THE RISK OF FAILURE IN FIRST-PRICE PROCUREMENT AUCTIONS: THE ROLE OF INFORMATIVE ENTRY

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ABSTRACT. Auctions with endogenous entry may fail if the entry is insufficient. Specifically, the government can cancel the contract when only a single bid is submitted. We study the implications of such a scenario on the expected cost of procurement using the framework of first-price auctions with endogenous entry. In this model, potential bidders do not know their private costs at the entry stage but receive imperfect signals. Based on her signal strength, a potential bidder decides to pay the entry cost, learn her true private cost, and participate in bidding. We compare the resulting expected procurement costs to those when the government commits to award the contract even if there is only one submitted bid but imposes a binding reserve price. We find that imposing a binding reserve price is a more efficient mechanism to address insufficient entry than canceling contracts. Using data on highway maintenance auctions from the Texas Department of Transportation (Li and Zheng, 2009), we find that the former mechanism can reduce the cost of procurement by up to 32% of the engineer's estimate relative to the latter. However, in both scenarios, the entry and the expected procurement cost can vary substantially with the informativeness of the signals. With more informative signals, entering bidders are likely to be more efficient. However, such signals may also deter entry and increase the risk of auction failure. Almost perfectly informative signals result in the lowest expected procurement cost in auctions with a binding reserve price. However, if at least two bids are required, such signals would result in zero entry.

KEYWORDS: First-Price Auctions, Endogenous Entry, Value of Information, Semi-parametric Estimation.
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1. Introduction

2. Insufficient entry and procurement auctions

We revisit the application from Li and Zheng (2009) on highway mowing procurement auctions run by the Texas Department of Transportation (TxDoT) between 2001–2003. One can find a detailed description of the data set in their paper; here, we discuss some important features that determine our modeling approach. Specifically, we discuss the issue of "missing" auctions with a single bid submitted.

The timing of the auction is as follows. First, TxDoT advertises an auction with a brief description of the project. Interested contractors must obtain detailed plans and official bidding proposals. Those contractors are treated as potential bidders. The number of requests is observed for each auction and is known to all potential bidders. Only some potential bidders choose to submit bids. Sealed bids must be submitted by a specific deadline. The number of bids submitted is recorded for each auction; however, it is unknown to potential bidders until the auction is concluded.

According to TxDoT rules, the contract is awarded to the lowest bid with several notable exceptions. For example, TxDoT will reject a contract if the lowest bid is higher than its estimate specified in the project's plans and "re-advertising for bids may result in a lower bid" (Texas Department of Transportation, 2014). As discussed in Li and Zheng (2009), the reserve price is not binding and many winning bids are above the published estimates. For example, in auctions with 9–14 potential bidders, the fraction of winning bids above the estimate is between 0.14–0.49.

Li and Zheng (2009) point out that in procurement (low-bid) sealed bid auctions with random entry and the absence of a binding reserve price, there is a strategy where an entering bidder bids infinity. Such a strategy is an equilibrium strategy as long as there is a nonzero probability of being the only active bidder. To address the issue, Li and Zheng (2009) assumes that the government would enter as another bidder in such situations.

The bidding rules also stipulate that a contract may be rejected if it is "in the best interest of the State" (Texas Department of Transportation, 2014). Thus, the infinite-bid strategy can be ruled out by requiring at least two bids for a contract to be awarded. I.e., an auction would be canceled and re-listed if only one bid is received. Such a course of action is plausible as receiving only a single bid may indicate a lack of competition or interest. Moreover, it is supported by the data. Li and Zheng (2009) indicates that there are only about 13 auctions (or 2.35% of the sample) with only one actual bidder. At the same time, given the estimated probabilities of entry, there should be a significant number of auctions with only one bidder. For example, in the cases of 9–12 potential bidders, 28%–30% of auctions should have only one active bidder. The sheer number of "missing" auctions with a single active bidder strongly suggests that contracts are not awarded in such cases. Hence, we model the auctions by assuming that an auction would be canceled unless at least two bids are received.

3. Model

In this section, we describe the auction model with endogenous entry. We follow Marmer, Shneyerov, and Xu (2013) while translating the model to low-bid procurement auctions and impose a requirement that at least two active bidders must participate in bidding for the contract to be awarded.

At the entry stage, each potential bidder independently draws a signal S from the same distribution. At that stage, a bidder does not know her private cost V of completing the contract, but can learn it after paying the entry cost κ. After paying the entry cost, she becomes an active bidder and proceeds to the bidding stage knowing her V. The decision to enter is based on the drawn signal S, the entry cost κ, the joint distribution of V and S, and the number of potential bidders. The joint distribution and the number of potential bidders are both known to all participants. Potential bidders are risk neutral, and their private costs and signals are private.

Only active bidders who paid the entry cost κ participate in the bidding. The number of active bidders is unknown to the participants. Sealed bids are submitted, and the contract is awarded to the lowest bidder provided that there are at least two active bidders. The contract is canceled if only one bidder or none submits bids.

Let $F_{V,S}(v,s)$ denote the joint CDF of private costs of completing the contract V and private signals S. It is convenient to express it in terms of the copula function: $F_{V,S}(v,s) = C(F(v), F_S(s))$, where $C(\cdot,\cdot)$ is the copula function, and $F(\cdot)$ and $F_S(\cdot)$ are the marginal CDFs of V and S respectively. Note that by copula properties, the conditional distribution of the private cost given the signal can be expressed as $F_{V|S}(v \mid s) = C_2(F(v), F_S(s))$, where $C_2(x,y) \equiv \partial C(x,y)/\partial y$. The following assumption is maintained throughout the paper.

Assumption 3.1.

(i) The copula $C(\cdot,\cdot)$ is continuously differentiable.

(ii) $C_{22}(x,y) \equiv \partial^2 C(x,y)/\partial y^2 \leq 0$ for all $x, y \in [0,1]$.

(iii) The CDF $F(\cdot)$ of the private costs V of completing the contract is absolutely continuous and has a compact support $[\underline{v}, \bar{v}]$.

Assumption 3.1(ii) is equivalent to a first-order stochastic dominance relationship for the conditional distribution of private costs given signals: for all $s_1 \leq s_2$,

$$F_{V|S}(\cdot \mid s_1) \geq F_{V|S}(\cdot \mid s_2).$$

The assumption was referred to as the "good news" assumption in Marmer, Shneyerov, and Xu (2013). When it holds strictly at least at some points, drawing a smaller signal corresponds to a stochastically smaller cost of completing the contract. There are many examples of copulas that satisfy the assumption.

Let $N \geq 2$ denote the number of potential bidders. Under Assumption 3.1(ii), the entry strategy is to enter if a sufficiently small signal is drawn. I.e., a bidder with a signal S enters when $S \leq s_N$, where s_N denotes the equilibrium cutoff in auctions with N potential bidders.

It is convenient to write expressions using the equilibrium entry probability $p_N \equiv \Pr(S \leq s_N)$. The distribution of V conditional on entry is given by

$$\Pr(V \leq v \mid S \leq s_N) = C(F(v), p_N)/p_N \equiv F^*(v \mid p_N). \tag{3.1}$$

Let $H(v \mid p, N)$ denote the probability of an active bidder with a private cost v winning the contract given an entry probability p:

$$\begin{aligned} H(v \mid p, N) &= \Lambda(v \mid p)^{N-1} - (1-p)^{N-1}, \\ \Lambda(v \mid p) &\equiv 1 - p + p \cdot (1 - F^*(v \mid p)) \\ &= 1 - C(F(v), p). \end{aligned}$$

The $\Lambda(\cdot \mid p)$ component of $H(\cdot \mid p, N)$ captures the probability of the event that a competitor does not enter or, if she does, draws a private cost above v. The $(1-p)^{N-1}$ term is due to the requirement that there has to be at least one other active bidder.

We consider only pure strategy symmetric equilibria, and using the standard arguments,[1] the equilibrium bidding strategy is given by

$$\beta(v \mid p, N) = v + \int_v^{\bar{v}} \frac{H(u \mid p, N)}{H(v \mid p, N)} du. \tag{3.2}$$

To determine the equilibrium cutoff s_N or, equivalently, the equilibrium entry probability p_N, consider the marginal bidder with a signal s_N. Her ex ante expected profit from entry is

$$\int_{\underline{v}}^{\bar{v}} (\beta(v \mid p_N, N) - v) H(v \mid p_N, N) dF_{V|S}(v \mid s_N) - \kappa,$$

[1]See, e.g., Krishna (2010, Section 2.3). In this case, the equilibrium bidding function $\beta(\cdot \mid p, N)$ solves the differential equation $d(\beta(v \mid p, N) \cdot H(v \mid p, N))/dv = v \cdot H'(v \mid p, N)$, where $f'(\cdot)$ denotes the derivative of a function $f(\cdot)$, with a boundary condition $\beta(\bar{v} \mid p, N) = \bar{v}$.

where recall that κ denotes the entry cost. In equilibrium, the bidder with a signal s_N should be indifferent between entering and not entering, i.e. her ex ante expected profit should be zero. We can now characterize the equilibrium entry probability p_N:

Proposition 3.1. *Let p_N be the entry probability in a pure-strategy symmetric equilibrium with N potential bidders. If $p_N > 0$ then it solves*

$$\int_{\underline{v}}^{\bar{v}} C_2(F(v), p_N) H(v \mid p_N, N) dv = \kappa. \tag{3.3}$$

Marmer, Shneyerov, and Xu (2013) shows that the marginal bidder's expected profit is nonincreasing in the entry probability p and, as a result, there is a unique symmetric entry equilibrium. The condition that the contract is canceled when there are fewer than two active bidders changes the situation in two ways. First, $p_N = 0$ (i.e. no entry) is always an equilibrium. Second, the marginal bidder's profit from entry now can be non-monotone in the entry probability,[2] which may create multiple non-trivial entry equilibria. Intuitively, the nonmonotonicity can be understood by observing that for the marginal bidder to win the contract, she needs at least one other active bidder to avoid the contract's cancellation. As a result, the expected profit from entry of the marginal bidder increases with the entry probability p when p is small. However, when the entry probability is sufficiently large, the marginal bidder will face more competition at the bidding stage and is more likely to lose.

More formally, the derivative of the marginal bidder's expected profit from entry with respect to p is given by

$$\begin{aligned}\int_{\underline{v}}^{\bar{v}} C_{22}(F(v), p) H(v \mid p, N) dv - (N-1) \int_{\underline{v}}^{\bar{v}} C_2^2(F(v), p) \Lambda^{N-2}(v \mid p) dv \\ +(N-1)(1-p)^{N-2} \int_{\underline{v}}^{\bar{v}} C_2(F(v), p) dv.\end{aligned}$$

By the "good news" assumption, $C_{22}(\cdot, \cdot) \leq 0$ and the expression in the first line is negative. However, the positive term in the second line may dominate the terms in the first line, especially for smaller values of p. In such cases, the marginal bidder's expected profit from entry increases in p. Note that the term in the second line is due to the minimum two bidders condition.

Figure 1 shows the estimated entry cost and the expected counterfactual revenue of the marginal bidder for different entry probabilities in TxDoT procurement auctions with 14 potential bidders; see the details in Section 7. The non-monotonicity of the expected profit of the marginal bidder creates multiple equilibria for entry. Besides the trivial

[2]This is due to the $(1-p)^{N-1}$ term in the probability of winning function $H(\cdot \mid p, N)$.

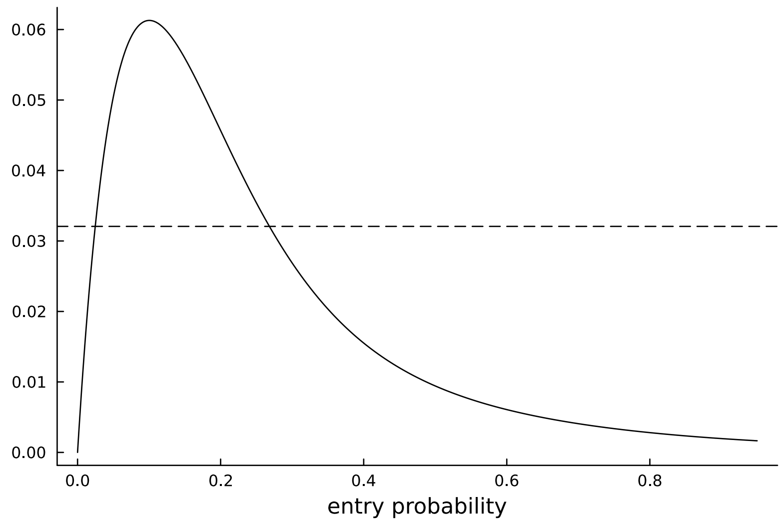

FIGURE 1. The entry cost (dashed line) and the marginal bidder's expected revenue from entry (solid line) for different entry probabilities estimated from the data for auctions with 14 potential bidders

zero-entry equilibrium, there are two other equilibria with $p_N = 0.025$ and $p_N = 0.269$. However, the left equilibrium ($p_N = 0.025$) is unstable as small negative shocks to p_N push the entry probability away from 0.025 and toward zero; similarly, small positive shocks push the entry probability toward one. Since there are no data from the trivial zero-entry equilibrium and after ruling out the unstable equilibrium, we can proceed under the assumption that all data in this example are generated by the stable equilibrium $p_N = 0.269$. Therefore, in what follows p_N denotes the entry probability in the non-trivial stable equilibrium.

Given the equilibrium entry and bidding strategies, we can now describe the expected cost of procurement, that is, the winning bid or price as in Li and Zheng (2009). Since the procurement cost is undetermined when an auction fails (when there are fewer than two active bidders), we condition on receiving at least two bids.

Proposition 3.2. *Let n denote the number of active bidders. Conditional on at least two active bidders, the expected equilibrium procurement cost is given by*

$$\begin{aligned} K(p_N, N \mid n \geq 2) =& \frac{1}{\Pr(n \geq 2 \mid p_N)} \left(N \int_{\underline{v}}^{\bar{v}} \Lambda(v \mid p_N)^{N-1} \left(1 - \frac{N-1}{N} \Lambda(v \mid p_N)\right) dv \right. \\ & \left. + \underline{v} - \bar{v} \Pr(n < 2 \mid p_N) \right), \end{aligned} \tag{3.4}$$

where $\Pr(n \geq 2 \mid p) \equiv 1 - (1-p)^N - Np(1-p)^{N-1}$ *is the probability of having at least two active bidders given the entry probability* p.

In practice, the auctioneer may be primarily concerned with the unconditional expected procurement cost, especially when the expenses associated with re-listing the contract, re-running the auction, or directly contracting with an outside party are significant. Suppose the auctioneer cannot delay the contract and, when the auction fails, they must hire a contractor without competition at the maximum cost $\bar{v}$. The corresponding unconditional expected procurement cost is given by

$$K(p_N, N) = N \int_{\underline{v}}^{\bar{v}} \Lambda(v \mid p_N)^{N-1} \left(1 - \frac{N-1}{N} \Lambda(v \mid p_N)\right) dv + \underline{v}. \tag{3.5}$$

4. Signals informativeness and procurement cost

Since the signals are not observed, the model is non-parametrically unidentified. Therefore, we adopt a semi-parametric approach, where a nonparametric distribution of private costs $F(\cdot)$ and a parametrically specified copula as discussed in Gentry and Li (2014, footnote 18):

$$C(F(v), p) = C(F(v), p; \theta_0),$$

where the function $C(\cdot, \cdot; \theta)$ is known up to the value of a single parameter $\theta \in \Theta \subset \mathbb{R}$. In addition to restoring identification, the semiparametric approach is convenient, as the single parameter θ now captures the dependence between the private costs V and the signals S. Therefore, θ can be viewed as a measure of the informativeness of the signals.

We make the following assumption on the copula function.

Assumption 4.1. $\partial C(x, y; \theta)/\partial \theta \geq 0$.

According to Assumption 4.1, the family of copulas $\{C(x, t; \theta) : \theta \in \Theta\}$ is positively ordered: for all $x, y \in [0, 1]$ and all $\theta_1 \leq \theta_2$, $C(x, y; \theta_1) \leq C(x, y; \theta_2)$. There are many families that satisfy the positive ordering assumption, including the Gaussian copula and important members of the class of Archimedean copulas such as Ali-Mikhail-Haq, Clayton, Frank, Gumbel, and Joe. For such copula functions, a higher value of θ corresponds to a stronger association between private costs and signals as measured by statistics such as Kendall's τ or Spearman's ρ (Nelsen, 2007, Chapter 5). Thus, in our auction context, the positive ordering property ensures that higher values of θ imply more informative signals.

The positive ordering property also has implications on the distribution of private costs conditional on entry defined in (3.1). Under more informative signals, the distribution of private costs conditional on entry is less stochastically dominant, and entering bidders tend to have smaller costs (for the same entry probability p).

We can now write $\Lambda(v \mid p, \theta) = 1 - C(F(v), p; \theta)$, and let $K(\theta, p_N, N \mid n \geq 2)$ denote the expected procurement cost conditional on having at least two active bidders as defined in Proposition 3.2, but now we explicitly indicate its dependence on θ in addition to p_N. Consider the effect of changing the informativeness of the signals on the expected procurement cost:

$$\frac{dK(\theta, p_N, N \mid n \geq 2)}{d\theta} = \underbrace{\frac{\partial K(\theta, p_N, N \mid n \geq 2)}{\partial \theta}}_{\text{information effect}} + \underbrace{\frac{\partial K(\theta, p_N, N \mid n \geq 2)}{\partial p} \frac{\partial p_N}{\partial \theta}}_{\text{cutoff effect}}.$$

There are two effects, which we refer to as the "information" and "cutoff" effects.

The information effect is the direct impact of having more informative signals. It operates through the $\Lambda(v \mid p, \theta)$ function and the distribution of private costs conditional on entry. Since the latter is less stochastically dominant under more informative signals due to the positive ordering condition, i.e. entering bidders tend to have smaller private costs, the information effect reduces the expected cost of procurement:

$$\frac{\partial K(\theta, p, N \mid n \geq 2)}{\partial \theta} = -\frac{N(N-1)}{\Pr(n \geq 2 \mid p)} \int_{\underline{v}}^{\bar{v}} \frac{\partial C(F(v), p; \theta)}{\partial \theta} \Lambda^{N-2}(v \mid p, \theta) C(F(v), p; \theta) dv \leq 0.$$

The cutoff effect is an equilibrium effect due to changes in the probability of entry and is ambiguous. The effect of θ on the equilibrium probability of entry p_N can be seen using Figure 1. Since we consider only the stable equilibrium with nonzero entry, $\partial p_N / \partial \theta$ is positive or negative depending on whether the marginal bidder's expected revenue (p-by-p) increases or decreases with θ, respectively. By the result of Proposition 3.1, the derivative of the marginal bidder's expected revenue with respect to θ is

$$\int_{\underline{v}}^{\bar{v}} \frac{\partial C_2(F(v), p; \theta)}{\partial \theta} H(v \mid p, N, \theta) dv - (N-1) \int_{\underline{v}}^{\bar{v}} C_2(F(v), p; \theta) \Lambda^{N-2}(v \mid p, N, \theta) \frac{\partial C(F(v), p; \theta)}{\partial \theta} dv,$$

where we write $H(v \mid p, N, \theta) = \Lambda^{N-1}(v \mid p, \theta) - (1-p)^{N-1}$. While the second term is negative, the sign of the first term is ambiguous because the derivative in the first integral can be positive or negative. Recall that $C_2(F(v), p; \theta)$ is the conditional CDF of the private costs given $S = s_N$. Suppose that the marginal distributions of private costs and signals are the same. Under more informative signals, the conditional distribution of V given $S = s_N$ is more concentrated around $v = s_N$, that is, the conditional CDFs of V given $S = s_N$ corresponding to different θ's cross at $v = s_N$. As a result, the derivative

$\partial C_2(F(v), p; \theta)/\partial\theta$ would be negative for $v < s_N$ and positive for $v > s_N$. However, note that the second term is more likely to be dominant in auctions with a larger number of potential bidders. Thus, in auctions with a sufficiently large N, the probability of entry is lower when the signals are more informative.

The sign of $\partial K(\theta, p, N \mid n \geq 2)/\partial p$ is also ambiguous. Similarly to (3.5), let $K(\theta, p_N, N)$ denote the unconditional expected procurement cost in auctions with N potential bidders corresponding to the signal informativeness θ and the equilibrium probability of entry p_N. Note that

$$K(\theta, p, N \mid n \geq 2) = \bar{v} + \frac{K(\theta, p, N) - \bar{v}}{\Pr(n \geq 2 \mid \theta, p, N)},$$

where $\Pr(n \geq 2 \mid \theta, p, N)$ is the probability that there are at least two active bidders when the informativeness of the signals is θ. One can easily see that a higher probability of entry reduces the expected unconditional cost of procurement:

$$\frac{\partial K(\theta, p, N)}{\partial p} = N(N-1)\int_{\underline{v}}^{\bar{v}} \Lambda^{N-2}(v \mid p, \theta) C_2(F(v), p; \theta)(\Lambda(v \mid p, \theta) - 1)dv \leq 0,$$

where the inequality holds because $\Lambda(v \mid p, \theta) \leq 1$. While $\partial K(\theta, p, N)/\partial p \leq 0$, the sign of $\partial K(\theta, p, N \mid n \geq 2)/\partial p$ can be positive or negative due to the derivative (with respect to p) of $\bar{v}/\Pr(n \geq 2 \mid \theta, p, N)$. The ambiguity of the effect of p on the expected procurement cost conditional on at least two active bidders can be traced back to that of the effect of p on the probability of winning $H(v \mid p, N)$. On the one hand, a larger probability of entry implies more competitors and a smaller probability of winning. On the other hand, a larger p reduces the probability of an auction failure due to insufficient entry.

The information effect for the unconditional procurement cost is negative by the same arguments as in the case of $K(\theta, p, N \mid n \geq 2)$. Thus, more informative signals reduce the unconditional expected procurement cost for the same probability of entry. However, the effect of having more informative signals on the entry cutoff remains undetermined together with the total effect on the expected unconditional procurement cost. If more informative signals result in less entry in the equilibrium, a positive cutoff effect may potentially dominate the information effect and increase the expected unconditional procurement cost. The ambiguity of the effect of θ on the equilibrium probability of entry is due to the same reasons as in the case of the expected conditional procurement cost: the derivative of $C_2(F(v), p; \theta)$ with respect to θ can be positive or negative.

In Section 7 below, we use TxDoT data on highway maintenance auctions to empirically study the effect of signal informativeness on the expected procurement cost.

5. Binding reserve price

In this section, we describe the results for a format with a binding reserve price that does not require at least two active bidders. In this scenario, a bidder is active if her signal is below the entry cutoff and her value is below the reserve price r. Since the contract is awarded even with a single bid below the reserve price, an active bidder with a value v wins the auction with probability $\Lambda^{N-1}(v \mid p, \theta)$, where $p = \Pr(S \leq s_N)$ is the probability of drawing a signal below the cutoff.[3] Note that in this format, the probability of winning decreases with p since it is not required that there are at least two bids submitted. The bidding function is given by

$$\beta(v \mid p, N, r, \theta) = v + \int_{\underline{v}}^{r} \frac{\Lambda^{N-1}(u \mid p, \theta)}{\Lambda^{N-1}(v \mid p, \theta)} du.$$

The expected revenue from entry of the marginal's bidder is

$$\int_{\underline{v}}^{r} C_2(F(v), p; \theta) \Lambda^{N-1}(v \mid p, \theta) dv.$$

In this case, there is a unique equilibrium for entry, as the expected revenue of the marginal's bidder is decreasing with p as in Marmer, Shneyerov, and Xu (2013). The CDF of values conditional on bidding is now

$$F^*(v \mid p, r, \theta) \equiv \frac{C(F(v), p; \theta)}{C(F(r), p; \theta)}.$$

The probability of auction failure, that is, not receiving any bids, is

$$(1 - C(F(r), p_N; \theta))^N.$$

Note that now it is directly dependent on θ, in addition to the indirect effect through p_N. As before, let n be the number of active bidders. By the same arguments as in the proof of Proposition 3.2, we can show that the expected cost of procurement conditional on $n \geq 1$ is given by

$$K(\theta, p_N, N, r \mid n \geq 1) = \left(N \int_{\underline{v}}^{r} \Lambda(v \mid p_N, \theta)^{N-1} \left(1 - \frac{N-1}{N} \Lambda(v \mid p_N, \theta) \right) dv \right.$$
$$\left. + \underline{v} - r \Pr(n = 0 \mid p_N, N, r, \theta) \right) \times \frac{1}{\Pr(n \geq 1 \mid p_N, N, r, \theta)}.$$

Because the probability of auction success or failure now directly depends on the informativeness of the signals θ, the information effect for the conditional expected cost of procurement is ambiguous. The cutoff effect remains ambiguous by the same arguments

[3]The probability of entry and bidding is $C(F(r), p)$.

as in Section 4 for the model with no binding reserve price that requires at least two active bidders.

Under the assumption that the government has to pay the maximum cost $\bar{v}$ when an auction fails, the unconditional expected cost of procurement with a binding reserve price and no two active bidders requirement is given by

$$K(\theta, p_N, N, r) = N \int_{\underline{v}}^{r} \Lambda(v \mid p_N, \theta)^{N-1} \left(1 - \frac{N-1}{N} \Lambda(v \mid p_N, \theta)\right) dv \\ + (\bar{v} - r) \Pr(n = 0 \mid p_N, N, r, \theta) + \underline{v}.$$

A binding reserve price is set below $\bar{v}$, and since the probability of auction failure decreases with θ, the information effect reduces the unconditional expected cost of procurement. However, the cutoff effect remains ambiguous.

The difference between the unconditional expected costs of procurement in the frameworks without and with reserve price, respectively, is given by

$$N \int_{r}^{\bar{v}} \Lambda(v \mid p_N, \theta)^{N-1} \left(1 - \frac{N-1}{N} \Lambda(v \mid p_N, \theta)\right) dv - (\bar{v} - r) \Pr(n = 0 \mid p_N, N, r, \theta). \quad (5.1)$$

The sign of the difference is ambiguous, that is, either of the two formats can result in a smaller unconditional expected cost of procurement. The comparison depends on the number of potential bidders N, informativeness of the signals θ, reserve price r, and the distribution of the private cost. However, the format with the binding reserve price is preferred when the probability of auction failure $\Pr(n = 0 \mid p_N, N, r, \theta)$ is sufficiently small. In the empirical section, we find that in the TxDOT data, the probability of auction failure is minimized for a large value of θ that corresponds to highly informative signals. However, such levels of informativeness will result in no entry in the case of the format without the binding reserve price.

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6. Identification and estimation

The model imposes the following restriction on the copula parameter θ and the CDF of private costs $F(\cdot)$: for all $v \in [\underline{v}, \bar{v}]$,

$$p_N \cdot F^*(v \mid p_N) = C(F(v), p_N; \theta).$$

The data provided by Li and Zheng (2009) contains only auctions with two or more potential bidders N, and we assume that an auction fails if it does not attract at least two bids. Hence, from the data, we can directly estimate the probability of entry conditional on the number of active bidders $n \geq 2$:

$$\Pr(S \leq s_N \mid n \geq 2) = \frac{p_N(1-(1-p_N)^{N-1})}{1-(1-p_N)^N - Np_N(1-p_N)^{N-1}}. \tag{6.1}$$

The probability of entry p_N can be recovered by solving the above equation.[4]

The CDF of private costs conditional on entry $F^*(\cdot \mid p)$ can be identified from data using a modified inverse-bidding-function approach of Guerre, Perrigne, and Vuong (2000). In the context of auctions with entry, the approach was used in Marmer, Shneyerov, and Xu (2013) and Xu (2013), but in our case it also requires an adjustment to have at least two active bidders.[5]

Proposition 6.1. *Let $G(\cdot \mid N)$ and $g(\cdot \mid N)$ denote the CDF and PDF of the bids, respectively, in auctions with N potential bidders. The inverse bidding function is given by*

$$\beta^{-1}(b \mid p_N, N) = b - \frac{1}{p_N(N-1)g(b \mid N)}\left(1 - p_N \cdot G(b \mid N) - \frac{(1-p_N)^{N-1}}{\left(1-p_N \cdot G(b \mid N)\right)^{N-2}}\right).$$

In practice, one can use the plugin estimator of the inverse bidding function, which replaces the unknown $G(\cdot \mid N)$ with the empirical CDF of bids in auctions with N potential bidders, $g(\cdot \mid N)$ with its kernel estimator, and p_N with the estimated equilibrium probability of entry recovered from the estimated probability of entry conditional on two active bidders using (6.1).

Let $\hat{\beta}^{-1}(\cdot \mid \hat{p}_N, N)$ denote the plugin estimator of the inverse bidding function, where $\hat{p}_N$ is the estimated equilibrium entry probability. Given bids data from T_N auctions with N potential bidders, $\{B_{t,i} : i = 1, \ldots, n_t, t = 1 : T_N\}$, where n_t is the number of active bidders in auction t, we can estimate the CDF of private costs conditional on entry by

$$\hat{F}^*(v \mid \hat{p}_N) = \frac{1}{T_N}\sum_{t=1}^{T_N}\frac{1}{n_t}\sum_{i=1}^{n_t}\mathbf{1}(\hat{\beta}^{-1}(B_{t,i} \mid \hat{p}_N, N) \leq v).$$

Since $F^*(\cdot \mid p_N)$ is identified, for a given value of the copula parameter θ, we can recover the implied CDF of private costs using (3.1):

$$F(v; p_N, \theta) \equiv C^{-1}(p_N F^*(v \mid p_N), p_N; \theta),$$

where $C^{-1}(\cdot, p; \theta)$ denotes the inverse function of $C(\cdot, p; \theta)$. To identify the CDF of private costs $F(\cdot)$ and the copula parameter θ, we assume that the private costs and signals are independent of the number of potential bidders N.

[4]The equation has unique numerical solutions with our data.

[5]The expression in (6.1) is the same as in Xu (2013) except for the last term in parentheses, which is due to the condition of at least two active bidders.

Assumption 6.1. The joint distribution of private costs and signals is independent of the number of potential bidders.

The copula parameter θ is now identified by the system of equations

$$p_N F^*(v \mid p_N) = C(F(v; p_N, \theta_0), p_N; \theta_0) \quad \text{for all } v, N.$$

The CDF of private costs $F(\cdot)$ is now identified by $F(v) = F(v; p_N, \theta_0)$ or all v and N.

We can estimate the copula parameter using a minimum distance approach. Let $\mathcal{N}$ denote a set of the number of potential bidders for which auction data is observed.

$$\hat{\theta} = \arg\min_{\theta\in\Theta} \sum_{N\in\mathcal{N}} \sum_{N'\in\mathcal{N}} \int\int \hat{\Delta}(v, N) W(v, N; v', N') \hat{\Delta}(v', N') dv dv',$$

where

$$\hat{\Delta}(v, N) \equiv \hat{p}_N \hat{F}^*(v \mid \hat{p}_N) - C(\hat{F}(v; \theta), \hat{p}_N; \theta),$$
$$\hat{F}(v; \theta) \equiv \frac{1}{|\mathcal{N}|} \sum_{N\in\mathcal{N}} C^{-1}(\hat{p}_N \hat{F}^*(v \mid \hat{p}_N), \hat{p}_N, \theta),$$

and $W(\cdot, \cdot; \cdot, \cdot)$ is a symmetric and positive definite weight function. The estimator of the CDF of private costs $F(\cdot)$ can now be constructed as

$$\hat{F}(v) \equiv \hat{F}(v; \hat{\theta}).$$

The boundaries of the support of the distribution of private costs can be estimated using the minimum and maximum estimated private costs. For example, the upper boundary $\bar{v}$ can be estimated using

$$\hat{\bar{v}} = \max_{N\in\mathcal{N}} \hat{\beta}^{-1}(\hat{\bar{b}}_N \mid \hat{p}_N, N),$$

where $\hat{\bar{b}}_N$ is the maximum observed bid in auctions in N potential bidders. The estimator for the lower boundary $\hat{\underline{v}}$ can be constructed similarly by replacing $\max$ with $\min$, and $\hat{\bar{b}}_N$ with the minimum bid in auctions with N potential bidders.

Once the primitives of the model are estimated, we can perform counterfactual experiments by changing the signal informativeness parameter θ or the format of the auctions. First, the entry cost for auctions with N potential bidders can be estimated using the estimated version of (3.3):

$$\hat{\kappa}_N = \int_{\hat{\underline{v}}}^{\hat{\bar{v}}} C_2(\hat{F}(v), \hat{p}_N; \hat{\theta}) \hat{H}(v \mid \hat{p}_N, N; \hat{\theta}) dv,$$

where $\hat{H}(v \mid p, N; \theta) \equiv (1 - C(\hat{F}(v), p; \theta))^{N-1} - (1-p)^{N-1}$. Let $\tilde{p}_N(\theta)$ denote the counterfactual equilibrium entry probability corresponding to the signals' informativeness θ. It can be computed by solving the estimated version of (3.3):

$$\int_{\hat{\underline{v}}}^{\hat{\bar{v}}} C_2(\hat{F}(v), \tilde{p}_N(\theta); \theta)\hat{H}(v \mid \tilde{p}_N(\theta), N; \theta)dv = \hat{\kappa}_N.$$

The counterfactual expected procurement cost in auctions with at least two active bidders can now be computed using the estimated version of (3.4):

$$\begin{aligned}\hat{K}(\theta, \tilde{p}_N(\theta) \mid n \geq 2) =& \frac{1}{\Pr(n \geq 2 \mid \tilde{p}_N(\theta))}\left(N \int_{\hat{\underline{v}}}^{\hat{\bar{v}}} \hat{\Lambda}(v \mid \tilde{p}_N(\theta); \theta)^{N-1}\right.\\ &\left.\times \left(1 - \frac{N-1}{N}\hat{\Lambda}(v \mid \tilde{p}_N(\theta); \theta)\right) dv + \hat{\underline{v}} - \hat{\bar{v}} \cdot \Pr(n < 2 \mid \tilde{p}_N(\theta))\right).\end{aligned}$$

One can similarly compute counterfactuals for the unconditional expected procurement cost using the estimated version of (3.5).

7. Empirical results

7.1. Data

The Li and Zheng (2009) data set for TxDoT "mowing highway right-of-way" auctions includes the following information on each auction: the number of potential bidders, submitted bids, engineer's estimate, number of items in a contract, and if it is a local, state, or interstate contract. While the number of potential bidders varies between 3–26, in many cases, the number of auctions and submitted bids is small.

The number of items in a project varies between 1–7. As explained in Li and Zheng (2009), the main item is "type-II full-width mowing". Additional tasks may include strip mowing, spot mowing, litter pickup and disposal, sign installation, etc. We can be confident that projects with one item involve the same main tasks. However, since the data does not contain information on the type of additional tasks, they may vary between auctions with the same number of items.

Li and Zheng (2009) also explains that there can be substantial differences between local, state, and interstate jobs. State jobs are auctioned by the state agency with potentially different requirements for preparing bid proposals. Interstate jobs can be more complicated because of a higher traffic volume.

To make our sample as homogeneous as possible, we focus only on local projects with one item. We further homogenize bids in our sample by the engineer's estimate; thus, bids are fractions of the engineer's estimate. We exclude the numbers of potential bidders

Potential bidders	**9**	**10**	**12**	**13**	**14**
Number of auctions	15	15	16	11	10
Number of bids	40	41	43	41	40
Engineer's estimate (dollars)					
mean	104,813	89,489	113,838	84,025	77,493
std	44,333	39,547	48,493	31,496	27,760
std.err	11,447	10,211	12,123	9,496	8,778
Bids (fraction of engineer's estimate)					
mean	1.068	1.004	1.106	1.037	1.057
std	0.165	0.172	0.167	0.204	0.169
min	0.815	0.721	0.799	0.703	0.722
max	1.445	1.471	1.470	1.556	1.530
Winning bids (fraction of engineer's estimate)					
mean	0.952	0.921	1.011	0.898	0.959
std	0.065	0.131	0.129	0.153	0.117
min	0.815	0.721	0.799	0.703	0.722
max	1.106	1.207	1.249	1.148	1.124
% above estimate	14.2	15.6	49.1	20.8	42.3
std.err for % above estimate	5.5	5.7	7.6	6.3	7.8

TABLE 1. Summary statistics for the sample of local projects with one item and at least 30 bids for each number of potential bidders

Ns that have fewer than 30 submitted bids to ensure that the CDFs of private costs conditional on entry are precisely estimated. Our final sample includes $N = 9, 10, 12, 13, 14$.

Table 1 reports the summary statistics for our sample. The average engineer's estimate is between \$77,493–\$104,813. There is a substantial variation in the estimate, with the standard deviations between \$27,760–\$48,493. The average submitted bid as a fraction of the engineer's estimate is between 1.0–1.11, with a minimum of 0.7 and a maximum of 1.56. The average winning bid as a fraction of the estimate ranges between 0.90–1.01. The percentage of winning bids above the estimate ranges between 14.2%–49.1%, confirming that the reserve price is not binding. The largest winning bid as a fraction of the estimate is 1.25, and the smallest is 0.70.

7.2. Entry probabilities, signals' informativeness, entry costs, and the distribution of private costs

We first discuss the estimated probability of entry conditional on having at least two active bidders and the implied estimated unconditional equilibrium probabilities of entry

	Estimate	**95% confidence interval**
Copula parameter θ	3.21 (0.39)	[2.44, 3.98]
Spearman correlation ρ	0.47 (0.09)	[0.38, 0.56]

TABLE 2. The estimates of the copula parameter and its implied Spearman rank correlation with their standard errors (in parentheses) and 95% confidence intervals

$\hat{p}_N$; both are displayed in Figure 2a. The estimated implied probability of entry varies between 15%–27% depending on the number of potential bidders. The difference between the estimated probability of entry conditional on at least two active bidders and the implied $\hat{p}_N$ can be substantial and should not be ignored. Both probabilities are non-monotone in the number of potential bidders. Note that due to the requirement of at least two active bidders, the relationship between the entry probability and the number of potential bidders can be non-monotone even for the same entry cost κ. This is unlike the case studied in Marmer, Shneyerov, and Xu (2013), where for the same entry cost, the equilibrium entry probability is monotone decreasing in the number of potential bidders.

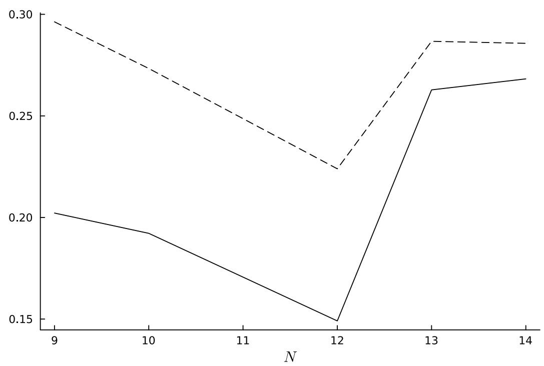
We use the triangular kernel to estimate the PDF of bids, which is required for the estimation of the inverse bidding function. We follow the rule of thumb for bandwidth selection with minor under-smoothing and set the bandwidth to $3.15 \times (\text{data std}) \times (\text{sample size})^{-1/5+\epsilon}$, with $\epsilon = 1/17$. [DISCUSS BOUNDARY CORRECTION?????] After the correction for at least two active bidders, we obtain monotone-increasing estimates of the inverse bidding functions. The estimated support of the distribution of private costs is $[0.47, 1.56]$.

We choose Frank's copula for our specification; however, we have also considered Clayton and Gumbel copulas, and our estimation results are not sensitive to the choice of the copula function.

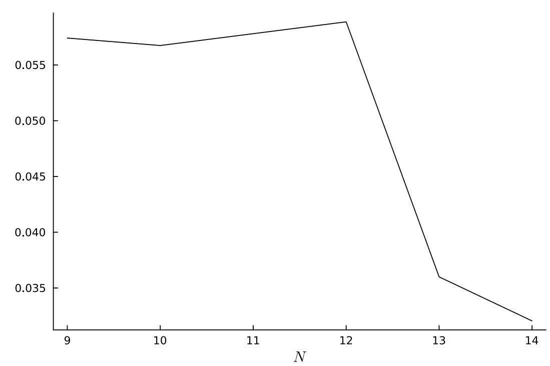
To estimate the copula parameter θ and the CDF of private costs $F(\cdot)$, we use a grid of values v between 0.4 and 1.6 with a step of 0.05. We use the efficient two-step GMM estimator. Table 2 shows the estimates for θ and the corresponding Spearman rank correlation coefficient ρ . According to our estimates, the signals are moderately informative with the 95% confidence interval for ρ between 0.38 – 0.56.

After estimating the copula parameter and the CDF of private costs $F(\cdot)$, see Figure 2c, we estimate the entry cost parameter κ for each value of the number of potential bidders N. The results are shown in Figure 2b. The estimated entry costs are between 3.2% –

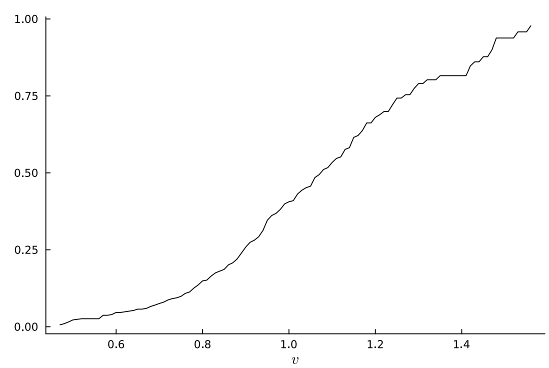
5.9% of the engineer's estimate (between \$2,485 – \$6,023). The entry costs are also tend to be negatively associated with the number of potential bidders.



(A) The estimated probabilities of entry conditional on at least two active bidders (dashed line) and the estimated unconditional entry probabilities p_N (solid line) for different numbers of potential bidders N



(B) The estimated entry costs κ for different numbers of potential bidders N

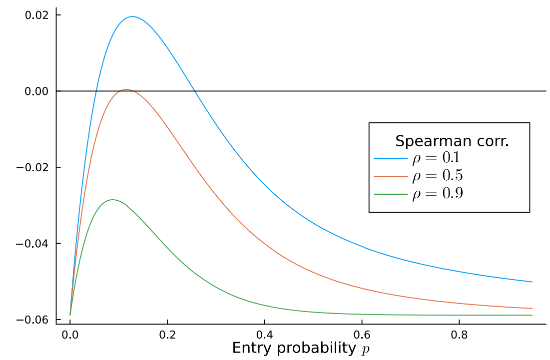


(C) The estimated CDF $F(v)$ of private costs

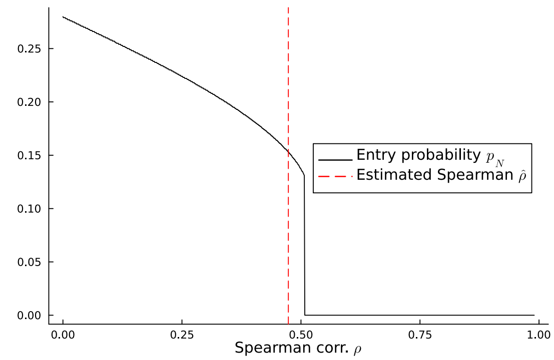
FIGURE 2. Estimation results for the entry probability p_N, entry cost κ, and CDF $F(\cdot)$ of private costs

7.3. Counterfactuals

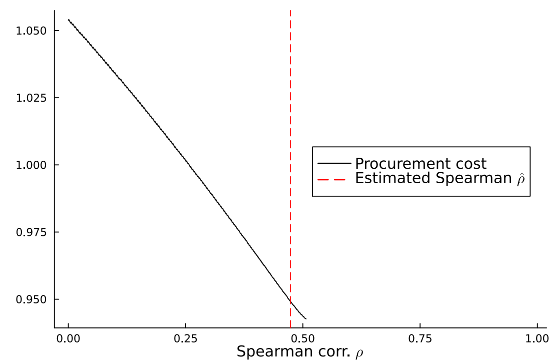
Equipped with the estimates for the model's primitives, we consider several counterfactual experiments. First, we consider how the change in signal informativeness affects the entry probabilities and the expected conditional and unconditional procurement costs. Second, we compare the two auction mechanisms used to address insufficient entry: imposing the minimum two-bids requirement or making the reserve price (the engineer's estimate binding). We focus on the auctions with $N = 12$ potential bidders. The results are typical for other N's except for a few specific cases discussed separately.



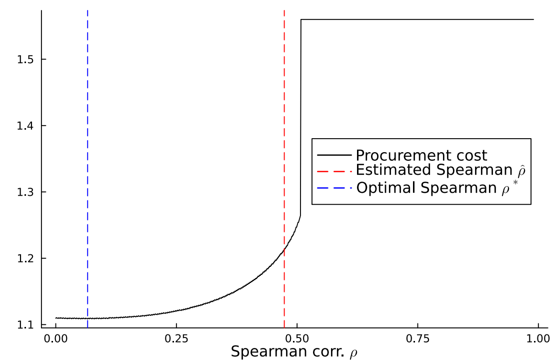
(A) Marginal bidder's expected profit from entry



(B) Entry probability p_N in the stable non-trivial equilibrium



(C) Conditional expected cost of procurement



(D) Unconditional expected cost of procurement

FIGURE 3. $N = 12$ potential bidders and no reserve price: The marginal bidder's expected profit from entry, equilibrium entry probability p_N, and conditional and unconditional expected costs of procurement as fractions of the engineer's estimate for different levels of signal informativeness as measured by the Spearman rank correlation ρ; the estimated ($\hat{\rho}$) and optimal (unconditional expected procurement cost minimizing, ρ^*) Spearman correlations

Figure 3a shows the expected profit from entry of the marginal bidder for three different levels of signal informativeness. Here, we consider the format without a binding reserve price but requiring at least two submitted bids. To make the result easier to interpret, signal informativeness is reported in terms of the Spearman rank correlation ρ between private costs and signals, as implied by the copula parameter θ. The figure shows that the expected marginal bidder's profit decreases with ρ. As a result, higher values of ρ imply lower entry probabilities in the stable equilibrium. Furthermore, the expected profit is negative for ρ's above the threshold of approximately 0.5 and, as Figure 3b shows, the equilibrium entry probability p_N drops from approximately 0.13 to zero once ρ exceeds that threshold.

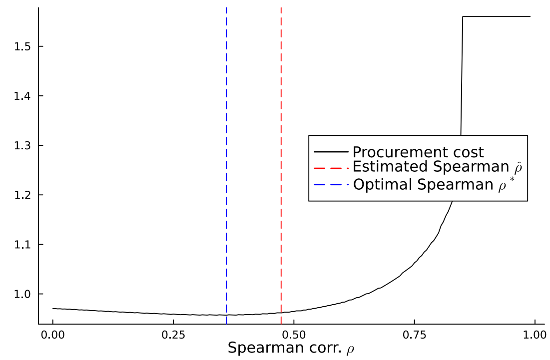
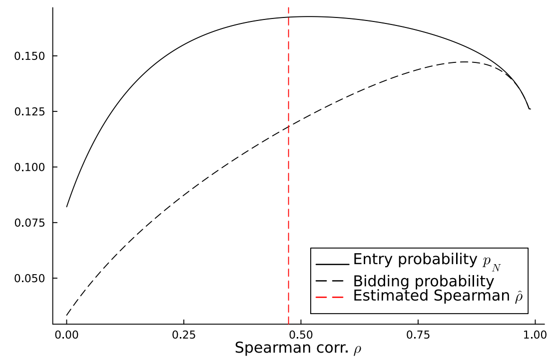


FIGURE 4. $N = 13$ potential bidders and no reserve price: The expected unconditional cost of procurement as fractions of the engineer's estimate, and estimated ($\hat{\rho}$) and optimal (ρ^*) Spearman rank correlations between private costs and signals

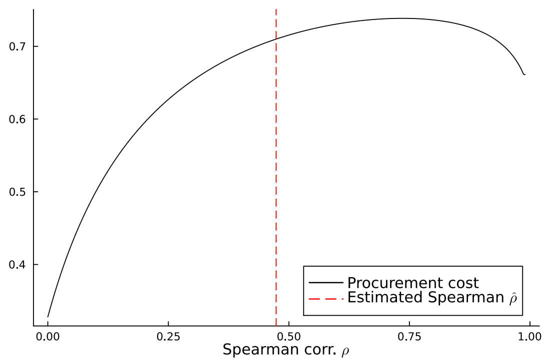
The conditional expected procurement cost is defined only for ρ's corresponding to non-zero entry probabilities. In this range, the expected conditional procurement cost decreases with the signal informativeness ρ, see Figure 3c. Note also that the estimated signal informativeness $\hat{\rho}$ is near the threshold where the entry probability drops to zero. Therefore, from the perspective of the conditional procurement cost, the estimated signal informativeness is nearly optimal.

Figure 3d shows the unconditional expected procurement cost. We computed it under the assumption that an auction fails when it attracts fewer than two bids and the auctioneer has to pay the maximum cost $\bar{v}$ estimated at 1.56. First, note that at the estimated level of signal informativeness, the unconditional expected procurement cost is higher than the conditional: approximately 1.2 vs. 0.95. Therefore, in a format that does not impose a binding reserve price but requires at least two bids, auction failure due to insufficient entry may increase the expected procurement cost by up to 25% of the engineer's estimate. However, we also compute the optimal level of signal informativeness, ρ^*, that minimizes the expected unconditional procurement cost. In this case, there is a non-trivial optimal level $\rho^* = 0.08$. By reducing the signal informativeness from $\hat{\rho} = 0.47$ to $\rho^* = 0.08$, the unconditional expected procurement cost can be reduced by approximately 10% of the engineer's estimate.

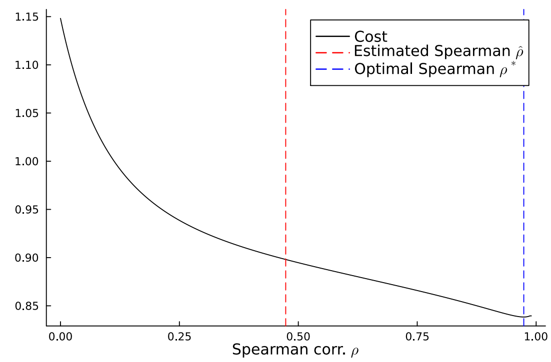
We draw the following conclusions from the counterfactual calculations shown in Figure 3. First, the format that requires at least two bids for the contract to be awarded may result in discontinuous behavior for the equilibrium entry probability: it can suddenly drop to zero if the informativeness of signals exceeds a certain threshold. Such a behavior is undesirable because small variations in informativeness may result in drastic changes in the outcome and auction failures. Second, ignoring auction failure may result



(A) Entry (p_N) and bidding probabilities



(B) Conditional expected procurement cost



(C) Unconditional expected procurement cost

FIGURE 5. $N = 12$ potential bidders and binding reserve price: The equilibrium probabilities of entry (p_N) and bidding, and conditional and unconditional expected costs of procurement as fractions of the engineer's estimate for different levels of signal informativeness as measured by the Spearman rank correlation ρ; the estimated ($\hat{\rho}$) and optimal (unconditional expected procurement cost minimizing, ρ^*) Spearman correlations

in a significant underestimation of the expected cost of procurement. Third, the optimal level of signal informativeness for the expected unconditional procurement cost may significantly differ from that for the expected conditional procurement cost. Lastly, one may see optimal non-trivial levels of signal informativeness. To emphasize the last point, Figure 4 shows the optimal Spearman rank correlation that minimizes the unconditional expected procurement cost when $N = 13$. The optimal level is $\rho^* = 0.36$ and away from perfectly informative or uninformative signals.

For our next counterfactual experiment, we consider a change of the format from at least two bidders with no binding reserve price to a format with no requirements on the minimum number of bids but with a binding reserve price. I.e., the contract is awarded even when only one bid is submitted as long as it is below the reserve price. Similarly to before, we consider the case of $N = 12$ and set the reserve price at one, that is the engineer's estimate. From the discussion in Section 3, we know that the expected profit from entry for the marginal bidder is now monotone. Therefore, in Figure 5 we show the probabilities of entry and bidding, and the expected conditional and unconditional procurement costs. Note that in the case of a binding reserve price, entry does not imply bidding because a potential bidder may discover that her private cost is above the reserve price after entering. Furthermore, an auction fails if no bidder enters or no active bidder draws a private cost below the reserve. To compute the unconditional expected cost of procurement, similarly to above, we assume that the auctioneer has to pay $\bar{v}$ when the auction fails.

Figure 5a shows that entry and bidding behavior is continuous, unlike in the case of auctions that require at least two submitted bids. Both the entry and bidding probabilities are continuous in the signal informativeness ρ. This is because the expected profit from entry of the marginal bidder is now monotone decreasing in the entry probability as discussed in Section 3. Note also that the bidding probability, which requires drawing a signal below the entry threshold and a private cost below the reserve price, is maximized at the signal informativeness level closer to one than that for the entry probability.

The conditional expected procurement cost in Figure 5b is computed conditional on the event of at least one bid submitted. Although its graph is non-monotone in ρ, the conditional expected procurement cost is maximized at $\rho = 0$, that is, completely non-informative signals. Making the signals completely non-informative saves up to 40% of the engineer's estimate in conditional procurement costs. However, it is important to indicate that the optimal level of informativeness with respect to the conditional expected procurement cost varies with the number of potential bidders. In auctions with $N \leq 12$, the optimal level is $\rho = 0$. However, in auctions with $N > 12$, the optimal level is $\rho = 1$.

Another striking finding is that the conditional expected procurement cost under a binding reserve price is uniformly lower than in the at least two bids format. Lastly, unlike the conditional procurement cost, the unconditional expected procurement cost in Figure 5c is monotone in ρ almost over its entire range and is minimized at a nearly perfect level of signal informativeness: $\rho^* = 0.97$. The result holds for all considered values of N.

It is also informative to compare the levels of procurement costs in the two formats: no binding reserve price with at least two submitted bids and a binding reserve price. Table 3 reports the expected cost of procurement for the two formats. In the case of a binding reserve price, we set it to 1.0, i.e., the engineer's estimate becomes a binding reserve price. For the same levels of signal informativeness, the binding reserve price format results in a lower conditional expected procurement cost: 0.95 vs. 0.70. That is,

Auction format	Signal' informativeness (Spearman corr. ρ)	Conditional expected procurement cost	Unconditional expected procurement cost
no reserve price at least two bids (data)	0.47	0.95	1.22
binding reserve price	0.47	0.70	0.90
binding reserve price	0.97		0.84
binding reserve price	0	0.32	

TABLE 3. Expected procurement cost as a fraction of the engineer's estimate for different auction formats and different levels of signal informativeness as measured by Spearman rank correlation (ρ) between private costs and signals

switching to the format with a binding reserve price saves 25% of the engineer's estimate in procurement costs. Furthermore, adjusting the signal informativeness to its optimal level saves additional 38%.

Similarly, under a binding reserve price, the unconditional expected procurement cost is lower than that in the at least two bids format: 0.90 vs. 1.22. Therefore, imposing a binding reserve price and removing the restriction of at least two bids saves 32% of the engineer's estimate in procurement costs. Making signals almost perfectly informative about private costs saves additional 6% in procurement costs.

Thus, one of the main findings of our paper is that imposing a binding reserve price is a more effective measure against the infinite bid strategy than requiring at least two submitted bids. In addition to higher expected procurement costs, the latter format generates discontinuous behavior for the entry stage, which may result in drastically different outcomes due to small variation in signal informativeness. Overall, we find the at least two bids format to be highly unfavorable for the auctioneer.

Another major finding of the paper is that the optimal level of signal informativeness depends on the format and the adopted measure, i.e., the conditional or unconditional expected procurement costs. If an auction failure may result in substantial additional costs, to minimize the unconditional expected procurement cost, the auctioneer prefers almost perfectly informed bidders. Note that the case of perfectly informed bidders describes the Samuelson model of entry (Samuelson, 1985).[6] Thus, according to our empirical results, in the case of TxDoT "mowing highway right-of-way" auctions, the Samuelson framework with perfectly informed bidders leads to least unconditional procurement costs.

Appendix A. Proofs

Proof of Proposition 3.1. By the copula properties, $F_{V|S}(v \mid s) = C_2(F(v), F_S(s))$. The expected profit from entry of the marginal bidder corresponding to the entry probability $p = F_S(s)$ is given by

$$\begin{aligned}
&\int_{\underline{v}}^{\bar{v}} (\beta(v \mid p, N) - v) H(v \mid p, N) dC_2(F(v), p) \\
=& \int_{\underline{v}}^{\bar{v}} \left(\int_{v}^{\bar{v}} H(u \mid p, N) du \right) dC_2(F(v), p) \\
=& \int_{\underline{v}}^{\bar{v}} C_2(F(v), p) H(v \mid p, N) dv,
\end{aligned}$$

where the equality in the second line holds by (3.2), and the equality in the last line holds by integration by parts and because $H(\bar{v} \mid p, N) = 0$. □

Proof of Proposition 3.2. Suppose there are $n \geq 2$ active bidders. The CDF of the minimum value among the n active bidders is

$$1 - (1 - F^*(v \mid p))^n,$$

and the corresponding expected winning bid when there are n active bidders is

$$\int_{\underline{v}}^{\bar{v}} \beta(v \mid p, N) d(1 - (1 - F^*(v \mid p))^n) = n \int_{\underline{v}}^{\bar{v}} \beta(v \mid p, N)(1 - F^*(v \mid p))^{n-1} dF^*(v \mid p).$$

Therefore, conditional on at least two active bidders, the expected winning bid is

$$\begin{aligned}
&\frac{1}{\Pr(n \geq 2)} \int_{\underline{v}}^{\bar{v}} \beta(v \mid p, N) \sum_{j=2}^{N} \binom{N}{j} j p^j (1-p)^{N-j} (1 - F^*(v \mid p))^{j-1} dF^*(v \mid p) \\
=& \frac{N}{\Pr(n \geq 2)} \int_{\underline{v}}^{\bar{v}} \beta(v \mid p, N) \sum_{j=2}^{N} \binom{N-1}{j-1} p^j (1-p)^{N-j} (1 - F^*(v \mid p))^{j-1} dF^*(v \mid p)
\end{aligned}$$

[6]As discussed in Marmer, Shneyerov, and Xu (2013), the Samuelson model is the limiting case of the endogenous entry model considered in the paper.

$$
\begin{aligned}
=&\frac{Np}{\Pr(n \geq 2)} \int_{\underline{v}}^{\bar{v}} \beta(v \mid p, N) \sum_{j=1}^{N-1} \binom{N-1}{j} p^j (1-p)^{N-1-j} (1 - F^*(v \mid p))^j dF^*(v \mid p) \\
=&\frac{Np}{\Pr(n \geq 2)} \int_{\underline{v}}^{\bar{v}} \beta(v \mid p, N) H(v \mid p, N) dF^*(v \mid p) \\
=&-\frac{N}{\Pr(n \geq 2)} \int_{\underline{v}}^{\bar{v}} \beta(v \mid p, N) H(v \mid p, N) d\Lambda(v \mid p), \qquad \text{(A.1)}
\end{aligned}
$$

where the first equality holds by the binomial property $j\binom{N}{j} = N\binom{N-1}{j-1}$, the equality in the third line holds by the binomial theorem $\sum_{j=1}^{N-1} \binom{N-1}{j} (p(1-F^*(v \mid p)))^j (1-p)^{N-1-j} = (1 - pF^*(v \mid p))^{N-1} - (1-p)^{N-1}$ and because $1 - pF^*(v \mid p) = \Lambda(v \mid p)$. Using integration by parts for the integral in (A.1),

$$
\begin{aligned}
&\int_{\underline{v}}^{\bar{v}} \beta(v \mid p, N) H(v \mid p, N) d\Lambda(v \mid p) \\
=&\beta(v \mid p, N) H(v \mid p, N) \Lambda(v \mid p) \Big|_{\underline{v}}^{\bar{v}} - \int_{\underline{v}}^{\bar{v}} \Lambda(v \mid p) d(\beta(v \mid p, N) H(v \mid p, N)) \\
=&-\underline{v} H(\underline{v} \mid p, N) - \int_{\underline{v}}^{\bar{v}} H(v \mid p, N) dv - \int_{\underline{v}}^{\bar{v}} \Lambda(v \mid p) d(\beta(v \mid p, N) H(v \mid p, N)) \\
=&-\underline{v} H(\underline{v} \mid p, N) + (1-p)^{N-1}(\bar{v} - \underline{v}) - \int_{\underline{v}}^{\bar{v}} \Lambda(v \mid p)^{N-1} dv \\
&\quad - \int_{\underline{v}}^{\bar{v}} \Lambda(v \mid p) d(\beta(v \mid p, N) H(v \mid p, N)) \\
=&\bar{v}(1-p)^{N-1} - \underline{v} - \int_{\underline{v}}^{\bar{v}} \Lambda(v \mid p)^{N-1} dv - \int_{\underline{v}}^{\bar{v}} \Lambda(v \mid p) d(\beta(v \mid p, N) H(v \mid p, N)). \qquad \text{(A.2)}
\end{aligned}
$$

where the second equality holds by $H(\bar{v} \mid p, N) = 0$ and the equilibrium bidding strategy in (3.2), and the last equality by $\Lambda(\underline{v} \mid p) = 1$. Using the first-order condition for the equilibrium bidding strategy

$$
d(\beta(v \mid p, N) \cdot H(v \mid p, N))/dv = v \cdot H'(v \mid p, N), \qquad \text{(A.3)}
$$

the second integral in (A.2) becomes

$$
\begin{aligned}
&(N-1) \int_{\underline{v}}^{\bar{v}} \Lambda(v \mid p)^{N-1} v d\Lambda(v \mid p) \\
=&\frac{N-1}{N} \int_{\underline{v}}^{\bar{v}} v d\Lambda(v \mid p)^N \\
=&\frac{N-1}{N} (\bar{v}(1-p)^N - \underline{v}) - \frac{N-1}{N} \int_{\underline{v}}^{\bar{v}} \Lambda(v \mid p)^N dv. \qquad \text{(A.4)}
\end{aligned}
$$

Combining (A.2) and (A.4) and multiplying by N, we obtain:

$$\begin{aligned}
&N \int_{\underline{v}}^{\bar{v}} \beta(v \mid p, N) H(v \mid p, N) d\Lambda(v \mid p) \\
=&\bar{v}((1-p)^N + Np(1-p)^{N-1}) - \underline{v} - N \int_{\underline{v}}^{\bar{v}} \Lambda(v \mid p)^{N-1} \left(1 - \frac{N-1}{N} \Lambda(v \mid p)\right) dv. \quad \text{(A.5)}
\end{aligned}$$

□

Proof of Proposition 6.1. By (A.3),

$$v = \beta(v \mid p, N) + \frac{H(v \mid p, N) \cdot \beta'(v \mid p, N)}{H'(v \mid p, N)}.$$

Substituting $v = \beta^{-1}(b \mid p_N, N)$ on the right-hand side and because $F^*(v \mid p_N) = G(\beta(v \mid p_N, N) \mid N)$, we obtain:

$$\beta^{-1}(b \mid p_N, N) = b - \frac{\left((1 - pG(b \mid N))^{N-1} - (1-p_N)^{N-1}\right)\beta'(\beta^{-1}(b \mid p_N, N) \mid p_N, N)}{p(N-1)(1-pG(b \mid N))^{N-2} f^*(\beta^{-1}(b \mid p_N, N) \mid p_N)},$$

where $f^*(\cdot \mid p_N)$ is the CDF of $F^*(\cdot \mid p_N)$. The result follows from $g(b \mid N) = f^*(\beta^{-1}(b \mid p_N, N) \mid p_N)/\beta'(\beta^{-1}(b \mid p_N, N) \mid p_N, N)$. □

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