Platform Competition and App Development*

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Abstract

We study the development of apps on competing platforms. We first show that, whenever an increase in the commission charged by a platform reduces the number of apps present on another platform, competition leads platforms to charge commission rates that exceed the level maximizing consumer surplus (and, a fortiori, total welfare). We then study a simple setting in which a fraction of developers can port their apps across platforms at no cost, and find that platform competition then always leads to excessive commissions; furthermore, as this fraction tends to one, commissions become so high that they deter any app development.

Key Words: Platform Competition, Ad-valorem Commissions, App Stores, App Development

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1 Introduction

The 30 percent commission charged by Apple's App Store and Google's Play Store has prompted major disputes, such as the battle led by Epic Games,[1] and triggered policy initiatives around the world. For instance, to reduce the commissions paid by app developers, in 2021 the South Korean parliament adopted in 2021 a bill banning major app store operators—such as Google and Apple—from requiring developers to only use the app stores' payment systems. Later on, the Indian Competition Commission issued a similar order.[2] In 2022, the European Union adopted the Digital Markets Act, which requires gatekeepers to apply fair, reasonable and non-discriminatory conditions of access to app stores, among others.[3]

The main concern raised by this 30 percent commission charged by Apple and Google is its impact on app development, well summarized by Brent Simmons, a Mac and iOS app developer, in his testimony before the U.S. Congress:

"[T]he more money Apple takes from developers, the fewer resources developers have. .... They decide not to make apps at all that they might have made were it easier to be profitable."[4]

In response, Apple and Google argue that platforms and consumers have a common interest in attracting apps, and moreover point to the disciplining role of platform competition. For instance, in its response to the investigation of the Dutch National Competition Authority (NCA), Google argues:

"The level of the commission fee charged is used by app stores to compete with each other, as a means to attract app providers on their platform."[5]

Some regulators have however expressed doubt about the extent to which the largest platforms are subject to competitive pressure, and emphasize instead the importance of switching costs and behavioral biases among consumers.

To shed some light on this debate, we study a setting in which two-sided platforms compete on prices for consumers and on ad valorem commission rates for apps. Consumers single-home and benefit from the platform's service and the available apps, whereas app developers, who face heterogeneous innovation costs, may single- or multihome, and derive their revenue from consumers.

[1] In August 2020, Epic started encouraging mobile-app users of its Fortnite game to adopt Epic's payment option, offering a 20% discount from Apple's or Google's in-app purchase. In response, Apple and Google removed Fortnite from their respective app stores, which led Epic to sue Apple and Google, with the backing of Microsoft, Facebook, Spotify, Match Group and ten other companies.

[2] See https://cci.gov.in/images/pressrelease/en/pr-no-562022-231666698260.pdf.

[3] See Article 6.12.

[4] See Subcommittee (2020), at p. 350. In the same vein, see Greg Bensinger's article, "What Apple's Fortnite Fee Battle Is Really About," the New York Times, https://www.nytimes.com/2020/09/24/opinion/apple-google-mobile-apps.html.

[5] Netherlands Authority for Consumers and Markets (2019), at p. 92.

Our main finding is that platform competition may not be a cure but, rather, an obstacle to app development. Specifically, competition generates higher commissions than what would maximize consumer surplus whenever an increase in one platform's commission *reduces* the number of apps present on the *rival* platform. This, in turn, is likely to be the case when apps multihome, as an increase in either commission then reduces their overall profitability.

Our analysis highlights a key factor, namely, the importance of multihoming on the app side: the greater the proportion of multihoming, the higher the commissions and the more limited is the app development. In practice, successful apps indeed tend to multihome; for instance, the U.K. NCA notes:

"Most large and popular third-party apps are present on both Apple's iOS and Google's Android. For example, we have estimated that 85% of the top 5,000 apps on the App Store also list on the Play Store and vice versa."[6]

In a similar vein, the Dutch NCA finds that consumers' initial choice between an iPhone and an Android phone does not depend on the availability of apps, because all popular and known apps are present in both smartphone platforms.[7]

Our analysis, along with the fact that most popular apps multihome, questions the role of competition as a disciplining device and provides a rationale for policy intervention. It suggests that platform competition is likely to generate excessively high commissions, compared to the level that would maximize consumer surplus (and, a fortiori, total welfare), leading to too little app development. Relatedly, platform competition may incentivize platforms to inhibit the development of cross-platform technology such as cloud gaming, which enables game developers to reach a larger user base without having to port their apps on multiple operating systems. CMA (2022) documents how Apple has used its control over app distribution to block the emergence of cloud gaming apps on its App Store.[8]

Our results also shed light on the level of commissions observed in the Chinese app store market. In that market, where Google Play Store is not available and Apple has typically less than 10 percent market share, there is vivid competition among multiple Chinese smartphone manufacturers, all based on the Android operating system. Yet, the major Chinese manufacturers (e.g., Xiaomi, Oppo, Vivo, and Huawei), who own their own app stores, charge a 50 percent commission to app developers.[9] Hence, the intense platform competition is associated with a commission that is even higher than

[6] CMA report (2022), at p. 121.

[7] See Netherlands Authority for Consumers and Markets (2019).

[8] See Appendix I of the CMA (2022) report.

[9] See for instance "China's App Store Fee's Make Apple's Look Cheap" by Zheping Huang, Bloomberg, 8 October, 2020, https://www.bloomberg.com/news/newsletters/2020-10-08/china-s-app-store-fees-make-apple-s-look-cheap.

the one charged by Google and Apple, which is consistent with our analysis: a larger number of competing platforms further exacerbates the negative externalities generated by multihoming apps, which induces the platforms to charge higher commissions.

Throughout the paper we model competition between the platforms as follows. In a first stage, platforms first set ad valorem commissions on the app side; app developers, facing heterogeneous innovation costs, then decide which platform(s), if any, to develop their apps for. In a second stage, first platforms set consumer prices and, simultaneously, active developers set app prices; consumers then decide which platform to join, if any, and which apps to buy. This timing, in which commissions are set ahead of consumer and app prices, is consistent with Apple charging a 30% commission since the launch of its App Store. Consumers are assumed to single-home and observe their valuations for the available apps after joining a platform. The distributions of valuations (for the platforms, and for each app) are ex ante symmetric; platforms are thus symmetrically differentiated and, in equilibrium, all apps are offered at the same price.

We first adopt a stylized approach in which the number of apps on a given platform is a function of both platforms' commissions. Competition for consumers is then similar to standard Bertrand competition, with the caveat that each platform receives a *subsidy* per consumer. This subsidy corresponds to the value—for the consumer and the platform—generated by the consumers' use of the apps, and therefore depends on the portfolio of apps available on the platform. It follows that the commission charged by a platform can affect its rival's subsidy as well as its own.its rival.

We first note that the joint interest of the platforms is indeed aligned with that of consumers, in that they both favor higher subsidies: raising the subsidies has a direct positive impact on platforms' profits, but also fosters competition for consumers, leading the platforms to transfer part of the additional subsidies to consumers. As a result, maximizing industry profit is the same as maximizing consumer surplus: it boils down to maximizing the subsidies. By contrast, app developers favor lower commission rates; hence, maximizing total welfare requires lower rates than those that maximize consumer surplus or the platforms' profits.

We then compare these benchmarks with the commission rates arising under platform competition. Increasing the subsidy of a given platform benefits that platform, as just noted, but also encourages it to compete more aggressively for consumers, which tends to reduce the profit of its rival. Hence, when choosing its commission rate, each platform has an incentive to increase its own subsidy but reduce its rival's. In the particular case in which one platform's commission has no impact on the number of apps available on the rival platform, it does not affect either the rival's subsidy; each platform then focuses on maximizing its own subsidy, and the competitive outcome thus maximizes consumer surplus as well as the platform's joint profit. However, if instead an increase in one commission reduces (resp., increases) the number of apps present on the other platform—and, thus, the rival's subsidy—, then the platforms have an incentive to raise (resp., lower) their commissions above (resp., below) the level maximizing their own subsidy—and, thus, consumer surplus.

To gain further insights, we then focus on a particular setting with horizontal differentiation à la Hotelling on the consumer side and two types of developers on the app side: *multihomers* can port their apps across platforms at no cost once they have developed their apps, whereas *single-homers* face platform-specific development costs. We find that as long as there is a positive fraction of multihomers, platform competition leads to a higher commission than what would maximize consumer surplus. Furthermore, as the fraction of single-homers tends to disappear, the commissions become so high that there is almost no app development. The intuition is that a platform has little incentive to encourage the development of apps when most of these apps become also available on the competing platform. By contrast, a monopolistic firm running both platforms would seek to encourage app development—and actually choose here the commissions that maximize consumer surplus.

Related literature. Our framework builds on the model of competitive bottlenecks developed by Armstrong (2006) and Armstrong and Wright (2007), by explicitly considering the innovation incentives of app developers. Belleflamme and Peitz (2010) extend Armstrong (2006)'s Hotelling model of platform competition and find that sellers invest less than is socially desirable when sellers multihome and buyers single-home. We emphasize instead that competition exacerbates this under-investment problem and can even eliminate any app development. Choi and Jeon (2022) study platform design in a model of competitive bottlenecks and identify the design biases (e.g., in technology adoption) generated by different platform business models.

Etro (2022) studies a setup that coincides with our second model in the absence of multihoming developers, and shows that platform competition leads to commission rates that maximize consumer surplus. We find that the commissions are instead above that level whenever there is a positive fraction of multihoming developers.[10]

Wright (2002) uses a model of competitive bottlenecks to study the market for fixed-to-mobile calls. Mobile network operators (MNOs) compete to attract consumers and charge fixed-to-mobile termination fees to a fixed-line network operator. If that operator is constrained to charge the same price for all fixed-to-mobile calls, then the

[10] Specifically, what matters is not whether apps end up being available on both platforms or only one, but whether developing the app for one platform affects the decision to develop it on the other platform.

MNOs set termination fees higher than the monopoly fee; in particular, if two MNOs compete à la Hotelling, the termination fees are so high that there is no fixed-to-mobile call, which is similar to our choke-off result. However, several differences can be noted. First, mobile subscribers obtain no utility from fixed-to-mobile calls and are thus insensitive to the level of the termination fees. By contrast, in our setting consumers enjoy the applications and thus indirectly care about the commissions charged on the app side as well as about the device prices on the consumer side. Second, the choke-off of fixed-to-mobile calls stems from a non-discrimination rule imposed on the fixed-line network, whereas in our setting, the choke-off of app development arises instead when the fraction of single-homing apps goes to zero.[11]

Roadmap. We describe the general setting in Section 2. In Section 3, we adopt a stylized approach to present our key insights. In Section 4, we illustrate them in the context of a fully specified model and emphasize the role of multihoming on the app side. We provide concluding remarks and policy implications in Section 5.

2 Setting

Two platforms (1 and 2) compete to attract (single-homing) consumers and (single- or multihoming) apps. On the app side, each platform sets an ad valorem commission rate; app developers, who face heterogeneous innovation costs, then decide whether to develop an app and, if they do, which platform(s) to join, if any. On the consumer side, each platform sets an access price; consumers then choose which platform to join, if any. This setting corresponds for example to the two leading mobile OS platforms (iOS and Android, with their app stores, App Store and Google Play), interpreting consumer prices as the prices of the devices (iPhone or Android phone), and treating for simplicity the Android platform as vertically integrated, like the iPhone platform.

We now present the model in more detail.

• *Consumers.* There is a continuum of consumers, each endowed with a stochastic value v for each app, drawn (independently across consumers and apps) from a distribution with c.d.f. $G(\cdot)$ over $\mathbb{R}_+$. A consumer's expected demand for an app offered at price p is therefore given by

$$d(p) \equiv 1 - G(p).$$

[11] In a previous version, we also found that a complete choke-off does not arise if platforms charge wholesale prices instead of ad valorem commissions on the app side.

Let

$$s\left(p\right) \equiv \int_{p}^{+\infty} d\left(\hat{p}\right) d\hat{p}$$

denote the associated surplus, and $\pi(p) \equiv pd(p)$ the per consumer profit, which is assumed to be maximal for some price p^m.

Consumers have also intrinsic utilities u_1 and u_2 for the two platforms, and they observe their valuations for the available apps only after joining a platform. Hence, if platform i $(= 1, 2)$ charges consumers a price p_i and attracts y_i apps, each offering an expected surplus s_i, then joining the platform gives consumers a net payoff equal to

$$u_i + s_i y_i - p_i = u_i - P_i,$$

where

$$P_i \equiv p_i - s_i y_i$$

denotes platform i's "quality-adjusted" price. We assume that consumers' intrinsic valuations for the two platforms are distributed in such a way that the demand for platform i is given by

$$D\left(P_i, P_j\right),$$

which features (imperfect) substitution: $\partial_1 D(\cdot) \leq -\partial_2 D(\cdot) < 0$.

• *Developers.* Apps being digital goods, the only costs that developers incur are fixed innovation costs, which vary across developers and possibly across platforms. Specifically, each developer faces three innovation costs: $k_i \geq 0$ for developing its app on platform i, for $i = 1, 2$, and k for developing it on both platforms. The costs $\boldsymbol{k} = (k_1, k_2, k)$ are independently and identically drawn from a joint distribution $\hat{F}\left(\boldsymbol{k}\right)$, which is symmetric in k_1 and k_2. If platform i charges an ad valorem commission a_i and attracts x_i consumers, then offering the app on platform i at price $\tilde{p}_i$ gives the developer a payoff equal to

$$\left(1 - a_i\right) \pi(\tilde{p}_i) x_i - k_i,$$

whereas the payoff from joining both platforms is given by:

$$\left(1 - a_1\right) \pi(\tilde{p}_1) x_1 + \left(1 - a_2\right) \pi(\tilde{p}_2) x_2 - k.$$

• *Platforms.* For the sake of exposition, we set the cost of servicing consumers to zero. Hence, if each platform i charges a commission rate a_i and a consumer price p_i, and attracts y_i developers generating an average profit π_i and consumer surplus s_i, then platform i obtains a payoff given by, for $i \neq j \in \{1, 2\}$:

$$(p_i + a_i \pi_i y_i) D (p_i - s_i y_i, p_j - s_j y_j).$$

- *Timing.* The timing is as follows:

  1. *Competition for apps*:

     (a) each platform $i = 1, 2$ sets its ad valorem commission rate $a_i \in [0, 1]$;

     (b) each app developer draws its innovation costs (k_1, k_2, k) and makes its investment decision;

     (c) those developers who invested develop their apps and then decide which platform(s) to join, if any.

  2. *Competition for consumers*:

     (a) each platform $i = 1, 2$ sets its price $p_i \in \mathbb{R}_+$ and each developer sets, for each platform it joined, the price at which consumers can buy its app on that platform;

     (b) each consumer then decides which platform to join, if any; upon joining a platform, each consumer learns her valuations for every available app and decides which apps to buy.

At each stage, decisions are simultaneous and publicly observed; hence, each stage determines a proper subgame. We will therefore look for the subgame-perfect equilibria of this game.

As is well-known, multi-sided markets are subject to network effects, which makes them prone to tipping; as a result, competition—even between equally efficient firms—may lead to monopolization. We highlight here another potential source of market failure, namely, that competition may lead to too little app development. For the sake of exposition, we will therefore ignore the possibility of tipping and focus instead on shared-market, symmetric equilibria, in which the two platforms eventually attract the same number of users on both sides of the market.

3 A stylized approach

We first adopt a *stylized approach* and assume that, once the commission rates have been set (in stage $1a$), there exists a well-behaved continuation equilibrium in the key next stages, namely, the app development stage $1b$ and the platform pricing stage $2a$.

Using backward induction, we first consider the second stage of the game. In stage $2b$, consumers' participation decisions determine the demand $D(P_i, P_j)$ described above. In stage $2a$, all developers charge the same price p^m—regardless of the commission rates a_1 and a_2, and of the expected number of consumers joining the platforms;[12] each developer present on a platform thus generates a profit $\pi^m \equiv \pi(p^m)$ per consumer, and each consumer obtains a surplus $s^m \equiv s(p^m)$ per app. For each platform i, choosing a price p_i thus amounts to choosing a quality-adjusted price $P_i = p_i - s^m y_i$ and the resulting profit, given by (1), can be expressed as:

$$\Pi_i = \Pi(P_i, P_j; \sigma_i) \equiv (P_i + \sigma_i) D(P_i, P_j),$$

where

$$\sigma_i \equiv (s^m + a_i \pi^m) y_i.$$

It follows that, given the commission rates (a_1, a_2) and the number of apps (y_1, y_2) developed in stage 1, in stage $2a$ the continuation subgame amounts to a classic price competition game, in which each platform i chooses its quality-adjusted price P_i and faces the demand $D(P_i, P_j)$, with the caveat that it benefits from a *subsidy* σ_i. In line with our stylized approach, we will suppose that, for any given subsidies σ_1 and σ_2, this game has a unique price equilibrium, in which platform i's price is given by

$$P_i = P^e(\sigma_i, \sigma_j).$$

Let

$$\Pi^e(\sigma_i, \sigma_j) \equiv \Pi(P^e(\sigma_i, \sigma_j), P^e(\sigma_j, \sigma_i); \sigma_i) \tag{1}$$

denote platform i's equilibrium profit. Intuitively, an increase in σ_i should benefit platform i, but also induce it to price more aggressively (i.e., charge a lower quality-adjusted price), thus harming the rival. We will therefore maintain the following assumptions, namely:

Assumption 1 (prices and profits). For any $\sigma \in \mathbb{R}_+$:

(a) $\partial_1 P^e(\sigma, \sigma) < \partial_2 P^e(\sigma, \sigma) < 0$;

(b) $\partial_1 \Pi^e(\sigma, \sigma) + \partial_2 \Pi^e(\sigma, \sigma) \geq 0$ and $\partial_1 \Pi^e(\sigma, \sigma) > \partial_2 \Pi^e(\sigma, \sigma)$.

Part (a) asserts that increasing a uniform subsidy reduces quality-adjusted prices. Part (b) asserts that such an increase can only improve profit, and that a unilateral

[12] In the boundary case where $a_i = 1$, developers obtain zero profit and are thus indifferent about joining and pricing decisions; for the sake of exposition, we assume that they still join platform i and charge p^m.

increase in one platform's subsidy has a more beneficial impact on that platform than on its rival.[13]

• *Competition for apps.* In stage 1c, every app joins any platform for which it was developed. In stage 1b, given the commission rates (a_1, a_2) set in stage 1a, developers base their innovation decisions on expected consumer participation in the two platforms, which in turn depends on app development, and thus on the commission rates a_1 and a_2. Sticking to our stylized approach, we will assume that the joint distribution $F(\boldsymbol{k})$ of the development costs $\boldsymbol{k}$ generates a unique app development equilibrium, in which the number of apps developed on platform i is given by

$$y^* (a_i, a_j),$$

which satisfies

$$y^* (0, 0) > 0 = y^* (1, 1).$$

The resulting expected subsidy for platform i is then given by

$$\sigma_i = \sigma^* (a_i, a_j) \equiv (s^m + a_i \pi^m) y^* (a_i, a_j), \tag{2}$$

and thus satisfies:

$$\partial_2 \sigma^* (a_i, a_j) = (s^m + a_i \pi^m) \partial_2 y^* (a_i, a_j). \tag{3}$$

As long as $s^m + a_i \pi^m$,[14] the sign of $\partial_2 \sigma^*$ is thus the same as the sign of $\partial_2 y^*$: an increase in the rival's commission rate a_j reduces platform i's subsidy if and only if it reduces the number of apps developed on that platform.

Summing up, our stylized approach postulates the existence of a price function $P^e (\sigma_1, \sigma_2)$ (with associated profit $\Pi^e (\sigma_1, \sigma_2)$ given by (1)) satisfying Assumption 1 and of an app supply function $y^* (a_i, a_j)$, such that, for any given commission rates (a_1, a_2), there is a unique continuation equilibrium, in which:

- the number of apps developed on platform i is given by $y_i = y^* (a_i, a_j)$ (with associated subsidy $\sigma_i = \sigma^* (a_i, a_j)$ given by (2));
- platform i charges consumers a quality-adjusted price

$$P_i = P^* (a_i, a_j) \equiv P^e (\sigma^* (a_i, a_j), \sigma^* (a_j, a_i)).$$

[13]The second condition in Assumption 1b is automatically satisfied if the rival is harmed (in which case $\partial_1 \Pi^e (\sigma, \sigma) > 0 > \partial_2 \Pi^e (\sigma, \sigma)$).

[14]As we will see, this is indeed the case for the commission rates that maximize consumer surplus – see Lemma 1.

The number of consumers joining platform i is therefore given by

$$x_i = D^* (a_i, a_j) \equiv D (P^* (a_i, a_j), P^* (a_j, a_i)),$$

and platform i's profit is

$$\Pi_i = \Pi^* (a_i, a_j) \equiv \Pi^e (\sigma^* (a_i, a_j), \sigma^* (a_j, a_i)). \tag{4}$$

We now show that platform competition can lead to commission rates exceeding those that would maximize consumer surplus or total welfare.

3.1 Benchmarks

We first characterize the optimal commission rate a that a regulator would seek to impose in stage $1a$,[15] assuming that the platforms, developers and consumers subsequently take their decisions according to the timing described above. We will denote by

$$y (a) \equiv y^* (a, a) \quad \text{and} \quad \hat{\sigma} (a) \equiv \sigma^* (a, a) = (s^m + a\pi^m) \, y (a)$$

the equilibrium number of apps on each platform and the associated subsidy, and by

$$\hat{P} (a) \equiv P^e (\hat{\sigma} (a), \hat{\sigma} (a)) \quad \text{and} \quad \hat{D} (a) \equiv 2D \left(\hat{P} (a), \hat{P} (a)\right)$$

the resulting quality-adjusted price and demand.

We distinguish two cases, depending on whether the regulator focuses on consumer surplus or total welfare.

3.1.1 Consumer surplus

Suppose first that the regulator sets the commission rate a so as to maximize consumer surplus, and let a^S denote the optimal rate. Consumer surplus can be expressed as:[16]

$$\hat{S} (a) \equiv \int_{\hat{P}(a)}^{+\infty} 2D (P, P) \, dP. \tag{5}$$

Maximizing $\hat{S} (a)$ amounts to minimizing the quality-adjusted price $\hat{P} (a)$, which, from Assumption 1a, amounts in turn to maximizing the subsidy $\hat{\sigma} (a)$. It follows from

[15] For the sake of exposition, we focus on symmetric rates (i.e., $a_1 = a_2$). Given the symmetry of the setting, it is natural to do so; moreover, the regulator may be constrained by non-discrimination provisions. That the continuation equilibrium is unique implies that it is also symmetric.

[16] For a uniform price $P_1 = P_2 = P$, total demand is $2D (P, P) = 1 - \tilde{G}(P)$, where $\tilde{G} (\tilde{u})$ denotes the distribution of the maximal intrinsic value $\tilde{u} \equiv \max \{u_1, u_2\}$, and consumer surplus is given by $\tilde{S} (P) = \int_P^{+\infty} (\tilde{u} - P) \, d\tilde{G} (\tilde{u})$, which satisfies $\tilde{S}'(P) = -[1 - \tilde{G}(P)] = -2D(P, P)$.

Assumption 1b that it also maximizes the profit of the platforms, given by

$$\hat{\Pi}_P(a) \equiv 2\Pi^e(\hat{\sigma}(a), \hat{\sigma}(a)). \tag{6}$$

Building on this leads to:

Lemma 1 *The commission rate that maximizes consumer surplus, a^S, also maximizes the platforms' subsidy, $\hat{\sigma}(a)$, as well as their profit, $\hat{\Pi}_P(a)$; it moreover satisfies:*

$$s^m + a^S \pi^m = \pi^m \frac{y(a^S)}{-y'(a^S)} > 0. \tag{7}$$

Proof. See Appendix A. ■

The interest of the platforms is therefore *aligned* with that of consumers: they both want to maximize the subsidy $\hat{\sigma}(a)$, as doing so maximizes the profit of the platforms *and* minimizes the quality-adjusted prices. For the sake of exposition, in what follows we will assume that a^S is uniquely characterized by (7).[17]

3.1.2 Total welfare

Suppose now that the regulator seeks to maximize total welfare, given by

$$\hat{W}(a) \equiv \hat{S}(a) + \hat{\Pi}_P(a) + \hat{\Pi}_D(a), \tag{8}$$

where the consumer surplus $\hat{S}(a)$ and the platforms' profit $\hat{\Pi}_P(a)$ are respectively given by (5) and (6), and the profit of the developers can be expressed as:

$$\hat{\Pi}_D(a) \equiv \int_{\mathbb{R}^3_+} \pi_D(\hat{r}(a), \mathbf{k})\, dF(\mathbf{k}), \tag{9}$$

where

$$\hat{r}(a) \equiv (1-a)\pi^m \hat{D}(a)$$

denotes the revenue that a developer can obtain by joining both platforms, and

$$\pi_D(r, \mathbf{k}) \equiv \max\left\{0, \frac{r}{2} - k_1, \frac{r}{2} - k_2, r - k\right\}$$

denotes the equilibrium profit of a developer with cost realization $\mathbf{k} = (k_1, k_2, k_3)$. As noted above, the commission rate a^S, which maximizes the subsidy $\hat{\sigma}(a)$, maximizes $\hat{S}(a)$ and $\hat{\Pi}_P(a)$ as well. Developers would instead favor lower rates, so as to boost

[17] In case of multiple solutions, Lemma 2 holds for any of them, including the lowest one.

their revenue, $\hat{r}(a)$. It follows that the welfare-maximizing commission rate, which we will denote by a^W, lies below a^S:[18]

Lemma 2 (total welfare) *The commission rate that maximizes total welfare, a^W, is such that $a^W < a^S$, and it satisfies:*

$$s^m + a^W \pi^m = \frac{\hat{P}\left(a^W\right) + (s^m + \pi^m) y(a^W)}{-y'\left(a^W\right)} \frac{\hat{D}'\left(a^W\right)}{\hat{D}\left(a^W\right)}. \tag{10}$$

Proof. See Appendix B. ■

3.2 Platform competition

We now show that competition between the platforms can lead to excessively high commission rates. To this end, we complete our stylized approach by assuming that, in stage 1a, the commission-setting game with payoffs $\Pi^*(a_i, a_j)$ (given by (4)) is "well-behaved", namely:

Assumption 2 (commission-setting game):

(a) $\Pi^*(a_1, a_2)$ is strictly quasi-concave in its first argument;

(b) $R(a) \equiv \arg\max_{\tilde{a}} \Pi^*(\tilde{a}, a)$ is differentiable and has a unique fixed point, a^C, which satisfies $\left|R'\left(a^C\right)\right| < 1$.

Part (a) ensures that the platforms have a unique best-response, $R(\cdot)$; part (b) ensures in turn that there exists a unique, locally stable equilibrium, in which $a_1 = a_2 = a^C$. Our first proposition shows that the comparison between this equilibrium commission rate and what would maximize consumer surplus hinges on a simple condition:

Proposition 1 (platform competition) *Platform competition yields higher (resp., lower) commissions than those maximizing consumer surplus whenever raising one commission reduces (resp., increases) the number of apps available on the rival platform. Formally, $a^C \gtreqless a^S$ if and only if $\partial_2 y^*\left(a^S, a^S\right) \lesseqgtr 0$.*

[18] The commission rates may also affect developers' revenue, $\hat{r}(a) = (1-a)\pi^m \hat{D}(a)$, through the consumer participation $\hat{D}(a)$; departing from a^S reduces the platforms' subsidies, which curbs the competition for consumers and depresses total participation whenever it is elastic. This additional effect further calls against raising a above a^S; by contrast, a slight reduction below a^S has only a second-order negative effect on subsidies and consumer participation, which is thus dominated by the direct positive impact on the developers' revenue.

Proof. See Appendix C. ■

Whether competition yields higher or lower commission rates than those maximizing consumer surplus thus simply depends on whether app development on the two platforms entails complementarity or substitutability. In case of *complementarity*, that is, if raising one commission *reduces* the number of apps present on the rival platform, competition generates higher rates than what would maximize consumer surplus—and welfare, as $a^S > a^W$. In case of *substitutability*, that is, if raising one commission *fosters* app development on the rival platform, competition generates lower rates than those maximizing consumer surplus—they may however still exceed those maximizing total welfare.

As the commission rate a^S maximizes both consumer surplus, $\hat{S}(a)$, *and* platforms' profits, $\hat{\Pi}_P(a)$, Proposition 1 shows that, as long as a platform's commission does not affect app development on the rival platform, competition induces each platform to maximize the sum of its profit and of the surplus of single-homing agents, as shown by Armstrong (2006, Proposition 4) for the case of competitive bottlenecks. However, this no longer holds when the commission charged by one platform exerts an externality on the apps available on the rival platform.

Remark 1 (robustness to pass-through) *It is easy to see that the result of the proposition is robust to introducing a positive marginal cost for apps such that developers pass through a higher commission rate into a higher app price. Introducing a positive marginal cost affects the subsidy of platform i, $\sigma_i \equiv (s^m + a_i \pi^m) y_i$, by making the consumer surplus s^m and the developer profit π^m depend on the platform's commission a_i. However, platform i's commission affects the rival's subsidy only through its number of apps. Therefore, introducing a positive marginal cost affects neither the result that a^S maximizes both consumer surplus $\hat{S}(a)$ and the platforms' profits $\hat{\Pi}_P(a)$ nor the result that $a^C \gtreqqless a^S$ if and only if $\partial_2 y^* (a^S, a^S) \lesseqqgtr 0$.*

3.3 Discussion

The above analysis highlights a key factor, namely, the effect that a commission has on the number of apps available on the rival platform.[19] To assess this effect, suppose that, starting from $a_1 = a_2 = a^S$, platform 1 slightly raises its commission by $da_1 > 0$. Initially, each platform attracts $x^S \equiv \hat{D}(a^S)$ consumers, offers developers a revenue equal to $r^S \equiv \hat{r}(a^S) = (1 - a^S)\pi^m x^S$, and as a result attracts $y^S \equiv y(a^S)$ apps.

[19] Within our stylized model, the consumer value generated by a platform's app base only depends on the number of available apps. More generally, we would expect the quality and diversity of apps to matter as well, both within and across app categories.

Raising a_1 can affect app development in two ways: directly, by reducing the revenue offered by platform 1, and indirectly, by altering consumer participation. However, as shown in Appendix D, the latter effect tends to reinforce the former.[20] We thus focus here on the direct impact, assuming that each platform keeps attracting x^S consumers.

Initially, a developer facing innovation costs $\mathbf{k} = (k_1, k_2, k)$ obtains a net payoff equal to $r^S - k_i$ if it develops its app specifically for platform $i = 1, 2$, and $2r^S - k$ if it develops it for both platforms; it therefore multihomes if $2r^S - k > \max\left\{r^S - k_1, r^S - k_2, 0\right\}$, single-homes on platform i if $r^S - k_i > \max\left\{2r^S - k, r^S - k_j, 0\right\}$, and otherwise refrains from developing the app. Raising a_1 reduces the revenue offered by platform 1 by $dr_1 \equiv \pi^m x^S da_1$, and has no direct impact on the revenue offered by platform 2; it therefore reduces the payoff from multihoming and single-homing on platform 1 by dr_1, and leaves unchanged the revenue from single-homing on platform 2. Hence, it cannot affect developers' choice between multihoming and single-homing on platform 1, but may discourage multihoming and/or induce a switch to single-homing on platform 2. Specifically:

- If the developer initially multihomes (which occurs if $k < r^S + \min\left\{k_1, k_2, r^S\right\}$), then either: (i) it keeps doing so (namely, if $k < r^S + \min\left\{k_1, k_2 - dr_1, r^S - dr_1\right\}$) or focuses on platform 2 (if $k_2 < \min\left\{k + dr_1 - r^S, k_1 + dr_1, r^S\right\}$), in which case there is no impact on y_2; or (ii) it stops developing entirely (if $\min\{k_1 + dr_1, k_2, (k + dr_1)/2\} > r^S$), in which case there is a *negative* impact on y_2.

- If instead the developer initially single-homes on platform 2 (which occurs if $k_2 < \min\left\{k_1, r^S, k - r^S\right\}$), then it keeps doing so (as its revenue is unchanged, and the revenues from multihoming or switching to platform 1 are reduced by dr_1); hence, there is no impact on y_2.

- Finally, if the developer initially single-homes on platform 1 (which occurs if $k_1 < \min\left\{k_2, r^S, k - r^S\right\}$), then either: (i) it keeps doing so (if $k_1 < \min\left\{k_2, r^S\right\} - dr_1$), in which case there is again no impact on y_2; or (ii) it switches to platform 2 (if $k_2 < \min\left\{k_1 + dr_1, r^S\right\}$), in which case there is a *positive* impact on y_2.

Summing up, raising platform 1's commission can have two conflicting effects on the app base of its rival: it may reduce y_2 by discouraging multihomers from developing their apps altogether, and may instead increase y_2 by inducing single-homers to switch

[20] The intuition is as follows. Suppose for example that raising a_1 has a negative direct impact on y_2 (i.e., $dy_2 < 0$). This reduces σ_2 by $d\sigma_2 = (s^m + a^S \pi^m) dy_2$ and increases σ_1 by $d\sigma_1 = -d\sigma_2$ (as by construction, starting from $a_1 = a_2 = a^S$, $d\sigma_1 + d\sigma_2 = 0$), which in turn induces platform 2 to increase its price, and platform 1 to lower its own price. As a result, consumer participation decreases on platform 2, and increases on platform 1, which further incentivizes some app developers to switch from platform 2 to platform 1.

from from platform 1 to platform 2. Hence, the overall net impact depends on the relative importance of multihoming versus single-homing apps. As already noted, in practice most popular apps are multihoming, which suggests that the negative effect is likely to prevail.

Remark 2 (scale economies) *It is worth noting that discouraging a multihoming developer requires $k < r^S + k_1$ (to induce multihoming for $a_1 = a^S$) and $r^S < k_2$ (to rule out single-homing on platform 2 for $a_1 > a^S$), which together implies:*

$$k < k_1 + k_2.$$

That is, the app development must in this case entail economies of scale. By contrast, inducing a single-homing developer to switch from platform 1 to platform 2 requires $k > k_1 + r^S$ (to prevent multihoming) and $r^S > k_2$ (to ensure that single-homing on platform 2 is profitable), which together implies:

$$k > k_1 + k_2.$$

That is, the app development must in that case entail diseconomies of scale.[21]

4 Illustration

We now illustrate the above insights using a classic horizontal differentiation setting on the consumer side, and specialized developers—in terms of single- or multihoming—on the app side.

• *Consumers.* The two platforms are located at the two ends of a unit-length Hotelling segment, along which consumers are uniformly distributed. Upon joining a platform, a consumer obtains an intrinsic utility $u_0 > 0$ and faces a transportation cost $t > 0$ per unit of distance. As in Armstrong (1998) and Laffont, Rey and Tirole (1998a,b), u_0 is supposed to be large enough to ensure that all consumers join a platform (full participation). Consumers know their locations before joining a platform and, from the above, anticipate an expected surplus s^m from each app present on the platform. Hence, if platform i charges consumers a price p_i and attracts y_i apps, joining that platform gives a consumer located at distance x_i a net payoff equal to:

$$u_0 + s^m y_i - p_i - tx.$$

[21] In practice, apps are often developed for one platform and then ported on the other platforms, at a cost that is presumably lower than the initial development cost; we would thus expect some scale economies. The portability decision is taken later on, and typically depends on the success of the app; we explore the implications in Section 4.3.

The consumer indifferent between the two platforms is located at a distance x from platform i (and, thus a distance $1-x$ from platform j, for $i \neq j \in \{1,2\}$) equal to:

$$\frac{1}{2} + \frac{s^m (y_i - y_j) - (p_i - p_j)}{2t}.$$

Using the quality-adjusted price $P_i = p_i - s^m y_i$, the demand for platform i is therefore given by:

$$D\left(P_i, P_j\right) = \frac{1}{2} - \frac{P_i - P_j}{2t}.$$

As before, its profit is equal to $\Pi_i = \left(P_i + \sigma_i\right) D\left(P_i, P_j\right)$, where $\sigma_i = \left(s^m + a_i \pi^m\right) y_i$. In stage 2, the platforms compete à la Hotelling with a marginal cost equal to $-\sigma_i$, which leads to:

Lemma 3 (Hotelling competition) *In stage 2, for any given subsidies (σ_1, σ_2), competition for consumers leads to $P^e\left(\sigma_i, \sigma_j\right) = P^H\left(\sigma_i, \sigma_j\right)$ and $\Pi^e\left(\sigma_i, \sigma_j\right) = \Pi^H\left(\sigma_i, \sigma_j\right)$, where*

$$P^H\left(\sigma_i, \sigma_j\right) \equiv t - \frac{2\sigma_i + \sigma_j}{3} \quad \textit{and} \quad \Pi^H\left(\sigma_i, \sigma_j\right) \equiv \frac{1}{2t}\left(t + \frac{\sigma_i - \sigma_j}{3}\right)^2.$$

Proof. See Appendix E. ■

It is straightforward to check that Assumption 1 is satisfied.[22]

• *Developers.* Some apps multihome whereas others single-home. To capture this in a simple way, we distinguish two types of app developers: a mass $\alpha \in (0,1)$ of *multihoming* developers and a mass $1-\alpha$ of *single-homing* developers for each platform—the maximal number of apps per platform is thus equal to 1.

Specifically, the multihoming developers can make their apps available on both platforms at cost k, drawn from a distribution with c.d.f. $H(\cdot)$ and density $h(\cdot) > 0$ over $[0, \infty)$.[23] The developers single-homing on platform i can instead develop their app at cost k_i, drawn from a distribution with c.d.f. $F(\cdot)$ and density $f(\cdot) > 0$ over $[0, \infty)$.

Remark 3 (interpretation) *This cost distribution is a particular case of the joint distribution $\hat{F}(\mathbf{k})$ introduced in Section 2, in which multihomers benefit from substantial scale economies (i.e., $k \ll k_1 + k_2$), and single-homers face instead substantially lower costs for a given platform (i.e., $k_i \ll k, k_j$). Alternatively, treating a "pair" of*

[22] We have $\partial_1 P^e(\cdot) = 2\partial_2 P^e(\cdot) = -2/3$ and $\partial_1 \Pi^e(\sigma, \sigma) = -\partial_2 \Pi^e(\sigma, \sigma) = 1/3$, and so $\partial_1 P^e(\cdot) + \partial_2 P^e(\cdot) < 0$, $\partial_1 \Pi^e(\sigma, \sigma) + \partial_2 \Pi^e(\sigma, \sigma) = 0$ and $\partial_1 \Pi^e(\sigma, \sigma) > (0 >) \partial_2 \Pi^e(\sigma, \sigma)$.

[23] Introducing a small cost of porting these apps from one platform to the other would not qualitatively affect the analysis.

single-homers dedicated to the two platforms as a single developer, this setting can be interpreted as featuring substantial scale economies for a fraction α of developers, and neither scale economies nor diseconomies (i.e., $k = k_1 + k_2$) for the other developers, who moreover face independently and symmetrically drawn platform-specific development costs. With the latter interpretation, the case $\alpha = 0$ corresponds to the model of Etro (2022).

We will denote by $\tilde{y}$ the number of multihoming apps, by $\hat{y}_i$ the number of apps single-homing on platform $i = 1, 2$, and by $y_i \equiv \tilde{y} + \hat{y}_i$ the total number of apps present on platform i.

To ensure the existence of a well-behaved equilibrium under competition, we will maintain the following assumption:

Assumption 3: The density functions are non-increasing: $h'(\cdot) \leq 0$ and $f'(\cdot) \leq 0$.

4.1 Benchmarks

If both platforms charge the same commission rate a, in the symmetric continuation equilibrium a developer can obtain $r(a) = (1-a)\pi^m$ when multihoming, and $r(a)/2$ when single-homing; hence, there are $y(a) = \hat{y}(a) + \tilde{y}(a)$ apps available on each platform, where

$$\tilde{y}(a) \equiv \alpha H\left((1-a)\pi^m\right) \tag{11}$$

is the number of multihoming apps, and

$$\hat{y}(a) \equiv (1-\alpha)F\left(\frac{(1-a)\pi^m}{2}\right) \tag{12}$$

is the number of single-homing apps. The resulting subsidy is

$$\hat{\sigma}(a) \equiv (s^m + a\pi^m)\, y(a),$$

leading to the (quality-adjusted) price and profit:

$$\hat{P}(a) \equiv P^H(\hat{\sigma}(a), \hat{\sigma}(a)) = t - \hat{\sigma}(a)$$

and

$$\hat{\Pi} \equiv \Pi^H(\hat{\sigma}(a), \hat{\sigma}(a)) = \frac{t}{2}.$$

Note that there is full pass-through: competition induces the platforms to pass on their entire app revenue to consumers; as a result, the platforms' profit does not depend on the commission rate.

As total consumer demand is inelastic (namely, $\hat{D}(a) = 1$), it follows from Lemma 2 that the welfare-maximizing commission rate a^W nullifies the subsidy: the first-order condition (10) boils down to $s^m + a^W \pi^m = 0$, implying $\hat{\sigma}\left(a^W\right) = 0$. It follows that $a^W = -s^m/\pi^m$; regardless of the proportion of multihoming, parameterized here by α, the social planner subsidizes the developers so as to align their profit per consumer, $(1 - a^W)\pi^m$, with the total surplus generated by their apps, $s^m + \pi^m$.

By contrast, the commission rate that maximizes consumer surplus, characterized by the first-order condition (7), may depend on α but always generates a positive subsidy.[24] Summing up, we have:

$$a^W = -\frac{s^m}{\pi^m} < a^S(\alpha) < 1 \text{ and } \hat{\sigma}\left(a^W\right) = 0 < \hat{\sigma}\left(a^S(\alpha)\right).$$

Example: uniform distribution. When development costs are uniformly distributed over $[0, 1]$ (i.e., $F(k) = H(k) = k$), the commission rate that maximizes consumer surplus is independent from α and equal to:

$$a^S = \frac{\pi^m - s^m}{2\pi^m} (\in (a^W, \frac{1}{2})). \tag{13}$$

Hence, $a^S \lesseqgtr 0$ if and only if $s^m \gtreqless \pi^m$.

Remark 4 (monopoly profit) *We have already noted that the commission rate a^S maximizes the profit of the platforms when they compete for consumers. Interestingly, in the Hotelling setting, $a = a^S$ also maximizes the monopoly profit that an integrated firm, operating both platforms, could obtain when exploiting consumers. This is because total demand is here inelastic, and therefore the same under competition and monopoly (as long as full participation remains optimal). It follows that the number of apps generated by a commission a is also the same in both situations; and as a monopolist can appropriate the consumer value generated by the apps, it finds it optimal to maximize $\hat{\sigma}(a)$.*[25]

4.2 Platform competition

We now show that competition between the platforms leads indeed to excessively high commission rates. As in Section 3, for any given commission rates (a_1, a_2) set in stage $1a$, let $y^*(a_i, a_j)$ denote the total number of apps available on platform i in

[24] See Appendix F.

[25] Given the unit demand and the absence of operating costs, the monopoly profit, which corresponds to the sum of the monopoly price on the consumer side and the platforms' revenue from apps, is equal to $u_0 - t/2 + \hat{\sigma}(a)$, and is thus maximal for a^S.

the continuation equilibrium, and $\sigma^*(a_i, a_j)$ denote the resulting expected subsidy for platform i, given by (2). Building on Lemma 3, platform i's expected demand satisfies

$$D^*(a_i, a_j) = \frac{1}{2} + \frac{\Delta^*(a_i, a_j)}{6t}, \tag{14}$$

where $\Delta^*(a_i, a_j) \equiv \sigma^*(a_i, a_j) - \sigma^*(a_j, a_i)$ denotes platform i's *subsidy advantage*. Furthermore, the number of single-homing apps on platform i is given by

$$\hat{y}^*(a_i, a_j) = (1-\alpha) F(\hat{\rho}^*(a_i, a_j)), \tag{15}$$

whereas the number of multihoming apps is equal to

$$\tilde{y}^*(a_1, a_2) = \alpha H(\hat{\rho}^*(a_1, a_2) + \hat{\rho}^*(a_2, a_1)), \tag{16}$$

where $\hat{\rho}^*(a_i, a_j)$ denotes the revenue from joining platform i, which satisfies

$$\hat{\rho}^*(a_i, a_j) = (1-a_i)\pi^m D^*(a_i, a_j). \tag{17}$$

Finally, the subsidy advantage can be expressed as

$$\Delta^*(a_i, a_j) = s^m[\hat{y}^*(a_i, a_j) - \hat{y}^*(a_j, a_i)] + \pi^m[a_i y^*(a_i, a_j) - a_j y^*(a_j, a_i)] \tag{18}$$

Together, equations (14) to (18) jointly characterize the continuation equilibrium.[26]

From Lemma 3, the continuation equilibrium profit of a platform increases with its subsidy advantage; hence, in stage $1a$, each platform i sets a_i so as to maximize $\Delta^*(a_i, a_j)$. Building on this, the following proposition establishes the existence of a unique equilibrium and highlights its key features.

Proposition 2 (illustration) *For t large enough, there exists a unique equilibrium, which is symmetric. Furthermore, whenever a symmetric equilibrium exists, it is unique and the equilibrium commission rate, $a^C(\alpha)$, is strictly increasing in α and such that:*

(i) $a^C(0) = a^S(0)$;

(ii) $a^C(\alpha) > a^S(\alpha)$ *for* $\alpha > 0$;

(iii) $a^C(1) = 1$.

[26] In particular, $y^*(a_i, a_j) = \hat{y}^*(a_i, a_j) + \tilde{y}^*(a_i, a_j)$; in addition, $\hat{y}^*(a, a) = \hat{y}(a)$ and $\tilde{y}^*(a, a) = \tilde{y}(a)$, where $\hat{y}(a)$ and $\tilde{y}(a)$ are respectively given by (12) and (11), $y^*(a, a) = y(a) = \hat{y}(a) + \tilde{y}(a)$ and $\sigma^*(a, a) = \hat{\sigma}(a) = (s^m + a\pi^m)\, y(a)$.

Proof. See Appendix G. ■

In the absence of multihoming (i.e., for $\alpha = 0$), as in Etro (2022), competition induces the platforms to adopt the commission rates that maximize consumer surplus. This is because the subsidy of rival platform j is then

$$\sigma_j = (s^m + a_j \pi^m) \hat{y}^* (a_j, a_i),$$

where, from (17) and (15), the number of apps $\hat{y}^* (a_j, a_i)$ of apps developed on the rival platform depends only on its own rate a_j and on its expected demand, $D_j^*(a_j, a_i)$, which, from (14), only depends on platform j's subsidy advantage $\Delta^* (a_j, a_i)$. However, by construction, in a symmetric equilibrium in which each platform maximizes its subsidy advantage, a small deviation in a_i has no first-order impact on $\Delta^* (a_j, a_i)$—and, thus, on $\hat{y}^* (a_j, a_i)$.[27] It follows that, when contemplating a deviation in a_i, platform i takes as given the rival's subsidy, and so maximizing its subsidy advantage boils down to maximizing its own subsidy $\sigma^* (a_i, a_j)$, leading in equilibrium to $a_1 = a_2 = a^S$.

This no longer holds in the presence of multihoming types. The rival's subsidy is then

$$\sigma_j = (s^m + a_j \pi^m) [\hat{y}^* (a_j, a_i) + \tilde{y}^* (a_j, a_i)],$$

where the number of multihoming apps, $\tilde{y}^* (a_j, a_i)$, depends not only on the rival's rate a_j and on its expected demand D_j, but is also directly affected by platform i's own commission rate. Each platform then has an additional incentive to raise its commission rate, as reducing the number of these apps decreases its rival's subsidy—all the more so when α is large.[28] In particular, when all developers multihome (i.e., $\alpha = 1$), platforms' incentives lead them to *choke off* entirely the development of (multihoming) apps: $a^C (1) = 1$, leading to $y_1 = y_2 = 0$—indeed, in that case, there cannot be any symmetric or asymmetric equilibrium without choke-off.[29] Interestingly, there is no longer a complete choke-off when platforms compete in wholesale prices or per-unit fees rather than commissions; this is because the former generate double-

[27] By construction, $\Delta^* (a_j, a_i) = -\Delta^* (a_i, a_j)$; hence, in a symmetric equilibrium in which each platform j maximizes its subsidy advantage $\Delta^* (a_j, a_i)$, we have $\partial_1 \Delta^* (a, a) = \partial_2 \Delta^* (a, a) = 0$.

[28] Evaluated at a symmetric equilibrium in which each platform maximizes its subsidy advantage (implying, as noted above, that a small change in a_i has no direct effect on market shares), the impact of a marginal increase in a_i on the number of multihoming apps is given by

$$\left. \frac{\partial \tilde{y}^* (a_1, a_2)}{\partial a_i} \right|_{a_1 = a_2 = a^C} = -\frac{\alpha}{2} \pi^m h (\tilde{\rho}^* (a, a)),$$

the absolute value of which increases with α.

[29] To see this, it suffices to note that platform i's advantage becomes $\Delta^* (a_i, a_j) = (a_i - a_j) \tilde{y} (a_1, a_2) \pi^m$. It follows that, starting from $a_i < a_j$, say, platform i would have an incentive to match its rival's subsidy; and starting from $a_1 = a_2 = a$, both platforms would have an incentive to raise their rates as long as their remains some app development, as $\partial_1 \Delta^* (a, a) = \tilde{y} (a, a) \pi^m$.

marginalization problems, which in turn act as a disciplining device.[30]

Recall that the commission rate that maximizes consumer surplus is always higher than the welfare-maximizing level; thus, for $\alpha = 0$ we have:

$$a^C(0) = a^S(0) > a^W,$$

and for any $\alpha > 0$, we have:

$$a^C(\alpha) > a^S(\alpha) > a^W.$$

The following example illustrates these insights.

Example: uniform distribution. When development costs are uniformly distributed over $[0, 1]$ (i.e., $F(k) = H(k) = k$), under competition the commission rate is equal to:

$$a^C(\alpha) = \frac{(1+\alpha)\pi^m - (1-\alpha)s^m}{2\pi^m},$$

which strictly increases from $a^S(0)$ to 1 as α increases from 0 to 1. Furthermore, the total number of apps on each platform is given by

$$y(a^C(\alpha)) = \frac{1-\alpha^2}{4}(\pi^m + s^m),$$

which strictly decreases to 0 as α increases. By contrast, as the commission rate that maximizes consumer surplus is independent of α (see (13)), the resulting number of apps, $y(a^S)$, *increases* with α. Figure 1 presents the values of interest for the commission rate for the case $\pi^m = 2s^m$. Figure 2 presents the corresponding number of apps.

These findings support neither the positions of Apple and Google nor the opposing views of competition authorities such as the CMA in the policy debate surrounding the commission rates charged by the app stores.[31] The two platforms argue that competition acts as a disciplining device, which prevents commission rates from being exces-

[30] Llobet and Padilla (2016) consider a setting in which upstream firms license their patents to a downstream firm. They compare ad valorem royalties with per-unit royalties and show that the former bring two welfare benefits. First, ad valorem royalties makes double marginalization less severe, not only between the downstream firm and its partners but also among upstream firms (the so-called royalty stacking problem), which leads to lower prices. This, in turn, gives all firms greater incentives to invest in complementary technologies. In our paper, ad valorem rates actually eliminate double marginalization entirely, because applications are digital goods with zero marginal costs. Interestingly, it is exactly this absence of double marginalization which harms consumers by choking off app development, thereby overturning the welfare comparison between ad valorem rates and wholesale prices.

[31] We thank Jorge Padilla for this comment.

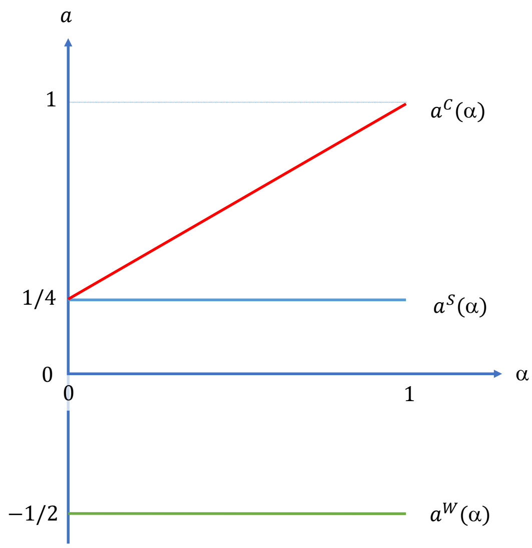

Figure 1: Commissions for a uniform distribution ($F(k) = H(k) = k$ and $\pi^m = 2s^m$)

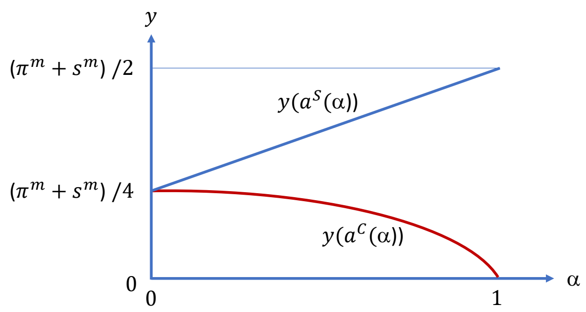

Figure 2: Number of apps for a uniform distribution ($F(k) = H(k) = k$ and $\pi^m = 2s^m$)

sive.[32] The CMA (2022) argues instead that, in practice, there is little competition.[33] Our analysis suggests instead that competition may actually create or exacerbate the

[32]For instance, Apple argued the 30 percent commission was determined in competitive conditions in 2008 and has not increased since then (CMA, 2022, p. 133). See also the responses of Apple and Google to the interim report of the CMA.

[33]"Overall, we consider that the lack of competition faced by the App Store and Play Store allows them to charge above a competitive rate of commission to app developers." (CMA, 2022, p.138)

problem.

4.3 Sequential development decisions

In practice, investing in app development is a risky venture, as the success of an app is uncertain. To mitigate this risk, developers initially develop their app for a given platform, and then port their app onto alternative platforms if it is sufficiently successful. We thus explore here a variant that accounts both for the uncertain nature of app success, and for the sequentiality of the development and porting decisions.

Specifically, we suppose that an app is successful with probability $\lambda \in (0,1)$. If successful, it generates a revenue of r_i on platform i, otherwise it generates a revenue ηr_i, where $\eta \in [0,1)$ is the fraction of consumers with a demand for the app. The success of an app is idiosyncratic and independent of the platform on which it is initially developed.[34] Let k_i denote the cost of developing an app for platform $i = 1, 2$, and δ denote the cost of porting an app to the other platform, which for the sake of exposition is set to be the same across developers and platforms. We assume that the development costs k_1 and k_2 are independently drawn from the same distribution with c.d.f. $F(\cdot)$ and density $f(\cdot)$, and that the porting cost δ always satisfies, for $i = 1, 2$:

$$\delta \in (\eta r_i, r_i) \, .$$

It follows that a developer ports its app if and only if it is successful.

If a developer facing development costs (k_1, k_2) develops an app for platform i and then ports it to platform j if successful, its profit is given by

$$\lambda(r_1 + r_2 - \delta) + (1 - \lambda)\eta r_i - k_i = R_i - k_i,$$

where $R_i \equiv \lambda(r_1 + r_2 - \delta) + (1 - \lambda)\eta r_i$. A developer thus chooses to develop its app for platform i if and only if:

$$R_i - k_i \geq \max \{R_j - k_j, 0\} \, .$$

To assess the impact of a platform's commission on its rival's app base, suppose that, starting from the equilibrium commissions (i.e., $a_1 = a_2 = a^C$), platform 1 slightly deviates and raises its commission by $da_1 > 0$. By construction, $a_1 = a^C$ maximizes platform 1's profit, given $a_2 = a^C$; in the Hotelling setting, this means that

[34] The analysis developed in the previous sections still applies, with the caveat that platform i's subsidy is now of the form $\sigma_i = (s^m + a_i \pi^m)\bar{y}_i$, where, denoting by $\check{y}_i$ the number of apps initially developed for platform i, $\bar{y}_i \equiv \lambda(\check{y}_1 + \check{y}_2) + (1 - \lambda)\eta \check{y}_i$ denotes the total number of apps eventually available on platform i, weighted by their popularity.

$a_1 = a^C$ maximizes platform 1's market share on the consumer side; it follows that the deviation has only a second-order effect on the platforms' consumer base. We can thus focus on the impact of da_1 on the platforms' app base through its direct impact on the revenues offered by the platforms—namely, r_1 is reduced by $dr_1 = \pi^m da_1/2$, whereas r_2 remains at its equilibrium value, $r^C \equiv a^C \pi^m/2$.

Consider first the polar cases where either η or λ is set to 0. If $\eta = 0$, then only successful apps generate value; all developed apps are therefore ported, and $R_1 = R_2 = \lambda(r_1 + r_2 - \delta)$. Hence, the number of apps available on both platforms is the same, and is given by $\tilde{F}(\lambda(r_1 + r_2 - \delta))$, where $\tilde{F}(\cdot)$ denotes the distribution of $\min\{k_1, k_2\}$. It follows that an increase in the commission of either platform has a *negative* impact on *both platforms'* app bases.

If instead $\lambda = 0$, then all apps single-home, and $R_i = \eta r_i$. A developer facing costs (k_1, k_2) then develops its app for platform i if and only if

$$k_i \leq \min\{k_j + \eta(r_i - r_j), \eta r_i\}.$$

Raising platform 1's commission, which lowers r_1 by dr_1, thus reduces the number of apps developed for platform 1, but *increases* the number of apps developed for *platform* 2.

Consider now the generic case where $\lambda\eta > 0$. In equilibrium, we have $R_1 = R_2 = R^C \equiv [2\lambda + (1-\lambda)\eta]\, r^C - \lambda\delta$. Hence, the number of apps initially developed on each platform is (see Figure 3):[35]

$$\bar{y}^C \equiv \int_0^{+\infty} F\left(\min\left\{k, R^C\right\}\right) f(k)\, dk.$$

Raising platform 1's commission reduces R_1 by $dR_1 = [\lambda + (1-\lambda)\eta]\, dr_1$ and R_2 by $dR_2 = \lambda dr_1$. This induces three changes, as illustrated by Figure 3:

a. $\left[1 - F\left(R^C\right)\right] f\left(R^C\right) dR_1$ developers, initially developing their apps for platform 1, drop out;

b. $\left[1 - F\left(R^C\right)\right] f\left(R^C\right) dR_2$ developers, initially developing their apps for platform 2, drop out as well;

c. finally, $\Phi\left(R^C\right)(dR_1 - dR_2)$ developers, where

$$\Phi(R) \equiv \int_0^R f^2(k)\, dk,$$

[35] For example, the app is developed for platform 1 for $k_1 < k_2$ if $k_2 \leq R^C$, and for $k_1 < R^C$ if instead $k_2 > R^C$.

which were initially developing their apps for platform 1, now switch to platform 2.

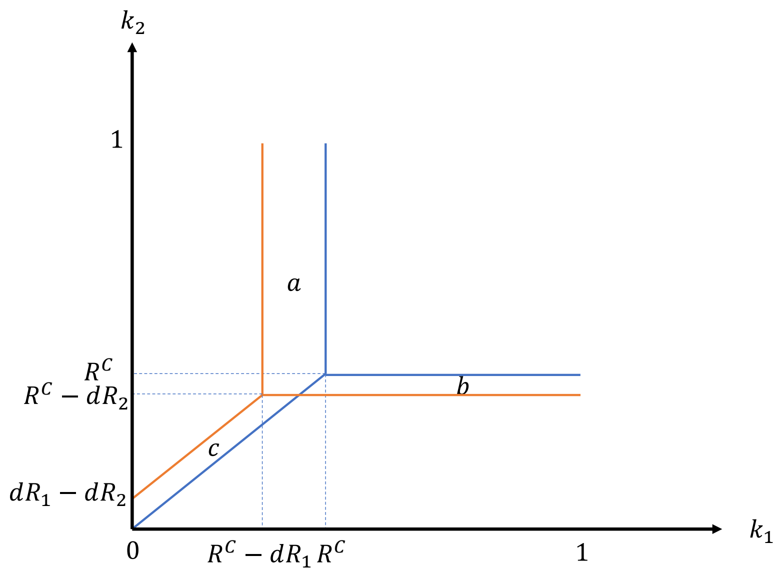

Figure 3: The effect of platform 1's commission on app development

Cases a and b represent a negative impact on platform 2, whereas case c represents instead a positive impact on that platform. In equilibrium, $2\lambda\bar{y}^C$ apps end up being successful and are eventually present on both platforms, and $(1-\lambda)\bar{y}^C$ unsuccessful apps have also been developed on each platform. Hence the total number of apps available on each platform, weighted by their popularity, is equal to

$$y^C \equiv [2\lambda + (1-\lambda)\eta]\,\bar{y}^C.$$

Building on the above, raising platform 1's by da_1 alters the weighted number of apps available on platform 2 by:

$$\begin{aligned} dy_2 &= \Phi\left(R^C\right)(dR_1 - dR_2) \times [\lambda + (1-\lambda)\,\eta] \\ &\quad - \left[1 - F\left(R^C\right)\right] f\left(R^C\right)\{dR_1 \times \lambda + dR_2 \times [\lambda + (1-\lambda)\,\eta]\} \\ &= \Phi\left(R^C\right)(1-\lambda)\eta dr_1 \times [\lambda + (1-\lambda)\,\eta] \\ &\quad - \left[1 - F\left(R^C\right)\right] f\left(R^C\right)\{[\lambda + (1-\lambda)\,\eta]\,dr_1 \times \lambda + \lambda dr_1 \times [\lambda + (1-\lambda)\,\eta]\} \\ &= \left\{(1-\lambda)\eta\Phi\left(R^C\right) - 2\lambda\left[1 - F\left(R^C\right)\right] f\left(R^C\right)\right\}[\lambda + (1-\lambda)\,\eta]\,dr_1. \end{aligned}$$

It follows that raising platform 1's commission reduces the weighted number of apps available on platform 2 if and only if:

$$\frac{2\lambda}{(1-\lambda)\,\eta} > \frac{\Phi\left(R^C\right)}{\left[1-F\left(R^C\right)\right] f\left(R^C\right)},$$

that is, if the rate of success λ is large enough and/or the revenue generated by unsuccessful apps is small enough (i.e., η low enough).

5 Concluding remarks

The main insight of this paper is that competition induces platforms to charge higher commissions than those maximizing consumer surplus—and, a fortiori, those maximizing total welfare—whenever raising the commission of one platform reduces the number of apps on rival platforms. Such negative externalities arise when a positive fraction of apps multihome—and in practice, a large majority of popular apps multihome on App Store and Google Play. This insight is illustrated in a particularly striking way in our model with a Hotelling-like full consumer participation, where platform competition leads to a complete choke-off of app development if all app developers multihome.

These insights suggest that platform competition between App Store and Google Play need not discipline the commission rates charged to developers. This is consistent with the remark by Judge Gonzalez Rogers to Apple's CEO Tim Cook in the legal battle between Epic and Apple:

"it doesn't seem to me that you feel under pressure or competition to actually change the manner in which you act to address the concerns of the developers".[36]

Indeed, as long as the cost of multihoming is smaller than the expected profit from joining an additional app store between App Store and Google Play, which should be the case for most popular apps, competition between the two platforms is likely to lead to excessively high commissions that stymie app development. This risk of underdevelopment is particularly strong in the case of cross-platform technology (such as cloud gaming), which is intrinsically a multihoming technology. Therefore, our results provide a rationale for regulating app store commissions as is specified by the recent Digital Markets Act in Europe, which requires FRAND access to app stores.[37]

[36] https://www.theverge.com/2021/5/21/22448023/epic-apple-fortnite-antitrust-lawsuit-judge-tim-cook-app-store-questions

[37] See Bisceglia and Tirole (2022) for a study of appropriate access prices.

Appendix

A Proof of Lemma 1

We already established that maximizing consumer surplus amounts to maximizing the subsidy $\hat{\sigma}(a)$. To conclude the proof, it suffices to note that the first-order condition yields:

$$0 = \hat{\sigma}'\left(a^S\right) = \left(s^m + a^S \pi^m\right) y'\left(a^S\right) + \pi^m y\left(a^S\right).$$

As $\hat{\sigma}(0) = s^m y(0) > 0$, the optimal commission is such that $\hat{\sigma}\left(a^S\right) = \left(s^m + a^S \pi^m\right) y\left(a^S\right) > 0$, thus implying $y\left(a^S\right) > 0$ (as by construction $y(\cdot) \geq 0$) and, thus, $s^m + a^S \pi^m > 0$. It then follows from the above equation that $y'\left(a^S\right) = -\pi^m y\left(a^S\right) / \left(s^m + a^S \pi^m\right) < 0$; dividing the above equation by $y'\left(a^S\right)$ then leads to (7).

B Proof of Lemma 2

Let

$$s_i(\mathbf{k}) \equiv 1_{\left\{\max\left\{0, \frac{r}{2} - k_1, \frac{r}{2} - k_2, r - k\right\} = \frac{r}{2} - k_i\right\}}$$

denote the indicator function for developers single-homing on platform i, and

$$m(\mathbf{k}) \equiv 1_{\left\{\max\left\{0, \frac{r}{2} - k_1, \frac{r}{2} - k_2, r - k\right\} = r - k\right\}}$$

denote that for multi-homing developers. The developers' profit, given by (9), can be expressed as:

$$\begin{aligned} \hat{\Pi}_D(a) &= \int_{\mathbb{R}^3_+} \left[\frac{\hat{r}(a)}{2} - k_1\right] s_1(\mathbf{k})\, dF(\mathbf{k}) + \int_{\mathbb{R}^3_+} \left[\frac{\hat{r}(a)}{2} - k_2\right] s_2(\mathbf{k})\, dF(\mathbf{k}) \\ &\quad + \int_{\mathbb{R}^3_+} \left[\hat{r}(a) - k\right] m(\mathbf{k})\, dF(\mathbf{k}), \end{aligned}$$

and its derivative is given by:[1]

$$\hat{\Pi}'_D(a) = \frac{\hat{r}'(a)}{2} \int_{\mathbb{R}^3_+} s_1(\mathbf{k})\, dF(\mathbf{k}) + \frac{\hat{r}'(a)}{2} \int_{\mathbb{R}^3_+} s_2(\mathbf{k})\, dF(\mathbf{k}) + \hat{r}'(a) \int_{\mathbb{R}^3_+} m(\mathbf{k})\, dF(\mathbf{k}),$$

[1]The expression captures the negative impact of an increase in a on the revenue $\hat{r}(a)$ of the infra-marginal developers (i.e., those who keep developing their apps). Raising a also induces some marginal developers to drop out, and may induce switches between multihoming and single-homing, and/or induce single-homing developers to switch platforms; however, these additional marginal impacts have zero first-order effect.

where the integrals are respectively equal to the number of developers single-homing on platforms 1 and 2, and to the number of multihoming developers. It follows that:

$$\hat{\Pi}_D'(a) = \hat{r}'(a)\left[\frac{1}{2}\times\hat{y}(a)+\frac{1}{2}\times\hat{y}(a)+\tilde{y}(a)\right] = \hat{r}'(a)\,y(a),$$

where $\hat{y}(a)$ denotes the number of single-homing apps, $\tilde{y}(a)$ the number of multihoming apps, and $y(a) = \hat{y}(a)+\tilde{y}(a)$. By construction $\hat{\sigma}'\left(a^S\right) = 0$, implying $\hat{D}'\left(a^S\right) = \hat{P}'\left(a^S\right) = 0$; hence:

$$\hat{\Pi}_D'\left(a^S\right) = -\pi^m y(a)\,\hat{D}(a) < 0. \tag{19}$$

Total welfare is given by (8), and its derivative is thus equal to:

$$\hat{W}'\left(a^S\right) = \hat{S}'\left(a^S\right)+\hat{\Pi}_P'\left(a^S\right)+\hat{\Pi}_D'\left(a^S\right) = \hat{\Pi}_D'\left(a^S\right) < 0, \tag{20}$$

where the second equality follows from a^S maximizing both $\hat{S}(a)$ and $\hat{\Pi}_P(a)$, and the inequality from (19). Furthermore, for any $a > a^S$:

- $\hat{S}(a) \le \hat{S}\left(a^S\right)$ and $\hat{\Pi}_P(a) \le \hat{\Pi}_P\left(a^S\right)$;
- $\hat{\sigma}(a) < \hat{\sigma}\left(a^S\right)$, implying $\hat{P}(a) > \hat{P}\left(a^S\right)$ and thus $\hat{D}(a) \le \hat{D}\left(a^S\right)$; hence:[2]

$$\hat{r}(a) = (1-a)\,\pi^m\hat{D}(a) < \left(1-a^S\right)\pi^m\hat{D}\left(a^S\right) = \hat{r}\left(a^S\right),$$

  which in turn implies $\hat{\Pi}_D(a) < \hat{\Pi}_D\left(a^S\right)$.

It follows that $\hat{W}(a) < \hat{W}\left(a^S\right)$ for any $a > a^S$; together with (20), this implies $a^W < a^S$.

The derivative of $\hat{W}(a)$ can be expressed as:

$$\begin{aligned}
\hat{W}'(a) &= \hat{S}'(a)+\hat{\Pi}_P'(a)+\hat{\Pi}_D'(a)\\
&= \left[-\hat{D}(a)\,\hat{P}'(a)\right]+\left\{\left[\hat{P}'(a)+\hat{\sigma}'(a)\right]\hat{D}(a)+\left[\hat{P}(a)+\hat{\sigma}(a)\right]\hat{D}'(a)\right\}+\hat{r}'(a)\,y(a)\\
&= \hat{\sigma}'(a)\,\hat{D}(a)+\left[\hat{P}(a)+\hat{\sigma}(a)\right]\hat{D}'(a)+\hat{r}'(a)\,y(a)\\
&= \left[\pi^m y(a)+(s^m+a\pi^m)\,y'(a)\right]\hat{D}(a)\\
&\quad+\left[\hat{P}(a)+(s^m+a\pi^m)\,y(a)\right]\hat{D}'(a)+\pi^m y(a)(1-a)\,\hat{D}'(a)-\pi^m y(a)\,\hat{D}(a)\\
&= (s^m+a\pi^m)\,y'(a)\,\hat{D}(a)+\left[\hat{P}(a)+(s^m+\pi^m)\,y(a)\right]\hat{D}'(a).
\end{aligned}$$

[2] If $a < 1$, the inequality follows from $1-a^S > 1-a$ and $\hat{D}\left(a^S\right) \ge \hat{D}(a)$. If instead $a \ge 1$, it follows from $\left(1-a^S\right)\pi^m\hat{D}\left(a^S\right) > 0 \ge (1-a)\,\pi^m\hat{D}(a)$.

The first-order condition therefore amounts to:

$$\left(s^m + a^W \pi^m\right) y'\left(a^W\right) \hat{D}\left(a^W\right) = -\left[\hat{P}\left(a^W\right) + \left(s^m + \pi^m\right) y\left(a^W\right)\right] \hat{D}'\left(a^W\right).$$

If $\hat{D}'\left(a^W\right) = 0$, then $y'\left(a^W\right) < 0$, as a slight increase in a has then no impact on demand, and strictly reduces the developers' share of revenue. And if instead $\hat{D}'\left(a^W\right) \neq 0$, then the first-order condition implies that $y'\left(a^W\right) \neq 0$. Dividing by $y'\left(a^W\right)$ then leads to (10).

C Proof of Proposition 1

Assumption 2b (namely, equilibrium uniqueness and local stability) implies that $a^* > a^S$ if and only $R\left(a^S\right) > a^S$; Assumption 2a (namely, strict quasi-concavity) ensures in turn that $R\left(a^S\right) > a^S$ if and only if $\partial_1 \Pi^*\left(a^S, a^S\right) > 0$. Furthermore (with $\sigma^S = \sigma^*\left(a^S, a^S\right)$):

$$\begin{aligned}
\partial_1 \Pi^*\left(a^S, a^S\right) &= \partial_1 \Pi^e\left(\sigma^S, \sigma^S\right) \partial_1 \sigma^*\left(a^S, a^S\right) + \partial_2 \Pi^e\left(\sigma^S, \sigma^S\right) \partial_2 \sigma^*\left(a^S, a^S\right) \\
&= -\left[\partial_1 \Pi^e\left(\sigma^S, \sigma^S\right) - \partial_2 \Pi^e\left(\sigma^S, \sigma^S\right)\right] \partial_2 \sigma^*\left(a^S, a^S\right) \\
&= -\left[\partial_1 \Pi^e\left(\sigma^S, \sigma^S\right) - \partial_2 \Pi^e\left(\sigma^S, \sigma^S\right)\right] \left(s^m + a^S \pi^m\right) \partial_2 y^*\left(a^S, a^S\right),
\end{aligned}$$

where the second equality stems from the first-order condition

$$0 = \hat{\sigma}'\left(a^S\right) = \partial_1 \sigma^*\left(a^S, a^S\right) + \partial_2 \sigma^*\left(a^S, a^S\right),$$

and the last equality follows from (3). From Assumption 1b, $\partial_1 \Pi^e\left(\sigma^S, \sigma^S\right) > \partial_2 \Pi^e\left(\sigma^S, \sigma^S\right)$. It follows that $\partial_1 \Pi^*\left(a^S, a^S\right) > 0$ if and only if $\partial_2 y^*\left(a^S, a^S\right) < 0$, which concludes the argument. A similar reasoning establishes that $a^* < a^S$ (resp., $a^* = a^S$) if and only if $\partial_2 y^*\left(a^S, a^S\right) > 0$ (resp., $\partial_2 y^*\left(a^S, a^S\right) = 0$).

D Impact of consumer participation on app development

We study here the indirect impact of consumer participation on app development. Specifically, starting from $a_1 = a_2 = a^S$, consider a slight increase in platform 1's commission by $da_1 > 0$. Recall that, initially, each platform attracts $x^S \equiv \hat{D}\left(a^S\right)$ consumers, and offers developers a revenue equal to $r^S \equiv \hat{r}\left(a^S\right)$; as a result, it attracts $y^S \equiv y\left(a^S\right)$ developers. Following the change in platform 1's commission, platform 1 offers a revenue $r_1 = (1 - a_1)\,\pi^m x_1$ and platform 2 offers a revenue $r_2 = \left(1 - a^S\right)\pi^m x_2$, where the platforms' consumer bases are given by

$$x_1 = D\left(P_1, P_2\right) \text{ and } x_2 = D\left(P_2, P_1\right),$$

where

$$\begin{aligned} P_1 &= P^*\left(a_1, a^S\right) = P^e\left(\sigma^*\left(a_1, a^S\right), \sigma^*\left(a^S, a_1\right)\right), \\ P_2 &= P^*\left(a^S, a_1\right) = P^e\left(\sigma^*\left(a^S, a_1\right), \sigma^*\left(a_1, a^S\right)\right). \end{aligned}$$

By construction,

$$d\sigma_2 = \left(s^m + a^S \pi^m\right) dy_2.$$

Furthermore, because a^S maximizes $\hat{\sigma}(a) = \sigma^*(a, a)$, we have:

$$d\sigma_1 = \partial_1 \sigma^*\left(a^S, a^S\right) da_1 = -\partial_2 \sigma^*\left(a^S, a^S\right) da_1 = -d\sigma_2.$$

It follows that:

$$dP_2 = \partial_1 P^e(\cdot)\, d\sigma_2 + \partial_2 P^e(\cdot)\, d\sigma_1 = \left[\partial_1 P^e(\cdot) - \partial_2 P^e(\cdot)\right] d\sigma_2,$$

where $\partial_i P^e(\cdot)$ is evaluated at $\sigma_1 = \sigma_2 = \hat{\sigma}\left(a^S\right)$, and

$$dP_1 = \partial_1 P^e(\cdot)\, d\sigma_1 + \partial_2 P^e(\cdot)\, d\sigma_2 = \left[\partial_2 P^e(\cdot) - \partial_1 P^e(\cdot)\right] d\sigma_2 = -dP_2.$$

Therefore:

$$\begin{aligned} dx_2 &= \partial_1 D(\cdot)\, dP_2 + \partial_2 D(\cdot)\, dP_1 \\ &= \left[\partial_1 D(\cdot) - \partial_2 D(\cdot)\right] dP_2 \\ &= \left[\partial_1 D(\cdot) - \partial_2 D(\cdot)\right]\left[\partial_1 P^e(\cdot) - \partial_2 P^e(\cdot)\right] d\sigma_2, \end{aligned}$$

and

$$dx_2 = -dx_1.$$

The overall impact of the change da_1 on platform 2's app base, dy_2, can be decomposed as $dy_2 = d\hat{y}_2 + d\tilde{y}_2$, where $d\hat{y}_2$ is the effect on platform 2's app base stemming from the direct impact of da_1 on the revenue offered by platform 1,

$$d\hat{r}_1 = -da_1 \pi^m x^S,$$

and $d\tilde{y}_2$ is the effect stemming from the indirect impact of da_1 on the revenues offered by the two platforms, through the induced change in the platforms' consumer bases:

$$d\tilde{r}_1 = \left(1 - a^S\right) \pi^m dx_1 \text{ and } d\tilde{r}_2 = \left(1 - a^S\right) \pi^m dx_2.$$

It follows from the above that $dx_1 + dx_2 = 0$: starting from $a_1 = a_2 = a^S$, increasing a_1 by da_1 has no first-order effect on total consumer participation; hence, $d\tilde{r}_1 + d\tilde{r}_2 = 0$: the impact of da_1 on the platforms' consumer base does not affect the developers' revenue from multihoming, as the total consumer base is unaffected. The impact on each platform's consumer base however affects the revenue from single-homing. We now argue that this change will reinforce the effect of da_1 on platform 2's app base through its direct impact on the revenue offered by platform 1, $d\hat{y}_2$. To see this, consider first the impact of $d\hat{y}_2$ on consumer participation. If $d\hat{y}_2 < 0$, then it decreases σ_2 by $d\hat{\sigma}_2 = \left(s^m + a^S \pi^m\right) d\hat{y}_2 \,(< 0)$ and increases σ_1 by $d\hat{\sigma}_1 = -d\hat{\sigma}_2 \,(> 0)$; this, in turn, induces platform 2 to increase its quality-adjusted price, and platform 1 to decrease its own price: as $\partial_1 P^e(\cdot) < \partial_2 P^e(\cdot)$ from Assumption 1(a), $d\hat{P}_2 = \left[\partial_1 P^e(\cdot) - \partial_2 P^e(\cdot)\right] d\hat{\sigma}_2 > 0$ and $d\hat{P}_1 = -d\hat{P}_2 < 0$. As a result, platform 2 attracts fewer consumers, whereas platform 1 expands instead its consumer base: $d\hat{x}_2 = \left[\partial_1 D(\cdot) - \partial_2 D(\cdot)\right] d\hat{P}_2 < 0$ and $d\hat{x}_1 = -d\hat{x}_2 > 0$. In other words, if the change $da_1 > 0$ has a *negative* direct impact on platform 2's app base (i.e., $d\hat{y}_2 < 0$), then the induced change in consumer participation *reduces* the revenue offered by platform 2 (as we then have $\left(1 - a^S\right) \pi^m d\hat{x}_2 < 0$) and *enhances* instead the revenue offered by platform 1 (as $\left(1 - a^S\right) \pi^m d\hat{x}_1 > 0$). This would further incentivize some of the developers initially single-homing on platform 2 (namely, those facing similar innovation costs for the two platforms) to switch to platform 1, thereby reinforcing the effect of da_1 on platform 2's app base through its direct impact on the revenue offered by platform 1.

E Proof of Lemma 3

The profit $\Pi_i = (P_i + \sigma_i) D(P_i, P_j)$ is strictly quasi-concave in P_i and maximal for

$$P_i = R(P_j) \equiv \frac{t - \sigma_i + P_j}{2}.$$

As this best-response has a slope lower than 1, the usual tâtonnement process converges towards a unique, stable equilibrium, in which each platform i charges $P_i = P^H(\sigma_i, \sigma_j)$, leading to

$$P_i + \sigma_i = 2tD(P_i, P_j) = t + \frac{\sigma_i - \sigma_j}{3},$$

and, thus, to $\Pi_i = \Pi^H(\sigma_i, \sigma_j)$.

F Illustration: consumer surplus benchmark

The first-order condition (7) amounts to:

$$\left(s^m + a^S\pi^m\right) y'\left(a^S\right) + \pi^m y\left(a^S\right) = 0,$$

where, from (12) and (11):

$$y\left(a\right) = (1-\alpha)F\left(\frac{(1-a)\pi^m}{2}\right) + \alpha H\left(\left(1-a\right)\pi^m\right),$$

and

$$y'\left(a\right) = -\frac{1-\alpha}{2} f\left(\frac{(1-a)\pi^m}{2}\right)\pi^m - \alpha h\left(\left(1-a\right)\pi^m\right)\pi^m.$$

Re-arranging, the first-order condition can be expressed as $\phi\left(a^S;\alpha\right) = 0$, where:

$$\begin{aligned}\phi\left(a;\alpha\right) \equiv{} & \left[F\left(\frac{(1-a)\pi^m}{2}\right) + \frac{\alpha}{1-\alpha}H\left(\left(1-a\right)\pi^m\right)\right] \\ & - \left(s^m + a^S\pi^m\right)\left[\frac{1}{2}f\left(\frac{(1-a)\pi^m}{2}\right) + \frac{\alpha}{1-\alpha}h((1-a)\pi^m)\right].\end{aligned}$$

For any $a < 1$, the two bracketed terms are both positive, and so the first-order condition implies $s^m + a^S\pi^m > 0$. Furthermore, the first one is strictly decreasing in a whereas the second one is strictly increasing in a under Assumption 3, and so $\phi\left(a;\alpha\right)$ is also strictly decreasing in a in the relevant range where $s^m + a\pi^m > 0$. In addition:

$$\begin{aligned}\phi\left(a^W;\alpha\right) &= F\left(\left(1-a^W\right)\frac{\pi^m}{2}\right) + \frac{\alpha}{1-\alpha}H\left(\left(1-a^W\right)\pi^m\right) > 0, \\ \phi\left(1;\alpha\right) &= -\left(s^m + \pi^m\right)\left(\frac{1}{2}f(0) + \frac{\alpha}{1-\alpha}h(0)\right) < 0,\end{aligned}$$

where (i) the first equality stems from $s^m + a^W\pi^m = 0$ and $a^W < 1$, and (ii) the second one from $F\left(0\right) = H\left(0\right) = 0$, $f\left(0\right) > 0$ and $h\left(0\right) > 0$ (from Assumption 3). Hence, there is a unique solution, which lies strictly between $a^W = -s^m/\pi^m$ and 1.

G Proof of Proposition 2

We first establish existence and uniqueness for t large enough (part 1), before characterizing the (unique) symmetric equilibrium (part 2).

Part 1. We first establish uniqueness, before turning to existence.

As noted in the text, in stage 1a each platform i seeks to maximize its subsidy advantage, $\Delta^*\left(a_i, a_j\right)$. Furthermore, as $t \longrightarrow +\infty$, the continuation equilibrium conditions (14) to (17) yield, up to $O(1/t)$:

$$\begin{aligned}
D^*(a_i, a_j) &\simeq \frac{1}{2}, \\
\hat{y}^*(a_i, a_j) &\simeq (1-\alpha)F\left(\frac{(1-a_i)\pi^m}{2}\right), \\
\tilde{y}^*(a_1, a_2) &\simeq \alpha H\left(\frac{(1-a_1)\pi^m}{2} + \frac{(1-a_2)\pi^m}{2}\right).
\end{aligned}$$

Plugging-in these expressions in (18) yields, up to $O(1/t)$, $\Delta^*(a_i, a_j) \simeq \hat{\Delta}^*(a_i, a_j)$, where:

$$\begin{aligned}
\hat{\Delta}^*(a_i, a_j) &= (s^m + a_i\pi^m)(1-\alpha)F\left(\frac{(1-a_i)\pi^m}{2}\right) - (s^m + a_j\pi^m)(1-\alpha)F\left(\frac{(1-a_j)\pi^m}{2}\right) \\
&\quad + \pi^m(a_i - a_j)\alpha H\left(\frac{(1-a_1)\pi^m}{2} + \frac{(1-a_2)\pi^m}{2}\right).
\end{aligned}$$

The first-order equilibrium conditions are therefore, up to $O(1/t)$:

$$\begin{aligned}
0 &= \frac{\partial \hat{\Delta}^*(a_1, a_2)}{\partial a_1} \\
&= \pi^m(1-\alpha)F\left(\frac{(1-a_1)\pi^m}{2}\right) - (s^m + a_1\pi^m)(1-\alpha)f\left(\frac{(1-a_1)\pi^m}{2}\right)\frac{\pi^m}{2} \\
&\quad + \pi^m\alpha H\left(\frac{(1-a_1)\pi^m}{2} + \frac{(1-a_2)\pi^m}{2}\right) \\
&\quad - \pi^m(a_1 - a_2)\alpha h\left(\frac{(1-a_1)\pi^m}{2} + \frac{(1-a_2)\pi^m}{2}\right)\frac{\pi^m}{2},
\end{aligned}$$

and:

$$\begin{aligned}
0 &= \frac{\partial \Delta^*(a_2, a_1)}{\partial a_2} \\
&= \pi^m(1-\alpha)F\left(\frac{(1-a_2)\pi^m}{2}\right) - (s^m + a_2\pi^m)(1-\alpha)f\left(\frac{(1-a_2)\pi^m}{2}\right)\frac{\pi^m}{2} \\
&\quad + \pi^m\alpha H\left(\frac{(1-a_1)\pi^m}{2} + \frac{(1-a_2)\pi^m}{2}\right) \\
&\quad + \pi^m(a_1 - a_2)\alpha h\left(\frac{(1-a_1)\pi^m}{2} + \frac{(1-a_2)\pi^m}{2}\right)\frac{\pi^m}{2}.
\end{aligned}$$

Subtracting the second condition from the first one yields:

$$\begin{aligned}
& (1-\alpha)\pi^m \left[F\left(\frac{(1-a_1)\,\pi^m}{2}\right) - F\left(\frac{(1-a_2)\,\pi^m}{2}\right)\right] \\
= \; & (1-\alpha)\frac{\pi^m}{2}\left[(s^m + a_1\pi^m)\, f\left(\frac{(1-a_1)\,\pi^m}{2}\right) - (s^m + a_2\pi^m)\, f\left(\frac{(1-a_2)\,\pi^m}{2}\right)\right] \\
& + 2\pi^m\,(a_1 - a_2)\,\alpha h\left(\frac{(1-a_1)\,\pi^m}{2} + \frac{(1-a_2)\,\pi^m}{2}\right)\frac{\pi^m}{2}.
\end{aligned}$$

If $a_1 \geq a_2$ (resp., $a_1 \leq a_2$), the left-hand side is weakly negative (resp., positive), as $F\left((1-a)\,\pi^m/2\right)$ is decreasing in a, whereas the right-hand side is weakly positive (resp., negative), as $s^m + a\pi^m$ and $f\left((1-a)\,\pi^m/2\right)$ are both non-negative and increasing in a, from Assumption 3. It follows that any equilibrium is symmetric: $a_1 = a_2$. Furthermore, any symmetric equilibrium satisfies (setting $a_1 = a_2 = a$ in the above conditions):

$$\begin{aligned}
0 \;=\; & \pi^m(1-\alpha)F\left(\frac{(1-a)\,\pi^m}{2}\right) + \pi^m\alpha H\left((1-a)\,\pi^m\right) \\
& - (s^m + a\pi^m)\,(1-\alpha)f\left(\frac{(1-a)\,\pi^m}{2}\right)\frac{\pi^m}{2}.
\end{aligned} \tag{21}$$

For $\alpha = 1$, this boils down to $H\left((1-a)\,\pi^m\right) = 0$, implying $a^C(\alpha) = 1$. For $\alpha < 1$, the above condition amounts to $\psi\left(a^C(\alpha);\alpha\right) = 0$, where:

$$\psi(a;\alpha) \equiv F\left(\frac{(1-a)\,\pi^m}{2}\right) + \frac{\alpha}{1-\alpha}H\left((1-a)\,\pi^m\right) - \frac{s^m + a\pi^m}{2}f\left(\frac{(1-a)\,\pi^m}{2}\right).$$

$\psi(a;\alpha)$ is strictly decreasing in a under Assumption 3, and it satisfies (using $s^m + a^W\pi^m = 0$)

$$\psi\left(a^W;\alpha\right) = F\left(\frac{\left(1-a^W\right)\pi^m}{2}\right) + \frac{\alpha}{1-\alpha}H\left(\left(1-a^W\right)\pi^m\right) > 0$$

and (using $F(0) = H(0) = 0$)

$$\psi(1;\alpha) = -\frac{s^m + \pi^m}{2}f(0) < 0,$$

where the inequalities respectively stem from $a^W = -s^m/\pi^m < 1$ and $f(0) > 0$ (from Assumption 3). It follows that there is a unique candidate symmetric equilibrium, which is moreover such that $a^C(\alpha) \in \left(a^W, 1\right)$ for $\alpha < 1$, and $a^C(\alpha) = 1$ for $\alpha = 1$.

To establish existence, we show that the function

$$\phi(a) \equiv \hat{\Delta}^*\left(a, a^C(\alpha)\right)$$

is indeed maximal for $a = a^C(\alpha)$. We first note that:

$$\begin{aligned}
\phi'(a) &= \pi^m(1-\alpha)F\left(\frac{(1-a)\pi^m}{2}\right) \\
&\quad -(s^m + a\pi^m)(1-\alpha)f\left(\frac{(1-a)\pi^m}{2}\right)\frac{\pi^m}{2} \\
&\quad +\pi^m\alpha H\left(\frac{(1-a)\pi^m}{2} + \frac{\left(1-a^C(\alpha)\right)\pi^m}{2}\right) \\
&\quad -\pi^m\left(a - a^C(\alpha)\right)\alpha h\left(\frac{(1-a)\pi^m}{2} + \frac{\left(1-a^C(\alpha)\right)\pi^m}{2}\right)\frac{\pi^m}{2},
\end{aligned}$$

which by construction satisfies $\phi'\left(a^C(\alpha)\right) = 0$, and

$$\begin{aligned}
\phi''(a) &\simeq -2\pi^m(1-\alpha)f\left(\frac{(1-a)\pi^m}{2}\right)\frac{\pi^m}{2} \\
&\quad +(s^m + a\pi^m)(1-\alpha)f'\left(\frac{(1-a)\pi^m}{2}\right)\left(\frac{\pi^m}{2}\right)^2 \\
&\quad -2\pi^m\alpha h\left(\frac{(1-a)\pi^m}{2} + \frac{\left(1-a^C(\alpha)\right)\pi^m}{2}\right)\frac{\pi^m}{2} \\
&\quad +\pi^m\left(a - a^C(\alpha)\right)\alpha h'\left(\frac{(1-a)\pi^m}{2} + \frac{\left(1-a^C(\alpha)\right)\pi^m}{2}\right)\left(\frac{\pi^m}{2}\right)^2.
\end{aligned}$$

For $a \geq a^C(\alpha)$, the second-order derivative is negative, as $f(\cdot) > 0 \geq f'(\cdot)$ and $h(\cdot) > 0 \geq h'(\cdot)$ under Assumption 3. Hence, in the range $a \geq a^C(\alpha)$, $\phi(a)$ is maximal for $a = a^C(\alpha)$.

Furthermore, for $a \leq a^C(\alpha)$, we have $\phi(a) \leq \hat{\phi}(a)$, where:

$$\begin{aligned}
\phi(a) \leq \hat{\phi}(a) \equiv \quad & (s^m + a\pi^m)(1-\alpha)F\left(\frac{(1-a)\pi^m}{2}\right) \\
& -\left[s^m + a^C(\alpha)\pi^m\right](1-\alpha)F\left(\frac{\left[1-a^C(\alpha)\right]\pi^m}{2}\right) \\
& +\pi^m\left[a - a^C(\alpha)\right]\alpha H\left(\left[1-a^C(\alpha)\right]\pi^m\right)
\end{aligned}$$

satisfies

$$\begin{aligned}\hat{\phi}'(a) &= \pi^m(1-\alpha)F\left(\frac{(1-a)\pi^m}{2}\right)-(s^m+a\pi^m)(1-\alpha)f\left(\frac{(1-a)\pi^m}{2}\right)\frac{\pi^m}{2}\\&\quad+\pi^m\alpha H\left(\left[1-a^C(\alpha)\right]\pi^m\right),\end{aligned}$$

By construction, $\hat{\phi}\left(a^C(\alpha)\right)=\phi\left(a^C(\alpha)\right)=0$ and $\hat{\phi}'\left(a^C(\alpha)\right)=\phi'\left(a^C(\alpha)\right)=0$. Furthermore:

$$\begin{aligned}\hat{\phi}''(a) &= -\pi^m(1-\alpha)f\left(\frac{(1-a)\pi^m}{2}\right)\pi^m\\&\quad+(s^m+a\pi^m)(1-\alpha)f'\left(\frac{(1-a)\pi^m}{2}\right)\left(\frac{\pi^m}{2}\right)^2\\&< 0,\end{aligned}$$

where the inequality stems from $f(\cdot)>0\geq f'(\cdot)$ under Assumption 3. It follows that, in the range $a\leq a^C(\alpha)$, $\phi(a)$ is again maximal for $a=a^C(\alpha)$.

Part 2. We now focus on symmetric candidate equilibria, in which both platforms thus set the same commission rate a. Suppose that platform 1, say, deviates to some $a_1\neq a$, and let $\hat{y}_i(a_1)\equiv\hat{y}^*(a_i,a_j)$(with the convention $a_2=a$), $\tilde{y}_i(a_1)\equiv\tilde{y}^*(a_i,a_j)$, $y_i(a_1)\equiv y^*(a_i,a_j)$. Let $D_i(a_1)\equiv D^*(a_i,a_j)$ and $\Delta_i(a_1)\equiv\Delta^*(a_i,a_j)$ denote the user base and the subsidy advantage of platform i following the deviation (by construction, $\tilde{y}_i=\tilde{y}_j$ and $D_1+D_2=1$), and $\hat{y}$, $\tilde{y}$, y, D and Δ denote their equilibrium values (by construction, $D=1/2$ and $\Delta=0$).

From (17) and (15), we have (using $D_1+D_2=1$):

$$\hat{y}_1 = (1-\alpha)F\left((1-a_1)\pi^m D_1\right), \tag{22}$$

$$\hat{y}_2 = (1-\alpha)F\left((1-a_2)\pi^m(1-D_1)\right), \tag{23}$$

leading to (evaluated at $a_2=a$):

$$\frac{d\hat{y}_1}{da_1} = (1-\alpha)f\left((1-a_1)\pi^m D_1\right)\pi^m[(1-a_1)\frac{dD_1}{da_1}-D_1], \tag{24}$$

$$\frac{d\hat{y}_2}{da_1} = -(1-\alpha)f\left((1-a)\pi^m(1-D_1)\right)\pi^m(1-a)\frac{dD_1}{da_1}. \tag{25}$$

Likewise, from (17) and (16), we have (evaluated at $a_2=a$ and using again $D_1+D_2=1$):

$$\tilde{y}_1=\alpha H\left((1-a)\pi^m+(a-a_1)\pi^m D_1\right), \tag{26}$$

leading to:

$$\frac{d\tilde{y}}{da_1} = \alpha h\left(\left(1-a\right)\pi^m + \left(a-a_1\right)\pi^m D_1\right)\pi^m[\left(a-a_1\right)\frac{dD_1}{da_1} - D_1]. \tag{27}$$

In addition, from (18), we have (using $\tilde{y}_2 = \tilde{y}_1$):

$$\Delta_1 = s^m(\hat{y}_1 - \hat{y}_2) + \pi^m(a_1\hat{y}_1 - a\hat{y}_2) + \pi^m\left(a_1 - a\right)\tilde{y}_1.$$

Differentiating leads to (using $\hat{y}_1 + \tilde{y}_1 = y_1$):

$$\begin{aligned} \frac{d\Delta_1}{da_1} &= s^m(\frac{d\hat{y}_1}{da_1} - \frac{d\hat{y}_2}{da_1}) + \pi^m(a_1\frac{d\hat{y}_1}{da_1} - a\frac{d\hat{y}_2}{da_1}) + \pi^m\left(a_1-a\right)\frac{d\tilde{y}}{da_1} + \pi^m\hat{y}_1 + \pi^m\tilde{y} \\ &= \left(s^m + a_1\pi^m\right)\frac{d\hat{y}_1}{da_1} - \left(s^m + a\pi^m\right)\frac{d\hat{y}_2}{da_1} + \left(a_1 - a\right)\pi^m\frac{d\tilde{y}}{da_1} + \pi^m y_1. \end{aligned} \tag{28}$$

Finally, differentiating (14) yields:

$$\frac{dD_1}{da_1} = \frac{1}{6t}\frac{d\Delta_1}{da_1}. \tag{29}$$

In equilibrium, we must have $d\Delta_1/da_1 = 0$. It then follows from (25) and (29) that

$$\frac{d\hat{y}_2}{da_1} = \frac{dD_1}{da_1} = 0$$

and from (24) that (using $D_1 = 1/2$ and $\hat{y}_1 = \hat{y}$)

$$\frac{d\hat{y}_1}{da_1} = -\frac{1-\alpha}{2}f\left(\frac{(1-a_1)\pi^m}{2}\right)\pi^m. \tag{30}$$

Using these observations and evaluating (28) at equilibrium, where $a_1 = a$, $D_1 = 1/2$ and $y_1 = y$, yields:

$$0 = \left[y_1 - \left(s^m + a\pi^m\right)\frac{1-\alpha}{2}f\left(\frac{(1-a_1)\pi^m}{2}\right)\right]\pi^m,$$

which, using (15) and (16), amounts to (21). It then follows from the analysis of part 1 that there is a unique symmetric equilibrium, where $a = a^C\left(\alpha\right)$.

Furthermore, $\psi\left(a;\alpha\right)$ is strictly increasing in α, implying that $a^C\left(\alpha\right)$ is also strictly increasing in α. In addition, comparing $\psi\left(a;\alpha\right)$ with the function $\phi\left(a;\alpha\right)$ characterizing $a^S\left(\alpha\right)$, we have:

$$\psi\left(a;\alpha\right) - \phi\left(a;\alpha\right) = \left(s^m + a^S\pi^m\right)\frac{\alpha}{1-\alpha}h((1-a)\pi^m) \geq 0,$$

where the inequality is strict for $\alpha > 0$. It follows that $a^C(0) = a^S(0)$ and $a^C(\alpha) > a^S(\alpha)$ for $\alpha > 0$.

References

[1] Armstrong, Mark. 1998. "Network Interconnection," *Economic Journal* , 108, pp. 545-564.

[2] Armstrong, Mark. 2006. "Competition in Two-Sided Markets," *The RAND Journal of Economics*, 37 (3), pp. 668-691.

[3] Armstrong, Mark and Julian Wright. 2007. "Two-Sided Markets, Competitive Bottlenecks and Exclusive Contracts," *Economic Theory*, 32, pp. 353-380.

[4] Belleflamme, Paul, and Martin Peitz. 2010. "Platform Competition and Seller Investment Incentives, " *European Economic Review*, 54, pp. 1059-1076.

[5] Bisceglia, Michele and Jean Tirole. 2022. "Fair Gatekeeping in Digital Ecosystems, " *Mimeo.*

[6] Choi, Jay Pil and Doh-Shin Jeon. 2022. "Platform Design Biases in Ad-Funded Two-Sided Markets," forthcoming, *The RAND Journal of Economics.*

[7] Competition and Markets Authority. 2022. *Mobile Ecosystems: Market Study Final Report.*

[8] Competition and Markets Authority. 2022. *Mobile Ecosystems: Market Study Final Report. Appendix I: Apple's restrictions on cloud gaming.*

[9] Etro, Federico. 2022. "Platform Competition with Free Entry of Sellers," forthcoming, *International Journal of Industrial Organization.*

[10] Laffont, Jean-Jacques, Patrick Rey and Jean Tirole, 1998a, "Network Competition I: Overview and Nondiscriminatory Pricing, " *The RAND Journal of Economics*, 29, pp.1-37.

[11] Laffont, Jean-Jacques, Patrick Rey and Jean Tirole, 1998b, "Network Competition II: Price Discrimination, " *The RAND Journal of Economics*, 29, pp.38-56.

[12] Llobet, Gerard and Jorge Padilla. 2016. "The Optimal Scope of the Royalty Base in Patent Licensing," *Journal of Law and Economics*, 59 (1), pp. 45-73.

[13] Netherlands Authority for Consumers and Markets. 2019. *Market Study into Mobile App Stores.*

[14] Subcommittee on Antitrust, Commercial and Administrative Law of the Committee on the Judiciary, U.S. House of Representatives, 2020. *Investigation of Competition in Digital Markets, Majority Staff Report and Recommendations.*

[15] Wright, Julian. 2002. "Access Pricing under Competition: An Application to Cellular Networks," *The Journal of Industrial Economics*, 50 (3), pp. 289-315.