Optimal task design: production and learning with moral hazard and adverse selection[1]

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January 11, 2023

This paper studies a principal-agent problem with learning before production. The learning stage is modeled as strategic experimentation with both dynamic moral hazard and adverse selection. We fully characterize effort off the equilibrium path in a mixed model to show that moral hazard requires the principal to reward success in learning, but adverse selection may induce the principal to reward failure. We find the principal uses the timing of failure as a screening instrument despite the presence of moral hazard. Therefore, both success and failure are rewarded with different payments and specific timing in the optimal contract. We also study whether the principal should hire the same agent for both the learning and production stages (integration), or different agents (separation). While separation is optimal under pure moral hazard, we show that adverse selection can make integration optimal. Having the same agent working on both stages enables the principal to use the adverse selection rent to address dynamic moral hazard. If adverse selection is severe, yielding a large rent, the principal can satisfy the moral hazard constraints by spreading the adverse selection rent over the duration of experimentation.

Keywords: Strategic Experimentation, Moral hazard, Adverse selection, Outsourcing, Integration and Separation.

JEL classification: D83, D86.

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1. Introduction

Many important tasks involve two stages: a preliminary stage of learning or experimentation before a production stage. For instance, a principal, who is hiring an agent to perform a new project, may want the agent to learn its profitability before starting production. Should the principal hire the same agent for both the learning and production stages (integration), or a different agent for each stage (separation)? This question has been studied in the endogenous information gathering literature, highlighting the interdependence between the private learning and production stages with an emphasis on the endogenous information rent in the production stage. This literature mostly ignores the dynamic aspect of learning by relying on a simple static learning stage.[2] The recent literature on strategic experimentation allows a rich and tractable context for dynamic learning, but the focus has been on motivating an agent to learn or experiment. Thus, the interdependence between the dynamic learning and production stages has been largely ignored.

We find that if there is only moral hazard at the learning stage, as is typically assumed in the strategic experimentation literature, separating the learning and production stages is optimal from an incentives point of view. If, however, the learning stage involves both adverse selection and moral hazard, then integrating the two stages may become optimal.[3] We show that the principal can use the adverse selection rent to address the dynamic moral hazard problem during learning. This requires solving a model with both adverse selection and moral hazard in a strategic experimentation setting, which can be challenging, as noted by Halac, Kartik, and Liu (2016).[4] One of our contributions is to offer a tractable mixed model and to characterize off-the-equilibrium path effort. Adverse selection in learning introduces a common value problem as the agent's type directly appears in the principal's objective function.[5] Then both upward and downward incentive constraints can be binding, leading the principal to reward failure to screen types. This result persists even in the presence of moral hazard in the learning stage, even though it increases the cost of inducing effort.

[2] See related literature section below.
[3] For instance, most large firms have in-house R&D departments.
[4] In particular, characterizing off the equilibrium path effort can be difficult.
[5] See, e.g., Laffont and Martimort (2002).

We introduce a principal-agent model where a production stage is preceded by a multi-period learning stage modeled as strategic experimentation.[6] Each period of experimentation is subject to shirking by an agent trying to learn the cost of production. Success in experimentation is assumed to take the form of uncovering "good news", i.e., the agent finds out whether production cost is low. Therefore, failure to uncover good news increases the expected cost of production. As in standard models of strategic experimentation, unobserved shirking makes the principal more pessimistic than the agent about the true cost leading to a moral hazard rent during experimentation.[7] Unlike in standard models of experimentation, we capture the impact of possibly different beliefs between the principal and agent after failure. We next explain the impact on the production stage of asymmetric beliefs after failure.

The possibility of shirking during experimentation implies a second moral hazard rent at the production stage as the principal, unaware of the shirking, overestimates the cost of production.[8] This second rent is novel and does not appear in models without a production stage or if we separate learning and production. Thus, we find that separation is optimal in pure moral hazard models of experimentation.

The learning stage also features adverse selection along with moral hazard. The agent has private information about the ex-ante probability that the cost is low, i.e., the probability that experimentation will succeed. Thus, updating beliefs after failure to uncover good news in each period depends on both the agent's private information and his effort.

Because of the common value problem mentioned above, both low- and high-cost agents may have incentives to misreport, leading the principal to reward failure.[9] The low-cost agent's incentive to overstate cost is resolved as usual by rewarding success, as the low-cost agent is more likely to succeed. The high-cost agent's incentive to understate cost is resolved by rewarding failure, as the high-cost agent is more likely to fail. Thus, rewarding this type after failure is an optimal screening instrument when both incentive compatibility constraints are

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[6] The exponential bandit model has been widely used as a canonical model of learning: see Bolton and Harris (1999) or Bergemann and Välimäki (2008).
[7] See, e.g., Bergemann and Hege (1998), Bergemann and Välimäki (2008), and Horner and Samuelson (2013).
[8] This rent *at the production stage* is similar to what we find in a standard procurement model. See, e.g., Lewis T. and Sappington D. (1997).
[9] For static models with adverse selection and moral hazard, see, e.g., Ollier and Thomas (2013), Chakraborty et al. (2021), Gottlieb and Moreira (2022), or Rodivilov et al. (2022).

binding. More precisely, the principal optimally pays the high-cost agent only if he fails in every period as the relative likelihood of failure between types is monotonic over time.

It may seem surprising that rewarding failure remains optimal in the presence of moral hazard. If the payment after failure is used as a screening instrument, the high-cost agent can guarantee himself rent by shirking in every period. Failing over the entire experimentation stage then becomes a profitable outside option for the high-cost agent, and it increases the cost of inducing the agent to work in each period. To prevent shirking, the principal must correspondingly increase the reward after success. For instance, consider the last period of learning. By shirking, the high-cost agent can guarantee himself the reward after failure. To induce him to work, the principal must provide a higher reward after success than after failure.[10] The high-cost agent is rewarded both after success and failure but gets a higher reward after success. Thus, having to reward failure is quite costly, and we find that it is optimal only when both incentive compatibility constraints are binding.

When the adverse selection rent is large, the principal can satisfy the moral hazard constraints 'for free' by spreading this rent across time. This provides the intuition for the optimality of integration. A large adverse selection rent must be paid under both integration and separation. But, under integration, the principal can choose to spread the adverse selection rent in every period of learning, satisfying the moral hazard constraints.[11] Under separation, the principal saves the aforementioned "second" moral hazard rent at the production stage. However, with two separate agents, she cannot use the adverse selection rent in the production phase to satisfy the standard moral hazard rent in the learning phase. Thus, there is a tradeoff, and the outcome depends on the relative size of the adverse selection and moral hazard problems.

Next, we briefly outline two examples to illustrate our insight regarding the optimality of separating learning and production. Consider first the case of drug approval trials, where a pharmaceutical company (principal) typically outsources to a clinical research organization (a separate agent) the clinical trials to demonstrate the effectiveness of a new drug. Moral hazard is

[10] In our dynamic model, each previous periods' moral hazard rewards must be increased to deter the agent from delaying success to the last period.
[11] Technically, it is because the relative probability of success over time is independent of type.

the more serious issue.[12] Adverse selection is less relevant as much information about the prospective efficacy of the drug is in the public domain.

In contrast, consider the case of a surgeon who must decide on an appropriate surgical procedure, and relies on their assessment of the prospect for success along with their diagnosis based on medical history and a series of diagnostic tests. The diagnosis is a dynamic learning process while the prospect for success depends largely on a surgeon's prior experience and ability. Adverse selection is likely to be a major issue depending on each surgeon's expertise and experience. Moral hazard is less of a concern given protocols and regulations for healthcare activities required by the health insurance company or HMO.[13] Integrating the two stages (a series of diagnostic tests and surgery) would be optimal as seems to be the observed practice.

Related Literature. Our paper builds on four strands of the literature. First, it is related to the extensive literature on the integration and separation of learning and production between agents. See, for instance, Lewis and Sappington (1997), Schmitz (2005), Khalil et al. (2006), Iossa and Martimort (2012), Li et al. (2015), Schmitz (2021), Hoppe and Schmitz (2013 and 2021). Unlike those papers, we model learning as a dynamic process with both moral hazard and adverse selection and show that their relative importance determines the optimal organization structure.

This paper is also related to the literature on endogenous information gathering before production. The standard model is static, where an agent privately exerts effort that increases the precision of the signal of the state (relevant for a production decision).[14] By modeling effort as experimentation, we contribute to this literature by introducing the dynamics of learning, and especially the possibility of asymmetric learning by different types of agents. In our model, the principal endogenously determines the degree of asymmetric information in the production stage by choosing the length of experimentation. Unlike the rest of the literature, we show that the principal may find it optimal to over-experiment to screen the types.

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[12] There are multiple examples of clinical research organizations shirking, for example, by creating fake patient profiles (see Lindblad et al. (2014), Anand et al. (2012), Pogue et al. (2013) and references therein).

[13] In addition, healthcare practitioners are required by law to record patient medical histories and retain detailed case histories. There is also little room for skipping tests or altering results since this behavior might be simply illegal and a surgeon might be subject to prosecution. Surgeons are of course also bound by the Hippocratic Oath.

[14] For early papers see Crémer and Khalil (1992), Lewis and Sappington (1997), and Crémer, Khalil, and Rochet (1998). For recent papers, see citations in Krähmer and Strausz (2011), Iossa and Martimort (2015), Rodivilov (2021), Downs (2021), Schmitz (2022), and Häfner and Taylor (2022).

In addition, our paper is related to the active literature on contracting for experimentation following Bergemann and Hege (1998). Most of the literature considers either moral hazard or adverse selection models in isolation.[15] Among the few exceptions that introduce both moral hazard and adverse selection are Gerardi and Maestri (2012), Guo (2016), and Halac et al. (2016). Unlike all those papers, we also consider a production stage and show how the rent in one stage echoes into the other stage.[16] In the pure moral hazard case, we find a new rent at the production stage, which implies that separation is optimal. This result justifies the standard assumption in the strategic experimentation literature which considers only the experimentation stage in isolation assuming pure moral hazard. Adding adverse selection in experimentation, we show that integration can be optimal. While the standard result in the literature is to reward success in the experimentation stage to address moral hazard, we find that adverse selection in experimentation may make rewarding failure in the experimentation stage optimal even in the presence of moral hazard. Moreover, the dynamic nature of the learning process allows the principal to use the timing of payments as a screening instrument.

Finally, we also contribute to the literature on the power of incentives for innovation. Manso (2011) shows that an optimal incentive scheme may exhibit reward for early failure for a risk averse agent. Benabou and Tirole (2003) show that using high-powered incentives may be detrimental to intrinsic motivation. In a laboratory experiment, Ederer and Manso (2013) find that a combination of rewards for both failure and success can be effective in incentivizing innovation. Sadler (2021) illustrates that high-powered incentives may discourage creativity. We contribute to this literature by showing theoretically that the coexistence of low- and high-powered incentive schemes can be optimal to mitigate the effect of adverse selection when failures to innovate are informative for the subsequent production decision.

2. The Model and the First Best

A principal hires an agent to implement a project. The cost of the project, c, is initially unknown to both the principal and the agent, but it is common knowledge that the cost can be low, $\underline{c}$, with probability $\beta_0 \in (0,1)$, or high, $\overline{c}$, with probability $1-\beta_0$. Both the principal and

[15] See, e.g., Horner and Samuelson (2013), Sadler (2021), Escobar and Zhang (2021), Rodivilov (2022), and Moroni (2022) for models of pure moral hazard, and Gomes et al. (2016) and Khalil et al. (2020) for models of adverse selection only.

[16] Except for Khalil et al. (2020) who introduce a production stage but with adverse selection only.

the agent are risk neutral, and, for simplicity, we assume that their discount factor is one. Before the actual production occurs, there is learning that we model as a standard *experimentation stage*, when the agent gathers information regarding the production cost.[17] In the *production stage*, the agent produces based on what is learned about cost during the experimentation stage.

2.1. The Experimentation Stage

The length of the experimentation T is chosen by the principal. In each period $t \in \{1, 2, \dots, T\}$, the principal must address a moral hazard problem. The agent privately chooses whether to perform an experiment, i.e., "work," $e_t = 1$, or not to experiment, "shirk," $e_t = 0$. Experimentation at t costs γe_t to the agent, where $\gamma > 0$.

The principal must also address an adverse selection problem. We assume that the agent is privately informed about the probability that the cost is low, which we refer to as the agent's type. Thus, we define the initial belief that the cost is low with the agent's type as the superscript $\theta \in \{H, L\}$:

$$\beta_0^\theta = Pr\left(c = \underline{c} \middle| \theta\right),$$

where $0 < \beta_0^L < \beta_0^H < 1$.[18] In other words, a high type is more optimistic that the cost is low before experimentation starts, i.e., the *high type has a relatively lower expected cost than the low type*. We assume that the agent is a high type ($\theta = H$) with probability $\nu \in (0,1)$ and a low type ($\theta = L$) with probability $(1 - \nu)$.

We assume that information gathering takes the form of looking for good news. If the experimentation reveals that the cost is low (*good news*), we will say that the experimentation was successful. If the cost is actually low and the agent works, success occurs with probability $0 < \lambda < 1$. Success is publicly observable. Once success occurs, the experimentation stage stops, and production takes place based on $c = \underline{c}$. Success cannot occur if the cost is high, or if the agent shirks. Thus, it is optimal to induce $e_t = 1$ in every period of the experimentation stage.

If the agent fails to learn that the cost is low in a period t, we say that experimentation failed in that period. Then, experimentation resumes if $t < T$, but both the agent and the

[17] See, e.g., Halac et al. (2016).

[18] If $\beta_0^\theta = 1$ for some type, there is no learning for that type.

principal become more pessimistic about the likelihood of the cost being low. Production takes place after the experimentation stage ends, either if the agent succeeds or if he fails all T times.

2.2. Updating Beliefs

Given that $e_t = 1$ for all t, and that experimentation has failed in previous periods, we denote by β_t^θ the updated belief of type θ that the cost is low at the beginning of period t (after $t-1$ failures). We have $\beta_t^\theta = \frac{\beta_{t-1}^\theta(1-\lambda)}{\beta_{t-1}^\theta(1-\lambda)+(1-\beta_{t-1}^\theta)}$, which can be re-written in terms of β_0^θ as follows:

$$\beta_t^\theta = \frac{\beta_0^\theta(1-\lambda)^{t-1}}{\beta_0^\theta(1-\lambda)^{t-1}+1-\beta_0^\theta}.$$

The expected cost for a type θ agent at the beginning of period t is then

$$c_t^\theta = \beta_t^\theta \underline{c} + \left(1-\beta_t^\theta\right)\overline{c}.$$

After each failure, a type θ agent becomes more pessimistic about the true cost being low (β_t^θ falls), and the expected cost rises. For the same number of failures during the experimentation stage, a low type always stays *relatively more pessimistic* than a high type with a relatively higher expected cost ($c_0^H < c_0^L$). However, both c_t^H and c_t^L approach $\overline{c}$ in the limit.

For future use, we also note that the difference in the expected cost,

$$\Delta c_t \equiv c_t^L - c_t^H = (\beta_t^H - \beta_t^L)\left(\overline{c} - \underline{c}\right) = (\beta_t^H - \beta_t^L)\Delta c > 0,$$

is either decreasing in time (if $\beta_0^H \leq 1 - \beta_0^L$) or is *non-monotonic* with one peak (if $\beta_0^H > 1 - \beta_0^L$). Two examples of Δc_t are presented in Figure 1 below. Sufficient conditions for binding incentive constraint rely on β_0^L small so that Δc_t is monotonically decreasing over time.

2.3. The Production Stage

If experimentation succeeds in some t or fails all T times, production takes place. Since success publicly reveals low cost, the output after success is produced under complete information. The interesting case occurs when the agent has failed to learn during the entire experimentation stage as production occurs under asymmetric information.[19] This is a significant departure from the standard literature on strategic experimentation, where the quantity

[19] We assume that the agent will learn the exact cost later, but it is not contractible.

after failure is implicitly assumed to be zero. In that case, asymmetric beliefs after failure between the principal and agent do not matter. A difference in beliefs matters whenever there is a decision to be made at this stage, and we capture it by assuming an explicit production stage even after failure. Thus, asymmetric information generated during the experimentation stage echoes into the production stage and, conversely, the anticipation of asymmetric information during production impacts the experimentation stage.

The simplest way to capture the impact of asymmetric beliefs in production after failure is to assume that the output after failure is fixed at $q_F > 0$. We relax this assumption in an extension.[20] To be consistent with the extension section, we assume that the principal's value of the project is given by $V(q)$, which is strictly increasing and strictly concave. The output after success, q_S, is determined by $V'(q_S) = \underline{c}$. We assume $V(q_S) > V(q_F) > 0$.

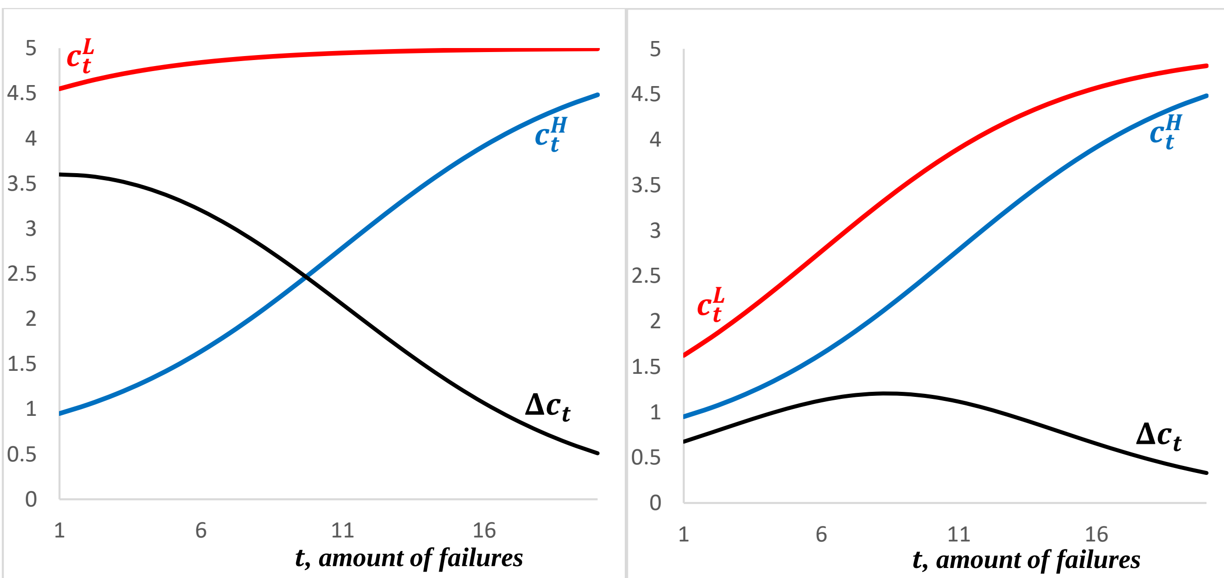

Figure 1. Expected cost with $\beta_0^H = 0.9$, $\lambda = 0.2$, $\underline{c} = 0.5$, $\overline{c} = 5$, $\beta_0^L = 0.1$ *(left)* and $\beta_0^L = 0.75$ *(right)*.

2.4. The Contract and Payoffs

Before the experimentation stage takes place, the principal offers the agent a menu of dynamic contracts. A contract specifies, for each type of agent, the length of the experimentation

[20] In the extension, the output is determined such that the marginal benefit of output equals its (expected) marginal cost. The key results are unaffected, except that variation in output after failure is an additional screening device (see, for instance, Khalil et al. (2020)).

stage, and the transfer and output as a function of whether experimentation is successful in any period.

Without loss of generality, we use a direct truthful mechanism, where the agent is asked to announce his type, denoted by $\hat{\theta}$. A contract is defined formally by

$$\varpi^{\hat{\theta}} = \left(T^{\hat{\theta}}, \left\{w_t^{\hat{\theta}}(\underline{c}), w_t^{\hat{\theta}}\left(c_{t+1}^{\hat{\theta}}\right)\right\}_{t=1}^{T^{\hat{\theta}}}\right),$$

where $T^{\hat{\theta}}$ is the (maximum) duration of the experimentation stage for the announced type $\hat{\theta}$, $w_t^{\hat{\theta}}(\underline{c})$ is the agent's wage if he observed $c = \underline{c}$ in period $t \leq T^{\hat{\theta}}$, and $w_t^{\hat{\theta}}\left(c_{t+1}^{\hat{\theta}}\right)$ is the agent's wage if he fails $t \leq T^{\hat{\theta}}$ times.

An agent of type θ, announcing his type as $\hat{\theta}$ and choosing effort profile $\vec{e}^{\theta} = \left\{e_t^{\theta}\right\}_{t=1}^{t=T^{\hat{\theta}}}$, receives expected utility $U^{\theta}\left(\varpi^{\hat{\theta}}\right)$ at time zero from a contract $\varpi^{\hat{\theta}}$:[21]

$$U^{\theta}\left(\varpi^{\hat{\theta}}, \vec{e}^{\theta}\right) = \left(1-\beta_0^{\theta}\right)\sum_{t=1}^{T^{\hat{\theta}}}\left[w_t^{\hat{\theta}}\left(c_{t+1}^{\hat{\theta}}\right) - 1_{\left\{t=T^{\hat{\theta}}\right\}} c^{\theta}_{\sum_{s=1}^{T^{\hat{\theta}}} e_s^{\theta}+1} q_F - \gamma e_t^{\theta}\right]$$

$$+\beta_0^{\theta}\sum_{t=1}^{T^{\hat{\theta}}}\left(\prod_{s=1}^{t-1}\left(1-\lambda e_s^{\theta}\right)\right)\left[e_t^{\theta}\left(\lambda\left(w_t^{\hat{\theta}}(\underline{c}) - \underline{c}q_S\right) - \gamma\right) + \left(1-\lambda e_t^{\theta}\right)\left(w_t^{\hat{\theta}}\left(c_{t+1}^{\hat{\theta}}\right) - 1_{\left\{t=T^{\hat{\theta}}\right\}} c^{\theta}_{\sum_{s=1}^{T^{\hat{\theta}}} e_s^{\theta}+1} q_F\right)\right].$$

Conditional on the actual cost being low, which happens with probability β_0^{θ}, the probability of succeeding for the first time in period $t \leq T^{\hat{\theta}}$ is given by $\left(\prod_{s=1}^{t-1}\left(1-\lambda e_s^{\theta}\right)\right)e_t^{\theta}\lambda$. If the agent succeeds, he will produce q_S and is paid $w_t^{\hat{\theta}}(\underline{c})$ by the principal. In addition, it is possible that the agent never observes that the cost is low. This is the case either if the cost is actually high ($c = \bar{c}$), which happens with probability $1-\beta_0^{\theta}$, or, if the agent fails $T^{\hat{\theta}}$ times despite $c = \underline{c}$, which happens with probability $\beta_0^{\theta}\prod_{s=1}^{T^{\hat{\theta}}}\left(1-\lambda e_s^{\theta}\right)$.[22] In this case, the agent produces q_F and is paid $w_{T^{\hat{\theta}}}^{\hat{\theta}}\left(c_{T^{\hat{\theta}}+1}^{\hat{\theta}}\right)$ based on the expected cost at period $T+1$, denoted by $c_{T^{\hat{\theta}}+1}^{\hat{\theta}}$.

Denote by $\vec{a}^{\theta}\left(\varpi^{\hat{\theta}}\right) \equiv argmax_{\vec{e}^{\theta}} U^{\theta}\left(\varpi^{\hat{\theta}}, \vec{e}^{\theta}\right)$ the optimal action profile for type θ facing a contract $\varpi^{\hat{\theta}}$ in all periods $t \leq T^{\theta}$. Denoting the equilibrium effort profile by $\vec{e}^{\theta} = \vec{1}$ (with $e_t^{\theta} = 1$ for all $t \leq T^{\theta}$), the optimal contract must satisfy the following (global) moral hazard constraint:

[21] Where a characteristic function $1_{\{t\in\mathbb{T}\}}$ is defined as $1_{\{t\in\mathbb{T}\}} = \begin{cases}1, & t \in \mathbb{T}\\ 0, & t \notin \mathbb{T}\end{cases}$ for any set $\mathbb{T}$.

[22] Recall that if the agent shirks, success will also never be achieved.

$(\boldsymbol{MH^\theta})$ $$\vec{1} \in \vec{a}^\theta(\varpi^\theta).$$

The optimal contract will have to satisfy the following incentive compatibility constraint for all θ and $\hat{\theta}$:

$(\boldsymbol{IC^{\theta,\hat{\theta}}})$ $$U^\theta(\varpi^\theta, \vec{1}) \geq U^\theta\left(\varpi^{\hat{\theta}}, \vec{a}^\theta(\varpi^{\hat{\theta}})\right).$$

For future convenience, we introduce some notation. We denote wage net of production cost (i.e., the rent) to the θ-type who succeeds in period t by y_t^θ, and that after failure until period t by x_t^θ:

$$y_t^\theta \equiv w_t^\theta(\underline{c}) - \underline{c}q_S,$$

$$x_t^\theta \equiv w_t^\theta(c_{t+1}^\theta) - 1_{\{t=T^\theta\}}c_{T^\theta+1}^\theta q_F.$$

We denote the probability that an agent of type θ does not succeed during the first t periods of the experimentation stage if $e_j^\theta = 1$ for all $j \leq t$ by:

$$P_t^\theta \equiv 1 - \beta_0^\theta + \beta_0^\theta(1-\lambda)^t.$$

Finally, we assume the agent must be paid his expected production cost whether experimentation succeeds or fails.[23] To account for this, we impose the following limited liability ($\boldsymbol{LL}$) constraints:

$(\boldsymbol{LLS_t^\theta})$ $$y_t^\theta \geq 0 \text{ for } t \leq T^\theta,$$

$(\boldsymbol{LLF_t^\theta})$ $$x_t^\theta \geq 0 \text{ for } t \leq T^\theta.$$

The principal's expected payoff from a contract ϖ^θ offered to an agent of type θ, that satisfies the above constraints, is given by

$$\pi^\theta(\varpi^\theta, \vec{1}) = \beta_0^\theta \sum_{t=1}^{T^\theta}(1-\lambda)^{t-1}\left[\lambda\left(V(q_S) - w_t^\theta(\underline{c})\right) - (1-\lambda)w_t^\theta(c_{t+1}^\theta)\right]$$

[23] Bankruptcy laws and minimum wage laws are well-known examples of legal restrictions on transfers that exemplify limited liability in contracts. See, e.g., Krähmer and Strausz (2015) for more examples. Note that the agent is not protected off the equilibrium path. Thus, this is not a constraint representing the agent's wealth. Without limited liability, the principal can receive first best profit since success during experimentation is a random event correlated with the agent's type (Crémer-McLean (1985)). To streamline the presentation, we assume the transfers must cover expected cost. This is reminiscent of the well-known cost-plus contracts in the procurement literature.

$$+\left(1-\beta_0^\theta+\beta_0^\theta(1-\lambda)^{T^\theta}\right)V(q_F)-\left(1-\beta_0^\theta\right)\sum_{t=1}^{T^\theta} w_t^\theta\left(c_{t+1}^\theta\right).$$

Thus, the principal's objective function is:

$$E_\theta \pi^\theta\left(\varpi^\theta, \vec{1}\right)=\nu\pi^H\left(\varpi^H, \vec{1}\right)+(1-\nu)\pi^L\left(\varpi^L, \vec{1}\right).$$

To summarize, the timing is as follows:

1. The agent learns his type θ.

2. The principal offers a contract to the agent. If the agent rejects the contract, the game is over and both parties get payoffs normalized to zero; if the agent accepts the contract, the game proceeds to the experimentation stage with maximum duration as specified in the contract.

3. The experimentation stage begins.

4. If the agent learns that $c=\underline{c}$, the experimentation stage stops, and the production stage occurs with output and transfers as specified in the contract. In case no success is observed during the entire experimentation stage, the production stage occurs with output and transfers as specified in the contract.

2.5. The First Best Benchmark

Suppose the agent's type θ is common knowledge *before* the principal offers the contract and, in addition, the agent's effort choice is publicly observable. The first-best solution is found by maximizing the expected surplus net of costs denoted by

$$\Omega^\theta=\beta_0^\theta\sum_{t=1}^{T^\theta}(1-\lambda)^{t-1}\lambda\left[V(q_S)-\underline{c}q_S-\sum_{s=1}^{t}\gamma\right]$$
$$+\left(1-\beta_0^\theta+\beta_0^\theta(1-\lambda)^{T^\theta}\right)\left[V(q_F)-c_{T^\theta+1}^\theta q_F-\sum_{s=1}^{T^\theta}\gamma\right].$$

Since the expected cost is rising until success is obtained, the first-best solution is characterized by a termination date T_{FB}^θ, the maximum number of periods an agent of type θ is allowed to experiment:

$$T_{FB}^\theta\in arg\max_{T^\theta}\Omega^\theta.$$

Note that T_{FB}^θ is bounded and it is the highest t^θ such that

$$\beta_{t^\theta}^\theta\lambda\left[V(q_S)-\underline{c}q_s\right]+\left(1-\beta_{t^\theta}^\theta\lambda\right)\left[V(q_F)-c_{t^\theta+1}^\theta q_F\right]\geq\gamma+\left[V(q_F)-c_{t^\theta}^\theta q_F\right].$$

The intuition is that, by extending the experimentation stage by one additional period, the agent of type θ can learn that $c = \underline{c}$ with probability $\beta_{t^\theta}^\theta \lambda$. If the agent succeeds in any t^θ, q_s is produced. The transfer to the agent covers the actual cost, and no rent is given to the agent. In case the agent fails in the entire experimentation stage, q_F is produced. The transfer covers the expected cost, and no expected rent is given to the agent.

Note that the first-best termination date of the experimentation stage T_{FB}^θ is a *monotonic* function of the agent's type.[24] The reason is that the high type is more likely to learn $c = \underline{c}$ (conditional on the actual cost being low) since $\beta_t^H \lambda > \beta_t^L \lambda$ for any t. This implies that the principal should allow the high type to experiment longer. As is standard, we assume that it is always optimal to experiment at least once in the first-best case, where the principal observes effort and knows β_0^θ.[25] This restriction does not apply in the optimal contract under asymmetric information, where the principal is free to choose not to experiment.

3. Optimal Contracts

3.1. Pure Adverse Selection (No Moral Hazard)

In this section, we consider the benchmark under pure adverse selection without moral hazard. Thus, we assume for now that the agent privately knows his type, but his effort choice is public and that $e_t^\theta = 1$ for all $t \leq T^\theta$. Since effort is observable, the principal pays for the cost of experimentation γ in each period directly as in the first best case. The key new feature relative to a standard adverse selection problem is that both incentive compatibility constraints can be binding because experimentation introduces a common value problem. As a result, the principal may choose to over-experiment and reward failure.

Next, we present the principal's optimization problem. Recalling that $P_t^\theta = 1 - \beta_0^\theta + \beta_0^\theta (1-\lambda)^t$, the principal chooses the contracts ϖ^H and ϖ^L to maximize

$$E_\theta \left\{ \Omega^\theta - \beta_0^\theta \sum_{t=1}^{T^\theta} (1-\lambda)^{t-1} \lambda y_t^\theta - \sum_{t=1}^{T^\theta} P_t^\theta x_t^\theta \right\} \textit{s.t.}$$

[24] This is different from Halac et al. (2016) and Khalil et al. (2020), where the first-best termination date is non-monotonic in type and plays a key role. The reason for the non-monotonicity in those papers is that agent's type is given by λ, and the conditional probability of success is higher for the high type early but becomes lower as the length of experimentation increases.

[25] In particular, we assume that the principal would not choose q_S without experimenting.

$(IC^{H,L})$ $\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H + \sum_{t=1}^{T^H} P_t^H x_t^H$

$$\geq \beta_0^H \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L + \sum_{t=1}^{T^L} P_t^H x_t^L + \ P_{T^L}^H \Delta c_{T^L+1} q_F,$$

$(IC^{L,H})$ $\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L + \sum_{t=1}^{T^L} P_t^L x_t^L$

$$\geq \beta_0^L \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H + \sum_{t=1}^{T^H} P_t^L x_t^H - P_{T^H}^L \Delta c_{T^H+1} q_F,$$

$\left(LLS_t^\theta\right)$ $y_t^\theta \geq 0$ for $t \leq T^\theta$,

$\left(LLF_t^\theta\right)$ $x_t^\theta \geq 0$ for $t \leq T^\theta$ for $\theta = L, H$.

In this benchmark without moral hazard, it is *only when experimentation fails all T^θ times* that the principal and a lying agent have asymmetric assessments of expected cost of production. If an agent experiences success before the terminal date, T^θ, the true cost $c = \underline{c}$ is publicly revealed.

3.1.1. Both (IC) may be binding

The $(IC^{H,L})$ is binding, which is typical in adverse selection models. Since a low type must be given at least his expected cost following failure, a high type will have lower expected costs if he lies and experimentation fails in all periods, that is, $c_{T^L+1}^H < c_{T^L+1}^L$. He will have to be given a rent of $P_{T^L}^H \Delta c_{T^L+1} q_F$ to truthfully report his type and $(IC^{H,L})$ is binding.

Less typical for adverse selection models is that $(IC^{L,H})$ might be binding as well. The reason is that experimentation leads to a common value problem since the agent's type β_0^θ directly enters the principal's objective function by determining the probability of success and failure.[26] As is well known, in a common value setting like ours, both incentive compatibility constraints can be binding because of a conflict between the principal's preference for the high type to experiment longer for pure efficiency reasons and the monotonicity condition imposed by asymmetric information.[27]

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[26] We would also have a common value problem if the asymmetric information was about the probability of success, λ, when the cost is low.

[27] See, e.g., Laffont and Martimort (2002), page 53. Experimentation introduces two key features to the standard adverse selection problem. Besides the common value problem mentioned above, note also that the difference in expected costs Δc_T, that determines the incentive to misreport, depends on the termination date T, which is endogenous in our model.

To understand why the low type might be interested in pretending to be the high type, i.e., $(IC^{L,H})$ might be binding, note that misreporting is a *gamble* for the low type with his payoff depending on the outcome of experimentation. If the low type lies about his type, he has a chance to obtain the high-type's rent $P^{H}_{T^L}\Delta c_{T^L+1}q_F$ if experimentation succeeds. This is the benefit of lying for the low type. But he will incur an expected loss in the production stage if experimentation fails T^H times since $c^L_{T^H+1} > c^H_{T^H+1}$. This is the cost of lying for the low type represented by $P^{L}_{T^H}\Delta c_{T^H+1}q_F$ on the RHS of $(IC^{L,H})$. The low type's gamble is positive if the benefit is higher than the cost.

We provide sufficient conditions for $(IC^{L,H})$ to be binding in Supplementary Appendix A. Specifically, we show that $(IC^{L,H})$ is binding if λ is high. To understand the reason, note that both the benefit and the cost of lying for the low type are decreasing in the duration of experimentation. That is, $P^{H}_{T^L}\Delta c_{T^L+1}$ is decreasing in T^L and $P^{L}_{T^H}\Delta c_{T^H+1}$ is decreasing in T^H. Then, if λ is high (each experiment changes Δc_t significantly), then the monotonicity condition implied by the two binding incentive compatibility constraints yields $T^L > T^H$.[28] This is in contradiction with the first-best order for the termination periods, $T^H \geq T^L$, as is often the case in common value problems.

3.1.2. The optimal contract under pure adverse selection

The optimal contract is presented in Proposition 1 and formally derived in Supplementary Appendix A. The principal has two tools to screen the agent: (*i*) the payment structure, and (*ii*) the length of the experimentation period. We first characterize them in the proposition below and then explain the intuition.

[28] To see the intuition why the gamble is positive for the low type when λ is high, consider first the case where $\lambda \to 0$. In that case, there is no experimentation, and our model becomes the standard second-best problem where the low type does not want to pretend to be the high type (see our sufficient condition for $(IC^{L,H})$ to be slack in Supplementary Appendix A). Next consider the other extreme case where $\lambda \to 1$. In Supplementary Appendix A, we show that the $(IC^{L,H})$ requires that

$0 \geq \beta^L_0\left[\frac{P^H_{T^L}\Delta c_{T^L+1}q_F}{\beta^H_0}\right] - P^L_{T^H}\Delta c_{T^H+1}q_F$, which can be rewritten as $(1-\lambda)^{T^H-T^L} \geq \frac{\frac{\beta^L_0}{(1-\beta^L_0)}}{\frac{\beta^H_0}{(1-\beta^H_0)}} \in (0,1)$. Maintaining the first-best order $T^H > T^L$, the gamble becomes positive when $\lambda \to 1$, and the low type wants to pretend to be a high type.

Proposition 1. ***The optimal contract under adverse selection (no moral hazard)***

(i) If both $(IC^{H,L})$ *and* $(IC^{L,H})$ *are binding, the high type is rewarded only after success (*$x_t^H = 0$ *for* $t \le T^H$ *and* $y_t^H > 0$ *for some* $t \le T^H$*), while the low type is rewarded only after failure in the last period (*$\boldsymbol{x_{T^L}^L > 0} = x_t^L$ *for* $t < T^L$ *and* $y_t^L = 0$ *for* $t \le T^L$*). If only* $(IC^{H,L})$ *is binding, the low type receives no rent and there is no restriction on when to reward the high type to pay his rent.*

(ii) Relative to the first best, the low type strictly over-experiments, while the high type weakly under-experiments (strictly, when both the incentive compatibility constraints are binding).

Proof: See Supplementary Appendix A.

The key findings regarding payments are derived when both incentive compatibility constraints bind, which we prove to be the case when λ is high. The principal must reward the high type only after *success* and the low type only after *failure* in the very last period. Therefore, in the absence of moral hazard, we find that dynamic screening of the agent's types can lead the principal to reward failure.

Screening requires rewarding each type for an event that is relatively more likely to occur given the type, which is success for the high-type and failure for the low-type, respectively.[29] Furthermore, because the relative probability of failure $\frac{P_t^L}{P_t^H}$ is increasing in t, it is optimal to postpone the low-type's reward to the very last period of the relationship, $x_{T^L}^L > 0$, making it less likely for a (misreporting) high type to obtain it. However, it does not matter when the principal rewards success as the relative likelihood of success is independent of time:

$$\frac{\beta_0^L(1-\lambda)^{t-1}\lambda}{\beta_0^H(1-\lambda)^{t-1}\lambda} = \frac{\beta_0^L}{\beta_0^H} \text{ for every } t \le T^L.$$

If only $(IC^{H,L})$ is binding, the rent to the low type is zero, and we show that the principal can use any combination of y_t^H and x_t^H to pay the rent to the high type as long as $(IC^{H,L})$ is satisfied, i.e.,

$$\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H + \sum_{t=1}^{T^H}\delta^t P_t^H x_t^H = P_{T^L}^H \Delta c_{T^L+1} q_F.$$

[29] The relative probability of failure is given by $\frac{P_t^L}{P_t^H} > 1$ for all t. The relative probability of success is given by $\frac{\beta_0^L(1-\lambda)^{t-1}\lambda}{\beta_0^H(1-\lambda)^{t-1}\lambda} = \frac{\beta_0^L}{\beta_0^H} < 1$, which is independent of t, for any $t \in N$.

The principal can also use the length of experimentation to screen the types. We first note that the positive part of rent for each type is determined by the high-type's rent, given by $P^H_{T^L}\Delta c_{T^L+1}q_F$, which we prove to be decreasing in T^L. Thus, it is optimal to distort T^L above the first-best level, which leads to over experimentation. When $(IC^{L,H})$ is also binding, the cost of lying for a low type is given by $P^L_{T^H}\Delta c_{T^H+1}q_F$, which is shown to be decreasing in T^H. Thus, the principal optimally under-experiments by distorting T^H below its first-best level.[30]

3.2. Pure Moral Hazard (No Adverse Selection)

In this section, we present a pure moral hazard benchmark by assuming that the agent's type θ is commonly known but his effort choice is private. To simplify notation, we omit the superscript θ in this section. The key new feature in our model relative to the standard dynamic moral hazard problem is that the agent receives rent during the production stage. Besides the standard moral hazard rent during experimentation, there will be a second moral hazard rent at the production stage since a shirking agent will have a lower expected cost compared to what the principal believes. This second moral hazard rent has two effects in this model. First, it makes the separation of experimentation and production optimal which we will explain in Section 4. Second, as we explain in this section, the second rent leads to a greater degree of under-experimentation than in moral hazard models without a production stage. Unsurprisingly, we also show that it is not optimal to reward failure in a pure moral hazard model.

The agent's expected utility from accepting contract ϖ at time zero while exerting an effort profile $\vec{e}$ is

$$U(\varpi,\vec{e}) = (1-\beta_0)\sum_{t=1}^{T}[x_t-\gamma e_t] + \beta_0\sum_{t=1}^{T}[\prod_{s=1}^{t-1}(1-\lambda e_s)][e_t(\lambda y_t-\gamma)+(1-\lambda e_t)x_t].$$

The principal's optimization problem in this case, denoted by $\mathbb{P}^M$, becomes the following:

$$[\mathbb{P}^M] \quad \max_{\varpi} \pi(\varpi,\vec{1}) \text{ subject to}$$

$$(MH) \qquad \vec{1} \in arg\max_{\vec{e}} U(\varpi,\vec{e}),$$

$$(LLS_t) \qquad y_t \geq 0 \text{ for } t \leq T,$$

$$(LLF_t) \qquad x_t \geq 0 \text{ for } t \leq T.$$

[30] As there is over-experimentation in T^L, we also require ν to be small when providing the sufficient conditions.

3.2.1. A production stage rent due to moral hazard

Before we present the solution to the principal's optimization problem, we argue that the global moral hazard constraint (MH) can be replaced by with a sequence of local one-period moral hazard constraints (MH_t). Consider the agent's incentives to engage in a one-shot deviation and shirk at period $t \leq T$, assuming that the agent has worked in all prior periods $j < t$ without success and will work in all subsequent periods $s > t$. The ensuing one-period moral hazard constraint (MH_t) at period t can be written as:

$$(MH_t)\ \ y_t - x_t \geq \frac{\gamma}{\lambda\beta_t} + \sum_{s=t+1}^{T}(1-\lambda)^{s-t-1}(\lambda y_s + (1-\lambda)x_s - \gamma) + \frac{(1-\beta_0)(1-\lambda)^{T-t}}{P_T}\Delta c q_F.$$

The principal can motivate the agent to work by paying a reward for success (y_t), but it should be intuitive from (MH_t) that there is no reason to pay the agent after failure ($x_t = 0$).

Next, we argue that if it is unprofitable for the agent to shirk only once, then it is not optimal to shirk in several periods either. If the (MH_t) constraint is satisfied in period t, the agent prefers to work in this period rather than shirk. If the agent has shirked in a preceding period, he can be only more optimistic as he has perceived fewer failures, so he strictly prefers to work. If he plans to deviate in a future period $\hat{t} > t$, his continuation value from shirking in period t can only go down since then he is more likely to get to period $\hat{t}$ without success and receive $x_{\hat{t}}$.

The first two terms on the *RHS* of (MH_t) capture a standard rent in a dynamic model of experimentation without production (see, e.g., Bergemann and Hege (1998)).[31] The third term is novel and represents the second moral hazard rent for the agent. The reason is that the shirking agent will be more optimistic than the principal that the cost of production is low when experimentation fails overall. The principal's belief is based on one more period of working compared to that of a shirking agent. Thus, the shirking agent has a lower expected cost: $c_T = c_{T+1} - (\beta_T - \beta_{T+1})\Delta c < c_{T+1}$, and he will receive an additional production stage rent as a result.

[31] The first term is a static moral hazard rent. The second term is an additional rent due to beliefs regarding the quality of the project being updated every period. When the agent deviates at period t, he knows that a failure at this period should not change beliefs regarding the project's quality. However, if this deviation is not observed by the principal, she will update her belief and become more pessimistic regarding the project. Thus, the agent must be compensated for his cost of effort and for the asymmetry of information (beliefs) he can create by shirking.

3.2.2. The optimal contract under pure moral hazard

In the optimal contract, the agent receives a positive moral hazard rent that is increasing in T. As a result, the principal terminates experimentation inefficiently early. We characterize the optimal contract in Proposition 2 below.

Proposition 2. ***The optimal contract with moral hazard (no adverse selection)***

(i) The agent receives two moral hazard rents: a standard rent in the experimentation stage and a second rent in the production stage. The agent is rewarded only after success, and the optimal reward y_t is constant for $t \leq T$:

$$y_t = \frac{\gamma}{\lambda \beta_T} + \frac{(1-\beta_0)}{P_T} \Delta c q_F, \text{ where } \frac{\gamma}{\lambda \beta_T} = \frac{\gamma}{\lambda \beta_t} + \gamma \sum_{s=1}^{T-t} \frac{(1-\beta_0)}{\beta_0 (1-\lambda)^{t+s-1}}.$$

(ii) The agent under-experiments relative to the first best.

Proof: See Supplementary Appendix B.

As noted above, the term $\frac{\gamma}{\lambda \beta_T}$ represents the standard moral hazard rent in a dynamic model of experimentation without production. The standard rent has two parts, where $\frac{\gamma}{\lambda \beta_t}$ addresses the static gain, and $\gamma \sum_{s=1}^{T-t} \frac{(1-\beta_0)}{\beta_0 (1-\lambda)^{t+s-1}}$ is the rent coming from a higher probability of collecting future moral hazard rents (than the principal expects in equilibrium).[32] The term $\frac{(1-\beta_0)}{P_T} \Delta c q_F$ is what we called the second moral hazard rent and it stems from the shirking agent having a lower expected cost of production than the principal expects in equilibrium.

3.3. General Case: Adverse Selection and Moral Hazard

We now return to the general case with both moral hazard and adverse selection. The optimal contract must satisfy the moral hazard $\left(MH_t^{\theta}\right)$ and incentive compatibility $\left(IC^{\theta,\hat{\theta}}\right)$ constraints for all θ and $\hat{\theta}$. Note that the moral hazard problem is also implicitly reflected in the $\left(IC^{\theta,\hat{\theta}}\right)$ constraints, which requires characterizing off-equilibrium effort on the RHS of these constraints. A key result of this section is that, even when the principal must address moral hazard, it remains optimal to reward the low type for failure when the impact of the adverse selection problem is significant.

[32] Note that y_t is constant for $t \leq T$ because we assume there is no discounting. Then, the optimal contract is unique up to payoff-irrelevant alteration. A similar reward structure holds in Halac et al. (2016) who argue that in the case of no discounting, the principal can be restricted to using constant bonus contracts. See also Rodivilov (2022).

3.3.1. Off the equilibrium path efforts

A key issue in a mixed model is characterizing off the equilibrium efforts, which we discuss next.[33] Consider the low type's off-equilibrium effort level $\vec{a}^L(\varpi^H)$. We find that it is not optimal for the low type to work in every period if he claims being a high type. We prove in Lemma C1 in Appendix C that if the low type misreports, he works for $t^{L,H} \leq T^H$ periods and shirks in other periods.[34] The first reason for this result is standard in a mixed model – a payment that makes a high-type agent work may be not sufficient to induce a low type to work as well. This is because, after misreporting his type, the low type is more pessimistic than the high type and less likely to collect promised rewards after success.[35] The second reason is specific to our model with a production stage with endogenous asymmetric information. Recall that the low type has a relatively higher expected cost than the high type if experimentation fails in all periods. Since rent at the production stage for the lying *low* type depends on the *high* type's expected cost after failure, it does not provide incentives for the lying low type to work.[36]

Next, consider the high type's off-equilibrium effort level $\vec{a}^H(\varpi^L)$. We show that the high type finds it optimal to never shirk if he claims being low. We prove this in Lemma C2 in Appendix C. Again, one reason is standard from a mixed model – a payment that makes a low type to work, will be enough to induce a high type to work as well. This is because, after misrepresenting his type, the high type is less pessimistic than the low type and more likely to collect promised rewards after success. The second reason also favors working by the high type off the equilibrium path as his expected cost is relatively lower compared to the low type's when experimentation fails in all periods. Since rent at the production stage for the lying *high* type

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[33] A similar result is not easily available in Halac et al. (2016) as the agent's private information is about λ the efficiency of learning parameter. In that case, the relative probability of success between the two types changes in ranking over time. As a result, the authors provide examples that it is possible to have multiple off-equilibrium paths for effort in the optimal contract.

[34] Note that we have weak inequality $t^{L,H} \leq T^H$ since time is discrete. It is possible that the low type shirks off-the-equilibrium in all periods, i.e., $t^{L,H} = 0$.

[35] It is without loss of generality to consider an off-the-equilibrium effort path where $\vec{a}^L(\varpi^H)$ is a stopping rule: the low type works up to period $t^{L,H} \leq T^H$ and shirks thereafter. The reason is that (i) all the rewards for success are identical and, (ii) the low type's probability of success in any period and the expected cost after failure depends on the total number of failures up to that period (not on when those failures occurred).

[36] The payment after failure for the high type must cover his expected cost $c^H_{T^H+1}$. If the low type works in every period after misrepresenting his type, he is more pessimistic than the high type if experimentation fails in all periods, i.e., $c^L_{T^H+1} > c^H_{T^H+1}$. Therefore, anticipating a smaller payment relative to his expected cost, the low type may not find it attractive working in every period off-equilibrium.

depends on the *low* type's expected cost after failure, it provides incentives for the lying high type to work.[37]

3.3.2. Optimal payment structure with both moral hazard and adverse selection

Having characterized off-the-equilibrium behavior for both types, we now discuss how the interaction between moral hazard and adverse selection affects optimal payments. We find that despite the presence of moral hazard, it may be optimal to reward the low type for failure due to the impact of adverse selection.

We highlight the impact of adverse selection on the optimal payments in the presence of moral hazard, and we outline our key findings in Proposition 3 below. Then, we discuss the optimal length of experimentation in Proposition 4.

If the moral hazard problem is relatively more important than the adverse selection problem, the reward required after success could be so high that neither incentive compatibility constraint binds. The agent is only rewarded for success as in the pure moral hazard benchmark of Proposition 2. Specifically, the agent is paid just enough to satisfy (MH_t^{θ}) as equalities.

At the other extreme, if adverse selection is the relatively more important problem and creates a common value problem, merely satisfying the (MH_t^{θ}) as equalities with only payments after success is not enough to satisfy either incentive constraint.[38] Then, both $(IC^{H,L})$ and $(IC^{L,H})$ are binding. An *additional screening rent* needs to be paid to each type on top of the rewards after success due to moral hazard.

The interesting question is when the principal should allocate this additional screening rent. Along with choosing the timing to minimize the screening cost, the principal has to ensure that each type has incentive to work in each period. Recall from the pure adverse selection case of Section 3.1 that the relative probability of success for the low type is independent of type

[37] The payment after failure for the low type must cover his expected cost $c_{T^L+1}^L$. If the high type works in every period after misrepresenting his type, he is more optimistic than the low type if experimentation fails in all periods, i.e., $c_{T^L+1}^H < c_{T^L+1}^L$. Therefore, the payment for failure that covers the low type's expected cost also covers the high type's cost.

[38] We present formal sufficient conditions for both *IC* binding in Supplementary Appendix F. They are analogous to those we found under pure adverse selection except that we must now account for moral hazard. Again, we show that both *IC* are binding when λ is high enough, but also that γ is relatively small (compared to Δc) to minimize the relative impact of moral hazard compared to adverse selection. We also derived sufficient conditions for the cases where only one or neither *IC* are binding and report them in Supplementary Appendix F.

while the relative probability of failure is increasing. Thus, as in the benchmark of Proposition 1, the additional screening rent is given to the high type only after success, while it is given to the low type only after failure in the last period:[39]

$$x_t^H = 0 \; for \; t \leq T^H,$$

$$y_t^H \geq \frac{\gamma}{\lambda \beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H} \Delta c q_F \; for \; t \leq T^H \text{ (strict inequality for some t),}$$

$$\boldsymbol{x_{T^L}^L > 0} = x_t^L \; for \; t \leq T^L.$$

Of course, rewarding failure in the last period requires that the reward after success also be raised (by the same amount) not only in that last period:

$$y_{T^L}^L = \boldsymbol{x_{T^L}^L} + \frac{\gamma}{\lambda \beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L} \Delta c q_F,$$

but also in the all the previous periods $t < T^L$:

$$y_t^L = \frac{\gamma}{\lambda \beta_t^L} + \sum_{s=t+1}^{T^L} (1-\lambda)^{s-t-1} (\lambda y_s^L - \gamma) + (1-\lambda)^{T^L - t} \boldsymbol{x_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L} \Delta c q_F,$$

which increases the payments y_t^L strictly above the optimal pure payments in each period under pure moral hazard: $\frac{\gamma}{\lambda \beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L} \Delta c q_F$. That is, the low-type agent must be given extra incentives to work in each period since he is now rewarded for failure in the last period. The benefit of rewarding failure comes from better screening of the two types.

The other two cases where only one incentive compatibility constraint binds are intermediate. In each case, rewards are paid only after success, and the additional screening rent for the type with the binding incentive constraint implies that his moral hazard constraints can be satisfied ‘for free’. For the type whose incentive constraint is not binding, all the moral hazard constraints are binding.[40]

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[39] For example, it is without loss of generality to pay the extra rent to the high type after the very first success, i.e., front load the extra rent.

[40] *If only* $(IC^{L,H})$ *binds* $x_t^H = 0$, $y_t^H = \frac{\gamma}{\lambda \beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H} \Delta c q_F$ *for* $t \leq T^H$. *If only* $(IC^{H,L})$ *binds* $x_t^L = 0$, $y_t^L = \frac{\gamma}{\lambda \beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L} \Delta c q_F$ *for* $t \leq T^L$.

To summarize, we find that rewarding failure can be optimal despite the presence of the moral hazard constraints. Because of moral hazard, success must always be rewarded but this reward may not be enough to satisfy the adverse selection constraints and additional rewards may be given after failure.

Proposition 3. ***The optimal contract with both moral hazard and adverse selection:***
To address moral hazard, the principal must reward each type after success in every period ($y_t^{\theta} > 0$ for $t \leq T^{\theta}$). Furthermore, each type is paid an additional ***screening rent*** *when his incentive compatibility constraint is binding. If both $(IC^{L,H})$ and $(IC^{H,L})$ are binding, the high type receives his additional screening rent only after success, while the low type receives this extra screening rent only after failure in all periods.*

Proof: See Appendix C.

Now we consider the optimal length of experimentation.

Proposition 4: Optimal length of experimentation:
Over-experimentation can occur whenever at least one incentive compatibility constraint is binding. If only moral hazard constraints are binding, both types under experiment.

Proof: See Appendix C.

When both $(IC^{L,H})$ and $(IC^{H,L})$ are slack, the moral hazard rent for each type is sufficiently high so that neither type is interested in misreporting their type. Such a case cannot occur under pure adverse selection. Note that if the principal was not allowed to use the length of the experimentation stage as a screening variable, i.e., if we exogenously imposed $T^H = T^L = T$, then $y_t^H < y_t^L$ for all t, and the high type would have incentives to misrepresent his type.[41] Because the high type would be attracted by the higher reward for success offered to the low type ($y_t^L > y_t^H$), $(IC^{H,L})$ can only be slack if he is allowed to experiment longer ($T^H > T^L$). As in the pure moral hazard case of Section 3.2, both types under-experiment to reduce the moral hazard rent.

[41] To see that $\frac{\gamma}{\lambda\beta_T^H} + \frac{(1-\beta_0^H)}{P_T^H}\Delta c q_F < \frac{\gamma}{\lambda\beta_T^L} + \frac{(1-\beta_0^L)}{P_T^L}\Delta c q_F$ note that $\beta_T^H > \beta_T^L$ and $\frac{(1-\beta_0^H)}{P_T^H} < \frac{(1-\beta_0^L)}{P_T^L}$.

When $(IC^{H,L})$ constraint is slack, but $(IC^{L,H})$ binding, only the low type is interested in pretending to be the high type. This is the case if T^H is sufficiently higher than T^L, so the low type benefits from lying since he has more chances to succeed during T^H periods rather than T^L periods only. Since the informational rent of the low-type agent is non-monotonic in T^H, it is possible, in general, to have over- or under-experimentation for the high type. The stopping time for the low type, T^L, does not affect information rents and, as a result, is not distorted.

When $(IC^{H,L})$ binds and $(IC^{L,H})$ is slack, only the high type is interested in pretending to be the low type. This is the case if T^H is not much higher than T^L, so only the high type benefits from misrepresenting. This scenario is similar to that under a standard adverse selection problem except that the low type also receives a rent due to the moral hazard problem. Since the informational rent of the high-type agent is non-monotonic in T^L, it is possible, in general, to have over- or under-experimentation for the low type. The stopping time for the high type, T^H, does not affect information rents and, as a result, is not distorted.

Finally, when both (IC) constraints are binding, it is possible to have under- or over-experimentation for both types since the informational rent of both types is non-monotonic in the termination date of the other type,

4. Is integrating experimentation and production optimal?

In our model with experimentation and production, the interaction of adverse selection and moral hazard creates interdependent rents. In a pure moral hazard model, the principal would prefer to employ two different agents, one for experimenting and one for producing. She would then save what we have called the second moral hazard rent at the production stage. This begs the question of whether integrating the two tasks, as in our main model, can be optimal due to the presence of adverse selection.

A key benefit to the principal of integrating the two tasks is being able to use the adverse selection rent to induce effort, i.e., pay for the moral hazard rent. We show that the principal is able to spread these rents across time to satisfy the dynamic moral hazard constraints.[42] This benefit must be balanced against the cost of the second moral hazard rent when integrating. We find that integration is optimal if the adverse selection problem (β_0^H far apart from β_0^L) is severe

[42] Since the relatively probability of success across types is time invariant, the distribution of the adverse selection rent does not impact the incentive to misreport.

enough relative to the moral hazard problem in experimentation. We present below sufficient conditions for separation/integration to be optimal.

Proposition 5: ***Sufficient Conditions for separation/integration*****:**

(i) Separation is optimal if the adverse selection problem is small enough (β_0^H is close to β_0^L).
(ii) Integration is optimal if the adverse selection problem is severe enough (β_0^H is close to one and β_0^L sufficiently close to zero) and ν is high enough.

Proof: See Appendix D.

To establish the above result, we can use a very simple extension of our model, where the principal outsources the experimentation task to a second agent (experimenter). An 'in-house' agent (producer) still produces output based on what is learned publicly in the experimentation stage, and his private information about the likelihood of low cost, β_0^θ. We discuss alternative models of separation at the end of this section. Before experimentation starts, the in-house producer is asked to publicly announce his type, based on which experimentation occurs. The principal pays an adverse selection rent to the producer to induce truthful reporting. However, the experimentation stage is a pure moral hazard problem, yielding only a standard moral hazard rent to the experimenter (based on a commonly known β_0^θ).

Since the principal saves the moral hazard rent at the production stage when she separates the two tasks, a key issue is the relative magnitude of this moral hazard rent at the production stage. When this rent is small relatively to the adverse selection rent, the principal's ability to use the adverse selection rent to satisfy moral hazard constraints dominates, and integration is optimal: for example, when the difference in cost Δc is small relative to the difference between β_0^H and β_0^L.[43]

A possible issue regarding the model of separation above is that we assume an in-house producer publicly pre-announces the type β_0^θ. We chose this benchmark for ease of comparison with the main model of integration. Instead, we could assume that the production agent is brought in after experimentation ends and therefore cannot announce his type before experimentation starts. Our key arguments regarding the optimality of integration would only

[43] As we show in our proofs, this basic intuition holds regardless of which *IC* is binding.

get stronger. This would also be the case if the in-house producer privately announced his type β_0^θ to the principal. We briefly discuss these two sub-extensions next.

Consider first that, under separation, experimentation occurs under a common prior, between the principal and the experimenter, that the cost of production is low with probability β_0. There is now an additional cost of separation as the length of experimentation can no longer be based on the private information about β_0^θ of the (integrated) agent. Next consider that the in-house producer privately announces it type to the principal, who contracts with an outside experimenter. In the interim, a principal's incentive constraint (PIC) would also have to be satisfied, which will again reduce the benefit of separation.

5. Endogenous Output

In this section, we allow the principal to choose output optimally after success and after failure, and she can now use output as another screening variable. While our main findings continue to hold, output after failure can now be used as a screening device. Thus, the key new results occur if the experimentation stage fails: the low type is asked to under-produce relative to the first best, while the high type might over-produce. Just like over-experimentation, over-production can be used to increase the cost of lying.

When output is optimally chosen by the principal in the contract, the main change from the base model is that output after failure, which is denoted by q_F^θ, can vary continuously depending on the expected cost. We can replace q_F by q_F^θ in the principal's problem.

We derive the formal output scheme in Supplementary Appendix E but present the intuition here. When experimentation is successful, there is no asymmetric information and no reason to distort the output. Both types produce the first best output. When experimentation fails to reveal the cost, asymmetric information will induce the principal to distort the output to limit the rent.

When both ($IC^{L,H}$) and ($IC^{H,L}$) are slack, both types under produce after failure. As would be the case in the pure moral hazard model with production (see Section 3.2), both types under-produce to reduce the moral hazard rent.

When the ($IC^{H,L}$) constraint is slack, but ($IC^{L,H}$) binding, it is possible, in general, to have over- or under-experimentation for the high type. The reason is that the informational rent of the low-type agent is non-monotonic in T^H. The output for the low type, q_F^θ, does not affect information rents and, as a result, is not distorted. The high type, however, might be asked to over produce. This is the case if the low type is more pessimistic after misreporting and failing experimentation. Over-production is, therefore, used to increase the cost of the lying low type.

When the $(IC^{H,L})$ binds and $(IC^{L,H})$ is slack, the low type is asked to under-produce in order to limit the rent of the high type. The output for the high type, q_F^θ, does not affect information rents and, as a result, is not distorted.

When both (IC) constraints are binding, the low type under produces to limit the rent of the high type. Like the case when only $(IC^{H,L})$ binds, the high type might be asked to over produce to increase the cost of the lying low type.

6. Conclusions

We presented a dynamic model of strategic experimentation with both moral hazard and adverse selection. Technically, such a mixed model of experimentation can become quickly intractable with off the equilibrium path effort hard to characterize.[44] We offer a tractable model to provide an explanation for the co-existence of both high and low-powered incentive schemes, which is used in practice to spur innovative activity. We find that, while moral hazard always leads the principal to reward success in experimentation, adverse selection may induce the principal to reward failure. The reason is that rewarding failure allows the principal to dynamically screen the agents, and it remains optimal even in the presence of moral hazard.

We also find that the principal may prefer to integrate experimentation and production by employing one agent for both. We show that the standard model of experimentation, where experimentation is studied in isolation without a production stage, is valid as long as adverse selection during experimentation is not a significant concern. Integration of experimentation and production allows the principal to use the adverse selection rent to incentivize the agent to work. By distributing the adverse selection rent optimally, the principal can alleviate the moral hazard constraints. This is the case if the adverse selection problem is severe enough relative to moral hazard or, equivalently, if the adverse selection rent is high.

[44] If the adverse selection lies in the probability of success, as in Halac et al. (2016), the relative probability of success between the two types changes in ranking over time.

Appendix C: Proof of Propositions 3 and 4

Proof: We first characterize the optimal payment structure (Proposition 3), and then the optimal length of experimentation (Proposition 4).

The principal's optimization problem is to choose contracts ϖ^θ for $\theta \in \{H, L\}$ to maximize

$$E_\theta \left\{ \Omega^\theta - \beta_0^\theta \sum_{t=1}^{T^\theta} (1-\lambda)^{t-1} \lambda y_t^\theta - \sum_{t=1}^{T^\theta} P_t^\theta x_t^\theta \right\} \text{ s.t.}$$

$(IC^{\theta,\hat{\theta}})$ $\quad U^\theta(\varpi^\theta, \vec{1}) \geq U^\theta\left(\varpi^{\hat{\theta}}, \vec{a}^\theta(\varpi^{\hat{\theta}})\right),$

(MH_t^θ) $\quad y_t^\theta - x_t^\theta \geq \frac{\gamma}{\lambda \beta_t^\theta} + \sum_{s=t+1}^{T^\theta} (1-\lambda)^{s-t-1} \left(\lambda y_s^\theta + (1-\lambda) x_s^\theta - \gamma\right) + \frac{(1-\beta_0^\theta)}{P_{T^\theta}^\theta} \Delta c q_F$ for $t \leq T^\theta$,

(LLS_t^θ) $\quad y_t^\theta \geq 0$ for $t \leq T^\theta$,

(LLF_t^θ) $\quad x_t^\theta \geq 0$ for $t \leq T^\theta$.

Note that with the moral hazard constraints all the (LLS_t^H) and (LLS_t^L) constraints are automatically satisfied and, therefore, can be ignored. When $\vec{e}^\theta = \vec{1}$, the notation for the expected costs in the $(IC^{\theta,\hat{\theta}})$ constraint is $c_{T^\theta+1}^\theta$. When the agent lies about his type and possibly shirks, we need to introduce a new notation. On the *RHS* of $(IC^{H,L})$ we label the effort chosen by the agent in each period s as $e_s^{H,L} \in \{0,1\}$, and the expected cost is $c^H_{\sum_{s=1}^{T^L+1} e_s^{H,L}}$. Similarly, on the *RHS* of $(IC^{L,H})$ we label the effort chosen by the agent in each period s as $e_s^{L,H} \in \{0,1\}$, and the expected cost is $c^L_{\sum_{s=1}^{T^H+1} e_s^{L,H}}$.

Labeling ξ^H, ξ^L, $\{\mu_t^H\}_{t=1}^{T^H}$, $\{\mu_t^L\}_{t=1}^{T^L}$, $\{\eta_t^H\}_{t=1}^{T^H}$, $\{\eta_t^L\}_{t=1}^{T^L}$ as the Lagrange multipliers of the constraints associated with $(IC^{H,L})$, $(IC^{L,H})$, (MH_t^H), (MH_t^L), (LLF_t^H) and (LLF_t^L) respectively, the optimization problem has the following Lagrangian:

$$\mathcal{L} = \nu\left[\Omega^H - \beta_0^H \sum_{t=1}^{T^H} (1-\lambda)^{t-1} \lambda y_t^H - \sum_{t=1}^{T^H} P_t^H x_t^H\right]$$

$$+(1-\nu)\left[\Omega^L - \beta_0^L \sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda y_t^L - \sum_{t=1}^{T^H} P_t^H x_t^L\right]$$

$$+\xi^H \left[\begin{array}{c} \beta_0^H \sum_{t=1}^{T^H} (1-\lambda)^{t-1} \lambda y_t^H + \sum_{t=1}^{T^H} P_t^H x_t^H - \sum_{t=1}^{T^H} P_{t-1}^H \gamma \\ -(1-\beta_0^H) \sum_{t=1}^{T^L} [x_t^L - \gamma e_t^{H,L}] - \beta_0^H \sum_{t=1}^{T^L} \left(\prod_{s=1}^{t-1} (1-\lambda e_s^{H,L})\right) [e_t^{H,L} \lambda y_t^L + (1-\lambda e_t^{H,L}) x_t^L - e_t^{H,L} \gamma] \\ -\left(1-\beta_0^H + \beta_0^H \left(\prod_{s=1}^{T^L} (1-\lambda e_s^{H,L})\right)\right) \left(c_{T^L+1}^L - c^H_{\sum_{s=1}^{T^L+1} e_s^{H,L}}\right) q_F \end{array}\right]$$

$$+\xi^L \left[\begin{array}{c} \beta_0^L \sum_{t=1}^{T^L} (1-\lambda)^{t-1} y_t^L + \sum_{t=1}^{T^L} P_t^L x_t^L - \sum_{t=1}^{T^L} P_{t-1}^L \gamma \\ -(1-\beta_0^L) \sum_{t=1}^{T^H} [x_t^H - \gamma e_t^{L,H}] - \beta_0^L \sum_{t=1}^{T^H} \left(\prod_{s=1}^{t-1} (1-\lambda e_s^{L,H})\right) [e_t^{L,H} \lambda y_t^H + (1-\lambda e_t^{L,H}) x_t^H - e_t^{L,H} \gamma] \\ -\left(1-\beta_0^L + \beta_0^L \left(\prod_{s=1}^{T^H} (1-\lambda e_s^{L,H})\right)\right) \left(c_{T^H+1}^H - c^L_{\sum_{s=1}^{T^H+1} e_s^{L,H}}\right) q_F \end{array}\right]$$

$$+ \sum_{t=1}^{T^H} \mu_t^H \left[y_t^H - x_t^H - \frac{\gamma}{\lambda \beta_t^H} - \sum_{s=t+1}^{T^H} (1-\lambda)^{s-t-1} (\lambda y_s^H + (1-\lambda) x_s^H - \gamma) - \frac{(1-\beta_0^H)}{P_{T^H}^H} \Delta c q_F\right]$$

$$+\sum_{t=1}^{T^L}\mu_t^L\left[y_t^L - x_t^L - \frac{\gamma}{\lambda\beta_t^L} - \sum_{s=t+1}^{T^L}(1-\lambda)^{s-t-1}(\lambda y_s^L + (1-\lambda)x_s^L - \gamma) - \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right]$$

$$+\sum_{t=1}^{T^H}\eta_t^H x_t^H + \sum_{t=1}^{T^L}\eta_t^L x_t^L.$$

The relevant Kuhn-Tucker conditions for the optimization problem are:

(C1) $\frac{\partial\mathcal{L}}{\partial y_t^H} = -\nu\beta_0^H(1-\lambda)^{t-1}\lambda + \xi^H\beta_0^H(1-\lambda)^{t-1}\lambda - \xi^L\beta_0^L\left(\prod_{s=1}^{t-1}(1-\lambda e_s^{L,H})\right)\lambda e_t^{L,H}$
$+\mu_t^H - \sum_{j=1}^{t-1}\mu_j^H(1-\lambda)^{t-j-1}\lambda = 0;$

(C2) $\frac{\partial\mathcal{L}}{\partial y_t^L} = -(1-\nu)\beta_0^L(1-\lambda)^{t-1}\lambda - \xi^H\beta_0^H\left(\prod_{s=1}^{t-1}(1-\lambda e_s^{H,L})\right)\lambda e_t^{H,L}$
$+\xi^L\beta_0^L(1-\lambda)^{t-1}\lambda + \mu_t^L - \sum_{j=1}^{t-1}\mu_j^L(1-\lambda)^{t-j-1}\lambda = 0;$

(C3) $\frac{\partial\mathcal{L}}{\partial x_t^H} = -\nu P_t^H + \xi^H P_t^H - \xi^L\left(1-\beta_0^L + \beta_0^L\prod_{s=1}^{t}(1-\lambda e_s^{L,H})\right)$
$-\mu_t^H - \sum_{j=1}^{t-1}\mu_j^H(1-\lambda)^{t-j} + \eta_t^H = 0;$

(C4) $\frac{\partial\mathcal{L}}{\partial x_t^L} = -(1-\nu)P_t^L - \xi^H\left(1-\beta_0^H + \beta_0^H\prod_{s=1}^{t}(1-\lambda e_s^{H,L})\right) + \xi^L P_t^L$
$-\mu_t^L - \sum_{j=1}^{t-1}\mu_j^L(1-\lambda)^{t-j} + \eta_t^L = 0.$

Optimal payment structure (Proposition 3)

We show next that both $(IC^{H,L})$ and $(IC^{L,H})$ may be slack, and either or both may be binding simultaneously. We examine each case below.

Case 1: *Both the $(IC^{H,L})$ and $(IC^{L,H})$ constraints are slack.*

If both the (IC) constraints are slack, it is because the moral hazard rent for each type is sufficiently high so that neither is interested in misreporting their type. The moral hazard constraints for both types are binding in every period. Each type is rewarded after success in every period to satisfy the moral hazard constraints, and none is rewarded after failure. This is Case 1.

Claim C1. $\xi^H = \xi^L = 0 \Rightarrow \eta_t^H, \eta_t^L, \mu_t^H, \mu_t^L > 0$ (it is optimal to set $x_t^\theta = 0$ and $y_t^\theta = \frac{\gamma}{\lambda\beta_{T^\theta}^\theta} + \frac{(1-\beta_0^\theta)}{P_{T^\theta}^\theta}\Delta c q_F$ for $t \le T^\theta$ and $\theta \in \{H, L\}$).

Proof:

$\boldsymbol{\mu_t^\theta} > \mathbf{0}$. We first prove that if the (IC) constraints are slack, then all the (MH_t^θ) constraints for $t \le T^\theta$ and $\theta \in \{H, L\}$ must be binding.

$\boldsymbol{\mu_t^H} > \mathbf{0}$. Given that $\xi^H = \xi^L = 0$, $(C1)$ at each period $t \le T^H$ can be rewritten as

$t = 1$: $-\nu\beta_0^H\lambda + \mu_1^H = 0 \implies \mu_1^H = \nu\beta_0^H\lambda > 0;$

$t = 2$: $-\nu\beta_0^H(1-\lambda)\lambda + \mu_2^H - \lambda\mu_1^H = 0 \implies \mu_2^H = \nu\beta_0^H\lambda > 0;$

Solving recursively for $t = 3, \dots, T^H$ we have

$$\mu_t^H = \nu\beta_0^H\lambda > 0 \text{ for } t \le T^H.$$

Thus, all the (MH_t^H) constraints are binding.

$\boldsymbol{\mu_t^L > 0}$. Given that $\xi^H = \xi^L = 0$, $(C2)$ at each period $t \leq T^L$ can be rewritten as

$t = 1$: $-(1-\nu)\beta_0^L \lambda + \mu_1^L = 0 \implies \mu_1^L = (1-\nu)\beta_0^L \lambda > 0$;

$t = 2$: $-(1-\nu)\beta_0^L(1-\lambda)\lambda + \mu_2^L - \lambda\mu_1^L = 0 \implies \mu_2^L = (1-\nu)\beta_0^L \lambda > 0$;

Solving recursively for $t = 3, \dots, T^L$ we have

$$\mu_t^L = (1-\nu)\beta_0^L \lambda > 0 \text{ for } t \leq T^L.$$

Thus, all the (MH_t^L) constraints are binding.

$\boldsymbol{x_t^\theta = 0}$. This follows immediately from $(C3)$ and $(C4)$ as $\xi^H = \xi^L = 0$ implies $\eta_t^\theta > 0$ for all $t \leq T^\theta$. Furthermore, given that $\mu_t^H = \nu\beta_0^H\lambda$, and $\mu_t^L = (1-\nu)\beta_0^L\lambda$, we can also show that $\eta_t^H = \nu(P_t^H + \beta_0^H\lambda) + \nu\beta_0^H\lambda\sum_{j=1}^{t-1}(1-\lambda)^{t-j} > 0$ for $t \leq T^H$, and that $\eta_t^L = (1-\nu)(P_t^L + \beta_0^L\lambda) + (1-\nu)\beta_0^L\lambda\sum_{j=1}^{t-1}(1-\lambda)^{t-j} > 0$ for $t \leq T^L$.

Thus, if the (IC) constraints are slack, both types rewarded only for success and all the (MH_t^θ) constraints for $t \leq T^\theta$ and $\theta \in \{H, L\}$ are be binding:

$$y_t^\theta = \frac{\gamma}{\lambda\beta_t^\theta} + \sum_{s=t+1}^{T^\theta}(1-\lambda)^{s-t-1}\left(\lambda y_s^\theta - \gamma\right) + \frac{(1-\beta_0^\theta)}{P_T^\theta}\Delta c q_F,$$

$$x_t^\theta = 0 \text{ for } t \leq T^\theta \text{ and } \theta \in \{H, L\}.$$

Finally, in Supplementary Appendix B, we proved that the unique sequence of y_t^θ that solves the system of binding (MH_t^θ) constraints is:

$$y_t^\theta = \frac{\gamma}{\lambda\beta_{T^\theta}^\theta} + \frac{(1-\beta_0^\theta)}{P_{T^\theta}^\theta}\Delta c q_F \text{ for } t \leq T^\theta \text{ and } \theta \in \{H, L\}.$$

This conclude the proof of Claim C1. *Q.E.D.*

Case 2: *The $(IC^{L,H})$ constraint binds and $(IC^{H,L})$ is slack.*

If $(IC^{H,L})$ is slack, it is because the high type's moral hazard rent is sufficiently high such that he is not interested in pretending to be the low type. It must be that (MH_t^H) is binding in each period. Moreover, the low type could now be interested in pretending to be the high type making $(IC^{L,H})$ binding. The adverse selection rent to the low type is sufficient to satisfy all moral hazard constraints for the low type at no extra cost. Thus, we have that (MH_t^L) are all slack. This is Case 2.

Claim C2. $\xi^H = 0$, $\xi^L > 0 \Rightarrow \eta_t^H, \mu_t^H > 0$ and $\mu_t^L = 0$, $\eta_t^L = 0$ (it is optimal to set $x_t^H = 0$ and $y_t^H = \frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F$ for $t \leq T^H$ and any combination of x_t^L and y_t^L such that $y_t^L - x_t^L \geq \frac{\gamma}{\lambda\beta_t^L} + \sum_{s=t+1}^{T^L}(1-\lambda)^{s-t-1}(\lambda y_s^L + (1-\lambda)x_s^L - \gamma) + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F$ for $t \leq T^L$ and

$(1-\beta_0^L)\sum_{t=1}^{T^L}[x_t^L - \gamma] + \beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}[(\lambda y_t^L - \gamma) + (1-\lambda)x_t^L] = U^L\left(\varpi^H, \vec{a}^L(\varpi^H)\right)$).

Proof:
H-type. We first prove that the high type is rewarded only for success and all the (MH_t^H) constraints are binding for $t \le T^H$.

$\boldsymbol{\mu_t^H > 0}$**.** Given that $\xi^H = 0$ and $\xi^L > 0$, condition $(C1)$ at each period $t \le T^H$ can be rewritten as

$$\mu_t^H = \nu\beta_0^H(1-\lambda)^{t-1}\lambda + \xi^L\beta_0^L\left(\prod_{s=1}^{t-1}(1-\lambda e_s^{L,H})\right)\lambda e_t^{L,H} + \sum_{j=1}^{t-1}\mu_j^H(1-\lambda)^{t-j-1}\lambda > 0 \text{ for } t \le T^H.$$

Thus, all the (MH_t^H) constraints are binding.

We next prove that the high type is rewarded only for success, i.e., $x_t^H = 0$ for $t \le T^H$.
$\boldsymbol{x_t^H = 0}$**.** Given that $\xi^H = 0$ and $\xi^L > 0$ condition $(C3)$ at each period $t \le T^H$ can be rewritten as

$$\eta_t^H = \nu P_t^H + \xi^L\left(1-\beta_0^L + \beta_0^L\prod_{s=1}^{t}(1-\lambda e_s^{L,H})\right) + \mu_t^H + \sum_{j=1}^{t-1}\mu_j^H(1-\lambda)^{t-j} > 0 \text{ for } t \le T^H.$$

Therefore, $\eta_t^H > 0$ for every $t \le T^H$ and, as a result, the high type is not rewarded for failures:

$$x_t^H = 0 \text{ for } t \le T^H.$$

Thus, the high type is rewarded only for success with the rewards given by:

$$y_t^H = \frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F \text{ for } t \le T^H.$$

L-type. We next characterize the optimal contract for the low type. In Lemma C1, we characterize the off-equilibrium effort level for the low type $\vec{a}^L(\varpi^H)$. We prove that if the low type claims being high, he works for $t^{L,H} \le T^H$ periods and shirks in other periods. For expositional ease, we consider that that agent works for the first $t^{L,H}$ periods without loss of generality.
Lemma C1. In Case 2, a lying low type works for $t^{L,H}$ periods and shirks otherwise: there exists $t^{L,H}$, such that $0 \le t^{L,H} \le T^H$ and $\sum_{s=1}^{T^H} e_s^{L,H} = t^{L,H}$.
Proof: First, it is without loss of generality to consider an off-the-equilibrium effort path where the low type works in consecutive periods. The reason is that (i) all the rewards for success are identical and, (ii) the low type's probability of success in any period and the expected cost after failure depends on the total number of failures up to that period (not on when those failures occurred).[45] Second, for the same reason, it is without loss of generality to consider an off-the-

[45] On the RHS of $(IC^{L,H})$, we focus on the middle term $\beta_0^L\sum_{t=1}^{T^H}\left(\prod_{s=1}^{t-1}(1-\lambda e_s^{L,H})\right)e_t^{L,H}[\lambda y_t^H - \gamma]$ to see that when shirking occurs does not matter to the total payoff since y_t^H is constant. To illustrate, consider the case of $T^H = 3$:

$$\beta_0^L\sum_{t=1}^{3}\left(\prod_{s=1}^{t-1}(1-\lambda e_s^{L,H})\right)e_t^{L,H}[\lambda y_t^H - \gamma] = \{e_1^{L,H} + (1-\lambda e_1^{L,H})e_2^{L,H} + (1-\lambda e_1^{L,H})(1-\lambda e_2^{L,H})e_3^{L,H}\}[\lambda y_t^H - \gamma].$$

Suppose that the agent works in any two periods. It is easy to see that we have identical payoffs no matter which period the agent chooses to shirk.

equilibrium effort path where $\vec{a}^L(\varpi^H)$ is a stopping rule: the low type works up to period $t^{L,H} \leq T^H$ and shirks after.[46]

Therefore, by replacing $x_t^H = 0$ for $t \leq T^H$ and using Lemma C1 on the *RHS* of the $(IC^{L,H})$, the low type's expected utility function when he pretends being the high type can be written as $U^L(\varpi^H, \vec{a}^L(\varpi^H)) =$

$$(1-\beta_0^L)\sum_{t=1}^{T^H}[x_t^H - \gamma e_t^{L,H}] + \beta_0^L \sum_{t=1}^{T^H}\left(\prod_{s=1}^{t-1}(1-\lambda e_s^{L,H})\right)[e_t^{L,H}\lambda y_t^H + (1-\lambda e_t^{L,H})x_t^H - e_t^{L,H}\gamma]$$

$$\left(1-\beta_0^L + \beta_0^L\left(\prod_{s=1}^{T^H}(1-\lambda e_s^{L,H})\right)\right)\left(c_{T^H+1}^H - c_{\sum_{s=1}^{T^H+1} e_s^{L,H}}^L\right)q_F$$

$$= \beta_0^L \sum_{t=1}^{t^{L,H}}(1-\lambda)^{t-1}\lambda y_t^H\,(1-\beta_0^L)\sum_{t=1}^{t^{L,H}}[-\gamma] + \beta_0^L\sum_{t=1}^{t^{L,H}}(1-\lambda)^{t-1}[-\gamma]$$
$$+P_{t^{L,H}}^L\left(c_{T^H+1}^H - c_{t^{L,H}+1}^L\right)q_F$$

$$= \beta_0^L \sum_{t=1}^{t^{L,H}}(1-\lambda)^{t-1}\lambda y_t^H - \sum_{t=1}^{t^{L,H}} P_{t-1}^L\gamma + P_{t^{L,H}}^L\left(c_{T^H+1}^H - c_{t^{L,H}+1}^L\right)q_F,$$

where we define $t^{L,H}$ as a time period that maximizes $U^L(\varpi^H, \vec{a}^L(\varpi^H))$:

$$t^{L,H} \coloneqq \arg\max_{0\leq t^{L,H}\leq T^H} U^L\left(\varpi^H, \vec{a}^L(\varpi^H)\right).$$

This concludes the proof of Lemma C1. *Q.E.D.*

To recap, given the definition of $t^{L,H}$ above the binding $(IC^{L,H})$ is given by,

$$\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}y_t^L + \sum_{t=1}^{T^L}P_t^L x_t^L - \sum_{t=1}^{T^L}P_{t-1}^L\gamma =$$
$$\beta_0^L\sum_{t=1}^{t^{L,H}}(1-\lambda)^{t-1}\lambda y_t^H - \sum_{t=1}^{t^{L,H}}P_{t-1}^L\gamma + P_{t^{L,H}}^L\left(c_{T^H+1}^H - c_{t^{L,H}+1}^L\right)q_F.$$

Next, we prove that the principal can use any combination of x_t^L and y_t^L such that

$$y_t^L - x_t^L \geq \frac{\gamma}{\lambda\beta_t^L} + \sum_{s=t+1}^{T^L}(1-\lambda)^{s-t-1}(\lambda y_s^L + (1-\lambda)x_s^L - \gamma) + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F \text{ for } t \leq T^L, \text{ and}$$

$$(1-\beta_0^L)\sum_{t=1}^{T^L}[x_t^L - \gamma] + \beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}[(\lambda y_t^L - \gamma) + (1-\lambda)x_t^L] = U^L\left(\varpi^H, \vec{a}^L(\varpi^H)\right).$$

$\boldsymbol{\mu_t^L = \eta_t^L = 0}$**.** Given that $\xi^H = 0$, conditions $(C2)$ and $(C4)$ can be rewritten as

(C2) $\frac{\partial\mathcal{L}}{\partial y_t^L} = -(1-\nu)\beta_0^L(1-\lambda)^{t-1}\lambda + \xi^L\beta_0^L(1-\lambda)^{t-1}\lambda + \mu_t^L - \sum_{j=1}^{t-1}\mu_j^L(1-\lambda)^{t-j-1}\lambda = 0;$

(C4) $\frac{\partial\mathcal{L}}{\partial x_t^L} = -(1-\nu)P_t^L + \xi^L P_t^L - \mu_t^L - \sum_{j=1}^{t-1}\mu_j^L(1-\lambda)^{t-j} + \eta_t^L = 0.$

There exists a solution to $(C2)$ and $(C4)$ for $t \leq T^L$ such that

$$\mu_t^L = \eta_t^L = 0 \text{ and } \xi^L = (1-\nu) \text{ for } t \leq T^L.$$

Therefore, the principal can use any combination of x_t^L and y_t^L such that

$$y_t^L - x_t^L \geq \frac{\gamma}{\lambda\beta_t^L} + \sum_{s=t+1}^{T^L}(1-\lambda)^{s-t-1}(\lambda y_s^L + (1-\lambda)x_s^L - \gamma) + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F \text{ for } t \leq T^L, \text{ and}$$

$$(1-\beta_0^L)\sum_{t=1}^{T^L}[x_t^L - \gamma] + \beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}[(\lambda y_t^L - \gamma) + (1-\lambda)x_t^L] = U^L\left(\varpi^H, \vec{a}^L(\varpi^H)\right).$$

This concludes the proof of Claim C2. *Q.E.D.*

[46] Alternatively, we could write that the agent worked for $t^{L,H}/T^H$ periods, but the notation would be cumbersome as we would need to indicate the periods he works in.

Case 3: *The* $(IC^{H,L})$ *constraint binds and* $(IC^{L,H})$ *is slack.*

The case where $(IC^{H,L})$ binds and $(IC^{L,H})$ is slack is similar to that under a standard adverse selection problem except that the low type also receives a rent due to moral hazard. The (MH_t^L) are binding in each period. As $(IC^{H,L})$ is binding, the high type's moral hazard rent is not sufficiently high to deter him from misreporting. We find that the high-type's adverse selection rent is sufficient to satisfy all moral hazard constraints for the high type at no extra cost. Thus, we have (MH_t^H) are all slack. This is Case 3.

Claim C3. $\xi^H > 0$, $\xi^L = 0 \Rightarrow \eta_t^L, \mu_t^L > 0$ and $\eta_t^H = \mu_t^H = 0$ (it is optimal to set $x_t^L = 0$ and $y_t^L = \frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F$ for $t \le T^L$ and any combination of x_t^H and y_t^H such that $y_t^H - x_t^H \ge \frac{\gamma}{\lambda\beta_t^H} + \sum_{s=t+1}^{T^H}(1-\lambda)^{s-t-1}(\lambda y_s^H + (1-\lambda)x_s^H - \gamma) + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F$ for $t \le T^H$ and

$$(1-\beta_0^H)\sum_{t=1}^{T^H}[x_t^H - \gamma] + \beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1}[(\lambda y_t^H - \gamma) + (1-\lambda)x_t^H] = U^H(\varpi^L, \vec{1})).$$

Proof:

L-type. We first prove that the low type is rewarded only for success and all the (MH_t^L) constraints are binding for $t \le T^L$.

$\boldsymbol{\mu_t^L > 0}$**.** Given that $\xi^L = 0$ and $\xi^H > 0$, condition $(C2)$ at each period $t \le T^L$ can be rewritten as

$$\mu_t^L = (1-\nu)\beta_0^L(1-\lambda)^{t-1}\lambda + \xi^H\beta_0^H\left(\prod_{s=1}^{t-1}\left(1-\lambda e_s^{H,L}\right)\right)\lambda e_t^{H,L} + \sum_{j=1}^{t-1}\mu_j^L(1-\lambda)^{t-j-1}\lambda > 0 \text{ for } t \le T^L.$$

Thus, all the (MH_t^L) constraints are binding.

We next prove that the low type is rewarded only for success, i.e., $x_t^L = 0$ for $t \le T^L$.

$\boldsymbol{x_t^L = 0}$**.** Given that $\xi^L = 0$ and $\xi^H > 0$ condition $(C4)$ at each period $t \le T^L$ can be rewritten as

$$\eta_t^L = (1-\nu)P_t^L + \xi^H\left(1-\beta_0^H + \beta_0^H\prod_{s=1}^{t}\left(1-\lambda e_s^{H,L}\right)\right) + \mu_t^L + \sum_{j=1}^{t-1}\mu_j^L(1-\lambda)^{t-j} > 0 \text{ for } t \le T^L.$$

Therefore, $\eta_t^L > 0$ for every $t \le T^L$ and, as a result, the low type is not rewarded for failures:

$$x_t^L = 0 \text{ for } t \le T^L.$$

Thus, the low type is rewarded only for success with the rewards given by:

$$y_t^L = \frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F \text{ for } t \le T^L.$$

H-type. We next characterize the optimal contract for the high type.

$\boldsymbol{\vec{a}^H(\varpi^L) = \vec{1}}$**.** In Lemma C2, we that the high type never shirks off-the-equilibrium path.

Lemma C2. In Case 3, a lying high type works off the equilibrium path: $e_t^{H,L} = 1$ for $t \le T^L$.

Proof: First, recall that the global moral hazard constraint is implied by the sequence of local ones (see Supplementary Appendix B for a formal proof). Second, consider the high type's incentives to engage in a one-shot deviation and shirk at period $t \le T^L$. Suppose the high type accepts a contract designed for the low type and deviates only at some period $t \le T^L$, then his continuation value from the relationship is

$$x_t^L + \beta_t^H \sum_{s=t+1}^{T^L} (1-\lambda)^{s-t-1}[\lambda y_s^L + (1-\lambda)x_s^L - \gamma] + (1-\beta_t^H)\sum_{s=t+1}^{T^L}(x_s^L - \gamma)$$
$$+\left(1-\beta_t^H + \beta_t^H(1-\lambda)^{T^L-t}\right)\left(c_{T^L+1}^L - c_{T^L}^H\right)q_F.$$

In contrast, if the lying high type decides to work at period t, his continuation value from the relationship becomes

$$-\gamma + \lambda\beta_t^H y_t^L + (1-\lambda\beta_t^H)x_t^L + \beta_t^H \sum_{s=t+1}^{T^L}(1-\lambda)^{s-t}[\lambda y_s^L + (1-\lambda)x_s^L - \gamma] +$$
$$(1-\beta_t^H)\sum_{s=t+1}^{T^L}(x_s^L - \gamma) + \left(1-\beta_t^H + \beta_t^H(1-\lambda)^{T^L-t+1}\right)\left(c_{T^L+1}^L - c_{T^L+1}^H\right)q_F.$$

By combining the two continuation values presented above, we can write a one-period moral hazard constraint at period t below for the lying high type, which we denote by $\left(MH_t^{H,L}\right)$:

$$\left(MH_t^{H,L}\right)\ y_t^L - x_t^L \geq \frac{\gamma}{\lambda\beta_t^H} + \sum_{s=t+1}^{T^L}(1-\lambda)^{s-t-1}(\lambda y_s^L + (1-\lambda)x_s^L - \gamma)$$
$$+\frac{(1-\lambda)^{T^L-t}(1-\beta_0^L)(\bar{c}-\underline{c})q_F}{1-\beta_0^L+\beta_0^L(1-\lambda)^{T^L}}.$$

Third, we prove that if $y_t^L = \frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F$ for $t \leq T^L$, then $\left(MH_t^{H,L}\right)$ is satisfied for every $t \leq T^L$. Replacing $x_s^L = 0$, the $\left(MH_t^{H,L}\right)$ constraint simplifies to:

$$y_t^L \geq \frac{\gamma}{\lambda\beta_t^H} + \sum_{s=t+1}^{T^L}(1-\lambda)^{s-t-1}(\lambda y_s^L - \gamma) + \frac{(1-\lambda)^{T^L-t}(1-\beta_0^L)(\bar{c}-\underline{c})q_F}{P_{T^L}^L},$$

$$\frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F \geq \frac{\gamma}{\lambda\beta_t^H} + \left(\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right] - \gamma\right)\sum_{s=t+1}^{T^L}(1-\lambda)^{s-t-1}$$
$$+\frac{(1-\lambda)^{T^L-t}(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F.$$

Given that $\sum_{s=t+1}^{T^L}(1-\lambda)^{s-t-1} = \sum_{j=0}^{T^L-t-1}(1-\lambda)^j = \frac{1-(1-\lambda)^{T^L-t}}{1-(1-\lambda)} = \frac{1-(1-\lambda)^{T^L-t}}{\lambda}$,

the $\left(MH_t^{H,L}\right)$ constraint can be rewritten as:

$$\frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F \geq \frac{\gamma}{\lambda\beta_t^H} + \left(\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right] - \gamma\right)\left(\frac{1-(1-\lambda)^{T^L-t}}{\lambda}\right)$$
$$+\frac{(1-\lambda)^{T^L-t}(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F,$$

$$\frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F \geq \frac{\gamma}{\lambda\beta_t^H} + \left(\left[\frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right] - \frac{\gamma}{\lambda}\right)\left(1-(1-\lambda)^{T^L-t}\right)$$
$$+\frac{(1-\lambda)^{T^L-t}(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F,$$

$$\frac{\gamma}{\lambda\beta_{T^L}^L} \geq \frac{\gamma}{\lambda\beta_t^H} + \left(\left[\frac{\gamma}{\lambda\beta_{T^L}^L}\right] - \frac{\gamma}{\lambda}\right)\left(1-(1-\lambda)^{T^L-t}\right)$$
$$+\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\left((1-\lambda)^{T^L-t} - 1 + 1 - (1-\lambda)^{T^L-t}\right),$$

$$\frac{1}{\beta_{T^L}^L} \geq \frac{1}{\beta_t^H} + \left(\frac{1}{\beta_{T^L}^L} - 1\right)\left(1-(1-\lambda)^{T^L-t}\right),$$

$$1 \geq \frac{\beta_{T^L}^L}{\beta_t^H} + \left(1-\beta_{T^L}^L\right)\left(1-(1-\lambda)^{T^L-t}\right),$$

$$\beta_t^H \geq \beta_{T^L}^L + \beta_t^H\left(1-\beta_{T^L}^L\right)\left(1-(1-\lambda)^{T^L-t}\right),$$

$$\frac{\beta_0^H(1-\lambda)^{t-1}}{1-\beta_0^H+\beta_0^H(1-\lambda)^{t-1}} \geq \frac{\beta_0^L(1-\lambda)^{T^L-1}}{1-\beta_0^L+\beta_0^L(1-\lambda)^{T^L-1}} + \frac{\beta_0^H(1-\lambda)^{t-1}}{1-\beta_0^H+\beta_0^H(1-\lambda)^{t-1}}\left(\frac{(1-\beta_0^L)\left(1-(1-\lambda)^{T^L-t}\right)}{1-\beta_0^L+\beta_0^L(1-\lambda)^{T^L-1}}\right),$$

$$\frac{\beta_0^H(1-\lambda)^{t-1}}{1-\beta_0^H+\beta_0^H(1-\lambda)^{t-1}} - \frac{\beta_0^L(1-\lambda)^{T^L-1}}{1-\beta_0^L+\beta_0^L(1-\lambda)^{T^L-1}} \geq \frac{\beta_0^H(1-\lambda)^{t-1}}{1-\beta_0^H+\beta_0^H(1-\lambda)^{t-1}}\left(\frac{(1-\beta_0^L)\left(1-(1-\lambda)^{T^L-t}\right)}{1-\beta_0^L+\beta_0^L(1-\lambda)^{T^L-1}}\right),$$

$$\frac{\beta_0^H(1-\lambda)^{t-1}\left(1-\beta_0^L+\beta_0^L(1-\lambda)^{T^L-1}\right)-\beta_0^L(1-\lambda)^{T^L-1}\left(1-\beta_0^H+\beta_0^H(1-\lambda)^{t-1}\right)}{\left(1-\beta_0^H+\beta_0^H(1-\lambda)^{t-1}\right)\left(1-\beta_0^L+\beta_0^L(1-\lambda)^{T^L-1}\right)}$$

$$\geq \frac{\beta_0^H(1-\lambda)^{t-1}}{\left(1-\beta_0^H+\beta_0^H(1-\lambda)^{t-1}\right)}\frac{(1-\beta_0^L)\left(1-(1-\lambda)^{T^L-t}\right)}{\left(1-\beta_0^L+\beta_0^L(1-\lambda)^{T^L-1}\right)},$$

$$\beta_0^H(1-\lambda)^{t-1} - \beta_0^L\beta_0^H(1-\lambda)^{t-1} - \beta_0^L(1-\lambda)^{T^L-1} + \beta_0^H\beta_0^L(1-\lambda)^{T^L-1}$$
$$\geq \beta_0^H(1-\lambda)^{t-1}\left(1-\beta_0^L-(1-\lambda)^{T^L-t}+\beta_0^L(1-\lambda)^{T^L-t}\right),$$

$$\beta_0^H(1-\lambda)^{t-1} - \beta_0^L\beta_0^H(1-\lambda)^{t-1} - \beta_0^L(1-\lambda)^{T^L-1} + \beta_0^H\beta_0^L(1-\lambda)^{T^L-1} \geq$$
$$\left(\beta_0^H(1-\lambda)^{t-1} - \beta_0^L\beta_0^H(1-\lambda)^{t-1} - \beta_0^H(1-\lambda)^{T^L-1} + \beta_0^H\beta_0^L(1-\lambda)^{T^L-1}\right),$$
$$-\beta_0^L(1-\lambda)^{T^L-1} \geq -\beta_0^H(1-\lambda)^{T^L-1},$$
$$\beta_0^H \geq \beta_0^L.$$

Therefore, all the $\left(MH_t^{H,L}\right)$ constraints are satisfied as strict inequalities and, as a result, the high type never shirks off-the-equilibrium if $y_t^L = \frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F$ for $t \leq T^L$.

This concludes the proof of Lemma C2. *Q.E.D.*

We next prove that the principal any combination of x_t^H and y_t^H such that all (MH_t^H) constraints are satisfied and the high type' expected rent is $U^H\left(\varpi^L,\vec{1}\right)$.

$\boldsymbol{\mu_t^H = \eta_t^H = 0}$**.** Given that $\xi^L = 0$, conditions $(C1)$ and $(C3)$ can be rewritten as

(C1) $\quad \frac{\partial\mathcal{L}}{\partial y_t^H} = -\nu\beta_0^H(1-\lambda)^{t-1}\lambda + \xi^H\beta_0^H(1-\lambda)^{t-1}\lambda + \mu_t^H - \sum_{j=1}^{t-1}\mu_j^H(1-\lambda)^{t-j-1}\lambda = 0;$

(C3) $\quad \frac{\partial\mathcal{L}}{\partial x_t^H} = -\nu P_t^H + \xi^H P_t^H - \mu_t^H - \sum_{j=1}^{t-1}\mu_j^H(1-\lambda)^{t-j} + \eta_t^H = 0.$

There exists a solution to $(C1)$ and $(C3)$ for $t \leq T^H$ such that
$\mu_t^H = \eta_t^H = 0$ and $\xi^H = \nu$ for $t \leq T^H$.

Therefore, the principal can use any combination of x_t^H and y_t^H such that

$$y_t^H - x_t^H \geq \frac{\gamma}{\lambda\beta_t^H} + \sum_{s=t+1}^{T^H}(1-\lambda)^{s-t-1}(\lambda y_s^H + (1-\lambda)x_s^H - \gamma) + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F \text{ for } t \leq T^H, \text{ and}$$

$$(1-\beta_0^H)\sum_{t=1}^{T^H}[x_t^H - \gamma] + \beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1}[(\lambda y_t^H - \gamma) + (1-\lambda)x_t^H] = U^H\left(\varpi^L,\vec{1}\right).$$

This concludes the proof of Claim C3. *Q.E.D.*

Case 4: *Both $(IC^{H,L})$ and $(IC^{L,H})$ bind.*

The case where both $(IC^{H,L})$ and $(IC^{L,H})$ are binding is similar to the Case B of the adverse selection only benchmark. The novel feature is that the principal has to pay an additional adverse selection rent on top of the moral hazard rent to both types.[47] Similarly, to the pure adverse selection benchmark, the low type receives his extra adverse selection rent after the very last failure. This is Case 4.

Claim C4. $\xi^H > 0$, $\xi^L > 0 \Rightarrow \eta_t^H > 0 = \mu_t^H$ for $t \le T^H$, $\mu_t^L > 0$ for $t \le T^L$, $\eta_t^L > 0 = \eta_{T^L}^L$ for $t < T^L$.

(it is optimal to set $\beta_0^H \sum_{t=1}^{T^H} (1-\lambda)^{t-1}(\lambda y_t^H - \gamma) - \gamma \sum_{t=1}^{T^H} P_t^H = U^H(\varpi^L, \vec{1})$,

$$y_{T^L}^L = x_{T^L}^L + \frac{\gamma}{\lambda \beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L} \Delta c q_F,$$

$$y_t^L = \frac{\gamma}{\lambda \beta_t^L} + \sum_{s=t+1}^{T^L} (1-\lambda)^{s-t-1}(\lambda y_s^L - \gamma) + (1-\lambda)^{T^L - t} x_{T^L}^L + \frac{(1-\beta_0^L)}{P_{T^L}^L} \Delta c q_F \text{ for } t < T^L,$$

and

$$\beta_0^L \sum_{t=1}^{T^L} (1-\lambda)^{t-1}(\lambda y_t^L - \gamma) - \gamma \sum_{t=1}^{T^L - 1} P_t^L + P_{T^L}^L (x_{T^L}^L - \gamma) = U^L(\varpi^H, \vec{a}^L(\varpi^H))).$$

Proof:

H-type. We first describe the optimal contract for the high type and prove that the high type is rewarded only for success with the rewards distributed such that all the (MH_t^H) constraints are satisfied (at no additional cost, i.e., $\mu_t^H = 0$ for $t \le T^H$).

$\boldsymbol{x_t^H = 0}$**.** First, the high type is rewarded only for success. The reason is that rewarding the high type for failures does not allow mitigating the low type's rent since he is relatively more likely to fail if he works off-the-equilibrium (the high type is not rewarded for failures in a benchmark without moral hazard, see Supplementary Appendix A for a formal proof) and, in addition, he is even more likely to fail if he shirks off-the-equilibrium. Therefore, rewarding the high type for failures does not mitigate the low type's rent. In addition, rewarding the high type for failures makes it only more difficult to satisfy the (MH_t^H) constraints. Therefore, the high type is not rewarded for failures:

$$x_t^H = 0 \text{ for } t \le T^H.$$

$\boldsymbol{y_t^H \ge 0}$**.** We next prove that the principal any combination of y_t^H such that all (MH_t^H) constraints are satisfied and the high type' expected rent is $U^H(\varpi^L, \vec{1})$.

$\boldsymbol{\mu_t^H = 0}$**.** Conditions $(C1)$ and $(C3)$ can be rewritten as

$$\textbf{(C1)}\ \frac{\partial \mathcal{L}}{\partial y_t^H} = -\nu \beta_0^H (1-\lambda)^{t-1}\lambda + \xi^H \beta_0^H (1-\lambda)^{t-1}\lambda - \xi^L \beta_0^L \left(\prod_{s=1}^{t-1}(1-\lambda e_s^{L,H})\right)\lambda e_t^{L,H}$$
$$+\mu_t^H - \sum_{j=1}^{t-1} \mu_j^H (1-\lambda)^{t-j-1}\lambda = 0;$$

$$\textbf{(C3)}\ \frac{\partial \mathcal{L}}{\partial x_t^H} = -\nu P_t^H + \xi^H P_t^H - \xi^L \left(1 - \beta_0^L + \beta_0^L \prod_{s=1}^{t}(1-\lambda e_s^{L,H})\right)$$
$$-\mu_t^H - \sum_{j=1}^{t-1} \mu_j^H (1-\lambda)^{t-j} + \eta_t^H = 0;$$

There exists a solution to $(C1)$ and $(C3)$ for $t \le T^H$ such that for all $t \le T^H$:

$$\mu_t^H = 0;$$

[47] We ignore the knife-edge case where both the (IC) constraints and all the (MH_t^H) are binding simultaneously. In that case, the adverse selection rent is exactly equal to the moral hazard rent and there is no extra rent to be paid.

$$\xi^L = \eta_t^H \frac{\beta_0^H(1-\lambda)^{t-1}\lambda}{\left(\left(1-\beta_0^L+\beta_0^L\prod_{s=1}^{t}\left(1-\lambda e_s^{L,H}\right)\right)\beta_0^H(1-\lambda)^{t-1}\lambda-\beta_0^L\left(\prod_{s=1}^{t-1}\left(1-\lambda e_s^{L,H}\right)\right)\lambda e_t^{L,H}P_t^H\right)} > 0;$$

$$\xi^H\beta_0^H(1-\lambda)^{t-1}\lambda P_t^H = \nu\beta_0^H(1-\lambda)^{t-1}\lambda P_t^H + \xi^L\beta_0^L\left(\prod_{s=1}^{t-1}\left(1-\lambda e_s^{L,H}\right)\right)\lambda e_t^{L,H}P_t^H > 0.$$

Therefore, the principal sets $x_t^H = 0$ and uses any combination of y_t^H such that

$$y_t^H \geq \frac{\gamma}{\lambda\beta_t^H} + \sum_{s=t+1}^{T^H}(1-\lambda)^{s-t-1}(\lambda y_s^H - \gamma) + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F \text{ for } t \leq T^H, \text{ and}$$

$$\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}(\lambda y_t^H - \gamma) - \gamma\sum_{t=1}^{T^H} P_t^H = U^H\left(\varpi^L, \vec{1}\right).$$

L-type. We now describe the optimal contract for the low type.

$\boldsymbol{\mu_t^L > 0}$ **for** $\boldsymbol{t \leq T^L}$**.** Combining $(C2)$ and $(C4)$ we have:[48]

$$\xi^H\lambda\begin{bmatrix}\beta_0^L(1-\beta_0^H)(1-\lambda)^{t-1} - \beta_0^H(1-\beta_0^L)e_t^{H,L}\prod_{s=1}^{t-1}\left(1-\lambda e_s^{H,L}\right) \\ +\beta_0^L\beta_0^H(1-\lambda)^{t-1}\left(1-e_t^{H,L}\right)\prod_{s=1}^{t-1}\left(1-\lambda e_s^{H,L}\right)\end{bmatrix} + \mu_t^L P_{t-1}^L$$
$$= \beta_0^L(1-\lambda)^{t-1}\lambda\eta_t^L + (1-\beta_0^L)\sum_{j=1}^{t-1}\mu_j^L(1-\lambda)^{t-j-1}\lambda \text{ for } t \leq T^L.$$

Since that $e_t^{H,L} = 1$ for $t \leq T^L$ by Lemma $C2$, the condition above simplifies to

$(\boldsymbol{C5})$
$$-\xi^H\lambda(1-\lambda)^{t-1}(\beta_0^H - \beta_0^L) + \mu_t^L P_{t-1}^L$$
$$= \beta_0^L(1-\lambda)^{t-1}\lambda\eta_t^L + (1-\beta_0^L)\sum_{j=1}^{t-1}\mu_j^L(1-\lambda)^{t-j-1}\lambda \text{ for } t \leq T^L.$$

Given that the *RHS* of $(C5)$ is non-negative, we have, $\mu_t^L > 0$ for every $t \leq T^L$ and, as a result, the (MH_t^L) constraints must be binding:

$$y_t^L - x_t^L = \frac{\gamma}{\lambda\beta_t^L} + \sum_{s=t+1}^{T^L}(1-\lambda)^{s-t-1}(\lambda y_s^L + (1-\lambda)x_s^L - \gamma) + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F \text{ for } t \leq T^L.$$

$\boldsymbol{x_{T^L}^L > x_t^L = 0}$ **for** $\boldsymbol{t < T^L}$**.** Since the low type gets a rent higher than the one determined by the binding (MH_t^L) constraint with $x_t^L = 0$ for $t \leq T^L$, we must have $x_t^L \geq 0$ with a strict inequality for some t. Following the same two steps as in the benchmark with adverse selection only (see Supplementary Appendix A), it follows that the low type is rewarded for failure in the very last period only.[49]

Therefore, the low type's payments are determined by

$$y_{T^L}^L = x_{T^L}^L + \frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F,$$

$$y_t^L = \frac{\gamma}{\lambda\beta_t^L} + \sum_{s=t+1}^{T^L}(1-\lambda)^{s-t-1}(\lambda y_s^L - \gamma) + (1-\lambda)^{T^L-t}x_{T^L}^L + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F \text{ for } t < T^L,$$

and

$$U^L\left(\varpi^H, \vec{a}^L(\varpi^H)\right) = \beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}(\lambda y_t^L - \gamma) - \gamma\sum_{t=1}^{T^L-1}P_t^L + P_{T^L}^L\left(x_{T^L}^L - \gamma\right).$$

[48] We multiply (C4) by $\beta_0^L(1-\lambda)^{t-1}\lambda$ and subtract it from (C2) multiplied by P_t^L.

[49] First, low type is rewarded for failure in only one period $s \leq T^L$. Second, it is optimal to reward the low type for the very last failure, i.e., $s = T^L$.

$\vec{a}^L(\varpi^H)$**.** Given the high type's rewards are front loaded, an off-the-equilibrium effort path $\vec{a}^L(\varpi^H)$ is a stopping rule: the low type works up to period $t^{L,H} \leq T^H$ and shirks thereafter. Therefore, the low type's expected payoff when he pretends being high can be written as:

$$U^L\left(\varpi^H, \vec{a}^L(\varpi^H)\right) = -\gamma(1-\beta_0^L)t^{L,H} + \beta_0^L \sum_{t=1}^{t^{L,H}} (1-\lambda)^{t-1} \left(\lambda y_t^H - \gamma\right) + P_{t^{L,H}}^L \left(c_{T^H+1}^H - c_{t^{L,H}+1}^L\right) q_F,$$

where $t^{L,H}$ is the number of time periods that maximizes $U^L\left(\varpi^H, \vec{a}^L(\varpi^H)\right)$:

$$t^{L,H} \coloneqq \arg \max_{0 \leq t^{L,H} \leq T^H} U^L\left(\varpi^H, \vec{a}^L(\varpi^H)\right).$$

This concludes the proofs of Claim C4 and part (i) of Proposition 3. *Q.E.D.*

Optimal length of experimentation (Proposition 4)

Case 1: Both the $(IC^{H,L})$ and $(IC^{L,H})$ constraints are slack (*under* experimentation for both types).

Information rents for both types are given by

$$U^\theta = \beta_0^\theta \sum_{t=1}^{T^\theta} (1-\lambda)^{t-1} \lambda y_t^\theta, \text{ where } y_t^\theta = \frac{\gamma}{\lambda \beta_{T^\theta}^\theta} + \frac{(1-\beta_0^\theta)}{P_{T^\theta}^\theta} \Delta c q_F \text{ for } t \leq T^\theta \text{ and } \theta \in \{H, L\}.$$

Since the agent's information rent is increasing in T^θ, there will be *under* experimentation for both types, that is, $T_{SB}^\theta < T_{FB}^\theta$ for $\theta \in \{H, L\}$.

Case 2: The $(IC^{L,H})$ binds and $(IC^{H,L})$ constraint is slack ($T_{SB}^L = T_{FB}^L$, *under* experimentation for the high type).

Information rents for both types are given by

$$U^H = \beta_0^H \sum_{t=1}^{T^H} (1-\lambda)^{t-1} \lambda y_t^H, \text{ and}$$

$$U^L\left(\varpi^H, \vec{a}^L(\varpi^H)\right) = -\gamma(1-\beta_0^L)t^{L,H} + \beta_0^L \sum_{t=1}^{t^{L,H}} (1-\lambda)^{t-1} \left(\lambda y_t^H - \gamma\right) + P_{t^{L,H}}^L \left(c_{T^H+1}^H - c_{t^{L,H}+1}^L\right) q_F,$$

where $y_t^H = \frac{\gamma}{\lambda \beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H} \Delta c q_F$ for $t \leq T^H$.

Since the informational rent of the low-type agent is non-monotonic in T^H, it is possible, in general, to have both $T_{SB}^H < T_{FB}^H$ and $T_{SB}^H > T_{FB}^H$. The stopping time for the low type, T^L, does not affect information rents and, as a result, is not distorted: $T_{SB}^L = T_{FB}^L$.

Case 3: The $(IC^{H,L})$ binds and $(IC^{L,H})$ constraint is slack ($T_{SB}^H = T_{FB}^H$, $T_{SB}^L < T_{FB}^L$ or $T_{SB}^L > T_{FB}^L$).

Information rents for both types are given by

$$U^L = \beta_0^L \sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda y_t^L, \text{ and}$$

$$U^H\left(\varpi^L, \vec{1}\right) = -\gamma(1-\beta_0^L)T^L + \beta_0^L \sum_{t=1}^{T^L} (1-\lambda)^{t-1} \left(\lambda y_t^L - \gamma\right) + P_{T^L}^H \left(c_{T^L+1}^L - c_{T^L+1}^H\right) q_F,$$

where $y_t^L = \frac{\gamma}{\lambda \beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L} \Delta c q_F$ for $t \leq T^L$.

Since the informational rent of the high-type agent is non-monotonic in T^L, it is possible, in general, to have both $T_{SB}^L < T_{FB}^L$ and $T_{SB}^L > T_{FB}^L$. The stopping time for the high type, T^H, does not affect information rents and, as a result, is not distorted: $T_{SB}^H = T_{FB}^H$.

Case 4: Both $(IC^{H,L})$ and $(IC^{L,H})$ bind ($T_{SB}^H < T_{FB}^H$ or $T_{SB}^H > T_{FB}^H$, $T_{SB}^L < T_{FB}^L$ or $T_{SB}^L > T_{FB}^L$).

Information rents for both types are given by

$$U^H(\varpi^L,\vec{1}) = (1-\beta_0^H)\sum_{t=1}^{T^L}[x_t^L - \gamma] + \beta_0^H\sum_{t=1}^{T^L}(1-\lambda)^{t-1}[(\lambda y_t^L - \gamma) + (1-\lambda)x_t^L]$$
$$+\left(1-\beta_0^H + \beta_0^H(1-\lambda)^{T^L}\right)\left(c_{T^L+1}^L - c_{T^L+1}^H\right)q_F;$$
$$U^L(\varpi^H,\vec{a}^L(\varpi^H)) = (1-\beta_0^L)\sum_{t=1}^{t=t^{L,H}}[x_t^H - \gamma] + \beta_0^L\sum_{t=1}^{t=t^{L,H}}(1-\lambda)^{t-1}[\lambda y_t^H + (1-\lambda)x_t^H - \gamma]$$
$$+(1-\beta_0^L)\sum_{t=t^{L,H}+1}^{T^H} x_t^H + (1-\lambda)^{t^{L,H}-1}\beta_0^L\sum_{t=t^{L,H}+1}^{T^H} x_t^H$$
$$-\left(1-\beta_0^L + \beta_0^L(1-\lambda)^{t^{L,H}}\right)\left(c_{t^{L,H}+1}^L - c_{T^H+1}^H\right)q_F.$$

Since the informational rent of the high-type agent is non-monotonic in T^L, it is possible, in general, to have both $T_{SB}^L < T_{FB}^L$ and $T_{SB}^L > T_{FB}^L$. Similarly, as the informational rent of the low-type agent is non-monotonic in T^H, it is possible, in general, to have $T_{SB}^H < T_{FB}^H$ and $T_{SB}^H > T_{FB}^H$. This completes part (ii) of the proof of Proposition 3.

Q.E.D.

Appendix D: Sufficient Conditions for Separation/Integration

Claim D1. Sufficient Conditions for Separation

Separation is optimal if the adverse selection problem is small enough (β_0^H is close to β_0^L).

Proof: We prove that separation is optimal in all 4 cases of the main model (depending on which IC are binding) if the adverse selection problem is small enough. That is, for any β_0^L there exists a value of β_0^H, called $\overline{\beta}_0^H(\beta_0^L)$, such that separation is optimal if $\beta_0^H < \overline{\beta}_0^H(\beta_0^L)$. We consider each of the four cases in turn and prove that in each of them the principal is better off with separating contracts for experimentation and production than under integration if the adverse selection problem is not severe.

From the principal's problem in Appendix C, the expected payment by the principal to both types under integration is given by:

$$E_\theta\left[\beta_0^\theta\sum_{t=1}^{T^\theta}(1-\lambda)^{t-1}\lambda y_t^\theta + \sum_{t=1}^{T^\theta}P_t^\theta x_t^\theta\right] = \nu\left[\beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H + \sum_{t=1}^{T^H}P_t^H x_t^H\right] +$$
$$(1-\nu)\left[\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L + \sum_{t=1}^{T^L}P_t^L x_t^L\right],$$

where the two (IC) constraints are

$(IC^{L,H})$ $\quad\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L + P_{T^L}^L x_{T^L}^L$

$$\geq \beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\gamma + \gamma\sum_{t=1}^{T^L}P_t^L - \gamma t^{L,H}(1-\beta_0^L) - \gamma\beta_0^L\sum_{t=1}^{t=t^{L,H}}(1-\lambda)^{t-1}$$
$$-\left(1-\beta_0^L+\beta_0^L(1-\lambda)^{t^{L,H}}\right)\left(c_{t^{L,H}+1}^L - c_{T^H+1}^H\right)q_F$$
$$+\beta_0^L\sum_{t=1}^{t=t^{L,H}}(1-\lambda)^{t-1}\lambda y_t^H,$$

$(IC^{H,L})$ $\quad\beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H$

$$\geq \gamma\sum_{t=1}^{T^H}P_t^H + \gamma\beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1} - \gamma T^L(1-\beta_0^H) - \gamma\beta_0^H\sum_{t=1}^{T^L}(1-\lambda)^{t-1}$$

$$+\left(1-\beta_0^H+\beta_0^H(1-\lambda)^{T^L}\right)\left(c_{T^L+1}^L-c_{T^L+1}^H\right)q_F$$

$$+\beta_0^H\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L+P_{T^L}^H x_{T^L}^L.$$

The expected payment by the principal to each type under separation is the sum of the standard moral hazard rent paid to the experimentation experimenter ($y_t^\theta=\frac{\gamma}{\lambda\beta_{T^\theta}^\theta}$ and $x_t^\theta=0$ for $t\leq T^\theta$) and the adverse selection rent paid to the producer. Recalling from the proof of part (i) of Proposition 1 in Supplementary Appendix A that this adverse selection rent depends on whether one or both IC are binding (Case A or B under separation):

Case A: $\nu U_A^H+(1-\nu)U_A^L=\nu P_{T^L}^H\Delta c_{T^L+1}q_F$ since $U_A^L=0$.

Case B: $\nu U_B^H+(1-\nu)U_B^L=\nu P_{T^L}^H\Delta c_{T^L+1}q_F+\left[\frac{\beta_0^L P_{T^L}^H\Delta c_{T^L+1}q_F-\beta_0^H P_{T^H}^L\Delta c_{T^H+1}q_F}{[\beta_0^H-\beta_0^L]}\right]EP_{T^L}^\theta,$

where $EP_{T^L}^\theta=\left(\nu P_{T^L}^H+(1-\nu)P_{T^L}^L\right)$.

Thus we have the expected payment by the principal under separation:

Case A: $(1-\nu)\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^L}^L}+\nu\beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^H}^H}+\nu P_{T^L}^H\Delta c_{T^L+1}q_F.$

Case B: $(1-\nu)\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^L}^L}+\nu\beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^H}^H}$

$$+\frac{\beta_0^L EP_{T^L}^\theta\, P_{T^L}^H\Delta c_{T^L+1}}{(\beta_0^H-\beta_0^L)}q_F-\frac{EP_{T^L}^\theta\,\beta_0^H P_{T^H}^L\Delta c_{T^H+1}}{(\beta_0^H-\beta_0^L)}q_F+\nu P_{T^L}^H\Delta c_{T^L+1}q_F.$$

Given that

$$(\boldsymbol{D1})\qquad \Delta c_{t+1}=(\beta_{t+1}^H-\beta_{t+1}^L)\Delta c=\frac{\beta_0^H(1-\lambda)^t}{P_t^H}-\frac{\beta_0^L(1-\lambda)^t}{P_t^L}\Delta c=\left(\frac{\beta_0^H(1-\lambda)^t}{P_t^H}-\frac{\beta_0^L(1-\lambda)^t}{P_t^L}\right)\Delta c$$

$$=\frac{\beta_0^H(1-\lambda)^t P_t^L-\beta_0^L(1-\lambda)^t P_t^H}{P_t^H P_t^L}\Delta c=\frac{(\beta_0^H-\beta_0^L)(1-\lambda)^t}{P_t^H P_t^L}\Delta c \text{ for any } t,$$

we rewrite the expected payment by the principal under separation:

Case A: $(1-\nu)\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^L}^L}+\nu\beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^H}^H}$

$$+\nu P_{T^L}^H\frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L}\Delta c q_F.$$

Case B: $(1-\nu)\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^L}^L}+\nu\beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^H}^H}$

$$+\frac{\beta_0^L E P_{T^L}^\theta P_{T^L}^H \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L}\Delta c}{(\beta_0^H-\beta_0^L)} q_F - \frac{E P_{T^L}^\theta \beta_0^H P_{T^H}^L \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^H}}{P_{T^H}^H P_{T^H}^L}\Delta c}{(\beta_0^H-\beta_0^L)} q_F +$$

$$\nu P_{T^L}^H \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L}\Delta c q_F.$$

Note that the standard MH payment during experimentation, denoted as MH_e:

$$(1-\nu)\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^L}^L} + \nu\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^H}^H}$$

is paid under both integration and separation.

We compare the adverse selection rent to the producer against the MH rent at the time of production under integration, denoted by MH_p:

$$E_\theta \sum_{t=1}^{T^\theta}(1-\lambda)^{t-1}\lambda\frac{(1-\beta_0^\theta)}{P_{T^\theta}^\theta}\Delta c q_F =$$

$$[\nu\beta_0^H \frac{(1-\beta_0^H)}{P_{T^H}^H}\left(1-(1-\lambda)^{T^H}\right) + (1-\nu)\beta_0^L \frac{(1-\beta_0^L)}{P_{T^L}^L}\left(1-(1-\lambda)^{T^L}\right)]\Delta c q_F.$$

We will check this for each of the Cases 1-4 depending on which IC is binding under integration.

Case 1 under integration: both IC are slack

In Case 1, when both ICs are slack, from Appendix C, we have $x_t^\theta = 0$ for $t \le T^\theta$ and $\theta \in \{H, L\}$ and $y_t^\theta = \frac{\gamma}{\lambda\beta_{T^\theta}^\theta} + \frac{(1-\beta_0^\theta)}{P_{T^\theta}^\theta}\Delta c q_F$ Thus, the expected rent under integration is given by:

$$(1-\nu)\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left(\frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right)$$

$$+\nu\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\left(\frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right).$$

Case A under separation (only $(IC^{H,L})$ ***is binding).*** We prove that separation is optimal if β_0^H is close to β_0^L. Separation is optimal if

$$\nu P_{T^L}^H \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L}\Delta c q_F < \left[\nu\frac{(1-\beta_0^H)\beta_0^H\left(1-(1-\lambda)^{T^H}\right)}{P_{T^H}^H} + (1-\nu)\frac{(1-\beta_0^L)\beta_0^L\left(1-(1-\lambda)^{T^L}\right)}{P_{T^L}^L}\right]\Delta c q_F,$$

(*D*1*A*) $$\nu(1-\lambda)^{T^L}(\beta_0^H-\beta_0^L)$$

$$< \nu(1-\beta_0^H)\left(1-(1-\lambda)^{T^H}\right)\frac{P_{T^L}^L}{P_{T^H}^H} + (1-\nu)(1-\beta_0^L)\left(1-(1-\lambda)^{T^L}\right).$$

Since the RHS stays strictly positive and the LHS goes to zero as $\beta_0^H \to \beta_0^L$, for any β_0^L there exists a value of β_0^H, called $\bar{\beta}_0^{H1a}(\beta_0^L)$, such that the inequality is satisfied if $\beta_0^H < \bar{\beta}_0^{H1a}(\beta_0^L)$.

Case B under separation (both (IC) ***are binding).*** We prove that separation is optimal if β_0^H is close to β_0^L. Separation is optimal if

$$\frac{\beta_0^L E P^\theta_{T^L} P^H_{T^L}\frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P^H_{T^L}P^L_{T^L}}\Delta c}{(\beta_0^H-\beta_0^L)}q_F - \frac{EP^\theta_{T^L}\beta_0^H P^L_{T^H}\frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^H}}{P^H_{T^H}P^L_{T^H}}\Delta c}{(\beta_0^H-\beta_0^L)}q_F + \nu P^H_{T^L}\frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P^H_{T^L}P^L_{T^L}}\Delta c q_F$$
$$< \left[\frac{\nu(1-\beta_0^H)\beta_0^H\left(1-(1-\lambda)^{T^H}\right)P^L_{T^L}+(1-\nu)(1-\beta_0^L)\beta_0^L\left(1-(1-\lambda)^{T^L}\right)P^H_{T^H}}{P^H_{T^H}P^L_{T^L}}\right]\Delta c q_F.$$

Or, equivalently

$$(\boldsymbol{D1B})\qquad \frac{\beta_0^L E P^\theta_{T^L}(1-\lambda)^{T^L}P^H_{T^H}-EP^\theta_{T^L}\beta_0^H(1-\lambda)^{T^H}P^L_{T^L}}{P^L_{T^L}P^H_{T^H}} + \nu P^H_{T^L}\frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P^H_{T^L}P^L_{T^L}}$$
$$< \frac{\nu(1-\beta_0^H)\beta_0^H\left(1-(1-\lambda)^{T^H}\right)P^L_{T^L}+(1-\nu)(1-\beta_0^L)\beta_0^L\left(1-(1-\lambda)^{T^L}\right)P^H_{T^H}}{P^H_{T^H}P^L_{T^L}},$$

Since the RHS stays strictly positive and the LHS goes to zero as $\beta_0^H \to \beta_0^L$, for any β_0^L there exists a value of β_0^H, called $\bar{\beta}_0^{H1b}(\beta_0^L)$, such that the inequality is satisfied if $\beta_0^H < \bar{\beta}_0^{H1b}(\beta_0^L)$.

Case 2 under integration: only $(\boldsymbol{IC^{L,H}})$ binding

We first determine the rent paid to the high type. In Case 2, Appendix C, we have $x_t^H = 0$ and $y_t^H = \frac{\gamma}{\lambda\beta^H_{T^H}} + \frac{(1-\beta_0^H)}{P^H_{T^H}}\Delta c q_F$ for all $t \le T^H$. Therefore, the rent paid to the high type is

$$\beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H + \sum_{t=1}^{T^H}P_t^H x_t^H = \beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\left(\frac{\gamma}{\lambda\beta^H_{T^H}} + \frac{(1-\beta_0^H)}{P^H_{T^H}}\Delta c q_F\right).$$

We next determine the rent paid to the low type. From Lemma C1 in Appendix C, Case 2, we have the binding $(IC^{L,H})$ constraint:

$$\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}y_t^L + \sum_{t=1}^{T^L}P_t^L x_t^L - \sum_{t=1}^{T^L}P^L_{t-1}\gamma =$$
$$\beta_0^L\sum_{t=1}^{t^{L,H}}(1-\lambda)^{t-1}\lambda y_t^H - \sum_{t=1}^{t^{L,H}}P^L_{t-1}\gamma + P^L_{t^{L,H}}\left(c^H_{T^H+1} - c^L_{t^{L,H}+1}\right)q_F,$$

and by moving $\sum_{t=1}^{T^L}P^L_{t-1}\gamma$ to the *RHS*, the rent paid to the low type can be written as:

$$\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L + \sum_{t=1}^{T^H}P_t^H x_t^L$$
$$= \beta_0^L\sum_{t=1}^{t^{L,H}}(1-\lambda)^{t-1}\lambda y_t^H + \sum_{t=1}^{T^L}P^L_{t-1}\gamma - \sum_{t=1}^{t^{L,H}}P^L_{t-1}\gamma + P^L_{t^{L,H}}\left(c^H_{T^H+1} - c^L_{t^{L,H}+1}\right)q_F.$$

Thus, the expected rent under integration is

$$\nu\left[\beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H + \sum_{t=1}^{T^H}P_t^H x_t^H\right] + (1-\nu)\left[\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L + \sum_{t=1}^{T^L}P_t^L x_t^L\right]$$

$$= \nu\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\left(\frac{\gamma}{\lambda\beta_{T^H}^H}+\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right)$$

$$+(1-\nu)\begin{pmatrix} \beta_0^L \sum_{t=1}^{t^{L,H}}(1-\lambda)^{t-1}\,\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H}+\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right] \\ +\sum_{t=1}^{T^L}P_{t-1}^L\gamma-\sum_{t=1}^{t^{L,H}}P_{t-1}^L\,\gamma+P_{t^{L,H}}^L\left(c_{T^H+1}^H-c_{t^{L,H}+1}^L\right)q_F \end{pmatrix}.$$

Case A under separation (only $(IC^{H,L})$ ***is binding).*** We prove that separation is optimal if β_0^H is close to β_0^L.

Separation is optimal if

$$(1-\nu)\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^L}^L}+\nu\beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^H}^H}+$$

$$\nu P_{T^L}^H\frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L}\Delta c q_F$$

$$< \nu\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\left(\frac{\gamma}{\lambda\beta_{T^H}^H}+\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right)$$

$$+(1-\nu)\begin{pmatrix} \beta_0^L \sum_{t=1}^{t^{L,H}}(1-\lambda)^{t-1}\,\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H}+\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right] \\ +\sum_{t=1}^{T^L}P_{t-1}^L\gamma-\sum_{t=1}^{t^{L,H}}P_{t-1}^L\,\gamma+P_{t^{L,H}}^L\left(c_{T^H+1}^H-c_{t^{L,H}+1}^L\right)q_F \end{pmatrix},$$

$(\boldsymbol{D2A})$

$$\nu(1-\lambda)^{T^L}\left(\frac{\beta_0^H-\beta_0^L}{P_{T^L}^L}\right)\Delta c q_F$$

$$< \nu\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\left(\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right)+$$

$$(1-\nu)\begin{pmatrix} \beta_0^L \sum_{t=1}^{t^{L,H}}(1-\lambda)^{t-1}\,\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H}+\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right] \\ +\sum_{t=1}^{T^L}P_{t-1}^L\gamma-\sum_{t=1}^{t^{L,H}}P_{t-1}^L\,\gamma+P_{t^{L,H}}^L\left(c_{T^H+1}^H-c_{t^{L,H}+1}^L\right)q_F-\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^L}^L} \end{pmatrix}.$$

The *LHS* goes to zero and as $\beta_0^H \to \beta_0^L$. Since $t^{L,H} \to T^H \to T^L$ as $\beta_0^H \to \beta_0^L$, the *RHS* goes to:[50]

$$\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\,\lambda\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F > 0.$$

Since the RHS stays strictly positive and the LHS goes to zero as $\beta_0^H \to \beta_0^L$, for any β_0^L there exists a value of β_0^H, called $\bar{\beta}_0^{H2a}(\beta_0^L)$, such that the inequality is satisfied if $\beta_0^H < \bar{\beta}_0^{H2a}(\beta_0^L)$.

Case B under separation (both (IC) ***are binding).*** We prove that separation is optimal if β_0^H is close to β_0^L. Separation is optimal if

[50] The value of $t^{L,H} \to T^H$ as $\beta_0^H \to \beta_0^L$ because the low type's disadvantage with the probability of success goes down as $\beta_0^H \to \beta_0^L$. To see this, recall that the payment y_t^H induces a lying low type to only work for $t^{L,H} \le T^H$ periods, and this difference in relative probabilities of success disappears as $\beta_0^H \to \beta_0^L$. Therefore, the value of $t^{L,H} \to T^H$ as $\beta_0^H \to \beta_0^L$.

$$(1-\nu)\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda \frac{\gamma}{\lambda\beta_{T^L}^L} + \nu\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda \frac{\gamma}{\lambda\beta_{T^H}^H}$$

$$+\frac{\beta_0^L E P_{T^L}^{\theta}\, P_{T^L}^H \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L}\Delta c}{(\beta_0^H-\beta_0^L)} q_F - \frac{E P_{T^L}^{\theta}\, \beta_0^H P_{T^H}^L \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^H}}{P_{T^H}^H P_{T^H}^L}\Delta c}{(\beta_0^H-\beta_0^L)} q_F +$$

$$\nu P_{T^L}^H \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L}\Delta c q_F$$

$$< \nu\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\left(\frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right)$$

$$+(1-\nu)\begin{pmatrix} \beta_0^L \sum_{t=1}^{t^{L,H}}(1-\lambda)^{t-1}\,\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right] \\ +\sum_{t=1}^{T^L} P_{t-1}^L\gamma - \sum_{t=1}^{t^{L,H}} P_{t-1}^L\,\gamma + P_{t^{L,H}}^L\left(c_{T^H+1}^H - c_{t^{L,H}+1}^L\right)q_F \end{pmatrix},$$

(D2B)

$$\frac{\beta_0^L E P_{T^L}^{\theta}(1-\lambda)^{T^L} P_{T^H}^H - E P_{T^L}^{\theta}\,\beta_0^H(1-\lambda)^{T^H} P_{T^L}^L}{P_{T^L}^L P_{T^H}^H}\Delta c q_F + \nu\frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^L}\Delta c q_F$$

$$< \nu\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F$$

$$+(1-\nu)\begin{pmatrix} \beta_0^L \sum_{t=1}^{t^{L,H}}(1-\lambda)^{t-1}\,\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right] \\ +\sum_{t=1}^{T^L} P_{t-1}^L\gamma - \sum_{t=1}^{t^{L,H}} P_{t-1}^L\,\gamma + P_{t^{L,H}}^L\left(c_{T^H+1}^H - c_{t^{L,H}+1}^L\right)q_F \\ -\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^L}^L} \end{pmatrix},$$

Since the *LHS* goes to zero and the *RHS* goes to $(1-\nu)\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F > 0$ as $\beta_0^H \to \beta_0^L$, for any β_0^L there exists a value of β_0^H, called $\bar{\beta}_0^{H2b}(\beta_0^L)$, such that the inequality is satisfied if $\beta_0^H < \bar{\beta}_0^{H2b}(\beta_0^L)$.

Case 3 under integration: only ($\boldsymbol{IC^{H,L}}$) binding

We first determine the expected payment to the low type. From Claim C3 in Case 3, Appendix C, we have $x_t^L = 0$, and $y_t^L = \frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F$ for all $t \leq T^L$. Therefore, the expected payment to the low type in the principal's objective function is

$$(1-\nu)\left[\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L + \sum_{t=1}^{T^L} P_t^L x_t^L\right]$$

$$= (1-\nu)\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left(\frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right).$$

We next determine the expected payment to the high type. Given $x_t^L = 0$, we have the binding ($IC^{H,L}$) constraint:

$$\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H + \sum_{t=1}^{T^H} P_t^H x_t^H - \sum_{t=1}^{T^H} P_{t-1}^H\gamma$$

$$= \beta_0^H \sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda y_t^L - \sum_{t=1}^{T^L} P_{t-1}^H \gamma + P_{T^L}^H \Delta c_{T^L+1} q_F,$$

and by moving $\sum_{t=1}^{T^H} P_{t-1}^H \gamma$ to the *RHS*, the expected payment to the high type in the principal's objective function can be written as:

$$\beta_0^H \sum_{t=1}^{T^H} (1-\lambda)^{t-1} \lambda y_t^H + \sum_{t=1}^{T^H} P_t^H x_t^H$$

$$= \beta_0^H \sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda y_t^L + \sum_{t=1}^{T^H} P_{t-1}^H \gamma - \sum_{t=1}^{T^L} P_{t-1}^H \gamma + P_{T^L}^H \Delta c_{T^L+1} q_F.$$

Thus, the expected payment by the principal to both types under integration in Case 3 is given by:

$$(1-\nu)\beta_0^L \sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda \left(\frac{\gamma}{\lambda \beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L} \Delta c q_F \right)$$

$$+\nu \left(\beta_0^H \sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda \left[\frac{\gamma}{\lambda \beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L} \Delta c q_F \right] + \sum_{t=1}^{T^H} P_{t-1}^H \gamma - \sum_{t=1}^{T^L} P_{t-1}^H \gamma + P_{T^L}^H \Delta c_{T^L+1} q_F \right).$$

Case A under separation (only $(IC^{H,L})$ ***is binding).*** We prove that separation is optimal if β_0^H is close to β_0^L. Separation is optimal if the expected rent in under Case A separation is smaller than that under Case 3 integration:

$$(1-\nu)\beta_0^L \sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda \frac{\gamma}{\lambda \beta_{T^L}^L} + \nu \beta_0^H \sum_{t=1}^{T^H} (1-\lambda)^{t-1} \lambda \frac{\gamma}{\lambda \beta_{T^H}^H} +$$

$$\nu P_{T^L}^H \frac{(\beta_0^H - \beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L} \Delta c q_F$$

$$< (1-\nu)\beta_0^L \sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda \left(\frac{\gamma}{\lambda \beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L} \Delta c q_F \right)$$

$$+\nu \left(\beta_0^H \sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda \left[\frac{\gamma}{\lambda \beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L} \Delta c q_F \right] + \sum_{t=1}^{T^H} P_{t-1}^H \gamma - \sum_{t=1}^{T^L} P_{t-1}^H \gamma + P_{T^L}^H \Delta c_{T^L+1} q_F \right),$$

$$(\boldsymbol{D3A}) \qquad \nu \beta_0^H \sum_{t=1}^{T^H} (1-\lambda)^{t-1} \lambda \frac{\gamma}{\lambda \beta_{T^H}^H} + \nu P_{T^L}^H \frac{(\beta_0^H - \beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L} \Delta c q_F$$

$$< (1-\nu)\beta_0^L \sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda \left(\frac{(1-\beta_0^L)}{P_{T^L}^L} \Delta c q_F \right)$$

$$+\nu \left(\beta_0^H \sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda \left[\frac{\gamma}{\lambda \beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L} \Delta c q_F \right] + \sum_{t=1}^{T^H} P_{t-1}^H \gamma - \sum_{t=1}^{T^L} P_{t-1}^H \gamma + P_{T^L}^H \Delta c_{T^L+1} q_F \right).$$

The *LHS* goes to $\nu \beta_0^L \sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda \frac{\gamma}{\lambda \beta_{T^L}^L}$ as $\beta_0^H \to \beta_0^L$ and the *RHS* goes to:

$$\beta_0^L \sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda \left(\frac{(1-\beta_0^L)}{P_{T^L}^L} \Delta c q_F \right) + \nu \beta_0^L \sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda \frac{\gamma}{\lambda \beta_{T^L}^L} \text{ as } \beta_0^H \to \beta_0^L.$$

Since $\beta_0^L \sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda \left(\frac{(1-\beta_0^L)}{P_{T^L}^L} \Delta c q_F \right) > 0$, the condition is satisfied as $\beta_0^H \to \beta_0^L$. Therefore, for any β_0^L there exists a value of β_0^H, called $\bar{\beta}_0^{H3a}(\beta_0^L)$, such that the inequality is satisfied if $\beta_0^H < \bar{\beta}_0^{H3a}(\beta_0^L)$.

Case B under separation (both (IC) are binding). We prove that separation is optimal if β_0^H is close to β_0^L. Separation is optimal if

$$(1-\nu)\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda \frac{\gamma}{\lambda\beta_{T^L}^L} + \nu\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda \frac{\gamma}{\lambda\beta_{T^H}^H}$$

$$+\frac{\beta_0^L E P_{T^L}^{\theta}\, P_{T^L}^H \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L}\Delta c}{(\beta_0^H-\beta_0^L)} q_F - \frac{E P_{T^L}^{\theta}\, \beta_0^H P_{T^H}^L \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^H}}{P_{T^H}^H P_{T^H}^L}\Delta c}{(\beta_0^H-\beta_0^L)} q_F + \nu P_{T^L}^H \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L}\Delta c q_F$$

$$< (1-\nu)\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left(\frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right)$$

$$+\nu\left(\beta_0^H \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right] + \sum_{t=1}^{T^H} P_{t-1}^H\gamma - \sum_{t=1}^{T^L} P_{t-1}^H\gamma + P_{T^L}^H \Delta c_{T^L+1} q_F\right),$$

$$(\boldsymbol{D3B}) \qquad \frac{\beta_0^L E P_{T^L}^{\theta}(1-\lambda)^{T^L} P_{T^H}^H - E P_{T^L}^{\theta}\,\beta_0^H(1-\lambda)^{T^H} P_{T^L}^L}{P_{T^L}^L P_{T^H}^H}\Delta c q_F + \nu\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^H}^H}$$

$$+\nu P_{T^L}^H \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L}\Delta c q_F$$

$$< (1-\nu)\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F$$

$$+\nu\left(\beta_0^H \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right] + \sum_{t=1}^{T^H} P_{t-1}^H\gamma - \sum_{t=1}^{T^L} P_{t-1}^H\gamma + P_{T^L}^H \Delta c_{T^L+1} q_F\right),$$

The *LHS* goes to $\nu\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^L}^L}$ as $\beta_0^H \to \beta_0^L$ and the *RHS* goes to:

$$\nu\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F + \beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F \text{ as } \beta_0^H \to \beta_0^L.$$

Since $\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F > 0$, the condition is satisfied as $\beta_0^H \to \beta_0^L$. Therefore, for any for any β_0^L there exists a value of β_0^H, called $\bar{\beta}_0^{H3b}(\beta_0^L)$, such that the inequality is satisfied if $\beta_0^H < \bar{\beta}_0^{H3b}(\beta_0^L)$.

Case 4 under integration: both IC are binding

Since the rent under integration in Case 4 is greater than in Cases 2 and 3, separation is optimal in Case 4 under the same parameters as in those two cases.

To complete the proof, define

$$\overline{\beta}_0^H(\beta_0^L) = \min\{\bar{\beta}_0^{H1a}(\beta_0^L), \bar{\beta}_0^{H1b}(\beta_0^L), \bar{\beta}_0^{H2a}(\beta_0^L), \bar{\beta}_0^{H2b}(\beta_0^L), \bar{\beta}_0^{H3a}(\beta_0^L), \bar{\beta}_0^{H3b}(\beta_0^L)\}$$

Q.E.D.

Claim D2. Sufficient Conditions for integration to be optimal

Integration is optimal if the adverse selection problem is severe enough (β_0^H is sufficiently close to one and β_0^L sufficiently close to zero) and ν is high enough.

Proof: We fix the experimentation lengths to the optimal values under separation. We will find conditions such that integration dominates separation given these experimentation lengths. Then, integration will also dominate (for the same parameter conditions) for the optimal T^θ under integration by revealed preference.

We argue first that we will only need to we consider only Case A under separation. As $\beta_0^L \to 0$ and $\beta_0^H \to 1$ then $T^L \to 0$, $T^H \to 1$, $t^{L,H} \to 0$, and $t^{H,L} \to 0$. Thus, the additional adverse selection rent in Case B under integration becomes negative as $\beta_0^L \to 0$ and $\beta_0^H \to 1$:

$$\frac{\beta_0^L E P_{T^L}^\theta (1-\lambda)^{T^L} P_{T^H}^H - E P_{T^L}^\theta \beta_0^H (1-\lambda)^{T^H} P_{T^L}^L}{P_{T^L}^L P_{T^H}^H} \Delta c q_F \to \frac{0-1\,(1-\lambda)^1 1}{1(1-\lambda)} \Delta c q_F = -\Delta c q_F < 0.$$

Therefore, Case B is not relevant as $\beta_0^L \to 0$ and $\beta_0^H \to 1$.

Case 1 under integration: both IC are slack

Case A under separation (only $(IC^{H,L})$ ***is binding).*** From condition $(D1A)$, integration is optimal if

$$\nu(1-\lambda)^{T^L}(\beta_0^H - \beta_0^L)$$

$$> \nu(1-\beta_0^H)\left(1-(1-\lambda)^{T^H}\right)\frac{P_{T^L}^L}{P_{T^H}^H} + (1-\nu)(1-\beta_0^L)\left(1-(1-\lambda)^{T^L}\right).$$

Since the LHS stays strictly positive as $\beta_0^L \to 0$ and $\beta_0^H \to 1$:

$$\nu(1-\lambda)^{T^L}(\beta_0^H - \beta_0^L) \to \nu(1-\lambda)^0(1-0) = \nu > 0,$$

and the RHS goes to zero as $\beta_0^L \to 0$ and $\beta_0^H \to 1$:

$$\nu(1-\beta_0^H)\left(1-(1-\lambda)^{T^H}\right)\frac{P_{T^L}^L}{P_{T^H}^H} + (1-\nu)(1-\beta_0^L)\left(1-(1-\lambda)^{T^L}\right)$$

$$\to \nu(1-1)\left(1-(1-\lambda)^{T^H}\right)\frac{P_{T^L}^L}{P_{T^H}^H} + (1-\nu)(1-0)\left(1-(1-\lambda)^0\right) = 0,$$

there exist $\bar{\beta}_0^{L1a} > 0$ and $\underline{\beta}^{H1a} < 1$, such that the inequality is satisfied if $\beta_0^L < \bar{\beta}_0^{L1a}$ and $\beta_0^H > \underline{\beta}^{H1a}$.

Case 2 under integration: only $(IC^{L,H})$ binding

Case A under separation (only $(IC^{H,L})$ ***is binding).*** From condition $(D2A)$, integration is optimal if

$$\nu(1-\lambda)^{T^L}\left(\frac{\beta_0^H-\beta_0^L}{P_{T^L}^L}\right)\Delta c q_F > \nu\beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\left(\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right)+$$

$$(1-\nu)\left(\begin{array}{c}\beta_0^L\sum_{t=1}^{t^{L,H}}(1-\lambda)^{t-1}\,\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H}+\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right]\\ +\sum_{t=1}^{T^L}P_{t-1}^L\gamma-\sum_{t=1}^{t^{L,H}}P_{t-1}^L\,\gamma+P_{t^{L,H}}^L\left(c_{T^H+1}^H-c_{t^{L,H}+1}^L\right)q_F-\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^L}^L}\end{array}\right),$$

Since the *LHS* goes to $\nu\Delta c q_F$ as $\beta_0^L\to 0$ and $\beta_0^H\to 1$:

$$\nu(1-\lambda)^{T^L}\left(\frac{\beta_0^H-\beta_0^L}{P_{T^L}^L}\right)\Delta c q_F\to\nu(1-\lambda)^0\left(\frac{1-0}{P_0^L}\right)\Delta c q_F=\nu\Delta c q_F,$$

and the *RHS* goes to $(1-\nu)(c_2^H-c_1^L)q_F$ as $\beta_0^L\to 0$ and $\beta_0^H\to 1$:

$$\nu\beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\left(\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right)+$$

$$(1-\nu)\left(\begin{array}{c}\beta_0^L\sum_{t=1}^{t^{L,H}}(1-\lambda)^{t-1}\,\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H}+\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right]\\ +\sum_{t=1}^{T^L}P_{t-1}^L\gamma-\sum_{t=1}^{t^{L,H}}P_{t-1}^L\,\gamma+P_{t^{L,H}}^L\left(c_{T^H+1}^H-c_{t^{L,H}+1}^L\right)q_F-\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^L}^L}\end{array}\right),$$

$$\to\nu\sum_{t=1}^{1}(1-\lambda)^{t-1}\lambda\frac{(1-1)}{P_1^H}\Delta c q_F+(1-\nu)P_0^L(c_{1+1}^H-c_{0+1}^L)q_F=(1-\nu)(c_2^H-c_1^L)q_F,$$

there exist $\bar{\beta}_0^{L2a}>0$ and $\underline{\beta}^{H2a}<1$, such that the inequality is satisfied if $\beta_0^L<\bar{\beta}_0^{L2a}$, $\beta_0^H>\underline{\beta}^{H2a}$ and $\nu>\frac{(c_2^H-c_1^L)}{\Delta c+c_2^H-c_1^L}$.

Case 3 under integration: only $(IC^{H,L})$ binding

Case A under separation (only $(IC^{H,L})$ ***is binding).*** From condition $(D3A)$, integration is optimal if

$$\nu\beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^H}^H}+\nu P_{T^L}^H\frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L}\Delta c q_F$$

$$>(1-\nu)\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left(\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right)$$

$$+\nu\left(\beta_0^H\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\,\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L}+\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right]+\sum_{t=1}^{T^H}P_{t-1}^H\gamma-\sum_{t=1}^{T^L}P_{t-1}^H\,\gamma+P_{T^L}^H\Delta c_{T^L+1}q_F\right).$$

Since the *LHS* goes to $\nu\frac{\gamma}{\beta_1^H}+\nu\Delta c q_F$ and the *RHS* goes to $\nu\gamma+\nu\Delta c q_F$ as $\beta_0^L\to 0$ and $\beta_0^H\to 1$, there exist $\bar{\beta}_0^{L3a}>0$ and $\underline{\beta}^{H3a}<1$, such that the inequality is satisfied if $\beta_0^L<\bar{\beta}_0^{L3a}$ and $\beta_0^H>\underline{\beta}^{H3a}$.

Case 4 under integration: both IC are binding

Case A under separation (only $(IC^{H,L})$ is binding).

Recalling from Appendix C that in Case 4, we have $y_t^H = \frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F$ for $t > 1$, and for all $t \le T^L$ $y_t^L = x_{T^L}^L + \frac{\gamma}{\lambda\beta_{T^L}^L} + (1+\lambda(T^L - t))\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F$, we can rewrite the binding (IC) constraints:

$$(IC^{L,H}) \qquad x_{T^L}^L\left[\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda + P_{T^L}^L\right] = \beta_0^L\lambda y_1^H$$

$$+\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\gamma + \gamma\sum_{t=1}^{T^L}P_t^L - \gamma t^{L,H}(1-\beta_0^L) - \gamma\beta_0^L\sum_{t=1}^{t=t^{L,H}}(1-\lambda)^{t-1}$$

$$-\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L} + \left(1+\lambda(T^L-t)\right)\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right]$$

$$+\beta_0^L\sum_{t=2}^{t=t^{L,H}}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right]$$

$$-\left(1-\beta_0^L+\beta_0^L(1-\lambda)^{t^{L,H}}\right)\left(c_{t^{L,H}+1}^L - c_{T^H+1}^H\right)q_F,$$

$$(IC^{H,L}) \qquad \beta_0^H\lambda y_1^H = x_{T^L}^L\left[\beta_0^H\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda + P_{T^L}^H\right]$$

$$+\gamma\sum_{t=1}^{T^H}P_t^H + \gamma\beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1} - \gamma T^L(1-\beta_0^H) - \gamma\beta_0^H\sum_{t=1}^{T^L}(1-\lambda)^{t-1}$$

$$-\beta_0^H\sum_{t=2}^{T^H}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right]$$

$$+\beta_0^H\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L} + (1+\lambda(T^L-t))\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right]$$

$$+\left(1-\beta_0^H+\beta_0^H(1-\lambda)^{T^L}\right)\left(c_{T^L+1}^L - c_{T^L+1}^H\right)q_F.$$

Solving y_1^H and $x_{T^L}^L$ from the two binding (IC) constraints we obtain:

$$x_{T^L}^L = \frac{\beta_0^L\lambda}{\left[\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda + P_{T^L}^L\right]}\, y_1^H$$

$$+\frac{1}{\left[\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda + P_{T^L}^L\right]}\Bigg(\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\gamma + \gamma\sum_{t=1}^{T^L}P_t^L - \gamma t^{L,H}(1-\beta_0^L) -$$

$$\gamma\beta_0^L\sum_{t=1}^{t=t^{L,H}}(1-\lambda)^{t-1} - \beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L} + \left(1+\lambda(T^L-t)\right)\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right] +$$

$$\beta_0^L\sum_{t=2}^{t=t^{L,H}}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right] - \left(1-\beta_0^L+\beta_0^L(1-\lambda)^{t^{L,H}}\right)\left(c_{t^{L,H}+1}^L - c_{T^H+1}^H\right)q_F\Bigg)$$

$$y_1^H =$$

$$\frac{\left[\beta_0^H \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda + P_{T^L}^H\right]}{\lambda\left(\beta_0^H P_{T^L}^L - \beta_0^L P_{T^L}^H\right)}\begin{bmatrix} \beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\gamma + \gamma \sum_{t=1}^{T^L} P_t^L - \gamma t^{L,H}(1-\beta_0^L) - \gamma\beta_0^L \sum_{t=1}^{t=t^{L,H}}(1-\lambda)^{t-1} \\ -\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L} + \left(1+\lambda(T^L-t)\right)\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right] \\ +\beta_0^L \sum_{t=2}^{t=t^{L,H}}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right] \\ -\left(1-\beta_0^L+\beta_0^L(1-\lambda)^{t^{L,H}}\right)\left(c_{t^{L,H}+1}^L - c_{T^H+1}^H\right)q_F \end{bmatrix}$$

$$+\frac{\left[\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda + P_{T^L}^L\right]}{\lambda\left(\beta_0^H P_{T^L}^L - \beta_0^L P_{T^L}^H\right)}\begin{bmatrix} \gamma \sum_{t=1}^{T^H} P_t^H + \gamma\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1} - \gamma T^L(1-\beta_0^H) - \gamma\beta_0^H \sum_{t=1}^{T^L}(1-\lambda)^{t-1} \\ +\beta_0^H \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L} + (1+\lambda(T^L-t))\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right] \\ -\beta_0^H \sum_{t=2}^{T^H}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right] \\ +\left(1-\beta_0^H+\beta_0^H(1-\lambda)^{T^L}\right)\left(c_{T^L+1}^L - c_{T^L+1}^H\right)q_F \end{bmatrix}$$

$$x_{T^L}^L =$$

$$\frac{\beta_0^H}{\left(\beta_0^H P_{T^L}^L - \beta_0^L P_{T^L}^H\right)}\begin{bmatrix} \beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\gamma + \gamma \sum_{t=1}^{T^L} P_t^L - \gamma t^{L,H}(1-\beta_0^L) - \gamma\beta_0^L \sum_{t=1}^{t=t^{L,H}}(1-\lambda)^{t-1} \\ -\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L} + \left(1+\lambda(T^L-t)\right)\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right] \\ +\beta_0^L \sum_{t=2}^{t=t^{L,H}}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right] \\ -\left(1-\beta_0^L+\beta_0^L(1-\lambda)^{t^{L,H}}\right)\left(c_{t^{L,H}+1}^L - c_{T^H+1}^H\right)q_F \end{bmatrix}$$

$$+$$

$$\frac{\beta_0^L}{\left(\beta_0^H P_{T^L}^L - \beta_0^L P_{T^L}^H\right)}\begin{bmatrix} \gamma \sum_{t=1}^{T^H} P_t^H + \gamma\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1} - \gamma T^L(1-\beta_0^H) - \gamma\beta_0^H \sum_{t=1}^{T^L}(1-\lambda)^{t-1} \\ +\beta_0^H \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L} + (1+\lambda(T^L-t))\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right] \\ -\beta_0^H \sum_{t=2}^{T^H}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right] \\ +\left(1-\beta_0^H+\beta_0^H(1-\lambda)^{T^L}\right)\left(c_{T^L+1}^L - c_{T^L+1}^H\right)q_F \end{bmatrix}.$$

Note that $y_1^H \to \frac{(c_2^H - c_1^L)q_F}{\lambda}$ and $x_{T^L}^L \to (c_2^H - c_1^L)q_F$ as $\beta_0^L \to 0$ and $\beta_0^H \to 1$.

Since $y_t^L = x_{T^L}^L + \frac{\gamma}{\lambda\beta_{T^L}^L} + (1+\lambda(T^L-t))\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F$ for $t \geq 1$and $y_t^H = \frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F \to$ for $t > 1$, the expected rent paid by the principal under integration converges to

$$(1-\nu)\left[\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L + P_{T^L}^L x_{T^L}^L\right] + \nu\left[\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H\right]$$

$$\to \nu\lambda y_1^H = \nu\lambda\frac{(c_2^H - c_1^L)q_F}{\lambda} = \nu(c_2^H - c_1^L)q_F.$$

Case A under separation (only $(IC^{H,L})$ ***is binding).*** The rent under separation converges to $\nu\gamma + \nu\Delta c q_F$ as $\beta_0^L \to 0$ and $\beta_0^H \to 1$. Therefore, integration is optimal as $\beta_0^L \to 0$ and $\beta_0^H \to 1$ if

$(\boldsymbol{D4A})$ $$\nu(c_2^H - c_1^L)q_F < \nu\gamma + \nu\Delta c q_F,\ (c_2^H - c_1^L)q_F < \gamma + \Delta c q_F,$$

which holds for any parameters. Thus, there exist $\bar{\beta}_0^{L4a} > 0$ and $\underline{\beta}^{H4a} < 1$, such that integration is optimal if $\beta_0^L < \bar{\beta}_0^{L4a}$, $\beta_0^H > \underline{\beta}^{H4a}$. *Q.E.D.*

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