Input Price Discrimination and Downstream Licensing*

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Abstract

The existing theory on input price discrimination assumes the absence of technology licensing between downstream firms with different levels of efficiency. This paper introduces technology licensing into the downstream market and investigates the effects of input price discrimination on technology transfer and welfare. We show that input price discrimination, which hurts the efficient firm as identified in the literature, may facilitate downstream licensing and improve the overall production efficiency. The property of the demand is important to the above result and we provide examples based on derived demands characterized by constant curvature. Further, we illustrate that enhanced licensing incentives could lead to higher consumer surplus and social welfare compared to uniform pricing. These results hold in the long run even if innovation is endogenous.

Key words: price discrimination, downstream licensing, demand curvature, welfare, innovation

JEL Classification: L13; L11; L24

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1 Introduction

Input price discrimination is widely observed in many markets with different final goods producers or retailers (see examples in Villas-Boas, 2009; Chen et al., 2022) and has raised increasing antitrust concerns in recent decades. The well-known debate centers on whether input price discrimination leads to welfare losses and thus should be regulated or not. Although the previous works generate important insights that help understand the welfare implications of input price discrimination, they generally assume the absence of technology licensing between downstream firms with different levels of efficiency, a phenomenon commonly observed in the reality and well-studied in IO literature.

The purpose of this paper is to investigate the welfare and policy implications of input price discrimination in the classic framework with an upstream monopolist and two asymmetric downstream producers. The novelty of this paper is that we consider technology licensing in our model. That is, we allow the technologically efficient firm to license it advanced technology to the inefficient firm. The consideration of technology licensing herein is relevant, because it is commonly observed that downstream firms with various production technologies may be motivated to engage in technology transfer as studied in extensive literature. The governments also strongly encourage the licensing of superior technologies to less efficient firms, aiming to reduce production costs. It is worth noting that the scheme of input pricing can significantly influence firms' incentives toward technology licensing and, therefore, affect market outcomes. This important aspect, however, has been neglected in the literature of input price discrimination.

In this paper, we adhere to the standard setting in the literature, deviating only in our consideration of downstream licensing. The less efficient firm has the opportunity to acquire advanced technology through a fixed licensing fee at the first stage of the game. In line with much of the literature, including Katz (1987) and DeGraba (1990), we assume linear input pricing by the upstream monopolist. At the second stage, the upstream monopolist determines the input prices. And then, at the last stage, downstream firms engage in Cournot competition. Our analyses are conducted within the framework of a general demand at the downstream level.

We demonstrate that allowing input price discrimination could facilitate downstream licensing, and therefore, benefit consumers and social welfare. The reasons behind our finding are that input price discrimination harms the efficient firm as identified in the literature, which could potentially induce it to license the advanced technology to the less efficient rival. The increase in production efficiency and competition under licensing may consequently lead to higher total output and social welfare. This result applies to a broad class of inverse demand functions, including those with a positive constant curvature (i.e., elasticity of the slope of inverse demand). To illustrate our main finding in a cleaner manner, we present the linear demand case as an example in Figure 1. The welfare-improving input price discrimination occurs in a wide range of parameter values. Even when innovation is endogenously determined in the long run, the licensing-enhancing effect of input price discrimination is also present. This, in turn, leads to higher consumer surplus and social welfare compared to uniform pricing under specific circumstances. These results, previously unexplored in the literature, are useful for evaluating the welfare implications of input price discrimination and formulating relevant policies.

After the literature review, the rest of the paper is organized as follows. In Section 3, we introduce the model in which an upstream monopolist sells an input to downstream duopolists which differ in their marginal costs of production. In Sections 4, we compare the two scenarios: input price discrimination and uniform pricing, and present our main findings. In section 5, we discuss the robustness of our results when innovation is endogenously determined in the long run. Concluding remarks are provided in Section 6. Proofs of all lemmas and propositions are relegated to the Appendix.

2 Literature Review

Input price discrimination is an important concern in competition policy. There is a vivid academic debate about this issue, which centers on whether input price discrimination leads to welfare losses and thus calls for antitrust regulations.

Researchers such as Katz (1987), DeGraba (1990), and Yoshida (2000) demonstrate that input price discrimination benefits inefficient firms by shifting production from efficient ones to inefficient ones, thereby generating production inefficiency and harming social welfare. By contrast, Inderst and Valletti (2009) and Chen (2017) show the opposite may occur if there exists input substitution or vertical product differentiation. Further, welfare-enhancing input price discrimination has been found in various contexts with the consideration of two-part tariffs contracts (Inderst and Shaffer, 2009), multiple markets (Arya and Mittendorf, 2010), endogenous entry (Herweg and Müller, 2012), bargaining (O'Brien, 2014; Pinopoulos, 2022), sequential contracting (Kim and Sim, 2015), non-linear demand forms (Valletti, 2003; Li, 2014, 2017; Mikl'os-Thal and Shaffer, 2021; Gaudin and Lestage, 2022) passive shareholding (Lestage, 2021; Li and Shuai, 2022) and downstream R&D (Lestage and Li, 2022), etc.

So far, this remains a longstanding question in economics that has only been partially answered. We contribute to the literature by examining the integration of downstream licensing into the conventional model of input price discrimination. This issue is particularly relevant, as recent empirical evidence suggests that licensing plays a crucial role across various industries when firms differ in production technologies. Moreover, different input pricing schemes will affect technology licensing between asymmetric downstream firms, thereby influencing market outcomes. We consider a general demand in the downstream market and provide conditions under which input price discrimination leads to greater social welfare than uniform pricing. The driving force behind this result is that input price discrimination facilitates downstream licensing, which holds true for a large class of demand forms. This result, along with the mechanism behind it, has not been found in the literature.

A closely related paper by Kao and Kwang (2017) also incorporate licensing to study the welfare implications of input price discrimination. Unlike our paper, the licensor is an outside innovator in their model and all downstream firms are homogeneous producers. Input price discrimination induces the outside innovator to grant more licenses to downstream firms, consequently improving social welfare. We, instead, consider the situation where one of the producers has an advanced technology and determines whether to licence it to the rival firm. Hence, the licensing incentives differ significantly between our model and theirs in that an outside licensor is only interested in the total licensing revenue, while an insider licensor is interested in the sum of licensing revenue plus profit and therefore faces a trade-off between gains from licensing and losses from higher competition.

3 The Model

Consider a vertical industry with an upstream supplier (firm 0) and two downstream producers (firm $i = 1, 2$). Each downstream producer (firm $i = 1, 2$) purchases inputs from the supplier at price w_i, and transforms one unit of inputs into one unit of homogeneous final goods. The production cost of inputs is normalized to zero, while that of final goods is denoted as c_i for each firm $i = 1, 2$. We assume that downstream producers have different constant marginal costs of production, where $c_1 < c_2$. Without loss of generality, we normalize $c_1 = 0$ and $c_2 = c$. The inverse demand function in the final goods market is $P(Q)$, where $Q = q_1 + q_2$

with q_i representing the individual output of firm i. We adopt the standard assumption that $P'(Q) < \min\{0, -QP''(Q)\}$, with $P(Q) > 0$.

We add to the existing literature by examining technology licensing within various input price regimes. Through licensing, the technologically less advanced firm (firm 2) can utilize the advanced technology obtained from the proficient firm (firm 1) to produce at zero marginal cost, incurring an implementation cost F. To get the advanced technology, the licensee has to pay a fixed-fee to the licensor.[1] If the joint profit of the two firms increases after licensing, a mutually agreeable price (i.e., the licensing fee) for transferring technology from firm 1 to firm 2 can be found, resulting in mutual benefits for both firms. In this paper, we are not focused on determining the optimal licensing fee, as it is not crucial to our findings, which may depend on the bargaining power of upstream and downstream firms. Under licensing, $c_1 = c_2 = 0$; otherwise, $c_1 = 0$ and $c_2 = c$. To ensure the relevance of our analysis, we assume a non-drastic cost difference, which ensures active participation in production by both firms across the diverse scenarios we are about to investigate.

Furthermore, we assume that the market participants are well-behaved under both price regimes. We introduce the following assumptions to ensure the concavity of firms' profits in various scenarios that we will discuss.[2]

Assumption 1: $3P' + 3QP'' + Q^2P'''/2 < 0.$

Assumption 2: $3P' + 3QP'' + Q^2P'''/2 + c^2 \left(P''' - 2\left(P''\right)^2/P'\right)/8\left(P'\right)^2 < 0.$

The timeline unfolds as follows: In the first stage, firm 1 decides whether to license its advanced technology to firm 2. In the second stage, the upstream supplier sets input prices. Finally, in the third stage, firm 1 and firm 2 engage in Cournot competition to determine quantities. The solution concept adopted in this paper is subgame perfect Nash equilibrium.

4 The Analysis and Results

We use backward induction to solve the market equilibrium. Our analysis starts at the third stage. In this stage, downstream firms compete with each other in a Cournot manner. Each

[1] We assume that the fee is paid in this form to avoid any distortion in the licensing incentive caused by contract frictions between downstream firms.

[2] The concavity assumption is standard in the literature (Li, 2017). In the following discussion, we will demonstrate that a large class of demand functions satisfies these assumptions.

firm i selects q_i to maximize profit[3]

$$\pi_i = P(Q)q_i - (c_i + w_i)q_i \quad \text{for } i = 1, 2.$$

Assumption 1 guarantees the concavity of each profit function in q_i. The optimal level of quantities are given by the first-order conditions, which are

$$c_i + w_i = P'(Q)q_i + P(Q) \quad \text{for } \ i = 1, 2. \tag{1}$$

The LHS of (1) is the total marginal cost of production which includes the cost for purchasing inputs and the cost in production of final goods. The RHS of (1) is the marginal benefit. By (1), we have

$$q_i = \frac{c_i + w_i - P(Q)}{P'(Q)} \quad \text{for } i = 1, 2.$$

Combine with the fact that $Q = q_1 + q_2$, we can obtain

$$q_1 = \frac{Q}{2} + \frac{c_1 + w_1 - c_2 - w_2}{P'(Q)} \quad \text{and} \quad q_2 = \frac{Q}{2} - \frac{c_1 + w_1 - c_2 - w_2}{P'(Q)}.$$

Under licensing, $c_1 = c_2 = 0$. Hence, $w_1 = w_2$ and it follows that $q_1 = q_2$. Otherwise, $c_1 = 0$ and $c_2 = c$. The input prices and therefore the equilibrium quantities are influenced by the prevailing price regime.

In the following, we explore two types of input price regimes: uniform pricing and price discrimination. The chosen input price regime is widely known among market participants and cannot be altered.

4.1 Uniform Pricing

When the upstream supplier charges uniform prices to all downstream firms, we denote the input price as w. We substitute w_1 and w_2 in the previous analysis with w, leading to

$$q_1 = \frac{Q}{2} + \frac{c_1 - c_2}{2P'(Q)} \quad \text{and} \quad q_2 = \frac{Q}{2} - \frac{c_1 - c_2}{2P'(Q)}.$$

[3]Throughout the paper, we use the term "profit" to denote the profit that the firm can obtain from production, excluding implementation costs as well as licensing fees.

In equilibrium, $q_1 \geq q_2$. The profits of downstream firms are

$$\pi_1^u = \left(\frac{c_2 - c_1}{2} - \frac{Q}{2}P'(Q)\right) q_1 \quad \text{and} \quad \pi_2^u = \left(\frac{c_1 - c_2}{2} - \frac{Q}{2}P'(Q)\right) q_2,$$

respectively. Here, the superscript "u" denotes "*uniform pricing*". It follows that the downstream industry profit is

$$\pi^u = \pi_1^u + \pi_2^u = -\frac{Q^2}{2}P'(Q) - \frac{(c_1 - c_2)^2}{2P'(Q)}. \tag{2}$$

In the second stage, the upstream supplier sets the input price w to maximize $\pi_0^u = wQ$. This is equivalent to choosing Q to maximize

$$\pi_0^u = wQ = \left(\frac{Q}{2}P'(Q) + P(Q) - \frac{c_1 + c_2}{2}\right) Q.$$

The first-order condition is

$$P(Q) + 2QP'(Q) + \frac{Q^2}{2}P''(Q) - \frac{c_1 + c_2}{2} = 0. \tag{3}$$

Assumption 2 ensures the concavity of the profit function π_0^u in Q. Define

$$H(X) \equiv P(X) + 2XP'(X) + \frac{X^2}{2}P''(X). \tag{4}$$

Assumption 2 indicates that $H(\cdot)$ is a decreasing function. Denote the inverse function of $H(\cdot)$ as $G(\cdot)$. The first-order condition in (3) can be rearranged as

$$Q = G(\frac{c_1 + c_2}{2}), \tag{5}$$

where $G(\cdot)$ is also a decreasing function.

In the first stage, firm 1 determines whether to grant a license to firm 2. In the absence of licensing, firm 2 produces with $c_2 = c$ while $c_1 = 0$. We denote the equilibrium total output as Q_{NL}^u, where the subscript "NL" indicates "*No Licensing*". Following (5), $Q_{NL}^u = G\left(c/2\right)$. According to (2), the equilibrium downstream industry profit is

$$\pi_{NL}^u = -\frac{\left(Q_{NL}^u\right)^2 P'(Q_{NL}^u)}{2} - \frac{c^2}{2P'(Q_{NL}^u)}. \tag{6}$$

If licensing occurs, the cost in the final goods production becomes zero: $c_2 = c_1 = 0$. We denote the equilibrium total output as Q_L^u. Then $Q_L^u = G(0)$. The equilibrium downstream industry profit is

$$\pi_L^u = -\frac{(Q_L^u)^2 P'(Q_L^u)}{2}. \tag{7}$$

Since $G'(\cdot) < 0$, it follows that $G(0) > G(c/2)$ and $Q_L^u > Q_{NL}^u$. That is, licensing leads to a higher total output.

In stage 1, licensing occurs if and only if the total profit for downstream firms, considering licensing and subtracting the implementation cost, exceeds the total profit under the scenario of no licensing, i.e., $\pi_L^u - F > \pi_{NL}^u$. Straightforward calculations lead to the following result on licensing.

Lemma 1. *Under uniform pricing, licensing occurs when $F < F^u$, where*

$$F^u = \frac{(Q_{NL}^u)^2 P'(Q_{NL}^u) - (Q_L^u)^2 P'(Q_L^u)}{2} + \frac{c^2}{2P'(Q_{NL}^u)}.$$

It is easy to check that $F^u = 0$, when $c = 0$ and $\frac{\partial F^u}{\partial c}|_{c=0} = \frac{Q_{NL}^u G'(0)}{2} \frac{2P'(Q_{NL}^u) + Q_{NL}^u P''(Q_{NL}^u)}{2} > 0$. This indicates that when c is sufficiently small, we can always find some $F^u > 0$. In this case, as long as the implementation cost of new technology for firm 2 is sufficiently small, it is mutually beneficial for firms to sign a licensing agreement.

When licensing leads to a reduction in input prices, the rationale is straightforward: it lowers the total production cost and improves output.[4]

In other cases, licensing introduces two opposing effects. On one hand, licensing reduces firm 2's production cost in final goods. One the other hand, upstream supplier raises input price when anticipating a higher demand of inputs caused by lower final goods production. Then the industrial average production cost depends on the combination of the two opposite effect.

Considering the cases with small c, the production distortions in the Cournot competition market are not severe, resulting in the technologically advanced firm producing slightly more

[4]Specifically, licensing raises the input price when

$$\frac{G(\frac{c}{2})}{2} P'\left(G(\frac{c}{2})\right) + P\left(G(\frac{c}{2})\right) - \frac{G(0)}{2} P'(G(0)) - P(G(0)) < \frac{c}{2},$$

and reduces the input price otherwise. For a large set of inverse demand functions satisfying a constant elasticity slope, expressed as $\frac{QP''(Q)}{P'(Q)} = E$ with a constant $E > -1$, licensing consistently leads to an increase in the input price.

than its disadvantaged counterpart. Consequently, licensing has a considerable influence on a large scale of production, contributing to a significant improvement in downstream final goods production. On the other hand, licensing leads to a marginal increase in input prices when c is small, resulting in a minor input price elevation. In this case, the overall production cost including inputs and final goods production is reduced and thus impacting positively on the overall downstream profit.

4.2 Price Discrimination

When price discrimination is allowed, the upstream supplier is able to set different input prices w_i for each downstream firm $i = 1, 2$. In the second stage, upstream supplier chooses w_1 and w_2 to maximize its profit

$$\pi_0^d = w_1 q_1 + w_2 q_2.$$

Here, the superscript "d" denotes "*price discrimination*". By substituting the first-order conditions of downstream firms into the upstream supplier's objective function, the pricing problem in stage 2 is once again equivalent to the problem of choosing quantities q_1 and q_2, i.e.,

$$\max_{q_1, q_2} \pi_0^d = q_1[P'(Q)q_1 + P(Q) - c_1] + q_2[P'(Q)q_2 + P(Q) - c_2].$$

When the profit function π_0^d is concave, the optimal choices of q_i are implied by the first-order conditions

$$[(q_1)^2 + (q_2)^2]P''(Q) + (2q_i + Q)P'(Q) + P(Q) - c_i = 0, \text{ for } i = 1, 2, \tag{8}$$

which lead to

$$q_1 = \frac{Q}{2} + \frac{c_1 - c_2}{4P'(Q)} \quad \text{and} \quad q_2 = \frac{Q}{2} - \frac{c_1 - c_2}{4P'(Q)}.$$

The overall marginal production cost of downstream firm 1 is

$$w_1 + c_1 = \frac{Q}{2}P'(Q) + P(Q) + \frac{c_1 - c_2}{4},$$

while that of firm 2 is

$$w_2 + c_2 = \frac{Q}{2}P'(Q) + P(Q) - \frac{c_1 - c_2}{4}.$$

The gap between the total marginal production costs $((w_1 + c_1) - (w_2 + c_2))$ narrows down to $(c_2 - c_1)/2$ compared with $c_2 - c_1$ in the uniform pricing case. This suggests that the competitive advantage of firm 1 in the Cournot competition is smaller under price discrimination compared with that under uniform pricing.

The first-order condition in (8) can be rewritten as

$$H(Q) + P''(Q)\frac{(c_1 - c_2)^2}{8[P'(Q)]^2} = \frac{c_1 + c_2}{2}. \tag{9}$$

In the first stage, firm 1 decides whether to license its technology to firm 2. Without licensing, firm 2 produces with $c_2 = c$. Following (9), the equilibrium output Q^d_{NL} solves

$$H(Q^d_{NL}) + \frac{c^2 P''(Q^d_{NL})}{8[P'(Q^d_{NL})]^2} = \frac{c}{2}. \tag{10}$$

The equilibrium downstream industry profit is:

$$\pi^d_{NL} = -\frac{\left(Q^d_{NL}\right)^2 P'(Q^d_{NL})}{2} - \frac{c^2}{8P'(Q^d_{NL})}. \tag{11}$$

Under licensing, firm 2 reduces production cost to zero. The equilibrium output is $Q^d_L = Q^u_L = G(0)$. The downstream industry profit is $\pi^d_L = \pi^u_L$, which is obtained in (7).

Mirroring the analysis in the previous case, licensing occurs when $\pi^d_L - F > \pi^d_{NL}$. The result is summarized as follows.

Lemma 2. *Under price discrimination, licensing occurs when $F < F^d$, where*

$$F^d = \frac{\left(Q^d_{NL}\right)^2 P'(Q^d_{NL}) - \left(Q^d_L\right)^2 P'(Q^d_L)}{2} + \frac{c^2}{8P'(Q^d_{NL})}.$$

As before, it is mutually beneficial for firms to sign a licensing agreement if the implementation cost of new technology is sufficiently small.

4.3 Uniform Licensing vs. Price Discrimination

Under licensing, both firms produce at equal marginal costs, rendering the two pricing schemes equivalent. Without licensing, price discrimination enables the monopolistic input supplier to charge different prices to downstream firms. This strategy changes the total output in accordance with the curvature of the final demand.

Proposition 1. *In equilibrium, (i)* $Q_L^d = Q_L^u$ *and (ii)* $Q_{NL}^u < (=,>) Q_{NL}^d$ *when* $P'' > (=,<) 0$.

This result was first stated by Li (2017), albeit relying on different modelling.[5] To grasp the underlying rationale, we begin by examining the total output Q_{NL}^u and analyze how adjustments in each quantity can enhance the upstream monopolist's profit. In the presence of input price discrimination, the upstream supplier sets a higher price for firm 1 and a lower price for firm 2, resulting in a higher output for firm 2 than firm 1. Assuming prices are adjusted such that firm 1 produces $q_1^d = q_1^u - \Delta$ and firm 2 produces $q_2^d = q_2^u + \Delta$, where q_1^u and q_2^u are the firm's output at the beginning.

When $P'' > 0$, the marginal benefit from increasing both firms output yields more profit to the upstream monopolist:

$$\begin{aligned} \frac{\partial \pi_0^d}{\partial q_1^d} + \frac{\partial \pi_0^d}{\partial q_2^d} &= 2[(q_1^d)^2 + (q_2^d)^2] P''(Q) + 4QP'(Q) + P(Q) - c \\ &> 2[(q_1^u)^2 + (q_2^u)^2] P''(Q) + 4QP'(Q) + P(Q) - c > 0. \end{aligned}$$

In contrast, when $P'' < 0$, the upstream supplier has an incentive to reduce the total output.

We next move to discuss the incentive for licensing. The following proposition presents the condition under which input price discrimination facilitates downstream licensing.

Proposition 2. *(i) When* $P'' = 0$, $F^d > F^u$*; (ii) When* $P'' \neq 0$, $F^d > F^u$ *if and only if*

$$(Q_{NL}^d)^2 P'(Q_{NL}^d) - (Q_{NL}^u)^2 P'(Q_{NL}^u) > \frac{c^2}{P'(Q_{NL}^u)} - \frac{c^2}{4P'(Q_{NL}^d)} \tag{12}$$

Intuitively, input price discrimination may hurt the efficient firm by charging a higher input price, which may motivate the efficient firm to license its technology to the rival to collect some benefit. Proposition 2 indicates that $F^u < F^d$ occurs either when $P'' = 0$ or when (12) holds. In such cases, licensing is more likely to occur under price discrimination. More specifically, when $F^u < F < F^d$, licensing exclusively occurs under input price discrimination, consequently enhancing the overall production efficiency.

Specific Demand Functions We have demonstrated in the proposition 2 that $F^d > F^u$

[5] The intuition behind is also well presented in Li (2017).

always holds when the inverse demand function is linear ($P'' = 0$). In the following, we show that $F^d > F^u$ consistently holds for some other demand functions.

For examples, let us consider a class of inverse demand functions whose elasticity of slope is constant and satisfies

$$\frac{QP''(Q)}{P'(Q)} \equiv E > 0 \quad \text{for all} \quad Q > 0.^6$$

It is easy to check that assumptions 1-2 hold for these demand functions. In this case, the total output is higher under uniform pricing than under price discrimination ($Q^u_{NL} > Q^d_{NL}$). To show that $F^d > F^u$, define

$$\tilde{\pi}^u(Q) = -[(q^u_1)^2 + (q^u_1)^2]P'(Q) = -\frac{Q^2P'(Q)}{2} - \frac{c^2}{2P'(Q)},$$

where $q^u_1 = \frac{Q}{2} - \frac{c}{2P'(Q)}$ and $q^u_2 = \frac{Q}{2} + \frac{c}{2P'(Q)}$. And,

$$\tilde{\pi}^d(Q) = -[(q^d_1)^2 + (q^d_1)^2]P'(Q) = -\frac{Q^2P'(Q)}{2} - \frac{c^2}{8P'(Q)},$$

where $q^d_1 = \frac{Q}{2} - \frac{c}{4P'(Q)}$ and $q^d_2 = \frac{Q}{2} + \frac{c}{4P'(Q)}$. It is easy to verify that $\tilde{\pi}^u(Q) > \tilde{\pi}^d(Q)$, then it follows that $\tilde{\pi}^u(Q^u_{NL}) > \tilde{\pi}^d(Q^u_{NL})$. Since $\tilde{\pi}^d(Q)$ is an increasing function, we have $\tilde{\pi}^d(Q^u_{NL}) > \tilde{\pi}^d(Q^d_{NL})$, which indicates $F^d > F^u$.

Consumer surplus is denoted as "CS" and social welfare is denoted as "SW", which can be calculated as

$$CS = \int_0^Q P(q) - P(Q)dq,$$

$$SW = \int_0^Q P(q)dq - F - c_1q_1 - c_2q_2 - \gamma F,$$

where γ denotes the licensing decision of firm 1: $\gamma = 1$ if firm 1 grants the license to firm 2 and $\gamma = 0$ if not.

Price discrimination could potentially improve consumer surplus as well as social welfare since it motivates the firm to license.[7]

[6] Linear demand is indeed a special case that falls into this category of demand functions, with $E = 0$.

[7] There are other cases in which social welfare and consumer surplus are higher under uniform pricing, as shown in the appendix. However, these cases are not our focus.

Proposition 3. *Licensing occurs exclusively under input price discrimination when* $F^u < F \leq F^d$. *Among these cases, consumer surplus and social welfare are improved when* $F^u < F \leq \min\{F^d, \int_{Q_{NL}^u}^{Q_L^u} P(Q)dQ + \frac{c}{2}Q_{NL}^u + \frac{c^2}{2P'(Q_{NL}^u)}\}$.

To illustrate the intuition behind Proposition 3, we take a linear demand function as an example. Figure 1 plots F^u and F^d under the linear inverse demand function $P(Q) = a - bQ$. The horizontal axis represents the marginal cost of the inefficient firm (c), and the vertical axis represents the implementation cost of the advanced technology (F). The two curves (F^u and F^d) divide the graph into three regions.

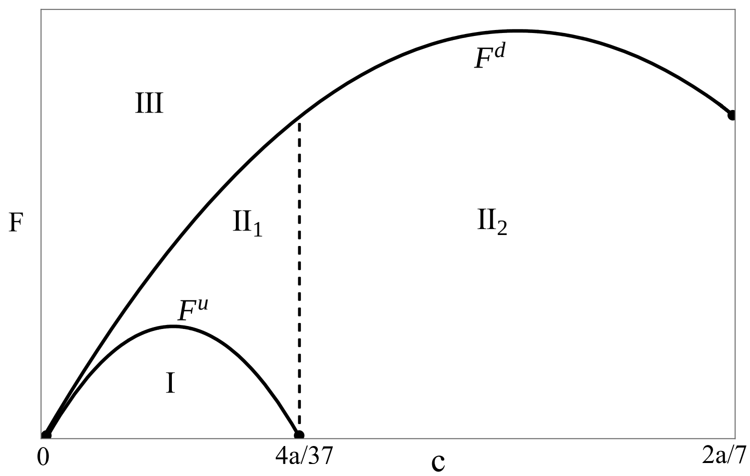

Figure 1: Welfare-implications of price discrimination

In **Regions I**, $F < F^u$. Firm 1 grants a license to firm 2 under both price regimes. Both firms produce at zero marginal cost, resulting an equivalent result under both price regimes. The total output, consumer surplus as well as social welfare are the same under these two regimes:

$$Q_L^u = Q_L^d = \frac{a}{3b}, CS_L^u = CS_L^d = \frac{a^2}{18b}, SW_L^u = SW_L^d = \frac{5a^2}{18b} - F.$$

Region III, situated above F^d curve, illustrates scenarios in which firm 1 does not license to firm 2 under both pricing regimes. The total output is unaffected by input price regimes when there is no licensing, as indicated by $Q_{NL}^u = Q_{NL}^d = \frac{2a-c}{6b}$. Consequently, consumer surplus remains constant ($CS_{NL}^u = CS_{NL}^d = \frac{(2a-c)^2}{72b}$). Downstream profit and social welfare, however, suffer under input price discrimination.[8]

[8]Similar results can also be found in studies by Katz (1987), DeGraba (1990), Yoshida (2000), Valletti (2003), and others.

This is because allowing input price discrimination elevates the efficient firm's input price, thereby reducing its production in the final goods market. The comparative advantage of the efficient firm, derived from utilizing a less costly production technology, is diminished. As a result, compared to the uniform pricing case, more products are produced by the less efficient firm, negatively impacting social welfare. More specifically, we can calculate social welfare as follows:

$$SW_{NL}^{u} = \frac{20a^2 - 20ac + 41c^2}{72b} > SW_{NL}^{d} = \frac{20a^2 - 20ac + 23c^2}{72b}$$

Regions II, situated between the two curves, contains the interesting observations shown in Proposition 3. In this area, firm 1 grants a license to firm 2 under input price discrimination but not under uniform pricing.

In the absence of licensing, the less efficient firm is offered a lower input price under price discrimination, compared to that under uniform pricing. This suggests that the inefficient firm produces more in the price discrimination case than in the uniform pricing case, which potentially hurts consumer surplus and social welfare. However, in **Regions II**, firm 1 is motivated to grant the license to firm 2 under price discrimination. Licensing reduces differences in downstream firms' production costs and mitigates the inefficiency in production caused by input price discrimination.[9] In linear demand case, the inefficiency is exactly canceled out by licensing. As a result, allowing input price discrimination enhances production efficiency through motivating licensing, contributing positively to both social welfare and consumer surplus. We can compare the consumer surplus and social welfare under the two price regimes as below.

$$Q_{NL}^{u} = \frac{2a - c}{6b} < Q_{L}^{d} = \frac{a}{3b}$$

$$CS_{NL}^{u} = \frac{(2a - c)^2}{72b} < CS_{L}^{d} = \frac{a^2}{18b}$$

$$SW_{NL}^{u} = \frac{20a^2 - 20ac + 41c^2}{72b} < SW_{L}^{d} - F = \frac{5a^2}{18b} - F$$

[9]Here, the "inefficiency" means that the efficient firm is charged higher input price than inefficient firm and consequently produce less under input price discrimination than under uniform pricing.

5 The Analysis with Innovation in the Long Run

Innovation constitutes a pivotal and strategic decision for a firm, influencing its competitive standing in the marketplace. In this section, we investigate the firm's incentives to innovate over the long run.

To maintain model tractability, we focus on the linear demand function in this section: $P(Q) = a - bQ$ with $a > 0$ and $b > 0$. In order to examine the incentive of the firm 1 to invest in innovation activities, we extend our framework by introducing a preliminary stage, denoted as stage 0, preceding stage 1. At this stage, firm 1 faces a critical decision: whether to invest a fixed amount K to enhance productivity and acquire advanced technology, as elucidated in earlier sections. For analytical convenience, we adopt a deterministic innovation process, positing that firm 1 secures advanced technology by incurring a cost of K during stage 0.

If firm 1 decides against investing in innovation during stage 0, it will subsequently operate using the same technology as firm 2 in later production stages. In this case, in the Cournot competition with firm 2, firm 1 expects to achieve

$$\pi_1 = \frac{(a-c)^2}{36b}.$$

The firm's innovation decision-making incorporates its expectations regarding future market policies and its understanding of the competitor's technology. In our model, firm 1 perfectly knows the price regime that the upstream supplier is allowed to adopt and the technology that firm 2 will use. The price regime cannot be altered.

If firm 1 chooses to make the investment in stage 0, the game proceeds to stages 1-3, as described in previous sections. We assume that, upon obtaining advanced technology, firm 1, with full bargaining power, decides whether to grant the license to firm 2. This assumption facilitates the identification of the largest set of parameters supporting firm 1's innovation incentive.

When the upstream supplier charges uniform price, firm 1 can either provide a "take-it-or-leave" offer to firm 2 or exclusively utilize the advanced technology. Firm 1 achieves a profit at

$$\pi_1^u = \frac{a^2}{18b} - F - K - \frac{(2a-7c)^2}{144b},$$

if it grants the licence to firm 2, and achieves

$$\pi_1^u = \frac{(2a+5c)^2}{144b} - K$$

if not. Compare the profit that firm 1 can earn in the above two scenarios, we can draw a conclusion that innovation occurs when

$$K < \max\{\frac{28ac+21c^2}{144b}, \frac{36ac-53c^2}{144b} - F\} \tag{13}$$

under uniform pricing.

Similarly, when the upstream supplier adopt input price discrimination, the firm 1 can either provides a "take-it-or-leave" offer to firm 2 or exlucively utilize the advanced techonology. Firm 1 acheives a profit at

$$\pi_1^d = \frac{a^2}{18b} - F - K - \frac{(a-2c)^2}{36b}$$

if it grants the licence to firm 2, and achieves

$$\pi_1^d = \frac{(a+c)^2}{36b} - K$$

if not. Under input price discrimination, innovation occurs when

$$K < \max\{\frac{ac}{9b}, \frac{6ac-5c^2}{36b} - F\} \tag{14}$$

When the innovation cost is sufficiently small or the technology improvement is significant, firm 1 is motivated to innovate. Our model further reveals that the possibility to sell licenses provides an additional incentive for the firm to innovate. Proposition 5 in Appendix B fully characterizes firm 1's innovation decision. We present the most interesting part of it in the following proposition.

Proposition 4. *When $F^u < F \leq F^d$, and $K \leq \frac{6ac-5c^2}{36b} - F$, firm 1 innovates regardless of the price regimes. Firm 1 licenses to firm 2 under input price discrimination, while exclusively using the advanced technology under uniform pricing.*

Proposition 4 indicates that, contrary to the expectation that input price discrimination might hinder innovation and hurt social welfare in the long run, there exist some cases where it can actually improve social welfare. We figure out the conditions for which input price discrimination does not distort the innovation incentives and simultaneously maintain the motivations for the efficient firm to share technology in the short run competition.

The results highly depends on the cost of implanting a new technology and the cost of innovation. Our result suggest that, the regulator should take more care of input price discrimination. In most of times, input price discrimination will hurt social welfare as it hinder innovation. However, when we take technology sharing into account, allowing input price discrimination can be beneficial to social welfare and consumer surplus in both short run and long run.

6 Conclusion

The paper conducts a comparison between input discrimination and uniform pricing in the context of innovation and licensing. In contrast to existing literature, input price discrimination is found to improve social welfare and consumer surplus by motivating licensing and enhancing production efficiency.

In the long run, input price discrimination may hinder innovation, as it grants the upstream supplier more pricing power, enabling the extraction of more surplus. Licensing, to some extent, mitigates this pricing power, especially in our model setting where price discrimination plays no role when licensing occurs.

It's worth noting that our analysis is limited to two downstream firms, which constrains the impact of input price discrimination when licensing occurs. Future research could extend this analysis to consider scenarios with more downstream firms and explore the effects of limited licensing quotas.

Appendix A

Proof of Proposition 1. $Q^d_{NL} = Q^u_{NL}$ follows directly from $H(Q^u_{NL}) = H(Q^d_{NL}) = 0$ and $H' < 0$.

Combining (5) and (10), we obtain

$$H\left(Q^d_{NL}\right) + P''(Q^d_{NL})\frac{c^2}{8\left[P'(Q^d_{NL})\right]^2} = \frac{c}{2} = H\left(Q^u_{NL}\right).$$

When $P'' > 0$, we have $H\left(Q_{NL}^d\right) < H\left(Q_{NL}^u\right)$. Since $H'(Q) < 0$, it follows that $Q_{NL}^d > Q_{NL}^u$. Similarly, when $P'' < 0$, $Q_{NL}^u > Q_{NL}^d$ and when $P'' = 0$, $Q_{NL}^u = Q_{NL}^d$. □

Proof of Proposition 2. By Proposition 1, $Q_{NL}^u = Q_{NL}^d$ and $Q_L^u = Q_L^d$ when $P'' = 0$. Since $P' < 0$, it follows straightforwardly that $F^d > F^u$.

Now we consider the cases with $P'' \neq 0$. According to the first order condition under price discrimination and $Q_L^d = Q_L^u$, we have

$$\begin{aligned} F^d - F^u &= \frac{(Q_{NL}^d)^2 P'(Q_{NL}^d) - (Q_L^d)^2 P'(Q_L^d)}{2} + \frac{c^2}{8P'(Q_{NL}^d)} \\ &\quad - \frac{\left(Q_{NL}^u\right)^2 P'(Q_{NL}^u) - \left(Q_L^u\right)^2 P'(Q_L^u)}{2} - \frac{c^2}{2P'(Q_{NL}^u)} \\ &= \frac{(Q_{NL}^d)^2 P'(Q_{NL}^d) - \left(Q_{NL}^u\right)^2 P'(Q_{NL}^u)}{2} + \frac{c^2}{8P'(Q_{NL}^d)} - \frac{c^2}{2P'(Q_{NL}^u)}. \end{aligned}$$

$F^d > F^u$ if and only if

$$(Q_{NL}^d)^2 P'(Q_{NL}^d) - \left(Q_{NL}^u\right)^2 P'(Q_{NL}^u) > \frac{c^2}{P'(Q_{NL}^u)} - \frac{c^2}{4P'(Q_{NL}^d)}.$$

□

Proof of Proposition 3. Firm 1 grants licenses to firm 2 exlusively under price discrimination when $F^U \leq F < F^D$. In this case,

$$SW_L^d - SW_{NL}^u = \int_0^{Q_L^d} P(Q)dQ - F - \int_0^{Q_{NL}^u} P(Q)dQ + \frac{c}{2}Q_{NL}^u + \frac{c^2}{2P'(Q_{NL}^u)}.$$

Firm 2 is actively involved when the condition $-\frac{c^2}{2P'(Q_{NL}^u)} < \frac{c}{2}Q_{NL}^u$ is satisfied. Consequently, $\int_{Q_{NL}^u}^{Q_L^u} P(Q)dQ + \frac{c}{2}Q_{NL}^u + \frac{c^2}{2P'(Q_{NL}^u)}$ is positive. When $F < \int_{Q_{NL}^u}^{Q_L^u} P(Q)dQ + \frac{c}{2}Q_{NL}^u + \frac{c^2}{2P'(Q_{NL}^u)}$, $W_L^d > W_{NL}^u$, indicating that social welfare is enhanced by allowing input price discrimination.

$$CS_L^d - CS_{NL}^d = -\int_0^{Q_L^d} qP'(q)dq + \int_0^{Q_{NL}^u} qP'(q)dq = \int_{Q_L^d}^{Q_{NL}^u} qP'(q)dq$$

According to Proposition 1, $Q_{NL}^u < Q_L^u = Q_L^d$, indicating that $CS_L^d > CS_{NL}^u$. Consumer surplus is also enhanced by permitting input price discrimination.

□

Appendix B

Proposition 5. *Firm 1's decision regarding innovation is outlined as follows:*

1. *When* $F \leq F^u$,

   - *if* $K \leq \frac{6ac-5c^2}{36b} - F$, *firm 1 will innovate under both uniform pricing and price discrimination;*
   - *if* $\frac{6ac-5c^2}{36b} - F < K \leq \frac{36ac-53c^2}{144b} - F$, *firm 1 will innovate under uniform pricing;*
   - *if* $K > \frac{36ac-53c^2}{144b} - F$, *firm 1 will not innovate.*

2. *When* $F^u < F \leq F^d$,

   - *if* $K \leq \frac{6ac-5c^2}{36b} - F$, *firm 1 will innovate under both uniform pricing and price discrimination;*
   - *if* $\frac{6ac-5c^2}{36b} - F < K \leq \frac{28ac+21c^2}{144b}$, *firm 1 will innovate under uniform pricing;*
   - *if* $K > \frac{28ac+21c^2}{144b}$, *firm 1 will not innovate.*

3. *When* $F > F^d$,

   - *if* $K \leq \frac{ac}{9b}$, *firm 1 will innovate under both uniform pricing and price discrimination;*
   - *if* $\frac{ac}{9b} < K \leq \frac{28ac+21c^2}{144b}$, *firm 1 will innovate under uniform pricing;*
   - *if* $K > \frac{28ac+21c^2}{144b}$, *firm 1 will not innovate.*

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