Personalized pricing when consumers can purchase multiple items*

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February 4, 2023

Abstract

We discuss the effect of personalized pricing on profits and welfare in a Hotelling model in which consumers can simultaneously purchase from both firms. We have the following results. If the additional gain from the second purchase (henceforth, the additional gain) is small, personalized pricing improves consumer welfare but harms firms' profits. If the additional gain is intermediate, personalized pricing improves consumer welfare and firms' profits. Finally, if the additional gain is large, personalized pricing improves firms' profits but harms consumer welfare. The latter results contrast with that under the single-unit purchase assumption in the literature: personalized pricing improves consumer welfare but harms firms' profits. We extend the model by assuming that firms can endogenously choose one of the pricing policies: uniform or personalized pricing. We show that both firms choose personalized pricing in any case and uniform pricing under some parameters; multiple equilibria can coexist in those parameters. Our results imply that when we discuss the impact of personalized pricing on profits and welfare, we need to consider the propensity of consumers' multistore shopping.

Keywords: Personalized pricing, Multi-unit purchase, Hotelling model

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*We thank Keizo Mizuno, Cong Pan, Andrew Rhodes, Shiva Shekhar, Mark J. Tremblay, and the seminar participants at Applied Regional Science Conference (Yamanashi University) and Japan Association for Applied Economics (Doshisha University) for their valuable discussions and comments. We gratefully acknowledge the financial support from the Japan Society for the Promotion of Science (JSPS), KAKENHI Grant Numbers JP19H01483, JP20H05631, JP21H00702, JP21K01452, and JP21K18430. Although Matsushima serves as a member of the Competition Policy Research Center (CPRC) at the Japan Fair Trade Commission (JFTC), the views expressed in this paper are solely ours and should not be attributed to the JFTC. The usual disclaimer applies.

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1 Introduction

We theoretically investigate the effect of personalized pricing on profits and welfare in a Hotelling duopoly model in which consumers can simultaneously purchase from both firms, given the current competitive environment explained below. Advances in information technology, particularly the rapid adoption of smartphones, have made personalized pricing a reality (Esteves and Resende, 2016), as exemplified by route-based pricing (Uber, a taxi platform) and the "JustforU" program (Safeway, a traditional retailer).[1] In addition to the anecdotal evidence, several academic articles have detected personalized offers and search discrimination (steering customers to particular product categories) even on regular e-commerce sites (Mikians et al., 2012, Hannak et al., 2014). Related to the impact of personalized pricing on profits, Shiller (2020) simulates counterfactual situations in which Netflix hypothetically engages in personalized pricing based on Web-browsing histories, using data about Web site visits and transactions during 2006. He shows that history-based personalized pricing will increase Netflix's profits by about 13%.[2]

Our assumption regarding consumers' multi-unit purchases from firms coincides with real-world purchasing behavior. The low cost of visiting online retailers and online services helps consumers purchase items from multiple online retailers and join multiple online services, including online music stores and games (e.g., Landsman and Stremersch (2011) for game consoles and Li and Zhu (2021) for a daily deals market). Even when consumers

[1] The following articles explain the details of the cases: Uber Testing New Policy: Charge What It Thinks You're Able to Pay (May 22, 2017) and Worth The Deal? Groceries Get a Personalized Price (August 20, 2012). The URLs of the articles are as follows:
http://www.thedrive.com/tech/10487/uber-testing-new-policy-charge-what-it-thinks-youre-able-to-pay
http://knkx.org/post/worth-deal-groceries-get-personalized-price

[2] Dubé and Misra (2023), Smith et al. (2022), and Shiller (2022) are further empirical investigations on targeted pricing.

purchase from offline retailers, more than half of consumers visit multiple offline retailers and consider whether to purchase from those offline stores (Gijsbrechts et al., 2008). Below, we explain two typical examples of consumers' multi-unit purchases.

A typical example of consumers' multi-unit purchases is the market for subscription video on demand (SVOD), in which more than one-third of consumers subscribe to multiple SVOD services (Ishihara and Oki, 2021a, p.15). One of the leading firms in the SVOD market, Netflix, provides personalized recommendations to customers (Kim et al., 2017), enabling it to potentially use personalized pricing based on its recommendation system as discussed in Shiller (2020). The same would apply to Amazon Prime Video because of its ability to provide personalized recommendations.[3]

Furthermore, playing multiple online games in the same genre (e.g., shooter video games) is common. For example, consider three famous shooter video game series: Call of Duty, Battlefield, and Halo. In a survey with 8,024 respondents in the US, UK, Germany, and France, almost half of the respondents have played at least two of the three (Melcher, 2021). Those game series sell some functionalities to users within the game applications. The nature of these products means that the games' producers can potentially use personalized pricing to sell personalized functionalities.

Following the recent market environments, we discuss the effect of personalized pricing on profits and welfare in a Hotelling duopoly model in which consumers can simultaneously purchase from both firms. We borrow the framework in Jeitschko et al. (2017) who investigate a Hotelling model in which consumers can purchase from multiple firms. The additional intrinsic utility from the second firm is smaller than the intrinsic utility from the first firm. Consumers observe prices proposed by firms and choose one of the options:

[3] Zhou and Zou (2022) theoretically investigate competitive personalized recommendations in online markets.

purchasing from (i) one of the firms or (ii) both firms. We compare the results when firms use uniform prices and when they use personalized prices.

We have the following results by analyzing the duopoly model. The consumer surplus under personalized pricing is higher than under uniform pricing if the additional intrinsic utility from the second product is not large. Firms benefit from personalized pricing if the additional intrinsic utility from the second product is large. The consumer surplus and firms' profits under personalized pricing are higher than under uniform pricing if the additional intrinsic utility from the second product is intermediate. In this case, personalized pricing expands the market demand because of the standard mechanism of first-degree price discrimination. The latter two results sharply contrast with those under the single-unit purchase assumption in the personalized pricing literature on Hotelling models: personalized pricing benefits consumers but harms firms.

We extend the model by endogenizing firms' pricing policies. At the beginning of the game, each firm chooses one of the pricing policies: uniform and personalized pricing. After the decisions, they compete in price. As a result, we obtain two types of equilibrium outcomes. First, both firms choose personalized pricing in any case; Second, both firms choose uniform pricing *only if* the following two hold: all consumers purchase from only one of the firms in the case where the firms employ uniform pricing; some consumers purchase from both firms in the case where one of the firms employs personalized pricing. The second type of equilibrium outcome is uncommon and novel in the context of personalized pricing, although several papers show that only one of the firms employs personalized pricing in asymmetric duopoly models (Ghose and Huang, 2009, Matsumura and Matsushima, 2015).

Our results contrast with the insights suggested by the earlier works. Some previous works show that the feasibility of personalized pricing intensifies competition, leading to a worse outcome for competing firms (e.g., Thisse and Vives, 1988, Choe et al., 2018).[4] Contrasting to the earlier finding, we show that the feasibility of personalized pricing does not always lead to worse outcomes when consumers are more likely to purchase multiple items. Our result implies that when we discuss the impact of personalized pricing on profits and welfare, we need to consider the propensity of consumers' multistore shopping (Gijsbrechts et al., 2008, Bell et al., 2011, Landsman and Stremersch, 2011).

Our paper contributes to two strands of literature on (i) multi-unit purchases in single-side product differentiation models and (ii) the effect of personalized pricing on profits and welfare.

In the first strand of literature, several papers discuss multi-unit purchases in standard vertical differentiation models (Mussa and Rosen, 1978, Shaked and Sutton, 1982) and horizontal differentiation models (d'Aspremont et al., 1979) since the early 21st century.[5] Gabszewicz et al. (2001) and Gabszewicz and Wauthy (2003) consider vertical differentiation models in which consumers can purchase multiple units and characterize price equilibria. Guo (2006) and Kim and Serfes (2006) independently investigate firms' location choices in Hotelling duopoly models in which consumers can purchase multiple units and show agglomeration of firms. Anderson et al. (2017) embed multiple product functionalities to multi-purchasing models based on the Hotelling duopoly framework. Those papers do not discuss personalized pricing.

Recently, Jeitschko et al. (2017) investigate a Hotelling model in which consumers can purchase multiple units and derive the condition that consumers purchase a single item or multiple items (Proposition 1 and Figure 3).[6] They also discuss discount offers to

[4] The negative effect of personalized pricing on profitability appears even in monopoly models (e.g., Hajihashemi et al., 2022).

[5] de Palma et al. (1999) consider multi-unit purchases in a Cournot model with network externality.

[6] Jeitschko and Tremblay (2020) provide a general discussion on consumers' multi-unit purchases.

consumers who purchase from both firms, although they do not discuss personalized pricing. Therefore, we complement their discussion by investigating the effect of personalized pricing on profits and welfare.

In the second strand of literature, following the growth in personalized pricing, researchers in the personalized pricing literature investigate the impact of personalized pricing on profitability and welfare by using the standard Hotelling model in which all consumers purchase from only one of the firms in equilibrium, the so-called full coverage assumption (e.g., Thisse and Vives, 1988, Chen and Iyer, 2002, Shaffer and Zhang, 2002, Zhang, 2011, Choe et al., 2018).[7] They show that personalized pricing tends to increase competition and improve consumer welfare.[8] Those papers also assume that consumers do not choose both firms' products. We relax this assumption and allow consumers to purchase from both firms.

In the literature of personalized pricing, several papers show that personalized pricing can be a profitable pricing strategy in contrast to the standard impact of personalized pricing on profits and welfare. Liu and Serfes (2013) consider two-sided markets to investigate the profitability of personalized pricing.[9] They show that personalized pricing is profitable

Several papers extend Jeitschko et al. (2017) to discuss a monopolistic content provider's optimal licensing to downstream firms (e.g., Jiang et al., 2019, Ishihara and Oki, 2021a,b, Lu, 2022). Wu and Chiu (2023) discuss content creation by a downstream firm. Carroni et al. (2020) also discuss a related issue in the context of two-sided markets.

[7] Many papers also discuss the impacts of personalized pricing on monopolists and consumers (e.g., Acquisti and Varian, 2005, Xu and Dukes, 2021, Hajihashemi et al., 2022). Arora et al. (2008) is a useful review article in the market literature.

[8] We refer to papers that show contrasting results later.

[9] Kodera (2015) considers discriminatory pricing for only one of the sides (advertisers' side) using the model in Liu and Serfes (2013). He shows that discriminatory pricing benefits the competing platforms if consumers' advertising aversion is strong. Furthermore, if discriminatory pricing harms the competing platforms, discriminatory pricing always increases advertisers' total profits. Adachi and Tremblay (2020) incorporate bilateral negotiations between a platform and firms in a two-sided market. They show that

but harms consumers if they can purchase from both firms.[10]

Chen et al. (2020) consider a static Hotelling duopoly model in which each firm has information about the location of consumers on the range from the firm's location to a particular point.[11] They assume, in contrast to the standard assumption in the literature on personalized pricing (e.g., Choe et al., 2018), that consumers can actively avoid personalized prices if those are higher than uniform prices. They show the possibility that personalized pricing can be an exploitative device. The key point in their result is the asymmetric distribution of customer information.

Jullien et al. (2022) investigate the optimal distribution strategy of a monopolistic manufacturer that initially distributes its product through an independent retailer. When the manufacturer opens its direct channel, the independent retailer and the direct channel compete in the downstream market. Jullien et al. (2022) show that personalized pricing can be an exploitative device if the manufacturer designs a proper wholesale tariff. The interaction between vertical contracts and personalized pricing is the key element of this result.

Laussel and Resende (2022) extend the two-period model in Choe et al. (2018) to investigate the interaction between product customization and personalized pricing based

such personalized negotiations are not always exploitative if the bargaining power of the platform over firms is strong. Shekhar (2022) extends Liu and Serfes (2013) and shows the conditions under which firms choose personalized pricing ("exclusive deal" in his paper) and/or uniform pricing.

[10] We can calculate the consumer surplus on each side using the result in Liu and Serfes (2013), and we mention that personalized pricing worsens consumer surplus in the main text of our paper.

[11] Esteves (2022) and Matsushima et al. (2022) discuss the conditions under which personalized pricing is more profitable for firms than uniform pricing in static Hotelling models by incorporating consumer heterogeneity (purchasing quantities, Esteves (2022); mismatch costs, Matsushima et al. (2022)). Furthermore, several studies show that personalized pricing does not necessarily result in a prisoner's dilemma in the case of firm asymmetry (e.g., quality difference (Shaffer and Zhang, 2002), quality choice (Choudhary et al., 2005, Ghose and Huang, 2009), and initial cost difference with R&D (Matsumura and Matsushima, 2015)).

on purchase histories in the first period.[12] They show that product-price personalization can be a profitable pricing strategy, contrasting with the finding in Choe et al. (2018).[13]

Rhodes and Zhou (2022) discuss generalized oligopoly models based on Perloff and Salop (1985) to investigate the effects of personalized pricing on profits and welfare.[14] They show that consumers are more likely to benefit from personalized pricing as the degree of market coverage increases, and the converse holds for firms (Figure 2 in their paper). Contrasting to their result, we show that consumers are more likely to benefit from personalized pricing as the degree of market coverage becomes lower (each consumer purchases from *only one of the two firms*), and the converse holds for firms.[15]

2 Model

We use the model in Jeitschko et al. (2017) and the same notations as theirs. Consumers are on the line segment of length one, $[0, 1]$ (Hotelling line). The mass of consumers is 1, and the distribution of consumers is uniform along the Hotelling line. Two firms (firms 1 and 2) are at the edges of the Hotelling line, 0 and 1, respectively. The utility from

[12] Chen et al. (2022) consider a two-market model in which one market deals with electric devices to gather consumer data and the other deals with data-applicable services (e.g., health care). A pair of firms in the former and the latter markets merge and use customer data gathered in the device market. They show the condition that the merger leads to the monopolization of the two markets.

[13] Choe et al. (2022) also extend the two-period model in Choe et al. (2018) to investigate firms' incentive to precommit to sharing customer information gathered at the end of the first period. They show that at the beginning of the game, firms agree to share customer information to mitigate competition in the first period.

[14] Zhou (2021) also use Perloff and Salop (1985) to investigate mixed bundling.

[15] As in the standard Hotelling model, we focus on situations in which each consumer purchases at least one item. In this sense, the market is fully covered in our model.

purchasing firm 1's product, firm 2's product, or both products is:

$$\begin{cases} w_1 - tx - p_1, & \text{purchasing from only firm 1,} \\ w_2 - t(1-x) - p_2, & \text{purchasing from only firm 2,} \\ w_1 + w_2 - V - t - p_1 - p_2, & \text{purchasing from firms 1 and 2,} \end{cases} \tag{1}$$

where w_i is the intrinsic utility of firm i's product, t is the per-length transportation cost, $x(\in [0,1])$ is the location of consumers at x on the Hotelling line, p_i is firm i's price, and V is the cross-effects of joint consumption.

We set several parametric assumptions to simplify the analysis. We assume that $w_i = w \geq 3t/2$ to ensure that each consumer purchases at least one of the products, $V \in (0, w)$, and $v \equiv w - V$, which is the gross utility of the second product.

We can rewrite (1) as follows:

$$\begin{cases} w - tx - p_1, & \text{purchasing from only firm 1,} \\ w - t(1-x) - p_2, & \text{purchasing from only firm 2,} \\ w + v - t - p_1 - p_2, & \text{purchasing from firms 1 and 2.} \end{cases} \tag{1'}$$

We discuss the case of asymmetric w_i in Section 4.

We consider two cases: (i) firms use uniform pricing, and (ii) they use personalized pricing. In the former case, the firms offer uniform prices to all consumers. In the latter case, the firms recognize the locations of all consumers and can offer personalized prices to them. That is, prices become a function of x, $p_i(x)$. We consider one-shot games in the two cases.

3 Results

We consider two cases: (i) firms use uniform pricing, and (ii) they use personalized pricing. Then, we compare the outcomes.

3.1 Uniform pricing

Using Proposition 1 in Jeitschko et al. (2017), we describe the results under uniform pricing. After that, we compare the results with those under personalized pricing.

All consumers purchase from only one of the firms (S: single unit) When all consumers purchase from one of the firms in equilibrium, the equilibrium prices, profit of each firm, and resulting consumer and total surpluses are the same as in the standard Hotelling model with unit demand. Concretey, those are

$$p_i^{US} = t, \quad \pi_i^{US} = \frac{t}{2}, \quad CS^{US} = w - \frac{5t}{4}, \quad TS^{US} = w - \frac{t}{4}.$$

The result is sustainable as an equilibrium outcome if and only if $v \leq \sqrt{2}t \simeq 1.414t$.

At least some consumers purchase from both firms (M: multiple units) When at least some consumers purchase from both firms in equilibrium, the equilibrium prices, profit of each firm, and resulting consumer and total surpluses are:

$$p_i^{UM} = \begin{cases} v - t & \text{if } 2t \leq v, \\ \dfrac{v}{2} & \text{if } v < 2t, \end{cases} \quad \pi_i^{UM} = \begin{cases} v - t & \text{if } 2t \leq v, \\ \dfrac{v^2}{4t} & \text{if } v < 2t, \end{cases}$$

$$CS^{UM} = \begin{cases} w - v + t & \text{if } 2t \leq v, \\ w + \dfrac{v(v-4t)}{4t} & \text{if } v < 2t, \end{cases} \quad TS^{UM} = \begin{cases} w + v - t & \text{if } 2t \leq v, \\ w + \dfrac{v(3v-4t)}{4t} & \text{if } v < 2t, \end{cases}$$

The outcome is effective if and only if $v > 2(2\sqrt{2}+1)t/7 \simeq 1.094t$.

Equilibrium multiplicity From the two cases, we find that the two equilibria are attainable if and only if $1.094t < v \leq 1.414t$. The profits in Case S are higher than those in

Case M in this parameter range of v. If we use the payoff dominance as the equilibrium refinement, we choose the result in Case S for all v such that $1.094t < v \leq 1.414t$.[16]

3.2 Personalized pricing

We derive the results when firms can use personalized pricing. We consider two cases regarding whether firm i can sell its product to consumers at x given that: (i) firm j sells to those consumers; (ii) firm j does not sell to those consumers.[17]

First, given that consumers at x purchase from firm j at a positive personalized price, they also buy from firm i if and only if

$$w + v - t - p_i(x) - p_j(x) \geq w - td_j(x) - p_j(x) \quad \Rightarrow \quad p_i(x) \leq v - t(1 - d_j(x)), \tag{2}$$

where $d_j(x)$ is the distance between firm j and the consumer at x.[18]

Second, given that firm j cannot attract consumers at x at a nonnegative personalized price and sets $p_j(x) = 0$, firm i's personalized price is acceptable for consumers at x if and

[16] Among the criteria of equilibrium selection, payoff dominance and risk dominance are well-known criteria (Harsanyi and Selten, 1988). Finding the risk-dominant equilibrium in our model with continuous strategic actions is difficult because we need to derive the best response of each firm to the rival's mixed strategy (van Damme and Hurkens, 2004, Section 3). In addition, we cannot adopt the reduced 2×2 game with the two equilibrium prices, p_i^{US} and p_i^{UM}, derived in the main text to derive the risk-dominant action pair because van Damme and Hurkens (2004, Section 5) caution against such a reduced form.

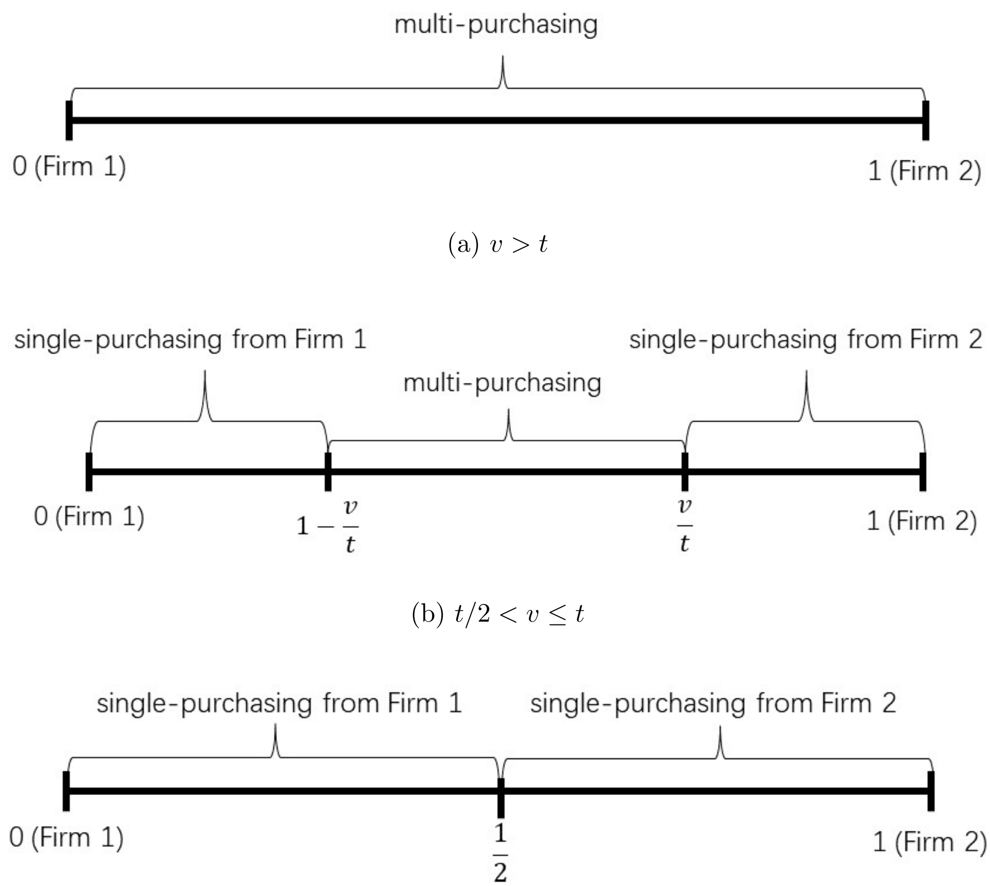
[17] The price competition at each point is similar to homogeneous good Bertrand competition between two asymmetric firms. Although Baye and Morgan (1999) and Kaplan and Wettstein (2000) show mixed strategy equilibria with non-zero profits under elastic demand functions in symmetric Bertrand competition, we guess that their results do not apply to our model because firms' profits in our model go to zero if prices are sufficiently large. Also, Jann and Schottmüller (2015, Theorem 2) show a correlated equilibrium under asymmetric Bertrand competition in which the equilibrium price is not larger than the second lowest marginal cost of the firms. The equilibrium property is similar to the equilibrium under personalized pricing in our paper when $w - v$ is small. Therefore, we focus on the pure strategy equilibrium derived here.

[18] When $v > w$ (the gross utility of the second product is larger than that of the first product), we also need to consider the condition, $w + v - t - p_i(x) - p_j(x) \geq 0$, which is redundant in the case of $v \leq w$. When $v > w$, the equilibrium personalized prices $p_1(x)$ and $p_2(x)$ are indeterminate such that $w + v - t - p_i(x) - p_j(x) = 0$ and all consumers purchase from both firms.

only if

$$w - td_i(x) - p_i(x) \geq w - td_j(x) - 0 \quad \Rightarrow \quad p_i(x) \leq t(d_j(x) - d_i(x)). \tag{3}$$

From (2) and (3), we obtain the following lemma (see also Figure 1):



(a) $v > t$

(b) $t/2 < v \leq t$

(c) $v \leq t/2$

Figure 1: Consumers' purchasing choice under different v

Lemma 1. *The schedules of personalized prices depend on v and t. Concretely,*

1. *both firms offer personalized prices that induce all consumers to purchase from both firms if and only if $v > t$;*

2. *firms 1 and 2 offer positive personalized prices for consumers on* $[0, v/t)$ *and* $(1 - v/t, 1]$*, respectively, if and only if* $t/2 < v \leq t$*. In this case, consumers on* $(1 - v/t, v/t)$ *purchase from both firms;*

3. *no firm offers personalized prices that induce consumers to buy from both firms if and only if* $v \leq t/2$*.*

The personalized prices of firms 1 and 2 are:

$$p_1(x) = \begin{cases} v - tx & \text{if } t < v, \\ \begin{array}{ll} \max\{t(1-2x), 0\} & \text{for } x \in [0, 1 - v/t], \\ \max\{v - tx, 0\} & \text{for } x \in [1 - v/t, 1], \end{array} & \text{if } t/2 < v \leq t, \\ \max\{t(1-2x), 0\} & \text{if } v \leq t/2, \end{cases}$$

$$p_2(x) = \begin{cases} v - t(1-x) & \text{if } t < v, \\ \begin{array}{ll} \max\{t(2x-1), 0\} & \text{for } x \in [v/t, 1], \\ \max\{v - t(1-x), 0\} & \text{for } x \in [0, v/t], \end{array} & \text{if } t/2 < v \leq t, \\ \max\{t(2x-1), 0\} & \text{if } v \leq t/2. \end{cases}$$

We derive the consumer surplus in the three cases of Lemma 1.

When $t < v$, all consumers purchase from both firms under $p_1(x) = v - tx$ and $p_2(x) = v - t(1-x)$. The total payments of the consumer at x are:

$$p_1(x) + p_2(x) = 2v - t.$$

The net utility of each consumer is

$$w + v - t - (2v - t) = w - v (> 0).$$

We summarize the outcome as a lemma:

Lemma 2. *Suppose that* $v > t$*. When firms use personalized pricing, firm* i *completely extracts the* <u>*additional*</u> *gross consumer surplus,* $p_i(x) = v - td_i(x)$*, from each consumer at* x*. Each consumer obtains the remaining consumer surplus,* $w - v$*.*

The lemma implies that as the cross-effects of joint consumption, $w - v(= V)$ become larger, the gains of consumers from personalized pricing are larger.

When $t/2 < v \leq t$, consumers on $[0, 1 - v/t]$ purchase from firm 1 under $p_1(x) = t(1 - 2x)$, consumers on $(1 - v/t, v/t)$ purchase from both firms under $p_1(x) = v - tx$ and $p_2(x) = v - t(1 - x)$, and consumers on $[v/t, 1]$ purchase from firm 2 under $p_2(x) = t(2x - 1)$. Consumer surplus when $t/2 < v \leq t$ is

$$(w - v)(2v/t - 1) + \int_0^{1-v/t} (w - tx - t(1 - 2x))dx + \int_{v/t}^1 (w - t(1 - x) - t(2x - 1))dx.$$

When $v \leq t/2$, consumers on $[0, 1/2]$ purchase from firm 1 under $p_1(x) = t(1 - 2x)$ and consumers on $(1/2, 1]$ purchase from firm 2 under $p_2(x) = t(2x - 1)$. Consumer surplus when $v \leq t/2$ is

$$\int_0^{1/2} (w - tx - t(1 - 2x))dx + \int_{1/2}^1 (w - t(1 - x) - t(2x - 1))dx.$$

In sum, consumer surplus is

$$CS^P = \begin{cases} w - v & \text{if } t < v, \\ w + v - t - \dfrac{v^2}{t} & \text{if } t/2 < v \leq t, \\ w - \dfrac{3t}{4} & \text{if } v \leq t/2. \end{cases}$$

We derive the profit of each firm in the three cases of Lemma 1. When $t < v$, the profit of each firm is

$$\pi_1^P = \pi_2^P = \int_0^1 (v - tx)dx = v - \frac{t}{2}.$$

When $t/2 < v \leq t$, the profit of each firm is

$$\pi_1^P = \pi_2^P = \int_0^{1-v/t} t(1 - 2x)dx + \int_{1-v/t}^{v/t} (v - tx)dx = \frac{t}{2} - \frac{v(t - v)}{t}.$$

When $v \leq t/2$, the profit of each firm is

$$\pi_1^P = \pi_2^P = \int_0^{1/2} t(1 - 2x)dx = \frac{t}{4}.$$

In sum, the profit of each firm is

$$\pi_1^P = \pi_2^P = \begin{cases} v - \dfrac{t}{2} & \text{if } t < v, \\ \dfrac{t}{2} - \dfrac{v(t-v)}{t} & \text{if } t/2 < v \le t, \\ \dfrac{t}{4} & \text{if } v \le t/2. \end{cases}$$

Total surplus is

$$TS^P = CS^P + \pi_1^P + \pi_2^P = \begin{cases} w + v - t & \text{if } t < v, \\ w - v + \dfrac{v^2}{t} & \text{if } t/2 < v \le t, \\ w - \dfrac{t}{4} & \text{if } v \le t/2. \end{cases}$$

3.3 Comparison of profits

We compare the outcome in the case of personalized pricing with those in the two cases of uniform pricing, Cases S and M.

First, we pick up the outcome in the case of uniform pricing such that v is small and then Case S occurs. The differences between the values under personalized pricing and those under uniform pricing in Case S are as follows:

$$\Delta CS^S = \begin{cases} \dfrac{5t}{4} - v & \text{if } t < v, \\ \dfrac{t}{4} + \dfrac{v(t-v)}{t} & \text{if } t/2 < v \le t, \\ \dfrac{t}{2} & \text{if } v \le t/2, \end{cases} \quad \Delta\pi_i^S = \begin{cases} v - t & \text{if } t < v, \\ -\dfrac{v(t-v)}{t} & \text{if } t/2 < v \le t, \\ -\dfrac{t}{4} & \text{if } v \le t/2, \end{cases}$$

$$\Delta TS^S = \begin{cases} v - \dfrac{3t}{4} & \text{if } t < v, \\ \dfrac{(t-2v)^2}{4t} & \text{if } t/2 < v \le t, \\ 0 & \text{if } v \le t/2. \end{cases}$$

We summarize the comparison as Proposition 1.

Proposition 1. *When v is small such that all consumers purchase from only one of the firms under uniform pricing ($v \leq \sqrt{2}t \simeq 1.414t$), compared with uniform pricing, personalized pricing increases the*

1. *profit of each firm if and only if $v > t$;*

2. *consumer surplus if and only if $v < 5t/4$;*

3. *total surplus if and only if $v > t/2$.*

In particular, firm profits and consumer surplus improve if and only if $t < v < 5t/4$.

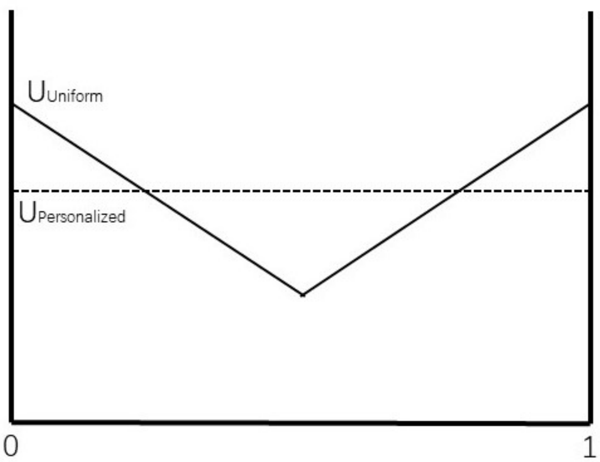


Figure 2: Consumer surplus comparison (Case S)
Note: The solid and dashed lines indicate consumer surplus under uniform and personalized pricing.

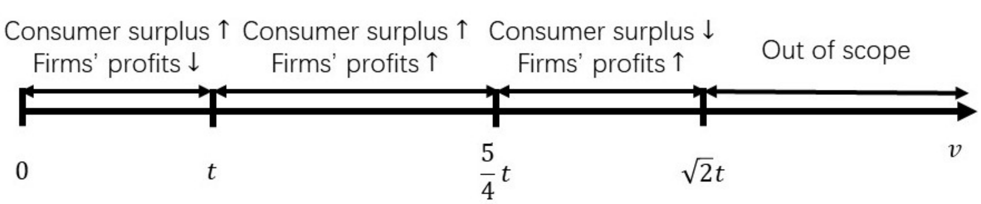


Figure 3: The changes in the consumer surplus and firms' profits (Case S)

We explain the reason that personalized pricing can improve profits and consumer surplus. Personalized pricing allows firms to expand their quantities supplied to consumers as in standard monopolistic personalized pricing. The gains of firms from personalized pricing are larger as the value of v increases (see Lemma 2). Furthermore, even when all consumers purchase from both firms ($v > t$) and firms fully extract the *additional* gross consumer surplus from the second product, consumers obtain the residual surplus $w - v$ (see Lemma 2). Therefore, when v satisfies $t < v < 5t/4$, that is, when $t < v < 5t/4$, personalized pricing improves firms' profits and consumer surplus. Figure 2 compares each consumer's surplus under Case S and the case of personalized pricing. Figure 3 shows the impacts of personalized pricing on the consumer surplus and firms' profits.

Note that $w - v$ must be larger than $t/4$ to obtain the result because we assume that $w \geq 3t/2$. Note also that the net surpluses of consumers around the edges decrease from $w - p_i^{US} - td_i(x) \simeq w - t$ to $w - v$ if $d_i(x)$ is sufficiently small ($t < v$ holds).

Second, we pick up the outcome in the case of uniform pricing such that v is large and then Case M occurs. The differences between the values under personalized pricing and those under uniform pricing in Case M are as follows:

$$\Delta CS^M = \begin{cases} -t & \text{if } 2t \leq v, \\ -\dfrac{v^2}{4t} & \text{if } \sqrt{2}t \leq v < 2t, \end{cases} \quad \Delta\pi_i^M = \begin{cases} \dfrac{t}{2} & \text{if } 2t \leq v, \\ \dfrac{t}{2} - \dfrac{(2t-v)^2}{4t} & \text{if } \sqrt{2}t \leq v < 2t \end{cases}$$

$$\Delta TS^M = \begin{cases} 0 & \text{if } 2t \leq v, \\ \dfrac{(2t-v)(3v-2t)}{4t} & \text{if } \sqrt{2}t \leq v < 2t \end{cases}$$

We summarize the comparison.

Proposition 2. *When v is large such that some consumers purchase from both firms under uniform pricing ($v > \sqrt{2}t \simeq 1.414t$) compared with uniform pricing, personalized pricing increases the*

1. profit of each firm for any v;

2. total surplus if and only if $v < 2t$.

However, personalized pricing worsens consumer surplus for any v and is irrelevant to total surplus for $v \geq 2t$.

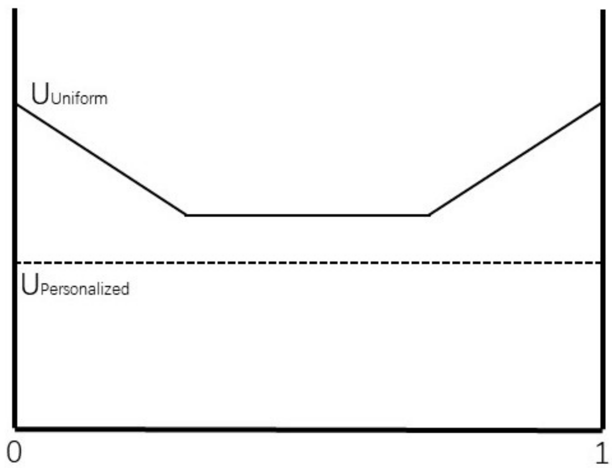


Figure 4: Consumer surplus comparison (Case M)

Note: The solid and dashed lines indicate consumer surplus under uniform and personalized pricing.

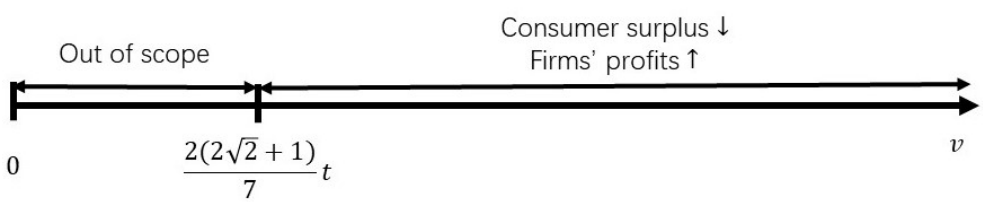


Figure 5: The changes in the consumer surplus and firms' profits (Case M)

The equilibrium uniform price in case M, p_i^{UM}, is smaller than that in case S, p_i^{US} if $v < 2t$ because firms need to induce some consumers around the center of the Hotelling line to consume both products by lowering their uniform prices. Actually, the total payment is $2p_i^{UM} = v$. Personalized pricing eliminates the downward pressure on uniform prices and increases prices. The total payment under personalized pricing is $2v - t$, which is larger than v if $v > \sqrt{2}t$ (see Lemma 2 and Figure 4). The difference in the starting points under the cases M and S means that the impacts of personalized pricing on consumers in the two cases are quite different. In case M, personalized pricing harms consumers but benefits firms (see Figure 5).

We compare our results with those in Rhodes and Zhou (2022). We show that consumers are more likely to benefit from personalized pricing as the degree of market coverage becomes lower (each consumer purchases from *only one of the two firms*), and the converse holds for firms. In a search theoretic model, Rhodes and Zhou (2022) show that consumers are more likely to benefit from personalized pricing as the degree of market coverage increases, and the converse holds for firms (Figure 2 in Rhodes and Zhou (2022)). Moreover, they also show that consumers and firms benefit from personalized pricing when the degree of market coverage is intermediate. The relationship between the degree of market coverage and the gains of consumers and firms from personalized pricing in Rhodes and Zhou (2022) differs from ours. The difference comes from the number of items each consumer can purchase (unit demand in Rhodes and Zhou (2022) and up to two items in our paper). Therefore, our results complement their findings by considering the standard Hotelling framework with multi-unit purchases.

3.4 Endogenous choices of pricing policies

We allow firms to choose one of the pricing policies endogenously: uniform and personalized pricing. We can discuss the endogenous choices by considering the asymmetric case in which one firm employs personalized pricing and the other employs uniform pricing. To discuss the asymmetric case, we assume that the latter firm (call it firm 2) sets its uniform price, and then observing the price, the former firm (call it firm 1) sets its personalized prices for consumers. The timing structure follows those in the related papers (e.g., Thisse and Vives, 1988, Shaffer and Zhang, 2002, Choe et al., 2018). The detail of the mathematical procedure is available in Appendix A.

We can classify the outcome of the asymmetric case into the following four: (i) all consumers purchase from both firms (if $v \geq 2t$); (ii) firm 1 serves all consumers but firm 2 serves the part of consumers (if $t \leq v < 2t$); (iii) consumers around the center purchase from both firms but those around the edges purchase from the closest firm (if $\sqrt{2}t/2 < v < t$); (iv) all consumers purchase from only one of the firms, which is the same as the asymmetric case in Thisse and Vives (1988) (if $v \leq \sqrt{2}t/2$). The following is the outcomes in the four cases:

1. When $v \geq 2t$, the prices of firms 1 and 2 are $p_1^*(x) = v - tx$ and $p_2^* = v - t$. The resulting demands for firms 1 and 2 are $N_1^* = N_2^* = 1$. The profits are $\pi_1^* = v - t/2$ and $\pi_2^* = v - t$. The adaptation of personalized pricing increases the profit of firm 1 from $v - t$ to $v - t/2$.

2. When $t \leq v < 2t$, the prices of firms 1 and 2 are

$$p_1^* = \begin{cases} t(1-2x) + v/2 & \text{for } x \leq 1 - v/(2t), \\ v - tx & \text{for } x \geq 1 - v/(2t), \end{cases} \quad \text{and} \quad p_2^* = v/2.$$

   The resulting demand for firms 1 and 2 are $N_1^* = 1$ and $N_2^* = v/(2t)$. The profits are $\pi_1^* = v(4t + v)/(8t)$ and $\pi_2^* = v^2/(4t)$. Firm 2's uniform price is higher than or equal to $t/2$, which is the uniform price under the asymmetric case in Thisse and Vives (1988). The higher uniform price of firm 2 implies that employing personalized pricing does not intensify competition, inducing firm 1 to choose personalized pricing.

3. When $\sqrt{2}t/2 \le v < t$, the prices of firms 1 and 2 are

$$p_1^* = \begin{cases} t(1-2x) + v/2 & \text{for } x < 1 - v/(2t), \\ v - tx & \text{for } 1 - v/(2t) \le x < v/t, \\ 0 & \text{for } x \ge v/t, \end{cases} \quad \text{and} \quad p_2^* = v/2.$$

   The resulting demands for firms 1 and 2 are $N_1^* = v/t$ and $N_2^* = v/(2t)$. The profits are $\pi_1^* = (4t^2 - 4tv + 5v^2)/(8t)$ and $\pi_2^* = v^2/(4t)$. Firm 2's uniform price is *lower* than $t/2$, contrasting with the previous case ($t \le v < 2t$). The lower uniform price of firm 2 implies that employing personalized pricing intensifies competition, diminishing firm 1's incentive to choose personalized pricing if v is small. In fact, the resulting profit of firm 1, $\pi_1^* = (4t^2 - 4tv + 5v^2)/(8t)$, is lower than that in case S in Section 3.2, $t/2$, if and only if $\sqrt{2}/2 \le v < 4t/5$ (note that the result in case S is the unique equilibrium outcome if $v < t$).

4. When $v \le \sqrt{2}t/2$, the prices of firms 1 and 2 are

$$p_1^* = \begin{cases} t(1-2x) + t/2 & \text{for } x < 3/4, \\ 0 & \text{for } x \ge 3/4, \end{cases}, \quad \text{and} \quad p_2^* = \frac{t}{2}.$$

   The resulting demand for firms 1 and 2 are $N_1^* = 3/4$ and $N_2^* = 1/4$. The profits are $\pi_1^* = 9t/16$ and $\pi_2^* = t/8$. As in Thisse and Vives (1988), employing personalized pricing increases the profit of firm 1.

Using the outcomes in the three cases in Sections 3.2, 3.1, and 3.4, we can derive the following proposition (see Figure 6).

Proposition 3. *Two types of pricing policy pairs can appear in equilibrium:*

1. *The following is always sustainable in equilibrium: both firms choose personalized pricing;*

2. *The following is also sustainable in equilibrium if and only if* $\sqrt{2}t/2 \leq v \leq 4t/5$*: both firms choose uniform pricing.*

That is, multiple equilibria co-exist if and only if $\sqrt{2}t/2 \leq v \leq 4t/5$.

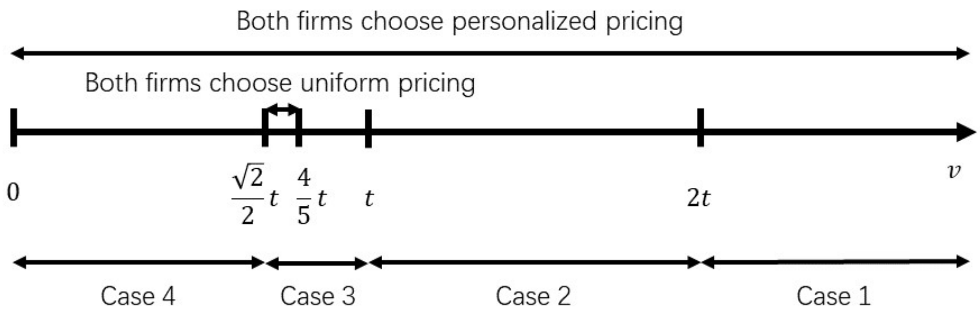


Figure 6: Endogeneous choices of prcing policies

In the context of personalized pricing, the second result is uncommon and novel. The key point of having the outcome is consumers' multi-unit purchases. Under the assumption of multi-unit purchases, each firm has an incentive to acquire consumers' second purchases if feasible. Because the market for consumers' second purchases is monopolistic, the demand for consumers' second purchases is more price elastic than when all consumers purchase from only one of the firms, which is a duopolistic case (see, Chen and Riordan, 2008, Cowan and Yin, 2008). Therefore, the uniform price of firm 2, $v/2$, is lower than that in Thisse and Vives (1988), $t/2$, in case 3 mentioned above ($v < t$).

We discuss the plausible outcome under the equilibrium multiplicity in the endogenous choices of pricing policies. We focus on the value of $v \in [\sqrt{2}t/2, 4t/5]$ that leads to the multiple equilibria in Proposition 3 (see Table 1). We find that 'U' is the payoff dominant action and 'P' is the risk dominant action. Also, following the result in Kendall (2022, pp.1130-1131), we can expect that 'U' is more likely to appear as the value of v decreases in the parameter range discussed here, although 'P' is the expectable action when v is

Table 1: The choices of prcing policies

Firm 2 / Firm 1	uniform (U)	personalized (P)
uniform (U)	$\frac{t}{2}, \frac{t}{2}$	$\frac{v^2}{4t}, \frac{t}{2} - \frac{v(4t-5v)}{8t}$
personalized (P)	$\frac{t}{2} - \frac{v(4t-5v)}{8t}, \frac{v^2}{4t}$	$\frac{t}{2} - \frac{v(t-v)}{t}, \frac{t}{2} - \frac{v(t-v)}{t}$

close to $4t/5$.[19]

We summarize the results and mention the implication. We show that personalized pricing is more likely to benefit firms if the additional gain from the second purchase, v, is larger than the threshold value, t (Propositions 1 and 2). We also show that firms can escape fierce price competition caused by personalized pricing if the additional gain from the second purchase is in the range where $\sqrt{2}t/2 \leq v \leq 4t/5$ because adopting personalized pricing induces its rival to set a sufficiently low uniform price (Proposition 3). Those results stem from incorporating the possibility of consumers' multi-unit purchases into the standard spatial competition model. Our result implies that when we discuss the impact of personalized pricing on profits and welfare, we need to consider the propensity of consumers' multistore shopping (Gijsbrechts et al., 2008, Bell et al., 2011, Landsman and Stremersch, 2011).

[19] Calculating the strategic values and the behavioral value in Kendall (2022, p.1113), we obtain $s_1 = (4t-5v)v/(16t)$, $s_2 = -(2t^2 - 4tv + 3v^2)/(8t) < 0$, and $b = (4t^2 + 4tv - 5v^2)/(32t)$. Also, applying the formulation in Kendall (2022, p.1115) to check the risk dominant action ((P, P) risk dominantes (U, U) if $|s_2| > s_1$), we find that $|s_2| - s_1 = (4t^2 - 12tv + 11v^2)/(16t) > 0$ for any $v \in [\sqrt{2}t/2, 4t/5]$. Thus, (P, P) risk dominantes (U, U). Also, Kendall (2022, p.1131) mentions "[o]ne should expect more payoff-dominant choices in games with larger s_1, s_2, and b values." We find that those values derived above are monotonically decreasing in v.

4 Heterogeneous firms

We relax the assumption on w_i and allow heterogeneous w_i ($v_i = w_i - V$). We assume that $w_1 + w_2 \geq 3t$ to ensure that all consumers purchase from at least one of the firms under uniform pricing. By a similar calculation procedure, we obtain the equilibrium prices and the players' benefits under personalized pricing and uniform pricing. As in the main model, when at least some consumers purchase from both firms, personalized pricing increases the profit of each firm but decreases the consumer surplus for any $v_i,\ i = 1,\ 2$. Combining these opposite effects, the total surplus improves if and only if $\exists i \in 1, 2,\ v_i < 2t$ (the detail is available in Appendix B).

Proposition 4. *Suppose that at least some consumers purchase from both firms under uniform pricing; equivalently, suppose that:*

$$\begin{cases} v_i > (1+\sqrt{2})(t - \frac{v_j}{2}) & \text{if } v_j > (6-4\sqrt{2})t, \\ v_i > 3t + \frac{1}{2}v_j - 2\sqrt{tv_j} & \text{if } v_j \leq (6-4\sqrt{2})t. \end{cases} \tag{4}$$

Compared with uniform pricing, personalized pricing increases the

1. *profit of each firm for any v_i;*

2. *total surplus if and only if $\exists i \in 1, 2,\ v_i < 2t$.*

However, personalized pricing worsens consumer surplus for any v_i and is irrelevant to total surplus when $\forall i \in 1, 2,\ v_i \geq 2t$.

When at least one of v_1 and v_2 is large (say, v_1 is large), some consumers purchase from both firms even under uniform pricing (see equation (4)). In this situation, if the pricing regime changes to personalized pricing, firm 2 can offer personalized prices in the monopoly market in which consumers, who purchase from firm 1, consider additional purchases from

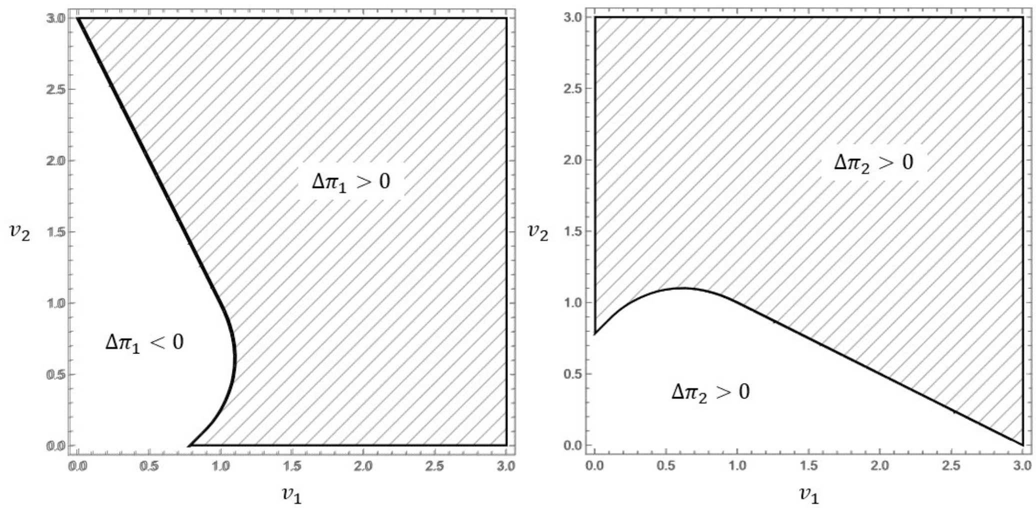


Figure 7: Difference in firm i's profits under personalized and uniform pricing ($\Delta\pi_i$)
Note: Lefthand-side: $\Delta\pi_1$; Righthand-side: $\Delta\pi_2$; Horizontal axis: v_1; Vertical axis: v_2.

firm 2. This pricing by firm 2 means that the interaction between the firms ceases, and the firms behave as if they are monopolists. Moreover, the elimination of the interaction vanishes firm 2's disadvantage over firm 1 when v_1 is larger than v_2. Because monopolistic personalized pricing expands the market supply, the regime of personalized pricing improves total surplus and profits. The converse holds for consumer surplus.

Then we show a parameter range of v_1 and v_2 such that personalized pricing improves profits and consumer welfare. Combining Figures 7 and 8, we divide the parameter area into seven parts. As a result, we obtain the following proposition:

Proposition 5. *Suppose that all consumers purchase from only one of the firms under uniform pricing; equivalently, suppose that:*

$$\begin{cases} v_i \leq 3t - (\frac{3\sqrt{2}}{2} - 1)v_j & \text{if } v_j \leq 2t, \\ v_i \leq 3t + v_j - 3\sqrt{2}\sqrt{t(v_j - t)} & \text{if } 2t < v_j \leq 3t. \end{cases} \tag{5}$$

Compared with uniform pricing, personalized pricing increases the

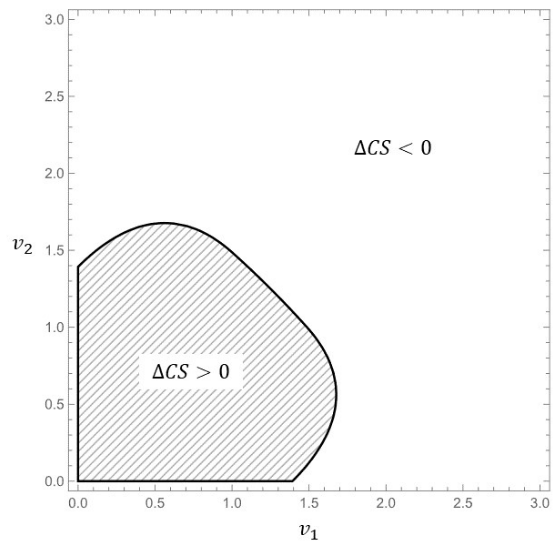


Figure 8: Difference in consumer surpluses under personalized and uniform pricing (ΔCS)
Note: Horizontal axis: v_1; Vertical axis: v_2.

1. profit of each firm if and only if

$$\begin{cases} 7(v_i - v_j) > (6\sqrt{2} - 3)t & \text{if } v_j \leq \dfrac{5 - 3\sqrt{2}}{7}t, \\ 8v_i + v_j - 3\sqrt{t^2 + 10tv_j - 7v_j^2} > 3t & \text{if } \dfrac{5 - 3\sqrt{2}}{7}t < v_j \leq \dfrac{1}{4}t, \\ v_i - v_j + 3\sqrt{3t^2 + v_j^2} > 6t & \text{if } \dfrac{1}{4}t < v_j \leq t, \\ 2v_i + v_j > 3t & \text{if } v_j > t; \end{cases}$$

2. consumer surplus if and only if

$$\begin{cases} v_i < v_j + 3\sqrt{2}\sqrt{6t^2 - v_j^2} - 9t & \text{if } v_j \leq t, \\ v_i < v_j + 3\sqrt{2}\sqrt{t(7t - v_j)} - 9t & \text{if } v_j > t; \end{cases}$$

3. total surplus unless $v_1 = v_2 \leq \dfrac{t}{2}$.

Figure 9 shows that when both v_1 and v_2 take intermediate values, personalized pricing improves consumer surplus and profits. In this area, the difference between the firms in terms of v_i is small. The following proposition summarizes the discussion.

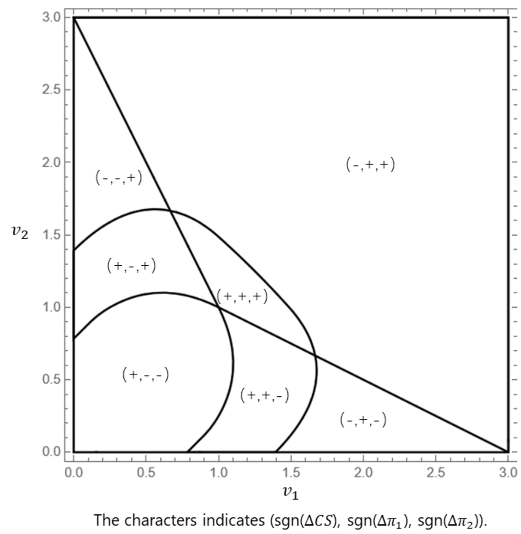


Figure 9: Parameter range when firm profits and consumer surplus improve
$(\cdot,\cdot,\cdot)$ indicates $(\mathrm{sgn}(\Delta CS), \mathrm{sgn}(\Delta\pi_1), \mathrm{sgn}(\Delta\pi_2))$. Horizontal axis: v_1; Vertical axis: v_2.

Proposition 6. *Personalized pricing improves profits and consumer surplus if and only if for each $i = 1, 2$,*

$$2v_i + v_j > 3t,$$

$$\begin{cases} v_i < v_j + 3\sqrt{2}\sqrt{6t^2 - v_j^2} - 9t & \text{if } v_j \le t, \\ v_i < v_j + 3\sqrt{2}\sqrt{t(7t - 2v_j)} - 9t & \text{if } v_j > t. \end{cases}$$

5 Conclusion

Given that advances in information technology have made personalized pricing a reality, we discuss the effect of personalized pricing on profits and welfare in a Hotelling duopoly model in which consumers can purchase from both firms. Our formulation complements Jeitschko et al. (2017) by considering personalized pricing and Rhodes and Zhou (2022) by adopting the standard Hotelling model.

We have the following results. Consumers benefit from personalized pricing only if no consumer purchases from both firms under uniform pricing. Under the necessary condition, consumer surplus improves if the additional intrinsic utility from the second product is smaller than a threshold value. Firms benefit from personalized pricing if at least some consumers purchase from both firms under uniform pricing or if the additional gain from the second product is larger than the transportation cost. There is a parameter range such that personalized pricing improves consumer surplus and firms' profits. The results contrast with the standard results in the personalized pricing literature based on Hotelling models and complement the findings in Jeitschko et al. (2017) and Rhodes and Zhou (2022).

Furthermore, we extend the model by endogenizing firms' pricing policies and obtain two types of equilibrium outcomes. First, both firms choose personalized pricing in any case; second, both firms choose uniform pricing when the additional intrinsic utility from the second product is in a parameter range in which the following two conditions hold (i) all consumers purchase from only one of the firms in equilibrium if the firms employ uniform pricing and (ii) some consumers purchase from both firms in equilibrium if only one of the firms employs personalized pricing. That is, in this parameter range, multiple equilibria can co-exist. The second type of equilibrium outcome is uncommon and novel in the context of personalized pricing, although several papers show that only one of the firms employs personalized pricing in asymmetric duopoly models (Ghose and Huang, 2009, Matsumura and Matsushima, 2015).

Those main results stem from the assumption that consumers purchase multiple items in the standard spatial competition model (Jeitschko et al., 2017). The key factors of those positive results are market demand expansions through personalized pricing. Therefore, we can conclude that the feasibility of personalized pricing is less likely to lead to worse outcomes for firms if personalized pricing expands the total demands of consumers. Also, we think that when we discuss the impact of personalized pricing on profits and welfare, we need to consider the propensity of consumers' multistore shopping (Gijsbrechts et al., 2008, Bell et al., 2011, Landsman and Stremersch, 2011).

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Appendix

A Endogenous choice of pricing policies

In the appendix, we consider the subgame in which firm 1 uses personalized pricing and firm 2 uses uniform pricing. The pricing sequence is that firm 2 sets its uniform pricing; after that, firm 1 sets personalized prices. We solve it by backward induction.

Firm 1's personalized pricing The utility from purchasing firm 1's product, firm 2's product, or both products is:

$$\begin{cases} U_1 = w - tx - p_1(x), & \text{purchasing from only firm 1,} \\ U_2 = w - t(1-x) - p_2, & \text{purchasing from only firm 2,} \\ U_{12} = 2w - V - t - p_1(x) - p_2, & \text{purchasing from firms 1 and 2.} \end{cases}$$

Let $v \equiv w - V$. Consumers purchase only from firm 1 if and only if $U_1 \geq U_2$ and $U_1 > U_{12}$; they purchase only from firm 2 if and only if $U_2 > U_1$ and $U_2 > U_{12}$; and they purchase from both firms if and only if $U_{12} \geq U_1$ and $U_{12} \geq U_2$. Substituting the utility functions into the inequalities, we get that 1) consumers purchase only from firm 1 when $p_1(x) \leq t(1-2x)+p_2$ and $p_2 > v - t(1-x)$; 2) consumers purchase only from firm 2 when $p_1(x) > t(1-2x)+p_2$ and $p_1(x) > v - tx$; 3) consumers purchase from both firms when $p_1(x) \leq v - tx$ and $p_2 \leq v - t(1-x)$. Firm 1 chooses the highest price that is acceptable for consumers locating at x, which means $p_1(x, p_2) = \max\{0, t(1-2x) + p_2, v - tx\}$. By organizing this price function, we derive the following conditions and firm 1's best response.

1. Firm 1 can offer personalized prices to all consumers if and only if $v > t$ or $p_2 > t$. In this case, consumers on $[0, \min\{(t + p_2 - v)/t, 1\})$ purchase only from firm 1, and consumers on $[\min\{(t + p_2 - v)/t, 1\}, 1]$ purchase from both firms. In the following, we omit the case in which $\min\{(t + p_2 - v)/t, 1\} = 1$ because this equation means that firm 2 is inactive.

2. Firm 1 offers positive personalized prices for consumers that locate on $[0,(t+p_2)/(2t)]$ if and only if $v \leq t$ and $2v - t < p_2 \leq t$. In this case, all consumers purchase from only one of the firms.

3. Firm 1 offers positive personalized prices for consumers that locate on $[0, v/t]$ if and only if $v \leq t$ and $p_2 \leq 2v - t$. In this case, consumers on $[0,(t+p_2-v)/t)$ purchase only from firm 1, consumers on $[(t+p_2-v)/t, v/t]$ purchase from both firms, and consumers on $(v/t, 1]$ purchase only from firm 2.

The personalized prices proposed by firms 1 are:

$$p_1(x,p_2) = \begin{cases} \begin{cases} t(1-2x)+p_2 & \text{for } x \in [0,(t+p_2-v)/t), \\ v - tx & \text{for } x \in [(t+p_2-v)/t,\ 1], \end{cases} & \text{if} \quad v > t \text{ or } p_2 > t, \\ \begin{cases} t(1-2x)+p_2 & \text{for } x \in [0,(t+p_2)/(2t)], \\ 0 & \text{for } x \in ((t+p_2)/(2t), 1], \end{cases} & \text{if} \quad \begin{array}{l} v \leq t \text{ and} \\ 2v-t < p_2 \leq t, \end{array} \\ \begin{cases} t(1-2x)+p_2 & \text{for } x \in [0,(t+p_2-v)/t], \\ v - tx & \text{for } x \in ((t+p_2-v)/t, v/t), \\ 0 & \text{for } x \in [v/t, 1], \end{cases} & \text{if} \quad \begin{array}{l} v \leq t \text{ and} \\ p_2 \leq 2v-t. \end{array} \end{cases}$$

Firm 2's uniform pricing We classify the case into the three subcases: (i) $v > t$ or $p_2 \geq t$; (ii) $v \leq t$ and $2v - t < p_2 \leq t$, and (iii) $v \leq t$ and $p_2 \leq 2v - t$.

When $v > t$ or $p_2 \geq t$ In this case, firm 1 offers positive personalized prices to all consumers, which means every consumer purchases from firm 1, and some of them also purchase from firm 2. Therefore, consumers at x who satisfy the condition that $U_{12} \geq U_1$, which means $x \geq (t - v + p_2)/t$, purchase from firm 2. The amount of consumers who purchase from firm 2 is

$$N_2 = \max\{\min\{\frac{v-p_2}{t}, 1\}, 0\}.$$

Then, firm 2 chooses its price p_2 to maximize its profit $\pi_2 = p_2 N_2$. Taking the first-order condition, $\partial\pi_2/\partial p_2 = 0$, we derive the optimal price of firm 2:

$$p_2^* = \begin{cases} \dfrac{v}{2} & \text{if } t < v < 2t, \\ v - t & \text{if } v \geq 2t. \end{cases}$$

When $v \leq t$ **and** $2v - t < p_2 \leq t$ In this case, all consumers purchase from only one firm of the firms. Firm 2 can serve consumers on $((t + p_2)/(2t), 1]$. The amount of consumers who purchase from firm 2 is

$$N_2 = \max\{\min\{\frac{t - p_2}{2t}, 1\}, 0\}.$$

Then, firm 2 chooses p_2 to maximize its profit $\pi_2 = p_2 N_2$. Taking the first-order condition, $\partial\pi_2/\partial p_2 = 0$, we derive the optimal price of firm 2:

$$p_2^* = \frac{t}{2}.$$

When $v \leq t$ **and** $p_2 \leq 2v - t$ In this case, consumers on $[0, (t + p_2 - v)/t)$ purchase only from firm 1, consumers on $[(t+p_2-v)/t, v/t]$ purchase from both firms, and consumers on $(v/t, 1]$ only purchase from firm 2. As $v \leq t$ and $p_2 \leq 2v - t$, p_2 must not be larger than v, so $1 - (t + p_2 - v)/t = (v - p_2)/t$ is always between 0 and 1. Similar to the first case, the amount of consumers who purchase from firm 2 is

$$N_2 = \frac{v - p_2}{t}.$$

Then, firm 2 chooses p_2 to maximize its profit $\pi_2 = p_2 N_2$. Taking the first-order condition, $\partial\pi_2/\partial p_2 = 0$, we derive the optimal prices of firm 2:

$$p_2^* = \frac{v}{2}.$$

To sum up, the equilibrium outcome is as follows:

1. When $v \geq 2t$, the prices of firms 1 and 2 are $p_1^*(x) = v - tx$ and $p_2^* = v - t$. The resulting demands for firms 1 and 2 are $N_1^* = N_2^* = 1$. The profits are $\pi_1^* = v - t/2$ and $\pi_2^* = v - t$.

2. When $t \leq v < 2t$, the prices of firms 1 and 2 are

$$p_1^* = \begin{cases} t(1-2x) + v/2 & \text{for } x \leq 1 - v/(2t), \\ v - tx & \text{for } x \geq 1 - v/(2t), \end{cases} \quad \text{and} \quad p_2^* = v/2.$$

   The resulting demand for firms 1 and 2 are $N_1^* = 1$ and $N_2^* = v/(2t)$. The profits are $\pi_1^* = v(4t + v)/(8t)$ and $\pi_2^* = v^2/(4t)$.

3. When $\sqrt{2}t/2 \leq v < t$, the prices of firms 1 and 2 are

$$p_1^* = \begin{cases} t(1-2x) + v/2 & \text{for } x < 1 - v/2, \\ v - tx & \text{for } 1 - v/(2t) \leq x < v/t, \\ 0 & \text{for } x \geq v/t, \end{cases} \quad \text{and} \quad p_2^* = v/2.$$

   The resulting demands for firms 1 and 2 are $N_1^* = v/t$ and $N_2^* = v/(2t)$. The profits are $\pi_1^* = (4t^2 - 4tv + 5v^2)/(8t)$ and $\pi_2^* = v^2/(4t)$.

4. When $v \leq \sqrt{2}t/2$, the prices of firms 1 and 2 are

$$p_1^* = \begin{cases} t(1-2x) + t/2 & \text{for } x < 3/4, \\ 0 & \text{for } x \geq 3/4, \end{cases}, \quad \text{and} \quad p_2^* = \frac{t}{2}.$$

   The resulting demand for firms 1 and 2 are $N_1^* = 3/4$ and $N_2^* = 1/4$. The profits are $\pi_1^* = 9t/16$ and $\pi_2^* = t/8$.

The choices of pricing policies In the first stage, the firms choose their pricing policies simultaneously. Let firm i's profit be $\pi_i(a_i, a_j)$ $(i, j = 1, 2,\ j \neq i)$, where $a_i \in \{p, u\}$ represents firm i's pricing policy, where p means personalized pricing and u means uniform pricing. Then, we can indicate the profit function of each firm as follows (when there are multiple equilibria under (u, u) (when $2(2\sqrt{2}+1)t/7 \leq v \leq \sqrt{2}t$, see Section 3.1), we choose the higher profit $\pi_i(u, u) = t/2$):

$$\pi_i(p,p) = \begin{cases} v - \frac{t}{2} & \text{if } t < v, \\ \frac{t}{2} - \frac{v(t-v)}{t} & \text{if } t/2 < v \leq t, \\ \frac{t}{4} & \text{if } v \leq t/2. \end{cases} \quad \pi_i(p,u) = \begin{cases} v - \frac{t}{2} & \text{if } 2t \leq v, \\ \frac{v(4t+v)}{8t} & \text{if } t < v < 2t, \\ \frac{4t^2-4tv+5v^2}{8t} & \text{if } \frac{\sqrt{2}}{2}t < v \leq t, \\ \frac{9}{16}t & \text{if } v \leq \frac{\sqrt{2}}{2}t. \end{cases}$$

$$\pi_i(u,p) = \begin{cases} v - t & \text{if } 2t \leq v, \\ \frac{v^2}{4t} & \text{if } \frac{\sqrt{2}}{2}t < v < 2t, \\ \frac{1}{8}t & \text{if } v \leq \frac{\sqrt{2}}{2}t. \end{cases} \quad \pi_i(u,u) = \begin{cases} w - t & \text{if } 2t \leq v, \\ \frac{v^2}{4t} & \text{if } \sqrt{2}t < v < 2t, \\ \frac{t}{2} & \text{if } v \leq \sqrt{2}t. \end{cases}$$

Table 2: The choices of prcing policies

Firm 1 \ Firm 2	personalized	uniform
personalized	$(\pi_1(p,p),\ \pi_2(p,p))$	$(\pi_1(p,u),\ \pi_2(u,p))$
uniform	$(\pi_1(u,p),\ \pi_2(p,u))$	$(\pi_1(u,u),\ \pi_2(u,u))$

We compare each firm's profits under different pricing policies. No matter what the parameters are, $\pi_i(p,p) > \pi_i(u,p)$. On the other hand, $\pi_i(u,u) \geq \pi_i(p,u)$ if and only if $\sqrt{2}t/2 \leq v \leq 4t/5$. The discussion leads to Proposition 3.

B Equilibrium with and without consumers who purchase from both firms

We slightly extend the range of exogenous parameters in Jeitschko et al. (2017). We derive the conditions: (i) some consumers purchase from both firms; (ii) all consumers purchase from one of the firms. The location of consumers who are indifferent between choosing only firm 1 and choosing both firms satisfies $w_1 - tx_1^* - p_1 = w_1 + w_2 - V - t - p_1 - p_2$. Let $v_i = w_i - V$ $(i = 1, 2)$. By organizing this equation, we obtain $x_1^* = 1 - (v_2 - p_2)/t$.

Similarly, $x_2^* = (v_1 - p_1)/t$ is the location of consumers who are indifferent between choosing only firm 2 and choosing both firms. When some consumers purchase from both firms, $x_1^* < x_2^*$, which implies that $\Sigma_k p_k < v_1 + v_2 - t$. When all consumers purchase from one of the firms, $x_1^* \geq x_2^*$, which implies that $\Sigma_k p_k \geq v_1 + v_2 - t$.

B.1 Uniform pricing

Below, first, we show the equilibrium outcome in which all consumers purchase from one of the firms, which implies that $\Sigma_k p_k \geq v_1 + v_2 - t$. Second, we show the equilibrium outcome in which some consumers purchase from both firms, which implies that $\Sigma_k p_k < v_1 + v_2 - t$.

B.1.1 Equilibrium when all consumers purchase from one of the firms

In this section, assume that $\Sigma_k p_k \geq \Sigma_k v_k - t$. The number of consumers who belong to firm i, N_i $(i = 1, 2)$, is:

$$N_i = \max\{\min\{\hat{N}_i, 1\}, 0\}, \text{ where } \hat{N}_i = \frac{1}{2} + \frac{v_i - v_j - (p_i - p_j)}{2t}. \tag{1}$$

Firm i $(i = 1, 2)$ chooses its price p_i to maximize its profit $\pi_i = p_i N_i$. Solving the first-order conditions, $\partial \pi_i / \partial p_i$ $(i = 1, 2)$, we obtain the optimal prices, number of consumers belonging to firm i, and resulting profits:

$$p_i^* = t + \frac{v_i - v_j}{3}, \quad N_i^* = \frac{1}{2} + \frac{v_i - v_j}{6t}, \quad \pi_i^* = 2t\left(\frac{1}{2} + \frac{v_i - v_j}{6t}\right)^2. \tag{2}$$

We need to check the condition that the firms have no incentive to deviate from the prices. After some calculus (available in Section B.1.2), we obtain the condition $(i, j = 1, 2,\ j \neq i)$:

$$\begin{cases} v_i \leq 3t - (\frac{3\sqrt{2}}{2} - 1)v_j & \text{if } v_j \leq 2t, \\ v_i \leq 3t + v_j - 3\sqrt{2}\sqrt{t(v_j - t)} & \text{if } 2t < v_j \leq 3t. \end{cases} \tag{3}$$

The condition is in equation (5) in Proposition 5.

B.1.2 Equilibrium when some consumers purchase from both firms

In this section, suppose that $\Sigma_k p_k < v_1 + v_2 - t$. The number of consumers who belong to firm i, $N_i, i = 1, 2$, is:

$$N_i = \max\{\min\{\hat{N}_i, 1\}, 0\}, \text{ where } \hat{N}_i = \frac{v_i - p_i}{t}. \tag{4}$$

Because some consumers purchase from both firms, $\Sigma_k N_k > 1$. Firm i chooses its price p_i to maximize its profit $\pi_i = p_i N_i$. Solving the first-order conditions, $\partial\pi_i/\partial p_i$ $(i = 1, 2)$, we obtain the optimal prices, number of consumers belonging to firm i, and resulting profits:

$$p_i^{**} = \begin{cases} \frac{v_i}{2} & \text{if } v_i < 2t, \\ v_i - t & \text{if } v_i \geq 2t, \end{cases} \quad N_i^{**} = \begin{cases} \frac{v_i}{2t} & \text{if } v_i < 2t, \\ 1 & \text{if } v_i \geq 2t, \end{cases} \quad \pi_i^{**} = \begin{cases} \frac{v_i^2}{4t} & \text{if } v_i < 2t, \\ v_i - t & \text{if } v_i \geq 2t. \end{cases} \tag{5}$$

We need to check the condition that the firms have no incentive to deviate from the prices. After some calculus (available in the last part of this section), we obtain the condition:

$$\begin{cases} v_i > (1 + \sqrt{2})(t - \frac{v_j}{2}) & \text{if } v_j > (6 - 4\sqrt{2})t, \\ v_i > 3t + \frac{1}{2}v_j - 2\sqrt{tv_j} & \text{if } v_j \leq (6 - 4\sqrt{2})t. \end{cases} \tag{6}$$

The condition is in equation (4) in Proposition 4.

Deviation incentives when all consumers purchase from one of the firms We show the condition that firms have no incentive to deviate from their prices. First, $p_i = p_i^*$ satisfies the first- and second-order conditions under the condition that $\Sigma_k p_k \geq \Sigma_k v_k - t$. Thus, p_i^* is always the optimal price for firm i when all consumers purchase from one of the firms. Therefore, we check the situation that firm i decreases its price to the level such that $p_i + p_j^* < \Sigma_k v_k - t$; that is, some consumers purchase from both firms. The demand for firm i is N_i in equation (4). Solving the first-order condition, the optimal deviation price of firm i is:

$$p_i^{D*} = \begin{cases} \frac{v_i}{2} & \text{if } v_i < 2t \text{ and } t < \frac{5}{12}v_i + \frac{1}{3}v_j, \\ \frac{4}{3}v_i + \frac{2}{3}v_j - 2t & \text{if } v_i < 2t \text{ and } t \geq \frac{5}{12}v_i + \frac{1}{3}v_j, \\ v_i - t & \text{if } v_i \geq 2t \text{ and } t < \frac{1}{3}v_i + \frac{2}{3}v_j, \\ \frac{4}{3}v_i + \frac{2}{3}v_j - 2t & \text{if } v_i \geq 2t \text{ and } t \geq \frac{1}{3}v_i + \frac{2}{3}v_j, \end{cases}$$

and the maximizing profit of firm i is

$$\pi_i^{D*} = \begin{cases} \frac{v_i^2}{4t} & \text{if } v_i < 2t \text{ and } t < \frac{5}{12}v_i + \frac{1}{3}v_j, \\ (\frac{4}{3}v_i + \frac{2}{3}v_j - 2t)(2 - \frac{v_i + 2v_j}{3t}) & \text{if } v_i < 2t \text{ and } t \geq \frac{5}{12}v_i + \frac{1}{3}v_j, \\ v_i - t & \text{if } v_i \geq 2t \text{ and } t < \frac{1}{3}v_i + \frac{2}{3}v_j, \\ \frac{4}{3}v_i + \frac{2}{3}v_j - 2t & \text{if } v_i \geq 2t \text{ and } t \geq \frac{1}{3}v_i + \frac{2}{3}v_j. \end{cases}$$

Comparing the deviation profit with the optimal profit π_i^*, we obtain the condition that firm i has no incentive to deviate from the optimal price p_i^* as in equation (2).

Deviation incentives when some consumers purchase from both firms We show the condition that the firms have no incentive to deviate from their prices. First, $p_i = p_i^{**}$ satisfies the first- and second-order conditions under the condition that $\Sigma_k p_k < v_1 + v_2 - t$. Thus, p_i^{**} is always the optimal price for firm i when some consumers purchase from both firms. Therefore, we check the situation that firm i raises its price to the level such that $p_i + p_j^{**} \geq v_1 + v_2 - t$; that is, all consumers purchase from one of the firms. The demand for firm i is N_i in equation (1). Solving the first-order condition, the optimal deviation price of firm i is:

$$p_i^{D**} = \begin{cases} v_i + \frac{1}{2}v_j - t & \text{if } v_j < 2t \text{ and } t < \frac{2v_i + 3v_j}{6}, \\ \frac{2v_i - v_j + 2t}{4} & \text{if } v_j < 2t \text{ and } t \geq \frac{2v_i + 3v_j}{6}, \\ v_i & \text{if } v_j \geq 2t. \end{cases}$$

and the deviation profit of firm i is

$$\pi_i^{D**} = \begin{cases} \left(v_i + \frac{1}{2}v_j - t\right)\left(1 - \frac{v_j}{2t}\right) & \text{if } v_j < 2t \text{ and } t < \frac{2v_i + 3v_j}{6}, \\ \frac{1}{2t}\left(\frac{2v_i - v_j + 2t}{4}\right)^2 & \text{if } v_j < 2t \text{ and } t \geq \frac{2v_i + 3v_j}{6}, \\ 0 & \text{if } v_j \geq 2t. \end{cases}$$

Comparing the deviation profit with the optimal profit π_i^{**}, we obtain the condition that firm i has no incentive to deviate from the optimal price p_i^{**} as in equation (5).

B.1.3 Welfare

All consumers purchase from one of the firms When no consumers purchase multiple units, consumer and total surpluses are:

$$\begin{aligned} CS^{U*} &= \frac{(v_1 - v_2)^2 + 18t(2V + v_1 + v_2) - 45t^2}{36t} \\ TS^{U*} &= \frac{5(v_1 - v_2)^2 + 18t(2V + v_1 + v_2) - 9t^2}{36t}. \end{aligned}$$

Some consumers purchase from both firms When some consumers purchase from both firms, consumer and total surpluses are:

$$CS^{U**} = \begin{cases} \frac{v_1^2 + v_2^2}{8t} + V & \text{if } v_1 < 2t \text{ and } v_2 < 2t, \\ \frac{v_2^2}{8t} + V + \frac{t}{2} & \text{if } v_1 \geq 2t \text{ and } v_2 < 2t, \\ \frac{v_1^2}{8t} + V + \frac{t}{2} & \text{if } v_1 < 2t \text{ and } v_2 \geq 2t, \\ V + t & \text{if } v_1 \geq 2t \text{ and } v_2 \geq 2t, \end{cases}$$

$$TS^{U**} = \begin{cases} \dfrac{3(v_1^2+v_2^2)}{8t} + V & \text{if } v_1 < 2t \text{ and } v_2 < 2t, \\ \dfrac{3v_2^2}{8t} + V + v_1 - \dfrac{t}{2} & \text{if } v_1 \geq 2t \text{ and } v_2 < 2t, \\ \dfrac{3v_1^2}{8t} + V + v_2 - \dfrac{t}{2} & \text{if } v_1 < 2t \text{ and } v_2 \geq 2t, \\ V + v_1 + v_2 - t & \text{if } v_1 \geq 2t \text{ and } v_2 \geq 2t. \end{cases}$$

B.2 Personalized pricing

First, we derive the condition that some consumers purchase from both firms. Given that consumers at x purchase from firm j at a positive personalized price, they also buy from firm i if and only if:

$$\begin{aligned} & w_i + w_j - V - t - p_i(x) - p_j(x) \geq w_j - td_j(x) - p_j(x) \\ \Rightarrow \quad & p_i(x) \leq v_i - t(1 - d_j(x)), \end{aligned}$$

where $d_j(x)$ is the distance between firm j and the consumer at x. If firm j cannot attract consumers at x at a nonnegative personalized price, it does not supply to them and sets $p_j(x) = 0$. Then, firm i's personalized price is acceptable for consumers at x if and only if:

$$w_i - td_i(x) - p_i(x) \geq w_j - td_j(x) - 0 \quad \Rightarrow \quad p_i(x) \leq v_i - v_j + t(d_j(x) - d_i(x)).$$

We obtain the following result: The schedules of personalized prices depend on v_i and t. Concretely,

1. both firms offer personalized prices that induce all consumers to purchase from both firms if and only if $v_i > t$, $\forall i = 1, 2$;

2. firm i offers positive personalized prices for consumers such that $d_i(x) < v_i/t$ if and only if $v_i \leq t$ and $v_i + v_j > t$. In this case, consumers on $(1 - v_2/t, v_1/t)$ purchase from both firms;

3. no firm offers personalized prices that induce consumers to buy from both firms if and only if $v_1 + v_2 \leq t$.

The personalized prices of firms 1 and 2 are:

$$p_1(x) = \begin{cases} \max\{v_1 - tx, 0\} & \text{if } v_2 > t, \\ \begin{array}{ll} \max\{v_1 - v_2 + t(1-2x), 0\} & \text{for } x \in [0, 1 - v_2/t] \\ \max\{v_1 - tx, 0\} & \text{for } x \in [1 - v_2/t, 1] \end{array} & \text{if } v_2 \leq t \text{ and } v_1 + v_2 > t, \\ \max\{v_1 - v_2 + t(1-2x), 0\} & \text{if } v_1 + v_2 \leq t, \end{cases}$$

$$p_2(x) = \begin{cases} \max\{v_2 - t(1-x), 0\} & \text{if } v_1 > t, \\ \begin{array}{ll} \max\{v_2 - v_1 + t(2x-1), 0\} & \text{for } x \in [v_1/t, 1] \\ \max\{v_2 - t(1-x), 0\} & \text{for } x \in [0, v_1/t] \end{array} & \text{if } v_1 \leq t \text{ and } v_1 + v_2 > t, \\ \max\{v_2 - v_1 + t(2x-1), 0\} & \text{if } v_1 + v_2 \leq t. \end{cases}$$

In this case, total surplus is:

$$TS^P = \begin{cases} v_1 + v_2 + V - t & \text{if } v_1 > t \text{ and } v_2 > t, \\ \dfrac{v_1^2}{2t} + v_2 + V - \dfrac{t}{2} & \text{if } v_1 \leq t \text{ and } v_2 > t, \\ v_1 + \dfrac{v_2^2}{2t} + V - \dfrac{t}{2} & \text{if } v_1 > t \text{ and } v_2 \leq t, \\ \dfrac{v_1^2 + v_2^2 + 2tV}{2t} & \text{if } v_1 \leq t,\ v_2 \leq t \text{ and } v_1 + v_2 > t, \\ \dfrac{(v_1 - v_2)^2 + 2t(v_1 + v_2 + 2V) - t^2}{4t} & \text{if } v_1 + v_2 \leq t. \end{cases}$$

The profit of each firm is:

$$\pi_1^P = \begin{cases} v_1 - \dfrac{t}{2} & \text{if } v_1 > t \text{ and } v_2 > t, \\ \dfrac{v_1^2}{2t} & \text{if } v_1 \leq t \text{ and } v_2 > t, \\ v_1 + \dfrac{v_2(v_2 - 2t)}{2t} & \text{if } v_1 > t \text{ and } v_2 \leq t, \\ \dfrac{v_1^2 + v_2^2 - 2tv_2 + t^2}{2t} & \text{if } v_1 \leq t,\ v_2 \leq t \text{ and } v_1 + v_2 > t, \\ \dfrac{(v_1 - v_2 + t)^2}{4t} & \text{if } v_1 + v_2 \leq t. \end{cases}$$

$$\pi_2^P = \begin{cases} v_2 - \dfrac{t}{2} & \text{if } v_1 > t \text{ and } v_2 > t, \\ v_2 + \dfrac{v_1(v_1 - 2t)}{2t} & \text{if } v_1 \le t \text{ and } v_2 > t, \\ \dfrac{v_2^2}{2t} & \text{if } v_1 > t \text{ and } v_2 \le t, \\ \dfrac{v_1^2 + v_2^2 - 2tv_1 + t^2}{2t} & \text{if } v_1 \le t,\, v_2 \le t \text{ and } v_1 + v_2 > t, \\ \dfrac{(v_2 - v_1 + t)^2}{4t} & \text{if } v_1 + v_2 \le t. \end{cases}$$

By calculating $CS = TS - \pi_1 - \pi_2$, we obtain consumer surplus as follows:

$$CS^P = \begin{cases} V & \text{if } v_1 > t \text{ and } v_2 > t, \\ v_1 + V - \dfrac{v_1^2}{2t} - \dfrac{t}{2} & \text{if } v_1 \le t \text{ and } v_2 > t, \\ v_2 + V - \dfrac{v_2^2}{2t} - \dfrac{t}{2} & \text{if } v_1 > t \text{ and } v_2 \le t, \\ v_1 + v_2 + V - \dfrac{v_1^2 + v_2^2}{2t} - t & \text{if } v_1 \le t,\, v_2 \le t \text{ and } v_1 + v_2 > t, \\ \dfrac{2t(v_1 + v_2 + 2V) - (v_1 - v_2)^2 - 3t^2}{4t} & \text{if } v_1 + v_2 \le t. \end{cases}$$

B.3 Comparison

We compare the outcomes in the cases of personalized pricing and uniform pricing. The differences between the values under personalized pricing and those under uniform pricing when all consumers purchase from only one of the firms are as follows:

$$\Delta TS^S = \begin{cases} \dfrac{v_1 + v_2}{2} - \dfrac{5(v_1 - v_2)^2}{36t} - \dfrac{3}{4}t & \text{if } v_1 > t \text{ and } v_2 > t, \\ \dfrac{13v_1^2 + 10v_1v_2 - 5v_2^2 - 18t(v_1 - v_2) - 9t^2}{36t} & \text{if } v_1 \le t \text{ and } v_2 > t, \\ \dfrac{13v_2^2 + 10v_1v_2 - 5v_1^2 - 18t(v_2 - v_1) - 9t^2}{36t} & \text{if } v_1 > t \text{ and } v_2 \le t, \\ \dfrac{13v_1^2 + 10v_1v_2 + 13v_2^2 - 18t(v_1 + v_2) + 9t^2}{36t} & \text{if } v_1 \le t,\, v_2 \le t \text{ and } v_1 + v_2 > t, \\ \dfrac{(v_1 - v_2)^2}{9t} & \text{if } v_1 + v_2 \le t, \end{cases}$$

$$\Delta CS^S = \begin{cases} -\dfrac{(v_1 - v_2)^2 + 18t(v_1 + v_2) - 45t^2}{36t} & \text{if } v_1 > t \text{ and } v_2 > t, \\ -\dfrac{19v_1^2 - 2v_1v_2 + v_2^2 - 18t(v_1 - v_2) - 27t^2}{36t} & \text{if } v_1 \leq t \text{ and } v_2 > t, \\ -\dfrac{19v_2^2 - 2v_1v_2 + v_1^2 - 18t(v_2 - v_1) - 27t^2}{36t} & \text{if } v_1 > t \text{ and } v_2 \leq t, \\ -\dfrac{19v_1^2 - 2v_1v_2 + 19v_2^2 - 18t(v_1 + v_2) - 9t^2}{36t} & \text{if } v_1 \leq t,\ v_2 \leq t \text{ and } v_1 + v_2 > t, \\ -\dfrac{5(v_1 - v_2)^2 - 9t^2}{18t} & \text{if } v_1 + v_2 \leq t, \end{cases}$$

$$\Delta \pi_1^S = \begin{cases} -\dfrac{(3t + v_1 - v_2)^2}{18t} + v_1 - \dfrac{t}{2} & \text{if } v_1 > t \text{ and } v_2 > t, \\ -\dfrac{(3t + v_1 - v_2)^2 - 9v_1^2}{18t} & \text{if } v_1 \leq t \text{ and } v_2 > t, \\ -\dfrac{v_1^2 - 2v_1v_2 - 8v_2^2 - 12t(v_1 - v_2) + 9t^2}{18t} & \text{if } v_1 > t \text{ and } v_2 \leq t, \\ \dfrac{4v_1^2 + v_1v_2 + 4v_2^2 - 3t(v_1 + 2v_2)}{9t} & \text{if } v_1 \leq t,\ v_2 \leq t \text{ and } v_1 + v_2 > t, \\ \dfrac{7(v_1 - v_2)^2 + 6t(v_1 - v_2) - 9t^2}{36t} & \text{if } v_1 + v_2 \leq t, \end{cases}$$

$$\Delta \pi_2^S = \begin{cases} -\dfrac{(3t + v_2 - v_1)^2}{18t} + v_2 - \dfrac{t}{2} & \text{if } v_1 > t \text{ and } v_2 > t, \\ -\dfrac{v_2^2 - 2v_1v_2 - 8v_1^2 - 12t(v_2 - v_1) + 9t^2}{18t} & \text{if } v_1 \leq t \text{ and } v_2 > t, \\ -\dfrac{(3t + v_2 - v_1)^2 - 9v_2^2}{18t} & \text{if } v_1 > t \text{ and } v_2 \leq t, \\ \dfrac{4v_2^2 + v_1v_2 + 4v_1^2 - 3t(v_2 + 2v_1)}{9t} & \text{if } v_1 \leq t,\ v_2 \leq t \text{ and } v_1 + v_2 > t, \\ \dfrac{7(v_2 - v_1)^2 + 6t(v_2 - v_1) - 9t^2}{36t} & \text{if } v_1 + v_2 \leq t. \end{cases}$$

Note that when at least some consumers purchase from both firms under uniform pricing, v_1 and v_2 cannot be less than t at the same time. Differences between the values under personalized pricing and those under uniform pricing are:

$$\Delta TS^M = \begin{cases} 0 & \text{if } v_1 \geq 2t \text{ and } v_2 \geq 2t, \\ v_2 - \dfrac{3v_2^2}{8t} - \dfrac{t}{2} & \text{if } v_1 \geq 2t \text{ and } t < v_2 < 2t, \\ \dfrac{v_2^2}{8t} & \text{if } v_1 \geq 2t \text{ and } v_2 \leq t, \\ v_1 - \dfrac{3v_1^2}{8t} - \dfrac{t}{2} & \text{if } t < v_1 < 2t \text{ and } v_2 \geq 2t, \\ v_1 + v_2 - \dfrac{3(v_1^2 + v_2^2)}{8t} - t & \text{if } t < v_1 < 2t \text{ and } t < v_2 < 2t, \\ \dfrac{v_2^2 - 3v_1^2 + 8tv_1 - 4t^2}{8t} & \text{if } t < v_1 < 2t \text{ and } v_2 \leq t, \\ \dfrac{v_1^2}{8t} & \text{if } v_1 \leq t \text{ and } v_2 \geq 2t, \\ \dfrac{v_1^2 - 3v_2^2 + 8tv_2 - 4t^2}{8t} & \text{if } v_1 \leq t \text{ and } t < v_2 < 2t, \end{cases}$$

$$\Delta CS^M = \begin{cases} -t & \text{if } v_1 \geq 2t \text{ and } v_2 \geq 2t, \\ -\dfrac{v_2^2 + 4t^2}{8t} & \text{if } v_1 \geq 2t \text{ and } t < v_2 < 2t, \\ v_2 - \dfrac{5v_2^2}{8t} - t & \text{if } v_1 \geq 2t \text{ and } v_2 \leq t, \\ -\dfrac{v_1^2 + 4t^2}{8t} & \text{if } t < v_1 < 2t \text{ and } v_2 \geq 2t, \\ -\dfrac{v_1^2 + v_2^2}{8t} & \text{if } t < v_1 < 2t \text{ and } t < v_2 < 2t, \\ -\dfrac{v_1^2 + 5v_2^2 - 8tv_2 + 4t^2}{8t} & \text{if } t < v_1 < 2t \text{ and } v_2 \leq t, \\ v_1 - \dfrac{5v_1^2}{8t} - t & \text{if } v_1 \leq t \text{ and } v_2 \geq 2t, \\ -\dfrac{v_2^2 + 5v_1^2 - 8tv_1 + 4t^2}{8t} & \text{if } v_1 \leq t \text{ and } t < v_2 < 2t, \end{cases}$$

$$\Delta\pi_1^M = \begin{cases} \frac{t}{2} & \text{if } v_1 \geq 2t \text{ and } v_2 > t, \\ -v_2 + \frac{v_2^2}{2t} + t & \text{if } v_1 \geq 2t \text{ and } v_2 \leq t, \\ v_1 - \frac{v_1^2}{4t} - \frac{t}{2} & \text{if } t < v_1 < 2t \text{ and } v_2 > t, \\ -\frac{v_1^2 - 2v_2^2 - 4t(v_1 - v_2)}{4t} & \text{if } t < v_1 < 2t \text{ and } v_2 \leq t, \\ \frac{v_1^2}{4t} & \text{if } v_1 \leq t \text{ and } v_2 > t, \end{cases}$$

$$\Delta\pi_2^M = \begin{cases} \frac{t}{2} & \text{if } v_1 > t \text{ and } v_2 \geq 2t, \\ v_2 - \frac{v_2^2}{4t} - \frac{t}{2} & \text{if } v_1 > t \text{ and } t < v_2 < 2t, \\ \frac{v_2^2}{4t} & \text{if } v_1 > t \text{ and } v_2 \leq t, \\ -v_1 + \frac{v_1^2}{2t} + t & \text{if } v_1 \leq t \text{ and } v_2 \geq 2t, \\ -\frac{v_2^2 - 2v_1^2 - 4t(v_2 - v_1)}{4t} & \text{if } v_1 \leq t \text{ and } t < v_2 < 2t. \end{cases}$$

By comparing social welfare, we obtain the condition that all players in this market are improved by personalized pricing. Suppose that all consumers purchase from only one of the firms under uniform pricing, then the profit of each firm improves if and only if, for each $i = 1, 2$,

$$\begin{cases} 7(v_i - v_j) > (6\sqrt{2} - 3)t & \text{if } v_j \leq \frac{5 - 3\sqrt{2}}{7}t, \\ 8v_i + v_j - 3\sqrt{t^2 + 10tv_j - 7v_j^2} > 3t & \text{if } \frac{5 - 3\sqrt{2}}{7}t < v_j \leq \frac{1}{4}t, \\ v_i - v_j + 3\sqrt{3t^2 + v_j^2} > 6t & \text{if } \frac{1}{4}t < v_j \leq t, \\ 2v_i + v_j > 3t & \text{if } v_j > t. \end{cases}$$

Consumer surplus improves if and only if

$$\begin{cases} v_i < v_j + 3\sqrt{2}\sqrt{6t^2 - v_j^2} - 9t & \text{if } v_j \leq t, \\ v_i < v_j + 3\sqrt{2}\sqrt{t(7t - 2v_j)} - 9t & \text{if } v_j > t. \end{cases}$$

Total surplus improves except when $v_1 = v_2 \leq \frac{t}{2}$. Those conditions are in Proposition 5.

In particular, both firm profits and consumer surplus improve if and only if for each $i = 1, 2$,

$$2v_i + v_j > 3t \quad and \quad \begin{cases} v_i < v_j + 3\sqrt{2}\sqrt{6t^2 - v_j^2} - 9t & \text{if } v_j \leq t, \\ v_i < v_j + 3\sqrt{2}\sqrt{t(7t - 2v_j)} - 9t & \text{if } v_j > t. \end{cases}$$

The condition is in Proposition 6.