

# Vertical Mergers: No Foreclosure, Yet Harm to Consumers\*

ALESSANDRO S. KADNER-GRAZIANO<sup>†</sup>

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## Abstract

*The typical concern about vertical mergers is the foreclosure of downstream rivals. In a vertically related industry where downstream firms have a common supplier, margins can reveal whether upstream competition constrains that supplier. I develop a test (based on margins) to identify whether the supplier is constrained pre-merger and, consequently, cannot raise input prices post-merger. However, even absent foreclosure, vertical mergers can harm consumers. A merger increases consumer prices and benefits all firms, including downstream rivals, when downstream (horizontal) competition weakens sufficiently. This theory of harm differs markedly from typical theories, which pit the merged entity against downstream rivals.*

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<sup>†</sup>Faculty of Law, Business and Economics, University of Bayreuth, Universitätsstr. 30, 95440 Bayreuth, Germany, a.kadner-graziano@uni-bayreuth.de.

# 1 Introduction

Vertical merger policy is hotly debated. In 2020, the Department of Justice (DOJ) & Federal Trade Commission (FTC) released new vertical merger guidelines but rescinded them in 2021 due to disagreements within the FTC.<sup>1</sup> In contrast, in the academic literature, there exists broad agreement on several points. First, vertical mergers can eliminate double marginalisation (Spengler, 1950; Waterson, 1980). The elimination of double marginalisation (EDM) benefits consumers via lower prices.<sup>2</sup> Second, a merged entity may harm its downstream rivals by raising their input costs (Ordoover, Saloner, and Salop, 1990, 1992; Salinger, 1988; Salop and Scheffman, 1983, 1987).<sup>3</sup> Raising rivals' costs (RRC) harms consumers via higher prices. It is possible that a merged entity has an incentive to, instead, lower its rivals' costs (Das Varma and De Stefano, 2020; Domnenko and Sibley, 2020). Intuitively: when the EDM effect is very large, the merged entity is more willing to accommodate lower prices of downstream rivals and lowers input prices. Third, vertical mergers can weaken horizontal competition downstream (Baker et al., 2019; Chen, 2001; Moresi and Salop, 2021; Riordan, 1998). After the merger, the merged entity supplies a downstream rival and benefits from its sales. Consequently, the merged entity competes less aggressively downstream. I call this the "stakeholder effect" because of the close link to the minority shareholding literature.<sup>4</sup> The stakeholder effect harms consumers. Finally, in the literature, the overall merger effect can mostly go either way: it is seldom clear.<sup>5</sup>

Much of the literature on vertical mergers focusses on RRC.<sup>6</sup> Similarly, foreclosure is often the primary concern of antitrust authorities when they review vertical mergers. Foreclosure can come in different forms. (1) The merged entity might set and obtain a higher price from downstream rivals. I call this "direct RRC" because the higher price is paid directly to the merged entity. Alternatively, the merged entity might stop supplying downstream rivals. Consequently, downstream rivals either (2) incur higher costs by

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<sup>1</sup>See Federal Trade Commission (2021). Salop (2021) lays out suggested guidelines.

<sup>2</sup>Salop (2021) argues, "Pass on of EDM should not be taken as automatic" and that EDM is not necessarily merger-specific. On this, see also Kwoka and Slade (2020).

<sup>3</sup>The post-Chicago literature formalised this argument. RRC can also result from partial vertical integration, see Levy, Spiegel, and Gilo (2018).

<sup>4</sup>This effect has no agreed-upon name in the literature. Chen (2001) first identified this effect. He called it the "collusive effect". Moresi and Schwartz (2017) call it the "input supply effect". Moresi and Schwartz (2021) call it the "Chen effect".

<sup>5</sup>For example, Riordan (1998) shows that harmful effects can dominate, whereas Loertscher and Reisinger (2014) show that the EDM effect can exceed foreclosure effects.

<sup>6</sup>I focus on input foreclosure, see Salop (2018) on customer foreclosure. In other theories of harm, vertical mergers facilitate downstream collusion (Biancini and Ettinger, 2017) or upstream collusion (Nocke and White, 2007, 2010). Moreover, a vertical merger can serve to limit arbitrage and better price discriminate across industries (see Perry (1978, 1980) on the classic ALCOA case).

sourcing inputs through other means or (3) exit the market.

Whereas much has been written on the *incentive* of the vertically integrated entity to raise rivals' costs, less has been written on the merged entity's *ability* to do so. The presence of upstream competitors may constrain the merged entity in the price it sets to non-integrated downstream rivals. Without the ability to engage in direct RRC, incentives to do so are inconsequential. My two main contributions focus on ability.

First, I show how to identify whether upstream competition constrains the supplier pre-merger. I use this finding to develop a novel merger test which answers the question: will a proposed merged entity be unable to obtain a higher price from downstream rivals? Even for complex industry structures, the test is practical, transparent, and simple. It uses observable data on margins to predict merger effects. The prediction in existing models that the merged entity can engage in RRC can be consistent or inconsistent with observable data. The test can serve to identify models that are consistent with observable facts of a particular industry. In addition, antitrust authorities can use the test for merger review (they "typically collect [profit margins] as part of merger investigations", see Sheu and Taragin, 2021).

Second, because I provide a test to dismiss particular antitrust concerns, I also show and emphasise that even absent foreclosure, vertical mergers can harm consumers and benefit all firms (not only the merged entity but also its rivals). Precisely when a merged entity wants to but cannot raise its rivals' costs, downstream rivals benefit from weakened downstream competition. In such cases, a vertical merger can increase all consumer prices. Opposite to the typical foreclosure concern, which pits the merged entity against its downstream rivals (raised, for example, in Chen, 2001; Rey and Tirole, 2007), I therefore lay out a theory of harm where the merger benefits all firms and unambiguously harms consumers.

The baseline model I consider is standard. In a vertically related industry, two downstream firms produce a differentiated consumer good. Each downstream firm needs one unit of an input per unit of output. There are one or more upstream producers of that input; each has a constant unit cost of production. In the first stage, each upstream producer set a unit price (a bid) to each downstream firm, at which price it offers to supply any quantity. Producers are permitted to discriminate across downstream firms (i.e. to engage in third-degree price discrimination). In the second stage, each downstream firm sets its consumer price. Finally, consumer demand is determined and downstream firms order inputs. In the baseline model, demand is log-concave. Results are robust to a series of extensions, e.g. to incomplete information and to heterogeneous inputs.

In this paper, the crux is to identify constraints. According to J. Lerner and Tirole (2004), a firm is either constrained by competition ("the competition margin binds") or limited by the demand function ("the demand margin binds"). Thus more specifically, the crux is to identify whether the supplier is constrained by competition (and thus cannot

raise price) or whether the supplier is unconstrained by competition, sets its monopoly price, and thus could raise price post-merger.

To construct a novel merger test, I build on a result that relates upstream to downstream margins. In Kadner-Graziano (2022), I show that an upstream monopolist earns a higher unit dollar margin than the downstream firm it supplies; this holds for any log-concave demand function.<sup>7</sup> Because the pass-through rate is lower than 100% for log-concave demand functions (Amir, Maret, and Troege, 2004), the elasticity of demand is lower upstream than downstream, and consequently, by the intuition for the Lerner markup (A. P. Lerner, 1934), the unit dollar margin upstream must be bigger than downstream.<sup>8</sup>

In the present paper, I find that this relation of margins also holds with downstream competition and when an upstream producer supplies competing downstream firms. With downstream competition, when a monopolist supplier increases the price it sets to some downstream firm A, that downstream firm increases its price and (in equilibrium) firm A's competitors raise their prices as well (downstream prices are strategic complements). Equilibrium price increases by competitors dampen downstream firm A's loss of quantity. Consequently, the elasticity of demand upstream is even lower than downstream.

Moreover, if a producer supplies several competing downstream firms, an input price increase to one downstream firm diverts some demand to other downstream firms it supplies. Consequently, the supplier's elasticity of total demand (of total quantity sold to downstream firms) decreases further. To summarise, the elasticity of demand upstream is lower than downstream due to three factors: downstream cost absorption (keeping prices at other downstream firms fixed, the pass-through rate is less than 100%); the strategic response of other downstream firms (equilibrium consumer price increases reduce the loss of quantity at downstream firm A); the profit earned on diverted sales (diverted sales reduce the total loss of quantity upstream relative to downstream).

A monopolist supplier, unconstrained by competition, earns a bigger margin than the margin of any downstream firm it supplies. The test I develop states: if the supplier, on its sales to a downstream firm, earns a smaller margin than that downstream firm, then the supplier is constrained by upstream competition pre-merger and the merged entity cannot obtain a higher price from that downstream firm post-merger. If the merged entity could obtain a higher price, the supplier would have already charged a higher price pre-merger. In this way, margins can reveal the existence of constraints sufficient to rule out direct RRC.

The test is robust to a series of extensions which portray the complexity and intertwined nature of real supply chains. Among others, the relation of margins also holds

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<sup>7</sup>E.g. the dollars of profit Apple versus one of its suppliers earns per iPhone sold.

<sup>8</sup>Adachi and Ebina (2014b) obtain this result for the case of successive monopolies in a Cournot setting.

when there are more than two downstream firms, when downstream firms use any number of different inputs, and when different downstream firms use different sets of inputs. A more general relation between upstream and downstream margins holds with log-convex demand and secret contracts.

Utilising margins to inform on the ability to engage in RRC is new. Bresnahan and Reiss (1985) and Adachi and Ebina (2014a) also compare upstream to downstream margins but not in a merger environment.<sup>9</sup> Inderst and Valletti (2011)<sup>10</sup> also analyse the pre-merger margins of merging parties. They show that these are insufficient to predict post-merger incentives.

Absent foreclosure, the merger can either benefit or harm consumers. The benefit from EDM is small when, pre-merger, the supplier earns a small margin on sales to the downstream firm with which it integrates. Harm from the stakeholder effect is large when, pre-merger and among other factors, the supplier earns a large margin on the non-integrated downstream firm. Consumer harm is clearest when a downstream firm acquires an upstream firm from which it does not source inputs but which supplies a downstream rival. This particular form of a vertical merger is called a “diagonal merger” (see Zenger, 2020). Such a merger generates no EDM; it is simply a credible commitment to compete less aggressively downstream. Absent foreclosure, the only merger effect is the stakeholder effect. Consequently, all firms benefit, and all consumer prices rise (to the unambiguous detriment of consumers). Contrary to Chen (2001), this theory of harm requires neither efficiency gains nor switching costs,<sup>11</sup> neither for merger profitability nor for consumer harm.

For example, suppose Samsung acquires a supplier of Apple without previously having had any relationship with that supplier. Apple should be worried about an increase in input prices if that supplier has pricing power. However, margin data indicates each Apple supplier is constrained,<sup>12</sup> such that Samsung could not raise prices post-merger,<sup>13</sup> and hence the acquisition would only render Samsung less aggressive on the smartphone market. Consequently, both smartphone makers would benefit, whereas consumers would lose out.<sup>14</sup>

A vertical merger with a constrained supplier is akin to a horizontal merger: it has two countervailing effects that resemble those of horizontal mergers. First, some form of efficiency gain (here, the EDM) versus second, a loss of horizontal competition (here,

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<sup>9</sup>Adachi and Ebina (2014a) analyse a setting with symmetric Cournot competition both upstream and downstream. They find that Cournot competition induces a ratio of margins which depends on the number of upstream and downstream firms.

<sup>10</sup>They use variable proportions, see their production cost function on p.822.

<sup>11</sup>To incorporate switching costs is non-standard in the literature on vertical mergers.

<sup>12</sup>See Kadner-Graziano (2022).

<sup>13</sup>Samsung could not raise prices, except if Samsung itself was the constraint on that supplier.

<sup>14</sup>This example applies irrespective of the parts Samsung already supplies to Apple (e.g. screens).

the stakeholder effect). On similarities to horizontal mergers, see also Riordan (1998), Chen (2001), Baker et al. (2019), Moresi and Salop (2021). Thus, a vertical merger can constitute yet another form of market concentration. This has significant implications for empirical research: estimates of concentration should encompass not only common ownership and cross-ownership but also ownership of suppliers to competitors. See, e.g., Azar, Schmalz, and Tecu (2018) on empirical studies of common ownership. Azar, Raina, and Schmalz (2019) analyse, in addition, cross-ownership.

The remainder of the paper is structured as follows. In section 2, I lay out the baseline model and develop a relation of upstream to downstream margins with complete information. In section 3, I analyse vertical mergers, develop a novel test, and discuss consumer welfare effects. In section 4, I discuss the robustness to extensions. In section 5, I discuss alternative models. Section 6 contains a brief antitrust discussion. Finally, I conclude in section 7 and defer proofs to the appendix.

## 2 A relation of upstream to downstream margins

### 2.1 Baseline model

I model a vertically related industry. There are two downstream firms. Each produces a consumer good. The two consumer goods are differentiated. Each downstream firm needs one unit of an essential input per unit of output (each has a Leontief production function). Downstream firm  $i \in \{A, B\}$  can produce the input in-house at constant unit marginal cost  $c_i \geq 0$ .

There are three upstream producers of the input. The second and third-most efficient producers have, respectively, constant unit marginal costs of production  $c^{[2]}$  and  $c^{[3]}$ , with  $c^{[3]} \geq c^{[2]} > 0$ . Without loss of generality, the most efficient has zero production costs.

There are two stages. In the first stage, all upstream producers simultaneously submit bids to the two downstream firms. To each downstream firm, each upstream producer submits a unit price at which it offers to supply any quantity demanded by that downstream firm. Upstream producers can price discriminate across downstream firms.

In the second stage, the two downstream firms simultaneously set their respective uniform consumer price,  $P_A$  and  $P_B$ . The product market clears and quantities  $Q_A$  and  $Q_B$  are determined. Each downstream firm then orders inputs from an upstream producer or produces inputs internally.<sup>15</sup> Downstream firm  $i \in \{A, B\}$  pays unit price  $v_i$  and earns profit  $\Pi_i = (P_i - v_i)Q_i$ . In equilibrium, the most efficient upstream producer supplies both downstream firms and earns profit  $\Pi_S = \sum_i v_i Q_i$ .

Downstream firm  $i$ 's demand strictly decreases and is weakly log-concave in consumer

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<sup>15</sup>In a tie between two producers, the upstream firm with the lowest cost is chosen.

price  $P_i$ :  $Q'_i < 0$  and  $(\ln Q_i)'' \leq 0$ . (In section 4, I develop results that allow for log-convex demands.) Downstream products are imperfect substitutes, with  $\frac{\partial Q_i}{\partial P_j} > 0$  for  $i \neq j$ . Moreover, I make the following three assumptions. Assumptions 1 and 2 are standard, see, e.g., Miklós-Thal and Shaffer (2021). Assumption 1 states: at the equilibrium, if all consumer prices rise by the same small amount, then demand for each consumer product decreases.<sup>16</sup>

**Assumption 1** (Gross substitutes). *At equilibrium prices  $P_i^*$  and  $P_j^*$ ,  $|Q'_i| > \frac{\partial Q_i}{\partial P_j}$ .*

Assumption 2 ensures that, for any  $v_j$ , the supplier's profit is strictly concave in  $v_i$ .<sup>17</sup>

**Assumption 2** (Concave profit). *The Hessian of  $\Pi_S(v_i, v_j)$  is negative definite.*

Assumption 3 is not necessary; it is made for conciseness of the algebraic exposition.<sup>18</sup>

**Assumption 3** (Additively separable).  *$\frac{\partial Q_i}{\partial P_j} = 0$  for  $i \neq j$ .*

Finally, I assume complete information in the baseline model (this is not necessary, as shown in section 4).

## 2.2 Equilibrium

The game is solved by backwards induction. Once consumer demand is determined, every downstream firm sources inputs from the cheapest source: either it produces internally at  $c_i$  or orders inputs from the supplier who offered the lowest unit price. In stage 2, downstream firm  $i \in \{A, B\}$  sets price  $P_i$  to maximise its profit. At the optimal consumer price  $P_i^*$ ,<sup>19</sup>

$$\frac{\partial \Pi_i}{\partial P_i} = Q_i(P_i^*, P_j) + [P_i^* - v_i] Q'_i(P_i^*, P_j) = 0 \quad (1)$$

In stage 1, upstream producers set offer prices to each downstream firm. The most efficient upstream producer bids unit price  $v_i$  to downstream firm  $i$ . Let  $v_i^u$  denote the monopoly price: the price the most efficient producer would set if it were unconstrained

<sup>16</sup>This assumption ensures sensible comparative statics (in particular, it ensures that a downstream firm raises its consumer price in response to an increase in its unit input cost).

<sup>17</sup>Assumption 2 is not necessary for the results of sections 2 and 3. It is sufficient to ensure that a constrained supplier prices at the constraint rather than below.

<sup>18</sup>Assumption 3 is not necessary: it could be replaced by a weaker though less transparent assumption. It removes a second-order effect. It is satisfied for any demand function where the price of a competing downstream firm enters as an additively separable term. E.g. as in Shubik-Levitan linear demand systems. However, unlike in linear demand systems, I allow for  $Q''_i \neq 0$ .

<sup>19</sup>The second-order condition is satisfied:  $Q_i^2 \geq Q_i Q''_i$  because demand is weakly log-concave, and therefore  $\frac{\partial^2 \Pi_i}{\partial P_i^2} \Big|_{P_i=P_i^*} = \frac{2Q_i^2 - Q_i Q''_i}{Q_i} < 0$ . Prices are strategic complements:  $\frac{\partial^2 \Pi_i}{\partial P_i \partial P_j} = \frac{\partial Q_i}{\partial P_j} > 0$ .

by any outside option of the downstream firm. Then, the most efficient producer sets unit price<sup>20</sup>

$$v_i^* \equiv \operatorname{argmax}_{v_i} \Pi_S = \min\{v_i^u, c^{[2]}, c_i\} \quad (2)$$

If the supplier cannot set its monopoly price, it is constrained either at  $c^{[2]}$  (the unit cost of the second-most efficient potential supplier) or at  $c_i$  (beyond which a downstream firm produces the input in-house).

**Definition 1.** *Supplier  $i$  is **unconstrained** if  $v_i^* = v_i^u$  but **constrained** if  $v_i^* \neq v_i^u$ .*

The most efficient producer has profit

$$\Pi_S = v_i Q_i(P_i^*(v_i, v_j), P_j^*(v_j, v_i)) + v_j Q_j(P_j^*(v_j, v_i), P_i^*(v_i, v_j)) \quad (3)$$

At the unconstrained (or monopoly) price  $v_i^u$ ,

$$\frac{\partial \Pi_S}{\partial v_i} = Q_i + v_i^u \left[ Q_i' \frac{dP_i^*}{dv_i} + \frac{\partial Q_i}{\partial P_j} \frac{dP_j^*}{dv_i} \right] + v_j \left[ Q_j' \frac{dP_j^*}{dv_i} + \frac{\partial Q_j}{\partial P_i} \frac{dP_i^*}{dv_i} \right] = 0 \quad (4)$$

### 2.3 Constrained or unconstrained – what margins reveal

I aim to identify whether a supplier is constrained. Rearranging (4), the supplier's unit dollar margin on sales to downstream firm  $i$ , when it is unconstrained,  $m_{S_i}^u$ , is<sup>21</sup>

$$v_i^u = - \frac{Q_i}{Q_i' \frac{dP_i^*}{dv_i} + \frac{\partial Q_i}{\partial P_j} \frac{dP_j^*}{dv_i}} - v_j \frac{Q_j' \frac{dP_j^*}{dv_i} + \frac{\partial Q_j}{\partial P_i} \frac{dP_i^*}{dv_i}}{Q_i' \frac{dP_i^*}{dv_i} + \frac{\partial Q_i}{\partial P_j} \frac{dP_j^*}{dv_i}} \quad (5)$$

A test that would say the supplier is constrained if and only if  $v_i^* < v_i^u$ , where  $v_i^u$  is given by (5), would not be helpful because the numerous derivatives in (5) are difficult to estimate in practice.

Instead, to develop a practical test I find a sufficient condition. My approach is to compare the margin of an unconstrained supplier with the downstream margin. Rearranging (1), the unit dollar margin of downstream firm  $i$ ,  $m_i^*$ , is

$$P_i^* - v_i = - \frac{Q_i(P_i^*, P_j)}{Q_i'(P_i^*, P_j)} \quad (6)$$

<sup>20</sup>Equation (2) can be written as  $v_i^* = \min\{v_i^u, \bar{c}\}$  where  $\bar{c} \equiv \min\{c^{[2]}, c_i\}$ . I distinguish between the two costs because the distinction matters for some merger results.

<sup>21</sup>Because the results relate upstream to downstream margins, the results do not change if the upstream firm has strictly positive unit costs of production and if the upstream firm has different unit costs of production for different downstream firms.



To compare margins in (5) and (6), one can rewrite (1) as

$$Q_i\left(P_i^*(v_i, v_j), P_j^*(v_j, v_i)\right) + [P_i^*(v_i, v_j) - v_i] Q'_i\left(P_i^*(v_i, v_j), P_j^*(v_j, v_i)\right) = 0 \quad (7)$$

and totally differentiate (7) with respect to  $v_i$  and  $v_j$ . Then, one can solve for  $\frac{dP_i^*}{dv_i}$  and  $\frac{dP_j^*}{dv_i}$ , and subsequently plug these two total derivatives into (5) and (6). Finally, comparing (5) with (6) yields the following key result on the relation of upstream to downstream margins. When unconstrained, the supplier's unit dollar margin on sales to downstream firm  $i$  weakly exceeds downstream firm  $i$ 's unit dollar margin.

**Theorem 1.** (*Relation of margins*) *The margin of an unconstrained supplier, on sales to downstream firm  $i$ , weakly exceeds downstream firm  $i$ 's margin:  $m_{S_i}^u \geq m_i^*$ .*

The relation of upstream to downstream margins in Theorem 1 is simple. Nevertheless, the intuition and effects which underlie the result are intricate. In stage 1, the supplier sets its profit-maximising unit input price to each downstream firm by accounting for stage 2 downstream equilibrium effects. Consider an increase in input price  $v_A$ . Keeping the price of downstream firm B fixed, downstream firm A passes through less than 100% to consumers (it absorbs some of the input price increase) because demand is log-concave. Holding  $P_B$  fixed, the elasticity of demand upstream is therefore lower than downstream and, by the intuition for the Lerner index,<sup>22</sup> the margin upstream must exceed the margin downstream.

In equilibrium, because downstream prices are strategic complements, downstream firm B raises its price. This dampens the supplier's loss of quantity sold to downstream firm A from an increase in  $v_A$ . The increase in  $P_B$  generates a feedback increase in  $P_A$ , but overall, the change in  $P_B$  and the feedback effect on  $P_A$  necessarily dampen (rather than exacerbate) the loss of demand at downstream firm A (because downstream firm A changes its price only "as a reaction" to downstream firm B's price change). As a result, after accounting for these changes in equilibrium downstream prices, the elasticity of demand upstream is lower still than downstream. And consequently, the unconstrained margin upstream is higher still compared to the margin downstream.

Moreover, because the supplier also supplies downstream firm B, it earns a profit on any sales diverted from downstream firm A to B. This effect further lowers the "total elasticity of demand" upstream (the loss of total quantity sold to downstream firms A and B) relative to the elasticity of demand of downstream firm A, and therefore further increases the unconstrained upstream margin relative to the downstream margin.

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<sup>22</sup>Whereas the Lerner index relates a firm's unit percentage margin to the elasticity of demand it faces, Theorem 1 can relate absolute margins because the quantities sold by the upstream and downstream firms are the same – or, more generally with fixed proportions, they are a fixed ratio of each other.

From Theorem 1, it directly follows that if  $m_{S_i}^* < m_i^*$ , then the supplier must be constrained in the price it sets to downstream firm  $i$ . In words: if the supplier earns a margin on sales to downstream firm  $i$  that is smaller than the margin of downstream firm  $i$ , then the supplier is constrained. The supplier would like to set a higher price but cannot because of some outside option of the downstream firm.

**Corollary 1** (Constrained supplier). *If  $m_{S_i}^* < m_i^*$ , then the supplier is constrained in the price it sets to downstream firm  $i$ .*

Corollary 1 provides a sufficient condition to identify a constrained supplier. The condition is transparent, simple, and practical. Even if the supplier earns a large margin (even if it has a large competitive advantage over other potential suppliers or in-house production by the downstream firm), the supplier may be constrained.

### 3 Vertical merger

I distinguish between two cases of foreclosure. The merged entity can engage in *direct RRC* if it can obtain a higher input price from the non-integrated downstream rival post-merger (relative to the price it obtained pre-merger). With direct RRC, the merged entity receives a higher price and thus benefits directly from its rival's higher cost. The merged entity can engage in *indirect RRC* if, by stopping to supply the non-integrated downstream firm, the latter's input cost increases (because, for example, it pays a higher price to another supplier or switches to more costly in-house production). With indirect RRC, the merged entity does not receive a direct payment from the downstream rival; instead, the merged entity benefits only indirectly from its rival's higher cost through weakened downstream competition. If stopping to supply causes a sufficiently large cost increase, the rival exits the market.<sup>23</sup>

In this section, I develop a test to identify whether the merged entity cannot extract a higher input price from the downstream rival. In other words, the test serves to answer the question: is the merged entity unable to engage in direct RRC?

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<sup>23</sup>This is sometimes referred to as “complete exclusion” (Rey and Tirole, 2007). No terms are used uniformly across the academic and legal antitrust literatures to distinguish between direct and indirect RRC. The first case is sometimes referred to as “foreclosure” or “RRC” or also as “partial foreclosure” (Moresi and Schwartz, 2021). The second case is also sometimes referred to as “foreclosure”, “RRC”, “complete foreclosure” (Rogerson, 2019), or also as “full foreclosure” (Baker et al., 2019). However, “full foreclosure” sometimes refers to the case where the downstream rival exits the market.

### 3.1 Post-merger equilibrium

Consider a merger between the supplier and downstream firm A. If the merged entity  $M$  continues to supply the non-integrated firm, its profit is

$$\Pi_M = P_A Q_A + v_B Q_B$$

The profit function of the non-integrated downstream firm B remains unchanged:

$$\Pi_B = (P_B - v_B) Q_B$$

The game is again solved by backwards induction. The non-integrated downstream firm's first-order condition is still given by (1). However, the merged entity's first-order condition differs from (1). At the optimal consumer price  $P_A^{*M}$ ,

$$\frac{\partial \Pi_M}{\partial P_A} = Q_A \left( P_A^{*M}, P_B \right) + P_A^{*M} Q'_A \left( P_A^{*M}, P_B \right) + v_B \frac{\partial Q_B}{\partial P_A} = 0$$

For a given  $v_B$ , there are two countervailing merger effects on the consumer price  $P_A$ . The merged entity competes more aggressively downstream because its unit cost is zero (the EDM effect). In contrast, the merged entity competes less aggressively because it earns  $v_B$  per unit sold by the non-integrated firm (the stakeholder effect).<sup>24</sup>

In stage 1, the merged entity sets the optimal input price to the non-integrated downstream firm

$$v_B^{*M} \equiv \operatorname{argmax}_{v_B} \Pi_M = \min \{ v_B^{uM}, c^{[2]}, c_B \}$$

where the merged entity sets  $v_B^{uM}$  if it is unconstrained (a monopolist supplier). At  $v_B^{uM}$ ,

$$\frac{\partial \Pi_M}{\partial v_B} = Q_B + v_B^{uM} \left( Q'_B \frac{dP_B^*}{dv_B} + \frac{\partial Q_B}{\partial P_A} \frac{dP_A^*}{dv_B} \right) + \frac{dP_A^*}{dv_B} Q_A + P_A^* \left( Q'_A \frac{dP_A^*}{dv_B} + \frac{\partial Q_A}{\partial P_B} \frac{dP_B^*}{dv_B} \right) = 0$$

The FOC of the merged entity differs from (4), the FOC of the supplier pre-merger, in two ways. First, the merged entity now earns  $P_A$  rather than  $v_A$  from each unit of the consumer product A. Second, an increase in  $v_B$  allows the merged entity to increase its own downstream price  $P_A$ , from which it benefits for any given quantity level  $Q_A$ . Algebraically, this refers to  $\frac{dP_A^*}{dv_B} Q_A$ .<sup>25</sup>

<sup>24</sup>Algebraically, the term  $v_B \frac{\partial Q_B}{\partial P_A} > 0$  stands for this effect. The merged entity does not earn a share of profits, its earnings are instead directly proportional to the quantity sold by the downstream rival and customer.

<sup>25</sup>The merged entity has an incentive to engage in direct RRC if and only if  $v_B^{uM} > v_B^*$ . It has an incentive to raise the input price beyond the pre-merger unconstrained price if and only if  $v_B^{uM} > v_B^u$ .

### 3.2 Ability to engage in direct RRC: the test

I now develop a new test for vertical mergers. The test follows from Theorem 1 and Corollary 1. It says: if pre-merger, the supplier's margin on sales to downstream firm B is smaller than downstream firm B's margin, then the supplier is constrained and cannot obtain a higher price post-merger. In other words, the merged entity cannot engage in direct RRC.

**Proposition 1 (Test).** *If  $m_{SB}^* < m_B^*$  pre-merger, the merged entity cannot obtain an input price  $v_B$  that is higher than pre-merger.*

The test provides a transparent, simple and practical sufficient condition. It is based on margin data because margins can reveal information on constraints. It suffices to know that the supplier's margin is smaller than that of the non-integrated downstream firm to conclude that the merged entity cannot extract a higher price from the non-integrated downstream firm. If the merged entity could do so, the supplier would have already raised the price pre-merger.<sup>26</sup>

### 3.3 Ability to engage in indirect RRC

If the supplier is constrained at  $c_B$  pre-merger and the merged entity stop supplying, the non-integrated downstream firm can switch to in-house production without incurring higher costs. Consequently, the merged entity cannot engage in indirect RRC and finds it profitable to continue to supply. The same holds if the supplier is constrained at  $c^{[2]}$ , with  $c^{[2]} = c^{[3]}$ . The condition  $c^{[2]} = c^{[3]}$  can be interpreted as the existence of a competitive fringe. Only if  $c^{[2]} < c_B$  and  $c^{[2]} < c^{[3]}$  does the merged entity have the ability to engage in indirect RRC. But, as is well-known, it may not have the incentive to halt supply.<sup>27</sup>

### 3.4 Consumer welfare

Consider a vertical merger with a constrained supplier. The merger can decrease consumer welfare even without foreclosure (absent direct and indirect RRC). Competition weakens post-merger when the merged entity benefits from sales of the non-integrated downstream firm (when  $v_B > 0$ ); because prices are strategic complements, both consumer prices can increase.

**Proposition 2 (Consumer Welfare).** *Absent a change in  $v_B$ , a vertical merger*

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<sup>26</sup>Even if the supplier is unconstrained pre-merger, a constraint could lie just above its pre-merger price, such that its ability to engage in RRC may be limited.

<sup>27</sup>“Vertical arithmetic” is an existing tool which practitioners use to assess the incentive of such foreclosure (Zenger, 2020).

- *increases consumer welfare whenever benefits from the EDM effect outweigh harm from the stakeholder effect but*
- *decreases consumer welfare whenever harm from the stakeholder effect outweighs benefits from the EDM effect.*

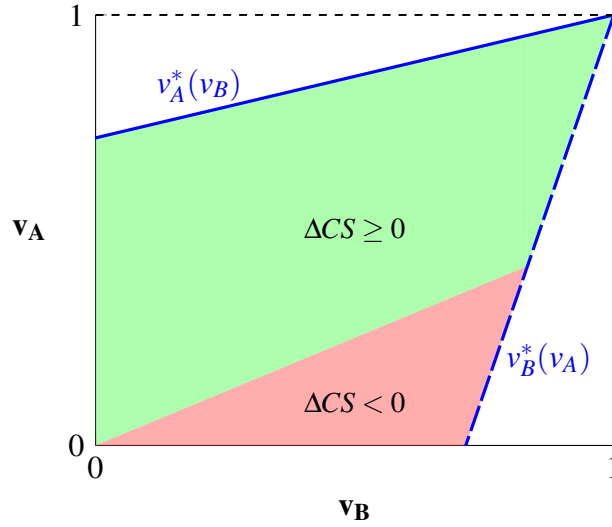
Intuitively, if pre-merger  $v_A > 0$  and  $v_B = 0$ , EDM is the only merger effect. Consequently, both consumer prices decrease and consumers benefit from the merger. Conversely, if pre-merger  $v_A = 0$  and  $v_B > 0$ , the stakeholder effect is the only merger effect. Consequently, both consumer prices increase and the merger harms consumers.

Figure 1 below illustrates cases where EDM and the stakeholder effect both materialise (in an example with Shubik-Levitan demand). Pre-merger, if the supplier is unconstrained, it sets  $(v_A^u, v_B^u) = (1, 1)$ . If the supplier is constrained in the price it can charge to downstream firm B, then its optimal price  $v_A$  lies below one. The blue line depicts  $v_A^u(v_B)$ , the supplier's unconstrained price to downstream firm A for a given level of  $v_B$ . Therefore, the supplier never sets  $v_A$  inside the top left white triangle. By symmetry, the supplier never sets  $v_B$  inside the bottom right white triangle pre-merger, and the dashed blue line depicts  $v_B^u(v_A)$ .<sup>28</sup> In the green and red areas, the supplier prices at the constraints pre-merger, e.g.  $(v_A^*, v_B^*) = (c_A, c_B)$ . In the green area, consumer prices  $P_A$  and  $P_B$  decrease post-merger: consumers benefit unambiguously. In the red area, consumer prices  $P_A$  and  $P_B$  increase post-merger: the merger unambiguously harms consumers.<sup>29</sup> Consumer harm occurs when the vertical elimination of double marginalisation (related to  $v_A$ ) is small enough relative to the loss of horizontal competition (related to  $v_B$ ).

<sup>28</sup>However, post-merger  $v_B$  can lie inside this bottom right triangle. The merged entity's unconstrained input price is  $v_B^M = 0.985$ . Therefore, foreclosure occurs when the supplier charges less than 0.985 pre-merger and when the constraint on  $v_B$  strictly exceeds the pre-merger unconstrained optimum. Finally, for any pre-merger price  $v_B > 0.985$ , the merged entity lowers its rival's cost.

<sup>29</sup>Absent a change in  $v_B$ ,  $P_B$  changes only as a reaction to a change in the best response function  $P_A^*(P_B)$ . Prices are strategic complements; therefore, consumer prices change in the same direction post-merger.

Figure 1: Post-Merger Change in Consumer Surplus, Without Foreclosure (example)



Note: Consumer surplus increases post-merger in the green area, whereas it decreases in the red area. Here, consumer demand is  $Q_i = 1 - P_i + \theta P_{-i}$  for  $i \in \{A, B\}$ , with  $\theta = 0.5$ . Higher  $\theta$  increases the slope of the cutoff line between the green and red areas and hence the region of consumer harm. Source: own workings.

When a downstream firm integrates with a competitor’s supplier without sourcing from that supplier (this corresponds to the case with  $v_A = 0$ ),<sup>30</sup> there is no EDM and the merged entity always wants to foreclose its downstream competitor. Absent foreclosure (when  $m_{SB}^* < m_B^*$  and when the merged entity continues to supply), the merger solely results in the stakeholder effect: it benefits all firms but harms consumers.

### 3.5 Merger profitability

The merged entity internalises the benefit of an increase in its consumer price  $P_A$  on the demand for the consumer product B. Therefore, the merged entity can find it profitable to lose both quantity- and sales-based market share (downstream). It is also possible that, due to the merger, the profit of the non-integrated downstream firm increases by more than the profit of the merging parties.<sup>31</sup>

**Proposition 3.** *Profitable vertical mergers can*

<sup>30</sup>See the discussion on “diagonal mergers” in Zenger (2020), and in the FTC & DOJ’s Vertical Merger Guidelines, U.S. Department of Justice & The Federal Trade Commission (2020), p.9-10.

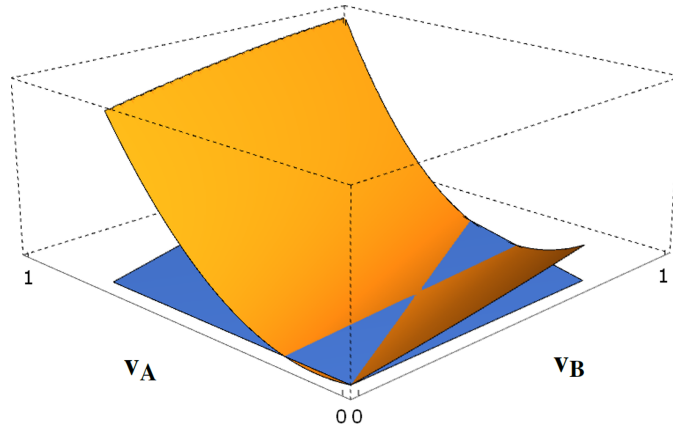
<sup>31</sup>Ordover, Saloner, and Salop (1990) and Hart and Tirole (1990) pioneered work on endogenous mergers: they analyse which downstream firm has, within their model, the greatest incentive to acquire the supplier. On endogenous mergers, see also Buehler and Schmutzler (2008). In practice, downstream firms A and B may be active on many markets rather than only one, such that there may be incentives to merge related to activities not captured within the present model.

- *lead the merged entity to reduce output and lose market share downstream. They can also*
- *profit the non-integrated firm more than the merging parties.*

Intuitively, the two points in Proposition 3 occur when the principal merger effect is to reduce the merged entity’s aggressiveness downstream. For example, without foreclosure and EDM (e.g. with  $m_{SB}^* < m_B^*$  and  $c_A = 0$ ), the only merger effect is to render the merged entity less aggressive downstream. As a result, both consumer prices rise in equilibrium. However, the first-order effect is an increase in  $P_A$  with a reduction in  $Q_A$ , whereas downstream firm B not only increases  $P_B$  but also enjoys an increase in  $Q_B$ .

Surprisingly perhaps, a vertical merger is not always profitable.<sup>32</sup> In contrast to the stakeholder effect, EDM commits the merged entity to compete more aggressively downstream. Because downstream firm B knows that the cost of its rival has fallen, downstream firm B behaves more aggressively too. More intense competition downstream is why the merger can be unprofitable. Figure 2 below plots the change in the merging parties’ profit  $\Pi_M - \Pi_S - \Pi_A$  on the z-axis, wherever the constraints bind pre-merger and for the same example demand system as in Figure 1. The merger is not profitable where the blue 0-profit plane lies above the orange profit surface.

Figure 2: Change in the Merging Parties’ Profit, Without Foreclosure (example)



Note: The orange surface depicts the post-merger change in profit  $\Pi_M - \Pi_S - \Pi_A$  (represented on the z-axis). The blue plane depicts the zero-plane: the merger is unprofitable where the blue plane lies above the orange surface. Here (as in Figure 1), consumer demand is  $Q_i = 1 - P_i + \frac{1}{2}P_{-i}$  for  $i \in \{A, B\}$ . Source: own workings.

<sup>32</sup>Loertscher and Riordan (2019) show that vertical mergers can be unprofitable when they reduce innovation incentives of non-integrated upstream producers.

**Remark.** *The vertical merger would always be profitable if either*

- *the merged entity sets both  $v_B$  and  $P_A$  in stage 1 and therefore acts as a Stackelberg leader in consumer prices or if*
- *the merged entity sets  $v_B$  and  $v_A$  in stage 1 and delegates the downstream profit maximisation problem. In stage 2, the downstream delegate chooses  $P_A$  to maximise  $(P_A - v_A)Q_A(P_A, P_B)$  whereas in stage 1, the merged entity sets  $v_B$  and  $v_A$  to maximise its total profit upstream and downstream.*<sup>33</sup>

*Delegation and Stackelberg leadership lead to identical profits and consumer prices (on this link, see Vickers, 1985 and Moresi and Schwartz, 2017). Seminal work by Schelling (1960) shows instances in which a firm finds it profitable to delegate maximisation problems to elicit less aggressive competition. Related work on delegation within a firm and strategic incentives includes Fershtman and Judd (1987).*

## 4 Robustness

Results of sections 2 and 3 are robust to each of the extensions below.

### 4.1 General model, with incomplete information

Consider the following extensions to the baseline model. First, a downstream firm's demand can now be log-convex in its price  $P_i$ . Second, there can be any number of competing downstream firms. Third, each downstream firm may need several different inputs, and the sets of inputs used by different downstream firms can differ. (For now, I maintain the Leontief production function: any input which a downstream firm uses is essential to that downstream firm.) Fourth, each upstream producer can have a different cost to supply each downstream firm (hence the efficiency ranking of producers may differ by downstream firm).

I now formally describe the general model. There is a set  $X$  of inputs. There are one or more producers of each input. Producers have a constant unit marginal cost of production. There is a number and a set  $D$  of downstream firms, with  $\frac{\partial Q_d}{\partial P_j} \geq 0$  for  $d \neq j$  and  $d, j \in D$ . Downstream firm  $d \in D$  has a Leontief production technology whereby it needs a set of

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<sup>33</sup>Intuitively, with delegation, the merged entity can at least replicate the pre-merger profit by charging its downstream firm the pre-merger input price  $v_A^*$ . With Stackelberg leadership, the merged entity can at least replicate the pre-merger profit by setting  $v_A$  and  $P_A$  to their pre-merger levels. Moresi and Schwartz (2017) explain that the result with delegation requires interim observability of the merged entity's internal price  $v_A$  by the non-integrated, and commitment power of the merged entity not to alter that internal price.



inputs  $x_d \subseteq X$ . (Different downstream firms can require different sets of inputs.) The unit cost of producer  $i$  to supply downstream firm  $d$  is  $c_{id}$ .

In stage 1, producers simultaneously submit unit price offers to downstream firms. Supplier  $i$  supplies the set of downstream firms  $S_i \subseteq D$ . In stage 2, downstream firms simultaneously set consumer prices. Let  $\mathbf{P}$  denote the  $D$ -dimensional vector of consumer prices. I assume (a) the profit  $\Pi_d$  of downstream firm  $d$ ,  $\forall d \in D$ , is twice differentiable and strictly quasi-concave in consumer price  $P_d$ ; (b) the pass-through rate  $\rho_d$  of any downstream firm  $d$  is non-decreasing in its unit cost; (c) the profit function of any supplier  $i$  is twice differentiable and strictly quasi-concave in the  $S_i$ -dimensional vector of prices  $\mathbf{v}_i$  it sets.

I depart from complete information. Let offers be secret (downstream firms do not observe the input prices of their competitors), let downstream firms form point beliefs about input prices of their competitors, and let these beliefs be correct in equilibrium. When a supplier sets its optimal input price  $v_i$  to downstream firm  $i$ , a change in  $v_i$  changes  $P_i$  but not the consumer prices of other downstream firms (who set their prices based on their point beliefs regarding  $v_i$ ). I use the “passive beliefs” assumption as laid out by Rey and Tirole (2007) in relation to “secret contracts” and as used by Moresi and Salop (2013). Accordingly, the supplier correctly believes that a change in  $v_i$  does not affect downstream firm  $i$ 's beliefs on the input prices of other firms. In this sense, there are passive beliefs.<sup>34</sup>

Downstream firm  $d \in D$  pays unit price  $v_{id}$  for input  $i \in x_d$ . It has profit

$$\Pi_d = (P_d - \sum_{i \in x_d} v_{id}) Q_d(\mathbf{P})$$

and first-order condition (with respect to  $P_d$ )

$$Q_d(\mathbf{P}) + m_d^* Q_d'(\mathbf{P}) = 0 \quad (8)$$

where  $m_d$  is its unit margin. Supplier  $i$  has profit

$$\Pi_i = \sum_{d \in S_i} (v_{id} - c_{id}) Q_d(\mathbf{P})$$

and first-order condition (with respect to  $v_{id}$ )

$$\begin{aligned} Q_d(\mathbf{P}) + (v_{id}^u - c_{id}) \frac{\partial Q_d(\mathbf{P})}{\partial v_{id}} + \sum_{j \neq d} (v_{ij} - c_{ij}) \frac{\partial Q_j(\mathbf{P})}{\partial v_{id}} &= 0 \\ Q_d(\mathbf{P}) + m_{id}^u \frac{\partial Q_d(\mathbf{P})}{\partial P_d} \frac{\partial P_d}{\partial v_{id}} + \sum_{j \neq d} (v_{ij} - c_{ij}) \frac{\partial Q_j(\mathbf{P})}{\partial P_d} \frac{\partial P_d}{\partial v_{id}} &= 0 \\ Q_d(\mathbf{P}) + m_{id}^u Q_d'(\mathbf{P}) \rho_d + \rho_d \sum_{j \neq d} (v_{ij} - c_{ij}) \frac{\partial Q_j(\mathbf{P})}{\partial P_d} &= 0 \end{aligned} \quad (9)$$

<sup>34</sup>This is the key technical and simplifying consequence of such beliefs.

Comparing (8) and (9),<sup>35</sup>

$$m_d^* = m_{id}^u \times \rho_d + \frac{1}{Q_d(\mathbf{P})} \rho_d \sum_{j \neq d} (v_{ij} - c_{ij}) \frac{\partial Q_j(\mathbf{P})}{\partial P_d}$$

$$\Rightarrow m_d^* \leq m_{id}^u \times \rho_d$$

**Theorem 2.** *With passive beliefs, if  $m_{id}^* \times \rho_d < m_d^*$ , then supplier  $i$  is constrained in the price it sets to downstream firm  $d$ , and it cannot raise the input price post-merger.*

The intuition for Theorem 2 is simpler than Theorem 1 because, with passive beliefs, downstream firms do not react to a change in a competitor's input price (there are no feedback effects downstream.) Intuitively, the monopoly margin of supplier  $i$  on sales to downstream firm  $d$  is higher (lower) than the margin of the downstream firm because, with log-concave (log-convex) demand, the pass-through rate is below (above) one and therefore, the elasticity of demand upstream is lower (higher) than downstream.

**Remark.** *Suppose upstream producers do not know the unit cost of their competitors. If each downstream firm uses a second-price auction (SPA) for every input  $i \in x_d$ , producers bid the offered unit price down to  $\min\{v_{id}^u, c_{id}^{[2]}\}$ . Then, each downstream firm has unit cost  $\min\{v_{id}^u, c_{id}^{[2]}, c_{id}\}$ . This outcome is the same as the complete information outcome of section 2.*

## 4.2 Input substitution

Suppose the input is differentiated. Product differentiation impacts the constraint: instead of being constrained at  $c^{[2]}$ , the supplier is constrained at another limit price (at the level of  $v_i$  beyond which downstream firm  $i$  would switch from its first-ranked to its second-ranked supplier).<sup>36</sup> Because Theorem 1 and Corollary 1 relate the downstream margins to the *unconstrained* upstream margin, both results are unrelated to the degree of upstream product or quality differentiation among producers of the given input. The test (Proposition 1) also remains unaffected.

<sup>35</sup>The weak inequality holds with equality if the supplier supplies only one downstream firm or if  $v_{ij} = c_{ij}$  for any other downstream firm  $j \neq d$  it supplies.

<sup>36</sup>The ranking is defined as follows. Suppose all suppliers bid a unit price equal to their unit production cost. At those offer prices, the downstream firm will have a (weak or strict) preference for one supplier. This supplier is the first-ranked supplier, with unit production cost  $c^{[1]}$ . The second-ranked supplier has unit production cost  $c^{[2]}$ . In the undifferentiated case, the limit price on the first-ranked is  $c^{[2]}$ . In the differentiated case, the limit price (greater than  $c^{[1]}$ ) may be greater or smaller than  $c^{[2]}$  (for, respectively, a quality advantage or disadvantage).

### 4.3 Multi-product suppliers

Suppose the supplier produces a range of different inputs (rather than quality differences, here I refer to inherently different inputs). Then, when applying the results, one considers the total unit margin a supplier earns on sales to a downstream firm. The results verify whether a supplier is constrained for however many inputs an upstream firm sells to a downstream firm.

### 4.4 Multi-product downstream firms

As is well-known, a vertical merger of single-product successive monopolies eliminates double marginalisation and necessarily decreases the consumer price. Salinger (1991) and Luco and Marshall (2020, 2021) build on Edgeworth’s taxation paradox (Edgeworth, 1925) and develop a counterintuitive result. They find that vertical integration by a multi-product downstream firm can lead all consumer prices to increase if some products benefit from EDM (“integrated products”) whereas others do not (“non-integrated products”). Surprisingly, costs decrease but prices increase. Intuitively, thanks to EDM, it becomes more profitable to sell integrated products and therefore, the merged entity raises the price of non-integrated products to divert sales to integrated products. The latter effect can be sufficiently large for all downstream prices to rise. This theory of harm does not feature a downstream rival and thus does not feature foreclosure.

In the present paper, all downstream prices can increase after a vertical merger due to the stakeholder effect. If the merged entity sells multiple consumer products,<sup>37</sup> where some benefit from EDM whereas others do not, the merged entity has additional tools (prices of non-integrated products) to raise not only demand for integrated products but also demand for the rival’s product. The insights of sections 2 and 3 continue to apply.

### 4.5 Production technology

The supplier’s optimal price function given in (2) contains a further limit price if the input is non-essential (i.e. downstream firms can do without the input and still produce a valuable consumer product). Beyond that limit price, a downstream firm chooses not to source the input. This adds a further explanation as to what may constrain a supplier but does not alter results.

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<sup>37</sup>Let those products be substitutes rather than complements.

## 5 Different settings

The test I develop applies when a supplier's margin reveals information on its pricing power. It does not apply to settings where a supplier's margin does not reflect its pricing power. Two such settings are two-part tariffs and bargaining, which I discuss below. Subsequently, I discuss a setting where the ability to engage in direct RRC is endogenous.

### 5.1 Two-part tariffs

With two-part tariffs and complete information, the supplier may not earn a positive margin and, instead, may extract all industry profits via fixed fees. However, with upstream contractual frictions, a supplier does not extract all profit via fixed fees and sets a price above marginal cost (see Calzolari, Denicolo, and Zanchettin, 2020). Similarly, when a producer supplies several competing downstream firms, it does not necessarily offer each downstream firm a price equal to marginal cost. Instead, the supplier might price discriminate (see Herweg and Müller, 2014, 2016; Inderst and Valletti, 2009). Generally, one should not apply the test to settings with two-part tariffs because comparing upstream to downstream margins does not necessarily reveal pricing (or “market”) power.<sup>38</sup>

### 5.2 Bargaining

The test should not be applied when upstream and downstream firms bargain over prices because, whilst margins do reflect bargaining power, input prices can increase post-merger even if a supplier has little bargaining power. The 2020 U.S. DOJ & FTC vertical merger guidelines explain how, with bargaining, disagreement payoffs change post-merger. The change allows the merged entity's upstream unit to demand a higher input price from non-integrated downstream firms, such that the merger results in RRC and can harm consumers (U.S. Department of Justice & The Federal Trade Commission, 2020, p.7).

Rogerson (2019) develops a tool to estimate the size of such effects, which he calls “bargaining leverage effect”, but in a partial rather than full equilibrium model (he leaves out EDM effects).<sup>39</sup> In contrast, Das Varma and De Stefano (2020) show that the merged entity can have the incentive to lower the input price of its downstream rival (and customer) such that the average equilibrium downstream consumer price decreases. Constraints, a focus of the present paper, are less relevant to bargaining but not superfluous. When suppliers are not monopolists, the EDM effect might be small and the merged entity is less likely to lower its rivals' costs. (See Das Varma and De Stefano, 2020 and

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<sup>38</sup>See Reisinger and Tarantino (2015) for a model of vertical integration with two-part tariffs and incomplete information.

<sup>39</sup>He simplifies the exercise by assuming downstream prices are fixed during input price negotiations. He motivates this assumption by arguing that upstream and downstream prices may be set simultaneously.

Zenger, 2020.) Hence, with competition upstream, bargaining models might indeed only yield RRC.

There is a growing literature on vertical mergers with bargaining and incomplete information. In such settings, merger effects can go either way (as with linear pricing and complete information). For example, Choné and Linnemer (2020) find vertical mergers can be anticompetitive even if they eliminate double marginalisation. Similarly, Loertscher and Marx (2022a) find there is no basis for assuming vertical mergers increase social surplus. With bargaining and incomplete information, another reason the test should not be applied is that margins reflect, in part, uncertainties about costs.

### 5.3 Endogenous ability: timing, commitment, and switching costs

In present paper, constraints  $c^{[2]}$  and  $c_i$  are exogenous. Chen (2001) shows that constraints can be endogenous to the merger. In his model, downstream firms choose their suppliers (and commit to their choice) before setting consumer prices. Pre-merger, downstream firms source from the cheapest supplier but post-merger, for any given  $v_B$ , the non-integrated downstream firm strictly prefers to source the input from the merged entity at cost  $v_B$  (rather than from an alternative supplier or in-house at cost  $v_B$ ) to elicit weakened downstream competition (the shareholder effect).<sup>40</sup> With this altered timing and the power to commit to a supplier, the merged entity can raise the input price above  $\min\{c^{[2]}, c_B\}$ . Hence, its ability to engage in RRC is endogenous. This is not foreclosure in the traditional sense: here, the non-integrated firm pays a higher input price to the merged entity due to a self-interested anticompetitive motivation rather than due to the lack of cheaper alternatives. Absent foreclosure and EDM, consumer prices increase not solely because of the stakeholder effect but also because the non-integrated downstream firm chooses to pay a higher input price. This merger effect harms consumers.

Furthermore, Chen (2001) shows that the merged entity can still engage in direct RRC if, instead of being fully committed to its chosen supplier, the non-integrated downstream firm can switch at some fixed cost after consumer prices are set. Similar results materialise if the non-integrated downstream firm has fixed costs to switch to in-house production<sup>41</sup> (such fixed costs are considered in Katz, 1987 and Inderst and Valletti, 2009). The lower the switching costs, the less the merged entity can raise rivals' costs. Results of sections 2 to 4 can be interpreted to hold when switching costs are negligible.

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<sup>40</sup>*Ceteris paribus*, the less aggressive the merged entity becomes through the stakeholder effect, the more the merged entity can raise the input price.

<sup>41</sup>In Chen (2001), a downstream firm incurs the switching cost only if it chooses one supplier at the end of stage 1 but orders from another after stage 2. Hence, if downstream firm B decides to source from the second-most efficient firm after the merger and does not change its choice at the end of stage 2 of the post-merger game, downstream firm B incurs no switching cost. This differs from downstream firm B incurring a fixed cost whenever it switches to in-house production.

## 6 Antitrust discussion

### 6.1 The test complements existing antitrust tools

The vGUPPI, developed by Moresi and Salop (2013), and vertical arithmetic (VA) are two other antitrust tools to assess vertical foreclosure concerns. The test developed herein complements these existing tools. A screening process for vertical mergers could incorporate the following steps. As a preliminary, an antitrust authority can verify whether the merger case under scrutiny fits with the modelling assumptions made herein. If the model fits, an antitrust authority can use the margins test. If the test indicates that the merged entity cannot raise rivals' costs, the merger may still be harmful (see Figure 1). Instead, if the merged entity can obtain higher input prices, one can then use the vGUPPI to estimate the extent to which the merged entity wants to raise rivals' costs.<sup>42</sup> In addition, and as is often the case in merger reviews, VA can be used to assess whether the merged entity would find it profitable to stop to supply downstream rivals (on VA, see Zenger, 2020). The test developed herein is relevant to VA because it clarifies whether RRC is possible in the alternative outcome to halting supply. As to data requirements, the vGUPPI and VA also need margin estimates. Therefore, the test developed herein requires less data than these two existing tools.

### 6.2 Remedies which do not prevent consumer harm

Suppose an antitrust authority imposes a remedy on a proposed merger whereby the merged entity cannot raise  $v_B$  beyond the pre-merger level. The remedy precludes RRC.<sup>43</sup> Nevertheless, the stakeholder effect materialises: downstream competition weakens. Consumer prices can increase despite the remedy (depending on the trade-off between EDM and the stakeholder effect). Therefore, such a remedy does not necessarily prevent consumer harm. Instead of such a remedy, if an antitrust review deems that the stakeholder effect would lead to overall consumer harm, then blocking the merger may be necessary.<sup>44</sup>

## 7 Conclusion

A primary focus in vertical merger theory and of antitrust agencies is RRC. Instead of competitors being harmed by RRC, this paper highlights that vertical mergers can benefit

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<sup>42</sup>The vGUPPI applies to settings with and without input substitution. As discussed in the introduction, the merged entity might want to lower its rivals' costs.

<sup>43</sup>And from any "choice of supplier" effect. On this effect, see section 5.

<sup>44</sup>For recent research on mergers that should be blocked, and on divestitures, see Loertscher and Marx (2022b).

all firms – including the non-integrated downstream firms – by reducing competition, increasing prices, and harming consumers. Whereas the underlying premise of numerous theoretical models and their use in practice is that the upstream supplier has pricing power (or “market power”) and can raise the input price post-merger, even if it has high margins (or “market power”) the merged entity may be unable to engage in RRC.

I develop a general relation between upstream and downstream margins. This relation holds when suppliers set unit prices. Using this relation, I develop a test for vertical mergers. One can use this test to determine whether the upstream firm cannot raise a rival’s cost. One can also use the test to identify models in the existing literature that are consistent with data on industries they seek to portray.

Finally, there are significant implications for empirical studies. As is recognised in the literature, a downstream firm which integrates with a rival’s supplier gains a stake in that rival and thus behaves less aggressively. Therefore, ownership of competitors’ suppliers can also contribute to rising margins. It would be interesting for further empirical work to augment the generalised HHI index to account for such ownership structures (on the GHHI index, see Azar, Raina, and Schmalz (2019)).

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## Appendix: Proofs

In the appendix, where convenient, I use the shorthand notation  $r_i = 2 - \frac{Q_i Q_i''}{Q_i'^2}$ , with  $r_i \geq 1$  because  $Q_i$  is log-concave.

*Proof of Theorem 1.* The theorem states that  $m_{S_i}^u$ , given in (5), weakly exceeds  $m_i^*$ , given in (6). In a first step, I show that

$$-v_j \frac{\frac{\partial Q_j}{\partial P_j} \frac{dP_j^*}{dv_i} + \frac{\partial Q_j}{\partial P_i} \frac{dP_i^*}{dv_i}}{Q_i' \frac{dP_i^*}{dv_i} + \frac{\partial Q_i}{\partial P_j} \frac{dP_j^*}{dv_i}} \geq 0$$

In a second step, I show that

$$-\frac{Q_i}{Q_i' \frac{dP_i^*}{dv_i} + \frac{\partial Q_i}{\partial P_j} \frac{dP_j^*}{dv_i}} \geq -\frac{Q_i}{Q_i'}$$

As preliminaries to both steps, I solve for  $\frac{dP_i^*}{dv_i}$  and  $\frac{dP_j^*}{dv_i}$ . I totally differentiate the first-order condition (7) with respect to  $v_i$ . This yields

$$\begin{aligned} Q_i' \frac{dP_i^*}{dv_i} + \frac{\partial Q_i}{\partial P_j} \frac{dP_j^*}{dv_i} + [P_i^* - v_i] Q_i'' \frac{dP_i^*}{dv_i} + \left[ \frac{dP_i^*}{dv_i} - 1 \right] Q_i' &= 0 \\ \Leftrightarrow \frac{dP_i^*}{dv_i} &= \frac{1 - \frac{1}{Q_i'} \frac{\partial Q_i}{\partial P_j} \frac{dP_j^*}{dv_i}}{2 - \frac{Q_i Q_i''}{Q_i'^2}} \end{aligned}$$

Similarly, one can calculate the total derivative

$$\frac{dP_j^*}{dv_i} = \frac{-\frac{1}{Q_j'} \frac{\partial Q_j}{\partial P_i} \frac{dP_i^*}{dv_i}}{2 - \frac{Q_j Q_j''}{Q_j'^2}}$$

Solving simultaneously yields<sup>45</sup>

$$\begin{aligned} \frac{dP_i^*}{dv_i} &= \frac{2 - \frac{Q_j Q_j''}{Q_j'^2}}{\left(2 - \frac{Q_j Q_j''}{Q_j'^2}\right) \left(2 - \frac{Q_i Q_i''}{Q_i'^2}\right) - \frac{1}{Q_i'} \frac{\partial Q_i}{\partial P_j} \frac{1}{Q_j'} \frac{\partial Q_j}{\partial P_i}} > 0 \\ \frac{dP_j^*}{dv_i} &= \frac{-\frac{1}{Q_j'} \frac{\partial Q_j}{\partial P_i}}{\left(2 - \frac{Q_j Q_j''}{Q_j'^2}\right) \left(2 - \frac{Q_i Q_i''}{Q_i'^2}\right) - \frac{1}{Q_i'} \frac{\partial Q_i}{\partial P_j} \frac{1}{Q_j'} \frac{\partial Q_j}{\partial P_i}} > 0 \end{aligned}$$

<sup>45</sup>As a specific example on the equilibrium pass-through rate: if downstream firms each face the same demand function  $Q_i = \alpha - P_i + \frac{1}{2}P_j$ , with  $\Pi_i = (P_i - v_i)Q_i$ , then  $\frac{dP_i^*}{dv_i} = \frac{2}{2 \times 2 - \frac{1}{2} \times \frac{1}{2}} = \frac{2}{4 - \frac{1}{4}} = \frac{8}{15} < 1$ .

Both denominators are positive, because  $Q_i'^2 > Q_i Q_i''$  (due to log concavity) and because  $|Q_i'| > \frac{\partial Q_i}{\partial P_j}$  (by Assumption 1).<sup>46</sup>

As further preliminaries, I show that  $\frac{dQ_i}{dv_i} < 0$  and that  $\frac{dQ_j}{dv_i} \geq 0$ . Plugging in  $\frac{dP_i^*}{dv_i}$  and  $\frac{dP_j^*}{dv_i}$  into  $\frac{dQ_i}{dv_i} = Q_i' \frac{dP_i^*}{dv_i} + \frac{\partial Q_i}{\partial P_j} \frac{dP_j^*}{dv_i}$  yields

$$\frac{Q_i' r_j - \frac{\partial Q_i}{\partial P_j} \frac{1}{Q_j'} \frac{\partial Q_j}{\partial P_i}}{\left(2 - \frac{Q_j Q_j''}{Q_j'^2}\right) \left(2 - \frac{Q_i Q_i''}{Q_i'^2}\right) - \frac{1}{Q_i'} \frac{\partial Q_i}{\partial P_j} \frac{1}{Q_j'} \frac{\partial Q_j}{\partial P_i}}$$

The fraction above has a positive denominator, because  $r_j \geq 1$ ,  $r_i \geq 1$  and, by Assumption 1,  $\frac{1}{Q_i'} \frac{\partial Q_i}{\partial P_j} \frac{1}{Q_j'} \frac{\partial Q_j}{\partial P_i} < 1$ . Therefore, the sign of the fraction above is equal to the sign of the nominator. Evaluating the sign of the nominator yields

$$\text{sign}\{\text{nominator}\} = -\text{sign}\left\{r_j - \frac{1}{Q_i'} \frac{\partial Q_i}{\partial P_j} \frac{1}{Q_j'} \frac{\partial Q_j}{\partial P_i}\right\} < 0$$

Therefore  $\frac{dQ_i}{dv_i} < 0$ .

Plugging in  $\frac{dP_i^*}{dv_i}$  and  $\frac{dP_j^*}{dv_i}$  into  $\frac{dQ_j}{dv_i} = Q_j' \frac{dP_j^*}{dv_i} + \frac{\partial Q_j}{\partial P_i} \frac{dP_i^*}{dv_i}$  yields

$$\frac{-\frac{\partial Q_j}{\partial P_i} + \frac{\partial Q_j}{\partial P_i} r_j}{\left(2 - \frac{Q_j Q_j''}{Q_j'^2}\right) \left(2 - \frac{Q_i Q_i''}{Q_i'^2}\right) - \frac{1}{Q_i'} \frac{\partial Q_i}{\partial P_j} \frac{1}{Q_j'} \frac{\partial Q_j}{\partial P_i}}$$

The sign of the above expression is equal to the sign of the nominator.

$$\text{sign}\{\text{nominator}\} = \text{sign}\left\{(r_j - 1) \frac{\partial Q_j}{\partial P_i}\right\} \geq 0$$

Therefore  $\frac{dQ_j}{dv_i} \geq 0$ .

From the preliminaries, it directly follows that

$$-v_j \frac{\frac{dQ_j}{dv_i}}{\frac{dQ_i}{dv_i}} \geq 0$$

because the denominator  $\frac{dQ_i}{dv_i} < 0$  and the nominator  $\frac{dQ_j}{dv_i} \geq 0$ .

Finally, I show

$$-\frac{Q_i}{Q_i' \frac{dP_i^*}{dv_i} + \frac{\partial Q_i}{\partial P_j} \frac{dP_j^*}{dv_i}} \geq -\frac{Q_i}{Q_i'}$$

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<sup>46</sup>We have  $|\frac{dP_i^*}{dv_i}| > |\frac{dP_j^*}{dv_i}|$  because the nominator of the former (weakly) exceeds 1 (due to log concavity of demand) whereas the nominator of the latter lies in  $(0, 1)$  (as the consumer products are gross substitutes, Assumption 1).

Rearranging,

$$Q'_i \leq Q'_i \frac{dP_i^*}{dv_i} + \frac{\partial Q_i}{\partial P_j} \frac{dP_j^*}{dv_i}$$

Plugging in  $\frac{dP_i^*}{dv_i}$  and  $\frac{dP_j^*}{dv_i}$ , and simplifying,

$$\begin{aligned} \Leftrightarrow r_i r_j Q'_i - \frac{\partial Q_i}{\partial P_j} \frac{1}{Q'_j} \frac{\partial Q_j}{\partial P_i} &\leq r_j Q'_i - \frac{\partial Q_i}{\partial P_j} \frac{1}{Q'_j} \frac{\partial Q_j}{\partial P_i} \\ \Leftrightarrow r_j Q'_i (r_i - 1) &\leq 0 \end{aligned}$$

because  $r_j \geq 1$ ,  $Q'_i < 0$  and  $r_i - 1 \geq 0$ . Therefore  $m_{Si}^u \geq m_i^*$ .  $\square$

*Proof of Corollary 1.* By Theorem 1, if  $v_i^* = v_i^u$  then  $m_{Si}^u \geq m_i^*$ . Therefore  $m_{Si}^u < m_i^* \Rightarrow v_i^* \neq v_i^u$ .  $\square$

*Proof of Proposition 1.* If  $m_{SB}^* < m_B^*$ , then by Corollary 1 the supplier is constrained pre-merger. If the supplier is constrained it sets  $v_B^* = \min\{c^{[2]}, c_B\}$  (rather than  $v_B^* = v_B^u$ ). Therefore, it cannot raise the price beyond  $\min\{c^{[2]}, c_B\}$ .  $\square$

*Proof of Proposition 2.* The following proves that, absent a change in  $v_B$ , the post-merger change in consumer welfare effect can go either way. Consumer welfare increases if the only merger effect is the EDM. This is the case when  $v_A > 0$  (the merger results in an EDM effect) and  $v_B = 0$  (there is no weakening of downstream competition). Conversely, consumer welfare decreases if the only merger effect is the weakening of downstream competition. This is the case when  $v_A = 0$  (no EDM) and  $v_B > 0$ .  $\square$

*Proof of Proposition 3.* To prove this Proposition, it suffices to show one example where

- the quantity-based and sales-based market shares of the merged entity's downstream unit decrease, and where
- the profit of downstream firm B rises by more than the combined profits of the supplier and downstream firm A.

I now provide such an example. Let  $Q_i = 1 - P_i + \frac{1}{2}P_j$ , for  $i = \{A, B\}$ . For concise workings, let  $c_A = 0$ . Let  $c_B = 0.1$ , with  $c^{[2]} \geq c_B$ .

Pre-merger, the supplier has profit  $\Pi_S = v_B Q_B$ . And each downstream firm has profit  $\Pi_i = (P_i - v_i) Q_i$ . The game is solved by backwards induction. The supplier's monopoly price is 0.71. This unconstrained price exceeds  $c_B$ , hence the supplier is constrained and sets  $v_B = 0.1$ . Post-merger, the merged entity unambiguously has an incentive to raise its rival's cost (because there is no EDM). But the merged entity cannot raise  $v_B$  beyond  $c_B$ . Thus,  $v_B$  remains at 0.1.

The table below provides pre- and post-merger figures on prices, quantities, sales (price times quantity), and profits.

Table 1: Pre-merger and post-merger values

Variable	Pre-merger	Post-merger	Change
$v_A$	0	0	none
$v_B$	0.1	0.1	none
$P_A$	0.68	0.71	increase
$P_B$	0.72	0.73	increase
$Q_A$	0.68	0.66	decrease
$Q_B$	0.62	0.63	increase
$\Pi_S$	0.062	0.063	+0.001
$\Pi_A$	0.462	0.464	+0.002
$\Pi_B$	0.384	0.393	+0.008

Source: own workings.

From the table,  $\frac{Q_A}{Q_A+Q_B}$  and  $\frac{P_A Q_A}{P_A Q_A + P_B Q_B}$  decrease, hence the respective market shares of the non-integrated downstream firm increase. Moreover, the quantity  $Q_A$  decreases post-merger.

From the table,  $\Pi_M - \Pi_S - \Pi_A$  is smaller than the post-merger increase in downstream firm B's profit. □

*Proof of Theorem 2.* This follows from the text. □