Consumer Tracking and Price Discrimination with Nested Consideration*

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Abstract

This paper analyses price discrimination, informed by consumer tracking, in an online retail oligopoly. Consumers consider sequentially products in a search outcome list and differ in their engagement levels. A model of nested consideration with limited product suitability underpins heterogeneity in consumers' consideration sets. When the market is sufficiently asymmetric, under both uniform and discriminatory pricing, duopolistic interactions can be supported in equilibrium. Under uniform pricing, these are driven by price segmentation. Under price discrimination, in contrast, they are driven by market segmentation, which intensifies price competition, harms industry profit, and benefits (imperfect) consumers (who gain from tracking).

Keywords: Oligopoly Pricing, Price Discrimination, Consideration Sets, Consumer Tracking, Segmentation

JEL classification: L11; L13; D43; D83

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1 Introduction

Digital retail markets have grown significantly over the last decade. A notable feature of these markets is that they facilitate the collection of consumer data. Online retailers can use 'tracking' technology (e.g., cookies, fingerprinting, or web bugs) to identify consumers' browsing behavior or purchase history. This allows them to refine their information on consumers' preferences or consideration sets. One practice, prevalent in retail markets, that is likely to be shaped by the increased availability of consumer data is price discrimination.

In many retail markets consumers differ in the set of products they consider before making a purchase. Differences in consumers' consideration sets (and so their degree of contestability) have an impact on firms' pricing strategies and market outcomes. When firms can access information on consumers' consideration sets, as revealed by their browsing or purchase history, this allows them to price discriminate between different consumer groups.[1] This paper analyses consideration-based price discrimination supported by consumer tracking, and compares it to uniform pricing, focusing on a homogeneous product oligopoly market with asymmetric (nested) consideration and limited product suitability or availability.

In online retail markets, consumers search for products and obtain a list of offers that they consider sequentially. Consumers differ in their levels of engagement (or rationality): given a search outcome list, at each consideration stage, some consumers drop out. All consumers consider the first product, some consider the first two products, some of the latter group consider the top three products and so on. A firm displayed closer to the top of the search list (a more prominent firm) is considered by a larger number of consumers than a firm displayed lower down. The firm at the top of the list has the largest reach, the second firm's reach is contained in the first firm's reach, the third firm's reach is contained in the second firm's reach and so on.

A model of nested consideration provides a natural representation of this environment, and limited product suitability or availability enriches and adapts it for the study of consideration-based price discrimination. Given a pattern of nested reach, a firm's product is available with some probability strictly below one: for instance, it may be 'temporarily out of stock' or have a long delivery time that makes it effectively unavailable or unsuitable. With limited availability, all firms have some captive consumers. In contrast, if all products were available for sure, only the firm with the largest reach would have some captive consumers.[2] Although some consumers consider at least two firms, they may end up being captive to one firm if any other product they consider is unavailable. With limited availability, a firm with a larger reach has strictly more captive consumers than a firm with a smaller reach, as its customers are contested by fewer rivals.

This analysis characterises mixed-strategy price equilibria under uniform pricing and under consideration-based price discrimination, supported by tracking, and compares these regimes. With uniform pricing, each firm charges the same price to all its customers. Under price discrimination,

[1] This type of price discrimination can be implemented by using discounts targeted at specific consumer groups. For related evidence, see for instance, Australian Energy Market Commission (2018).

[2] This is the case in the model of nested reach proposed in Armstrong and Vickers (2022). This analysis uses a variant of their model with limited availability.

a firm may use tracking technology to discriminate between consumers in its 'strong market' and consumers in its 'weak market'. A firm's strong market is the group of consumers who drop out after considering it. A firm's weak market is the group of consumers who consider this firm and also consider at least one other firm listed below it. Consumers in a firm's strong market are more likely to be captive than those in a firm's weak market. The last firm in the list cannot distinguish between the two groups and charges only one price regardless of the pricing regime. The analysis focuses on sufficiently asymmetric markets in the sense that a firm with a relatively wider reach also has a sufficiently wide incremental reach.[3]

All consumers in the strong market of the firm with the largest reach are captive. Not all consumers in the strong market of any other firm are captive as they consider at least two firms and there is a positive probability that the product of a rival that is considered is available. Due to limited availability, there are captive consumers in any firm's weak market. Some consumers in a firm's weak market are as contestable as the consumers in the firm's strong market and, together with consumers in the firm's strong market, they are referred to as a firm's *availability-adjusted* strong market. However, a consumer in a firm's strong market is more likely to be captive than a consumer in the firm's weak market.

Firms can use consumer browsing data to distinguish between their weak and strong markets but, as they do not know rivals' realised availability, they cannot identify their captive consumers using tracking. In either pricing regime, there is a conflict between firms' incentive to extract all surplus from captive consumers and their incentive to undercut and compete for contested consumers. Due to this tension, in any price equilibrium firms use strategies and choose prices randomly according to a distribution function.

Under uniform pricing, there exists an equilibrium characterised by price segmentation and 'duopolistic interactions': the supports of firms' price distributions are interlocked and exactly two firms use prices in a given price range. Under full availability, Armstrong and Vickers (2022) identify equilibrium price segmentation if a firm's strong market (or incremental reach) is strictly larger than the strong market of the next firm in the search list. Under limited availability, their results are qualitatively robust if a firm's strong market is larger than the availability-adjusted strong market of the next firm on the list. This condition guarantees that there are more captive consumers in a firm's strong market than in its weak market.

Under price discrimination, each firm except from the firm with the smallest reach, chooses two prices. When there are more captives in a firm's strong market than in its weak market, there exists an equilibrium where each firm specializes in attracting the least contested customers in its weak market - that is, the consumers in the strong market of the next firm in the list - and serves more contested customers only if the products of the other firms these consumers consider are unavailable. Each firm's strong market forms a separate market segment characterised by duopolistic interaction. In triopoly equilibrium, there is complete market segmentation (the intervals of prices used in the strong markets of consecutive firms are adjacent). However, in more fragmented markets, there is

[3]This is, for example, the case if the consumer drop out rate is constant and the suitability probability is high enough.

only partial market segmentation, as the equilibrium patterns are more intricate and only a weak form of specialisation emerges.

In both pricing regimes, all firms make strictly positive expected profits that are weakly larger than their monopoly profit from captive consumers, with equality only for the firm with largest reach. When a firm's strong market is larger than the availability adjusted strong market of the next firm on the list, a firm's expected profit increases in its reach. This is consistent with markets where firms pay more to be displayed in a more prominent position (e.g., sponsored search).

The firm with the largest reach is indifferent between uniform and discriminatory pricing. Conditions are presented for all other firms to make lower expected equilibrium profits under price discrimination than under uniform pricing. In sufficiently asymmetric markets, in equilibrium, under uniform pricing firms can support price segmentation, while under price discrimination, they can support market segmentation. Both forms of segmentation result in duopolistic interactions, but compared to price segmentation, market segmentation intensifies competition and harms industry profit. As a result, this analysis identifies the possibility that price discrimination based on tracking benefits imperfect consumers and harms the firms.

1.1 Related Literature

This paper is related to the literature on price discrimination; see, for instance, Armstrong (2006) and Stole (2007) for reviews. Thisse and Vives (1988) and Corts (1998) present oligopoly models of product differentiation with deterministic pricing and show that, compared to uniform pricing, price discrimination intensifies competition and lowers firms' profits. In contrast to their work, the analysis here focuses on a homogeneous product market with stochastic pricing where consideration set heterogeneity and limited availability or suitability underpin the differences between uniform and discriminatory pricing regimes.

Armstrong and Vickers (2019) analyse the role of price discrimination on consumer welfare in duopoly markets where firms face both captive and contested consumers. When firms discriminate between these groups, they charge captive consumers the monopoly price, while charging contested consumers a price equal to marginal cost as they compete à la Bertrand in this segment. An increase in aggregate profit or in profit variance across consumer groups harms consumer welfare. As a result, discrimination is worse (better) for consumers than uniform pricing if firms are symmetric (sufficiently asymmetric). Here, in contrast, firms cannot identify their captive consumers and, instead, use tracking to price discriminate between their strong and weak markets. Like in their setting, when firms are sufficiently asymmetric, discrimination harms firms and benefits consumers.

In an oligopoly model where retailers are equally likely to be considered, Chioveanu (2023) analyses and compares three price discrimination regimes that differ in the granularity of the firms' information, and assesses them against a uniform pricing benchmark. Firms use purchase history data to refine their consideration set information and discriminate on this basis. Coarse price discrimination leads to higher expected industry profit compared to uniform pricing, and granular price discrimination further benefits the firms and harms consumers. In a hybrid price discrimination regime, one retailer is affiliated to the digital competition platform and has access to more refined data than its rivals. Under mild conditions, the platform affiliate is indifferent between hybrid and granular discrimination and, under more stringent conditions, this may be the case for all retailers.

This analysis contributes to a recent literature that examines the role of consideration set heterogeneity for price competition in oligopoly markets. Considering a general structure of consumer choice sets, Armstrong and Vickers (2022) analyse competition under uniform pricing. They generalize existing models by studying a 'symmetric interactions' pattern of consideration and fully characterise mixed pricing strategy equilibria in triopoly.[4] When examining asymmetric interactions, they propose a model of nested reach and show that, when a firm with larger reach also has (weakly) larger incremental reach, in price equilibrium firms' price supports are interlocked and there is duopolistic interaction. The analysis here builds on their model of nested reach and shows that, under uniform pricing, their results are qualitatively robust under limited product availability.[5]

A stream of marketing research has explored the role of consideration sets, also known as evoked sets in consumer choice, with the introduction of the concept being attributed to Howard and Sheth (1969); see, for instance, Roberts and Lattin (1997) for a review. The central idea is that consumers, especially when faced with a large number of alternatives, use a sequential decision making process: they start by selecting a subset of alternatives which are perceived as relevant and then make purchase decisions from this restricted subset.[6] This literature focused on specific cases and mainly on the managerial implications of restricted consideration sets.

From an economics perspective, Eliaz and Spiegler (2011) analyse firms' strategic attempts to manipulate consumers' consideration sets in markets where consumers are boundedly rational, i.e. they may have irrational perceptions of what products are relevant to them. In the current model, which is motivated by observed features of online markets, consideration set heterogeneity is driven the interaction between differences in consumers' engagement level (which can be regarded as a consumer 'imperfection') and an ordered search outcome list.

Given its focus on price discrimination supported by tracking, this analysis is related to a growing literature on the economics of consumer data; see, for instance, the reviews in Acquisti et al. (2016) - which focuses on consumer privacy - and Bergemann and Bonatti (2019).[7] Much of the recent work examines the market for information, (e.g. the price of data) and the design of information structures.[8] In contrast, the focus here is on the impact of consumers information on firms' pricing strategies, especially price discrimination, and market outcomes. A natural interpretation of the nested reach model is that firms compete on an online platform. In this case, the platform can track consumers' browsing behavior and provide information to firms.

[4]The 'interaction' between a group of firms measures the correlation between firms' reaches.

[5]For models with limited product availability/suitability, see, for instance, Ireland (1993), McAfee (1994), and Chen and He (2011). As firms with nested reach differ in their level of salience, this framework can also be related to work on prominence, e.g. Armstrong et al. (2009), Armstrong and Zhou (2011), and Rhodes (2011).

[6]Bronnenberg and Vanhonacker (1996) find empirical support that price variation responses are limited to the brands in the choice set using scanner data from the French powder detergent market. See also the references therein and Hauser and Wernerfelt (1990) for related discussion.

[7]For an early analysis of information sharing in oligopoly, see Vives (1984).

[8]Armstrong and Zhou (2022), for instance, highlight the limits of competition when consumers observe a private signal of their preferences, by examining firm-optimal and consumer-optimal information structures. Bergemann and Bonatti (2015) study market outcomes and optimal pricing when a monopoly data provider sells informative signals (cookies) on consumers' match values to advertisers (so the market is segmented based on users' browsing behavior).

This work is related to the substantial literature on price competition in homogeneous product markets with information frictions, e.g. to the 'one or all' models in Varian (1980), Rosenthal (1980), Narasimhan (1988), Baye et al. (1992), to the model with consideration heterogeneity in Burdett and Judd (1983), and to independent reach models, like those in Butters (1977), Ireland (1993) and McAfee (1994).

The paper is organized as follows. Next section introduces the framework. Section 3 presents a preliminary triopoly analysis with uniform pricing (3.1), price discrimination (3.2), and a comparison between these regimes (3.3). Section 4 analyses a general oligopoly model, using the same structure. section 5 analyses a more refined form of price discrimination and is followed by conclusions. All proofs missing from the text are relegated to a technical appendix (section 7).

2 Model

Consider an online market for a homogeneous product where $n \geq 3$ firms compete for consumers. Firms have constant marginal costs of production normalized to zero. There is a unit mass of consumers with unit demands and a common valuation for the product normalized to one. Each firm's product is suitable (matches a consumer's preferences) or is available for purchase with a given probability, $\alpha \in (0, 1)$. With probability $(1-\alpha)$ a firm's product is unsuitable or unavailable.[9]

Consumers search online for the product, obtain a product list, and consider the products on the list sequentially, starting from the top. They differ in their levels of engagement or rationality and some drop out after each item on the list. As a result, there is heterogeneity in consumers' consideration sets and firms have 'nested reach' as described in Armstrong and Vickers (2022). Firm i reaches a larger share of consumers than firm $i-1$, all consumers who consider $i-1$ consider i, but not all consumers who consider i consider $i-1$, for $i \geq 2$.

Let firm i's reach be $\sigma_i \leq 1$, for any i, with $\sigma_n = 1$. With nested reach, for any $i \geq 2$, $\sigma_i > \sigma_{i-1}$. Let $\beta_i \equiv \sigma_i - \sigma_{i-1} > 0$ denote the incremental reach of firm i. Let, for consistency, $\beta_1 \equiv \sigma_1$. Figure 1 illustrates this pattern of consideration for $n = 3$.

Firms compete in prices and each consumer either buys the cheapest available product in her consideration set or defers purchase. Due to limited product suitability/availability, all firms have some captive consumers.[10] If firms do not have any information on consumers' consideration sets, they compete in uniform prices: a firm charges the same price to all its customers. Information on browsing history (for instance, cookie tracking) allows firms to identify their least contestable consumers (their strong markets) and to price discriminate. In this case, firm $i \geq 2$ charges one price to consumers in its strong market - segment β_i, and a (possibly) different price to consumers in its weak market - segment $\sigma_{i-1} = \Sigma_{j=1}^{i-1} \beta_j$.

[9] A product may be out of stock or unavailable for delivery in the time frame relevant to consumers. Alternatively, it may not fit a consumer's idiosyncratic preferences: e.g. an environmentally conscious consumer might find a product with a large carbon footprint unsuitable.

[10] However, if all products were available, only the firm with the largest reach (the most prominent firm on the list) would have captive consumers.

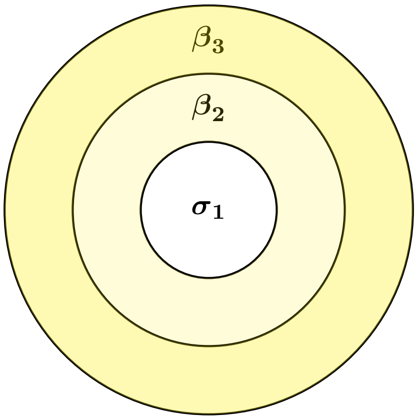

Figure 1: Nested Consideration Sets for $n = 3$

Denote

$$\gamma_i \equiv \sum_{j=1}^{j=i} \beta_j (1-\alpha)^{i-j} \quad \text{for} \quad i = \{1, 2, ...n\} \quad \text{where } \beta_1 = \sigma_1 \ .$$

While $\beta_i = \sigma_i - \sigma_{i-1}$ is firm i's strong market, γ_i is firm i's *availability-adjusted* strong market. A firm's availability-adjusted strong market is formed of consumers in the firm's strong market together with consumers in the firm's weak market who, due to limited availability, are as contestable as consumers in the firm's strong market. For instance, in Figure 1, consumers in group β_2 are firm 2's least contested consumers (i.e., they are contested by firm 3 but not by firm 1). However, while consumers in group $\sigma_1 = \beta_1$ also consider firm 1, with probability $(1 - \alpha)$ firm 1's product is not available so that these consumers are as contestable as those in group β_2. Then, firm 2's availability-adjusted strong market is $\gamma_2 = \beta_2 + (1 - \alpha)\sigma_1$.

Firm i has a group of captive consumers of measure

$$(1-\alpha)^{n-i} \left[\sum_{j=1}^{j=i} \beta_j (1-\alpha)^{i-j} \right] = (1-\alpha)^{n-i}\beta_i + (1-\alpha)^{n-i+1} \left[\sum_{j=1}^{j=i-1} \beta_j (1-\alpha)^{i-1-j} \right] \ ,$$

where the first and second terms on the RHS give the captive consumers in this firm's strong market and in its weak market, respectively. The share of captives in firm i's strong market (which is $(1-\alpha)^{n-i}$) is higher than the share of captives in firm i's weak market (which is $(1-\alpha)^{n-i+1}\gamma_{i-1}/\sigma_{i-1}$ where $\sigma_{i-1} = \sum_{j=1}^{j=i-1} \beta_j$).

Under price discrimination, firms use cookie tracking to identify consumers in their strong markets (who are more likely to be captive) but they cannot identify their captive consumers or their availability-adjusted strong markets as they does not know if rivals' products are suitable/available.

Under uniform pricing, firms compete for both captive and contestable consumers. The tension between the incentives to undercut rivals to attract contested consumers and to charge the monopoly price to captive consumers rules out the existence of pure strategy equilibria. Under price discrimination, firms segment their strong markets from their weak markets, but this tension is still present with the exception of one segment. All consumers in firm n's strong market are captive and this firm can charge them the monopoly price. This analysis characterises mixed strategy equilibria in the two regimes and provides a comparison to explore the impact of tracking-based price discrimination in markets with nested consideration.

Assumption 1. *$\beta_i > \gamma_{i-1}$ for $\forall i \geq 3$.*

Assumption 2. *$\beta_i > (1-\alpha)\gamma_{i-1}$ for $\forall i \geq 2$.*

Assumption 3. *$\beta_i \geq (1-\alpha)\gamma_{i-1}\Phi$ where $\Phi = \frac{\gamma_i}{\gamma_{i+1}} / \frac{\gamma_{i-1}}{\gamma_i}$.*

Assumption 1 guarantees that a firm's strong market is sufficiently large and so that the market is asymmetric enough. This is sufficient to characterise mixed strategy price equilibrium that displays price segmentation under uniform pricing. Assumption 2 is also an asymmetry condition that is sufficient to characterise mixed strategy price equilibrium under price discrimination.

The analysis in the following sections includes illustrations that use the example below, which corresponds to a situation where consumers drop out at a constant rate $(1-\delta) \in (0,1)$.

Example 1. *Let $\sigma_i = \delta^{n-i}$. For $i \geq 2$,*

$$\beta_i = \sigma_i(1-\delta) \text{ and } \gamma_i = \delta^{n-i}(1-\delta)\left[\frac{1-(1-\alpha)^{i-1}\delta^{i-1}}{1-\delta+\alpha\delta} + \frac{(1-\alpha)^{i-1}\delta^{i-1}}{1-\delta}\right] .$$

Assumptions 1 and 2 both hold if $(1-\alpha) < (1-\delta)^2/[\delta(1-\delta+\alpha\delta^2)]$, which is always the case if $\delta < 0.5$ or $\alpha = \delta < 0.618$. Here, $\Phi < 1$, as

$$\frac{\gamma_i}{\gamma_{i+1}} = \frac{\delta[1-\delta+\alpha\delta(\delta(1-\alpha))^{i-1}]}{1-\delta+\alpha\delta(\delta(1-\alpha))^i}$$

and it decreases in i. Therefore, when Assumption 2 holds, so does Assumption 3.

3 Triopoly Analysis

This section focuses on a triopoly market and illustrates the derivation of mixed strategy equilibria under uniform pricing and under price discrimination. It then provides a comparison of firms' expected equilibrium profits in the two regimes.

3.1 Uniform Pricing

Let $n = 3$ and suppose that firms compete in uniform prices. Let $(p_i^U)_{i=0}^{i=4}$ be a sequence of prices, with $0 < p_0^U = p_1^U < p_2^U < p_3^U = p_4^U = 1$. Consider a candidate mixed strategy equilibrium where firm i chooses its price randomly according to a price distribution function $F_i^U(p)$, defined on $[p_{i-1}^U, p_{i+1}^U]$, which is atomless at prices in its support, except possibly for $p = 1$ when $i = 3$.

At a price $p \in [p_1^U, p_2^U]$, conditional on availability, firm 1's expected profit is given by

$$E\pi_1^U(p) = p\sigma_1(1-\alpha F_2^U(p))(1-\alpha F_3^U(p)) = p\sigma_1(1-\alpha F_2^U(p)) = p_1^U\sigma_1 \equiv E\pi_1^U .$$

Firm 1 can only attract consumers in its reach (group σ_1, who consider all available offers) and does so provided that firm 2 and firm 3's products are either not suitable or more expensive. At this price, $F_3^U(p) = 0$ and the expected profit level obtains by evaluating at $p = p_1^U$, where $F_2(p_1^U) = 0$.

Firm 1's constant profit condition implies that firm 2's price distribution ($F_2^U(p)$) satisfies

$$1 - \alpha F_2^U(p) = \frac{p_1^U}{p} \text{ for } p \in [p_1^U, p_2^U] \ , \tag{1}$$

it is strictly increasing in p with $\left(1 - \alpha F_2^U(p_2^U)\right) = p_1^U / p_2^U < 1$. For $F_2^U(p)$ to be well defined it must hold that $(1 - \alpha) < p_1^U / p_2^U$, which is verified in expression (2) below.

At a price $p \in [p_1^U, p_3^U]$, conditional on availability, firm 2's expected profit is:

$$\begin{aligned} E\pi_2^U(p) &= p[\beta_2(1 - \alpha F_3^U(p)) + \sigma_1(1 - \alpha F_1^U(p))(1 - \alpha F_3^U(p))] \equiv E\pi_2^U \\ &= \begin{cases} p[\beta_2 + \sigma_1(1 - \alpha F_1^U(p))] = p_1^U(\beta_2 + \sigma_1) \text{ for } p \in [p_1^U, p_2^U] \\ p(1 - \alpha F_3^U(p))[\beta_2 + \sigma_1(1 - \alpha)] = p(1 - \alpha F_3^U(p))\gamma_3 \text{ for } p \in (p_2^U, p_3^U] \ . \end{cases} \end{aligned}$$

At price p, firm 2 can attract consumers in its strong reach (group β_2, who also consider firm 3 but do not consider firm 1), provided that firm 3's product is not suitable or it is more expensive. At price p, firm 2 can also attract consumers in firm 1's reach (group σ_1, who consider all available offers), provided that firm 1's and firm 3's offers are not suitable or are more expensive. For $p \in [p_1^U, p_2^U]$, $F_3^U(p) = 0$ and the expected profit level obtains by evaluating at $p = p_1^U$ where $F_1(p_1^U) = 0$. Firm 2's equilibrium constant profit condition requires that $E\pi_2^U(p_1^U) = E\pi_2^U(p_2^U)$ and it must hold that $F_1(p_2^U) = 1$. Therefore,

$$\frac{p_1^U}{p_2^U} = \frac{\beta_2 + \sigma_1(1 - \alpha)}{\beta_2 + \sigma_1} = \frac{\gamma_2}{\sigma_2} \in ((1 - \alpha), 1) \ . \tag{2}$$

Firm 2's equilibrium constant profit conditions identify firm 1's and firm 3's price distributions

$$1 - \alpha F_1^U(p) = \frac{p_1^U}{p}\frac{(\beta_2 + \sigma_1)}{\sigma_1} - \frac{\beta_2}{\sigma_1} \text{ and } 1 - \alpha F_3^U(p) = \frac{p_1^U}{p}\frac{(\beta_2 + \sigma_1)}{[\beta_2 + \sigma_1(1 - \alpha)]} = \frac{p_2^U}{p} \ . \tag{3}$$

It is easy to see that $F_1^U(p)$ is strictly increasing on $[p_1^U, p_2^U]$, $F_3^U(p)$ is strictly increasing on $[p_2^U, 1)$, and $F_3(p_2^U) = 0$. As $F_3(1) = (1 - p_2^U)/\alpha$, consistency requires that $p_2^U \geq (1 - \alpha)$.

At a price $p \in [p_2^U, 1]$, conditional on availability, firm 3's expected profit is given by

$$\begin{aligned} E\pi_3^U(p) &= p[\beta_3 + \beta_2(1 - \alpha F_2^U(p)) + \sigma_1(1 - \alpha F_1^U(p))(1 - \alpha F_2^U(p))] \\ &= p\left[\beta_3 + \gamma_2(1 - \alpha F_2^U(p))\right] = p_2^U\left(\beta_3 + \frac{p_1^U}{p_2^U}\gamma_2\right) = p_2^U\beta_3 + p_1^U\gamma_2 \equiv E\pi_3^U \ . \end{aligned}$$

At price p, firm 3 can attract: consumers in its strong market (group β_3, who are captive); consumers in firm 2's strong market, provided that this firm's product is not suitable or it is more expensive; and consumers in firm 1's reach, provided that the rivals' products are unsuitable or are more expensive. In this price range, $(1 - \alpha) = (1 - \alpha F_1^U(p))$. The expected profit level obtains by evaluating at $p = p_2^U$ and using the continuity of $F_2^U(p)$ at this point.

Firm 3's equilibrium constant profit condition identifies firm 2's price distribution on $(p_2^U, p_3^U]$:

$$1 - \alpha F_2^U(p) = \frac{1}{\gamma_2}\left(\frac{p_2^U\beta_3 + p_1^U\gamma_2}{p} - \beta_3\right) , \tag{4}$$

and it is easy to check that $F_2^U(p)$ is strictly increasing in p. As $F_2^U(1) = 1$, it must hold that

$$\beta_3 + \beta_2(1-\alpha) + \sigma_1(1-\alpha)^2 = p_2^U\beta_3 + p_1^U[\beta_2 + \sigma_1(1-\alpha)] \Leftrightarrow \gamma_3 = p_2^U\beta_3 + p_1^U\gamma_2 .$$

Combining (2) with the expression above, the cut-off prices p_1^U and p_2^U obtain:

$$(1-\alpha) < p_1^U = \frac{\gamma_3\gamma_2}{\beta_3(\beta_2+\sigma_1) + (\gamma_2)^2} < p_2^U = \frac{\gamma_3(\beta_2+\sigma_1)}{\beta_3(\beta_2+\sigma_1) + (\gamma_2)^2} < 1 . \tag{5}$$

Appendix 7.1 shows that this is indeed an equilibrium by ruling out profitable unilateral deviations when Assumption 1 holds. Next result follows.

Proposition 1. *Suppose that Assumption 1 holds and consider a uniform pricing regime. Take threshold prices $(p_i^U)_{i=0}^{i=4}$, satisfying $(1-\alpha) < p_0^U = p_1^U < p_2^U < p_3^U = p_4^U = 1$, with p_1^U and p_2^U defined in (5). There exists a mixed strategy price equilibrium where firm i's price distribution is $F_i^U(p)$ defined on $[p_{i-1}^U, p_{i+1}^U]$ - given in expressions (1), (3), and (4). The distributions $F_1^U(p)$ and $F_2^U(p)$ are atomless, while $F_3^U(p)$ has an atom at $p = 1$. Firm i's expected profit, conditional on participation, is strictly positive and given by $E\pi_i^U = p_i^U\gamma_i$. Unconditional expected industry profit is $E\pi_T^U = \alpha\sum_{i=1}^{i=3} E\pi_i^U$.*

In an analysis of price competition with consideration set heterogeneity, Armstrong and Vickers (2022) propose a nested reach model where all products are available for sure and firms compete in uniform prices. They show that when firms are sufficiently asymmetric, so that their reaches are sufficiently spread out, only two firms compete at any given price that is assigned positive probability in equilibrium. They refer to this pricing pattern, where a firm competes directly only with its closest rivals, as 'overlapping duopolies'. In their case where $\alpha = 1$, the asymmetry condition requires that $\beta_i \geq \beta_{i-1}$ for $i \geq 3$, and in triopoly it reduces to $\beta_3 \geq \beta_2$.

Assumption 1 is an extension of their asymmetry condition that guarantees the existence of a qualitatively robust equilibrium in a market where product suitability/availability is probabilistic. The price equilibrium in Proposition 1 is qualitatively similar to their related result: there are duopolistic interactions and the market is effectively segmented through pricing. Firms' price supports are interlocked. Any price that is assigned positive probability in equilibrium is used by exactly two firms - any price $p \in [p_1^U, p_2^U]$ is used only by firms 1 and 2, while any price $p \in [p_2^U, 1]$ is used only by firms 2 and 3.

Consider the values in Example 1. For $\alpha = \delta = 0.6$, firms' equilibrium price distributions are illustrated in Figure 2, where $p_1^U = 0.55$ and $p_2^U = 0.86$. In this case, $\gamma_3 = 0.56$, $\gamma_2 = 0.38$, and $\gamma_1 = 0.36$.

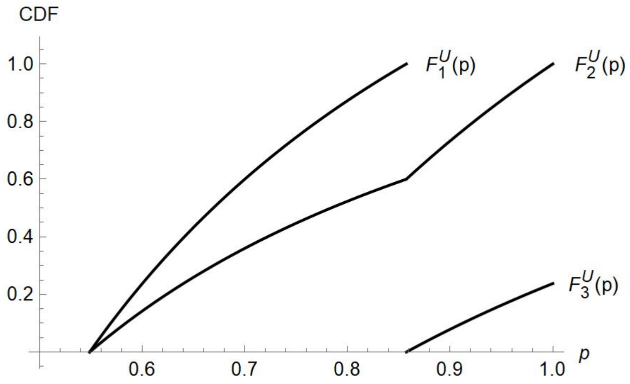

Figure 2: Firms' Equilibrium Price Distributions Under Uniform Pricing ($\alpha = \delta = 0.6$)

3.2 Price Discrimination

Suppose now that firms use tracking technology to identify consumers in their strong markets (who are more likely to be captive). They use this information to price discriminate between these consumers and those in their weak markets.

Let $(p_i^D)_{i=1}^{i=3}$ be a sequence of prices, with $0 < p_1^D < p_2^D < p_3^D = 1$. Consider the following candidate mixed strategy equilibrium. Firm i charges a price drawn according to the distribution function $F_i^s(p)$ to consumers in its strong market (group β_i, who do not consider firm $j < i$). $F_3^s(p)$ is degenerate at $p = 1$ as group β_3 are captive. Firm i for $i \geq 2$ charges a price p drawn according to the distribution function $F_i^w(p)$ to consumers in its weak market (group σ_{i-1}, who consider at least one rival). Only firms 2 and 3 can discriminate between consumers in their strong and weak markets. Suppose that $F_1^s(p)$ and $F_2^w(p)$ are defined on $[p_1^D, p_2^D]$, while $F_2^s(p)$ and $F_3^w(p)$ are defined on $[p_2^D, 1]$, with $F_2^s(1) = 1$.

Below are firm 1's expected profit and firm 2's expected profit in its weak market at $p \in [p_1^D, p_2^D]$.

$$\begin{aligned} E\pi_1^s(p) &= p\sigma_1(1-\alpha F_2^w(p))(1-\alpha F_3^w(p)) = p\sigma_1(1-\alpha F_2^w(p)) = p_1^D\sigma_1 \equiv E\pi_1^s \\ E\pi_2^w(p) &= p\sigma_1(1-\alpha F_1^s(p))(1-\alpha F_3^w(p)) = p\sigma_1(1-\alpha F_1^s(p)) = p_1^D\sigma_1 \equiv E\pi_2^w \end{aligned}$$

Firm 1 cannot price discriminate and the only difference from its expected profit under uniform pricing is that under price discrimination this firm's consumers compare its price to rivals' weak market prices, which are drawn from c.d.f.s $F_2^w(p)$ and $F_3^w(p)$. Consumers in firm 2's weak market (group σ_1) consider all available offers. In this price range $F_3^w(p) = 0$ and firm 2's expected profit level follows from evaluating the expression at p_1^D, where $F_1^s(p_1^D) = 0$.

The constant profit conditions above imply that:

$$(1-\alpha F_1^s(p)) = (1-\alpha F_2^w(p)) = \frac{p_1^D}{p} \text{ for } p \in [p_1^D, p_2^D] \quad \text{and} \quad \frac{p_1^D}{p_2^D} = (1-\alpha) \ . \tag{6}$$

Firm 1's strong market price distribution and firm 2's weak market price distribution are well-defined. As they must be atomless - otherwise, there would be a positive probability of a tie and a profitable unilateral deviation to a slightly lower price - the cut-off price requirement follows.

Firm 2's constant profit condition in its strong market at price $p \in [p_2^D, 1]$ is presented below and identifies firm 3's price distribution in its weak market

$$E\pi_2^s(p) = p\beta_2(1-\alpha F_3^w(p)) = p_2^D\beta_2 \equiv E\pi_2^s \Rightarrow (1-\alpha F_3^w(p)) = \frac{p_2^D}{p} \quad \text{for} \quad p \in [p_2^D, 1] \ . \tag{7}$$

$F_3^w(p)$ is strictly increasing, $F_3^w(p_2) = 0$ and $(1-\alpha F_3^w(1)) = p_2^D$. Consumers in firm 2's strong market (group β_2) consider both firm 2's and firm 3's products and buy from firm 2 if firm 3's product is either unsuitable or more expensive. The equilibrium profit $E\pi_2^s$ is obtained by evaluating the expected profit at price p_2^D, as $F_3^w(p_2^D) = 0$.

Firm 3's profit in its strong market, where this firm sets $p = 1$, is $\pi_3^s = \beta_3$. Its expected profit in its weak market at a price $p \in [p_2^D, 1]$ is

$$\begin{aligned} E\pi_3^w(p) &= p[\beta_2(1-\alpha F_2^s(p)) + \sigma_1(1-\alpha F_1^s(p))(1-\alpha F_2^w(p))] \\ &= p[\beta_2(1-\alpha F_2^s(p)) + \sigma_1(1-\alpha)^2] = p_2^D[\beta_2 + \sigma_1(1-\alpha)^2] = \gamma_2(1-\alpha) \equiv E\pi_3^w \ , \end{aligned}$$

where constant profit levels are obtained by evaluating the expression at the end points of the price interval, using $F_2^s(p_2^D) = 0$ and $F_2^s(1) = 1$.

Firm 3's weak market includes consumers in firm 2's strong market (group β_2), for whom this firm competes only with firm 2, and consumers in firm 1's reach (group σ_1), for whom it competes with both firm 1 and firm 2. Firm 3 serves group β_2 provided that firm 2's product is unsuitable or more expensive. Firm 3 serves group σ_1 if firm 1's and firm 2's products are unsuitable or more expensive.

The constant profit condition above implicitly defines firm 2's strong market price distribution

$$1 - \alpha F_2^s(p) = \frac{p_2^D}{p}\frac{[\beta_2 + \sigma_1(1-\alpha)^2]}{\beta_2} - \frac{\sigma_1(1-\alpha)^2}{\beta_2} \ , \tag{8}$$

with $F_2^s(p)$ strictly increasing on $[p_2^D, 1)$ and $F_2^s(p_2^D) = 0$. Moreover, $F_2^s(1) = 1$ implies that

$$(1-\alpha)^2 < p_1^D = \frac{\gamma_2(1-\alpha)^2}{\beta_2 + \sigma_1(1-\alpha)^2} < (1-\alpha) < p_2^D = \frac{\gamma_2(1-\alpha)}{\beta_2 + \sigma_1(1-\alpha)^2} < 1 \ . \tag{9}$$

To confirm that the price distributions above are indeed part of an equilibrium, unilateral deviations are ruled out in Appendix 7.2, using Assumption 2. Next result follows.

Proposition 2. *Suppose that Assumption 2 holds and consider a price discrimination regime. There exists a mixed strategy price equilibrium where firm 1's price distribution is $F_1^s(p)$ and firm 2's price distribution in its weak market is $F_2^w(p)$, both defined on $[p_1^D, p_2^D]$ and given in (6), firm 2's price distribution in its strong market is $F_2^s(p)$ and firm 3's price distribution in its weak market is $F_3^w(p)$, both defined on $[p_2^D, 1]$, and given in (8) and, respectively, (7). Firm 3 charges $p = 1$ in its strong market. The threshold prices are defined in (9). Firms' expected profits, conditional on participation, are strictly positive and given by $E\pi_i^D = p_i^D\gamma_i$. Unconditional expected industry profit is $E\pi_T^D = \alpha\sum_{i=1}^{i=3} E\pi_i^D$.*

The equilibrium in Proposition 2, like the equilibrium presented in Proposition 1, is characterised by duopolistic interactions: any price that is assigned positive probability is used by exactly two firms. However, in price discrimination equilibrium, firms use tracking information to segment the market: each strong market forms a different sub-market. Price segmentation is a by-product of market segmentation in this case. Both firms' price distributions in their strong markets ($F_1^s(p)$, $F_2^s(p)$, and $F_3^s(p)$ - the latter of which is degenerate at $p = 1$) and their price distributions in their weak markets ($F_2^w(p)$ and $F_3^w(p)$) display an 'echeloned' pattern and have adjacent supports.

Besides from consumers in its strong market, firm 2 can only attract consumers from firm 1's strong market, and as a result $F_1^s(p)$ and $F_2^w(p)$ are identical. In contrast, firm 3 can attract consumers from both firm 1's and firm 2's strong markets. However, in equilibrium, firm 3 'specializes' in targeting consumers in firm 2's strong market and attracts consumers from firm 1's strong market only if neither this firm's product nor firm 2's product are available. When competing head-to-head with firm 2 for consumers in firm 2's strong market, firm 3 is a softer competitor: while firm 2 targets its strong market when setting a price according to $F_2^s(p)$, firm 3 targets firm 2's availability-adjusted strong market (which also includes residual consumers from firm 1's strong market if firm 1's and firm 2's products are unavailable) when setting a price according to $F_3^w(p)$. For this reason, $F_3^w(p)$ first order stochastically dominates $F_2^s(p)$.

Consider the values in Example 1. For $\alpha = \delta = 0.6$, firms' equilibrium price distributions are presented in Figure 3, where $p_1^D = 0.206$ and $p_2^D = 0.516$ (with $\gamma_3 = 0.56$, $\gamma_2 = 0.38$, and $\gamma_1 = 0.36$).

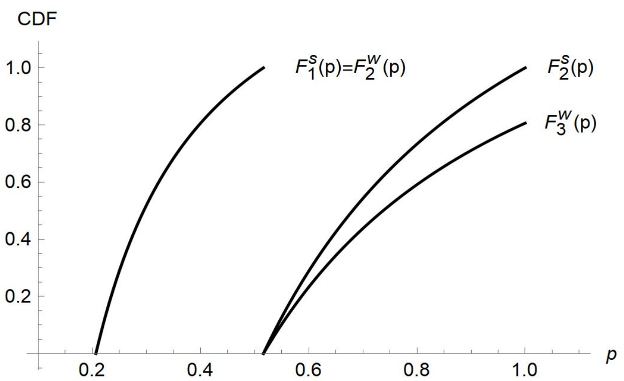

Figure 3: Firms' Equilibrium Price Distributions Under Price Discrimination ($\alpha = \delta = 0.6$)

Section 4.2 analyses mixed strategy pricing equilibrium under price discrimination in an arbitrarily fragmented market. The results there indicate that price equilibrium pattern in triopoly is simpler than that in more fragmented markets, although the findings are qualitatively robust.

3.3 Uniform vs Discriminatory Pricing in Triopoly

Firms' expected profits in the uniform pricing equilibrium presented in Proposition 1 are given by $E\pi_i^U = p_i^U \gamma_i$, while in the discriminatory pricing equilibrium in Proposition 2 are $E\pi_i^D = p_i^U \gamma_i$. To assess the desirability of price discrimination from firms' perspective, it is sufficient to compare the equilibrium cut-off prices across the two regimes.

Suppose that Assumptions 1 and 2 hold (that is, $\beta_3 > \gamma_2$ and $\beta_2 > \sigma_1(1-\alpha)$). As shown in (i) and (ii) below, the threshold prices defined in (5) and (9) satisfy

$$p_2^U > p_2^D \text{ and } p_1^U > (1-\alpha)p_2^U > (1-\alpha)p_2^D = p_1^D \ .$$

(i) Expression (5) implies that $dp_2^U/d\beta_3 > 0$, so that

$$p_2^U > p_2^U \mid_{\beta_3=0} = \frac{\left[\beta_2(1-\alpha)+\sigma_1(1-\alpha)^2\right](\beta_2+\sigma_1)}{\left[\beta_2+\sigma_1(1-\alpha)\right]^2} \equiv p_2^U(0) \ .$$

Combining with (9),

$$p_2^U(0) = \frac{(1-\alpha)(\beta_2+\sigma_1)}{[\beta_2+\sigma_1(1-\alpha)]} > \frac{(1-\alpha)\left[\beta_2+\sigma_1(1-\alpha)\right]}{[\beta_2+\sigma_1(1-\alpha)^2]} = p_2^D \ .$$

(ii) The second inequality follows from (2) and (6).

When the market is sufficiently asymmetric, firm 1's, firm 2's, and industry's expected profits are greater under uniform pricing than under price discrimination, while firm 3's expected profit is the same in both regimes. Market asymmetry allows firms to soften competition through price segmentation under uniform pricing. Discrimination based on consumer tracking leads to market segmentation and intensifies price competition in each segment.

Consumers are better off under price discrimination than under uniform pricing: total welfare is normalized to $\left[\sigma_n - (1-\alpha)\sum_{i=1}^{i=n}\beta_i(1-\alpha)^{n-i}\right]$ and so greater expected industry profit under uniform pricing leads to smaller expected consumer surplus in this regime, compared to price discrimination. Under nested consideration, imperfect consumers may benefit from tracking as data availability intensifies price competition.

In both uniform pricing equilibrium in Proposition 1 and discriminatory pricing equilibrium in Proposition 2, the more prominent a firm is (that is, the higher up it is displayed on the search outcome list), the larger its expected profit is. The results in both regimes are consistent with a market where firms pay for prominence.

4 Oligopoly Markets

4.1 Uniform Pricing

This section generalizes the results presented in Section 3.1 to an oligopoly market with $n \geq 3$ firms. Like in the triopoly analysis, in mixed strategy price equilibrium characterised in this section, there is price segmentation and any price that is assigned positive probability is used by exactly two firms.

Consider a sequence of prices $(p_i^U)_{i=0}^{i=n}$ with $p_0^U = p_1^U < p_2^U < ... < p_n^U = p_{n+1}^U = 1$. Suppose that firm i chooses its price randomly from $[p_{i-1}^U, p_{i+1}^U]$ according to a cumulative distribution function $F_i^U(p)$. Except for $F_n^U(p)$, all distribution functions are atomless.

Take an arbitrary price interval $[p_i^U, p_{i+1}^U]$. At this price only firms i and $i+1$ compete head-to-head, as $F_i^U(p)$ and $F_{i+1}^U(p)$ are strictly increasing, $F_j^U(p) = 1$ for any $j \leq i-1$, and $F_l^U(p) = 0$

for any $l \geq i+2$. Firm i's and firm $i+1$'s expected profits and the corresponding constant profit conditions are presented below.

$$\begin{aligned} E\pi_i^U(p) &= p\gamma_i(1-\alpha F_{i+1}^U(p)) = p_i^U\gamma_i \equiv E\pi_i^U \\ E\pi_{i+1}^U(p) &= p\left[\gamma_i(1-\alpha F_i^U(p))+\beta_{i+1}\right] = p_{i+1}^U\gamma_{i+1} \equiv E\pi_{i+1}^U \end{aligned}$$

Firm i sells to consumers in its reach provided that firm $i+1$'s product is either unsuitable or more expensive. The constant profit level is obtained by evaluating this firm's expected profit at the lower bound of the price interval where $F_{i+1}^U(p_i^U) = 0$. Firm $i+1$ sells to consumers in firm i's availability-adjusted strong market (group γ_i) provided that firm i's product is either unsuitable or more expensive.[11] Firm $i+1$ sells to consumers in its strong market (group β_{i+1}) for sure. The constant profit level is obtained by evaluating this firm's expected profit at the upper bound of the price interval where $F_i^U(p_{i+1}^U) = 1$.

The constant profit conditions identify firms' price distributions on any interval $[p_i^U, p_{i+1}^U]$. These are used below to obtain the expressions that implicitly define firms' price distributions on their supports.

$$\begin{aligned} (1-\alpha F_1^U(p)) &= \frac{p_1^U(\sigma_1+\beta_2)}{p\sigma_1} - \frac{\beta_2}{\sigma_1} \quad \text{for} \quad p\in[p_1,p_2] \\ (1-\alpha F_i^U(p)) &= \begin{cases} \dfrac{p_{i-1}^U}{p} & \text{for} \quad p\in[p_{i-1}^U,p_i^U] \\ \dfrac{p_{i+1}^U\gamma_{i+1}}{p\gamma_i} - \dfrac{\beta_{i+1}}{\gamma_i} & \text{for} \quad p\in(p_i^U,p_{i+1}^U] \end{cases} \qquad \text{for} \quad i\in\{2,...n-1\} \\ (1-\alpha F_n^U(p)) &= \frac{p_{n-1}^U}{p} \quad \text{for} \quad p\in[p_{n-1},p_n] \end{aligned} \tag{10}$$

Using (10), the requirement that $F_i^U(p)$ be continuous at p_i, leads to the following expression:

$$p_{i+1}^U\gamma_{i+1} = p_{i-1}^U\gamma_i + p_i^U\beta_{i+1} \; . \tag{11}$$

The boundary prices are defined iteratively by (11) and are pinned down by the condition $p_n^U = 1$. The price sequence is well defined if

$$p_{i+1}^U = \frac{p_{i-1}^U\gamma_i + p_i^U\beta_{i+1}}{\gamma_{i+1}} > p_i^U \Leftrightarrow p_{i-1}^U\gamma_i > p_i^U(\gamma_{i+1}-\beta_{i+1}) \Leftrightarrow \frac{p_{i-1}^U}{p_i^U} > (1-\alpha) \; . \tag{12}$$

It is shown by induction in Appendix 7.3 that this is indeed the case. Appendix 7.4 uses Assumption 1 to rule out profitable unilateral deviations from the price distributions derived above. These findings are summarised below.

[11] For $i \geq 2$, firm i's availability-adjusted strong market is larger than the firm's strong market as it also includes, due to product unavailability, residual consumers from the strong markets of firms with smaller reach. In the case of firm 1, as it is the firm with the smallest reach, there is no difference between its strong market and its availability-adjusted strong market.

Proposition 3. *Suppose that Assumption 1 holds and consider a uniform pricing regime. There exists a mixed strategy price equilibrium where firm i's price distribution is $F_i^U(p)$ defined on $[p_{i-1}^U, p_{i+1}^U]$ and given implicitly by (10). The threshold prices satisfy $0 < p_0^U = p_1^U$, $p_n^U = p_{n+1}^U = 1$, $p_{i-1}^U < p_i^U$, and $p_{i-1}^U > (1-\alpha)p_i^U$ for $2 \leq i \leq n$ and are defined iteratively in (11). All firms' expected profits, conditional on participation, are strictly positive and given by $E\pi_i^U = p_i^U \gamma_i$.*

Consider the values in Example 1. For $\alpha = \delta = 0.6$, firms' equilibrium price distributions are illustrated in Figure 4.

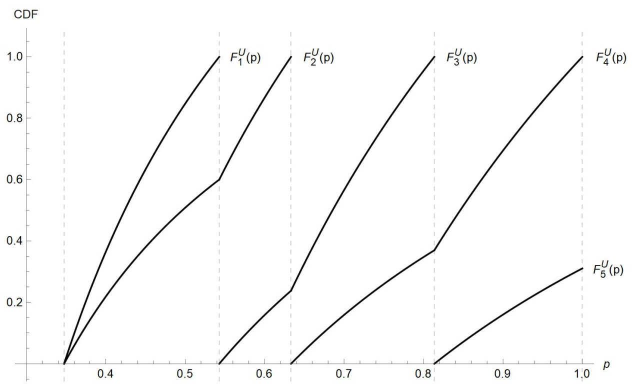

Figure 4: Firms' Equilibrium Price Distributions Under Uniform Pricing ($n = 5$ and $\alpha = \delta = 0.6$)

Corollary 1. *Suppose that Assumption 1 holds. Consider mixed strategy equilibrium in Proposition 3. Firm $i+1$'s expected profit is larger than firm i's expected profit.*

Proof. By (12), $p_{i+1}^U > p_i^U$. By Assumption 1, $\gamma_{i+1} = \beta_{i+1} + (1-\alpha)\gamma_i > \gamma_i + (1-\alpha)\gamma_i > \gamma_i$. Then, $E\pi_{i+1}^U = p_{i+1}^U \gamma_{i+1} > E\pi_i^U = p_i^U \gamma_i$. □

4.2 Price Discrimination

Consider now an oligopoly market where $n \geq 3$ and firms use consumer tracking to price discriminate between their strong and weak markets. This part generalizes the results from the triopoly analysis in Section 3.2. When $n \geq 4$ mixed strategy price equilibrium is characterised by duopolistic interactions but, in contrast to the triopoly case, by a weaker form of market segmentation.

Consider price sequences $(p_i^D)_{i=1}^{i=n+1}$, with $p_1^D < p_2^D < ... < p_n^D = p_{n+1}^D = 1$, and $(H_i)_{i=2}^{i=n}$, with $H_2 = p_2^D$, $H_i \in (p_i^D, p_{i+1}^D)$ for $i \in \{3, ...n-1\}$, and $H_n = 1$. Suppose that firms use the following mixed pricing strategies. Firm i charges a price p drawn according to the distribution function $F_i^s(p)$ to consumers in its strong market (group β_i, who do not consider any firm $j \leq i-1$). Firm i for $i \geq 2$ charges a price p drawn according to the distribution function $F_i^w(p)$ to consumers in its weak market (group $\sigma_{i-1} = \Sigma_{j=1}^{j=i-1}\beta_j$, who consider at least one firm $j \leq i-1$). Firm n's price distribution in its strong market (group β_n) is degenerate at price $p = 1$, as these consumers are captive (they only consider firm n's product). Suppose that $F_i^s(p)$ is defined on $[p_i^D, p_{i+1}^D]$ while $F_i^w(p)$ is defined on $[p_{i-1}^D, H_i]$. Except for $F_n^w(p)$, all distribution functions are atomless.

The detailed equilibrium analysis is relegated to Appendix 7.5 and underpins the identification of firms' pricing distributions and of the boundary prices. For these price distributions to be part of a mixed strategy equilibrium, unilateral deviations should not be profitable. This is verified under Assumption 2 in Appendix 7.6 and proves the next result.

Proposition 4. *Suppose that Assumption 2 holds and consider a price discrimination regime. There exists a mixed strategy oligopoly price equilibrium with the following properties. For $i \geq 2$, firm $i-1$'s strong market price distribution is $F^s_{i-1}(p)$, defined on $[p^D_{i-1}, p^D_i]$ and given in (14), and firm i's strong market price distribution is $F^w_i(p)$, defined on $[p^D_{i-1}, H_i]$ and given in (13). Firm n charges $p = 1$ in its strong market. The sequence of threshold prices, $(p^D_i)^{i=n}_{i=1}$ and $(H_i)^{i=n}_{i=2}$, are defined in (15), with $p^D_n = H_n = 1$, $p^D_{i-1} < p^D_i$, and $p^D_{i-1} > (1-\alpha)p^D_i$ for $i \in \{2, \ldots n\}$. All firms' expected profits, conditional on participation, are strictly positive and given by $E\pi^D_i = p^D_i \gamma_i$.*

$$(1 - \alpha F^w_i(p)) = \frac{p^D_{i-1}}{p} \tag{13}$$

$$(1 - \alpha F^s_{i-1}(p)) = \begin{cases} \dfrac{p^D_{i-1}}{p} & \text{for } p \in [p^D_{i-1}, H_{i-1}] \\ \dfrac{p^D_i}{p}\dfrac{(1-\alpha)\gamma_{i-1}}{\beta_{i-1}} - \dfrac{(1-\alpha)^2\gamma_{i-2}}{\beta_{i-1}} & \text{for } p \in (H_{i-1}, p^D_i] \end{cases} \tag{14}$$

$$p^D_{i-2} = H_{i-1}(1-\alpha) \quad \text{and} \quad p^D_i(1-\alpha)\gamma_{i-1} = p^D_{i-1}\beta_{i-1} + p^D_{i-2}(1-\alpha)\gamma_{i-2} \tag{15}$$

When $n \geq 4$, firm 2's weak market price distribution ($F^w_2(p)$) is defined on $[p^D_1, p^D_2]$, consistent with the triopoly analysis. Firm 3's weak market price distribution ($F^w_3(p)$) is defined on $[p^D_2, H_3]$ with $H_3 > p^D_3$. In contrast, when $n = 3$, $F^w_3(p)$ is defined on $[p^D_2, p^D_3] = [p^D_2, 1]$ and has an atom at the upper bound of its support. When $n \geq 4$, p^D_3 is an interior price and so a mass point there cannot be part of an equilibrium. Firm 3 and firm 4 compete head to head for some of the customers in their weak markets: if firm 3 placed an atom at p^D_3, firm 4 would have an incentive to undercut slightly as this would lead to a jump up in demand.[12] As a result, in its weak market firm 3 offers with positive probability prices that are strictly higher than those offered by firm 2 in its strong market. At these prices, firm 3 targets 'residual' consumers from firm 1's and firm 2's strong markets, that is, consumers who found those firms' products unsuitable or unavailable, although they considered them and they were cheaper. In general, for $n \geq 4$, this leads to a weaker form of segmentation. However, the mixed strategy equilibrium presented in Proposition 4 is still characterised by duopolistic interactions: prices in the interval $[H_i, p^D_i]$ are only offered by firms i and $i+1$, although firm i offers these prices both to customers in its strong market (according to F^s_i) and to customers in its weak market (according to F^w_i).

Consider the values in Example 1. For $\alpha = \delta = 0.6$, firms' equilibrium price distributions are illustrated in Figure 5.

Corollary 2. *Suppose that Assumption 1 holds. Consider mixed strategy equilibrium in Proposition 4. Firm $i+1$'s expected profit is larger than firm i's expected profit.*

[12] See Lemma A5 in Johnen and Ronayne (2021) for a related argument.

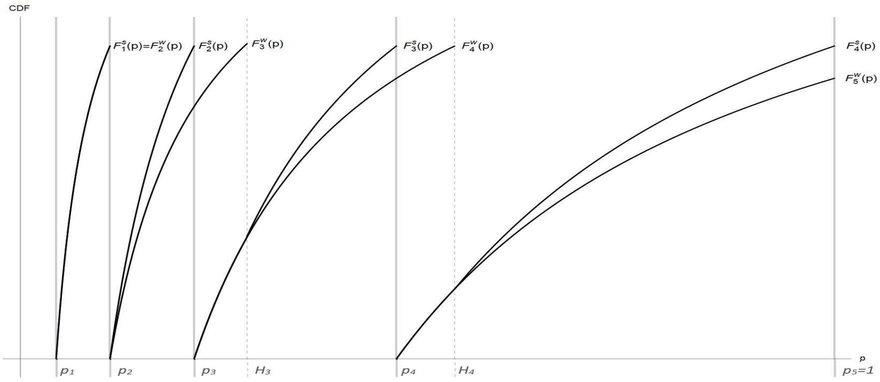

Figure 5: Firms' Equilibrium Distributions Under Price Discrimination ($n = 5$ and $\alpha = \delta = 0.6$)

Proof. By (15), $p_{i+1}^D > p_i^D$. By Assumption 1, $\gamma_{i+1} = \beta_{i+1} + (1-\alpha)\gamma_i > \gamma_i + (1-\alpha)\gamma_i > \gamma_i$. Then, $E\pi_{i+1}^D = p_{i+1}^D \gamma_{i+1} > E\pi_i^D = p_i^D \gamma_i$. □

Both under uniform pricing (Corollary 1) and under price discrimination (Corollary 2), regardless of market structure, the more prominent a firm is in the search result list, the larger its expected equilibrium profit. This makes the results in Propositions 3 and 4 consistent with paid placement in a market with homogenous products and imperfect consumers.

4.3 Uniform vs Discriminatory Pricing in Oligopoly

This part examines the relative desirability of uniform pricing and of tracking based price discrimination in the general oligopoly model.

Suppose that Assumptions 1, 2, and 3 hold. Consider mixed strategy equilibria in Propositions 3 and 4. Re-writing expressions (12) and (15), equilibrium boundary prices satisfy:

$$\frac{p_{i+1}^U}{p_i^U} = \frac{1}{\gamma_{i+1}}\left(\frac{p_{i-1}^U}{p_i^U}\gamma_i + \beta_{i+1}\right) \text{ and } \frac{p_{i+1}^D}{p_i^D} = \frac{1}{(1-\alpha)\gamma_i}\left[\frac{p_{i-1}^D}{p_i^D}(1-\alpha)\gamma_{i-1} + \beta_i\right]$$

As $p_{i-1}^U < p_i^U$ and $p_{i-1}^D > (1-\alpha)p_i^D$, this leads to the following inequalities.

$$\frac{p_{i+1}^U}{p_i^U} - 1 = \frac{p_{i-1}^U}{p_i^U}\frac{\gamma_i}{\gamma_{i+1}} - \frac{(1-\alpha)\gamma_i}{\gamma_{i+1}} = \frac{\gamma_i}{\gamma_{i+1}}\left[\frac{p_{i-1}^U}{p_i^U} - (1-\alpha)\right] < \frac{\alpha\gamma_i}{\gamma_{i+1}}$$

$$\frac{p_{i+1}^D}{p_i^D} - 1 = \frac{p_{i-1}^D}{p_i^D}\frac{\gamma_{i-1}}{\gamma_i} + \frac{\beta_i - (1-\alpha)\gamma_i}{(1-\alpha)\gamma_i} = \frac{\gamma_{i-1}}{\gamma_i}\left[\frac{p_{i-1}^D}{p_i^D} - (1-\alpha)\right] + \frac{\alpha\beta_i}{(1-\alpha)\gamma_i} > \frac{\alpha\beta_i}{(1-\alpha)\gamma_i}$$

Then, using Assumption 3,

$$\frac{\alpha\gamma_i}{\gamma_{i+1}} \leq \frac{\alpha\beta_i}{(1-\alpha)\gamma_i} \Rightarrow \frac{p_{i+1}^U}{p_i^U} < \frac{p_{i+1}^D}{p_i^D}\ .$$

However, as $p_n^U = p_n^D = 1$,

$$\frac{p_{i+1}^U}{p_i^U} < \frac{p_{i+1}^D}{p_i^D} \Leftrightarrow \frac{p_i^U}{p_{i+1}^U} > \frac{p_i^D}{p_{i+1}^D} \Rightarrow p_i^U > p_i^D \text{ for any } i \leq n-1 \ .$$

The boundary prices determine market outcomes in the two pricing regimes. Firm i's expected profit in equilibrium is $E\pi_i^U = p_i^U \gamma_i$ under uniform pricing - see Proposition 3 - and it is $E\pi_i^D = p_i^D \gamma_i$ under price discrimination - see Proposition 4. As total welfare is fixed, larger expected industry profit corresponds to lower expected consumer surplus.

These findings are summarized below.

Proposition 5. *Suppose that Assumptions 1, 2, and 3 hold. Consider mixed strategy pricing equilibria in Proposition 3 and 4. The sequences of boundary prices $(p_i^U)_{i=1}^{i=n}$ and $(p_i^D)_{i=1}^{i=n}$, and firms' expected profits satisfy:*

$$p_i^U > p_i^D \textit{ and } E\pi_i^U > E\pi_i^D \textit{ for any } i.$$

Firms (consumers) are better off (worse off) under uniform pricing than under price discrimination.

In triopoly markets, Assumption 3 is not necessary for the result in Proposition 5. Subsection 3.3 provides a comparison of boundary prices using closed form solutions. In Example 1, which corresponds to a setting where consumers drop out at a constant rate, Assumption 3 holds whenever Assumption 2 does. Therefore, in these cases, the result in Proposition 5 obtains whenever the equilibrium in Proposition 4 exists.

In arbitrarily fragmented oligopoly markets, as the boundary prices are defined recursively, there are no closed form solutions and the comparison of equilibrium boundary prices (and, implicitly, of expected profits) relies on the sufficient condition in Assumption 3.

This analysis identifies the possibility that price discrimination based on consumer tracking harms the firms and benefits consumers whose behavior is imperfect in a homogeneous product oligopoly market. When a market with nested reach is sufficiently asymmetric, firms can soften competition through price segmentation under uniform prices. In this case, price discrimination based on consumer tracking leads to market segmentation and intensifies competition. While this is beneficial for consumers, despite heterogeneity in their levels of engagement with the market, it harms the firms.

5 Granular Price Discrimination

The main analysis of price discrimination assumes that firm $i \geq 2$ offers the same price to all consumers in its weak market. For example, when $n = 3$, firm 3 identifies the consumers in its strong market (group β_3) and those in its weak market (group σ_2) but does not know if a consumer in its weak market compares its offer only to firm 2's offer or compares it to both firm 2's and firm 1's offers.

Suppose instead that firm $i \geq 2$ knows what firms a consumer in its weak market considers and can price discriminate accordingly. Firm i charges one price to group β_{i-1}, a random draw from price distribution $F_i^{w\ i-1}(p)$, a (possibly) different price to group β_{i-2}, a random draw from price distribution $F_i^{w\ i-2}(p)$, and so on. Like in the main analysis, firm j charges one price in its strong market (β_j) drawn from $F_j^s(p)$, with $F_n^s(p)$ degenerate at the monopoly price ($p = 1$). Firm 1 charges one price only (drawn from $F_1^s(p)$).

In this case, there is a complete segmentation of the market.

(a) In sub-market β_n, firm n is a monopolist and makes profit $E\pi_n^s = \beta_n$.

(b) In sub-market β_{n-1}, there is a symmetric duopoly where firm $n-1$ competes with firm n. Due to limited availability, a firm's customers come in two types: they are either captive (if the rival's product is unavailable) or they compare both offers (if the rival's product is available).

(c) In sub-market β_i, for $i \leq n-2$ all firms compete and are symmetric. Due to limited availability, a firm's customers are either captive or they compare (at least two) offers. Moreover, with positive probability, a firm's offer is compared to exactly one rival's offer.

Any segment β_i for $i \leq n-1$ is a symmetric sub-market with independent reach where $n+1-i$ firms compete and, when $i \leq n-2$, each pair of firms has some mutually contested consumers. By Proposition 1 in Armstrong and Vickers (2022), there is a unique equilibrium where each firm chooses prices randomly from an interval $[p_i^G, 1]$ for $p_i^G = (1-\alpha)^{n-i} < 1$ and makes expected profit $\beta_i p_i^G$.

For an illustration, let $n = 3$.

Take sub-market σ_1. Firms' price distributions are $F_3^{w\ 1}(p)$, $F_2^{w\ 1}(p)$, and $F_1^s(p)$ defined on $[p_1^G, 1]$. Firms' expected profits at price $p \in [p_1^G, 1]$ are

$$\begin{aligned}
E\pi_1^s &= p\sigma_1(1-\alpha F_2^{w\ 1}(p))(1-\alpha F_3^{w\ 1}(p)) = p_1^G\sigma_1 = \sigma_1(1-\alpha)^2 \ , \\
E\pi_2^{w\ 1} &= p\sigma_1(1-\alpha F_1^s(p))(1-\alpha F_3^{w\ 1}(p)) = p_1^G\sigma_1 = \sigma_1(1-\alpha)^2 \ , \\
E\pi_3^{w\ 1} &= p\sigma_1(1-\alpha F_1^s(p))(1-\alpha F_2^{w\ 1}(p)) = p_1^G\sigma_1 = \sigma_1(1-\alpha)^2 \ .
\end{aligned}$$

It follows that $p_1^G = (1-\alpha)^2$ and the price distributions are

$$(1-\alpha F_1^s(p)) = (1-\alpha F_2^{w\ 1}(p)) = (1-\alpha F_3^{w\ 1}(p)) = p^{-\frac{1}{2}}(1-\alpha) \ .$$

Take sub-market β_2. Firms' price distributions are $F_3^{w\ 2}(p)$ and $F_2^s(p)$, defined on $[p_2^G, 1]$. Firms' expected profits at $p \in [p_2^G, 1]$ are

$$\begin{aligned}
E\pi_2^s &= p\beta_2(1-\alpha F_3^{w\ 2}(p)) = p_2^G\beta_2 = \beta_2(1-\alpha) \ , \\
E\pi_3^{w\ 2} &= p\beta_2(1-\alpha F_2^s(p)) = p_2^G\beta_2 = \beta_2(1-\alpha) \ .
\end{aligned}$$

Then, $p_2^G = (1-\alpha)$ and the distributions are

$$(1 - \alpha F_2^s(p)) = (1 - \alpha F_3^{w\ 2}(p)) = p^{-1}(1-\alpha)\ .$$

Take sub-market β_3. Only firm 3 is active in this segment and makes profit $E\pi_3^s = \beta_3$.

Aggregating over the three market segments, it follows that firms' expected profits are

$$\begin{aligned} E\pi_1^G &= \sigma_1(1-\alpha)^2 = \gamma_1(1-\alpha)^2\ , \\ E\pi_2^G &= \beta_2(1-\alpha) + \sigma_1(1-\alpha)^2 = \gamma_2(1-\alpha)\ , \\ E\pi_3^G &= \beta_3 + \beta_2(1-\alpha) + \sigma_1(1-\alpha)^2 = \gamma_3\ . \end{aligned}$$

Returning to the general case (arbitrary $n \geq 3$), next result follows.

Proposition 6. *Under granular price discrimination, in unique symmetric price equilibrium, firms' expected profits are equal to their monopoly profits from captive consumers,* $E\pi_i^G = (1-\alpha)^{n-i}\gamma_i$.

Firms' expected profits in the (coarse) price discrimination equilibrium in Proposition 4, and implicitly those in the uniform pricing equilibrium in Proposition 3, are strictly larger than under granular price discrimination. This follows as under (coarse) price discrimination the boundary price $p_i^D > (1-\alpha)^{n-i}$ for $i \leq n-1$.

6 Conclusions

When consumers search for products or shop online, they leave a digital footprint. Platforms and retailers collect data on their browsing behaviour or purchase history. Consumer information can be used to price discriminate between customer groups that differ, for instance, in their degree of contestability. This raises concerns that dominant platforms' or strategic retailers' use of data might harm consumers, especially when there are demand side imperfections.

This paper aims to contribute to a better understanding of the implications of price discrimination informed by consumer tracking. It adapts a model of nested consideration that provides a natural representation of an online retail market for the study of price discrimination. Consumers differ in their levels of engagement and there is heterogeneity in their consideration sets. By using tracking technology, firms can identify their least contestable consumers and price discriminate.

The analysis identifies the possibility that, in sufficiently asymmetric markets with nested consideration, imperfect consumers benefit from price discrimination based on consumer tracking. Both under uniform pricing and under price discrimination, duopolistic interactions can be supported in equilibrium. However, while under uniform pricing, there is price segmentation, under discriminatory pricing, there is market segmentation. Compared to price segmentation, market segmentation increases price competition, decreases expected industry profit, and increases expected consumer surplus. These findings complement extant work that identifies potentially anticompetitive effects of price discrimination based on consumer data in online markets.

7 Appendix

7.1 Triopoly: Uniform Pricing - No Unilateral Deviations

Suppose that firm 1 deviates to a price $p \in (p_2^U, 1]$. Its expected deviation profit, conditional on availability, is then

$$\begin{aligned} E\pi_1^{Ud}(p) &= p\sigma_1(1-\alpha F_2^U(p))(1-\alpha F_3^U(p)) \\ &= p\sigma_1 \frac{1}{[\beta_2+\sigma_1(1-\alpha)]}\left\{\frac{p_2^U\beta_3+p_1^U[\beta_2+\sigma_1(1-\alpha)]}{p}-\beta_3\right\}\frac{p_2^U}{p} \\ &= \frac{p_2^U\sigma_1}{[\beta_2+\sigma_1(1-\alpha)]}\left\{\frac{p_2^U\beta_3+p_1^U[\beta_2+\sigma_1(1-\alpha)]}{p}-\beta_3\right\} \\ &< \frac{p_2^U\sigma_1}{[\beta_2+\sigma_1(1-\alpha)]}\left\{\frac{p_2^U\beta_3+p_1^U[\beta_2+\sigma_1(1-\alpha)]}{p_2^U}-\beta_3\right\} = p_1^U\sigma_1 = E\pi_1^U \ , \end{aligned}$$

and so the deviation is not profitable.

Suppose that firm 3 deviates to $p \in [p_1^U, p_2^U)$. Its expected deviation profit, conditional on availability, is

$$\begin{aligned} E\pi_3^{Ud}(p) &= p[\beta_3+\beta_2(1-\alpha F_2^U(p))+\sigma_1(1-\alpha F_1^U(p))(1-\alpha F_2^U(p))] \\ &= p\left\{\beta_3+\beta_2\frac{p_1^U}{p}+\sigma_1\left[\frac{p_1^U}{p}\frac{(\beta_2+\sigma_1)}{\sigma_1}-\frac{\beta_2}{\sigma_1}\right]\frac{p_1^U}{p}\right\} \\ &= p\beta_3+\frac{(p_1^U)^2}{p}(\beta_2+\sigma_1) = p\beta_3+\frac{p_1^U p_2^U}{p}[\beta_2+\sigma_1(1-\alpha)] = p\beta_3+\frac{p_1^U p_2^U}{p}\gamma_2 \ . \end{aligned}$$

The deviation is not profitable iff for any $p \in [p_1^U, p_2^U)$,

$$p\beta_3+\frac{p_1^U p_2^U}{p}\gamma_2 < p_2^U\beta_3+p_1^U\gamma_2 \Leftrightarrow (p_2^U-p)\beta_3 > p_1^U\left(\frac{p_2^U-p}{p}\right)\gamma_2 \Leftrightarrow p\beta_3 > p_1^U\gamma_2 \ .$$

The inequality holds for any $p \in [p_1^U, p_2^U)$ iff Assumption 1 holds (that is, $\beta_3 > \gamma_2 = \beta_2+\sigma_1(1-\alpha)$).

It is trivial to rule out deviations by any firm at a price $p > 1$, as it makes zero profit, or at a price $p < p_1^U$, as this is dominated by setting $p = p_1^U$.

7.2 Triopoly: Price Discrimination - No Unilateral Deviations

Firm 1's expected profit if it deviates to a price $p \in [p_2^D, 1]$ is

$$\begin{aligned} E\pi_1^{sd}(p) &= p\sigma_1(1-\alpha F_2^w(p))(1-\alpha F_3^w(p)) \\ &= p\sigma_1(1-\alpha)\frac{p_2^D}{p} = p_2^D(1-\alpha)\sigma_1 = p_1^D\sigma_1 = E\pi_1^s \ , \end{aligned}$$

so there is no gain from this deviation.

Firm 2's expected profit if it deviates to a price $p \in [p_2^D, 1]$ in its weak market is

$$E\pi_2^{wd}(p) = p\sigma_1(1-\alpha F_1^s(p))(1-\alpha F_3^w(p))$$

$$= \quad p\sigma_1(1-\alpha)\frac{p_2^D}{p} = p_1^D\sigma_1 = E\pi_2^w \ ,$$

so there is no gain from this deviation.

Firm 2's expected profit if it deviates to a price $p \in [p_1^D, p_2^D)$ in its strong market is

$$E\pi_2^{sd}(p) = p\beta_2(1-\alpha F_3^w(p)) = p\beta_2 < p_2^D\beta_2 = E\pi_2^s \ ,$$

so there is no incentive for such deviation.

Firm 3's expected profit if it deviates to a price $p \in [p_1^D, p_2^D)$ in its weak market is

$$\begin{aligned} E\pi_3^{wd}(p) &= p[\beta_2(1-\alpha F_2^s(p)) + \sigma_1(1-\alpha F_1^s(p))(1-\alpha F_2^w(p))] \\ &= p\left[\beta_2 + \sigma_1\left(\frac{p_1^D}{p}\right)^2\right] \ , \end{aligned}$$

which is a convex function of p minimized at $p = p_1^D\left(\frac{\sigma_1}{\beta_2}\right)^{\frac{1}{2}} \equiv \bar{p}_3^{wd}$.

(a) If $\bar{p}_3^{wd} < p_1^D \Leftrightarrow \beta_2 > \sigma_1$, then $E\pi_3^{wd}(p)$ is strictly increasing in p and the deviation is not profitable

$$E\pi_3^{wd}(p) < p_2^D\left[\beta_2 + \sigma_1\left(\frac{p_1^D}{p_2^D}\right)^2\right] = p_2^D\left[\beta_2 + \sigma_1(1-\alpha)^2\right] = E\pi_3^w \ .$$

(b) If $p_1^D < \bar{p}_3^{wd} < p_2^D \Leftrightarrow \sigma_1(1-\alpha)^2 < \beta_2 < \sigma_1$, then $E\pi_3^{wd}(p)$ is U-shaped in p and the deviations is not profitable iff $\lim_{p\to p_2^D} E\pi_3^{wd}(p) < E\pi_3^w$ - which holds as $E\pi_3^{wd}(p_2^D) = E\pi_3^w$ (see above) - and $E\pi_3^{wd}(p_1^D) < E\pi_3^w$ which requires

$$\begin{aligned} p_1^D(\beta_2+\sigma_1) &< p_2^D\left[\beta_2 + \sigma_1(1-\alpha)^2\right] \Leftrightarrow \\ \beta_2 &> \sigma_1(1-\alpha) \ . \end{aligned}$$

Therefore, the deviation is not profitable iff $\sigma_1(1-\alpha) < \beta_2 < \sigma_1$. However, the deviation cannot be ruled out when $\sigma_1(1-\alpha)^2 < \beta_2 < \sigma_1(1-\alpha)$.

(c) If $\bar{p}_3^{wd} > p_2^D \Leftrightarrow \beta_2 < \sigma_1(1-\alpha)^2$, then $E\pi_3^{wd}(p)$ is strictly decreasing in p and the deviation is not profitable iff $E\pi_3^{wd}(p_1^D) < E\pi_3^w \Leftrightarrow \beta_2 > \sigma_1(1-\alpha)$. A contradiction. So the deviation cannot be ruled out when $\beta_2 < \sigma_1(1-\alpha)^2$.

Combining (a), (b), and (c), there are no deviation incentives iff $\beta_2 > \sigma_1(1-\alpha)$.

7.3 Uniform Pricing in Oligopoly: Well-defined Boundary Prices

Step 1. Expression (11) implies that $p_1^U/p_2^U = \gamma_2/(\sigma_1+\beta_2) = [\sigma_1(1-\alpha)+\beta_2]/(\sigma_1+\beta_2) \in ((1-\alpha),1)$.
Step 2. It can be verified that the two price ratios below satisfy (11).

$$\frac{p_{i-1}^U}{p_i^U} = \frac{x}{z} \quad \text{and} \quad \frac{p_i^U}{p_{i+1}^U} = \frac{\gamma_{i+1}z}{\beta_{i+1}z + \gamma_i x}$$

Step 3. Suppose that $p_i^U > p_{i-1}^U \Leftrightarrow x < z$. Then, indeed

$$\begin{aligned}
\frac{p_i^U}{p_{i+1}^U} &= \frac{\gamma_{i+1} z}{\beta_{i+1} z + \gamma_i x} > (1-\alpha) \Leftrightarrow \\
\gamma_{i+1} z &> (1-\alpha)\beta_{i+1} z + (1-\alpha)\gamma_i x \Leftrightarrow \\
\left[\beta_{i+1} + (1-\alpha)\gamma_i\right] z &> (1-\alpha)\beta_{i+1} z + (1-\alpha)\gamma_i x
\end{aligned}$$

and, by (12), $p_{i+1}^U > p_i^U$.

7.4 Proof of Proposition 3

Take firm i. The firm is indifferent between prices $p \in [p_{i-1}^U, p_{i+1}^U]$, where it makes expected profit $E\pi_i = p_i^U \gamma_i$.

(a) Deviations to higher prices. Let $i \leq n-2$ and consider a unilateral deviation to a price $p \in [p_{i+k}^U, p_{i+k+1}^U]$ for $1 \leq k \leq n-i-1$. Firm i's competitors other than firms $i+k$ and $i+k+1$ come in two types. Type (i) includes all firms j for $j \leq i+k-1$, which choose prices lower than p with probability 1. Type (ii) includes all firms j for $j \geq i+k+2$, which choose prices higher than p with probability 1. Firms $i+k$ and $i+k+1$ both choose prices in $[p_{i+k}^U, p_{i+k+1}^U]$ and, using (10), in this range their c.d.f.s satisfy

$$(1-\alpha F_{i+k+1}^U(p)) = \frac{p_{i+k}^U}{p} \text{ and } (1-\alpha F_{i+k}^U(p)) = \frac{p_{i+k+1}^U \gamma_{i+k+1}}{p\gamma_{i+k}} - \frac{\beta_{i+k+1}}{\gamma_{i+k}} .$$

Firm i competes for customers in submarkets $\sigma_1 = \beta_1$, β_2,...β_{i-1}, and β_i.

In submarket β_i it competes with all the firms with larger reach. Type (ii) firms can only attract these consumers if they do not consider firm i, firm $i+k$, or firm $i+k+1$. Type (i) firms (i.e., firms $i+1, ...i+k-1$) undercut firm i and so attract these consumers if they are considered (there are $k-1$ such firms). As a result, at prices in this range, firm i competes with firms $i+k$ and $i+k+1$ for $\beta_i(1-\alpha)^{k-1}$ consumers.

In submarket β_{i-1}, firm i competes with all firms j for $j \geq i-1$. Type (ii) firms can only attract these consumers if they do not consider firm i, firm $i+k$, or firm $i+k+1$. Type (i) firms (i.e., firms $i-1$, $i+1, ...i+k-1$) undercut firm i if they are considered (and there are k such firms). As a result, at prices in this range, firm i competes with firms $i+k$ and $i+k+1$ for $\beta_{i-1}(1-\alpha)^k$ consumers.

Taking into account all submarkets, firm i's deviation profit is

$$\begin{aligned}
E\pi_i^d &= p(1-\alpha F_{i+k}(p))(1-\alpha F_{i+k+1}(p)) \left[\Sigma_{j=1}^{j=i} \beta_j (1-\alpha)^{i+k-1-j}\right] \\
&= p(1-\alpha F_{i+k}(p))(1-\alpha F_{i+k+1}(p))(1-\alpha)^{k-1} \left[\Sigma_{j=1}^{j=i} \beta_j (1-\alpha)^{i-j}\right] \\
&= \left[\frac{p_{i+k+1}^U \gamma_{i+k+1}}{p\gamma_{i+k}} - \frac{\beta_{i+k+1}}{\gamma_{i+k}}\right] p_{i+k}^U (1-\alpha)^{k-1} \gamma_i \\
&\leq \left[\frac{p_{i+k+1}^U \gamma_{i+k+1}}{p_{i+k}^U \gamma_{i+k}} - \frac{\beta_{i+k+1}}{\gamma_{i+k}}\right] p_{i+k}^U (1-\alpha)^{k-1} \gamma_i
\end{aligned}$$

$$= \quad p^U_{i+k-1}(1-\alpha)^{k-1}\gamma_i$$

where the inequality follows as $E\pi^d_i$ decreases over this price range and the last step uses (11) to simplify the term in square brackets. Then, (12) implies that this deviation is not profitable as

$$E\pi^d_i = p^U_{i+k-1}(1-\alpha)^{k-1}\gamma_i < p^U_{i+k-2}(1-\alpha)^{k-2}\gamma_i < \dots < p^U_i\gamma_i = E\pi_i \quad .$$

(b) Deviations to lower prices. Let $i \geq 3$ and consider a unilateral deviation to a price $p \in [p^U_{i-k-1}, p^U_{i-k}]$ for $i-2 \geq k \geq 1$. Firm i's competitors other than firms $i-k$ and $i-k-1$ come in two types. Type (i) includes all firms j for $j \leq i-k-2$, which choose prices lower than p with probability 1. Type (ii) includes all firms j for $j \geq i-k+1$, which choose prices higher than p with probability 1. Firms $i-k$ and $i-k-1$ both choose prices in $[p^U_{i-k-1}, p^U_{i-k}]$ and, using (10), in this range their c.d.f.s satisfy

$$(1-\alpha F^U_{i-k}(p)) = \frac{p^U_{i-k-1}}{p} \text{ and } (1-\alpha F^U_{i-k-1}(p)) = \frac{p^U_{i-k}\gamma_{i-k}}{p\gamma_{i-k-1}} - \frac{\beta_{i-k}}{\gamma_{i-k-1}} \quad .$$

Firm i competes for customers in submarkets $\sigma_1 = \beta_1$, β_2,...β_{i-1}, and β_i.

In submarket β_j for $i-k+1 \leq j \leq i$ it offers the lowest price and attracts all the consumers who consider it.

In submarket β_{i-k} it offers a strictly lower price than any type (ii) firm and competes only with firm $i-k$ as these consumers do not consider firm $i-k-1$ or any type (i) firm.

In submarket j for $1 \leq j \leq i-k-1$, there is a measure $\beta_j(1-\alpha)^{i-k-1-j}$ of consumers who do not consider any type (i) firms and firm i competes for these consumers with both firm $i-k$ and firm $i-k-1$.

Considering all submarkets, firm i's expected deviation profit at price $p \in [p^U_{i-k-1}, p^U_{i-k}]$ is

$$\begin{aligned}
E\pi^d_i(p) &= p\left\{\left[\Sigma^{j=i-k-1}_{j=1}\beta_j(1-\alpha)^{i-k-1-j}(1-\alpha F_{i-k-1}(p)) + \beta_{i-k}\right](1-\alpha F_{i-k}(p)) + \Sigma^{j=i}_{j=i-k+1}\beta_j\right\} \\
&= p\left\{\left[\gamma_{i-k-1}(1-\alpha F_{i-k-1}(p)) + \beta_{i-k}\right](1-\alpha F_{i-k}(p)) + \Sigma^{j=i}_{j=i-k+1}\beta_j\right\} \\
&= \frac{p^U_{i-k-1}p^U_{i-k}\gamma_{i-k}}{p} + p\Sigma^{j=i}_{j=i-k+1}\beta_j \; .
\end{aligned}$$

As $E\pi^d_i(p)$ is convex in p, it is maximized at a cut-off price. As a result, it is sufficient to rule out deviations to cut-off prices p^U_j for $1 \leq j \leq i-2$.

Consider a deviation to $p = p^U_{i-k}$. The expected deviation profit becomes

$$\begin{aligned}
E\pi^d_i(p^U_{i-k}) &= p^U_{i-k}\left[\gamma_{i-k}\left(\frac{p^U_{i-k+1}\gamma_{i-k+1}}{p^U_{i-k}\gamma_{i-k}} - \frac{\beta_{i-k+1}}{\gamma_{i-k}}\right) + \beta_{i-k+1} + \Sigma^{j=i}_{j=i-k+2}\beta_j\right] \\
&= p^U_{i-k-1}\gamma_{i-k} + p^U_{i-k}\beta_{i-k+1} + p^U_{i-k}\Sigma^{j=i}_{j=i-k+2}\beta_j \; .
\end{aligned}$$

Then, using Assumption 1 and $p_{i-k}^U > p_{i-k-1}^U$, it follows that

$$\begin{aligned} E\pi_i^d(p_{i-k}^U) &> p_{i-k-1}^U\gamma_{i-k} + p_{i-k}^U\beta_{i-k+1} + p_{i-k}^U\Sigma_{j=i-k+2}^{j=i}\beta_j - (p_{i-k}^U - p_{i-k-1}^U)(\beta_{i-k+1} - \gamma_{i-k}) \\ &= p_{i-k}^U\gamma_{i-k} + p_{i-k-1}^U\beta_{i-k+1} + p_{i-k}^U\Sigma_{j=i-k+2}^{j=i}\beta_j \\ &> p_{i-k}^U\gamma_{i-k} + p_{i-k-1}^U\Sigma_{j=i-k+1}^{j=i}\beta_j = E\pi_i^d(p_{i-k-1}^U) \ . \end{aligned}$$

This implies that the most profitable deviation is at $p = p_{i-1}^U$ but there deviation profit is equal to expected equilibrium profit.

7.5 The Analysis of Price Discrimination in Oligopoly

Consider first a price $p \in [p_1^D, p_2^D]$. This price is in the support of firm 1's strong market price distribution and in the support of firm 2's weak market price distribution. These firms' expected profits at p and constant profit levels are presented below.

$$\begin{aligned} E\pi_1^s(p) &= p\sigma_1(1-\alpha F_2^w(p))\Pi_{j\geq 3}(1-\alpha F_j^w(p)) = p\sigma_1(1-\alpha F_2^w(p)) = p_1^D\sigma_1 \equiv E\pi_1^s \\ E\pi_2^w(p) &= p\sigma_1(1-\alpha F_1^s(p))\Pi_{j\geq 3}(1-\alpha F_j^w(p)) = p\sigma_1(1-\alpha F_1^s(p)) = p_1^D\sigma_1 = p_2^D\sigma_1(1-\alpha) \equiv E\pi_2^w \end{aligned}$$

The logic behind the expected profit expressions is similar to that in triopoly. These expressions simplify as, for $j \geq 3$, $F_j^w(p) = 0$ and so $\Pi_{j\geq 3}(1-\alpha F_j^w(p)) = 1$ in this price range. The expected profit levels obtain by using $F_1^s(p_1^D) = F_2^w(p_1^D) = 0$ and $F_1^s(p_2^D) = 1$.

Then, expression (6) gives $F_1^s(p)$ and $F_2^w(p)$ and the boundary price ratio p_1^D/p_2^D.

Consider now a price $p \in [p_2^D, p_3^D)$. This price is in the support of firm 2's strong market price distribution and in the support of firm 3's weak market price distribution. These firms' expected profits at price p and constant profit conditions are presented below.

$$\begin{aligned} E\pi_2^s(p) &= p\beta_2(1-\alpha F_3^w(p))\Pi_{j\geq 4}(1-\alpha F_j^w(p)) = p\beta_2(1-\alpha F_3^w(p)) = p_2^D\beta_2 \equiv E\pi_2^s \\ E\pi_3^w(p) &= p[\beta_2(1-\alpha F_2^s(p)) + \sigma_1(1-\alpha F_1^s(p))(1-\alpha F_2^w(p))]\Pi_{j\geq 4}(1-\alpha F_j^w(p)) \\ &= p_3^D(1-\alpha)\gamma_2 \equiv E\pi_3^w \end{aligned}$$

These expression simplify as $F_j^w(p) = 0$ for $j \geq 4$ and so $\Pi_{j\geq 4}(1-\alpha F_j^w(p)) = 1$, while $F_1^s(p) = F_2^w(p) = 1$ and so $(1-\alpha F_1^s(p)) = (1-\alpha F_2^w(p)) = (1-\alpha)$ in this price range. Constant profit levels obtain by using $F_2^s(p_2^D) = F_3^w(p_2^D) = 0$ and $F_2^s(p_3^D) = 1$.

The constant profit conditions above imply that $F_3^w(p)$ and $F_2^s(p)$ are implicitly defined by (7) and (8), while the boundary prices satisfy

$$\frac{p_2^D}{p_3^D} = \frac{\beta_2(1-\alpha) + \sigma_1(1-\alpha)^2}{\beta_2 + \sigma_1(1-\alpha)^2} \in ((1-\alpha), 1) \ .$$

The distribution $F_3^w(p)$ is defined on $[p_2^D, H_3]$ with $F_3^w(p_3^D) = (p_3^D - p_2^D)/\alpha p_3^D = \beta_2/[\beta_2 + (1-\alpha)^2\sigma_1] \in (0,1)$. In triopoly, $p_3^D = H_3 = 1$ and $F_3^w(p)$ has an atom at this price, $\phi = 1 - F_3^w(p_3^D) =$

$1 - F_3^w(1) = [p_2^D - (1-\alpha)]/\alpha$. In contrast, when $n \geq 4$, $H_3 \in (p_3^D, p_4^D)$, with $p_4^D \leq 1$.

Consider a price $p \in [p_3^D, H_3]$. This price is in the support of firm 3's strong market price distribution and in the supports of firm 3's and firm 4's weak market price distributions. These firms' expected profits at price p and constant profit conditions are presented below. This step uses the conjecture that $(1 - \alpha F_3^w(p)) = p_2^D/p$ at any price in its support, which is then verified.

$$
\begin{aligned}
E\pi_3^s(p) &= p\beta_3(1-\alpha F_4^w(p))\Pi_{j\geq 5}(1-\alpha F_j^w(p)) = p\beta_3(1-\alpha F_4^w(p)) = p_3^D\beta_3 \equiv E\pi_3^s \\
E\pi_4^w(p) &= p\left[\beta_3(1-\alpha F_3^s(p)) + \gamma_2(1-\alpha)(1-\alpha F_3^w(p))\right]\Pi_{j\geq 5}(1-\alpha F_j^w(p)) \\
&= p[\beta_3(1-\alpha F_3^s(p)) + \gamma_2(1-\alpha)(1-\alpha F_3^w(p))] \\
&= p\beta_3(1-\alpha F_3^s(p)) + p_2^D\gamma_2(1-\alpha) = p_3^D\beta_3 + p_2^D\gamma_2(1-\alpha) \equiv E\pi_4^w \\
E\pi_3^w(p) &= p\gamma_2(1-\alpha)(1-\alpha F_4^w(p))\Pi_{j\geq 5}(1-\alpha F_j^w(p)) \\
&= p\gamma_2(1-\alpha)(1-\alpha F_4^w(p)) = p_3^D\gamma_2(1-\alpha) \equiv E\pi_3^w
\end{aligned}
$$

In its strong market, firm 3 competes for consumers in group β_3 with any firm j for $j \geq 4$. However, in this price range, for $j \geq 5$ firm j's weak market distribution $F_j^w(p) = 0$, so that firm 3 competes head to head only with firm 4. Firm 3's constant profit level in its strong market is obtained by evaluating the expected profit at p_3^D, where $F_4^w(p_3^D) = 0$.

Firm 4 competes in its weak market with firm 3 for consumers in firm 3's strong market but it can also attract 'residual' consumers in firm 1's and firm 2's strong markets, provided that these firms' products are not available. All these consumers are also contested by any firm j for $j \geq 5$, but these firms' weak market price distributions in this price range are $F_j^w(p) = 0$. Residual consumers from firm 1's and firm 2's strong markets are also contested by firm 3. In the expression for $E\pi_4^w(p)$,

$$
\gamma_2(1-\alpha) = [\beta_2(1-\alpha F_2^s(p)) + \sigma_1(1-\alpha F_1^s(p))(1-\alpha F_2^w(p))] \ ,
$$

as in this price range $F_1^s(p) = F_2^w(p) = F_2^s(p) = 1$. The expression for firm 4's expected profit in its weak market further simplifies by using the conjecture that $(1 - \alpha F_3^w(p)) = p_2^D/p$. Firm 4's constant profit level in its weak market is obtained by evaluating its expected profit at price p_3^D, where $F_3^s(p_3^D) = 0$.

In its weak market, firm 3 competes for residual consumers from firm 1's and firm 2's strong market and the simplification above is used.

These constant profit conditions imply that

$$
1 - \alpha F_3^s(p) = 1 - \alpha F_4^w(p) = \frac{p_3^D}{p} \ .
$$

Firm 4's weak-market constant profit condition $E\pi_4^w(p_3) = E\pi_4^w(H_3)$ and the requirement that $F_3^w(H_3) = 1$ imply that

$$
p_3^D = H_3(1 - \alpha F_3^s(H_3)) \text{ and } p_2^D = (1-\alpha)H_3 \ .
$$

Consider now a price $p \in [H_3, p_4^D]$. This price is in the support of firm 3's strong market price distribution and in the support of firm 4's weak market price distribution. These firms' expected profits at price p and constant profit conditions are presented below.

$$\begin{aligned} E\pi_3^s(p) &= p\beta_3(1-\alpha F_4^w(p))\Pi_{j\geq 5}(1-\alpha F_j^w(p)) = p\beta_3(1-\alpha F_4^w(p)) \\ &= H_3\beta_3(1-\alpha F_4^w(H_3)) = p_3^D\beta_3 \equiv E\pi_3^s \\ E\pi_4^w(p) &= p\left[\beta_3(1-\alpha F_3^s(p)) + \gamma_2(1-\alpha)(1-\alpha F_3^w(p))\right]\Pi_{j\geq 5}(1-\alpha F_j^w(p)) \\ &= p[\beta_3(1-\alpha F_3^s(p)) + \gamma_2(1-\alpha)^2] = H_3[\beta_3(1-\alpha F_3^s(H_3)) + \gamma_2(1-\alpha)^2] \\ &= p_3^D\beta_3 + H_3\gamma_2(1-\alpha)^2 = p_4^D\gamma_3(1-\alpha) \equiv E\pi_4^w \end{aligned}$$

These expressions simplify as, in this price range, $\Pi_{j\geq 5}(1-\alpha F_j^w(p)) = 1$, $F_3^w(p) = 1$, $1-\alpha F_3^s(H_3) = p_3^D/H_3$, and $F_3^s(p_4^D) = 1$. The constant profit conditions then imply that

$$\begin{aligned} 1-\alpha F_4^w(p) &= \frac{p_3^D}{p} \\ 1-\alpha F_3^s(p) &= \frac{p_4^D}{p}\frac{\gamma_3(1-\alpha)}{\beta_3} - \frac{\gamma_2(1-\alpha)^2}{\beta_3} \\ H_3 &= \frac{p_4^D\gamma_3(1-\alpha) - p_3^D\beta_3}{\gamma_2(1-\alpha)^2} = \frac{p_4^D\gamma_2(1-\alpha)^2 - [p_3^D - (1-\alpha)p_4^D]\beta_3}{\gamma_2(1-\alpha)^2} \end{aligned}$$

The cut off price H_3 is well defined. It follows from the continuity of $F_3^w(H_3)$ that $H_3 > p_3^D$, as:

$$p_2^D = (1-\alpha)H_3 \text{ and } \frac{p_2^D}{p_3^D} > (1-\alpha)\ .$$

Moreover, $H_3 < p_4^D$ iff $p_3^D > (1-\alpha)p_4^D$ but this inequality follows from firm 4's constant profit condition in its weak market at prices p_3^D and p_4^D. Re-arranging the terms of $E\pi_4^w(p_3^D) = E\pi_4^w(p_4^D)$ and using $p_2^D/p_3^D \in ((1-\alpha), 1)$, it follows that:

$$\frac{p_3^D}{p_4^D} = \frac{\beta_3(1-\alpha) + \gamma_2(1-\alpha)^2}{\beta_3 + \frac{p_2^D}{p_3^D}\gamma_2(1-\alpha)} \in ((1-\alpha), 1)\ .$$

If $n = 4$, the steps above complete the derivation of firms' equilibrium strategies, $p_4^D = 1$ and firm 4's price distribution in its weak market has an atom at the upper bound of its support.

To complete the analysis for any n, consider now an arbitrary price interval $[p_{i-1}^D, p_i^D]$ for $i \geq 4$, where firms $i-1$ and i compete head to head. As before, suppose that in this interval $(1-\alpha F_{i-1}^w(p)) = p_{i-2}^D/p$. Figure 6 provides an illustration of firms' price distributions in the interval $[p_{i-1}^D, H_i]$ and supports the derivations below.

Consider first firm $i-1$ in its strong market. Its expected profit at a price $p \in [p_{i-1}^D, p_i^D]$ and the corresponding profit condition are

$$E\pi_{i-1}^s(p) = p\beta_{i-1}\Pi_{j=i}^n(1-\alpha F_j^w(p)) = p\beta_{i-1}(1-\alpha F_i^w(p)) = p_{i-1}^D\beta_{i-1} \equiv E\pi_{i-1}^s\ .$$

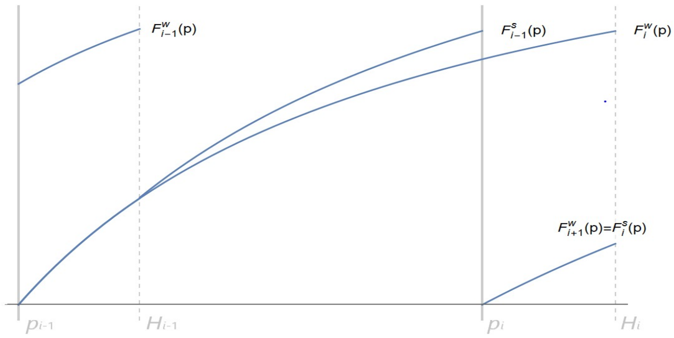

Figure 6: Firms' Distributions for $p \in [p^D_{i-1}, H_i]$

Firm $i-1$ competes for consumers in its strong market (group β_{i-1}) with any firm j for $j \geq n$, whose price is a random draw from its weak market price distribution. The expected profit simplifies as $F^w_j(p)) = 0$ for $j \geq i+1$, so that $\Pi^n_{j=i+1}(1-\alpha F^w_j(p)) = 1$ in this range. The constant profit condition then leads to (13) and implies that $F^w_i(p)$ is strictly increasing, with $F^w_i(p^D_{i-1}) = 0$ and $1-\alpha F^w_i(p_i) = p^D_{i-1}/p^D_i$. It is easy to verify that the c.d.f. in expression (13) is consistent with firm $i-1$'s constant profit condition in its weak market at price $p \in [p^D_{i-1}, H_{i-1}]$.

$$\begin{aligned}
E\pi^w_{i-1}(p) &= p\left[\sum_{j=1}^{j=i-2}\beta_j(1-\alpha F^s_j(p))\prod_{l=j+1,l\neq i-1}^{l=n}(1-\alpha F^w_l(p))\right] \\
&= p(1-\alpha F^w_i(p))\left[\beta_{i-2}(1-\alpha)+\sum_{j=1}^{j=i-3}\beta_j(1-\alpha)\prod_{l=j+1}^{l=i-2}(1-\alpha)\right] \\
&= p(1-\alpha F^w_i(p))(1-\alpha)\gamma_{i-2} = p^D_{i-1}(1-\alpha)\gamma_{i-2} \equiv E\pi^w_{i-1}\ .
\end{aligned}$$

Consider firm i in its weak market at price $p \in [p^D_{i-1}, p^D_i]$. Its expected profit is given below.

$$E\pi^w_i(p) = p\left[\beta_{i-1}(1-\alpha F^s_{i-1}(p)) + \sum_{j=1}^{j=i-2}\beta_j(1-\alpha F^s_j(p))\prod_{l=j+1}^{l=i-1}(1-\alpha F^w_l(p))\right]\prod_{l=i+1}^{l=n}(1-\alpha F^w_l(p))$$

The firm targets consumers in firm $i-1$'s strong market (group β_{i-1}), for whom it competes only with firm $i-1$. Moreover, it targets consumers in the strong market of any firm j for $j \leq i-2$, for whom it competes with firm j (which offers its strong market price) and with any firm l, for $l \geq j+1$ (which offers its weak market price). In this price range, $F^w_l(p) = 1$ for $j+1 \leq l \leq i-2$ and $F^w_l(p) = 0$ for $l \geq i+1$. For $p \in [p^D_{i-1}, H_{i-1}]$, firm i competes head to head with firm $i-1$ and $(1-\alpha F^w_{i-1}(p)) = p^D_{i-2}/p$ (which implies that $p^D_{i-2} = H_{i-1}(1-\alpha)$). For $p \in [H_{i-1}, p^D_i]$, firm i's price is for sure higher than firm $i-1$'s weak-market price. These cases are examined separately.

Firm i's weak-market expected profit at price $p \in [p^D_{i-1}, H_{i-1}]$ is simplified below and the constant profit conditions obtain by evaluating the expected profit at p^D_{i-1} and H_{i-1}.

$$\begin{aligned}
E\pi^w_i(p) &= p\beta_{i-1}(1-\alpha F^s_{i-1}(p)) + p^D_{i-2}(1-\alpha)\gamma_{i-2} = p^D_{i-1}\beta_{i-1} + p^D_{i-2}(1-\alpha)\gamma_{i-2} \\
&= H_{i-1}\beta_{i-1}(1-\alpha F^s_{i-1}(H_{i-1})) + H_{i-1}(1-\alpha)^2\gamma_{i-2} \equiv E\pi^w_i
\end{aligned}$$

These constant profit conditions identify firm $i-1$'s strong market price distribution:

$$(1-\alpha F_{i-1}^s(p)) = \frac{p_{i-1}^D}{p} \quad \text{for} \quad p \in [p_{i-1}^D, H_{i-1}] \ .$$

Firm i's weak market expected profit at price $p \in [H_{i-1}, p_i^D]$ is simplified below and the constant profit conditions obtain by evaluating the expected profit at H_{i-1}, where $(1-\alpha F_{i-1}^s(H_{i-1})) = p_{i-1}^D/H_{i-1}$, and at p_i^D.

$$\begin{aligned} E\pi_i^w(p) &= p\left[\beta_{i-1}(1-\alpha F_{i-1}^s(p)) + (1-\alpha)^2\gamma_{i-2}\right] \\ &= H_{i-1}\beta_{i-1}(1-\alpha F_{i-1}^s(H_{i-1})) + H_{i-1}(1-\alpha)^2\gamma_{i-2} \\ &= p_{i-1}^D\beta_{i-1} + H_{i-1}(1-\alpha)^2\gamma_{i-2} = p_i^D(1-\alpha)\gamma_{i-1} \equiv E\pi_i^w \ . \end{aligned}$$

These constant profit conditions identify the boundary prices presented in (15) and firm $i-1$'s distribution in its strong market for $p \in [H_{i-1}, p_i^D]$:

$$(1-\alpha F_{i-1}^s(p)) = \frac{p_i^D}{p}\frac{(1-\alpha)\gamma_{i-1}}{\beta_{i-1}} - \frac{(1-\alpha)^2\gamma_{i-2}}{\beta_{i-1}} = \frac{p_{i-1}^D\beta_{i-1} + H_{i-1}(1-\alpha)^2\gamma_{i-2}}{p\beta_{i-1}} - \frac{(1-\alpha)^2\gamma_{i-2}}{\beta_{i-1}} \ .$$

Expression (15) defines, together with $p_n^D = 1$, the sequences of cut-off prices $(p_i^D)_{i=1}^{i=n}$ and $(H_i)_{i=3}^{i=n-1}$. As $p_{i-1}^D > p_{i-2}^D$, (15) implies that $p_{i-1}^D/p_i^D > (1-\alpha)$. This confirms that $\left(1-\alpha F_i^w(p_i^D)\right) = p_{i-1}^D/p_i^D$ is well defined, implying also that $H_{i-1} = p_{i-2}^D/(1-\alpha) > p_{i-1}^D$. Note that the second expression in (15) can be re-written as:

$$p_i^D(1-\alpha)\gamma_{i-1} = p_{i-1}^D\beta_{i-1} + p_{i-2}^D(1-\alpha)\gamma_{i-2} \Leftrightarrow \frac{p_{i-1}^D}{p_i^D} = \frac{(1-\alpha)\gamma_{i-1}}{\beta_{i-1} + \dfrac{p_{i-2}^D}{p_{i-1}^D}(1-\alpha)\gamma_{i-2}} \ .$$

Using (15), it can also be verified that

$$H_{i-1} = \frac{p_i^D(1-\alpha)\gamma_{i-1} - p_{i-1}^D\beta_{i-1}}{\gamma_{i-2}(1-\alpha)^2} = \frac{p_i^D(1-\alpha)^2\gamma_{i-2} - p_i^D\beta_{i-1}\left[\dfrac{p_{i-1}^D}{p_i^D} - (1-\alpha)\right]}{\gamma_{i-2}(1-\alpha)^2} < p_i^D \ .$$

Combining the results above, firm i draws its weak market from the interval $[p_{i-1}^D, H_i]$ according to the c.d.f. $F_i^w(p)$ defined implicitly by (13). Firm $i-1$ draws its strong market price from the interval $[p_{i-1}^D, p_i^D]$ according to the c.d.f. $F_{i-1}^s(p_i)$ defined implicitly by (14), whose continuity at $p = H_{i-1}$ is verified using (15). Firm n's price distribution in its weak market is defined on $[p_{n-1}^D, 1]$ and has an atom at the upper bound $F_n^w(1) = (1-p_{n-1}^D)/\alpha < 1$, where the inequality follows as $p_{n-1}^D/p_n^D > (1-\alpha)$ and $p_n^D = 1$.

Finally notice that, as $p_i^D/p_{i+1}^D > (1-\alpha)$ and $p_n^D = 1$, $p_i^D > (1-\alpha)^{n-i}$ for $i \leq n-1$.

7.6 Proof of Proposition 4

(a) Consider first deviations by firm $i-1$ in its strong market. This firm's equilibrium price distribution is defined on $[p^D_{i-1}, p^D_i]$.

(a-1) A deviation to price $p < p^D_{i-1}$ is dominated by setting $p = p^D_{i-1}$:

$$E\pi^{sd}_{i-1}(p) = p\beta_{i-1}\Pi^n_{j=i}(1-\alpha F^w_j(p)) = p\beta_{i-1} < p^D_{i-1}\beta_{i-1} = E\pi^s_{i-1} \; .$$

(a-2) A deviation to price $p \in [p^D_{i+k}, p^D_{i+k+1}]$ for $k \geq 0$ yields:

$$\begin{aligned} E\pi^{sd}_{i-1}(p) &= p\beta_{i-1}\Pi^n_{j=i}(1-\alpha F^w_j(p)) \\ &= p\beta_{i-1}(1-\alpha F^w_{i+k}(p))(1-\alpha F^w_{i+k+1}(p))\Pi^{i+k-1}_{j=i}(1-\alpha) \\ &= p\beta_{i-1}(1-\alpha F^w_{i+k}(p))(1-\alpha F^w_{i+k+1}(p))(1-\alpha)^k \; , \end{aligned}$$

where the simplifications follow as $F^w_j(p) = 1$ for $j \leq i+k-1$ and $F^w_j(p) = 0$ for $j \geq i+k+2$.

When $p \in [p^D_{i+k}, H_{i+k}]$,

$$\begin{aligned} E\pi^{sd}_{i-1}(p) &= p\beta_{i-1}(1-\alpha)^k \frac{p^D_{i+k-1}p^D_{i+k}}{p^2} \leq p^D_{i+k-1}\beta_{i-1}(1-\alpha)^k \\ &= \Pi^{j=k}_{j=1}\frac{p^D_{i-1+j}}{p^D_{i-2+j}} p^D_{i-1}\beta_{i-1}(1-\alpha)^k < p^D_{i-1}\beta_{i-1} = E\pi^s_{i-1} \; , \end{aligned}$$

as by (15) $p^D_{i-1+j}/p^D_{i-2+j} < 1/(1-\alpha)$.

When $p \in [H_{i+k}, p^D_{i+k+1}]$,

$$\begin{aligned} E\pi^{sd}_{i-1}(p) &= p\beta_{i-1}(1-\alpha)^{k+1}\frac{p^D_{i+k}}{p} = p^D_{i+k}\beta_{i-1}(1-\alpha)^{k+1} \\ &= \prod_{j=1}^{j=k+1}\frac{p^D_{i-1+j}}{p^D_{i-2+j}} p^D_{i-1}\beta_i(1-\alpha)^{k+1} < p^D_{i-1}\beta_{i-1} = E\pi^s_{i-1} \; , \end{aligned}$$

as by (15) $p^D_{i-1+j}/p^D_{i-2+j} < 1/(1-\alpha)$.

(b) Consider now deviations by firm i in its weak market. This firm's equilibrium price distribution is defined on $[p^D_{i-1}, H_i]$.

(b-1) A deviation by firm i to a price $p \in [H_{i+k}, H_{i+k+1}]$ for $k \geq 0$ yields:

$$\begin{aligned} E\pi^{wd}_i(p) &= p\left[\sum_{j=1}^{j=i-1}\beta_j(1-\alpha F^s_j(p))\prod_{l=j+1,l\neq i}^{l=n}(1-\alpha F^w_l(p))\right] \\ &\overset{(1)}{=} p\left[\sum_{j=1}^{j=i-1}\beta_j(1-\alpha)\prod_{l=j+1,l\neq i}^{l=i+k+2}(1-\alpha F^w_l(p))\right] \\ &= p(1-\alpha F^w_{i+k+1}(p))(1-\alpha F^w_{i+k+2}(p))\left[\sum_{j=1}^{j=i-1}\beta_j(1-\alpha)\prod_{l=j+1,l\neq i}^{l=i+k}(1-\alpha F^w_l(p))\right] \end{aligned}$$

$$\begin{aligned}
&\overset{(2)}{=} \quad p(1-\alpha F^w_{i+k+1}(p))(1-\alpha F^w_{i+k+2}(p))(1-\alpha)^k \left[\sum_{j=1}^{j=i-1} \beta_j (1-\alpha)^{i-j}\right] \\
&= \quad p(1-\alpha F^w_{i+k+1}(p))(1-\alpha F^w_{i+k+2}(p))(1-\alpha)^{k+1}\gamma_{i-1} \ .
\end{aligned}$$

Simplification (1) uses that $F^s_j(p) = 1$ for $j \leq i-1$ and that $F^w_l(p) = 0$ for $l \geq i+k+2$ at this price. Simplification (2) uses that $F^w_l(p) = 1$ for $l \leq i+k$ where $l \neq i$.

When $p \in [H_{i+k}, p^D_{i+k+1}]$, expected deviation profit simplifies to

$$E\pi^{wd}_i(p) = p\frac{p^D_{i+k}}{p}(1-\alpha)^{k+1}\gamma_{i-1} = (1-\alpha)^{k+1}\prod_{j=1}^{l=k}\frac{p^D_{i+j}}{p^D_{i+j-1}}p^D_i\gamma_{i-1} < p^D_i(1-\alpha)\gamma_{i-1} = E\pi^w_i \ ,$$

as by (15) $p^D_{i+j}/p^D_{i+j-1} < 1/(1-\alpha)$.

When $p \in [p^D_{i+k+1}, H_{i+k+1}]$, expected deviation profit simplifies to the same expression as above and so it is strictly below expected equilibrium profit:

$$E\pi^{wd}_i(p) = p\frac{p^D_{i+k}p^D_{i+k+1}}{p^2}(1-\alpha)^{k+1}\gamma_{i-1} \leq p^D_{i+k}(1-\alpha)^{k+1}\gamma_{i-1} < E\pi^w_i \ .$$

(b-2) A deviation by firm i to a price $p \in [p^D_{i-k}, p^D_{i-k+1}]$ for $k \geq 2$ and $i-k \geq 1$ results in expected deviation profit

$$\begin{aligned}
E\pi^{wd}_i(p) \quad &= \quad p\left[\sum_{j=1}^{j=i-1}\beta_j(1-\alpha F^s_j(p))\prod_{l=j+1,l\neq i}^{l=n}(1-\alpha F^w_l(p))\right] \\
&\overset{(1)}{=} \quad p\left[\sum_{j=1}^{j=i-1}\beta_j(1-\alpha F^s_j(p))\prod_{l=j+1,l\neq i}^{l=i-k+1}(1-\alpha F^w_l(p))\right] \\
&\overset{(2)}{=} \quad p\left[\sum_{j=1}^{j=i-k}\beta_j(1-\alpha F^s_j(p))\prod_{l=j+1}^{l=i-k+1}(1-\alpha F^w_l(p)) + \sum_{j=i-k+1}^{j=i-1}\beta_j(1-\alpha F^s_j(p))\right] \\
&\overset{(3)}{=} \quad p(1-\alpha F^w_{i-k+1}(p))\left[\sum_{j=1}^{i-k-1}\beta_j(1-\alpha F^s_j(p))\prod_{l=j+1}^{i-k}(1-\alpha F^w_l(p)) + \beta_{i-k}(1-\alpha F^s_{i-k}(p))\right] \\
&+ \quad p\sum_{j=i-k+1}^{j=i-1}\beta_j \\
&\overset{(4)}{=} \quad p^D_{i-k}\left[\sum_{j=1}^{i-k-1}\beta_j(1-\alpha)^{i-k-j}(1-\alpha F^w_{i-k}(p)) + \beta_{i-k}(1-\alpha F^s_{i-k}(p))\right] + p\sum_{j=i-k+1}^{j=i-1}\beta_j \ .
\end{aligned}$$

Simplification (1) uses that $F^w_l(p) = 0$ for $l \geq i-k+2$ at this price. As a result, for consumers in the strong market of firm j for $i-k+1 \leq j \leq i-1$, firm i only competes with firm j, which sets its price according to $F^s_j(p)$ - this is reflected in simplification (2) - and at this price $F^s_j(p) = 0$ - as reflected in the last term in simplification (3). The first term in the (3) is obtained by factoring out $(1-\alpha F^w_{i-k+1}(p))$, which is strictly decreasing over this interval. Simplification (4) uses that $(1-\alpha F^w_{i-k+1}(p)) = p^D_{i-k}/p$ and also that $F^s_j(p) = F^w_l(p) = 1$ for $j \leq i-k-1$ and $l \leq i-k-1$.

When $p \in [p^D_{i-k}, H_{i-k}]$, by (13) and (14), $(1-\alpha F^w_{i-k}(p)) = p^D_{i-k-1}/p$ and $(1-\alpha F^s_{i-k}(p)) = p^D_{i-k}/p$

and the expected deviation profit becomes

$$
\begin{aligned}
E\pi_i^{wd}(p) &= p_{i-k}^D\left[\sum_{j=1}^{j=i-k-1}\beta_j(1-\alpha)^{i-k-j}\frac{p_{i-k-1}^D}{p}+\beta_{i-k}\frac{p_{i-k}^D}{p}\right]+p\sum_{j=i-k+1}^{j=i-1}\beta_j \\
&= \frac{\left(p_{i-k}^D\right)^2}{p}\left[\sum_{j=1}^{j=i-k-1}\beta_j(1-\alpha)^{i-k-j}\frac{p_{i-k-1}^D}{p_{i-k}^D}+\beta_{i-k}\right]+p\sum_{j=i-k+1}^{j=i-1}\beta_j\ ,
\end{aligned}
$$

which is convex in p and so maximised at either $p=p_{i-k}^D$ or at $p=H_{i-k}$.

When $p\in[H_{i-k},p_{i-k+1}^D]$, by (13) as $F_{i-k}^w(p)=1$ and $\sum_{j=1}^{i-k-1}\beta_j(1-\alpha)^{i-k-j}(1-\alpha F_{i-k}^w(p))=(1-\alpha)^2\gamma_{i-k-1}$. Then, using (14), expected deviation profit is

$$
\begin{aligned}
E\pi_i^{wd}(p) &= p_{i-k}^D\left\{(1-\alpha)^2\gamma_{i-k-1}+\beta_{i-k}\left[\frac{p_{i-k+1}^D(1-\alpha)\gamma_{i-k}}{p\beta_{i-k}}-\frac{(1-\alpha)^2\gamma_{i-k-1}}{\beta_{i-k}}\right]\right\}+p\sum_{j=i-k+1}^{j=i-1}\beta_j \\
&= \frac{p_{i-k}^Dp_{i-k+1}^D(1-\alpha)\gamma_{i-k}}{p}+p\sum_{j=i-k+1}^{j=i-1}\beta_j\ ,
\end{aligned}
$$

which is convex in p and so maximised at either $p=H_{i-k}$ or at $p=p_{i-k+1}^D$. It it easy to verify, using (15), that expected deviation profit is continuous at $p=H_{i-k}$.

These deviations are ruled out below in two steps, using Assumption 2. Step 1 shows that $E\pi_i^{wd}(p_{i-k+1}^D)>E\pi_i^{wd}(p_{i-k}^D)$ and step 2 shows that $E\pi_i^{wd}(p_{i-k+1}^D)>E\pi_i^{wd}(H_{i-k})$. Together, they imply that expected deviation profit is highest at $p=p_{i-1}^D$ but at that price it is equal to expected equilibrium profit.

Step 1. Expected deviation profits $E\pi_i^{wd}(p_{i-k}^D)$ and $E\pi_i^{wd}(p_{i-k+1}^D)$ are presented below.

$$
\begin{aligned}
E\pi_i^{wd}(p_{i-k}^D) &= p_{i-k-1}^D\left[\sum_{j=1}^{j=i-k-1}\beta_j(1-\alpha)^{i-k-j}\right]+p_{i-k}^D\sum_{j=i-k}^{j=i-1}\beta_j \\
&= p_{i-k-1}^D(1-\alpha)\gamma_{i-k-1}+p_{i-k}^D\sum_{j=i-k}^{j=i-1}\beta_j \\
&\overset{(1)}{=} p_{i-k+1}^D(1-\alpha)\gamma_{i-k}-p_{i-k}^D\beta_{i-k}+p_{i-k}^D\sum_{j=i-k}^{j=i-1}\beta_j \\
&= p_{i-k+1}^D(1-\alpha)\gamma_{i-k}+p_{i-k}^D\sum_{j=i-k+1}^{j=i-1}\beta_j \\
E\pi_i^{wd}(p_{i-k+1}^D) &= p_{i-k}^D(1-\alpha)\gamma_{i-k}+p_{i-k+1}^D\sum_{j=i-k+1}^{j=i-1}\beta_j
\end{aligned}
$$

Simplification (1) uses (15), which implies that $p_{i-k-1}^D(1-\alpha)\gamma_{i-k-1}=p_{i-k+1}^D(1-\alpha)\gamma_{i-k}-p_{i-k}^D\beta_{i-k}$. Then, if Assumption 2 holds:

$$
E\pi_i^{wd}(p_{i-k+1}^D)>E\pi_i^{wd}(p_{i-k}^D)\Leftrightarrow(p_{i-k+1}^D-p_{i-k}^D)\left[\sum_{j=i-k+1}^{j=i-1}\beta_j-(1-\alpha)\gamma_{i-k}\right]>0\ .
$$

Step 2.

$$
\begin{gathered}
E\pi_i^{wd}(H_{i-k}) < E\pi_i^{wd}(p_{i-k+1}^D) \Leftrightarrow \\
\frac{p_{i-k}^D p_{i-k+1}^D (1-\alpha)\gamma_{i-k}}{H_{i-k}} + H_{i-k} \sum_{j=i-k+1}^{j=i-1} \beta_j < p_{i-k}^D (1-\alpha)\gamma_{i-k} + p_{i-k+1}^D \sum_{j=i-k+1}^{j=i-1} \beta_j \\
\sum_{j=i-k+1}^{j=i-1} \beta_j \left[p_{i-k+1}^D - H_{i-k}\right] > (1-\alpha)\gamma_{i-k} \frac{p_{i-k}^D}{H_{i-k}} \left[p_{i-k+1}^D - H_{i-k}\right] \Leftrightarrow \\
\sum_{j=i-k+1}^{j=i-1} \beta_j > (1-\alpha)\gamma_{i-k} \frac{p_{i-k}^D}{H_{i-k}} \ ,
\end{gathered}
$$

which holds as, under Assumption 2, $\sum_{j=i-k+1}^{j=i-1} \beta_j > (1-\alpha)\gamma_{i-k} > (1-\alpha)\gamma_{i-k} p_{i-k}^D / H_{i-k}$.

As no firm has an incentive to deviate to prices above $p = 1$, there are no profitable unilateral deviations.

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