Strategic capacity investment with overlapping ownership

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Abstract

We study how overlapping ownership affects capacity investments when firms invest sequentially. Whereas followers react less aggressively, leaders act more aggressively by choosing larger capacities or by shifting from accommodation to deterrence. Consumer surplus and welfare may increase when a leader's strategy shifts. To endogenize leader and follower roles, we allow demand to fluctuate over time and show that in a preemption equilibrium with internalization leader entry occurs earlier but at a smaller scale.

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1 Introduction

Horizontal ownership concentration, where a small number of institutional investors hold significant minority stakes in competing firms, is an increasingly pervasive phenomenon which has raised regulatory concern because of its potential to foster anticompetitive behavior (Backus et al. 2021, Posner et al. 2016). Horizontal minority shareholding, where firms take non-controlling stakes in product market rivals, has also risen since the turn of the millennium. Such forms of overlapping ownership have attracted a wave of academic interest, kindled notably by empirical evidence of pricing distortions in the airline industry due to common ownership (Azar et al. 2018). New types of evidence like natural and laboratory experiments have emerged (Heim et al. 2022, Hariskos et al. 2022), lending broad credence to the thesis that managers account for ownership structure in their decision-making by internalizing some of the effects they exert on rival firms.

In the discussion surrounding common ownership, the causal mechanism linking owners to the managerial decisions that determine product market outcomes is a central theme (Hemphill and Kahan 2019, Anton et al. 2021). Case evidence suggests that institutional investors regularly engage with the management of their portfolio firms (Shekita 2022), and chief among the strategic decisions that top management makes is the exercise of a firm's real options (Smit and Trigeorgis 2017). So far the study of strategic effects of common ownership has centered mostly around R&D investments, but firms in many industries concerned by common ownership hold other options which are equally important if not more, like the option to build production capacity.

When firms exercise such investment options sequentially, the consequences of overlapping ownership are diverse. Internalization generally drives followers to behave less aggressively by entering markets at a smaller scale if they do enter, or not entering at all. Leaders are then subject to two opposing forces. They may take advantage of softer follower behavior to improve their position in the market, but they are limited in this endeavor by their own internalization of follower profits. Most often, the former effect dominates and the overall response of leaders to internalization is to act more aggressively. From a normative standpoint though, more aggressive leader behavior may generate higher consumer surplus and welfare, which means that evidence of delayed or deterred follower entry (e.g. Newham et al. 2018) does not alone suffice to draw normative conclusions about overlapping ownership.

In this paper, we study how leader and follower strategies are affected if two firms have overlapping ownership (either because of common shareholders or because of cross-holding) which induces their management to internalize rival value when making capacity investment decisions. We first study sequential capacity choices in the standard Stackelberg model. In the case of a static market, we verify that internalization softens the follower's quantity reaction and show that this drives the leader to choose a larger capacity under accommodation. In addition sufficient internalization allow the leader to shut the follower out of the market without exercising deterrence in the conventional sense. We then introduce an entry cost and characterize equilibrium capacities, showing that either a higher fixed cost or greater internalization shift the leader's strategy from accommodation to deterrence. When the leader's strategy shifts with internalization however, total output jumps up which implies that consumers can benefit from more aggressive leader behavior.

The following example from Section 3 below illustrates this idea. Take a standard entry deterrence framework with sequential capacity choices by a leader and a follower, linear demand $Q = 1 - P$, zero variable cost, and a fixed cost of entry $f = .0025$ for the follower. With this setup the leader ordinarily prefers to accommodate follower entry.

Suppose instead that there is symmetric cross-ownership with each firm i holding a 100λ percent stake in the rival with $\lambda \in \{0, .1, .2\}$. Up to a normalization, each firm therefore maximizes a weighed sum of profits, $\pi_i + \lambda\pi_{-i}$. The equilibrium capacities of the leader and follower and resulting consumer surplus and welfare are:

Table 1: Procompetitive strategic shift (static case)

	Q_L^*	$Q_F^*\,(Q_L^*)$	Q^{Total}	CS	W
no cross-ownership	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	.28125	.46625
10% cross-ownership	$\frac{9}{11}$	0	$\frac{9}{11}$	.33	.48
20% cross-ownership	$\frac{3}{4}$	0	$\frac{3}{4}$	.28125	.46875

Table 1 shows how a small degree of ownership overlap can be procompetitive. This happens because internalization (due to mutual 10% stakes in the second row) induces the leader to adopt a deterrence strategy which significantly increases output, to an extent that sufficient to outweigh the follower's absence. With more overlap (mutual 20% stakes), internalization still shifts the leader's strategy from accommodation to deterrence, but there is no longer a procompetitive output effect (total welfare is marginally higher though due to the economy of the follower's entry cost). We get the same finding in our dynamic model, providing a similar example of a procompetitive strategic shift involving both follower capacity and timing (see Table 3, page 31).

A central criticism of the Stackelberg model is that it assumes firm roles are exogenous. To address this shortcoming we expand our analysis by allowing demand to fluctuate over time and firms to compete for leadership by determining the timing as well as the scale of their investment. We show that in this setting both the follower's timing and capacity choice are less aggressive with internalization.[1] The follower's less aggressive timing and

[1] This finding contrasts with work involving fixed investments which shows that the follower can enter

quantity reactions benefit the leader, which enjoys a protracted monopoly period followed by less intense duopoly competition, but the leader's choices are also affected by its own internalization of the follower. In the dynamic setting it is inefficient to deter the follower permanently, but the leader can exercise strategic deterrence by delaying the follower's entry. We show the leader's capacity decision problem is qualitatively unaltered by internalization if the degree of internalization is not too high, which means that the leader chooses to deter the follower in low demand states and to accommodate in high demand states. At higher levels of internalization, we also show however that the leader behaves more aggressively by invariably deterring the follower.

When firms compete for the leadership role, we find that in a preemption equilibrium the leader chooses a capacity that delays the follower's entry rather than accommodating it. The follower's less aggressive reaction makes leading relatively more attractive, so internalization exacerbates positional competition between firms which has a procompetitive effect on entry timing. This procompetitive effect is offset however by the leader's lower equilibrium capacity. As in the static case, we also find in the dynamic model that if leader investment occurs at an intermediate demand state, a moderate degree of internalization can exert a procompetitive effect by shifting the leader's strategy from accommodation to deterrence.

We complement our analytical results with a numerical analysis which bears out these insights, i.e. an anticompetitive effect of internalization for the follower but also a procompetitive effect on the timing of investment in preemption equilibrium. We show moreover that the other procompetitive effect, due to the shift in the leader's strategy, effectively occurs and that the resulting increase in capacity can be large enough that consumer surplus and welfare increase.

earlier if product market profits are extremely sensitive to internalization, which is not the case with the quantity choice specification in the present model (Zormpas and Ruble 2021, Vives and Vravosinos 2022).

Our paper contributes to the study of how overlapping ownership affects strategic behavior. Many contributions so far have focused on the key dimension of innovation, where overlapping ownership has been shown to affect welfare positively if R&D spillovers are important (Vives 2020), in innovation contests (Stenbacka and Van Moer 2022), and by facilitating technology transfer (Papadopoulos et al. 2019). Our analysis complements this stream by identifying procompetitive effects which arise more broadly, e.g. in industries that are not R&D intensive.

The contributions that relate most closely to ours are those that study the effect of internalization on entry. Li et al. (2015) find that an incumbent may transfer ownership to a potential entrant in order to deter it, though there are no capacity commitments in their setup, and in symmetric entry models various authors find an inverted-U relationship between internalization and entry (Sato and Matsumura 2020, Vives and Vravosinos 2022). Relative to these papers, our analysis highlights how internalization affects capacity commitments and provides a complete characterization of accommodation and deterrence in the standard duopoly entry model. Our analysis also relates to an entirely different strain or work, exemplified by Etro (2008)'s study of Stackelberg commitment with endogenous followers, which shows that more aggressive leader behavior can improve welfare.

The dynamic version of our model also contributes to the literature on strategic investment with timing and capacity choice (Huisman and Kort 2015) by complementing other studies which have allowed for pre-existing capacities or introduced time-to-build considerations (Huberts et al. 2019, Jeon 2021), and also complements prior work with fixed-size investments (Zormpas and Ruble 2021), notably by showing how leader strategy shifts with internalization and how lower equilibrium capacities mitigate the procompetitive effect of preemption.

The rest of the paper is organized as follows. Section 2 states the main assumptions of our model. Section 3 characterizes leader and follower behavior in the static market case, first in a standard Stackelberg setup and then in an entry deterrence framework. Section 4 studies follower behavior and leader behavior capacity choice in a dynamic market. Section 5 endogenizes firm roles describes equilibrium investment. In Section 6, we study a numerical example and discuss implications for welfare.

2 Model

An industry consists of two firms which are inactive in the product market initially. Their ownership structures overlap and are symmetric. Each firm therefore maximizes an objective

$$\Omega_i = V_i + \lambda V_{-i} \tag{1}$$

where V_i denotes the value of the firm's own assets and V_{-i} the value of its rival's assets. The parameter $\lambda \in [0, 1]$ measures the weight each firm's objective places on rival value. This weight is referred to as the degree of internalization, with $\lambda = 0$ representing purely self-interested behavior and $\lambda = 1$ representing joint value maximization. Vives (2020) discusses common and cross-ownership structures that yield Eq. (1) up to possible normalization. Estimates of λ vary widely over industries and countries, with average values for U.S. firms possibly reaching .7 (Backus et al. 2021). The degree of internalization need not vary much across firms however, particularly in the case of common ownership.[2] To motivate our assumption that the degree of internalization is symmetric, in the U.S. pharmaceutical industry the combined ownership of the top three institutional shareholders (BlackRock, State Street and Vanguard) in the top three firms (Johnson & Johnson, Merck, and Pfizer) amounted to 19, 18, and 18% respectively (as of August 2022).

[2] Ownership structures and hence internalization are less likely to be symmetric in situations of cross-ownership, as with the minority share acquisitions reported by Heim et al. (2022).

We suppose that at any time $t \geq 0$, inverse demand in the product market is linear with

$$P(t) = X(t)\,(1 - Q(t)) \tag{2}$$

where $Q(t) \in [0, 1]$ is industry capacity and $X(t)$ is an exogenous shock. The exogenous shock evolves according to a geometric Brownian motion

$$dX(t) = \mu X(t)dt + \sigma X(t)d\omega(t) \tag{3}$$

where μ is the drift, $\sigma \geq 0$ the volatility, and $\omega(t)$ is a standard Wiener process.

The firms choose when and at what capacity to enter the market. Market entry involves a single capacity investment. The incremental cost of capacity is a constant $\delta > 0$, and capacity can be neither altered nor resold once it is installed. There are no production costs and firms invariably operate at capacity.

Finally the discount rate r is constant and we suppose $r > \mu$ to focus on the case where the expected revenue stream is bounded.

3 Sequential capacity choice with internalization (static market)

Suppose $\mu = \sigma = 0$ so the demand shock is a constant, which means the market is static. Denote the demand shock level by X and suppose that $\delta < \frac{X}{r}$ so the capacity cast is not prohibitive compared to capitalized demand. For compactness, let $\delta' = \frac{r\delta}{X}$ denote normalized capacity cost. We start the section by studying how internalization affects Stackelberg equilibrium and then discuss entry deterrence.

Suppose that the firms choose capacities sequentially and denote leader and follower capacities by Q_L and Q_F respectively. We proceed by backward induction and begin by analyzing the follower's problem.

By Eq. (1) the follower's perceived profit function is

$$\begin{aligned}\Omega_F(Q_F, Q_L) &= V_F(Q_F, Q_L) + \lambda V_L(Q_L, Q_F) \\ &= \left(\frac{X}{r}(1 - Q_L - Q_F) - \delta\right)(Q_F + \lambda Q_L),\end{aligned} \tag{4}$$

$Q_F \in [0, 1 - Q_L]$. Its optimal capacity is therefore $Q_F^*(Q_L) = \max\left\{0,\ \frac{1}{2}(1 - \delta' - (1+\lambda) Q_L)\right\}$. As the follower's reaction insofar as the optimal capacity is weakly decreasing in λ, internalization predictably softens the follower's reaction. Moreover the slope of the follower's reaction is decreasing in λ, so internalization also has a negative incentive effect.

By Eq. (1), the leader's perceived profit function is

$$\begin{aligned}\Omega_L(Q_L) &= V_L(Q_L, Q_F^*(Q_L)) + \lambda V_F(Q_F^*(Q_L), Q_L) \\ &= \begin{cases} \frac{1}{4}\frac{X}{r}\left(-(2+\lambda)(1-\lambda)^2 Q_L^2 + 2(1-\lambda)(1-\delta') Q_L + \lambda(1-\delta')^2\right), & Q_L < \frac{1-\delta'}{1+\lambda} \\ \left(\frac{X}{r}(1 - Q_L) - \delta\right) Q_L, & Q_L \geq \frac{1-\delta'}{1+\lambda}. \end{cases}\end{aligned} \tag{5}$$

The first line shows how internalization affects the leader's payoff both directly, through the weight that the leader attributes to the follower's payoff $V_F(Q_F^*(Q_L), Q_L)$, and indirectly, through the follower's softened quantity reaction $Q_F^*(Q_L)$. The leader's payoff in the second line is defined piecewise due to the follower's reaction. Over the first piece the leader's capacity is low enough that the follower chooses to be active, and over the second piece the leader drives the follower to choose zero capacity. The leader's optimum capacity can lie on either piece depending on the degree of internalization.

If the degree of internalization is not too high (if $\lambda < \sqrt{2} - 1$), the leader's perceived profit has an interior maximum on its first piece and is decreasing on the second so the interior maximum is global. This interior maximum is referred to as the accommodation capacity, and we denote it by Q_L^{a}. For all positive λ, the accommodation capacity is larger than the capacity the leader would choose in the absence of internalization, and which we denote by Q_M as it corresponds to the monopoly capacity.[3] Intuitively, the follower's softer and steeper reaction raises the leader's incentive to build capacity while the leader's internalization of its negative effect on the follower works in the opposite direction, and with linear demand it is the former of these effects which dominates.[4] Moreover Q_L^{a} is increasing in λ so greater internalization makes the leader more aggressive.

If the degree of internalization is high enough (if $\lambda \geq \sqrt{2}-1$), the leader's perceived profit is increasing over the first piece and decreasing over the second. The leader's optimal capacity is then $\frac{1-\delta'}{1+\lambda}$. This capacity is sufficient to drive the follower out of the market, even though it does not face a positive entry cost (aside from its capacity cost) and could profitably produce a positive output. As in the low internalization case, the leader's optimal capacity is more aggressive than without internalization. Its optimal capacity decreases locally with the level of internalization however, as less scale is needed to discourage the follower from producing.

To summarize:

[3] We have $Q_L^{\mathrm{a}} = \frac{1-\delta'}{(2+\lambda)(1-\lambda)}$ and $Q_M = \frac{1-\delta'}{2}$. $Q_L^{\mathrm{a}} = Q_M$ for $\lambda = 0$ and Q_L^{a} is increasing in λ, so $Q_L^{\mathrm{a}} > Q_M$ for $\lambda > 0$.

[4] Consider the general case of logconcave demand, so $P(Q) + QP'(Q)$ is decreasing. The leader's marginal perceived profit is $\frac{dV_L}{dQ_L}\left(Q_L, Q_F^*(Q_L)\right) + \lambda \frac{dV_F}{dQ_L}\left(Q_F^*(Q_L), Q_L\right)$.

The first term in this expression is the standard marginal profit of a Stackelberg leader, up to the follower reaction $Q_F^*(Q_L)$ which incorporates the follower's internalization of the leader's payoff. Developing this first term gives $P\left(Q_L + Q_F^*(Q_L)\right) + Q_L P'\left(Q_L + Q_F^*(Q_L)\right)\left(1 + \frac{dQ_F^*}{dQ_L}\right) - \delta$, which is higher with internalization because of the softer and steeper follower reaction (lowering $Q_F^*(Q_L)$ increases $P\left(Q_L + Q_F^*(Q_L)\right) + Q_L P'\left(Q_L + Q_F^*(Q_L)\right)$ because demand is logconcave, and $Q_L P'\left(Q_L + Q_F^*(Q_L)\right)\frac{dQ_F^*}{dQ_L}$ increases if $\frac{dQ_F^*}{dQ_L}$ becomes more negative).

The second term the leader's marginal perceived profit is $\lambda\left(P\left(Q_L + Q_F^*(Q_L)\right) + Q_F^* P'\left(Q_L + Q_F^*(Q_L)\right) - \delta\right)\frac{dQ_F^*}{dQ_L}$ $+\lambda Q_F^* P'\left(Q_L + Q_F^*(Q_L)\right)$. The follower's first-order condition implies that $P\left(Q_L + Q_F^*(Q_L)\right) + Q_F^* P'\left(Q_L + Q_F^*(Q_L)\right) - \delta = -\lambda Q_L P'\left(Q_L + Q_F^*(Q_L)\right) > 0$ and $\frac{dQ_F^*}{dQ_L} < 0$, so the entire term, which does not appear in the absence of internalization, is negative.

Proposition 1. *In Stackelberg equilibrium the leader's capacity choice is more aggressive with internalization than without. Moreover,*

i) if $\lambda < \sqrt{2}-1$, *the leader's optimal capacity* Q_L^{a} *increases with internalization;*

ii) if $\lambda \geq \sqrt{2}-1$, *the leader's optimal capacity* $\frac{1-\delta'}{1+\lambda}$ *shuts the follower out of the market.*

Though we have not labeled it this way so far, part *ii*) of Proposition 1 describes how a leader can deter a follower if there is sufficient internalization, even in the absence of an entry cost. In order to understand this possibility further, we next discuss strategic deterrence in the conventional sense. To do this it suffices to generalize to an affine capacity cost with positive intercept. For the rest of this section, suppose therefore that the follower incurs an additional flat cost of entry f, and let $f' = \frac{rf}{X}$ denote its normalized value.

In this situation, the follower's entry decision factors in both the internalized profit if the leader were to operate as a monopolist and the entry cost. The follower accordingly enters if

$$\Omega_F\left(Q_F^*(Q_L), Q_L\right) > \Omega_F\left(0, Q_L\right) + f \tag{6}$$

where $\Omega_F\left(Q_F, Q_L\right)$ refers to the follower's post-entry payoff (Eq. 4). The internalized leader monopoly profit represents an additional opportunity cost of entering, which along with the capacity reaction $Q_F^*(Q_L)$ further softens the follower's entry decision. Substituting the follower payoff expressions into Eq. (6) gives the capacity level beyond which the leader deters the follower, which we denote by Q_L^{d}.[5] Q_L^{d} is decreasing in λ, i.e. the greater the degree of internalization, the less capacity the leader needs to install in order

[5]Setting $\left(\frac{X}{r}(1-Q_L-Q_F^*(Q_L))-\delta\right)\left(Q_F^*(Q_L)+\lambda Q_L\right) = \lambda\left(\frac{X}{r}(1-Q_L)-\delta\right)Q_L + f$, substituting for $Q_F^*(Q_L)$, and solving for Q_L gives $Q_L^{\mathrm{d}} = \frac{1-\delta'-2\sqrt{f'}}{1+\lambda}$. The capacity $\frac{1-\delta'}{1+\lambda}$ in part *ii*) of Proposition 1 therefore represents a limiting case of deterrence in the conventional sense as the entry cost f goes to zero.

to deter the follower.

Incorporating the follower's entry decision into the leader's perceived profit (Eq. 5) gives

$$\Omega_L(Q_L) = \begin{cases} \frac{1}{4}\frac{X}{r}\left(-(2+\lambda)(1-\lambda)^2 Q_L^2 + 2(1-\lambda)(1-\delta')Q_L + \lambda(1-\delta')^2\right) - \lambda f, & Q_L < Q_L^{\mathrm{d}} \\ \left(\frac{X}{r}(1-Q_L) - \delta\right) Q_L, & Q_L \geq Q_L^{\mathrm{d}}. \end{cases} \tag{7}$$

Over the first piece of Eq. (7), the leader's payoff has an interior maximum at the accommodation capacity Q_L^{a} if λ and f are not too large. Conversely, if λ and f are large enough the leader's payoff has an interior maximum over the second piece at the monopoly capacity Q_M. For intermediate values of λ and f, $Q_L^{\mathrm{a}} < Q_L^{\mathrm{d}}$ and $Q_L^{\mathrm{d}} > Q_M$ so the leader chooses between accommodation and deterrence by comparing the respective profits associated with the two local optima, $\Omega_L(Q_L^{\mathrm{a}})$ and $\Omega_L\left(Q_L^{\mathrm{d}}\right)$.

Figure 1 plots the regions in $\left(\frac{\sqrt{f'}}{1-\delta'}, \lambda\right)$-space where the leader accommodates ($Q_L^* = Q_L^{\mathrm{a}}$), deters ($Q_L^* = Q_L^{\mathrm{d}}$), or naturally blocks ($Q_L^* = Q_M$) the follower's entry. The horizontal axis ($\lambda = 0$) is the standard entry deterrence model, where the leader chooses these three strategies successively as the fixed cost increases. The vertical axis ($f = 0$) is the Stackelberg equilibrium analyzed at the beginning of the section, where the leader successively accommodates and deters as the degree of internalization increases, with blockade occurring only for $\lambda = 1$. The solid black curves extend the accommodation, deterrence and blockade regions to situations where both the entry cost and degree of internalization are positive.[6] Starting from $f = \lambda = 0$, increasing either the fixed cost or the degree of internalization shifts the leader's strategy from accommodation to deterrence, and finally to blockade. Within the range of parameter values where the leader deters, we also can

[6] Though it is difficult to distinguish visually, the boundary between accommodation and deterrence only intersects the vertical axis at $\lambda = \sqrt{2} - 1$.

distinguish two subregions depending upon whether the leader foregoes an interior optimum (accommodation) to shut the follower out strategically ($\lambda < \sqrt{2} - 1$, below the dotted line) or has a unique optimum ($\lambda \geq \sqrt{2} - 1$, above the dotted line).

By evaluating the total output that results from the leader's optimal capacity choice, we get the following set of results.

Proposition 2. *The leader is more likely to deter the follower rather than accommodate as the degree of internalization increases. Moreover, greater internalization reduces output for a given leader strategy (accommodation or deterrence), but output jumps up if greater internalization switches the leader's strategy from accommodation to deterrence.*

Proof. See Appendix A.1.

Proposition 2 has a number of normative implications. First of all, because consumer surplus is directly related to output, consumer surplus decreases with internalization provided that the leader's strategy (accommodation or deterrence) is unchanged. Second, because industry output is above the monopoly level, industry variable profit increases with internalization, provided again that the leader's strategy is unchanged.

In addition to these first two effects, because output jumps up at points where greater internalization shifts the leader's strategy from accommodation to deterrence, so does consumer surplus. We also show in the proof of Proposition 2 that industry profit decreases at such points. If λ is not too large therefore, the jump in consumer surplus can be large enough that consumers are better off than if there were no internalization at all, as illustrated by our introductory example (Table 1), and the same is true of total welfare. In Figure 1, the area where internalization shifts the leader's strategy (i.e. where the leader deters but would accommodate in the absence of overlapping ownership) is the wedge bounded above and below by the two black curves and to the right by the dashed segment. Within this region, the area below the dark gray curve describes where consumer surplus is higher with internalization than without because it induces sufficiently aggressive deterrence by the leader instead of accommodation, and the area below the light gray curve corresponds to higher total welfare for similar reasons.[7]

In this section we have shown how internalization leads to more aggressive leader behavior, but also that such a switch can generate higher consumer surplus and welfare. Our analysis so far suffers from the standard criticism of Stackelberg models however, namely that the order of moves is imposed upon firms exogenously. In the next sections, we relax this assumption by turning to the dynamic version of the model and endogenizing firm roles, which also allows us to identify which outcome (accommodation or deterrence) is likelier to emerge in equilibrium in an evolving market.

4 Sequential capacity choice with internalization (dynamic market)

In this and the following sections, we suppose that the demand shock has positive drift μ or volatility σ so the market evolves over time. In low demand states therefore, there is an incentive to wait for more opportune conditions rather than invest immediately. Because demand may increase without bound, it is also more difficult for the leader to shut the follower out permanently, because demand may increase without bound. However the leader can exercise a dynamic form of deterrence where it delays the follower's entry

[7] The condition for consumer surplus to be higher under internalization/deterrence is $Q_L^{\mathrm{d}} > \frac{3}{4}\left(1-\delta'\right) = Q^{\mathrm{a}}\big|_{\lambda=0}$ where $Q^{\mathrm{a}} = Q_L^{\mathrm{a}} + Q_F^{*}\left(Q_L^{\mathrm{a}}\right)$. For welfare (consumer surplus and industry profit, net of any follower entry cost) the condition is $\frac{Q_L^{\mathrm{d}\,2}}{2} + Q_L^{\mathrm{d}}\left(1-\delta'-Q_L^{\mathrm{d}}\right) > \frac{15}{32}\left(1-\delta'\right)^2 - f'$ $= \left(\frac{1}{2}\left(Q^{\mathrm{a}}\right)^2 + Q^{\mathrm{a}}\left(1-\delta'-Q^{\mathrm{a}}\right)\right)\Big|_{\lambda=0} - f'$.

strategically, which we return to in the the latter part of the section.[8]

Even if demand evolves in continuous time, from a decision perspective the investment problem remains essentially a sequential one. In this section we assume that the firms have exogenous leader or follower roles, so as to study the leader and follower decision problems individually. We suppose that the leader can choose its capacity freely but must invest at the initial state. Capacity choice is definitive so the leader cannot postpone investment by initially choosing $Q_L = 0$, and we accordingly restrict attention to demand states where investment is profitable.[9] The follower chooses both the timing and the size of its investment. We proceed by backward induction and study follower investment problem first before turning to the leader's problem.

4.1 Follower investment

Suppose that when the demand state is $X > 0$ the leader builds capacity Q_L. Then the follower holds a real option on a perceived duopoly profit flow. It chooses when to invest and what capacity level Q_F to install when it does.

We start by studying the follower's capacity choice problem. Let X' denote the value of the demand state when the follower invests. Although the demand process fluctuates perpetually after the follower has invested, its expected behavior looking forward is straightforward. Because $X(t)$ follows a geometric Brownian motion, the capitalized expected demand state starting from X' is $E_{X'}\left[\int_0^\infty X(s)e^{-rs}ds\right] = \frac{X'}{r-\mu}$.[10] The follower's perceived

[8] There would be little to gain in the analysis therefore by incorporating a positive entry cost f as in the latter part of Section 3.

[9] I.e., $X > \delta(r-\mu)$. The monopoly value of capacity Q in state X is

$$E_X\left[\int_0^\infty X(s)(1-Q)\,Qe^{-rs}ds - \delta Q\right] = \frac{X}{r-\mu}\left(1 - \frac{\delta(r-\mu)}{X} - Q\right)Q$$

so this condition is necessary for positive monopoly investment.

[10] See Dixit et al. (1994), p. 316.

payoff function upon entry is therefore $\Omega_F(Q_F, Q_L, X') = \frac{X'}{r-\mu}(1-(Q_L+Q_F))(\lambda Q_L + Q_F) - \delta Q_F$. Aside from the additional μ term which reflects expected growth of the demand process, this payoff function is the same as in the static case (Eq. 4), and the follower's capacity decision is entirely analogous. Optimizing the perceived payoff yields a quantity reaction $Q_F^*(Q_L, X') = \max\left\{0,\ \frac{1}{2}\left(1-\frac{\delta(r-\mu)}{X'}-(1+\lambda)Q_L\right)\right\}$. This reaction is softened by internalization, and it determines the follower's perceived payoff upon entry $\Omega_F(Q_F^*(Q_L, X'), Q_L, X')$.

We next turn to the follower's timing decision. To economize on notation, we use $\Omega_F(\cdot)$ to denote both the terminal payoff resulting from capacities Q_F and Q_L in state X, $\Omega_F(Q_F, Q_L, X)$ and the follower's option value in a given demand state X, $\Omega_F(X)$. Letting T denote its choice of stopping time, the follower's perceived value is

$$\Omega_F(X) = \sup_{T\geq 0} E_X\left[\int_0^T \lambda X(s)(1-Q_L)Q_L e^{-rs}ds + \Omega_F(Q_F^*(Q_L, X(T)), Q_L, X(T))\, e^{-rT}\right]. \tag{8}$$

In Eq. (8), the integral term is the internalized value of the leader's profit while it is operating as a monopolist. When the follower chooses its timing the leader has sunk its capacity, so its cost does not appear here. The second term in Eq. (8) is the expected discounted value of the perceived payoff the follower gets upon stopping.

If $Q_L < \frac{1}{1+\lambda}$, the follower's option is valuable and its optimal policy is to set a finite threshold. It thus invests when the demand shock first reaches

$$X_F^*(Q_L) = \frac{\beta+1}{\beta-1}\frac{\delta(r-\mu)}{1-(1+\lambda)Q_L} \tag{9}$$

where

$$\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{1}{2}-\frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1 \tag{10}$$

is a constant that reflects discounting in a stochastic environment.[11] We denote the lower bound of follower thresholds, below which follower entry cannot occur, by $\underline{X}_F^* = \frac{\beta+1}{\beta-1}\delta\left(r-\mu\right)$.

If $Q_L \geq \frac{1}{1+\lambda}$, the follower's option is worthless and its optimal policy is to never enter, which amounts to setting an infinite threshold ($X_F^* = \infty$).

Eq. (9) shows that the follower's threshold X_F^* is positively related to λ. A greater degree of internalization is therefore associated with later follower entry. Intuitively, with internalization the follower's option involves a perceived dividend, $\lambda X(t)\left(1-Q_L\right)Q_L$. Like in the static case (Eq. 6), this dividend represents an additional opportunity cost of entry. The follower's net payoff from entry is therefore lower with internalization, which softens its timing decision. In the extreme the leader can block the follower from entering at any demand state by choosing sufficient capacity, though we show further down that this form of deterrence is never actually chosen.

4.2 Leader investment

Because the market is dynamic, the leader's capacity choice has a strategic effect on both the timing of the follower's entry and, indirectly, the size of its investment.

Letting $X'(Q_L) = \max\left\{X, X_F^*(Q_L)\right\}$ denote the demand state when the follower enters, the leader's perceived payoff when the follower enters is $\Omega_L\left(Q_L, Q_F^*(Q_L, X'), X'\right) = \frac{X'}{r-\mu}\left(1-\left(Q_L+Q_F^*(Q_L,X')\right)\right)\left(Q_L+\lambda Q_F^*\left(Q_L,X'\right)\right) - \lambda\delta Q_F$. Up to the additional drift term μ, the leader's perceived payoff is similar to the static case (Eq. 5). At its own

[11] See Appendix A.2.

capacity decision stage, the leader's instantaneous perceived value is therefore

$$\Omega_L(X) = \max_{Q_L \in [0,1]} E_X \left[\int_0^T X(s) \left(1 - Q_L\right) Q_L e^{-rs} ds + \Omega_L \left(Q_F^*(Q_L, X'(Q_L)), Q_L, X'(Q_L)\right) e^{-rT} \right] - \delta Q_L \tag{11}$$

where $T = \inf \{t \geq 0 \,|\, X(t) \geq X'(Q_L)\}$ is the follower's stopping time. The follower invariably reacts to the leader's capacity through $Q_F^*(X)$, even if $T = 0$ so both firms effectively enter at the same moment with the follower choosing its capacity "just after" the leader.

Through its capacity choice, the leader affects the follower's entry timing by determining whether the follower enters immediately ($T = 0$, provided the current demand state allows it), after a finite time ($T \in (0, \infty)$), or never ($T = \infty$). For states $X \geq \underline{X}_F^*$, setting $X_F^*(Q_L) = X$ gives the capacity beyond which the leader drives the follower to delay entering,

$$\widehat{Q}_L(X) = \frac{1}{1+\lambda} \left(1 - \frac{\beta+1}{\beta-1} \frac{\delta(r-\mu)}{X}\right). \tag{12}$$

Finally we can rule out capacities beyond $\frac{1}{1+\lambda}$ when studying the leader's capacity choice, as these are unnecessary to deter the follower and lie on the decreasing part of the leader's profit.

The leader's perceived payoff in the capacity choice stage is then defined piecewise as follows. For $X < \underline{X}_F^*$ (so the follower invariably enters later immediately),

$$\Omega_L(Q_L, X) =$$
$$\begin{cases} \left(\frac{X}{r-\mu}(1-Q_L)-\delta\right)Q_L + \left(\frac{X}{X_F^*(Q_L)}\right)^\beta \left(\Omega_L\left(Q_L, Q_F^*(Q_L, X_F^*(Q_L)), X_F^*(Q_L)\right) - \frac{X_F^*(Q_L)}{r-\mu}(1-Q_L)Q_L\right), \\ \hfill \text{if } 0 \leq Q_L < \frac{1}{1+\lambda}, \\ \left(\frac{X}{r-\mu}(1-Q_L)-\delta\right)Q_L, \text{ if } Q_L = \frac{1}{1+\lambda}, \end{cases} \tag{13}$$

and for $X \geq \underline{X}_F^*$ (so the follower may enter immediately),

$$\Omega_L(Q_L, X) =$$
$$\begin{cases} \Omega_L\left(Q_L, Q_F^*(Q_L, X), X\right), \text{ if } 0 \leq Q_L \leq \widehat{Q}_L(X), \\ \left(\frac{X}{r-\mu}(1-Q_L)-\delta\right)Q_L + \left(\frac{X}{X_F^*(Q_L)}\right)^\beta \left(\Omega_L\left(Q_L, Q_F^*(Q_L, X_F^*(Q_L)), X_F^*(Q_L)\right) - \frac{X_F^*}{r-\mu}(1-Q_L)Q_L\right), \\ \hfill \text{if } \widehat{Q}_L(X) < Q_L < \frac{1}{1+\lambda}, \\ \left(\frac{X}{r-\mu}(1-Q_L)-\delta\right)Q_L, \text{ if } Q_L = \frac{1}{1+\lambda}. \end{cases} \tag{14}$$

Eq. (13) states that at low demand states, the leader's capacity choice involves two pieces corresponding to strategic delay and the limiting case of permanent deterrence (where the follower has infinite threshold and stopping time). Eq. (14) describes the payoff at higher demand states and involves an additional piece at low capacity which corresponds to accommodation.

Under accommodation (over the first piece of Eq. 14), the existence of a local optimum and the resulting payoff proceed similarly to the static case aside from the constraint that the leader's capacity is consistent with immediate follower entry ($Q_L \leq \widehat{Q}_L(X)$). Although accommodation is possible for all states above $\underline{X}_F^*$, at lower states within this range the leader's payoff is increasing over $\left(0, \widehat{Q}_L(X)\right)$, whereas accommodation may occur only if there is an interior maximum. If $\lambda < \sqrt{2} - 1$, then there exists a finite demand state threshold $X_1^{\mathrm{a}} > \underline{X}_F^*$ beyond which $\Omega_L(Q_L, X)$ has an interior maximum Q_L^{a} over its first piece.[12]

Under strategic delay (over the first piece of Eq. (**??** and the second piece of Eq. 14), the interior maximum is defined only implicitly. Even without an analytic solution, we can provide a partial characterization. To begin with, we can show that the second-order condition holds if β and λ are not too large.[13] We maintain throughout our discussion that this sufficient condition holds. Next, there exists a demand state threshold $X_1^{\mathrm{d}} < \frac{\beta}{\beta-1}\delta(r-\mu) < X_1^{\mathrm{a}}$ below which no interior maximum exists. There also exists a finite demand state threshold X_2^{d} above which there is no interior maximum if $\lambda \notin \left[\sqrt{\frac{\beta+1}{\beta}} - 1, \frac{1}{\beta}\right]$ (X_2^{d} is infinite if $\lambda \in \left[\sqrt{\frac{\beta+1}{\beta}} - 1, \frac{1}{\beta}\right]$). Finally, because the threshold function $X_F^*(Q_L)$ is continuous, we can directly rule out the limiting case of permanent deterrence ($Q_L = \frac{1}{1+\lambda}$) whenever there is an interior optimum Q_L^{d}, i.e. over $\left[X_1^{\mathrm{d}}, X_2^{\mathrm{d}}\right]$.

The following proposition describes the optimal payoff that results from the capacity choice problem $\max_{Q_L \in \left[0, \frac{1}{1+\lambda}\right]} \Omega_L(Q_L, X)$ provided the degree of internalization is not too high.

[12] Differentiating the first piece gives $Q_L^{\mathrm{a}}(X) = \frac{1 - \frac{\delta(r-\mu)}{X}}{(2+\lambda)(1-\lambda)}$ which results in a payoff $\Omega_L(Q_L^{\mathrm{a}}, X) = \frac{1}{4}\frac{X}{r-\mu}\left(-(2+\lambda)(1-\lambda)^2 Q_L^2 + 2(1-\lambda)\left(1 - \frac{\delta(r-\mu)}{X}\right) Q_L + \lambda\left(1 - \frac{\delta(r-\mu)}{X}\right)^2\right)$ similar to Eq. (5), provided $X \geq X_1^{\mathrm{a}}$ where $X_1^{\mathrm{a}} = \frac{\beta(1-\lambda(2+\lambda))+3-\lambda^2}{(\beta-1)(1-\lambda(2+\lambda))}\delta(r-\mu)$ solves $Q_L^{\mathrm{a}}(X_1^{\mathrm{a}}) = \widehat{Q}_L(X_1^{\mathrm{a}})$ if $\lambda < \sqrt{2} - 1$ or $\lambda > \sqrt{2} - 1$ and $\beta > \frac{3-\lambda^2}{(\lambda+1-\sqrt{2})(\lambda+1+\sqrt{2})}$ (this second possibility is ruled out however by the restriction we impose on β further below in footnote 13), or is infinite otherwise.

[13] The specific condition $\beta < \frac{2}{\lambda(3-\lambda^2)}$ (see Appendix A.3).

Proposition 3. *If* $\lambda < \sqrt{\frac{\beta+1}{\beta}} - 1$ *the leader's payoff is*

$$\Omega_L(X) = \begin{cases} \Omega_L(0,X), & \text{if } X \leq X_1^{\mathrm{d}} \\ \Omega_L(Q_L^{\mathrm{d}},X), & \text{if } X_1^{\mathrm{d}} < X < X_1^{\mathrm{a}} \\ \max\left\{\Omega_L(Q_L^{\mathrm{a}},X), \Omega_L(Q_L^{\mathrm{d}},X)\right\}, & \text{if } X_1^{\mathrm{a}} \leq X < X_2^{\mathrm{d}} \\ \Omega_L(Q_L^{\mathrm{a}},X), & \text{if } X \geq X_2^{\mathrm{d}}. \end{cases} \tag{15}$$

where $\Omega_L(0,X) = \lambda\frac{(\beta-1)^{\beta-1}X^\beta}{(\beta+1)^{\beta+1}\delta^{\beta-1}(r-\mu)^\beta}$, $\Omega_L(Q_L^{\mathrm{a}},X) = \frac{(1+\lambda)^2 X}{4(2+\lambda)(r-\mu)}\left(1 - \frac{\delta(r-\mu)}{X}\right)^2$, *and* $\Omega_L(Q_L^{\mathrm{d}},X) = \left(\frac{X}{r-\mu}\left(1-Q_L^{\mathrm{d}}\right) - \delta\right)Q_L^{\mathrm{d}} + \left(\frac{X}{X_F^*\left(Q_L^{\mathrm{d}}\right)}\right)^\beta \frac{\delta\left(\lambda-(1+\lambda)(\beta+1-\lambda\beta)Q_L^{\mathrm{d}}\right)}{\beta^2-1}$.

Proof. See Appendix A.4.

Proposition 3 asserts that the solution of the leader's capacity choice problem at low enough levels of internalization resembles the situation without internalization. That is to say, its only optimal capacities are either those which induce delay (0 or Q_L^{d}) or an interior accommodation solution (Q_L^{a}) and the demand states that determine when these solutions arise satisfy the ranking $X_1^{\mathrm{d}} < X_1^{\mathrm{a}} < X_2^{\mathrm{d}}$. Based on numerical simulations, Huisman and Kort (2015) observe that the accommodation and delay payoff cross at a single threshold $\overline{X}_L^{\mathrm{d}} \in \left(X_1^{\mathrm{a}}, X_2^{\mathrm{d}}\right)$, so that the leader chooses to delay the follower at lower demand states and to accommodate at higher demand states (where deterrence is more costly). Because internalization softens both the follower's timing and capacity choice, we expect internalization to favor deterrence (which leverages both dimensions of the follower's reaction) relative to accommodation (which leverages only its quantity reaction) so that $\overline{X}_L^{\mathrm{d}}$ increases. In Section 6, we conduct a numerical analysis which bears out these different ideas.

Provided that the second-order condition holds, the restriction on λ in Proposition 3 can be relaxed to $\lambda < \sqrt{2} - 1$. In this case (if $\sqrt{\frac{\beta+1}{\beta}} \le \lambda + 1 < \sqrt{2}$), X_2^{d} is infinite and the last piece of Eq. (15) is irrelevant but the rest of the payoff function is unaltered. A more interesting situation arises if $\lambda \ge \sqrt{2} - 1$ and $\beta < \frac{2}{\lambda(3-\lambda^2)}$ (so Q_L^{d} is well-defined). In this case, both X_1^{a} and X_2^{d} are infinite. Accommodation remains a possibility for the leader at any demand state $X > \underline{X}_F^*$, but because there is never a maximum on the first piece we can conclude that the leader chooses to strategically delay the follower's entry at all demand states $X \ge X_1^{\mathrm{a}}$. This situation, which does not arise without internalization, lends further support to the idea that internalization generally induces more aggressive leader behavior.

There is a counterexample however, where internalization leads to less aggressive follower behavior. Because the lower bound X_1^{d} is an increasing function of λ, for positive degrees of internalization there is a range of demand states $\left(X_1^{\mathrm{d}}(0), X_1^{\mathrm{d}}(\lambda)\right]$ where the leader sets $Q_L^{\mathrm{d}}(X) = 0$ instead of a positive capacity without internalization. The reason for this unusual behavior is that if the leader must invest at a low enough demand state, it is better off leaving the market entirely to the follower, who is free to enter optimally, and reaping its perceived share of the follower's monopoly profits. In such instances internalization pushes the leader acts less aggressively rather than more, though this is due in large part to the constraint that the leader invest immediately.

At the other extreme, for sufficiently high demand states ($X \ge X_1^{\mathrm{a}}$), the outcome of the capacity decision resembles accommodation in the static case. As internalization increases, the leader's capacity decreases, i.e. internalization invariably leads to more aggressive leader behavior in this case.

5 Equilibrium investment

To characterize equilibrium investment, we suppose that the initial demand state is low enough that no firm invests immediately and that the roles of each firm (leader or follower) are determined noncooperatively (implying $\lambda < 1$).[14,15] At any demand state X at which no investment has yet occurred, the firms have the choice to invest or to wait. The instantaneous payoff from investing as a leader in demand state X is $\Omega_L(X)$ (Eq. **??**). The instantaneous payoff for the remaining firm is the follower payoff $\Omega_F(X, Q_L^*)$ (Eq. **??**) net of the internalized leader investment cost $\lambda\delta Q_L^*$, where Q_L^* denotes the leader's optimal capacity at X. The incentive of each firm to preempt its rival by investing first is therefore given by the difference

$$f(X) = \Omega_L(X) - (\Omega_F(X, Q_L^*) - \lambda\delta Q_L^*)\,. \tag{16}$$

The set of demand states over which firms prefer to lead rather than follow is called the preemption range. In equilibrium, the first investment in the industry takes place at the lower bound of this range. We denote this lower bound by X_P. Intuitively, since X_P is the smallest demand state at which firms prefer to lead rather than follow and as the payoff to following is non-negative, if one of the firms were to set a higher investment threshold $X' > X_P$ its rival would have an incentive to enter before it at a lower threshold in (X_P, X'). If the initial state is low enough therefore ($X < X_P$), one of the firms must invest as a leader at X_P in equilibrium.[16] The preemption threshold does not have an explicit expression, but the following proposition narrows down the range within which it

[14] The demand state is low enough for firms to wait if $X \leq X_P$, where X_P is the preemptionnthreshold defined further below in the section. A sufficient condition is $X \leq \delta(r - \mu)$.

[15] If $\lambda = 1$ joint profit is maximized by having a single firm invest as a monopoly, i.e. with capacity $\frac{1}{\beta+1}$ at the demand state threshold $\frac{\beta+1}{\beta-1}\delta(r - \mu)$.

[16] See Appendix A.5 for a more detailed discussion of strategies and outcomes in this game.

lies and establishes that that firms invest sequentially.

Proposition 4. *For initial states $X \leq X_1^{\mathrm{d}}$, in a preemption equilibrium the leader invests at the demand state threshold $X_P = \inf\{X > 0,\ s.t.\ f(X) > 0\}$ with $X_P \in \left(X_1^{\mathrm{d}}, \min\left\{X_1^{\mathrm{a}}, X_2^{\mathrm{d}}\right\}\right)$ and chooses capacity $Q_L^{\mathrm{d}}(X_P)$. The follower invests at the demand state threshold $X_F^*\left(Q_L^{\mathrm{d}}\right) > X_P$.*

Proof. See Appendix A.6.

If $\lambda < \sqrt{\frac{\beta+1}{\beta}} - 1$ for example, Propositions 4 asserts that the first equilibrium investment occurs at a threshold in the range of demand states $\left(X_1^{\mathrm{d}}, X_F^*\left(Q_L^{\mathrm{d}}\right)\right)$ where the leader's optimum capacity is $Q_L^* = Q_L^{\mathrm{d}}$, which implies the follower's investment is delayed. In this range the preemption incentive takes the form

$$f(X) = (1-\lambda)\left(\frac{\left(1-Q_L^{\mathrm{d}}\right)Q_L^{\mathrm{d}}X}{r-\mu} - \delta Q_L^{\mathrm{d}} - \left(\frac{X}{X_F^*\left(Q_L^{\mathrm{d}}\right)}\right)^{\beta}\left(\frac{\left(1-Q_L^{\mathrm{d}}\right)Q_L^{\mathrm{d}}X_F^*\left(Q_L^{\mathrm{d}}\right)}{r-\mu} - \delta Q_L^{\mathrm{d}}\right)\right.$$
$$\left. + \left(\frac{X}{X_F^*\left(Q_L^{\mathrm{d}}\right)}\right)^{\beta}\left(\frac{\left(1-\left(Q_L^{\mathrm{d}}+Q_F^*\left(X_F^*\right)\right)\right)X_F^*\left(Q_L^{\mathrm{d}}\right)}{r-\mu} - \delta\right)\left(Q_L^{\mathrm{d}} - Q_F^*\left(X_F^*\right)\right)\right). \tag{17}$$

Up to scaling by $(1-\lambda)$, Eq. (17) breaks the preemption incentive down into two parts. The first part consists of the terms in the first line which represent the rent that the leader obtains from the industry's monopoly phase by entering ahead of the follower. The second part consists of the terms in the second line, which represent the leader's relative profit during the industry's duopoly phase. This relative profit may be either positive or negative depending on whether is the leader has a larger capacity than the follower or not. For sufficiently small values of λ moreover, the upper bound on the preemption threshold can be tightened to $\underline{X}_F^*$, which means that competition between the firms to enter first is intense enough to drive the leader's entry down to states where it is no longer possible to accommodate the follower, in which case the leader's capacity satisfies $Q_L^{\mathrm{d}} < \frac{1}{\beta+1}$ and is therefore lower than the monopoly capacity.[17]

To see how internalization affects the preemption incentive, recall from Section 4.1 that internalization softens the follower's investment timing and quantity reactions, and lowers its asset value. Because the follower is less aggressive internalization the leader's asset value increases, both through the lengthier monopoly phase and through the higher duopoly share the leader gets once the follower does enter. These positive effects of internalization are offset by the leader's internalization of the follower's lower value. Similarly, the decrease in the follower's own value is partially offset by its internalization of the leader's higher value value. The effect of internalization on the preemption incentive depends on the difference of these two sets of effects. We expect own value effects to dominate cross value effects at low levels of internalization, so a small increase in internalization should raise the incentive to preempt and result in earlier initial investment. In equilibrium therefore, the anticompetitive effect of overlapping ownership on the follower should lead to more competitive behavior ex-ante as firms vie for leadership more aggressively. The next proposition bears out these ideas by characterizing the effect of internalization on the preemption equilibrium.

Proposition 5. *For small λ, the preemption threshold and leader capacity decrease with internalization ($dX_P/d\lambda, dQ_L^{\mathrm{d}}(X_P)/d\lambda < 0$).*

Proof. See Appendix A.7.

The negative effect of internalization on the preemption threshold is similar to a result in the fixed investment size case, whereby softer follower timing due to internalization accelerates preemptive investment (Zormpas and Ruble 2021). The more novel assertion

[17] See Appendix A.7.

in Proposition 5 is therefore that earlier preemptive investment is accompanied by lower leader capacity when capacities are endogenized. This negative capacity effect is the result of several effects. First of all, there is a direct effect of the demand state which is common to models of timing and capacity choice. All else equal, capacity is relatively more expensive at lower demand states. As preemption drives firms to enter at lower demand states, this effect drives the leader to install less capacity. In addition to this, internalization also has a number of strategic effects on the leader's first order condition, e.g. through the follower's softer reaction and through the leader's internalization of follower profits. The overall sign of these strategic effects is indeterminate.[18] At demand states near the lower bound X_1^{d}, the leader's capacity approaches zero, and greater internalization reduces the leader's capacity as it places greater weight on the follower's profit. Conversely at demand states near $\underline{X}_F^*$, the leader's capacity approaches $\frac{1}{\beta+1}$ and greater internalization increases the leader's capacity. Proposition 5 establishes that either the direct effect or the follower internalization effect must dominate. In either case, the leader's lower capacity offsets the procompetitive effect of earlier entry.

6 Numerical analysis

As the leader's optimal capacity and the preemption threshold are defined only implicitly, we use numerical methods in this section to further examine the consequences of internalization. We use the parameter values $r = .1$, $\mu = .06$, $\sigma = .1$, $\delta = .1$ in order to replicate existing results in Huisman and Kort (2015) and measure the effects of moderate internalization levels ($\lambda = .1$ or $\lambda = .2$). We conducted the same computations varying the values for the discount rate, drift, and volatility parameters and obtained similar results

[18] See Eq. 53 in the Appendix.

to those we report here. Besides corroborating the main insights of the preceding sections, e.g. an anticompetitive effect of internalization on follower behavior and a procompetitive effect on equilibrium investment, the numerical analysis also serves to highlight a novel procompetitive effect of internalization at demand states above the preemption threshold, whereby moderate internalization drives a leader to opt for strategic deterrence by choosing significantly larger capacity.

To visualize first how internalization affects equilibrium investment timing, Figure 3 plots the leader and follower payoffs as functions of the demand state. The leader payoff $\Omega_L(X)$ is the upper envelope of the payoffs under accommodation and delay, i.e. of the local maxima $\Omega_L\left(\min\left\{Q_L^{\mathrm{a}}(X), \widehat{Q}(X)\right\}, X\right)$ (dashed curve) and $\Omega_L(Q_L^{\mathrm{d}}(X), X)$ (dotted curve). With internalization, the leader perceives a positive payoff even below X_1^{d} because it accounts for the follower's positive option value. The follower's ex-ante payoff lies above the leader payoff initially, and crosses below it at the preemption threshold X_P. The figure indicates there is a single demand state $\overline{X}_L^{\mathrm{d}}$ at which the leader shifts from deterrence to accommodation. This shift creates an upward kink in the leader payoff and an upward jump in the follower payoff. The effect of internalization is gauged by comparing with the benchmark no-internalization case which is plotted in gray. With respect to the two critical demand states, X_P decreases with internalization consistently with Proposition 5, whereas $\overline{X}_L^{\mathrm{d}}$ increases so the leader chooses to deter the follower over a broader range of demand states.

The effect of internalization on firm capacities is represented in Figure 4, which plots optimal leader and follower capacities against the demand state at which the leader invests. For either of the strategies that a leader can adopt (deterrence or accommodation), higher demand states result in higher leader capacity. At the demand state $\overline{X}_L^{\mathrm{d}}$ where the leader shifts from deterrence to accommodation however, its optimal capacity jumps downward.

The follower's capacity is decreasing in the demand state if the leader opts to delay its entry, but increasing if both firms invest simultaneously. The effect of internalization on capacities is non-monotonic. Internalization decreases the leader's optimal capacity at low demand states though the effect is slight, and increases it at higher demand states. The follower's equilibrium capacity on the other hand increases with internalization at low demand states, albeit very slightly, and decreases at higher demand states.

For a given leader strategy, total capacity (and hence instantaneous consumer surplus) is invariably lower with internalization as in the static model. However, with the moderate internalization levels considered here, over the range of demand states where internalization shifts the leader's strategy from accommodation to deterrence, the procompetitive effect of internalization on leader capacity outweighs the anticompetitive effect on follower capacity and total capacity increases. Thus, if a leader is brought to invest at a moderately high demand state (e.g. because the initial value of $X(t)$ is sufficiently high), moderate internalization can shift its strategy towards deterrence so as to induce a sufficiently higher leader capacity that total capacity increases. Instantaneous welfare therefore increases during the industry's duopoly phase, but there is also a countervailing dynamic effect because the follower's entry is delayed until its optimal threshold X_F^* is reached.

To evaluate the effect of internalization on welfare, we assume that consumers have the same discount rate as firms and use a consumer surplus welfare standard, which is stricter than total surplus as internalization invariably raises firm values in equilibrium. The consumer surplus at the moment that the leader invests is[19]

$$S(X) = E_X \left[\int_0^T \frac{1}{2} X(s) \eta Q_L^2 e^{-rs} ds + \int_T^\infty \frac{1}{2} X(s) \eta \left(Q_L + Q_F^* \left(X(T) \right) \right)^2 e^{-rs} ds \right]. \quad (18)$$

[19] In the analysis of this section, we add a slope parameter $\eta = .05$ to inverse demand as Huisman and Kort (2015) do.

Inside the conditional expectation in Eq. (18), the first term is the discounted consumer surplus during the industry's monopoly phase and the second term is the discounted consumer surplus during the industry's duopoly phase, evaluated at an optimal leader capacity. If the demand state is sufficiently high for the leader to choose positive capacity and low enough for it to opt for delay, taking the expectation gives

$$S^{\mathrm{d}}(X) = \frac{\eta}{2}\frac{\left(Q_L^{\mathrm{d}}\right)^2 X}{r-\mu} + \frac{\eta}{2}\left(\frac{X}{X_F^*}\right)^{\beta} \frac{\left(\left(Q_L^{\mathrm{d}} + Q_F^*\left(X_F^*\right)\right)^2 - \left(Q_L^{\mathrm{d}}\right)^2\right) X_F^*}{r-\mu}. \tag{19}$$

At demand states which are high enough that the leader accommodates and follower entry is immediate, substituting values for $Q_F^*(X)$ and Q_L gives a consumer surplus expression

$$S^{\mathrm{a}}(X) = \frac{(3+\lambda)^2}{8\eta\,(2+\lambda)^2}\left(1 - \frac{\delta\,(r-\mu)}{X}\right)^2 \frac{X}{r-\mu}. \tag{20}$$

We first study the preemption equilibrium and resulting welfare. Table 2 reports the values which we obtain. Consistently with Proposition 5 in the preceding section internalization has a procompetitive effect on equilibrium entry timing, but also results in lower leader capacity. The effect on the follower's capacity is negative and its threshold increases. Internalization therefore raises instantaneous consumer surplus because the monopoly phase starts earlier but has countervailing consequences on leader capacity and on the timing and surplus associated with the duopoly phase. To gauge the overall effect, we take three different degrees of internalization (0, .1, and .2) and determine the preemption equilibrium in each case. To compare consumer surplus values we evaluate these at a common demand state $X = X_P\,(.2) = .0100$, which is the smallest of the preemption equilibria. The countervailing effects dominate here so the effect of moderate internalization levels on consumer surplus and welfare is negative.

Table 2: Preemption equilibrium

λ	$X_P(\lambda)$	Q_L^*	Q_F^*	Q^{Total}	X_F^*	$S(X)\vert_{X=X_P(.2)}$	$W(X)\vert_{X=X_P(.2)}$
0	.0105	5.53	5.59	11.12	0.0243	.5269	.9873
.1	.0102	5.38	5.44	10.82	0.0250	.4967	.9684
.2	.0100	5.22	5.31	10.52	0.0256	.4678	.9495

We focus next on the shift in leader strategy. To this end we take initial demand states around the range where internalization alters the leader's strategy and suppose that one of the firms invests immediately as a leader in the initial state, either exogenously or as a result of competition with the follower. A shift in leader strategy has contrasting effects on overall consumer surplus, as it raises capacity which mitigates the harm induced by the monopoly phase (which would not be incurred if the leader accommodated), but also raises capacity during the industry's duopoly phase. To assess the balance of these effects, we take three different degrees of internalization (0, .1, and .2) and evaluate consumer surplus and welfare at the three demand states to the right of which the leader's strategy shifts ($\overline{X}_L^{\text{d}}(0) = .0325$, $\overline{X}_L^{\text{d}}(.1) = .0397$, and $\overline{X}_L^{\text{d}}(.2) = .0559$) The resulting capacities and surplus values are reported in Table 3.

In the top part of the table which corresponds to a demand state just to the left of $\overline{X}_L^{\text{d}}(0)$, the leader chooses deterrence for all the internalization levels, as evidenced by the fourth column (X_F^* is invariably larger than X). As described above, with the leader's strategy held constant, leader capacity increases with internalization whereas the follower's capacity decreases. Total duopoly output and consumer surplus decrease with internalization, as does total welfare.

The middle part of the table is the most similar to Table 1 in the introduction, though in the dynamic model the effect of internalization involves both capacities and follower timing. The leader chooses accommodation if $\lambda = 0$ and deterrence otherwise, so internalization produces a strategic shift here. As a result, the leader's capacity and total capacity increase sharply between $\lambda = 0$ and $\lambda = .1$, yielding in an increase in consumer surplus of roughly 5% as well as a smaller total welfare increase of 1%. Going from $\lambda = .1$ to $\lambda = .2$, the increase in leader capacity is much smaller and total capacity decreases. Total capacity is still higher than without internalization, but the follower's investment is significantly delayed and consumer surplus and welfare both decrease.

Table 3: Procompetitive strategic shift (dynamic)

			$X = .0325$			
λ	Q_L^*	Q_F^*	Q^{Total}	X_F^*/X	$S(X)$	$W(X)$
0	9.75	3.96	13.71	1.06	3.76	5.94
.1	10.09	3.44	13.53	1.22	3.54	5.89
.2	10.29	2.96	13.24	1.41	3.30	5.82
			$X = .0397$			
λ	Q_L^*	Q_F^*	Q^{Total}	X_F^*/X	$S(X)$	$W(X)$
0	8.99	4.50	13.49	1	4.51	7.52
.1	10.77	3.15	13.92	1.09	4.72	7.60
.2	10.96	2.65	13.60	1.29	4.37	7.50
			$X = .0559$			
λ	Q_L^*	Q_F^*	Q^{Total}	X_F^*/X	$S(X)$	$W(X)$
0	9.28	4.64	13.93	1	6.78	11.29
.1	9.82	3.88	13.71	1	6.56	11.13
.2	12.04	2.14	14.19	1.13	6.89	11.34

In the bottom part of the table, the leader's strategy does not shift at $\lambda = .1$ so a low level of internalization only leads to reduced total capacity and consumer surplus. At $\lambda = .2$ however, the shift to deterrence does occur and the sharp increase in leader capacity that ensues results in slightly higher consumer surplus and welfare than without internalization, again because the increase in the follower's threshold is not too large.

We take from these results that increased competition for the market due to internalization may not be beneficial to consumers if capacities are endogenous, but also that if competition occurs at a moderately high demand state some degree of common ownership or cross holding can generate a procompetitive shift in the leader's strategy which is beneficial for consumers.

7 Conclusion

In this article, we study how either common ownership or symmetric cross holdings affect strategic capacity decisions in an evolving market by driving managers to internalize effects on rival firms. Greater internalization predictably makes a follower to react less aggressively. Because softer timing and capacity reactions are particularly beneficial to the leader if the follower delays entry, internalization drives leaders to pursue deterrence over a broader range of demand states. We show through an example that such a shift in leader strategy can be procompetitive, both in the static and in the dynamic versions of the model. If firms compete for industry leadership, then the follower's softer timing and capacity reactions raise the attractiveness of leading. Internalization therefore has a procompetitive effect on the timing of entry, though we show this is mitigated by lower leader capacities.

Because they do not hinge on the presence of R&D spillovers or high R&D intensity, the effects of overlapping ownership we identify may emerge in a broad range of industries. Our analysis implies, for example, that common ownership of retailers like Wal-Mart and Costco[20] should accelerate their initial entry into local markets, but also lower store sizes and lengthen the gap between initial and follower entry.

[20]Combined ownership of the top three institutional shareholders was respectively 9% and 17% for these two firms (as of March 2023).

Our analysis relies on several assumptions which could be relaxed in future work. To begin with, the assumption of symmetric ownership structures may closely reflect common ownership in certain industries but not in others. In the case of unilateral minority share acquisitions for example, cross holdings are naturally asymmetric. A closer representation of these situations would therefore account for asymmetric ownership and result in an asymmetric preemption game, whose equilibrium outcome generalizes the one which we describe here. In addition, we have restricted our attention to new markets where neither firm operates initially, but further effects of overlapping ownership could be expected to arise in markets where firms have preexisting capacities or the ability to make multiple capacity additions.

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A Appendix

A.1 Proof of Proposition 2

The conditions referred to in the text for the leader to have local maxima involving accommodation and deterrence are $Q_L^{\mathrm{a}} < Q_L^{\mathrm{d}}$ and $Q_L^{\mathrm{d}} < Q_M$, or respectively $\frac{\sqrt{f'}}{1-\delta'} < \frac{1}{2}\frac{\left(\sqrt{2}+1+\lambda\right)\left(\sqrt{2}-1-\lambda\right)}{(2+\lambda)(1-\lambda)}$ and $\frac{\sqrt{f'}}{1-\delta'} > \frac{1-\lambda}{4}$. Note that $\lambda < \sqrt{2}-1$ is necessary for accommodation to occur at some fixed cost.

We have $\Omega_L\left(Q_L^{\mathrm{a}}\right) = \frac{(1+\lambda)^2(1-\delta')^2}{4(2+\lambda)} - \lambda f'$ and $\Omega_L\left(Q_L^{\mathrm{d}}\right) = \frac{\left(1-\delta-2\sqrt{f'}\right)\left(\lambda(1-\delta)+2\sqrt{f'}\right)}{(1+\lambda)^2}$. The leader prefers deterrence over accommodation if $\Omega_L\left(Q_L^{\mathrm{d}}\right) > \Omega_L\left(Q_L^{\mathrm{a}}\right)$, or

$$-\left(1-\lambda\right)\left(4+3\lambda+\lambda^2\right)\frac{f'}{\left(1-\delta'\right)^2} + 2\left(1-\lambda\right)\frac{\sqrt{f'}}{1-\delta'} - \left(\frac{\left(1+\lambda\right)^4}{4\left(2+\lambda\right)} - \lambda\right) > 0 \qquad (21)$$

after rearrangement.

The left-hand side has a lower root $\frac{\sqrt{f_0'}}{1-\delta'} = \frac{1}{4+3\lambda+\lambda^2}\left(1-\sqrt{\frac{(1+\lambda)^3\left(4-3\lambda-4\lambda^2-\lambda^3\right)}{4(1-\lambda)(2+\lambda)}}\right)$ which is positive and decreasing for $\lambda < \sqrt{2}-1$ (which is necessary for $Q_L^{\mathrm{a}} < Q_L^{\mathrm{d}}$), and accommodation is no longer feasible as $Q_L^{\mathrm{d}} < Q_L^{\mathrm{a}}$ once $\frac{\sqrt{f'}}{1-\delta'}$ reaches the upper root. The leader therefore prefers deterrence for $f' \geq f_0'\left(\lambda\right)$.

Under accommodation, total output is $Q_L^{\mathrm{a}} + Q_F^{*}\left(Q_L^{\mathrm{a}}\right) = \frac{(1-\delta')(3+\lambda)}{2(2+\lambda)}$, which decreases with λ. Under deterrence, total output is Q_L^{d}, which is decreasing in λ

Because total output lies above the monopoly output, it is negatively related to short run industry profit. To show that output jumps up when the leader's strategy shifts, consider a degree of internalization where the leader is indifferent between accommodation and deterrence, i.e. λ such that $\Omega_L\left(Q_L^{\mathrm{a}}\right) = \Omega_L\left(Q_L^{\mathrm{d}}\right)$. Expanding gives $V_L\left(Q_L^{\mathrm{a}}, Q_F^{*}\left(Q_L^{\mathrm{a}}\right)\right) + \lambda V_F\left(Q_F^{*}\left(Q_L^{\mathrm{a}}\right), Q_L^{\mathrm{a}}\right) - \lambda f' = V_L\left(Q_L^{\mathrm{d}}, 0\right)$, where V_F denotes the follower's profit without accounting for entry cost. Then $V_L\left(Q_L^{\mathrm{a}}, Q_F^{*}\left(Q_L^{\mathrm{a}}\right)\right) + V_F\left(Q_F^{*}\left(Q_L^{\mathrm{a}}\right), Q_L^{\mathrm{a}}\right) = V_L\left(Q_L^{\mathrm{d}}, 0\right) +$

$\lambda f' + (1-\lambda)V_F\left(Q_F^*\left(Q_L^{\mathrm{a}}\right), Q_L^{\mathrm{a}}\right) > V_L\left(Q_L^{\mathrm{d}}, 0\right)$, which implies that output is higher under deterrence.

We can also establish that (long run) industry profit jumps down at such a point. This requires showing that $V_L\left(Q_L^{\mathrm{a}}, Q_F^*\left(Q_L^{\mathrm{a}}\right)\right) + V_F\left(Q_F^*\left(Q_L^{\mathrm{a}}\right), Q_L^{\mathrm{a}}\right) - f' > V_L\left(Q_L^{\mathrm{d}}, 0\right)$. Substituting again for $V_L\left(Q_L^{\mathrm{d}}, 0\right)$ and rearranging, the desired inequality is equivalent to $(1-\lambda)\left(V_F\left(Q_F^*\left(Q_L^{\mathrm{a}}\right), Q_L^{\mathrm{a}}\right) - f'\right) > 0$. To verify that the second term on the left hand side is positive, observe that under accommodation the follower prefers to pay the entry cost, i.e. $\Omega_F\left(Q_F^*(Q_L^{\mathrm{a}}), Q_L^{\mathrm{a}}\right) > \Omega_F\left(0, Q_L^{\mathrm{a}}\right) + f'$, so $V_F\left(Q_F^*\left(Q_L^{\mathrm{a}}\right), Q_L^{\mathrm{a}}\right) + \lambda V_L\left(Q_L^{\mathrm{a}}, Q_F^*\left(Q_L^{\mathrm{a}}\right)\right) > \lambda V_L\left(Q_L^{\mathrm{a}}, 0\right) + f'$ and therefore $V_F\left(Q_F^*\left(Q_L^{\mathrm{a}}\right), Q_L^{\mathrm{a}}\right) - f' > \lambda\left(V_L\left(Q_L^{\mathrm{a}}, 0\right) - V_L\left(Q_L^{\mathrm{a}}, Q_F^*\left(Q_L^{\mathrm{a}}\right)\right)\right) > 0$. □

A.2 Follower value

As stated in the text, the follower's option is worthless if $Q_L \geq \frac{1}{1+\lambda}$ in which case $\Omega_F(X) = \lambda\frac{X}{r-\mu}\left(1-Q_L\right)Q_L$. Hereafter suppose that $Q_L < \frac{1}{1+\lambda}$.

The follower will only stop in states where its optimal capacity $Q_F^*(Q_L, X)$ is positive, i.e. where $X > \frac{\delta(r-\mu)}{1-(1+\lambda)Q_L}$. In such states its terminal payoff is

$$\Omega_F\left(Q_F^*(Q_L, X), Q_L, X\right) = \frac{1}{4}\left(\frac{X}{r-\mu}\left(1-(1-\lambda)Q_L\right)^2 - 2\delta\left(1-(1+\lambda)Q_L\right) + \frac{r-\mu}{X}\delta^2\right). \tag{22}$$

A sufficient condition for optimality of an upper threshold policy is if $\lambda X\left(1-Q_L\right)Q_L - r\Omega_F\left(Q_F^*(Q_L, X), Q_L, X\right)$ is decreasing in X (Dixit et al. 1994, p. 128). Developing and differentiating gives a condition

$$-\left(4\lambda\frac{r-\mu}{r} + (1-\lambda)^2\right)Q_L^2 + 2\left(2\lambda\frac{r-\mu}{r} + (1-\lambda)\right)Q_L - 1 + \frac{\delta^2(r-\mu)^2}{X^2} < 0. \tag{23}$$

Substituting the lower bound on X gives a further sufficient condition which holds trivially, $-4\lambda\frac{\mu}{r}\left(1-Q_L\right)Q_L \leq 0$.

The follower's value satisfies the no-arbitrage condition $r\Omega_F(X)dt = \lambda X\left(1-Q_L\right)Q_L dt + E_X\left[d\Omega_F(X)\right]$. Applying Itô's lemma and taking the expectation gives a second-order ordinary differential equation

$$r\Omega_F(X) = \lambda X\left(1-\eta Q_L\right)Q_L + \mu X\Omega_F'(X) + \frac{1}{2}\sigma^2X^2\Omega_F''(X) \tag{24}$$

over the inaction region $(0, X_F^*)$, with boundary conditions

$$\Omega_F(0) = 0 \tag{25}$$

and

$$\Omega_F\left(X_F^*\right) = \Omega_F\left(Q_F^*(Q_L, X_F^*), Q_L, X_F^*\right). \tag{26}$$

The optimal threshold X_F^* satisfies the smooth pasting condition

$$\Omega_F'\left(X_F^*\right) = \frac{\partial\Omega_F}{\partial X}\left(Q_F^*(Q_L, X_F^*), Q_L, X_F^*\right). \tag{27}$$

The solution has the form $\Omega_F(X) = \lambda\frac{X}{r-\mu}\left(1-Q_L\right)Q_L + A_FX^\beta$ where $\beta > 1$ is the upper root of $\frac{1}{2}\sigma^2b(b-1)+\mu b - r = 0$ (see Eq. 10), and solving yields the threshold X_F^* in the text and the payoff

$$\Omega_F\left(X, Q_L\right) = \begin{cases} \lambda\frac{X}{r-\mu}\left(1-Q_L\right)Q_L + \left(\frac{X}{X_F^*}\right)^\beta\left(\Omega_F\left(Q_F^*(Q_L, X_F^*), Q_L, X_F^*\right) - \lambda\frac{X_F^*}{r-\mu}\left(1-Q_L\right)Q_L\right), & \text{if } X < X_F^* \\ \Omega_F\left(Q_F^*(Q_L, X), Q_L, X\right), & \text{if } X \geq X_F^*. \end{cases}$$

$\square$

A.3 Leader payoff with strategic delay

Evaluating $\Omega_L(Q_L, X)$ over the range $Q_L \in \left(\max\left\{0, \widehat{Q}_L(X)\right\}, \frac{1}{1+\lambda}\right)$ where $X_F^*(Q_L)$ is finite gives

$$\left(\frac{X}{r-\mu}(1-Q_L)-\delta\right)Q_L+\left(\frac{X}{X_F^*(Q_L)}\right)^{\beta}\frac{\delta\left(\lambda-(1+\lambda)(\beta+1-\lambda\beta)Q_L\right)}{\beta^2-1}. \tag{28}$$

Differentiating with respect to Q_L gives the first-order condition for an interior optimum Q_L^{d},

$$\frac{X}{r-\mu}\left(1-2Q_L^{\mathrm{d}}\right)-\delta-\left(\frac{X}{X_F^*\left(Q_L^{\mathrm{d}}\right)}\right)^{\beta}\frac{\delta(1+\lambda)\left(1-(1+\lambda)(\beta+1-\lambda\beta)Q_L^{\mathrm{d}}\right)}{(\beta-1)\left(1-(1+\lambda)Q_L^{\mathrm{d}}\right)}=0. \tag{29}$$

The second-order condition is $\frac{X}{r-\mu}\left(\left(\frac{X}{X_F^*\left(Q_L^{\mathrm{d}}\right)}\right)^{\beta-1}\frac{\beta(1+\lambda)^2\left(2-\lambda-(1+\lambda)(\beta+1-\lambda\beta)Q_L^{\mathrm{d}}\right)}{(\beta+1)\left(1-(1+\lambda)Q_L^{\mathrm{d}}\right)}-2\right)<0$. Observe that $\frac{2-\lambda-(1+\lambda)(\beta+1-\lambda\beta)Q_L^{\mathrm{d}}}{1-(1+\lambda)Q_L^{\mathrm{d}}}=1+(1-\lambda)\frac{1-\beta(1+\lambda)Q_L^{\mathrm{d}}}{1-(1+\lambda)Q_L^{\mathrm{d}}}<2-\lambda$. Since $X<X_F^*(Q_L)$, the second-order condition holds if $(1+\lambda)^2(2-\lambda)<2\frac{\beta+1}{\beta}$, which gives the condition in footnote 13.

We next determine the range of demand states for which Eq. (29) has a solution. First, setting $Q_L^{\mathrm{d}}=0$ gives a condition

$$\frac{X}{\delta(r-\mu)}-1-\frac{(1+\lambda)(\beta-1)^{\beta-1}}{(\beta+1)^{\beta}}\left(\frac{X}{\delta(r-\mu)}\right)^{\beta}=0. \tag{30}$$

The left-hand side is strictly concave in X and negative at zero. Evaluating it at $X=\frac{\beta}{\beta-1}\delta(r-\mu)$ gives $\frac{1}{\beta-1}\left(1-\left(\frac{\beta}{\beta+1}\right)^{\beta}(1+\lambda)\right)>0$ so there is a lower root X_1^{d} which satisfies $X_1^{\mathrm{d}}<\frac{\beta}{\beta-1}\delta(r-\mu)$. Evaluating the left-hand side at $\underline{X}_F^*>\frac{\beta}{\beta-1}\delta(r-\mu)$ gives $\frac{1-\lambda}{\beta-1}>0$, so the $Q_L \geq \widehat{Q}_L(X) > 0$ constraint binds at the upper root.

To sign $\frac{dQ_L^{\mathrm{d}}}{dX}$, differentiate the first-order condition with respect to X and substitute it back in for the second set of terms so as to get $\frac{\partial^2 \Omega_L}{\partial Q \partial X}(Q_L^{\mathrm{d}}, X) = -(\beta - 1)\frac{1-2Q_L^{\mathrm{d}}}{r-\mu} + \beta\frac{\delta}{X}$. By the implicit function theorem therefore, $\frac{dQ_L^{\mathrm{d}}}{dX}$ is zero if $X = \frac{\beta}{\beta-1}\frac{\delta(r-\mu)}{1-2Q_L^{\mathrm{d}}}$. Supposing that such a point exists, substitute back into Eq. (29) to get

$$\frac{\delta}{\beta - 1}\left(1 - \left(\frac{\beta}{\beta+1}\right)^{\beta}\left(\frac{1-(1+\lambda)Q_L^{\mathrm{d}}}{1-2Q_L^{\mathrm{d}}}\right)^{\beta-1}\left(1-(1+\lambda)(\beta+1-\lambda\beta)Q_L^{\mathrm{d}}\right)\right). \tag{31}$$

Because $Q_L^{\mathrm{d}} < \frac{1}{1+\lambda}$ at any interior solution, $\left(\frac{\beta}{\beta+1}\right)^{\beta}\left(\frac{1-(1+\lambda)Q_L^{\mathrm{d}}}{1-2Q_L^{\mathrm{d}}}\right)^{\beta-1} = \left(\frac{\beta\left(1-(1+\lambda)Q_L^{\mathrm{d}}\right)}{(\beta+1)\left(1-2Q_L^{\mathrm{d}}\right)}\right)^{\beta}\frac{1-2Q_L^{\mathrm{d}}}{1-(1+\lambda)Q_L^{\mathrm{d}}} <$ 1 so the expression above is positive. Because there is no X at which the derivative cancels, $\frac{dQ_L^{\mathrm{d}}}{dX} > 0$.

Next, provided $\lambda \notin \left[\sqrt{\frac{\beta+1}{\beta}} - 1, \frac{1}{\beta}\right]$ there exists a finite demand state at which setting $Q_L^{\mathrm{d}} = \widehat{Q}_L(X)$ solves Eq. (29),

$$X_2^{\mathrm{d}} = \frac{(\beta+1)(2+\lambda)(1-\beta\lambda)}{(\beta-1)(1-2\beta\lambda(2+\lambda))}\delta(r-\mu). \tag{32}$$

which represents the upper bound of the demand states at which an interior optimum exists. To verify that X_2^{d} (if it exists) is an upper bound and not just a tangency point with the $\widehat{Q}_L(X)$ locus, we check that $\frac{dQ_L^{\mathrm{d}}}{dX}(X_2^{\mathrm{d}}) < \frac{d\widehat{Q}_L}{dX}(X_2^{\mathrm{d}})$. From Eq. (29)

$$\frac{dQ_L^{\mathrm{d}}}{dX}\left(X_2^{\mathrm{d}}\right) = -\frac{1-2Q_L^{\mathrm{d}} - \frac{\beta(1+\lambda)}{\beta+1}\left(1-(1+\lambda)(\beta+1-\lambda\beta)Q_L^{\mathrm{d}}\right)}{X_2^{\mathrm{d}}\left(\frac{\beta(1+\lambda)^2}{\beta+1}\frac{\left(2-\lambda-(1+\lambda)(\beta+1-\lambda\beta)Q_L^{\mathrm{d}}\right)}{1-(1+\lambda)Q_L^{\mathrm{d}}} - 2\right)}, \tag{33}$$

and from Eq. (12)

$$\frac{d\widehat{Q}_L}{dX}\left(X_2^{\mathrm{d}}\right) = \frac{\beta+1}{\beta-1}\frac{\delta(r-\mu)}{X_2^{\mathrm{d}2}(1+\lambda)}. \tag{34}$$

Comparing the two, we find that $\frac{d\widehat{Q}_L}{dX}\left(X_2^{\mathrm{d}}\right) > \frac{dQ_L^{\mathrm{d}}}{dX}\left(X_2^{\mathrm{d}}\right)$ if $\beta\lambda\left(2-\lambda(1+\lambda)\right) < 1-\lambda$ which holds whenever X_2^{d} is finite. $\square$

A.4 Proof of Proposition 3

The payoff $\Omega_L(0,X)$ is the leader's share of the follower payoff $\Omega_L(X,0)$ (Eq. **??**), $\Omega_L(Q_L^{\mathrm{a}},X)$ is a straight application of $\Omega_L\left(Q_L,Q_F^*(Q_L,X),X\right)$ in the text (see also footnote 12), and $\Omega_L(Q_L^{\mathrm{d}},X)$ is given by Eq. (28). If the leader shuts the follower out permanently, its payoff is $\Omega_L\left(\frac{1}{1+\lambda},X\right) = \left(\frac{X}{r-\mu}\frac{\lambda}{1+\lambda}-\delta\right)\frac{1}{1+\lambda}$.

We first show that the leader choose zero capacity rather than $\frac{1}{1+\lambda}$ if $X < X_1^{\mathrm{d}}$. Setting $\Omega_L\left(0,X\right) > \Omega_L\left(\frac{1}{1+\lambda},X\right)$ gives

$$\frac{(1+\lambda)\left(\beta-1\right)^{\beta-1}}{\left(\beta+1\right)^{\beta}}\left(\frac{X}{\delta(r-\mu)}\right)^{\beta} - \frac{\beta+1}{1+\lambda}\frac{X}{\delta(r-\mu)} + \frac{\beta+1}{\lambda} > 0. \tag{35}$$

By Eq. (30), X_1^{d} is the lower root of

$$\frac{(1+\lambda)\left(\beta-1\right)^{\beta-1}}{\left(\beta+1\right)^{\beta}}\left(\frac{X}{\delta\left(r-\mu\right)}\right)^{\beta} - \frac{X}{\delta\left(r-\mu\right)} + 1 = 0. \tag{36}$$

The left hand sides of the two expressions above have a unique intersection at $X^* = \frac{1+\lambda}{\lambda}\frac{\beta+1-\lambda}{\beta-\lambda}\delta(r-\mu)$ where Eq. (35) cuts Eq. (36) from above. We suppose that $Q_L^* = \frac{1}{1+\lambda}$ for some $X \leq X_1^{\mathrm{d}}$ and argue by contradiction. For the left-hand side of Eq. (35) to have a lower root at X_1^{d} or below, it is necessary that X^* lie to the left of minimum of Eq. (36), $X_0 = \frac{(\beta+1)^{\frac{\beta}{\beta-1}}}{(\beta-1)\beta^{\frac{1}{\beta-1}}(1+\lambda)^{\frac{1}{\beta-1}}}\delta(r-\mu)$, i.e. that

$$\frac{\beta\left(\beta-1\right)^{\beta-1}\left(1+\lambda\right)^{\beta}\left(\beta+1-\lambda\right)^{\beta-1}}{\left(\beta+1\right)^{\beta}\lambda^{\beta-1}\left(\beta-\lambda\right)^{\beta-1}} < 1, \tag{37}$$

and also that the left-hand side of Eq. (36) be negative at X^*, i.e. that

$$\frac{(\beta-1)^{\beta-1}(1+\lambda)^{\beta+1}(\beta+1-\lambda)^{\beta}}{(\beta+1)^{\beta+1}\lambda^{\beta-1}(\beta-\lambda)^{\beta-1}} \geq 1. \tag{38}$$

Dividing Eq. (38) by Eq. (37) gives $\frac{(1+\lambda)(\beta+1-\lambda)}{\beta(\beta+1)} > 1$, which cannot hold as $\lambda \in [0,1)$ and $\beta > 1$. Therefore, $Q_L = \frac{1}{1+\lambda}$ is strictly suboptimal up to X_1^{d}.

Next, for $X \in \left(X_1^{\mathrm{d}}, X_2^{\mathrm{d}}\right]$, the leader's capacity choice problem has a local maximum at Q_L^{d} which is interior. The restriction $\lambda \leq \sqrt{\frac{\beta+1}{\beta}} - 1$ implies $\lambda \leq \sqrt{2} - 1$, so X_1^{a} is finite. Moreover, $X_1^{\mathrm{a}} < X_2^{\mathrm{d}}$. To establish this last inequality, is is enough to show that the denominator terms satisfy $1 - \lambda(2+\lambda) > 1 - 2\beta\lambda(2+\lambda)$ (which holds trivially) whereas the numerator terms satisfy $(\beta+1)(2+\lambda)(1-\beta\lambda) > \beta(1-\lambda(2+\lambda)) + 3 - \lambda^2$. This last inequality is equivalent to $-(1+\beta)\lambda^2 + \left(\beta + \frac{1}{2}\right)\lambda + 1 > 0$, and the quadratic expression on the left-hand side is positive at $\lambda = 0$ and $\lambda = \sqrt{\frac{\beta+1}{\beta}} - 1$, and hence for all $\lambda < \sqrt{\frac{\beta+1}{\beta}} - 1$

Finally, to show that $Q_L = \frac{1}{1+\lambda}$is suboptimal for $X > X_1^{\mathrm{a}}$ (and hence $> X_2^{\mathrm{d}}$), we compare $\Omega_L(Q_L^{\mathrm{a}}, X)$ and $\Omega_L\left(\frac{1}{1+\lambda}, X\right)$ over this range. After rearrangement, blockade is therefore ruled out if

$$\left(\frac{2-(1+\lambda)^2}{(1+\lambda)^2}\right)^2\left(\frac{X}{\delta(r-\mu)}\right)^2 - 2\frac{\lambda^3+3\lambda^2+\lambda-3}{(1+\lambda)^3}\frac{X}{\delta(r-\mu)} + 1 > 0, \tag{39}$$

and the left-hand side is positive and increasing at X_1^{a}. □

A.5 Preemption game specification

The text gives an intuitive account of the preemption game, but the firms' strategies and resulting outcomes have a more formal description. If firms choose when to stop in continuous time, then in any subgame starting at a given time t_0 the strategies consist of pairs of real-valued functions $\left(G_i^{t_0}, \alpha_i^{t_0}\right)$ for each player i, where $G_i^{t_0}$ is a conditional distribution function representing that player's investment probability and $\alpha_i^{t_0}$ is an intensity parameter which augments the strategy space to allow limiting outcomes of discrete time strategies to be represented in continuous time, subject to a set of regularity and consistency requirements. The subgame perfect equilibrium strategies which support the equilibrium described in Proposition 4 are $\left(\left(G_1^t, \alpha_1^t\right), \left(G_2^t, \alpha_2^t\right)\right)_{t \in R_+}$ where, for $i \in \{1, 2\}$ and any $t \geq 0$,

$$G_i^t(u) = \begin{cases} 0, & \text{if } u < T_P^t \\ \frac{\Omega_L\left(X_{T_P^t}\right) - \Omega_M\left(X_{T_P^t}\right)}{\Omega_L\left(X_{T_P^t}\right) - 2\Omega_M\left(X_{T_P^t}\right) + \Omega_F\left(X_{T_F^t}\right)} & \text{if } T_P^t \leq u < T_F^t \\ 1, & \text{if } u \geq T_F^t \end{cases} \tag{40}$$

and

$$\alpha_i^t(u) = \begin{cases} 0, & \text{if } u < T_P^t \\ \frac{\Omega_L\left(X_{T_P^t}\right) - \Omega_F\left(X_{T_P^t}\right)}{\Omega_L\left(X_{T_P^t}\right) - \Omega_M\left(X_{T_P^t}\right)} & \text{if } T_P^t \leq u < T_F^t \\ 1, & \text{if } u \geq T_F^t \end{cases} \tag{41}$$

where $T_P^t = \inf\{\tau \geq t | \, X_\tau \geq X_P\}$ and $T_F^t = \inf\{\tau \geq t | \, X_\tau \geq X_F^*\}$ are stopping times, and

$$\Omega_M(X) = (1 + \lambda)\left(\frac{X}{r - \mu}\max\{1 - 2Q_L^*, 0\} - \delta\right)Q_L^* \tag{42}$$

is the perceived payoff simultaneous stopping (Thijssen et al. 2012, Theorem 1). At the threshold X_P, $\Omega_L\left(X_{T_P^t}\right) = \Omega_F\left(X_{T_P^t}\right)$ implying that either firm invests, with equal probability.[21]

[21]The main difference relative to Thijssen et al. (2012) pertains to variable investment size rather than internalization. Inside the preemption range, there is a positive probability of simultaneous invest-

A.6 Proof of Proposition 4

First take demand states $X \leq X_1^{\mathrm{d}}$. If a firm invests as a leader at such demand states, it sets $Q_L^* = 0$. The value of the preemption incentive is then $f(X) = -(1-\lambda)\,\Omega_F(X,0)$, which is negative because $\Omega_F(X,0) > 0$. The preemption threshold is therefore bounded below by X_1^{d}.

For the upper bound, suppose first that X_1^{a} is finite. It is convenient to express the preemption incentive as $f(X) = \Omega_L(Q_L^*, X) - (\lambda V_L(Q_L^*, X) + V_F(Q_L^*, X)) = \Omega_L(Q_L^*, X) - \left(\lambda \Omega_L(Q_L^*, X) - \lambda^2 V_F(Q_L^*, X) + V_F(Q_L^*, X)\right) = (1-\lambda)\,\Omega_L(Q_L^*, X) - \left(1-\lambda^2\right) V_F(Q_L^*, X)$, where $V_F(Q_L^*, X)$ denotes the value of the follower's own assets (with timing and capacity choices $X_F^*(Q_L^*)$ and Q_F^* being those in Section 4.1). At X_1^{a}, $Q_L^* = Q_L^{\mathrm{d}}$ is optimal for the leader so $(1-\lambda)\,\Omega_L(Q_L^{\mathrm{d}}, X) \geq (1-\lambda)\,\Omega_L(Q_L^{\mathrm{a}}, X)$. Because greater leader capacity lowers the follower's residual demand and its value, $-\left(1-\lambda^2\right) V_F(Q_L^{\mathrm{a}}, X) < -\left(1-\lambda^2\right) V_F(Q_L^{\mathrm{d}}, X)$ is increasing. Hence, $f(X_1^{\mathrm{a}}) > (1-\lambda)\,\Omega_L(Q_L^{\mathrm{a}}, X_1^{\mathrm{a}}) - \left(1-\lambda^2\right) V_F(Q_L^{\mathrm{a}}, X_1^{\mathrm{a}}) = \Omega_L(Q_L^{\mathrm{a}}, X_1^{\mathrm{a}}) - (\Omega_F(Q_L^{\mathrm{a}}, X_1^{\mathrm{a}}) - \lambda\delta Q_L^{\mathrm{a}})$ where the right-hand side is the value of the preemption incentive if leader capacity were set suboptimally at Q_L^{a}. At $Q_L = Q_L^{\mathrm{a}}$ however, investments are simultaneous with the follower acting as a Stackelberg quantity follower, so payoffs are those of the static Stackelberg game with internalization (see Section 3). The leader's perceived payoff is therefore higher than the follower's ($\Omega_L(Q_L^{\mathrm{a}}, X_1^{\mathrm{a}}) > \Omega_F(Q_L^{\mathrm{a}}, X_1^{\mathrm{a}}) - \lambda\delta Q_L^{\mathrm{a}}$), which implies that $f(X_1^{\mathrm{a}}) > 0$.

If X_1^{a} is infinite, then a similar argument can be made at X_2^{d} (provided X_2^{d} is finite) with $\widehat{Q}_L\left(X_2^{\mathrm{d}}\right)$ instead of Q_L^{a}. Otherwise, both X_1^{a} and X_2^{d} are infinite, and in this case $Q_L^* = Q_L^{\mathrm{d}}$ for arbitrarily high demand states. The monopoly rent term in Eq. (17) is then positive for large enough X as is the relative profit term because $\lim_{X\to\infty}\left(Q_L^{\mathrm{d}}(X) - Q_F^*\right) \geq$

ment which implies that the optimal capacity Q_L^* must maximize the expected payoff $\frac{1-\alpha}{2-\alpha}\Omega_L(X, Q_L) + \frac{\alpha}{2-\alpha}\Omega_M(X, Q_L)$.

$\lim_{X\to\infty}\left(\widehat{Q}_L(X)-Q_F^*\right)=\frac{1}{1+\lambda}$, so $f(X)$ is positive for sufficiently large X.

We conclude that the preemption range is nonempty, with lower bound $X_P\in\left(X_1^{\mathrm{d}},\min\left\{X_1^{\mathrm{a}},X_2^{\mathrm{d}}\right\}\right)$.

□

A.7 Proof of Proposition 5

The preemption equilibrium is characterized by the equilibrium condition $f\left(X_P\right)=0$ (where X_P is restricted to be the lower root) along with the first-order condition defining $Q_L^{\mathrm{d}}\left(X_P\right)$. To express these compactly, define $Z=\frac{X_P}{\delta(r-\mu)}$ to get a the system of equations

$$f\left(X,Q\right)=\left(1-Q\right)Z-1-Z^{\beta}\frac{\left(\beta-1\right)^{\beta-1}}{\left(\beta+1\right)^{\beta+1}}\frac{\left(1+\beta\left(1+\lambda\right)Q\right)\left(1-\left(1+\lambda\right)Q\right)^{\beta}}{Q}=0 \quad (43)$$

(Eq. 17) and

$$g\left(Z,Q\right)=\left(1-2Q\right)X-1-X^{\beta}\frac{\left(\beta-1\right)^{\beta-1}}{\left(\beta+1\right)^{\beta}}\left(1+\lambda\right)\left(1-\left(1+\lambda\right)\left(\beta+1-\lambda\beta\right)Q\right)\left(1-\left(1+\lambda\right)Q\right)^{\beta-1}=0 \quad (44)$$

(for Eq. 29).

By the implicit function theorem, in the preemption equilibrium described by Eqs. (43) and (44), the sensitivities of Z and Q with respect to λ are given by

$$\frac{dZ}{d\lambda}=\frac{\frac{\partial f}{\partial Q}\frac{\partial g}{\partial\lambda}-\frac{\partial f}{\partial\lambda}\frac{\partial g}{\partial Q}}{\frac{\partial f}{\partial Z}\frac{\partial g}{\partial Q}-\frac{\partial f}{\partial Q}\frac{\partial g}{\partial Z}}\text{ and }\frac{dQ}{d\lambda}=\frac{\frac{\partial f}{\partial\lambda}\frac{\partial g}{\partial Z}-\frac{\partial f}{\partial Z}\frac{\partial g}{\partial\lambda}}{\frac{\partial f}{\partial Z}\frac{\partial g}{\partial Q}-\frac{\partial f}{\partial Q}\frac{\partial g}{\partial Z}}. \quad (45)$$

We first evaluate the partial derivatives with respect to λ,

$$\frac{\partial f}{\partial\lambda}=Z^{\beta}\frac{\beta\left(\beta-1\right)^{\beta-1}}{\left(\beta+1\right)^{\beta}}\left(1+\lambda\right)Q\left(1-\left(1+\lambda\right)Q\right)^{\beta-1} \quad (46)$$

and

$$\frac{\partial g}{\partial \lambda} = -X^{\beta}\frac{(\beta-1)^{\beta-1}}{(\beta+1)^{\beta}}\left(1-(1+\lambda)Q\right)^{\beta-2} \left(\left(\beta^2(1-\lambda)+\beta(1-2\lambda)+1\right)(1+\lambda)^2Q^2-(2\beta+2-3\lambda\beta)(1+\lambda)Q+1\right). \tag{47}$$

To establish the proposition we determine the signs of $\frac{dZ}{d\lambda}$ and $\frac{dQ}{d\lambda}$ at $\lambda=0$ and argue that these hold for small λ by continuity. Evaluated at $\lambda=0$, the system of equilibrium conditions is

$$\begin{cases} (1-Q)Z-1-Z^{\beta}\frac{(\beta-1)^{\beta-1}}{(\beta+1)^{\beta+1}}\frac{(1+\beta Q)(1-Q)^{\beta}}{Q}=0 \\ (1-2Q)Z-1-Z^{\beta}\frac{(\beta-1)^{\beta-1}}{(\beta+1)^{\beta}}\left(1-(\beta+1)Q\right)(1-Q)^{\beta-1}=0. \end{cases} \tag{48}$$

We establish that a solution to this system satisfies $Q<\frac{1}{\beta+1}$ and $Z<\frac{\underline{X}_F^*}{\delta(r-\mu)}$. At $Z=\frac{X_1^{\mathrm{d}}}{\delta(r-\mu)}$ the optimal leader capacity is $Q_L^{\mathrm{d}}=0$. The first-order condition for Q_L^{d} (second line in Eq. 48) implies that $Z=\frac{1}{1-2Q}=\frac{\underline{X}_F^*}{\delta(r-\mu)}>\frac{X_1^{\mathrm{d}}}{\delta(r-\mu)}$ is the only demand state at which the optimal leader capacity is $Q_L^{\mathrm{d}}=\frac{1}{\beta+1}$. Continuity of $Q_L^{\mathrm{d}}(X)$ then implies $Q<\frac{1}{\beta+1}$ for any $Z<\frac{\underline{X}_F^*}{\delta(r-\mu)}$. Furthermore, at $Z=\frac{\underline{X}_F^*}{\delta(r-\mu)}$ (hence $Q=Q_L^{\mathrm{d}}=\frac{1}{\beta+1}$), the preemption incentive (the first line in Eq. 48) is positive, since[22]

$$(1-Q)Z-1-Z^{\beta}\frac{(\beta-1)^{\beta-1}}{(\beta+1)^{\beta+1}}\frac{(1+\beta Q)(1-Q)^{\beta}}{Q} > 0 \Leftrightarrow 1-\frac{2\beta+1}{\beta+1}\left(\frac{\beta}{\beta+1}\right)^{\beta}>0 \tag{49}$$

Therefore, $X_P<\underline{X}_F^*$, implying that a solution to Eq. (48) satisfies $Q<\frac{1}{\beta+1}$. Observe

[22] To verify the last inequality, denote the left-hand side by $\Lambda(\beta)$. Then $\Lambda(1)=.75<1$, and $\Lambda'(\beta)=\frac{\beta^{\beta}}{(\beta+1)^{\beta+1}}\left(2+(2\beta+1)\ln\left(\frac{\beta}{\beta+1}\right)\right)$. Taking the first terms of the Maclaurin series, $\ln\left(\frac{\beta}{\beta+1}\right)<-\frac{1}{\beta+1}-\frac{1}{2(\beta+1)^2}-\frac{1}{3(\beta+1)^3}$, and substituting back and simplifying yields $\Lambda'(\beta)<-\frac{1}{6}\frac{\beta^{\beta}(\beta-1)}{(\beta+1)^{\beta+4}}<0$.

also that at a solution (Z,Q) to Eq. (48),

$$Z=\frac{(\beta^2+\beta+1)Q^2-2Q+1}{(1-Q)\left((\beta^2+1)Q^2-3Q+1\right)} \tag{50}$$

and

$$Z^\beta\frac{(\beta-1)^{\beta-1}}{(\beta+1)^{\beta+1}}(1-Q)^{\beta-1}=\frac{Q^2}{(1-Q)\left((\beta^2+1)Q^2-3Q+1\right)}. \tag{51}$$

For $\lambda=0$, the partial derivatives with respect to λ above (Eqs. 46 and 47) become

$$\frac{\partial f}{\partial\lambda}=Z^\beta\frac{\beta(\beta-1)^{\beta-1}}{(\beta+1)^\beta}Q(1-Q)^{\beta-1} \tag{52}$$

and

$$\frac{\partial g}{\partial\lambda}=-Z^\beta\frac{(\beta-1)^{\beta-1}}{(\beta+1)^\beta}\left(\left(\beta^2+\beta+1\right)Q^2-2(\beta+1)Q+1\right)(1-Q)^{\beta-2}. \tag{53}$$

Using Eqs. (50) and (51) to substitute for Z and Z^β gives

$$\frac{\partial f}{\partial Z}=\frac{(1-Q)^2(1-(\beta+1)Q)}{(\beta^2+\beta+1)Q^2-2Q+1}>0 \tag{54}$$

and

$$\frac{\partial f}{\partial Q}=\frac{(\beta+1)Q}{(\beta^2+1)Q^2-3Q+1}>0. \tag{55}$$

Also,

$$\begin{aligned}\frac{\partial g}{\partial Z} &= 1-2Q-X^{\beta-1}\frac{\beta(\beta-1)^{\beta-1}}{(\beta+1)^\beta}(1-(\beta+1)Q)(1-Q)^{\beta-1} \\ &= \frac{\beta}{Z}-(\beta-1)(1-2Q)=\frac{\partial f}{\partial Z}+(\beta-1)Q>0\end{aligned} \tag{56}$$

where the second line uses the first-order condition $g(Z,Q)=0$ to substitute for the last term. Using Eqs. (50) and (51) once again to substitute for Z and Z^{β},

$$\frac{\partial g}{\partial Q}=-\frac{(\beta^{3}-\beta-2)Q^{3}+6Q^{2}-6Q+2}{(1-Q)^{2}\left((\beta^{2}+1)Q^{2}-3Q+1\right)}<0, \tag{57}$$

which is the leader's second-order condition.

We show first that the denominator of $\frac{dZ}{d\lambda}$ and $\frac{dQ}{d\lambda}$ is negative. Using Eq. (56), the denominator can be expressed as $\frac{\partial f}{\partial Z}\frac{\partial g}{\partial Q}-\frac{\partial f}{\partial Q}\frac{\partial g}{\partial Z}=\frac{\partial f}{\partial Z}\left(\frac{\partial g}{\partial Q}-\frac{\partial f}{\partial Q}\right)-\frac{\partial f}{\partial Q}\left(\beta-1\right)Q$. Because $\frac{\partial f}{\partial Z}$ and $\frac{\partial f}{\partial Q}$ are positive, it is enough to show that $\frac{\partial g}{\partial Q}-\frac{\partial f}{\partial Q}$ is negative. The sign of $\frac{\partial g}{\partial Q}-\frac{\partial f}{\partial Q}$ is that of the cubic expression $-(\beta^{3}-1)Q^{3}+2(\beta-2)Q^{2}-(\beta-5)Q-2$, which is negative because the quadratic $2(\beta-2)Q^{2}-(\beta-5)Q-2$ is negative for $Q<\frac{1}{\beta+1}$.

To establish the proposition, all that remains is to show that the numerators of $\frac{dZ}{d\lambda}$ and $\frac{dQ}{d\lambda}$ are positive. Starting with the latter, using Eq. (56) to substitute for $\frac{\partial g}{\partial Z}$ gives $\frac{\partial f}{\partial \lambda}\frac{\partial g}{\partial Z}-\frac{\partial f}{\partial Z}\frac{\partial g}{\partial \lambda}=\frac{\partial f}{\partial Z}\left(\frac{\partial f}{\partial \lambda}-\frac{\partial g}{\partial \lambda}\right)+\frac{\partial f}{\partial \lambda}\left(\beta-1\right)Q$. Because $\frac{\partial f}{\partial X}$ and $\frac{\partial f}{\partial \lambda}$ are positive, it is enough to show that $\frac{\partial \widetilde{f}}{\partial \lambda}-\frac{\partial \widetilde{g}}{\partial \lambda}$ is also positive. Evaluating gives

$$\frac{\partial f}{\partial \lambda}-\frac{\partial g}{\partial \lambda}=Z^{\beta}\frac{(\beta-1)^{\beta-1}}{(\beta+1)^{\beta}}\left(1-Q\right)^{\beta-2}\left((\beta^{2}+1)Q^{2}-(\beta+2)Q+1\right). \tag{58}$$

The sign is that of the last bracket. Express this term as a quadratic in β, $Q^{2}\beta^{2}-Q\beta+(1-Q)^{2}$, which is positive for $\beta=1$ and has a negative discriminant $Q^{2}\left(1-4\left(1-Q\right)^{2}\right)$ because $Q<.5$, so $\frac{\partial f}{\partial \lambda}-\frac{\partial g}{\partial \lambda}>0$.

The other numerator is

$$\frac{\partial f}{\partial Q}\frac{\partial g}{\partial \lambda}-\frac{\partial f}{\partial \lambda}\frac{\partial g}{\partial \widetilde{Q}}=$$
$$Z^{\beta}\frac{(\beta-1)^{\beta-1}}{(\beta+1)^{\beta}}\left(1-Q\right)^{\beta-2}\left(-\frac{\partial f}{\partial Q}\left(\left(\beta^{2}+\beta+1\right)Q^{2}-2\left(\beta+1\right)Q+1\right)-\frac{\partial g}{\partial Q}\beta Q\left(1-Q\right)\right).$$

The sign is that of the last bracketed term. Developing and simplifying by $Q(1-Q)$ leaves a cubic expression, $\left(\beta^4+\beta^3+\beta^2+1\right)Q^3-\left(\beta^3+4\beta^2+3\right)Q^2+\left(2\beta^2-\beta+3\right)Q+\beta-1$. This is greater than $-\left(\beta^3+4\beta^2+3\right)Q^2+\left(2\beta^2-\beta+3\right)Q+\beta-1$, which takes the value $\beta-1>0$ at $Q=0$ and $\frac{\left(2\beta^2+1\right)(\beta-1)}{(\beta+1)^2}>0$ at $Q=\frac{1}{\beta+1}$, and is hence positive for all $Q\in\left[0,\frac{1}{\beta+1}\right]$ implying $\frac{\partial f}{\partial Q}\frac{\partial g}{\partial\lambda}-\frac{\partial f}{\partial\lambda}\frac{\partial g}{\partial Q}>0$. $\square$

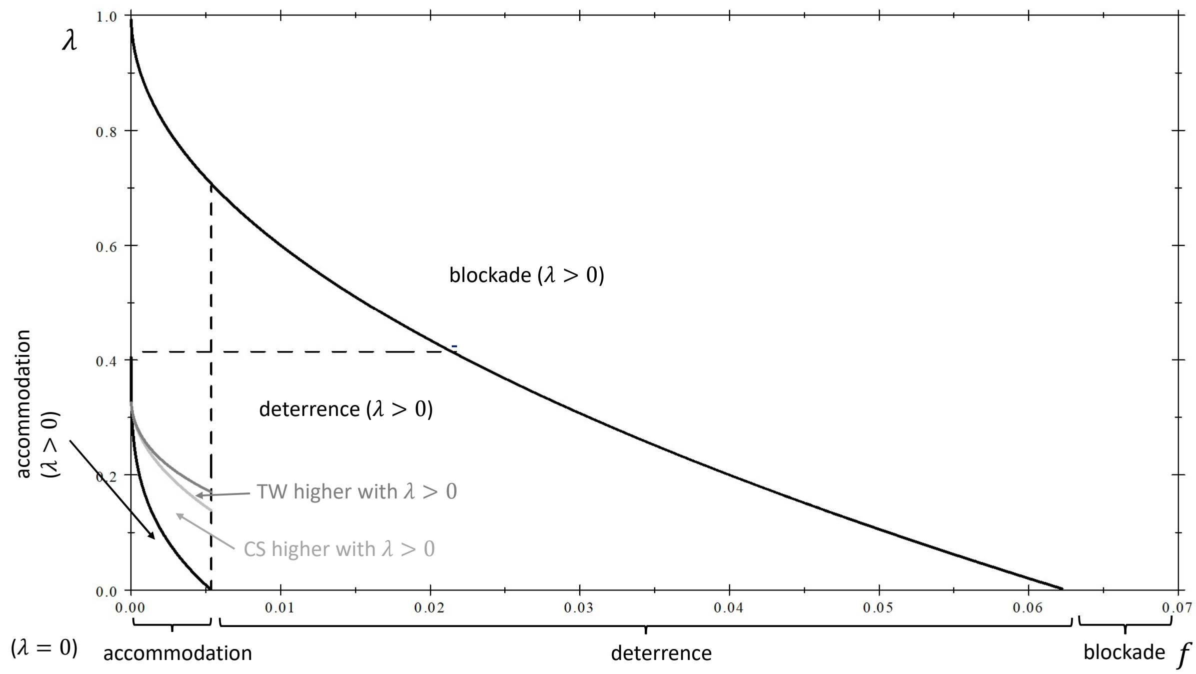

Figure 1: In $\left(\frac{\sqrt{f'}}{1-\delta'}, \lambda\right)$-space, the accommodation ($Q_L^* = Q_L^{\mathrm{a}}$), deterrence (Q_L^{d}), and blockade (Q_M) regions are delimited by the black curves. To the left of the dashed segment in the deterrence region, the leader would accommodate in the absence of internalization. Within this region consumer surplus is higher with internalization due to the leader's strategic shift up to the dark grey curve, and up to the light gray curve total welfare is higher.

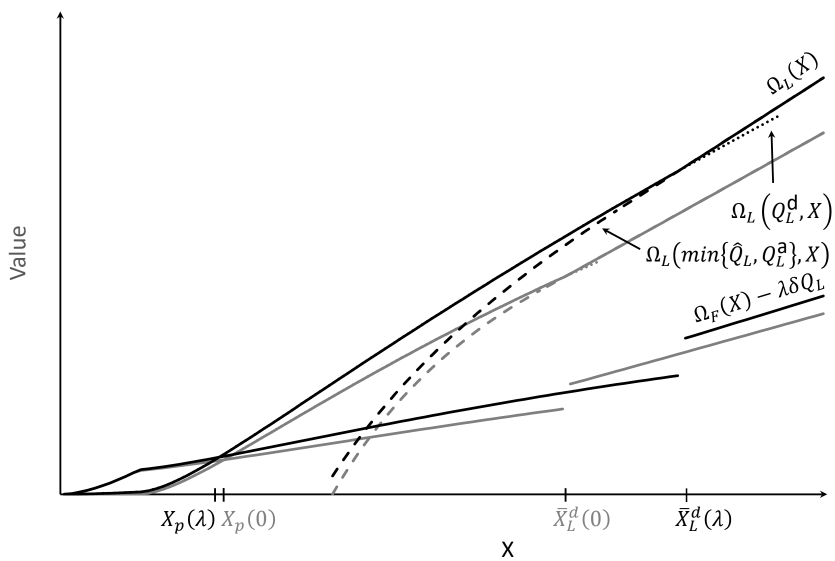

Figure 2: Leader and follower payoffs for $r = .1$, $\mu = .06$, $\sigma = .1$, $\delta = .1$, $\eta = .05$ and $\lambda = 0$ (gray) or .1 (black). At the demand state $\overline{X}_L^{\mathrm{d}}$ the leader shifts from deterrence to accommodation, resulting in a kink in the leader payoff and an upward jump in the follower payoff. The preemption equilibrium X_P lies at the intersection of the leader and follower payoffs. Greater internalization results in earlier equilibrium investment (lower X_P) and drives the leader to pursue deterrence over a broader range (higher $\overline{X}_L^{\mathrm{d}}$).

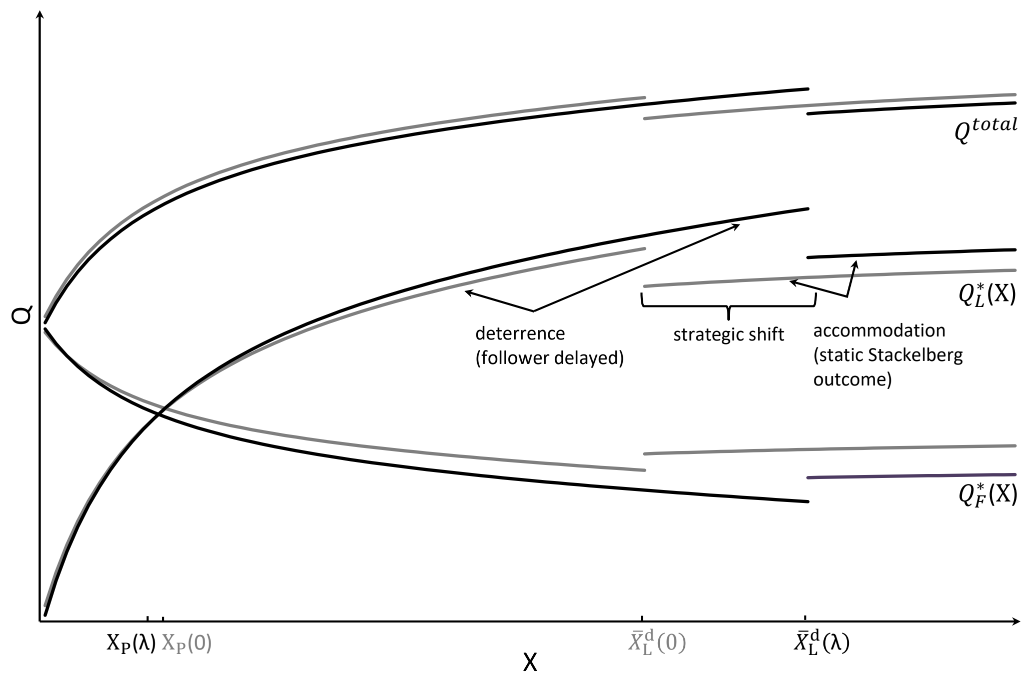

Figure 3: Optimal leader and follower capacities for $r = .1$, $\mu = .06$, $\sigma = .1$, $\delta = .1$, $\eta = .05$ and $\lambda = 0$ (gray) or $.1$ (black). For demand states between $\overline{X}_L^{\mathrm{d}}(0)$ and $\overline{X}_L^{\mathrm{d}}(\lambda)$ internalization shifts the leader's strategy from accommodation to deterrence, which results here in higher total output.