Informational Autocrats, Diverse Societies*

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This paper presents a theoretical model of an autocrat who controls the media in an attempt to persuade a society of his competence. We base our analysis on a Bayesian persuasion framework, in which citizens have heterogeneous preferences and beliefs about the autocrat. We characterize the autocrat's information manipulation strategy when the society is monolithic and when it is divided. In equilibrium, when the preferences and beliefs in the society are more diverse, the autocrat engages in less information manipulation. Our findings thus suggest that the diversity of attitudes and opinions in a society can act as a bulwark against information manipulation by hostile actors.

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1 Introduction

Over the past two decades, many democracies have devolved into hybrid regimes and outright autocracies. From Venezuela's Hugo Chávez to Hungary's Victor Orbán and Russia's Vladimir Putin, politicians who came to power through democratic means have consolidated their control and undermined democratic institutions. Unlike the dictators of the 20th century, this new breed of autocrat does not resort to overt violence. Instead, they maintain power by building support among the masses and winning elections that appear to be democratic. To cultivate their image as competent leaders, they manipulate information by controlling state media (Rozenas and Stukal, 2019), co-opting or pressuring independent media outlets (McMillan and Zoido, 2004; Szeidl and Szucs, 2021), and covertly censoring unfavorable news (Lorentzen, 2014).[1] They are, as Guriev and Treisman (2019, 2020) put it, *informational autocrats*.

But not all autocracies are alike. In addition to those that still adhere to the 20th-century playbook and completely control the media (such as North Korea), there is a wide variation in media freedom across informational autocracies. As Egorov and Sonin (2022) note, "media freedom varies a lot across nondemocratic regimes, from levels comparable to mature democracies, to that of totalitarian regimes."[2] There is also substantial within-country variation in information manipulation over time. For instance, Tai (2014) shows that from 2007 to 2013, the Chinese "propaganda apparatus has banned fewer reports and guided more of them." As Guriev and Treisman (2019) observe, "in Taiwan, an overt dictatorship under Chiang Kai-Shek evolved into an informational autocracy under his son, Chiang Ching-Kuo, in his later years, before transitioning to full democracy in the 1990s."[3] This raises the question of why some societies are capable of preserving a degree of media freedom under autocratic rule while others are totally dominated by information manipulation.

This paper establishes a theoretical link between the distribution of attitudes and opinions in a society and its vulnerability to information manipulation. It posits that autocrats find it harder to manipulate information in their favor in more diverse societies. This is because the optimal manipulation strategy has to be fine-tuned to the attitudes and opinions of the citizens, and an increase in diversity makes this more challenging for autocrats. As attitudes and opinions become more dispersed, it becomes harder for autocrats to convince their opponents without alienating

[1] Attempts to control media and manipulate information extend beyond autocracies. Examples in democracies abound, from Argentina (Di Tella and Franceschelli, 2011) to Italy (Durante and Knight, 2012), and from Mexico (Stanig, 2015) to the United States (Qian and Yanagizawa-Drott, 2017; Gentzkow and Shapiro, 2008).

[2] A close look at the Global Media Freedom Dataset (GMFD) of Whitten-Woodring and Van Belle (2017) reveals the extent of variation. Among the 196 countries from 1948 to 2010 included in GMFD, 5275 observations are labeled as "non-democracies" based on their Polity Score. Interestingly, 4456 of such observations are classified as having "Not Free Media" by the authors, whereas the remaining observations have "Imperfectly Free/Free Media" (Whitten-Woodring and Van Belle, 2017, Fig.1, p.184).

[3] Yet another example is the immense pressure on media outlets during Rafael Correa's regime in Ecuador from 2007 to 2017, and the backtracking of these policies by his vice-president and successor, Lenín Moreno: `https://www.cjr.org/analysis/ecuador-moreno-correa-supercom-press-freedom.php`.

their supporters. They respond optimally by manipulating information less and allowing for a more free media landscape.

We present this insight using a Bayesian persuasion model à la Kamenica and Gentzkow (2011) with a population of heterogeneous citizens. The autocrat commits to a public communication policy. The citizens observe the message drawn according to the policy and decide whether to support the autocrat or not. The extent to which the autocrat manipulates information depends on the distribution of the citizens' attitudes and opinions. We analyze the autocrat's decision and characterize the optimal information manipulation policy.

At the model's core is the trade-off faced by an informational autocrat. To gain support, the autocrat must convince citizens of his competence through manipulation. He would do so by sending the message that things are "good" as frequently as possible. However, the citizens understand that information is manipulated and only act based on the autocrat's communication when they find it informative. As the autocrat manipulates information more, fewer people act based on the autocrat's messages but those who do lend him a higher support. The optimal policy balances these two effects.

The paper's main result establishes that when the citizens' attitudes and opinions are more dispersed, the autocrat finds it optimal to engage in less information manipulation. The intuition for this result is best understood by introducing a small amount heterogeneity to a standard Bayesian persuasion model, in which citizens all have identical attitudes and opinions. In the model with identical citizens, the optimal strategy entails sending the "bad" message just frequently enough that citizens are indifferent between supporting the autocrat and not when they receive the "good" message. Compare this with a society where the citizens' attitudes and opinions are dispersed but centered around those in the homogeneous society. If the autocrat were to follow the communication policy that was optimal in the homogeneous case, he would only get the support of about half of the citizens by sending the "good" message. To receive more support, the autocrat needs to reach out to citizens who are more skeptical than the median citizen. This can only be done by reducing the extent of information manipulation.

Our main contribution is to illustrate how this reasoning generalizes under a novel partial order on distributions, which captures the idea of dispersion in attitudes and opinions. We start with a model in which citizens are heterogeneous along two dimensions: preferences and initial beliefs about the autocrat. We collapse these two dimensions of heterogeneity into a one-dimensional distribution, which we call the *virtual density*. The virtual density is a sufficient statistic for the distribution of opinions and attitudes when it comes to the citizens' support for the autocrat. When the virtual density is single-peaked, the opinions and attitudes in the society are similar to each other, i.e., the society is *monolithic*. Conversely, when the virtual density is single-dipped, there are two large groups with opposing attitudes toward the autocrat, i.e., the society is *divided*. We characterize the optimal persuasion policy in the cases where the virtual density is single-peaked and single-dipped. This allows us to consistently define what it means for the autocrat to engage in *more/less information manipulation*. It also allows us to introduce a partial order on one-dimensional distributions that captures the idea of *dispersion*. Our comparative statics results establish that when the virtual density is more dispersed, the autocrat engages in less information manipulation under the optimal policy. This result holds in both monolithic and divided societies.

Related Literature. First and foremost, our model contributes to the growing literature on informational autocrats (Guriev and Treisman, 2019, 2020, 2022; Egorov and Sonin, 2022; Gehlbach, Luo, Shirikov and Vorobyev, 2022).[4] A closely related literature studies *media capture*, the idea that politicians exert control over media by co-opting private media (Besley and Prat, 2006), controlling state media (Gehlbach and Sonin, 2014), censoring news (Shadmehr and Bernhardt, 2015; Boleslavsky, Shadmehr and Sonin, 2021), or controlling media's access to information (Ozerturk, 2022); see Prat (2015) and Enikolopov and Petrova (2015) for two comprehensive reviews.[5] We contribute to this literature by establishing that the vulnerability of a society to media capture depends not only on the opinion of the median citizen but also on the dispersion of opinions in the society.

A stream of papers focus on understanding the variation in information manipulation and studying its limits. As a source of variation, Egorov, Guriev and Sonin (2009) study the natural resource endowment, VonDoepp and Young (2013) study the threats that governments face, while McGreevy-Stafford (2020) study protests. Another literature identifies factors that limit information manipulation. Di Tella, Galiani and Schargrodsky (2012) emphasize first-hand experiences, Durante and Knight (2012), Gläßel and Paula (2020), Knight and Tribin (2022), and Enikolopov, Rochlitz, Schoors and Zakharov (2023) emphasize the existence of alternative media outlets, Qin, Strömberg and Wu (2018) emphasize market competition, and Knight and Tribin (2019) emphasize the citizens' ability to "tune out." Our findings contribute to this literature by highlighting that the diversity of citizens' attitudes and opinions can a put a limit on the extent of information manipulation.

Our model is a Bayesian persuasion model à la Kamenica and Gentzkow (2011) with a heterogeneous audience. We incorporate both heterogeneous preferences (Wang, 2015; Alonso and Câmara, 2016a; Kolotilin, Mylovanov, Zapechelnyuk and Li, 2017; Bardhi and Guo, 2018; Chan, Gupta, Li and Wang, 2019; Arieli and Babichenko, 2019; Kerman, Herings and Karos, 2022; Sun, Schram and Sloof, 2022) and heterogeneous priors (Alonso and Câmara, 2016b; Laclau

[4] Also related is the literature on *democratic authoritarianism* (Brancati, 2014) and *competitive authoritarianism* (Levitsky and Way, 2002) in political science. However, those works are less focused on information manipulation and more on the dismantling of democratic institutions.

[5] Corneo (2006), Petrova (2008), Petrova (2012), and Alonso and Padró i Miquel (2022) present models of media capture by special interest groups.

and Renou, 2017; Kosterina, 2022).[6] To collapse our two-dimensional primitives onto a single dimension, we introduce an object that we call *virtual density*. Our comparative statics results rely on the partial order we introduce on the virtual density. Kolotilin (2015), Kolotilin, Mylovanov and Zapechelnyuk (2022), Sun, Schram and Sloof (2022), and Curello and Sinander (2022) also conduct comparative statics exercises in Bayesian persuasion settings. Whereas Kolotilin (2015) focuses on changes in welfare, we analyze how the optimal policy changes with parameters of the model. Sun, Schram and Sloof (2022) derive comparative statics results with respect to the sender's preferences. Our comparative statics result complement theirs by focusing on changes with respect to the receivers' characteristics. Kolotilin, Mylovanov and Zapechelnyuk (2022)'s comparative statics are with respect to the receivers' inclination to be persuaded, whereas ours is with respect to the receivers' heterogeneity. Finally, Curello and Sinander (2022) study the effect of changes in the sender's value function on the sender's optimal policy. Our focus on how changes in the *heterogeneity* of receivers affect the optimal persuasion policy sets this paper apart from those mentioned above.

A crucial assumption in the Bayesian persuasion literature is commitment by the sender. In our model, the sender can commit to a public communication policy, which is observed by all receivers. This assumption can be defended on several grounds. First of all, in our setup, persuasion satisfies the credibility assumption of Lin and Liu (2022).[7] Second, the autocrat's policy can viewed as an "editorial policy," which describes the general attitude of media sources, with the details of the coverage to be decided by reporters and editors (Gehlbach and Sonin, 2014). Finally, the outcome under commitment can be seen as a benchmark that describes the best-case scenario for the sender. Under this interpretation, our results characterize an "ideal media landscape" for a politician in a diverse society.

2 Setup

2.1 The Model

There are two types of agents: an autocrat and a unit measure of citizens, indexed by $r \in [0,1]$. Each citizen takes an action $a_r \in \{0,1\}$, the action being whether she supports the autocrat.

There is an underlying state of the world: $\theta \in \{0,1\}$. We call the $\theta = 1$ state the "good" state. When the state is good, supporting the autocrat is in the citizens' best interest. The $\theta = 0$ state is the "bad" state where it is optimal for citizens not to support the autocrat. Citizen r's payoff when

[6] Also related is the literature on information design, which studies the optimal information structure in a game to be played among multiple players (Bergemann and Morris, 2019; Taneva, 2019; Mathevet, Perego and Taneva, 2020; Inostroza and Pavan, 2022).

[7] In particular, we can allow for undetectable deviations by the sender. Since the sender's payoff in our model is additively separable, there is no profitable deviation that gives the same message distribution as the optimal policy. It should be noted that, with heterogeneous priors, the set of undetectable deviations is different for each agent. In such a case, one has to require deviations to be undetectable given the sender's prior.

she chooses action a_r and the state is θ is given by

$$u_r(a_r, \theta) = a_r(\theta - c_r), \tag{1}$$

where $c_r \in [0, 1]$ is citizen r's cost of supporting the autocrat. If citizen r knew the state, she would support the autocrat (i.e., $a_r = 1$) in the good state and not support her (i.e., $a_r = 0$) in the bad state. Hence, the good state may be interpreted as the state where the autocrat is competent, and the bad state is where the autocrat is incompetent.

The autocrat wants to persuade the citizens to support him regardless of the state of the world. His payoff when citizen r provides support a_r and the state is θ is given by

$$u_s(\{a_r\}_r) = \int_0^1 a_r dr. \tag{2}$$

When the state is good, the autocrat and citizens have common interests, whereas when the state is bad, their interests are opposed. We denote the autocrat's prior that the state is good by $p_s \in (0, 1)$.

The citizens do not learn the state of the world until after they have decided on whether to support the autocrat. Since the citizens do not observe the state, they can only act based on their beliefs. The autocrat can influence those beliefs (and the resulting actions) by sending informative messages. To simplify the analysis, we assume that the autocrat can commit to a public communication strategy $\sigma : \{0, 1\} \to \Delta(M)$, where $\sigma(\theta)[m]$ is the probability that public message $m \in M$ is generated when the state is θ. The communication strategy represents the policies followed by media controlled by the autocrat and used by him to influence the views of the citizens.

The citizens are heterogeneous both in their preferences and their prior beliefs. The heterogeneity of priors captures the idea that even people with identical payoffs may have different perspectives about the likelihood that the autocrat is competent. We let p_r denote citizen r's prior that the state is good and, let $f(c, p)$ denote the joint density of costs and priors in the population. We take f as a primitive of the model and study how changing the distribution affects the autocrat's optimal policy. We assume that f is common knowledge and continuously differentiable and bounded over its support.

The heterogeneity of perspectives poses a challenge for an autocrat who wants to garner broad support. Convincing different citizens with different preferences and beliefs requires different communication strategies. Yet, communication is public, so the autocrat cannot tailor his messaging strategy to the citizens' diverse perspectives. The main characterization result of the paper concerns the optimal way of resolving the inherent tension in convincing different segments of the population.

Timing. The timing of the communication game is as follows:

1. The prior and cost of each citizen is drawn, and citizen r observes (p_r, c_r).
2. The autocrat commits to a strategy σ, which is observed by all citizens.
3. The state is realized, and the autocrat sends the message drawn according to σ.
4. Each citizen r updates her prior and chooses an action a_r.
5. Payoffs are realized.

The solution concept we adopt is the Perfect Bayesian Equilibrium.

2.2 An Equivalent Representative-Citizen Problem

The fact that the autocrat is communicating with a population of heterogeneous citizens complicates her problem. However, the autocrat's optimal strategy can be found by solving a related persuasion problem with a *representative citizen* whose prior coincides with the autocrat's prior.

The key simplification comes from Proposition 1 of Alonso and Câmara (2016b). Consider citizens r and r' with priors p_r and $p_{r'} = p_s$. Since the two citizens observe the same (public) message, their posteriors are related through the following expression:

$$\mu_r = \frac{\mu_{r'} \frac{p_r}{p_{r'}}}{\mu_{r'} \frac{p_r}{p_{r'}} + (1 - \mu_{r'}) \frac{1-p_r}{1-p_{r'}}}, \tag{3}$$

where μ_r and $\mu_{r'}$ denote the posteriors of r and r', respectively.[8] This coupling of posteriors holds regardless of the communication strategy employed by the autocrat. It uniquely pins down the posterior μ_r of *every* citizen r as a function of the posterior of citizen r'—who will be our representative citizen.

Citizen r supports the autocrat if and only if her posterior that the state is good is at least as large as his cost of action; that is, $a_r = 1$ if and only if

$$c_r \leq c(\mu_s, p_r) \equiv \frac{\mu_s \frac{p_r}{p_s}}{\mu_s \frac{p_r}{p_s} + (1 - \mu_s) \frac{1-p_r}{1-p_s}}, \tag{4}$$

where μ_s denotes the posterior of the representative citizen (who has the same prior as the autocrat). The payoff to the autocrat is the fraction of the population who supports the autocrat:

$$v(\mu_s) = \int_0^1 \int_0^{c(\mu_s, p)} f(p, c) dc dp. \tag{5}$$

The autocrat's problem is thus equivalent to a standard Bayesian persuasion problem with a representative citizen. The autocrat and the citizen share the common prior p_s that the state is good. The payoff to the autocrat when he induces a posterior of μ_s for the representative citizen is given by $v(\mu_s)$, defined in equation (5). Following Kamenica and Gentzkow (2011), we refer

[8] Throughout the paper, we use posterior to mean subjective posterior probability of state $\theta = 1$ given an agent's information.

to $v(\mu_s)$ as the autocrat's *value function*. Whenever there is no risk of confusion, we drop the s subscript and simply write $v(\mu)$ for the value to the autocrat of inducing posterior μ for the representative citizen.

The value function has several useful properties. First, $v(\mu)$ is increasing in μ. Inducing a higher posterior for the representative citizen results in a higher posterior for every citizen, thus increasing the share of citizens who support the autocrat. Second, $v(0) = 0$ and $v(1) = 1$. When the representative citizen is certain that the state is bad, so is every other citizen. Therefore, no citizen supports the autocrat. Likewise, when the representative citizen is certain that the state is good, every other citizen is also certain that the state is good and supports the autocrat. Finally, $v(\mu)$ is differentiable in μ due to the differentiability of f.

The value function can thus be seen as a differentiable cumulative distribution function. We let $h(\mu) \equiv v'(\mu)$ denote the corresponding density and refer to it as the *virtual density* of the persuasion problem with heterogeneous citizens. The virtual density has an intuitive interpretation: $h(\mu)$ is the density of citizens who are indifferent between taking the two actions whenever the representative citizen's posterior is equal to μ. Importantly, construction of h allows us to reduce a two-dimensional object into a one-dimensional one. That is, it defines a *belief threshold* to every citizen, so that the citizen supports the autocrat if and only if the autocrat's beliefs are above her threshold. Note that the virtual density is a primitive of the problem: characteristics of f translate into characteristics of h. For example, when the citizens have common prior, the virtual density reduces to the distribution of costs in the population.

3 Information Manipulation in Monolithic Societies

3.1 Single-Peaked Distributions

The solution to the autocrat's persuasion problem takes a particularly simple form when the distribution of citizen types satisfies the following condition:

Definition 1. The virtual density $h(\mu)$ is *single-peaked* if there exists some $\tilde{\mu} \in [0,1]$ such that $h'(\mu) > 0$ for all $\mu < \tilde{\mu}$ and $h'(\mu) < 0$ for all $\mu > \tilde{\mu}$.

Single-peakedness is an assumption on the joint distribution of citizens' costs and prior beliefs. It requires a large share of citizens to have moderate preferences and beliefs, with fewer and fewer people having extreme preferences or beliefs. We thus consider single-peaked virtual densities to be representative of *monolithic* societies.

The significance of Definition 1 rests on the following observation: When the virtual density is single-peaked, the autocrat's value function is first convex and then concave. Figure 1 illustrates the value function in this case. Corollary 2 of Kamenica and Gentzkow (2011) implies the following characterization of the optimal strategy when the virtual density is single-peaked:

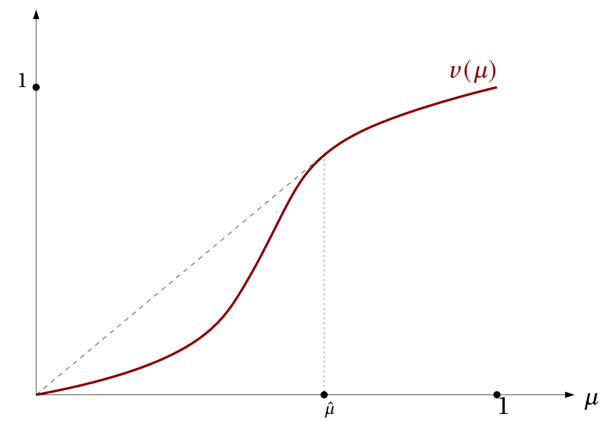

Figure 1. The value function under the assumption that the virtual density is single-peaked.

Proposition 1. *If the virtual density is single-peaked, the optimal strategy uses only two messages, and one of the messages fully reveals the bad state.*

We maintain the assumption of single-peakedness throughout this section. We do so in part for tractability. However, single-peaked distributions also constitute a natural and widely used class of distribution functions. In Section 4, we show that optimal strategy in the case where the virtual density is instead single-dipped is the mirror image of the optimal strategy in the single-peaked case.

Whether the virtual density is single-peaked only depends on the distribution of types, f, and the autocrat's prior, p_s. In the remainder of this subsection, we find a set of easy-to-check sufficient conditions for the virtual density to the single-peaked. If citizens have a common prior that coincides with the autocrat's prior, then the single-peakedness of the virtual density is equivalent to the single-peakedness of the density of costs:

Proposition 2. *Suppose $p_r = p_s$ for all r. The virtual density $h(\mu)$ is single-peaked in μ if and only if the density of costs $f(c)$ is single-peaked in c.*

If citizens have a common cost, on the other hand, then the single-peakedness of the virtual density is implied by a condition that is weaker than the log-concavity of the density of priors:

Proposition 3. *Suppose $c_r = c \in (0,1)$ for all r. The virtual density $h(\mu)$ is single-peaked if the density of priors $f(p)$ is strictly positive for all $p \in (0,1)$ and satisfies*

$$\frac{d^2}{dp^2} \log f(p) < 2(\gamma - 1)^2 \min\left\{1, \frac{1}{\gamma^2}\right\} \quad \text{for all } p \in (0,1), \tag{6}$$

where $\gamma \equiv \frac{1-c}{c}\frac{1-p_s}{p_s} \geq 0$.

The following corollary of Proposition 3 is a straightforward consequence of the facts that the left-hand side of equation (6) is negative if $f(p)$ is log-concave, while its right-hand side is always non-negative:[9]

[9]See Bagnoli and Bergstrom (2005) for a list of well-known distributions satisfying log-concavity.

Corollary 1. *Suppose $c_r = c$ for all r. The virtual density $h(\mu)$ is single-peaked in μ if the density of priors $f(p)$ is strictly log-concave in p.*

3.2 A Measure of Information Manipulation

In light of Proposition 1, we can assume without loss that the autocrat uses only two messages. We label the messages $m \in M = \{0,1\}$, with $m = 1$ the "good" message, which is suggestive of $\theta = 1$, and $m = 0$ the "bad" message, which is suggestive of $\theta = 0$. The autocrat's strategy can be represented by a pair of numbers:

$$\sigma = (\sigma^0, \sigma^1) \in [0,1]^2,$$

where $\sigma^\theta \equiv \sigma(\theta)[m = 1]$ is the probability of sending the good message in state $\theta \in \{0,1\}$. Throughout, we assume without loss of generality that $\sigma^1 \geq \sigma^0$.

The autocrat *manipulates information* if he sends the good message when the state is bad or sends the bad message when the state is good. By Proposition 1, when the virtual density is single-peaked, the bad message fully reveals the bad state; this entails sending the good message whenever the state is good. Therefore, manipulation of information is conveniently summarized in the single-peaked case by the probability σ^0 of sending the good message when the state is bad. We use the following notion of information manipulation in this case:

Definition 2. Consider single-peaked virtual densities h_1 and h_2 with the corresponding optimal strategies $\sigma_1 = (\sigma_1^0, \sigma_1^1)$ and $\sigma_2 = (\sigma_2^0, \sigma_2^1)$ for the autocrat. The autocrat *manipulates information less* given h_1 than given h_2 if $\sigma_1^0 \leq \sigma_2^0$.

3.3 A Measure of Dispersion

As our main motivation is studying how dispersion affects information manipulation, we need to introduce a measure of dispersion. Our measure of dispersion is a novel partial order on probability distributions:

Definition 3. Consider single-peaked densities f_1 and f_2 supported on a common compact set and satisfying

$$f_2(x) = \frac{(f_1(x))^\alpha}{\kappa} \qquad \text{for all } x, \tag{7}$$

some $\alpha > 0$, and a normalization constant $\kappa > 0$. If $0 < \alpha \leq 1$, then f_2 is *more dispersed* than f_1. If $\alpha \geq 1$, then f_2 is *less dispersed* than f_1.

The partial order has an intuitive interpretation. Consider densities f_1 and f_2 satisfying (7) for some $\alpha > 1$. Going from f_1 to f_2 moves mass from parts of the distribution that initially have smaller mass to parts with larger initial mass. In other words, f_2 looks like f_1, but with higher peaks and deeper troughs. On the other hand, since f_1 is single-peaked, most of its mass is concentrated

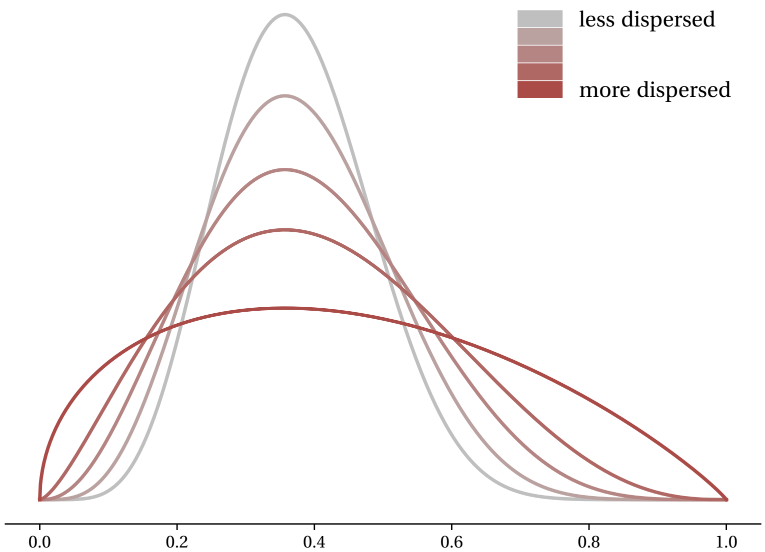

Figure 2. The dispersion order on single-peaked densities.

around its peak. Therefore, f_2 has even more mass in the center and even less mass in the periphery than f_1; that is, f_2 is less dispersed than f_1. Figure 2 illustrates the probability density functions for a set of single-peaked Beta distributions that are ranked in the dispersion order.

It is instructive to consider the extremes of equation (7). In the $\alpha \to \infty$ limit, f_2 becomes a point mass at the mode of f_1. Therefore, for any distribution f_1 with a unique mode, the degenerate distribution f_2 with a point mass on the mode of f_1 is less dispersed than f_1. Conversely, in the $\alpha \to 0$ limit, f_2 becomes the uniform distribution on the support of f_1. Thus, the uniform distribution is more dispersed than any single-peaked distribution with the same support.

Members of many parametric families of distributions can be ordered in the dispersion order. Two examples follow:

Example 1. Consider two single-peaked Beta distributions:

$$f_1 = \text{Beta}(\alpha_1, \beta_1),$$
$$f_2 = \text{Beta}(\alpha_2, \beta_2),$$

where $\frac{\alpha_2 - 1}{\alpha_1 - 1} = \frac{\beta_2 - 1}{\beta_1 - 1} = \alpha$ for some $\alpha \geq 0$. If $\alpha \leq 1$, then f_2 is more dispersed than f_1, while if $\alpha \geq 1$, then f_2 is less dispersed than f_1. In particular, any two symmetric and single-peaked Beta distributions are ranked according to the dispersion partial order.

Example 2. Consider the following truncated normal distributions on $[0, 1]$:

$$f_1 = \text{TruncatedNormal}(\mu, \sigma_1^2),$$
$$f_2 = \text{TruncatedNormal}(\mu, \sigma_2^2).$$

If $\sigma_2^2 \geq \sigma_1^2$, then f_2 is more dispersed than f_1, while if $\sigma_2^2 \leq \sigma_1^2$, then f_2 is less dispersed than f_1.

3.4 Dispersion and Information Manipulation in Monolithic Societies

We are now ready to examine how dispersion affects information manipulation. Our main result establishes that information manipulation is less severe in more diverse societies:

Theorem 1. *Let h_1 and h_2 be two single-peaked virtual densities. If h_1 is more dispersed than h_2, then the autocrat manipulates information less given h_1 than h_2.*

Why does more dispersion lead to less information manipulation? The mechanical answer to this question lies in the shape of the value function in Figure 1: in a more dispersed society, the value function is less rapidly increasing around the modal citizen, which means $\hat{\mu}$ shifts to the right. To gain some intuition, recall that our construction of virtual density assigns a *belief threshold* to every citizen. This allows us to rank the citizens in terms of their willingness to support the autocrat: a citizen with a lower threshold is more inclined to support. Under the optimal strategy, the autocrat targets a marginal citizen (a citizen with a threshold of $\hat{\mu}$). Citizens with thresholds below the marginal citizen find it optimal to take the action aligned with the message; in other words, they *comply* with the message. Citizens with thresholds above $\hat{\mu}$ find it optimal to never support the autocrat. The optimal strategy chooses $\hat{\mu}$ such that there are a sufficient number of citizens who comply *and* the good message is produced sufficiently frequently (so that the compliers support the autocrat with high probability).

In a less dispersed society, all citizens are alike, so the thresholds are tightly concentrated around the modal citizen. Therefore, targeting a citizen whose threshold is slightly above the mode ensures the support of almost all citizens. On the other hand, when society is more dispersed, the citizens' thresholds are more dispersed, which means targeting the same citizen generates too few compliers. To address this, the autocrat increases the informativeness of the news, so that $\hat{\mu}$ is higher and there are more compliers.

Theorem 1 describes the impact of dispersion on information manipulation while maintaining the assumption that the society is monolithic, and so, the virtual density is single-peaked. In the next section, we study persuasion in highly divided societies in which there are more people in the extremes of preference and belief distribution than are at its center.

4 Divided Societies

Throughout this section, we study the properties of the optimal persuasion strategy when the virtual density is the polar opposite of single-peaked:

Definition 4. The virtual density $h(\mu)$ is *single-dipped* if there exists some $\tilde{\mu} \in [0,1]$ such that $h'(\mu) < 0$ for all $\mu < \tilde{\mu}$ and $h'(\mu) > 0$ for all $\mu > \tilde{\mu}$.

In a society with a single-dipped virtual density, there are fewer moderates than those with extreme preferences or beliefs. We therefore consider single-dipped virtual densities to be representative of *divided* societies.[10]

When the virtual density is single-dipped, the autocrat's value function is first concave and then convex. Our next result characterizes the optimal persuasion strategy in this case.

Proposition 4. *If the virtual density is single-dipped, the optimal strategy uses only two messages, and the good message perfectly reveals the good state.*

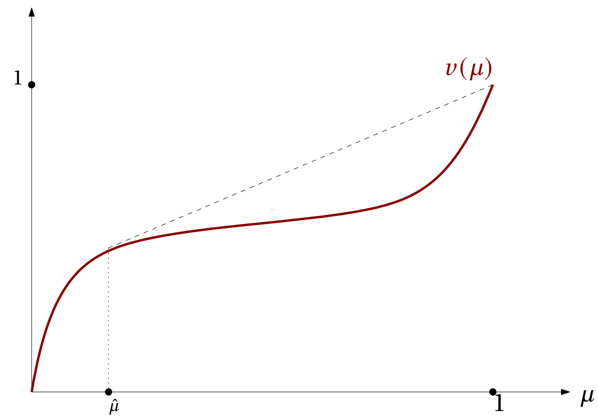

Figure 3. The value function under the assumption that the virtual density is single-dipped.

The argument for the proposition is easiest to see by examining the autocrat's value function. Figure 3 illustrates the value function in this case. When the distribution is single-dipped, the optimal policy induces only two values for the posterior of the representative citizen, one of which is $\mu = 1$.[11] For the good message to induce the $\mu = 1$ posterior, it must perfectly reveal the good state.

Most notably, a comparison of Proposition 1 and 4 reveal that the optimal persuasion strategy is qualitatively different in a divided society compared to a monolithic one. Intuitively, in a divided society, there are many extreme supporters (i.e., those with belief thresholds close to 0) and many extreme skeptics (i.e., those with belief thresholds close to 1). The autocrat faces a challenge: he wants to convince extreme skeptics without losing the support of extreme supporters. The optimal persuasion strategy turns out to be creating a media source that frequently sends the bad message, so that the rare but credible occurrence of good messages is sufficient to convince the extreme skeptics.[12] Moreover, under such a persuasion strategy, extreme supporters are incentivized not to comply with the news, because they support the autocrat even when the bad message is sent.

[10] Following Fiorina and Abrams (2008, Figure 1), one may also call such a society *polarized*. We refrain from adopting this terminology, because polarization is typically visualized as the society having a small number of groups, with high homogeneity within groups and high heterogeneity across groups (Esteban and Ray, 1994).

[11] When the autocrat's prior is low enough that $p_s < \hat{\mu}$ in Figure 3, the optimal policy reveals no information, and any policy that satisfies $\sigma^0 = \sigma^1$ is optimal. In that case, we choose the policy according to which $\sigma^0 = \sigma^1 = 0$ and set the citizens' posterior following the (zero probability) good message to $\mu = 1$.

[12] Baum and Groeling (2009), Ladd and Lenz (2009), and Chiang and Knight (2011) document evidence of the persuasive power of communication when messages are sent by actors least expected to send them.

One may interpret the optimal persuasion strategy as the existence of an independent media source that may occasionally support the autocrat, and in such times, the autocrat may enjoy the credible message sent by the usually-opposing source. Such strategies are indeed employed by informational autocrats from time to time. For instance, following the anti-government protests and riots in Zhanaozen in December 2011, Kazakhstan's President Nursultan Nazarbayev suffered from the lack of credibility of state broadcasting outlets. When all else failed to calm the mass public, the government invited six well-known bloggers, most labeling themselves as "independent," to make a two-day visit to Zhanaozen. The bloggers carried a sense of credibility that the government sources lacked, and they were "quite effective at reassuring readers that the city was outwardly calm, that rumours of morgues or hospitals full of corpses were unfounded and that shops were well-stocked and inhabitants able to buy food and drink" (Lewis, 2016, p.267, also see Guriev and Treisman, 2022, p.79). In a similar episode, Vladimir Putin utilized the liberal Russian radio station Echo of Moscow to cover a credible account of a large pro-government demonstration in the capital in early 2012, thereby discouraging participation in opposition rallies elsewhere (Sobolev, 2023).

We continue with introducing a set of sufficient conditions for the virtual density to be single-dipped. If citizens have a common prior which coincides with the autocrat's prior, the single-dippedness of the virtual density is equivalent to the single-dippedness of the density of costs:

Proposition 5. *Suppose $p_r = p_s$ for all r. The virtual density $h(\mu)$ is single-dipped in μ if and only if the density of costs $f(c)$ is single-dipped in c.*

If citizens have a common cost, the single-dippedness of the virtual density is implied by a condition that is slightly stronger than the log-convexity of the density of priors:

Proposition 6. *Suppose $c_r = c$ for all r. The virtual density $h(\mu)$ is single-dipped if*

$$\frac{\partial^2}{\partial p^2} \log f(p) > 2(\gamma - 1)^2 \max\left\{1, \frac{1}{\gamma^2}\right\} \quad \text{for all } p \in [0, 1], \tag{8}$$

where $\gamma \equiv \frac{1-c}{c}\frac{1-p_s}{p_s} \geq 0$.

Note that the right-hand side of equation (8) is positive, and the left-hand side is positive if $f(p)$ is log-convex. Therefore, condition (8) can be interpreted as "$f(p)$ being sufficiently log-convex." If $p_s + c = 1$, then $\gamma = 1$, and condition (8) reduces to the log-convexity of the distribution of priors.

When the virtual density is single-dipped, the autocrat's optimal strategy entails sending the bad message whenever the state is bad. Then, the extent of information manipulation is summarized by the probability σ^1 of sending the good message when the state is good:

Definition 5. Consider single-dipped virtual densities h_1 and h_2 with the corresponding optimal strategies $\sigma_1 = (\sigma_1^0, \sigma_1^1)$ and $\sigma_2 = (\sigma_2^0, \sigma_2^1)$ for the autocrat. The autocrat *manipulates information less* given h_1 than given h_2 if $\sigma_1^1 \geq \sigma_2^1$.

We now examine the impact of increased dispersion on information manipulation. The following partial order is the appropriate adaptation of the partial order defined in Section 3.3 for single-peaked densities to the set of single-dipped virtual densities:

Definition 6. Consider single-dipped densities f_1 and f_2 supported on a common compact set and satisfying

$$f_2(x) = \frac{(f_1(x))^\alpha}{\kappa} \qquad \text{for all } x, \tag{9}$$

some $\alpha > 0$, and a normalization constant $\kappa > 0$. If $\alpha \geq 1$, then f_2 is *more dispersed* than f_1. If $0 < \alpha \leq 1$, then f_2 is *less dispersed* than f_1.

Figure 4 illustrates the dispersion partial order on a set of a single-dipped Beta distributions. As the distribution becomes more dispersed, mass is moved from the center of the distribution to its extremes.

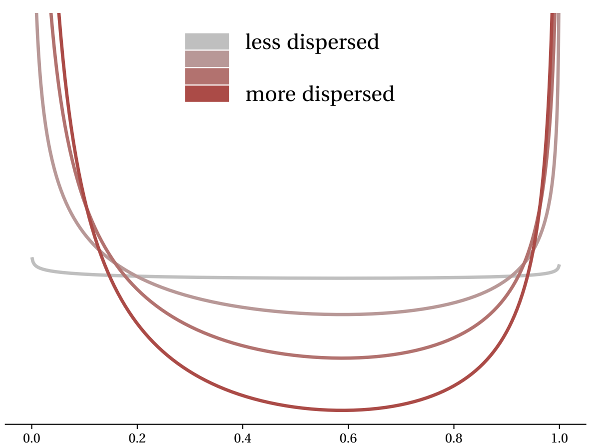

Figure 4. The dispersion order on single-dipped densities.

It is, once again, instructive to consider the limits of equation (9). In the $\alpha \to \infty$ limit, f_2 becomes two point masses at the bounds of the support. Our measure identifies such distributions as extremely dispersed. Conversely, in the $\alpha \to 0$ limit, f_2 becomes the uniform distribution. Thus, the uniform distribution is less dispersed than any single-dipped distribution with the same support.

Our next result establishes that, as in the single-peaked case, dispersion reduces information manipulation in the single-dipped case:

Theorem 2. *Let h_1 and h_2 be two single-dipped virtual densities. If h_1 is more dispersed than h_2, then the autocrat manipulates information less given h_1 than h_2.*

Theorem 2 shows that the main message of Theorem 1 continues to apply for single-dipped virtual densities: Dispersion of attitudes and opinions tends to reduce the extent of information manipulation.

5 Dispersion and Information Manipulation

Theorems 1 and 2 paint a consistent picture across the board: dispersion reduces information manipulation. This observation can be succinctly summarized in a single figure by extending the dispersion partial order.

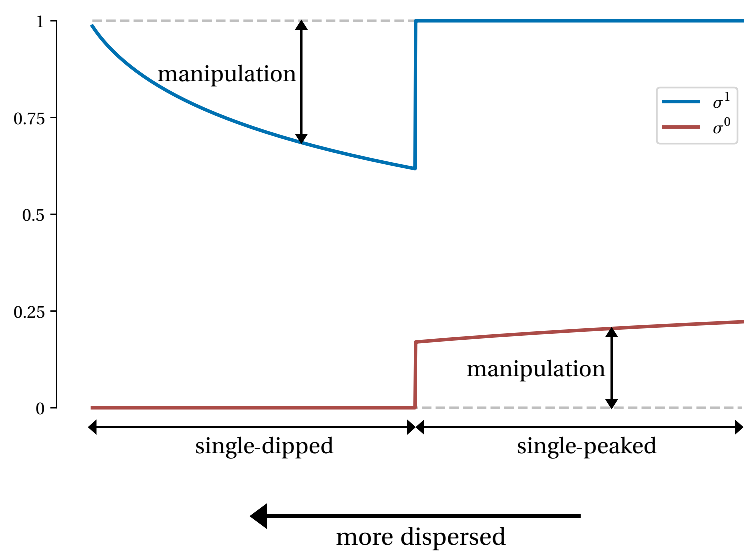

Figure 5. Autocrat's information manipulation as a function of dispersion in society.

Take a single-peaked virtual density h_1 supported on $[0, 1]$, and consider the parametric family of distributions $\{h_\alpha\}_\alpha$ parameterized by the scalar $\alpha \in \mathbb{R}$:

$$h_\alpha(x) = \frac{(h_1(x))^\alpha}{\kappa(\alpha)} \qquad \text{for all } x,$$

where $\kappa(\alpha)$ is a normalization constant. For positive values of α, h_α is a single-peaked distribution. It becomes more dispersed as α decreases to zero. As $\alpha \to 0$, h_α converges to the uniform distribution, which is more dispersed than any single-peaked distribution. For negative values of α, h_α is a single-dipped density, which is more dispersed than the uniform distribution. It becomes more dispersed as α becomes more negative. The upshot is that a lower value of α—be it positive or negative—corresponds to a more dispersed society.

Figure 5 illustrates the effect of dispersion on information manipulation. The virtual density is a (symmetric) Beta$(1 + \alpha, 1 + \alpha)$ distribution, and the autocrat's prior is given by $p_s = 0.4$. The figure plots how the autocrat's optimal strategy changes as α ranges from -1 to $+1$. In the right half of the figure, $\alpha > 0$, the distribution is single-peaked, and so, by Proposition 1, the optimal policy has the form $(\sigma^0_\alpha, \sigma^1_\alpha) = (\sigma^0_\alpha, 1)$. As the society becomes more dispersed, by Theorem 1, σ^0_α decreases and the autocrat manipulates information less. On the left half of the figure, $\alpha < 0$, the distribution is single-dipped, the optimal policy has the form $(\sigma^0_\alpha, \sigma^1_\alpha) = (0, \sigma^1_\alpha)$ (by Proposition 4), and σ^1_α increases and information manipulation decreases with dispersion (by Theorem 2).

Transitioning from a single-peaked to a single-dipped virtual density changes the nature of autocrat's optimal policy. This makes it hard to compare the extent of information manipulation in the single-peaked and single-dipped cases. We continue using our simple measure of information manipulation while acknowledging that any such measure will be imperfect. The change in the nature of information manipulation as we transition from a single-peaked to a single-dipped density manifests itself in a possibly discontinuous change in our measure of information manipulation, as can be seen in Figure 5.

Regardless of the discontinuity at uniform distributions, the overall message is clear: in more diverse societies, one would expect to observe less information manipulation and more free media. To provide empirical support for this prediction, one needs to find variables that capture heterogeneity of attitudes and opinions in a society. One such measure is the Gini coefficient, as income inequality is considered to be related to social conflict (Rodrik, 1999) and lack of social cohesion (Easterly, Woolcock and Ritzen, 2006). The supporting evidence for the link between income inequality and media freedom in autocracies comes from Figure 2 of Petrova (2008). Within autocracies (classified as countries with Democracy score ≤ 1 in Polity IV dataset), there is a positive association between Gini coefficient and Freedom House media freedom index in 1994-2003. Reassuringly, the corresponding association is negative for coutries classified as democracies (Figure 1 of Petrova, 2008), which suggests that the lack of functioning democratic institutions is an essential part of this story.

6 Conclusion

The growing literature on the rise of informational autocrats (Guriev and Treisman, 2022) discusses the modern autocrats' tendency to manipulate information. A natural question that follows from this research is about the conditions that make a society more susceptible to information manipulation. In this paper, we show that the dispersion of opinions puts an inherent limit on an informational autocrat's ability to manipulate information.

Throughout our analysis, we considered the distribution of opinions and attitudes to be exogenous, and we remained agnostic about the forces that may increase its dispersion. Two channels that may lead to increased dispersion are independent media and online media. In a recent working paper, Enikolopov, Rochlitz, Schoors and Zakharov (2023) demonstrate that access to independent online TV in Russia before the 2016 elections increased polarization among those who rely on news from social media. Motivated by their findings, and in light of the discussion here, one can argue that online media not only affect the attitudes of citizens but also have an impact on the effectiveness of traditional state-controlled media. In particular, online media do not have to convince every citizen; even if they convince *some* citizens, they would make it harder for the informational autocrat to engage in information manipulation.

In this paper, we focused on information manipulation as the only tool available to an autocrat. In reality, many autocrats have other tools at their disposal, such as repression and indoctrination (Gitmez and Sonin, 2022; Gehlbach, Luo, Shirikov and Vorobyev, 2022), even if they do not always use them. The question of how the mix of tools used by autocrats is affected by the distribution of opinions is a fruitful avenue for future research.

Appendices

A Proofs for Section 3

Because f is continuously differentiable over its support and bounded, $h'(\mu)$ exists and is continuous. We begin by noting that the virtual density is single-peaked if and only if $h'(\mu)$ satisfies the *strict single-crossing-from-above property*. The strict single-crossing property is adapted from (Milgrom and Shannon, 1994, p.160) and is as follows:

$$\text{If } h'(\mu) \geq 0 \text{ for some } \mu \in [0,1], \text{ then } h'(\tilde{\mu}) > 0 \text{ for all } \tilde{\mu} < \mu.$$

In our proofs, we rely on the equivalence of this condition with single-peakedness of h.

Proof of Proposition 1. If $h'(\mu)$ satisfies the strict single-crossing-from-above condition, by definition, so does $v''(\mu)$. Therefore, whenever $v(\mu)$ is convex at μ, it is strictly convex at any $\hat{\mu} < \mu$. This means that $v(\mu)$ is first strictly convex and then strictly concave. Therefore, the set where the concave closure of $v(\mu)$—call it $V(\mu)$—coincides with $v(\mu)$ has the following form:

$$\{\mu \in [0,1] : V(\mu) = v(\mu)\} = \{0\} \cup [\hat{\mu}, 1],$$

for some $\hat{\mu} \in [0,1]$.

When $p_s < \hat{\mu}$, by Corollary 2 of Kamenica and Gentzkow (2011), the optimal policy generates two posteriors: $\mu \in \{0, \hat{\mu}\}$. This is achieved by two messages, with one perfectly revealing the bad state.

When $p_s \geq \hat{\mu}$, the optimal policy is not revealing any information. This can also be achieved by two messages, $m \in \{0,1\}$, and an information structure where $\Pr(m = 1|\theta = 0) = \Pr(m = 1|\theta = 1)$. Message $m = 0$ will occur with probability zero, and the posterior beliefs following $m = 0$ will be free in a Perfect Bayesian Equilibrium. One can assign the posterior $\Pr_r(\theta = 0|m = 0) = 1$ assign $m = 0$ as the message that perfectly reveals the bad state. □

Proof of Proposition 2. Since $p_r = p_s$ for all r, equation (5) simplifies to

$$v(\mu) = \int_0^{c(\mu, p_s)} f(c)dc.$$

On the other hand, by definition, $c(\mu, p_s) = \mu$ for all μ. Therefore, $h(\mu) = v'(\mu) = f(c(\mu, p_s)) = f(\mu)$, and so, h is single-peaked if and only if f is single-peaked. □

Proof of Proposition 3. When $c_r = c$ for all r, equation (5) simplifies to

$$v(\mu) = \int_{p(\mu,c)}^{1} f(p)dp, \tag{10}$$

where

$$p(\mu,c) \equiv \frac{1-\mu}{(1-\mu)+\mu\frac{1-c}{c}\frac{1-p_s}{p_s}}. \tag{11}$$

The virtual density $h(\mu)$ is then given by

$$h(\mu) = v'(\mu) = -f(p(\mu,c)) \cdot \frac{\partial}{\partial\mu}p(\mu,c),$$

and so,

$$h'(\mu) = -f'(p(\mu,c))\left(\frac{\partial}{\partial\mu}p(\mu,c)\right)^2 - f(p(\mu,c)) \cdot \frac{\partial^2}{\partial\mu^2}p(\mu,c).$$

Therefore, the sign of $h'(\mu)$ is the same as the sign of

$$-\frac{f'(p(\mu,c))}{f(p(\mu,c))} - \frac{\frac{\partial^2}{\partial\mu^2}p(\mu,c)}{\left(\frac{\partial}{\partial\mu}p(\mu,c)\right)^2},$$

where $f(p(\mu,c)) > 0$ by assumption and $\partial p(\mu,c)/\partial\mu > 0$ follows $\gamma > 0$. Using (11) and substituting $\gamma = \frac{1-c}{c}\frac{1-p_s}{p_s}$, we get

$$-\frac{f'(p(\mu,c))}{f(p(\mu,c))} - \frac{\frac{\partial^2}{\partial\mu^2}p(\mu,c)}{\left(\frac{\partial}{\partial\mu}p(\mu,c)\right)^2} = -\frac{\partial}{\partial p}\log f(p(\mu,c)) - 2\frac{\gamma-1}{\gamma}(1+(\gamma-1)\mu). \tag{12}$$

Substituting the value of γ into equation (11) gives: $p(\mu,c) = \frac{1-\mu}{1+(\gamma-1)\mu}$. Solving for μ,

$$\mu = \frac{1-p(\mu,c)}{1+(\gamma-1)p(\mu,c)}. \tag{13}$$

Substituting for μ in equation (12), we get

$$\begin{aligned}
-\frac{f'(p(\mu,c))}{f(p(\mu,c))} - \frac{\frac{\partial^2}{\partial\mu^2}p(\mu,c)}{\left(\frac{\partial}{\partial\mu}p(\mu,c)\right)^2} &= -\frac{\partial}{\partial p}\log f(p(\mu,c)) - 2\frac{\gamma-1}{1+(\gamma-1)p(\mu,c)} \\
&= -\frac{\partial}{\partial p}\log f(p(\mu,c)) + g\left(p(\mu,c)\right),
\end{aligned}$$

where $g(p) \equiv -2\frac{\gamma-1}{1+(\gamma-1)p}$. Note that $g(p)$ is increasing in p, is convex in p if $\gamma \le 1$, and is concave in p if $\gamma \ge 1$. Therefore,

$$\begin{aligned}
\min_{p\in[0,1]} g'(p) &= \begin{cases} g'(0) & \text{if } \gamma \le 1, \\ g'(1) & \text{if } \gamma \ge 1 \end{cases} \\
&= \begin{cases} 2(\gamma-1)^2 & \text{if } \gamma \le 1, \\ 2\frac{(\gamma-1)^2}{\gamma^2} & \text{if } \gamma \ge 1 \end{cases} \\
&= 2(\gamma-1)^2 \min\left\{1, \frac{1}{\gamma^2}\right\}.
\end{aligned} \tag{14}$$

If condition (6) holds, then

$$\frac{\partial^2}{\partial p^2}\log f(p) < \min_{p\in[0,1]} g'(p),$$

which implies

$$\frac{\partial^2}{\partial p^2}\log f(p) < g'(p) \quad \forall p \in [0,1]. \tag{15}$$

Our claim is that, under condition (6), $h'(\mu)$ satisfies the strict single-crossing-from-above condition. To see this, take any two $\mu, \hat{\mu}$ with $\hat{\mu} < \mu$ and $h'(\mu) \geq 0$. Because $p(\mu, c)$ is strictly decreasing in μ, $p(\mu, c) < p(\hat{\mu}, c)$. Since $h'(\mu) \geq 0$, $\frac{\partial}{\partial p}\log f(p(\mu,c)) - g(p(\mu,c)) \leq 0$. Then,

$$\begin{aligned}
&\frac{\partial}{\partial p}\log f(p(\hat{\mu},c)) - g(p(\hat{\mu},c)) \\
&\quad= \frac{\partial}{\partial p}\log f(p(\mu,c)) - g(p(\mu,c)) + \int_{p(\mu,c)}^{p(\hat{\mu},c)} \underbrace{\left(\frac{\partial^2}{\partial p^2}\log f(p) - g'(p)\right)}_{<0 \text{ by } (15)} dp \\
&\quad< \frac{\partial}{\partial p}\log f(p(\mu,c)) - g(p(\mu,c)) \leq 0.
\end{aligned}$$

Therefore, $h'(\hat{\mu}) > 0$. The result follows. □

We continue with some notation and preliminary results for the proof of Theorem 1.

Lemma 1. *The value function $v(\mu)$ satisfies*

$$\lim_{\mu\to 0}\mu v'(\mu) = \lim_{\mu\to 1}(1-\mu)v'(\mu) = 0.$$

Proof. First, note that

$$\begin{aligned}
v'(\mu) &= \int_0^1 f(p, c(\mu,p)).\frac{\partial}{\partial\mu}c(\mu,p)dp \\
&= \int_0^1 f(p, c(\mu,p))\frac{p(1-p)p_s(1-p_s)}{(p_s(1-p)+\mu(p-p_s))^2}dp,
\end{aligned}$$

But since f is bounded, there exist some $C > 0$ such that

$$\begin{aligned}
|v'(\mu)| &\leq C\int_0^1 \frac{p(1-p)p_s(1-p_s)}{(p_s(1-p)+\mu(p-p_s))^2}dp \\
&= C\frac{p_s(1-p_s)}{(p_s-\mu)^3}\left[2(\mu-p_s) - (\mu(1-p_s)+p_s(1-\mu))\log\left(\frac{\mu(1-p_s)}{p_s(1-\mu)}\right)\right].
\end{aligned}$$

On the other hand,

$$\begin{aligned}
&\lim_{\mu\to 0}\mu\cdot\frac{p_s(1-p_s)}{(p_s-\mu)^3}\left[2(\mu-p_s) - (\mu(1-p_s)+p_s(1-\mu))\log\left(\frac{\mu(1-p_s)}{p_s(1-\mu)}\right)\right] \\
&\quad= \lim_{\mu\to 0}\frac{-(1-p_s)}{p_s}\mu\log(\mu) = 0,
\end{aligned}$$

and

$$\lim_{\mu\to 1}(1-\mu)\cdot\frac{p_s(1-p_s)}{(p_s-\mu)^3}\left[2(\mu-p_s)-(\mu(1-p_s)+p_s(1-\mu))\log\left(\frac{\mu(1-p_s)}{p_s(1-\mu)}\right)\right]$$
$$=\lim_{\mu\to 1}\frac{-p_s}{1-p_s}(1-\mu)\log(1-\mu)=0.$$

Therefore,

$$\lim_{\mu\to 0}\mu v'(\mu)=\lim_{\mu\to 1}(1-\mu)v'(\mu)=0.$$

This completes the proof of the Lemma. □

Consider a single-peaked virtual density $h(\mu)$. As discussed in the proof of Proposition 1, $\{\mu\in[0,1]:V(\mu)=v(\mu)\}=\{0\}\cup[\hat{\mu},1]$ for some $\hat{\mu}\in[0,1]$. Note that:

- $v'(\mu)\mu<v(\mu)$ for all $\mu\in(0,1)$ if and only if $\hat{\mu}=0$.
- $v'(\mu)\mu>v(\mu)$ for all $\mu\in(0,1)$ if and only if $\hat{\mu}=1$.
- When $\hat{\mu}\in(0,1)$, it satisfies:

$$v'(\hat{\mu})\hat{\mu}=v(\hat{\mu}). \tag{16}$$

Let

$$y(\mu)\equiv v'(\mu)\mu-v(\mu)=h(\mu)\mu-\int_0^{\mu}h(\tilde{\mu})\tilde{\mu},\qquad\forall\mu\in[0,1]. \tag{17}$$

Then, $\hat{\mu}\in(0,1)$ is characterized by the equation: $y(\hat{\mu})=0$. We start with some remarks that will be used in the proof of Theorem 1.

Remark 1. $\lim_{\mu\to 0}y(\mu)=0$. This follows Lemma 1 and the fact that $v(0)=0$.

Remark 2. *$y(\mu)$ is continuous in μ over* $(0,1)$. This is because f is continuous over its support.

Remark 3. *$y(\mu)$ is first strictly increasing and then strictly decreasing.* This is because $y'(\mu)=v''(\mu)\mu+v'(\mu)-v'(\mu)=v''(\mu)\mu=h'(\mu)\mu$. Since $h'(\mu)$ satisfies strict single crossing from above, so does $y'(\mu)$, and the remark follows.

Remark 4. *If $h_1(\mu)$ is a single-peaked distribution, then any distribution with density*

$$h_2(\mu)=\frac{(h_1(\mu))^{\alpha}}{\kappa}\qquad\textit{for all }\mu\in[0,1],\textit{ where }\alpha\geq 1,\kappa>0$$

is single-peaked. To see this, suppose $h_1(\mu)$ is single-peaked. Then, $h_1'(\mu)$ satisfies the strict single-crossing-from-above condition:

$$\text{If } h_1'(\mu)\geq 0 \text{ for some } \mu\in[0,1], \text{ then } h_1'(\hat{\mu})>0 \text{ for all } \hat{\mu}<\mu.$$

Note that

$$h_2'(\mu) = \alpha \frac{(h_1(\mu))^{\alpha-1}}{\kappa} h_1'(\mu),$$

which implies that the sign of $h_2'(\mu)$ is the same as the sign of $h_1'(\mu)$. The remark follows.

Remark 5. *If $h(\mu)$ is a single-peaked distribution, then $h(\mu) > 0$ for all $\mu \in (0,1)$.* This is a simple consequence of the fact that, for any single-peaked distribution, there exists some $\hat{\mu}$ such that $h'(\mu) > 0$ for all $\mu \in [0, \hat{\mu})$ and $h'(\mu) < 0$ for all $\mu \in (\hat{\mu}, 1]$.

Proof of Theorem 1. Take a single-peaked distribution $h(\mu)$. Consider a family of distributions $\{h_\alpha\}_{\alpha \geq 1}$ characterized by

$$h_\alpha(\mu) = \frac{(h(\mu))^\alpha}{\kappa(\alpha)}, \qquad \text{for all } \mu \in [0,1], \alpha \geq 1,$$

where $\kappa(\alpha)$ is the normalization constant given by

$$\kappa(\alpha) \equiv \int_0^1 (h(t))^\alpha dt.$$

The corresponding cdf's are given by:

$$H_\alpha(\mu) \equiv \int_0^\mu h_\alpha(x)dx = \frac{\int_0^\mu (h(x))^\alpha dt}{\kappa(\alpha)}.$$

By Remark 4, any distribution in this family is single-peaked. Take any such distribution h_α, and let

$$y_\alpha(\mu) \equiv h_\alpha(\mu)\mu - H_\alpha(\mu).$$

Based on Remarks 1, 2 and 3, the set $\mathcal{U}_{y_\alpha} \equiv \{\mu \in [0,1] : y_\alpha(\mu) \geq 0\}$ has the following form:

$$\mathcal{U}_{y_\alpha} = [0, \hat{\mu}_\alpha].$$

The proof goes through showing that $\hat{\mu}_\alpha$ is decreasing in α. We continue with two important remarks.

Remark 6. $y_\alpha'(\hat{\mu}_\alpha) < 0$. This follows from the fact that $\mathcal{U}_{y_\alpha} = [0, \hat{\mu}_\alpha]$. Then, $y_\alpha(\mu)$ crosses zero from above at $\hat{\mu}_\alpha$. Since $y_\alpha(\mu)$ is differentiable, the remark follows.

Remark 7. *If $\hat{\mu}_\alpha \in (0,1)$, then $y_\alpha(\hat{\mu}_\alpha) = 0$, or equivalently,*

$$h_\alpha(\hat{\mu}_\alpha)\hat{\mu}_\alpha = \int_0^{\hat{\mu}_\alpha} h_\alpha(x)dx. \tag{18}$$

By Remark 7, $\hat{\mu}_\alpha \in (0,1)$ satisfies

$$y_\alpha(\hat{\mu}_\alpha) = 0.$$

Implicitly differentiate with respect to α to get

$$\frac{\partial}{\partial \alpha} y_\alpha(\hat{\mu}_\alpha) + y'_\alpha(\hat{\mu}_\alpha)\frac{\partial \hat{\mu}_\alpha}{\partial \alpha} = 0.$$

By Remark 6, $y'_\alpha(\hat{\mu}_\alpha) < 0$. Then, $\frac{\partial \hat{\mu}_\alpha}{\partial \alpha} \leq 0$ if and only if

$$\left.\frac{\partial}{\partial \alpha} y_\alpha(\mu)\right|_{\mu=\hat{\mu}_\alpha} \leq 0.$$

Note that, for any $\mu \in [0,1]$,

$$\frac{\partial}{\partial \alpha} y_\alpha(\mu) \leq 0 \iff \frac{\partial}{\partial \alpha} h_\alpha(\mu)\mu \leq \int_0^\mu \frac{\partial}{\partial \alpha} h_\alpha(x)dx.$$

Recall that $h_\alpha(\mu) = \frac{(h(\mu))^\alpha}{\kappa(\alpha)}$. Therefore, for any $x \in (0,1)$, $\frac{\partial}{\partial \alpha} h_\alpha(x) = h_\alpha(x)\left(\log h(x) - \frac{\kappa'(\alpha)}{\kappa(\alpha)}\right)$, and so,

$$\left.\frac{\partial}{\partial \alpha} y_\alpha(\mu)\right|_{\mu=\hat{\mu}_\alpha} \leq 0 \iff h_\alpha(\hat{\mu}_\alpha)\log h(\hat{\mu}_\alpha)\hat{\mu}_\alpha \leq \int_0^{\hat{\mu}_\alpha} h_\alpha(x)\log h(x)dx.$$

Using (18) to substitute $h_\alpha(\hat{\mu}_\alpha)\hat{\mu}_\alpha = \int_0^{\hat{\mu}_\alpha} h_\alpha(x)dx$ on the left hand side of the above inequality, we have

$$\begin{aligned}
\left.\frac{\partial}{\partial \alpha} y_\alpha(\mu)\right|_{\mu=\hat{\mu}_\alpha} \leq 0 &\iff \log h(\hat{\mu}_\alpha)\int_0^{\hat{\mu}_\alpha} h_\alpha(x)dx \leq \int_0^{\hat{\mu}_\alpha} h_\alpha(x)\log h(x)dx \\
&\iff \int_0^{\hat{\mu}_\alpha} h_\alpha(x)\log h(\hat{\mu}_\alpha)dx \leq \int_0^{\hat{\mu}_\alpha} h_\alpha(x)\log h(x)dx \\
&\iff \alpha\int_0^{\hat{\mu}_\alpha} h_\alpha(x)\log h(\hat{\mu}_\alpha)dx \leq \alpha\int_0^{\hat{\mu}_\alpha} h_\alpha(x)\log h(x)dx \\
&\iff \int_0^{\hat{\mu}_\alpha} h_\alpha(x)\log(h(\hat{\mu}_\alpha))^\alpha dx \leq \int_0^{\hat{\mu}_\alpha} h_\alpha(x)\log(h(x))^\alpha dx \\
&\iff \int_0^{\hat{\mu}_\alpha} h_\alpha(x)\log\left(\frac{(h(\hat{\mu}_\alpha))^\alpha}{(h(x))^\alpha}\right)dx \leq 0 \\
&\iff \int_0^{\hat{\mu}_\alpha} h_\alpha(x)\log\left(\frac{h_\alpha(\hat{\mu}_\alpha)}{h_\alpha(x)}\right)dx \leq 0.
\end{aligned}$$

For any real number $z > 0$, $\log(z) \leq z - 1$, with a strict inequality for any $z \neq 1$. Therefore,

$$\int_0^{\hat{\mu}_\alpha} h_\alpha(x)\log\left(\frac{h_\alpha(\hat{\mu}_\alpha)}{h_\alpha(x)}\right)dx \leq \int_0^{\hat{\mu}_\alpha} h_\alpha(x)\left(\frac{h_\alpha(\hat{\mu}_\alpha)}{h_\alpha(x)} - 1\right)dx.$$

Therefore, $\left.\frac{\partial}{\partial \alpha} y_\alpha(\mu)\right|_{\mu=\hat{\mu}_\alpha} \leq 0$ as long as

$$\begin{aligned}
\int_0^{\hat{\mu}_\alpha} h_\alpha(x)\left(\frac{h_\alpha(\hat{\mu}_\alpha)}{h_\alpha(x)} - 1\right)dx \leq 0 &\iff \int_0^{\hat{\mu}_\alpha} \left(h_\alpha(\hat{\mu}_\alpha) - h_\alpha(x)\right)dx \leq 0 \\
&\iff \int_0^{\hat{\mu}_\alpha} h_\alpha(\hat{\mu}_\alpha)dx \leq \int_0^{\hat{\mu}_\alpha} h_\alpha(x)dx
\end{aligned}$$

$$\iff h_\alpha(\hat{\mu}_\alpha)\hat{\mu}_\alpha \le \int_0^{\hat{\mu}_\alpha} h_\alpha(x)dx,$$

which is guaranteed by (18). We conclude that for any $\hat{\mu}_\alpha \in (0,1)$, $\frac{\partial \hat{\mu}_\alpha}{\partial \alpha} \le 0$.

Since h_1 and h_2 are within the family we considered (with h_1 corresponding to $\alpha = 1$ and h_2 corresponding to some $\alpha \ge 1$), $\hat{\mu}_2 \le \hat{\mu}_1$. To conclude the proof, consider three cases:

1. If $p_s \ge \hat{\mu}_1$, the optimal policy does not reveal any information in either case, and we pick $\sigma_1^0 = \sigma_2^0 = 1$.
2. If $\hat{\mu}_1 > p_s \ge \hat{\mu}_2$, the optimal policy under $h_2(\mu)$ does not reveal any information. In this case, we pick $\sigma_2^0 = 1$ and $\sigma_1^0 < 1$.
3. If $p_s < \hat{\mu}_2$, the optimal policies σ_1^0 and σ_2^0 satisfy:
$$\frac{p_s}{p_s + (1-p_s)\sigma_1^0} = \hat{\mu}_1, \qquad \frac{p_s}{p_s + (1-p_s)\sigma_2^0} = \hat{\mu}_2.$$
   Then, $\hat{\mu}_1 \ge \hat{\mu}_2$ implies $\sigma_1^0 \le \sigma_2^0$.

In any case, $\sigma_1^0 \le \sigma_2^0$, and the result follows. □

B Proofs for Section 4

Note that single-dippedness of the virtual density is equivalent to the following *strict single-crossing-from-below property* for $h'(\mu)$:

$$\text{If } h'(\mu) \ge 0 \text{ for some } \mu \in [0,1], \text{ then } h'(\tilde{\mu}) > 0 \text{ for all } \tilde{\mu} > \mu.$$

Proof of Proposition 4. If $h'(\mu)$ satisfies the strict single-crossing-from-below condition, by definition, so does $v''(\mu)$. Therefore, whenever $v(\mu)$ is convex at μ, it is strictly convex at any $\hat{\mu} \ge \mu$. This means that $v(\mu)$ is first strictly concave and then strictly convex. Therefore, the set where the concave closure of $v(\mu)$ coincides with $v(\mu)$ has the following form:

$$\{\mu \in [0,1] : V(\mu) = v(\mu)\} = [0, \hat{\mu}] \cup \{1\}.$$

When $p_s < \hat{\mu}$, the optimal policy is not revealing any information. This can be achieved by two messages, $m \in \{0,1\}$, and an information structure where $\Pr(m = 1|\theta = 0) = \Pr(m = 1|\theta = 1) = 0$. Message $m = 1$ will occur with probability zero, and the posterior beliefs following $m = 1$ will be free in a Perfect Bayesian Equilibrium. One can assign posteriors $\Pr_r(\theta = 1|m = 1) = 1$ to make $m = 1$ as the message that perfectly reveals the good state.

When $p_s \ge \hat{\mu}$, by Corollary 2 of Kamenica and Gentzkow (2011), the optimal policy generates two posteriors: $\mu \in \{\hat{\mu}, 1\}$. This is achieved by two messages, $m \in \{0,1\}$, where message $m = 1$ perfectly reveals the good state.

□

Proof of Proposition 5. The proof of Proposition 5 is identical to the proof of Proposition 2. □

Proof of Proposition 6. The proof follows identical steps to that of Proposition 3 until equation (14). The rest of the argument is provided below.

Recall that $g(p) \equiv -2\frac{\gamma-1}{1+(\gamma-1)p}$ and $g(p)$ is increasing in p, convex in p if $\gamma \leq 1$, and concave in p if $\gamma \geq 1$. Therefore,

$$\max_{p\in[0,1]} g'(p) = \begin{cases} g'(1) & \text{if } \gamma \leq 1 \\ g'(0) & \text{if } \gamma \geq 1 \end{cases} = \begin{cases} 2\frac{(\gamma-1)^2}{\gamma^2} & \text{if } \gamma \leq 1 \\ 2(\gamma-1)^2 & \text{if } \gamma \geq 1 \end{cases} = 2(\gamma-1)^2 \max\left\{1, \frac{1}{\gamma^2}\right\}. \tag{19}$$

If condition (8) holds,

$$\frac{\partial^2}{\partial p^2}\log f(p) > \max_{p\in[0,1]} g'(p),$$

and so

$$\frac{\partial^2}{\partial p^2}\log f(p) > g'(p) \quad \forall p \in [0,1]. \tag{20}$$

Our claim is that, under condition (8), $h'(\mu)$ satisfies the strict single-crossing-from-below condition. To see this, take any two $\mu, \hat{\mu}$ with $\hat{\mu} > \mu$ and $h'(\mu) \geq 0$. Because $p(\mu, c)$ is strictly decreasing in μ, $p(\hat{\mu}, c) < p(\mu, c)$. Since $h'(\mu) \geq 0$, $\frac{\partial}{\partial p}\log f(p(\mu,c)) - g(p(\mu,c)) \leq 0$. Then,

$$\begin{aligned}
&\frac{\partial}{\partial p}\log f(p(\hat{\mu},c)) - g(p(\hat{\mu},c)) \\
&\quad= \frac{\partial}{\partial p}\log f(p(\mu,c)) - g(p(\mu,c)) - \int_{p(\hat{\mu},c)}^{p(\mu,c)} \underbrace{\left(\frac{\partial^2}{\partial p^2}\log f(p) - g'(p)\right)}_{>0 \text{ by } (20)} dp \\
&\quad< \frac{\partial}{\partial p}\log f(p(\mu,c^*)) - g(p(\mu,c)) \leq 0.
\end{aligned}$$

Therefore, $h'(\hat{\mu}) > 0$. The result follows. □

We continue by introducing some notation and preliminary results for the remaining proofs.

Consider a single-dipped virtual density $h(\mu)$. As discussed in the proof of Proposition 4, $\{\mu \in [0,1] : V(\mu) = v(\mu)\} = [0,\hat{\mu}] \cup \{1\}$ for some $\hat{\mu} \in [0,1]$. Note that:

- $v'(\mu)(1-\mu) > 1 - v(\mu)$ for all $\mu \in (0,1)$ if and only if $\hat{\mu} = 1$.
- $v'(\mu)(1-\mu) < 1 - v(\mu)$ for all $\mu \in (0,1)$ if and only if $\hat{\mu} = 0$.
- When $\hat{\mu} \in (0,1)$, it satisfies:

$$v'(\hat{\mu})(1-\hat{\mu}) = 1 - v(\hat{\mu}). \tag{21}$$

Let

$$z(\mu) \equiv v'(\mu)(1-\mu) - (1 - v(\mu)) = h(\mu)(1-\mu) - \int_{\mu}^{1} h(\tilde{\mu})\tilde{\mu}, \qquad \forall \mu \in [0,1]. \tag{22}$$

Then, $\hat{\mu} \in (0,1)$ is characterized by the equation: $z(\hat{\mu}) = 0$. We start with some remarks.

Remark 8. $\lim_{\mu\to 1} z(\mu) = 0$. This follows Lemma 1 and the fact that $1 - v(1) = 0$.

Remark 9. *$z(\mu)$ is continuous in μ over $(0,1)$.* This is because f is continuous over its support.

Remark 10. *$z(\mu)$ is first strictly decreasing and then increasing.* This is because $z'(\mu) = v''(\mu)(1-\mu) - v'(\mu) + v'(\mu) = v''(\mu)(1-\mu) = h'(\mu)(1-\mu)$. Since $h'(\mu)$ satisfies strict single crossing from below, so does $z'(\mu)$, and the remark follows.

Remark 11. *If $h_1(\mu)$ is a single-dipped distribution, then any distribution with density*

$$h_2(\mu) = \frac{(h_1(\mu))^\alpha}{\kappa} \qquad \text{for all } \mu \in [0,1], \text{ where } \alpha \geq 1, \kappa > 0$$

is single-dipped. To see this, suppose $h_1(\mu)$ is single-dipped. Then, $h_1'(\mu)$ satisfies the strict single-crossing-from-below condition:

$$\text{If } h_1'(\mu) \geq 0 \text{ for some } \mu \in [0,1], \text{ then } h_1'(\hat{\mu}) > 0 \text{ for all } \hat{\mu} > \mu.$$

Note that

$$h_2'(\mu) = \alpha \frac{(h_1(\mu))^{\alpha-1}}{\kappa} h_1'(\mu),$$

which implies that the sign of $h_2'(\mu)$ is the same as the sign of $h_1'(\mu)$. The remark follows.

Remark 12. *If $h(\mu)$ is a single-dipped distribution, then $h(\mu) > 0$ for almost all μ.* This is a simple consequence of the fact that for any single-dipped distribution, there exists some $\hat{\mu}$ such that $h'(\mu) < 0$ for all $\mu \in [0,\hat{\mu})$ and $h'(\mu) > 0$ for all $\mu \in (\hat{\mu}, 1]$. The only point μ at which $h(\mu)$ could be zero is $\hat{\mu}$.

Proof of Theorem 2. Take a single-dipped distribution $h(\mu)$. Consider a family of distributions $\{h_\alpha\}_{\alpha \geq 1}$ characterized by

$$h_\alpha(\mu) = \frac{(h(\mu))^\alpha}{\kappa(\alpha)}, \qquad \text{for all } \mu \in [0,1], \alpha \geq 1,$$

where $\kappa(\alpha)$ is the normalization constant given by

$$\kappa(\alpha) \equiv \int_0^1 (h(t))^\alpha dt.$$

The corresponding cdf's are given by:

$$H_\alpha(\mu) \equiv \int_0^\mu h_\alpha(x)dx = \frac{\int_0^\mu (h(x))^\alpha dt}{\kappa(\alpha)}.$$

By Remark 11, any distribution in this family is single-dipped. Take any such distribution h_α, and let

$$z_\alpha(\mu) \equiv h_\alpha(\mu)(1-\mu) - (1 - H_\alpha(\mu)).$$

Based on Remarks 8, 9 and 10, the set $\mathcal{L}_{z_\alpha} \equiv \{\mu \in [0,1] : z_\alpha(\mu) \leq 0\}$ has the following form:

$$\mathcal{L}_{z_\alpha} = [\hat{\mu}_\alpha, 1].$$

The proof goes through showing that $\hat{\mu}_\alpha$ is decreasing in α. We continue with two important remarks.

Remark 13. $z'_\alpha(\hat{\mu}_\alpha) < 0$. This follows from the fact that $\mathcal{L}_{z_\alpha} = [\hat{\mu}_\alpha, 1]$. Then, $z_\alpha(\mu)$ crosses zero from above at $\hat{\mu}_\alpha$. Since $z_\alpha(\mu)$ is differentiable, the remark follows.

Remark 14. *If* $\hat{\mu}_\alpha \in (0,1)$, *then* $z_\alpha(\hat{\mu}_\alpha) = 0$, *or equivalently,*

$$h_\alpha(\hat{\mu}_\alpha)(1-\hat{\mu}_\alpha) = \int_{\hat{\mu}_\alpha}^1 h_\alpha(x)dx. \tag{23}$$

By Remark 14, $\hat{\mu}_\alpha \in (0,1)$ satisfies

$$z_\alpha(\hat{\mu}_\alpha) = 0.$$

Implicitly differentiate with respect to α to get

$$\frac{\partial}{\partial \alpha} z_\alpha(\hat{\mu}_\alpha) + z'_\alpha(\hat{\mu}_\alpha)\frac{\partial \hat{\mu}_\alpha}{\partial \alpha} = 0.$$

By Remark 13, $z'_\alpha(\hat{\mu}_\alpha) < 0$. Then, $\frac{\partial \hat{\mu}_\alpha}{\partial \alpha} \leq 0$ if and only if

$$\left.\frac{\partial}{\partial \alpha} z_\alpha(\mu)\right|_{\mu=\hat{\mu}_\alpha} \leq 0.$$

Note that, for any $\mu \in [0,1]$,

$$\frac{\partial}{\partial \alpha} z_\alpha(\mu) \leq 0 \iff \frac{\partial}{\partial \alpha} h_\alpha(\mu)(1-\mu) \leq \int_\mu^1 \frac{\partial}{\partial \alpha} h_\alpha(x)dx.$$

Recall that $h_\alpha(\mu) = \frac{(h(\mu))^\alpha}{\kappa(\alpha)}$. Therefore, for any x for which $h_\alpha(x) > 0$, we have $\frac{\partial}{\partial \alpha} h_\alpha(x) = h_\alpha(x)\left(\log h(x) - \frac{\kappa'(\alpha)}{\kappa(\alpha)}\right)$, and so,

$$\left.\frac{\partial}{\partial \alpha} z_\alpha(\mu)\right|_{\mu=\hat{\mu}_\alpha} \leq 0 \iff h_\alpha(\hat{\mu}_\alpha)\log h(\hat{\mu}_\alpha)(1-\hat{\mu}_\alpha)) \leq \int_{\hat{\mu}_\alpha}^1 h_\alpha(x)\log h(x)dx,$$

where we are using the fact that, by Remark 12, $h_\alpha(x) > 0$ almost everywhere. Using (23) to substitute $h_\alpha(\hat{\mu}_\alpha)(1-\hat{\mu}_\alpha) = \int_{\hat{\mu}_\alpha}^1 h_\alpha(x)dx$ on the left hand side of the above inequality, we have

$$\begin{aligned}
\left.\frac{\partial}{\partial \alpha} z_\alpha(\mu)\right|_{\mu=\hat{\mu}_\alpha} \leq 0 &\iff \log h(\hat{\mu}_\alpha)\int_{\hat{\mu}_\alpha}^1 h_\alpha(x)dx \leq \int_{\hat{\mu}_\alpha}^1 h_\alpha(x)\log h(x)dx \\
&\iff \int_{\hat{\mu}_\alpha}^1 h_\alpha(x)\log h(\hat{\mu}_\alpha)dx \leq \int_{\hat{\mu}_\alpha}^1 h_\alpha(x)\log h(x)dx \\
&\iff \alpha\int_{\hat{\mu}_\alpha}^1 h_\alpha(x)\log h(\hat{\mu}_\alpha)dx \leq \alpha\int_{\hat{\mu}_\alpha}^1 h_\alpha(x)\log h(x)dx
\end{aligned}$$

$$
\begin{aligned}
\iff & \int_{\hat{\mu}_\alpha}^{1} h_\alpha(x)\log(h(\hat{\mu}_\alpha))^\alpha dx \le \int_{\hat{\mu}_\alpha}^{1} h_\alpha(x)\log(h(x))^\alpha dx \\
\iff & \int_{\hat{\mu}_\alpha}^{1} h_\alpha(x)\log\left(\frac{(h(\hat{\mu}_\alpha))^\alpha}{(h(x))^\alpha}\right) dx \le 0 \\
\iff & \int_{\hat{\mu}_\alpha}^{1} h_\alpha(x)\log\left(\frac{h_\alpha(\hat{\mu}_\alpha)}{h_\alpha(x)}\right) dx \le 0.
\end{aligned}
$$

For any real number $z > 0$, $\log(z) \le z - 1$, with a strict inequality for any $z \neq 1$. Therefore,

$$
\int_{\hat{\mu}_\alpha}^{1} h_\alpha(x)\log\left(\frac{h_\alpha(\hat{\mu}_\alpha)}{h_\alpha(x)}\right) dx \le \int_{\hat{\mu}_\alpha}^{1} h_\alpha(x)\left(\frac{h_\alpha(\hat{\mu}_\alpha)}{h_\alpha(x)} - 1\right) dx.
$$

Therefore, $\frac{\partial}{\partial\alpha} z_\alpha(\mu)\Big|_{\mu=\hat{\mu}_\alpha} \le 0$ as long as

$$
\begin{aligned}
\int_{\hat{\mu}_\alpha}^{1} h_\alpha(x)\left(\frac{h_\alpha(\hat{\mu}_\alpha)}{h_\alpha(x)} - 1\right) dx \le 0 \iff & \int_{\hat{\mu}_\alpha}^{1} \left(h_\alpha(\hat{\mu}_\alpha) - h_\alpha(x)\right) dx \le 0 \\
\iff & \int_{\hat{\mu}_\alpha}^{1} h_\alpha(\hat{\mu}_\alpha) dx \le \int_{0}^{\hat{\mu}_\alpha} h_\alpha(x) dx \\
\iff & h_\alpha(\hat{\mu}_\alpha)(1-\hat{\mu}_\alpha) \le \int_{\hat{\mu}_\alpha}^{1} h_\alpha(x) dx,
\end{aligned}
$$

which is guaranteed by (23). We conclude that for any $\hat{\mu}_\alpha \in (0,1)$, $\frac{\partial \hat{\mu}_\alpha}{\partial \alpha} \le 0$.

Since h_1 and h_2 are within the family we considered (with h_2 corresponding to $\alpha = 1$ and h_1 corresponding to some $\alpha \ge 1$), $\hat{\mu}_1 \le \hat{\mu}_2$. Repeating the same argument in the proof of Theorem 1, we conclude that $\sigma_2^1 \le \sigma_1^1$. □

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