Mergers and R&D investment: A Unified Approach

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March 2024

P R E L I M I N A R Y and I N C O M P L E T E

Comments welcome!

Abstract

We formulate a general two-stage game to study the implications of mergers on R&D. We identify three channels through which a merger affects R&D investment: anticipation of price coordination that enhances payoffs, internalisation of a direct innovation externality stemming from an enhanced chance of innovation success, and internalisation of an indirect innovation externality arising from business-stealing in the product market. Under price regulation, only the latter two effects play a role and mergers result in an unambiguous decrease in R&D. Our model captures the results of existing models with deterministic innovation intended to reduce costs or improve quality (Motta and Tarantino, 2021), stochastic innovation intended to enter a new market (Federico et al., 2017; Denicólo and Polo, 2018; Jullien and Lefouili, 2020), and stochastic innovation in settings where the payoffs from innovation failure are not equal to zero and hence the *Arrow replacement effect* is present (Mukherjee, 2022). We illustrate the importance of the pre-merger level of innovation, and hence the magnitude of investment costs.

JEL Classification: K21, L13, L40

Keywords: Merger Policy, R&D Investments, Innovation

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1 Introduction

The impact of mergers on innovation has been an important concern for antitrust authorities for at least two decades. For example, before the acquisition of *Sun Microsystems* by *Oracle* was cleared in the US and the European Commission in 2010, the latter investigated innovation concerns in areas such as cloud computing, database management, and open-source software development. Another example is the acquisition of *Monsanto* by *Bayer* in 2018, which underwent extensive antitrust investigation due to concerns about its potential impact on research and development of genetically modified seeds, agricultural chemicals, and digital farming technologies. Finally, the acquisition of *Celgene Corporation* by *Bristol-Myers Squibb* is also a case in point. Before the merger was approved in 2019, the Federal Trade Commission and the European Commission conducted thorough investigations to assess the risk of innovation loss resulting from discontinuation, delay or redirection of overlapping drug development pipelines.

In this paper, we study the implications of mergers on R&D in a general reduced-form two-stage game in which firms invest in R&D and then compete in the market place. Our model captures the results of existing models with stochastic innovation intended for entry into a new market (Federico et al., 2017; Denicólo and Polo, 2018; Jullien and Lefouili, 2020; Mukherjee, 2022) and deterministic innovation intended to reduce costs or improve quality (Motta and Tarantino, 2021).

We identify three channels through which a merger affects R&D investment: (i) anticipation of price coordination that enhances payoffs, (ii) internalisation of a direct innovation externality stemming from an enhanced chance of innovation success, and (iii) internalisation of an indirect innovation externality arising from business-stealing in the product market. Whether each of these channels bears on the incentives to invest or not depends on the setting and we provide conditions under which R&D increases or decreases for various relevant settings, including markets where there is price regulation and hence no price effects, deterministic R&D intended to reduce marginal costs or to enhance demand, and stochastic R&D intended for entry or to improve quality. We demonstrate how our general model is able to reproduce existing results from various articles in the literature after imposing their (restrictive) assumptions. More importantly, we show that new results obtain in richer settings where innovation is stochastic and the payoffs from innovation failure are not equal to zero (i.e. an *Arrow replacement effect* is present) and illustrate the importance of the pre-merger level of innovation, and hence the magnitude of investment costs. Our model ultimately shows that existing results tend to be model-specific and hardly apply to general settings and provides indications on how to assess the likely impact of mergers on R&D by properly accounting for the above mentioned effects.

We now describe the results we obtain for the different markets we study. We first show that in settings where the price effects are zero for example due to price regulation (or bounded due to extreme concavity of demand), only the two externality effects play a (significant) role and mergers result in an unambiguous decrease in R&D. This result is quite general and holds no matter whether the innovation process is stochastic or deterministic, and irrespective of whether both acquirer and target are active in the innovation market or just one of them. Hence, in the absence of other (efficiency) gains from mergers, mergers in price-regulated markets should be blocked.

The second class of models we study examines the impact of mergers in settings where the R&D process is deterministic. In these models success probabilities are independent from investment efforts (and moreover set equal to 1), while payoffs are affected by the investment efforts either through reduction in marginal costs or through quality improvements. Examples of such models are Motta and Tarantino (2021), Bourreau and Jullien (2018), Federico, Langus, Valletti (2018) and Bourreau, Jullien, Lefouili (2021). In these settings, there are two effects at play operating in opposite directions. On the one hand, the anticipated price effects after a merger tend to increase the incentives to invest. On the other hand, the internalization of the indirect innovation externality reduces the incentives to invest after a merger. We provide a condition under which a merger may increase or decrease investment. Verification of the condition requires to compare the R&D sensitivity of the gains a merging partner makes from price coordination with that of the total merger profits of its partner firm. In a standard model of price competition and demand-enhancing R&D with Singh-Vives demands we verify that the slope of one of the merging parties' gains from price coordination is greater than the slope of its partner's profits so that equilibrium R&D effort of the merged entity is higher than pre-merger. In a standard model of price competition and cost-reducing R&D with Singh-Vives demands we obtain the opposite. This is akin to the result Motta and Tarantino (2021) obtain in models of price competition and cost-reducing R&D and price competition with Shubik-Levitan and Salop demands. See also Bourreau, Jullien and Lefouili (2021).

The third class of models we study are models with stochastic R&D intended for entry. Models of R&D intended for entry have the special feature that failure to innovate results in a payoff of zero. Examples of such models are Federico et al. (2017), Denicolo and Polo (2018), and Jullien and Lefouili (2020). For this kind of models, we identify a necessary and sufficient condition under which a merger leads to less R&D. The condition requires to compare the incremental gains from price coordination a merging firm gets conditional on the partner successfully innovating with the reduction in profits the merging firm causes on the partner firm. We verify that this condition is always satisfied in the specific models of Federico et al. (2017), Denicolo and Polo (2018); by contrast in Jullien and Lefouili (2020) parameters exist for which a merger leads to more R&D.

Finally, we study models with stochastic R&D in which failure to innovate does not result in a payoff of zero. An examination of such models reveals that R&D for entry is special because an *Arrow replacement effect* is absent. In such a richer model, the conditions that lead to an increase or decrease in R&D investment after a merger are more complex, since they involve payoffs from successful innovation and payoffs from unsuccessful innovation. In different words, the magnitude of the Arrow replacement effect is important. More notably, our results make it clear that under more general assumptions the nature of the cost function may be crucial for the evaluation of the effects of mergers on innovation incentives. This is because there are circumstances in which the impact of a merger on R&D investment depends on whether the pre-merger level of investment is low or high, which ultimately depend on whether the marginal cost of investment is very steep or otherwise quite flat. We provide an example based on the model of price competition with horizontally and vertically differentiated products of Sutton (2001) that illustrate the various results that may be obtained depending on parameters and the slope of the cost of R&D investment.

Related literature

Our work is related to the growing theoretical literature on the impact of mergers on innovation. As discussed above, this literature can be divided into two classes. The first class of models looks at innovation where R&D process is uncertain and firms succeed in R&D with some probability. Federico et al. (2017), Denicolo and Polo (2018) and Jullien and Lefouili (2020) study the effects of horizontal mergers on the incentives to develop a new product in a model where investments in R&D affect the probability of success but not the payoff from the innovation. These papers assume that if a firm did not succeed to discover the new product it gets a zero payoff. So that the "replacement effect" is absent and the firms would not be operating in case of a failure to develop a new product. First two papers assume that products are homogeneous, while Jullien and Lefouili (2020) allow for horizontal product differentiation. Mukherjee (2022) also analyzes the market with horizontally differentiated products and relaxes the assumption that the firms would not be operating in case of a failure to implement the innovation. In Mukherjee (2022) the investments in R&D affect the probability of success, while conditional payoffs are affected only through exogenous reduction in marginal cost of production in case of success. Our model allows for richer environments with both horizontal and vertical product differentiation, where both price effects and the "replacement effect" can play a role.

Federico et al. (2017) show in homogeneous product setting that in the absence of price effects[1] a merger between two symmetric duopolists results in a reduction of R&D efforts to develop a new product. Denicolo and Polo (2018) in similar homogeneous products settings confirm the results of Federico et al. (2017) for sharply increasing marginal cost of R&D. But they also noticed that merger can lead to more innovation when the returns to R&D decrease

[1]Meaning that single innovator's profits are identical to monopoly profit in case joint entity successfully implements innovation. This holds in homogeneous products price competition setting, but does not have to hold in other setting, e.g. differentiated products quantity competition. This would not hold also when failing firms can stay in the market with low quality product, which is ruled out in Federico et al. (2017).

moderately (or when the probability of failure in innovation is log-concave in R&D investments), so that the merged entity will focus all its efforts on one research lab.We show that this new result, where mergers enhance innovation incentives, can be obtained also for sharply increasing marginal cost in environments where a failure to develop a new higher quality product does not result in a zero payoff and price effects are present. Finally, Jullien and Lefouili (2020) in a model with horizontally differentiated products identify the conditions, under which a merger can spur innovation. These conditions require that the merged entity's incremental gain from a second innovation is greater than the individual profit of a firm when both firms innovate in the no-merger scenario. This is possible in differentiated products setting of Jullien and Lefouili (2020), but is not the case in homogeneous products models of Federico et al. (2017), Denicolo and Polo (2018). Mukherjee (2022) extends the model of Federico et al. (2017) to analyze process innovation where success in R&D stage also results in an exogenous reduction in marginal costs. This extension allows identifying a new set of conditions under which mergers can enhance innovation incentives.[2]

The work by Federico, Langus, Valetti (2018) is the only paper that bridges the two classes of models. They identify the usual pricing externality and the innovation externality. However, due to tractability issues, the authors provide a numerical simulation to study the relationship between the innovation externality and price coordination. They numerically find that the negative innovation externality dominates, which leads to a reduction in the post-merger innovation efforts of the merged entity. Looking at the industry level investment and taking into account the response of the non-merging firms, they find that while the merged entity reduces its investment in R&D, non-merging parties respond by raising their R&D effort. To identify which effect dominates, the authors rely on a numerical simulation and, in the absence of efficiency gains, find that the net effect is negative. We add to this debate by identifying more subtle differences in the direct and indirect innovation externalities. We derive conditions, which guide the direction of innovation effects of mergers. We clarify the importance of price effects and "replacement" effect for the impact of mergers on investment incentives in both classes of models. We also clarify how the direction of the impact of mergers on R&D investments is affected by the shape of the R&D cost function.

Our work is also related to earlier literature on the relation between competition intensity and investments. Chen and Schwartz (2013) analyze the ranking of incentives to introduce new products in the absence of mergers. They show that the gain from such an innovation for a monopolist can be bigger than that for a competitive firm facing competition from sellers of the old product. Greenstein and Ramey (1998) analyze the effects of market structure on the returns from process innovation in a model where new products are vertically differentiated from older products. They show that under conditions competition and monopoly in the old product

[2]However, Mukherjee (2022) identifies these conditions for a specific example with quantity competition, horizontally differentiated products and specific functional form of the probability of success function.

market can provide identical returns and if monopolist is threatened with entry, monopoly provides strictly greater incentives to innovate.

The remainder of the paper is structured as follows. Section 2 presents the general model and characterizes general solutions for equilibrium investment levels in the non-cooperative benchmark and in the merger scenario, identifies the key externalities and provides the intuition. Section 3 examines the case in which the price effects of mergers are bounded, either because of price regulation or because demand is very concave. Section 4 studies the case of deterministic R&D. Section 5 looks at the case of stochastic R&D intended for entry. Section 6 examines the more general case of stochastic R&D and describes the importance of the Arrow replacement effect. Section 7 concludes.

2 The model and preliminary intuition

2.1 Model description and assumptions

We consider a duopoly market with symmetric firms, which we denote by i and j. Firms interact in the market during two stages. In the first stage, firms invest in R&D. Let x_i and x_j the amounts firms i and j put in R&D. In the second stage, upon observing the outcomes of their R&D investments, firms compete in the market.

In the innovation stage, we assume that if a firm, say i, invests in R&D an amount $x_i > 0$, then it costs the firm $C(x_i)$, with $C' > 0$ and $C'' < 0$. Investment need not result in innovation success and we denote by $\beta(x_i) \in (0, 1]$ the probability of successful innovation, with $\beta' > 0$ and $\beta'' < 0$. In the competition stage, firms compete in the market to sell their products. To keep the model as general as possible, we formulate reduced-form payoffs corresponding to the possible subgames that ensue after the innovation stage is over. The following table describes the possible subgames and the corresponding firms' payoffs:

Firm 2

		$Success(s)$	$Failure(f)$
Firm 1	$Success(s)$	$\pi_i^{ss}(x_i; x_j)$, $\pi_j^{ss}(x_j; x_i)$	$\pi_i^{sf}(x_i; x_j)$, $\pi_j^{fs}(x_j; x_i)$
	$Failure(f)$	$\pi_i^{fs}(x_i; x_j)$, $\pi_j^{sf}(x_j; x_i)$	$\pi_i^{ff}(x_i; x_j)$, $\pi_j^{ff}(x_j; x_i)$

Table 1: Firms' conditional payoffs under product competition

Note that the super-indices that describe a given subgame are ordered by player. For example, in the subgame where firm i's innovation is successful while firm j's is not, we index the payoff corresponding to firm i by "sf" to indicate that this is the payoff of a successful innovator competing with a failing one, and, likewise, the payoff corresponding to firm j is indexed by "fs". Hence, the first entry of the super-index refers to the innovation outcome of the player in question and the second entry to the innovation outcome of the rival player.

An important aspect of this formulation is that it allows for the conditional payoffs to depend on firms' investments. This is often the case in models of cost-reducing innovations as well as product-innovations. We next make some natural assumptions on these conditional payoffs.[3]

Assumption 1. Firms' conditional payoffs.

i. $\frac{\partial \pi_i^{ss}(\cdot)}{\partial x_i} > 0$ and $\frac{\partial \pi_i^{sf}(\cdot)}{\partial x_i} > 0$. This assumption captures either the impact of cost-reducing innovations, quality-improvements or marketing-related efficiency impropvements.

ii. $\frac{\partial \pi_i^{fs}(\cdot)}{\partial x_i} = 0$ and $\frac{\partial \pi_i^{ff}(\cdot)}{\partial x_i} = 0$. This says that the investment of a firm does not affect its own payoff if its R&D effort proves unsuccessful.

iii $\frac{\partial \pi_i^{ss}(\cdot)}{\partial x_j} < 0$ and $\frac{\partial \pi_i^{fs}(\cdot)}{\partial x_j} < 0$. This assumption captures firm rivalry in the product market: conditional on the rival being a successful innovator, an increase in the rival's investment lowers firm i's payoff no matter whether firm i was successful or not.

iv $\frac{\partial \pi_i^{sf}(\cdot)}{\partial x_j} = 0$ and $\frac{\partial \pi_i^{ff}(\cdot)}{\partial x_j} = 0$. This says that when a firm fails to get an innovation, its investment does not bear on the conditional payoff of the rival.

v. Conditional payoffs rank as follows: $\pi_i^{sf}(x_i) \geq \pi_i^{ss}(x_i, x_j) > \pi_i^{ff} > \pi_i^{fs}(x_j)$, where we have employed the above assumptions to describe the dependency of payoffs on investments x_i and x_j.

vi. $\pi_i^{sf}(x_i)$ and $\pi_i^{ss}(x_i, x_j)$ are strictly concave in x_i.

Assumption 1 imposes some restrictions on the conditional payoffs but these restrictions are rather mild. In particular, the ranking of payoffs in part (v) of Assumption 1 holds in standard models of R&D competition.

2.2 Pre-merger market equilibrium

The innovation stage payoff of a firm i investing x_i in R&D is given by:

$$\begin{aligned}\mathbb{E}\pi_i(x_i; x_j) = \beta_i(x_i) &\left[\beta_j(x_j)\pi_i^{ss}(x_i, x_j) + (1 - \beta_j(x_j))\pi_i^{sf}(x_i)\right] \\ &+(1 - \beta_i(x_i)) \left[\beta_j(x_j)\pi_i^{fs}(x_j) + (1 - \beta_j(x_j))\pi_i^{ff}\right] - C(x_i),\end{aligned}$$

The bracket in the first line of this expression is firm i's payoff conditional on its innovation project being successful; this payoff depends on the rival's innovation outcome. The bracket in the second line gives firm i's payoff conditional on failing to innovate.

[3] As an example, suppose we have price competition in the second-stage. Then, anticipating the second-stage Nash equilibrium $\mathbf{p}(\mathbf{x}) = (p_i(x_i; x_j), p_j(x_i; x_j))$, the first-stage payoff is $\pi_i(x_i; \mathbf{p}(\mathbf{x}))$.

The strict concavity of the success probabilities and the conditional payoffs ensure that a pure-strategy equilibrium of the investment subgame exists and is unique. Therefore, assuming the equilibrium is interior, it is given by the solution to the system of first order conditions (FOCs) for profits-maximization:

$$\underbrace{\frac{\partial \beta_i(\cdot)}{\partial x_i}\left[\beta_j(\cdot)\left[\pi_i^{ss}(x_i,x_j)-\pi_i^{fs}(x_j)\right]+(1-\beta_j(\cdot))\left[\pi_i^{sf}(x_i)-\pi_i^{ff}\right]\right]}_{\text{marginals gains from increasing success probability}} \tag{1}$$
$$+\underbrace{\beta_i(\cdot)\left[\beta_j(\cdot)\frac{\partial \pi_i^{ss}(x_i,x_j)}{\partial x_i}+(1-\beta_j(\cdot))\frac{\partial \pi_i^{sf}(x_i)}{\partial x_i}\right]}_{\text{marginals gains from increasing conditional payoffs}}-C'(x_i)=0, \quad \text{and similarly for firm } j.$$

This FOC says that a firm should continue to increase its R&D investment till the marginal revenue equals the marginal cost of investment. An increase in x_i has two effects on the expected payoff of a firm. On the one hand, it increases the probability of innovation. The first line of this FOC describes this effect, keeping constant conditional payoffs. On the other hand, it increases a firm's payoff conditional on innovation. The second line of this FOC describes this second effect, keeping constant the probability of innovation.[4] For later use, let x^* denote the pre-merger symmetric equilibrium R&D effort.

To the best of our knowledge, the literature has not presented models in which these two effects of increasing innovation effort are in place. There is a group of papers, namely, Federico et al. (2017), Denicolo and Polo (2018), Jullien and Lefouili (2020) and Mukherjee (2022), focusing on stochastic R&D in which the first effect is examined but, owing to their assumptions on the constancy of the conditional payoffs, the second effect is shut down. Likewise, there is a second group of papers of deterministic R&D, namely Motta and Tarantino (2021), Bourreau and Jullien (2018), and Bourreau, Jullien, Lefouili (2021) where success probabilities are exogenously set to 1 and hence the first effect is shut down by construction; the focus then lies on how investment effort impacts conditional payoffs.[5]

2.3 Mergers

We now examine the impact of mergers on R&D investment. Consider now that firms i and j merge. A merger has two important implications. On the one hand, a merger results in the monopolisation of the product market, thereby, as it is by now well known, creating negative

[4] It is also useful to remark that an increase in the rival's R&D investment steals business from firm i on two accounts. First, it increases the rival probability of success, which lowers firm i's payoff because $\pi_i^{ss}(\cdot)-\pi_i^{sf}(\cdot)<0$ and $\pi_i^{fs}(\cdot)-\pi_i^{ff}<0$. Second, it reduces firm i's conditional payoffs. Hence, from the point of view of the firms as a collective, firms shall invest too much in R&D.

[5] Federico, Langus, Valletti (2018) has both effects. However, their simulations do not identify positive effects of mergers on innovation efforts.

price effects. On the other hand, a merger results in the monopolisation of the innovation market. As we shall see, these two effects are related to one another and a complete understanding of the impact of mergers on innovation ought to take both of them into account.

We capture the price effects of mergers by specifying reduced-form conditional payoffs that are higher than pre-merger and to avoid notation confusion we label the monopoly payoffs with the "hat" symbol. Specifically, the merged-entity's conditional payoffs are given in the following table:

		Firm 2	
		$Success(s)$	$Failure(f)$
Firm 1	$Success(s)$	$\hat{\pi}_i^{ss}(x_i; x_j) + \hat{\pi}_j^{ss}(x_j; x_i)$	$\hat{\pi}_i^{sf}(x_i) + \hat{\pi}_j^{fs}(x_i)$
	$Failure(f)$	$\hat{\pi}_i^{fs}(x_j) + \hat{\pi}_j^{sf}(x_j)$	$\hat{\pi}_i^{ff} + \hat{\pi}_j^{ff}$

Table 2: Merged entity's conditional payoffs

In describing these payoffs in Table 2, we have already assumed assumptions similar to Assumption 1 but for the monopoly conditional payoffs, in particular $\frac{\partial \hat{\pi}_i^{fs}(\cdot)}{\partial x_i} = \frac{\partial \hat{\pi}_i^{ff}}{\partial x_i} = \frac{\partial \hat{\pi}_j^{sf}(\cdot)}{\partial x_i} = \frac{\partial \hat{\pi}_j^{ff}}{\partial x_j} = 0$. The fact that the merged entity coordinates prices implies that $\hat{\pi}_i^{ss}(x_i, x_j) \geq \pi_i^{ss}(x_i, x_j)$, $\hat{\pi}_i^{sf}(x_i) \geq \pi_i^{sf}(x_i)$, $\hat{\pi}_i^{fs}(x_j) \geq \pi_i^{fs}(x_j)$, $\hat{\pi}_i^{ff} \geq \pi_i^{ff}$ for both i and j. Further, we make the following assumptions:

Assumption 2. Monopoly conditional payoffs.

i. $\frac{\partial \hat{\pi}_i^{ss}(\cdot)}{\partial x_i} > 0$ and $\frac{\partial \hat{\pi}_i^{sf}(\cdot)}{\partial x_i} > 0$.

ii. $\frac{\partial \hat{\pi}_j^{ss}(\cdot)}{\partial x_i} < 0$ and $\frac{\partial \hat{\pi}_j^{fs}(\cdot)}{\partial x_i} < 0$.

iii. $\hat{\pi}_i^{ss}(x_i, x_j)$ is strictly concave in x_i and x_j, $\hat{\pi}_i^{sf}(x_i)$ is strictly concave in x_i, and $\hat{\pi}_i^{fs}(x_j)$ is strictly concave in x_j.

Consider now that firms i and j merge and assume that it is optimal for the merged entity to keep the two research labs of the constituent firms running. In such a case, the merged entity chooses investments x_i and x_j to maximize the (joint) payoff:

$$\begin{aligned}\pi^m(x_i, x_j) = \beta_i(x_i) &\left[\beta_j(x_j)\hat{\pi}_i^{ss}(x_i, x_j) + (1 - \beta_j(x_j))\hat{\pi}_i^{sf}(x_i)\right] \\ &+(1 - \beta_i(x_i)) \left[\beta_j(x_j)\hat{\pi}_i^{fs}(x_j) + (1 - \beta_j(x_j))\hat{\pi}_i^{ff}\right] - C(x_i) \\ &+\beta_j(x_j) \left[\beta_i(x_i)\hat{\pi}_j^{ss}(x_i, x_j) + (1 - \beta_i(x_i))\hat{\pi}_j^{sf}(x_j)\right] \\ &+(1 - \beta_j(x_j)) \left[\beta_i(x_i)\hat{\pi}_j^{fs}(x_i) + (1 - \beta_i(x_i))\hat{\pi}_j^{ff}\right] - C(x_j)\end{aligned}$$

The merger payoff is constructed as the sum of the payoffs of the merging parties. Our assumptions imply that the merged entity's payoff is strictly concave in x_i and x_j. Hence, assuming that an interior maximum exists, it is given by the solution of the system of FOCs. The FOC for the maximization of the profits of the merged entity with respect to x_i is given by:

$$FOC_i^m(x_i,x_j) \equiv \underbrace{\frac{\partial \beta_i(\cdot)}{\partial x_i}\left[\beta_j(\cdot)\left[\hat{\pi}_i^{ss}(x_i,x_j)-\hat{\pi}_i^{fs}(x_j)\right]+(1-\beta_j(\cdot))\left[\hat{\pi}_i^{sf}(x_i)-\hat{\pi}_i^{ff}\right]\right]}_{\text{marginal gains from increasing success probability}} \tag{2}$$

$$\underbrace{+\frac{\partial \beta_i(\cdot)}{\partial x_i}\left[\beta_j(\cdot)\left[\hat{\pi}_j^{ss}(x_i,x_j)-\hat{\pi}_j^{sf}(x_j)\right]+(1-\beta_j(\cdot))\left[\hat{\pi}_j^{fs}(x_i)-\hat{\pi}_j^{ff}\right]\right]}_{\text{direct innovation externality}}$$

$$+\underbrace{\beta_i(\cdot)\left[\beta_j(\cdot)\frac{\partial \hat{\pi}_i^{ss}(x_i,x_j)}{\partial x_i}+(1-\beta_j(\cdot))\frac{\partial \hat{\pi}_i^{sf}(x_i)}{\partial x_i}\right]}_{\text{marginal gains from increasing conditional payoffs}}$$

$$+\underbrace{\beta_i(\cdot)\left[\beta_j(\cdot)\frac{\partial \hat{\pi}_j^{ss}(x_i,x_j)}{\partial x_i}+(1-\beta_j(\cdot))\frac{\partial \hat{\pi}_j^{fs}(x_i)}{\partial x_i}\right]}_{\text{indirect innovation externality}}-C'(x_i)=0, \quad \text{and similarly for } x_j.$$

The FOC for profits maximization of the merged entity with respect to x_j is similar and, to save space, has been omitted.

A comparison between the FOCs pre-merger and post-merger is central to the understanding of the complexity of the impact of mergers on R&D investment. Moreover, it is key to understand why the different assumptions in the literature have led to distinct results. Comparing the post-merger FOC (2) to the pre-merger one in equation (1) leads to three important observations.

- First, because of price coordination in the market stage, the post-merger FOC involves monopoly payoffs rather than competitive payoffs. This means that when the merged-entity picks its R&D effort, it factors different conditional payoffs compared to pre-merger. This issue in isolation has a bearing on the choice of R&D effort post-merger.
- Second, the post-merger FOC reflects the internalization of *two* externalities that operate on the incentives to invest in the same direction.
  - The first is a direct and *negative*externality and arises because when the merged-entity increases its R&D effort x_i, it decreases the returns from investment of its partner firm (this is the second line of FOC (2)).
  - The second is an indirect and *negative* externality and arises because an increase in R&D effort x_i reduces the conditional payoffs of the partner firm j (this is the last line of FOC (2)).

The standard approach to address the question whether a merger leads to more or less investment compared to the pre-merger equilibrium consists of studying the sign of the FOC (2) evaluated at the pre-merger symmetric equilibrium x^*. This gives:

$$\begin{aligned}
FOC_i^m(x^*) =& \frac{\partial \beta_i(x^*)}{\partial x_i}\left[\beta_j(x^*)\left[\hat{\pi}_i^{ss}(x^*,x^*)-\hat{\pi}_i^{fs}(x^*)\right]+(1-\beta_j(x^*))\left[\hat{\pi}_i^{sf}(x^*)-\hat{\pi}_i^{ff}\right]\right] \\
&+\frac{\partial \beta_i(x^*)}{\partial x_i}\left[\beta_j(x^*)\left[\hat{\pi}_j^{ss}(x^*,x^*)-\hat{\pi}_j^{sf}(x^*)\right]+(1-\beta_j(x^*))\left[\hat{\pi}_j^{fs}(x^*)-\hat{\pi}_j^{ff}\right]\right] \\
&+\beta_i(x^*)\left[\beta_j(x^*)\frac{\partial \hat{\pi}_i^{ss}(x^*,x^*)}{\partial x_i}+(1-\beta_j(x^*))\frac{\partial \hat{\pi}_i^{sf}(x^*)}{\partial x_i}\right] \\
&+\beta_i(x^*)\left[\beta_j(x^*)\frac{\partial \hat{\pi}_j^{ss}(x^*,x^*)}{\partial x_i}+(1-\beta_j(x^*))\frac{\partial \hat{\pi}_j^{fs}(x^*)}{\partial x_i}\right]-C'(x^*).
\end{aligned} \tag{3}$$

Since the FOC (1) holds (with equality) at the pre-merger market symmetric equilibrium x^*, equation (3) can be simplified to:

$$\begin{aligned}
FOC_i^m(x^*) =& \frac{\partial \beta_i(x^*)}{\partial x_i}\left[\beta_j(x^*)\left[\Delta_i^{ss}(x^*,x^*)-\Delta_i^{fs}(x^*)\right]+(1-\beta_j(x^*))\left[\Delta_i^{sf}(x^*)-\Delta^{ff}\right]\right] \\
&+\frac{\partial \beta_i(x^*)}{\partial x_i}\left[\beta_j(x^*)\left[\hat{\pi}_j^{ss}(x^*,x^*)-\hat{\pi}_j^{sf}(x^*)\right]+(1-\beta_j(x^*))\left[\hat{\pi}_j^{fs}(x^*)-\hat{\pi}_j^{ff}\right]\right] \\
&+\beta_i(x^*)\left[\beta_j(x^*)\frac{\partial \Delta_i^{ss}(x^*,x^*)}{\partial x_i}+(1-\beta_j(x^*))\frac{\partial \Delta_i^{sf}(x^*)}{\partial x_i}\right] \\
&+\beta_i(x^*)\left[\beta_j(x^*)\frac{\partial \hat{\pi}_j^{ss}(x^*,x^*)}{\partial x_i}+(1-\beta_j(x^*))\frac{\partial \hat{\pi}_j^{fs}(x^*)}{\partial x_i}\right].
\end{aligned} \tag{4}$$

where $\Delta_i^{ss}(\cdot) \equiv \hat{\pi}_i^{ss}(\cdot) - \pi_i^{ss}(\cdot)$ denotes the extra profits that accrue to the partner firm i in the subgame where both research labs are successful purely stemming from price coordination. Let $\Delta_i^{sf}(\cdot)$, $\Delta_i^{fs}(\cdot)$ and $\Delta_i^{ff}(\cdot)$ be defined anologously.

Evaluating the sign of (4) in general is quite difficult because there are conflicting effects. Even in restricted settings where for example only one of the players has research capabilities, it is quite hard to sign equation (4). We now report on some useful results that arise in special settings.

3 Mergers in price-regulated markets

The first case of interest yields a clear-cut result. Consider a merger in which there aren't price effects, due to for example price regulation, like it occurs for some pharmaceutical products. Alternatively, it could be that while the R&D departments manage to coordinate investments after a merger the sales departments do not. In such a case, $\hat{\pi}_i^{ss}(\cdot) = \pi_i^{ss}(\cdot)$, $\hat{\pi}_i^{sf}(\cdot) = \pi_i^{sf}(\cdot)$, $\hat{\pi}_i^{fs}(\cdot) = \pi_i^{fs}(\cdot)$ and $\hat{\pi}_i^{ff} = \pi_i^{ff}$, and hence $\Delta_i^{ss}(\cdot) = \Delta_i^{sf}(\cdot) = \Delta_i^{fs}(\cdot) = \Delta_i^{ff} = 0$. As a result, (4) would clearly be negative. Hence:

Proposition 1 *Consider a merger in which there aren't price effects, due to for example price regulation. Then, irrespective of whether the innovation process is stochastic or deterministic, and irrespective of whether both target and acquirer are active in the innovation market or just one of them, a merger results in a fall in R&D effort.*

Proof. Evaluating the FOC of the merged entity at the pre-merger symmetric equilibrium gives:

$$FOC_i^m(x^*) = \frac{\partial \beta_i(x^*)}{\partial x_i}\left[\beta_j(x^*)\left[\pi_j^{ss}(x^*,x^*) - \pi_j^{sf}(x^*)\right] + (1-\beta_j(x^*))\left[\pi_j^{fs}(x^*) - \pi_j^{ff}\right]\right] + \beta_i(x^*)\left[\beta_j(x^*)\frac{\partial \pi_j^{ss}(x^*,x^*)}{\partial x_i} + (1-\beta_j(x^*))\frac{\partial \pi_j^{fs}(x^*)}{\partial x_i}\right] < 0.$$

This implies that, evaluated at the pre-merger symmetric equilibrium, the gradient of the merged-entity is negative. Hence, post-merger symmetric equilibrium must have an investment level $x^m < x^*$. ∎

In the absence of gains from price/quantity coordination in the market stage, the merger would alter its investment in order to internalize the direct and indirect innovation externalities (which are negative), thereby reducing investment.

4 Deterministic R&D

The second case of interest is that in which the innovation process is deterministic. Our model can easily accommodate two cases. In the first case, both firms have research capabilities and investment always leads to a successful outcome, i.e. $\beta_i(x_i) = \beta_j(x_j) = 1$. In the second case, only one firm has research capabilities, say firm i, i.e. $\beta_i(x_i) = 1$ and $\beta_j(x_j) = 0$.

Proposition 2 *Consider a merger between symmetric firms and suppose their innovation processes are deterministic so that investment surely results in success. Then:*

(i) If the acquirer, say i, has research capabilities while the target has not, and if $\frac{\partial \Delta_i^{sf}(x^*,x^*)}{\partial x_i} > (<) - \frac{\partial \hat{\pi}_j^{fs}(x^*,x^*)}{\partial x_i}$ *a merger leads to an increase (decrease) in R&D effort.*

(ii) If both acquirer and target have research capabilities, and if $\frac{\partial \Delta_i^{ss}(x^*,x^*)}{\partial x_i} > (<) - \frac{\partial \hat{\pi}_j^{ss}(x^*,x^*)}{\partial x_i}$ *a merger leads to an increase (decrease) in R&D effort.*

An example of an article in which the innovation process is deterministic is Motta and Tarantino (2021). In their model, all firms have research capabilities. Proposition 2(ii) shows that whether a merger results in more or less investment by the merging parties is ambiguous. The condition in the proposition says that a merger will result in more investment if the profit boost stemming from price coordination of one of the divisions increases more rapidly in investment compared to the (indirect) innovation externality. Proposition 2(i) shows that the mergers of individual firms, where one firm is research active and the other is not, can easily be captured by our model. The condition for this types of mergers to lead to an increase in investment is analogous to the setting in which both firms are active in the research market. Next we present two examples. In both examples, firms compete in prices to sell the differentiated products. In the first example, we consider R&D that enhances demand and we show that there is more R&D after the merger. The second example is in line with the examples in Motta and Tarantino (2021): we look at cost-reducing R&D and show that there is less R&D after the merger.

Example 4.1: Price competition and demand-enhancing R&D. Consider a standard symmetric model of price competition and demand-enhancing R&D with Sighn and Vives' (1985) demands given by: $q_1 = ax_1 - bp_1 + dp_2$ and $q_2 = ax_2 - bp_2 + dp_1$, with $b > d$. Firms have marginal costs equal to c. The variables x_1 and x_2 represent R&D investment that enhances the demands of firms 1 and 2. To obtain a demand-enhancement of x, an individual firm have to invest an amount $C(x) = x^2/2$. It is straightforward to derive the payoffs that enable us to check the condition in Proposition 2. In the pre-merger market the equilibrium price of firm 1 is $p_1 = \frac{(2ax_1+2bc+cd)b+adx_2}{4b^2-d^2}$, and similarly for firm 2. The reduced firm (gross) profits equal $\pi_1 = \frac{b\left(2abx_1+adx_2-2b^2c+bcd+cd^2\right)^2}{(d^2-4b^2)^2}$, and similarly for firm 2. As we can see, a firm's (gross) profits increase in own R&D effort and in the rival's R&D effort. Equating the marginal revenue from an increase in R&D effort to the marginal cost and applying symmetry gives the symmetric equilibrium R&D investment in the pre-merger market: $x_1^* = x_2^* = x^* = \frac{4ab^2c(b-d)}{4a^2b^2-(2b-d)^2(2b+d)}$.

Consider now the merged entity that chooses its prices to maximize the joint payoff. Easy calculations reveal that the merged entity will pick a price equal to $p_1 = \frac{abx_1+adx_2+b^2c-cd^2}{2b^2-2d^2}$ for product 1, and symmetrically for product 2. To evaluate the condition in 2(ii), we need to derive separately the profits of the two divisions of the merged entity: $\hat{\pi}_1(\cdot) = \frac{(ax_1-bc+cd)\left(abx_1+d(ax_2+cd)-b^2c\right)}{4(b^2-d^2)}$, and similarly for division 2. With the information provided, we can now compute $\Delta_1^{ss}(\cdot) = \hat{\pi}_1(\cdot) - \pi_1(\cdot)$. Taking the derivative with respect to x_1 gives: $\frac{\partial \Delta_1^{ss}}{\partial x_1} = \frac{ad^2\left(8b^2+d^2\right)\left(2abx1+adx2-2b^2c+bcd+cd^2\right)}{4(b^2-d^2)(d^2-4b^2)^2}$. Evaluating this derivative at the pre-merger equilibrium we get $\frac{\partial \Delta_1^{ss}(x^*,x^*)}{\partial x_1} = -\frac{acd^2\left(8b^2+d^2\right)}{4(b+d)((d-2b)^2(2b+d)-4a^2b^2)}$. The next is to compute $\frac{\partial \hat{\pi}_2^{ss}(\cdot)}{\partial x_1} = \frac{ad(ax2-bc+cd)}{4(b^2-d^2)}$, which evaluated at the at the pre-merger equilibrium gives $\frac{\partial \hat{\pi}_2^{ss}(x^*,x^*)}{\partial x_1} = -\frac{acd(d-2b)^2(2b+d)}{4(b+d)((d-2b)^2(2b+d)-4a^2b^2)}$. To conclude, we evaluate the condition in the proposition: $\frac{\partial \Delta_1^{ss}(x^*,x^*)}{\partial x_1} + \frac{\partial \hat{\pi}_2^{ss}(x^*,x^*)}{\partial x_1} = \frac{acd\left(4b^2-2bd+d^2\right)}{8a^2b^2-2(d-2b)^2(2b+d)} > 0$, so a merger must lead to an increase in R&D. In fact, if we derive the equilibrium R&D effort of the merged entity we get $x_1^m = x_2^m = \frac{ac(b-d)}{a^2-2b+2d}$, which is higher than pre-merger.

Example 4.2: Price competition and cost-reducing innovation. In this example, we pick values for the parameters in order to make it less cumbersome. Consider a standard symmetric model of price competition and cost-reducing investments with Sighn and Vives' (1985) demands: $q_1 = 1 - 2p_1 + p_2$ and $q_2 = 1 - 2p_2 + p_1$. Initially firms have marginal costs equal to c. Suppose that to obtain a cost-reduction of x, an individual firm has to invest an amount $C(x) = x^2/2$. Then, it is straightforward to derive the payoffs that enable us to check the condition in Proposition 2. In the pre-merger market the equilibrium prices are $p_1 = \frac{1}{15}(5+10c-8x_1-2x_2)$, and similarly for firm 2, with corresponding firms' (gross) profits equal to $\pi_1 = \frac{2}{225}(5-5c+7x_1-2x_2)^2$, and similarly for firm 2. As we can see, a firm's (gross) profits increase in own R&D effort but decline in the effort of the rival firm. Equating the marginal revenue from an increase in R&D effort to the marginal cost and applying symmetry gives the symmertric equilibrium R&D investment in the pre-merger market: $x_1^* = x_2^* = x^* = \frac{28}{17}(1-c)$.

Consider now the merged entity that chooses its prices to maximize the joint payoff. Easy calculations reveal that the merged entity will pick prices equal to $p_1 = \frac{1}{2}(1+c-x_1)$ and $p_2 = \frac{1}{2}(1+c-x_2)$. To evaluate the condition in 2(ii), we need to derive separately the profits of the two divisions of the merged entity: $\hat{\pi}_1(\cdot) = \frac{1}{4}(1-c+x_1)(1-c+2x_1-x_2)$, and similarly for division 2. With the information provided, we can now compute $\Delta_1^{ss}(\cdot) = \hat{\pi}_1(\cdot) - \pi_1(\cdot)$. Taking the derivative with respect to x_1 gives: $\frac{\partial \Delta_1^{ss}}{\partial x_1} = \frac{1}{900}(115(1-c) + 116x_1 - x_2)$. Evaluating this derivative at the pre-merger equilibrium we get $\frac{\partial \Delta_1^{ss}(x^*,x^*)}{\partial x_1} = \frac{23}{68}(1-c)$. The next is to compute $\frac{\partial \hat{\pi}_2^{ss}(\cdot)}{\partial x_1} = -\frac{1}{4}(1-c+x_2)$, which evaluated at the pre-merger equilibrium gives $\frac{\partial \hat{\pi}_2^{ss}(x^*,x^*)}{\partial x_1} = -\frac{45}{68}(1-c)$. To conclude, because $\frac{\partial \Delta_1^{ss}(x^*,x^*)}{\partial x_1} < -\frac{\partial \hat{\pi}_2^{ss}(x^*,x^*)}{\partial x_1}$, a merger must lead to a cut in R&D. In fact, if we derive the equilibrium R&D effort of the merged entity we get $x_1^m = x_2^m = 1-c$, which is less than pre-merger.

5 R&D for entry

A third class of models that our general formulation captures are models where market entry can only occur upon successful innovation. We model this idea by assuming that conditional payoffs in case of innovation failure are equal to zero: $\hat{\pi}_i^{fs}(\cdot) = \pi_i^{fs}(\cdot) = 0$, and $\hat{\pi}_i^{ff}(\cdot) = \pi_i^{ff}(\cdot) = 0$, and similarly for firm j. Moreover, because a failing innovation project does not lead to entry, $\hat{\pi}_i^{sf}(\cdot) = \pi_i^{sf}(\cdot)$, hence $\Delta_i^{sf} = 0$. Then we have the following implications of mergers:

Proposition 3 *Consider a merger and suppose that unsuccessful innovation results in entry failure so that conditional payoffs are equal to zero. Then:*

(i) If the acquirer, say firm i, has research capabilities while the target has not, then a merger results in an increase in R&D investment if and only if:

$$\frac{\partial \beta_i(x^*)}{\partial x_i}\Delta_i^{sf}(x^*) + \beta_i(x^*)\frac{\partial \Delta_i^{sf}(x^*)}{\partial x_i} > 0$$

(ii) If both acquirer and target have research capabilities, and if conditional payoffs are independent of R&D effort, then a merger results in an increase in R&D investment if and only if:

$$\Delta_i^{ss} + \hat{\pi}_j^{ss} > \hat{\pi}_j^{sf}.$$

Proof. In the first setting we have:

$$FOC^m(x^*) = \frac{\partial \beta_i(x^*)}{\partial x_i}\Delta_i^{sf}(x^*) + \beta_i(x^*)\frac{\partial \Delta_i^{sf}(x^*)}{\partial x_i} > 0.$$

In the second setting:

$$FOC^m(x^*) = \frac{\partial \beta_i(x^*)}{\partial x_i}\beta_j(x^*)\left[\Delta_i^{ss} + \hat{\pi}_j^{ss} - \hat{\pi}_j^{sf}\right] \tag{5}$$

■

Part (ii) of this proposition captures the results of Jullien and Lefouili (2020). They consider horizontally differentiated products and also assume that unsuccessful innovation results in entry failure. In Jullien and Lefouili (2020) Π_2 is the monopoly payoff of a two-product seller, thus in our notation $\Pi_2 = \hat{\pi}_i^{ss} + \hat{\pi}_j^{ss}$. Π_1 is the monopoly payoff of a single-product seller (because the other innovation has failed), thus in our notation $\Pi_1 = \hat{\pi}_i^{sf}$. π_2 is the profit of a duopolistic firm when both innovate, thus $\pi_2 = \pi_i^{ss} = \pi_j^{ss}$. The payoffs are summarized in Tables $1'$ and $2'$. Hence, the condition for merger to unambiguously increase the R&D in Proposition 1 of Jullien and Lefouili (2020), $\Pi_2 - \Pi_1 > \pi_2$, is identical to $\hat{\pi}_i^{ss} + \hat{\pi}_j^{ss} - \hat{\pi}_j^{sf} > \pi_i^{ss}$ in our notation. Which can be rewritten as condition in Proposition 3 (ii) $\Delta_i^{ss} + \hat{\pi}_j^{ss} > \hat{\pi}_j^{sf}$.

Firm 2

		$Success(s)$	$Failure(f)$
Firm 1	$Success(s)$	π_2, π_2	$\Pi_1, 0$
	$Failure(f)$	$0, \Pi_1$	$0, 0$

Table $1'$: Firms' conditional payoffs under product competition: Jullien and Lefouili (2020)

Firm 2

		$Success(s)$	$Failure(f)$
Firm 1	$Success(s)$	Π_2	Π_1
	$Failure(f)$	Π_1	0

Table $2'$: Merged entity's conditional payoffs: Jullien and Lefouili (2020)

Results in Federico et al. (2017) are also captured by part (ii) of Proposition 3. With homogeneous products, price competition and no entry in case of failure the relevant profits, using notation of Federico et al. (2017), are given by expressions in Tables $1''$ and $2''$ below.

		Firm 2	
		$Success(s)$	$Failure(f)$
Firm 1	$Success(s)$	δ,δ	1,0
	$Failure(f)$	0,1	0,0

Table 1″: Firms' conditional payoffs under product competition: Federico et al. (2017)

		Firm 2	
		$Success(s)$	$Failure(f)$
Firm 1	$Success(s)$	$\frac{1}{2}+\frac{1}{2}$	1+0
	$Failure(f)$	0+1	0,0

Table 2″: Merged entity's conditional payoffs: Federico et al. (2017)

Hence, the condition $\Delta_i^{ss}+\hat{\pi}_j^{ss} > \hat{\pi}_j^{sf}$ can easily be checked after noting that $\Delta_i^{ss} = \frac{1}{2}-\delta$, which can be positive or negative, $\hat{\pi}_j^{ss} = \frac{1}{2}$ and $\hat{\pi}_j^{sf} = 1$. It follows that the condition in Proposition 3(ii) never holds and a merger always results in less R&D, which is the result Federico et al. (2017) get.

Furthermore, part (ii) of Proposition 3 also captures Denicolo and Polo's (2018) interior equilibrium. They have a similar homogeneous products price competition setting and no entry in case of innovation failure. The relevant profits before and after the merger are given in Tables 1‴ and 2‴.

		Firm 2	
		$Success(s)$	$Failure(f)$
Firm 1	$Success(s)$	$\frac{V}{2}$,$\frac{V}{2}$	V,0
	$Failure(f)$	0,V	0,0

Table 1‴: Firms' conditional payoffs under product competition: Denicolo and Polo (2018)

		Firm 2	
		$Success(s)$	$Failure(f)$
Firm 1	$Success(s)$	$\frac{V}{2}+\frac{V}{2}$	V+0
	$Failure(f)$	0+V	0,0

Table 2‴: Merged entity's conditional payoffs: Denicolo and Polo (2018)

Again, the condition $\Delta_i^{ss} + \hat{\pi}_j^{ss} > \hat{\pi}_j^{sf}$ can be worked out: $\Delta_i^{ss} = \frac{V}{2} - \frac{V}{2} = 0$, $\hat{\pi}_j^{ss} = \frac{V}{2}$ and $\hat{\pi}_j^{sf} = V$. This implies that condition in part (ii) of Proposition 3 never holds and a merger always results in less R&D.

XXXX

Next, we show that more general results obtain in richer settings where the payoffs from innovation failure are not equal to zero (an *Arrow replacement effect* is present) and illustrate the importance of the pre-merger level of innovation, and hence the magnitude of investment costs.

6 A more general model of R&D competition

A fourth class of models that our general formulation captures are models with vertical and horizontal differentiation, where market entry with low quality product can also occur when innovation fails. In this case conditional payoffs in case of innovation failure are not equal to zero: $\hat{\pi}_i^{fs}(\cdot) > \pi_i^{fs}(\cdot) > 0$, and $\hat{\pi}_i^{ff} > \pi_i^{ff} > 0$, and similarly for firm j.[6]

As will be shown in part (i) of Proposition 4, in the symmetric model, where conditional payoffs are independent of R&D effort, conditions for merger to unambiguously increase R&D investment will be

$$\hat{\pi}_i^{ss} + \hat{\pi}_j^{ss} - \left(\hat{\pi}_i^{fs} + \hat{\pi}_j^{sf}\right) > \pi_i^{ss} - \pi_i^{fs} \tag{6}$$

$$\hat{\pi}_i^{sf} + \hat{\pi}_j^{fs} - \left(\hat{\pi}_i^{ff} + \hat{\pi}_j^{ff}\right) > \pi_i^{sf} - \pi_i^{ff} \tag{7}$$

Note that in the LHS of the inequality in (6) we have the return on innovation for the joint entity conditional on rival's success, while on the RHS we have the benefit from unilateral success in R&D for independent firm conditional on rival's success. Inequality (7) can be interpreted similarly but conditional on rival's failure. The LHSs and the RHSs can also be viewed as replacement effects for the merged entity and for the independent innovator, respectively. In this setting the implications of mergers depend on the pre-merger level of innovation and the magnitude of investment costs.

For stating the next result, it is convenient to define $\hat{x}$ as the investment level that solves:

$$\beta(x) - \frac{\hat{\pi}_i^{sf} + \hat{\pi}_j^{fs} - \left(\hat{\pi}_i^{ff} + \hat{\pi}_j^{ff}\right) - \left(\pi_i^{sf} - \pi_i^{ff}\right)}{\hat{\pi}_i^{sf} + \hat{\pi}_j^{fs} - \left(\hat{\pi}_i^{ff} + \hat{\pi}_j^{ff}\right) - \left(\pi_i^{sf} - \pi_i^{ff}\right) - \left[\hat{\pi}_i^{ss} + \hat{\pi}_j^{ss} - \left(\hat{\pi}_i^{fs} + \hat{\pi}_j^{sf}\right) - \left(\pi_i^{ss} - \pi_i^{fs}\right)\right]} = 0.$$

Proposition 4 *If both acquirer and target have research capabilities, and if conditional payoffs are independent of R&D effort, then a merged entity can either increase or reduce its investments in each lab compared to the pre-merger investments:*

(i) if both (7) and (6) hold, then $x^m > x^*$.

(ii) if (7) holds and (6) does not hold, then $x^m > x^*$ *if and only if* $x^* < \hat{x}$.

(iii) if (7) does not hold and (6) holds, then $x^m > x^*$ *if and only if* $x^* > \hat{x}$.

(iv) if both (7) and (6) do not hold, then $x^m < x^*$.

Proof. In this setting, where conditional payoffs are independent of R&D effort, evaluating

[6] A similar set-up is analized in Mukherjee (2022) in the setting with horizontally differentiated products in the absence of vertical differentiation, while conditional payoffs are affected only through exogenous reduction in marginal cost of production in case of sucess.

post-merger FOC (3) at the symmetric pre-merger equilibrium, we obtain:

$$\frac{\partial \beta_i(x^*)}{\partial x_i}\beta_j(x^*)\left[\left(\hat{\pi}_i^{ss}+\hat{\pi}_j^{ss}-\left(\hat{\pi}_i^{fs}+\hat{\pi}_j^{sf}\right)\right)-\left(\pi_i^{ss}-\pi_i^{fs}\right)\right]$$
$$+\frac{\partial \beta_i(x^*)}{\partial x_i}(1-\beta_j(x^*))\left[\left(\hat{\pi}_i^{sf}+\hat{\pi}_j^{fs}-\left(\hat{\pi}_i^{ff}+\hat{\pi}_j^{ff}\right)\right)-\left(\pi_i^{sf}-\pi_i^{ff}\right)\right]\gtrless 0.$$

Denote

$$\begin{aligned}K_1 &= \left[\left(\hat{\pi}_i^{ss}+\hat{\pi}_j^{ss}-\left(\hat{\pi}_i^{fs}+\hat{\pi}_j^{sf}\right)\right)-\left(\pi_i^{ss}-\pi_i^{fs}\right)\right]\\ K_2 &= \left[\left(\hat{\pi}_i^{sf}+\hat{\pi}_j^{fs}-\left(\hat{\pi}_i^{ff}+\hat{\pi}_j^{ff}\right)\right)-\left(\pi_i^{sf}-\pi_i^{ff}\right)\right].\end{aligned}$$

Then since $\beta' > 0$, the inequality is equivalent to

$$\begin{aligned}\beta(x^*)K_1+(1-\beta(x^*))K_2 &\gtrless 0\\ \text{or}\quad \beta(x^*)(K_1-K_2)+K_2 &\gtrless 0.\end{aligned}$$

Suppose $K_1 > 0$ and $K_2 > 0$, then the FOC is positive. This proves result in (*i*).
Suppose $K_1 < 0$ and $K_2 < 0$, then the FOC is negative. This proves result in (*iv*).
Suppose $K_1 < 0$ and $K_2 > 0$, then the FOC is positive for all $x^* < \hat{x}$. This proves result in (*ii*).
Suppose $K_1 > 0$ and $K_2 < 0$, then the FOC is positive for all $x^* > \hat{x}$. This proves result in (*iii*).

Note $\hat{x}=\beta^{-1}\left(\frac{K_2}{K_2-K_1}\right)=\beta^{-1}\left(\frac{\left[\left(\hat{\pi}_i^{sf}+\hat{\pi}_j^{fs}-\left(\hat{\pi}_i^{ff}+\hat{\pi}_j^{ff}\right)\right)-\left(\pi_i^{sf}-\pi_i^{ff}\right)\right]}{\left[\left(\hat{\pi}_i^{sf}+\hat{\pi}_j^{fs}-\left(\hat{\pi}_i^{ff}+\hat{\pi}_j^{ff}\right)\right)-\left(\pi_i^{sf}-\pi_i^{ff}\right)\right]-\left[\left(\hat{\pi}_i^{ss}+\hat{\pi}_j^{ss}-\left(\hat{\pi}_i^{fs}+\hat{\pi}_j^{sf}\right)\right)-\left(\pi_i^{ss}-\pi_i^{fs}\right)\right]}\right).$

■

Condition in part (i) of the proposition indicates that when the return on innovation for joint entity is bigger than the benefit from unilateral success in R&D for independent firm both in case of rival's success in R&D stage and in case of rival's failure, the merger will always result in higher investments in each research lab.[7] Condition in part (iv) of the proposition implies that the return on innovation for joint entity is smaller than the benefit from unilateral success in R&D for independent firm both when the rival succeeds in R&D stage and when the rival fails. This excludes the possibility of positive effects of mergers on innovation independent of the R&D cost structure. Condition in part (ii) of the proposition implies that the return on innovation for joint entity is bigger than the benefit from unilateral success in R&D for independent firm in case of rival's failure, but not in case of rival's success in R&D stage. In such a scenario merger can result in higher investments in each lab, but only when x^* is sufficiently low, which can occur

[7] As we explained above, a special case of this condition was analyzed in Jullien and Lefouili (2020). There due to absence of vertical product differentiation and assumption of no entry in case of failure (which excludes possibility of replacement effect) we have $\hat{\pi}_i^{sf}+\hat{\pi}_j^{fs}-(\hat{\pi}_i^{ff}+\hat{\pi}_j^{ff})=\pi_i^{sf}-\pi_i^{ff}=\Pi_1$, which is single product monopoly profit in terminology of Jullien and Lefouili (2020), and $\hat{\pi}_i^{ss}+\hat{\pi}_j^{ss}-(\hat{\pi}_i^{fs}+\hat{\pi}_j^{sf})=\Pi_2-\Pi_1$, while $\pi_i^{ss}-\pi_i^{fs}=\pi_2$, where Π_2 is two-product monopoly profit and π_2 is duopoly profit if each firm succeeds in terminology of Jullien and Lefouili (2020). Note that the result of Jullien and Lefouili (2020) is different from Federico et al. (2017), Denicolo and Polo (2018) due to presence of the price effects, i.e. $\Pi_2 > \Pi_1$ (or $\Pi_2 > \pi_2$)

when investment is costly, i.e. marginal cost of R&D is increasing sufficiently fast or R&D cost function is sufficiently convex. Condition in part (iii) of the proposition identifies the scenario where returns on innovation for merged entity are higher in case of rival's success, but not in case of rival's failure. In that case merger could potentially increase the incentives to innovate when x^* is sufficiently high, which can occur when investment is less costly or marginal cost of R&D is sufficiently flat.

This proposition makes it clear that under more general assumptions the nature of the cost is crucial for evaluation of the effects of mergers on innovation incentives. This effect has been overlooked in the literature. Our more general model highlights this effect and helps to identify a richer set of results, which result from interaction of replacement effects and price effects, and a broader range of environments, where mergers can spur innovation.

6.1 A micro-founded example

In this section we use several micro-founded examples to illustrate the results of the general reduced form model described above in the setting where both firms have research capabilities and conditional payoffs are independent of R&D effort. In this section to optimize notation we set $\hat{\pi}_i^{ss} + \hat{\pi}_j^{ss} = \hat{\pi}_m^{ss}$, $\hat{\pi}_i^{fs} + \hat{\pi}_j^{sf} = \hat{\pi}_m^{fs}$, $\hat{\pi}_i^{sf} + \hat{\pi}_j^{fs} = \hat{\pi}_m^{sf}$, $\hat{\pi}_i^{ff} + \hat{\pi}_j^{ff} = \hat{\pi}_m^{ff}$. Note also that $\hat{\pi}_m^{ss} \geq \hat{\pi}_m^{sf} = \hat{\pi}_m^{fs} > \hat{\pi}_m^{ff}$

6.1.1 Linear Probability of Innovation

We start with the example where probability of innovation is linear function of investment levels, i.e. $p(x_i) = x_i$ and $1 - p(x_i) = 1 - x_i$. This set up is similar to Federico et al. (2017) and Jullien and Lefouili (2020) and requires restricting the domain of x_i to $[0, 1]$ interval. Further, we assume quadratic investment costs $c(x_i) = \frac{\gamma x_i^2}{2}$, where γ is the parameter which reflects the steepness of the cost function. The assumptions about functional forms imply that $p'(x) = 1$ and $c'(x) = \gamma x$. To characterize the profits in the second stage we use the demand structure which stems from the model with the quality-augmented quadratic utility function and Cournot competition.[8] For the sake of exposition, we assume away horizontal product differentiation by setting $\sigma = 2$ and set marginal cost of production to zero.[9] The basic product has low quality $s_f > 0$. If the firm's investment turns out successful, we assume that the firm is able to offer a product of

[8]See Sutton (1997, 2001):
$U = \sum_{i=1}^{2} \left[\alpha q_i - \left(\frac{\beta q_i}{s_i}\right)^2\right] - \sigma \sum_{i=1}^{2} \sum_{i<j} \frac{\beta q_i}{s_i} \frac{\beta q_j}{s_j} - \sum_{i=1}^{2} p_i q_i$

[9]In a more general setting with both vertical and horisontal product differentiation we would have more complicated expressions for corresponding profits (see (8)). However, this will not effect the main insights

higher quality s_s, with $s_f < s_s < 2s_f$.[10] Otherwise, the firm continues offering basic low quality product. We normalize the marginal cost of production to zero. Under these assumptions the corresponding profits are given by

$$\pi_i^{ss} = \frac{\alpha^2 s_s^2}{18\beta^2},\ \pi_i^{sf} = \frac{\alpha^2(2s_s - s_f)^2}{18\beta^2},\ \pi_i^{fs} = \frac{\alpha^2(2s_f - s_s)^2}{18\beta^2},\ \pi_i^{ff} = \frac{\alpha^2 s_f^2}{18\beta^2} \tag{9}$$
$$\hat{\pi}_m^{ss} = \hat{\pi}_m^{sf} = \ \hat{\pi}_m^{fs} = \frac{\alpha^2 s_s^2}{8\beta^2},\ \hat{\pi}_m^{ff} = \frac{\alpha^2 s_f^2}{8\beta^2}.$$

Now (1) evaluated at the pre-merger symmetric equilibrium, x^*, and (2) valuated at the post-merger symmetric equilibrium, x^m,can be rewritten as follows

$$x^* \left(\pi_i^{ss} - \pi_i^{fs}\right) + (1 - x^*) \left(\pi_i^{sf} - \pi_i^{ff}\right) = \gamma x^* \tag{10}$$

$$x^m \left(\hat{\pi}_m^{ss} - \hat{\pi}_m^{fs}\right) + (1 - x^m) \left(\hat{\pi}_m^{sf} - \hat{\pi}_m^{ff}\right) = \gamma x^m \tag{11}$$

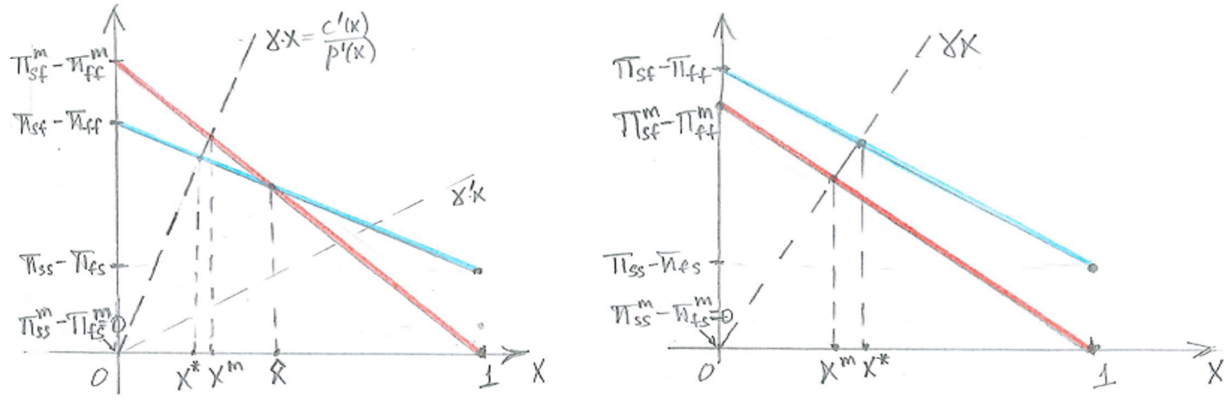
This system can be represented graphically in the diagram, where on the horizontal axis we place the amount of investment x and plot the values of the LHS and the RHS of these equations as functions of x. The LHSs of (10) and (11) are given by the blue and red decreasing lines, respectively, and the RHS is given by the dashed increasing line.[11] Analysis of the profits in (9) implies that the following two scenarios are possible in this micro-founded example. Either we have $\hat{\pi}_m^{sf} - \hat{\pi}_m^{ff} > \pi_i^{sf} - \pi_i^{ff}$ for $1 < \frac{s_s}{s_f} < \frac{9}{7}$, which corresponds to Figure 1(a), where (7) holds. Alternatively, we can have $\hat{\pi}_m^{sf} - \hat{\pi}_m^{ff} < \pi_i^{sf} - \pi_i^{ff}$ for $\frac{9}{7} < \frac{s_s}{s_f} < 2$, which corresponds to Figure 1(b), where (7) does not hold. Note that in this example in the absence of horizontal differentiation $\hat{\pi}_m^{ss} - \hat{\pi}_m^{fs} = 0 < \pi_i^{ss} - \pi_i^{fs}$ for all parameters, i.e. (6) does not hold. Also due to $\hat{\pi}_m^{sf} - \hat{\pi}_m^{ff} - \left(\hat{\pi}_m^{ss} - \hat{\pi}_m^{fs}\right) > \pi_i^{sf} - \pi_i^{ff} - \left(\pi_i^{ss} - \pi_i^{fs}\right)$ the LHS in (11) decreases faster then LHS in (10). Hence, in the second scenario we always have $x^m < x^*$. While in the first scenario the outcome depends on the shape of the investment cost function. In particular, on how steep the R&D marginal cost is, which in this example is determined by the parameter γ. We can see that for sufficiently large γ, we have $x^m > x^*$. This happens for $x < \hat{x}$. These two scenarios illustrate the differences between Proposition 4(ii) and Proposition 4(iv).

presented in this example.

$$\pi_i^{ss} = \frac{2\alpha^2 s_s^2}{(4+\sigma)^2\beta^2},\ \pi_i^{sf} = \frac{2\alpha^2(4s_s - \sigma s_f)^2}{(16-\sigma^2)^2\beta^2},\ \pi_i^{fs} = \frac{2\alpha^2(4s_f - \sigma s_s)^2}{(16-\sigma^2)^2\beta^2},\ \pi_i^{ff} = \frac{2\alpha^2 s_f^2}{(4+\sigma)^2\beta^2} \tag{8}$$
$$\hat{\pi}_m^{ss} = \frac{\alpha^2 s_s^2}{2(2+\sigma)\beta^2},\ \hat{\pi}_m^{sf} = \ \hat{\pi}_m^{fs} = \frac{\alpha^2(s_s^2 + s_f^2 - \sigma s_s s_f)^2}{2(4-\sigma^2)\beta^2},\ \hat{\pi}_m^{ff} = \frac{\alpha^2 s_f^2}{2(2+\sigma)\beta^2}.$$

[10]The restriction $s_s < 2s_f$ rules out drastic innovations.

[11]Note that under the assumptions in Federico et al. (2017) and Denicolo and Polo (2018) the blue decreasing line will always be above the red decreasing line. This holds because $\hat{\pi}_m^{sf} - \hat{\pi}_m^{ff} = \pi_i^{sf} - \pi_i^{ff}$ and $\hat{\pi}_m^{sf} - \hat{\pi}_m^{ff} - \left(\hat{\pi}_m^{ss} - \hat{\pi}_m^{fs}\right) > \pi_i^{sf} - \pi_i^{ff} - \left(\pi_i^{ss} - \pi_i^{fs}\right)$ so that LHS in (11) decreases faster then LHS in (10). This immediately explains the impossibility of the result with R&D enhancing merger in those contributions.



(a) (b)

Figure 1

In this example we can compute closed form solutions for both pre-merger and post-merger investment levels. They are given by

$$x^* = \frac{\left(\pi_i^{sf} - \pi_i^{ff}\right)}{\gamma + \left(\pi_i^{sf} - \pi_i^{ff}\right) - \left(\pi_i^{ss} - \pi_i^{fs}\right)}$$

$$x^m = \frac{\left(\hat{\pi}_m^{sf} - \hat{\pi}_m^{ff}\right)}{\gamma + \left(\hat{\pi}_m^{sf} - \hat{\pi}_m^{ff}\right) - \left(\hat{\pi}_m^{ss} - \hat{\pi}_m^{fs}\right)}$$

Comparing these two expressions gives lower bound on parameter γ, for which merger can result in higher R&D investments:

$$\overline{\gamma} = \frac{\left(\hat{\pi}_m^{sf} - \hat{\pi}_m^{ff}\right)\left(\pi_i^{ss} - \pi_i^{fs}\right) - \left(\hat{\pi}_m^{ss} - \hat{\pi}_m^{fs}\right)\left(\pi_i^{sf} - \pi_i^{ff}\right)}{\left(\hat{\pi}_m^{sf} - \hat{\pi}_m^{ff}\right) - \left(\pi_i^{sf} - \pi_i^{ff}\right)}$$

For all $\gamma < \overline{\gamma}$ merger will result in a reduction in R&D investments compared to pre-merger equilibrium. We summarize the analysis of this example in the following proposition.

Proposition 5 *The merged entity increases its investments in each lab compared to the pre-merger investments if and only if:*

- $\hat{\pi}_m^{sf} - \hat{\pi}_m^{ff} > \pi_i^{sf} - \pi_i^{ff}$ *(i.e.* $1 < \frac{s_s}{s_f} < \frac{9}{7}$*) and*
- $\gamma > \overline{\gamma}$ *(i.e. R&D cost function is sufficiently convex)*

Otherwise, the merged entity decreases R&D investments.

This example illustrates the scenarios identified in Proposition 4, parts (ii) and (iv). To the best of our knowledge, a situation like part (ii) has not been identified in previous contributions.

It shows the importance of the shape of the R&D cost function in combination with conditions on the second stage profits. We show that the new result, where mergers can enhance innovation incentives, can be obtained also for sharply increasing marginal cost in environments where a failure to develop a new higher quality product does not result in a zero payoff and price effects are present.

7 Conclusions

We formulate a general two-stage game to study the implications of mergers on R&D. We identify three channels through which a merger affects R&D investment: anticipation of price coordination that enhances payoffs, internalisation of a direct innovation externality stemming from an enhanced chance of innovation success, and internalisation of an indirect innovation externality arising from business-stealing in the product market. Under price regulation, only the latter two effects play a role and mergers result in an unambiguous decrease in R&D. Our model captures the results of existing models with stochastic innovation intended to enter a new market and deterministic innovation intended to reduce costs or improve quality. We argue that the literature so far has restricted the analysis to settings where the payoffs from innovation failure are equal to zero (an hence the Arrow replacement effect is absent) and illustrate the importance of the pre-merger level of innovation, and hence the magnitude of investment costs.

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