It takes two to tango: Interlockings and Partial Equity Ownership

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Abstract

We study the relation between partial equity ownership and interlocking directorates among rival companies. Partial equity ownership between rivals in the product market, raises profits of both companies by internalizing competition. The gain from the acquisition, however, depends on the value of the target company. When this value is private information, the bidder has to elicit the true value of the equity stake through a proper design of the offer to the target in the context of asymmetric information. The bidder might ask the target to host one of his executives on the board, so as to be able to observe the private value. To succeed, however, the bidder has to convince the target to accept this interlocking directorate. We build a novel framework to analyze the choice to interlock together with the acquisition of a minority equity share. In the case the value of the target is private information, we find the conditions for an interlock to occur at the equilibrium. In this setup, interlocking directorates may be ancillary to a minority acquisition.

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1 Introduction

In Europe, more than 10% of the companies share their executives with other companies, with this percentage raising up to 19% among the largest EU companies (see van Veen and Kratzer, 2011; Heemskerk, 2013). The literature of management and sociology explains the occurrence of these strategic alliances for their impact in terms of resource-seeking, monitoring, reducing market uncertainty, internalizing potential conflicts and assessing human capital (see e.g. Lamb andRoundy, 2016). Companies, through interlocks, may gather private information and therefore gain competitive advantages (see e.g. Larcker and Tayan, 2010). It is also common to observe, worldwide, partial equity ownership among rival companies where the control remains firmly in the hands of the target company: for instance, in the acquisition of non-voting stocks by Gilette in Wilkinson Sword in 1989, in the acquisition by Microsoft of about 7% of the non-voting stock in Apple in 1997, in the passive investment by TCI in Time Warner in 1999 or in the acquisition of minority stock holdings of 25% by Ryanair in Air Lingus in 2006, rising to 29.8% in 2008 (see Gilo, 2000; Ezrachi and Gilo, 2006; OECD, 2008).

Although interlocking directorate and cross-ownership are considered equivalent form of ties between rival companies from an anti-competitive perspective, they play a different role.¹ In the present paper, we study the choice to interlock together with partial equity acquisition, as we believe they are intertwined aspects of the same business strategy, although each of them plays a specific role. In particular, we believe that an interlocking directorate may be ancillary to a partial acquisition. This intuition finds support in the empirical evidence showing that partial acquisitions are more likely among companies directly connected through interlocking directorates at the time of the deal (see Stuart and Yim, 2010; Renneboog and Zhao, 2014). Also, the price paid in the acquisition is significantly lower when the interlocking director comes from the acquiring company: "*Having a board connection between two firms may improve information flow and communication between the firms, and increase each firm's knowledge and understanding of the other firm's operations and corporate culture.... The information advantage may also affect the takeover premium and hence the transaction price of the deal. This is because acquirers with a board connection to the target may enjoy a bargaining advantage in deal negotiations due to their private information about the target firm relative to outside bidders with no connection to the target." (Cai and Seviril, 2012, p.327).*

We focus on minority acquisitions, that is, on cases where the target does not transfer any control over its strategic variables, contrary to what happens in the case of full mergers. Still, the involved companies gain from the deal as they internalize the effect of their actions on profits, thus softening competition (see, for instance, the empirical evidence in Nain and Wang, 2018). In this framework, we assume that the bidder does not observe the degree of efficiency of the target, because the marginal cost is private information, and thus, the bidder benefits from interlocking with the target before launching the deal. Through interlocking, the external director observes the true marginal cost of the target company. Interlocking may become, therefore, instrumental to the corporate deal. For the interlocking to occur, however, the target company has to accept hosting the executive of the future acquiring company on its board. The choice to interlock is the outcome of strategic interaction between the two companies in the context of imperfect information. By interlocking the bidder improves the accuracy of the design of

¹Khanna and Thomas (2009) show that both these ties increase the synchronicity of companies' stock prices, although their intensity is different. This evidence leaves unexplained the specific role of each of these two ties.

the bid, while the target, by disclosing its private information, gains more favourable conditions in the acquisition.

We discuss the characteristics of the solution in which the interlocking directorate is associated to a minority acquisition. We model this problem as a three-stage game: in the first stage, the bidder and the target agree on the interlocking, in the second stage the bidder acquires a non-controlling block in the target company and finally, in the last stage, the two companies compete à la Cournot in the product market. To anticipate our results, we show that depending upon the type of the target company (either efficient or inefficient), the bidder company will succeed in having one of its executives sitting on the board of the target, before launching the bid. Without interlocking, the bidder cannot condition the deal to the true type of the target since it faces adverse selection. Hence, the bidder has to induce truth-telling by the inefficient target. By accepting to host on its board one of the directors of the acquiring company, the target company discloses its private type. While the efficient type might gain from disclosure when competing in the product market, the inefficient target gives up the rent from asymmetric information. Hence, the inefficient target will refuse to interlock. The paper shows that the proposal to interlock can be used by the bidder as a screening device in the market for corporate control. In this respect, interlocking can be interpreted as a preliminary step in the acquisition of the equity stake.

It is well known that interlocking ties undermine the independence of board decisions,² might facilitate the extraction of private benefits to the detriment of minority shareholders and reduce competition in the product market. We suggest an alternative explanation to the simple representation of the new shareholding once the acquisition is completed. We believe, in line with empirical evidence, that interlocking is ancillary to the corporate deal. Interlocking is the result of a strategy to reduce the cost of the acquisition, where the crucial element driving the interlocking decision is information acquisition. This result might have implications for the antitrust analysis where interlocking ties and partial equity ownership are considered equivalent since they both foster collusion in the product market (see Petersen, 2016). Adding to this concern, we point to the fact that interlocking may not only limit competition in the product market, but more importantly, it affects the market for corporate control.

The structure of the paper is the following. After discussing the literature in section 2, we present the setup of the model in section 3. Section 4 solves the competition in the product market, while in section 5 we analyze the result in the market for corporate control. In section 6 we derive the Perfect Bayesian equilibrium of the game. Section 7 concludes the paper.

2 Related Literature

The paper is related to different strands of the literature.

A first group of papers in the sociological and managerial literature analyzes the effects of interlocking directorates (ID) starting from Mizruchi (1996). Some papers suggest that ID reduces the informative gap between companies and that the spillover sourcing from those ties may benefit managerial practices (see *e.g.* Loderer and Peyer, 2002; Ozmel et al., 2013; Lamb and Roundy, 2016; Mazzola et al., 2016). To the best of our knowledge, there are no theoretical papers analyzing the strategic reasons why companies

²See Adams (2017) for the many instances in which board decisions are not independent.

form ID, except Battaggion and Cerasi (2020) where the choice to interlock between rival companies is modeled. Following on that contribution, this paper shows that interlocking is the outcome of a strategic decision of the two companies and, in addition, that it is instrumental to the acquisition of a minority block-holding when the level of efficiency of the target company is privately known.

Another relevant group of papers studies the effect of partial equity ownership (PEO) in oligopoly models. Among these, Reynolds and Snapp (1986) and Flath (1991) study a model with quantity competition, while Shalegia and Spiegel (2012) a model with price competition. They all show that PEO softens competition in equilibrium. Ezrachi and Gilo (2006) analyzes the relation between PEO and the incentive to collude, while Jovanovic and Wey (2014) consider PEO as a preliminary step in the direction of a full merger. Our paper shares the same results on product market competition. In particular, focusing on static models with Cournot competition, the acquisition of a non-controlling block in the rival, raises the profits of both companies by internalizing the strategic externalities caused by competition. However, since we are not focusing on collusion, we restrict our analysis to a static game and to the acquisition of a partial equity stake. Liu et al.(2018) analyze a model where firms competing à la Cournot with PEO face adverse selection due to private information about their marginal costs, a framework similar to our paper. They show that both firms prefer at the equilibrium information sharing. In our paper, the existence of benefits from PEO is the motivation as to why the bidder launches the acquisition of a stake in the rival company. We aim to study the asymmetry of information in the deal and how the ID helps in overcoming it by letting the bidder observe the true value of the target company. Notice that in our model, for the ID to succeed, the target company must be willing to host the director of the rival company.

The paper is also close to the literature on takeovers when the bidder does not know the intrinsic value of the target company. Typically, this literature assumes a contest among bidders addressing directly the shareholders of the company. We simplify the analysis of the market for corporate control, as we assume there are no dispersed shareholders of the target company nor other bidders interested in participating in the acquisition. Our framework is similar to Schnitzer (1996), who studies the conditions by which a raider may decide to launch a friendly takeover when the value of the target is private information.

Finally, the paper is related to the empirical evidence on the acquisitions of companies (see *e.g.* Renneboog and Zhao, 2014; Stuart and Yim, 2010; Chikh and Filbien, 2011; Cai and Seviril, 2012) as it uncovers a role for ID in the market of corporate control. This literature, although mainly empirical provides the ground on which we measure our theoretical predictions.

3 Setup

We consider a simple market structure with two firms, 1 and 2, selling a homogeneous good. The demand for the product is:

$$P(q_1, q_2) = 1 - q_1 - q_2 \tag{1}$$

The level of efficiency of a company is measured by its marginal cost, which can take two values either $c^L = 0$ or $c^H = c$. The parameter $c \in (0, 1]$ captures the difference between an efficient company L and an inefficient one H.

The setup is one of asymmetric information since we assume that the marginal cost of the company 1

is common knowledge and set equal to $c_1^L = 0$. In contrast, company 2's marginal cost is private information. Company 1 forms belief about the rival being efficient by assigning probability $\mu \equiv \Pr \{c_2^L = 0\} \in [0, 1]$.

Company 1 can acquire an equity stake σ of company 2, assuming $0 \le \sigma \le 0.5$ to focus only on minority acquisitions. With a minority shareholding, company 1 chooses quantity q_1 to maximize its consolidated profit, that is, individual profit plus the value of its share of profits in company 2. With a minority shareholding company 1 does not control the quantity of the rival, which is set by company 2 alone. Still, having an equity stake in the rival, softens competition, so that both companies enjoy greater profits. This explains why company 2 might be willing to tender a share of its equity to company 1. To acquire an equity stake in company 2, company 1 has to make an offer (bid); we assume that company 1 (the "bidder") has full bargaining power. Since the bidder does not observe the degree of efficiency of company 2 (the "target"), she might end up paying an excessive price for this minority equity stake. One possible way to circumvent the asymmetric information is, for the bidder to propose an interlocking directorate to the target. If the target accepts to host an executive of the bidder, the bidder would gain fully disclosure on the marginal cost of the rival. However, for this agreement to take place, the target company has to accept the proposal in the first place, i.e. "it takes two to tango".

The timing of the game is the following:

- at t = 0, Nature selects the type of company 2, either $c_2^L = 0$ or $c_2^H = c$; the type remains private information;
- at t = 1, company 1 may propose an interlocking;
- at t = 2, once the proposal is public, company 2 decides whether to accept it and, in case of acceptance, opens the board to an executive of company 1;
- at t = 3, company 1 (the bidder) offers a bid B to acquire a minority equity stake σ in company 2 (the target);
- at t = 4, the two companies compete in the product market.

The game is solved backward, using the concept of Bayesian Perfect Equilibrium.

4 Product Market

We begin by unraveling and solving the game from the last stage: at t = 4, where the two companies compete in quantities, there are two possible cases:

- case *ID*, (all equilibrium variables are denoted by *ID*): when the two companies form an interlocking and, as a consequence, the type of company 2 is common knowledge;
- case *N*, (all equilibrium variables are denoted by *N*) when there is no interlocking and therefore company 1 does not observe the type of company 2.

For a given σ , the consolidated profit of each of the two companies is given by:

$$V_1(\sigma, q_1, q_2) \equiv \pi_1(q_1, q_2) + \sigma \pi_2(q_1, q_2)$$
(2)

$$V_2(\sigma, q_1, q_2) \equiv (1 - \sigma) \pi_2(q_1, q_2)$$
(3)

where π_1 and π_2 are the individual profits of the two companies.

4.1 Case ID

By interlocking, company 1 chooses quantity q_1 to maximize its consolidated profit in (2) observing the marginal cost of the rival; company 2 chooses quantity q_2 to maximize the consolidated profit in (3). Solving for the Nash equilibrium, we derive the quantities:

$$q_1^{ID}(\sigma|c_2) = \frac{1 - \sigma + (1 + \sigma)c_2}{3 - \sigma}$$
(4)

$$q_2^{ID}(\sigma|c_2) = \frac{1 - 2c_2}{3 - \sigma}$$
(5)

and the equilibrium price

$$P^{ID}(\sigma|c_2) = \frac{1 + c_2(1 - \sigma)}{3 - \sigma}$$
(6)

where c_2 can take either of the two values, $c_2^L = 0$ or $c_2^H = c \in (0, 1]$, according to the type of company 2. Notice that for the quantity $q_2^{ID}(\sigma|c_2)$ to be positive, the marginal cost of company 2 is bound to be smaller than 0.5.

At the equilibrium the individual profits are:

$$\pi_1^{ID}(\sigma|c_2) \triangleq \frac{(1+c_2(1-\sigma))(1-\sigma+c_2(1+\sigma))}{(3-\sigma)^2}$$
(7)

and

$$\pi_2^{ID}(\sigma|c_2) \stackrel{\scriptscriptstyle \Delta}{=} \left(\frac{1-2c_2}{3-\sigma}\right)^2 = \left(q_2^{ID}(\sigma|c_2)\right)^2 \tag{8}$$

For a given σ , individual profit of company 1 in (7) is affected by the efficiency of company 2. The more efficient is company 2, the lower the quantity produced by company 1, while leaving the rival to expand its production: its own losses in profit are compensated by the gains in value of the equity stake in the rival.

4.2 Case N

Assume now that the two companies do not interlock. In this case company 1 chooses the quantity q_1 to maximize the consolidated profit in (2), without observing the quantity set by company 2. Solving for the Nash equilibrium, we obtain the following quantities:

$$q_1^N(\sigma|c_2;\mu) = \frac{1-\sigma}{3-\sigma} + \frac{c(1+\sigma)(1-\mu)}{3-\sigma}$$
(9)

$$q_2^N(\sigma|c_2;\mu) = -\frac{c_2}{2} + \frac{-c\left(1+\sigma\right)\left(1-\mu\right)+2}{2\left(3-\sigma\right)}$$
(10)

and the equilibrium price

$$P^{N}(\sigma|c_{2};\mu) = \frac{c_{2}}{2} + \frac{-c(1-\mu)(1+\sigma)+2}{2(3-\sigma)}$$
(11)

where c_2 can take either of the two values, $c_2^L = 0$ or $c_2^H = c \in (0, 1]$, according to the type of company 2. Notice that, as the level of inefficiency of the target increases, i.e. c, the quantity produced by company 2 is reduced and partially re-assigned to company 1. In addition all the quantities and the price are now affected by the parameter μ , representing the belief of company 1 about the type of company 2.

The equilibrium profit of company 1 without interlocking is:

$$\pi_1^N(\sigma|c_2;\mu) \stackrel{\scriptscriptstyle \Delta}{=} \frac{(c\,(1+\sigma)\,(1-\mu)-\sigma+1)\,((3-\sigma)\,c_2-(1-\mu)\,c\,(1+\sigma)+2)}{2\,(3-\sigma)^2} \tag{12}$$

while the equilibrium profit of company 2 is

$$\pi_2^N(\sigma|c_2;\mu) \stackrel{\scriptscriptstyle \Delta}{=} \left(q_2^N(\sigma|c_2;\mu)\right)^2 \tag{13}$$

As expected, the profit of company 2 is decreasing in c_2 , while the profit of company 1 increases in the level of inefficiency of the target.

5 Market for corporate control

Now we turn to the deal in the market for corporate control. Company 1 (the bidder) launches bid B to acquire a minority equity stake in company 2 (the target). In exchange, the target relinquishes a share $\sigma \leq 0.5$ of his profits to the bidder. As before, there are two possible cases, according to the occurrence of the interlocking.

5.1 Case ID

By having an executive sitting on the board, the bidder observes the marginal cost of the target, hence she will offer a contract conditional on the type of the target, solving the following program:

$$\begin{cases} \max \left[V_1^{ID}(\sigma|c_2) - B_{c_2}^{ID} \right] \\ s.t. \quad (IR_2) \quad V_2^{ID}(\sigma|c_2) + B_{c_2}^{ID} \ge V_2^{ID}(0|c_2) \end{cases} \end{cases}$$

Given that the bidder has full bargaining power, the (IR_2) is saturated.

Proposition 1 When the bidder (company 1) interlocks with the target (company 2), the deals at the equilibrium are:

• if $c_2^L = 0$,

$$\sigma_0^{ID} = 0.5 \text{ and } B_0^{ID} = \frac{7}{225};$$
 (14)

• if
$$c_2^H = c \leq \frac{1}{5}$$

$$\sigma_c^{ID} = \frac{5c-1}{3c-1} \text{ and } B_c^{ID} = \frac{35}{18}c^2 - \frac{17}{18}c + \frac{1}{9};$$
 (15)

Proof. (See A.1 in the Appendix). \blacksquare

The bidder acquires an increasing equity stake in the efficiency of the target. Notice when $c > \frac{1}{5}$, the bidder abstains from making an offer.

5.2 Case N

In this case the bidder does not observe the marginal cost of the target and faces the risk of adverse selection. With asymmetric information, the inefficient target will pretend to be the efficient one, since the equity share of the efficient is valued more than that of the inefficient target. The bidder has to prevent the inefficient from mimicking the efficient one. As it is usual in these class of models, the two binding constraints are the participation constraint of the efficient type, (IR_{2L}) , and the incentive compatibility constraint of the inefficient type, (IC_{2H}) .

Company 1 solves the following program:

$$\max \quad \mu \left[V_1^N(\sigma_0|0;\mu) - B^N(\sigma_0|0;\mu) \right] + (1-\mu) \left[V_1^N(\sigma_c|c;\mu) - B^N(\sigma_c|c;\mu) \right] \\s.t. \quad (IR_{2L}): \quad V_2^N(\sigma_0|0;\mu) + B^N(\sigma_0|0;\mu) \ge V_2^N(0|0;\mu) \\ (IC_{2H}): \quad V_2^N(\sigma_c|c;\mu) + B^N(\sigma_c|c;\mu) \ge V_2^N(\sigma_0|c;\mu) + B^N(\sigma_0|c;\mu)$$
(16)

From the two binding constraints, we derive the two bids:

$$B_0^N(\mu) \triangleq B^N(\sigma_0|0;\mu) = V_2^N(0|0;\mu) - V_2^N(\sigma_0|0;\mu)$$
(17)

$$B_{c}^{N}(\mu) \triangleq B^{N}(\sigma_{c}|c;\mu) = B^{N}(\sigma_{0}|0;\mu) + V_{2}^{N}(\sigma_{0}|c;\mu) - V_{2}^{N}(\sigma_{c}|c;\mu)$$
(18)

The two bids reflect the distortion induced by the asymmetry of information. The efficient target is left at his reservation value as in the interlocking case (although the reservation value now involves an expected term, given by the bidder's belief about the type of the target, μ).

We can now state the result in the following proposition.

Proposition 2 The bidder offers the following couple of contracts with the objective to induce each type of target to self-select at the equilibrium:

$$\sigma_0^N(\mu) = 0.5, \quad B_0^N(\mu) = \frac{1}{9} - \frac{1}{2} \left(\frac{3}{10} c \left(\mu - 1\right) + \frac{2}{5} \right)^2$$

and $\sigma_c^N(\mu) = \frac{5c + c\mu - 1}{3c - c\mu - 1}, \quad B_c^N(\mu) = \left(\frac{1}{9} - \frac{1}{2} \left(\frac{3}{10} c \left(\mu - 1\right) + \frac{2}{5} \right)^2 \right) + \frac{(3c\mu - 8c + 4)^2}{200} - \frac{c(\mu + 1)(3c - 1)^2}{2c\mu - 6c + 2}.$

Proof. (See A.2 in the Appendix). \blacksquare

In order to induce truth-telling, the bidder leaves a rent to the inefficient target, to compensate him for the loss incurred when tendering $\sigma_c^N(\mu)$ instead of $\sigma_0^N(\mu)$.

We can show that this informational rent is decreasing in μ .

Corollary 3 *The rent is decreasing in the initial belief* μ *.*

Proof. (See A.4 in the Appendix). ■

The more likely it is that the target is efficient, the smaller the rent paid to the inefficient target to convince him to abide to the incentive compatibility constraint.

6 The Equilibrium

In this section, we solve for the equilibrium of the game.

First we solve for stage t = 2 where the target has to decide whether to accept or reject the proposal to interlock by the bidder, accepting to host an executive on the board. If the target accepts the proposal, the interlock takes place. Then we move back to stage t = 1, asking whether the bidder is willing to launch the proposal to interlock.

At stage t = 2, a target of type c_2 accepts the interlock whenever

$$V_2^{ID}(\sigma^{ID}|c_2;\mu) + B_{c_2}^{ID}(\mu) \ge V_2^N(\sigma^N|c_2;\mu) + B_{c_2}^N(\mu)$$

The following proposition states our result.

Proposition 4 At stage t = 2, once the proposal has been made,

- the efficient target ($c_2^L = 0$) is indifferent between accepting or rejecting the interlocking;
- the inefficient target $(c_2^H = c)$) always rejects the interlocking.

Proof. (See A.3 in the Appendix). \blacksquare

Proposition 4 shows that an interlocking may only occur when the target is efficient.

Let's now turn to stage t = 1, when the bidder has to decide whether to propose the interlocking to the target. By proposing the interlocking, the bidder may gain insights on the type of the target.

As a matter of fact, when the bidder observes "acceptance", she knows with certainty that the target is efficient, since according to Proposition 4, the inefficient target will never accept the proposal. In this case the bidder updates her initial belief μ to a posterior equal to 1. Conversely, when the bidder observes "rejection", there is still some uncertainty about the true type of the target. The bidder will update her initial belief μ using the Bayes rule, given the observed actions. Notice that, the efficient target is indifferent between accepting or rejecting the ID proposal, hence we assume he tosses a coin. The posterior probabilities become:

$$\nu = prob(c_2 = 0 | No) = \frac{\mu}{2 - \mu}$$

and $\gamma = prob(c_2 = 0 | yes) = 1$. The following Proposition states our result. **Proposition 5** For a given c, the bidder proposes the interlocking agreement, whenever the initial belief of facing an efficient target is sufficiently large, that is when $\mu > \hat{\mu}$ with

$$\widehat{\mu} \equiv \frac{1}{9c} \left(6c + \sqrt{-102c - 66c^2 + 25} - 2 \right)$$

Proof. (See A.5 in the Appendix). ■

This result shows when the bidder benefits from proposing an interlocking if planning to acquire a minority equity stake in a rival company.

Here is the intuition about a lower bound on μ . Recall that the rent paid to the target is decreasing in the initial belief μ , according to Corollary 3. Hence, when μ is large, that is when the initial probability of facing an efficient target is high, the bidder prefers to make the proposal: by making the proposal the bidder may acquire new information about the type of the target. Precisely, it is the interplay between c and μ that affects the bidder's choice. If c is relatively small -meaning that even the inefficient target is not too inefficient - for whatever probability μ the bidder prefers to pop the proposal to interlock. As c increases, the bidder proposes an interlocking agreement only if the probability that the target is efficient is sufficiently large. Furthermore, even if the target rejects the agreement, the bidder acquires new information about the type of target.

We can now solve the game for the sub-game perfect Bayesian equilibrium, focusing on the case where the bidder proposes the interlocking agreement and acquires a partial equity ownership.

Proposition 6 Whenever $\mu \ge \hat{\mu}$ and $c \le \frac{1}{5}$, at the sub-game perfect Bayesian equilibrium the bidder proposes the interlocking:

- the target agrees to host the interlocking director; with posterior probability $\gamma = 1$, the bidder proposes $\sigma_0^{ID} = 0.5$ and $B(\sigma_0^{ID}|0)$;
- the target refuses the proposal; with posterior probability $\nu = \frac{\mu}{2-\mu}$ the bidder offers a menu σ_j^N and $B(\sigma_j^N|c_2)$ with j = 0, c.

Proof. The proof follows from the equilibrium values of the shares and from the results in Propositions 4 and 5. ■

Notice that we only observe the occurrence of an interlocking directorate in association with partial equity ownership in the first of the two cases, i.e. when the bidder proposes the agreement and the target accepts it. Still, the result in Proposition 6 proves that interlocking and partial equity ownership is one possible outcome in the sub-game perfect Bayesian equilibrium.

Finally, the following Corollary shows the benefit for the bidder of interlocking since she saves money in the deal.

Corollary 7 For $\mu < 0.5$ and $c < \frac{1}{5}$ the bidder who interlocks pays a lower bid.

Proof. (See A.6 in the Appendix). \blacksquare

The corollary shows that the bidder, by interlocking, pays a lower price for the acquisition provided that μ is not too large. As the probability of meeting an efficient target increases the bids in both cases, interlocking or not interlocking, converge.

7 Conclusions

In this paper, we study interlocking as a mechanism to screen the target in an acquisition of a minority stake among rival companies. Before the acquisition, when the level of efficiency of the rival is private information, the bidder might ask to interlock to elicit information about the true level of efficiency of the target. We show that the efficient target has indeed an incentive to accept, therefore disclosing his type. In contrast, the inefficient target rejects the proposal to interlock in order to extract an informational rent from the bidder. However, by rejecting the proposal to interlock the inefficient target releases information about his type. When there is asymmetric information, interlocking might be an important preliminary step in the acquisition as it affects the terms of the deal. We show that, without interlocking, the price paid is almost always higher, due to a lower accuracy in the evaluation of the target.

Our results contribute to the literature along two dimensions. On the one hand, our model provides a theoretical justification for the empirical evidence that ID and PEO can be observed in association. On the other hand, it points to a new implication for the antitrust analysis. From the antitrust perspective, ID and PEO are considered almost equivalent ties, with a potential for collusive agreements in the product market. In contrast, we suggest that ID is the result of a strategy to reduce the cost of the acquisition. More precisely, interlocking may reduce competition in the market for corporate control since the bidder benefits from accessing private information about the target company, thus excluding other potential bidders from the acquisition. We leave it for future research to elaborate on the antitrust implications of interlocking for the market of corporate control. In this respect, we see this paper as an initial contribution on the role of interlocking in limiting competition both in the product market and in the market for corporate control.

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A Appendix

A.1 Proof of Proposition 1.

Given that company 1 has full bargaining power, the (IR_2) is binding. We obtain the expression of the bid as a function of σ^{ID} :

$$B_{c_2}^{ID} = \pi_2^{ID}(0|c_2) - (1 - \sigma^{ID})\pi_2^{ID}(\sigma^{ID}|c_2) = \frac{(1 - 2c_2)^2}{9} - (1 - \sigma^{ID})\left(\frac{1 - 2c_2}{3 - \sigma^{ID}}\right)^2$$

We consider first the case company 2 is inefficient, $c_2 = c$. Substituting the expression of the bid in the payoff, we derive:

$$V_1^{ID}(\sigma^{ID}|c) - B_c^{ID} = \pi_1^{ID}(\sigma^{ID}|c) + \pi_2^{ID}(\sigma^{ID}|c) - \pi_2^{ID}(0|c) = \frac{\left(1 + c\left(1 - \sigma_c^{ID}\right)\right)\left(1 - \sigma_c^{ID} + c\left(1 + \sigma_c^{ID}\right)\right)}{(3 - \sigma^{ID})^2} + \left(\frac{1 - 2c}{3 - \sigma_c^{ID}}\right)^2 - \frac{(1 - 2c)^2}{9}$$

Taking the derivative w.r.t. σ_c^{ID} we obtain $\sigma_c^{ID} = \frac{5c-1}{3c-1}$, which is positive if $c \leq \frac{1}{5}$, therefore

$$\sigma^{ID}(c) = \min\left(\left(\max\frac{5c-1}{3c-1}, 0\right), \frac{1}{2}\right)$$

Analogously, considering the case company 2 is efficient, $c_2 = 0$ and substituting the expression of the bid in the payoff, we obtain:

$$V_1^{ID}(\sigma^{ID}|0) - B_0^{ID} = \pi_1^{ID}(\sigma^{ID}|0) + \pi_2^{ID}(\sigma^{ID}|0) - \pi_2^{ID}(0|0) = \frac{1 - \sigma_0^{ID}}{(3 - \sigma_0^{ID})^2} + \left(\frac{1}{3 - \sigma_0^{ID}}\right)^2 - \frac{1}{9}$$

Taking the derivative w.r.t. σ_0^{ID} we derive the following (FOC):

$$\frac{\sigma_0^{ID} - 1}{\left(\sigma_0^{ID} - 3\right)^3} = 0$$

from which we derive the equilibrium stake:

$$\sigma_0^{ID} = 1 \longrightarrow \overline{\sigma}_0^{ID} = \frac{1}{2}$$

Finally, we calculated the optimal bids in the two cases, B_c^{ID} when company 2 is inefficient and B_0^{ID} when company 2 is efficient:

$$B_c^{ID} = (1 - 2c)^2 \left(\frac{1}{9} - \frac{(1 - \sigma_c^{ID})}{(3 - \sigma_c^{ID})^2} \right) = \frac{35}{18}c^2 - \frac{17}{18}c + \frac{1}{9}$$

$$B_0^{ID} = \frac{1}{9} - \frac{(1 - \sigma_0^{ID})}{(3 - \sigma_0^{ID})^2} = \frac{1}{9} - \frac{(1 - \frac{1}{2})}{(3 - \frac{1}{2})^2} = \frac{7}{225}$$

A.2 Proof of Proposition 2

Substituting the two bids from (17) and (18) into the objective function, we derive the relaxed optimization problem:

$$\max_{\sigma_0^N, \sigma_c^N} EV_1^N(\mu) = \mu \left[V_1^N(\sigma_0|0;\mu) + V_2^N(\sigma_0|0;\mu) \right] + (1-\mu) \left[V_1^N(\sigma_0|c;\mu) + V_2^N(\sigma_0|0;\mu) - V_2^N(\sigma_0|c;\mu) + V_2^N(\sigma_c|c;\mu) \right] - V_2^N(0|0;\mu)$$
(19)

where the element $V_2^N(\sigma_0|0;\mu) - V_2^N(\sigma_0|c;\mu)$ is the cost of the distortion, as the profit that firm 1 let go to the firm 2, to prevent the inefficient type to mimic the efficient in σ_0 . Plugging (12) and (13) into (19) we obtain:

$$EV_1^N(\mu) = \mu \left(\pi_1^N(\sigma_0|0;\mu) + \pi_2^N(\sigma_0|0;\mu) \right) + (1-\mu) \left(\pi_1^N(\sigma_c|c;\mu) + \pi_2^N(\sigma_c|c;\mu) + (1-\sigma_0) \left(\pi_2^N(\sigma_0|0;\mu) - \pi_2^N(\sigma_0|c;\mu) \right) \right) - \pi_2^N(0|0;\mu)$$
(20)

The first order condition with respect to σ_c is given by:

$$(FOC_{\sigma_c}): \quad (1-\mu)\frac{\partial}{\partial\sigma_c}\left(\pi_1^N(\sigma_c|c;\mu) + \pi_2^N(\sigma_c|c;\mu)\right) = 0 \tag{21}$$

which becomes:

$$(1-\mu)\frac{1}{(3-\sigma_c)^3}(-2c\mu+2c-1)(5c+\sigma_c+c\mu-3c\sigma_c+c\sigma_c\mu-1) = 0$$

Therefore, the optimal stake is:

$$\sigma_c^N(\mu) = \frac{5c + c\mu - 1}{3c - c\mu - 1}$$
(22)

Analogously, the first order condition with respect to σ_0 is given by:

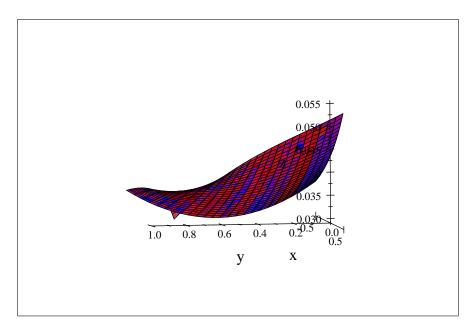
$$(FOC_{\sigma_0}) : \mu \frac{\partial}{\partial \sigma_0} \left(\left(\pi_1^N(\sigma_0|0;\mu) + \pi_2^N(\sigma_0|0;\mu) \right) \right) + \left(1 - \mu \right) \frac{\partial}{\partial \sigma_0} \left((1 - \sigma_0) \left(\pi_2^N(\sigma_0|0;\mu) - \pi_2^N(\sigma_0|c;\mu) \right) \right) = 0$$

$$(23)$$

which becomes:

$$\mu \left(\frac{1}{(3-\sigma_0)^3} \left(2c\mu - 2c + 1 \right) \left(c - \sigma_0 - c\mu + c\sigma_0 - c\sigma_0\mu + 1 \right) \right) + -(1-\mu) \left(\frac{1}{4} \frac{c}{(\sigma_0 - 3)^2} \left(7c + 2c\mu - c\left(\sigma_0\right)^2 + 6c\sigma_0 - 12c\sigma_0\mu + 2c\left(\sigma_0\right)^2\mu - 8 \right) \right) = 0$$
(24)

(23) does not provide a close solution for $\sigma_0^N(\mu)$. However, we can show graphically, see Figure 1, that the derivative 24 is always positive for $\mu \in [0, 1]$ and $c \in [0, 0.5]$,



Therefore, the bidder would like to buy the maximum stake $\sigma_0^N(\mu) = 1$, but, given the constraint $\sigma_0^N(\mu) = \overline{\sigma} = \frac{1}{2}$.

Now we can calculate the bids:

$$B_c^N(\mu) = B^N(\sigma_0^N|0;\mu) + V_2^N(\sigma_0^N|c;\mu) - V_2^N(\sigma_c^N|c;\mu) = \left(\frac{1}{9} - \frac{1}{2}\left(\frac{3}{10}c\left(\mu - 1\right) + \frac{2}{5}\right)^2\right) + \frac{1}{200}\left(3c\mu - 8c + 4\right)^2 - c\left(\mu + 1\right)\frac{\left(3c - 1\right)^2}{2c\mu - 6c + 2}$$

and

$$B_0^N(\mu) = V_2^N(0|0;\mu) - V_2^N(\sigma_0^N|0;\mu) = \frac{1}{9} - \frac{1}{2} \left(\frac{3}{10}c(\mu-1) + \frac{2}{5}\right)^2$$

A.3 **Proof of Proposition 4**

We need to check whether the payoff of the two types of company 2 when accepting the interlocking is greater than without interlocking.

• Assume that company 2 is efficient. When the target accepts the interlock, his payoff is given by:

$$V_2^{ID}(\sigma^{ID}|0) + B_0^{ID} = \frac{1}{9}$$

while when it refuses the interlock, his payoff is given by:

$$V_2^N(\sigma_0^N(\mu)|0;\mu) + B_0^N(\mu) = (1 - \sigma_0^N(\mu))\pi_2^N(\sigma_0^N(\mu)|0;\mu) + B_0^N(\mu) = \frac{1}{9}$$

It is immediate to observe that:

$$V_2^{ID}(\sigma^{ID}|0) + B_0^{ID} = \frac{1}{9} = V_2^N(\sigma_0^N(\mu)|0;\mu) + B_0^N(\mu)$$

Therefore, when the company 2 is efficient she is indifferent between ID and N.

• Assume that company 2 is inefficient. When the target accepts the interlock, his payoff is given by:

$$V_2^{ID}(\sigma^{ID}|c) + B_c^{ID} = \frac{1}{9}(2c-1)^2$$

while when it refuses the interlock, his payoff is the following:

$$V_2^N(\sigma_c^N(\mu) | c; \mu) + B_c^N(\mu) = \left(1 - \sigma_c^N(\mu)\right) \pi_2^N(\sigma_c^N(\mu) | c; \mu) + B_c^N(\mu)$$
$$= \frac{11}{40}c^2 - \frac{3}{20}c^2\mu - \frac{1}{5}c + \frac{1}{9}$$

The comparison between the payoff with and without interlocking depends on the value of μ . We can prove that, for $\mu = 1$,

$$V_2^{ID}(\sigma^{ID}|c) + B_c^{ID} = \frac{1}{9}\left(2c-1\right)^2 < \frac{1}{8}c^2 - \frac{1}{5}c + \frac{1}{9} = V_2^N(\sigma_c^N(\mu)|c;\mu) + B_c^N(\mu)$$

and for $\mu = 0$,

$$V_2^{ID}(\sigma^{ID}|c) + B_c^{ID} = \frac{1}{9} \left(2c - 1\right)^2 < \frac{11}{40}c^2 - \frac{1}{5}c + \frac{1}{9} = V_2^N(\sigma_c^N(\mu)|c;\mu) + B_c^N(\mu)$$

Then, given that the R.H.S. is continuos in the interval and its derivative with respect to μ is negative, it is a monotonic decreasing. Therefore $\forall \mu V_2^N(\sigma_c^N(\mu) | c; \mu) + B_c^N(\mu) > V_2^{ID}(\sigma^{ID} | c) + B_c^{ID}$. Hence, we can conclude that company 2 when of inefficient rejects the invitation. \Box

A.4 Proof of Corollary 3

From the incentive compatibility constraint, IC_{2H} , in (16) the cost of the distortion is given by $V_2(\sigma_0^N(\mu) | 0; \mu) - V_2(\sigma_0^N(\mu) | c; \mu)$. Therefore the rent for the target is:

$$R_{2} = V_{2}(\sigma_{0}^{N}(\mu)|0;\mu) - V_{2}(\sigma_{0}^{N}(\mu)|c;\mu) = (1 - \sigma_{0}^{N}(\mu)) (\pi_{2}^{N}(\sigma_{0}^{N}(\mu)|c;\mu) - \pi_{2}^{N}(\sigma_{0}^{N}(\mu)|0;\mu))$$

Substituting $\sigma_0^N(\mu)$ and rearrenging, we obtain:

$$R_2 = \frac{1}{40}c\left(11c - 6c\mu - 8\right)$$

Taking the derivative with respect μ , we obtain the result:

$$\frac{\partial R_2}{\partial \mu} = -\frac{3}{20}c^2 < 0.$$

A.5 **Proof of Proposition 5.**

The bidder has to decide between proposing to interlock or not.

• If the bidder does not propose the interlocking, she does not elicit any new information about the

type of the target, therefore the payoff is conditional on the prior μ .

$$EV_{1}^{N}(\mu,c) \triangleq \mu \left[V_{1}^{N}(\sigma_{0}^{N}(\mu)|0;\mu) - B_{0}^{N}(\mu) \right] + (1-\mu) \left[V_{1}^{N}(\sigma_{c}^{N}(\mu)|c;\mu) - B_{c}^{N}(\mu) \right] = \mu \left(\pi_{1}^{N}(\sigma_{0}^{N}(\mu)|0;\mu) + \pi_{2}^{N}(\sigma_{c}^{N}(\mu)|c;\mu) + (1-\sigma_{0}^{N}(\mu)) \left(\pi_{2}^{N}(\sigma_{0}^{N}(\mu)|0;\mu) - \pi_{2}^{N}(\sigma_{0}^{N}(\mu)|c;\mu) \right) \right) - (1-\mu) \left(\pi_{1}^{N}(\sigma_{c}^{N}(\mu)|c;\mu) + \pi_{2}^{N}(\sigma_{c}^{N}(\mu)|c;\mu) + (1-\sigma_{0}^{N}(\mu)) \left(\pi_{2}^{N}(\sigma_{0}^{N}(\mu)|0;\mu) - \pi_{2}^{N}(\sigma_{0}^{N}(\mu)|c;\mu) \right) \right) - (25)$$

Substituting, we can rewrite (25) as follows:

$$EV_1^N(\mu,c) = -\frac{27}{200}c^2\mu^3 + \frac{63}{200}c^2\mu^2 - \frac{7}{20}c^2\mu + \frac{17}{100}c^2 - \frac{3}{50}c\mu^2 - \frac{7}{100}c\mu + \frac{13}{100}c + \frac{7}{200}\mu + \frac{169}{1800}$$
(26)

• If the bidder proposes the interlocking, we have two possible circumstances, the target can accept or refuse the proposal. According to Proposition 4, the efficient target might accept the proposal, while the inefficient one always refuses. Therefore, the bidder should update her probability, given the observed actions. Notice that, the efficient target is indifferent between accepting or rejecting the ID proposal, hence we assume he tosses a coin. The posterior probabilities become:

$$\nu = prob(c_2 = 0 | No) = \frac{prob(No|c_2 = 0) \mu}{prob(No|c_2 = 0) \mu + prob(No|c_2 = c)(1 - \mu)} = \frac{\mu}{2 - \mu}$$

and

$$1 - \nu = prob (c_2 = c | No) = \frac{2(1 - \mu)}{2 - \mu}$$
$$\gamma = prob (c_2 = 0 | yes) = \frac{prob (yes | c_2 = 0) \mu}{prob (yes | c_2 = 0) \mu + prob (yes | c_2 = c) (1 - \mu)} = \frac{\frac{1}{2}\mu}{\frac{1}{2}\mu} = 1$$
$$(1 - \gamma) = prob (c_2 = c | Yes) = 0$$

If the target accepts the ID, the bidder knows that he is efficient and updats her probability with posterior $\gamma = 1$, therefore the expected payoff is:

$$V_1^{ID}(\sigma^{ID}|0) - B_0^{ID} = \frac{29}{225}$$
(27)

If the target does not accept the interlocking, we have two different payoffs according to the target's type. If $c_2 = 0$,

$$V_1^N(\sigma_0^N(\nu) | 0; \nu) - B_0^N(\nu)$$

If $c_2 = c$,

$$V_{1}^{N}(\sigma_{c}^{N}\left(\nu\right)|c;\nu) - B_{c}^{N}\left(\nu\right)$$

To solve for the optimal decision of the bidder, we first compare (26) with (27). The comparison depends on the value of μ . There exists a threshold value $\mu = \hat{\mu} = \frac{1}{9c} \left(6c + \sqrt{-102c - 66c^2 + 25} - 2 \right)$ such that $V_1^{ID}(\sigma^{ID}|0) - B_0^{ID} = EV_1^N(\mu, c)$. Then, for $1 \ge \mu > \hat{\mu}$, $EV_1^N(\mu, c) < V_1^{ID}(\sigma^{ID}|0) - B^{ID}(\sigma^{ID}|0)$.

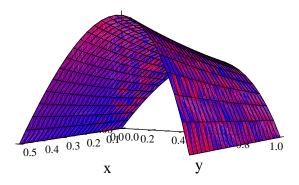


Figure 1: $D_1(\mu, c)$

If the target refuses the interlocking, N, The comparison reduces to: $V_1^N(\sigma^N | Ec | no) - B^N$ and $EV_1^N(\mu, c) - B^N$, where,

$$V_1^N(\sigma^N | Ec| no) = (\nu) \left(\pi_1(\sigma_0^N | 0) + \pi_2(\sigma_0^N | 0) \right) + (1-\nu) \left(\pi_1(\sigma_c^N | c) + \pi_2(\sigma_c^N | c) + (1-\sigma_0^N) \left(\pi_2(\sigma_0^N | 0) - \pi_2(\sigma_0^N | c) \right) \right) - \pi_2(0|0)$$
(28)

(28) becomes,

$$V_1^N(\sigma^N | Ec| no) = \frac{\left(162c^2\mu^3 - 405c^2\mu^2 + 549c^2\mu - 306c^2 + 162c\mu^2 + 72c\mu - 234c + 53\mu - 169\right)}{900(\mu - 2)}$$
(29)

We define $D_1(\mu, c) = V_1^N(\sigma^N | Ec| no) - EV_1^N(\mu, c)$ and we plot it, Comparison between B_c^{ID} and $B_c^N(\mu = 1)$, $B_c^N(\mu = \frac{1}{2})$. \Box

A.6 Proof of Corollary 7

In the case the target is efficient, we compare $B_0^N(\mu) = \frac{1}{9} - \frac{1}{2} \left(\frac{3}{10}c(\mu-1) + \frac{2}{5}\right)^2$ and B_0^{ID} . $B_0^N(\mu)$ is decreasing in μ , with minimum $B_0^N(\mu = 1) = \frac{7}{225} = B_0^{ID}$. Therefore, for $\mu \in (0, 1]$, $B_0^{ID} < B_0^N(\mu)$.

In the case the target is inefficient, we compare

$$B_c^{ID} = \frac{35}{18}c^2 - \frac{17}{18}c + \frac{1}{9}$$
(30)

for $c \in \left(0, \frac{1}{5}\right)$, and

$$B_c^N(\mu) = \left(\frac{1}{9} - \frac{1}{2}\left(\frac{3}{10}c\left(\mu - 1\right) + \frac{2}{5}\right)^2\right) + \frac{(3c\mu - 8c + 4)^2}{200} - \frac{c(\mu + 1)(3c - 1)^2}{2c\mu - 6c + 2}$$
(31)

Plotting (30) in bold, (31) for $\mu = 0$ (dotted) and for $\mu = \frac{1}{2}$ (dashed) we show the result:

