpha = 1/M$). Then, for a given $\alpha$, the candidate equilibrium punishment phase conditional on the signal $s_i \in \{s_0, s_1, \ldots, s_{k-1}\}$ under ex post information provision lasts for a number of periods equal to

$$T_i^p = 0, \text{ for } i \in \{0, ..., k-1\}. \tag{S14}$$

Substituting (S14) into (S13), we find that, for any given $\alpha$, the candidate equilibrium punishment phase conditional on the signal $s_i \in \{s_k, s_{k+1}, \ldots, s_{M-1}\}$ lasts for a number of periods such that

$$\begin{aligned} T_i^p(\alpha) = (\ln\delta)^{-1}\ln\Bigg(&\frac{M(M-1)\left[1-(1-\delta)N\right] - \delta k\left[M-1+MN(1-\alpha)\right]}{M-1-k(1-\alpha)N} \\ &+\frac{\delta k^2(1-\alpha)N}{M-1-k(1-\alpha)N} - \sum_{j=k,\ j\neq i}^{M-1}\delta^{T_j+1}\Bigg) - 1 > 0, \text{ for } i \in \{k, ..., M-1\}. \end{aligned} \tag{S15}$$

Replacing (S14) and (S15) into (S11), we obtain that, for any given $\alpha$, the firms' candidate equilibrium aggregate present discounted collusive profits under ex post information provision are given by

$$\Pi^p(\alpha) = \frac{(M-k)\left[M-1-k(1-\alpha)N\right]}{M(1-\delta)\left[M-1-k(1-\alpha)\right]}\theta_k. \quad \blacksquare \tag{S16}$$

## 4 Equilibrium information provision

The information provider designs the accuracy of the experiment $\alpha \in [\alpha_0, 1]$ and extracts through the price $\rho$ the firms' aggregate collusive profits net of their outside option. Ignoring the outside option without loss of insights, the information provider's profit maximization problem writes as

$$\max_{\alpha\in[\alpha_0,1]} \Pi^c(\alpha) - C(\alpha), \tag{S17}$$

where $\Pi^c(\alpha) \in \{\Pi^a(\alpha), \Pi^p(\alpha)\}$ denotes the firms' aggregate collusive profits under ex ante and ex post information provision respectively, and $C(\alpha) = c(\alpha - \alpha_0)$ represents the cost of information provision. In the following proposition, we compare the two modes of information provision. It can be easily shown that with $M = 2$ demand states, the two modes of information provision are equally profitable for $N \geq 3$ firms (the case $N = 2$ is formalized in Proposition 3 of the paper). Then, we consider the setting with $M \geq 3$ demand states.

**Proposition 7** *Suppose that the state of demand can assume $M \geq 3$ values — i.e., $\theta_i \in \Theta_M \triangleq \{\theta_0, \theta_1, \ldots, \theta_{M-1}\}$ — and $N \geq 2$ firms operate in the market. Also, suppose that $\theta_k \in \{\theta_1, \theta_2, \ldots, \theta_{M-1}\}$ is the collusive price charged under ex post information provision. Then, for $\sum_{j=0,\ j\neq k}^{M-1} \theta_j \geq \Phi_k$, ex ante information provision is more profitable than ex post information provision. For $\sum_{j=0,\ j\neq k}^{M-1} \theta_j < \Phi_k$, ex ante information provision is more profitable than ex post information provision if and only if $c < c_k$. For $N > N_k$, ex ante information provision is more profitable than ex post information provision.*

**Proof of Proposition 7.** First, we consider the case of ex ante information provision. Let $\Psi^a(\alpha) \triangleq \Pi^a(\alpha) - C(\alpha)$ be the information provider's profit under ex ante information provision, where $\Pi^a(\alpha)$ is given by (S8) and $C(\alpha) = c(\alpha - \alpha_0)$. The information provider's profit maximization problem is described in (S17) for $\Pi^c(\alpha) = \Pi^a(\alpha)$. Taking the derivative of $\Psi^a(\alpha)$ with respect to $\alpha$ yields

$$\frac{N}{M(M-1)(1-\delta)} \sum_{j=0}^{M-1} j\theta_j - c.$$

As $\Psi^a(\alpha)$ increases with $\alpha$ for $c < N/[M(M-1)(1-\delta)] \sum_{j=0}^{M-1} j\theta_j$ and decreases with $\alpha$ otherwise, the equilibrium value for $\alpha$ is either $\alpha^a = 1$ or $\alpha^a = \alpha_0$, where $\alpha_0 = 1/M$. The information provider's equilibrium profit at $\alpha^a = 1$ under ex ante information provision is given by

$$\Psi^a|_{\alpha^a=1} = \frac{\sum_{j=0}^{M-1} \theta_j - c(M-1)(1-\delta)}{M(1-\delta)}. \tag{S18}$$

Now, we turn to the case of ex post information provision. Let $\Psi^p(\alpha) \triangleq \Pi^p(\alpha) - C(\alpha)$ be the information provider's profit under ex post information provision, where $\Pi^p(\alpha)$ is given by (S16) and $C(\alpha) = c(\alpha - \alpha_0)$. The information provider's profit maximization problem is described in (S17) for $\Pi^c(\alpha) = \Pi^p(\alpha)$. The first-order condition for $\alpha$ associated with an interior solution is

$$\frac{k(M-1)(M-k)(N-1)\theta_k}{M(1-\delta)[M-1-k(1-\alpha)]^2} - c = 0.$$

Differentiating the left-hand side of this expression with respect to $\alpha$ yields

$$\frac{2k^2(M-1)(M-k)(N-1)\theta_k}{M(1-\delta)[M-1-k(1-\alpha)]^3} < 0,$$

which ensures that the second-order condition for a maximum is satisfied (the inequality follows from the assumptions on the parameters of the model). The equilibrium value for $\alpha$ in an interior solution under ex post information provision is given by

$$\alpha^p(\theta_k) = \sqrt{\frac{(M-1)(M-k)(N-1)\theta_k}{ckM(1-\delta)}} - \frac{M-1-k}{k}. \tag{S19}$$

This constitutes an interior solution, i.e., $\alpha^p(\theta_k) \in (\alpha_0, 1)$, where $\alpha_0 = 1/M$, if and only if $c > k(M-k)(N-1)\theta_k / [M(M-1)(1-\delta)]$ and $c < kM(N-1)\theta_k / [(M-1)(M-k)(1-\delta)]$. This implies that, for $c < kM(N-1)\theta_k / [(M-1)(M-k)(1-\delta)]$, the equilibrium value for $\alpha$ is $\alpha^p = 1$ and the information provider's equilibrium profit is given by

$$\Psi^p|_{\alpha^p=1} = \frac{(M-k)\theta_k - c(M-1)(1-\delta)}{M(1-\delta)}. \tag{S20}$$

The information provider's equilibrium profit at $\alpha^p(\theta_k) \in (\alpha_0, 1)$ in (S19) is given by

$$\begin{aligned} \Psi^p|_{\alpha^p(\theta_k)\in(\alpha_0,1)} = & \frac{kN(M-k)\theta_k + c(M-1)(M-k)(1-\delta)}{kM(1-\delta)} \\ & - \frac{2\sqrt{ckM(M-1)(M-k)(N-1)(1-\delta)\theta_k}}{kM(1-\delta)}. \end{aligned} \tag{S21}$$

It follows from (S18) and (S20) that $\Psi^a|_{\alpha^a=1} > \Psi^p|_{\alpha^p=1}$. Furthermore, we find from (S18) and (S21) that there exists a threshold

$$\Phi_k \triangleq \frac{M(M-1) - k(2M - kN - 1)}{M-k}\theta_k$$

such that it holds $\Psi^a|_{\alpha^a=1} > \Psi^p|_{\alpha^p(\theta_k)\in(\alpha_0,1)}$ for $\sum_{j=0,\ j\neq k}^{M-1}\theta_j \geq \Phi_k$. For $\sum_{j=0,\ j\neq k}^{M-1}\theta_j < \Phi_k$, it holds $\Psi^a|_{\alpha^a=1} > \Psi^p|_{\alpha^p(\theta_k)\in(\alpha_0,1)}$ if and only if

$$c < c_k \triangleq \frac{\sum_{j=0}^{M-1}\theta_j + (N-2)(M-k)\theta_k + 2\sqrt{(M-k)(N-1)\left[\sum_{j=0}^{M-1}\theta_j - (M-k)\theta_k\right]\theta_k}}{M(M-1)(1-\delta)}k.$$

Taking the derivative of $\Psi^p|_{\alpha^p(\theta_k)\in(\alpha_0,1)}$ in (S21) with respect to $N$ (and ignoring the integer constraint) yields

$$\frac{(M-k)\theta_k}{M(1-\delta)} - \sqrt{\frac{c(M-1)(M-k)\theta_k}{kM(N-1)(1-\delta)}} < 0,$$

which implies that $\Psi^p|_{\alpha^p(\theta_k)\in(\alpha_0,1)}$ decreases with $N$ (the inequality follows from the assumptions on the parameters of the model). As $\Psi^a|_{\alpha^a=1}$ in (S18) is independent of $N$, there exists a threshold

$$N_k \triangleq \frac{k\sum_{j=0}^{M-1}\theta_j + cM(M-1)(1-\delta) - 2\sqrt{ckM(M-1)(1-\delta)\left[\sum_{j=0}^{M-1}\theta_j - (M-k)\theta_k\right]}}{k(M-k)\theta_k}$$

such that, for $N > N_k$, it holds $\Psi^a|_{\alpha^a=1} > \Psi^p|_{\alpha^p(\theta_k)\in(\alpha_0,1)}$. ■