Optimal regulation with loopholes

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Abstract

We study a static environment where two competing firms are subject to a regulation which increases their cost of production. The regulation has a loophole and the firms can exert private effort to find it. A firm which finds the loophole can lower their cost of production. Given the loophole-finding effort of a firm, the stricter the regulation, the less likely it is that the firm finds the loophole. We demonstrate a new channel via which stricter regulation reduces welfare. We show that stricter regulation increases the reward for finding the loophole. This lead to higher loophole-finding effort which offsets the stricter regulation. We further show that even if the strictness of the regulation is endogenously determined by lobbying, it may still be welfare superior to the regime in which there is no regulation. This is because it is optimal for the firms to have some regulations.

Keywords - regulations, loopholes, competition, moral hazard

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"America's financial system has been highly innovative, but to a great degree innovation has recently been directed at circumventing laws and regulations [...]" – Stiglitz (2009)

"[...] Complex rules have generated both the incentives and the means to exploit regulatory loopholes." – Haldane (2013)

1 Introduction

Regulators are commonly tasked with devising suitable regulations in line with government objectives. In practice, however, regulations can be circumvented via loopholes.[1] This could be done legally, like in the case of *The Clean Air Act Amendments* of 1977 in the USA, which states that only new factories and power plants have to meet tighter emission standards imposed by the act. Existing plants are regulated under the pre-existing standards unless they were 'substantially modified.' Unfortunately, the regulation did not specify exactly what 'substantially modified' means. Several old firms took advantage of this loophole by building practically new plants at pre-existing locations. Additionally, firms can get around a regulation illegally, like in the case of the German car manufacturer Volkswagen that fitted its cars with software that would cheat NOx testing under laboratory conditions.

Obviously, a firm that finds a loophole in the regulation obtains a competitive edge over its rivals. Thus, there could be significant incentives to find loopholes in regulations.[2] Regulators are aware that their regulations may have loopholes, and they constantly strive to make it difficult for firms to find the loopholes by strengthening regulations [3] We take the 'strengthening' of regulations to refer to any act which makes it more difficult for firms to find loopholes. Since regulations are usually put in place to curb something undesirable,

[1]In this paper, we use the word 'loophole' to describe a practice or method to get around a regulation or set of regulations.

[2]In his 1977 paper (Kane (1977)), Edward Kane spoke about the 'regulatory dialectic', a phrase he used to describe the cycle of formulation of regulations, and the market's response to maintain/increase profits by finding and exploiting loopholes in the same.

[3]In the USA, the Great Depression spawned the Glass-Steagall Act in 1933, which was about 37 pages in length. The global financial crisis 2007/2008 and subsequent recession has spawned the Dodd-Frank Act in 2010, spanning 848 pages . Once completed, Dodd-Frank might run to 30,000 pages of rule making.

it would appear intuitive that regulators should jump at any chance to reduce the number of loopholes in regulations. This brings us to our research question - If a regulator can strengthen a regulation at no cost, should she always do it?

We propose a new channel via which increasing the regulation strength could reduce welfare. We show that strengthening regulations can make the reward for finding loopholes bigger. This could lead to higher levels of loophole finding effort which undermines the strengthening of the regulation. The intuition is that when two firms compete with each other, the reward for finding the loophole in a regulation is highest when the other firm does not find the loophole. Thus, under some conditions, when regulation strength is increased, two opposing forces shape the firm's incentive to exert effort to find loopholes. One, the increase in regulation strength makes it less likely that a firm will find a loophole. This reduces the return from loophole finding effort. Two, the increase in regulation strength makes it more likely that if a firm finds the loophole, it may be the only firm which does so (that is, it becomes more likely that the other firm does not find the loophole). This increases the returns from loophole finding effort. We find conditions under which the latter effect dominates the former leading to the counter-intuitive result that - when regulation strength is increased, the probability that the regulation will be violated is also increased. Our theoretical results are consistent with the empirical findings of Hu et al. (2017) who find that as the permissible levels of Nitrogen Oxide emissions went down in Europe (between 2000-2014), the number of violations (as captured by an on the road sensor) went up.

We go on to show that an increase in competition can also increase the probability of one of the firms finding a loophole. In this way, the impact of an increase in regulation strength and the impact of increased competition is similar. If the latter can be affected by the regulator, then the regulator may think of regulatory strength and competition level as substitutes. Finally, we endogenize the regulation strength in the model by allowing the firms to put in lobbying effort which directly influences the level of regulation strength. We show that even though regulation strength is determined by the lobbying effort of firms, the society may be better off with influenced regulations as compared to no regulations. The key idea is that even though regulation strength is determined by lobbying effort, it is not in the interest of the firms to make regulation very weak (else the rival firm also has a high chance of finding the loophole). This means that there is a high enough probability that no firm finds the loophole, and this benefits the society.

Our model shows that well-intentioned regulations may produce undesirable effects if policy makers ignore how the relevant players will react to a change in regulation. Regulators often try to patch old regulations to make them 'stronger'. This could be because politicians want to score brownie points with their constituents, or because they truly believe that this is in public interest. However, we show that the mindless strengthening of old regulations can actually have the opposite of the desired effect. The policy implication of our result is that under some conditions, it is better to *reduce* the strength of the regulation (or remove the regulation entirely). Given how costly (both time and money) regulations are, any theory which prescribes lower regulations is important and must be examined carefully. Our result is in the spirit of the theory of the second best (see Lipsey and Lancaster (1956)).

The rest of the paper is structured as follows. Section 2 describes the relevant literature and section 3 describes our baseline model. Section 4 presents the analysis for our baseline model. In section 5.2, we introduce endogenous regulation strength determined by the lobbying efforts of the two firms. We find the equilibrium regulation strength under lobbying, and show the regime with lobby determined regulation strength can be better than one without regulations. Section 6 concludes the paper.

2 Literature

Our paper highlights a new channel via which stricter regulations can worsen welfare. We identify conditions under which stricter regulations increase the returns from circumventing the regulation in an environment where firms compete on prices. This is related to several branches of the Economics and Management literature.

Several papers speak about the economics of raising the cost of rivals (Oster (1982), Salop and Scheffman (1983), Michaelis (1994)). A central argument in such papers is that in an environment with asymmetric firms, it may be beneficial to raise the cost of rivals (even if it means facing higher costs oneself) if there is comparative advantage to be gained. This idea has a flavour of our arguments, but our paper contributes to the literature by demonstrating a new channel via which even symmetric firms may prefer an increase in regulation (see section 5.2). This is because they want to get away from the zero profits that accrue to them when they are competing with a symmetric firm, to an environment where they can get a competitive edge over their rival by finding a loophole and subsequently earn positive profits.[4]

Two papers that are somewhat close to ours is Branco and Villas-Boas (2015) and Hu et al. (2017). They are primarily interested in the impact of increasing competition on the effort made towards regulatory compliance, but they also consider the impact of stricter standards on compliance. They find conditions under which stricter regulatory standards can reduce welfare by reducing the incentives to be compliant. The idea is that stricter standards increase the marginal cost of production and this can reduce the incentive to be compliant as it reduces profits/revenue. This is because they assume that the cost of being non-compliant is directly proportional to the profits, as a firm found to be non-compliant is assumed to lose all its profits. Thus, lower profits reduce the cost of non-compliance. These papers differ from our paper in several ways, the most important of which are as follows. First, both Branco and Villas-Boas (2015) and Hu et al. (2017) consider quantity competition, whereas we consider price competition in our model. We believe our model is more reflective of an environment where one firm can significantly corner a market by finding a regulatory loophole and undercutting all its rivals. The angle of competition is, in fact, completely missing from Hu et al. (2017) where the compliance effort of a rival firm has no impact on the incentive to exert compliance effort for a given firm. Secondly, we allow the firms to make their pricing decision after they learn of the success of the loophole finding

[4] In an environment where firms trade with each other, Glode et al. (2012) warn against the risk of becoming too asymmetric though, since asymmetric information can lead to no trade inefficiencies.

effort. Thus, the pricing decision is made after a significant asymmetry is realized. We believe pricing decisions must be a function of the outcome of loophole finding effort. This aspect of our model is not present in Branco and Villas-Boas (2015).

A key result in our paper is that when firms are competing with each other, an increase in regulation strength can lead to more violations as firms exert greater effort in finding loopholes. Other papers have also spoken about how firms facing high competition are more likely to break the rules (Shleifer (2004) gives five examples). Another example is that of Bennett et al. (2013) who argue that private car testing facilities may illegally pass several polluting cars in the hope of retaining customers who can provide them with income for years to come (else the customers can go to a rival testing facility). Using data from Chinese firms, Cai and Liu (2009) find that firms in more competitive environments or firms facing higher tax rates are more likely to evade taxes. This has a similar flavour as our results. Tax avoidance can be seen as the firms finding a loophole in the regulation (tax laws) and this results in the society facing costs (losing out on additional taxes). However, our channel focuses on how higher strength regulations can increase the incentives to find the loophole because it reduces the probability of a rival firm also finding the loophole at the same time. Whereas, in Cai and Liu (2009) firms are more likely to avoid taxes when tax rates go up because of the higher savings per dollar announced as revenue.

There have been several papers which discuss other negative side effects of regulations. Going back to the work of Stigler and Posner in the 1970's (Stigler (1971), Posner et al. (1974)), where they postulate that the political process of regulation is typically captured by the industry. In this case, they argue that "regulation not only fails to counter monopoly pricing, but is actually used to sustain it through state intervention". More recently, Glaeser and Shleifer (2001) and Cheng and Lai (2012) argue that regulations may work better when they impose a small cost only. The idea is that as regulations become more stringent, it may become efficient for firms to bribe, or to exert greater political pressure (via lobby groups for example) to reduce the impact of the regulation or worse, reverse the intended impact of the regulation. Harstad and Svensson (2011) develop a model where firms have the option to either comply with regulations or bribe to bypass regulations or lobby to change the regulation. They use this model to explain how firms transition from bribing when they are small to lobbying when they are large because the cost of bribing goes up fast as they grow. A key difference from our paper is that firms are not competing with each other in Harstad and Svensson (2011). Getting the competitive edge by finding a loophole is the main incentive for exerting loophole finding effort in our model. In our result with lobbying, firms lobby non cooperatively and the outcome of lobbying is not an end to regulation for all the firms, but to achieve just the right level of regulation to maximize the probability of gaining a competitive edge over their rivals.

It has also been demonstrated in the finance and economics literature that an increase in regulation strength can lead to more risk-taking behaviour by the players who seek to maximize short-run profits - Laffont (1995), Gonzalez (2005). Finally, increasing regulation strength can be welfare reducing if the incentives of the political agents/regulators are not aligned with those of the society. A case in point is New York's Martin Act, the state law that gives the state Attorney general broad powers to investigate and press charges against alleged financial fraud. The purpose of the act was to deter and fight financial fraud. However, it has been argued that this law has been misused to gain political points.[5]

3 Model

There are two firms, a unit mass of consumers distributed on the interval $[0,1]$, and one regulator. In our baseline static model all actors are risk neutral. Firms can produce one of two goods: A high type good (H) and a low type good (L). Each consumer demands only one unit of output (irrespective of the type). The marginal cost of production for the high type good (c_h) is higher as compared to the marginal cost of production for the low type good (c_l). Furthermore, we assume that $0 < c_l < c_h < 1$.

[5] "Attorney General Eric Schneiderman went after several large energy firms for allegedly over-optimistic financial comments on up-state gas 'fracking,' a charge conveniently aligned with his own anti-fracking stance." - *Devil's bargain: Wall St. & the Martin act - New York Post August 30, 2011*

While production of the high type good results in no social cost, producing the low type good inflicts a cost of $C \in \mathbb{R}_+$ on each consumer. Consumers are indifferent between the two goods and obtain utility $u > c_h$ from consuming either. Consumers choose based on the price alone. Thus, if one firm charges a lower price for their good, then all consumers purchase from this firm.

The regulator cares about the total utility of consumers and can introduce regulation to ban the production of any type of good to increase welfare (the case where the regulator maximizes the utility of consumers plus the utility of the firms is discussed in Section 4.4.1). A key assumption in our model—and deviation from the literature—is that we assume that regulation is imperfect, i.e. that firms can find loopholes in the regulation. This means that the firms can exert costly effort and circumvent the regulation if their effort is successful. Formally, firms can exert effort $e \in [0,1]$ to circumvent the regulation and find a loophole. Let $p \in [0,1]$ denote the "strength" of the regulation, which is understood as the probability of a firm finding a loophole if it exerts full effort. More generally, if a firm exerts effort e, then the probability of the firm discovering the loophole to circumvent the regulation is pe. We assume that the cost of exerting effort e for firm i is $\frac{M_i e^2}{2}$ where M_i is a positive real number denoting a firm's intrinsic ability to find a loophole.[6] M_i is common knowledge. Without loss of generality, we assume that firm 1 is more capable of finding loopholes by assuming that $M_1 < M_2$. If a firm discovers a loophole, then the firm may produce any good it wants, and in particular it can also produce the banned good.

3.1 Timing

First, firms learn p and then simultaneously make the effort choice to discover loopholes in the regulation. Then, firms learn which of them—if any—were successful at finding a loophole. Next, firms simultaneously decide which good to produce and what price to sell it at. Finally, consumers decide which good to buy.

[6]Quadratic costs ensure bounded interior solutions for optimal effort levels.

3.2 Strategies and Equilibrium

First, we define a feasible action set for the firms given the outcome of their loophole finding effort. Let F denote the feasible set. If a firm's loophole finding effort is successful, $F = \{H, L\} \times \mathbb{R}^+$ where the first argument denotes the good choice and the second argument denotes the price choice. If a firm's loophole finding effort is unsuccessful, $F = X \times \mathbb{R}^+$ where X denotes the set of goods which are not banned by the regulator and the second argument denoted the price choice of the firm for its chosen good. The strategy for any firm is given by a function S which goes from the strength of regulation to loophole finding effort, and good and price choice. Thus, $S : [0,1] \to [0,1] \times F$. We are interested in the Nash equilibria of this game.

4 Analysis

Main model: model with imperfect regulator; Benchmarks: laissez-faire and model with perfect regulator; Then we study welfare and compare two notions of welfare; types of equilibria; comparative statics;

4.1 Laissez-faire

Consider the model without the regulator first. In this case, no type of good is banned and therefore there is no need to exert effort to find loopholes. The action set for both firms is $\{H, L\} \times \mathbb{R}$ i.e. both firms simply choose any combination of type of good and price.

Remark 1. *The unique Nash equilibrium in the static game without the regulator is one in which both firms choose the strategy* (L, c_l).

Since the proof of this claim is trivial, we omit it here. Consumers don't care about the type of good (since they don't internalize the cost to others), and will purchase from the firm with the lowest price. This forces both firms to produce only the low type good in equilibrium since it has a lower marginal cost of production which allows them to charge lower prices. Thus, in the game without regulator, both firms will split the consumers (any split of consumers is permitted). Competition in this symmetric environment ensures that both firms receive zero profits, and the total payoff for all consumers (including social cost) is $(u - c_l - C)$.

4.2 Model with perfect regulator

Next, consider the case of a "perfect" regulator, i.e. a regulator who is able to craft a regulation that has no loophole. We call a regulator perfect if $p = 0$, i.e. there is no chance of any firm finding a loophole, irrespective of the effort exerted. In this case, there will be no effort exerted by the firms in equilibrium because they can never be successful. The perfect regulator observes that if it bans the L type good from being produced, then both firms will have to produce the H type good. In this case, competition between symmetric firms implies that both receive a payoff of zero. The total payoff for all consumers, including social cost, is $(u - c_h)$.

Clearly, from $C > (c_h - c_l)$ it follows that $u - c_h > u - c_l - C$. Thus, if the marginal social cost of producing the low type good is higher than the difference in the marginal costs of production of the two good types, then a perfect regulator maximizing consumer payoff will ban the low type good. Note that banning the low type good means that all customers will pay a higher price c_h. But the loss in utility is more than made up by the gain from reduced social costs.

4.3 Model with imperfect regulator

Next consider a version of our model where the regulator is imperfect, i.e. where $p > 0$. Let $C > c_h - c_l$. Suppose that the regulator bans the L type good and can choose any p in a fixed interval $[p_1, 1]$ without incurring a cost, where $p_1 > 0$. The lower bound to the regulator's choice set can be justified by the efforts of a lobby group trying to lower regulations, or through restrictions placed on the cost of regulation (it may be prohibitively expensive to draft and implement a perfect regulation with $p = 0$). We are interested in the question whether there are any conditions under which the regulator will not prefer the highest feasible regulation strength, i.e. the lowest p_1? Given that perfect regulations maximize consumer utility (because $C > c_h - c_l$) and choosing p_1 is not more costly than choosing any other lower level of regulation strength, it may seem optimal to choose p_1. Furthermore, note that if there was only one firm in the market, it is trivial to show that choosing p_1 is optimal for the regulator. However, we consider a model with two competing firms, and find conditions under which choosing the highest feasible regulation strength is *not* optimal.

Suppose the regulator has chosen a fixed regulation strength p. First, we show that optimal loophole finding effort can be a non-monotone function of regulation strength. Subsequently, we will use this result to show that welfare can actually decrease when regulation strength is increased. Our model admits four different types of equilibria. There is one equilibrium where the effort choice game has an internal solution while all others have a border solution[7]. From here on, unless mentioned otherwise, when we speak of an equilibrium, we will mean the former kind of equilibrium. We want to show a possibility result (that higher regulation strength can lead to higher loophole finding effort and lower welfare, and for that showing the existence of such an equilibrium is our objective.

Let $Y = c_h - c_l$. Thus, Y denotes the difference between the marginal cost of production of the two goods. With this, we can now formulate:

Proposition 1. *Let e_i^* be the equilibrium loophole finding effort exerted by firm i. Furthermore, let p, Y, M_2 be such that $3M_2 < Y$, then there exists a p'' such that if $p \in (p'', 1]$, then there exists an equilibrium in which $\frac{de_i^*}{dp} \leq 0 \; \forall i$.*

Proof. In the appendix. □

From this proposition and from expression 5—where it is clear that $\frac{de_1^*}{dp} > 0$ when p

[7]Border solutions allow for equilibria if the forms: one adversary puts in full effort while the other exerts an internal effort ($(e_1, e_2) = (1, \frac{pY(1-p)}{M_2})$, $(e_1, e_2) = (\frac{pY(1-p)}{M_1}, 1)$), and both adversaries exert full effort.

is small—it follows directly that optimal effort is non-monotone in regulation strength. But why should optimal effort increase when regulation strength has also increased? The intuition is that when p is reduced—the regulation strength is increased—there are two effects. One, the increase in regulation strength makes it less likely that the loophole finding effort is successful for any given effort level. This reduces returns to effort. Two, with decreasing p it becomes less likely that a firm's competitor finds the loophole, which is good if the firm finds the loophole itself. This increases returns from effort as a firm can only get non-zero profits when it finds the loophole, but the other firm does not. When the regulation strength is low enough (p is high enough), and the cost of loophole finding effort is not very high compared to the reward ($Y > 3M_2$), the latter effect dominates the former. On the other hand, when regulation strength is high enough ($p \ll 1$), a further increase in regulation strength lowers the returns to effort to such an extent that optimal loophole finding effort falls, thereby reducing the probability that the loophole will be found by any firm.

4.4 Welfare

While the above result is interesting, the regulator is more concerned with how welfare changes with increasing regulation strength. It is possible that the optimal loophole finding effort goes up when regulation strength is weakened, but the impact is not large enough to reduce welfare. In this section, we determine conditions under which consumer welfare goes down when the regulation strength is increased. Let welfare—the total consumer utility—be denoted by W_{IPR}, where IPR stands for imperfect regulator. When neither firm finds the loophole, total consumer utility is $(u - c_h)$. When one firm finds the loophole and the other firm does not, total consumer utility is $(u - c_h - C)$. Finally, when both firms find the loophole, consumer utility is $(u - c_l - C)$. Therefore, when the regulation strength is p and the corresponding equilibrium loophole finding efforts are given by e_1^*, e_2^*, we have:

$$\begin{aligned} W_{IPR} \;=\;& (1 - pe_1^*)(1 - pe_2^*)(u - c_h) + pe_1^*(1 - pe_2^*)(u - c_h - C) + pe_2^*(1 - pe_1^*)(u - c_h - C) \\ & + pe_1^* pe_2^*(u - c_l - C) \end{aligned}$$

Proposition 2. *Let $Y > M_2$. If $\frac{8M_1M_2}{M_1+M_2} < C < 1$, there exists a $\bar{Y}, \bar{p}$ such that if $Y \in (\bar{Y}, C)$ and $p \in (\bar{p}, 1)$, then $\frac{dW_{IPR}}{dp} > 0$*

Proof. In the appendix. □

The idea is that if the prior regulation strength is weak, and the reward for finding loopholes is high, then a strengthening of regulation causes a large enough increase in loophole finding effort to more than compensate for the increased regulation strength. This causes a reduction in welfare because the probability of the loophole being found is higher. The effect on welfare is not only via increasing the probability of social cost C (and the social cost of loophole discovery (C) is high enough because $C > \frac{8M_1M_2}{M_1+M_2}$), but also by increasing the price one would have to pay for the good (high probability of c_h as compared to c_l).

4.4.1 An alternative notion of welfare

In the analysis before, we have taken the regulator's utility function to mean the aggregate utility of all consumers. While this is a useful benchmark, we also consider other welfare criteria which are more common in economic analysis. Specifically, define welfare to be the sum of payoffs of all players, firms and consumers. This welfare function eliminates the effect of prices (since it cancels out across producers and consumers), and will focus on efficiency in terms of net surplus. This takes into account only three variables: the benefit to the consumers (sum of social cost and utility of consumption), cost of production of the firm(s), and the cost of loophole finding effort for the firms. Formally, the welfare is given by W where:

$$\begin{aligned} W &= (1-pe_1^*)(1-pe_2^*)(u-c_h) + pe_1^*(1-pe_2^*)(u-c_l-C) + pe_2^*(1-pe_1^*)(u-c_l-C) \\ &\quad + pe_1^*pe_2^*(u-c_l-C) - \frac{M_1e_1^{*2}}{2} - \frac{M_2e_2^{*2}}{2} \\ \Rightarrow W &= (u-c_l-C) + (1-pe_1^*)(1-pe_2^*)(C-Y) - \frac{M_1e_1^{*2}}{2} - \frac{M_2e_2^{*2}}{2}. \end{aligned}$$

Next, we specify the conditions under which this notion of welfare is decreasing in regulation strength:

Proposition 3. *Let M_2, C satisfy $M_2 \leq \frac{C^2}{3(C-M_1)}$, there exists a $\bar{p}$ and a $\bar{Y}$ such that if $p \in (\bar{p}, 1)$ and $Y \in (\bar{Y}, C)$, then $\frac{dW}{dp} > 0$.*

Proof. In the appendix. □

The intuition here is pretty simple. If Y is close enough to C then the payoff gains to the firms from finding a loophole are negated by the societal cost (C) of a firm finding a loophole. In this case, the change in welfare when the regulation becomes tighter is a function of the cost of change in loophole finding efforts. We already know from proposition 1 that if Y is large enough and the regulation strength is sufficiently weak, then the loophole finding effort can increase when regulation strength is increased.

5 Discussion

5.1 Firm heterogeneity

The analysis so far is for heterogenous firms, but we have not quantified how heterogeneity affects the equilibrium outcomes of the model. Specifically, we are interested in the question of how optimal loophole finding effort changes with increasing firm similarity, i.e. when M_1 becomes closer to M_2? In this case, firm 1 loses some of its comparative advantage.

$$\frac{de_1^*}{dM_1} = \frac{pYM_2(p^2Y - M_2)}{((p^2Y)^2 - M_1M_2)^2} \qquad \frac{de_1^*}{dM_2} = \frac{p^3Y^2[M_1 - p^2Y]}{((p^2Y)^2 - M_1M_2)^2}$$
$$\frac{de_2^*}{dM_1} = \frac{p^3Y^2[M_2 - p^2Y]}{((p^2Y)^2 - M_1M_2)^2} \qquad \frac{de_2^*}{dM_2} = \frac{pYM_1(p^2Y - M_1)}{((p^2Y)^2 - M_1M_2)^2}$$

It is trivial to see that in all but the internal efforts equilibrium, equilibrium effort e_i is weakly decreasing in M_i (and is unaffected by M_j). In the internal efforts equilibrium, when p is large enough, we have $\frac{de_1^*}{dM_1} \geq 0$ and $\frac{de_1^*}{dM_2} \leq 0$. Similarly, $\frac{de_2^*}{dM_1} \leq 0$ and $\frac{de_2^*}{dM_2} \geq 0$. Thus, when

p is large and the regulation is weak, a firm exerts less effort in equilibrium when its cost of loophole finding effort goes down, whereas its rival puts in more effort. So, keeping M_1 fixed, if we reduce M_2, we expect firm 1 to exert more effort in equilibrium and firm 2 to exert less. Does this increase the probability of finding a loophole? Let p^o denote the equilibrium probability that at least one firm finds a loophole. Then we can show that the probability that at least one firm finds a loophole increases when M_2 comes down. Thus, as the competing firms become more similar, the probability that one of them finds the loophole increases. We formalize this in the proposition below:

Proposition 4. *If* $Y > 2M_2$ *then* $\frac{dp^o}{dM_2} < 0$.

Proof. In the Appendix. □

In other words, the impact of an increase in regulation strength and decrease in firm heterogeneity are similar. The general rule of thumb that perfect competition (where there is no firm with an advantage) always improves welfare does not always work.

On the other hand, when p is very large, but Y is smaller and close to M_2, we can show that $\frac{dp^o}{dM_2} > 0$. When regulation strength is low and the reward from loophole discovery is not too high, an increase in firm similarity can reduce the probability of a loophole being found. Thus, whether increasing firm homogeneity increases or decreases welfare crucially depends upon the size of the reward for finding loopholes.

5.2 Lobbying

Hitherto, we have taken the regulation strength p as given. However, in several environments, the firms in an industry not only try to bypass regulation with loophole finding effort, they actively try to influence the regulation strength as well with lobbying effort. In this section, we modify our baseline model to allow for firms to exert lobbying effort e_i^L to influence p. The modification to game structure is simple: We introduce a period 0 when both firms exert lobbying effort ($\in [0,1]$). The regulation strength chosen by the regulator is assumed to be a function of the total lobbying effort. In particular, if firm i exerts effort level e_i^L, then the regulation strength is assumed to be $\frac{e_1^L+e_2^L}{2}$. Thus, the regulation strength is assumed to be decreasing in total lobbying effort, which is a natural assumption. The more lobbying effort an industry as a whole puts in collectively, the weaker we expect the regulations governing this industry to be.

For ease of exposition, we restrict ourselves to the simple and tractable case of symmetric firms. Thus, for this subsection $M_1 = M_2 = M$. In this case, optimal loophole effort in the symmetric equilibrium, given regulation strength p, is given by $e^* = \frac{p}{p^2+\frac{M}{Y}}$. Before any further analysis, we make a simple assumption which makes sure that too much lobbying effort is undesirable. In particular, we assume that if the two firms put in full lobbying effort then the welfare of the society is lower than what it would be in an environment without regulations. When firms put in full lobbying effort, the regulation strength is at its weakest i.e. $p = 1$, because $p = \frac{e_1^L+e_2^L}{2}$, and optimal effort is given as $e^* = \frac{p}{p^2+\frac{M}{Y}} = \frac{Y}{Y+M}$. This assumption is formalized by:

$$\left[(1-p^*e^*(p^*))^2\right](u-c_h)+2\left[pe^*(p^*)(1-p^*e^*(p^*))\right](u-c_h-C)$$
$$+\left[(p^*e^*(p^*))^2\right](u-c_l-C) < u-c_l-C.$$

which is equivalent to:

$$C < Y + \frac{2Y^2}{M},$$

or in words: Welfare when lobbying effort is maximum is lower than welfare when there is no regulation. We assume $C < Y + \frac{2Y^2}{M}$ from here on.

Next, we determine the regulation strength in a symmetric equilibrium and obtain the following Lemma:

Lemma 1. *The symmetric equilibrium regulation strength is given by* $p^* = \sqrt{\frac{M}{Y}}$

Proof. In the Appendix. □

There are two things to note immediately. One, the firms prefer to have some regulation. This is obvious, since without any regulation, the firms necessarily get zero profits because they are symmetric. However, the introduction of imperfect regulations allows for the possibility of any firm separating from the other by finding a loophole and earning positive profits. The other interesting aspect of this result is that firms do not want the weakest possible regulation strength. Notice that lobbying effort is costless. And yet, firms choose an equilibrium effort level below the maximum effort level of 1. Firms face a tradeoff when choosing lobbying effort. If they put in a lot of effort then the resulting regulation will be weak which will increase the probability of any firm finding the loophole. However, a weak regulation will also increase the probability of the other firm finding the loophole. The profits go to zero if both firms are successful in finding the loophole as they become completely symmetric. On the other hand, if they put in very little lobbying effort, the regulation is of very high strength which would make it very difficult for any firm to find a loophole. Once again, if neither firm finds a loophole, they will be symmetric and competition will drive their payoffs to zero. Thus, the optimal lobbying effort is interior.

Next, we show that the society may actually benefit from imperfect regulations even when they are biased due to the lobbying effort. Formally:

Proposition 5. *If $Y + \frac{2Y^2}{M} > C > 3Y$ then the welfare under the symmetric equilibrium under the lobbying model is greater than the symmetric equilibrium when there in no regulation.*

Proof.

$$\begin{aligned}
p^o &= 1-(1-pe_1^*)(1-pe_2^*) \\
&\Rightarrow \frac{dp^o}{dM_2} < 0 \Leftrightarrow \frac{de_2^*}{dM_2}(1-pe_1^*) < -\left[\frac{de_1^*}{dM_2}(1-pe_2^*)\right] \\
&\Leftrightarrow M_2(p^2Y - M_1) - p^2Y(p^2Y - M_2) < 0 \\
&\Leftrightarrow Y^2 + M_1M_2 - 2YM_2 > 0 \text{ when } p \approx 1.
\end{aligned}$$

Note that the LHS is increasing in Y (because $Y > M_2$) and the LHS is positive at $Y = 2M_2$. Thus, when p is large enough and $Y > 2M_2$, an increase in competition increases the probability of one of the firms finding a loophole. □

The intuitive idea here is that there is a trade-off. With no regulation, society has to bear the entire social cost C of the low type good, but the competition means the price of the product is low (c_l). On the other hand, even though regulation strength is determined by lobbying effort, it is not in the interest of the firms to make regulation very weak, which would reduce welfare than under no regulation by our assumption $C < Y + \frac{2Y^2}{M}$. This means that there is high enough probability that no firm finds the loophole and there is no social cost C, but the prices would be higher if any firm finds the loophole. When $C > 3Y$, the lower probability of suffering the social cost under the lobbying with biased regulation regime (which stems from their own self-interested desire to keep regulation from becoming too lax) compensates for the higher prices which may arise if a firm finds a loophole.

5.3 Increasing Y

An increase in Y increases the reward for finding the loophole for the firms. The change in equilibrium effort as Y changes for the internal efforts equilibrium is given by:

$$\begin{aligned}\frac{de_i}{dY} &= \frac{((p^2Y)^2 - M_1M_2)(2p^3Y - M_jp) - (p^3Y^2 - pYM_j)2p^4Y}{((p^2Y)^2 - M_1M_2)^2} \\ &= \frac{pM_j((p^2Y)^2 - 2p^2YM_i + M_1M_2)}{((p^2Y)^2 - M_1M_2)^2} \\ &= \frac{pM_j((p^2Y - M_i)^2 + M_i(M_j - M_i))}{((p^2Y)^2 - M_1M_2)^2}\end{aligned}$$

Clearly $\frac{de_1}{dY} \geq 0$ since $M_2 - M_1 > 0$. The sign of $\frac{de_2}{dY}$ is also positive when p is high enough. Note that the internal efforts equilibrium always exists when p is large. When both efforts are not internal, equilibrium efforts are clearly always weakly increasing in Y (since effort is either constant 1 or $\frac{pY(1-p)}{M_i}$). Thus, an increase in the reward for finding loopholes leads to high equilibrium efforts to do so. This, in turn, reduces welfare.

6 Conclusion

Our paper presents a very general model of regulations with competitive firms. We show that increasing the strength of regulations can increase the incentives to find loopholes in the regulation, and this is a new channel through which strengthening regulations may hurt welfare. In light of the fact that framing, amending and implementing regulations is costly, it is imperative that we understand exactly when we should be strengthening regulations. Politicians often want to make regulations stronger just to score brownie points with their constituents. We show that such acts may actually hurt their constituents. We go on to show that though lobby groups have the welfare of the firms in mind (and not the consumers), it is in their interest to not make regulations too lax. In fact, under some conditions, a regime with no regulation is actually worse for the consumers than one where regulation strength is determined by lobbying effort.

There are natural extensions of our paper which would be very interesting to study. For example, it is clear from our model that regulations with loopholes give competing symmetric firms a chance to earn higher profits by finding a loophole. What is the impact of this on long-run competition in the market? If a firm finds a loophole, then it corners the market till the time the other firm also finds the loophole. Surely, this kind of competitive advantage can cause the other firm to exit the market if it does not find the loophole quickly, thereby making the market less competitive. Additionally, it would be interesting to explore whether it is optimal for the regulator to make regulations stronger by decreasing p (making it harder for firms to find loopholes), or to increase monitoring of regulation compliance given a fixed level of regulation strength. We hope to study such questions in the future.

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A Appendix

A.1 Proofs

A.1.1 Proof of Proposition 1

Proof. We will solve the game by backward induction. Consider the subgames beginning after we know the outcome of the loophole finding efforts. If both firms find a loophole or if neither finds a loophole then competition between symmetric firms will push their payoffs to zero (they will produce at marginal cost in the unique Nash equilibrium). Let us look at the subgame starting at the node where one firm finds a loophole, but the other does not. Without loss of generality, suppose firm 1 finds a loophole and firm 2 does not. The strategy of a firm in this subgame simply consists of a tuple describing the good to produce and the price for this good. Consider the following strategies in this subgame:

$$\text{Firm 1} - (L, c_h)$$

$$\text{Firm 2} - (H, c_h)$$

Lemma 2. *The above strategies constitute a Nash equilibrium of the subgame beginning after firm 1 finds a loophole but firm 2 does not. All customers buy from firm 1. Moreover, all Nash equilibria of the subgame are payoff equivalent to this one.*

Proof. It is easy to show that the above is a Nash equilibrium of the subgame. Firm 1 chooses the good with the lower cost of production, and the price is chosen to maximize the payoff. Clearly, at any price strictly lower than c_h, firm 1 gets the entire market (firm 2 has not found the loophole, so its cost of production is c_h, thereby making the equilibrium price charged by firm 2 at least as much) and this price is individually rational for firm 1 as long as it is greater than or equal to c_l. The payoff is maximized at the highest possible price - c_h. Firm 2 must produce only good H, and can choose any price greater than or equal to c_h in equilibria (to get non-negative payoffs). However, this price choice will not change the payoff for any player as all customers will buy from firm 1 since it offers lower prices. Note that while the customers are indifferent between the two firms when they both charge the price c_h, there is no equilibrium in which some consumers buy from firm 2. This is because if this were the case, then firm 1 could offer a slightly lower price and attract these customers. □

Thus, we have the following equilibrium payoffs in the subgame beginning after the effort outcome stage. The first argument represents firm 1's payoff and the second argument represents firm 2's payoff:

Both firms find loophole – $(0,0)$

Firm 1 finds loophole, Firm 2 does not find loophole – $(c_h - c_l, 0) = (Y, 0)$

Firm 1 does not find loophole, Firm 2 finds loophole – $(0, c_h - c_l) = (0, Y)$

Neither firm find loophole – $(0,0)$

Now, we solve the optimization problem at the beginning of the game. Both firms need to choose an effort. We know the equilibrium payoffs after the effort outcome stage. Suppose firm 1 puts in effort e_1^* in equilibrium and firm 2 puts in effort e_2^*. Firm 1's optimization problem:

$$\max_{x \in [0,1]} (xp)(1 - e_2^* p)Y - \frac{M_1 x^2}{2} \tag{1}$$

$$\Rightarrow x = e_1^* = \frac{p(1 - e_2^* p)Y}{M_1} \tag{2}$$

Similarly, we get $e_2^* = \frac{p(1-e_1^* p)Y}{M_2}$. Solving for e_1^* and e_2^*, we get:

$$e_1^* = \frac{pY(p^2 Y - M_2)}{(p^2 Y)^2 - M_1 M_2} \tag{3}$$

$$e_2^* = \frac{pY(p^2 Y - M_1)}{(p^2 Y)^2 - M_1 M_2} \tag{4}$$

The above is for parameter values p, Y, M_1, M_2 such that $e_i^* \in (0,1)$. Border solutions allow for two more types of equilibria: one adversary puts in full effort while the other exerts an internal effort $((e_1,e_2)=(1,\frac{pY(1-p)}{M_2}),\ (e_1,e_2)=(\frac{pY(1-p)}{M_1},1))$, and both adversaries exert full effort. For all our analysis we will focus on the internal efforts equilibrium. We want to show a possibility result (that higher regualtion strength can lead to higher loophole finding effort and lower welfare, and for that showing the existence of such an equilibrium is our objective.

We know that $Y > M_2$ (the proposition demands $Y > 3M_2$), and it is clear that when p is high enough, then e_i^* is between zero and one i.e. the internal efforts equilibrium exists. Additionally, it is easy to check that if p is high then for any $i,j \in \{1,2\}$ where $i \neq j$, e_i^* is a higher payoff response to e_j^* as compared to the bounds $(e=1,\ e=0)$. So, e_1^*, e_2^* represent equilibrium efforts. Note that $e_1^* < e_2^*$, that is, the firm with the lower cost of loophole finding effort exerts lower effort in equilibrium. The intuition for this is that in equilibrium, the marginal cost of effort is balanced with the marginal benefit from effort. Since the firm with the lower M_i has a low marginal cost of effort, it must also get a low marginal benefit from effort in equilibrium. This is achieved when the firm with the lower M_i exerts sufficiently lower effort than the other firm. Furthermore, in this equilibrium:

$$\frac{de_1^*}{dp} \leq 0$$

$$\Leftrightarrow ((p^2Y)^2 - M_1M_2)(3p^2Y^2 - YM_2) - 4p^4Y^3(p^2Y - M_2) \leq 0$$

$$\Leftrightarrow M_1M_2[M_2 - 3p^2Y] \leq (p^2Y)^2[p^2Y - 3M_2] \tag{5}$$

$$\Leftrightarrow M_1M_2[M_2 - 3Y] \leq Y^2[Y - 3M_2]\ ;\ when\ p \approx 1 \tag{6}$$

Now, the LHS is negative (since $Y > M_2$), so it is easy to check that 6 holds if $Y \geq 3M_2$ and $p \approx 1$. So we have that if $3M_2 < Y$ and p high enough, then $\frac{de_1}{dp} \leq 0$. Similarly, we can show that if $3M_1 < Y$ and p high enough, then $\frac{de_2}{dp} \leq 0$. Since $M_1 < M_2$, we can pick $3M_2 < Y$, and p large enough to get that $\frac{de_i}{dp} \leq 0\ \forall i$. □

A.1.2 Proof of proposition 2

Proof. We will show that the result holds when $Y \to C$ and $p \to 1$. The proposition would hold by continuity in Y, p. We know that:

$$W_{IPR} = (1-pe_1^*)(1-pe_2^*)(u-c_h) + pe_1^*(1-pe_2^*)(u-c_h-C) + pe_2^*(1-pe_1^*)(u-c_h-C)$$

$$+ pe_1^* pe_2^*(u-c_l-C)$$

$$\Leftrightarrow W_{IPR} = (u-c_h) + e_1^* e_2^* p^2(Y+C) - pC[e_1^*+e_2^*]$$

substituting the optimal values of e_1, e_2 from equations 3,4 and simplifying with $x = p^2$:

$$W_{IPR} = (u-c_h) + \frac{(xY)^2(xY-M_1)(xY-M_2)(Y+C)}{((xY)^2-M_1M_2)^2} - \frac{xYC(2xY-M_1-M_2)}{(xY)^2-M_1M_2}$$

taking $Y \to C$, we get:

$$W_{IPR} = (u-c_h) + C^2[\frac{4x^2CM_1M_2-(M_1+M_2)(x^3C^2+xM_1M_2)}{(x^2C^2-M_1M_2)^2}]$$

and

$$\frac{dW_{IPR}}{dx} \geq 0 \Leftrightarrow (x^2C^2-M_1M_2)^2[8xCM_1M_2-(M_1+M_2)(3x^2C^2+M_1M_2)]$$

$$-4xC^2(x^2C^2-M_1M_2)[4x^2CM_1M_2-(M_1+M_2)(x^3C^2+xM_1M_2)] \geq 0$$

when $x \to 1$, we get:

$$\frac{dW_{IPR}}{dx} \geq 0 \Leftrightarrow [C^2-M_1M_2][(M_1+M_2)(C^4+6C^2M_1M_2+M_1^2M_2^2)-8CM_1M_2(C^2+M_1M_2)] \geq 0$$

$Y > M_2$ and we know that $C \geq Y$, then $C^2 - M_1M_2 \geq 0$, and:

$$\frac{dW_{IPR}}{dx} \geq 0 \Leftrightarrow C^3[C(M_1+M_2)-8M_1M_2] + M_1M_2[C(6C(M_1+M_2)-8M_1M_2)+M_1M_2(M_1+M_2)] \geq 0$$

Clearly, the above holds when $C > \frac{8M_1M_2}{M_1+M_2}$. Thus, a sufficient condition for welfare to go down with increasing regulation strength is that the initial regulation strength is low ($p \approx 1$), Y high, and the cost of loophole discovery for the society high enough $C > \frac{8M_1M_2}{M_1+M_2}$). Note that $C < 1$ is required to make the assumption $Y \to C$ possible, since $Y = c_h - c_l$ is always less than 1. Also note that the condition $C > \frac{8M_1M_2}{M_1+M_2}$ is easily satisfied as one of the firms becomes sufficiently better than the other firm at loophole finding (as $M_1 \to 0$). □

A.1.3 Proof of Proposition 3

Proof. We have that:

$$W = (u - c_l - C) + (1 - pe_1^*)(1 - pe_2^*)(C - Y) - \frac{M_1 e_1^{*2}}{2} - \frac{M_2 e_2^{*2}}{2}$$

$$\Rightarrow \frac{dW}{dp} \geq 0 \Leftrightarrow \frac{d}{dp}((1 - pe_1^*)(1 - pe_2^*)(C - Y)) - \frac{d}{dp}(\frac{M_1 e_1^{*2}}{2}) - \frac{d}{dp}(\frac{M_2 e_2^{*2}}{2}) \geq 0 \tag{7}$$

Now, let us look at each part of this expression one by one:

$$\frac{d}{dp}((1 - pe_1^*)(1 - pe_2^*)(C - Y))$$
$$= \frac{2M_1 M_2 pY(C - Y)((p^2 Y)^2 - M_1 M_2)[((p^2 Y)^2 - M_1 M_2)(2p^2 Y - M_1 - M_2) - 4p^2 Y(p^2 Y - M_1)(p^2 Y - M_2)]}{((p^2 Y)^2 - M_1 M_2)^4} \tag{8}$$

$$as\ Y \to C,\ \frac{d}{dp}(\frac{d}{dp}((1 - pe_1^*)(1 - pe_2^*)(C - Y))) \to 0 \tag{9}$$

&

$$\frac{d}{dp}(\frac{M_1 e_1^{*2}}{2})$$
$$= \frac{M_1 p^3 Y^3 (p^2 Y - M_2)[-(p^2 Y)^2 - Y M_2 - 3M_1 M_2 + 4p^2 Y M_2]}{((p^2 Y)^2 - M_1 M_2)^3} \tag{10}$$

$$as\ p \to 1$$
$$= \frac{M_1 Y^3 (Y - M_2)[-Y^2 - Y M_2 - 3M_1 M_2 + 4Y M_2]}{(Y^2 - M_1 M_2)^3}$$
$$as\ Y \to C,\ \frac{d}{dp}(\frac{M_1 e_1^{*2}}{2}) \leq 0\ if\ M_2 \leq \frac{C^2}{3(C - M_1)} \tag{11}$$

&

$$\frac{d}{dp}(\frac{M_2 e_2^{*2}}{2})$$
$$= \frac{M_2 p^3 Y^3 (p^2 Y - M_1)[-(p^2 Y)^2 - Y M_1 - 3M_1 M_2 + 4p^2 Y M_1]}{((p^2 Y)^2 - M_1 M_2)^3} \tag{12}$$
$$as\ p \to 1$$
$$= \frac{M_2 Y^3 (Y - M_1)[-Y^2 - Y M_1 - 3M_1 M_2 + 4Y M_1]}{(Y^2 - M_1 M_2)^3}$$
$$as\ Y \to C,\ \frac{d}{dp}(\frac{M_2 e_2^{*2}}{2}) \leq 0\ if\ M_1 \leq \frac{C^2}{3(C - M_2)} \tag{13}$$

Since $M_1 > M_2$, $M_2 \leq \frac{C^2}{3(C-M_1)} \implies M_1 \leq \frac{C^2}{3(C-M_2)}$. Therefore, from 9,11 and 13, we have that there exists a $\bar{Y}, \bar{p}$ such that if $M_2 \leq \frac{C^2}{3(C-M_1)}$ and $p \in (\bar{p}, 1)$, $Y \in (\bar{Y}, C)$ then $\frac{dW}{dp} \geq 0$. □

A.1.4 Proof of Proposition 5

Proof of Lemma 1

Proof. If the regulation strength is p, the ex-ante expected payoff of each firm is given by:

$$pe^*(1 - pe^*)Y - \frac{M(e^*)^2}{2}$$
$$= \frac{p^2 Y}{p^2 + \frac{M}{Y}}(1 - \frac{p^2}{p^2 + \frac{M}{Y}}) - \frac{M}{2}(\frac{p}{p^2 + \frac{M}{Y}})^2$$
$$= \frac{Mp^2}{2(p^2 + \frac{M}{Y})^2}$$

Now, in stage 0, when the firms choose the lobbying effort, if firm 2 chooses a lobbying effort of e_2^L and firm 1 chooses a lobbying effort of e_1^L, then the resultant regulation strength is given by $\frac{e_1^L + e_2^L}{2}$. Therefore, the maximization problem for firm 1 is:

$$\max_{x \in [0,1]} \frac{(\frac{x + e_2^L}{2})^2 M}{2((\frac{x + e_2^L}{2})^2 + \frac{M}{Y}^2)}$$

Taking first order conditions and substituting $e_2^L = x$ to obtain the strategy choices for a symmetric equilibrium gives us optimal lobbying effort as $e_1^L = e_2^L = \sqrt{\frac{M}{Y}}$. This implies an equilibrium regulation strength of $\frac{\sqrt{\frac{M}{Y}}+\sqrt{\frac{M}{Y}}}{2} = \sqrt{\frac{M}{Y}} = p^*$. This gives each firm an expected ex-ante payoff of $(\frac{Y}{8})$ in equilibrium. □

Proof of Proposition 5

Proof. The welfare level when there is no regulation is $u - c_l - C$. When we allow for lobbying, let the payoff for the society in the symmetric equilibrium be denoted by W_{sym_L} where:

$$W_{sym_L} = [(1-p^*e^*(p^*))^2](u-c_h) + 2[pe^*(p^*)(1-p^*e^*(p^*))](u-c_h-C) + [(p^*e^*(p^*))^2](u-c_l-C)$$

$$\Leftrightarrow W_{sym_L} = (u-c_h-C) + (1-p^*e^*(p^*))^2 C + (p^*e^*(p^*))^2 Y$$

since the symmetric effort level $e^*(p) = \frac{p}{p^2+\frac{M}{Y}}$:

$$\Leftrightarrow W_{sym_L} = (u-c_h-C) + (\frac{\frac{M}{Y}}{\frac{M}{Y}+(p^*)^2})^2 C + (\frac{(p^*)^2}{\frac{M}{Y}+(p^*)^2})^2 Y$$

In the symmetric equilibrium under lobbying, the equilibrium $p^* = \sqrt{\frac{M}{Y}}$, so:

$$\Leftrightarrow W_{sym_L} = u - \frac{3c_h}{4} - \frac{c_l}{4} - \frac{3C}{4} \tag{14}$$

Now, the welfare from the symmetric equilibrium with lobbying effort is higher if:

$$W_{sym_L} > u - c_l - C$$

$$\Leftrightarrow u - \frac{3c_h}{4} - \frac{c_l}{4} - \frac{3C}{4} > u - c_l - C$$

$$\Leftrightarrow C > 3(c_h - c_l)\ i.e.\ C > 3Y$$

This is the condition required by the proposition. □