The Role of Entry in a Mixed Oligopoly with Licensing

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Abstract

We consider a mixed quantity-setting oligopoly model in which two private and partially public incumbent firms face further competition from an entrant. The incumbents may acquire the quality-improving license from the innovator. The innovator, an outsider to the market, may decide whether to make the quality-improving license exclusive to one of the incumbents or non-exclusive and decide whether to make such a transaction via an upfront fee or per unit royalty fee. In this environment, we first show that even without quality-improving licensing, a semi-public firm always benefits from the entry, even if the entrant's quality is higher than its own. Although an entrant makes the competition fiercer and reduces the incumbents' private profits, a partially public firm can internalize this externality through increasing social welfare. When firms are innovative and constrained to exclusive contracts, it is optimal for the innovator to sell the license exclusively to the semi-public firm. However, with a new entrant in the production market, in contrast to the existing literature, royalty licensing outperforms fixed-fee licensing regardless of the innovation size and the entrant's quality.

PRELIMINARY DRAFT

1 Introduction

A mixed economy, in which a public or semipublic (partially state-owned) firm competes against one or more private profit-maximizing firms, is very common in many crucial sectors around the world, such as the energy, steel, banking, telecommunications, broadcasting, education, airline and postal services (see e.g.:Matsumura, 1998; Ishida and Matsushima, 2009). For instance, in the US railway industry, the leader of long-distance intercity passenger railroads is Amtrak (quasi-public corporation). At the same time, multiple private competitor rail systems include Genesee & Wyoming, Norfolk Southern, Canadian National Railway, Kansas City Southern, BNSF Railway, and Arriva. Conventional wisdom assumes that less efficient public firms compete with private firms in the same market. The empirical evidence partially supports this rendition, indicating the greater efficiency of private firms relative to comparable public firms. However, there is also empirical evidence supporting that differences in efficiency can go either way (Martin and Parker, 1997; Willner, 2001). On the theoretical side, there is extensive literature on mixed oligopolies, but more attention needs to be paid to the determinants of production efficiency driven by innovations and technological activity. In this respect, on the one hand, the expenditures on R&D and the research activity performed intra-muros, and on the other hand, the transfer of a superior technology by signing licensing contracts with outside innovators are crucial instruments to foster efficiency.

To fill this gap, we investigate the role of licensing strategy on private and public firms' performance and social welfare using a mixed oligopoly model. In particular, we investigate the licensing behavior between the firms with one welfare-maximizing public firm and a profit-maximizing private firm. We consider an outside innovator licensing a quality-enhancing innovation (product innovation), while the two firms compete in a vertical differentiation setting. Furthermore, we attempt to provide a better understanding of technology licensing with the consideration of entry. The entry of potential competitors and their relative technological efficiency crucially affects the intensity of licensing, transaction characteristics, private revenues, and social surplus. We will answer the following relevant questions: will the entry of a high quality competitor affect the incentive to acquire a quality-improving technology by the private and/or the public firm? From a policy maker's perspective, is the newcomer good or bad news for the public firm, the private one, the licensing transfer, and the whole sector? Shall we go for an exclusive license or a transfer to both the public and the private firms?

To anticipate our results, we show that entry and the technological quality of the entrant deeply affect the type of licensing deal and the equilibrium fees. In this respect, we show that the entry of a high quality competitor is good news for the outside innovator and the whole sector. Furthermore, we show that with exclusive licensing under fixed-fee and per unit royalty regimes, the benefit for the outside innovator changes according to the quality level of the entrant. In both fixed fee and per unit royalty deals, the innovator prefers to license to a private firm. However, the outside innovator maximizes the extracted rent with a per unit royalty contract.

The structure of the paper is as follows: after discussing the literature in the following subsection, we present the setup of the model in section 2. Section 3 introduces the equilibrium analysis for the benchmark case without licensing. In section 4 we discuss both the case of fixed fee licensing and per unit royalty. In section 5 we derive the equilibrium with non-exclusive licensing. Section 6 concludes the paper.

1.1 Literature

There is a vast literature on patent licensing, discussing the nature of the contract (fixed fee vs royalties and exclusive vs non-exclusive transfer) that should take place between the patentee and the licensee(s): e.g. Kamien and Tauman (1986); Katz and Shapiro (1986); Wang (1998), Denicolò (2000), (2002); Kamien and Tauman (2002); Wang (2002); Forfuri and Roca (2004) and Poddar and Sinha (2010). However, the above quoted contributions examine the licensing behavior among firms in an oligopoly market structure, but not in a mixed oligopoly market. Analogously, in the literature on mixed oligopoly, many issues have been explored, such as privatization, efficiency, quality, product differentiation, market integration, R&D, and welfare: see e.g. De Fraia and Delbono (1990); Cremer et al. (1991); Delbono et al. (1996); Matsushima and Matsumura (2004); Nishimori and Ogawa (2004), (2005). Nevertheless, the technology licensing strategy in the mixed market has seldom been analyzed; a few exceptions are Chen et al. (2014); Mukherjee and Sinha (2014); Yang and Huang (2023). Chen et al. developed a mixed oligopoly model to explore the licensing strategy by the innovative private firm. They analyze different types of licensing contracts (royalty, fixed fee, and two-part tariff) and show that, if the public firm accepts the licensing, all of the three different types of licensing contracts can be optimal. Differently from our model, they provide a framework of quantity competition, a là Cournot, without any kind of products in which the technology licensed is a process innovation. In a similar framework, Mukherjee and Sinha (2014) focus on the relation between technology transfer and privatization. Different from the conventional opinion that privatization helps to reduce inefficiency, they show that technology licensing might help to reduce production inefficiency due to cost asymmetry between public (less efficient) and private (more efficient) firms. If the profit maximising private firm is technologically superior to that of the welfare maximising public firm, the society and the private firm benefit from technology licensing; therefore cost asymmetry between the public and the private firms alone may not justify privatization. Again, different from our framework, the considered innovation is a process one and the market competition is over quantity with homogeneous products. Very recently Yang and Huang (2023) proposed a mixed Cournot duopoly to study an outside innovator's optimal licensing strategy for a quality-improving innovation. They show that exclusive contracts with fixed-fee licensing to the public firm are always optimal. Then, with non-exclusive contracts, fixed-fee licensing outperforms royalty licensing when the innovation size and the private share of the public firm are sufficiently large. We use the same framework as Yang and Huang, but we add the potential competition, considering the entry of the newcomer. This ingredient shakes the results in terms of the optimal licensing strategy according to the type of innovation and the private share of public firm. Also, Wang and Zeng (2019) analyze a mixed oligopoly model with licensing and entry. However, they examine how technology licensing, of a process innovation, by a private innovator affects privatization with exante cost asymmetry. They focus on the incentives for privatization, comparing domestic or foreign entry of a private firm.

2 The model

We extend the Yang and Huang (2023) model by allowing the presence of an entrant firm. We consider a mixed oligopoly model with one partially public firm, say firm 1, one private firm, say firm 2, and an entrant E; they compete in the product market by choosing quantities. Firm 1 is a jointly owned company both by the private sector by a share $\lambda \in (0,1)$, and by the state with the complementary share. The private firms are only concerned with their profits, while the public firm seeks to maximize a weighted average of its profit and social welfare, with the weight being measured by the share of private ownership. On the demand side of the market, there is a mass of consumers and each patronizes, at most, one unit of the quality-differentiated product at price p_i, with $i = 1, 2, E$. Consumers utility is then

$$\mathcal{U} \triangleq \begin{cases} \theta s_i - p_i & \text{if a consumer buys a good with quality } s_i, \\ 0 & \text{if a consumer does not buy the good at all,} \end{cases}$$

where θ is a (random) taste variable and uniformly distributed on $[0,1]$, with density $f(\theta) = 1$, and s_i and p_i denote the firm i's quality and price, respectively. In the absence of a license, the incumbent firms 1 and 2 produce the same quality $s_1 = s_2 = s \in (0,1)$. The new entrant, instead, enters into the underlying market with a higher quality vis-à-vis the incumbent players and is equal to $\sigma \in (0,1)$, with $\sigma > s$. There are no entry costs. The cost of production is given by $C_i(s_i) = \frac{1}{2} s_i q_i$, for the incumbent firms $i = 1, 2$ and is equal to $C_E(\sigma) = \frac{1}{2}\sigma q_E$, where q_i

with $i = 1, 2, E$ denotes quantity.

The timing is the following three-stage process. In stage 1, the outside innovator decides the licensing contract and makes the offers accordingly. In stage 2, the

firms decide whether to accept the offer, by comparing the profits with and without the license. Finally, in stage 3, the two firms compete in quantities. We solve this process through backward induction starting from a no-licensing benchmark.

3 Entry under no licensing

Without licensing the incumbent firms, semi-private firm 1 and private firm 2 produce at the same quality s at a cost $c_i = \frac{1}{2} s q_i$, $i \in \{1, 2\}$. The two incumbents face competition from an entrant with the quality equal to $\sigma \in (s, 1]$. Hence, in our setting entrant's quality upon entry is higher than than those of the incumbents already operating in the market. Notice, however, that an entrant with a higher initial quality implies a higher cost of production for the entrant as $\frac{\partial c_E(\sigma)}{\partial \sigma} > 0$.

The marginal consumer who is indifferent between buying the variety s from the incumbents and nothing is given by $\underline{\theta} = \frac{P}{s}$. Instead, the marginal consumer who is indifferent between purchasing quality s from the incumbent firms and quality σ from E has the taste parameter $\overline{\theta}$ such that

$$\overline{\theta} = \frac{p_E - P}{\sigma - s}.$$

Therefore, the demands for the two varieties, respectively, are

$$q_1 + q_2 = \int_{\underline{\theta}}^{\overline{\theta}} d\theta = \frac{p_E - P}{\sigma - s} - \frac{P}{s}, \quad \text{and} \quad q_E = \int_{\overline{\theta}}^{1} d\theta = 1 - \frac{p_E - P}{\sigma - s}.$$

Inverting, we can derive the inverse demand functions as follows:

$$P = s\left(1 - q_1 - q_2 - q_E\right),$$

and

$$p_E = \sigma\left(1 - q_E\right) - s\left(q_1 + q_2\right).$$

Using the superscript N to denote the no-licensing regime, the private profits of the incumbent firms are given by

$$\Pi_i^N = s\left(1 - q_1 - q_2 - q_E\right) q_i - \frac{1}{2} s q_i, \qquad i = 1, 2,$$

and the profit of E is

$$\Pi_E^N = C = \left(\sigma\left(1 - q_E\right) - s\left(q_1 + q_2\right)\right) q_E - \frac{1}{2} \sigma q_E.$$

Consequently, the corresponding consumer surplus is given by

$$CS^N = \int_{\underline{\theta}}^{\overline{\theta}} (\theta s - P)\, d\theta + \int_{\overline{\theta}}^{1} (\theta\sigma - p_E)\, d\theta,$$

and the social welfare is

$$\mathcal{W}^N = \sum_{i=1}^{2} \Pi_i^N + \Pi_E^N + CS^N.$$

Notice that the public firm maximizes the mixed profits

$$\mathbb{V}_1^N = \lambda \Pi_1^N + (1-\lambda)\, \mathcal{W}^N.$$

The following lemma characterizes the firms' equilibrium outputs, social welfare, and firms' profit when a new entrant enters the market with a higher initial quality vis-à-vis incumbents and incumbents who do not hold a quality improving license. A detailed equilibrium analysis can be found in the Appendix.

Lemma 1 *In the case of no licensing,*

- *The private incumbent produces the lowest quantity* $q_2^N < \min\left\{q_1^N, q_E^N\right\}$. *Moreover, the public firm produces more than the entrant only if* $\lambda \leqslant \frac{s}{2\sigma - s}$.
- *Social welfare is decreasing in* λ *only if* $\sigma \leqslant 2s$ *and* $\lambda > \frac{\sigma - s}{s}$. *Otherwise, social welfare is always increasing in* λ.
- *Let* $\mathbb{V}_1^N$ *be the public firm's mixed profit and* Π_2^N *and* Π_E^N *denote the private firm's and entrant's profit under no licensing. Then, we have* $\mathbb{V}_1^N > \Pi_E^N > \Pi_2^N$ *for all* $\lambda \in [0,1]$.

First, this lemma allows us to see if E enters the market with a higher initial quality than those of the incumbents, the private firm produces less than both public firm with an initial quality s and that of the incumbent with an initial quality $\sigma > s$:

$$q_2^N < \min\left\{q_1^N, q_E^N\right\}.$$

However, even if the public firm has an initial quality level lower than that of E, still it may produce more than the entrant under no-licensing. The reason is that, different than E, in equilibrium the public firm 1 produces by taking into account the social welfare with a share of $1 - \lambda$. That is, while having a new entrant who is more efficient in terms of initial quality decreases its private profits, at the same time, such an entrant is increasing the public firm's profits since entry increases social welfare. Consequently, equilibrium production translates these two contrasting effects. Specifically, if firm 1 is more public oriented (λ sufficiently small) so that social welfare counts more on its mixed profits then even he produces with a quality level s can produce more than the entrant who enters into the mixed market with a higher quality $\sigma > s$. That is

$$q_1^N - q_E^N = \frac{s - (2\sigma - s)\lambda}{2(4\sigma - s)\lambda + 2(2\sigma - s)} > 0 \Longleftrightarrow \lambda < \frac{s}{2\sigma - s}.$$

Second, the entrant's initial quality vis-à-vis incumbents may have some ambiguous effect on social welfare. Interestingly, for $\sigma \leqslant 2s$, the social welfare is actually decreasing in λ only if $\lambda > \frac{\sigma - s}{s}$, is increasing in λ otherwise. Hence, the effect of a new entry on social welfare depends on how large is the entrant's quality relative to the incumbents. If E enters the market with a moderate initial quality $\sigma \leqslant 2s$, then social welfare is affected by such entry if firm 1 is a more private oriented firm (that is λ is sufficiently large). In other words, if firm 1 is more private oriented and when the firms are alike in terms of their initial quality, this makes product market competition fiercer and hence, social welfare is affected negatively by such entry.

Finally, Lemma 1 shows us firm 1's profit is always higher than that of the entrant even if it enters with a higher initial quality and this is regardless of whether firm 1 is more public or private oriented. This result tells us even if the social welfare is decreasing, the negative externality E creates in the production phase, is always internalized by firm 1 through its mixed structure.

4 Exclusive Licensing

4.1 Fixed Fee Licensing

We first consider a fixed fee strategy where one of the incumbent firms becomes a licensee by paying a fixed fee F tto the outsider innovator. In what follows, we analyze where the outside innovator sells the license exclusively to either public firm 1 or private firm 2 both with an initial quality s and they face competition by an high-quality entrant whose quality is $\sigma > s$. Unlike the case of no-licensing, the firm that signs exclusively the fixed-fee contract increases its product quality from s to 1.

When the quality-improving license is exclusive to the incumbent for $i \in \{1, 2\}$, then the consumer who is indifferent between buying from the incumbent $j \neq i$ selling the lowest quality s in the product market and not buying is identified by $\underline{\theta} = \frac{p_{-i}}{s}$. The consumer who is indifferent between s from firm j and or quality σ from E is identified by

$$\widehat{\theta} = \frac{p_E - p_{-i}}{\sigma - s},$$

and the consumer who is indifferent between getting the product with quality σ from E and or with quality 1 from from the licensed firm i is given by[1]

$$\overline{\theta} = \frac{p_i - p_E}{1 - \sigma}.$$

By the same calculation we used in the previous subsection 3, we calculate the inverse demand functions, the profits, the consumer welfare and the equilibrium quantities. We distinguish two cases in the first the license goes exclusively to the public firm, while in the second the license goes to the private firm 2. Therefore, in the former case, we calculate the mixed profit of the public firm,in which the superscript EF_i denotes the license is exclusive to firm i and the contract is a fixed fee one:

$$\mathbb{V}_1^{EF_2} = \lambda \Pi_1^{EF_2} + (1 - \lambda) W^{EF_2}$$

Given this, the outside innovator designs the exclusive fixed fee contract with the public firm 1 by extracting its profit relative to no-licensing case, that is

$$\mathbb{V}_1^{EF_1} - \mathbb{V}_1^N = 0,$$

which yields,

$$F_1^{EF_1}(s, \sigma, \lambda) = \frac{4\left(1-\lambda+2\lambda^2\right)\sigma^4 - 4\left(4+\lambda^2+3\lambda^3\right)\sigma^3 + \left(5s\lambda^3 - s\lambda^2 + 16\lambda + (16+4s)\right)\sigma^2}{8\lambda((4\sigma-s)(1+\lambda)-2\sigma^2)^2} - \frac{s\left(8(1+\lambda) - s\lambda^2(1-\lambda)\right)\sigma + s^2(1+\lambda)}{8\lambda((4\sigma-s)(1+\lambda)-2\sigma^2)^2} + \\ + \frac{\left(12\sigma^3 - 5s\sigma^2 + s^2\sigma\right)\lambda^3 - s\left(5\sigma^2 - s\sigma\right)\lambda^2 - \left(9\sigma^3 - 5s\sigma^2 + s^2\sigma\right)\lambda - \left(3\sigma^3 - 3s\sigma^2 + s^2\sigma\right)}{8\lambda(2\sigma - s + (4\sigma - s)\lambda)^2}.$$

In the latter case, the outside innovator designs the fixed fee contract by charging firm 2 its maximum willingness to pay for the license by setting

$$\Pi_2^{EF_2} - \Pi_2^N = 0.$$

Solving this expression for F_2, we find the optimal fixed fee to be paid by firm 2 to outside innovator as

[1] Notice that this case becomes negligible when we consider E with an initial quality $\sigma = 1$.

$$F_2^{EF_2}(s,\sigma,\lambda) = \frac{\left(s-2\sigma+\sigma^2-2\sigma\lambda+\sigma^2\lambda\right)^2}{4(\sigma(4-\sigma)\lambda+(\sigma(4-\sigma)-2s))^2} - \frac{s\sigma^2\lambda^2}{4(2\sigma-s+\lambda(4\sigma-s))^2}.$$

We can replicate the above analysis, considering that before the exclusive licensing deal is set, the entrant firm has the same quality as firm i (with $i = 1, 2$).

Finally, we investigate the behavior of the outside innovator, comparing the exclusive fees to determine whether it is more convenient to sell the license exclusively to public firm 1 or private firm 2.

Lemma 2 *When the fixed license is exclusive, regardless of the entrant's product quality it is optimal for the outside innovator to sell the exclusive license to the public firm* 1*.*

4.2 Per Unit Royalty

We consider the case in which the license on the quality improving innovation is still exclusive, but now the outside innovator charges the incumbent firm, public firm 1,or private firm 2, a royalty fee per unit of the good produced, $r_1 \geq 0$. Let us start with exclusive licensing to public firm 1 with per unit royalty. We can write down the expressions for the firms' private profits, the consumer surplus and the social welfare. Finally we obtain the public firm 1's mixed profit, where the superscript ER_1 denotes the license is exclusive to public firm 1 :

$$\mathbb{V}_1^{ER_1} = \lambda \Pi_1^{ER_1} + (1-\lambda) W^{ER_1}.$$

Given this, the outside innovator designs the royalty contract with the public firm 1 by extracting its profit relative to no-licensing case, that is

$$\mathbb{V}_1^{ER_1} - \mathbb{V}_1^{N} = 0,$$

which yields to the optimal $r_1(\sigma,\lambda)$. As before, we divide the analysis into two parts regarding the entrant's quality. First, suppose E enters the market with the same quality as the incumbent players — i.e., such that $\sigma = s$. In this case, denoting r_i^I and r_i^C, with $i = 1, 2$, the interior equilibrium value of the royalty and the value, respectively, we show the following results,

Proposition 3 *Suppose the contract is exclusive to the public firm with a royalty fee. In that case, if the entrant enters with $s_E = s$, when the size of the quality improving innovation:*

- *is small, the solution is interior and is equal to $r_1^I = \frac{3-2s}{12\lambda}$;*

- *is large, the solution is corner and is equal to* r_1^C*;*
- *is intermediate, the solution is corner (if* $\lambda < \lambda^C(s)$*) or interior (if* $\lambda \geqslant \lambda^C(s)$*) depending on the public share of the mixed firm.*

Conversely, if E enters the market with the higher quality - i.e., such that $\sigma = 1$, then the following result holds,

Proposition 4 *Suppose the contract is exclusive to the public firm with a royalty fee. In that case, if the entrant enters with* $\sigma = 1$*, regardless of the size of the innovation, a corner solution is preferred.*

Furthermore, when the solution is interior, the outside innovator extracts a higher fee when the market competition is tougher, given the high quality of the entrant.

Corollary 5 *Suppose the contract is exclusive to the public firm with a royalty fee and the interior solution, the outside innovator charges the public firm* 1*:* $r_1^I(s_E = \sigma) > r_1^I(s_E = s)$*.*

The entry of a high-quality competitor increases the value of the quality-enhancing innovation for the incumbent firms, and it benefits the outside innovator who can extract a higher fee.

Finally, denoting

$$\pi_0^{ER_i}(s, \lambda) = rq_i^{ER_i},$$

the innovator's profit, we can show that the outside innovator is better off charging the public firm 1 a per-unit royalty fee instead of a fixed fee.

Proposition 6 *When the contract is exclusive to the public firm* 1 *the outside innovator prefers a royalty fee for any quality level of the entrant:* $\pi_0^{ER_1}(s, \lambda) > \pi_0^{EF_1}(s, \lambda)$*.*

Let us turn to the licensing with per unit royalty to the private firm 2. Analogous to the previous analysis with the public firm, we calculate the private firm profit when it obtains the exclusive license, ER_2, and the optimal royalty fee considering the outside innovator designs the royalty contract by extracting its profit relative to no-licensing case:

$$\Pi_2^{ER_2} - \Pi_2^N = 0.$$

which yields to the optimal $r_2(\sigma, \lambda)$. The following Proposition illustrates the optimal values, interior and corner, of the royalty.

Proposition 7 *Suppose the contract is exclusive to the private firm with a royalty fee. In that case, if the entrant enters with $s_E = \sigma = s$, when the size of the quality improving innovation:*

- *is large, then the solution is interior;*
- *is small, then there exists λ^{Crit} such that for $\lambda < \lambda^{Crit}$ the solution is interior.*

Given the previous Proposition 7, we can compare the royalties paid by the private and the public firms when the solution is the interior one.

Proposition 8 *Suppose the contract is exclusive. The innovator prefers a royalty fee with the public firm, regardless of the entrant's initial quality: $r_1 > r_2$.*

5 Non-exclusive Licensing

In the non-exclusive case, the innovator licenses the quality-enhancing innovation to the incumbent firms — both public firm 1 and private firm 2 — with a lump-sum fee F_i or with royalty r_i.

To be completed.

6 Conclusions

In a mixed oligopoly framework with a quality-enhancing innovation, we illustrate the optimal licensing contracts according to the entry of a new competitor. We show that entry and the technological quality of the entrant deeply shake the equilibrium results. In this respect, the standard result of the literature that fixed fees are always optimal is not robust according to a change in the market conditions due to a newcomer's entry. Conversely, we show that with exclusive licensing under fixed-fee and per unit royalty regimes, the benefit for the outside innovator changes according to the quality level of the entrant. In both fixed fee and per unit royalty deals, the innovator prefers to license to a private firm. However, the outside innovator maximizes the extracted rent with a per unit royalty contract.

To be completed.

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8 Appendix.

Proof of Lemma 1. The indifferent consumer between buying quality s from the incumbents and not buying is,

$$\underline{\theta} s - P = 0 \Leftrightarrow \underline{\theta} = \frac{P}{s}.$$

By the same token, the indifferent consumer between buying the good with quality s from the incumbents or buying the good from the entrant with quality σ with $\sigma > s$ is,

$$\overline{\theta} s - P = \overline{\theta} \sigma - p_E \Leftrightarrow \overline{\theta} = \frac{p_E - P}{\sigma - s}.$$

Hence, the demand for E's product is equal to

$$q_E = \int_{\overline{\theta}}^{1} d\theta = 1 - \frac{p_E - P}{\sigma - s},$$

while the total demand for the incumbents' product is given by

$$q_1 + q_2 = \int_{\underline{\theta}}^{\overline{\theta}} d\theta = \int_{\frac{P}{s}}^{\frac{T-P}{\sigma - s}} d\theta = \frac{p - P}{\sigma - s} - \frac{P}{s}.$$

Converting the demand function, it is easy to find the inverse demand functions as follows

$$P = s\left(1 - q_1 - q_2 - q_E\right),$$
$$p_E = \sigma\left(1 - q_E\right) - s\left(q_1 + q_2\right).$$

The corresponding consumer surplus is

$$\rightarrow CS^N = \int_{\underline{\theta}}^{\overline{\theta}} \left(\theta s - P\right) d\theta + \int_{\overline{\theta}}^{1} \left(\theta \sigma - p_E\right) d\theta = \frac{s\left(q_1 + q_2\right)^2 + q_E\left(q_E \sigma + 2q_1 s + 2q_2 s\right)}{2}.$$

Moreover, firms' private profits are then given by

$$\Pi_1^N = s\left(1 - q_1 - q_2 - q_E\right) q_1 - \frac{1}{2} s q_1, \tag{1}$$
$$\Pi_2^N = s\left(1 - q_1 - q_2 - q_E\right) q_2 - \frac{1}{2} s q_2, \tag{2}$$
$$\Pi_E^N = \left(\sigma\left(1 - q_E\right) - s\left(q_1 + q_2\right)\right) q_E - \frac{1}{2} \sigma q_E. \tag{3}$$

While purely private firm 2 and the entrant firm E maximizes their private profits Π_2 and Π_E, respectively, the public firm 1 maximizes the mixed profits which takes into account the private profits with a share of λ, as well as the the social welfare with a complementary share. The social welfare is given by the sum of all operating firms' private profits and the consumer surplus. That is

$$W^N = \Pi_1^N + \Pi_2^N + \Pi_3^N + CS^N = \frac{1}{2}\left(q_1 + q_2\right)\left(1 - q_1 - q_2 - 2q_E\right)s + q_E + \sigma\left(1 - q_E\right).$$

Public firm's optimization program than takes into account its mixed profits

$$\mathbb{V}_1^N = \lambda \Pi_1^N + (1-\lambda) W^N. \tag{4}$$

Maximizing 4), (2) and (3) with respect to q_1, q_2 and q_E respectively, yields the equilibrium production under no licensing as follows

$$q_1^N = \frac{\sigma}{2\left(4\sigma - s\right)\lambda + 2\left(2\sigma - s\right)},$$
$$q_2^N = \frac{\sigma\lambda}{2\left(4\sigma - s\right)\lambda + 2\left(2\sigma - s\right)},$$
$$q_E^N = \frac{\left(\sigma - s\right) + \left(2\sigma - s\right)\lambda}{2\left(4\sigma - s\right)\lambda + 2\left(2\sigma - s\right)}.$$

Direct comparison of these quantities yields

$$q_1^N = \frac{\sigma}{2\left(4\sigma - s\right)\lambda + 2\left(2\sigma - s\right)} > q_2^N = \frac{\sigma\lambda}{2\left(4\sigma - s\right)\lambda + 2\left(2\sigma - s\right)},$$

as expected. The entrant produces

$$q_E^N = \frac{\left(\sigma - s\right) + \left(2\sigma - s\right)\lambda}{2\left(4\sigma - s\right)\lambda + 2\left(2\sigma - s\right)},$$

where

$$\min\left\{q_1^N, q_E^N\right\} > q_2^N.$$

Notice that

$$q_1^N - q_E^N = \frac{s - \left(2\sigma - s\right)\lambda}{2\left(4\sigma - s\right)\lambda + 2\left(2\sigma - s\right)} > 0 \iff \lambda < \frac{s}{2\sigma - s}.$$

Hence, without licensing public company can produce more than the entrant even it enters the market a higher quality. Of course this depends on the weight of the social welfare function on the public firm's mixed profits. In other words, the public firm with a quality level s produces more than the entrant with a quality $\sigma > s$ only if the private share of the public company is sufficiently small — i.e., such that $1 - \lambda$ is sufficiently large.

Now, given the equilibrium quantities, we can calculate the social welfare

$$W^N = \sigma \frac{\left(s^2 - 5s\sigma + 12\sigma^2\right)\lambda^2 + 2\left(6\sigma^2 - 4s\sigma + s^2\right)\lambda + \left(s^2 - 3s\sigma + 3\sigma^2\right)}{8\left(s - 2\sigma + s\lambda - 4\sigma\lambda\right)^2}$$

and

$$\frac{\partial W^N}{\partial \lambda} = \frac{s\sigma^2\left((\sigma - s) - s\lambda\right)}{4\left((4\sigma - s)\lambda + 2\sigma - s\right)^3},$$

which is positive only if

$$\lambda < \overline{\lambda}^N \triangleq \frac{\sigma - s}{s}.$$

Notice further that

$$1 - \overline{\lambda}^N = \frac{2s - \sigma}{s},$$

which is positive only if $\sigma < 2s$.

Interestingly, for $\sigma \leqslant 2s$, the social welfare is actually decreasing in λ only if $\lambda > \overline{\lambda}^N = \frac{\sigma - s}{s}$. If instead, $\sigma > 2s$ social welfare is always increasing in λ. Given this, we now have a look at the firms' profits. So entrant's initial quality vis-à-vis incumbents matters.

Now, let us calculate firms' profits. Public firm's mixed profits are

$$\mathbb{V}_1^N = \sigma \frac{\left(5s\sigma - s^2 - 12\sigma^2\right)\lambda^3 + \left(5s\sigma - s^2\right)\lambda^2 + \left(s^2 - 5s\sigma + 9\sigma^2\right)\lambda + \left(s^2 - 3s\sigma + 3\sigma^2\right)}{8\left((4\sigma - s)\lambda + (2\sigma - s)\right)^2}$$

private firm's profit is

$$\Pi_2^N = \frac{s\sigma^2\lambda^2}{4\left((4\sigma - s)\lambda + (2\sigma - s)\right)^2},$$

and entrant's profits are

$$\Pi_E^N = \sigma \frac{\left(\sigma - s + 2\sigma\lambda - s\lambda\right)^2}{4\left((4\sigma - s)\lambda + (2\sigma - s)\right)^2}.$$

where

$$\Pi_E^N - \Pi_2^N = \sigma(\sigma - s)\frac{\sigma - s + \lambda\left(4\sigma - 2s + \lambda(4\sigma - s)\right)}{4\left((4\sigma - s)\lambda + (2\sigma - s)\right)^2} > 0,$$

and

$$\Pi_1^N - \Pi_2^N = \frac{1}{8}\sigma(1-\lambda)\frac{(s^2-5s\sigma+12\sigma^2)\lambda^2+(2s^2-8s\sigma+12\sigma^2)\lambda+(s^2-3s\sigma+3\sigma^2)}{((4\sigma-s)\lambda+(2\sigma-s))^2} > 0,$$

Hence, $\Pi_2^N < \min\left\{\mathbb{V}_1^N, \Pi_E^N\right\}$ irrespective of the public firm's composition and the entrant's quality level upon entry provided that $\sigma > s$.

Now let us do the same by comparing the entrant's profit upon entry with the profit of the public firm. Direct comparison of these profits yields:

$$\Pi_E^N - \mathbb{V}_1^N = \sigma\frac{(s^2-5s\sigma+12\sigma^2)\lambda^3+(3s^2-13s\sigma+8\sigma^2)\lambda^2+(3s^2-7s\sigma-\sigma^2)\lambda+(s^2-s\sigma-\sigma^2)}{8(s-2\sigma+s\lambda-4\sigma\lambda)^2},$$

whose sign depends on the sign of the numerator

$$\varphi = \left(s^2-5s\sigma+12\sigma^2\right)\lambda^3+\left(3s^2-13s\sigma+8\sigma^2\right)\lambda^2+\left(3s^2-7s\sigma-\sigma^2\right)\lambda+\left(s^2-s\sigma-\sigma^2\right).$$

In the case,

$$\lim_{\lambda\to 0}\varphi = -s\sigma-\sigma^2+s^2<0,$$
$$\lim_{\lambda\to 1}\varphi = 2(4s-9\sigma)(s-\sigma)<0.$$

Moreover,

$$\frac{\partial\varphi}{\partial\lambda} = 3\left(s^2-5s\sigma+12\sigma^2\right)\lambda^2+2\left(3s^2-13s\sigma+8\sigma^2\right)\lambda+\left(3s^2-7s\sigma-\sigma^2\right).$$

Setting this equation equal to 0 and solving for λ yields the critical points

$$\lambda_1 = \frac{13s\sigma-8\sigma^2+\sqrt{7s^2\sigma^2+100\sigma^4+29s\sigma^3-12s^3\sigma}-3s^2}{3(-5s\sigma+12\sigma^2+s^2)} > 0,$$
$$\lambda_2 = \frac{13s\sigma-8\sigma^2-\sqrt{7s^2\sigma^2+100\sigma^4+29s\sigma^3-12s^3\sigma}-3s^2}{3(-5s\sigma+12\sigma^2+s^2)} < 0,$$

Since

$$\lim_{\lambda\to\lambda_1}\frac{\partial^2\varphi}{\partial\lambda^2} = \lim_{\lambda\to\left(\frac{13s\sigma-8\sigma^2+\sqrt{7s^2\sigma^2+100\sigma^4+29s\sigma^3-12s^3\sigma}-3s^2}{3\left(-5s\sigma+12\sigma^2+s^2\right)}\right)}\frac{\partial^2\varphi}{\partial\lambda^2} = 2\sqrt{7s^2\sigma^2+100\sigma^4+29s\sigma^3-12s^3\sigma} > 0,$$

φ has a relative minimum at λ_1. Finally, for $\lambda\in[0,1]$, the critical point lies inside the region of interest since

$$\lambda_1 > 0,\ \forall s\in(0,1), \sigma\in(s,\sigma),$$

and

$$\lambda_1 < 1 \text{ for any } \sigma \in [s, 1].$$

Hence, φ never crosses the x-axis and hence $\varphi < 0$ for all λ. Hence, public companies profit is always higher than that of the entrant even if it enters with a higher initial quality. This result is regardless of the degree of private shares λ and regardless of the entrant's level of innovation upon entry.

Proof of Lemma 2

When license is exclusive to the public firm 1, we have three firms with different qualities: firm 1 has quality 1, firm 2 has quality s, and E with quality $\sigma \geq s$. In that case, the consumer who is indifferent between purchasing quality s from firm 2 or nothing is identified by $\underline{\theta} = \frac{p_2}{s}$. The marginal consumer who is indifferent between purchasing quality s from firm 2 and quality $\sigma > s$ from E is identified by

$$\widehat{\theta} = \frac{p_E - p_2}{\sigma - s},$$

and the consumer who is indifferent between purchasing quality σ from E and or with quality 1 from the public firm 1 is given by

$$\overline{\theta} = \frac{p_1 - p_E}{1 - \sigma}.$$

Straightforward calculations yield the demand functions for the three firms as follows:

$$\begin{aligned}
q_1^{EF_1} &= \int_{\overline{\theta}}^{1} d\theta = 1 - \frac{p_1 - p_E}{1 - \sigma}, \\
q_2^{EF_1} &= \int_{\underline{\theta}}^{\widehat{\theta}} d\theta = \frac{p_E - p_2}{\sigma - s} - \frac{p_2}{s}, \\
q_E^{EF_1} &= \int_{\widehat{\theta}}^{\overline{\theta}} d\theta = \left(\frac{p_1 - p_E}{1 - \sigma}\right) - \left(\frac{p_E - p_2}{\sigma - s}\right).
\end{aligned}$$

where the superscript EF_1 denotes the license is exclusive to firm 1 and the contract is a fixed fee one.

Inversting the demand function, we derive the inverse demand functions as follows:

$$p_1^{EF_1} = 1 - \sigma q_E - s q_2 - q_1, \tag{5}$$
$$p_2^{EF_1} = s\left(1 - q_1 - q_2 - q_E\right), \tag{6}$$
$$p_E^{EF_1} = \sigma - \sigma q_1 - \sigma q_E - s q_2. \tag{7}$$

The consumer surplus is equal to

$$CS = \int_{\underline{\theta}}^{\widehat{\theta}} (\theta s - p_2)\, d\theta + \int_{\widehat{\theta}}^{\overline{\theta}} (\theta\sigma - p_E)\, d\theta + \int_{\overline{\theta}}^{1} (\theta - p_1)\, d\theta, \qquad (8)$$

The private profits of the public firm 1 who has the quality improving license can be written as

$$\Pi_1^{EF_1} = (1 - \sigma q_E - s q_2 - q_1)\, q_1 - \frac{1}{2} q_1 - F_1,$$

where we use the fact that the cost of producing a high quality good is more costly for the licencee firm $c_1(s_1 = 1) = 1$.

Instead, the private profits of the private firm 2 with quality s and E with quality $\sigma \geq s$, are respectively,

$$\Pi_2^{EF_1} = s\,(1 - q_1 - q_2 - q_E)\, q_2 - \frac{1}{2} s q_2, \qquad (9)$$

$$\Pi_E^{EF_1} = (\sigma - \sigma q_1 - \sigma q_E - s q_2)\, q_E - \frac{1}{2}\sigma q_E. \qquad (10)$$

As a result, the social welfare can be written as the sum of firms' private profits and the consumers surplus:

$$W^{EF_1} = \Pi_1^{EF_1} + \Pi_2^{EF_1} + \Pi_E^{EF_1} + CS.$$

Now, while firm 2 and and E maximize their private profits, (9) and (10), respectively, the licensee public firm 1 maximizes its mixed profits,

$$\mathbb{V}_1^{EF_1} = \lambda \Pi_1^{EF_1} + (1 - \lambda)\, W^{EF_1}. \qquad (11)$$

Maximizing (9), (10) and (11), with respect to q_1, q_2 and q_E yields the equilibrium quantities

$$q_1^{EF_1} = \frac{2\sigma(2 - \sigma) - s}{2\left((4\sigma - s)(1 + \lambda) - 2\sigma^2\right)}, \qquad (12)$$

$$q_2^{EF_1} = \frac{\lambda\sigma}{2\left((4\sigma - s)(1 + \lambda) - 2\sigma^2\right)}, \qquad (13)$$

$$q_E^{EF_1} = \frac{\lambda(2\sigma - s)}{2\left((4\sigma - s)(1 + \lambda) - 2\sigma^2\right)}. \qquad (14)$$

Notice that, for any $\sigma \in (s, 1)$, the equilibrium quantites are such that

$$q_1^{EF_1} - q_2^{EF_1} = \frac{\sigma - s + \sigma\left(3 - 2\sigma - \lambda\right)}{2\left(\left(4\sigma - s\right)\left(1+\lambda\right) - 2\sigma^2\right)} > 0,$$

$$q_1^{EF_1} - q_E^{EF_1} = \frac{2\sigma\left(2 - \sigma\right) - \lambda\left(2\sigma - s\right) - s}{2\left(\left(4\sigma - s\right)\left(1+\lambda\right) - 2\sigma^2\right)} > 0, \quad \forall \lambda \in (0,1).$$

$$q_E^{EF_1} - q_2^{EF_1} = \lambda\frac{\sigma - s}{2\left(\left(4\sigma - s\right)\left(1+\lambda\right) - 2\sigma^2\right)} > 0.$$

Using the equilibrium quantities from the expressions (12), (13) and (14), the firms' private profits in equilibrium are given by

$$\Pi_1^{EF_1} = \frac{\lambda\left(s - 4\sigma + 2\sigma^2\right)^2}{4\left(\left(4\sigma - s\right)\left(1+\lambda\right) - 2\sigma^2\right)^2} - F,$$

$$\Pi_2^{EF_1} = \frac{s\sigma^2\lambda^2}{4\left(\left(4\sigma - s\right)\left(1+\lambda\right) - 2\sigma^2\right)^2},$$

$$\Pi_E^{EF_1} = \frac{\sigma\lambda^2\left(s - 2\sigma\right)^2}{4\left(\left(4\sigma - s\right)\left(1+\lambda\right) - 2\sigma^2\right)^2}.$$

The consumer surplus and the social welfare are given by

$$CS^{FE_1} = \frac{\sigma\left(s\sigma + 4\sigma^2 - s^2\right)\lambda^2 + 4\sigma^2\left(4\sigma - 2\sigma^2 - s\right)\lambda + \left(4\sigma - 2\sigma^2 - s\right)^2}{8\left(\left(4\sigma - s\right)\left(1+\lambda\right) - 2\sigma^2\right)^2} > 0,$$

and

$$W^{EF_1} = \frac{\sigma\left(12\sigma^2 - 5s\sigma + s^2\right)\lambda^2 + 2\left(4\sigma - s\right)\left(4\sigma - 2\sigma^2 - s\right)\lambda + \left(4\sigma - 2\sigma^2 - s\right)^2}{8\left(\left(4\sigma - s\right)\left(1+\lambda\right) - 2\sigma^2\right)^2} > 0.$$

Given this, the outside innovator designs the exclusive fixed fee contract with the the public firm 1 by extracting its profit relative to no-licensing case, that is

$$\mathbb{V}_1^{EF_1} - \mathbb{V}_1^N = 0,$$

which yields,

$$F_1^{EF_1} = \frac{4\left(1 - \lambda + 2\lambda^2\right)\sigma^4 - 4\left(4 + \lambda^2 + 3\lambda^3\right)\sigma^3 + \left(5s\lambda^3 - s\lambda^2 + 16\lambda + (16 + 4s)\right)\sigma^2 - s\left(8\left(1+\lambda\right)\right.}{8\lambda\left(\left(4\sigma - s\right)\left(1+\lambda\right) - 2\sigma^2\right)^2}$$
$$+\sigma\frac{\left(12\sigma^2 - 5s\sigma + s^2\right)\lambda^3 - s\left(5\sigma - s\right)\lambda^2 - \left(9\sigma^2 - 5s\sigma + s^2\right)\lambda - \left(3\sigma^2 - 3s\sigma + s^2\right)}{8\lambda\left(2\sigma - s + \left(4\sigma - s\right)\lambda\right)^2}$$

with

$$\lim_{\sigma \to s} F_1^{EF_1} = (1 - s)\frac{24s\lambda^4 + \left(4s - 32s^2 + 81\right)\lambda^3 + \left(16s^2 - 112s + 135\right)\lambda^2 + \left(20s^2 - 72s + 63\right)\lambda + (}{8\lambda\left(1 + 3\lambda\right)^2\left(3\left(1+\lambda\right) - 2s\right)^2}$$

with

$$\frac{\partial F_1^{EF_1}(\sigma = s)}{\partial s} = \frac{\left(32\lambda^2 - 64\lambda^3 + 40\lambda + 8\right) s^3 + 36(\lambda + 1)\left(-5\lambda - 4\lambda^2 + 8\lambda^3 - 1\right) s^2}{8\lambda(3\lambda + 1)^2 (3 + 3\lambda - 2s)^3} +$$
$$+ \frac{-2\left(-189\lambda - 413\lambda^2 - 199\lambda^3 + 156\lambda^4 + 72\lambda^5 - 27\right) s + 3(\lambda + 1)\left(-63\lambda - 139\lambda^2 - 77\lambda^3 + 24\lambda^4 - \right.}{8\lambda(3\lambda + 1)^2 (3 + 3\lambda - 2s)^3}$$

and

$$\lim_{\sigma \to 1} F_1^{EF_1} = (1 - s) \frac{1 + 3\lambda + 2\lambda^2 (2 - s)}{8\lambda(2 - s + \lambda(4 - s))^2},$$

with

$$\frac{\partial F_1^{EF_1}(\sigma = 1)}{\partial s} = \frac{\left(10\lambda^3 - \lambda^2 - 4\lambda - 1\right) s - \left(16\lambda^3 + 10\lambda^2 + 2\lambda\right)}{8\lambda(2 - s + (4 - s)\lambda)^3} < 0,$$

within our region of interest.

Fixed Fee License to the Private Firm When license is exclusive to the private firm, the analysis is similar to the one derived above for the public firm. Still we have three firms with different qualities but now the public firm 1 has the lowest quality s in the product market. Therefore, a consumer who is indifferent between purchasing quality s from the public firm 1 or nothing is identified by $\underline{\theta} = \frac{p_1}{s}$, the marginal consumer who is indifferent between purchasing quality s from firm 1 and quality $\sigma > s$ from E is identified by

$$\widehat{\theta} = \frac{p_E - p_1}{\sigma - s},$$

and the consumer who is indifferent between purchasing quality σ from E and or with quality 1 from the incumbent firm 2 is given by

$$\overline{\theta} = \frac{p_2 - p_E}{1 - \sigma}.$$

Demand functions for the incumbents are then

$$q_1 = \int_{\overline{\theta}}^{1} d\theta = 1 - \frac{p_2 - p_E}{1 - \sigma}, \qquad q_2 = \int_{\underline{\theta}}^{\widehat{\theta}} d\theta = \frac{p_E - p_1}{\sigma - s} - \frac{p_1}{s},$$

and for the entrant is

$$q_E = \int_{\widehat{\theta}}^{\overline{\theta}} d\theta = \frac{p_E - p_1}{\sigma - s} - \frac{p_2 - p_E}{1 - \sigma}.$$

Adopted the same logic developed above, we immediately derive the inverse demand functions

$$\begin{aligned} p_1 &= s\left(1 - q_1 - q_2 - q_E\right), \\ p_2 &= 1 - sq_1 - \sigma q_E - q_2, \\ p_E &= \sigma - sq_1 - \sigma q_2 - \sigma q_E. \end{aligned}$$

The consumer surplus when the license is exclusive to the private incumbent is then equal to

$$CS^{EF_2} = \int_{\underline{\theta}}^{\widehat{\theta}} \left(\theta s - p_1\right) d\theta + \int_{\widehat{\theta}}^{\overline{\theta}} \left(\theta \sigma - p_E\right) d\theta + \int_{\overline{\theta}}^{1} \left(\theta - p_2\right) d\theta,$$

where we used superscript EF_2 to denote license contact is a fixed fee one and is exclusive for the firm 2. In that case, firm 2's private profits after paying the fixed fee F_2 and using the quality-improving license writes as

$$\Pi_2^{EF_2} = \left(1 - sq_1 - \sigma q_E - q_2\right) q_2 - \frac{1}{2} q_2 - F_2, \tag{15}$$

where producing a high quality good is more costly for the licencee firm $c_2\left(s_2 = 1\right) = 1$. Instead, the private profits of the mixed firm 1 with quality s and E with quality $\sigma \geq s$, are respectively,

$$\Pi_1^{EF_2} = s\left(1 - q_1 - q_2 - q_E\right) q_1 - \frac{1}{2} s q_1, \tag{16}$$

$$\Pi_E^{EF_2} = \left(\sigma - sq_1 - \sigma q_2 - \sigma q_E\right) q_E - \frac{1}{2} \sigma q_E. \tag{17}$$

Hence, the social welfare can be written as the sum of firms' private profits and the consumers surplus:

$$W^{EF_2} = \Pi_1^{EF_1} + \Pi_2^{EF_2} + \Pi_E^{EF_2} + CS^{EF_2}.$$

Moreover, firm 1's mixed profits are then

$$\mathbb{V}_1^{EF_2} = \lambda \Pi_1^{EF_2} + \left(1 - \lambda\right) W^{EF_2}. \tag{18}$$

Maximizing (18), (15) and (17), with respect to q_1, q_2 and q_E yields the equilibrium quantities as

$$q_1^{EF_2} = \frac{\sigma}{2\sigma\left(4 - \sigma\right)\lambda + 2\left(\sigma\left(4 - \sigma\right) - 2s\right)}, \tag{19}$$

$$q_2^{EF_2} = \frac{\sigma\left(2 - \sigma\right)\lambda + \sigma\left(2 - \sigma\right) - s}{2\sigma\left(4 - \sigma\right)\lambda + 2\left(\sigma\left(4 - \sigma\right) - 2s\right)}, \tag{20}$$

$$q_E^{EF_2} = \frac{\sigma\left(1 + \lambda\right) - s}{2\sigma\left(4 - \sigma\right)\lambda + 2\left(\sigma\left(4 - \sigma\right) - 2s\right)}. \tag{21}$$

For any $\sigma \in (s, 1)$, the equilibrium quantites are such that

$$q_1^{EF_2} - q_2^{EF_2} = \frac{-\sigma\left(\left(2-\sigma\right)\lambda + \left(1-\sigma\right)\right) + s}{2\sigma\left(4-\sigma\right)\lambda + 2\left(\sigma\left(4-\sigma\right) - 2s\right)} < 0,$$
$$q_2^{EF_2} - q_E^{EF_2} = \frac{\sigma\left(1-\sigma\right)\left(1+\lambda\right)}{2\sigma\left(4-\sigma\right)\lambda + 2\left(\sigma\left(4-\sigma\right) - 2s\right)} > 0.$$

Using the equilibrium quantities, we can calculate the firms' private profits

$$\Pi_1^{EF_2} = \frac{s\sigma^2\lambda}{4\left(\sigma\left(4-\sigma\right)\lambda + \left(\sigma\left(4-\sigma\right) - 2s\right)\right)^2},$$
$$\Pi_2^{EF_2} = \frac{\left(\left(1+\lambda\right)\left(\sigma\left(2-\sigma\right)\right) - s\right)^2}{4\left(\sigma\left(4-\sigma\right)\lambda + \left(\sigma\left(4-\sigma\right) - 2s\right)\right)^2} - F,$$
$$\Pi_E^{EF_2} = \frac{\sigma\left(s - \sigma\left(1+\lambda\right)\right)^2}{4\left(\sigma\left(4-\sigma\right)\lambda + \left(\sigma\left(4-\sigma\right) - 2s\right)\right)^2}.$$

Using the equilibrium quantities from the expressions (19), (20) and (21), the social welfare and firm 1's mixed profits are equal to

$$W^{EF_2} = \frac{\sigma^2\left(12 - 5\sigma + \sigma^2\right)\lambda^2 + \left(24\sigma^2 - 12s\sigma - 10\sigma^3 + 2\sigma^4 + 2s\sigma^2\right)\lambda + \left(s^2\sigma + 3s^2 + s\sigma^2 - 12s\sigma + \right.}{8\left(\sigma\left(4-\sigma\right)\lambda + \left(\sigma\left(4-\sigma\right) - 2s\right)\right)^2}$$

Public firms objective function is then

$$\mathbb{V}_1^{EF_2} = \frac{\sigma^2\left(5\sigma - \sigma^2 - 12\right)\lambda^3 + \left(5\sigma^3 - \sigma^4 - 12\sigma^2 + 12s\sigma\right)\lambda^2}{8\left(\sigma\left(4-\sigma\right)\lambda + \left(\sigma\left(4-\sigma\right) - 2s\right)\right)^2} +$$
$$+ \frac{\left(s\sigma^2 - 3s^2 - s^2\sigma + \sigma^4 - 5\sigma^3 + 12\sigma^2\right)\lambda + \left(s^2\sigma + 3s^2 + s\sigma^2 - 12s\sigma + \sigma^4 - 5\sigma^3 + 12\sigma^2\right)}{8\left(\sigma\left(4-\sigma\right)\lambda + \left(\sigma\left(4-\sigma\right) - 2s\right)\right)^2}.$$

Now the outside innovator desing the fixed fee contract by charging firm 2 its maximum willingness to pay for the license by setting

$$\Pi_2^{EF_2} - \Pi_2^N = 0.$$

Solving this expression for F_2, we find the optimal fixed fee to be paid by firm 2 to outside innovator as

$$F_2^*\left(s, \sigma, \lambda\right) = \frac{\left(s - 2\sigma + \sigma^2 - 2\sigma\lambda + \sigma^2\lambda\right)^2}{4\left(\sigma\left(4-\sigma\right)\lambda + \left(\sigma\left(4-\sigma\right) - 2s\right)\right)^2} - \frac{s\sigma^2\lambda^2}{4\left(2\sigma - s + \lambda\left(4\sigma - s\right)\right)^2}.$$

Notice that different from the exclusive licensing to public firm, it can be shown that the fixed fee F_2^* is decreasing in s,regardless of the entrant's quality.

Finally, we need to is it convenient for the outside innovator to sell the license exclsuively to the public firm 1 or the private firm 2? To answer this we need to sign the following difference

$$F_1^*(s,\sigma,\lambda) - F_2^*(s,\sigma,\lambda).$$

Now, suppose that entrant's initial quality is equal to $\sigma = s$. Then

$$F_1^*(s,\sigma=s,\lambda) - F_2^*(s,\sigma=s,\lambda) = (1-\lambda)(1-s)\frac{\chi(s,\lambda)}{8\lambda(3\lambda+1)^2(2s-3\lambda-3)^2(s-4\lambda+s\lambda-2)^2},$$

where

$$\chi(s,\lambda) = 4\left(6\lambda+10\lambda^2+1\right)(\lambda+1)^2 s^4 - 4(12\lambda+7)(\lambda+1)\left(6\lambda+10\lambda^2+\lambda^3+1\right)s^3 + \left(18\lambda^6+666
\\ -2(2\lambda+1)\left(357\lambda+1085\lambda^2+1375\lambda^3+645\lambda^4+72\lambda^5+42\right)s + 18(\lambda+2)(\lambda+1)(2\lambda+1$$

The sign of this expression depends on the sign of $\chi(s,\lambda)$, with

$$\frac{\partial\chi(s,\lambda)}{\partial\lambda} = 108\left(36-16s+s^2\right)\lambda^5 + 30\left(504-454s+111s^2-8s^3\right)\lambda^4 + 4\left(5202-6790s+3013s^2
\\ +6\left(-3545s+2094s^2-538s^3+52s^4+2169\right)\lambda^2 + 4\left(-1799s+1264s^2-392s^3+46s^4+945\right)\lambda + 2$$

in the relevant region of parameters. Since

$$\chi(s,\lambda=0) = (3-2s)^2(2-s)^2 > 0,$$
$$\chi(s,\lambda=1) = 16(3-s)\left(324-339s+120s^2-17s^3\right) > 0,$$

the numerator is positive $\chi(s,\lambda)$ for any $\lambda \in [0,1]$. Therefore, when an entrant enters the market with a quality equal to those of the incumbents' initial quality, it is optimal to sell the license exclusively to the public firm.

Suppose, instead, that entrant is of high quality and enters the market with $\sigma = 1$. Then

$$F_1^*(s,\sigma=1,\lambda) - F_2^*(s,\sigma=1,\lambda) = \frac{(1-\lambda)(1-s)\eta(s,\lambda)}{8\lambda(2s-3\lambda-3)^2(s-4\lambda+s\lambda-2)^2},$$

where

$$\eta(s,\lambda) = 2(16-s)\lambda^4 + 4\left(23-11s+s^2\right)\lambda^3 + \left(97+14s^2-2s^3-74s\right)\lambda^2 + 2\left(23-22s+3s^2+s^3\right)\lambda$$

with

$$\frac{\partial\eta(s,\lambda)}{\partial\lambda} = 8(16-s)\lambda^3 + 12\left(23-11s+s^2\right)\lambda^2 + 2\left(97+14s^2-2s^3-74s\right)\lambda + 2\left(23-22s+3s^2+s$$

Since

$$\eta\left(s,\lambda=0\right)=\left(3-2s\right)^{2}>0,$$

and

$$\eta\left(s,\lambda=1\right)=4\left(3-s\right)\left(23-7s\right)>0,$$

the numerator $\eta\left(s,\lambda\right)$ is strictly positive for any $\lambda\in[0,1]$, and hence, even if the entrant enters the market with the highest quality, it still is optimal to sell the license exclusively to the public firm.

Proof of Proposition 3.

We consider the case in which the license is still exclusive but now the outside innovator charges the public firm 1 a royalty payment per unit of the good produced, $r_1\geq 0$, for the quality improvement through innovation. Using the inverse demand function from the expressions (5)-(7), the firms' private profits can be written as follows:

$$\begin{aligned}\Pi_1^{ER_1}&=\left(1-q_1-sq_2-q_E\sigma\right)q_1-\frac{1}{2}q_1-r_1q_1,\\ \Pi_2^{ER_1}&=s\left(1-q_1-q_2-q_E\right)q_2-\frac{1}{2}sq_2, &(22)\\ \Pi_E^{ER_1}&=\left(\sigma-q_1\sigma-sq_2-q_E\sigma\right)q_E-\frac{1}{2}\sigma q_E. &(23)\end{aligned}$$

where the superscript ER_1 denotes the license is exclusive to public firm 1 with per unit royalty. As we did above, in that case the cost of producing a higher quality good is normalized to $c_1\left(s_1=1\right)=1$ for the licensee firm 1.

The expression for the consumer surplus is still given by the expression (8), while the social welfare function changes due to per unit royal and writes as

$$W^{ER_1}=rq_1^{ER_1}+\Pi_2^{ER_1}+\Pi_2^{ER_1}+\Pi_E^{ER_1}+CS^{ER_1}.$$

Hence, the public firm 1's mixed profit is then

$$\mathbb{V}_1^{ER_1}=\lambda\Pi_1^{ER_1}+\left(1-\lambda\right)W^{ER_1}. \qquad (24)$$

As before, the public firm maximizes its mixed profit (24) with respect to q_1, the private firm 2 and E maximize their private profits (22) and (23) with respect to q_2 and q_E, respectively. The equilibrium outputs are then

$$
\begin{aligned}
q_1^{ER_1} &= \frac{2\sigma(2-\sigma) - s - 2\lambda r(4\sigma - s)}{2\left((4\sigma - s)\lambda + 2\sigma(2-\sigma) - s\right)}, \\
q_2^{ER_1} &= \frac{\sigma\lambda(1+2r)}{2\left((4\sigma - s)\lambda + (4\sigma - 2\sigma^2 - s)\right)}, \\
q_E^{ER_1} &= \frac{\lambda(2\sigma - s)(1+2r)}{2\left((4\sigma - s)\lambda + 2\sigma(2-\sigma) - s\right)},
\end{aligned}
$$

where $q_2^{ER_1} > 0$ and $q_E^{ER_1} > 0$, while $q_1^{ER_1} > 0$ requires no-shut down by firm 1 that is $r < \overline{r}(\sigma, s) \triangleq \frac{2\sigma(2-\sigma)-s}{2\lambda(4\sigma-s)}$.

Let $\pi_0^{ER_1}(s, \sigma = s, \lambda)$ be the innovator's profit. As before, in what follows we divide the analysis into two parts regarding entrant's quality. First suppose E enters the market with the same quality as of incumbent players — i.e., such that $\sigma = s$. In that case, the outside innovator's profit from selling the innovation is equal to

$$
\pi_0^{ER_1}(s, \lambda) = r q_1^{ER_1} = r \times q_1^{ER_1}(s, \lambda) = \frac{(2\sigma(2-\sigma) - s) r - 2\lambda r^2 (4\sigma - s)}{2\left((4\sigma - s)\lambda + 2\sigma(2-\sigma) - s\right)},
$$

which is positive only if $r < \widehat{r}(s) = \frac{3-2s}{6r}$.

Innovator's optimization program, yields the royalty fee that maximizes his profit as an interior solution. That is

$$
\frac{\partial \pi_0^{ER_1}(s, \lambda)}{\partial r} = 0 \rightarrow r_I^{ER_1} = \frac{3-2s}{12\lambda},
$$

where the interior solution satisfies the no shut down by the public firm since $r_I^{ER_1} < \widehat{r}(\sigma = s)$. Moreover, we need to find the conditions where the participation constraint holds for the licensee firm 1. This condition is given by the profit that the the licensee firm 1 obtains from paying the per unit royalty and get the quality improving license is equal to its profit producing without license:

$$
\mathbb{V}_1^{ER_1}(s, \sigma = s, \lambda) = \mathbb{V}_1^{N}(s, \sigma = s, \lambda).
$$

Solving this equality for the per unit royalty fee r, it yields only one eligible corner solution that satisfies no-shut down condition. That is

$$
r_C^{ER_1} = \frac{(2s - 3(1+\lambda))\sqrt{s(\lambda+1)\left(4s\lambda^3 + 4(3-s)\lambda^2 + 5\lambda + 1\right)} + (3\lambda+1)\left(2s\lambda^2 + (9 - 2s - 4s^2\right.}{2\lambda(3\lambda+1)(9(1+\lambda) - 4s(1-\lambda))}
$$

which is strictly positive in the relevant region of parameters.

The optimal royalty rate is just $\min\left\{r_I^{ER_1}, r_C^{ER_1}\right\}$. By direct comparison we have

$$r_C^{ER_1} - r_I^{ER_1} = \frac{6\left(2s-3\lambda-3\right)\sqrt{\left(\left(4s^2\right)\lambda^4+12s\lambda^3+\left(17s-4s^2\right)\lambda^2+6s\lambda+s\right)}+36s\lambda^3+3\left(27-2s\right.}{12\lambda\left(3\lambda+1\right)\left(-4s+9\lambda+4s\lambda+9\right.}$$

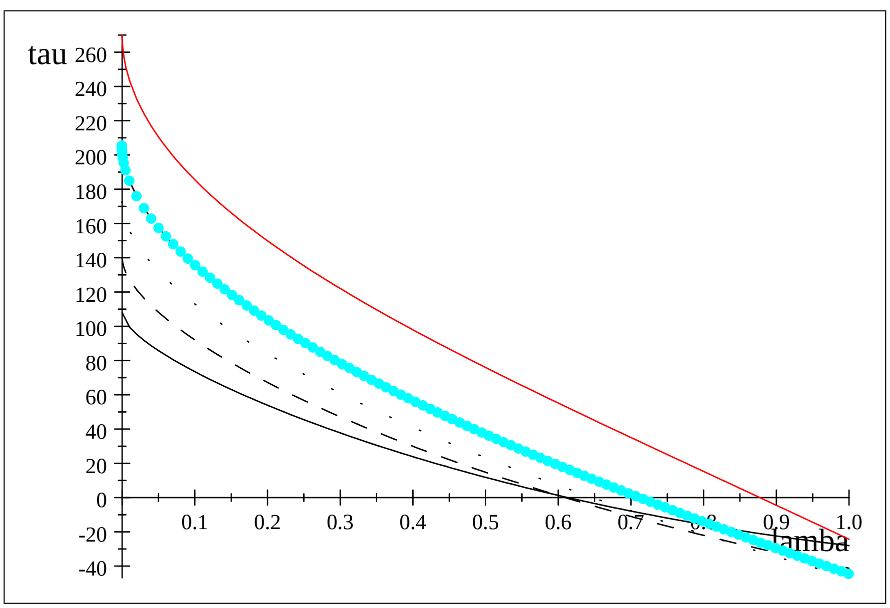
whose sign depends on the sign of the numerator

$$\tau\left(s,\lambda\right)=q=6\left(2s-3\lambda-3\right)\sqrt{\left(\left(4s^2\right)\lambda^4+12s\lambda^3+\left(17s-4s^2\right)\lambda^2+6s\lambda+s\right)}+36s\lambda^3+3\left(27-2s-\right.$$

with

$$\frac{\partial\tau\left(s,\lambda\right)}{\partial\lambda}=2\frac{\left(54s\lambda^2+\left(81-6s-48s^2\right)\lambda+\left(16s^2-66s+54\right)\right)\sqrt{s\left(\lambda+1\right)\left(5\lambda+12\lambda^2-4s\lambda^2+4s\lambda^3\right.}}{\sqrt{s\left(\lambda+1\right)\left(5\lambda+12\lambda^2-4s\lambda^2+4s\lambda^3+1\right)}}$$
$$+2\frac{-108s^2\lambda^4-s\left(45+12s-8s^2\right)\lambda^3-36s\left(13-5s\right)\lambda^2-6s\left(39-23s+4s^2\right)\lambda-18s\left(2\right.}{\sqrt{s\left(\lambda+1\right)\left(5\lambda+12\lambda^2-4s\lambda^2+4s\lambda^3+1\right)}}$$

where in what follows we plot how the $\frac{\partial\tau(s,\lambda)}{\partial\lambda}$ changes for various values of λ (solid line captures $\lambda=0$ and solid red line captures $\lambda=1$).



Now, notice that

$$\tau\left(s,\lambda=0\right)=\left(2\sqrt{s}+3\right)\left(4\sqrt{s}-3\right)\left(2s-3\right)>0 \text{ only if } s<0.56,$$
$$\tau\left(s,\lambda=1\right)=12\left(18-12s-3\sqrt{36s}+s\sqrt{36s}\right)>0 \text{ only if } s<0.58.$$

Hence, when $s < 0.56$ then the optimal fee is $r_I^{ER_1}$, when $s \geq 0.58$, the optimal fee is $r_C^{ER_1}$,while when $0.56 \leq s < 0.58$, then by the mean value theorem there exists $\widetilde{\lambda}$ such that for $\lambda < \lambda^C$, the optimal fee is equal to $r_C^{ER_1}$. Otherwise, innovator charges firm 1 its profit maximizing royalty fee $r_I^{ER_1}$

Instead now, consider the case in which E enters the market with the highest quality — i.e., such that $\sigma = 1$. In that case, innovator's optimization program, yields the royalty fee that maximizes his profit as an interior solution. That is

$$\frac{\partial \pi_0^{ER_1}(s, \sigma = 1, \lambda)}{\partial r} = 0 \rightarrow r_I^{ER_1}(\sigma = 1) = \frac{2 - s}{4\lambda (4 - s)},$$

where $r_I^{ER_1}(\sigma = 1) < \widehat{r}(\sigma = 1)$ so that there is never shut down in firm 1's production. Together with the interior solution, we must verify firm 1's participation to accept the contract proposed by the outside innovator. That is we need to find the per unit royalty fee such that the licensee firm 1's from paying the per unit royalty get the quality improving license is equal to its profit producing without license. This condition is

$$\mathbb{V}_1^{ER_1}(s, \sigma = 1, \lambda) = \mathbb{V}_1^N(s, \sigma = 1, \lambda).$$

Solving this equality for the per unit royalty fee r, it yields only one eligible corner solution that satisfies no-shut down condition. That is

$$r_C^{ER_1}(s, \sigma = 1, \lambda) = \frac{\zeta(s, \lambda)}{2\lambda \left((20 - 7s)\lambda + (12 - 9s + 2s^2)\right)},$$

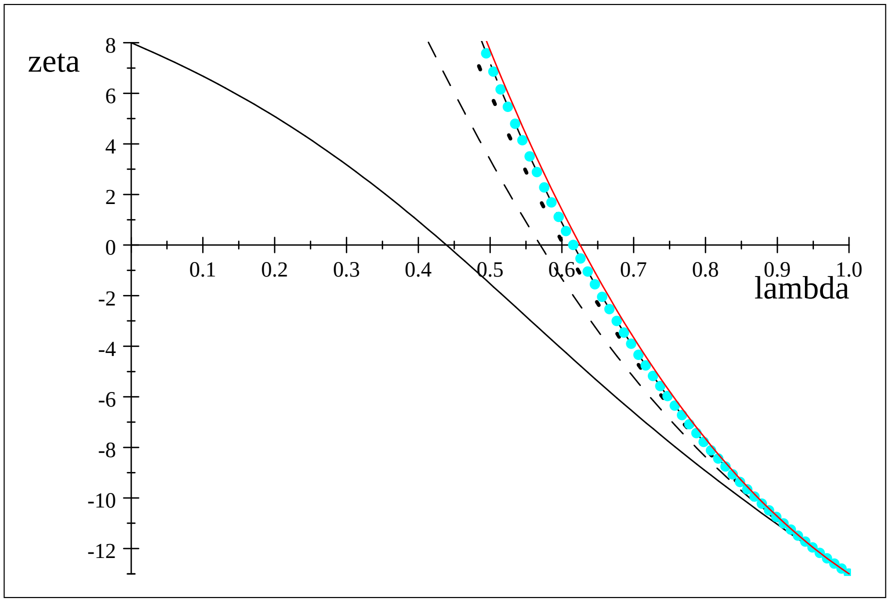
where

$$\zeta(s, \lambda) = 2(4 - s)\left(4 - 3s + s^2\right)\lambda^2 + (4 - 3s)(6 - s)\lambda + (4 - 3s)(2 - s) + \\ -(4 - s)\sqrt{4(2\lambda + 1)^2(1 - \lambda)^2 + (1 - \lambda)^2(1 + \lambda)^2 s^4 + \left(4\lambda^3 - 6\lambda^4 + 16\lambda^2 - 4\lambda - 6\right)s^3 +$$

As we did before, the optimal fee is determined according to $\min\left\{r_I^{ER_1}(\sigma = 1), r_C^{ER_1}(\sigma = 1)\right\}$. Hence, comparing the corner solution with interior solution we have

$$r_C^{ER_1}(\sigma = 1) - r_I^{ER_1}(\sigma = 1) = \frac{\zeta(s, \lambda)}{4\lambda (4 - s)\left((20 - 7s)\lambda + (2s^2 - 9s + 12)\right)},$$

In what follows, we plot $\frac{\partial \zeta(s,\lambda)}{\partial \lambda}$ for various values of λ (solid line captures $\lambda = 0$ and solid red line captures $\lambda = 1$).



Now, notice that

$$\zeta\left(s,\lambda=0\right)=\lim_{\lambda\to0}\varphi=3s^{2}+8-10s+\left(\left(2s-8\right)\sqrt{-11s+13s^{2}-6s^{3}+s^{4}+4}\right)<0,$$

$$\zeta\left(s,\lambda=1\right)=\lim_{\lambda\to1}\varphi=2\left(4-s\right)\left(-6s-\sqrt{64s-32s^{2}+4s^{3}}+s^{2}+8\right)<0\text{ only if }s>0.53$$

Therefore, when $s>0.53$, the numerator $\zeta\left(s,\lambda\right)$ is always negative and hence the optimal fee is $r_{C}^{ER_{1}}\left(\sigma=1\right)$. Instead, when $s\leq0.53$, then since

$$\zeta\left(s,\lambda=0\right)<0,$$
$$\zeta\left(s,\lambda=1\right)>0,$$

then there exists λ such that for $\lambda>\widetilde{\lambda}$ the solution is interior. Otherwise, it is corner.

Moreover, notice that when the entrant enters with the highest techonology royalty fee is prefered by the outside innovator. In fact,

$$F_{1}^{*}\left(s,\sigma=1,\lambda\right)-r_{1}^{I}=\frac{\left(36s-6s^{2}-48\right)\lambda^{2}+\left(4s^{3}-29s^{2}+65s-52\right)\lambda+\left(2s^{3}-11s^{2}+19s-12\right)}{8\lambda\left(4-s\right)\left(s-4\lambda+s\lambda-2\right)^{2}},$$

with

$$\lim_{\lambda\to0}\left(\left(36s-6s^{2}-48\right)\lambda^{2}+\left(4s^{3}-29s^{2}+65s-52\right)\lambda+\left(2s^{3}-11s^{2}+19s-12\right)\right)<0$$

$$\lim_{\lambda\to1}\left(\left(36s-6s^{2}-48\right)\lambda^{2}+\left(4s^{3}-29s^{2}+65s-52\right)\lambda+\left(2s^{3}-11s^{2}+19s-12\right)\right)<0$$

$$\frac{\partial\left(\left(36s-6s^{2}-48\right)\lambda^{2}+\left(4s^{3}-29s^{2}+65s-52\right)\lambda+\left(2s^{3}-11s^{2}+19s-12\right)\right)}{\partial\lambda}=2\left(36s-6s^{2}-\right.$$

This result is a sharp contrast with the existing literature.

Suppose that entrant is not verticlly integrated with the innovator so that it enters with $\sigma = s$. In that case,

$$F_1^* (s, \sigma = s, \lambda) = (1 - s) \frac{24s\lambda^4 + (4s - 32s^2 + 81) \lambda^3 + (16s^2 - 112s + 135) \lambda^2 + (20s^2 - 72s + 63)}{8\lambda (3\lambda + 1)^2 (2s - 3\lambda - 3)^2}$$

In that case, suppose that the size of the innovation is sufficiently small such that $s > 0.58$, so the solution is always interior. This is because

$$F_1^* (s, \sigma = s, \lambda) - r_1^I = \frac{\mu (s, \lambda)}{24\lambda (3\lambda + 1)^2 (2s - 3\lambda - 3)^2},$$

where

$$\begin{aligned} \mu (s, \lambda) = & \left(396s - 72s^2 - 486\right) \lambda^4 + \left(96s^3 - 540s^2 + 1281s - 1053\right) \lambda^3 + \left(96s^3 - 552s^2 + 1131s - 78 \right. \\ & + \left(36s^3 - 204s^2 + 387s - 243\right) \lambda + \left(4s^3 - 24s^2 + 45s - 27\right). \end{aligned}$$

Note that

$$\begin{aligned} \mu (s = 0.58, \lambda) &= -280.54\lambda^4 - 472.95\lambda^3 - 293.98\lambda^2 - 80.142\lambda - 8.1932 < 0, \\ \mu (s = 1, \lambda) &= -2 (3\lambda + 1)^4 < 0. \end{aligned}$$

with

$$\frac{\partial \mu (s, \lambda)}{\partial s} = 12 \left(9\lambda + 24\lambda^2 + 24\lambda^3 + 1\right) s^2 - 24 \left(17\lambda + 46\lambda^2 + 45\lambda^3 + 6\lambda^4 + 2\right) s + 3 \left(129\lambda + 377\lambda^2 + 4 \right.$$

Suppose that the outside innovator charges a royalty fee r to the public firm and licensing is exclusive to public firm only. In that case, if the entrant enters without an innovation (or technology) —i.e., such that $\sigma = s$, in that case

- the interior solution $r^I = \frac{3-2s}{12\lambda}$ is chosen by the outside innovator for any λ if $s > 0.58$
- the corner solution $r^C = \frac{3-2s}{12\lambda}$ is chosen by the outside innovator for any λ if $s < 0.56$
- When $0.56 < s < 0.57$, then there exists λ^C such that for $\lambda < \lambda^C$, the corner solution is chosen by the innovator, and the interior solution otherwise, where

$$\begin{aligned} r^C = & \frac{-12s + 36\lambda - 12s^2\lambda^2 + -38s\lambda + 27\lambda^2 - 3 (1 + \lambda) \sqrt{s (\lambda + 1) \left(5\lambda + 12\lambda^2 - 4s\lambda^2 + 4s\lambda^3 \right.}}{(24s + 54) \lambda^3 + (72 - 16s) \lambda^2 + (18 - 8s) \lambda} \\ & + \frac{9 - 4s\lambda^2 + 8s^2\lambda + 6s\lambda^3 + 4s^2 + 2s\sqrt{s (\lambda + 1) \left(5\lambda + 12\lambda^2 - 4s\lambda^2 + 4s\lambda^3 + 1\right)}}{(24s + 54) \lambda^3 + (72 - 16s) \lambda^2 + (18 - 8s) \lambda}. \end{aligned}$$

Suppose, instead, the entrant enters with the highest technology (perhaps this may be case when the innovator is integrated with the private firm) i.e., such that $\sigma = 1$, in that case, the solution is always interior and is equal to

$$r^I = \frac{2-s}{4\lambda(4-s)}.$$

Now notice that suppose that the solution is interior — i.e., $s > 0.58$ — in that case,

$$\left(\frac{2-s}{4\lambda(4-s)} - \frac{3-2s}{12\lambda}\right) < 0$$

This implies that when the entrant enters with an innovation the outside innovator is able to charge a higher fee to public firm when the entrant does not have the innovation (provided that their quality is sufficiently high).

- This can be confirmed that when the entrant's equality is $\sigma < 1$, the general expression for the interior solution is

$$\frac{\partial(rx)}{\partial r} = 0 \rightarrow r^{EL_1} = \left(\frac{-s+4\sigma-2\sigma^2}{4\lambda(4\sigma-s)}\right) \text{ interior solution}$$

  which is decreasing in σ, s and λ

$$\frac{\partial\left(\frac{-s+4\sigma-2\sigma^2}{4\lambda(4\sigma-s)}\right)}{\partial\sigma} < 0$$
$$\frac{\partial\left(\frac{-s+4\sigma-2\sigma^2}{4\lambda(4\sigma-s)}\right)}{\partial\lambda} < 0$$
$$\frac{\partial\left(\frac{-s+4\sigma-2\sigma^2}{4\lambda(4\sigma-s)}\right)}{\partial s} < 0$$

- Moreover, notice that when the entrant enters with the highest techonology royalty fee is prefered by the outside innovator. In fact,

$$F_1^*(s, \sigma = 1, \lambda) = \frac{1}{8}(s-1)\frac{-3\lambda - 4\lambda^2 + 2s\lambda^2 - 1}{\lambda(s - 4\lambda + s\lambda - 2)^2}$$

  then

$$F_1^*(s, \sigma = 1, \lambda) - r_1^I = \frac{(36s - 6s^2 - 48)\lambda^2 + (4s^3 - 29s^2 + 65s - 52)\lambda + (2s^3 - 11s^2 + 19s - 12)}{8\lambda(4-s)(s - 4\lambda + s\lambda - 2)^2},$$

with

$$\lim_{\lambda\to 0}\left(\left(36s-6s^2-48\right)\lambda^2+\left(4s^3-29s^2+65s-52\right)\lambda+\left(2s^3-11s^2+19s-12\right)\right)=-\left(3-s\right)\left(-5s\right.$$
$$\lim_{\lambda\to 1}\left(\left(36s-6s^2-48\right)\lambda^2+\left(4s^3-29s^2+65s-52\right)\lambda+\left(2s^3-11s^2+19s-12\right)\right)=-2\left(56-60s+\right.$$
$$\frac{\partial\left(\left(36s-6s^2-48\right)\lambda^2+\left(4s^3-29s^2+65s-52\right)\lambda+\left(2s^3-11s^2+19s-12\right)\right)}{\partial\lambda}=-2\left(48-36s+\right.$$

Hence, it is optimal for the innovator to sell the license with per unit royalty fee rather than through fixed fee. This result is a sharp contrast with the Yang and Huang (2023) paper.