Behavior-based price discrimination and elastic demand

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Abstract

I investigate behavior-based price discrimination with three purchase histories: having bought from Firm A, having bought from Firm B, and having bought nothing. I relax the assumption that the market is fully covered in equilibrium by introducing low-value consumers. I indicate that firms offer high prices for old customers since firms recognize low-value consumers who buy goods in the first period as loyal consumers in the second period. However, the demand from low-value consumers is elastic. Some low-value consumers forgo buying goods in the first period to avoid being identified as old consumers in the second period. Moreover, the higher the prices for old consumers, the more high-value consumers switch to another firm. As a result, behavior-based price discrimination decreases consumer surplus and worsens the firm's profitability.

1 Introduction

Some firms offer different prices to consumers according to their purchase histories in markets, such as web retailers, electric companies, and telecommunication. They collect consumer data to keep track of their customers and utilize them for price discrimination. This price discrimination is called behavior-based price discrimination (BBPD).

After the seminal works by Chen (1997), Fudenberg and Tirole (2000), and Villas-Boas (1999), many studies have discussed BBPD. In their models, two firms compete over two periods. In the first period, each firm offers a single price for all consumers since there are no purchase histories, and all consumers buy either one good. After the first period, firms can recognize whether a consumer bought from the firm or its rival firm. In the second period, each firm offers a price for consumers who bought from the firm and a price for consumers who bought from the rival firm, respectively. Existing studies show that BBPD usually intensifies competition and causes welfare loss by consumers' switching.

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The BBPD models assume that the market is fully covered in equilibrium. If a consumer has not bought from one firm, it must have bought from another firm. In other words, firms identify the consumers who have not bought their goods as rival consumers. However, consumers who have not bought from the firm are not always rival customers in the real world since there is a possibility that some consumers have not purchased anything. Some firms ask rival consumers to provide evidence of their purchase of rival goods and offer different prices for rival consumers and consumers who have not purchased anything.

It is well known that mobile phone companies and some software companies engage in BBPD. Furthermore, some firms engage in BBPD by clearly distinguishing their rival consumers and consumers who have not bought anything in these markets. Japanese cell phone company, Docomo, offers a discount for consumers who switch to it from other cell phone companies and a discount for consumers who have a smartphone contract for the first time. T-Mobile offers discounts for new customers and pays the previous carrier's remaining device payment balance for consumers who switch from several mobile companies if they submit evidence. New users can try Shed, which is an event scheduling software, for free before they buy. In addition to this, if users contract with Whova, EventMobi, vFairs, Webex, and others, they can receive a switching providers discount by sending an invoice or a screenshot of these applications. Reclaimai.ai, which is a calendar application, offers a free trial of a paid version and also offers a twenty percent discount for 6 months for users who switch from Clockwise, Motion, or Calendly.

Existing studies on BBPD assume that consumers' willingness to pay is sufficiently large and the market is fully covered in the equilibrium. Then, there are two kinds of purchase histories: "bought from a firm" and "bought from another firm". However, some consumers do not buy goods depending on prices offered by firms since there are consumers whose willingness to pay is small and they don't buy goods if there is no favorite good. This creates three purchase histories: "bought from a firm", "bought from another firm", and "bought nothing". How does one more purchase history affect the impact of BBPD on competition and welfare? This paper aims to analyze BBPD with these three purchase histories.

This paper extends Fudenberg and Tirole (2000). I assume that half of consumers' willingness to pay is low. This relaxes the full coverage assumption and then there are three purchase histories after the first period. I call consumers with a high willingness to pay high-value consumers and those with low low-value consumers in this paper. I also assume that firms can distinguish rival consumers from consumers who have not bought their goods by requiring them to submit things that indicate their purchase histories, for example, invoices or screenshots of rival goods, and offer prices for each of the three purchase histories in the second period.

I found that firms offer discounts not only for consumers who bought rival goods but also for consumers who have not bought anything in the second period. The discounts for consumers who have not bought anything are smaller than those for consumers who have bought rival goods despite all consumers who have not bought anything are low type. In contract, firms charge consumers who have bought their own goods high prices. The consumers who buy same goods over the two periods pay more money in the second period than the first period. Fudenberg and Tirole (2000), in which all consumers are high type, show that BBPD intensifies the second period's competition by lowering the prices. However, BBPD does not necessarily intensify competition if there are low-value consumers.

The reason for these results is that low-value consumers' purchases in the first period reveal their relatively higher preference for one of the goods, and then firms have incentives to raise prices in the market segment where they exist. This implies that low-value consumers can buy goods cheaper by forgoing buying goods in the first period, and high type can avoid high prices by switching in the second period. Therfore, BBPD reduces total demands over two periods and worsens the firm's profitability and consumer surplus.

This paper is related to the strand of the literature on BBPD. There is extensive literature on BBPD (See Chen(2005), Fudenberg and Villas-voas (2006), and Esteves (2009b) for review). My paper is closely related to Chen and Zhang (2009). Chen and Zhang (2009) assume that loyal consumers and switchers. Loyal consumers are price insensitive and always purchase from a particular firm and switchers are price-sensitive consumers and purchase from a firm that offers a lower price. Firms compete for switchers. Chen and Zhang (2009) show that BBPD moderates competition and benefits for firms since firms set prices high to screen out switchers in the first period and engage on BBPD with distinguishing loyal consumers and switchers. We can say that low-value consumers who have relatively high preferences for particular goods in my model are loyal consumers. Their existence moderates the second period competition.

This paper is also related to Villas-Boas (2004), Acquisti and Varian (2005), Laussel and Resende (2020). These studies show that price discrimination is not beneficial for firms in monopoly markets. This is because low-value consumers have incentives to forgo buying goods today not to be recognized as old customers tomorrow. If they are recognized as old customers, they are offered higher prices. In my model, low-value consumers forgo buying in the first period to be recognized as consumers who have bought nothing, too.

This paper is organized as follows. Section 2 is preliminary, Section 3 discusses uniform pricing and BBPD, and Section 4 is the conclusion.

2 Preliminary

Two firms, Firm A and Firm B, are located at the extremes of a unit interval $[0, 1]$. Firm A is located at 0 and Firm B is located at 1. They produce horizontally differentiated goods with a constant marginal cost, which is normalized to zero. The locations of the firms are fixed.

A unit of consumers is uniformly distributed on the interval. Consumers evaluate the products differently. Half of the consumers are the high-value type whose willingness to pay is v_H. The other half is the low-value type whose willingness to pay is v_L ($v_H > v_L$). The location of each consumer represents her preference. A consumer located at x incurs a disutility of tx from buying good A and a disutility of $t(1-x)$ from buying good B. The parameter $t > 0$ measures the disutility per unit of distance of purchasing away from the ideal product.

Consumer buys at most one unit. The utility for a consumer located at x who purchases from Firm A at price p is given by $u_A(x, p) = v_i - tx - p$ and that of Firm B is given by $u_B(x, p) = v_i - t(1-x) - p$ ($i = H, L$). I assume that v_H is sufficiently large so that all high-type consumers buy products in equilibrium. However, v_L is not large enough so that all low-type consumers buy goods in equilibrium ($2v_L < t$). When a consumer buys nothing, her utility is defined to be zero.

There are two periods. Firms choose prices simultaneously to maximize their profits in each period. In the first period, there are no purchased histories. Firm A offers a_1 and Firm B offers b_1 for all consumers. Consumers observe the offered prices and decide to buy good A, good B, or nothing. Then, there are three kinds of purchase histories: "bought good A", "bought good B", and "bought nothing" at the end of this period. In the second period, firms can set different prices depending on the purchase histories. I assume that Firm A offers a_o for consumers who bought good A, a_n for consumers who bought good B, and α for consumers who bought nothing. This is because consumers who bought good A in the previous period are "old" consumers for firm A and consumers who bought good B are "new"consumers for firm A if they buy good A in the second period. Similarly, Firm B offers b_o for consumers who bought good A, b_n for consumers who bought good B, and β for consumers who bought nothing.

Firm A distinguishes consumers who bought its goods from others by observing collected purchase histories. Firm A also distinguishes consumers who bought Firm B's goods from consumers who did not buy good A by requiring them to certifications of previous contracts, for example, receipts or goods themselves. The same is true for Firm B. It should be noted that there is a possibility that consumers who bought goods in the first period buy goods at the prices α or β by purposely not submitting the certifications.

I assume that all agents discount their future by the common factor, which is normalized to one. I derive subgame perfect equilibrium by backward induction.

3 Analysis

3.1 Uniform pricing

I consider a case in which the firms can not engage in BBPD as a benchmark case. The two-period model can be reduced to two replications of a static model. I solve the static model.

Firm A and Firm B offer p_A and p_B for all consumers, respectively. I assume that the high-type consumer who is indifferent between buying goods A and B is located at x^*. The low-type consumer who is indifferent between buying goods A and buying nothing is located at $\underline{y}$, and that who is is indifferent betlween buying goods B and buying nothing is located at $\underline{y}$. These indifferent consumers identified by the condition $v_H - tx^* - p_A = v_H - t(1-x^*) - p_B$, $v_L - t\underline{y} - p_A = 0$, $v_L - t(1-\overline{y}) - p_B = 0$. From these, we have

$$\underline{y} = \frac{v_L - p_A}{t}, \quad x^* = \frac{t - p_A + p_B}{2t}, \quad \overline{y} = \frac{-v_L + t + p_B}{t}. \tag{1}$$

Since the high-type consumers on $[0, x^*]$ and the low-type consumers on $[0, \underline{y}]$ buy good A and the high-type consumers on $[x^*, 1]$ and the low-type consumers $[\overline{y}, 1]$ buy good B, and others buy nothing, the firms' profit are

$$\pi_A^U = \frac{1}{2}p_A(x^* + \underline{y}), \quad \pi_B^U = \frac{1}{2}p_B\{(1-x^*) + (1-\overline{y})\}. \tag{2}$$

where the superscript "U" stand for uniform pricing. Firm A choose p_A to maximize π_A^U and Firm B choose p_B to maximize π_B^U. First-order conditions are

$$\frac{\partial \pi_A^U}{\partial p_A} = \Big(\frac{t - p_A + p_B}{2t} - \frac{p_A}{2t}\Big) + \Big(\frac{v_L - p_A}{t} - \frac{p_A}{t}\Big) = 0, \tag{3}$$

and

$$\frac{\partial \pi_B^U}{\partial p_B} = \Big(\frac{t - p_B + p_A}{2t} - \frac{p_B}{2t}\Big) + \Big(\frac{v_L - p_B}{t} - \frac{p_B}{t}\Big) = 0, \tag{4}$$

Solving the maximization problems, we have Solving the equations, we have

$$p_A = p_B = \frac{1}{5}(2v_L + t). \tag{5}$$

Introducing the equilibrium prices into (1), we have

$$\underline{y} = \frac{3v_L - t}{5t}, \quad x^* = \frac{1}{2}, \quad \overline{y} = \frac{-3v_L + 6t}{5t}. \tag{6}$$

If firms could distinguish between the high-type consumers and the low-type consumers and offer different prices for each type, they would offer t for the former and $\frac{v_L}{2}$ for the latter. We can easily find that $\frac{v_L}{2} < \frac{1}{5}(2v_L + t) < t$.

3.2 Price discrimination

In this section, I consider a case in which the firms engage in BBPD.

3.2.1 The second period

Suppose that the high-type consumer who is indifferent between buying goods A and B is located at x_1, the low-type consumer who is indifferent between buying goods A and buying nothing is located at y_A, and the low-type consumer who is indifferent between buying goods B and buying nothing is located at y_B in the first period. The high-type consumers on $[0, x_1]$ and the low-type consumers on $[0, y_A]$ bought good A, and the high-type consumers on $[x_1, 1]$ and the low-type consumers on $[y_B, 1]$ buy good B in the period.

In the second period, consumer on $[0, x_1]$ continues to buy from Firm A again if $v - tx - a_o \geq v - t(1-x) - b_n$. Otherwise, she or he switches to Firm B. Consumer on $[x_1, 1]$ continues to buy from Firm B again if $v - t(1-x) - b_o \geq v - tx - a_n$. Otherwise, she or he switches to Firm A. Then, the consumer who is indifferent between buying from Firm A and switching to Firm B and the consumer who is indifferent between buying from Firm B and switching to Firm A are located at

$$x_A = \frac{t - a_o + b_n}{2t}, \quad x_B = \frac{t - a_n + b_o}{2t}. \tag{7}$$

Consumers who bought nothing in the first period buy good A if $v - tx - \alpha \geq 0$ or buy good B if $v - t(1-x) - \beta \geq 0$. Otherwise, she or he does not buy again. Let z_A be the consumer who is indifferent between buying from Firm A and buying nothing, and z_B be the consumer who is indifferent between buying from Firm B and buying nothing. Those indifferent consumers are located at

$$z_A = \frac{v_L - \alpha}{t}, \qquad z_B = \frac{-v_L + t + \beta}{t}. \tag{8}$$

The second-period profits of Firm A and Firm B can be written as

$$\pi_A^{D2} = \frac{1}{2}\{a_o(x_A + y_A) + a_n(x_B - x_1) + \alpha(z_A - y_A)\}, \tag{9}$$

$$\pi_B^{D2} = \frac{1}{2}\{b_o(1 - x_B + 1 - y_B) + b_n(x_1 - x_A) + \beta(y_B - z_B)\}, \tag{10}$$

where "D" stands for price discrimination and the number "2" stands for period two[1]. Firm A chooses a_o, a_n, and α to maximize (9), and Firm B chooses b_o, b_n, and β to

[1] If $y_A \geq z_A$, the indifferent consumer, y_A satisfies $v + ty_A - a_1 = 0$. Hence, $v + y_A \geq 0$ unless a_1 is negative. When $v + y_A \geq 0$, firm A can increase its profit by offering α, no matter how low the price is. Therefore, $y_A \leq z_A$.

maximize (10). First-order conditions are as follows:

$$\frac{\partial \pi_A^{D2}}{\partial a_o} = \frac{t + b_n - a_o}{2t} + y_A - \frac{a_o}{2t} = 0, \tag{11}$$

$$\frac{\partial \pi_A^{D2}}{\partial a_n} = \frac{t + b_o - a_n}{2t} - x_1 - \frac{a_n}{2t} = 0, \tag{12}$$

$$\frac{\partial \pi_A^{D2}}{\partial \alpha} = \frac{v_L - \alpha}{t} - y_A - \frac{\alpha}{t} = 0, \tag{13}$$

$$\frac{\partial \pi_B^{D2}}{\partial b_o} = 1 - \frac{t + b_o - a_n}{2t} + 1 - y_B - \frac{b_o}{2t} = 0, \tag{14}$$

$$\frac{\partial \pi_B^{D2}}{\partial a_n} = x_1 - \frac{t - a_o + b_n}{2t} - \frac{b_n}{2t} = 0, \tag{15}$$

$$\frac{\partial \pi_B^{D2}}{\partial \alpha} = y_B - \frac{-v_L + t + \beta}{t} - \frac{\beta}{t} = 0, \tag{16}$$

By solving these maximization problems, we can obtain equilibrium prices as follows:

$$a_o = \frac{t}{3}(1 + 2x_1 + 4y_A), \quad a_n = \frac{t}{3}\{3 - 4x_1 - 2(1 - y_B)\}, \tag{17}$$

$$b_o = \frac{t}{3}\{3 - 2x_1 + 4(1 - y_B)\}, \; b_n = \frac{t}{3}(-1 + 4x_1 + 2y_A), \tag{18}$$

and

$$\alpha = \frac{1}{2}(v_L - ty_A), \quad \beta = \frac{1}{2}\{(v_L - t(1 - y_B)\}. \tag{19}$$

BBPD divides the market into two market segments according to their purchase histories: having bought goods in the first period and having bought nothing. Firms set prices for the former segment as duopolists and set prices in the latter segment as if they were monopolist[2]. An increase in y_A and a decrease in y_B means an expansion of the segment of consumers who have bought and a shrinkage of the segment of consumers who have not bought. Equilibrium prices in (17), (18), and (19) show that firms set higher prices as the segment size is expanded in each segment. Introducing these equilibrium prices into (9) and (10), we have

$$\pi_A^{D2} = \frac{t}{36}(1 + 2x_1 + 4y_A)^2 + \frac{t}{36}(5 - 4x_1 - 2y_B)^2 + \frac{1}{8t}(v_L - ty_A)^2, \tag{20}$$

$$\pi_B^{D2} = \frac{t}{36}(7 - 2x_1 - 4y_B)^2 + \frac{t}{36}(-1 + 4x_1 + 2y_A)^2 + \frac{1}{8t}(v_L - t + ty_B)^2. \tag{21}$$

[2]If it were not for low-type consumers, firms would set $a_o = \frac{t}{3}(1 + 2x_1)$, $a_n = \frac{t}{3}(3 - 4x_1)$, $b_o = \frac{t}{3}(3 - 2x_1)$, $b_n = \frac{t}{3}(-1 + 4x_1)$.

3.3 The first period

In the first period, consumers make decisions to purchase goods anticipating the second period's prices. The indifferent high-type consumer located at x_1 is indifferent between buying from Firm A in the first period at a price a_1 and then buying from Firm B in the second period at a price b_n, or buying from Firm B in the first period at a price b_1 and then buying from firm A in the second period at a price a_n. Thus, x_1 satisfies $v_H - tx_1 - a_1 + v_H - t(1-x_1) - b_n = v_H - t(1-x_1) - b_1 + v_H - tx_1 - a_n$. Then, we have

$$x_1 = \frac{6t - 2t(y_A + y_B) - 3a_1 + 3b_1}{8t}. \tag{22}$$

The indifferent low-type consumer who is located at y_A is indifferent between buying from Firm A in the first period at a price a_1 and buying from Firm A again in the second period at a price a_o, or buying nothing in the first period and then buying from Firm A in the second period at a price α. Thus, y_A satisfies $v_L - ty_A - a_1 + v_L - ty_A - a_o = v_L - ty_A - \alpha$. In the same way, y_B satisfies $v_L - t(1-y_B) - b_1 + v_L - t(1-y_B) - b_o = v_L - t(1-y_B) - \beta$. Then, we also have

$$y_A = \frac{9v_L - 2t - 4tx_1 - 6a_1}{17t}, \quad y_B = \frac{-9v_L + 23t - 4tx_1 + 6b_1}{17t}. \tag{23}$$

Since variables x_1, y_A, and y_B satisfies equations in (22) and (23), we can obtain x_1 as a function of prices, a_1 and b_1, as

$$x_1 = \frac{20t - 13a_1 + 13b_1}{40t}. \tag{24}$$

Furthermore, we also obtain

$$y_A = \frac{90v_L - 40t - 47a_1 - 13b_1}{170t}, \quad y_B = \frac{-90v_L + 210t + 13a_1 + 47b_1}{170t}. \tag{25}$$

The first period's profits of Firm A and Firm B are given by

$$\pi_A^{D1} = \frac{1}{2}a_1(x_1 + y_A), \quad \pi_B^{D1} = \frac{1}{2}b_1(1 - x_1 + 1 - y_B). \tag{26}$$

where the number " 1 " stands for period one. Firm A chooses a_1 to maximize $\Pi_A^D = \pi_A^{D1} + \pi_A^{D2}$. and Firm B chooses b_1 to maximize $\Pi_B^D = \pi_B^{D1} + \pi_B^{D2}$ in the first period. Solving these maximization problems, we have

$$a_1 = b_1 = \frac{1828}{8325}(2v_L + t) \simeq 0.219580(2v_L + t). \tag{27}$$

Introducing the equilibrium prices into (22) and (25), we have

$$x_1 = \frac{1}{2}, \quad y_A = \frac{3117v_L - 2604t}{8325t}, \quad y_B = 1 - y_A. \tag{28}$$

From (7), (8), (17) and (18), we can derive the second period's equilibrium prices as

$$a_n = b_n = \frac{1039}{8325}(2v_L + t),\ a_o = 2a_n,\ b_o = 2b_n,\ \alpha = \beta = \frac{1302}{8325}(2v_L + t), \tag{29}$$

and the locations of the indifferent consumers as

$$x_A = \frac{-1039v + 3643t}{8325t}, \quad x_B = 1 - x_A, \quad z_A = \frac{5721v_L - 1302t}{8325t}, \quad z_B = 1 - z_A. \tag{30}$$

By comparing (5) and (29), we can see that $a_o > p_A$ and $b_o > p_B$, which can be sumaraized the following propositon.

Proposition 1 *Firm A offers higher prices for consumers who bought its goods in the previous period than the price under uniform pricing in the second period under BBPD.*

The reason is explained as follows. Equation (3) shows that both the demand of low-value consumers and that of high-value consumers decrease if Firm A raises p_A under uniform pricing. On the other hand, equation (11) shows that only the demand of high-value consumers decreases when Firm A raises a_o under BBPD. The difference between equations (3) and (11) is attributed to the fact that Firm A can separately set a_o for low-value consumers on $[0, y_A]$, who strongly prefer good A, and α for other low-value consumers. All of the consumers on $[0, y_A]$ repurchase good A even if Firm A offers a higher price since they strongly prefer good A under BBPD in equilibrium. Therefore, Firm A offers a higher price for consumers who bought its goods under BBPD than under uniform pricing, for a given price chosen by the rival.

The converse of Proposition 1, that is $a_o < p_A$ and $b_o < p_B$, would be established if all of the consumers were high-value types in this model.[3] Standard BBPD models, for instance, Fudenberg and Tirole (2000), conclude that, in the second period, firms lower prices for consumers who bought rival goods under BBPD compared to the uniform pricing since the market of consumers who bought their rival goods is a weak market for their goods under BBPD. The rival firms lower the prices for consumers who bought their goods in response to this. Thus, the prices for consumers who bought its goods in the second period are lower than the prices under uniform pricing in the standard BBPD models in this model.

This is partially true for this paper. Return to the case when both high-value types and low-value types exist. Equation (4) and (15) indicate that firm B is aggressive in the pricing of b_n under BBPD. However, this negative effect on a_o is smaller than the positive effect on a_o, which I mentioned under Proposition 1 in equilibrium. Therefore, Proposition 1 is established.

From (29), we also can find that $a_o > \alpha > a_n$, $b_o > \beta > b_n$. Firms offer lower prices for consumers who bought rival goods than those who bought nothing in the second period under BBPD. Therefore,

[3] Firms offer $p_A = p_B = t$ under uniform pricing, and $a_1 = b_1 = \frac{4}{3}t$, $a_o = b_o = \frac{2}{3}t$, $a_n = b_n = \frac{1}{3}t$ if all of the consumers are high-value types.

Proposition 2 *Firms offer discounts for consumers who bought rival goods and who bought nothing in the second period. Furthermore, they offer larger discounts for consumers who bought rival goods than consumers who bought nothing.*

It is apparent that all of the consumers who bought nothing are low-value types. As I said earlier, BBPD divides the market into the market segment of consumers who bought goods in the previous period and of consumers who bought nothing. There are both high-value consumers and low-value consumers in the former market segment while there are only low-value consumers in the latter market segment. It could be said that firms set prices lower in the market segment of consumers who bought nothing than in the market segment of consumers who bought goods. Why don't firms offer greater discounts for consumers who bought nothing, even though all of the consumers are low-value types? Equations (13) and (16) show that firms set prices α and β as if they were monopolists in the market segment of consumers who bought nothing although they set other prices as duopolists in another market segment. That is, BBPD relaxes competition in the market segment of consumers who bought nothing. Then, firms do not lower these prices so much.

Lastly, it should be noted that this proposition indicates that no consumers who bought goods A or B in the first period have incentives to buy goods at α or β by pretending not to buy anything. The reason is that they can buy goods at lower prices a_n or b_n by revealing their purchase histories correctly.

4 Total demand and welfare

In this paper, the total demand is elastic, unlike the standard BBPD models. This is because I assume that half of consumers are low-value consumers and their willingness to pay is less than $\frac{t}{2}$. From (6), the total demand under uniform pricing can be derived as

$$1 + (\underline{y} + 1 - \overline{y}) = 1 + \frac{6v_L - 2t}{5t}. \tag{31}$$

Low-value consumers on $[\underline{y}, \overline{y}]$ do not buy goods in each period, despite all high-value consumers buy goods. By using (28), and (30), we can derive the first period's total demand and the second period's total demand as

$$1 + (y_A + 1 - y_B) = 1 + \frac{6234v_L - 5208t}{8325t}, \tag{32}$$

and

$$1 + (z_A + 1 - z_B) = 1 + \frac{11442v_L - 2604t}{8325t}. \tag{33}$$

These results indicate that BBPD shrinks the first period's total demand and expands the second period's total demand since $y_A+1-y_B > \underline{y}+1-\overline{y} > z_A+1-z_B$. Moreover, BBPD shrinks total demand over period since $(y_A+1-y_B)+(z_A+1-z_B) < 2(\underline{y}+1-\overline{y})$.

This is because low-value consumers on $[y_A, z_A]$ and $[z_B, y_B]$ do not buy goods in the first period but buy goods in the second period under BBPD. They anticipate that they face higher prices a_o or b_o in the second period if they buy goods in the first period. They do not buy goods to avoid establishing purchase histories of buying goods in the first period and buy goods at lower prices α or β in the second period. We can easily check that consumers who buy the same goods over two periods pay more money in the second period than in the first period.

We define per-period equilibrium consumer surplus under uniform pricing as welfare

$$cs^U = \int_{\underline{y}}^{x^*} (v - t|x| - p_A)dx + \int_{x^*}^{\overline{y}} (v - t|x-1| - p_B)dx. \tag{34}$$

From this and equation (2), (5), and (6), per-period equilibrium consumer surplus and per-period equilibrium profits under uniform pricing can be obtained as

$$cs^U = \frac{9}{100}\frac{(2v+t)^2}{t} - \frac{t}{2}, \tag{35}$$

and

$$\pi_A^U = \pi_B^U = \frac{3}{50}\frac{(2v+t)^2}{t}. \tag{36}$$

Since social welfare is the sum of consumer surplus and profits, per-period equilibrium social welfare under uniform pricing is

$$sw^U = cs^U + \pi_A^U + \pi_B^U = \frac{21}{100}\frac{(2v+t)^2}{t} - \frac{t}{2}. \tag{37}$$

Total discounted consumer surplus, total discounted profits and total discounted social welfare are given by $CS^U = 2cs^U$, $\Pi_i^U = 2\pi_i^U$ $(i = 1, 2)$, and $SW^U = 2sw^U$ since the discount rate is one.

As a same matter, from (20), (21), (26), and (27)-(30), consumer surplus and the profits under BBPD are

$$cs^{D1} = \frac{25,213,715}{4(8325)^2}\frac{(2v+t)^2}{t} - \frac{t}{2}, \tag{38}$$

$$cs^{D2} = \frac{19,390,857}{4(8325)^2}\frac{(2v+t)^2}{t} - \frac{t}{2}, \tag{39}$$

$$\tag{40}$$

$$CS^D = \frac{44,604,572}{4(8325)^2}\frac{(2v+t)^2}{t} - t, \tag{41}$$

and

$$\pi_i^{D1} = \frac{5697876}{2(8325)^2}\frac{(2v+t)^2}{t}, \tag{42}$$

$$\pi_i^{D2} = \frac{8788013}{2(8325)^2}\frac{(2v+t)^2}{t}, \tag{43}$$

$$\Pi_i^{D} = \frac{14485889}{2(8325)^2}\frac{(2v+t)^2}{t}. \tag{44}$$

Then, social welfare under BBPD can be obtained as follows.

$$sw^{D1} = \frac{42182361}{4(8325)^2}\frac{(2v+t)^2}{t} - \frac{t}{2}, \tag{45}$$

$$sw^{D2} = \frac{60,365,767}{4(8325)^2}\frac{(2v+t)^2}{t} - \frac{t}{2}, \tag{46}$$

$$SW^{D} = \frac{102,548,128}{4(8325)^2}\frac{(2v+t)^2}{t} - t. \tag{47}$$

Comparing consumer surplus and profits under BBPD with those under uniform pricing, we can find that $CS^U > CS^D$, $\Pi_i^U > \Pi_i^D$. The total discounted profit per firm under BBPD is smaller than that under uniform pricing. Furthermore, the consumer surplus under BBPD is smaller than that under uniform pricing. Therefore, BBPD worsens social welfare. Now, we can state the following proposition.

Proposition 3 *BBPD decreases the total demand and consumer surplus.*

The standard BBPD models show that BBPD reduces profits but increases consumer surplus. The reason is that BBPD increases consumers' transportation costs and reduces the second period's prices. However, standard BBPD models assume that demand is inelastic. In this model, BBPD reduces total demand and does not necessarily reduce all second period's prices.

5 Conclusion

This paper analyzes BBPD in a market with elastic aggregate demand by assuming half of the consumers are low-value type and the others are high-value type. The willingness to pay of low-value consumers is sufficiently low and the market is not fully covered in equilibrium. There are three kinds of purchase histories: "bought good A", "bought good B", and "bought nothing" after the first period. Firms set different prices depending on consumers' purchase histories in the second period.

This paper shows the following results. Firstly, firms offer higher prices for consumers who bought their goods in the previous period than equilibrium prices under uniform pricing in the second period under BBPD. This is because all low-value consumers who bought their goods buy their goods again even if firms raise the prices since they strongly prefer the goods. Secondly, firms offer lower prices for consumers who bought their rival goods in the previous period than consumers who bought nothing, The reason is that firms can offer their prices to low-value consumers who bought nothing as if they were monopolists. Thirdly, total demand and consumer surplus under BBPD are smaller than those under uniform pricing. Some low-value consumers refrain from purchasing goods in the first period since firms offer high prices to the consumers who buy the same goods over two periods.

I analyze the case where two firms are symmetry and always engage in BBPD. I assume that product qualities and marginal costs are equal across firms. Introducing asymmetry and endogenizing the firms' decisions to engage in BBPD remain for future research.

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