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Marco Buso, Cesare Dosi, Michele Moretto

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By **Marco Buso**
(Department of Economics and Management, University of Padova and Interuniversity Centre for Public Economics - CRIEP)
Cesare Dosi
(Department of Economics and Management, University of Padova and Interuniversity Centre for Public Economics - CRIEP)
Michele Moretto
(Department of Economics and Management, University of Padova and Interuniversity Centre for Public Economics - CRIEP)

Summary

In a continuous-time setting, we study the design of a dynamic contract between a government and a private entity, wherein the latter commits to pay the government in return for the exclusive right to sell a service by operating a public facility. Private revenues are modelled as depending on the unobservable ability to seize market opportunities and on imperfectly correlated changes in consumers' preferences. We show that optimal regulation requires an appropriate combination of fixed and variable payments to the government, acting together both as an information revelation mechanism and as a risk sharing device.

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JEL Classification: D81, D82, D86, H54

Address for correspondence:

Marco Buso
Assistant Professor
Department of Economics and Management
University of Padova and Interuniversity Centre for Public Economics (CRIEP)
via del Santo 33 - 35123 Padova (Italy)
E-mail: marco.buso@unipd.it

Dynamic Regulation of Public Franchises with Imperfectly Correlated Demand Shocks

Marco Buso* Cesare Dosi† Michele Moretto‡

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Abstract

In a continuous-time setting, we study the design of a dynamic contract between a government and a private entity, wherein the latter commits to pay the government in return for the exclusive right to sell a service by operating a public facility. Private revenues are modelled as depending on the unobservable ability to seize market opportunities and on imperfectly correlated changes in consumers' preferences. We show that optimal regulation requires an appropriate combination of fixed and variable payments to the government, acting together both as an information revelation mechanism and as a risk sharing device.

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*Department of Economics and Management, University of Padova and Interuniversity Centre for Public Economics (CRIEP), via del Santo 33 35123 Padova, Italy (marco.buso@unipd.it)

†Department of Economics and Management, University of Padova and Interuniversity Centre for Public Economics (CRIEP), via del Santo 33 35123 Padova, Italy (cesare.dosi@unipd.it)

‡Department of Economics and Management, University of Padova and Interuniversity Centre for Public Economics (CRIEP), via del Santo 33 35123 Padova, Italy (michele.moretto@unipd.it)

1 Introduction

Outsourcing of traditionally public sector activities has become a common practice in several countries. This has occurred in various ways and with different results, impacted by the governments' ability to transform themselves from direct providers into effective regulators (OECD, 2012; European Commission, 2017).

The increased use of public-private partnerships (PPPs) for infrastructure services somehow symbolizes the wave that started in the 1980s. As it is currently used, the term PPP covers a wide spectrum of contractual arrangements, which can be classified with different criteria (Delmon, 2010), such as the tasks assigned (e.g. financing, building and operating new infrastructures, or simply operating existing public facilities) and the source of private cash-flows (drawing revenues from taxpayers or from service users), as well as the kind of risks projects are exposed to and the way in which they are distributed between the contracting parties.

Risks identification and allocation are indeed at the heart of PPP projects (Yescombe and Farquharson, 2018) which, because of their usually long-term horizons, are almost inevitably exposed to more or less severe stressors. This calls for well defined risk sharing rules, enabling the parties to rapidly react to changing circumstances while keeping the partnership alive (Demirel et al., 2017; Beuve et al., 2019). At the same time, another policy challenge emerges from the exclusivity granted to "concessionaires", which requires appropriate measures to avoid potential inefficiencies due to market power and information imbalances.

In the economic literature, monopoly regulation under adverse selection was first addressed by using static models where private information parameters (e.g. production costs) are assumed not to vary over time (Baron and Myerson, 1982; Laffont and Tirole, 1986; Riordan and Sappington, 1987). Further contributions introduced some dynamics, mostly using two-period models (Baron and Besanko, 1984; Laffont and Tirole, 1993).

Assuming perfect commitment, there are three possible environments for such a discrete-time setup (Laffont and Martimort, 2002). First, if private information parameters change with time but are perfectly correlated between the periods, it is optimal for the government to commit to the repetition of the static contract. Second, if the realizations in each period are completely independent, the parties sign a long-term contract before the second-period information is disclosed, thus only the first-period private information is costly to the principal. Finally, if private information parameters are weakly correlated, the government uses the first report to update his beliefs on the agent's second-period type. In this case, an intertemporal incentive-compatibility constraint needs to be added to the principal's maximization problem, such that the agent has no incentives to misreport his type in both periods.

However, when moving from the two-period to a multiperiod-time setting, things become more difficult. Indeed, while the first and the second cases are still solved as described above[1], when private information parameters are more realistically modelled as subject to imperfectly correlated shocks, the solution is not anymore straightforward, because, since the space for deviations by the agent could be very rich, the standard incentive-compatible mechanism is in general hard to be implemented (Pavan et al., 2014; Bergemann and Valimaki, 2018). Yet, this intertemporal adverse selection problem has been technically overcome by Bergemann and Strack (2015) who derive, in a continuous-time setting, necessary conditions for a direct incentive-compatible mechanism and use their findings to obtain a close form solution for a dynamic contract between a revenue-maximizing seller and a privately informed buyer.

Building on Bergemann and Strack (2015), in this paper we study the optimal design of a contract wherein a private entity is granted with the exclusive right to supply a good or service and to collect all revenues thereof, by operating an existing public facility.[2] In exchange, the awarding authority shall

[1]See, for instance, Auriol and Picard (2013) who study in a continuous-time model the optimal regulation of a monopoly firm holding private information on production costs which are assumed not to vary over time.

[2]Our results can be generalized to other types of PPPs, namely to greenfield projects involving investment obligations, where the design features and the timing of investment are predetermined and enforceable by contract. On endogeneous investment timing in PPPs, see for instance, Dosi and Moretto (2015) and Buso et al. (2021).

receive continued compensation. For instance, public franchises of this kind, in some legal system referred to as "lease contracts",[3] generally provide for the payment of a "rental price", which often takes on the form a flat fee (World Bank, 2022).

Uncertainty in our model comes from the demand side, as we assume that revenues from sales depends on the firm's ability to seize market opportunities, which is private information, as well as on exogeneous changes in consumers' preferences evolving as a Brownian motion.

As our main result, we derive conditions under which both risk sharing and monopoly regulation issues, which are often treated separately in the PPP literature, can be dealt with together within a dynamic contract. Specifically, we find that optimal regulation requires an appropriate combination of fixed and variable transfers between the parties, with the latter linked to both the predicted revenue potential and the actual sales. Moreover, we study how each component of the two-part schedule can be affected by the firm's type, the expected volatility of private revenues and the importance attached by the awarding authority to public income. In so doing, we show that the use of flat fees, rather than time-adjusted transfers, that is, the repetition of a standard static contract, may find justification only where there is little uncertainty about future private benefits or, alternatively, when the contracting authority gives higher priority to public income relative to other welfare concerns.

The remainder is organized as follows. In the next Section we set up the model and assumptions. In Section 3 we derive the optimal regulatory scheme, whose main features are illustrated in Section 4 through numerical examples. Section 5 concludes. The proofs are presented in the appendices.

[3] Although they are not completely overlapping, in the literature on PPPs the terms lease contracts, *affermages* and management contracts -which share the feature of not providing for substantial investment obligations- are sometimes used interchangeably. On the differences between these contract types, see World Bank (2022).

2 Set up

Consider a public authority (henceforth, "the government", he) offering a private firm (she) a take-it-or-leave-it contract that gives her the exclusive right to supply a specific good or service ("good") by operating an existing government-owned facility.

Under the contract, signed at $t = 0$, the firm commits to pay a sum of money (a "tax") for using the asset and for cashing monopolistic revenues. For the sake of simplicity, we assume that the firm will not bear any additional costs and that the franchise term is sufficiently long to be approximated as infinite.[4]

The firm's activity generates in every period $t \geq 0$ a consumer surplus denoted by $S(Q_t, x_t)$, where Q_t is the output level and x_t is the "demand shifter" (Auriol and Picard, 2013).

$S(Q_t, x_t)$ has the standard properties, namely: $S_Q > 0$, $S_{QQ} < 0$, and $S_x > 0$, $S_{Qx} > 0$. The willingness to pay for an extra unit of the good and, therefore, the per-unit-of-time (henceforth, the "instantaneous" or "current") surplus increases with x_t.

Consumers cannot store and transfer goods to the next time periods and the whole production is sold at the market equilibrium price $P(Q_t, x_t) \equiv S_Q(Q_t, x_t)$ that defines the inverse demand function.

In order to highlight the characteristics of the regulatory scheme and without loosing in generality, we shape the surplus $S(Q_t, x_t)$ as a quadratic function, with a linear demand shifter: $S(Q_t, x_t) = Q_t(x_t - \frac{Q_t}{2})$. Therefore, $S_Q(Q_t, x_t) \equiv P(Q_t, x_t) = x_t - Q_t > 0 > S_{QQ}(Q_t, x_t) = -1$.

The demand shifter is assumed to evolve stochastically according to the following trendless geometric Brownian process:[5]

[4] The inclusion of operating costs, minimized for all production levels, would not qualitatively alter our results and conclusions. The same applies to the assumption of an infinitely long contract period, which allows us to get a closed form solution for the optimal regulation problem.

[5] The assumption of a trendless random walk allows us to focus on the pure effect of the uncertainty. Notice, however, that by the Markov property of Eq. (1), our general results would not be altered by using a non-zero trend for x_t.

$$dx_t = \sigma x_t dZ_t \qquad\qquad x_{t=0} = x_0 \tag{1}$$

where $\sigma > 0$ is the constant instantaneous volatility and $Z_t \sim N(0,t)$ is a standard Wiener process having a normal distribution with zero mean and variance t.

By solving (1), the demand shifter can be written as follows:

$$x_t \equiv \phi(t, x_0, Z_t) = x_0 \exp\left(-\frac{\sigma^2}{2}t + \sigma Z_t\right) \tag{2}$$

Eq. (2), which highlights that x_t depends on the initial value x_0, on the uncertainty parameter σ and on the contemporaneous shock Z_t, has several interesting properties.[6]

First, the higher is x_0 the higher will, ceteris paribus, be the future demand of the good, i.e. $\frac{\partial\phi(t,x_0,Z_t)}{\partial x_0} > 0$. Second, the relative impact of x_0 versus the $Z_{t,}$, i.e. $\frac{\partial\phi(t,x_0,Z_t)}{\partial x_0} / \frac{\partial\phi(t,x_0,Z_t)}{\partial Z_t} = \frac{1}{\sigma x_0}$, is decreasing in x_0.[7]

In short, Eq. (2) says that a high (low) value of x_0 is less (more) informative for predicting the future values of x_t, because future demands will be more (less) subject to contemporaneous shocks Z_t. Similarly, the information potential of x_0 decreases as the uncertainty parameter σ increases.

We assume that while σ is public knowledge, the firm is better informed than the government about x_t ($t \geq 0$). The initial value x_0, reflecting the firm's innate ability to seize market opportunities ("the firm's type")[8], is dis-

[6]For a trendless process like (1), the autocorrelation between two values of the willingness-to-pay is given by:

$$\rho_{s,t} = \frac{cov(x_s, x_t)}{\sqrt{V(x_s)}\sqrt{V(x_t)}} = \left(\frac{e^{\sigma^2 s} - 1}{e^{\sigma^2 t} - 1}\right)^{1/2} < 1$$

where $s < t$. Notice that, for any given s, $\rho_{t,s} \to 0$ either when $t \to \infty$ or when $\sigma \to \infty$. In all other cases we get a partial autocorrelation. In other words, the values of x_t at neighboring time points are more and more strongly and positively correlated as time goes by. On the other hand, the values of x_t at distant time points are less and less correlated.

[7]Although the impact of x_0 on x_t reduces over time, it never fades. See Bergemann and Strack (2015) for a more in-depth discussion of these properties.

[8]In a standard (static) adverse selection problem, or in a dynamic model with perfectly correlated shocks, this would actually be the only private information parameter of interest.

tributed on $[x^l, x^h]$, according to the cumulative distribution function $G(x_0)$, with density $g(x_0) \geq 0$ and $g(x^l)$, $g(x^h) > 0$, which is common knowledge.[9] The function $G(x_0)$ is such that $\frac{1-G(x_0)}{g(x_0)x_0}$ is monotone and decreasing, with $x^l g(x^l) \geq k > 0$.[10]

Since the tax that she has committed to pay is the only cost incurred by the firm, her instantaneous utility function can be written as:

$$u_t = \pi(Q_t, x_t) - T_t, \tag{3}$$

where $\pi(Q_t, x_t) = P(Q_t, x_t)Q_t$ are the firm's gross profits and T_t is the current tax.

Thus, the firm's expected intertemporal utility at $t = 0$ is given by:

$$U = E_0 \left[\int_0^{\infty} e^{-rt} u(Q_t, x_t) dt \right] \tag{4}$$

where r is the discount rate.

The government is assumed to be benevolent and utilitarian, in the sense that he maximizes the sum of the expected present value of the intertemporal consumer and producer surpluses, plus the welfare gains arising from tax receipts:

$$\begin{aligned} W &= E_0 \left[\int_0^{\infty} e^{-rt} (S(Q_t, x_t) - \pi(Q_t, x_t) + (1+\lambda)T_t) dt \right] + U(x_0) \qquad (5) \\ &= E_0 \left[\int_0^{\infty} e^{-rt} (S(Q_t, x_t) + \lambda T_t) dt \right] \end{aligned}$$

where $\lambda > 0$ indicates that a unit of tax revenue from the firm yields a net welfare gain, by saving an excess burden of taxation in other markets.

See, for instance, Baron and Besanko (1984), Auriol and Picard (2009; 2013).

[9] As in Arve and Zwart (2014), Skrzypacz and Toikka (2015) and Buso et al. (2021), this is equivalent to assuming that the firm's private information is represented by two stochastic processes, where the one representing the initial value is constant after time zero, but influences the transitions of the second one.

[10] Note that this condition is strictly weaker than the standard increasing hazard rate assumption (see, e.g., Guesnerie and Laffont 1984; Jullien 2000).

We assume that government and the firm share the same time preferences, i.e. the same discount rate r.

3 The optimal contract

3.1 Incentive-compatibility conditions

The initial value x_0 is private information. Moreover, Eq. (1) implies that, even if the government could get x_0 revealed, this information would not be sufficient to infer the ex-post values of x_t. Hence, the government's problem consists of finding a mechanism capable of inducing the firm to truthfully report x_t ($t \geq 0$).

This intertemporal adverse selection problem can be addressed by restricting attention to a smaller class of deviations called "consistent deviations" (Bergemann and Strack, 2015).[11]

Borrowing this approach, the government's problem can be solved in two steps.

1. For any given initial value x_0, the government will find it optimal to commit himself to the repetition of a standard static regulatory contract where, at each $t > 0$, the firm reports x_t truthfully.

2. Since each future realization x_t depends on the initial value x_0 and the contemporaneous shock Z_t, i.e. $x_t = \phi(x_0, Z_t)$, the government's problem reduces to induce the firm to report x_0 truthfully.[12]

[11] The concept of "consistent deviations" can be summarized as follows. If a firm, whose true initial type is x_0, misreports by reporting $\hat{x}_0$, then she will continue to misreport, by reporting $\hat{x}_t = \phi(\hat{x}_0, Z_t)$ instead of the true value $x_t = \phi(x_0, Z_t)$ in all $t > 0$. In other words, since x_t ($t > 0$) depends on x_0, a firm misreporting with a "consistent deviation", continues to misreport her type x_t in all future periods. This means that the type reported at time t, $\hat{x}_t$, would have been be the true type if the initial value was $\hat{x}_0$, given the actual contemporaneous shock, Z_t. Notice that this definition is well suitable with Eq. (2): each new realization of Z_t determines a new realization of x_t which depends only on time and x_0.

[12] Thereafter, we drop the direct dependence on time in $\phi(x_0, Z_t)$ for simplicity of the notation.

Thus, by the separability of the problem, the optimal tax schedule comprises two components. First, an annuitized fixed tax $F(x_0)$ for the revelation of x_0.[13] Second, a time-varying transfer $TV(x_0, x_t)$ for the revelation of x_t $(t > 0)$.

As usual, it is convenient to work backward, starting from $t > 0$. Assuming that the government has already obtained a truthful report of x_0, the firm's intertemporal utility at $t > 0$ becomes the sum over time of single standard problems.

Specifically, by (3) and (4) , we get:

$$U(x_0, \hat{x}_t, x_t) = E_0 \left[\int_0^{\infty} e^{-rt} (\underbrace{\pi(Q(x_0, \hat{x}_t), x_t) - TV(x_0, \hat{x}_t)} - F(x_0)) dt \right] \quad (6)$$

where $TV(x_0, \hat{x}_t)$ is such that the firm truthfully reports $\hat{x}_t = x_t$, for all $t > 0$.

Defining $\tilde{u}(x_0, \hat{x}_t, x_t) = \pi(Q(x_0, \hat{x}_t), x_t) - TV(x_0, \hat{x}_t)$, the necessary and sufficient conditions for incentive-compatibility are the following:[14]

$$\frac{d\tilde{u}(Q(x_0, x_t), x_t))}{dx_t} = Q(x_0, x_t) \text{ for all } t > 0 \quad (7)$$

$$\frac{dQ(x_0, x_t)}{dx_t} > 0 \quad \text{for all } t > 0 \quad (8)$$

$$Q(x_0, x_t) \geq 0 \quad \text{for all } t > 0 \quad (9)$$

Once $TV(x_0, x_t)$ has been determined, it remains to determine the fixed part $F(x_0)$, that is, the government's problem reduces to a new single standard adverse selection problem.

The firm's utility becomes:

$$U(x_0, \hat{x}_0) = E_0 \left[\int_0^{\infty} e^{-rt} (\pi(Q(\hat{x}_0, \phi(x_0, Z_t)), \phi(x_0, Z_t))) - TV(\hat{x}_0, \phi(x_0, Z_t) - F(\hat{x}_0)) dt \right], \quad (10)$$

[13] Alternatively, the fixed part could be charged up front, instead of being annuitized.

[14] The SOC for the problem is presented in Appendix A.

where $F(\hat{x}_0)$ is determined in such a way so as to induce the firm to report truthfully the initial value x_0.

As $x_t = \phi(x_0, Z_t)$, the necessary and sufficient condition for incentive-compatibility is now:

$$\frac{dU(x_0)}{dx_0} = E_0\left[\int_0^\infty e^{-rt}Q(x_0, \phi(x_0, Z_t))\frac{\partial\phi(x_0, Z_t)}{\partial x_0}dt\right] \tag{11}$$

whereas the second order sufficient condition is:

$$\frac{dQ(x_0, x_t)}{dx_0} \geq 0 \tag{12}$$

with $\frac{\partial(\pi Q(x_0,\phi(x_0,Z_t)),\phi(x_0,Z_t))}{\partial\phi(x_0,Z_t)} = Q(x_0, \phi(x_0, Z_t))$.[15]

3.2 The two-part tax

Let's consider the first step. If x_0 has already been revealed by the firm, the government can determine the quantity to be produced at each time t and the optimal variable tax amount by maximizing the following function:

$$\max_{Q(x_0,x_t),TV(x_0,x_t)} \int_{x^l}^{x^h} \left\{E_0\left[\int_0^\infty e^{-rt}(S(Q(x_0, x_t), x_t) + \lambda T(x_0, x_t))dt\right]\right\} g(x_0)dx_0, \tag{13}$$

where $T(x_0, x_t) = F(x_0) + TV(x_0, x_t)$, subject to (7) and (8) and the following intertemporal participation constraint:

$$U(x_0) \geq 0 \tag{14}$$

The participation constraint (14) stipulates that the total utility, and not, therefore, necessarily the instantaneous utility, must be positive. That is, we assume that the firm commits to stay in the contract, even if, in some phases of the project, her instantaneous (after tax) profits turn out to be negative.[16]

[15] The SOC for the problem is presented in Appendix A.

[16] Indeed, as in Martimort and Straub (2016), we assume that profits in each period are redistributed as dividends to the firm's owners in the same period. However, unlike them,

Since $F(x_0)$ does not depend on x_t, we have a standard adverse selection problem of regulation under asymmetric information (Baron and Myerson, 1982; Laffont and Tirole, 1993).

The general solutions for Q_t^* and TV_t^* are (see Appendix B):

$$S_Q(Q^*(x_0,x_t),x_t)+\lambda\pi_Q(Q^*(x_0,x_t),x_t))-\lambda\frac{(1-G(x_0))}{g(x_0)}\frac{\partial\phi(x_0,Z_t)}{\partial x_0}=0 \text{ for all } t>0 \tag{15}$$

$$TV^*(x_0,x_t)=\pi(Q^*(x_0,x_t),x_t))-\int_0^{x_t}Q(x_0,z)dz \quad \text{for all } t>0 \tag{16}$$

Given the optimal values of $TV^*(x_0,x_t)$ and $Q^*(x_0,x_t)$, substituting (16) into (6) gives:

$$U(x_0)=E_0\left[\int_0^{\infty}e^{-rt}[\int_0^{x_t}Q^*(x_0,z)dz-F(x_0)]dt\right] \tag{17}$$

Since $TV^*(x_0,x_t)$ is such that the firm, given her initial report x_0, will reveal x_t truthfully, the government's problem reduces to a static design problem where (11)-(12) are the first and second order conditions for incentive-compatibility.

By the Envelope theorem, Eq. (11) implies that:

$$U(x_0)=\int_{x^l}^{x_0}E_0\left[\int_0^{\infty}e^{-rt}Q^*(y,x_t)\frac{\partial\phi(y,Z_t)}{\partial y}dt\right]dy \tag{18}$$

Using (17) and (18), we get the general solution for $F^*(x_0)$:

$$F^*(x_0)=r\left\{E_0\left[\int_0^{\infty}e^{-rt}[\int_0^{x_t}Q^*(x_0,z)dz]dt\right]-\int_{x^l}^{x_0}E_0\left[\int_0^{\infty}e^{-rt}Q^*(y,x_t)\frac{\partial\phi(y,Z_t)}{\partial y}dt\right]dy\right\} \tag{19}$$

The following proposition characterizes the optimal contract, under the assumptions presented in Section 2.

Proposition 1 *For any given x_0, the quantity supplied at each time t that*

we assume that any current losses are covered via new equity. This simplifying assumption implies that the firm does not need to build a liquidity buffer to maintain the firm afloat.

solves problem (13) is given by:

$$Q^*(x_0, x_t) = \frac{1+\lambda}{1+2\lambda}\left[x_0 - \frac{\lambda}{1+\lambda}\frac{(1-G(x_0))}{g(x_0)}\right]\frac{x_t}{x_0} \text{ for all } t > 0 \tag{20}$$

where $\frac{\partial\phi(x_0,Z_t)}{\partial x_0} = \frac{x_t}{x_0}$.

The time-dependent variable tax (16) becomes:

$$TV^*(x_0, x_t) = (x_t - Q^*(x_0, x_t))Q^*(x_0, x_t) - Q^*(x_0, x_t)\frac{x_t}{2} \text{ for all } t > 0 \tag{21}$$

while the fixed tax (19) is:

$$F^*(x_0) = rE_0\left\{\int_0^\infty e^{-rt}[Q^*(x_0, x_t)\frac{x_t}{2} - \int_{x^l}^{x_0} Q^*(y, x_t)\frac{x_t}{y}dy]dt\right\} \tag{22}$$

Proof. Proof: See Appendix B ■

The LHS of (20) includes the information rents, $\frac{1-G(x_0)}{g(x_0)}$, which depend on the initial value x_0. As usual, there is no output distortion for the highest type firm, i.e. $Q^*(x^h, x_t) = \frac{1+\lambda}{1+2\lambda}x_t$, $t > 0$.

Notice that Eq. (20) can be written as:

$$Q^*(x_0, x_t) = Q^*(x_0)\frac{x_t}{x_0}, \tag{23}$$

where $Q^*(x_0) = \frac{1+\lambda}{1+2\lambda}\left[x_0 - \frac{\lambda}{1+\lambda}\frac{1-G(x_0)}{g(x_0)}\right]$ is the optimal quantity at time zero.[17]

Eq. (23) indicates that the optimal quantity supplied at each time $t > 0$ is given by the time zero quantity multiplied by an impulse response function, i.e. $\frac{\partial\phi(x_0,Z_t)}{\partial x_0} = \frac{x_t}{x_0}$, capturing the effect of a small change of x_0 on x_t.[18] In addition, both $\frac{\partial Q^{**}(x_0,x_t)}{x_t}$ and $\frac{\partial Q^*(x_0,x_t)}{\partial x_0} = \frac{\partial Q^*(x_0,x_t)}{x_0} + \frac{\partial Q^*(x_0,x_t)}{x_t}\frac{\partial\phi(x_0,Z_t)}{\partial x_0}$ are positive.

To avoid corner solutions we make the following assumption, stating that

[17]Notice that $Q^*(x_0)$ can be written as $\frac{x_0}{2} + \frac{1}{1+2\lambda}\left[x_0 - \lambda\frac{1-G(x_0)}{g(x_0)}\right]$, where $\frac{x_0}{2}$ is the profit-maximizing quantity and $\frac{1}{1+2\lambda}\left[x_0 - \lambda\frac{1-G(x_0)}{g(x_0)}\right]$ is the adjustment due to regulation.

[18]On the "impulse response function" see Pavan et al. (2014).

the consumers' willingness-to-pay for the first unit of the good is sufficiently large so as to allow even the lowest type firm to earn a non-negative revenue.[19]

Assumption 1. $k \geq \frac{\lambda}{1+\lambda}$

Let's look more in detail at the instantaneous total tax payments:

$$\begin{aligned} T^*(x_0, x_t) &= F^*(x_0) + TV^*(x_0, x_t) \\ &= rE_0 \left[\int_0^\infty e^{-rt} [Q^*(x_0, x_t) \frac{x_t}{2} - \int_{x^l}^{x_0} Q^*(y, x_t) \frac{x_t}{y} dy] dt \right] \\ &\quad + (x_t - Q^*(x_0, x_t)) Q^*(x_0, x_t) - Q^*(x_0, x_t) \frac{x_t}{2} \end{aligned} \quad (24)$$

The third line of Eq. (24) indicates that, like in any standard adverse selection problem, the variable component is given by the firm's revenues, i.e. $(x_t - Q^*(x_0, x_t))Q^*(x_0, x_t)$, minus the information rents, $Q^*(x_0, x_t)\frac{x_t}{2}$, paid to the firm to reveal x_t.

Notice, however, that the same rents also appear, with positive sign, in the fixed component (first term in the second line). This means that the government determines the fixed tax by anticipating the amount of information rents that he will have to pay to induce the firm to disclose x_t. However, since the information initially obtained about x_0 provides some elements for predicting x_t, that amount is reduced by the second term, which represents the rents that the government expects to save in the future.

Using (23), $F^*(x_0)$ can be simplified as follows (see Appendix B):

$$F^*(x_0) = \frac{r}{r - \sigma^2} \left[\frac{Q^*(x_0)x_0}{2} - \int_{x^l}^{x_0} Q^*(y) dy \right] > 0 \quad (25)$$

Eq. (25) indicates that the fixed tax is always positive and increasing with x_0, $(\frac{\partial F^*(x_0)}{\partial x_0} > 0)$. The reason is that, being a high value of x_0 not so informative about future demand levels, the higher is the revealed value of

[19] Since $\frac{x_0 g(x_0)}{1-G(x_0)}$ is increasing in x_0, the quantity of the lowest type is $Q^*(x^l) = \frac{1+\lambda}{1+2\lambda}\left[k - \frac{\lambda}{1+\lambda}\right]\frac{x^l}{k}$, which is always positive if $k > \frac{\lambda}{1+\lambda}$ and null in the case of equality.

x_0, the greater the information rents the government expects to pay over the contract term and, therefore, the higher is the fixed tax amount.

For the same reason, all other things being equal, the fixed tax increases with the uncertainty parameter σ, i.e. $\frac{\partial F^*(x_0)}{\partial \sigma} > 0$.

From (24), it is easy to show that the discounted value at time zero of the intertemporal public revenues is given by:

$$\begin{aligned} TT^*(x_0) &= E_0\left[\int_0^\infty e^{-rt}T^*(x_0,x_t)\right]dt \\ &= \frac{(x_0-Q^*(x_0))Q^*(x_0)}{r-\sigma^2} - \frac{\int_{x^l}^{x_0} Q^*(y)dy}{r-\sigma^2} \end{aligned} \tag{26}$$

where the second term on the RHS indicates the discounted value of the information rents paid to a firm of type x_0. As $Q^*(x_0)$ is lower then the time zero profit-maximizing quantity $\frac{x_0}{2}$, the intertemporal revenues $TT^*(x_0)$ are decreasing in x_0.

By substituting (26) into (4), we obtain the firm's intertemporal utility:

$$U^*(x_0) = \int_{x^l}^{x_0} \frac{Q^*(y)}{r-\sigma^2}dy \tag{27}$$

with $U^*(x^l) = 0$.

Finally, from (5) and (26), we obtain the government's expected payoff:

$$\begin{aligned} W^*(x_0) &= \frac{S(Q_0^*,x_0)}{r-\sigma^2} + \lambda E_0\left[\int_0^\infty e^{-rt}T^*(x_0,x_t)\right]dt \\ &= \frac{S(Q_0^*,x_0)+\lambda\pi(Q_0^*,x_0)}{r-\sigma^2} - \lambda U^*(x_0) \end{aligned} \tag{28}$$

3.3 Permanent shocks

The tax schedule presented above is derived in a continuous-time setting where private revenues are driven by demand conditions which are private information to the firm.

In the traditional regulatory literature, a simplifying solution of this model is derived by assuming that the realizations of the state variable are perfectly correlated over time.[20] For instance, this is the framework used by Auriol and Picard (2013), who argue that the optimal regulatory process consists of the repetition of a static contract with time-independent transfers.

Indeed, the same result can be replicated in our model by assuming that there is no uncertainty. Defining $T_{\sigma=0}(x_0)$ as the instantaneous tax payments under $\sigma = 0$, from (26) it is easy to get the Auriol and Picard's time-independent transfer:

$$T^*_{\sigma=0}(x_0) = rTT_{\sigma=0}(x_0) = (x_0 - Q^*(x_0)Q^*(x_0) - \int_{x^l}^{x_0} Q^*(y)dy \qquad (29)$$

Eq. (29) implies that the firm's intertemporal utility is given by:

$$U_{\sigma=0}(x_0) = \frac{\int_{x^l}^{x_0} Q^*(y)dy}{r} \qquad (30)$$

while the government's payoff is:

$$W_{\sigma=0}(x_0) = \frac{S(Q^*_0, x_0) + \lambda\pi(Q^*_0, x_0)}{r} - \lambda U_{\sigma=0}(x_0) \qquad (31)$$

Notice, however, that if, despite $\sigma > 0$, the government used only fixed levies, without subsequent adjustments, both sides would be worse off, that is:

$$\begin{aligned} U^*(x_0) &> U_{\sigma=0}(x_0) \\ W^*(x_0) &> W_{\sigma=0}(x_0) \end{aligned}$$

The reason is that, when the signals that the firm observes are imperfectly correlated over time, the repetition of a static contract, only based on the initial report x_0, would lead to persistent and potentially increasing distortions.

[20] An exception is Baron and Besanko (1985), who analyze a continuing relationship between a regulator and a firm, where costs are private information and change over time following an AR(1) process. In this case, they show that the distortions from the efficient allocation vanish as $t \to \infty$.

4 A numerical example

4.1 Tax payments

For illustration purposes, we assume a uniform distribution $G(x_0) = \frac{x_0 - x^l}{x^h - x^l}$, with $g(x_0) = \frac{1}{x^h - x^l}$, which implies that $\frac{1-G(x_0)}{g(x_0)} = x^h - x_0$. Moreover, we assume (unless otherwise indicated) that $x^l = 1$, $x^h = 3$, $\lambda = 0.5$, $r = 0.05$ and $\sigma = 0.20$.

Figure 1 shows that the annuitized fixed tax $F^*(x_0)$ monotonically increases with the firm's type x_0. The reason is that, as already pointed out, the fixed-tax amount positively depends on the information rents that the government expects to pay over the franchise term. Since the higher is x_0 the lower is its informational value, the higher is x_0 the greater is $F^*(x_0)$.

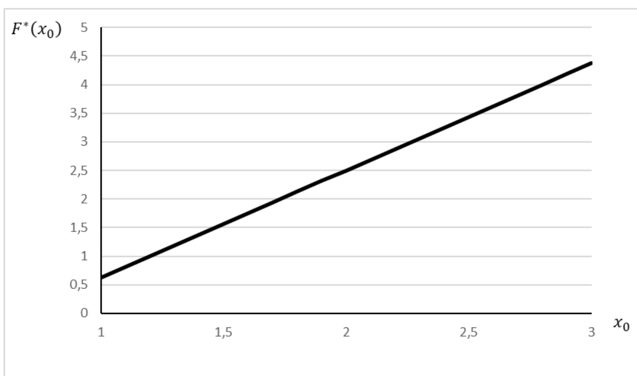


Fig. 1: Change of $F^*(x_0)$ with x_0

Let's now consider the instantaneous variable tax $TV^*(x_0, x_t)$, which can be rewritten as a levy linked to the quantity supplied in each period:

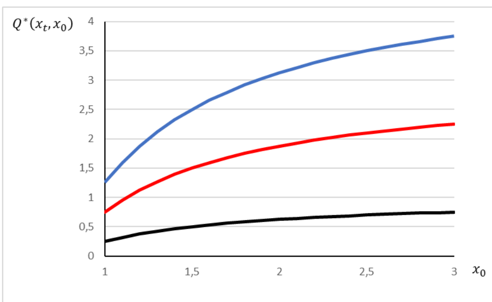
$$TV^*(x_0, x_t) = \beta(x_0, x_t) Q^*(x_0, x_t), \tag{32}$$

where $\beta(x_0, x_t) = \frac{x_t}{2} - Q^*(x_0, x_t) = \frac{x_t}{1+2\lambda}\left[\lambda \frac{1-G(x_0)}{g(x_0)x_0} - \frac{1}{2}\right] x_t$ is the excise tax rate.

Figure 2a (where black $x_t = 1$, red $x_t = 3$, blue $x_t = 5$; this legend will remain the same in all following figures) shows that the tax base Q^*

monotonically increases with the firm's type and, for any given x_0, with the current realization x_t . Regarding the tax rate, Figure 2b shows how β varies with x_0 and, for any given x_0, with x_t.

Three comments are in order. First, for any given realization x_t, the higher is x_0, the lower will be the excise tax rate. Second, beyond a threshold value of x_0 (in our example, $x_0 = 1.5$), β turns out to be negative, i.e. the excise tax becomes a subsidy. Third, beyond the threshold, the higher is x_t the greater will be the subsidy rate.



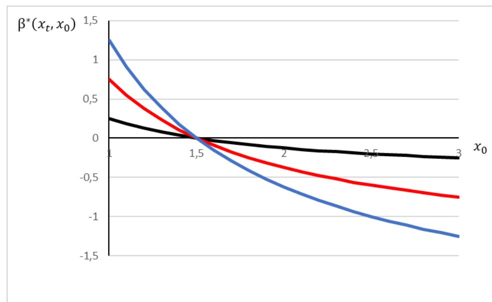


Fig. 2a: Change of Q^* with x_0 and x_t Fig. 2b: Change of β with x_0 and x_t

Taken together, Figure 1 and Figure 2b indicate that the two-part tax schedule can be regarded as a risk sharing device whereby firms with a greater revenue potential assume a risk, by signing a contract that involves high fixed payments, in exchange of the government's commitment to subsequently adjusting the variable tax rate. Indeed, when x_0 is high, subsequent upward demand shifts require a benevolent government to pay more information rents, by reducing the tax rate β so that the firm will have no incentives to misreport x_t and to reduce the output at the detriment of the consumer surplus.

Notice that, as shown by Figure 3, the instantaneous net total amount of government revenue T^* can turn out to be negative. For instance, this could occur at values of x_t significantly higher than the initial value x_0 (e.g., $x_0 = 2.3$ and $x_t = 5$), in which case the subsidies received by the firm would be greater than the fixed fee.

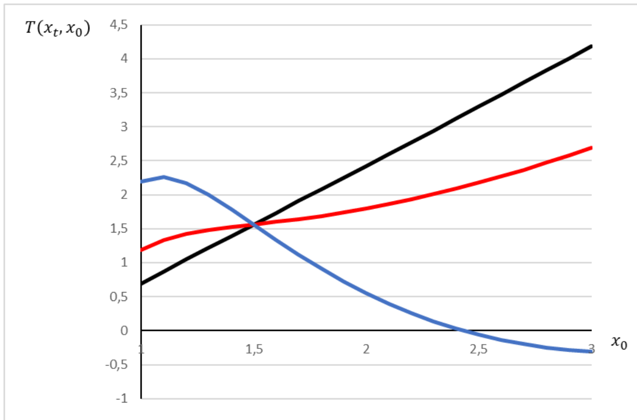


Fig. 3: Change of T^* with x_0 and x_t

This highlights how the project's riskiness is shared between the parties. For instance, from the firm's point of view, the worst scenario is that in which, although market expectations were very promizing at the time of contracting (high x_0), the firm is subsequently hit by demand shocks that significantly reduces profit margins (very low x_t). Conversely, the impact of a negative shock is relatively less significant for firms with lower profit expectations.

4.2 The effect of λ

The regulatory scheme, and thus the tax schedule, is affected among other by the parameter λ that captures the weight attached by the government to public revenues. For instance, an increase in λ causes the government to prioritize budgetary issues at the expense of the output level, in so doing sacrificing part of the consumer surplus. That said, it is worth analysing the effect of λ on the composition of tax receipts, namely on the optimal amount of fixed and variable transfers.

From (24) and (26), the present value of total expected variable payments is given by:

$$TTV^*(\lambda, x_0) = \frac{1}{r - \sigma^2}\left[(x_0 - Q^*(\lambda, x_0))Q^*(\lambda, x_0) - \frac{Q^*(\lambda, x_0)x_0}{2}\right] \qquad (33)$$

while, from (25), the present value of total fixed payments is given by:[21]

$$TF^*(\lambda, x_0) = \frac{1}{r-\sigma^2}\left[\frac{Q^*(\lambda, x_0)x_0}{2} - \int_{x^l}^{x_0} Q^*(\lambda, y)dy\right] \quad (34)$$

Figures 4a and 4b describe the effect of $\lambda \in [0.4, 1]$ for a "high-type" ($x_0 = 2.2$) firm (red line) and a "low-type" firm ($x_0 = 1.2$) (black line) on (33) and on (34) respectively.

The effect of an increase of λ on the variable component is similar for both types (Figure 4a), insofar as an increase of the shadow cost of public funds leads to a reduction of the intertemporal subsidies received by the high-type firm or, equivalently, to an increase of tax payments for a low-type firm. The simple intuition is that a government attaching more weight to budgetary aspects will find it convenient to reduce the information rents paid during the franchise period.

On the contrary, the effect of an increase λ on the fixed payments is not univocal (Figure 4b): while they always increase with λ for the high-type firm, they decrease for the low-type firm.[22] The reason is that the fixed tax must be calibrated against the rents that the government expects to save in the future by inducing the firm to reveal her type x_0.

As a low value of x_0 is relatively more informative about future demand levels, a government attaching more importance to budgetary issues can find it more "productive" to gain information on x_0 when it is low (i.e. to reduce the fixed tax) instead of adjusting (i.e. lowering) later the variable tax rate to gain information on the current demand levels.

[21]In the formulas we add λ to highlight how this parameter impacts on the optimal tax payments.

[22]This result is analytically proven in the Appendix C, where it is shown that, when x_0 is low, the derivative of TF with respect to λ can be negative if the rents that the government expects to pay in the future (first term in square brackets of Eq. (34) decreases more with λ than the rents paid to induce the firm to truthfully reveal x_0 (second term in square brackets of Eq. (34)).

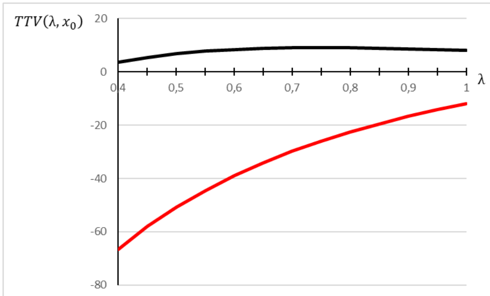


Fig. 4a: Change of TTV^* with λ and x_0

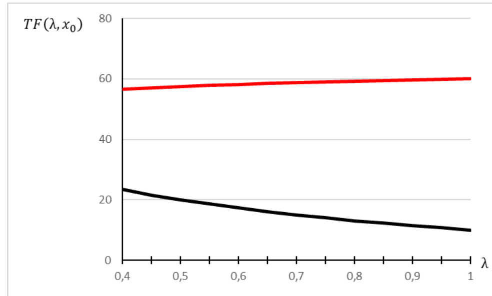


Fig. 4b: Change of TF^* with λ and x_0

4.3 Implementing the contract

Instead of using a truthful direct mechanism, the government can use an indirect mechanism which does not require exchange of information between the parties (Laffont and Tirole, 1993). Specifically, in our model, an indirect mechanism would involve letting the firm to decide the production level and determining the tax payments on the basis of the observable output.

As under our assumptions all functions are invertible, i.e. $x_0 = Q^{-1}(Q_0)$ and $x_t = \frac{Q^{-1}(Q_0)Q_t}{2}$, substituting in Eq. (24) and rearranging we get the optimal tax schedule as a function of the time zero and the current production level, which replicates the same choice of output as with the direct revelation mechanism:

$$\begin{aligned} T^*(Q_0, Q_t) &= F^*(Q^{-1}(Q_0)) + TV^*(Q^{-1}(Q_0), \frac{Q^{-1}(Q_0)Q_t}{2}), \quad \text{for } \; t \geq 0 \\ &= F^*(Q^{-1}(Q_0)) + \gamma(Q_0)Q_t^2 \end{aligned} \tag{35}$$

with $\gamma(Q_0) = \frac{Q^{-1}(Q_0)}{2Q_0} - 1$.

Notice that Eq. (35) can be reformulated as an equivalent adaptive regulatory mechanism capable of adjusting the levies that the firm will bear over time in response to (and to be suitable for) new situations of demand. Indeed, after some algebraic steps and approximating the tax schedule in discrete time, we get:

$$T_t^* = T_{t-1}^* + \gamma(Q_0)\left(Q_t^2 - Q_{t-1}^2\right) \quad \text{for } t \geq 0 \tag{36}$$

with $T_0^* = F^*(Q_0) + \left(\frac{Q^{-1}(Q_0)}{2} - Q_0\right) Q_0$, and where $\gamma(Q_0)$ now represents an adjustment coefficient that applies to the difference between time t and time $t-1$ production.

The firm chooses at time zero the adjustment coefficient $\gamma(Q_0)$ and, consequently, the payment T_0^*, and then the adaptive regulatory policy can start. For instance, using the same parameters and the uniform distribution as in section 4.1, if $Q_0 = \frac{9}{8}$ (i.e. the firm's type is $x_0 = 1.5$), Eq. (36) becomes:

$$T_t^* = T_{t-1}^* - \frac{1}{3}\left(Q_t^2 - Q_{t-1}^2\right), \quad \text{with } \gamma = -\frac{1}{3} \text{ and } T_0^* = 1.56$$

Instead, if $Q_0 = 3$ (i.e. $x_0 = 3$), the adaptive mechanism becomes:

$$T_t^* = T_{t-1}^* - \frac{1}{2}\left(Q_t^2 - Q_{t-1}^2\right), \qquad \text{with } \gamma = -\frac{1}{2} \text{ and } T_0^* = 4.38$$

Eq. (36) also allows to further highlight the impact of the government's objectives, namely the effect of the parameter λ on the optimal tax schedule.

As already pointed out, an increase of λ brings about a reduction of the role that time-adjusted transfers play within the regulatory mechanism. Indeed, Eq. (36) implies that it would be optimal to simply use a flat tax if $\gamma(Q_0) = 0$, which happens only when $x_0 = \frac{3\lambda}{1+\lambda}$.

Thus, whereas it would never be optimal to use a flat tax if $\lambda < 0.5$, it is always possible to find a firm's type such that $\gamma = 0$ if $\lambda \geq 0.5$. For instance, if $\lambda = 0.5$, as assumed in our numerical example, it would be optimal to use a flat tax only if the firm happened to be the one with the lowest possible revenue potential, i.e. $x_0 = 1$.

5 Final remarks

An important issue when designing PPPs is the correct assignment of risks. Moreover, as these contracts often consist of exclusive right-to-sell agreements, public authorities need to take into account potential inefficiencies arising from market power and information imbalances.

In this paper we have addressed these related issues, by focusing on contracts under which a private firm agrees to pay a sum of money for the right to sell a good or service, within an exclusive market, by managing an existing public asset.

As a main result of our analysis, we have shown that when contracting authorities face agents holding private information on the project's returns, evolving in such a way that the information obtained at the award stage is not fully informative about future earnings, optimal regulation requires an appropriate combination of fixed and variable payments to the government. For instance, firms with a greater revenue potential should be charged with relatively high fixed fees. However, incentive-compatibility requires subsequent downward adjustments of the variable pay, namely in the case where revenues perform beyond expectations. Indeed we find that it might be optimal, in some circumstances, to subsidize the firm. On the other hand, agents with lower potential must be charged with low fixed payments, in exchange of the commitment to pay a greater share of their proceeds, should revenues increase during the contract period.

It is not unusual to observe public franchises where concessionaires are charged with flat fees. However, when private returns are affected by imperfectly correlated shocks, this leads to inefficient outcomes, as both parties would be better off under a well calibrated two-part schedule. Indeed, a flat pay system could be justified only when there is little uncertainty about private proceeds. Alternatively, a system tending to favor the fixed component over the variable one could find justification in the importance attributed by the government to public revenues, in which case the adaptive function of time-dependent transfers would be sacrificed, favoring instead the receipts provided by fixed charges.

Appendix A

Neglecting to indicate the dependence on x_0, we can write:

$$\tilde{u}(x_t, \hat{x}_t) = (x_t - Q(\hat{x}_t))Q(\hat{x}_t) - TV(\hat{x}_t) \tag{A.1}$$

where $\hat{x}_t$ is the report by the firm's type x_t

The FOC with respect to $\hat{x}_t$ is:

$$\frac{\partial \tilde{u}(x_t, \hat{x}_t)}{\partial \hat{x}_t} = (x_t - Q(\hat{x}_t))\frac{dQ(\hat{x}_t)}{d\hat{x}_t} - Q(\hat{x}_t)\frac{dQ(\hat{x}_t)}{d\hat{x}_t} - \frac{dTV(\hat{x}_t)}{d\hat{x}_t} = 0 \tag{A.2}$$

A truthful report is optimal if, at $\hat{x}_t = x_t$:

$$\left.\frac{\partial \tilde{u}(x_t, \hat{x}_t)}{\partial \hat{x}_t}\right|_{\hat{x}_t = x_t} = 0$$

Further, the local SOC is:

$$\frac{\partial^2 \tilde{u}(x_t, \hat{x}_t)}{\partial \hat{x}_t^2} = (x_t - Q(\hat{x}_t))\frac{d^2Q(\hat{x}_t)}{d\hat{x}_t^2} - 2(\frac{dQ(\hat{x}_t)}{d\hat{x}_t})^2 - Q(\hat{x}_t)\frac{d^2Q(\hat{x}_t)}{d\hat{x}_t^2} - \frac{d^2TV(\hat{x}_t)}{d\hat{x}_t^2}\ |_{\hat{x}_t = x_t} \leq 0 \tag{A.3}$$

Since, when $\hat{x}_t = x_t$, Eq. (A.2) is an identity, the derivative is zero, i.e.:

$$\frac{dQ(\hat{x}_t)}{d\hat{x}_t} + (x_t - Q(\hat{x}_t))\frac{d^2Q(\hat{x}_t)}{d\hat{x}_t^2} - 2(\frac{dQ(\hat{x}_t)}{d\hat{x}_t})^2 - Q(\hat{x}_t)\frac{d^2Q(\hat{x}_t)}{d\hat{x}_t^2} - \frac{d^2TV(\hat{x}_t)}{d\hat{x}_t^2}\ |_{\hat{x}_t = x_t} = 0 \tag{A.4}$$

By replacing (A.4) in (A.3), we obtain:

$$\left.\frac{\partial^2 \tilde{u}(x_t, \hat{x}_t)}{\partial \hat{x}_t^2}\right|_{\hat{x}_t = x_t} = \frac{dQ(\hat{x}_t)}{d\hat{x}_t} = \frac{dQ(x_t)}{dx_t} \geq 0 \tag{A.5}$$

that is condition (8) in the text.

Once $x_t > 0$ is known, the firm's intertemporal utility becomes:

$$U(x_0, \hat{x}_0) = E_0 \left[\int_0^\infty e^{-rt} (\pi(Q(\hat{x}_0, \phi(\hat{x}_0, W_t)), \phi(\hat{x}_0, W_t))) - TV(\hat{x}_0, \phi(\hat{x}_0, W_t) - F(\hat{x}_0)) dt \right] \tag{A.6}$$

where $\hat{x}_0$ is the report by the firm's type x_0 and the time-varying x_t depends on $\hat{x}_0$ through the function $x_t = \phi(\hat{x}_0, W_t)$.

Therefore, by using Eq. (A.3), the FOC with respect to $\hat{x}_0$ is:

$$\frac{\partial U(x_0, \hat{x}_0)}{\partial \hat{x}_0} = E_0 \left[\int_0^\infty e^{-rt} (\frac{\partial \pi_t}{\partial Q_t} \frac{dQ_t}{d\hat{x}_0} - \frac{dTV_t}{d\hat{x}_0} - \frac{dF}{d\hat{x}_0}) dt \right] = 0 \tag{A.7}$$

while the local SOC is:

$$\frac{\partial^2 U(x_0, \hat{x}_0)}{\partial \hat{x}_0^2} = E_0 \left[\int_0^\infty e^{-rt} (\frac{\partial^2 \pi_t}{\partial Q_t^2} \frac{dQ_t}{d\hat{x}_0} + \frac{\partial \pi_t}{\partial Q_t} \frac{d^2 Q_t}{d\hat{x}_0^2} - \frac{d^2 TV_t}{d\hat{x}_0^2} - \frac{d^2 F}{d\hat{x}_0^2}) dt \right] \leq 0 \tag{A.8}$$

A truthful report is optimal if at $\hat{x}_0 = x_0$:

$$\left. \frac{\partial U(x_0, \hat{x}_0)}{\partial \hat{x}_0} \right|_{\hat{x}_0 = x_0} = 0$$

and:

$$\left. \frac{\partial^2 U(x_0, \hat{x}_0)}{\partial \hat{x}_0^2} \right|_{\hat{x}_0 = x_0} \leq 0$$

Totally differentiating Eq. (A.7), we get:

$$E_0 \left[\int_0^\infty e^{-rt} \left(\frac{\partial^2 \pi_t}{\partial Q_t^2} \frac{dQ_t}{d\hat{x}_0} + \frac{\partial \pi_t}{\partial Q_t} \frac{d^2 Q_t}{d\hat{x}_0^2} + \frac{dx_t}{d\hat{x}_0} \frac{dQ_t}{\partial \hat{x}_0} - \frac{d^2 TV_t}{d\hat{x}_0^2} - \frac{d^2 F}{d\hat{x}_0^2} \right) dt \right] |_{\hat{x}_0 = x_0} = 0 \tag{A.9}$$

Finally, by replacing Eq. (A.9) in Eq. (A.8), we obtain:

$$\frac{\partial^2 U(x_0, \hat{x}_0)}{\partial \hat{x}_0^2} = -E_0 \left[\int_0^\infty e^{-rt} \left(\frac{dx_t}{d\hat{x}_0} \frac{dQ_t}{\partial \hat{x}_0} \right) dt \right] |_{\hat{x}_0 = x_0} \leq 0$$

A sufficient condition for the previous equation to hold is:

$$\frac{dQ_t}{d\hat{x}_0} = \frac{dQ_t}{dx_0} \geq 0 \tag{A.10}$$

that is condition (12) in the text.

Appendix B

Proof of Proposition 1

The standard approach to solve Eq. (13) is to ignore, for the moment, the the second order conditions, Eqs. (8) and (12), and to solve the relaxed problem. By the Envelope theorem (see Milgrom and Segal, 2002, Theorem 1 and Theorem 2), Eq. (11) implies that:

$$U(x_0) = \int_{x^l}^{x_0} E_0 \left[\int_0^\infty e^{-rt} Q(y, x_t) \frac{\partial \phi(y, Z_t)}{\partial y} dt \right] dy \tag{B.1}$$

where the lowest demand gets zero utility, i.e. $U(x^l) = 0$. Integrating B.1 by part we get:

$$\begin{aligned} &\int_{x^l}^{x^h} U(x_0) g(x_0) dx \\ &= \int_{x^l}^{x^h} E_0 \left[\int_0^\infty e^{-rt} Q(x_0, x_t) \frac{\partial \phi(x_0, Z_t)}{\partial x_0} dt \right] \frac{(1 - G(x_0))}{g(x_0)} g(x_0) dx_0 \end{aligned} \tag{B.2}$$

From Eq. (4) we get:

$$\begin{aligned} &\int_{x^l}^{x^h} \left\{ E_0 \left[\int_0^\infty e^{-rt} T(x_0, x_t) dt \right] \right\} g(x_0) dx_0 \\ &= - \int_{x^l}^{x^h} U(x_0) g(x_0) dx + \int_{x^l}^{x^h} \left\{ E_0 \left[\int_0^\infty e^{-rt} (\pi(Q(x_0, x_t), x_t)) dt \right] \right\} g(x_0) dx_0 \\ &= - \int_{x^l}^{x^h} \left\{ E_0 \int_0^\infty e^{-rt} [Q(x_0, x_t) \frac{(1 - G(x_0))}{g(x_0)} \frac{\partial \phi(x_0, Z_t)}{\partial x_0} + \pi(Q(x_0, x_t), x_t))] dt \right\} g(x_0) dx_0 \end{aligned} \tag{B.3}$$

Substituting (B.3) in the objective function (13), we obtain:

$$\max_{Q(x_0,x_t)} \int_{x^l}^{x^h} \left\{ E_0 \left[\begin{array}{c} \int_0^\infty e^{-rt}[S(Q(x_0,x_t),x_t) + \lambda\pi(Q(x_0,x_t),x_t)) \\ -\lambda Q(x_0,x_t)\frac{(1-G(x_0))}{g(x_0)}\frac{\partial\phi(x_0,Z_t)}{\partial x_0}]dt \end{array} \right] \right\} g(x_0)dx_0 \tag{B.4}$$

Differentiating Eq. (B.4) with respect to Q_t we obtain the first order condition for the optimal output:

$$S_Q(Q^{**}(x_0,x_t),x_t) + \lambda u_Q(Q^{**}(x_0,x_t),x_t)) - \lambda\frac{(1-G(x_0))}{g(x_0)}\frac{\partial\phi(x_0,Z_t)}{\partial x_0} = 0 \tag{B.5}$$

Since $\frac{\partial\phi(x_0,Z_t)}{\partial x_0} = \frac{x_t}{x_0}$ and given the the assumption on $\frac{(1-G(x_0))}{g(x_0)}$, both the second order conditions, Eqs. (8) and (12), are satisfied, i.e.:

$$\frac{dQ^*(x_0,x_t)}{dx_t} > 0 \quad \text{and} \quad \frac{dQ^*(x_0,x_t)}{dx_0} > 0 \tag{B.6}$$

Let's now derive the time-variant payment contract $TV^*(x_0,x_t)$. For each time $t > 0$, integrating Eq. (7), we obtain:

$$\tilde{u}(Q^*(x_0,x_t),x_t)) = \int_0^{x_t} Q(x_0,z)dz \tag{B.7}$$

where $\tilde{u}(Q^*(x_0,0),0)) = 0$. Substituting Eq. (B.7) into Eq. (A.1) we obtain:

$$TV^*(x_0,x_t) = \pi(Q^*(x_0,x_t),x_t)) - \int_0^{x_t} Q^*(x_0,z)dz, \tag{B.8}$$

while substituting (B.8) in (6) we get:

$$U(x_0,x_t) = E_0\left[\int_0^\infty e^{-rt}[\int_0^{x_t} Q^*(x_0,z)dz - F(x_0)]dt\right] \tag{B.9}$$

We now turn to the second problem (10)-(12). Since by construction of $TV^*(x_0,x_t)$, independently of his initial report x_0, the firm finds it optimal to report x_t truthfully, the firm's value can be rewritten as:

$$U(x_0, \hat{x}_0) = E_0 \left[\int_0^\infty e^{-rt}((\pi(Q^*(\hat{x}_0, x_t), \phi(x_0, Z_t)) - TV^*(\hat{x}_0, x_t)) - F(\hat{x}_0))dt \right] \tag{B.10}$$

where x_0 is the true initial shock and $\hat{x}_0$ is the one reported. In addition, as it is optimal to report x_t truthfully for all t, we have that $\frac{\partial}{\partial x_t} \left(\pi(Q^*(\hat{x}_0, x_t), x_t) - TV^*(\hat{x}_0, x_t) \right) = Q^*(x_0, x_t)$. Thus, as $x_t = \phi(x_0, Z_t)$, the derivative of Eq. (B.10) with respect to the initial shock x_0 reduces to Eq. (11), while the integral of Eq. (11) with respect to x_0 is simply Eq. (B.1):

$$U(x_0) = \int_{x^l}^{x_0} E_0 \left[\int_0^\infty e^{-rt} Q^*(y, x_t) \frac{\partial \phi(y, Z_t)}{\partial y} dt \right] dy$$

Finally, equalizing (B.1) and (B.9) we obtain:

$$\begin{aligned} U(x_0) &= \int_{x^l}^{x_0} E_0 \left[\int_0^\infty e^{-rt} Q^*(y, x_t) \frac{\partial \phi(y, Z_t)}{\partial y} dt \right] dy \\ &= E_0 \left[\int_0^\infty e^{-rt} [\int_0^{x_t} Q^*(x_0, z) dz - F(x_0)] dt \right] \\ &= U(x_0, x_t) \end{aligned}$$

and solving for $F^*(x_0)$, we get:

$$F^*(x_0) = r \left\{ E_0 \left[\int_0^\infty e^{-rt} \left[\int_0^{x_t} Q^*(x_0, z) dz \right] dt \right] - \int_{x^l}^{x_0} E_0 \left[\int_0^\infty e^{-rt} Q^*(y, x_t) \frac{\partial \phi(y, W_t)}{\partial y} dt \right] dy \right\} \tag{B.11}$$

Proof of formula (25)

Recalling that $Q^*(y, x_t) = Q^*(y) \frac{x_t}{y}$ and $\frac{\partial \phi(y, W_t)}{\partial y} = \frac{x_t}{y}$, we get:

$$E_0 \left[Q^*(y, x_t) \frac{x_t}{y} \right] = Q^*(y) \frac{1}{y^2} E_0 \left[x_t^2 \right] = Q^*(y) e^{\sigma^2 t}$$

where $E_0(x_t^2) = y^2 e^{\sigma^2 t}$. Therefore, the second term on the R.H.S. of Eq, (B.11) reduces to:

$$\begin{aligned}
&\int_{x^l}^{x_0} E_0 \left[\int_0^{\infty} e^{-rt} Q^*(y, x_t) \frac{\partial \phi(y, Z_t)}{\partial y} dt \right] dy \\
&= \int_{x^l}^{x_0} \left[\int_0^{\infty} e^{-rt} E_0 [Q^*(y, x_t) \frac{x_t}{y}] dt \right] dy \\
&= \frac{1}{r - \sigma^2} \int_{x^l}^{x_0} Q^*(y) dy
\end{aligned} \tag{B.12}$$

Let consider now the first term of Eq. (B.11). Applying the same approach as before, we can write:

$$\begin{aligned}
E_0 \int_0^{x_t} Q^*(x_0, z) dz &= \frac{Q^*(x_0)}{x_0} E_0 \int_0^{x_t} z dz \\
&= Q^*(x_0) \frac{x_0 e^{\sigma^2 t}}{2}
\end{aligned} \tag{B.13}$$

Therefore, the first term on the R.H.S. of Eq. (B.11) reduces to:

$$\begin{aligned}
&E_0 \left[\int_0^{\infty} e^{-rt} [\int_0^{x_t} Q^*(x_0, z) dz] dt \right] \\
&= \frac{x_0}{2(r - \sigma^2)} Q^*(x_0)
\end{aligned} \tag{B.14}$$

Putting together (B.12) and (B.14) we obtain:

$$F^*(x_0) = \frac{r}{(r - \sigma^2)} \left[\frac{Q^*(x_0) x_0}{2} - \int_{x^l}^{x_0} Q^*(y) dy \right] \tag{B.15}$$

In addition, since $\frac{dQ^*(x_0)}{dx_0} > 0$, the fixed part $F^*(x_0)$ is always positive.

Appendix C

By some algebra, we can write $TF^*(x_0)$ as:

$$TF^*(x_0) = \frac{1}{(r-\sigma^2)}\left[\frac{Q^*(x_0)x^l}{2} + \frac{1}{2}\left(\int_{x^l}^{x_0}(Q^*(x_0) - 2Q^*(y))dy\right)\right] \tag{C.1}$$

where $Q^*(x_0) = \frac{x_0}{2} + \frac{1}{1+2\lambda}\left[x_0 - \lambda\frac{1-G(x_0)}{g(x_0)}\right]$.

Thus, substituting $Q^*(x_0)$ in (C.1), the sign of $\frac{\partial F^*}{\partial \lambda}$ is driven by the sign of the following term:

$$sign\frac{\partial F^*}{\partial \lambda} = sign\left\{\left[x_0^2 - (x^l)^2 - \frac{1}{2}\frac{1-G(x_0)}{g(x_0)}x_0\right] + \int_{x^l}^{x_0}\frac{1-G(y)}{g(y)}dy\right\}$$

The second term of the previous equation is always positive, while the first term is negative when x_0 is sufficiently low.

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Fondazione Eni Enrico Mattei
Corso Magenta 63, Milano – Italia

Tel. +39 02 403 36934

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