Changing Tastes, Projection Bias, and Consumer Search

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Abstract

We consider search markets where consumers' preferences may change during the search process. Some consumers are unbiased, i.e. they are fully aware that their preferences will change in the future. However, other consumers are (fully) projection biased, i.e., they believe that their current preferences will remain the same in the future. We show that if the fraction of unbiased consumers is sufficiently large, higher search costs can lead to lower market prices. When search costs become higher, both types of consumers search less, which increases the market power of firms, but the unbiased consumers who do not search are more price sensitive than the projection-biased consumers. Thus, higher search costs may motivate firms to lower their prices to capture the demand of unbiased consumers. Furthermore, when consumers in a current "hot" state decide to buy immediately or to continue searching and buy in a future "cold" state, higher search costs can increase both consumer welfare and social welfare by correcting for excessive search by projection-biased consumers.

JEL-Classification:
Keywords: future taste, projection bias, consumer search, social welfare

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1 Introduction

Consumers are often unable to search frequently due to time or physical constraints, and by the next time they have the opportunity to search, their preferences may have changed. In addition, evidence from various disciplines (see the literature review below) suggests that people have *projection bias*: they tend to overestimate the degree to which their future preferences will be similar to their current preferences.

To gain insight into the impact of changing preferences and projection bias on consumer search behavior and firm pricing, we develop a consumer search model in which consumers' preferences change during the search process. A fraction α of consumers are unbiased in that they correctly perceive that their future preferences will change. However, the remaining $1 - \alpha$ of consumers are (fully) projection biased, i.e., they wrongly believe that their future preferences will be the same as their current preferences.

When consumers in a current "cold" state decide whether to buy immediately or to continue their search and purchase in a future "cold" state, only those consumers whose match value with the product they have learned about is sufficiently low will continue their search, as in the standard search model (e.g. Wolinsky, 1986; Anderson and Renault, 1999). In contrast to the standard model, we find that if the proportion of unbiased consumers is sufficiently large, higher search costs can lead to lower market prices. The main difference is that if the state becomes colder after the search process, consumers receive less benefit from the product if they search and return to the first product. This is an additional cost from searching that would not exist if preferences did not change. In contrast to projection-biased consumers, unbiased consumers correctly perceive this additional cost from searching; hence, their equilibrium demand is less price sensitive. Moreover, since this effect is larger for consumers whose match value with the first product is higher, the higher the search cost, the less price sensitive the demand of unbiased consumers will be. Thus, higher search costs may motivate firms to lower prices if there are enough unbiased consumers.

Furthermore, we find that if there are enough projection-biased consumers and if the post-search state is sufficiently cold relative to the pre-search state, an increase in search costs improves both consumer welfare and social welfare. Projection-biased consumers search too much relative to the socially efficient level because they incorrectly predict that the post-search state will be the same as the pre-search state. Thus, if this over-search problem is severe enough for projection-biased consumers, increasing search costs improves welfare.

When the post-search state is hotter than the pre-search state, the search behavior itself may differ significantly from the classical search model for unbiased consumers, as opposed to (fully) projection-biased consumers who are completely unaware of the difference between the post-search and pre-search states. In fact, we find that consumers' search decisions may not be monotone in the match value with their first product. As the post-state becomes hotter after the search process, consumers receive more benefits from the product if they search and return to the first product. This is the benefit from searching that would not exist if preferences did not change, and this benefit is greater for consumers whose match value with the first product is higher. In other words, if the post-search state is hot enough relative to the pre-search state, consumers with a lower match value to the first product will have less incentive to continue searching.

Increasing preferences also introduce stronger incentives for firms to raise prices as search costs fall. Lower search costs reduce the match value to the first product of marginal consumers who are indifferent between searching and not searching. Thus, if the post-search state is hot enough relative to the pre-search state, their demand becomes less price sensitive, thus motivating firms to raise prices.

A central research question in the consumer search literature is how a change in consumers' search costs affects competition outcomes, including firms' equilibrium profits and the resulting social welfare. In classical models of consumer search (e.g. Wolinsky, 1986; Anderson and Renault, 1999), an increase in search costs generally leads to higher equilibrium prices, higher firm profits, and lower social welfare. The basic logic is this: because consumers must incur costs to learn the price of each firm, those who visit one firm and stay (the fresh demand) are less price-sensitive than those who also visit its competitor(s) and learn their prices. A higher search cost discourages consumers from searching for another firm, thus creating more fresh demand for each firm, which raises its price to further capture that demand. Moreover, although a higher search cost reduces the total volume of searchers, it increases the cost of each search, which is dominant and causes a welfare loss.

The standard argument that higher search costs lead to higher equilibrium prices is challenged by several subsequent papers that focus on the order of search. Among others, Haan, Moraga-González and Petrikaitė (2018) examine a duopoly model in which products have two attributes of match value, one of which is revealed before search and the other after costly search. Choi, Dai and Kim (2018) consider a more general model in which firms' prices are announced in advance and products' match values are to be discovered. The basic idea is that if firms reveal some product attributes before the search starts, the search order will be directed. Then, higher search costs can lower the equilibrium prices because less future search motivates each firm to lower its price before the search starts. In addition, Bar-Isaac, Caruana and Cuñat (2012) incorporate firms' product design (whether a broad market design or a niche design) and find a similar result. A lower search cost may increase firms' prices by motivating firms to choose the niche design and target the fringe market. These papers have similar results that equilibrium prices increase with search costs, but they are based on Varian (1980), which assumes that consumers know some of the attributes (the price or the matching value) of the product. This paper, on the other hand, is based on Wolinsky (1986), which assumes that consumers do not know either the price or the matching value of a product.

2 The Model

The model is based on Wolinsky (1986). The market consists of a unit mass of consumers and two firms, indexed by $i \in \{A, B\}$. Each firm produces at zero cost and there is no discounting.

The utility of each consumer (excluding search costs) from buying firm $i \in \{A, B\}$ product at price p_i is given by

$$u_i = \lambda_k v_i - p_i.$$

The coefficient λ_k captures each consumer's taste for products in $k \in \{n, l\}$, i.e., "now"or "later."Moreover, v_i is the match value attached by this consumer to firm i's product, which distributes according to a distribution $G(\cdot)$ with a differentiable density $g(\cdot)$ over $[\underline{v}, \overline{v}]$. The consumers are assumed to have sufficiently negative outside options $u_0 < 0$ so that they always buy either product (i.e., the market is fully covered).

There are two types of consumers. A fraction α of the consumers are projection-biased, i.e., they wrongly believe in period n that they will continue to have $\lambda_l = \lambda_n$ in period l. The remaining $(1 - \alpha)$ of the consumers are unbiased, i.e., they correctly believe in period n that their preference will be based on λ_l in period l. We assume that although α is publicly known, the firms cannot distinguish which consumers are projection-biased.

Each consumer visits firm $i \in A, B$ with equal probability in period n with equal probability and learns its match value v_i and price p_i. Then the consumer decides whether to visit the next firm in period l to discover its product's match value and price, which incurs a cost $s > 0$. The consumers can return to the firm they have previously visited without incurring any returning cost. We assume that each consumer stops searching if indifferent.

2.1 Benchmark: Consistent Taste

We first consider a benchmark case and let consumers have a consistent taste. That is, $\lambda_n = \lambda_l = 1$. As well known from a standard search model, consumers' decision-making follows the stopping rule that anyone with v_i larger than the reservation value $\dot{v}(p_i, \dot{p})$ buys firm i's product immediately given firm j's equilibrium price $\dot{p}$, where

$$\dot{v}(p_i, \dot{p}, s) = \left\{ v_i | s = \int_{v_i - (p_i - \dot{p})}^{\overline{v}} (v - (v_i - (p_i - \dot{p})))dG(v) \right\}.$$

Suppose that $\dot{v}(\dot{p},\dot{p},s) \in (\underline{v},\overline{v})$, firm i's demand function is given by

$$\begin{aligned} D_i(p_i,\dot{p},s) = \frac{1}{2}\Bigg\{ &(1-G(\dot{v}(p_i,\dot{p},s))) + \int_{\underline{v}}^{\dot{v}(\dot{p},\dot{p},s)} (1-G(v_j+(p_i-\dot{p})))\,dG(v_j) \\ &+ \int_{\underline{v}}^{\dot{v}(p_i,\dot{p},s)} G(v_i-((p_i-\dot{p})))\,dG(v_i)\Bigg\}. \end{aligned} \tag{1}$$

The first line denotes the demand of those who visit firm i and immediately buy there without a further search (the fresh demand), and the second line denotes the demand of those who have visited both firms and know both prices (the return demand). Firm i's profit maximization problem is characterized by $\max_{p_i} \pi_i(p_i,\dot{p},s) = p_i D(p_i,\dot{p},s)$. The increasing hazard rate assumption (i.e., $1-G(v)$ is logconcave in v) is sufficient to guarantee the quasi-concavity of firms' profit function (Anderson and Renault, 1999). In the symmetric equilibrium, the equilibrium price is derived from the first-order condition as follows:[1,2]

$$\begin{aligned} &D_i(\dot{p},\dot{p},s) + \dot{p}\partial_{p_i} D_i(\dot{p},\dot{p},s) = 0 \\ \textit{where}\quad &\partial_{p_i} D_i(\dot{p},\dot{p},s) = \underbrace{-\int_{\underline{v}}^{\dot{v}(\dot{p},\dot{p},s)} g(v)dG(v)}_{\text{the price effect}} \underbrace{-\frac{1}{2}(1-G(\dot{v}))g(\dot{v})\partial_{p_i}\dot{v}(\dot{p},\dot{p},s)}_{\text{the search effect}} < 0. \end{aligned}$$

Some important properties regarding consumers' equilibrium reservation value $\dot{v}(\dot{p},\dot{p},s)$ are summarized as follows:

Remark 1 *(1)* $\partial_{p_i}\dot{v}(\dot{p},\dot{p},s) = 1$; *(2)* $\partial_s \dot{v}(\dot{p},\dot{p},s) = -1/(1-G(\dot{v})) < 0$.

Looking at the slope of the demand function, an increase in firm i's price reduces its demand via two effects:
(1) *The price effect*: an increase p_i discourages the purchase of those who know both firms' prices (those who first visit firm j or firm i);
(2) *The search effect*: an increase p_i increases consumers' reservation value for an additional search, which reduces firm i's fresh demand.[3]
The search effect comprises *the price sensitivity of the reservation value* ($\partial_{p_i}\dot{v}(\dot{p},\dot{p},s)$) and *the reservation value sensitivity of the fresh demand* ($G'(\dot{v}) = g(\dot{v})$).

Processing comparative statics of search cost on the symmetric equilibrium price

[1] For exposition simplicity, we sometimes drop the variables of a function when doing so does not bring ambiguity.

[2] We will denote by $\partial x_i f$ the partial derivative of function f with respect to its argument x_i: if $f = f(x_1...X_n)$ $\partial_{x_i} f = \partial f/\partial x_i$; Similarly, $\partial^2_{x_i x_j} f = \partial^2 f/\partial x_i \partial x_j$, and so forth.

[3] Of course, an increase in $\dot{v}$ would mean a larger return demand for firm i, but since some of the return demand ultimately buys from firm j, the aggregate effect of the increase of $\dot{v}$ on firm i's 'demand is negative. Therefore, the overall decrease in the fresh demand is discounted by $1-G(\dot{v})$.

yields:

$$\frac{d\dot{p}}{ds} = -\frac{\dot{p}\partial^2_{p_i s} D_i(\dot{p},\dot{p}) + \partial_s D_i(\dot{p},\dot{p})}{\partial^2_{p_i p_i}\pi_i(\dot{p},\dot{p})}.$$

We then have[4]

$$\begin{aligned} Sign[\frac{d\dot{p}}{ds}] &= Sign[\partial^2_{p_i s} D_i(\dot{p},\dot{p},s)], \\ where \quad \partial^2_{p_i s} D_i(\dot{p},\dot{p},s) &= -\underbrace{\partial_s \dot{v}(\dot{p},\dot{p},s)}_{<0} g(\dot{v})^2 \\ &\quad -\frac{1}{2}\left\{ \left[-g(\dot{v})^2 + (1-G(\dot{v})g'(\dot{v}))\right] \underbrace{\partial_s \dot{v}(\dot{p},\dot{p},s)}_{<0} \underbrace{\partial_{p_i} \dot{v}(\dot{p},\dot{p},s)}_{=1} \right. \\ &\quad \left. +(1-G(\dot{v}))g(\dot{v}) \underbrace{\partial^2_{p_i s}\dot{v}(\dot{p},\dot{p},s)}_{=0} \right\}. \end{aligned}$$

The way an increase in search cost affects the equilibrium price is in the same direction as how it affects the demand function's slope. First, an increase in search cost does not change the price sensitivity of the reservation value since $\partial_{p_i}\dot{v}(\dot{p},\dot{p},s) = 1$. However, since there are less consumers who go searching and know both firms' prices, the price effect is weakened. Moreover, when $g'(\cdot)$ is sufficiently large (as implied by the increasing hazard rate of $G(\cdot)$), a higher search cost reduces consumers' reservation value of searching, which weakens the search effect. To summarize, a higher search cost weakens both the price effect and the search effect, making the demand faced by firm i less price sensitive.

Observation 1 *When consumers have a consistent taste, a higher search cost increases the symmetric equilibrium price.*

Regarding social welfare, we first derive the total surplus as follows:

$$\dot{TS}(\dot{v},s) = \int_{\dot{v}}^{\overline{v}} v_i dG(v_i) + \int_{\underline{v}}^{\dot{v}} \left(\int_{\underline{v}}^{v_i} v_i dG(v_j) + \int_{v_i}^{\overline{v}} v_j dG(v_j) \right) dG(v_i) - G(\dot{v})s.$$

Processing comparative statics of the search cost s on $\dot{TS}$ yields

$$\frac{d\dot{TS}}{ds} = \underbrace{\partial_{\dot{v}}\dot{TS}(\dot{v},s)}_{=0} \partial_s \dot{v}(\dot{p},\dot{p},s) + \partial_s \dot{TS}(\dot{v},s) = -G(\dot{v}).$$

Since consumers can always optimally adjust their stopping rule when confronting a change in search cost, the envelope theorem suggests that a higher search cost does not affect the total surplus by changing the reservation value. Therefore, the impact a higher search cost has on the total surplus is negative: It causes a higher social loss given a

[4] $\partial^2_{p_i p_i}\pi_i(\dot{p},\dot{p},s) < 0$ from the second-order condition and $\partial_s D_i(\dot{p},\dot{p},s) = 0$.

certain number of searchers (i.e., $G(\dot{v})$).

Observation 2 *When consumers have a consistent taste, a higher search cost decreases the total surplus.*

3 Changing Taste and Projection Bias

We now discuss how consumers' changing taste and their projection-bias issue affects the well-known results of a standard search model. In the symmetric equilibrium, all firms set the same price p^*, and consumers expect all firms which they have not visited will charge p^*. Thus, after visiting firm i and learning its price and match value (p_i, v_i), when expecting firm j to charge p^*, the consumer stops searching if and only if the payoff from buying product i immediately is no less than the expected payoff from visiting firm j, i.e.,

$$\underbrace{\lambda_n v_i - p_i}_{\text{buys from } i \text{ immediately}} \geq \underbrace{-s + \int_{v_i - \frac{\Delta}{\lambda_l}}^{\overline{v}} (\lambda_l v_j - p^*) dG(v_j)}_{\text{visits } j \text{ and buys from } j} + \underbrace{G\left(v_i - \frac{\Delta}{\lambda_l}\right)(\lambda_l v_i - p_i)}_{\text{visits } j \text{ but returns to } i}$$

if the consumer is unbiased, where $\Delta \equiv p_i - p^*$, and

$$\underbrace{\lambda_n v_i - p_i}_{\substack{\text{buys from } i \\ \text{immediately}}} \geq \underbrace{-s + \int_{v_i - \frac{\Delta}{\lambda_n}}^{\overline{v}} (\lambda_n v_j - p^*) dG(v_j)}_{\text{visits } j \text{ and buys from} j} + \underbrace{G\left(v_i - \frac{\Delta}{\lambda_n}\right)(\lambda_n v_i - p_i)}_{\text{visits } j \text{ but returns to } i}$$

if the consumer has projection bias.

Notice that the projection-biased consumers in period n wrongly evaluates their payoff in period l, based on their current taste λ_n instead of their true future taste λ_l (Loewenstein, O'Donoghue and Rabin, 2003). For simplicity, we let $\lambda_n = 1$ and denote λ_l by $\lambda > 0$ but $\neq 1$. Then we can rearrange the above inequalities as follows:

$$s \geq \lambda \int_{v_i - \frac{\Delta}{\lambda}}^{\overline{v}} \left(v_j - \left(v_i - \frac{\Delta}{\lambda}\right)\right) dG(v_j) - (1-\lambda)v_i \equiv h_i(v_i, \Delta) \tag{2}$$

for unbiased consumers and

$$s \geq \int_{v_i - \Delta}^{\overline{v}} (v_j - (v_i - \Delta)) dG(v_j) \equiv \hat{h}_i(v_i, \Delta) \tag{3}$$

for project-biased consumers, where we denote the expected net gain from searching by $h_i(v_i, \Delta)$ for unbiased consumers and $\hat{h}_i(v_i, \Delta)$ for projection-biased consumers. For the unbiased consumers, their utility of searching one more firm consists of their expected incremental value (the first term) being adjusted by their changing future taste (the second term). When $\lambda < 1$, consumers downward adjust their future taste; When $\lambda > 1$, they upward adjust it. For the projection-biased consumers, because they wrongly predict their future taste would stay the same as now, their expected utility of searching for one more firm consists of their expected incremental value only and disregard the adjustment term.

Since $\hat{h}_i(v_i, \Delta)$ is strictly decreasing in v_i,[5] the boundary condition $\hat{h}_i(\overline{v}, \Delta) < s < \hat{h}_i(\underline{v}, \Delta)$ ensures an (interior) cutoff match value (denoted by $\hat{v}^*$) exists for the projection-biased consumer such that anyone with $v_i < \hat{v}^*$ searches. We, therefore, say $\hat{v}^*$ is the reservation value for a projection-biased consumer. For the unbiased consumers, because of the adjustment term, $h_i(v_i, \Delta)$ is decreasing in v_i for $\lambda < 1$ but is U-shaped when $\lambda > 1$.[6] Let $\underline{v}^*$ and $\overline{v}^*$ (with $\underline{v}^* < \overline{v}^*$) be the two roots of $h_i(v_i, \Delta) = s$, which represent the lower and higher reservation values for the unbiased consumers. Based on whether one or both of these two reservation values are within the support $(\underline{v}, \overline{v})$, we have three types of stopping rules, which are summarized by the lemma below:

Lemma 1 *For the projection biased consumers, suppose $\hat{h}_i(\overline{v}, \Delta) < s < \hat{h}_i(\underline{v}, \Delta)$. Then, we have the stopping rule that anyone with match value $v_i < \hat{v}^*(p_i, p^*, s) \in (\underline{v}, \overline{v})$ continues to search, where $\hat{v}^*(p_i, p^*, s)$ solves $s = \hat{h}_i(v_i, \Delta)$.*

For the unbiased consumers, we have three types of stopping rules:

Type-1: if $h_i(\overline{v}, \Delta) < s < h_i(\underline{v}, \Delta)$, anyone with $v_i \in (\underline{v}, \underline{v}^*(p_i, p^*, s))$ continues to search;
Type-2: if $\lambda > 1$ and $h_i(\underline{v}, \Delta) < s < h_i(\overline{v}, \Delta)$, anyone with $v_i \in (\overline{v}^*(p_i, p^*, s), \overline{v})$ continues to search;
Type-3: if $\lambda > 1$ and $\min_{v_i \in (\underline{v}, \overline{v})} h_i(v_i, \Delta) < s < \min\{h_i(\underline{v}, \Delta), h_i(\overline{v}, \Delta)\}$, anyone with $v_i \in (\underline{v}, \underline{v}^*(p_i, p^*, s)) \cup (\overline{v}^*(p_i, p^*, s), \overline{v})$ continues to search.

$\underline{v}^*(p_i, p^*, s)$ and $\overline{v}^*(p_i, p^*, s)$ is the two roots that solve $s = h_i(v_i, \Delta)$ for $v_i \in (-\infty, \infty)$.

The type-1 stopping rule for the unbiased consumers is consistent with the projection-biased consumers and with the benchmark model. That is, anyone with a sufficiently high match value of firm i stops. The type-2 stopping rule is the opposite. That is, anyone with a sufficiently high match value keeps searching. This irregular case happens when consumers' future tastes grow higher. The net benefit of search would always increase because search one more time would bring a consumer at least $(1 - \lambda)v_i$ (the incremental benefit when that consumer finally returns to firm i), which increases in v_i.

[5]Differentiating $\hat{h}_i(v_i, \Delta)$ with respect to v_i yields $-(1 - G(v_i - \Delta)) < 0$.
[6]When $\lambda > 1$, $\partial^2_{v_i v_i} h_i(v_i, \Delta) = \lambda g(v_i, \Delta) > 0$.

3.1 Market Equilibrium

Next, we the symmetric equilibrium. Using $\underline{v}^*(p_i, p^*, s)$, $\overline{v}^*(p_i, p^*, s)$ and $\hat{v}^*(p_i, p^*, s)$, the demands of the projection-biased and unbiased consumers faced by firm i are given by

$$d_i(p_i, p^*, s)$$

$$= \begin{cases} \frac{1}{2}\Big\{(1 - G(\underline{v}^*(p_i, p^*, s))) + \int_{\underline{v}}^{\underline{v}^*(p^*,p^*,s)} \left(1 - G\left(v_j + \frac{\Delta}{\lambda}\right)\right) dG(v_j) \\ + \int_{\underline{v}}^{\underline{v}^*(p_i,p^*,s)} G\left(v_i - \frac{\Delta}{\lambda}\right) dG(v_i)\Big\}, \text{ under the type-1 stopping rule,} \\ \frac{1}{2}\Big\{G(\overline{v}^*(p_i, p^*, s) + \int_{\overline{v}^*(p^*,p^*,s)}^{\overline{v}} \left(1 - G\left(v_j + \frac{\Delta}{\lambda}\right)\right) dG(v_j) \\ + \int_{\overline{v}^*(p_i,p^*,s)}^{\overline{v}} G\left(v_i - \frac{\Delta}{\lambda}\right) dG(v_i)\Big\}, \text{ under the type-2 stopping rule,} \\ \frac{1}{2}\Big\{(G(\overline{v}^*(p_i, p^*, s) - G(\underline{v}^*(p_i, p^*, s)) + \int_{\overline{v}^*(p^*,p^*,s)}^{\overline{v}} \left(1 - G\left(v_j + \frac{\Delta}{\lambda}\right)\right) dG(v_j) \\ + \int_{\overline{v}^*(p_i,p^*,s)}^{\overline{v}} G\left(v_i - \frac{\Delta}{\lambda}\right) dG(v_i) + \int_{\underline{v}}^{\underline{v}^*(p^*,p^*,s)} \left(1 - G\left(v_j + \frac{\Delta}{\lambda}\right)\right) dG(v_j) \\ + \int_{\underline{v}}^{\underline{v}^*(p_i,p^*,s)} G\left(v_i - \frac{\Delta}{\lambda}\right) dG(v_i)\Big\}, \text{ under the type-3 stopping rule,} \end{cases}$$

$$\hat{d}_i(p_i, p^*) = \frac{1}{2}\Big\{(1 - G(\hat{v}^*(p_i, p^*, s))) + \int_{\underline{v}}^{\hat{v}^*(p^*,p^*,s)} \left(1 - G\left(v_j + \frac{\Delta}{\lambda}\right)\right) dG(v_j)$$
$$+ \int_{\underline{v}}^{\hat{v}^*(p_i,p^*,s)} G\left(v_i - \frac{\Delta}{\lambda}\right) dG(v_i)\Big\}$$

and the total demand faced by firm i is $D_i(p_i, p^*, s) = \alpha \hat{d}(p_i, p^*, s) + (1 - \alpha) d_i(p_i, p^*, s)$. Hereafter are some explanations of the compositions of the demand function. We take the projection-biased consumers' demand function $\hat{d}_i(p_i, p^*, s)$ as an example. The composition of the unbiased consumers' demand function $d_i(p_i, p^*, s)$ can be explained similarly. The first term in the curly brackets represents the fresh demand. A project-biased consumer visits firm i first with probability 1/2 and buys its product immediately if $v_i \geq \hat{v}^*(p_i, p^*, s)$. Moreover, a project-biased consumer visits firm j first with probability 1/2 and chooses to search if $v_j < \hat{v}^*(p^*, p^*, s)$ (recall that firm j charges p^*, and consumers expect all firms that they have not visited to charge p^*), and then chooses to buy from firm i after learning (v_i, v_j, p_i, p^*, s) if $\lambda v_i - p_i \geq \lambda v_j - p^*$ or $v_i \geq v_j + \frac{\Delta}{\lambda}$. The third term represents the demand by a projection-biase consumer who returns to firm i. A consumer visits firm i first with probability 1/2 and chooses to search if $v_i < \hat{v}^*(p_i, p^*, s)$, and then chooses to buy from firm i after learning (v_i, v_j, p_i, p^*) if $\lambda v_i - p_i \geq \lambda v_j - p^*$ or $v_j \leq v_i - \frac{\Delta}{\lambda}$.

Given the price p^* charged by the other firm, firm i sets p_i to maximize $\pi_i = p_i D_i(p_i, p^*, s)$. The first-order condition is given by

$$D_i(p_i, p^*, s) + p_i \partial_{p_i} D_i(p_i, p^*, s) = 0, \tag{4}$$

from which we have the symmetric equilibrium price p^*. The following assumptions ensure the existence of the symmetric equilibrium.

Assumption A1.

(1) When $\lambda < 1$, $s \in (0, h_i(\underline{v}, 0))$;

(2) When $\lambda > 1$, $s \in \Big(\max\{0, \min_{v_i \in (\underline{v}, \overline{v})} h_i(v_i, 0)\}, \min\{\hat{h}_i(\underline{v}, 0), \max\{h(\underline{v}, 0), h_i(\overline{v}, 0)\}\}\Big)$.

Assumption A2.

Define

$$\gamma(v) \equiv \left(1 - \frac{1 - \lambda G(v)}{1 - G(v)} \frac{2}{\lambda} - \frac{(1-\lambda)^2}{\lambda(1 - \lambda G(v))(1 - G(v))}\right) \frac{g(v)^2}{1 - G(v)}.$$

- For small α,

  (1) Under the type-1 stopping rule for the unbiased consumers (which prevails when $s \in (h_i(\overline{v}, 0), h_i(\underline{v}, 0))$), $\forall v \in (\underline{v}, \overline{v})$, $g'(v) > \gamma(v)$;

  (2) Under the type-2 stopping rule for the unbiased consumers (which prevails when $\lambda > 1$ and $s \in (h_i(\underline{v}, 0), h_i(\overline{v}, 0))$), $\forall v \in (\underline{v}, \overline{v})$, $g'(v) < \gamma(v)$;

  (3) Under the type-3 stopping rule for the unbiased consumers (which prevails when $\lambda > 1$ and $\min_{v_i \in (\underline{v}, \overline{v})} h_i(v_i, 0) < s < \min\{h_i(\underline{v}, 0), h_i(\overline{v}, 0)\}$), $\forall v \in (\underline{v}, \overline{v})$, $g''(v) < \gamma'(v)$;

- For large α, $v \in (\underline{v}, \overline{v})$, $g'(v) > \left(1 - \frac{2}{\lambda}\right) \frac{g(v)^2}{1 - G(v)}$;

A1 ensures that the equilibrium reservation values $\underline{v}^*$, $\overline{v}^*$ and $\hat{v}^*$ satisfy the boundary conditions provided in Lemma 1. **A2** guarantees the quasi-concavity of firms' profit function in both cases where the fraction of projection-biased consumers is either small or large. We establish that p^* is the symmetric equilibrium price and v^* and $\hat{v}^*$ are the interior reservation values under the following two assumptions:

Lemma 2 *Under **A1** and **A2**, the symmetric equilibrium price is given by p^* in (**??**), and the equilibrium reservation value for projection-biased consumers is given by $\hat{v}^* \in (\underline{v}, \overline{v})$, and that for unbiased consumers is given by (1) $\underline{v}^* \in (\underline{v}, \overline{v})$ under the type-1 stopping rule, (2) $\overline{v}^* \in (\underline{v}, \overline{v})$ under the type-2 stopping rule, and (3) $\underline{v}^*, \overline{v}^* \in (\underline{v}, \overline{v})$ under the type-3 stopping rule, respectively.*

Proof. See Appendix.

Some important properties regarding consumers' equilibrium reservation values are summarized as follows:

Lemma 3 *Consider a symmetric equilibrium in which the equilibrium reservation values for unbiased and projection-biased consumers are given by Lemma 1. Then, these reservation values have the following properties:*

(1) For the projection biased consumers: $\partial_{p_i}\hat{v}^*(p^*, p^*, s) = 1$, $\partial_s\hat{v}^*(p^*, p^*, s) = \frac{-1}{1-G(\hat{v}^*)} < 0$, $\partial^2_{p_i s}\hat{v}^*(p^*, p^*, s) = 0$.

(2) For the unbiased consumers: $\partial_{p_i}\underline{v}^*(p^*, p^*, s) = \frac{1-G(\underline{v}^*)}{1-\lambda G(\underline{v}^*)} > 0$, $\partial_s\underline{v}^*(p^*, p^*, s) = \frac{-1}{1-\lambda G(\underline{v}^*)} < 0$, $\partial^2_{p_i s}\underline{v}^*(p^*, p^*, s) = \frac{(1-\lambda)g(\underline{v}^*)}{[1-\lambda G(\underline{v}^*)]^3} > 0$ *if* $\lambda < 1$; $\partial_{p_i}\overline{v}^*(p^*, p^*, s) = \frac{1-G(\overline{v}^*)}{1-\lambda G(\overline{v}^*)} < 0$, $\partial_s\overline{v}^*(p^*, p^*, s) = \frac{-1}{1-\lambda G(\overline{v}^*)} > 0$, $\partial^2_{p_i s}\overline{v}^*(p^*, p^*, s) = \frac{(1-\lambda)g(\overline{v}^*)}{[1-\lambda G(\overline{v}^*)]^3} > 0$.

Recall that the equilibrium reservation match values in (2) and (3) are such that the search cost s equals the search benefit, i.e., h_i for unbiased consumers and $\hat{h}_i$ for projection-biased consumers. For the projection-biased consumers, since they wrongly ignore their changing future tastes, the search benefit $\hat{h}_i$ decreases in the match value from the current visiting firm, v_i. Then, as consistent with the benchmark model, firm i's higher price p_i or a lower search cost s, both of which increase the search benefit, which requires firm i to offer a higher match value v_i to compensate the consumers to make them stop searching. Moreover, consistent with the benchmark model, a change in search cost does not affect the price sensitivity of reservation value.

However, the unbiased consumers correctly anticipate their changing future tastes. As summarized in Lemma 1, when the stopping rule is characterized by the reservation value $\underline{v}^*$, a sufficiently high match value of firm i leads the unbiased consumers to stop searching. Then, either firm i's higher price or a lower search cost leads to a higher reservation value such that a consumer who initially visits firm i must be compensated with a higher match value v_i to make he/her stop. On the contrary, when the stopping rule is characterized by the reservation value $\overline{v}^*$, consumers' future tastes are higher. Then, an unbiased consumer stops searching when firm i offers a sufficiently low match value.[7] Then, either firm i's higher price or a lower search cost strengthens consumers' incentives to search for one more firm, so the reservation value must become lower such that only the one with a sufficiently low value v_i is willing to stop.

Moreover, different from the benchmark model and the case of projection-biased consumers, for the unbiased consumers, a change in search cost affects the price sensitivity of reservation value. When the stopping rule is characterized by $\underline{v}^*$, a higher search cost

[7]As mentioned, in this case, when firm i offers a sufficiently high match value, the benefit of search stems from the fact that not buying immediately leads to one's higher taste. Then, a higher match value means a larger search benefit because all consumers can at least costlessly return to firm i they have visited.

leads to a lower reservation value, making the marginal consumer indifferent between searching or stopping more sensitive to a change in firm i's price. On the contrary, when the stopping rule is characterized by $\overline{v}^*$, a higher search cost leads to a higher reservation value, making the marginal consumer less sensitive to a change in firm i's price.

3.2 Symmetric Equilibrium Price

We now discuss how a change in search cost affects the symmetric equilibrium prices. Following the logic of the benchmark model, we have[8]

$$Sign[\frac{dp^*}{ds}] = Sign[\partial^2_{p_i s} D_i(p^*, p^*, s)].$$

That is, a higher search cost increases (decreases) the symmetric equilibrium price if and only if it makes the demand less(more) price-sensitive.

The slope of firm i's demand function comprises the price sensitivity of the projection-

[8]From (4), $\frac{dp^*}{ds} = -\frac{\partial^2_{p_i s} D_i(p^*,p^*,s)+\partial_s D_i(p^*,p^*,s)}{\partial^2_{p_i p_i}\pi(p^*,p^*,s)}$. Substiting $\partial_s D_i(p^*, p^*, s) = 0$ and $\partial^2_{p_i p_i}\pi(p^*, p^*, s) < 0$ leads to our arguments below.

biased consumers and the unbiased ones, which is given by

$$\partial_{p_i} D_i(p^*, p^*, s) = \alpha \partial_{p_i} \hat{d}_i(p^*, p^*, s) + (1-\alpha)\partial_{p_i} d_i(p^*, p^*, s),$$

$$\text{where} \qquad \partial_{p_i} \hat{d}_i(p^*, p^*, s) = \underbrace{-\frac{1}{\lambda}\int_{\underline{v}}^{\hat{v}^*} g(v)^2 dv}_{\text{the price effect}} \underbrace{-\frac{1}{2}\left\{(1-G(\hat{v}^*))g(\hat{v}^*)\underbrace{\partial_{p_i}\hat{v}^*(p^*, p^*, s)}_{=1}\right\}}_{\text{the search effect}},$$

$$\partial_{p_i} d_i(p^*, p^*, s) = \begin{cases} \underbrace{-\frac{1}{\lambda}\int_{\underline{v}}^{\underline{v}^*} g(v)^2 dv}_{\text{the price effect}} \underbrace{-\frac{1}{2}\{(1-G(\underline{v}^*))g(\underline{v}^*)\partial_{p_i}\underline{v}^*(p^*, p^*, s)\}}_{\text{the search effect}} \\ \text{under the type-1 stopping rule,} \\ \underbrace{-\frac{1}{\lambda}\int_{\overline{v}^*}^{\overline{v}} g(v)^2 dv}_{\text{the price effect}} \underbrace{+\frac{1}{2}\{(1-G(\overline{v}^*))g(\overline{v}^*)\partial_{p_i}\overline{v}^*(p^*, p^*, s)\}}_{\text{the search effect}} \\ \text{under the type-2 stopping rule} \\ \underbrace{-\frac{1}{\lambda}\left(\int_{\underline{v}}^{\underline{v}^*} g(v)^2 dv + \int_{\overline{v}^*}^{\overline{v}} g(v)^2 dv\right)}_{\text{the price effect}} \\ \underbrace{-\frac{1}{2}\left\{\begin{array}{c} (1-G(\underline{v}^*))g(\underline{v}^*)\partial_{p_i}\underline{v}^*(p^*, p^*, s) \\ +(1-G(\overline{v}^*))g(\overline{v}^*)\partial_{p_i}\overline{v}^*(p^*, p^*, s) \end{array}\right\}}_{\text{the search effect}} \\ \text{under the type-3 stopping rule.} \end{cases} \tag{5}$$

As in the benchmark model, an increase in firm i's price generates the *price effect* and *the search effect*, which reduces the demand of the two types of consumers. Specifically, in the price effect, a higher p_i discourages both types of consumers from buying firm i's products after searching both firms; in the latter effect, a higher p_i increases consumers' reservation value for an additional search, which discourages both types of consumers from buying immediately from firm i. In the presence of consumers' changing tastes, an increase in search cost would subtly alter the strength of these two effects, which leads to findings contradicting our benchmark model.

We find that the symmetric equilibrium price does not necessarily increase in the search cost.

Proposition 1 *Suppose **A1** and **A2** hold. Define*

$$\Gamma(v) \equiv \frac{1-\lambda G(v)}{(1-G(v))^2}\left\{\frac{(1-G(v))[2-\lambda(1+G(v))]}{(1-\lambda G(v))^2} - \frac{2}{\lambda}\right\} g(v)^2.$$

The symmetric equilibrium prices fall with the search cost ($dp^*/ds < 0$) if the fraction of

unbiased consumers is sufficiently large (α is sufficiently small) and if any of the following conditions holds:
(1) when the unbiased consumers follow the type-1 stopping rule, they have a lower future taste ($\lambda < 1$) and $g(v)$ falls sufficiently fast at $v = \underline{v}^*$ such that $g'(\underline{v}^*) < \Gamma(\underline{v}^*) < 0$;
(2) when the unbiased consumers follow the type-2 stopping rule, they have a higher future taste ($\lambda > 1$) and $g(v)$ increases sufficiently slow at $v = \overline{v}^*$ such that $g'(\overline{v}^*) > \Gamma(\overline{v}^*) > 0$;
(3) when the unbiased consumers follow the type-3 stopping rule, they have a higher future taste ($\lambda > 1$) and $g(v)$ falls sufficiently fast at $v = \underline{v}^*$ but increases sufficiently slow at $v = \overline{v}^*$ such that $g'(\underline{v}^*) < \Gamma(\underline{v}^*) < 0$ and $g'(\overline{v}^*) > \Gamma(\overline{v}^*) > 0$.

If none of these conditions holds, the symmetric equilibrium price rises with the search cost.

Proof. See Appendix.

Our benchmark model (as well as other standard search models such as Wolinsky (1986) and Anderson and Renault (1999)) demonstrates that when consumers have consistent future tastes, firms' symmetric equilibrium price increases in the search cost if and only if the distribution function of consumers' match value has an increasing hazard rate property.[9] Proposition 1 differs from the following two aspects: (1) for $\lambda < 1$, $\Gamma(\underline{v}^*) < -\frac{g(\underline{v}^*)^2}{(1-G(\underline{v}^*))}$, so a higher search cost *increases* the symmetric equilibrium price even when $G(v)$ has a decreasing hazard rate property; (2) for $\lambda > 1$, $\Gamma(\overline{v}^*) > -\frac{g(\overline{v}^*)^2}{(1-G(\overline{v}^*))}$, so a higher search cost *decreases* the symmetric equilibrium price even when $G(v)$ has an increasing hazard rate property.

The intuition behind Proposition 1 stems from how the projection-biased and unbiased consumers' price sensitivities are affected by the increase in search cost. We first see the case of unbiased consumers. When the future taste becomes lower ($\lambda < 1$), the unbiased consumers follow the type-1 stopping rule. An increase in search cost reduces the number of searchers who ultimately know both firms' prices, weakening each firm's incentive to undercut the rival. Hence, *the price effect* becomes weaker, which motivates each firm to increase its price.[10] However, the increase in search cost affects the *search effect* in the opposite direction. It first increases *the price-sensitivity of the reservation value (i.e., $d|\partial_{p_i}\underline{v}^*(p^*,p^*,s)|/ds = \partial^2_{p_i s}\underline{v}^*(p^*,p^*,s) > 0$).* The reason is that a higher search cost lowers consumers' reservations for search, making them care more about a marginal change in price. Moreover, under condition (1) in Proposition 1, a lower reservation value $\underline{v}^*$, caused by a higher search cost, increases $g(\underline{v}^*)$, meaning that the marginal effect a change in the reservation value has on the fresh demand grows stronger (i.e., $d|\partial_{\underline{v}^*}(1 - G(\underline{v}^*))|/ds = d|g(\underline{v}^*)|/ds > 0$). Therefore, an increase in search cost would,

[9]As mentioned by Zhou (2011), although a symmetric equilibrium price may decrease in search cost when the match value's PDF has a decreasing hazard rate property, the symmetric equilibrium itself will fail to exist.

[10]In mathematics, we have $d\left(-\frac{1}{\lambda}\int_{\underline{v}}^{\underline{v}^*} g(v)^2 dv\right)/ds > 0$.

adversely, strengthen *the search effect* brought by a marginal change in price, which dominates the weakened *price effect*, motivating firm i to reduce its price.

When the future taste becomes higher ($\lambda > 1$) and the unbiased consumers follow the type-2 stopping rule, as in the case of consumers' lower future tastes discussed above, an increase in search cost reduces the number of searchers who know both firms' prices, therefore weakening *the price effect* and motivating each firm to increase its price.[11] Notice that *the price effect* is less important here compared with the case of consumers' lower future tastes because the future match value is more focused, so the searchers care less about a marginal change in price.[12] Regarding *the search effect*, an increase in search cost now lowers *the price-sensitivity of the reservation value (i.e.,* $d|\partial_{p_i}\overline{v}^*(p^*, p^*, s)|/ds = -\partial^2_{p_i s}\overline{v}^*(p^*, p^*, s) < 0$). The reason is that under the type-2 stopping rule, a higher search cost increases consumers' reservations for search, making them care less about a marginal change in price. However, under condition (2) in Proposition 1, a higher reservation value $\overline{v}^*$, caused by a higher search cost, would lead to a large increase in $g(\underline{v}^*)$, meaning that the change in fresh demand is sufficiently drastic when the reservation value changes marginally (i.e., $d|\partial_{\overline{v}^*}(1-G(\underline{v}^*))|/ds = d|g(\overline{v}^*)|/ds > 0$). Therefore, an increase in search cost would, adversely, strengthen *the search effect* brought by a marginal change in price, which dominates the weakened *price effect*, motivating firm i to reduce its price.[13]

For the projection-biased consumers, since they wrongly neglect their change in future tastes, their stopping rule is consistent with the benchmark model wherein there are no changing tastes in the first place. Therefore, a higher search cost increases the symmetric equilibrium price. To summarize the intuition behind Proposition 1, an increase in search cost makes both the projection-biased and unbiased consumers search less and buy earlier. The increased fresh demand of the projection-biased consumers is less price-sensitive, whereas that of the unbiased consumers could be more price sensitive. Proposition 1 provides the condition under which a higher search cost motivates firms to lower their prices to capture the fresh demand of the increasingly price-sensitive unbiased consumers.

3.3 Welfare Analysis

Define the total surplus TS as

[11] In mathematics, we have $d\left(-\frac{1}{\lambda}\int_{\overline{v}^*}^{\overline{v}} g(v)^2 dv\right)/ds > 0$.

[12] In mathematics, the coefficient of the price effect is larger than unit when $\lambda < 1$ but less than unit when $\lambda > 1$.

[13] The intuition of Proposition 1 when the unbiased consumers follow the type-3 stopping rule is a combination of the type-2 and type-3 stopping rules.

$$
\begin{aligned}
TS^* &= \alpha \hat{ts} + (1-\alpha) ts^*,\\
\textit{where} \qquad \hat{ts}^*(\hat{v}^*(p^*,p^*,s),s) &= \int_{\hat{v}^*}^{\overline{v}} v dG(v) + \lambda \int_{\underline{v}}^{\hat{v}^*} \left(\int_{\underline{v}}^{v} v dG(x) + \int_{v}^{\overline{v}} x dG(x) \right) dG(v) \\
&\quad - G(\hat{v}^*) s,
\end{aligned}
$$

$$
ts^*(p^*,p^*,s) = \begin{cases}
= ts^{*1}(\underline{v}^*(p^*,p^*,s),s) = \int_{\underline{v}^*}^{\overline{v}} v dG(v) \\
\quad + \lambda \int_{\underline{v}}^{\underline{v}^*} \left(\int_{\underline{v}}^{v} v dG(x) + \int_{v}^{\overline{v}} x dG(x) \right) dG(v) - G(\underline{v}^*) s \\
\text{under the type-1 stopping rule,} \\
\\
= ts^{*2}(\overline{v}^*(p^*,p^*,s),s) = \int_{\underline{v}}^{\overline{v}^*} v dG(v) + \\
\lambda \int_{\overline{v}^*}^{\overline{v}} \left(\int_{\underline{v}}^{v} v dG(x) + \int_{v}^{\overline{v}} x dG(x) \right) dG(v) - (1 - G(\overline{v}^*)) s \\
\text{under the type-2 stopping rule} \\
\\
= ts^{*3}(\underline{v}^*(p^*,p^*,s), \overline{v}^*(p^*,p^*,s),s) = \int_{\underline{v}^*}^{\overline{v}^*} v dG(v) \\
+ \lambda \int_{\overline{v}^*}^{\overline{v}} \left(\int_{\underline{v}}^{v} v dG(x) + \int_{v}^{\overline{v}} x dG(x) \right) dG(v) \\
+ \lambda \int_{\underline{v}}^{\underline{v}^*} \left(\int_{\underline{v}}^{v} v dG(x) + \int_{v}^{\overline{v}} x dG(x) \right) dG(v) - (G(\underline{v}^*) + (1 - G(\overline{v}^*)) s \\
\text{under the type-3 stopping rule.}
\end{cases} \tag{6}
$$

The total surplus comprises the convex combination of the projection-biased consumers' gross utility ($\hat{ts}^*(\cdot)$) and that of the unbiased consumers ($ts^*(\cdot)$). The first and second terms in $\hat{ts}^*(\cdot)$ respectively represent projection-biased consumers' gross utility from not searching (the first term) and from searching (the second term). The third term represents the total search cost. The explanation for $ts^*(\cdot)$ follows a similar logic.

How an increase in s affects the total surplus is determined by how it affects the projection-biased and unbiased consumers, respectively. Regarding the unbiased consumers, processing comparative statics of s on $ts^*(\cdot)$ yields,

$$\frac{dts^*(p^*,p^*,s)}{ds} = \begin{cases} = \underbrace{\partial_{\underline{v}^*} ts^{*1}(\underline{v}^*(p^*,p^*,s),s)}_{=0} \partial_s \underline{v}^*(p^*,p^*,s) + \partial_s ts^{*1}(\cdot) = -G(\underline{v}^*) \\ \text{under the type-1 stopping rule,} \\ = \underbrace{\partial_{\overline{v}^*} ts^{*1}(\overline{v}^*(p^*,p^*,s),s)}_{=0} \partial_s \overline{v}^*(p^*,p^*,s) + \partial_s ts^{*2}(\cdot) = -(1-G(\overline{v}^*)) \\ \text{under the type-2 stopping rule} \\ = \underbrace{\partial_{\underline{v}^*} ts^{*3}(\underline{v}^*(p^*,p^*,s), \overline{v}^*(p^*,p^*,s),s)}_{=0} \partial_s \underline{v}^*(\cdot) \\ + \underbrace{\partial_{\overline{v}^*} ts^{*3}(\underline{v}^*(p^*,p^*,s), \overline{v}^*(p^*,p^*,s),s) \partial_s}_{=0} \overline{v}^*(\cdot) + \partial_s ts^{*3}(\cdot) \\ = -(G(\underline{v}^*) + (1-G(\overline{v}^*)) \\ \text{under the type-3 stopping rule.} \end{cases} \tag{7}$$

A change in search cost affects the unbiased consumers' gross utility by two channels: first, it motivates consumers to change their reservation values in response, and second, it changes the total loss stemming from the search. Notice that whichever stopping rule the unbiased consumers are following, since they can correctly anticipate the future changing tastes, they can adjust the reservation value of search in response to the change in search cost. In other words, an increase in search cost induces an optimal adjustment of reservation value by unbiased consumers, which does not affect the total surplus. However, an increase in search cost induces a higher total loss, which is the only negative impact on the total surplus.

However, the projection-biased consumers wrongly neglect their changing future taste, so they cannot optimally adjust their reservation value when confronting an increase in search cost. Processing comparative statics of s on $\hat{ts}^*(\cdot)$ yields

$$\frac{d\hat{ts}^*(\hat{v}^*(p^*,p^*,s),s)}{ds} = \partial_{\hat{v}^*} ts^*(\hat{v}^*(p^*,p^*,s),s)\partial_s \hat{v}^*(p^*,p^*,s) + \partial_s ts^*(\cdot),$$
$$where \quad \partial_{\hat{v}^*} ts^*(\hat{v}^*(p^*,p^*,s),s) = -(1-\lambda)g(\hat{v}^*)\left(\hat{v}^*G(\hat{v}^*) + \int_{\hat{v}^*}^{\overline{v}} x dG(x)\right).$$

Notice that $\partial_{\hat{v}^*} ts^*(\cdot) < 0 (> 0)$ when $\lambda < 1 (> 1)$, meaning that when consumers' future taste becomes lower, the reservation value $\hat{v}^*$ is above the welfare-maximizing level and the projection-biased consumers search too much, whereas when the future taste grows higher, the reservation value $\hat{v}^*$ is below the welfare-maximizing level, and the projection-biased consumers search too less. Following this logic, a higher search cost lowers the reservation value $\hat{v}^*$, which discourages projection-biased consumers from searching too much when they should buy now (i.e., when $\lambda < 1$), which generates a welfare-improving effect. We summarize our findings in the following proposition:

Proposition 2 *suppose* **A1** *and* **A2** *hold. The total surplus increases with the search cost if (1) the fraction of projection-biased consumes is sufficiently large, (2) the consumers' future taste grows lower, and (3) the search cost is either sufficiently small or sufficiently large. Specifically, for a sufficiently large α and for any $\lambda < 1$, there exists $\underline{s}$ and $\overline{s}$ such that $dTS^*/ds > 0$ for $s < \underline{s}$ and $s > \overline{s}$.*

Proof. See Appendix.

When the future taste grows lower, a projection-biased consumer who eventually buys the first product after exploring the second product will have reduced preferences in the future and will regret not buying that product earlier. A higher search cost benefits some of such kind of consumers to correct their delayed purchase, at the expense of the remaining projection-biased consumers who still over-search and pay higher costs. The social benefit outweighs the social loss when s is sufficiently small such that an increase in s drastically reduces the reservation value $\hat{v}^*$ such that a sufficiently large number of projection-biased consumers are made to purchase earlier,[14] or when s is sufficiently large such that the number of projection-biased consumers who still over-search is sufficiently small.

4 Conclusion

In everyday life, consumers often do not have easy access to product categories and therefore cannot frequently search for their favorite products. Due to this lack of search opportunities, consumers may change their tastes during the search process. Moreover, while some consumers can correctly predict their taste change, others may incorrectly project their current taste onto their future taste due to psychological failures.

In this study, we develop a search model to capture consumers' changing tastes for products and their projection bias when search opportunities are scarce. In the standard search model based on Wolinsky (1986), higher search costs increase the market power of firms, raising their equilibrium prices and reducing social welfare.

However, we found that in the presence of a change in consumer preferences, an increase in search costs does not necessarily lead to higher prices or a decrease in social welfare. The logic is that higher search costs reduce search and increase each firm's fresh demand, but if consumers correctly anticipate changes in their preferences (i.e., are unbiased), their increased fresh demand may be more price-sensitive. Thus, if the proportion of unbiased consumers is sufficiently large, higher search costs may motivate

[14]From Lemma 3, $\partial^2_{ss}|\hat{v}^*(p^*, p^*, s)| = -1/(1 - G(\hat{v}^*))^3 < 0$.

firms to lower prices to target price-sensitive consumers. On the other hand, projection-biased consumers are aware of changing preferences and may search too much or too little relative to the socially optimal level. Thus, if the proportion of projection-biased consumers is sufficiently large, and if consumers' future preferences decline sufficiently, high search costs can correct for the social losses due to over-search.

Conventional wisdom from standard search models suggests that changes in search efficiency always have opposite effects on the firm and the consumers. That is, if a change in search efficiency benefits one, it must harm the other. Our results suggest otherwise. If the fraction of unbiased consumers is sufficiently large, improving search efficiency can lead to a Pareto improvement, but if the fraction of projection-biased consumers is sufficiently large, it can reduce the welfare of all parties in society.

Appendix

Derivation of the slope of the demand function.

$$\begin{aligned}
&\frac{\partial D_i(p_i, p^*)}{\partial p_i} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(A1)}\\
&= -\frac{\alpha}{2}\left\{g(\hat{v}^*(p_i, p^*))\frac{d\hat{v}^*(p_i, p^*)}{dp_i} + \frac{1}{\lambda}\int_{\underline{v}}^{\hat{v}^*(p^*, p^*)} g\left(v_j + \frac{\Delta}{\lambda}\right) dG(v_j)\right\}\\
&\quad - \frac{1-\alpha}{2}\left\{g(v^*(p_i, p^*))\frac{dv^*(p_i, p^*)}{dp_i} + \frac{1}{\lambda}\int_{\underline{v}}^{v^*(p^*, p^*)} g\left(v_j + \frac{\Delta}{\lambda}\right) dG(v_j)\right\}\\
&\quad + \frac{\alpha}{2}\left\{G\left(\hat{v}^*(p_i, p^*) - \frac{\Delta}{\lambda}\right) g(\hat{v}^*(p_i, p^*))\frac{d\hat{v}^*(p_i, p^*)}{dp_i} - \frac{1}{\lambda}\int_{\underline{v}}^{\hat{v}^*(p_i, p^*)} g\left(v_i - \frac{\Delta}{\lambda}\right) dG(v_i)\right\}\\
&\quad + \frac{1-\alpha}{2}\left\{G\left(v^*(p_i, p^*) - \frac{\Delta}{\lambda}\right) g(v^*(p_i, p^*))\frac{dv^*(p_i, p_j^*)}{dp_i} - \frac{1}{\lambda}\int_{\underline{v}}^{v^*(p_i, p^*)} g\left(v_i - \frac{\Delta}{\lambda}\right) dG(v_i)\right\}.
\end{aligned}$$

Proof of Lemma 2. We show that p^* satisfies the second-order condition: $\frac{\partial^2 D_i(p_i,p^*)}{\partial(p_i)^2}p_i + \frac{\partial D_i(p_i,p^*)}{\partial p_i} < 0$ at $p_i = p^*$, which is always satisfied if $\left.\frac{\partial^2 D_i(p_i,p^*)}{\partial(p_i)^2}\right|_{p_i=p^*} < 0$. It follows that

$$
\begin{aligned}
&\frac{\partial^2 D_i(p_i,p^*)}{\partial(p_i)^2} \\
&= -\frac{\alpha}{2}\left\{g'(\hat{v}^*)(\partial_{p_i}\hat{v}^{c*})^2 + g(\hat{v}^*)\frac{d\partial_{p_i}\hat{v}^*}{dp_i}\right\} - \frac{1-\alpha}{2}\left\{g'(v^*)(v_1^*)^2 + g(v^*)\frac{dv_1^*}{dp_i}\right\} \\
&\quad + \frac{\alpha}{2}\left\{g(\hat{v}^*)^2\partial_{p_i}\hat{v}^{c*}\left(\partial_{p_i}\hat{v}^* - \frac{2}{\lambda}\right) + G(\hat{v}^*)\, g'(\hat{v}^*)(\partial_{p_i}\hat{v}^*)^2 + G(\hat{v}^*)\, g(\hat{v}^*)\frac{d\hat{v}_1^*}{dp_i}\right\} \\
&\quad + \frac{1-\alpha}{2}\left\{g(v^*)^2\, v_1^*\left(v_1^* - \frac{2}{\lambda}\right) + G(v^*)\, g'(v^*)(v_1^*)^2 + G(v^{c*})\, g(v^*)\frac{dv_1^*}{dp_i}\right\} \\
&= -\frac{\alpha}{2}\left\{(1-G(\hat{v}^*))\left(g'(\hat{v}^*)(\partial_{p_i}\hat{v}^*)^2 + g(\hat{v}^*)\frac{d\partial_{p_i}\hat{v}^*}{dp_i}\right) - g(\hat{v}^*)^2\partial_{p_i}\hat{v}^*\left(\partial_{p_i}\hat{v}^* - \frac{2}{\lambda}\right)\right\} \\
&\quad - \frac{1-\alpha}{2}\left\{(1-G(v^*))\left(g'(v^*)(v_1^*)^2 + g(v^*)\frac{dv_1^*}{dp_i}\right) - g(v^*)^2\, v_1^{c*}\left(v_1^* - \frac{2}{\lambda}\right)\right\} \\
&= -\frac{\alpha}{2}\left\{(1-G(\hat{v}^{c*}))\, g'(\hat{v}^{c*}) + g(\hat{v}^{c*})^2\left(\frac{2}{\lambda} - 1\right)\right\} \\
&\quad - \frac{1-\alpha}{2}\frac{(1-G(v^{c*}))^2}{(1-\lambda G(v^{c*}))^2}\left\{(1-G(v^{c*}))g'(v^{c*}) + \frac{(1-\lambda)^2 g(v^{c*})^2}{\lambda(1-G(v^{c*}))(1-\lambda G(v^{c*}))}\right. \\
&\qquad\qquad \left. + \frac{(1-\lambda G(v^{c*}))g(v^{c*})^2}{(1-G(v^{c*}))}\left(\frac{2}{\lambda} - \frac{1-G(v^{c*})}{1-\lambda G(v^{c*})}\right)\right\}
\end{aligned}
\tag{A2}
$$

where the last equality holds by $\frac{d\partial_{p_i}\hat{v}^{c*}}{dp_i} = 0$ and $\frac{dv_1^{c*}}{dp_i} = \frac{(1-\lambda)^2 g(v^{c*})}{\lambda(1-\lambda G(v^{c*}))^3}$.[15] It follows that

$$
\begin{aligned}
&(1-G)g' + \frac{(1-\lambda)^2 g^2}{\lambda(1-G)(1-\lambda G)} + \frac{(1-\lambda G)g^2}{(1-G)}\left(\frac{2}{\lambda} - \frac{1-G}{1-\lambda G}\right) \\
&> (1-G)g' + \frac{(1-\lambda G)g^2}{(1-G)}\left(\frac{2}{\lambda} - \frac{1-G}{1-\lambda G}\right) \\
&> (1-G)g' + g^2\left(\frac{2}{\lambda} - 1\right)
\end{aligned}
$$

where the second inequality holds by $1 - \lambda G > 1 - G$. Therefore, $\frac{\partial^2 D_i(p^*,p^*)}{\partial(p_i)^2} < 0$ holds under **A2** because the expressions in both of the curly brackets in (A2) are positive. □

[15]By Footnote **??**, $\frac{d\partial_{p_i}\hat{v}^*(p_i,p^*)}{dp_i} = 0$ and $\frac{dv_1^*(p_i,p^*)}{dp_i} = \frac{\partial v_1^*}{\partial v^*}\frac{\partial v^*(p_i,p^*)}{\partial p_i} + \frac{\partial v_1^*}{\partial p_i} = \left.\frac{(1-\lambda)^2 g(v_i - \frac{\Delta}{\lambda})}{\lambda\left(1-\lambda G(v_i - \frac{\Delta}{\lambda})\right)^3}\right|_{v_i = v^*(p_i,p_j^*)}$.

Proof of Proposition 1. We have

$$\begin{aligned}
&\frac{d\delta(v^*,\hat{v}^*,v_1^*)}{ds}\\
&= -\frac{\alpha}{2}\Big\{(1-G(\hat{v}^*))g'(\hat{v}^*)+\Big(\frac{2}{\lambda}-1\Big)g(\hat{v}^*)^2\Big\}\frac{d\hat{v}^*}{ds}\\
&\quad -\frac{1-\alpha}{2}\Big\{\frac{2}{\lambda}g(v^*)^2+[(1-G(v^*))g'(v^*)-g(v^*)^2]v_1^{c*}\Big\}\frac{dv^*}{ds}-\frac{1-\alpha}{2}(1-G(v^*))g(v^*)\frac{dv_1^*}{ds}\\
&= \frac{\alpha}{2}\Big\{g'(\hat{v}^*)+\Big(\frac{2}{\lambda}-1\Big)\frac{g(\hat{v}^*)^2}{1-G(\hat{v}^*)}\Big\}\\
&\quad +\frac{1-\alpha}{2}\Big\{\frac{2}{\lambda}g(v^*)^2+[(1-G(v^*))g'(v^*)-g(v^*)^2]\frac{1-G(v^*)}{1-\lambda G(v^*)}\Big\}\frac{1}{1-\lambda G(v^*)}\\
&\quad -\frac{1-\alpha}{2}\frac{(1-\lambda)(1-G(v^*))g(v^*)^2}{(1-\lambda G(v^*))^3}\\
&= \frac{\alpha}{2}\Big\{g'(\hat{v}^*)-\overline{\Gamma}(\hat{v}^*)\Big\}+\frac{1-\alpha}{2}\Big\{g'(v^*)-\Gamma(v^*)\Big\}\frac{(1-G(v^*))^2}{(1-\lambda G(v^*))^2}
\end{aligned}$$

where $\overline{\Gamma}(v)\equiv\big(1-\frac{2}{\lambda}\big)\frac{g(v)^2}{1-G(v)}$ and the second equality holds by $v_1^*=\frac{1-G(v^*)}{1-\lambda G(v^*)}$, $\frac{d\hat{v}^*}{ds}=-\frac{1}{1-G(\hat{v}^*)}$, $\frac{dv^*}{ds}=-\frac{1}{1-\lambda G(v^*)}$, $\frac{d\hat{v}_1^*}{ds}=0$, and $\frac{dv_1^*}{ds}=\frac{-(1-\lambda)g(v^*)}{(1-\lambda G(v^*))^2}\frac{dv^*}{ds}=\frac{(1-\lambda)g(v^*)}{(1-\lambda G(v^*))^3}$.

First, $g'(\hat{v}^*)-\hat{\Gamma}(\hat{v}^*)>0$ holds by **A2**. Second, $\hat{v}^*$ and v^* are independent of α by (2) and (3). Thus, if $g'(v^*)-\Gamma(v^*)<0$, $\frac{dp^*}{ds}<0$ holds for sufficiently small α. Finally, **A2** and $g'(v^*)-\Gamma(v^*)<0$ require $\overline{\Gamma}(v^*)<\Gamma(v^*)$, which holds if $G(v^*)<\frac{2+\lambda-\sqrt{4+4\lambda-7\lambda^2}}{4\lambda}$. $\square$

Proof of Proposition 2. It follows from (2) and (3) that $\hat{v}^*$ is independent of λ and

$$\frac{dv^*}{d\lambda}=-\frac{\frac{\partial h_i(v^*,p^*,p^*)}{\partial\lambda}}{\frac{\partial h_i(v_i,p^*,p^*)}{\partial v_i}\Big|_{v_i=v^*}}=\frac{v^*G(v^*)+\int_{v^*}^{\overline{v}}xdG(x)}{1-\lambda G(v^*)}>0.$$

$\frac{dTS^*}{ds}>0$ if and only if

$$\underbrace{s+\hat{v}^*-\lambda\left(\hat{v}^*G(\hat{v}^*)+\int_{\hat{v}^*}^{\overline{v}}xdG(x)\right)}_{\equiv L(\hat{v}^*,\lambda)}>\underbrace{\frac{1-G(\hat{v}^*)}{\alpha g(\hat{v}^*)}\left\{\alpha G(\hat{v}^*)+(1-\alpha)G(v^*)\right\}}_{\equiv R(\hat{v}^*,v^*(\lambda))}.$$

Since $\frac{dR(\hat{v}^{c*},v^*(\lambda))}{d\lambda}=\frac{\partial R(\hat{v}^{c*},v^*(\lambda))}{\partial v^*}\frac{dv^*}{d\lambda}=(1-\alpha)g(v^*)\frac{1-G(\hat{v}^*)}{\alpha g(\hat{v}^*)}\frac{dv^*}{d\lambda}>0$, it follows that $L(\hat{v}^*,\lambda)$ is decreasing in λ, whereas $R(\hat{v}^{c*},v^*(\lambda))$ is increasing in λ. Thus, it remains to show $L(\hat{v}^*,\lambda=\underline{\lambda}(s))>R(\hat{v}^{c*},v^*(\lambda=\underline{\lambda}(s)))$ and $L(\hat{v}^*,\lambda=1)<R(\hat{v}^{c*},v^*(\lambda=1))$ to ensure the existence of $\overline{\lambda}(s)\in(\underline{\lambda}(s),1)$ stated in the proposition.

We then have

$$L(\hat{v}^*,\lambda)=(1-\lambda)\left(\hat{v}^*G(\hat{v}^{c*})+\int_{\hat{v}^*}^{\overline{v}}xdG(x)\right).$$

At $\lambda = 1$, we have $\hat{v}^* = v^*$ and $L(\hat{v}^*, 1) = 0 < R(\hat{v}^*, \hat{v}^*) = \frac{1-G(\hat{v}^*)}{\alpha g(\hat{v}^*)} G(\hat{v}^*)$.

At $\lambda = \underline{\lambda}(s)$, we have $v^*(\underline{\lambda}(s)) = \underline{v}$, yielding $R(\hat{v}^*, \underline{v}) = \frac{1-G(\hat{v}^*)}{g(\hat{v}^*)} G(\hat{v}^*)$. It follows from Lemma 3 that $\hat{v}^*$ is continuous and decreasing in s and $\hat{v}^* = \overline{v}$ at $s = 0$. Moreover, we have

$$L(\hat{v}^* = \overline{v}, \underline{\lambda}(s)) = (1 - \underline{\lambda}(s))\overline{v} > R(\hat{v}^* = \overline{v}, v^*(\underline{\lambda}(s))) = 0.$$

Therefore, together with continuity of $L(\hat{v}^*, \underline{\lambda}(s))$ and $R(\hat{v}^{c*}, v^*(\underline{\lambda}(s)))$ in $\hat{v}^*$ and s, it follows that $L(\hat{v}^*, \underline{\lambda}(s)) > R(\hat{v}^*, v^*(\underline{\lambda}(s)))$ for sufficiently small s. $\square$

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