Piercing Complex Corporate Veils: Theory and Evidence*

Sunghoon Hong†

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Abstract

Under the doctrine of piercing the corporate veil, a court may invalidate limited liability protection and hold parent companies liable for a subsidiary's debt within a corporate group. This paper develops a model of corporate veil piercing. A court chooses a piercing rate to specify how often it pierces a corporate veil. A corporate group chooses the length of an ownership chain to specify how many veils the group builds into the chain. The comparative statics of the Nash equilibrium with respect to the bargaining weight predict a hump-shaped relationship between piercing rate and ownership length, consistent with empirical evidence.

JEL classification: D23, K22, L22

Keywords: corporate group, limited liability, piercing rate, ownership length, hump-shaped relationship

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†Department of Science in Taxation, University of Seoul, Seoul 02504, South Korea, sunghoonhong@uos.ac.kr.

1 Introduction

In principle, a company's shareholders, individual or corporate, are protected by limited liability, which ensures that the shareholders are responsible for the company's debt only up to the capital they invested in the company.[1] However, under the doctrine of piercing the corporate veil, a court can decide to invalidate limited liability protection and to hold the shareholders liable for the company's debt without any limit.[2]

In a modern economy, where value creation is divided among multiple sectors and regions, a company often takes the form of a corporate group, which consists of a parent company and its subsidiaries. The parent company is a corporate shareholder of its subsidiaries. Even if they are separate legal entities, the parent company may be held liable for its subsidiaries' debt by a court under the veil-piercing doctrine.[3] Expecting the court's decision, the corporate group, or the parent company, can choose to organize a more complex structure to own and operate its subsidiaries to reduce the likelihood of liability. In turn, the court can choose to pierce complex corporate veils between the parent and subsidiary companies more easily and frequently. Therefore, the internal ownership structures of corporate groups are influenced by the veil-piercing decisions of courts, or vice versa.

[1]Section 6.22(b) of the Model Business Corporation Act stipulates that a corporation's shareholders are not personally liable for the acts or debts of the corporation unless the shareholders may become personally liable by reason of their own acts or conduct. Section 102(b)(6) of the Delaware General Corporation Law also provides for limited liability.

[2]Legal tests for piercing the corporate veil vary in exact terms across states, but include similar elements. As stated in *Van Dorn Co. v. Future Chemical and Oil Corp.*, 753 F.2d 565, 570 (7th Cir. 1985), for instance, courts disregard a corporate entity and pierce the veil if (i) there is unity of interest and ownership between a company and its shareholders and (ii) it would sanction a fraud or promote injustice not to pierce the veil. To check the first part, the courts consider the following factors: failure to maintain adequate corporate records or to comply with corporate formalities; commingling of funds or assets; undercapitalization; one corporation treating another corporation's assets as its own.

[3]For example, see *In re Silicone Gel Breast Implants Products Liability Litigation*, 887 F.Supp. 1447 (1995). Bristol-Myers Squibb Company (Bristol) was the sole shareholder of Medical Engineering Corporation (MEC), which supplied breast implants. Bristol had never itself produced or distributed breast implants. Plaintiffs, injured from using MEC's breast implants, sued Bristol for damages. The court emphasized the fact-intensive nature of corporate veil piercing and denied Bristol's motion for summary judgment.

In this paper I develop a game-theoretic model of corporate veil piercing between a court and a corporate group. The court chooses a piercing rate to specify how often the court pierces a corporate veil. The group chooses the length of an ownership chain to specify how many veils the group builds into the chain. I characterize the Nash equilibrium when the court and the group move simultaneously, as well as the subgame-perfect Nash equilibrium when the court observes the group's ownership length, with the parameters of the model, such as bargaining weight, agency cost convexity, agency cost level, and net liability. Interestingly, the comparative statics of the Nash equilibrium with respect to the bargaining weight predict a hump-shaped relationship between piercing rate and ownership length, whereas the comparative statics of the subgame-perfect Nash equilibrium with respect to the bargaining weight predict no relationship.[4]

In addition I examine the empirical relationship between piercing rate and ownership length.[5] I combine veil-piercing data from Oh (2010) with data on internal ownership, state incorporation, and financial accounting, respectively from the Orbis database of Bureau van Dijk, the EDGAR system of the U.S. Securities and Exchange Commission, and the Eikon database of Refinitiv. By using quadratic regressions, I find a significant hump-shaped relationship between piercing rate and ownership length. This finding is consistent with the theoretical prediction based on the comparative statics of the Nash equilibrium. The peak of mean length is estimated to be 2.34 at a piercing rate around 0.47. If courts raise piercing rates from 0.26, which equals the piercing rate in Maryland, to 0.50, which equals the piercing rate in New York, corporate groups appear to increase the mean length of ownership chains by about 0.56. If courts raise piercing rates from 0.50 to 0.68, which

[4]The bargaining weight is given exogenously to the court, and may be determined by corporate laws and veil-piercing precedents in the court's jurisdiction. The bargaining weight may vary across state courts due to variations in corporate laws. Stricter state laws for piercing imply that the court is given a lower bargaining weight for plaintiff-creditors.

[5]In the main empirical analysis, ownership length is averaged over all ownership chains for each group, and called mean length. Later I will also use ownership data at the chain level to check the robustness of the results at the group level.

equals the piercing rate in Tennessee, corporate groups decrease the mean length of ownership chains by 0.51. If courts reduce piercing rates from 0.50 to 0.34, which equals the piercing rate in Delaware, corporate groups decrease the mean length of ownership chains by 0.20. A typical corporate group can reduce the mean length by 0.20 by removing an intermediate subsidiary from each of 264 ownership chains among its 1,306 chains.

Veil piercing has been a central topic in corporate law. Most of legal studies focus on the conceptual analysis of veil piercing.[6] Thompson (1991) first conducts an empirical investigation and finds variations in piercing rates depending on the characteristics of courts, plaintiffs, or causes of action. Matheson (2009) studies 360 veil-piercing cases involving parent and subsidiary companies. Most broadly, Oh (2010) collects 2,908 cases in the United States and confirms variations in piercing rates across states.

Internal ownership structures of corporate groups have attracted increasing attention from researchers in economics.[7] Almeida and Wolfenzon (2006) develop a model of pyramidal ownership to show that corporate groups may organize pyramidal structures to exploit payoff and financing advantages for controlling shareholders.[8] Hong (2022) finds that multinational corporate groups reduce effective tax rates by using indirect ownership chains with foreign equity holding companies in countries with favorable tax treaties.[9]

[6]Blumberg (1986, 2005) study the concepts of limited liability and veil piercing within corporate groups. Easterbrook and Fischel (1985) discuss the economic rationale for limited liability and the circumstances in which courts may waive limited liability. Hansmann and Kraakman (1991) argue for unlimited shareholder liability in tort cases.

[7]The theory of the firm deals with the issues on the organization and operation of business entities. Previous studies tend to focus on the relationship between shareholders and managers. Jensen and Meckling (1976) analyze the agency problem due to the conflict of interests between shareholders (owners) and managers. Aghion et al. (2013) examine the role of institutional shareholders on innovation.

[8]Riyanto and Toolsema (2008) suggest that corporate groups may choose pyramidal structures to boost tunneling (upward cash flows) and propping (downward cash flows). Almeida et al. (2011) find that Korean groups (chaebols) choose pyramidal ownership when they acquire companies with low pledgeable income and high acquisition premiums. Bena and Ortiz-Molina (2013) examine the financing advantage of pyramidal structures.

[9]Dyreng et al. (2015) also discover that corporate groups use foreign holding companies to obtain treaty benefits, such as reduced withholding taxes on dividends. Lewellen and Robinson (2013) and Mintz and Weichenrieder (2010) study the tax motives of American

Incorporation choices of firms under various state laws have been studied in law and economics. Bebchuk and Cohen (2003) study the effects of anti-takeover laws on incorporation choices. Dammann and Schündeln (2011) use veil-piercing data from Thompson (1991) and examine the relationship between veil-piercing risks and out-of-state incorporations.[10]

To my knowledge, there have been no studies on how veil-piercing decisions influence corporate ownership structures, with the only exception, Belenzon et al. (2023), who focus on horizontal structures, or asset partitioning. They find a significant negative relationship between country-level veil-piercing scores and the number of subsidiaries. In other words, corporate groups in countries with higher piercing scores tend to incorporate less subsidiaries, i.e., tend not to partition their assets into new subsidiaries. In contrast to Belenzon et al. (2023), I aim to understand how veil-piercing decisions influence vertical (pyramidal) corporate ownership structures.

The rest of this paper is organized as follows. Section 2 develops a game-theoretic model of corporate veil piercing. Section 3 analyzes equilibrium behavior. Section 4 examines the empirical relationship between veil piercing and ownership structures. Section 5 concludes.

2 Model

A corporate group, or simply a group, consists of a parent company and its subsidiaries.[11] Within the group, an ownership chain is a series of legal entities with direct ownership relations from the parent company to a terminal subsidiary. Each ownership chain describes how the parent company owns a terminal subsidiary. The length of an ownership chain, denoted by l, is defined as the number of distinct direct ownership relations in the chain.

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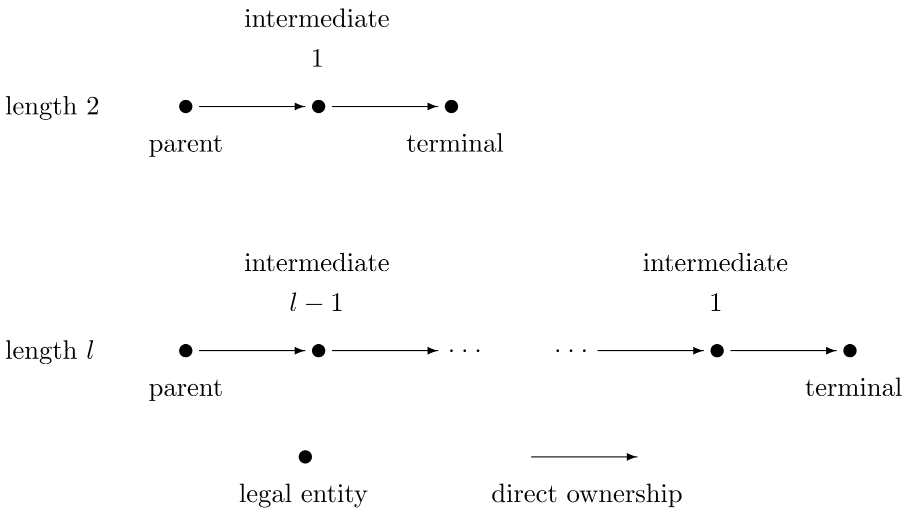
and German corporate groups organizing foreign ownership chains.

[10]Dammann and Schündeln (2012) investigate the formation choices of limited liability companies. Moon (2020) examines the legal grounds of offshore incorporations and discusses their implications for corporate law.

[11]Within the group, a legal entity is called a subsidiary if it is owned directly or indirectly by the parent company. A subsidiary is called terminal if it owns no other subsidiaries. A subsidiary is called intermediate if it is not terminal, i.e., if it owns another subsidiary.

Figure 1. Ownership chains



The parent company may own a terminal subsidiary directly or indirectly through intermediate subsidiaries. If $l = 1$, the parent company directly owns a terminal subsidiary. If $l = 2$, the parent company owns an intermediate subsidiary, which owns a terminal subsidiary. Generally, if $l \geq 2$, the parent company indirectly owns a terminal subsidiary through a series of $l - 1$ intermediate subsidiaries. Figure 1 illustrates examples of ownership chains.

The group plans to invest capital $k > 0$ to own and operate a terminal subsidiary, which will generate income m. Before it is realized, income m is a random variable. It may be realized as a profit $m \geq 0$ or a loss $m < 0$.

If the terminal subsidiary incurs a loss that exceeds the capital, i.e., if $m + k < 0$, a court decides whether to pierce a corporate veil. The likelihood of the veil-piercing decision, denoted by p, is referred to as the piercing rate.[12]

[12] When dealing with an actual case, a court examines relevant facts, including transaction and ownership structures, and then makes the veil-piercing decision. In my model, I have not included such litigation stages, possibly involving strategic plaintiff-creditors.

The group, or the parent company, is liable for the terminal subsidiary's loss $m + k$ with probability p^l, where l is the length of the ownership chain from the parent company to the terminal subsidiary. Length l can be regarded as the number of corporate veils, piercing rate p as the probability of piercing one veil at a time, and p^l as the probability of piercing all l veils.[13]

With probability p^l, the terminal subsidiary's creditors are allowed to reach the parent company's assets. With probability $1 - p^l$, the group is protected by limited liability, and thus not liable for the loss. To balance these outcomes, as a moderator, the court is given a bargaining weight b for the creditors and $1 - b$ for the group, where $0 < b < 1$. The court chooses the piercing rate p to maximize the following payoff function:

$$(1 - p^l)^{1-b}(p^l)^b$$

When organizing an ownership chain with length l, the group incurs an agency cost cl^a, where $c > 0$ determines the overall level of the agency cost, and $a \geq 1$ specifies its convexity. If $a = 1$, the agency cost is linear in length l and c is the constant marginal cost. If $a > 1$, the agency cost is convex in l. Given a net liability $n < 0$, the group chooses l, the length of the ownership chain, to maximize the following payoff function:

$$np^l - cl^a$$

These payoff functions are actually derived from more general settings, as Remarks 1 and 2 show. I introduce some mathematical notations for the purpose of demonstrating them. Let $\mathrm{E}_m[\cdot]$ denote the expectation over m. Let $\Pr(m+k < 0)$ denote the probability that the terminal subsidiary incurs a loss exceeding the capital. Let $\Pr(m + k \geq 0)$ denote the probability of no such loss. Let $n = \Pr(m + k < 0)\mathrm{E}_m\left[(m + k) \mid m + k < 0\right]$ denote the net liability. Note that $n < 0$.

[13] For concise modeling, I assume that the court investigates all the veils (direct ownership relations) in the ownership chain from the parent company to the terminal subsidiary, and decides a piercing rate, which applies equally to each veil, irrespective of its order in the chain. This assumption seems plausible because veil-piercing tests in actual cases, like the *Van Dorn* test, do not stipulate the order of ownership relations as their element.

Remark 1. The court's payoff function can be derived from a bargaining model where creditors (plaintiffs) contend with the group (defendant) to recover the terminal subsidiary's debt $m + k$. Without the court's decision, both parties get their disagreement payoffs, which are assumed to be 0. Upon a decision of the court, the group may have to pay the debt $m + k$ to the creditors. With probability $1 - p^l$, the group does not pay the debt and the court regards that its decision is worth $0 - (m + k)$ to the group. With probability p^l, the group pays the debt to the creditors and the court regards that its decision is worth $0 - (m + k)$ to the creditors. To balance these outcomes, the court is given the bargaining weight b for the creditors and $1 - b$ for the group. A bargaining solution can be written as follows:

$$\mathrm{E}_m \left[(1 - p^l)^{1-b}(0 - (m + k))^{1-b}(p^l)^b(0 - (m + k))^b \mid m + k < 0\right]$$

Because p and l are independent of the random variable m, maximizing the court's payoff function is equivalent to maximizing the bargaining solution.[14]

Remark 2. The group's payoff function can be derived from an expected payoff function of the following form:

$$\begin{aligned}
&\Pr(m + k \geq 0)\mathrm{E}_m [m \mid m + k \geq 0] \\
&\qquad + \Pr(m + k < 0)\mathrm{E}_m \left[0(1 - p^l) \mid m + k < 0\right] \\
&\qquad\qquad + \Pr(m + k < 0)\mathrm{E}_m \left[(m + k)p^l \mid m + k < 0\right] - cl^a
\end{aligned}$$

Here the first term shows the expected profit or loss when $m + k \geq 0$ and it is independent of length l. The second term captures the limited liability protection when $m + k < 0$ and equals zero. The third term, showing the expected net liability when $m + k < 0$, can be simplified to np^l because p and l are independent of the random variable m. The last term is the agency cost. Therefore, maximizing the group's payoff function is equivalent to maximizing the expected payoff function.

[14]This bargaining solution is an application of the weighted Nash bargaining solution, which originated from the seminal work of Harsanyi and Selten (1972). Peters (1986) studies its axiomatic properties, such as individual rationality, Pareto optimality, scale invariance, and independence of irrelevant alternatives.

3 Equilibrium Analysis

In this section I analyze strategic interactions between the court and the corporate group. My analysis is based on two types of games. When the court and the group move simultaneously, each player does not know or observe the other player's strategic choice. In such simultaneous-move games, I examine Nash equilibria to predict the strategic behavior of the players. When the group moves first by organizing its ownership chains, the court observes the group's ownership length before deciding the piercing rate. In such sequential-move games, I examine subgame-perfect Nash equilibria.[15]

First, suppose that the court and the group move simultaneously. When deciding the piercing rate, the court does not know the length of the ownership chain. When organizing the ownership chain, the group does not know the piercing rate. In this simultaneous-move game, the Nash equilibrium (p^*, l^*) arises when each player maximizes its payoff function given the other player's strategy, i.e., when no player can gain by changing its strategy unilaterally. More precisely, given the group's ownership length l^*, the court maximizes its payoff by choosing the piercing rate p^*. At the same time, given p^*, the group maximizes its payoff by choosing l^*. I characterize the Nash equilibrium.

Let $\ln x$ denote the natural logarithm of x. Let e denote the base of the natural logarithm. Let $\exp(x) = e^x$ denote the exponential function of x.

Proposition 1. *In the Nash equilibrium (p^*, l^*) of the simultaneous-move game, the court chooses the piercing rate $p^* = \exp((nb \ln b)^{-1/a}(ac)^{1/a} \ln b)$, and the group chooses the ownership length $l^* = (nb \ln b)^{1/a}(ac)^{-1/a}$.*

Proof. By differentiating the court's payoff function with respect to p, we

[15] Simultaneous-move games do not require players to choose their strategies exactly at the same time, but allow some difference in the timing of strategic choices as far as the players do not know or observe their opponents' strategies. Hence, the players cannot plan their own strategies conditional on the knowledge of the opponents' strategies. However, in sequential-move games, subsequent players exactly know preceding players' strategies and choose their own strategies conditional on the knowledge.

obtain the first order condition:

$$(1-b)(1-p^l)^{-b}(-lp^{l-1})(p^l)^b + (1-p^l)^{1-b}b(p^l)^{b-1}l(p^{l-1}) = 0$$

By rearranging the terms, we get the following equation:

$$(1-p^l)(p^l)^{-1} = (1-b)b^{-1}$$

Hence, in equilibrium, it holds that $p^l = b$ and $\ln p = l^{-1} \cdot \ln b$. Next, by differentiating the group's payoff function with respect to l, we obtain the first order condition:

$$n \cdot p^l \cdot \ln p - a \cdot c \cdot l^{a-1} = 0$$

By plugging $p^l = b$ and $\ln p = l^{-1} \cdot \ln b$ into the above condition, after rearranging the terms, we get the following equation:

$$n \cdot b \cdot \ln b = a \cdot c \cdot l^a$$

By solving for l, we obtain the ownership length $l^* = (nb \ln b)^{1/a}(ac)^{-1/a}$ of the group in equilibrium. By plugging $l = l^*$ into $\ln p = l^{-1} \cdot \ln b$, we get the following equation:

$$\ln p = (nb \ln b)^{-1/a}(ac)^{1/a} \ln b$$

By taking the exponential function on both sides, we obtain the piercing rate $p^* = \exp((nb \ln b)^{-1/a}(ac)^{1/a} \ln b)$ of the court in equilibrium. In Appendix A, we see that the second order conditions are satisfied. □

If the agency cost is linear in length l, i.e., if $a = 1$, the Nash equilibrium (p^*, l^*) can be simplified as $p^* = \exp(c/nb)$ and $l^* = (nb/c) \ln b$.

The equilibrium may shift when the value of a parameter, such as bargaining weight b, agency cost convexity a, agency cost level c, or net liability n, changes. The comparative statics of the equilibrium with respect to the bargaining weight lead to the following result.

Proposition 2. *In equilibrium (p^*, l^*), the piercing rate p^* is increasing in the bargaining weight b, and the ownership length l^* is increasing in b if $b < 1/e$ and decreasing otherwise.*

Proof. Let $p^* = \exp((nb\ln b)^{-1/a}(ac)^{1/a}\ln b)$ and $l^* = (nb\ln b)^{1/a}(ac)^{-1/a}$ denote the equilibrium. First, by differentiating p^* with respect to b, we get:

$$\frac{\partial p^*}{\partial b} = -(ab)^{-1}((nb\ln b)^{-1/a}(ac)^{1/a}\ln b)(1 + 1/\ln b - a/\ln b) \times p^* > 0$$

This is because $0 < b < 1$, $\ln b < 0$, $a \geq 1$, $c > 0$, $n < 0$, and $p^* > 0$. Thus, p^* is increasing in b. Next, by differentiating l^* with respect to b, we get:

$$\frac{\partial l^*}{\partial b} = (ab)^{-1}(nb\ln b)^{1/a}(ac)^{-1/a}(1 + 1/\ln b)$$

If $b < 1/e$, then $1+1/\ln b > 0$ and $\partial l^*/\partial b > 0$. If $b > 1/e$, then $1+1/\ln b < 0$ and $\partial l^*/\partial b < 0$. If $b = 1/e$, then $1 + 1/\ln b = 0$ and $\partial l^*/\partial b = 0$. Thus, l^* is increasing in b if $b < 1/e$ and decreasing otherwise. □

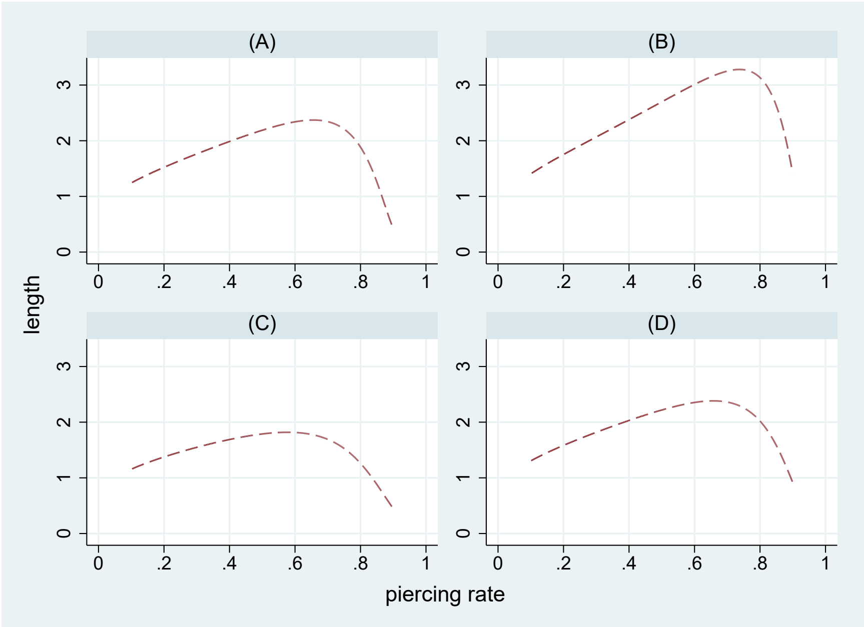
When the bargaining weight increases, the court pierces corporate veils more frequently. The group organizes a longer ownership chain until the bargaining weight reaches the threshold $1/e$, which is approximately 0.37. However, if the bargaining weight increases above the threshold, the group organizes a shorter ownership chain.

Additionally, the comparative statics of the equilibrium with respect to each of the other parameters lead to the following remark, whose proof is presented in Appendix A.

Remark 3. In equilibrium: (i) p^* and l^* are decreasing in the agency cost convexity a if $a < (e/c)(nb\ln b)$ and increasing otherwise. (ii) p^* and l^* are decreasing in the agency cost level c. (iii) p^* and l^* are increasing in $|n|$, the absolute value of the net liability n.

When the agency cost convexity increases, but remains below a certain threshold, the court pierces corporate veils less frequently and the group organizes a shorter ownership chain. However, when the agency cost convexity increases above the threshold, the court pierces corporate veils more frequently and the group organizes a longer ownership chain. When the agency cost level increases, the court pierces corporate veils less frequently and the

Figure 2. Comparative statics



group organizes a shorter ownership chain. When the net liability increases in absolute value, the court pierces corporate veils more frequently and the group organizes a longer ownership chain.[16] Therefore, as each of the three parameters varies, in equilibrium, the piercing rate and the ownership length move toward the same direction, which implies a positive linear relationship.

In contrast, the comparative statics of the equilibrium with respect to the bargaining weight suggest a non-linear relationship between piercing rate and ownership length. Figure 2 illustrates the relationship using numerical examples. For the comparative statics, the bargaining weight b varies between 0 and 1. The agency cost level is normalized at $c = 1.00$.

[16]Suppose that the corporate group considers an insulation plan by investing more capital into the terminal subsidiary. Even if the capital k increases, it is unclear how the net liability $n = \Pr(m+k<0)\mathrm{E}_m[(m+k) \mid m+k<0]$ changes, because $\Pr(m+k<0)$ decreases and $\mathrm{E}_m[(m+k) \mid m+k<0]$ increases. By further assuming that the increase in k leads to the increase in n and the decrease in $|n|$, one can expect the decrease in p^* and l^* from Remark 3. Hence, if the group's insulation plan reduces the net liability in absolute value, then the court pierces corporate veils less frequently, and the group organizes a shorter ownership chain.

Across the panels of Figure 2, the comparative statics of the equilibrium with respect to the bargaining weight show a hump-shaped relationship between piercing rate and ownership length. In panel (A) the agency cost convexity is set at $a = 1.25$ and the net liability is set at $n = -10.00$. The peak of the ownership length $l^* = 2.37$ is reached at the piercing rate $p^* = 0.66$. In panel (B) the parameters are set at $a = 1.25$ and $n = -15.00$. The peak length of 3.28 is reached at the piercing rate 0.74. In panel (C) with $a = 1.50$ and $n = -10.00$, the peak length of 1.82 is reached at the piercing rate 0.58. In panel (D) with $a = 1.50$ and $n = -15.00$, the peak length of 2.38 is reached at the piercing rate 0.66.

Let us imagine a situation where various courts are given different values for the bargaining weight. For instance, a state court is obliged to apply the laws of the state regarding the doctrine of piercing the corporate veil. Stricter state laws for piercing imply that the court is given a lower bargaining weight for the creditors who seek to pierce the veil. As Propositions 1 and 2 suggest, when the bargaining weight varies across courts, there is a hump-shaped relationship between piercing rate and ownership length in equilibrium. The following result characterizes the relationship.

Proposition 3. *As the bargaining weight b changes, in equilibrium, there is a hump-shaped relationship between piercing rate $p^*(b)$ and ownership length $l^*(b)$ such that $l^*(b) = \ln b / \ln p^*(b)$.*

Proof. From Proposition 1, in equilibrium (p^*, l^*), it holds that $p^* = \exp((nb \ln b)^{-1/a}(ac)^{1/a} \ln b)$ and $l^* = (nb \ln b)^{1/a}(ac)^{-1/a}$. For the comparative statics of the equilibrium with respect to the bargaining weight b, take the other parameters a, c, and n as given. We can rewrite the equilibrium piercing rate as:

$$p^*(b) = \exp((nb \ln b)^{-1/a}(ac)^{1/a} \ln b)$$

By taking the natural logarithm function on both sides, we obtain $\ln p^*(b) = (nb \ln b)^{-1/a}(ac)^{1/a} \ln b$. After rearranging the terms, we have $\ln b / \ln p^*(b) = (nb \ln b)^{1/a}(ac)^{-1/a}$, which is equal to $l^*(b)$. Hence, $l^*(b) = \ln b / \ln p^*(b)$.

From Proposition 2, if $b < 1/e$, both $p^*(b)$ and $l^*(b)$ are increasing in b. However, if $b > 1/e$, $p^*(b)$ is increasing but $l^*(b)$ is decreasing in b. Thus, in equilibrium, there is a hump-shaped relationship between piercing rate $p^*(b)$ and ownership length $l^*(b)$ such that $l^*(b) = \ln b / \ln p^*(b)$. $\square$

In short, the comparative statics of the equilibrium with respect to the bargaining weight predict a hump-shaped relationship between piercing rate and ownership length. This theoretical prediction is testable with data on state-level piercing rates because the bargaining weight may vary across state courts, which govern corporate laws and veil-piercing decisions.

Next, suppose that the group and the court move sequentially. In the first stage, the corporate group organizes its ownership chains. Subsequently, in the second stage, the court observes the group's ownership length and decides the piercing rate, which is conditional on the length. In this sequential-move game, the subgame-perfect Nash equilibrium (l^{**}, p^{**}) arises when the second mover (court) maximizes its payoff, in every subgame given the strategy of the first mover (group), by choosing $p^{**}(l)$ as a function of l, and the first mover maximizes its payoff by choosing l^{**} while expecting $p^{**}(l)$ in the second stage. This process is called backward induction. I characterize the subgame-perfect Nash equilibrium.

Proposition 4. *In the subgame-perfect Nash equilibrium (l^{**}, p^{**}) of the sequential-move game where the court observes the group's ownership length, the group chooses the direct ownership chain with length $l^{**} = 1$, and the court chooses the piercing rate $p^{**}(l) = b^{1/l}$ with $p^{**}(1) = b$.*

Proof. The subgame-perfect Nash equilibrium is constructed by backward induction. Suppose that the group chose an ownership chain with length $l \geq 1$ in the first stage. In the second stage, the court observes the ownership length l and chooses the piercing rate $p(l)$ as a function of the length l.

Given the length l, the court chooses $p(l)$ to maximize its payoff. As in the proof of Proposition 1, the court's payoff is maximized when $p^l = b$. Thus, after observing the length l, the court chooses the piercing rate $p^{**}(l) = b^{1/l}$ in the second stage.

By backward induction, in the first stage, the group's payoff function can be rewritten as $np^l - cl^a = nb - cl^a$ because $p = p^{**}(l) = b^{1/l}$. The group's payoff is maximized when $l = 1$ because $n < 0$, $b > 0$, $c > 0$, and $a \geq 1$. Therefore, in the subgame-perfect Nash equilibrium, the group chooses $l^{**} = 1$ in the first stage, and the court chooses the piercing rate $p^{**}(l) = b^{1/l}$ with $p^{**}(1) = b$ in the second stage. □

Remark 4. In the subgame-perfect Nash equilibrium, the piercing rate is increasing in the bargaining weight, whereas the ownership length remains constant. Even if the bargaining weight changes, there is no relationship between piercing rate p^{**} and ownership length l^{**} in equilibrium.

The comparative statics of the subgame-perfect Nash equilibrium with respect to the bargaining weight predict no relationship between piercing rate and ownership length. This theoretical prediction contrasts with that of Proposition 3 showing a hump-shaped relationship and with that of Remark 3 implying a positive linear relationship, both of which are derived from the comparative statics of the Nash equilibrium.

A court may be able to observe the internal ownership structure of a corporate group before deciding whether to pierce a corporate veil in a specific individual case. To explain the court's behavior in such a circumstance, it seems plausible to examine subgame-perfect Nash equilibria in a sequential-move game. According to Remark 4, there may be no significant relationship between veil-piercing decisions and ownership structures.

However, in circumstances where a court, probably an appellate court, sets down a legal test, such as the *Van Dorn* test, affecting the difficulty and frequency of piercing the corporate veil in its jurisdiction, the court may not be able to know the internal ownership structures of corporate groups in generic cases. Moreover, a corporate group may not be able to know the exact piercing rate, even if it knows the generic legal test for piercing, before organizing its internal ownership structure. To explain the players' behavior in the context, it seems reasonable to examine Nash equilibria in a simultaneous-move game. According to Proposition 3, when the bargaining weight varies across courts, there may appear a hump-shaped relationship between piercing rate and ownership length. According to Remark 3, when either of the other parameters, agency cost convexity, agency cost level, or net liability, varies across companies, there may also appear a positive linear relationship.[17]

Depending on whether the court and the group move simultaneously, and depending on which parameter varies, one can expect various patterns of relationship between piercing rate and ownership length. Notably, in the Nash equilibrium of the simultaneous-move game, as the bargaining weight varies, a hump-shaped relationship is expected between piercing rate and ownership length, and it can be tested with data on state-level piercing rates.

4 Empirical Analysis

In this section I examine empirical evidence on the relationship between veil-piercing decisions and ownership structures. To this end I combine four main sources of data with additional state-level variables.

First I use the Orbis database of Bureau van Dijk to obtain data on internal ownership structures of corporate groups.[18] I focus on a sample of corporations that are constituents of the S&P 500 index and that are incorporated in the United States. This sample includes 458 corporations, each of which forms a corporate group. Table 1 presents summary statistics. For each corporate group, LENGTH denotes the mean length of ownership chains, and TOTAL denotes the total number of ownership chains. On average, a corporate group operates 1,306 terminal subsidiaries and organizes the same number of ownership chains with length of 2.22. In addition, for

[17] The net liability may be affected by the standard of care, which is set by a subsidiary company's manager. Ganuza and Gómez (2008) study the optimal standard of care in a model of unilateral accidents by injurers with limited assets.

[18] Accessed in October 2017 at orbis.bvdinfo.com

each corporate group, LENGTHD denotes the mean length of domestic ownership chains, and TOTALD denotes the total number of domestic ownership chains.[19] On average, a corporate group organizes 432 domestic ownership chains with length of 1.96.

Second I use the EDGAR system of the U.S. Securities and Exchange Commission (SEC) to obtain data on states of incorporation and principal executive offices.[20] INCDE is an indicator variable specifying whether the parent company of a corporate group is incorporated in Delaware (DE), and PEOUT is an indicator variable specifying whether the parent company of a corporate group has principal executive offices outside the state of incorporation.[21] As Table 1 shows, 64% of corporate groups have parent companies incorporated in Delaware, and 74% have principal executive offices outside the state of incorporation. From the SEC's EDGAR system, I also obtain data on the Standard Industrial Classification codes to set up industry dummies, each of which corresponds to either of the following eight industries: mining; construction; manufacturing; transportation and utilities; wholesale trade; retail trade; finance, insurance and real estate; services.

Third I obtain data on veil-piercing decisions from Oh (2010), who collects a set of 2,908 cases in the United States between the years 1658 and 2006 from the Westlaw database.[22] Table 6 of Oh (2010) provides state-level piercing rates. For each corporate group, PIERCE denotes the piercing rate in the state of incorporation. In Table 1, the piercing rate is 0.38 on average. The piercing rate is the lowest at 0.26 in Maryland and the highest at 0.68 in Tennessee, while it is 0.34 in Delaware and 0.50 in New York.

[19] An ownership chain is called domestic if the parent company and the terminal subsidiary are incorporated in the same country.

[20] Accessed in December 2018 at sec.gov/edgar/searchedgar/companysearch.html

[21] Bebchuk and Cohen (2003) examine firms' choice of locations to incorporate and find a significant home-state advantage. However, about 59% of Fortune 500 firms are incorporated in Delaware (Table 2). Out-of-state incorporations among Fortune 500 firms take more than 70% of the total (Table 4).

[22] In this dataset there are 2,929 observations because cases involving different co-defendants are divided into separate entries. The overall piercing rate is 0.4851 or 48.51%. Since the 1970s, the number of veil-piercing cases has increased sharply, but the piercing rate by decade has remained around the historical mean.

Table 1. Summary statistics

Variable	Description	Obs	Mean	SD	Min	Max
LENGTH	Mean length of ownership chains	458	2.22	1.18	1.00	8.51
LENGTHD	Mean length of domestic ownership chains	456	1.96	0.94	1.00	7.43
PIERCE	Piercing rate in the state of incorporation	458	0.38	0.09	0.26	0.68
INCDE	1 if incorporated in Delaware (DE)	458	0.64	0.48	0	1
PEOUT	1 if PEO is located outside the state of incorporation	458	0.74	0.44	0	1
NIBT	Net income before taxes in hundred billions USD	458	0.03	0.05	-0.04	0.60
ASSET	Total assets in hundred billions USD	458	0.66	2.15	0.01	24.73
DAR	Ratio of total debt to total assets	458	0.30	0.18	0.00	1.10
LEAR	Ratio of litigation expenses to total assets	458	0.004	0.04	-0.43	0.38
SCIT	State corporate income tax rate	458	0.08	0.02	0.00	0.10
BUFR	Business friendliness measure from CNBC	458	0.51	0.17	0.00	0.92
LECL	Legal climate measure from Institute for Legal Reform	458	0.66	0.21	0.00	0.92
TOTAL	Total number of ownership chains in hundreds	458	13.06	51.83	0.01	349.64
TOTALD	Total number of domestic ownership chains in hundreds	456	4.32	11.91	0.02	118.44

Table 2. Correlations

	LENGTH	LENGTHD	PIERCE	INCDE	PEOUT	TOTAL	TOTALD
LENGTH	-	0.92***	-0.02	0.03	0.02	0.76***	0.64***
LENGTHD	0.88***	-	-0.02	0.02	0.01	0.74***	0.64***
PIERCE	0.05	0.07	-	-0.63***	-0.67***	-0.07	-0.09*
INCDE	0.02	-0.02	-0.56***	-	0.77***	0.04	0.03
PEOUT	0.02	-0.01	-0.65***	0.77***	-	0.01	0.02
TOTAL	0.43***	0.37***	-0.06	0.07	0.14***	-	0.88***
TOTALD	0.37***	0.41***	-0.03	-0.01	0.07	0.90***	-

Note: Pearson coefficients are presented above the diagonal and Spearman coefficients below. ***, **, and * denote significance at 1%, 5%, and 10%, respectively.

Fourth I use the Eikon database of Refinitiv, previously of Thomson Reuters, to obtain financial accounting data of the corporate groups in the sample for the period 2013 to 2017.[23] For each corporate group, NIBT denotes net income before taxes, and ASSET denotes total assets, averaged for the five-year period and expressed in hundred billion US dollars. On average, a corporate group earns about 3 billion US dollars a year and owns about 66 billion US dollars. For each corporate group, DAR denotes the ratio of total debt to total assets, averaged for the five-year period. On average, a corporate group maintains its debt-to-asset ratio at 0.30. For each corporate group, LEAR denotes the ratio of historical litigation expenses to total assets. Historical litigation expenses mean the sum of litigation expenses for the twenty-year period from 1998 to 2017. Negative litigation expenses imply net compensation from lawsuits and settlements. On average, a corporate group retains its litigation-expenses-to-asset ratio at 0.004.

Last I use additional sources to obtain data on some state-level variables. SCIT denotes the state corporate income tax rate, which is obtained from the Tax Foundation for the year 2017.[24] For a state with more than one tax bracket, I take the tax rate for the highest bracket. BUFR denotes the business friendliness measure from CNBC for 2017.[25] CNBC's business friendliness, based on lawsuit and liability climates, regulatory regimes, and overall bureaucracy, is originally presented as a ranking for each state, with the most business friendly state ranked 1 and the least ranked 50. I convert the CNBC measure by taking 1-rank/50. Thus, the most business friendly state gets 0.98 and the least gets zero, while the converted measure is increasing in terms of business friendliness. LECL denotes the legal climate measure from the U.S. Chamber Institute for Legal Reform for 2017.[26] The original legal climate measure is presented as a ranking for each state. By

[23] Accessed in September 2019 and in April 2023 at eikon.refinitiv.com

[24] Accessed in December 2018 at taxfoundation.org/state-corporate-income-tax-rates-brackets-2017

[25] Accessed in December 2018 at cnbc.com/americas-top-states-for-business-2017

[26] Accessed in April 2023 at instituteforlegalreform.com/research/2017-lawsuit-climate-survey-ranking-the-states

taking 1-rank/50, I convert the original measure into the new one, which is increasing in perceived fairness and reasonableness of state court systems.

Table 2 presents the Pearson and Spearman correlations. The mean length of ownership chains is positively correlated with the total number of ownership chains. However, the mean length of ownership chains is not significantly correlated with the piercing rate, while the Pearson and Spearman correlation coefficients show opposite signs. In addition, the mean length of domestic ownership chains is positively correlated with the total number of domestic ownership chains. The mean length of domestic ownership chains is not significantly correlated with the piercing rate.

As Proposition 3 predicts, in equilibrium, there is a hump-shaped relationship between piercing rate and ownership length. To test this prediction, I set up regression models with the quadratic term for the piercing rate.

$$\text{LENGTH}_{ijh} = \beta_0 + \beta_1 \text{PIERCE}_h + \beta_2 \text{PIERCE}_h^2 + \mathbf{S}_h^{'} \boldsymbol{\eta} + \mathbf{G}_i^{'} \boldsymbol{\gamma} + \delta_j + \varepsilon_{ijh}$$

Here LENGTH_{ijh} denotes the mean length of ownership chains for group i in industry j in state h. PIERCE_h denotes the piercing rate in state h. $\mathbf{S}_h$ is a vector of state-specific variables, such as SCIT, BUFR, and LECL. $\mathbf{G}_i$ is a vector of group-specific variables, such as TOTAL, INCDE, PEOUT, and the interaction of the last two. $\mathbf{G}_i$ also includes the financial accounting variables, such as NIBT, ASSET, and DAR. δ_j is the fixed effect of industry j. ε_{ijh} is the error term. When LENGTHD is the dependent variable, TOTAL is replaced with TOTALD.

A positive estimate for β_1 and a negative estimate for β_2 may be viewed as consistent with the hump-shaped relationship between piercing rate and ownership length.

Table 3 presents regression results with the piercing rate in the state of incorporation, PIERCE. All regressions include industry dummies. Standard errors clustered by state of incorporation are in parentheses. The dependent variable is the mean length of ownership chains, LENGTH in columns (1) and (2) while it is the mean length of domestic ownership chains, LENGTHD in (3) and (4).

Table 3. Regression results

	(1) LENGTH	(2) LENGTH	(3) LENGTHD	(4) LENGTHD
PIERCE	11.42***	0.68	11.43***	0.69
	(3.31)	(0.54)	(3.54)	(0.58)
$PIERCE^2$	-12.03***		-12.02***	
	(3.68)		(4.02)	
INCDE	-0.68***	-0.50**	-0.48**	-0.30*
	(0.21)	(0.20)	(0.18)	(0.18)
PEOUT	0.06	0.01	0.01	-0.03
	(0.11)	(0.11)	(0.07)	(0.08)
INCDE×PEOUT	0.65***	0.66***	0.45***	0.46***
	(0.18)	(0.17)	(0.13)	(0.13)
NIBT	1.32	1.36	0.39	0.43
	(1.22)	(1.23)	(0.80)	(0.82)
ASSET	0.04	0.04	0.06***	0.06**
	(0.04)	(0.04)	(0.02)	(0.02)
DAR	0.12	0.09	0.09	0.06
	(0.18)	(0.17)	(0.18)	(0.17)
LEAR	0.60**	0.63***	0.35***	0.39***
	(0.25)	(0.22)	(0.13)	(0.11)
SCIT	2.07	1.08	1.01	0.02
	(1.96)	(1.74)	(1.88)	(1.60)
BUFR	0.35	0.24	0.64**	0.53*
	(0.25)	(0.31)	(0.24)	(0.29)
LECL	-0.15	-0.41	-0.04	-0.30
	(0.30)	(0.27)	(0.27)	(0.25)
TOTAL	0.02***	0.02***		
	(0.001)	(0.001)		
TOTALD			0.04***	0.04***
			(0.004)	(0.004)
Constant	-1.01	1.50***	-1.25	1.19***
	(0.84)	(0.36)	(0.83)	(0.35)
Observations	458	458	456	456
R^2	0.61	0.60	0.46	0.46

Note: All observations are at the group level. All regressions include industry dummies. Standard errors clustered by state of incorporation are in parentheses. ***, **, and * show significance at 1%, 5%, and 10%, respectively.

In column (1) of Table 3, PIERCE is positively and significantly related to LENGTH, and the quadratic term, $PIERCE^2$ is negatively and significantly related to LENGTH. This result implies that the piercing rate in the state of incorporation shows a hump-shaped relationship with the mean length of ownership chains. According to the estimates in column (1), the mean length of ownership chains peaks at the piercing rate of 0.47. The peak of mean length is estimated to be 2.34 at the means of the other independent variables. If courts raise piercing rates but keep them below the threshold 0.47, corporate groups tend to organize longer ownership chains. However, if courts raise piercing rates above the threshold, corporate groups tend to organize shorter ownership chains.

For instance, if courts raise piercing rates from 0.26, which equals the piercing rate in Maryland courts, to 0.50, which equals the piercing rate in New York courts, corporate groups appear to increase the mean length of ownership chains by about 0.56, using additional corporate veils and incurring greater agency costs. A typical corporate group can increase the mean length of ownership chains by 0.56 by inserting an additional intermediate subsidiary into each of 731 ownership chains among its 1,306 chains.[27] If courts raise piercing rates from 0.50 to 0.68, which equals the piercing rate in Tennessee courts, corporate groups decrease the mean length of ownership chains by about 0.51. If courts reduce piercing rates from 0.50 to 0.34, which equals the piercing rate in Delaware courts, corporate groups decrease the mean length of ownership chains by about 0.20. A typical corporate group can reduce the mean length by 0.20 by removing an intermediate subsidiary from each of 264 ownership chains among its 1,306 chains.

One may see an interesting relationship between corporate locations and ownership structures in column (1) of Table 3. A corporate group appears to organize shorter ownership chains by 0.68 than the other groups do when its parent company is incorporated in Delaware and has principal executive

[27] This corporate group may set up one common intermediate subsidiary, or 731 distinct intermediate subsidiaries, along the 731 ownership chains. Thus, the agency cost of longer (more complex) ownership chains depends on the group's overall structure.

offices in the state, i.e., when INCDE = 1 and PEOUT = 0. However, a corporate group tends to have slightly longer ownership chains by $0.03 = 0.65 + 0.06 - 0.68$ than the other groups do when its parent company is incorporated in Delaware and has principal executive offices outside the state, i.e., when INCDE = 1 and PEOUT = 1, though the coefficient for PEOUT is not significant.

In column (3) of Table 3, PIERCE is positively and significantly related to LENGTHD, and PIERCE^2 is negatively and significantly related to LENGTHD. The piercing rate in the state of incorporation shows a hump-shaped relationship with the mean length of domestic ownership chains.

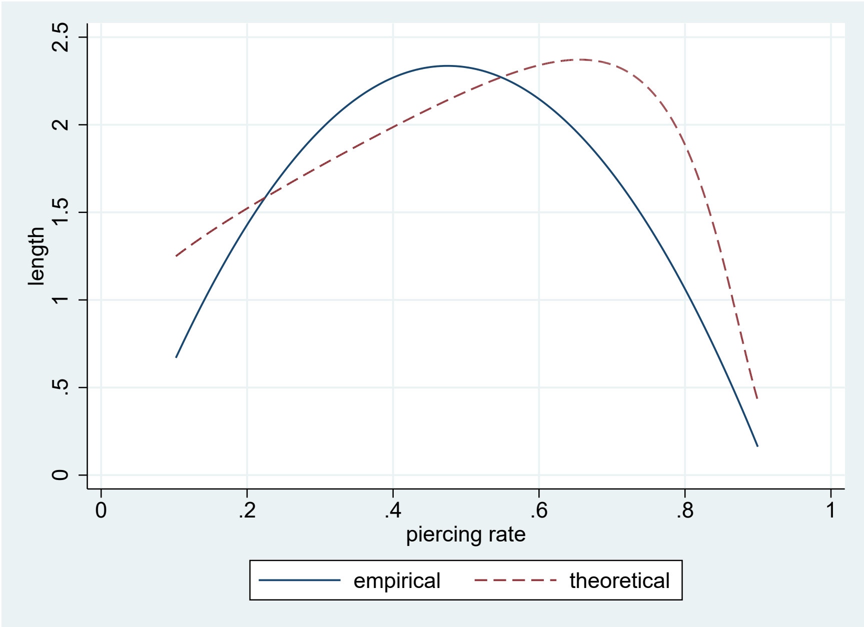
Columns (2) and (4), which do not include the quadratic term, show insignificant results. PIERCE is not significantly related to LENGTH in column (2) or LENGTHD in column (4).

Across the columns of Table 3, the total number of ownership chains is positively and significantly related to the mean length of ownership chains. Likewise, the total number of domestic ownership chains is positively and significantly related to the mean length of domestic ownership chains. This finding suggests that corporate groups organize more complex ownership chains as they own more terminal subsidiaries in number.

Interestingly, among the financial accounting variables in Table 3, the litigation-expenses-to-asset ratio, denoted by LEAR, is positively and significantly related to both of the ownership lengths. Corporate groups appear to organize more complex ownership chains as they have spent more litigation expenses over the past years relative to their total assets. The total assets are positively and significantly related to the mean length of domestic ownership chains in columns (3) and (4) though their coefficients are not significant for the ownership length in columns (1) and (2). The net income before taxes, NIBT, and the debt-to-asset ratio, DAR, are not significantly related to either of the ownership lengths.

Notably, the regressions in Table 3 include state-specific variables, such as the state corporate income tax rate, SCIT, the business friendliness measure,

Figure 3. Empirical and theoretical predictions



BUFR, and the legal climate measure, LECL. These state-specific variables are presumed to be proxies for the bargaining weight, which may vary across state courts.[28] However, their effects appear to be insignificant. SCIT and LECL are not significantly related to either of the ownership lengths. BUFR is positively and significantly related to the domestic ownership length in columns (3) and (4) while it is not significantly related to the ownership length in columns (1) and (2).

Overall, I find a significant hump-shaped relationship between piercing rate and ownership length. This finding is consistent with the theoretical

[28]Because my empirical analysis is based on cross-sectional data, it is hard to conclude a causal effect of veil-piercing decisions on ownership structures. An unobserved factor can affect both piercing rate and ownership length. For instance, the bargaining weight may not be directly observable at the court level, or even at the state level, while influencing the two. To deal with this endogeneity issue, I use SCIT, BUFR, and LECL as proxy variables for the bargaining weight. The Pearson correlation coefficients are 0.08 between SCIT and BUFR, 0.46 between SCIT and LECL, 0.48 between BUFR and LECL. Because these correlation coefficients are in a moderate range, and because the VIF (variance inflation factor) values are below 5, multicollinearity is not likely a problem in my regressions.

prediction in Proposition 3.

Figure 3 illustrates empirical and theoretical predictions within the range of the piercing rates in the data. The empirical prediction in the solid blue line is based on the estimates in column (1) of Table 3. The peak length of 2.34 is reached at a piercing rate around 0.47. The theoretical prediction in the dashed red line is based on the comparative statics of the equilibrium with respect to the bargaining weight b from Proposition 3 when $a = 1.25$, $c = 1.00$, and $n = -10.00$. The peak length of 2.37 is reached at a piercing rate around 0.66. Both predictions show hump-shaped relationships between piercing rate and ownership length.

In Appendix B, I examine ownership data at the chain level to check the robustness of the results in Table 3. Again, there appears a hump-shaped relationship between piercing rate and ownership length. Additionally, a positive linear relationship turns out to be significant in the chain-level data. This finding may be viewed as consistent with the predictions in Remark 3 focusing on changes in subsidiary characteristics.

5 Conclusion

In this paper I develop a game-theoretic model of corporate veil piercing. A court chooses a piercing rate to specify how often the court pierces a corporate veil. A corporate group chooses the length of an ownership chain to specify how many veils the group builds into the chain. I characterize the Nash equilibrium when the court and the group move simultaneously, as well as the subgame-perfect Nash equilibrium when the court observes the group's ownership length, with parameters, such as bargaining weight, agency cost convexity, agency cost level, and net liability. The comparative statics of the Nash equilibrium with respect to the bargaining weight predict a hump-shaped relationship between piercing rate and ownership length.

I also examine the empirical relationship between piercing rate and ownership length by combining data on veil piercing, internal ownership, state incorporation, and financial accounting. Empirical results, based on quadratic regression models, support a hump-shaped relationship between piercing rate and ownership length. This relationship is consistent with my theoretical prediction.

For future studies, it will be interesting to study the relationship between piercing rate and ownership length across countries. In this paper, I focused on variations in piercing rates across American states to explain the patterns of ownership structures. However, as the veil-piercing doctrine and its application differ across countries, there may exist variations in national piercing rates, which can influence internal ownership structures of corporate groups.

It will also be interesting to consider the role of a plaintiff-creditor as a strategic decision-maker. In my model, I assumed that a court would act as a moderator by assigning the bargaining weights for plaintiffs, who were not decision-makers. In more realistic circumstances, plaintiffs may choose their own litigation strategies against corporate groups, depending on the characteristics of lawsuits. Anticipating such litigation strategies, corporate groups can organize ownership structures to reduce liability.

Appendix A. Proofs

Proof of Proposition 1. Let $u(p,l) = (1-p^l)^{1-b}(p^l)^b$ denote the court's payoff function. By differentiating $u(p,l)$ with respect to p, and by rearranging the terms, we get:

$$\frac{\partial u}{\partial p} = -l(1-b)(1-p^l)^{-b}p^{lb+l-1} + lb(1-p^l)^{1-b}p^{lb-1}$$

By differentiating the first derivative with respect to p, and by rearranging the terms, we get:

$$\begin{aligned}\frac{\partial^2 u}{\partial p^2} &= lb(1-b)(1-p^l)^{-b-1}(-lp^{l-1})p^{lb+l-1}\\ &\quad - l(1-b)(1-p^l)^{-b}(lb+l-1)p^{lb+l-2}\\ &\quad + lb(1-b)(1-p^l)^{-b}(-lp^{l-1})p^{lb-1}\\ &\quad + lb(1-p^l)^{1-b}(lb-1)p^{lb-2}\\ &= (1-p^l)^{-b}p^{lb}p^{-2} \times [lb(1-b)(1-p^l)^{-1}(-lp^l)p^l\\ &\quad - l(1-b)(lb+l-1)p^l + lb(1-b)(-lp^l) + lb(1-p^l)(lb-1)]\end{aligned}$$

By evaluating the second derivative at $p = b^{1/l}$, and by rearranging the terms, we get:

$$\begin{aligned}\left.\frac{\partial^2 u}{\partial p^2}\right|_{p=b^{1/l}} &= (1-b)^{-b}b^b b^{-2/l} \times [lb(1-b)(1-b)^{-1}(-lb)b\\ &\quad - l(1-b)(lb+l-1)b + lb(1-b)(-lb) + lb(1-b)(lb-1)]\\ &= (1-b)^{-b}b^b b^{-2/l} \times lb \times [(-lb)b\\ &\quad - (1-b)(lb+l-1) + (1-b)(-lb) + (1-b)(lb-1)]\\ &= (1-b)^{-b}b^b b^{-2/l} \times lb \times [-l] < 0\end{aligned}$$

This is because $0 < b < 1$ and $l > 0$. Thus, the second order condition for maximizing the court's payoff function is satisfied.

Next, let $u(p,l) = np^l - cl^a$ denote the group's payoff function. By differentiating $u(p,l)$ with respect to l, we get:

$$\frac{\partial u}{\partial l} = n \cdot p^l \cdot \ln p - a \cdot c \cdot l^{a-1}$$

By differentiating the first derivative with respect to l, we get:

$$\frac{\partial^2 u}{\partial l^2} = n \cdot p^l \cdot (\ln p)^2 - a(a-1)c \cdot l^{a-2} < 0$$

This is because $n < 0$ and $a \geq 1$. Thus, the second order condition for maximizing the group's payoff function is satisfied. □

Proof of Remark 3. Let $p^* = \exp((nb\ln b)^{-1/a}(ac)^{1/a}\ln b)$ and $l^* = (nb\ln b)^{1/a}(ac)^{-1/a}$ denote the equilibrium.

(i) By differentiating p^* with respect to a, we get:

$$\frac{\partial p^*}{\partial a} = a^{-2}((nb\ln b)^{-1/a}(ac)^{1/a}\ln b)(\ln(nb\ln b) - \ln ac + 1) \times p^*$$

If $a < (e/c)(nb\ln b)$, then $\ln(nb\ln b) - \ln ac + 1 > 0$ and $\partial p^*/\partial a < 0$. To the contrary, if $a > (e/c)(nb\ln b)$, then $\partial p^*/\partial a > 0$. If $a = (e/c)(nb\ln b)$, then $\partial p^*/\partial a = 0$. Thus, p^* is decreasing in a if $a < (e/c)(nb\ln b)$ and increasing otherwise. Also, by differentiating l^* with respect to a, we get:

$$\frac{\partial l^*}{\partial a} = a^{-2}(nb\ln b)^{1/a}(ac)^{-1/a}(\ln ac - \ln(nb\ln b) - 1)$$

If $a < (e/c)(nb\ln b)$, then $\ln ac - \ln(nb\ln b) - 1 < 0$ and $\partial l^*/\partial a < 0$. To the contrary, if $a > (e/c)(nb\ln b)$, then $\partial l^*/\partial a > 0$. If $a = (e/c)(nb\ln b)$, then $\partial l^*/\partial a = 0$. Thus, l^* is decreasing in a if $a < (e/c)(nb\ln b)$ and increasing otherwise.

(ii) By differentiating p^* and l^* with respect to c, we get:

$$\frac{\partial p^*}{\partial c} = (ac)^{-1}((nb\ln b)^{-1/a}(ac)^{1/a}\ln b) \times p^* < 0$$

$$\frac{\partial l^*}{\partial c} = -(ac)^{-1}(nb\ln b)^{1/a}(ac)^{-1/a} < 0$$

This is because $0 < b < 1$, $\ln b < 0$, $a \geq 1$, $c > 0$, $n < 0$, and $p^* > 0$. Thus, p^* and l^* are decreasing in c.

(iii) By differentiating p^* and l^* with respect to n, we get:

$$\frac{\partial p^*}{\partial n} = -(an)^{-1}((nb\ln b)^{-1/a}(ac)^{1/a}\ln b) \times p^* < 0$$

$$\frac{\partial l^*}{\partial n} = (an)^{-1}(nb\ln b)^{1/a}(ac)^{-1/a} < 0$$

This is because $0 < b < 1$, $\ln b < 0$, $a \geq 1$, $c > 0$, $n < 0$, and $p^* > 0$. Thus, p^* and l^* are decreasing in n. As $|n| = -n$, they are increasing in $|n|$. □

Appendix B. Chain-Level Analysis

In this appendix I use ownership data at the chain level to analyze the relationship between piercing rate and ownership length. The data include 598,167 ownership chains of 458 corporate groups, which are constituents of the S&P 500 index, incorporated in the United States. Note that an ownership chain is defined for each terminal subsidiary of a corporate group. A corporate group with larger total assets and net income tends to own more terminal subsidiaries with longer ownership chains. Table B.1 presents summary statistics. CHLENGTH denotes the length of each ownership chain. Its mean is 5.76 and greater than the mean of LENGTH in Table 1.

Table B.2 shows the Pearson and Spearman correlations. The length of each ownership chain is negatively correlated with the piercing rate and positively correlated with the total number of ownership chains.

Table B.3 presents regression results at the chain level. All regressions include industry dummies. Standard errors clustered by state of incorporation are in parentheses. The dependent variable is the length of each ownership chain, CHLENGTH, across the four columns. Columns (1) and (2) show the results with all ownership chains, domestic or foreign. Columns (3) and (4) show the results with domestic ownership chains.

In columns (1) and (3) of Table B.3, PIERCE is positively and significantly related to CHLENGTH, and the quadratic term, PIERCE^2 is negatively and significantly related to CHLENGTH. These results suggest that the piercing rate in the state of incorporation shows a hump-shaped relationship with the ownership length, which are consistent with the regression results at the group level in Table 3.

In columns (2) and (4) without the quadratic term, PIERCE is positively and significantly related to CHLENGTH. Thus, the piercing rate in the state of incorporation exhibits a positive linear relationship with the ownership length. This finding contrasts with those in Table 3 showing insignificant results and deserves an explanation.

Note that the hump-shaped relationship is derived from the comparative statics of the Nash equilibrium with respect to the bargaining weight b. In addition, according to Remark 3, the comparative statics of the Nash equilibrium with respect to each of the other parameters, agency cost convexity a, agency cost level c, and net liability n suggest a positive linear relationship between piercing rate and ownership length. The bargaining weight b is a parameter that reflects the characteristics of a jurisdiction, or a state court. The other parameters a, c, and n reflect the characteristics of a subsidiary company, or an ownership chain. The variations in the data at the chain (subsidiary) level may allow us to observe a positive linear relationship. However, as such variations tend to average out at the group level, the positive linear relationship fades out, and only the hump-shaped relationship remains significant.

Table B.1. Summary statistics at the chain level

Variable	Description	Obs	Mean	SD	Min	Max
CHLENGTH	Length of each ownership chain	598,167	5.76	3.17	1	10
PIERCE	Piercing rate in the state of incorporation	598,167	0.36	0.06	0.26	0.68
INCDE	1 if incorporated in Delaware (DE)	598,167	0.72	0.45	0	1
PEOUT	1 if PEO is located outside the state of incorporation	598,167	0.77	0.42	0	1
NIBT	Net income before taxes in hundred billions USD	598,167	0.08	0.11	-0.04	0.60
ASSET	Total assets in hundred billions USD	598,167	5.29	7.54	0.01	24.73
DAR	Ratio of total debt to total assets	598,167	0.25	0.21	0.00	1.10
LEAR	Ratio of litigation expenses to total assets	598,167	0.003	0.01	-0.43	0.38
SCIT	State corporate income tax rate	598,167	0.08	0.01	0.00	0.10
BUFR	Business friendliness measure from CNBC	598,167	0.53	0.12	0.00	0.92
LECL	Legal climate measure from Institute for Legal Reform	598,167	0.69	0.20	0.00	0.92
TOTAL	Total number of ownership chains in hundreds	598,167	218.31	147.65	0.01	349.64
TOTALD	Total number of domestic ownership chains in hundreds	598,139	45.61	33.12	0.02	118.44

Table B.2. Correlations at the chain level

	CHLENGTH	PIERCE	INCDE	PEOUT	TOTAL	TOTALD
CHLENGTH	-	-0.13***	0.08***	-0.03***	0.69***	0.45***
PIERCE	-0.05***	-	-0.46***	-0.37***	-0.24***	-0.36***
INCDE	0.08***	-0.44***	-	0.88***	0.01***	-0.13***
PEOUT	-0.03***	-0.39***	0.88***	-	-0.13***	-0.26***
TOTAL	0.62***	-0.14***	-0.08***	-0.21***	-	0.77***
TOTALD	0.46***	-0.24***	-0.08***	-0.21***	0.74***	-

Note: Pearson coefficients are presented above the diagonal and Spearman coefficients below. ***, **, and * denote significance at 1%, 5%, and 10%, respectively.

Table B.3. Regression results at the chain level

	(1) CHLENGTH	(2) CHLENGTH	(3) CHLENGTH	(4) CHLENGTH
PIERCE	20.70***	4.58***	56.89***	4.46*
	(3.19)	(0.87)	(9.05)	(2.51)
$PIERCE^2$	-20.46***		-63.63***	
	(3.90)		(10.42)	
INCDE	-0.31**	-0.12	-1.34***	-0.39
	(0.13)	(0.13)	(0.41)	(0.75)
PEOUT	0.14	0.11	-0.17	-0.27
	(0.15)	(0.16)	(0.21)	(0.30)
INCDE×PEOUT	0.92***	0.94***	0.72***	0.81**
	(0.15)	(0.15)	(0.25)	(0.34)
NIBT	1.51*	1.50*	-4.31***	-4.39***
	(0.81)	(0.83)	(0.68)	(0.64)
ASSET	-0.01	-0.01	0.10***	0.10***
	(0.01)	(0.01)	(0.01)	(0.01)
DAR	0.59**	0.65***	-1.88***	-1.81***
	(0.24)	(0.21)	(0.40)	(0.46)
LEAR	-0.47	-0.07	-1.75**	-0.69
	(1.71)	(1.46)	(0.84)	(0.92)
SCIT	2.21	4.42	5.57	8.65
	(2.92)	(3.19)	(3.88)	(7.34)
BUFR	-0.58	-0.31	1.35***	1.64*
	(0.36)	(0.40)	(0.46)	(0.94)
LECL	-0.79**	-1.05***	1.00	-0.53
	(0.32)	(0.32)	(0.71)	(1.27)
TOTAL	0.01***	0.01***		
	(0.001)	(0.001)		
TOTALD			0.02***	0.02***
			(0.002)	(0.002)
Constant	-2.77***	0.004	-10.86***	-0.57
	(0.60)	(0.39)	(2.24)	(2.10)
Observations	598,167	598,167	197,199	197,199
R^2	0.50	0.50	0.35	0.33

Note: All observations are at the chain level. All regressions include industry dummies. Standard errors clustered by state of incorporation are in parentheses. ***, **, and * show significance at 1%, 5%, and 10%, respectively.

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