Ownership and Information Exchange in a Duopoly*

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Abstract

I analyze effects of common ownership or cross-ownership on the incentives of Cournot competitors to exchange perfectly correlated information. Consistent with empirical findings, firms have stronger incentives to exchange information for greater degrees of cross-ownership. There is a trade-off for consumers. On the one hand, more cross-ownership relaxes competition, which hurts consumers. On the other hand, more cross-ownership may yield more information exchange, which benefits consumers. Consequently, expected consumer surplus may be non-monotonic in the degree of cross-ownership. Possibly, positive degrees of cross-ownership may maximize the expected consumer (and total) surplus.

Keywords: common ownership, cross-ownership, duopoly, information exchange, common value
JEL Codes: D21, D43, D83, L13, L33, M41, M48

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1 Introduction

Common ownership is on the rise. In US data of S&P 500 index companies, Backus *et al.* (2021) find a steady increase of implied profit weights among firms between 1980 and 2017 regardless of weighing. Gibbon and Schain (2023) analyze a panel of European manufacturing firms during the period 2005-2016. They find that the share of markets with common ownership increases, and that common owners hold a greater share on average in 2016 than in 2005.

Firms do not always know the demand or cost factors in their market. For example, the effects of a pandemic, energy crisis, or less dramatic events, on a good's demand or the cost production may not be fully predictable. In those cases, information about the demand or production cost can be valuable. In this paper, I analyze the incentives of firms to exchange such information in markets with common ownership.

Empirical analyses find a positive relation between common ownership or cross-ownership and information disclosure: Boone and White (2015), Park, Sani, Shroff and White (2019), Pawliczek and Skinner (2018), and Pawliczek, Skinner and Zechman (2022). I intend to contribute to this empirical literature by providing a theoretical analysis of the relation between common ownership and information exchange.

There exists an extensive literature on information exchange in oligopoly without common ownership (e.g., see Raith, 1996, and Vives, 2000, for excellent reviews). In particular, Dye (1985), Jung and Kwon (1988), Malueg and Tsutsui (1998), and Jansen (2008) analyze information exchange where it is uncertain whether a firm receives information. I contribute to this literature by studying the effects of common ownership on the incentives to exchange information and their economic consequences.

There is a growing literature about the effects of common and cross-ownership on market power (e.g., see Schmalz, 2018, for a review). Early literature (e.g., Farrell and Shapiro, 1990) focuses on the direct effects of cross-ownership on conduct and outcomes in oligopolistic markets. More recent studies extend the analysis by including the indirect effects of common and cross-ownership. For example, López and Vives (2019) show that common ownership may increase incentives of firms to invest in process innovations, because firms partially internalize knowledge spillovers. In turn, this may raise the rate of innovation which may be beneficial for consumers.[1] In line with the recent literature, I consider indirect effects of common or cross-ownership through incentives of firms to exchange information about demand (or a common

[1]Likwise, Li *et al* (2023) show that common ownership may stimulate product innovation by a high-quality firm, which may make consumers better off.

cost factor). I show that common ownership strengthens the incentives for information exchange, which may benefit consumers on average.

The paper is organized as follows. Section 2 describes the model. Section 3 derives the equilibrium output strategies of the firms. Section 4 characterizes the incentives of firms to share information. In Section 5, I characterize the effects of common ownership and information sharing on consumer surplus and welfare. Finally, Section 6 concludes the paper. The Appendix contains proofs of the paper's propositions.

2 The Model

Two firms, i.e., firm 1 and firm 2 compete in a market with an unknown common demand or cost parameter $\theta \in [\underline{\theta}, \overline{\theta}]$, which is drawn from the distribution $F : [\underline{\theta}, \overline{\theta}] \to [0, 1]$. Each firm receives an imperfect private signal about the parameter, Θ_i for firm i with $i = 1, 2$. With probability $0 < p < 1$, firm i receives a perfectly informative signal, i.e., $\Theta_i = \theta$, whereas the firm receives the uninformative signal $\Theta_i = \varnothing$ with probability $1 - p$ (e.g., see Dye, 1985, Jung and Kwon, 1988, and Malueg and Tsutsui, 1998). The firms' signal draws are independent (conditional on the draw of θ).

Before learning their signals, the firms commit to either exchange their signals or keep them secret from each other. For example, the firms choose whether to establish a trade association which facilitate information exchange between its members. Then, each firm privately learns its cost signal. The signal stays private information if the firms committed to secrecy, whereas both firms learn both signals if firms committed to exchange information.

After information has been received and possibly exchanged, the firms simultaneously choose the output levels of perfectly substitutable goods, i.e., $q_i \geq 0$ for $i = 1, 2$.[2] For $i, j = 1, 2$ with $i \neq j$ and parameter θ, if firm i chooses $q_i \geq 0$ units and firm j chooses q_j units, then firm i earns the following profit:

$$\pi_i(q_i, q_j; \theta) = (\alpha - \theta - q_i - q_j)\, q_i. \tag{1}$$

The parameter θ has at least two interpretations. First, it can be seen as a common cost parameter for firms in a market with the inverse demand $P(q_i, q_j) = \alpha - q_i - q_j$. Alternatively, the parameter can be interpreted as a common demand parameter for the inverse demand function $P(q_i, q_j) = (\alpha - \theta) - q_i - q_j$.[3]

[2] I assume perfect substitutability for simplicity. Similar results emerge for close substitutes.

[3] Although the marginal cost would seem to be equal to zero here, the same results hold if firms would have contant identical marginal cost $c \geq 0$ and constant demand intercept $\alpha + c$ instead of α.

For simplicity, I assume that firms have symmetric stakes in each other, which yields the following payoff for the manager of firm i with parameter θ (for $i, j = 1, 2$ with $i \neq j$):

$$V_i(q_i, q_j; \theta) \equiv \pi_i(q_i, q_j; \theta) + \lambda \pi_j(q_j, q_i; \theta). \tag{2}$$

The parameter $0 \leq \lambda < 1$ reflects the degree of cross-ownership or common ownership (e.g., see López and Vives, 2019). The bigger is λ, the bigger is the stake each firm has in the other.[4] Firms are risk-neutral, which means that firm i maximizes the expected value of its payoff $V_i(q_i, q_j; \theta)$ for $i = 1, 2$.

I solve the model backwards and look for the perfect Bayesian equilibrium (PBE).

3 Equilibrium output strategies

In this section, I derive the firms' equilibrium output strategies with information exchange and with secrecy.

First, suppose that firms committed to exchange their information. With information exchange, there is symmetric information. There are two cases. If at least one of the firms received the informative signal (i.e., $\Theta_i = \theta$ for some $i = 1, 2$), then both firms know the parameter θ and choose their output under complete information. I denote these events by $\Theta = \theta$. Else, if $\Theta_1 = \Theta_2 = \varnothing$, then the firms do not know the parameter θ and they hold prior beliefs. I denote this event by $\Theta = \varnothing$. Given the compounded signal Θ, the firms' expectations are as follows:

$$E\{\theta|\Theta\} = \begin{cases} \theta, & \text{if } \Theta = \theta \\ E\{\theta\}, & \text{if } \Theta = \varnothing \end{cases}$$

For $i, j = 1, 2$ with $i \neq j$ and $\Theta \in \{\theta, \varnothing\}$, firm i chooses $q_i(\Theta)$ to maximize the payoff $E\{V_i(q_i, q_j; \theta)|\Theta\}$. This yields the following best reply for firm i:

$$q_i(\Theta) = \frac{1}{2}\left(\alpha - E\{\theta|\Theta\} - (1+\lambda)q_j(\Theta)\right). \tag{3}$$

In equilibrium, firm i chooses the following output level (for $i = 1, 2$ and $\Theta \in \{\theta, \varnothing\}$):

$$q^I(\Theta) = \frac{\alpha - E\{\theta|\Theta\}}{3 + \lambda}. \tag{4}$$

Given the compounded signal $\Theta \in \{\theta, \varnothing\}$ and symmetry, firm i expects to earns the following profit $\pi^I(\Theta) = (1 + \lambda)q^I(\Theta)^2$ in equilibrium. With information exchange, a

[4]For example, if $\lambda = 0$, then each firm maximizes its own profit. If $\lambda \to 1$, then the firms tend towards maximizing their joint profits.

firm expects to earn the following equilibrium profit:[5]

$$\begin{aligned}\Pi^I &\equiv [1-(1-p)^2]E\{\pi_i(q^I(\theta),q^I(\theta);\theta)\}+(1-p)^2E\{\pi_i(q^I(\varnothing),q^I(\varnothing);\theta)\}\\ &= (1+\lambda)\left(q^I(\varnothing)^2+p(2-p)\frac{\text{Var}\{\theta\}}{(3+\lambda)^2}\right).\end{aligned} \tag{5}$$

Second, suppose that firms committed to secrecy. Without information sharing, each firm does not know which signal the other firm received when it choose its output level. Firm i anticipates that firm j with signal Θ_j chooses the output level $q_j(\Theta_j)$. If firm i received the signal $\Theta_i \in \{\theta,\varnothing\}$, it chooses $q_i(\Theta_i)$ to maximize $E\{pV_i(q_i,q_j(\theta);\theta)+(1-p)V_i(q_i,q_j(\varnothing);\theta)|\Theta_i\}$. The maximization problem yields the following best-reply equations for $\Theta_i \in \{\theta,\varnothing\}$ and $i,j=1,2$ with $i\neq j$:

$$q_i(\Theta_i)=\frac{1}{2}\left(\alpha-E\{\theta|\Theta_i\}-(1+\lambda)[pE\{q_j(\theta)|\Theta_i\}+(1-p)q_j(\varnothing)]\right). \tag{6}$$

It follows from equation (6) that $q_i(\varnothing)=E\{q_i(\theta)\}$ in equilibrium. Hence, equation (6) for $\Theta_i=\varnothing$ yields the equilibrium output of $q^N(\varnothing)=q^I(\varnothing)$ as in equation (4) for an uninformed firm. Then, equation (6) for $\Theta_i=\theta$ yields the following equilibrium output for an informed firm:

$$q^N(\theta)=q^I(\varnothing)+\frac{E\{\theta\}-\theta}{2+p(1+\lambda)}.$$

Given the signal $\Theta_i \in \{\theta,\varnothing\}$, firm i earns the expected equilibrium profit $\pi^N(\Theta_i)=(1+\lambda)q^N(\Theta_i)^2$. With secrecy, a firm expects to earn the following profit in equilibrium:

$$\begin{aligned}\Pi^N &\equiv pE\{p\pi_i(q^N(\theta),q^N(\theta);\theta)+(1-p)\pi_i(q^N(\theta),q^N(\varnothing);\theta)\}\\ &\quad +(1-p)E\{p\pi_i(q^N(\varnothing),q^N(\theta);\theta)+(1-p)\pi_i(q^N(\varnothing),q^N(\varnothing);\theta)\}\\ &= (1+\lambda)q^I(\varnothing)^2+p(1+p\lambda)\frac{\text{Var}\{\theta\}}{[2+p(1+\lambda)]^2}.\end{aligned} \tag{7}$$

4 Information exchange choices

Now, I look into the incentives of firms to exchange information. Although each firm maximizes a weighed sum of profits, it is sufficient to compare expected profits. This is due to the fact that expected equilibrium profits are identical. This means that a firm's payoff (2) simplifies to $E\{V_i\}=(1+\lambda)E\{\Pi\}$. Hence, the incentive to exchange information depends on the marginal expected equilibrium profit from information exchange, $\Pi^I-\Pi^N$, which is the difference between equations (5) and (7). Analysis of this marginal expected profit gives the following result.

[5] I define the variance of parameter θ as $\text{Var}\{\theta\}\equiv E\{[\theta-E\{\theta\}]^2\}$.

Proposition 1 *There exists a unique critical probability $p^*(\lambda)$, with $0 < p^* < 1$, such that firms keep information secret in equilibrium iff $p \leq p^*(\lambda)$, whereas they exchange their information in equilibrium iff $p \geq p^*(\lambda)$.*

Without common ownership (i.e., $\lambda = 0$), Malueg and Tsutsui (1998, Example 1) find the critical probability $p^*(0)$. Proposition 1 contributes to this result by showing that it continues to hold in markets with common ownership (i.e., $\lambda > 0$).

The following proposition shows that common ownership strengthens the incentive to exchange information.

Proposition 2 *The critical probability $p^*(\lambda)$ from Proposition 1 is decreasing in λ (i.e., $dp^*/d\lambda < 0$). In particular, $p^*(0) = \frac{1}{2}\left(\sqrt{13} - 3\right) \approx 0.30$ and $p^*(1) = 0$.*

In other words, with a greater profit share λ, the firms have a stronger incentive to exchange information. Intuition for this result is as follows. Information exchange may help a firm's competitor to make better output choices, which is beneficial for the competitor. If a firm has a greater stake in the competitor's profit, then the firm internalizes this positive externality to a greater extent. This gives the firm a stronger incentive to share information.

Fig. 1(a) illustrates the equilibrium information-exchange choices of firms in a market. The bold downward-sloping curve sketches the combinations of p and λ where the firms are indifferent between sharing their information and keeping it secret. The curve traces the threshold probability p^* for different values of λ. In the region above the bold curve, firms choose to share information in equilibrium. By contrast, below the bold curve, the firms choose to keep information secret in equilibrium.

The theoretical result of Proposition 2 is in line with empirical findings. For example, Boone and White (2015), Park *et al.* (2019), Pawliczek and Skinner (2018), and Pawliczek *et al.* (2022) empirically find that firms tend to share more information with common ownership.

5 Effects of ownership on welfare

The consumer surplus is $CS = \frac{1}{2}(q_1 + q_2)^2$ for given output levels (q_1, q_2). With information exchange and equilibrium output choices, the expected consumer surplus

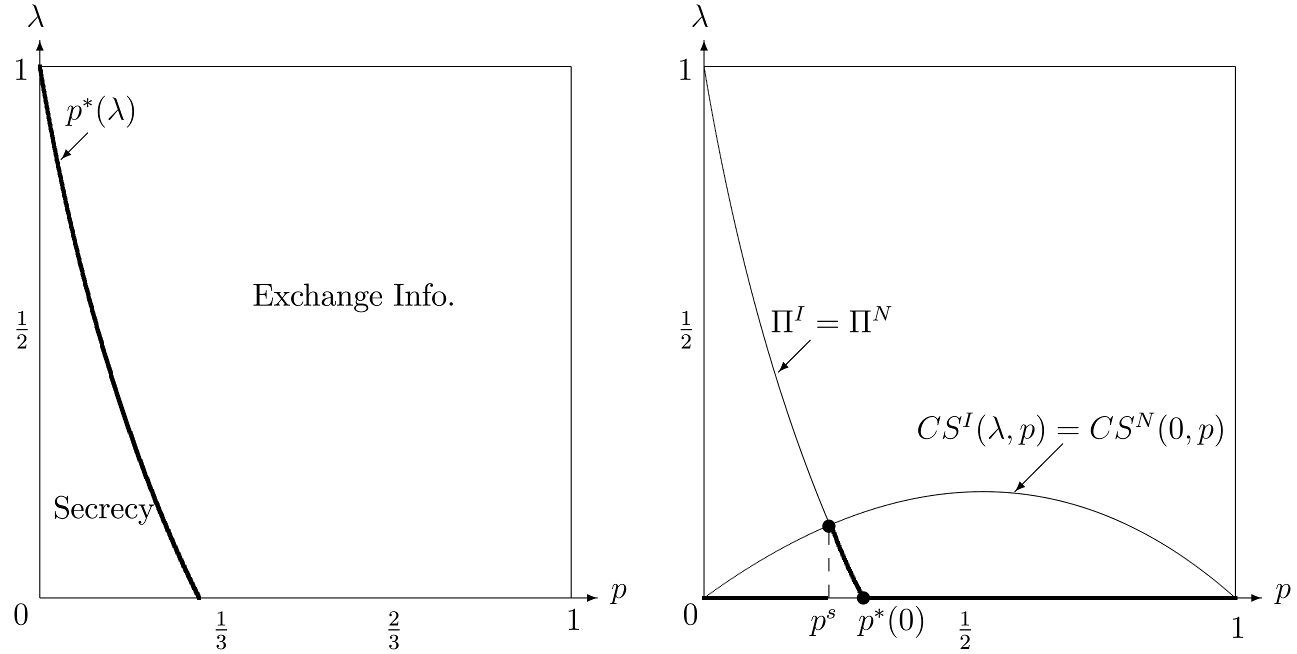

Fig. 1(a): Profit maximization | Fig. 1(b): Consumer surplus maximization

Figure 1: *Equilibrium choices.*

is as follows:

$$\begin{aligned} CS^I(\lambda, p) &\equiv 2p(2-p)E\left\{q^I(\theta)^2\right\} + 2(1-p)^2 q^I(\varnothing)^2 \\ &= 2\left(\frac{\alpha - E\{\theta\}}{3+\lambda}\right)^2 + 2p(2-p)\frac{\mathrm{Var}\{\theta\}}{(3+\lambda)^2}. \end{aligned} \tag{8}$$

With commitment to secrecy, the expected equilibrium consumer surplus is as follows:

$$\begin{aligned} CS^N(\lambda, p) &\equiv 2p^2 E\left\{q^N(\theta)^2\right\} + p(1-p)E\left\{\left[q^N(\theta) + q^N(\varnothing)\right]^2\right\} + 2(1-p)^2 q^N(\varnothing)^2 \\ &= 2\left(\frac{\alpha - E\{\theta\}}{3+\lambda}\right)^2 + p(1+p)\frac{\mathrm{Var}\{\theta\}}{(2+p+p\lambda)^2}. \end{aligned} \tag{9}$$

The proposition below establishes some basic properties of the expected equilibrium consumer surpluses CS^I and CS^N.

Proposition 3 ***(a)*** *In equilibrium, the expected consumer surplus is decreasing in parameter λ with or without information exchange (i.e., $\partial CS^k/\partial\lambda < 0$ for $k \in \{I, N\}$).* ***(b)*** *For all λ and p, the expected equilibrium consumer surplus is greater with information exchange than without information exchange (i.e., $CS^I > CS^N$).*

Proposition 3**(a)** shows that, all else equal, the expected consumer surplus is decreasing in the degree of common ownership, λ. The greater the firms' stakes in each other, the friendlier each firm becomes in the product market. With friendlier firms, the expected equilibrium price increases and consumers are worse off.

In markets with Cournot competition, but without common ownership (i.e., $\lambda = 0$), Vives (1984) shows that information exchange yields a higher expected consumer surplus than secrecy. Proposition 3**(b)** shows that this result also holds in markets with common ownership (i.e., $\lambda \geq 0$).

Propositions 2 and 3 may have interesting economic policy implications. An increase in the degree of common ownership, λ, yields a trade-off for consumers. On the one hand, an increase in λ makes consumers worse off for a given amount of information (Proposition 3**(a)**). On the other hand, an increase in λ may result in more information sharing (Proposition 2) and this tends to make consumers better off (Proposition 3**(b)**). The following proposition shows that the latter effect may outweigh the former effect locally. As a result, there may be a non-monotonic relation between the degree of common ownership, λ, and the expected consumer surplus.

Proposition 4 *There exists a critical probability p^s, with $0 < p^s < p^*(0)$, such that a positive degree of common ownership (i.e., $\lambda > 0$) maximizes expected consumer surplus if $p^s \leq p < p^*(0)$, whereas no common ownership (i.e., $\lambda = 0$) maximizes expected consumer surplus otherwise.*

If $p \geq p^*(0)$, then the consumers expect the consumer surplus $CS^I(\lambda, p)$, because the firms exchange information no matter what size λ has (Proposition 1). Then, Proposition 3**(a)** implies that consumers are on average best off without shared ownership ($\lambda = 0$). If $p < p^*(0)$, then the effect of λ on the expected consumer surplus is less clear-cut. There are two local maxima. The expected consumer surplus reaches the first local maximum if there is no cross ownership (i.e., $\lambda = 0$). For $p < p^*(0)$ and $\lambda = 0$, firms keep their information secret (Proposition 1) and consumers expect the surplus $CS^N(0, p)$. The second local maximum of the expected consumer surplus emerges for the degree of common ownership where the firms switch from secrecy to information exchange. The switch happens for ownership degree $\lambda = \lambda^*(p)$, which is the inverse of the critical probability $p^*(\lambda)$ from Proposition 1. The firms' switch yields an upward jump of the expected consumer surplus from $CS^N(\lambda^*(p), p)$ to the level $CS^I(\lambda^*(p), p)$. Whether $CS^N(0, p)$ or $CS^I(\lambda^*(p), p)$ is the global maximum depends on the probability p. If p is close to zero, then λ^* is close to one. Consequently, the consumers can obtain the expected benefits of information exchange only if the firms' ownership structures are so intertwined that they almost act like a cartel. This is not worthwhile for consumers, and consumers are best off without cross ownership (i.e., $CS^N(0,p) > CS^I(\lambda^*,p)$ if p is small). Conversely, if p is close to $p^*(0)$, then λ^* is close to zero. Then, reaching an equilibrium with information exchange requires only a minor distortion of market power through a small degree of cross ownership, $\lambda^* > 0$. Hence, if p is slightly below $p^*(0)$, then the expected consumer surplus reaches the global maximum at $\lambda = \lambda^*$ (i.e., $CS^I(\lambda^*,p) > CS^N(0,p)$ for p close to $p^*(0)$).

Fig. 1(b) illustrates the finding from Proposition 4. The hump-shaped thin curve sketches the parameter values (p,λ) where consumers are indifferent between secrecy without common ownership, on the one hand, and information exchange with a degree of common ownership, on the other hand (i.e., $CS^N(0,p) = CS^I(\lambda,p)$). For those degrees of common ownership, the consumers' gain from information sharing exactly offsets the consumers' loss from common ownership.[6] Below the curve, consumers are on average best off with information exchange (i.e., $CS^I(\lambda,p) > CS^N(0,p)$).[7] Conversely, above the hump-shaped curve, expected consumer surplus is highest with secrecy and no common ownership (i.e., $CS^N(0,p) > CS^I(\lambda,p)$). The downward-sloping thin curve connects the parameter values where firms are indifferent between information exchange and secrecy (i.e., $\Pi^I = \Pi^N$ as along the bold curve in Fig. 1(a)).

The discontinuous bold curve in Fig. 1(b) traces the degrees of common ownership which maximize consumer surplus. For low and high probabilities p (i.e., $p < p^s$ and $p \geq p^*(0)$), the consumers are best off without common ownership. For intermediate probabilities (i.e., $p^s \leq p < p^*(0)$), a positive degree of common ownership maximizes the expected consumer surplus. In the latter case, the surplus-maximizing degree of common ownership is the minimal degree necessary to induce the firms to exchange their information. In Fig. 1(b), the bold curve coincides with the firms' indifference curve on the interval $[p^s, p^*(0)]$ for this reason.

What is the effect of common ownership on the expected total surplus? Total surplus is a weighted sum of consumer surplus and profits, i.e., $W^k \equiv CS^k + \omega \cdot 2\Pi^k$ for weight $0 \leq \omega \leq 1$ and $k \in \{N, I\}$. Above, I have characterized how the expected equilibrium consumer surplus depends on the degree of common ownership. Now, I focus on the effect of the degree of common ownership on the expected equilibrium profits.

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[6] Explicitly, by equations (8) and (9), consumers are indifferent if the following holds:

$$\lambda = 3\left((2+p)\sqrt{\frac{2\left(\alpha - E\{\theta\}\right)^2 + 2p(2-p)\mathrm{Var}\{\theta\}}{2(2+p)^2\left(\alpha - E\{\theta\}\right)^2 + 9p(1+p)\mathrm{Var}\{\theta\}}} - 1\right).$$

[7] For example, along the horizontal axis, there is no common ownership (i.e., $\lambda = 0$), and consumer surplus is then highest with information exchange (i.e., $CS^I(0,p) > CS^N(0,p)$ by Proposition 3**(b)**).

The following proposition shows that expected equilibrium profits are increasing in λ.

Proposition 5 *In equilibrium, the expected profits are increasing in parameter λ with or without information sharing (i.e., $\partial\Pi^k/\partial\lambda > 0$ for $k \in \{I, N\}$).*

With more common ownership, firms relax competition, and this raises their expected profits. Monotonicity of the expected equilibrium profits has the following immediate implication for the expected total surplus in equilibrium.

Proposition 6 *The degree of common ownership that maximizes expected equilibrium welfare is at least as big as the degree that maximizes expected equilibrium consumer surplus.*

Hence, there are instances where a benevolent regulator prefers some degree of common ownership instead of no common ownership regardless of the weights she assigns to the consumer and producer surpluses.

6 Conclusion

Common ownership has interesting effects on information aggregation in oligopolistic markets. In turn, these effects have interesting economic policy implications.

A greater degree of common or cross-ownership strengthens the firms' incentives to exchange information about the demand (or common cost) in their market. Common ownership has two conflicting effects on the expected consumer surplus. On the one hand, common or cross-ownership relaxes competition between firms and this makes consumers worse off. On the other hand, common or cross-ownership may yield more information exchange in the market and this makes consumers better off on average. I have shown that the latter, indirect effect may dominate the former, direct effect.

The paper's result may contribute to developing a nuanced economic policy towards common and cross-ownership in oligopolistic markets.

A Appendix

This Appendix contains the proofs of Propositions 1-6.

Proof of Proposition 1

By using equations (5) and (7), I rewrite the marginal expected profit from information exchange as $\Pi^I - \Pi^N = p(1-p)\text{Var}\{\theta\} \cdot \Phi^f(\lambda, p)$, where:

$$\begin{aligned}
\Phi^f(\lambda,p) &\equiv \frac{1}{1-p}\left(\frac{(2-p)(1+\lambda)}{(3+\lambda)^2} - \frac{1+p\lambda}{[2+p(1+\lambda)]^2}\right) \\
&= \frac{1+2\lambda}{(3+\lambda)^2} - \frac{(1+p\lambda)(1+\lambda)[4+(1+p)(1+\lambda)]}{(3+\lambda)^2[2+p(1+\lambda)]^2} \\
&= \frac{N^f(\lambda,p)}{(3+\lambda)^2[2+p(1+\lambda)]^2},
\end{aligned} \tag{A.1}$$

where the numerator is as follows

$$N^f(\lambda,p) \equiv (1+2\lambda)[2+p(1+\lambda)]^2 - (1+p\lambda)(1+\lambda)[4+(1+p)(1+\lambda)]. \tag{A.2}$$

The sign of of the numerator of Φ^f determines the sign of the marginal expected profit from information exchange $\Pi^I - \Pi^N$, because the denominator of Φ^f is positive and $p(1-p)\text{Var}\{\theta\} > 0$. The numerator $N^f(\lambda, p)$ is convex in p, because $\partial^2 N^f/\partial p^2 = 2(1+\lambda)^3 > 0$. For $p = 0$, the numerator simplifies as follows:

$$N^f(\lambda, 0) = 4(1+2\lambda) - (1+\lambda)(5+\lambda) < 0,$$

for all $\lambda < 1$. For $p = 1$ the numerator (A.2) is positive, because:

$$N^f(\lambda, 1) = (1+2\lambda)(3+\lambda)^2 - 2(1+\lambda)^2(3+\lambda) = (1+3\lambda)(3+\lambda) > 0.$$

Hence, there exists a unique critical probability p^*, with $0 < p^* < 1$ such that $N^f(\lambda, p) < 0$ iff $p < p^*$, whereas $N^f(\lambda, p) > 0$ iff $p > p^*$. $\square$

Proof of Proposition 2

The proof of Proposition 1 finds that the marginal profit $\Pi^I - \Pi^N$ crosses the p-axis from below. Hence, the root $p^*(\lambda)$ decreases if $\Pi^I - \Pi^N$ is increasing in λ. By equation (A.1), the partial derivative of $\Pi^I - \Pi^N$ is as follows:[8]

$$\frac{\partial(\Pi^I - \Pi^N)}{\partial\lambda} = p\text{Var}\{\theta\}(1-\lambda)\left(\frac{2-p}{(3+\lambda)^3} - \frac{p^2}{[2+p(1+\lambda)]^3}\right) > 0.$$

The root of numerator N^f in (A.2) yields the root of marginal profit $\Pi^I - \Pi^N$. Extreme degrees of ownership yield: $N^f(0,p) = p^2 + 3p - 1$ with root $p^*(0) = \frac{1}{2}\left(\sqrt{13} - 3\right)$ and $N^f(1,p) = 8p(1+p)$ with root $p^*(1) = 0$. $\square$

[8]The inequality follows from the observations that both terms $(2-p)/(3+\lambda)^3$ as well as $-p^2/[2+p(1+\lambda)]^3$ are decreasing in p, and their sum equals zero for $p \to 1$.

Proof of Proposition 3

(a) Equations (8) and (9) yield $\partial CS^I/\partial\lambda < 0$ and $\partial CS^N/\partial\lambda < 0$, respectively.
(b) Information exchange has the following effect on expected consumer surplus:

$$CS^I - CS^N = \frac{p(1-p)\mathrm{Var}\{\theta\}}{(3+\lambda)^2(2+p+p\lambda)^2} N^s(\lambda,p), \tag{A.3}$$

where I define the term N^s as follows

$$\begin{aligned}
N^s(\lambda,p) &\equiv \frac{1}{1-p}\Big(2(2-p)(2+p+p\lambda)^2 - (1+p)(3+\lambda)^2 \Big) \\
&= \frac{1}{1-p}\Big((1-p)\left[2(2+p+p\lambda)^2 - (3+\lambda)^2\right] + 2\left[(2+p+p\lambda)^2 - p(3+\lambda)^2\right] \Big) \\
&= 2\left[2+p(1+\lambda)\right]^2 - (3+\lambda)^2 + 2\left[4 - p(1+\lambda)^2\right].
\end{aligned}$$

The sign of $N^s(\lambda,p)$ determines the sign of $CS^I - CS^N$. It is easily verified that $N^s(\lambda,p)$ is increasing in p. Hence, for all $0 < p < 1$, the inequality $N^s(\lambda,p) > N^s(\lambda,0) = 16 - (3+\lambda)^2 \geq 0$ holds. As $N^s(\lambda,p) > 0$ this implies that $CS^I > CS^N$. □

Proof of Proposition 4

For $p > p^*(0)$, the firms exchange information for all λ (Proposition 1). Then, $\lambda = 0$ maximizes the expected consumer surplus by Proposition 3**(a)** for $k = I$.

The remainder of the proof covers $0 < p < p^*(0)$. Propositions 1 and 2 show that for any p, with $0 < p < p^*(0)$, there exists a unique λ, with $0 < \lambda < 1$, such that $p^*(\lambda) = p$, where Proposition 1 defines $p^*(\lambda)$. Hence, it is without loss of generality to consider $p = p^*(\lambda)$ for some $0 < \lambda < 1$.

For $p = p^*(\lambda)$, the expected consumer surplus has the local maxima $CS^N(0, p^*(\lambda))$ and $CS^I(\lambda, p^*(\lambda))$ by Propositions 1 and 3. Define the difference between the two local maxima of expected surplus as follows:

$$M(\lambda) \equiv CS^I(\lambda, p^*(\lambda)) - CS^N(0, p^*(\lambda)).$$

Evaluating M for extreme values gives $M(0) \equiv CS^I(0,p^*(0)) - CS^N(0,p^*(0)) > 0$ by Proposition 3**(b)** and $M(1) \equiv CS^I(1,0) - CS^N(0,0) = \left(\frac{2}{16} - \frac{2}{9}\right)(\alpha - E\{\theta\})^2 < 0$ by equations (8) and (9). Further, M is continuous in λ. Next, I show that M is monotonic (decreasing) in λ. I decompose the effect of λ on M as follows:

$$\frac{dM}{d\lambda} = \frac{\partial CS^I(\lambda, p^*(\lambda))}{\partial\lambda} + \frac{\partial\left[CS^I(\lambda,p^*(\lambda)) - CS^N(0,p^*(\lambda))\right]}{\partial p} \cdot \frac{dp^*}{d\lambda} < 0.$$

By Proposition 3**(a)**, the first term is negative, i.e., $\partial CS^I/\partial\lambda < 0$. The last term is negative too. First, Proposition 2 shows that $dp^*/d\lambda < 0$. Second, a Supplementary Appendix shows that $\partial[CS^I(\lambda, p^*(\lambda)) - CS^N(0, p^*(\lambda))]/\partial p > 0$ for all λ.

Hence, there exists a unique degree λ^s, with $0 < \lambda^s < 1$, such that $M(\lambda^s) = 0$. As M is decreasing, $M(\lambda) > 0$ for all $\lambda < \lambda^s$, i.e., $CS^I(\lambda, p^*(\lambda)) > CS^N(0, p^*(\lambda))$ for all $\lambda < \lambda^s$. Define $p^s \equiv p^*(\lambda^s)$. By Proposition 2, the inequality $0 < \lambda < \lambda^s$ corresponds to $p^s < p^*(\lambda) < p^*(0)$. Hence, for all $p^s < p < p^*(0)$, the expected consumer surplus reaches a global maximum for $\lambda > 0$. Conversely, $M(\lambda) < 0$ for all $\lambda > \lambda^s$, which implies that the expected consumer surplus is maximal without cross ownership (i.e., $\lambda = 0$) for all $0 < p < p^s$. $\square$

Proof of Proposition 5

With information exchange, an increase of λ has the following effect on equation (5) for all $\lambda < 1$:

$$\begin{aligned}\frac{\partial \Pi^I}{\partial \lambda} &= [(\alpha - E\{\theta\})^2 + p(2-p)\mathrm{Var}\{\theta\}]\frac{\partial\left(\frac{1+\lambda}{(3+\lambda)^2}\right)}{\partial\lambda} \\ &= [(\alpha - E\{\theta\})^2 + p(2-p)\mathrm{Var}\{\theta\}]\frac{(1-\lambda)}{(3+\lambda)^3} > 0.\end{aligned}$$

With secrecy, increasing λ has the following effect on equation (7) for all $\lambda < 1$:

$$\begin{aligned}\frac{\partial \Pi^N}{\partial \lambda} &= (\alpha - E\{\theta\})^2\frac{\partial\left(\frac{1+\lambda}{(3+\lambda)^2}\right)}{\partial\lambda} + p\mathrm{Var}\{\theta\}\frac{\partial\left(\frac{1+p\lambda}{[2+p(1+\lambda)]^2}\right)}{\partial\lambda} \\ &= \frac{(1-\lambda)(\alpha - E\{\theta\})^2}{(3+\lambda)^3} + \frac{p^3(1-\lambda)\mathrm{Var}\{\theta\}}{[2+p(1+\lambda)]^3} > 0.\end{aligned}$$

$\square$

Proof of Proposition 6

Distinguish two cases. First, if the expected equilibrium consumer surplus is maximal without common ownership (i.e., $\lambda = 0$), then, by definition (i.e., $\lambda \geq 0$), the welfare-maximizing degree of common ownership is at least zero. Second, suppose that the expected equilibrium consumer surplus is maximal with common ownership, say, for degree λ^* with $0 < \lambda^* < 1$. Then, for all $0 \leq \lambda < \lambda^*$, firms keep their information secret, and $CS^N(\lambda, p) < CS^I(\lambda^*, p)$ and, by Proposition 5, $\Pi^N(\lambda, p) < \Pi^I(\lambda^*, p)$. Hence, $W^N(\lambda, p) < W^I(\lambda^*, p)$ for all $0 \leq \lambda < \lambda^*$ and all weights ω. In other words, the welfare-maximizing degree of common ownership is at least λ^*. $\square$

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Ownership and Information Exchange in a Duopoly

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Supplementary Appendix: not for publication

Here, I show that for any degree of ownership λ, the following inequality holds:

$$\frac{\partial\left[CS^I(\lambda, p^*(\lambda)) - CS^N(0, p^*(\lambda))\right]}{\partial p} > 0, \tag{A.4}$$

where equation (8) defines CS^I, equation (9) defines CS^N, and Proposition 1 defines $p^*(\lambda)$.

I take the following two steps to show that the inequality (A.4) holds for all λ:

$$\begin{aligned}
\frac{\partial\left[CS^I(\lambda, p^*(\lambda)) - CS^N(0, p^*(\lambda))\right]}{\partial p} &> \frac{\partial\left[CS^I(\lambda, \frac{1-\lambda}{3}) - CS^N(0, \frac{1-\lambda}{3})\right]}{\partial p} && \text{(A.5)} \\
&\geq 0. && \text{(A.6)}
\end{aligned}$$

Equations (8) and (9) yield the following partial derivative for any probability p:

$$\frac{\partial\left[CS^I(\lambda, p) - CS^N(0, p)\right]}{\partial p} = \mathrm{Var}\{\theta\}\left(\frac{4(1-p)}{(3+\lambda)^2} - \frac{2+3p}{(2+p)^3}\right). \tag{A.7}$$

For $p = \frac{1}{3}(1-\lambda)$, the derivative (A.7) is as follows:

$$\begin{aligned}
\frac{\partial\left[CS^I(\lambda, \frac{1-\lambda}{3}) - CS^N(0, \frac{1-\lambda}{3})\right]}{\partial p} &= \mathrm{Var}\{\theta\}\left(\frac{4(2+\lambda)}{3(3+\lambda)^2} - \frac{27(3-\lambda)}{(7-\lambda)^3}\right) \\
&\geq \frac{\partial\left[CS^I(1,0) - CS^N(0,0)\right]}{\partial p} = 0,
\end{aligned}$$

where the inequality follows from the fact that derivative (A.7) evaluated at $p = \frac{1}{3}(1-\lambda)$ is decreasing in λ.[9] This proves the inequality (A.6).

[9] The term $4(2+\lambda)/[3(3+\lambda)^2]$ as well as the term $-27(3-\lambda)/(7-\lambda)^3$ are decreasing in λ.

To prove inequality (A.5), I show that $p^*(\lambda) < \frac{1}{3}(1-\lambda)$ for all λ, and $\partial^2[CS^I(\lambda,p)-CS^N(0,p)]/\partial p^2 < 0$. Evaluating the numerator (A.2) for probability $p = \frac{1}{3}(1-\lambda)$ yields the following for all $0 < \lambda < 1$:

$$\begin{aligned} N^f(\lambda, \frac{1-\lambda}{3}) &\equiv \frac{1}{9}(1+2\lambda)(7-\lambda^2)^2 - \frac{1}{9}\left[3+\lambda(1-\lambda)\right](1+\lambda)[12+(4-\lambda)(1+\lambda)] \\ &= \frac{1}{9}(1-\lambda)\left(1+26\lambda+3\lambda^2-5\lambda^3-\lambda^4\right) > 0. \end{aligned}$$

Proposition 1 defines $p^*(\lambda)$ as the probability with $N^f(\lambda, p^*(\lambda)) = 0$, where $N^f(\lambda,p) > 0$ for all $p > p^*(\lambda)$. Hence, $p^*(\lambda) < \frac{1}{3}(1-\lambda)$. For any p, the second-order partial derivative is:

$$\begin{aligned} \frac{\partial^2[CS^I(\lambda,p) - CS^N(0,p)]}{\partial p^2} &= -\mathrm{Var}\{\theta\}\left(\frac{4}{(3+\lambda)^2} - \frac{6p}{(2+p)^4}\right) \\ &\leq -\mathrm{Var}\{\theta\}\left(\frac{1}{4} - \frac{6\left(\frac{2}{3}\right)}{\left(2+\frac{2}{3}\right)^4}\right) < 0. \end{aligned}$$

This finding and $p^*(\lambda) < \frac{1}{3}(1-\lambda)$ yield inequality (A.5). $\square$