Joint Elimination of Low-End Products

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P R E L I M I N A R Y - A N D - I N C O M P L E T E

Abstract

In a number of instances, producers of differentiated products have agreed on the joint elimination of their respective low-end products, often with the argument that these are environmentally unfriendly. We analyze the competitive effects of this practice. We analyze a price-setting duopoly with both horizontal and vertical product differentiation. Each firm offers a high quality product and a low-end version. The four products are in inter- and intrabrand competition. We show that for some parameter values, it is profitable for the two firms to jointly eliminate their low-end products, even though they have no incentive to do so unilaterally. Prices of the high quality products, which remain in interbrand competition, are then likely to increase and consumers are hurt. Our analysis applies to the landmark CECED (2000), in which producers of washing machine producers were allowed to jointy take their least energy-efficient models off the market, as well as to a recent collaboration by truck manufacturers to phase out their diesel engines.

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1 Introduction

There is growing support for the idea that competitors can collaborate in developing more sustainable and socially responsible business practices Corporations claim to want to make the transition to cleaner and fairer ways of producing, but also that they are only able to do so jointly On these grounds, competition authorities consider whether and under what conditions they might exempt such socially responsible collaborations from the antitrust laws Cartel laws offer concrete possibilities for this, in essence requiring that consumers receive a fair share of the benefits of such an agreement, to compensate for any anticompetitive effects.

One example that is viewed favorably is the joint elimination of low-end varieties that are often cheaply produced and cause negative production externalities. Such low-end products are likely to be less durable, use older technologies that generate higher emissions, or involve labor and raw materials that are cheap because of unfair trade terms, exploited labor, or poorly treated farm animals. In such circumstances producers may want to eliminate their low-end product, but may also be reluctant to do so unilaterally if their rivals would continue to produce their low-end offerings. Joint elimination of such products then sounds like a good idea. It may stimulate the purchase of more responsibly produced products, such as more full-efficient or electric cars, higher energy-label household appliances, more responsibly manufactured products, such as fair trade clothing, coffee and chocolate, and healthier better-life meat.

A favorable treatment was given on these grounds by the European Commission to an agreement of manufacturers of washing machines. In *CECED* (2000), the Commission decided to allow manufacturers to take their least energy-efficient models off the market as an exemption from the cartel prohibition, Article 101 TFEU.[1] The case set a precedent by

[1]CECED (Case IV.F.1/36.718) Commission Decision C(1999) 5064 [2000] OJ L187/47. (19) The parties agreed to cease producing and/or importing into the Community machines of the heavier categories D and

considering the avoided emissions of carbon dioxide, sulphur dioxide and nitrous oxide as sustainability benefits that could off-set the anticompetitive effects of the agreement. While the cases was not decided on the projected environmental benefits—instead the European Commission concluded that a typical consumer would be compensated for the increased purchase costs of more energy-efficient washing machines by saving more on electricity bills in use alone—the collective sustainability benefits for all of Europe were valued at more than seven times the higher costs of purchasing a more energy-efficient washing machine.[2] Shortly after the agreement was extended to dishwashers and domestic electric water heaters as well.[3] Crucially, in the CECED (2000) case, the European Commission assumed that competition for high-end products would not be affected and hence high-end prices would not be adversely affected due to the joint elimination of low-end offering.[4] In fact, the Commission argued that, if anything, prices were likely to decrease.[5]

E, and of all categories F and G.

[2]CECED (2000), recital 56.

[3]IP/01/1659, 26 November 2001.

[4]Essentially, the Commission argued that prices would only temporarily increase for products for which the production standards had to be raised due to the agreement:

> "The agreement will inevitably raise the production costs of those manufacturers that used to produce machines which are no longer allowed. Estimates of costs for adapting washing machines to the new minimum standard would suggest that production and unit costs would increase appreciably, albeit not excessively, for those models which need upgrading. Therefore, in the short term, the agreement is likely to increase the price of those models, and hence the prices of some manufacturers' product ranges, thereby raising their costs and bringing their prices closer to those of competitors, thereby distorting price competition."

see CECED (2000), recital 34.

[5]This is witnessed by the following quote:

> "[I]t is not possible to determine in advance its effect on the average selling price of those models of washing machines which are not directly affected. Indeed, the restriction in one product dimension, energy consumption, may increase competition on other product characteristics, including price. Therefore, while the minimum price of washing machines is likely to increase, it cannot be ruled out that products in categories A and B may become available at a lower price. In a market characterised by strong competition amongst manufacturers and bargaining power from distributors, these benefits are likely to accrue to consumers."

see CECED (2000), recital 53. Essentially, the Commission only looked at the need for (otherwise or previous) consumers of a low-end washing to be compensated. By taking the savings on electricity and water that those buyers would have from now (having no choice but) buying a high-end washing machine as sufficient to cover the extra expense of buying a high-end machine at the going price (hopefully even

In the so-called 'green antitrust' debate, the CECED case is a landmark and rather unique case, which set a precedent for proponents and companies arguing for being allowed to make sustainability agreements. The *Horizontal Guidelines*, which are currently under review and in the previous version actually offered a chapter with guidance on sustainability agreements, are expected to offer more possibilities for cooperation between competitors to fade-out unsustainable product lines. In the meantime, there have been several initiatives, that have generally been greeted with approval by the wider public and the media. For example have truck manufacturers recently published joint plans to phase out their diesel engine technology by 2040 under the umbrella of automaker association ACEA (see www.acea.be).

In this paper, we analyze the potential anticompetitive effects of the joint elimination of low-end products. Was the Commission right in assuming that such elimination would not affect competition for the high-end product? Analyzing this question is not straightforward. It requires a model that has both horizontal and vertical product differentiation, and each firm potentially selling multiple varieties. We propose a model that has exactly that.

We propose a model along the lines of Perloff and Salop (1985). In that model. two firms are active on a market. The extent to which consumers like the product of a firm is captured by a match value. These match values are independent across products and consumers. Hence, for each individual consumer, its valuations for the two products are uncorrelated, different from e.g. a Hotelling model in which they are perfectly negatively correlated. To this model of horizontal product differentiation, we add vertical product differentiation, We assume that each firm sells to varieties of the product: a high-end and a low-end version. We assume that each consumers' valuation for the high-end product

a lower price), the Commission implicitly assumed that that going high-end price would not be affected and, if so, only downwards. If the Commission had realized the high-end prices could have gone up too, it would have had to consider compensation for consumers that would have bought the high-end machine anyhow, which necessarily would have needed to be the appreciation of the environmental benefits—as for those buyers there were no further savings on electricity or water.

of a firm is always equal to their valuation of the low-end variety, but multiplied by some constant term greater than 1, and equal for all consumers. Hence, the four products are in inter- and intrabrand competition.

In our baseline analysis, we assume that the market is always fully covered, i.e. that all consumers buy a washing machine in equilibrium.[6]. In an extension, we also study a model in which that is not the case—but a closed-form solution then becomes infeasible. In each model, we study how the joint elimination of low-end products would affect prices for the high-end variety, firms profits, and consumer surplus. Crucially, we cannot take the negative externalities of the production of low-end varieties into account. For that, we require more information about the case at hand. We can only focus on the competitive effects. Anticompetitive joint agreements of this kind may still be exempted from cartel law for their sustainability benefits to society at large.

In essence, the elimination of low-end products has two separate effects. On the one hand, there is a price-discrimination effect. Offering a high and a low quality product allows firms to price discriminate. In the case of monopolist, this implies that elimination of the low-end product would unambiguously lower firm profits and decrease prices of the high-end product. We show this in Section 6. But, on top of that, there is also a competition-softening effect. Suppose that both firms offer two varieties. Each variety will then target a particular segment of the market. That also implies that competition for that segment will be much fiercer compared to a case in which each firm only offers one variety.

Which of these two effects dominates depend on the parameters of the model. In the

[6] In essence, this is also what the Commission seems to assume:

> "The agreement does not directly impose any reduction of output, since more efficient machines would, in principle, replace those being phased out. Limited effects on output, if any, may only arise indirectly through reduced demand, depending on price elasticity, which is low for washing machines, when viewed separately from other factors."

See CECED (2000), recital 35.

baseline model, if the extent of vertical product differentiation and the additional cost of producing the high-end variety are both relatively high, joint elimination of the low-end product increases prices and profits and hurts consumers. For other values of the parameters prices of the high-end product will decrease, and profits may either decrease or increase as a result. It is even feasible that profits and consumer surpluses both increase, although the set of parameter values for which this holds is relatively small.

In our analysis of an uncovered market, the results are more straightforward. We then find that the joint elimination of low-end products always raises prices of the high-end product, and always lowers consumer surplus. Hence, the softening-competition effect now always dominates. In this scenario, there is an additional welfare loss due to the fact that fewer consumers buy in equilibrium. As a result, the effect on total profits is ambiguous, as fewer consumers buy in equilibrium. Needless to say, the firms only want to jointly eliminate their low-end products if it profitable to do see. In our model, it is never profitable for firms to unilaterally eliminate their low-end product.

Our model—apart from being a contribution to the literature by combining horizontal and vertical product differentiation with multiproduct firms—allows for a fuller analysis of relevant policy cases. In the CECED decision, it is implicitly assumed that high-end prices would not be affected by the elimination of the low-end washing machines. A drastic intervention in the product spectrum with which firm compete, both interbrand and intrabrand, should be expected to have an appreciable effect on competition however. In the CECED case, product variety was reduced by almost two-thirds: 1718 of the 2730 washing machines models that were on offer in 1995 would no longer be available by 2001.[7] A better understanding of the competitive effects of this can also shed light on possible ulterior motivations of the firms involved.

The remainder of this paper is organized as follow. In the next section, we review related

[7]CECED (2000), recital 45.

literature. In Section 3, we introduce a novel model of inter- and intrabrand competition and characterize the competitive equilibrium. In Section 4 we study the implications of price agreements on the low-end product. Section 5 studies when the joint elimination of low-end products is a profitable proposition for the firms, as well its effects on prices and consumer surplus. Section 6 shows that the firms' incentives to eliminate their low-end products disappear if they manage to extend coordination to their other strategic choice variables as well. In Section 7 we study the case of an uncovered market. Section 7 concludes. Derivations and proofs are given in an Appendix.

2 Related Literature

To be added.

3 A Model of Inter- and Intrabrand Competition

Consider two firms, A and B, that each offer two vertically differentiated versions of their product: a high quality version H and a low quality version L. The high-end product is better than the low-end product in an objective sense, for example in performance, energy efficiency in use or sustainability of production. The cost of manufacturing the high-end product are also higher: marginal cost of production are constant and normalized to $c_L = 0 < c_H$.

There is a unit mass of consumers. They differ in their valuation for each firm. Consider one representative consumer. Her taste for the products of firm $i \in \{A, B\}$ is reflected by v_i, and is a random draw from the uniform distribution on $[0, 1]$. Her utility from consuming a low quality product is given by

$$U_{iL} = v + \mu v_i,$$

while purchase of the high quality product gives her

$$U_{iH} = v + \gamma \cdot \mu v_i,$$

with $\gamma > 1$. The constant $v > 0$ reflects the utility of having any functioning product. For now, we assume it is high enough such that the market is always fully covered in equilibrium. The parameter μ reflects the extent of horizontal product differentiation; a higher value of μ implies that the offerings of firm A are more highly differentiated from those of firm B. The parameter γ reflects the extent of vertical product differentiation, that is, the extent to which the high-end product is perceived to be better than the low-end product. A higher value of γ implies that consumers have a stronger preference for the high-end product of a firm.

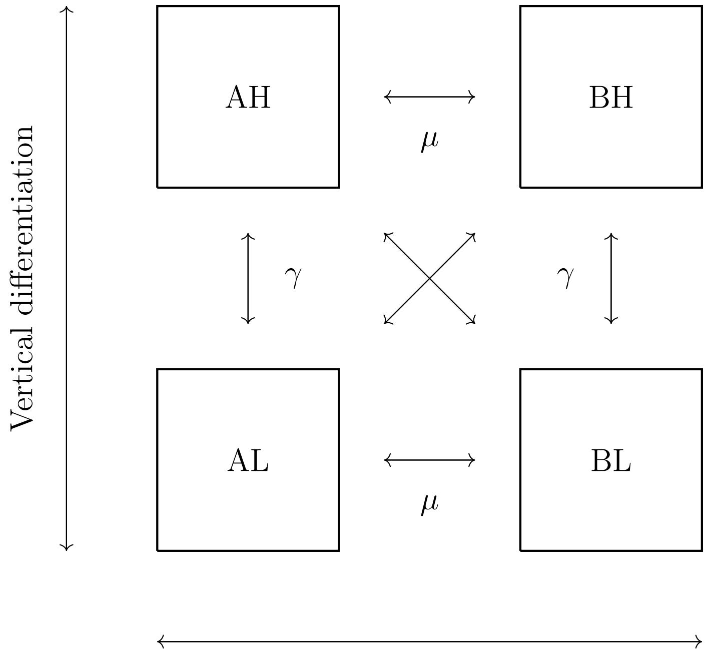
The products on offer are thus horizontally differentiated between firms, as valuations v_A and v_B are independent draws, as well as vertically differentiated within firms, as each consumer has a valuation for the high quality product of any given firm that is a factor γ higher than her valuation of the low quality product. Higher values of γ increase the relative demand for the high-end product, expressing for example increasing consumers awareness and willingness to pay for more sustainably manufactured goods.

The four products AL, AH, BL, BH compete with each other for consumers, so we have both interbrand and intrabrand competition. Figure 1 sketches the set-up of the model. All four products compete with each other. There is horizontal differentiation between AL and BL, and between AH and BH, while there is vertical differentiation between AL and AH and between BL and BH.

Denote by p_{ij} the price of product ij. A consumer will buy product AH if

$$\gamma \mu v_A - p_{AH} \geq \max \{\mu v_A - p_{AL}, \gamma \mu v_B - p_{BH}, \mu v_B - p_{BL}\}.$$

Figure 1: Set-up of the model



The left-hand side reflects the net utility she obtains from buying AH, the right-hand side that from buying AL, BH and BL respectively. For the condition to hold we need

$$\begin{aligned} v_A &\geq \frac{p_{AH} - p_{AL}}{(\gamma - 1)\,\mu} \\ v_A &\geq v_B + \frac{p_{AH} - p_{BH}}{\gamma\mu} \\ v_A &\geq \frac{v_B}{\gamma} + \frac{p_{AH} - p_{BL}}{\gamma\mu}, \end{aligned} \tag{1}$$

where the colors will correspond to those in the figures below.

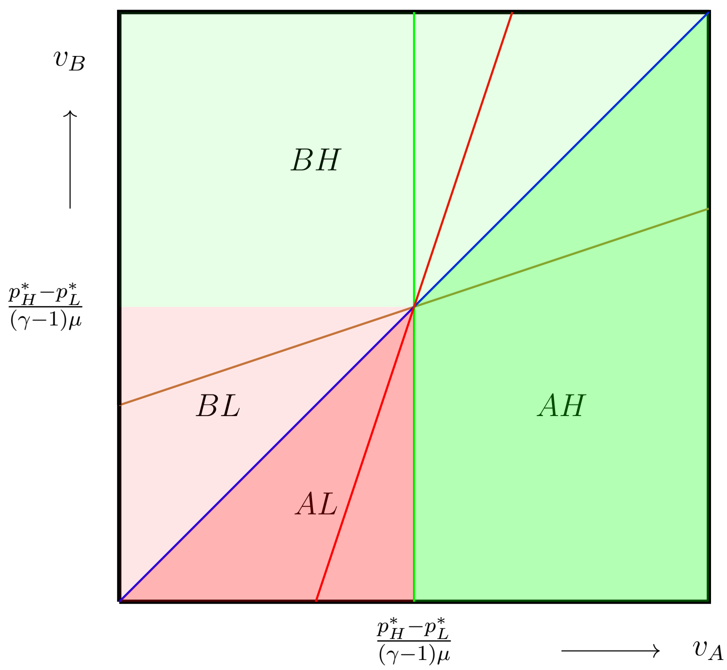
Similarly, a consumer will buy AL if she prefers AL over AH, over BH, and over BL.

This implies

$$
\begin{aligned}
v_A &\leq \frac{p_{AH} - p_{AL}}{(\gamma - 1)\,\mu}, \\
v_A &\geq \gamma v_B + \frac{p_{AL} - p_{BH}}{\mu}, \\
v_A &\geq v_B + \frac{p_{AL} - p_{BL}}{\mu}, \qquad (2)
\end{aligned}
$$

respectively.

Figure 2: Both qualities – equilibrium



In a symmetric equilibrium, both firms set some p_H^* for the high-end product, and some p_L^* for the low-end product. In such an equilibrium purchase decisions are given by Figure 2. The colors of the lines reflect the colors of the corresponding inequalities in (1) and (2). The darker green area depicts the consumers that buy AH, the darker red area those that buy AL. The lighter areas are the consumers that buy BH (green) or BL

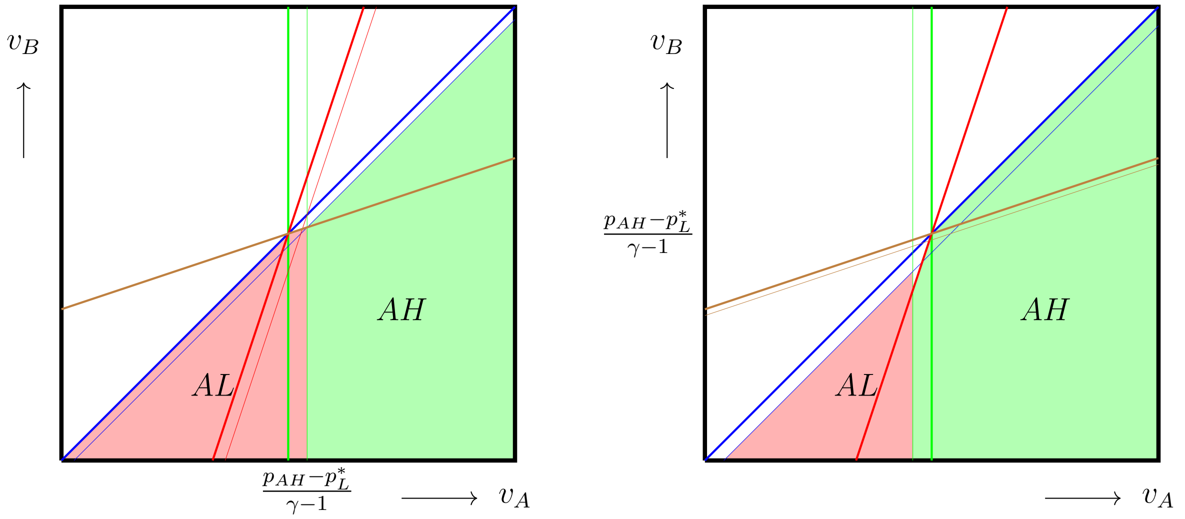
(red). From the Figure, a consumer buys AL if $v_A > v_B$ and $v_A \geq \frac{p_H^* - p_L^*}{(\gamma-1)\mu}$, while she buys AH if $v_A > v_B$ and $v_A < \frac{p_H^* - p_L^*}{(\gamma-1)\mu}$. The line $v_A = v_B$ thus reflects the marginal consumers in interbrand competition, while those with $v_A = \frac{p_H^* - p_L^*}{(\gamma-1)\mu}$ are the marginal consumers in intrabrand competition. Equilibrium sales are given by:

$$q_L^* = \int_0^{\frac{p_H^* - p_L^*}{(\gamma-1)\mu}} v_A dv_A = \frac{1}{2}\left(\frac{p_H^* - p_L^*}{(\gamma-1)\mu}\right)^2 \quad (3)$$

$$q_H^* = \int_{\frac{p_H^* - p_L^*}{(\gamma-1)\mu}}^1 v_A dv_A = \frac{1}{2} - \frac{1}{2}\left(\frac{p_H^* - p_L^*}{(\gamma-1)\mu}\right)^2. \quad (4)$$

Figure 3: Both qualities: out-of-equilibrium sales.

(a) $p_{AH} > p_H^*$. (b) $p_{AL} > p_L^*$.



To derive equilibrium prices we proceed as follows. First, suppose that firm A defects from a tentative equilibrium by charging a $p_H^A \neq p_H^*$. As profit functions are continuous and twice differentiable, we can restrict attention to $p_H^A \geq p_H^*$. The new situation is depicted in Figure 3a: the green, blue and red curves given in (1) all shift to the right. As can be seen, such a price increase has three effects on the sales of firm A. First, it decreases sales of AH, as marginal consumers with $v_A = v_B$ now prefer BH over AH – an interbrand effect.

Second, it decreases sales of AH as marginal consumers with $v_A = \frac{p_H^* - p_L^*}{(\gamma-1)\mu}$ now prefer AL over AH – an intrabrand effect. Third, it increases sales of AL for the exact same reason. Sales of q_{AL} and q_{AH} are now given by

$$\begin{aligned} q_{AL}\left(p_{AH}; p_L^*, p_H^*\right) &= \int_0^{\frac{p_H^* - p_L^*}{(\gamma-1)\mu}} v_A dv_A + \int_{\frac{p_H^* - p_L^*}{(\gamma-1)\mu}}^{\frac{p_{AH} - p_L^*}{(\gamma-1)\mu}} \left(\frac{v_A}{\gamma} + \frac{p_L^* - p_H^*}{\gamma\mu}\right) dv_A & (5) \\ q_{AH}\left(p_{AH}; p_L^*, p_H^*\right) &= \int_{\frac{p_{AH} - p_L^*}{(\gamma-1)\mu}}^{1} \left(v_A + \frac{p_H^* - p_{AH}}{\gamma\mu}\right) dv_A. & (6) \end{aligned}$$

Next consider a deviation for the low price to some $p_{AL} \geq p_L^*$. The new situation is depicted in Figure 3b: the green curve given in (2) shifts to the left while the brown and blue curves shift to the right. As can be seen, such a price increase has three effects on the sales of firm A. First, it decreases sales of AL, as marginal consumers with $v_A = v_B$ now prefer BL over AL – an interbrand effect. Second, it decreases sales of AL as marginal consumers with $v_A = \frac{p_H^* - p_L^*}{(\gamma-1)\mu}$ now prefer AH over AL – an intrabrand effect. Third, it increases sales of AH for the exact same reason. Sales of q_{AL} and q_{AH} are now given by

$$\begin{aligned} q_{AL}\left(p_{AL}; p_L^*, p_H^*\right) &= \int_{\frac{p_{AL} - p_L^*}{\mu}}^{\frac{p_H^* - p_{AL}}{(\gamma-1)\mu}} \left(v_A + \frac{p_L^* - p_{AL}}{\mu}\right) dv_A & (7) \\ q_{AH}\left(p_{AL}; p_L^*, p_H^*\right) &= \int_{\frac{p_H^* - p_L^*}{(\gamma-1)\mu}}^{1} v_A dv_A + \int_{\frac{p_H^* - p_{AL}}{(\gamma-1)\mu}}^{\frac{p_H^* - p_L^*}{(\gamma-1)\mu}} \left(\gamma v_A + \frac{p_L^* - p_H^*}{\mu}\right) dv_A, & (8) \end{aligned}$$

To derive the equilibrium, first note that profits of firm A are given by

$$\pi_A = p_{AL} q_{AL} + \left(p_{AH} - c\right) q_{AH}. \tag{9}$$

Taking the first-order conditions with respect to p_{AH} and p_{AL} using (5)–(8) and imposing symmetry then yields the following two conditions on equilibrium prices: (details in

Appendix A):

$$\begin{aligned}\left(3\gamma p_L^* - (2+\gamma)\, p_H^* + 2c\right)\left(p_H^* - p_L^*\right) - \mu\,(\gamma-1)^2\left(2p_H^* - 2c - \gamma\mu\right) &= 0. &(10)\\ 3p_H^* - 2\gamma p_L^* - p_L^* - 2c &= 0. &(11)\end{aligned}$$

Solving (11) and plugging the result into (10) allows us to pin down the equilibrium.

Theorem 1 *In competition, when $\gamma > 1 + c/\mu$, both firms will each offer both varieties, at equilibrium prices:*

$$\begin{aligned} p_H^* &= \frac{(\gamma-1)(\gamma+2)^2\mu - 3c(2-\gamma) - (\gamma+2)\sqrt{R}}{(\gamma-1)(4-\gamma)} &(12)\\ p_L^* &= \frac{3(\gamma-1)(\gamma+2)\mu - c(5-2\gamma) - 3\sqrt{R}}{(\gamma-1)(4-\gamma)}, &(13)\end{aligned}$$

in which

$$R = c^2 + 2c(\gamma-1)\left(\gamma^2-2\gamma-2\right)\mu + 2\mu^2\left(\gamma^2+2\right)(\gamma-1)^2.$$

When $\gamma < 1 + c/\mu$, both firms will only offer the low quality product.

Proof See Appendix A.

Note that the condition $\gamma > 1+c/\mu$ is intuitive. Consumers with the highest willingness-to-pay are willing to pay a premium $(\gamma-1)\,\mu$ for the high quality product. With $(\gamma-1)\,\mu < c$, this premium would not be sufficient for the firms to be able to afford the cost premium on the high quality product. Hence, when $\gamma < 1 + c/\mu$, firms would only offer the low quality product in this scenario.

4 Fixing the Price of the Low-end Products

In this section, we take the free-market equilibrium we derived in the previous section as a starting point, and consider whether firms would have an incentive to fix the price of the low-end product, even if they would still have to compete on the high-end product. They could argue, for example, that a price floor on an environmentally product could be welfare-improving, as it incentivizes consumers to consume less of that product. Alternatively, they could agree not to use the low-end product as a loss-leader, which would have a similar effect. We then have the following:

Theorem 2 *Suppose that firms fix prices for the low-end product, while competing on the high-end product. Starting from the competitive outcome, an increase in the price of the low-end product then has the following effects:*

1. *Prices for the high-end products increase, but by less than those of the low-end products.*
2. *Sales of the high-end products increase, while those of the low-end products decrease.*
3. *Profits increase.*

Proof See Appendix B.

Thus, firms are indeed better off by agreeing to increase the price of the low-end product—even if they still compete on the high-end product. Moreover, increasing the price of the low-end product relaxes intrabrand competition, which implies that prices of the high-end product will now also be higher. As a result, profits increase. Sales of the low-end product do decrease, however.[8]

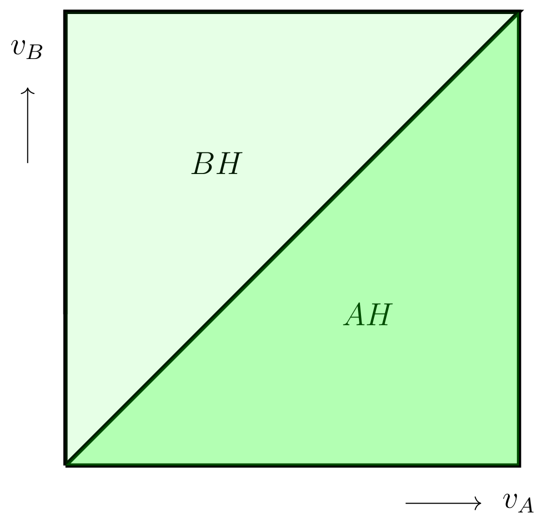
[8] Finding the profit-maximizing low-end cartel price is beyond the scope of this paper. It is a tedious exercise: clearly the cartel wants to set the price of the low-end product, and hence that of the high-end product, as high as possible. Yet, at some point, with the price of the low-end product sufficiently high, firms have an incentive to essentially undercut that price with their high-end product, such that sales of the low-end product drop to zero. This makes it hard to derive the equilibrium.

5 Elimination of the Low-end Products

We now derive the market equilibrium in the case that the firms coordinate to both eliminate the low-end product from their product lines, and then compete in prices for the high-end product. In the resulting subgame, both firms thus only offer high quality. A consumer now buys from firm A if and only if

$$\mu\gamma v_A - p_{AH} \geq \mu\gamma v_B - p_{BH}.$$

Figure 4: Only high qualities



In a symmetric equilibrium, both firms set some p_H^*. In such an equilibrium purchase decisions are given by Figure 4. The darker green area depicts the consumers that buy AH, the lighter green area those that buy BH. Total demand for AH is given by

$$q_{AH} = \int_{\frac{p_{AH}-p_{BH}}{\mu\gamma}}^{1} \left(v_A + \frac{p_{BH} - p_{AH}}{\mu\gamma} \right) dv_A = \frac{\left(\mu\gamma + p_{BH} - p_{AH}\right)^2}{2\gamma^2\mu^2}. \quad (14)$$

Since profits now are simply $\pi_A = (p_A - c)\, q_{AH}$, maximization with respect to p_{AH} and imposing symmetry then yields an equilibrium price of

$$p_H^c = c + \gamma\mu/2.$$

Note therefore that equilibrium prices are higher as γ increases. From (14), elasticity of demand is lower when γ is higher. As taste becomes more important it becomes harder to attract additional consumers by charging a lower price. Hence equilibrium prices will be higher.

We can now analyze for what combinations of (c, μ, γ) elimination of the low-quality product would lead to higher equilibrium prices for the high quality product.

Theorem 3 *Joint elimination of the low-quality product leads to higher equilibrium prices of the high quality product whenever* $\gamma > 4 - 2c/\mu$.

Proof See Appendix C.

The joint elimination of low-end products essentially has two effects. On the one hand, there is a **price-discimination effect**. Offering two products would allow a monopolist to price discriminate between consumers. By eliminating the low-end product this is no longer feasible. This effect when viewed in isolation would lead to a lower price for the high-end product (as firms now try to cater to a wider range of consumers) and lower profits. On the other hand, there is also a **competition-softening effect**. Suppose that both firms offer two varieties. Each variety will then target a particular segment of the market. That also implies that competition for that segment will be much fiercer compared to a case in which each firm only offers one variety, and thus competes for a much broader range of consumers. This effect when viewed in isolation would lead to a higher price for the high-end product. The Theorem suggests that the competition-softening effect dominates for high values of γ, while the price-discrimination effect dominates for low values.

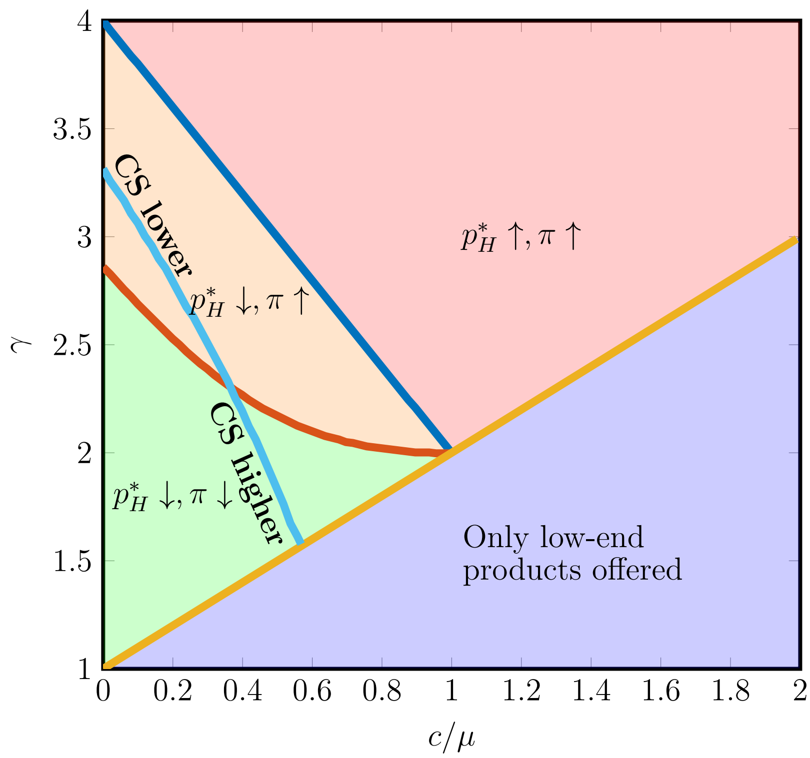
It is easy to see that if joint elimination of the low quality leads to a higher p_H^*, then it also leads to higher profits. In both scenarios, total sales for each firm equal 1/2. In the two-variety case, with $p_H^* > p_L^*$, total profits are thus smaller than $p_H^*/2$. As profits with one variety are equal to $p_H^*/2$, sufficient for joint elimination of the low quality to increase profits, is for the equilibrium price for the high quality to increase.

Indeed, there are more parameter values for which profits increase due to joint elimination of low quality. It is cumbersome to find an analytical solution, but Figure 5 gives a numerical solution. The red line gives the combinations of $(c/\mu, \gamma)$ for which profits are equal in both scenarios. The yellow line gives the lower bound on γ for the firms to be willing to offer two qualities, while the blue line gives the combinations of $(c/\mu, \gamma)$ for which the prices of the high quality are equal in both scenarios. Hence, in the green area, elimination of the low qualities would lead to lower profits; in the orange area it would lead to higher profits but lower prices for the high quality good, while in the red area it would lead to higher profits and higher prices for the high quality good. In the green area, profit-maximizing firms would not voluntarily eliminate the low-end product.

We thus have that for relatively low values of γ, joint elimination of the low-end product is not profitable. For higher γ it is profitable, and for even higher γ it even implies an increase of the price of the high-end product. Indeed, for $c > \mu$, elimination of low-end products always increases prices of high-end products. This can be understood as follows. Note that having higher γ also implies having effectively more product differentiation. Rather than having consumers differ in their valuation between 0 and μ, they now differ between 0 and $\gamma\mu$.[9] Without the low-end product, more product differentiation would imply more market power and hence higher prices. As long as γ is sufficiently low, the presence of the low-quality product increases profits for both firms, as it allows them to price discriminate. Yet, as γ increases, intrabrand competition from the low quality is

[9] This also implies a higher average willingness to pay - but that effect is competed away.

Figure 5: Elimination of low quality products



increasingly constraining the possibility to the exercise the higher market power on the high-end product. For γ sufficiently high, firms are thus better off by agreeing to eliminate the low-end products altogether.

In Figure 5, we also look at the effect on consumer surplus. Note that, when each firm offers two varieties, consumer surplus is given by

$$CS^* = 2\int_0^{q_L^*} (\mu v - p_L^*)\, dv + 2\int_{q_L^*}^1 (\gamma\mu v - p_H^*)\, dv.$$

With one variety, we have

$$CS^c = 2\int_0^1 (\gamma v - p_H^c)\, dv.$$

We refrain from giving the full expressions, as these are hardly informative. Instead,

Figure 5 also gives these effects. The lighter blue line gives the parameter values for which consumer surplus is not affected by elimination of the low-end product. To the right of this line, consumer surplus increases while to the right, it decreases. Thus, for most parameters where firms find it profitable to jointly eliminate their low-end products, consumer welfare decreases. It always does when such an elimination leads to an increase in the price of the high-end product. But even when joint elimination leads to lower prices of the high-end product, consumers are often worse off. The absence of the low-end product implies that many consumers are no longer able to consumer their more preferred product, which may yield a utility loss than is not fully compensated by the lower price of the high-end product.

6 Monopoly

To study to what extent the effects we find are driven by competition, it is instructive to show what the incentives to eliminate the low-end products are in the case of a monopoly. To that end, we have to slightly change our model, as having a fully covered market would imply that the monopolist could set an infinitely high price. We thus assume that the utility a consumer obtains from consuming the monopolist's low-end product is given by $U_L = \mu v_i$, and the utility from consuming the high-end product is given by $U_H = \gamma\mu v_i$, with $\gamma > 1$ and v_i a draw from the uniform distribution on $[0, 1]$.

With only the high-end product, a consumer would buy whenever $\gamma\mu v - p > 0$, so demand would be $1 - p/\gamma\mu$. Maximizing profits $\pi = (p - c)(1 - p/\gamma\mu)$ then yields

$$\begin{aligned} p_H^* &= \frac{1}{2}(\gamma\mu + c) \\ \pi &= \frac{1}{4}\frac{(\gamma\mu - c)^2}{\gamma\mu} \end{aligned}$$

Now suppose that the monopolist offers both qualities. A consumer buys H if $v > \frac{p_H - p_L}{\gamma\mu - 1}$, and L if $\frac{p_H - p_L}{(\gamma-1)\mu} > v > \frac{p_L}{\mu}$. Profits thus equal

$$\pi = \left(1 - \frac{p_H - p_L}{(\gamma - 1)\mu}\right)(p_H - c) + \left(\frac{p_H - p_L}{(\gamma - 1)\,\mu} - \frac{p_L}{\mu}\right)p_L$$

Maximizing

$$\begin{aligned}
\frac{\partial \pi}{\partial p_H} &= \frac{1}{(\gamma - 1)\,\mu}\left(c + (\gamma - 1)\,\mu - 2p_H + 2p_L\right) = 0 \\
\frac{\partial \pi}{\partial p_L} &= \frac{1}{(\gamma - 1)\,\mu}\left(2p_H - 2\gamma p_L - c\right) = 0
\end{aligned}$$

Hence, from the latter

$$p_L = \frac{2p_H - c}{2\gamma}.$$

Plugging this into the former yields

$$c + (\gamma - 1)\,\mu - 2p_H + 2\left(\frac{2p_H - c}{2\gamma}\right) = 0$$

Solving yields

$$\begin{aligned}
p_H^* &= \frac{1}{2}(\gamma\mu + c) \\
p_L^* &= \frac{1}{2}\mu.
\end{aligned}$$

Note that these are exactly the monopoly prices of the two products when offered in isolation.

Theorem 4 *A monopolist always wants to maintain both product varieties.*

Proof See Appendix D.

Hence, in the absence of competition it is *never* profitable for to discontinue the low-end products. In this case, there is only a price-discrimination effect, so elimination of the low-end product yields lower prices and profits. The softening-competition effect is eliminated, This also suggests that if the firms were able to fully collude, they would lose their incentive to eliminate the low-end products.

7 Extension: the case of an uncovered market

When we assume that the market is not fully covered, the analysis becomes more complicated. We use the same specification as above but now assume that v is low enough such that not all consumers buy in equilibrium. In our numerical analysis, we will set $v = 0.5$: this is sufficient for the market to not be fully covered, but also for both varieties to be offered in equilibrium.

We go through the same steps as in the analysis above. All results regarding the choice between products still go through, but consumers only buy if their utility from buying is better than their outside option – which is not buying at all. The equilibrium is now given by Figure 6: for consumers, the trade-off between products does not change, but consumers with a very low v for both firms now refrain from buying: to make consumption of AL worthwhile, for example, we need $v + \mu v_A > p_A$, hence $v_A > (p_A - v)/\mu$.

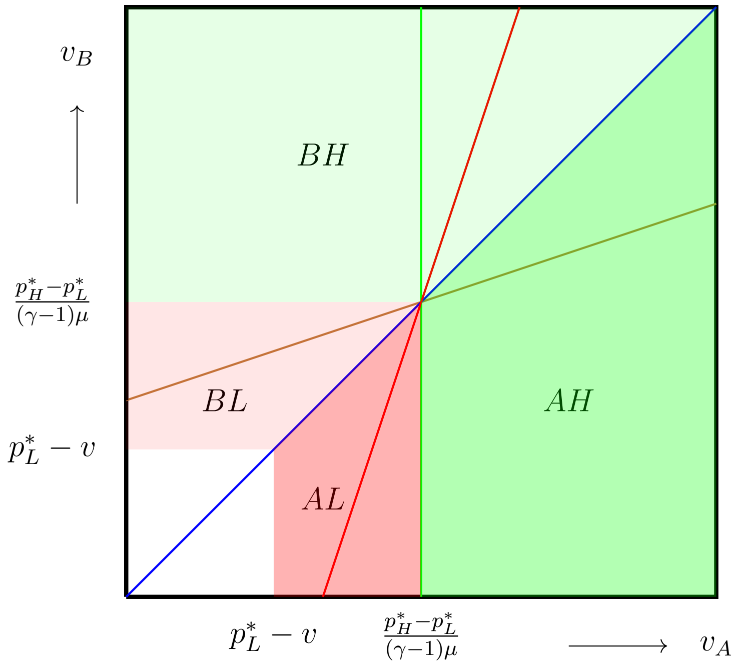
Hence, equilibrium sales are now given by:

$$q_L^* = \int_{\frac{p_L^* - v}{\mu}}^{\frac{p_H^* - p_L^*}{(\gamma-1)\mu}} v_A dv_A = \frac{1}{2}\left(\frac{p_H^* - p_L^*}{(\gamma-1)\mu}\right)^2 - \frac{1}{2}\left(\frac{p_L^* - v}{\mu}\right)^2 \tag{15}$$

$$q_H^* = \int_{\frac{p_H^* - p_L^*}{(\gamma-1)\mu}}^{1} v_A dv_A = \frac{1}{2} - \frac{1}{2}\left(\frac{p_H^* - p_L^*}{(\gamma-1)\mu}\right)^2. \tag{16}$$

Consider a defection by firm A to some $p_H^A > p_H^*$. Sales of q_{AL} and q_{AH} are now given

Figure 6: Both qualities – equilibrium



by

$$q_{AL}\left(p_{AH};p_L^*,p_H^*\right) = \int_{\frac{p_L^*-v}{\mu}}^{\frac{p_H^*-p_L^*}{(\gamma-1)\mu}} v_A dv_A + \int_{\frac{p_H^*-p_L^*}{(\gamma-1)\mu}}^{\frac{p_{AH}-p_L^*}{(\gamma-1)\mu}} \left(\frac{v_A}{\gamma}+\frac{p_L^*-p_H^*}{\gamma\mu}\right) dv_A \tag{17}$$

$$q_{AH}\left(p_{AH};p_L^*,p_H^*\right) = \int_{\frac{p_{AH}-p_L^*}{(\gamma-1)\mu}}^{1} \left(v_A+\frac{p_H^*-p_{AH}}{\gamma\mu}\right) dv_A. \tag{18}$$

Note that the expression for sales of q_{AH} are identical to those in (6). Sales of q_{AL} are different, as the lower bound on the first integral is now $\left(p_L^*-v\right)/\mu$, but this change does not affect $\partial q_{AL}/\partial p_{AH}$. Taking the first-order condition for profit maximization and imposing symmetry, equilibrium again requires

$$p_L^* \cdot \left.\frac{\partial q_{AL}}{\partial p_{AH}}\right|_{p_H^*} + q_H^* + \left(p_H^*-c\right)\left.\frac{\partial q_{AH}}{\partial p_{AH}}\right|_{p_H^*} = 0,$$

but all these terms are unaffected by whether the market is covered. Hence this yields the same condition as before, i.e.

$$\left(3\gamma p_L^* - (2+\gamma)\, p_H^* + 2c\right)\left(p_H^* - p_L^*\right) - \mu\left(\gamma - 1\right)^2\left(2p_H^* - 2c - \gamma\mu\right) = 0. \tag{19}$$

Next consider a deviation for the low price to some $p_{AL} \geq p_L^*$. Sales of q_{AL} and q_{AH} are now given by

$$q_{AL}\left(p_{AL}; p_L^*, p_H^*\right) = \int_{\frac{p_{AL}-v}{\mu}}^{\frac{p_H^*-p_{AL}}{(\gamma-1)\mu}} \left(v_A + \frac{p_L^* - p_{AL}}{\mu}\right) dv_A \tag{20}$$

$$q_{AH}\left(p_{AL}; p_L^*, p_H^*\right) = \int_{\frac{p_H^*-p_L^*}{(\gamma-1)\mu}}^{1} v_A dv_A + \int_{\frac{p_H^*-p_{AL}}{(\gamma-1)\mu}}^{\frac{p_H^*-p_L^*}{(\gamma-1)\mu}} \left(\gamma v_A + \frac{p_L^* - p_H^*}{\mu}\right) dv_A. \tag{21}$$

This implies that q_{AH} and hence $\partial q_{AH}/\partial q_{AL}$ is identical to that in the case of a covered market. The only way in which the analysis differs from that with a covered market is that the expression for q_L^* changes in the first-order condition. Going through the analysis now yields the condition (see Appendix E)

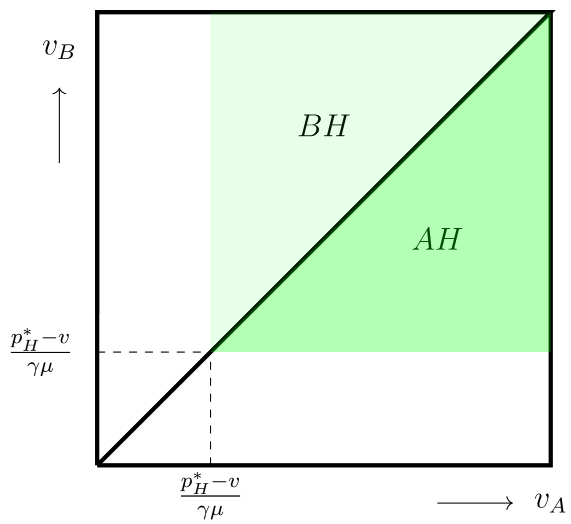
$$\left(p_H^* - p_L^*\right)\left(3p_H^* - 2c - p_L^*\left(1+2\gamma\right)\right) - \left(p_L^* - v\right)^2\left(\gamma - 1\right)^2 = 0. \tag{22}$$

Finding the equilibrium values (p_L^*, p_H^*) now requires solving the system of non-linear equations given by (19) and (22). As both expressions are quadratic, there are in principal four solutions to the system, but only one of these is feasible. Finding an analytical solution is not feasible either, but we can solve numerically.[10]

To find the solution for the case with only the high-end product, we proceed as follows. First note that consumers are now only willing to buy, say, product AH if $v + \gamma\mu v_A > p_H^A$, so $v_A > (p_H^A - v)/\gamma\mu$. The equilibrium now looks like Figure 7.

[10]This is done in MATLAB. The code is available upon request. More details in Appendix E.

Figure 7: Only high qualities, non-covered market



Equilibrium sales are thus given by

$$q_H^* = \int_{\frac{p_H^* - v}{\gamma\mu}}^{1} v_A dv_A = \frac{1}{2} - \frac{1}{2}\left(\frac{p_H^* - v}{\gamma\mu}\right)^2.$$

Sales from charging some $p_{AH} \geq p_H^*$ are given by

$$q_{AH} = \int_{\frac{p_{AH} - v}{\gamma\mu}}^{1} \left(v_A - \frac{p_{AH} - p_H^*}{\mu\gamma}\right) dv_A.$$

This implies

$$\left.\frac{\partial q_{AH}}{\partial p_{AH}}\right|_{p_H^*} = -\frac{1}{\gamma\mu}.$$

Maximizing profits and imposing symmetry then yields

$$p_H^* = c + \gamma\mu q_L^*,$$

which implies

$$p_H^* = v - \gamma\mu + \sqrt{2\gamma\mu\left(c - v + \gamma\mu\right)}.$$

Consumer surplus with both varieties is now given by

$$CS = \int_{\frac{p_L^*-v}{\mu}}^{\frac{p_H^*-p_L^*}{(\gamma-1)\mu}} \left(v + \mu v_A - p_L^*\right) dv_A + \int_{\frac{p_H^*-p_L^*}{(\gamma-1)\mu}}^{1} \left(v + \gamma\mu v_A - p_H^*\right) dv_A.$$

In the elimination case:

$$CS = \int_{\frac{p_H^*-v}{\gamma\mu}}^{1} \left(v + \gamma\mu v_A - p_H^*\right) dv_A$$

Figure 8: Elimination of low quality products, uncovered market

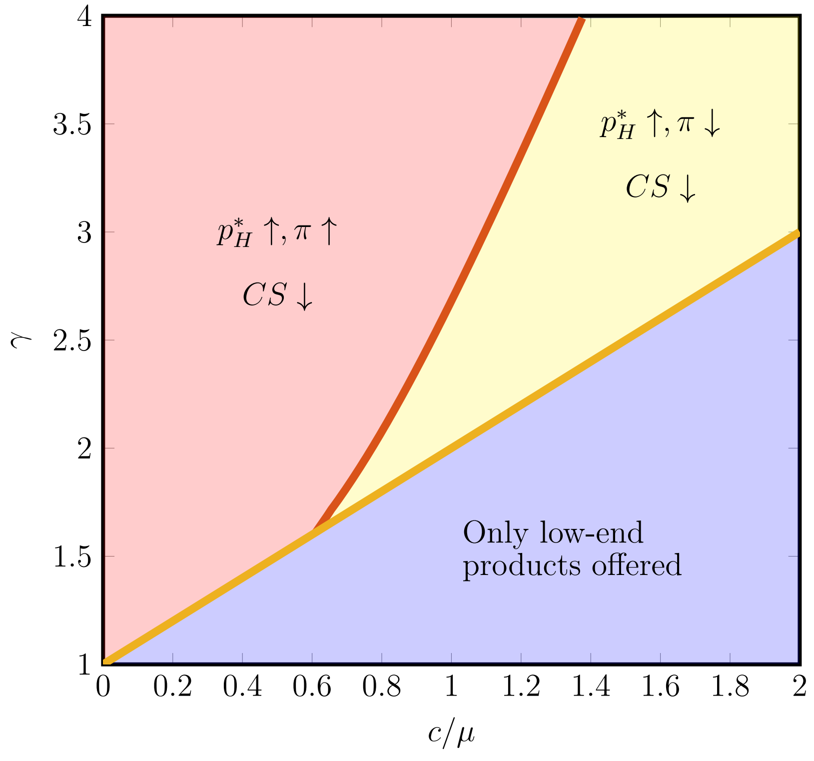


Figure 8 summarizes the effects of the elimination of low-end products in this case. Again, in the blue area, firms find it most profitable to only offer the low-end products.

The set of feasible parameter values now consists of only two separate areas. In the yellow area, profits increase, while in the red area, they increase, But prices of the high-end product now *always* increase due to the elimination of low-end products, and consumer surplus always decreases.

Hence, the softening-competition effect now always dominates. Price discrimination in the two-variety case is less effective as consumers with a low willingness to pay are likely to drop out of the market. With price discrimination less effective, the price=discrimination effect also loses much of its bite. Moreover, there is now an additional adverse effect on consumer surplus, as the elimination of the low-end product implies that fewer consumers will be served in equilibrium.

8 Concluding Remarks

In this paper, we proposed a new model that intra- and interbrand competition that combines elements of horizontal and vertical product differentiation. Using that model, we studied the extent to which firms have an incentive to jointly agree to eliminate their low-end offerings, a practice that has been observed and condoned in some recent antitrust cases, most notably the CECED case in the EU.

We found that an agreement to jointly eliminate low-end products may increase equilibrium prices of the high-end products and hence benefit firms but hurt consumers. This is especially the case if the market is not fully covered, in other words, if total demand for the product is elastic. The elimination of low-end products has two separate effects. On the one hand, there is a price-discrimination effect. Offering a high and a low quality product allows firms to price discriminate. Viewed in isolation, the elimination of one product would then lead to lower prices for the other, and lower profits. But on the other hand, there is also a competition-softening effect. With varieties, firms compete on two segments with relatively homogeneous consumers. That implies that competition for that segment will be much fiercer compared to a case in which each firm only offers one variety.

Our model also has other applications. In a recent paper, Bourreau et al. (2021) study the entry of a low-end incumbent on the French mobile telecommunications market. As a result the three incumbent firms also started offering "'fighting brands", i.e. a low-end variety of their product. The authors conclude that "their strategies are consistent with a breakdown of tacit semi-collusion: before entry, the incumbents could successfully coordinate on restricting product variety", Such behavior, which has also been witnessed in the airline industry, is entirely consistent with our model. Rather than jointly eliminating a low-end product, these firms semi-colluded not to offer a low-end product in the first place.

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A Proof of Theorem 1

First take the first-order condition of profits with respect to p_{AH}. This yields

$$\frac{\partial \pi_A}{\partial p_{AH}} = p_{AL}\frac{\partial q_{AL}}{\partial p_{AH}} + q_{AH} + (p_{AH} - c)\frac{\partial q_{AH}}{\partial p_{AH}} = 0. \tag{23}$$

From (5):

$$\frac{\partial q_{AL}}{\partial p_{AH}} = \left(\frac{p_{AH} - p_L^*}{\gamma(\gamma - 1)\mu} + \frac{p_H^* - p_L^*}{\gamma\mu}\right)\frac{1}{(\gamma - 1)\mu}.$$

Evaluated at p_H^*:

$$\left.\frac{\partial q_{AL}}{\partial p_{AH}}\right|_{p_H^*} = \frac{p_H^* - p_L^*}{(\gamma - 1)^2\mu^2}.$$

From (6):

$$\frac{\partial q_{AH}}{\partial p_{AH}} = -\int_{\frac{p_{AH} - p_L^*}{(\gamma-1)\mu}}^{1} \frac{1}{\gamma\mu}dv_A - \left(\frac{p_{AH} - p_L^*}{(\gamma - 1)\mu} + \frac{p_H^* - p_{AH}}{\gamma\mu}\right)\frac{1}{(\gamma - 1)\mu}.$$

Evaluated at p_H^* we then have

$$\left.\frac{\partial q_{AH}}{\partial p_{AH}}\right|_{p_H^*} = -\frac{1}{\gamma\mu} - \frac{p_H^* - p_L^*}{\gamma\mu^2(\gamma - 1)^2}.$$

Evaluating (23) in p_H^* then yields

$$\left.\frac{\partial \pi_A}{\partial p_{AH}}\right|_{p_H^*} = p_L^* \cdot \frac{p_H^* - p_L^*}{(\gamma - 1)^2\mu^2} + \left(\frac{1}{2} - \frac{1}{2}\frac{(p_H^* - p_L^*)^2}{(\gamma - 1)^2\mu^2}\right) - (p_H^* - c)\left(\frac{1}{\gamma\mu} + \frac{p_H^* - p_L^*}{\gamma\mu^2(\gamma - 1)^2}\right) = 0.$$

Multiplication with $2\gamma(\gamma - 1)^2\mu^2$ and further simplification yields (10) in the main text.

Next take the first-order condition with respect to p_{AL}:

$$\frac{\partial \pi_A}{\partial p_{AL}} = p_{AL}\frac{\partial q_{AL}}{\partial p_{AL}} + q_{AL} + (p_H^* - c)\frac{\partial q_{AH}}{\partial p_{AL}} = 0, \tag{24}$$

From (5),

$$\frac{\partial q_{AH}}{\partial p_{AL}} = \left(\gamma \frac{p_H^* - p_{AL}}{(\gamma-1)\,\mu} + \frac{p_L^* - p_H^*}{\mu}\right) \frac{1}{(\gamma-1)\,\mu},$$

Evaluated at p_L^* we have

$$\left.\frac{\partial q_{AH}}{\partial p_{AL}}\right|_{p_L^*} = \frac{p_H^* - p_L^*}{(\gamma-1)^2\,\mu^2}.$$

From (7),

$$\frac{\partial q_{AL}}{\partial p_{AL}} = -\frac{\gamma\left(p_H^* - p_L^* + \gamma\left(p_L^* - p_{AL}\right)\right)}{(\gamma-1)^2\,\mu^2},$$

or evaluated at p_L^*

$$\left.\frac{\partial q_{AL}}{\partial p_{AL}}\right|_{p_L^*} = -\frac{\gamma\left(p_H^* - p_L^*\right)}{(\gamma-1)^2\,\mu^2}. \tag{25}$$

From (24) we thus have

$$\frac{\partial \pi_A}{\partial p_{AL}} = p_{AL}\frac{\partial q_{AL}}{\partial p_{AL}} + q_{AL} + \left(p_H^* - c\right)\frac{\partial q_{AH}}{\partial p_{AL}},$$

so that

$$\begin{aligned} \left.\frac{\partial \pi_A}{\partial p_{AL}}\right|_{p_L^*} &= -p_L^* \frac{\gamma\left(p_H^* - p_L^*\right)}{(\gamma-1)^2\,\mu^2} + \frac{1}{2}\left(\frac{p_H^* - p_L^*}{(\gamma-1)\,\mu}\right)^2 + \left(p_H^* - c\right)\frac{p_H^* - p_L^*}{(\gamma-1)^2\,\mu^2} \\ &= \left(p_H^* - \gamma p_L^* - c\right)\frac{p_H^* - p_L^*}{(\gamma-1)^2\,\mu^2} + \frac{1}{2}\left(\frac{p_H^* - p_L^*}{(\gamma-1)\,\mu}\right)^2 = 0, \end{aligned}$$

which by dividing by $p_H^* - p_L^*$ and multiplying by $2(\gamma-1)^2\mu^2$ yields (11). From that equality, we have

$$p_L^* = \frac{3p_H^* - 2c}{2+\gamma}.$$

Plugging this into (10), we get

$$\left(3\gamma\frac{3p_H^* - 2c}{2+\gamma} - (2+\gamma)\,p_H^* + 2c\right)\left(p_H^* - \frac{3p_H^* - 2c}{2+\gamma}\right) - \mu(\gamma-1)^2\left(2p_H^* - 2c - \gamma\mu\right) = 0. \tag{26}$$

which, after multiplying by $(2+\gamma)^2/(\gamma-1)$ reduces to

$$\left(p_H^*\left(4-\gamma\right)-4c\right)\left(p_H^*\left(\gamma-1\right)+2c\right)-\left(\gamma+2\right)^2\mu\left(\gamma-1\right)\left(2p_H^*-2c-\mu\gamma\right)=0. \qquad (27)$$

Solving this yields, after some tedious algebra, the (p_L^*, p_H^*) in the Theorem.

For (p_L^*, p_H^*) to be an equilibrium, we need that equilibrium prices and resulting equilibrium quantities are well-defined. In particular, we need that $q_L^* \in \left(0, \frac{1}{2}\right)$. Clearly, the equilibrium sales of the low quality product cannot be smaller than 0, and if it would be bigger than 1/2, that would imply from (3) and (4) that equilibrium sales of the high quality product would be negative. From (11), we have

$$p_H^* - p_L^* = p_H^* - \frac{3p_H^* - 2c}{2+\gamma} = \frac{2c + p_H^*\left(\gamma-1\right)}{\gamma+2} > 0,$$

which implies from (3) that $q_L^* > 0$. Hence, in equilibrium, if the firms are able to offer two varieties, the low quality will *always* be sold, regardless of c and λ.

To derive conditions such that $q_L^* \leq 1/2$ is more involved. Note from (12) and (13) that

$$p_H^* - p_L^* = \frac{\left(\gamma+2\right)\left(\gamma-1\right)\mu + c - \sqrt{R}}{\left(4-\gamma\right)}.$$

From (3), for $q_L^* = 1/2$, we need that $p_H^* - p_L^* = (\gamma-1)\mu$, which then implies

$$\left(\gamma+2\right)\left(\gamma-1\right)\mu + c - \sqrt{R} = \left(4-\gamma\right)\left(\gamma-1\right)\mu$$

or $2\left(\gamma-1\right)^2\mu + c = \sqrt{R}$. Squaring both sides, using the definition of R and cancelling terms this simplifies to $c/\mu - \gamma + 1 = 0$, or $\gamma = c + 1$. Thus $q_L^* = 1/2$ if and only if $\gamma = c/\mu + 1$. Take $c = 0$ and $\gamma = 2$, which is above this line. We then have $R = 12\mu^2$ and $q_L^* = (2-\sqrt{3})^2/2 < 1/2$. As $q_L^* = 1/2$ if and only if $\gamma = c/\mu + 1$, this immediately implies that $q_L^* < 1/2$ for any $\gamma > c/\mu + 1$.

A final thing left to check is whether $R > 0$ so $\sqrt{R}$ is real-valued. With $(\gamma - 1)\mu > c$, we immediately have $R > c^2 (2\gamma - 1)^2 > 0$, so this is satisfied.

B Proof of Theorem 2

From the analysis leading up to (10), we have that for a given p_L, competition for the high-end product yields a p_H^* that is implicitly defined by $H(p_H^*, p_L) = 0$, with

$$H(p_H^*, p_L) = (3\gamma p_L - (2+\gamma) p_H^* + 2c)(p_H^* - p_L) - \mu (\gamma - 1)^2 (2p_H^* - 2c - \gamma\mu).$$

Hence, using the implicit function theorem,

$$\frac{\partial p_H^*}{\partial p_L} = -\frac{\partial H/\partial p_L}{\partial H/\partial p_H^*} = \frac{(4\gamma + 2) p_H^* - 6\gamma p_L - 2c}{(4 + 2\gamma) p_H^* - (4\gamma + 2) p_L + 2\mu (\gamma - 1)^2 - 2c} \tag{28}$$

If we subtract the numerator from the denominator, we get

$$-2(\gamma - 1)(p_H^* - p_L - \mu(\gamma - 1)) > 0, \tag{29}$$

as the second bracketed term is negative: in equilibrium we need $q_L^* < 1/2$ which, from (3), implies $p_H^* - p_L < \mu (\gamma - 1)$. Hence, the denominator in (28) is larger than the numerator.

Next, we evaluate (28) at the market equilibrium. Using (11), the numerator of (28) is then given by

$$(4\gamma + 2) p_H^* - 6\gamma \frac{3p_H^* - 2c}{2 + \gamma} - 2c = \frac{4(\gamma - 1)^2 p_H^* + 2(5\gamma - 2) c}{2 + \gamma} > 0.$$

Hence, in that case, both the numerator and the denominator of (28) are positive while the denominator is larger. This implies

$$0 < \left.\frac{\partial p_H^*}{\partial p_L}\right|_{p_L = p_L^*} < 1.$$

As p_H^* increases by less than p_L, we immediately have from (3) that q_L^* decreases, so q_H^* increases. This in turn implies that profits increase if the profit margin on the high-end product is higher than that on the low-end product. But at the market equilibrium, this is always the case. This can be seen as follows. We have that

$$(p_H^* - c) - p_L^* = \frac{\mu(\gamma+2)(\gamma-1) + c(\gamma-3) - \sqrt{R}}{(4-\gamma)}.$$

This is strictly positive if

$$\frac{[\mu(\gamma+2)(\gamma-1) + c(\gamma-3)]^2 - R}{(4-\gamma)} > 0.$$

Plugging in R this simplifies to

$$\gamma\mu^2(\gamma-1)^2 - c^2(\gamma-2) - 2c\mu(\gamma-1) > 0.$$

Using $c < (\gamma-1)\mu$, we have that

$$\begin{aligned} & \gamma\mu^2(\gamma-1)^2 - c^2(\gamma-2) - 2c\mu(\gamma-1) \\ > \; & \gamma\mu^2(\gamma-1)^2 - ((\gamma-1)\mu)^2(\gamma-2) - 2(\gamma-1)\mu\mu(\gamma-1) = 0 \end{aligned}$$

which establishes the result.

C Proof of Theorem 3

We first check whether such elimination would lead to the same price in both scenarios. The equilibrium price with only the high variety is $p_H^* = c + \gamma/2$. We thus want to know for which (c, γ) this price exactly satisfies the first-order condition for profit maximization with two varieties, given by (27). Plugging $p_H^* = c + \gamma/2$ into that equality yields

$$\frac{1}{4}\left(\left(2c+\gamma\right)\left(4-\gamma\right)-8c\right)\left(\left(2c+\gamma\right)\left(\gamma-1\right)+4c\right)=0,$$

as the second term in (27) vanishes. This simplifies to

$$\left(2c\left(\gamma+1\right)+\gamma\left(\gamma-1\right)\right)\gamma\left(2c+\gamma-4\right)=0.$$

The first term is always positive, hence prices in the two scenarios are equal if and only if $\gamma = 4 - 2c$. Consider the case $c = 0$ and $\gamma < 4$. It is then easy to see that p_H^* collapses to

$$p_H^* = \frac{\left(\gamma+2\right)^2 - \left(\gamma+2\right)\sqrt{2\left(\gamma^2+2\right)}}{4-\gamma}$$

This is higher than in the scenario with one variety if

$$2\left(\gamma+2\right)^2 - \gamma\left(4-\gamma\right) > \left(\gamma+2\right)\sqrt{2\left(\gamma^2+2\right)}$$

Squaring both sides, this is satisfied if $48\gamma + 52\gamma^2 + 16\gamma^3 + 7\gamma^4 + 48 > 0$, which is clearly the case. Hence, joint elimination of the low quality product leads to a higher p_H^* if and only if $\gamma > 4 - 2c$.

D Proof of Theorem 4

Total profits are equal to

$$\pi = \frac{1}{4}\left(\gamma\mu - 2c + \frac{c^2}{(\gamma-1)\mu}\right)$$

Comparing this to the profits when only offering the high end product:

$$\frac{1}{4}\left(\gamma\mu - 2c + \frac{c^2}{(\gamma-1)\mu}\right) - \frac{1}{4}\frac{(\gamma\mu - c)^2}{\gamma\mu} = \frac{1}{4}\frac{c^2}{\gamma\mu\,(\gamma-1)} > 0.$$

E Analysis non-covered market

Note that

$$\begin{aligned} \left.\frac{\partial q_{AL}}{\partial p_{AL}}\right|_{p_L^*} &= -\frac{1}{(\gamma-1)\,\mu}\left(\frac{p_H^* - p_L^*}{(\gamma-1)\,\mu}\right) - \int_{\frac{p_L^*-v}{\mu}}^{\frac{p_H^*-p_L^*}{(\gamma-1)\mu}} \frac{1}{\mu}dv - \frac{1}{\mu}\left(\frac{p_L^* - v}{\mu}\right) \\ &= -\frac{\gamma\,(p_H^* - p_L^*)}{\mu^2\,(\gamma-1)^2}, \end{aligned}$$

which is the exact same expression as in the case of a covered market, given in (25). The first-order condition thus requires

$$p_L^* \cdot \left.\frac{\partial q_{AL}}{\partial p_{AL}}\right|_{p_L^*} + q_L^* + (p_H^* - c)\left.\frac{\partial q_{AL}}{\partial p_{AL}}\right|_{p_L^*} = 0.$$

Both derivatives are the same as in the case of a covered market, but q_L^* is different and now given by (15). We thus require

$$-\frac{\gamma\,(p_H^* - p_L^*)}{\mu^2\,(\gamma-1)^2}p_L^* + \frac{1}{2}\left(\frac{p_H^* - p_L^*}{(\gamma-1)\,\mu}\right)^2 - \frac{1}{2}\left(\frac{p_L^* - v}{\mu}\right)^2 + (p_H^* - c)\,\frac{p_H^* - p_L^*}{(\gamma-1)^2\,\mu^2} = 0,$$

which, after multiplying by $2\mu^2\,(\gamma-1)^2$, simplifies to (22).

As noted in the main text, finding the equilibrium values (p_L^*, p_H^*) requires solving a system quadratic equations given by (19) and (22). There are in principal four solutions to the system. Figure 9 illustrates for the case that $v = 0.5$, $\mu = 1$, $\gamma = 3$, and $c = 1$. The black curves show in (p_H, p_L)-space the combinations of prices for which (22) is satisfied, while the red curves show those for which (19) is satisfied. There are indeed four points where the curves intersect. The equilibrium is clearly the intersection where both prices are positive, and where $p_H > p_L$, which is the far-right intersection in the graph, where $p_L^* = 0.566$ and $p_H^* = 1.990$.

Figure 9: Solving the case of an non-covered market